8. Creating Plant Models for Analyzing System Robustness

Robustness is the ability of the control system to tolerate external disturbances and also variations or uncertainties in the vehicle parameters. In this section we shall develop dynamic models that can be used to analyze robustness to parameter uncertainties and also to synthesize robust control laws using μ-tools. Sensitivity is evaluated by the ability of the control system to tolerate the effect of external disturbances on some sensitive outputs, for example, an optical sensor or a structural load sensor. Well-known sensitivity methods (using SV) are employed for analyzing the sensitivity of a system between certain inputs and outputs. The question is how do we analyze a system's robustness to internal parameter variations? How much parameter variations is a system able to tolerate before it becomes unstable, or stops performing properly? Parameter uncertainties can be seen as imprecise knowledge of the plant model parameters, such as: the mass, moments of inertia, aerodynamic coefficients, vehicle altitude, dynamic pressure, center of gravity, etc. The uncertainties in a model are defined in terms of variations in the actual plant parameters, above or below their nominal values. These uncertainties are called *"Structured"*, in contrast with the *"Unstructured"* uncertainties which are described in the frequency domain in terms of maximum amplitude error in the transfer function model.
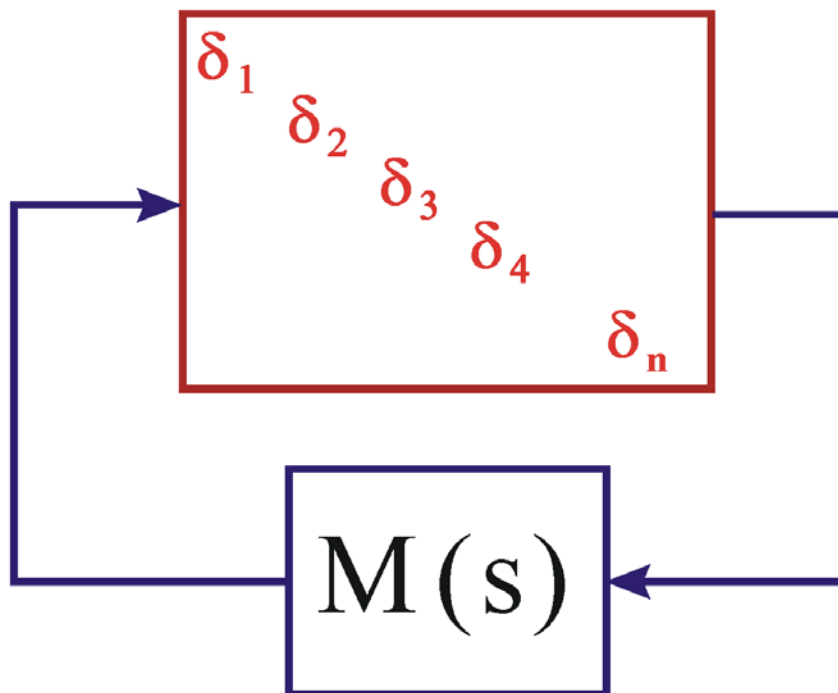

Figure 1 Uncertainties are extracted from the plant M(s) and are placed in a diagonal Δ block

In this section we are presenting a method that is used for creating state-space models for analyzing the robustness of systems that have bounded and known uncertainties in their parameters. Each parameter variation is "pulled out" of the uncertain plant model and it is placed inside a diagonal block Δ that contains only the uncertainties. The remaining plant is assumed to be known (best guess). The Δ block is attached to the known plant M(s) by means of (n) input/ output "wires", where (n) is the number of plant uncertainties, as shown in Figure 1. In essence, if M(s) is the plant model representing the flight vehicle, we are creating (n) additional inputs and outputs to M(s) that connect with the uncertainties block Δ, which is a block diagonal matrix $\Delta = \mathrm{diag}(\delta_1, \delta_2, \delta_3, \ldots \delta_n)$. The individual elements δ_i of the matrix block Δ may be scalars or matrices and each element represents a real uncertainty in the plant. The magnitude of δ_i represents the maximum variation of the corresponding parameter above or below its nominal value p_i. The parameters may be aerodynamic coefficient variations from the specified values, moment of inertia variations, thrust variations, etc. Note that M(s) in addition to the plant model, also includes the control system in closed-loop form and M(s) is assumed to be closed-loop stable. In essence the internal uncertainty Δ block is "pulled out" of the closed-loop plant M(s) and is connected to it by the additional inputs and outputs.

The nominally stable closed-loop system M(s) in Figure 1 is defined to be robust to a set of parameter variations δ_i which are included in the Δ block if it remains stable regardless of all possible variations of those parameters as long as the magnitude of each variation from nominal does not exceed the maximum defined uncertainty δ_i. The control system Robustness and Performance are analyzed in the frequency domain, similar to sensitivity analysis, using the structured singular value (SSV) or $\mu-$method. Three types of μ-analysis criteria are used for analyzing three different types of closed-loop system situations: (a) "nominal performance", that is sensitivity to external disturbances alone without any parameter variations, (b) "robustness" to parameter variations, and (c) "robust performance" which is simultaneously satisfying robustness to uncertainties while maintaining an acceptable sensitivity to external disturbances. In this section we will describe a method used for extracting the uncertainties out of the plant, and it is called the Internal Feedback Loop (IFL) method. The augmented state-space model (without the Δ block) is then used to analyze robustness using μ-analysis methods. To simplify the analysis, the plant M(s) inputs and outputs are scaled so that the individual elements of the diagonal uncertainty block Δ can now vary between +1 and -1. This is simple scaling. The gains of the parameter variations δ_i are absorbed in M(s) and the magnitudes of the new uncertainties are now bounded to be less than 1. The value of $1/\mu(M)$ represents the magnitude of the smallest perturbation that will destabilize the normalized closed-loop system M(s). According to the small gain theorem, the closed-loop system is robust as long as $\mu(M)$ across the normalized block Δ is less than one at all frequencies.

But the question is how do we extract the uncertainties out of the model? In the following section we will present an algorithm that uses the Flixan vehicle modeling program for generating dynamic models that can be used for analyzing robustness to parameter uncertainties. The vehicle dynamic model created includes additional inputs and outputs that represent connections with the uncertainty parameter block Δ. The magnitudes of the uncertainties are defined in the input data file together with the vehicle input data. The augmented models are then used to analyze robustness as we shall demonstrate using four real vehicle analysis examples.

8.1 The Internal Feedback Loop (IFL) Structure

The IFL method allows internal parameter perturbations in a plant to be treated like external disturbances in the system by means of fictitious inputs and outputs. This representation allows us to use μ-tools for analyzing robustness to uncertainties, or to apply H_∞ and other robust methods to design control systems that can tolerate a certain amount of parameter variations. To utilize the IFL concept the system must be expressed in the following form, where [ΔA, ΔB, ΔC, ΔD] are variations in the state-space system matrices as a result of variation in one of the parameters.

$$\begin{bmatrix} \dot{x} \\ y \end{bmatrix} = \left\{ \begin{bmatrix} A & B \\ C & D \end{bmatrix} + \begin{bmatrix} \Delta A & \Delta B \\ \Delta C & \Delta D \end{bmatrix} \right\} \begin{bmatrix} x \\ u \end{bmatrix}$$

Suppose that they are (l) independently perturbed parameters: p_1, p_2, … p_l with bounded parameter variations δp_i, where their magnitude $|\delta p_i| \le 1$. The perturbation matrix ΔP= [ΔA, ΔB; ΔC, ΔD] can be decomposed with respect to each parameter variation as follows:

$$\Delta_i = -\sum_{i=1}^{l} \delta p_i \begin{pmatrix} \alpha_x^{(i)} \\ \alpha_y^{(i)} \end{pmatrix} \begin{pmatrix} \beta_x^{(i)} & \beta_u^{(i)} \end{pmatrix}$$

Where for each parameter p_i
$\alpha_x^{(i)}$ and $\alpha_y^{(i)}$ are column vectors
$\beta_x^{(i)}$, and $\beta_u^{(i)}$ are row vectors

The plant uncertainty matrix ΔP due to all perturbations can be written in the following form, where the perturbation block ΔP is assumed to have a rank-1 dependency with respect to each parameter p_i.

$$\Delta P = -\begin{pmatrix} M_x \\ M_y \end{pmatrix} \Delta \begin{pmatrix} N_x & N_u \end{pmatrix} = -M \Delta N$$

Where M_x and M_y are stacks of column vectors, and N_x and N_u are stacks of row vectors as shown below

$$M_x = \begin{bmatrix} \alpha_x^{(1)} & \alpha_x^{(2)} & \dots & \alpha_x^{(l)} \end{bmatrix}; \qquad M_y = \begin{bmatrix} \alpha_y^{(1)} & \alpha_y^{(2)} & \dots & \alpha_y^{(l)} \end{bmatrix}$$

$$N_x = \begin{bmatrix} \beta_x^{(1)} \\ \vdots \\ \beta_x^{(l)} \end{bmatrix}; \qquad\qquad N_u = \begin{bmatrix} \beta_u^{(1)} \\ \vdots \\ \beta_u^{(l)} \end{bmatrix} \qquad\qquad \text{and}$$

Where Δ = diag [δp_1, δp_2, δp_3,…. δp_l] is the diagonal block of Figure-1 containing the uncertainties. Notice, that in order to simplify the implementation, the columns of matrices M_x and M_y and the rows of matrices N_x and N_u are scaled, so that the elements of the diagonal block Δ have unity upper bound. Now let us introduce two new variables (z_p and w_p) and rewrite the equations in the following system form in order to express it as a block diagram.

$$z_p = N_x x + N_u u \quad and \quad w_p = -\Delta z_p$$

The perturbed state-space system can be expressed by the following augmented representation which is the same as the original system in the upper left side, with some additional input and output vectors, an input and an output for each parameter uncertainty.

$$\begin{pmatrix} \dot{x} \\ y \\ z_p \end{pmatrix} = \begin{bmatrix} A & B & M_x \\ C & D & M_y \\ N_x & N_u & 0 \end{bmatrix} \begin{pmatrix} x \\ u \\ w_p \end{pmatrix}$$

If we further separate the plant inputs (u) into disturbances (w) and controls (u_c). That is: u=[w, u_c], and if we also separate the plant outputs (y) into performance criteria (z) and control measurements (y_m), the above system is augmented as shown below.

$$\begin{pmatrix} \dot{x} \\ z \\ y_m \\ z_p \end{pmatrix} = \begin{bmatrix} A & B_1 & B_2 & M_x \\ C_1 & D_{11} & D_{12} & M_w \\ C_2 & D_{21} & D_{22} & M_{ym} \\ N_x & N_w & N_{uc} & 0 \end{bmatrix} \begin{pmatrix} x \\ w \\ u_c \\ w_p \end{pmatrix}$$

The above formulation is useful for μ-synthesis or robustness/ performance analysis using μ-methods. It is also shown in block diagram form in Figure 2. The uncertainties block Δ is connected to the plant by means of the inputs w_p and the outputs z_p. The columns in the M_x, M_w, and M_{ym} matrices and the rows in the N_x , N_w, and N_{uc} matrices are scaled by dividing with the square root of the corresponding singular value in order to allow the elements of the uncertainty block Δ to be normalized to unity.

The control system K(s) is designed to stabilize the nominal plant P(s). When the feedback loop is closed between y_m and u_c the control system is also expected to keep the plant stable despite all possible variations in the elements of the block Δ which are allowed to vary between -1 and +1. This property is defined as Robust Stability. In addition to robust stability the control system must also be able to satisfy "Nominal Performance" requirements. That is a bounded and well-behaved response between the disturbances w and the criteria z.

We also have third property for the perturbed plant which is called "Robust Performance". The plant P(s) has the control loop closed and also the uncertainty loop closed via the Delta block. The closed-loop system satisfies the Robust Performance property when it remains stable, and it is also able to satisfy the expected performance criteria between w and z despite all possible variations in the internal parameters represented in the normalized uncertainties block Δ, where the individual magnitudes δ_i cannot exceed 1.
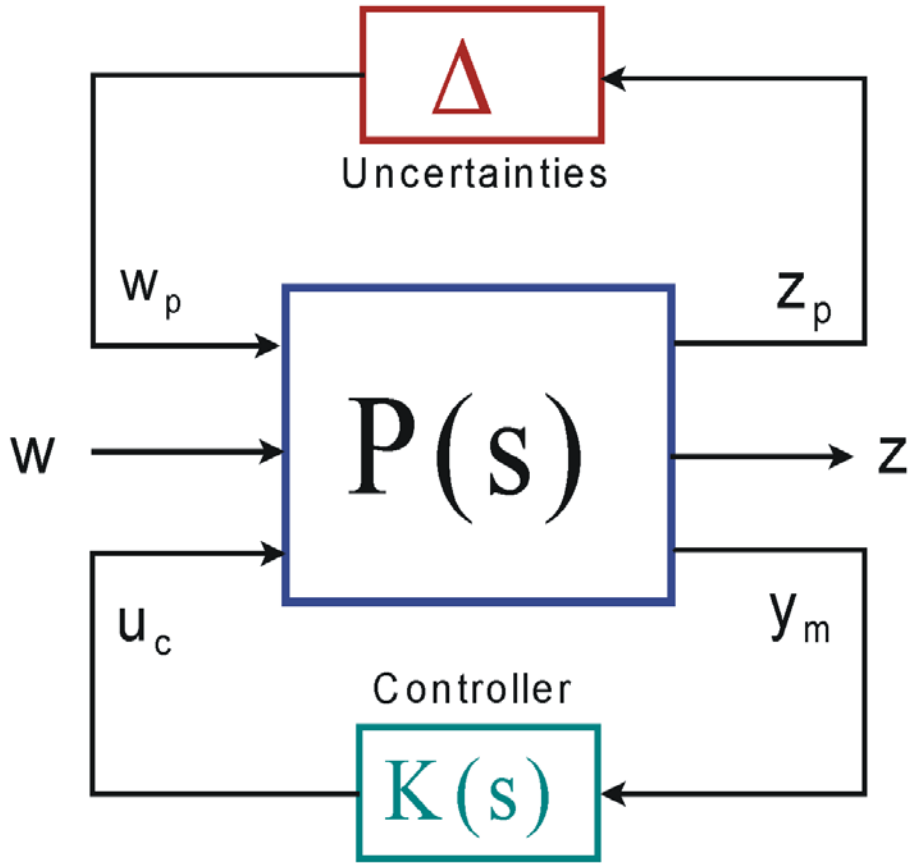
Figure 2 Robustness Analysis Block showing the Uncertainties IFL loop, the control feedback loop, the disturbances (w), and performance outputs (z)

This system can also be represented in matrix transfer function form as follows

$$\begin{pmatrix} z_p \\ z \\ y_m \end{pmatrix} = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \begin{pmatrix} w_p \\ w \\ u_c \end{pmatrix} \qquad \text{where:}$$

$$w_p = -\Delta z_p \qquad and \qquad u_c = -K(s) y_m$$

After closing the loop with a stabilizing controller K(s) the closed loop system is represented with the following transfer function matrix

$$\begin{pmatrix} z_p \\ z \end{pmatrix} = \begin{bmatrix} T_{11}(s) & T_{12}(s) \\ T_{21}(s) & T_{22}(s) \end{bmatrix} \begin{pmatrix} w_p \\ w \end{pmatrix} \quad \text{and} \quad w_p = -\Delta z_p$$

where

$$T_{11}(s) = G_{11} - G_{13}K(I + G_{33}K)^{-1}G_{31}; \qquad T_{12}(s) = G_{12} - G_{13}K(I + G_{33}K)^{-1}G_{32}$$

$$T_{21}(s) = G_{21} - G_{23}K(I + G_{33}K)^{-1}G_{31}; \qquad T_{22}(s) = G_{22} - G_{23}K(I + G_{33}K)^{-1}G_{32}$$

The above transfer functions are used to evaluate system robustness and performance of the closed loop system

Robust Stability: Stability robustness with respect to parameter uncertainty is determined by the transfer function $T_{11}(s)$. Smaller $\|T_{11}\|_\infty$ allows larger parameter uncertainty for closed loop stability. The closed loop system is considered to be robustly stable with respect to the parameter perturbations block Δ, where $\|\Delta\| \leq 1$, when the $\mu\{T_{11}(\omega)\} < 1$ at all frequencies (ω).

Nominal Performance: Nominal performance is used to calculate the system's sensitivity to excitations and it is obtained from the transfer function $T_{22}(s)$. This transfer function must be scaled by multiplying its inputs with the max magnitude of the excitations and by dividing its outputs with the max allowable error. The system meets Nominal Performance when the scaled $\|T_{22}(\omega)\|_\infty < 1$ at all frequencies (ω). For example, maximum wind-gust velocity disturbance must not exceed the maximum allowable angle of attack dispersion.

Robust Performance: is achieved when the system meets the performance and robustness requirements together. This happens when the following condition is satisfied at all frequencies.

$$\mu \begin{bmatrix} T_{11}(s) & T_{12}(s) \\ T_{21}(s) & T_{22}(s) \end{bmatrix} < 1$$

8.2 Parameter Uncertainties Modeling Program

The parameter uncertainties modeling program in Flixan implements the IFL method of extracting the parameter uncertainties and creating an augmented state-space vehicle model that includes the additional fictitious inputs and outputs that connect with the normalized uncertainty block Δ, as shown in Figure 2 and described in the previous section. The program calls the flight vehicle modeling program that processes the vehicle data and generates state-space systems. In addition to the vehicle data, the program also reads the uncertainties data from the input data file (.Inp). The algorithm calls the vehicle modeling program multiple times. It begins by processing the nominal vehicle dataset and repeats the data processing for each parameter variation. It eventually generates the uncertainty state-space vehicle model, which is similar to the nominal model, but it includes the additional input/output pairs that connect with the extracted uncertainties.

The parameter variations from their nominal values are included in a separate uncertainties dataset which is located in the input data file (.Inp) together with the vehicle data. The title of the uncertainties dataset must be included with the vehicle input data in order to associate the variations with the actual vehicle parameters. The program reads and processes the uncertainties together with the vehicle data by calling the vehicle modeling program. It uses the following process to calculate the uncertainty model:

1. The modeling program is used initially to process the nominal set of vehicle data and to create the "known" plant state-space model [A, B; C, D].

2. One (and only one) of the vehicle data parameters must be modified at a time, either increased or decreased from its nominal value by an amount that is equal to the maximum expected variation (δp_1) and the vehicle data is reprocessed by the vehicle modeling program to create a new state-space system [A_1, B_1, C_1, D_1] that corresponds to parameter #1 variation. The matrix difference between the nominal and the perturbed state-space models is calculated:

$$\begin{bmatrix} \Delta A_1 & \Delta B_1 \\ \Delta C_1 & \Delta D_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} - \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

3. This matrix is decomposed using SVD to calculate the column vectors $\alpha_x^{(1)}$ and $\alpha_y^{(1)}$ and the row vectors $\beta_x^{(1)}$, and $\beta_u^{(1)}$, as shown in the equation.

4. Restore the previous parameter to its original value and modify another parameter #2 in the vehicle input data by an amount δp_2 that represents the maximum variation of this parameter, as in step-2. Repeat steps 2 and 3 and calculate the vectors $\alpha_x^{(2)}$, $\alpha_y^{(2)}$, $\beta_x^{(2)}$, and $\beta_u^{(2)}$.

5. Select another parameter to perturb and repeat steps 2, and 3 until there are no more uncertain parameters to vary. Stack the row and column vectors as shown to create the stacks of column vectors: M_x and M_y and the stacks of row vectors: N_x and N_u.

6. These matrices are then used to create the additional inputs and outputs in the state-space model. The columns of matrices M_x and M_y and the rows of matrices N_x and N_u must also be scaled according to the magnitude of the uncertainties δp_i so that the interconnections correspond to a unity normalized Δ-block.

The uncertainty model is then used in combination with the flight control system to analyze the closed-loop system performance and robustness to uncertainties by calculating the μ-frequency response of the plant across the interconnections with the Δ block, as shown in Figure 2. That is, between w_p and z_p, with the control loop K(s) closed.

The parameter uncertainties data-set in the input file appears similar to the vehicle data-set. It includes variations from the nominal vehicle data and a title above the data. The uncertainties should correspond to the parameters in the vehicle dataset. That is, in terms of number of aerosurfaces, engines, slosh tanks, etc. Only the variations in the uncertain parameters are non-zero. Obviously, the variations in the parameters which are known and do not vary must be set to zero. An additional input/ output pair is created in the system for each uncertainty. The Flixan program identifies the data-sets that contain parameter uncertainties from this label: "*UNCERTAIN PARAMETER VARIATIONS FROM NOMINAL ...*" which is located above the uncertainties data set. There is also a title below this label which identifies a particular uncertainties dataset, similar to other types of Flixan datasets. This title associates the uncertainties data with a vehicle input data. A typical flight vehicle set for a lifting body aircraft and the corresponding parameter variations set are shown below.

```
FLIGHT VEHICLE INPUT DATA ......
Lifting-Body at Mach 0.8, Alpha 20, Qbar 400, h=3000 (ft) with 35 Uncertainties
! The following Lifting-Body vehicle has 6 control surfaces and 3 RCS jets
Body Axes Output,Attitude=Euler Angles,Without GAFD, No Turn Coordination

Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft)   : 550.0  32.17 0.20896E+08
Moments and products of Inertias Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)         : 7000.0  30000.0  33000.0
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)        : -28.32    0.0    0.0
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)      : 0.8  700.0  400.00  3000.0
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)        : -98.0  0.0  -50.00  -60.0
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                        : 20.0  0.0  -10.0  0.000
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)  : -0.0  20.0  0.0 00 0.0 0.0
Wind Gust Vel wrt Vehi (Azim & Elev) angles (deg), or Force(lb), Torque(ft-lb), locat:xyz:  Gust -135.0  90.0
Surface Reference Area (feet^2), Mean Aerodynamic Chord (ft), Wing Span in (feet)       : 300.0  30.0   15.0
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial_rho/ Partial_H} : -27.9  0.0  0.0  0.0000
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}: -0.0021, -0.0045, 0.00035,
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}: 0.0, -0.014, 1.7,0.0,
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}: -0.7, -0.03,-5.0, 0.0, 0.0,
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}: 0.0, -0.01, 0.0,-0.3, 0.4,
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}: -0.012,-0.0006, 0, 0, -0.5
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}: 0.0, 0.0033, 0.0, 0.184,

Number of Control Surfaces, With or No TWD (Tail-Wags-Dog and Hinge Moment Dynamics) ?  : 4   NO TWD

Control Surface No: 1                                                                     Elevon
Trim Angle, Max/Min Deflection Angles from Trim, Hinge Line Angles: phi_h, lamda_h  (deg): 0.0  30.0 -30.0 +0.0000
Surface Mass, Inertia about Hinge, Moment Arm (Hinge to Surface CG), Surface Chord, Area : 0.0  0.0 0.0 0.0 0.0
Hinge Moment Derivatives (1/deg), { Chm_Alpha, Chm_Beta, Chm_Delta, Chm_Mach }          : -0.00 0.0 0.0 0.0
Location of the Hinge Line Center with respect to Vehicle Reference (feet), {Xcs,Ycs,Zcs}: 0.0  0.0 0.0
Forces (-x,y,z) due to Deflect. and Rates {Ca_del,Cy_del,Cz_del, Ca_deld,Cy_deld,Cz_deld}: -0.0011 0.0 -0.003 0.0 0.0
Moments due to Deflections and Rates {Cl_del,Cm_del,Cn_del,Cl_deldot,Cm_deldot,Cn_deldot}: 0.0 -0.0015 0.0 0.0 0.0
Control Surface No: 2                                                                     Aileron
Trim Angle, Max/Min Deflection Angles from Trim, Hinge Line Angles: phi_cs,lamda_cs (deg): 0.0  30.0  -30.0
Surface Mass, Inertia about Hinge, Moment Arm (Hinge to Surface CG), Surface Chord, Area : 0.0  0.0   0.0  0.0  0.0
Hinge Moment Derivatives (1/deg), { Chm_Alpha, Chm_Beta, Chm_Delta, Chm_Mach }          : 0.0  0.0   0.00   0.0
Location of the Hinge Line Center with respect to Vehicle Reference (feet), {Xcs,Ycs,Zcs}: 0.0  0.0   -0.0
Forces (-x,y,z) due to Deflect. and Rates {Ca_del,Cy_del,Cz_del, Ca_deld,Cy_deld,Cz_deld}: 0.0  0.0001 0.0  0.0000
Moments due to Deflections and Rates {Cl_del,Cm_del,Cn_del,Cl_deldot,Cm_deldot,Cn_deldot}: 0.0016 0.0 -0.00002 0. 0.0
Control Surface No: 3                                                                     Rudder-1
Trim Angle, Max/Min Deflection Angles from Trim, Hinge Line Angles: phi_h, lamda_h  (deg): 0.0  30.0  -30.0 +0.0  0.0
Surface Mass, Inertia about Hinge, Moment Arm (Hinge to Surface CG), Surface Chord, Area : 0.0  0.0  0.0  0.0  0.0
Hinge Moment Derivatives (1/deg), { Chm_Alpha, Chm_Beta, Chm_Delta, Chm_Mach }          : 0.0  0.0  0.0  0.0
Location of the Hinge Line Center with respect to Vehicle Reference (feet), {Xcs,Ycs,Zcs}: 0.0   0.0    0.0
Forces (-x,y,z) due to Deflect. and Rates {Ca_del,Cy_del,Cz_del, Ca_deld,Cy_deld,Cz_deld}: 0.0 0.004 0.0 0.0 0.0  0.0
Moments due to Deflections and Rates {Cl_del,Cm_del,Cn_del,Cl_deldot,Cm_deldot,Cn_deldot}: 0.0005 0.0 -0.002 0.0  0.0
Control Surface No: 4                                                                     Rudder-2
Trim Angle, Max/Min Deflection Angles from Trim, Hinge Line Angles: phi_h, lamda_h  (deg): 0.0   30.0  -30.0 +0.0
Surface Mass, Inertia about Hinge, Moment Arm (Hinge to Surface CG), Surface Chord, Area : 0.0    0.0   0.0   0.0
Hinge Moment Derivatives (1/deg), { Chm_Alpha, Chm_Beta, Chm_Delta, Chm_Mach }          : 0.0    0.0   0.0   0.0
Location of the Hinge Line Center with respect to Vehicle Reference (feet), {Xcs,Ycs,Zcs}: 0.0    0.0   0.0
Forces (-x,y,z) due to Deflect. and Rates {Ca_del,Cy_del,Cz_del, Ca_deld,Cy_deld,Cz_deld}: 0.0    0.006 0.0    0.0
Moments due to Deflections and Rates {Cl_del,Cm_del,Cn_del,Cl_deldot,Cm_deldot,Cn_deldot}: 0.004   0.0  0.005  0.0
```

Number of Thruster Engines, Include or Not the Tail-Wags-Dog and Load-Torque Dynamics ? : 3

RCS Jet No: 1   (Gimbaling Throttling   Single_Gimbal)   : Jet #1   Throttling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)   :   0.0   30.0
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg):   0.0 -90.0   0.0   0.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):   0.00 0.0   0.0
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :   4.5   0.0   0.0
RCS Jet No: 2   (Gimbaling Throttling   Single_Gimbal)   : Jet #2   Throttling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)   :   0.0   30.0
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): -90.0   0.0 0.0   0.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):   0.00   0.0 0.0
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :   4.2 -4.0 0.0
RCS Jet No: 3   (Gimbaling Throttling   Single_Gimbal)   : Jet #3   Throttling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)   :   0.0   30.0
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): -90.0   0.0   0.0 0.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):   0.00   0.0   0.0
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :   4.2   4.0   0.0

Parameter Uncertainties Data
Uncertainties at Mach=0.8, Alpha=20

Number of Bending Modes   : 0

UNCERTAIN PARAMETER VARIATIONS FROM NOMINAL ......
Uncertainties at Mach=0.8, Alpha=20
! The following data are not actual vehicle parameters but they represent uncertainties
! of the corresponding parameters for the vehicle above. The title above these comments
! defines the flight condition where they apply. The uncertainty values represent a +ve
! or -ve additive variation above or below the nominal values shown in the set above.

Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft)   : 0.0   0.0   0.0
Moments and products of Inertias Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)   : 200.0 1000.0 1000.0 0.0
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)   : 0.5   0.5   0.5
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)   : 0.0   0.0   0.0   0.0
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)   : 0.0   0.0   0.0   0.0
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)   : 2.0   2.0   0.0   0.0
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)   : -0.0   0.0 0.0   0.0   0.0
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}: 0.005, -0.001, 0.0, 0.0,
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}: 0.0,   -0.002, 0.0, 0.0,
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}: 0.02, -0.003, 1.0, 0.0,
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}: 0.0, -0.002, 0.0, 0.1, 0.1
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}: 0.006, -0.001, 0.0, 0.0,
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}: 0.0,   0.001, 0.0, 0.1,

Number of Control Surfaces, (Variations from Nominal Aero Parameters)   : 4

Control Surface No: 1   Elevon
Surface Mass, Inertia about Hinge, Moment Arm (Hinge to Surface CG), Surface Chord, Area : 0.0 0.0   0.0 0.0 0.0
Forces (-x,y,z) due to Deflect. and Rates {Ca_del,Cy_del,Cz_del, Ca_deld,Cy_deld,Cz_deld}: -0.0002 0.0 -0.0005 0.0
Moments due to Deflections and Rates {Cl_del,Cm_del,Cn_del,Cl_deldot,Cm_deldot,Cn_deldot}: 0.0 -0.0002 0.0 0.0
Control Surface No: 2   Aileron
Surface Mass, Inertia about Hinge, Moment Arm (Hinge to Surface CG), Surface Chord, Area : 0.0 0.0 0.0 0.0   0.0
Forces (-x,y,z) due to Deflect. and Rates {Ca_del,Cy_del,Cz_del, Ca_deld,Cy_deld,Cz_deld}: 0.0 0.0005 0.0 0.0   0.0
Moments due to Deflections and Rates {Cl_del,Cm_del,Cn_del,Cl_deldot,Cm_deldot,Cn_deldot}: 0.00024 0.0 0.00013   0.0
Control Surface No: 3   Rudder-1
Surface Mass, Inertia about Hinge, Moment Arm (Hinge to Surface CG), Surface Chord, Area : 0.0 0.0 0.0 0.0   0.0
Forces (-x,y,z) due to Deflect. and Rates {Ca_del,Cy_del,Cz_del, Ca_deld,Cy_deld,Cz_deld}: 0.0 0.00043 0.0 0.0 0.0
Moments due to Deflections and Rates {Cl_del,Cm_del,Cn_del,Cl_deldot,Cm_deldot,Cn_deldot}: 0.0002 0.0 0.0002 0.0
Control Surface No: 4   Rudder-2
Surface Mass, Inertia about Hinge, Moment Arm (Hinge to Surface CG), Surface Chord, Area : 0.0 0.00 0.0 0.0 0.0
Forces (-x,y,z) due to Deflect. and Rates {Ca_del,Cy_del,Cz_del, Ca_deld,Cy_deld,Cz_deld}: 0.0 0.0003 0.0 0.0 0.0
Moments due to Deflections and Rates {Cl_del,Cm_del,Cn_del,Cl_deldot,Cm_deldot,Cn_deldot}: 0.00025 0.0 0.0003 0.0 0.0

Number of Thruster Engines, (Variations from Nominal Parameters)   : 3

TVC Engine No: 1   : Jet #1
Engine Thrust Additive Variation, (lb)   : 0.0
Engine Mounting Angles Variations from Nominal Angles (Dyn,Dzn) (deg)   : 0.0   0.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm: engine CG to gimbal (ft): 0.0   0.0   0.0
TVC Engine No: 2   : Jet #2
Engine Thrust Additive Variation, (lb)   : 0.0
Engine Mounting Angles Variations from Nominal Angles (Dyn,Dzn) (deg)   : 0.0   0.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft): 0.0   0.0   0.0
TVC Engine No: 3   : Jet #3
Engine Thrust Additive Variation, (lb)   : 0.0
Engine Mounting Angles Variations from Nominal Angles (Dyn,Dzn) (deg)   : -0.0   -0.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft): 0.0   0.0   0.0

8.3 Running the Program

There are many ways to run the vehicle uncertainties modeling program. It can be executed either from the vehicle modeling program option when the uncertainties dataset title is already included in the input file, and the corresponding uncertainties title is also included in the vehicle data, as already described, or as a separate option from the Flixan main menu. The second option is used when there are more than one set of uncertainties datasets in the input file to choose from, and the associated uncertainties title is not included in the vehicle data.

To demonstrate the second option, start the Flixan program, select the project directory, and from the Flixan main menu go to "*Program Functions*", and then select *"Robust Control Synthesis Tools"*, and then *"Modeling Vehicle Parameter Uncertainties"*, as shown below.
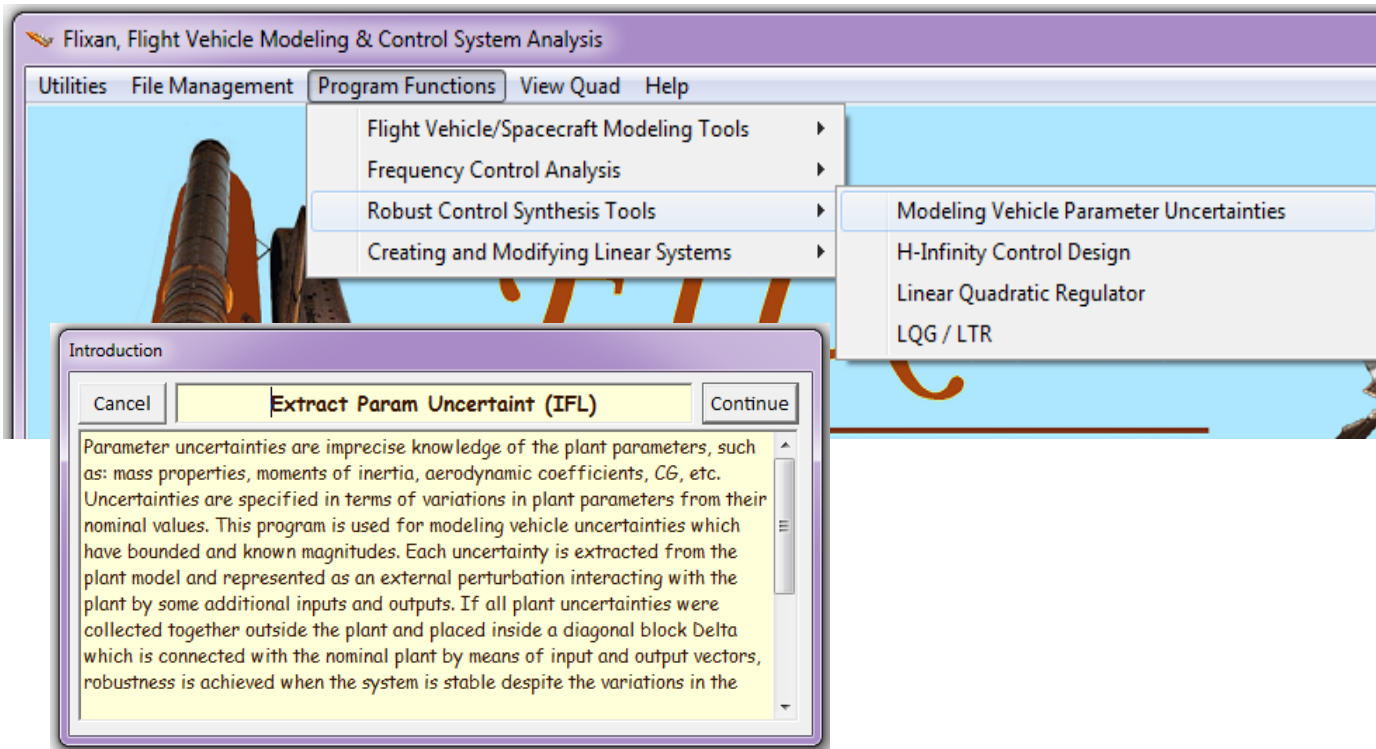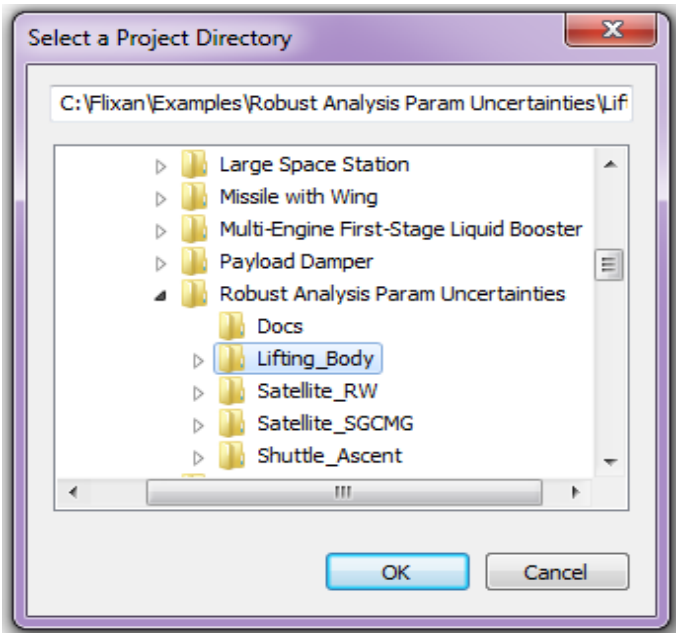
From the filename selection menu shown, select an input file name (.Inp), and a systems file name (.Qdr), and click on "*Process Files*". The systems file will include the vehicle state-space model augmented with the additional inputs and outputs that would connect to the uncertainties block Δ.
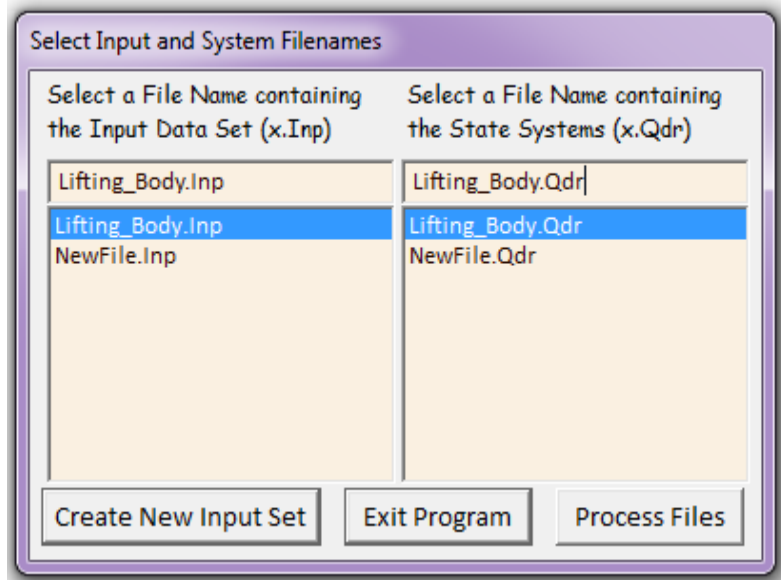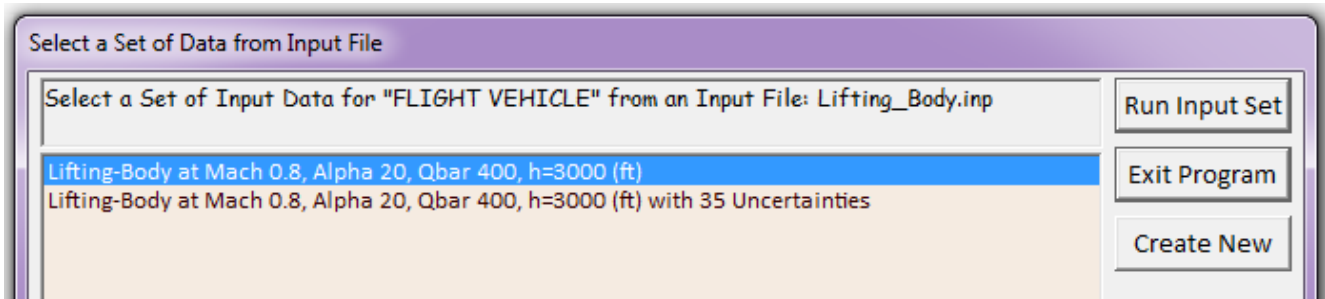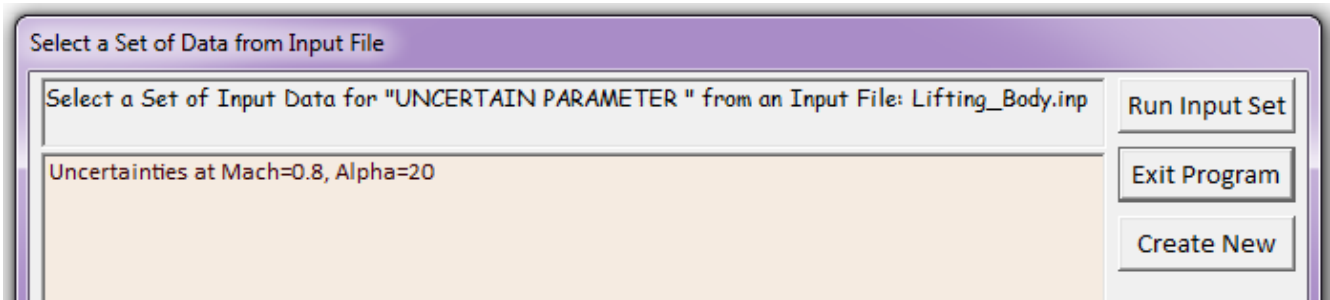
From the next menu select the title of the dataset that contains the vehicle data and click on "*Run Input Set*", assuming that the title of the associated uncertainties is not included in the dataset.

Select Input and System Filenames

Select a File Name containing the Input Data Set (x.Inp)

Lifting_Body.Inp

Lifting_Body.Inp
NewFile.Inp

Select a File Name containing the State Systems (x.Qdr)

Lifting_Body.Qdr

Lifting_Body.Qdr
NewFile.Qdr

Create New Input Set   Exit Program   Process Files

Select a Set of Data from Input File

Select a Set of Input Data for "FLIGHT VEHICLE" from an Input File: Lifting_Body.inp | Run Input Set

Lifting-Body at Mach 0.8, Alpha 20, Qbar 400, h=3000 (ft)
Lifting-Body at Mach 0.8, Alpha 20, Qbar 400, h=3000 (ft) with 35 Uncertainties | Exit Program

Create New

Select also from the next menu below, the title of the parameter uncertainties dataset and click on *"Run Input Set"*. The parameter uncertainties dataset contains the maximum variations of the vehicle parameters from their nominal values.

Select a Set of Data from Input File

Select a Set of Input Data for "UNCERTAIN PARAMETER " from an Input File: Lifting_Body.inp | Run Input Set

Uncertainties at Mach=0.8, Alpha=20 | Exit Program

Create New

The program will then process the vehicle data and the uncertainties data, create the uncertain vehicle state-space model, which is augmented with extra inputs and outputs that connect with the uncertainties block, and it will save the augmented system in the systems file. In the situation where the uncertainties dataset is not yet available and saved in the input file (.Inp), a utility is activated that allows the user to create the uncertainties dataset interactively, and it will save it there for future use, as described in the next section. Note that, the program can also be executed via the Input File Manager Utility. This implementation is much faster especially when processing the data in batch mode. See the examples for more details.

8.4 Creating a Parameter Uncertainties Data-Set

There is also an interactive utility for generating the uncertainties data from scratch which is very similar to the interactive utility that creates the flight vehicle data. It assumes that the vehicle data already exist in the input file and the input file also includes the selected modal data set when the vehicle model is flexible. This utility is activated by the Uncertainties Modeling Program when the user fails to provide a title for the uncertainties data. Begin the Flixan program, select the directory and go to "*Modeling Vehicle Parameter Uncertainties*", and select the input and system files, as before. In this example select the Shuttle Stage-2 files: "*Shuttle_Stg2.Inp*", and "*Shuttle_Stg2.Qdr*".
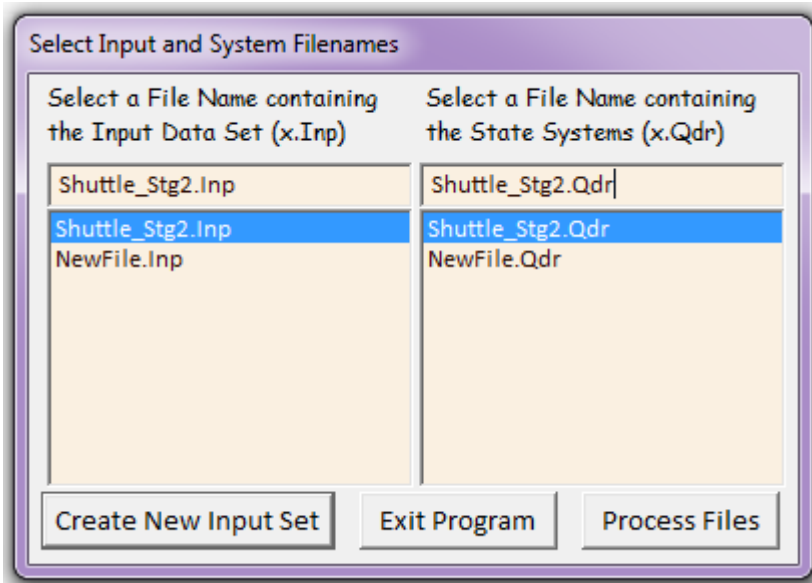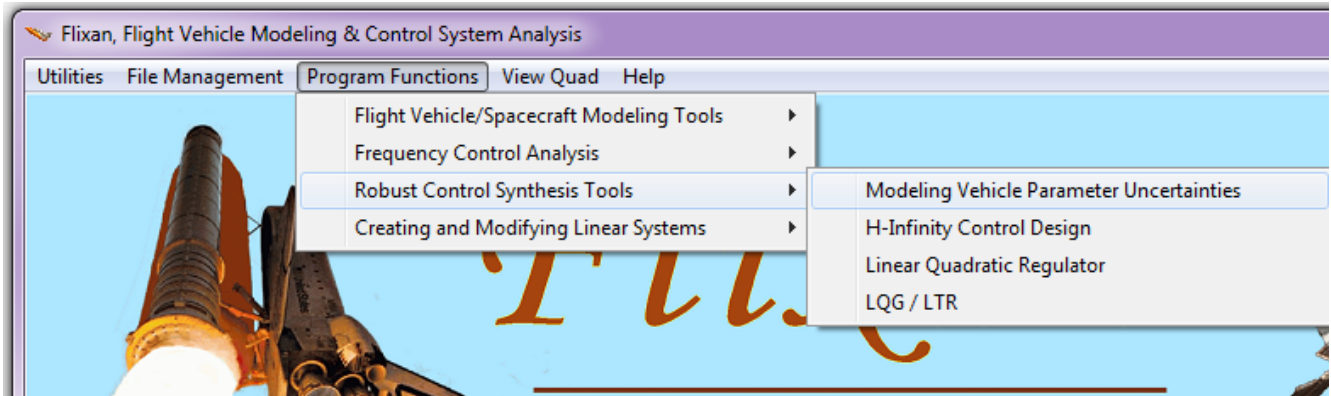
Then from the following menu select the title of the flight vehicle input data-set and click on "*Run Input Set*". The vehicle data is already in file. It is the uncertainties data set that is missing and we will create them interactively.
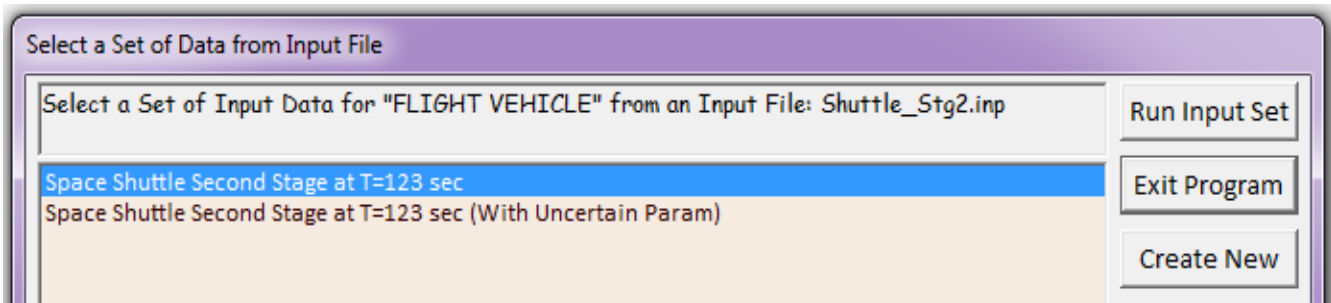
If the program cannot find an uncertainties set of data it will display the following dialog, expecting a confirmation from the user to begin the interactive process that will create the parameter variations dataset by interrogating the user. Otherwise, if the program detects uncertainties datasets in the input file, it will display a menu similar to the one shown earlier prompting the user to select one of the uncertainty titles. The user may still refuse to select an uncertainties title by clicking on the "*Create New*" button, in which case the same dialog appears expecting a confirmation to proceed with the interactive utility.

New Data

A New Set of "UNCERTAIN PARAMETER " Data Will be Created in File Shuttle_Stg2.inp

OK

The interactive utility dialog consists of various sub-dialogs which are selectable from the tabs. Each tab contains different sets of data, similar to the vehicle modeling dialog utility. At the top of the main dialog you must enter a title for the new uncertainties set. In this case we enter "*Uncertainties for Shuttle Ascent Second Stage (b)*". The dialog already knows from the vehicle data that this vehicle has 3 engines, 2 sloshing tanks, and 20 flex modes. Start by selecting one of the tabs at a time, and entering the maximum variation of each vehicle parameter. You are not obligated to enter data in every field. Parameters which are known and do not vary should have their variations set zero. The fields with light blue background show the nominal vehicle parameters for user reference and they are not to be used for entering data. You should only enter data in the parameter variation fields which are on the right and have white background. After entering data in one of the tabs, click on the "*Update Data*" button before opening another tab. In this example, if we click on the "Mass Properties" tab, the bottom part of the dialog shows: the vehicle mass, inertias, and CG locations in the light blue fields on the left. It expects the user to enter the corresponding uncertainty values on the right. In this case we enter uncertainty values for Ixx, Iyy, Izz, and the Xcg location, and we click on "*Update Data*" button.

Flight Vehicle Parameter Uncertainties

Uncertainties Title

Uncertainties for Shuttle Ascent Second Stage (b)

Edit Input File   Exit

Update Data   Run

Save in File

| Gimbaling Engines or Jets. Include Tail-Wags-Dog? | 3 | Reaction Wheels? | 0 | Structure Bending Modes | 20 |
| Rotating Control Surfaces. Include Tail-Wags-Dog? | 0 | Single Gimbal CMGs? | 0 | Fuel Sloshing Modes | 2 |

Reaction Wheels | Single Gimbal CMGs | Double Gimbal CMG System | Fuel Slosh | Flex Modes | User Comments

Mass Properties | Trajectory Data | Aero Force Coeffs | Aero Moment Coeffs | Control Surfaces | Gimbal Engines/ RCS

Vehicle Mass Properties

		Nominal Data	Param. Variations
Vehicle Mass in (slugs)		41820.90	0.000000
Moments of Inertia in (slugs-ft^2)	Ixx	6047220.	400000.0
	Iyy	0.8733540E+08	3000000.
	Izz	0.8295620E+08	3000000.
Products of Inertia in (slugs-ft^2)	Ixy	64808.00	0.000000
	Ixz	0.1405210E+08	0.000000
	Iyz	19942.00	0.000000
CG Location in (feet)	Xcg	-95.19400	1.000000
	Ycg	0.6190000E-01	0.000000
	Zcg	-38.84700	0.000000

There are also tabs for trajectory data, basic aero-force coefficients, aero-moment coefficients, engine TVC or RCS, aerosurface data, reaction wheels, CMGs, fuel sloshing parameters, flex mode frequency and damping, and a text area for entering user comments. The following dialog tab is used for entering the variations in the aero moment coefficients. The actual aero moment coefficients are shown for reference in the light blue fields on the left. The user must enter the corresponding parameter variations in the white fields on the right and then click on the "*Update Data*" button before selecting another tab. The aero force coefficients tab is very similar to the aero moments.

Flight Vehicle Parameter Uncertainties

Uncertainties Title

Uncertainties for Shuttle Ascent Second Stage (b)

Edit Input File	Exit
Update Data	Run
Save in File	

| Gimbaling Engines or Jets. Include Tail-Wags-Dog? | 3 | Reaction Wheels? | 0 | Structure Bending Modes | 20 |
| Rotating Control Surfaces. Include Tail-Wags-Dog? | 0 | Single Gimbal CMGs? | 0 | Fuel Sloshing Modes | 2 |

Tabs: Reaction Wheels | Single Gimbal CMGs | Double Gimbal CMG System | Fuel Slosh | Flex Modes | User Comments
Mass Properties | Trajectory Data | Aero Force Coeffs | Aero Moment Coeffs | Control Surfaces | Gimbal Engines/ RCS

Basic Aerodynamic Moment Coefficients, about vehicle axes: X, Y, and Z

Rolling Aero Moments

	Nominal Aero	Aero Variations
Clo= Cl (at nominal beta)	-0.1730000E-02	0.000000
Cl_ beta (1/deg)	-0.4512500E-02	0.2000000E-02
Cl_ p (1/deg/sec)	0.000000	0.000000
Cl_beta-dot (1/deg/sec)	0.000000	0.000000
Cl_alpha (1/deg)	0.000000	0.000000
Cl_ r (1/deg/sec)	0.000000	0.000000

Yawing Aero Moments

	Nominal Aero	Aero Variations
Cno= Cn (at nominal beta)	0.1520000E-02	0.000000
Cn_beta (1/deg)	0.8675680E-02	0.2000000E-02
Cn_ p (1/deg/sec)	0.000000	0.000000
Cn_beta-dot (1/deg/sec)	0.000000	0.000000
Cn_alpha (1/deg)	0.000000	0.000000
Cn_ r (1/deg/sec)	0.000000	0.000000

Pitching Aero Moments

	Nominal Aero	Aero Variations
Cmo= Cm (at nominal alpha)	0.5586000E-01	0.2000000E-01
Cm_ alpha (1/deg)	-0.1484560E-01	0.2000000E-02
Cm_ q (1/deg/sec)	0.000000	0.000000
Partial Cm/ Patial V (1/ft/sec)	0.000000	0.000000
Cm_alpha-dot (1/deg/sec)	0.000000	0.000000
Cm_beta (1/deg)	0.000000	0.000000
Partial Cm/ Patial H (1/feet)	0.000000	0.000000

The dialog below is used for entering parameter variations for the two sloshing propellant tanks. The light blue fields on the left show the slosh frequencies in (rad/sec) at one g acceleration. The program will adjust the slosh frequency according to the vehicle acceleration. It shows also the modal damping coefficients, the slosh mass (slugs), and slosh mass location, x, y, z in (feet). The white fields on the right are for entering the parameter variations. In this case we enter uncertainties in the slosh frequencies (both pitch and yaw because the slosh mass can oscillate in two orthogonal directions that may be at different frequencies), uncertainties in the slosh mass, and uncertainties in the x and z locations. When you complete the first tank, click on "Next Slosh" to enter data for the next fuel tank. You must also click on "Update Data" before selecting another tab.

The "Flex Mode" tab is used for entering variations in flex mode frequency (in the 3rd column) and in the damping coefficients (in the 5th column). The actual flex mode frequencies and damping are shown for reference in the 2nd and 4th columns. After entering the data, click on "*Update Data*" button. When you finish entering data using the dialog click on "*Save in File*" to save the uncertainties set in file "*Shuttle_Stg2.Inp*". You may also click on "*Run*" to create the state-space model which will be saved in file "*Shuttle_Stg2.Qdr*".

Flight Vehicle Parameter Uncertainties

Uncertainties Title
Uncertainties for Shuttle Ascent Second Stage (b)

Edit Input File | Exit

| Gimbaling Engines or Jets. Include Tail-Wags-Dog? | 3 | Reaction Wheels? | 0 | Structure Bending Modes | 20 |
| Rotating Control Surfaces. Include Tail-Wags-Dog? | 0 | Single Gimbal CMGs? | 0 | Fuel Sloshing Modes | 2 |

Update Data | Run
Save in File

Mass Properties | Trajectory Data | Aero Force Coeffs | Aero Moment Coeffs | Control Surfaces | Gimbal Engines/ RCS
Reaction Wheels | Single Gimbal CMGs | Double Gimbal CMG System | Fuel Slosh | Flex Modes | User Comments

This Vehicle has 20 Flexible Modes

Enter the Max Frequency and Damping Variations for some of the strongest Flexible Modes for Robustness Analysis Modeling

Mode Num	Mode Frequency	Frequ. Variation	Mode Damping	Damp. Variation
1	9.0752	0.50000	0.20000E-01	0.20000E-02
2	9.4117	0.0000	0.10000E-01	0.0000
3	9.7279	0.70000	0.10000E-01	0.20000E-02
4	15.901	0.70000	0.20000E-01	0.20000E-02
5	16.599	0.80000	0.10000E-01	0.30000E-02
6	19.502	0.0000	0.10000E-01	0.0000
7	21.689	0.0000	0.10000E-01	0.0000
8	23.028	0.90000	0.50000E-02	0.20000E-02
9	29.714	0.0000	0.50000E-02	0.0000
10	30.196	0.0000	0.50000E-02	0.0000
11	35.047	0.0000	0.50000E-02	0.0000
12	36.204	0.0000	0.50000E-02	0.0000
13	37.357	0.0000	0.50000E-02	0.0000
14	38.970	0.0000	0.50000E-02	0.0000
15	44.884	0.0000	0.50000E-02	0.0000
16	45.938	0.0000	0.50000E-02	0.0000
17	49.589	1.2000	0.50000E-02	0.30000E-02
18	50.285	0.0000	0.50000E-02	0.0000
19	52.381	0.0000	0.50000E-02	0.0000
20	54.582	0.0000	0.50000E-02	0.0000

The following examples illustrate the methodology described in Section 8, demonstrate the Flixan uncertainty modeling program and analyze control system's robustness to parameter variations using μ-analysis. The examples consist of the following flight vehicles:

1. Lifting-Body Glider Aircraft
2. Space Shuttle during Second Stage Ascent
3. Satellite with Reaction Wheels
4. Agile Spacecraft with Single Gimbal CMGs

8.5 Robustness Analysis of a Lifting Body Aircraft

In this example we have a lifting body glider vehicle at 0.8 Mach and 20º angle of attack. The vehicle has four aero-surfaces, an Elevon for pitch control, and a combination of Aileron and two Rudders for lateral control. It also has RCS jets which are not used in this robustness analysis. The dynamic model is excited by a wind gust input that affects both pitch and lateral dynamics. The plant model has 33 parameters which are uncertain. They are defined by an estimated value and a maximum variation expected from the estimated value. We are going to generate a dynamic model for this flight vehicle and simulate its response to a wind gust. We will also create an uncertainty model that includes connections with the normalized uncertainties block Δ, and analyze robustness with respect to uncertainties using μ-analysis.

Modeling

The analysis files for this example are in folder *"C:\Flixan\Examples\Robust Analysis Param Uncertainties\ Lifting-Body"*. The vehicle data is in file *"Lifting_Body.Inp"*. This file contains two sets of flight vehicle input data for implementing the vehicle dynamic model using two different approaches: a standard flight vehicle model with a title: *"Lifting-Body at Mach 0.8, Alpha 20, Qbar 400, h=3000 (ft)"*, and a set of vehicle data that includes the parameter uncertainties which are bounded in magnitude and saved in a separate data-set. The title of the second vehicle data set is *"Lifting-Body at Mach 0.8, Alpha 20, Qbar 400, h=3000 (ft) with 35 Uncertainties"*. The vehicle data-set points to the title of the uncertainties data-set which includes magnitudes of maximum variations of each parameter from its nominal value. It is assumed that the vehicle parameters may vary plus or minus from their nominal values, but the variation of each parameter cannot exceed the specified maximum magnitude. The uncertainties are processed by the vehicle modeling program together with the vehicle data, and the resulting dynamic model includes additional input/ output pairs that connect to the "pulled out" uncertainties block Δ.

The Flixan program recognizes the uncertainties data-sets by the label "*UNCERTAIN PARAMETER VARIATIONS FROM NOMINAL …*" which is located at the top of the data set. There is also a title for the uncertainties set below this label: "*Uncertainties at Mach=0.8, Alpha=20*", which is used to reference and relate the uncertainties data with the vehicle input data. The uncertainties title must be included in the vehicle data-set that below the line: "*Parameter Uncertainties Data*", otherwise, the dynamic model will not include the IFL input/ output connections. The variations of the parameters which are known and do not vary are set to zero in the uncertainties data-set and they are not extracted in the Δ block. In this example the uncertainties data contains 33 parameter variations. However, two of the uncertainties, the Xcg and the Zcg are of rank-2 dependency and they couple to both: pitch and lateral states. They are extracted, therefore, as two uncertainties each, a total of four uncertainties and input/ output interconnections.

In this example the input file "*Lifting_Body.Inp*" will be processed in batch mode. At the top of the file there is a batch set of data that we can run to generate the vehicle state-space models in file "*Lifting_Body.Qdr*". It also converts them to Matlab functions that can be loaded into Matlab to perform the μ-analysis. Start the Flixan program, and select the project folder "*…\Examples\Robust Analysis Param Uncertainties\Lifting_Body*" as shown below. Then, from the Flixan main menu, go to "*File Management*", "*Manage Input Files*", and "*Edit/ Process Input Data*", as shown below.
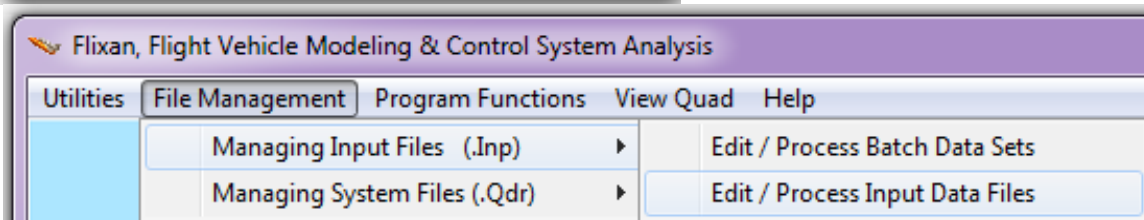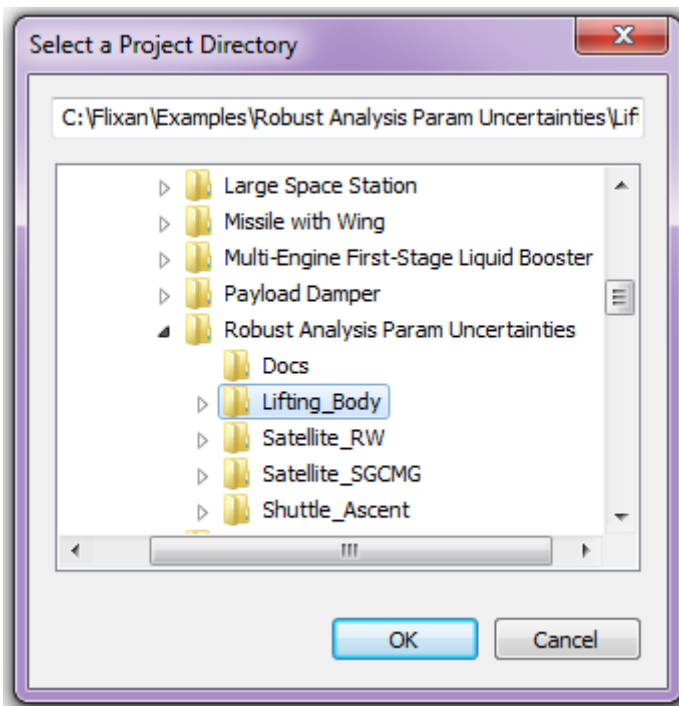
The following dialog is used for processing input data files and it has two menus. From the file selection menu on the left select the input file "*Lifting_Body.Inp*" and click on "Select Input File". The menu on the right shows the titles of the data-sets which are included in this input file. Select the first title which is a batch "*Batch Set for Creating Glider Design Models*" and click on "*Process Input Data*" button to process the batch set. The Flixan program will generate the systems and it will save them in systems file "*Lifting_Body.Qdr*".
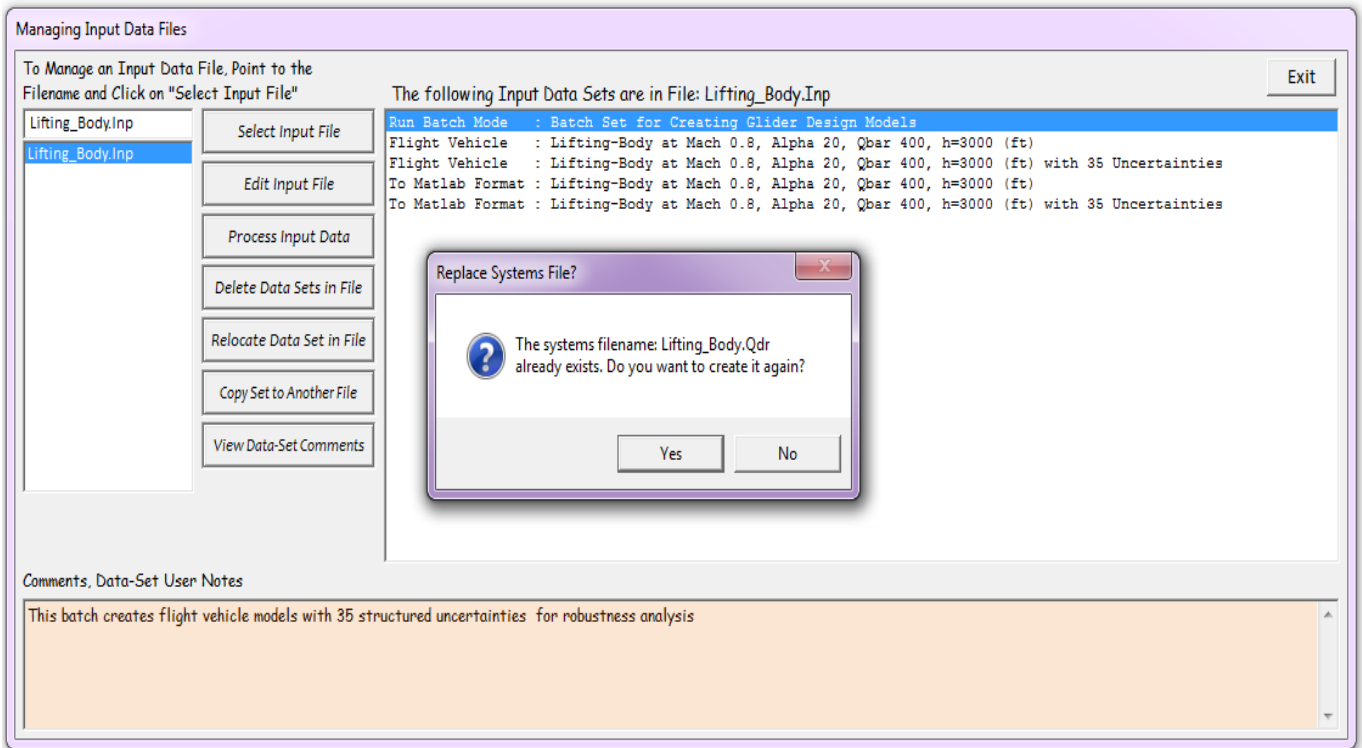
It will also convert the systems to Matlab function format for the μ-analysis. The vehicle system without uncertainties is saved in file "*Vehicle_Nom.m*", and the system with uncertainties is saved in file "*Vehicle_35Unc.m*". Both systems contain pitch and lateral coupled vehicle dynamics. The second system in addition to the standard inputs and outputs of the first system, it includes also the 35 pairs of inputs and outputs that connect to the uncertainties Δ block. This is the scaled diagonal block that contains the normalized uncertainties which vary between -1 and +1. Some of the uncertainties couple mainly with the pitch states, and some uncertainties couple with the lateral states. The CG variations, however, couple with both: pitch and lateral states, as expected.

μ–Analysis

The Matlab script file "run.m" performs the analysis in Matlab. It loads the coupled system "*Vehicle_35Unc.m*", and decouples it into pitch and lateral subsystems using the Matlab file "*Split.m*". It separates also the uncertainties input/output pairs into pitch and lateral uncertainties. It synthesizes also an LQR state-feedback control law for the pitch and the lateral systems independently using the Matlab m-file "*K_des.m*". The folder also contains two Simulink models for simulation and μ-analysis, "*Closed_Loop_Pitch.Mdl*" for pitch, and "*Closed_Loop-Lateral.Mdl*" for lateral analysis. They are shown in figures (5.1 & 5.2) below.

Pitch Closed-Loop Sim

Inputs = 2 + 14
4 Elevon Deflection   (radians)
8 Wind Gust Azim, Elev Angles=(-135, 90) (deg)

Pitch Variations= 14
10 Cm_alfa : -166.667 % Variation
12 Ca_alpha : -22.222 % Variation
13 Cz_alpha : -10.000 % Variation
15 Cm_0   : -50.000 % Variation
16 CZ_0   : -2.857 % Variation
17 Cm_q   : -40.000 % Variation
18 Cz_q   : -20.000 % Variation
23 CA_0   : -952.381 % Variation
25 I_yy   : 3.333 % Variation
27 Xcg locat: -1.766 % Variation
31 Zcg locat: 0.5000 Addit. Variat.
32 Ca_surf 1: -18.182 % Variation
33 Cz_surf 1: -16.667 % Variation
34 Cm_surf 1: 13.333 % Variation

K_ptch   K*u

xp stat
Pitch states

elev

gust   1

D_out   1

Vehicl/Uncert
x' = Ax+Bu
y = Cx+Du

% 5 States .....
% 3 Pitch Attitude (rad)
% 4 Pitch Rate (rad/sec)
% 7 Alpha (rad)
% 9 Delta_Altitude (ft)
% 10 Delta_veloc (ft/sec)

thet
q
alfa
dh
Vdot   Accel

D_in   1

% Outputs = 7 + 14
% 3 Pitch Attitude (thet-bdy) (radians)
% 4 Pitch Rate   (q-body)   (rad/sec)
% 7 Angle of attack, alfa, (radians)
% 9 Change in Altitude, delta-h, (feet)
% 10 Forward Acceleration (V-dot) (ft/sec)
% 12 CG Acceleration along X axis, (ft/sec^2)
% 14 CG Acceleration along Z axis, (ft/sec^2)

% Pitch Variations= 14
% 16 Cm_alfa : -166.667 % Variation
% 18 Ca_alpha : -22.222 % Variation
% 19 Cz_alpha : -10.000 % Variation
% 21 Cm_0   : -50.000 % Variation
% 22 CZ_0   : -2.857 % Variation
% 23 Cm_q   : -40.000 % Variation
% 24 Cz_q   : -20.000 % Variation
% 29 CA_0   : -952.381 % Variation
% 31 I_yy   : 3.333 % Variation
% 33 Xcg locat: -1.766 % Variation
% 37 Zcg locat: 0.5000 Addit. Variat.
% 38 Ca_surf 1: -18.182 % Variation
% 39 Cz_surf 1: -16.667 % Variation
% 40 Cm_surf 1: 13.333 % Variation

Lateral Closed-Loop Sim

Inputs = 25
5 Aileron Deflection   (radians)
6 Rudder1 Deflection   (radians)
7 Rudder2 Deflection   (radians)
8 Wind Gust Azim, Elev Angles=(-135.0 90.0) (deg)

Lateral Variations= 21
9   Cl_beta : -20.000 % Variation
11 Cn_beta : 30.303 % Variation
14 Cy_beta : -14.286 % Variation
19 Cn_p   : 54.348 % Variation
20 Cn_r   : -20.408 % Variation
21 Cl_p   : -33.333 % Variation
22 Cl_r   : 25.000 % Variation
24 I_xx   : 2.857 % Variation
26 I_zz   : 3.030 % Variation
28 Xcg locat: -1.766 % Variation
29 Ycg locat: 0.5000 Addit. Variat.
30 Zcg locat: 0.5000 Addit. Variat.
35 Cy_surf 2: 500.000 % Variation
36 Cl_surf 2: 15.000 % Variation
37 Cn_surf 2: -650.000 % Variation
38 Cy_surf 3: 10.750 % Variation
39 Cl_surf 3: 40.000 % Variation
40 Cn_surf 3: -10.000 % Variation
41 Cy_surf 4: 5.000 % Variation
42 Cl_surf 4: 6.250 % Variation
43 Cn_surf 4: 6.000 % Variation

K_lat   K*u

Lat States
Y   U

ailer
rudd

gust   1

D_out   1

Vehicl/Uncert
x' = Ax+Bu
y = Cx+Du

% 5 Lateral States .....
% 1 Roll Attitude (rad)
% 2 Roll Rate (rad/sec)
% 5 Yaw Attitude (rad)
% 6 Yaw Rate (rad/sec)
% 8 Beta (rad)

phi
P
psi
R
beta
Vcr   Y Accel
D_in   1

% Outputs = 28
% 1 Roll Attitude (phi-body) (radians)
% 2 Roll Rate   (p-body) (rad/sec)
% 5 Yaw Attitude (psi-body) (radians)
% 6 Yaw Rate   (r-body) (rad/sec)
% 8 Angle of sideslip, beta, (radian)
% 11 Cross Range Velocity (Vcr) (ft/sec)
% 13 CG Acceleration along Y axis, (ft/sec)

% Lateral Variations= 21
% 15 Cl_beta : -20.000 % Variation
% 17 Cn_beta : 30.303 % Variation
% 20 Cy_beta : -14.286 % Variation
% 25 Cn_p   : 54.348 % Variation
% 26 Cn_r   : -20.408 % Variation
% 27 Cl_p   : -33.333 % Variation
% 28 Cl_r   : 25.000 % Variation
% 30 I_xx   : 2.857 % Variation
% 32 I_zz   : 3.030 % Variation
% 34 Xcg locat: -1.766 % Variation
% 35 Ycg locat: 0.5000 Addit. Variat.
% 36 Zcg locat: 0.5000 Addit. Variat.
% 41 Cy_surf 2: 500.000 % Variation
% 42 Cl_surf 2: 15.000 % Variation
% 43 Cn_surf 2: -650.000 % Variation
% 44 Cy_surf 3: 10.750 % Variation
% 45 Cl_surf 3: 40.000 % Variation
% 46 Cn_surf 3: -10.000 % Variation
% 47 Cy_surf 4: 5.000 % Variation
% 48 Cl_surf 4: 6.250 % Variation
% 49 Cn_surf 4: 6.000 % Variation

Figures 5.1 and 5.2, Closed-Loop Models Used for μ-Analysis

The two systems have their control loops closed via the state-feedback gains K_p and K_l. The 35 uncertainties were separated to 14 pitch and 21 lateral uncertainties, as shown above. The uncertainty inputs and outputs are combined into vectors which are supposedly connected to the diagonal uncertainty block Δ (not shown).

Robustness analysis is performed by calculating the μ frequency response between the input and the output uncertainty vectors for the pitch and the lateral systems separately. The Matlab file "run.m" calculates the μ frequency response, assuming real and not complex parameter variations (complex are easier to calculate but they are too conservative), and plots μ versus frequency, as shown in Figure 5.3. Robustness is guaranteed when μ is less than 1 at all frequencies for both the pitch and the lateral systems. It means that the pitch and lateral systems will be stable despite all the parameter uncertainties which have bounded and known magnitudes.
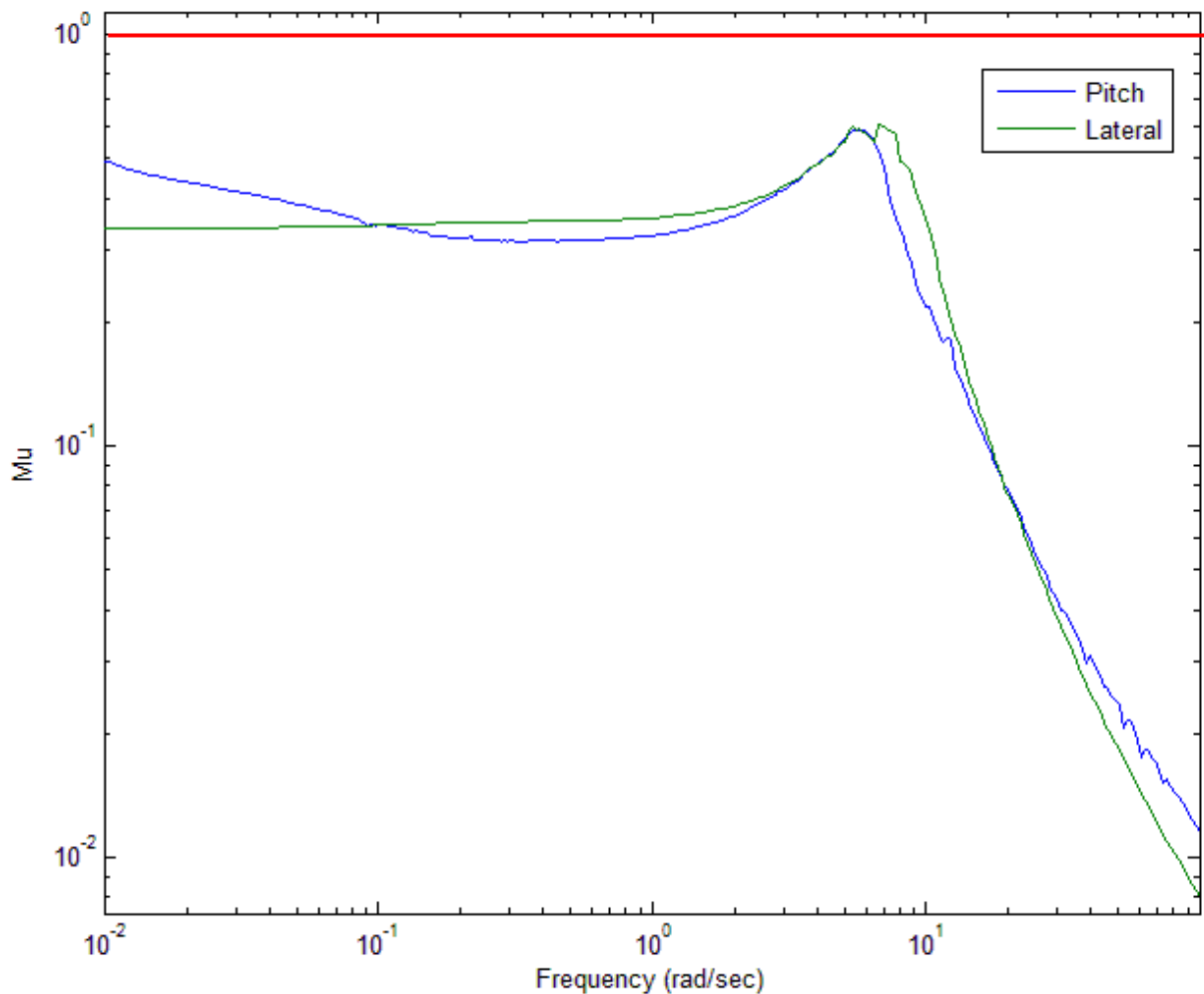

Figure 5.3, Mu frequency response for 14 pitch and 21 lateral uncertainties, showing that the control system is robust to the parameter variations.

Simulation Model

The Simulink model "*Closed_Loop_Sim.mdl*", shown in Figure 5.4, is used for the pitch and lateral coupled simulation. The system "*Lifting-Body at Mach 0.8, Alpha 20, Qbar 400, h=3000 (ft) with 35 Uncertainties*" is loaded into Matlab from file "*Vehicle-35Unc.m*" and included as a state-space model. The inputs and outputs are shown in the Figure, and the control loops are closed via the derived state-feedback gains Kp and Kl. The system also has the 35 uncertainty input and output pairs at the bottom of the list. Actually, there are 33 uncertainties but as we already mentioned the Xcg and the Zcg couple into both: pitch and lateral dynamics, and are extracted, therefore, as two separate uncertainties each.
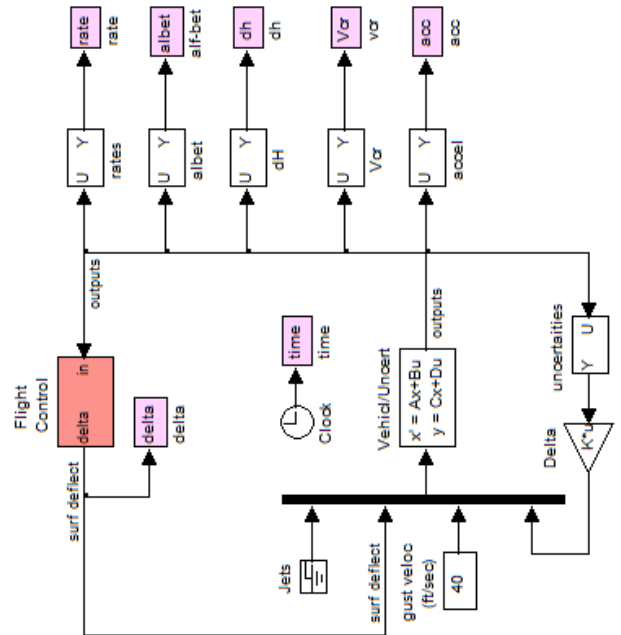
The parameter uncertainties loop via the diagonal matrix "Delta" that represents the normalized uncertainties, is shown closed in the model. When the uncertainties loop is opened the system behaves like the nominal system which is closed-loop stable since the gains were "tailored" around this original system. According to our μ-analysis, the closed-loop system is robust to the uncertainties, and therefore, it should remain stable after the uncertainties loop is closed, as long as the absolute values of the diagonal elements of matrix "Delta" are less than "1". Remember, that Delta is normalized because the actual parameter variations were absorbed inside the plant model. This simulation model can be used to verify system robustness to uncertainties by varying the diagonal elements of the Delta matrix and checking stability (as long as they vary between -1 and +1). The following figures show the nominal system responses to a 40 ft/sec wind gust step. The gust is applied in a direction that excites both, pitch and lateral states. After running the simulation model use the Matlab script "pl.m" to plot the data.

Simulation Model

Figure 5.4 Time Domain Simulation Model with Control and Uncertainties Loops Closed

% Inputs = 8 + 35
% 1 Throttle Input dTh/Th for Engine No 1 (-)
% 2 Throttle Input dTh/Th for Engine No 2 (-)
% 3 Throttle Input dTh/Th for Engine No 3 (-)
% 4 Elevon Surface No 1 Deflect. (radians)
% 5 Aileron Surface No 2 Deflect. (radians)
% 6 Rudder-1 Surface No 3 Deflect. (radians)
% 7 Rudder-2 Surface No 4 Deflect. (radians)
% 8 Wind Gust Azim, Elev Angles=(-135, 90) (deg)

% Param Variations = 35
% 9 Cl_beta : 20.000 % Variation
% 10 Cm_alfa : 166.667 % Variation
% 11 Cn_beta : 30.303 % Variation
% 12 Ca_alpha : 22.222 % Variation
% 13 Cz_alpha : 10.000 % Variation
% 14 Cy_beta : 14.286 % Variation
% 15 Cm_0 : -50.000 % Variation
% 16 CZ_0 : -2.857 % Variation
% 17 Cm_q : -40.000 % Variation
% 18 Cz_q : -20.000 % Variation
% 19 Cn_p : 54.348 % Variation
% 20 Cn_r : -20.408 % Variation
% 21 CLp : -33.333 % Variation
% 22 CLr : 25.000 % Variation
% 23 CA_0 : -238.095 % Variation
% 24 Lxx : 2.857 % Variation
% 25 Lyy : 3.333 % Variation
% 26 Lzz : 3.030 % Variation
% 27 Xcg locat: -1.766 % Variation
% 28 Xcg locat: -1.766 % Variation
% 29 Ycg locat: 0.5000 Addit. Variat.
% 30 Zcg locat: 0.5000 Addit. Variat.
% 31 Zcg locat: 0.5000 Addit. Variat.
% 32 Ca_surf 1: 18.182 % Variation
% 33 Cz_surf 1: 16.667 % Variation
% 34 Cm_surf 1: 13.333 % Variation
% 35 Cy_surf 2: 500.000 % Variation
% 36 Cl_surf 2: 15.000 % Variation
% 37 Cn_surf 2: -650.000 % Variation
% 38 Cy_surf 3: 10.750 % Variation
% 39 Cl_surf 3: 40.000 % Variation
% 40 Cn_surf 3: -10.000 % Variation
% 41 Cy_surf 4: 5.000 % Variation
% 42 Cl_surf 4: 6.250 % Variation
% 43 Cn_surf 4: 6.000 % Variation

% Outputs = 14 + 35
% 1 Roll Attitude (phi-body) (radians)
% 2 Roll Rate (p-body) (rad/sec)
% 3 Pitch Attitude (thet-bdy) (radians)
% 4 Pitch Rate (q-body) (rad/sec)
% 5 Yaw Attitude (psi-body) (radians)
% 6 Yaw Rate (r-body) (rad/sec)
% 7 Angle of attack, alfa, (radians)
% 8 Angle of sideslip, beta, (radian)
% 9 Change in Altitude, delta-h, (feet)
% 10 Forward Acceleration (V-dot) (ft/sec)
% 11 Cross Range Velocity (Vcr) (ft/sec)
% 12 CG Acceleration along X axis, (ft/sec^2)
% 13 CG Acceleration along Y axis, (ft/sec^2)
% 14 CG Acceleration along Z axis, (ft/sec^2)

% Param Variations= 35
% 15 Cl_beta : 20.000 % Variation
% 16 Cm_alfa : 166.667 % Variation
% 17 Cn_beta : 30.303 % Variation
% 18 Ca_alpha : 22.222 % Variation
% 19 Cz_alpha : 10.000 % Variation
% 20 Cy_beta : 14.286 % Variation
% 21 Cm_0 : -50.000 % Variation
% 22 CZ_0 : -2.857 % Variation
% 23 Cm_q : -40.000 % Variation
% 24 Cz_q : -20.000 % Variation
% 25 Cn_p : 54.348 % Variation
% 26 Cn_r : -20.408 % Variation
% 27 CLp : -33.333 % Variation
% 28 CLr : 25.000 % Variation
% 29 CA_0 : -238.095 % Variation
% 30 Lxx : 2.857 % Variation
% 31 Lyy : 3.333 % Variation
% 32 Lzz : 3.030 % Variation
% 33 Xcg locat: -1.766 % Variation
% 34 Xcg locat: -1.766 % Variation
% 35 Ycg locat: 0.5000 Addit. Variat.
% 36 Zcg locat: 0.5000 Addit. Variat.
% 37 Zcg locat: 0.5000 Addit. Variat.
% 38 Ca_surf 1: 18.182 % Variation
% 39 Cz_surf 1: 16.667 % Variation
% 40 Cm_surf 1: 13.333 % Variation
% 41 Cy_surf 2: 500.000 % Variation
% 42 Cl_surf 2: 15.000 % Variation
% 43 Cn_surf 2: -650.000 % Variation
% 44 Cy_surf 3: 10.750 % Variation
% 45 Cl_surf 3: 40.000 % Variation
% 46 Cn_surf 3: -10.000 % Variation
% 47 Cy_surf 4: 5.000 % Variation
% 48 Cl_surf 4: 6.250 % Variation
% 49 Cn_surf 4: 6.000 % Variation

% 10 States .....
% 1 Roll Attitude (rad)
% 2 Roll Rate (rad/sec)
% 3 Pitch Attitude (rad)
% 4 Pitch Rate (rad/sec)
% 5 Yaw Attitude (rad)
% 6 Yaw Rate (rad/sec)
% 7 Alpha (rad)
% 8 Beta (rad)
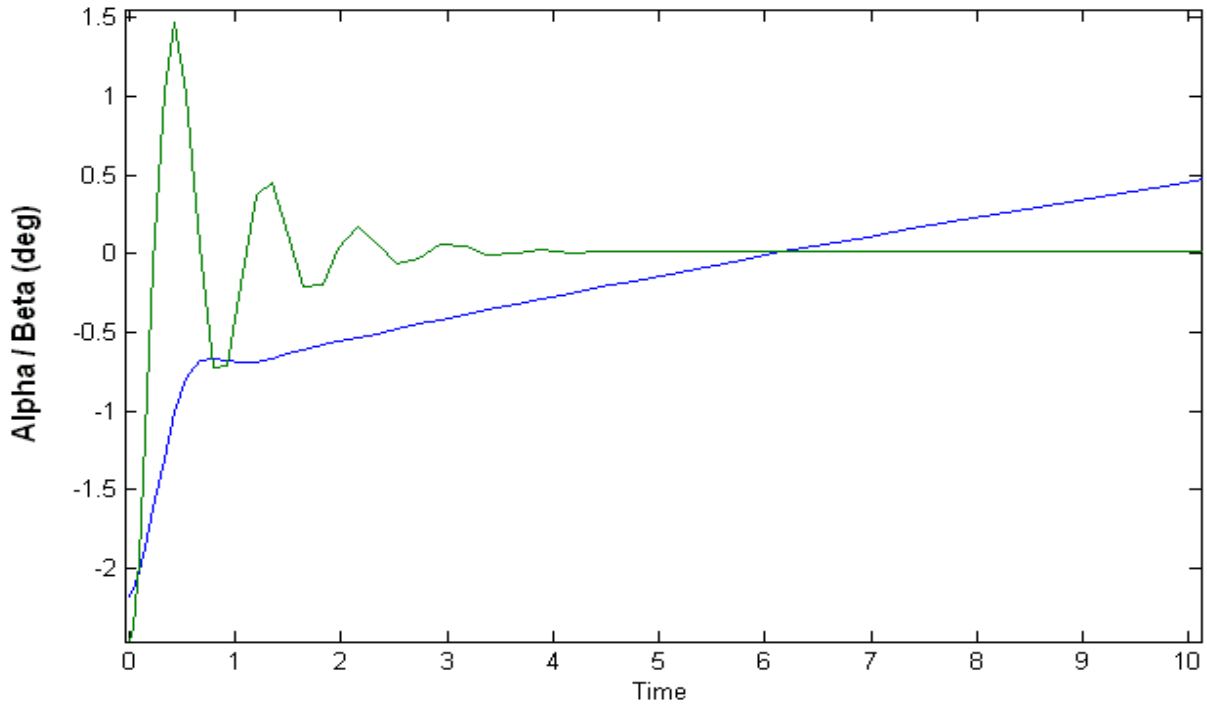% 9 Delta_Altitude (ft)
% 10 Delta_veloc (ft/sec)

Angle of Attack/ Sideslip Response to 40 ft/sec Gust Shear

Lifting Body Aircraft Simulation

Lifting Body Aircraft Simulation

Lifting Body Aircraft Simulation

8.6 Space Shuttle Robustness Analysis during Second Stage

This example analyzes the Space Shuttle vehicle during second stage, immediately after the solid rocket booster separation. Although in this time period the Shuttle is almost outside the atmosphere, there is still a considerable amount of dynamic pressure for not ignoring the aerodynamics. We will develop a dynamic model of the vehicle including structural flexibility and propellant sloshing dynamics. The model is uncertain with 45 parameter variations. In addition to the standard state variables it includes IFL inputs and outputs that connect to the normalized uncertainty block Δ. It will be used to perform robustness analysis.

The analysis data files are in folder "*\Flixan\Examples\Robust Analysis Param Uncertainties\ Shuttle_Ascent*", and the vehicle input data is in file: "*Shuttle_Stg2.Inp*". The vehicle is controlled by the 3 Shuttle main engines which are gimbaling in p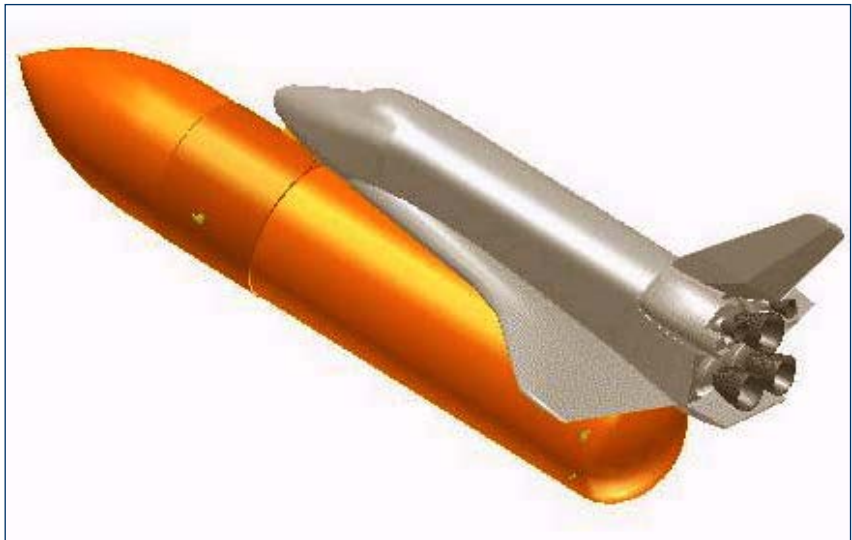itch and yaw. The tail-wag-dog effect is included in the dynamic model by setting the flag in the engine data "WITH TWD". A linear main engine actuator model is also included in the input data file. A gust disturbance is applied in the model perpendicular to the vehicle x axis, skewed to excite both: pitch and lateral directions. The input data file "*Shuttle_Stg2.Inp*" contains two sets of vehicle data, a set that generates the standard flight vehicle model with title: "*Space Shuttle Second Stage at T=123 sec*", and a set of vehicle data that generates a similar vehicle model with additional inputs and outputs for the uncertainty model. The title of the second data-set is "*Space Shuttle Second Stage at T=123 sec (With Uncertain Param)*", and requires also a set of parameter uncertainties data which is also included in the input file "*Shuttle_Stg2.Inp*". Its title is "*Uncertainties for Shuttle Ascent Second Stage*". It contains maximum variations in vehicle parameters, such as: moments of inertia, angles of attack, sideslip, aero coefficients, engine thrusts, slosh uncertainties, and flex mode frequency and damping variations. The title of the uncertainties data is included in the vehicle data, below the line "*Parameter Uncertainties Data*".

The dynamic models include 20 flexible structural modes. The modes are already preselected from a big finite elements model and a smaller number of selected modes are included in file "*Shuttle_Stg2.Inp*". The title of the selected modes set is: "*Shuttle Second Stage at T=123 sec, Mixed Modes*". The mode selection process is not shown in this example. The Flight Vehicle Modeling program will process the uncertainties data set together with the vehicle data and the slosh and flex modes to create the augmented state-space model. In this example, the vehicle, flex modes, and the uncertainties data-sets already exist and they are saved in the input file "*Shuttle_Stg2.Inp*". The Flixan program will process the input data and it will create a state-space model of the vehicle augmented with the additional inputs and outputs necessary to connect with the uncertainty block Δ. The augmented system will be saved in systems file "*Shuttle_Stg2.Qdr*". Notice, that the input/ output pairs that connect to the uncertainty block Δ is greater than the number of parameter variations. This is

because some of the uncertainties, such as: engine thrusts, slosh masses, and slosh positions are higher than rank-1 dependency and they couple to both pitch and lateral dynamics. Those parameters are extracted more than once because they couple in both pitch and lateral equations. This model will be used to perform robustness analysis using μ methods.

Processing the Data Files in Batch Mode

The input file for this example "*Shuttle_Stg2.Inp*" is already prepared. It includes vehicle data, structural data, uncertainties, actuators, sensor models, the flight control system and systems interconnection data. It will be processed interactively in batch mode in order to generate the systems for the analysis. At the top of the file there is a batch set that automates the processing of the data-sets. It creates the state-space models and converts them to Matlab format for further analysis using Matlab. It begins by creating the two vehicle models: a standard model and an uncertainty model that includes the parameter variations data. The uncertain model is dynamically identical to the regular model but it includes one additional input/ output pair for each uncertainty. The batch generates also actuator models for the three SSME engines that include acceleration outputs for coupling the actuators with the acceleration inputs in the vehicle model. The acceleration inputs generate the tail-wag-dog dynamics. The actuator modeling program is used for generating the actuator state-space model that has a title: "*Shuttle Main Engine Hydraulic Actuator (Type-I)*". This system is duplicated six times for the 3 main engines, 3 pitch and 3 yaw actuators. It is also combined with the TVC gains to form a single system "*Actuators/TVC (Second Stage)*".

The batch also creates dynamic models for the gyro and IMU sensors using transfer-function block interconnections, its title is "*Sensor Dynamics*". It combines the vehicle, actuators and sensor subsystems together into a single system "*Plant Model, Vehicle/ Actuators/ Sensors*", and generates also the flight control system from transfer-function blocks "*Shuttle Stage-2 Flight Control System*". The systems generated are saved in the systems file "*Shuttle_Stg2.Qdr*". The systems from the systems file are also re-formatted to Matlab m-functions to be analyzed for robustness in Matlab. The vehicle models are saved in files "*Vehicle.m*", and "*Vehicle_Unc.m*". The sensors system is saved in "*Sensors.m*". The combined actuator and TVC system is saved in "*Actor_tvc.m*". The flight control system is saved in "*Controller.m*". The combined plant model is saved in "*Plant.m*". These systems are loaded into Matlab by executing the script file "run.m".
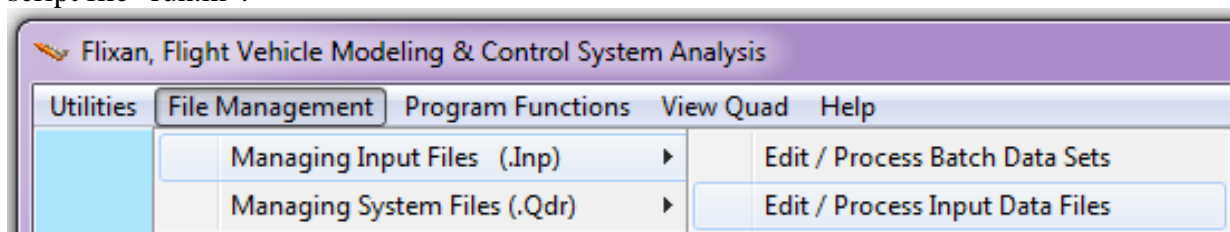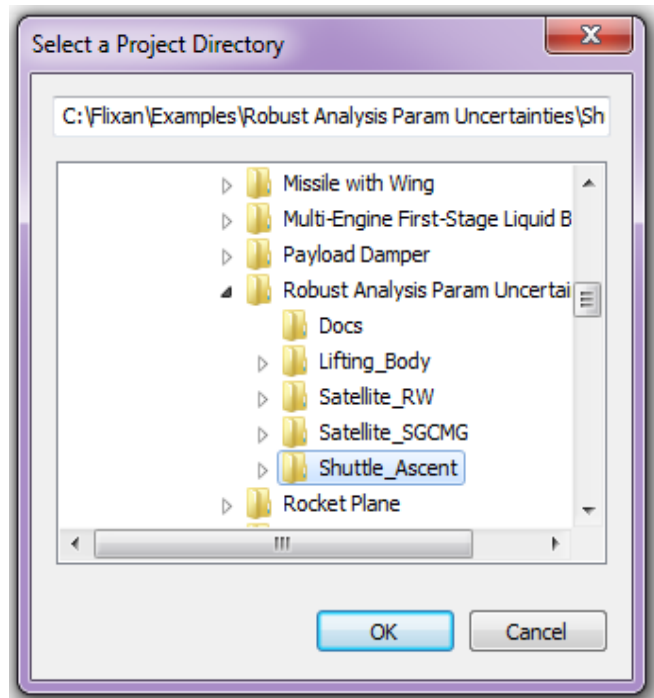
From the Flixan main menu go to "*Edit/ Process Input Data Files*". The following dialog comes up and it is used for processing input data files. It has two menus. From the file selection menu on the left select the input file "*Shuttle_Stg2.Inp*" and click on "*Select Input File*". The menu on the right shows the titles of the data-sets which are included in this input file. Select the first title which is a batch "*Batch Set for Creating a Space Shuttle Model During Second Stage*" and click on "*Process Input Data*" button to process the batch set. The Flixan program will generate the systems and it will save them in systems file "*Shuttle_Stg2.Qdr*". A display window momentarily pops up highlighting the batch commands as they execute.
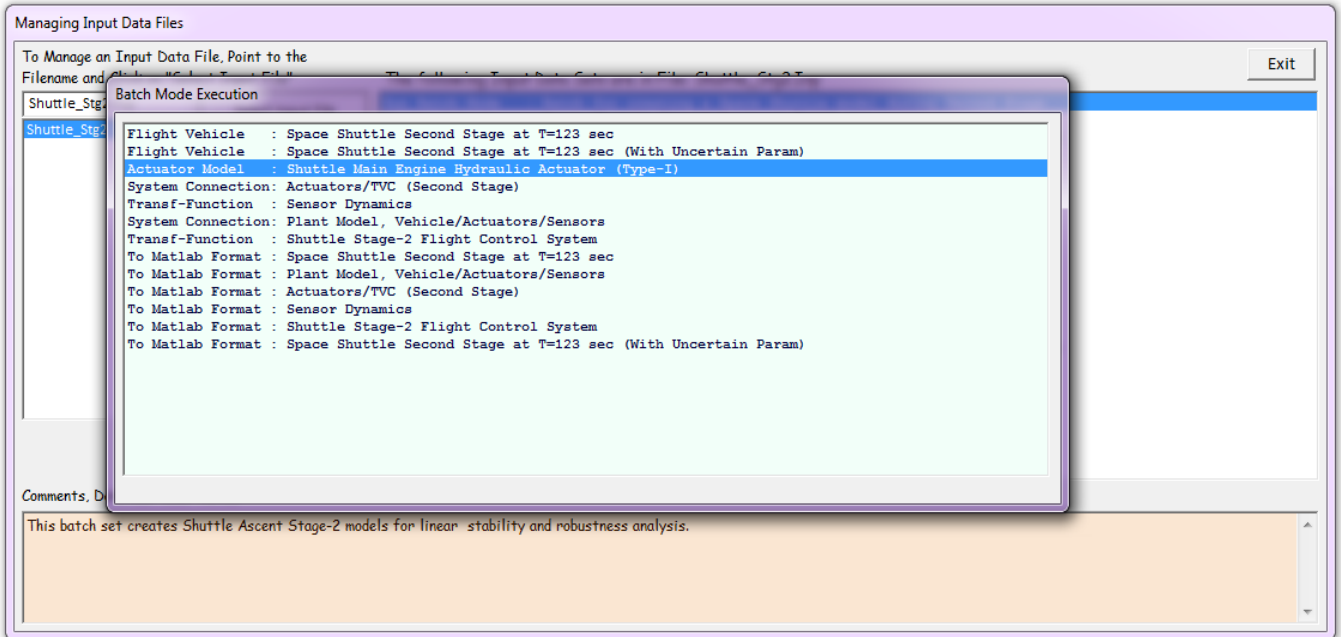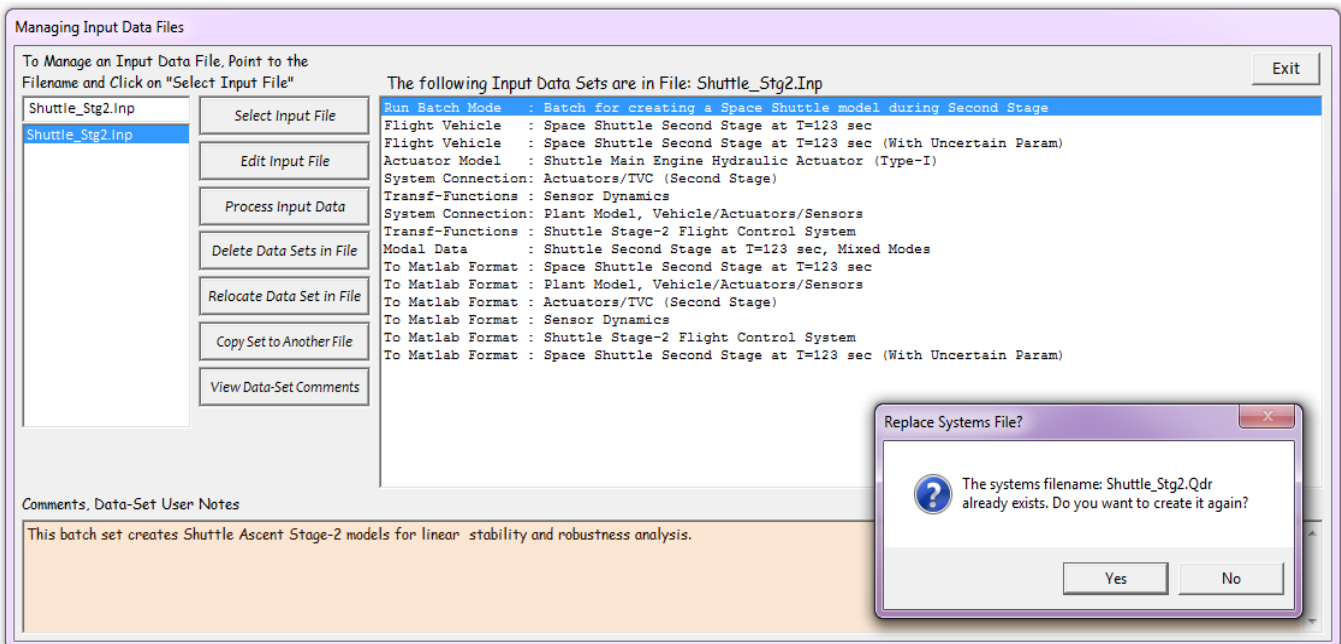
Control Analysis

The folder "*Examples\Robust Analysis Param Uncertainties\Shuttle_Ascent*" includes several Simulink models used in this robustness analysis. The file "run.m" below loads the systems, and performs linear stability and robustness μ-analysis. One of the simulation models is "*Closed_Loop_Plant.mdl*", shown in Figure 6.1a. It uses the combined plant model "*Plant.m*", and the flight control system "*Controller.m*" in closed-loop form. The plant subsystem and the controller are shown in Figures (6.1b and 6.1c). Figure 6.2 shows the system's transient responses to a wind gust step.

```matlab
% Computes the Open-Loop Frequency Response for Flight Control Analysis
% Loop is opened at the Flight Control System Output
d2r=pi/180; r2d=180/pi;
Npv=45;                                      % Number of Param Variations
% Load the sontrol loop subsystems
[Av, Bv, Cv, Dv]= vehicle;                   % Vehicle Nominal Model
[Au, Bu, Cu, Du]= vehicle_unc;               % Vehicle Model with Uncertainties
[At, Bt, Ct, Dt]= actor_tvc;                 % Actuator + TVC Model
[As, Bs, Cs, Ds]= sensors;                   % Sensors Model
[Ac, Bc, Cc, Dc]= controller;                % Controller
[Ap, Bp, Cp, Dp]= plant;                      % Combined Plant Model

label='Stage-2 Yaw Axis Stability, (Pitch & Roll Closed)';
[Ao,Bo,Co,Do]= linmod('Open_Loop');          % Frequ Response Model (z-domain)
sys=ss(Ao,Bo,Co,Do);                          % Perform Linearization
w=logspace(-2,2,12000);                       % and Frequ domain analysis
figure(1); Nichols(sys,w); title(label)
figure(2); Bode(sys,w); title(label)

w=logspace(-2,2,500);
[Acp,Bcp,Ccp,Dcp]=linmod('Closed_Loop_Unc');
sys=ss(Acp,Bcp,Ccp,Dcp);
sysf= FRD(sys,w);
blk=[-ones(Npv,1), zeros(Npv,1)];
[bnd,muinfo]= mussv(sysf,blk);

ff= get(muinfo.bnds, 'frequency');
muu=get(muinfo.bnds, 'responsedata');
muu=squeeze(muu);
muu=muu(1,:);
loglog(ff,muu)
xlabel('Frequency (rad/sec)')
ylabel('ssv')
```
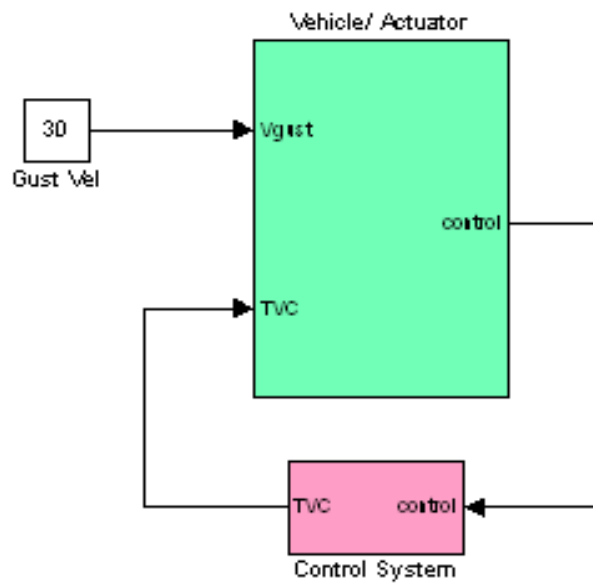
Figure 6.1a Plant Model and Flight Control System in Closed-Loop Form

% Inputs = 4
% 1 Roll FCS Command (DP-TVC)
% 2 Pitch FCS Command (DQ-TVC)
% 3 Yaw FCS Command (DR-TVC)
% 4 Wind-Gust Velocity (ft/sec)

% Outputs = 12
% 1 Roll Attitude to FCS
% 2 Roll Rate to FCS
% 3 Pitch Attitude to FCS
% 4 Pitch Rate to FCS
% 5 Yaw Attitude to FCS
% 6 Yaw Rate to FCS
% 7 Angle of Attack, alpha (rad)
% 8 Angle of Sideslip, beta (rad)
% 9 Normal Accelerometer, Nz (ft/sec^2)
% 10 Lateral Accelerometer, Ny (ft/sec^2)
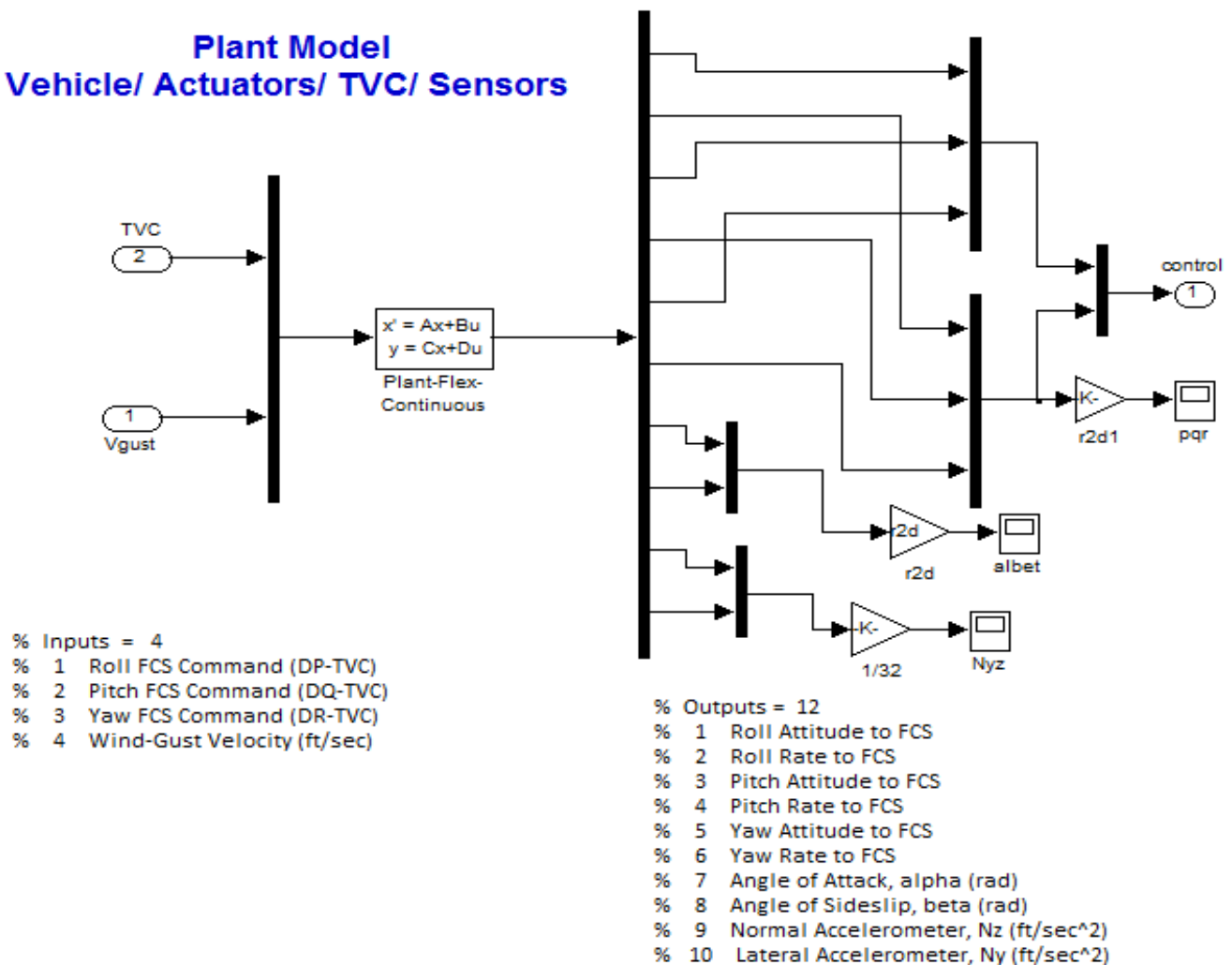
Figure 6.1b Plant model from file "plant.m" consisting of Vehicle, Actuators, TVC, and Sensor dynamics

8-30

Attitude and Rate Errors

Stage-2
Controller
Continuous

control
(1)

$x' = Ax+Bu$
$y = Cx+Du$

TVC
(1)

```
%  Inputs =  6
%   1    Roll Attitude Error  (phi-err)
%   2    Pitch Attitude Error (theta-err)
%   3    Yaw Attitude Error   (psi-err)
%   4    Roll Rate Error  (p-err)
%   5    Pitch Rate Error (q-err)
%   6    Yaw Rate Error   (r-err)
```

```
%   1    Roll FCS (DP-TVC)
%   2    Pitch FCS (DQ-TVC)
%   3    Yaw FCS (DR-TVC)
```
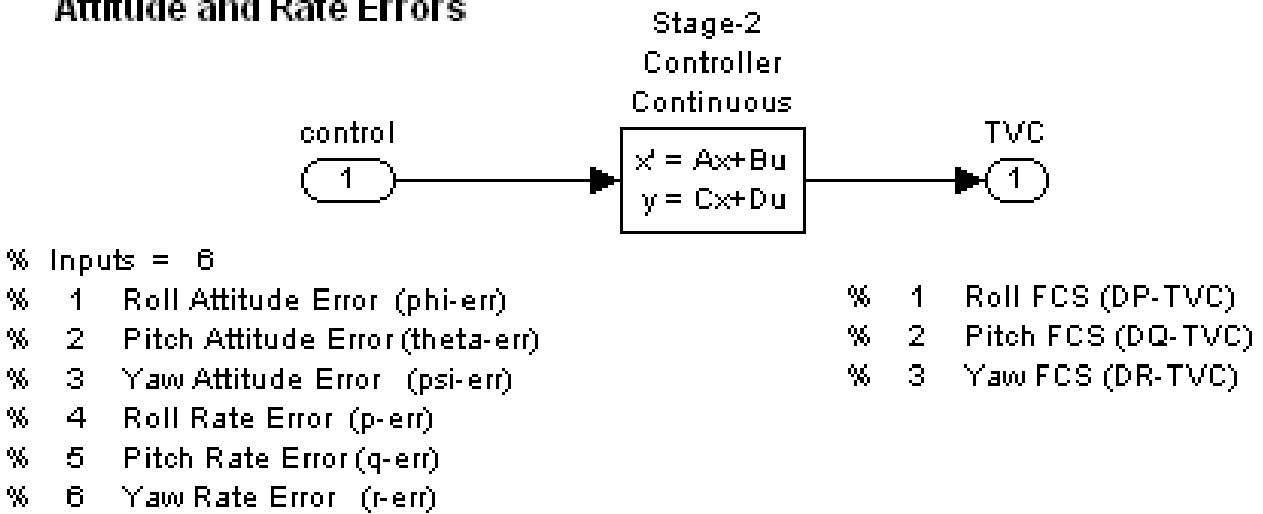
Figure 6.1c Shuttle Second Stage flight control system converted to State-Space form
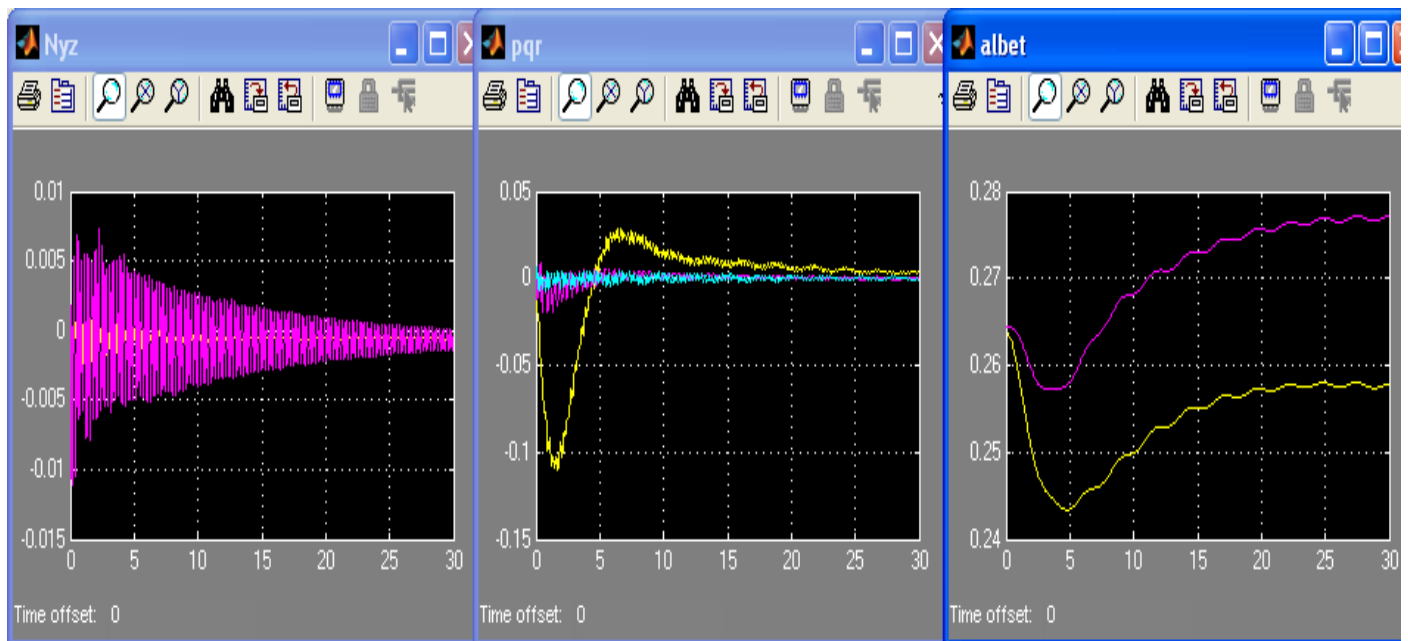
Figure 6.2 Vehicle Responses to a Wind-Gust Step

Robustness Analysis Model

Figure 6.3 shows the closed-loop Simulink model "*Closed_Loop_Unc.mdl*" that it is used for analyzing robustness and it includes the input/output attachments to the uncertainty block. It consists of the following subsystems:

1. Vehicle model "*Space Shuttle Second Stage at T=123 sec (With Uncertain Param)*" which is loaded into Matlab from file "*Vehicle_Unc.m*", and it is shown in detail in Figure 6.3b.
2. Actuators/TVC model is loaded from file "*Actor_tvc.m*" and it shown in detail in Figure 6.3c
3. Sensors, and
4. Control System
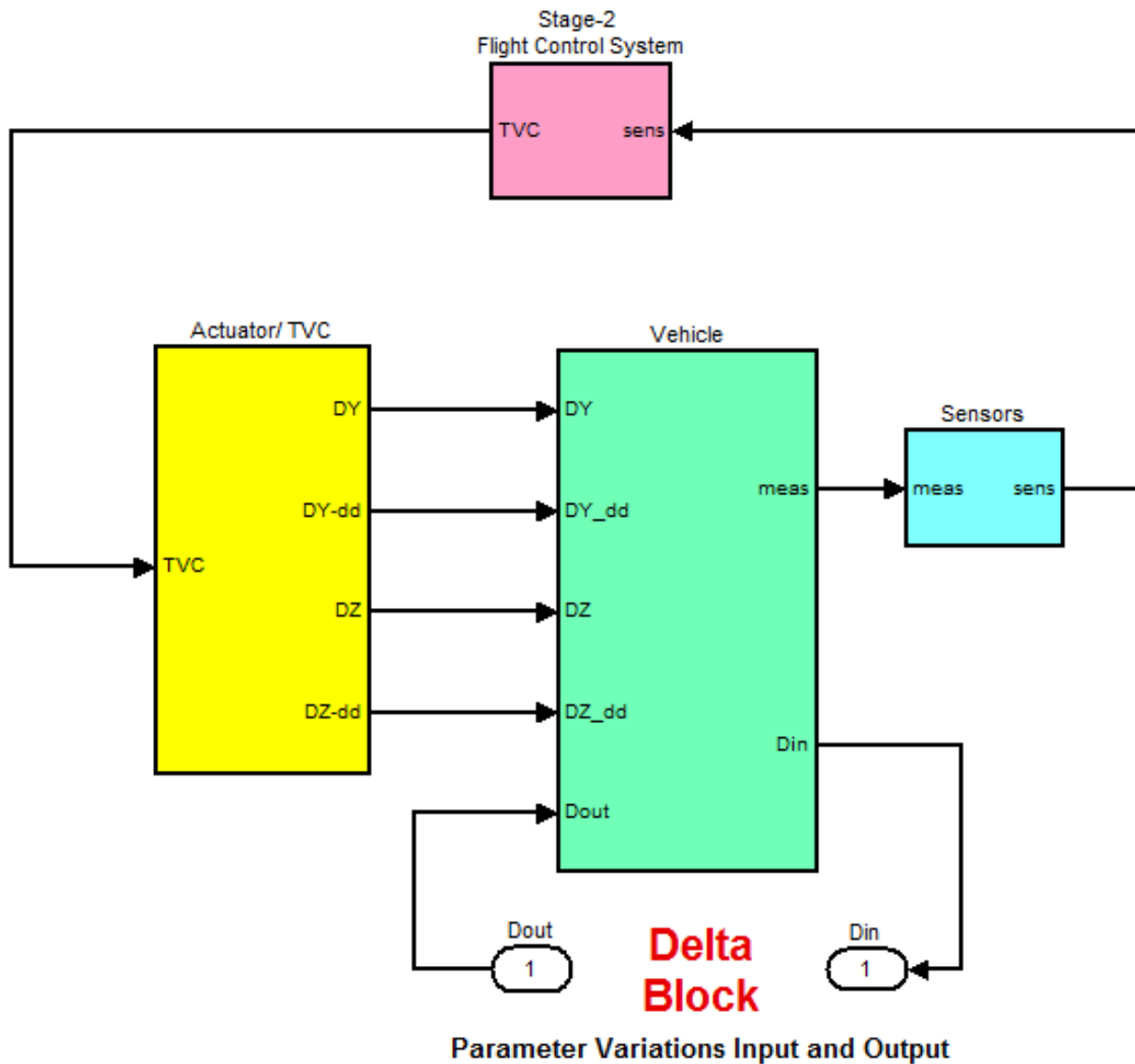
Figure 6.3, Shuttle vehicle model with the flight control system loop closed. It includes the input and output vectors that connect to the uncertainty block Δ.

The dynamic model includes the input and output attachments that connect to the uncertainty block Δ. The system's uncertainty inputs and outputs are scaled so that the elements (δ_i) of the Δ block vary between -1 and +1. This model is used in robustness analysis by calculating the (μ) structural singular

value (SSV) frequency response between the uncertainty inputs and outputs. According to small gain theory, the system is robust to the uncertainties (defined by the parameter variations) when the SSV of the closed-loop system, as seen across Δ, is less than one at all frequencies.

```
% Outputs = 67
%  1   Roll Attitude  (phi-body) (radians)
%  2   Roll Rate     (p-body)  (rad/sec)
%  3   Pitch Attitude (thet-bdy) (radians)
%  4   Pitch Rate    (q-body)  (rad/sec)
%  5   Yaw Attitude (psi-body) (radians)
%  6   Yaw Rate      (r-body)  (rad/sec)
%  7   Angle of attack, alfa, (radians)
%  8   Angle of sideslip, beta, (radian)
%  9   Change in Altitude, delta-h, (feet)
% 10   Forward Acceleration (V-dot) (ft/sec)
% 11   Cross Range Velocity (Vcr) (ft/sec)
% 12   Rate-Gyro # 1, Roll Rate  (Body) (rad/sec)
% 13   Rate-Gyro # 2, Pitch Rate (Body) (rad/sec)
% 14   Rate-Gyro # 3, Yaw Rate   (Body) (rad/sec)
% 15   Accelerom # 1, (along Z), (ft/sec^2) Translat. Acceler
% 16   Accelerom # 2, (along Y), (ft/sec^2) Translat. Acceler
% 17   Ptch Load-Torque Tly for Engine: 1,(ft-lb)
% 18   Ptch Load-Torque Tly for Engine: 2,(ft-lb)
% 19   Ptch Load-Torque Tly for Engine: 3,(ft-lb)
% 20   Yaw  Load-Torque Tlz for Engine: 1,(ft-lb)
% 21   Yaw  Load-Torque Tlz for Engine: 2,(ft-lb)
% 22   Yaw  Load-Torque Tlz for Engine: 3,(ft-lb)
```

```
% Inputs = 58
%  1   Engine No  1 Pitch Deflect. (rad), Dymax= 10.0000 deg
%  2   Engine No  2 Pitch Deflect. (rad), Dymax= 10.0000 deg
%  3   Engine No  3 Pitch Deflect. (rad), Dymax= 10.0000 deg
%  4   Engine No  1 Pitch Acceleration  (rad/sec^2)
%  5   Engine No  2 Pitch Acceleration  (rad/sec^2)
%  6   Engine No  3 Pitch Acceleration  (rad/sec^2)
%  7   Engine No  1 Yaw Deflect. (rad), Dzmax= 10.0000 deg
%  8   Engine No  2 Yaw Deflect. (rad), Dzmax= 10.0000 deg
%  9   Engine No  3 Yaw Deflect. (rad), Dzmax= 10.0000 deg
% 10   Engine No  1 Yaw Acceleration  (rad/sec^2)
% 11   Engine No  2 Yaw Acceleration  (rad/sec^2)
% 12   Engine No  3 Yaw Acceleration  (rad/sec^2)
% 13   Wind Gust Azim, Elev Angles=(  45.0  90.0) (deg)

Variations = 45
% 14   Cl_beta  :  -44.321 % Variation
% 15   Cm_alpha :  -13.472 % Variation
% 16   Cn_beta  :   23.053 % Variation
% 17   Ca_alpha :   31.831 % Variation
% 18   Cz_alpha :   -6.829 % Variation
% 19   Cy_beta  :  -25.185 % Variation
% 20   Cm_0     :   35.804 % Variation
% 21   CZ_0     :  230.769 % Variation
% 22   CA_0     :   13.937 % Variation
% 23   I_xx   :   6.615 % Variation
% 24   I_yy   :   3.435 % Variation
% 25   I_zz   :   3.616 % Variation
% 26   Xcg locat:  -1.050 % Variation
% 27   Xcg locat:  -1.050 % Variation
% 28   Thr_eng 1:   2.046 % Variation
% 29   Thr_eng 1:   2.046 % Variation
% 30   Thr_eng 2:   2.046 % Variation
% 31   Thr_eng 2:   2.046 % Variation
% 32   Thr_eng 3:   2.046 % Variation
% 33   Thr_eng 3:   2.046 % Variation
% 34   M_slosh 1:   1.660 % Variation
% 35   M_slosh 1:   1.660 % Variation
% 36   X_slosh 1:  -3.093 % Variation
% 37   X_slosh 1:  -3.093 % Variation
% 38   Z_slosh 1:  -1.500 % Variation
% 39   Z_slosh 1:  -1.500 % Variation
% 40   Wslsh_Y 1:   2.257 % Variation
% 41   Wslsh_Z 1:   2.257 % Variation
% 42   M_slosh 2:  10.762 % Variation
% 43   M_slosh 2:  10.762 % Variation
% 44   M_slosh 2:  10.762 % Variation
% 45   X_slosh 2:  -1.596 % Variation
% 46   X_slosh 2:  -1.596 % Variation
% 47   X_slosh 2:  -1.596 % Variation
% 48   Z_slosh 2:  -1.500 % Variation
% 49   Z_slosh 2:  -1.500 % Variation
% 50   Z_slosh 2:  -1.500 % Variation
% 51   Wslsh_Y 2:   2.379 % Variation
% 52   Wslsh_Z 2:   2.379 % Variation
% 53   W_flex 1: 0.5000   Addit. Variat.
% 54   W_flex 2: 0.7000   Addit. Variat.
% 55   W_flex 3: 0.7000   Addit. Variat.
% 56   W_flex 4: 0.8000   Addit. Variat.
% 57   W_flex 5: 0.9000   Addit. Variat.
% 58   W_flex 6: 1.200    Addit. Variat.
```
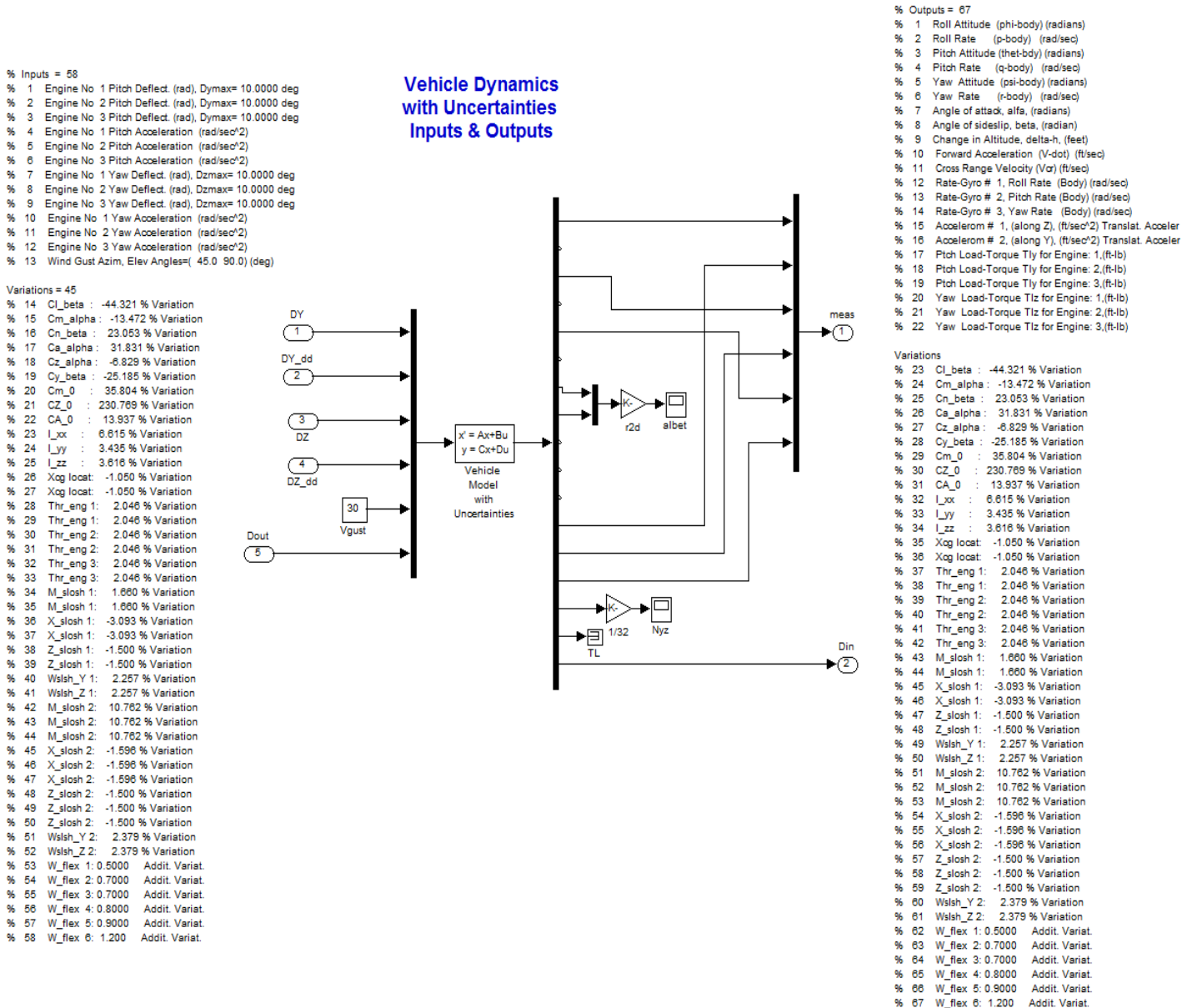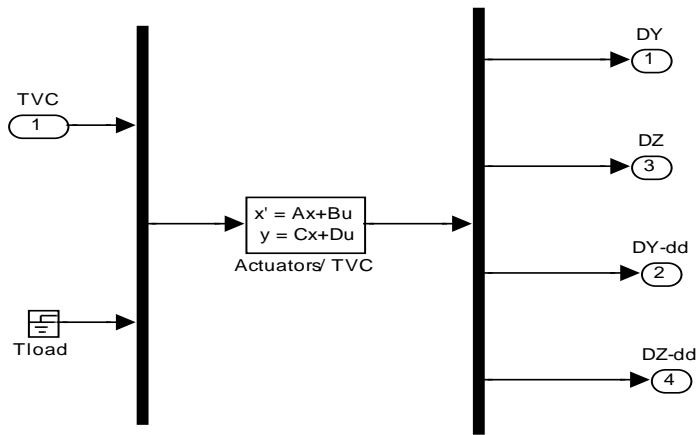
```
Variations
% 23   Cl_beta  :  -44.321 % Variation
% 24   Cm_alpha :  -13.472 % Variation
% 25   Cn_beta  :   23.053 % Variation
% 26   Ca_alpha :   31.831 % Variation
% 27   Cz_alpha :   -6.829 % Variation
% 28   Cy_beta  :  -25.185 % Variation
% 29   Cm_0     :   35.804 % Variation
% 30   CZ_0     :  230.769 % Variation
% 31   CA_0     :   13.937 % Variation
% 32   I_xx   :   6.615 % Variation
% 33   I_yy   :   3.435 % Variation
% 34   I_zz   :   3.616 % Variation
% 35   Xcg locat:  -1.050 % Variation
% 36   Xcg locat:  -1.050 % Variation
% 37   Thr_eng 1:   2.046 % Variation
% 38   Thr_eng 1:   2.046 % Variation
% 39   Thr_eng 2:   2.046 % Variation
% 40   Thr_eng 2:   2.046 % Variation
% 41   Thr_eng 3:   2.046 % Variation
% 42   Thr_eng 3:   2.046 % Variation
% 43   M_slosh 1:   1.660 % Variation
% 44   M_slosh 1:   1.660 % Variation
% 45   X_slosh 1:  -3.093 % Variation
% 46   X_slosh 1:  -3.093 % Variation
% 47   Z_slosh 1:  -1.500 % Variation
% 48   Z_slosh 1:  -1.500 % Variation
% 49   Wslsh_Y 1:   2.257 % Variation
% 50   Wslsh_Z 1:   2.257 % Variation
% 51   M_slosh 2:  10.762 % Variation
% 52   M_slosh 2:  10.762 % Variation
% 53   M_slosh 2:  10.762 % Variation
% 54   X_slosh 2:  -1.596 % Variation
% 55   X_slosh 2:  -1.596 % Variation
% 56   X_slosh 2:  -1.596 % Variation
% 57   Z_slosh 2:  -1.500 % Variation
% 58   Z_slosh 2:  -1.500 % Variation
% 59   Z_slosh 2:  -1.500 % Variation
% 60   Wslsh_Y 2:   2.379 % Variation
% 61   Wslsh_Z 2:   2.379 % Variation
% 62   W_flex 1: 0.5000   Addit. Variat.
% 63   W_flex 2: 0.7000   Addit. Variat.
% 64   W_flex 3: 0.7000   Addit. Variat.
% 65   W_flex 4: 0.8000   Addit. Variat.
% 66   W_flex 5: 0.9000   Addit. Variat.
% 67   W_flex 6: 1.200    Addit. Variat.
```

Vehicle Dynamics with Uncertainties Inputs & Outputs

DY
DY_dd
DZ
DZ_dd
Vgust
Dout

meas

x' = Ax+Bu
y = Cx+Du
Vehicle Model with Uncertainties

r2d   albet
K-

Nyz
1/32
TL
K-

Din

Figure 6.3b, State-space model consisting of vehicle dynamics plus 45 additional input/ output pairs that connect across the uncertainty block Δ.

```
% Inputs =  9
%   1   Roll FCS Command (DPTVC)
%   2   Pitch FCS Command (DQTVC)
%   3   Yaw FCS Command (DRTVC)
%   4   Engine 1 Pitch Load-Torque TLY1
%   5   Engine 2 Pitch Load-Torque TLY2
%   6   Engine 3 Pitch Load-Torque TLY3
%   7   Engine 1 Yaw Load-Torque TLZ1
%   8   Engine 2 Yaw Load-Torque TLZ2
%   9   Engine 3 Yaw Load-Torque TLZ3
```

```
% Outputs = 12
%   1   Engine 1 Pitch Deflection (rad)
%   2   Engine 2 Pitch Deflection (rad)
%   3   Engine 3 Pitch Deflection (rad)
%   4   Engine 1 Yaw  Deflection (rad)
%   5   Engine 2 Yaw  Deflection (rad)
%   6   Engine 3 Yaw  Deflection (rad)
%   7   Engine 1 Pitch Accelerat (rad/sec^2)
%   8   Engine 2 Pitch Accelerat (rad/sec^2)
%   9   Engine 3 Pitch Accelerat (rad/sec^2)
%   10  Engine 1 Yaw  Accelerat (rad/sec^2)
%   11  Engine 2 Yaw  Accelerat (rad/sec^2)
%   12  Engine 3 Yaw  Accelerat (rad/sec^2)
```

Figure 6.3c, State-space system consisting of a combination of 6 engine actuators, 3 pitch and 3 yaw, including the TVC. The gimbal acceleration outputs drive the tail-wag-dog dynamics. The Load-Torque feedback is ignored.

Open-Loop Frequency Response Analysis

The following Simulink model "*Open_Loop.mdl*", shown in Figure 6.4, is used for calculating open-loop frequency responses and analyzing classical stability by measuring the gain and phase margins. The configuration shown below (yaw loop opened, roll and pitch loops closed) is used for calculating the frequency response and stability margins in the yaw direction, as shown by the Nichols plot in Figure 6.5. The block diagram must be modified accordingly for calculating the pitch or roll frequency responses and stability margins.
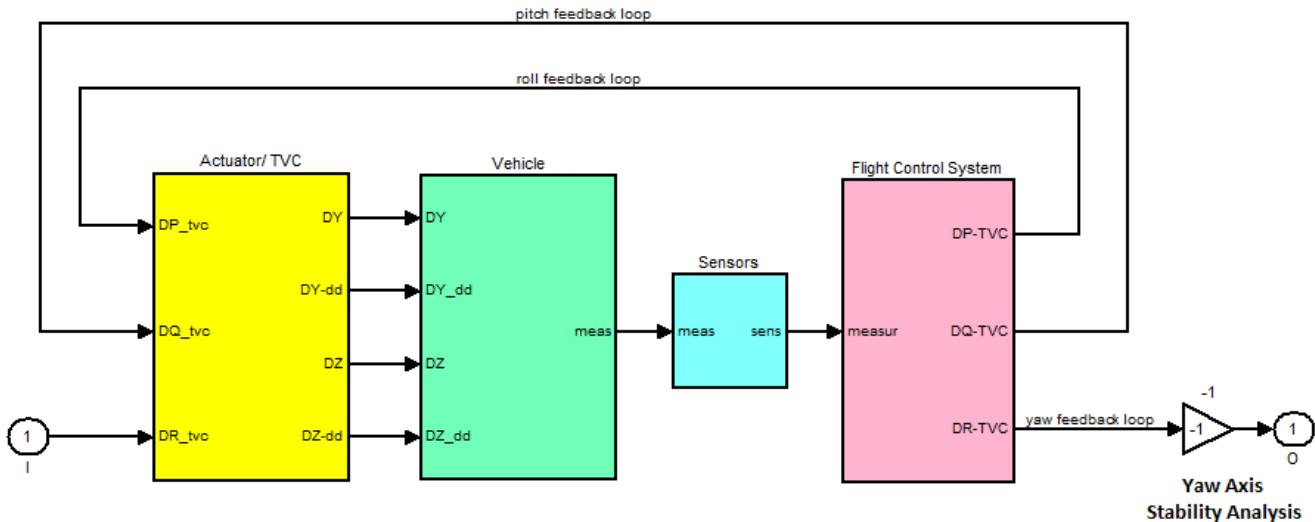
Figure 6.4, Open-Loop Model for Linear Stability Analysis

The Matlab file "run.m" uses the open-loop model "*Open_Loop.mdl*" shown in Figure 6.4 to calculate the frequency response and the Nichols chart in Figure 6.5 that is used to analyze classic stability.

Yaw Nichols Plot

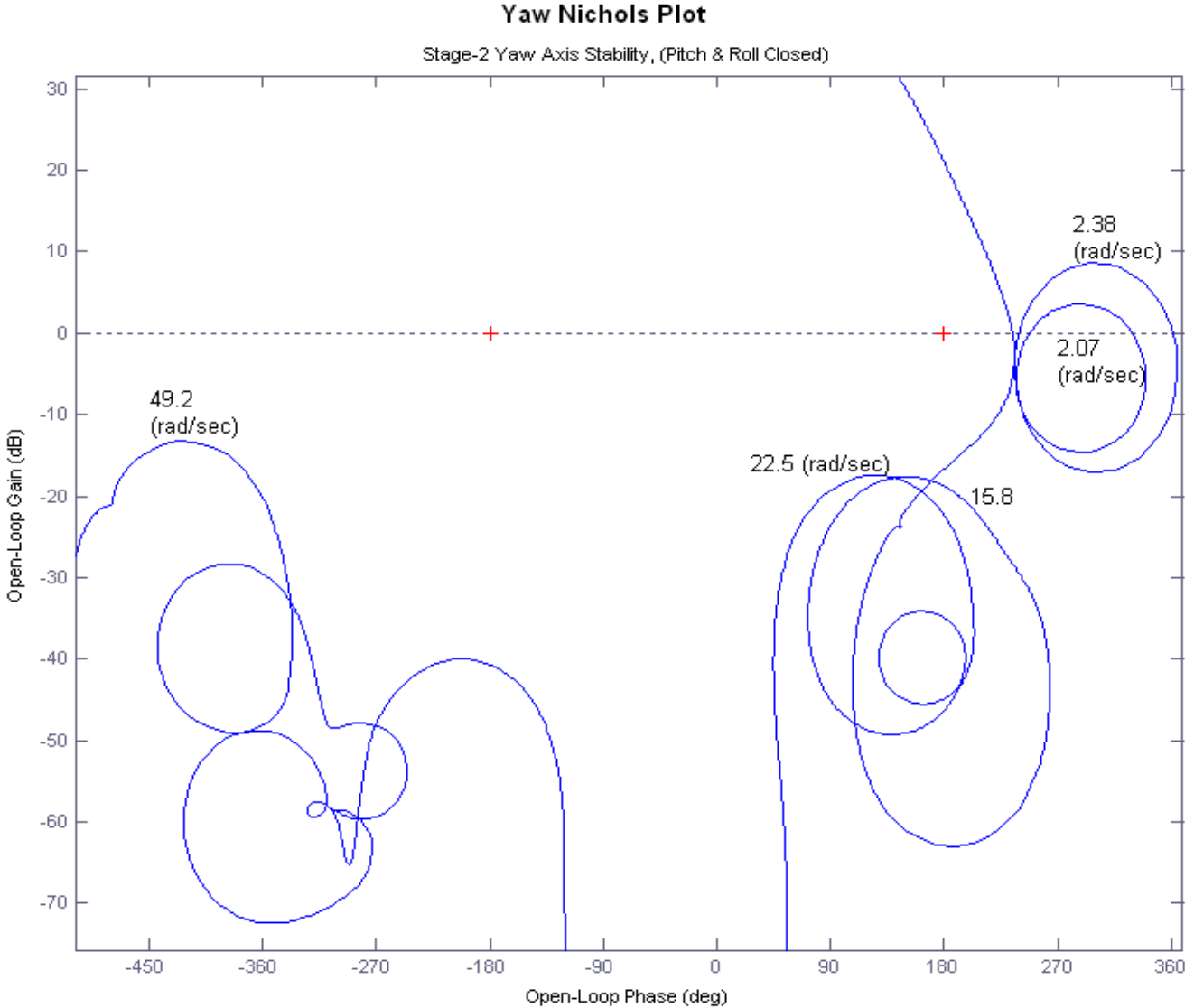Stage-2 Yaw Axis Stability, (Pitch & Roll Closed)

Figure 6.5, Yaw axis Nichols plot shows slosh and flex resonances. It also shows that the flight control system has sufficient phase and gain margins

Mu Analysis

The Matlab script file "run.m" also uses the closed-loop model with uncertainties "*Closed_Loop-Unc.mdl*" to calculate the μ frequency response across the scaled Δ block, shown in Figure 6.6. The structural singular value plot demonstrates that the closed-loop system barely satisfies the μ[M(ω)]<1 requirement for robustness. It means that the control system (which is nominally stable) is capable of tolerating the specified uncertainties without becoming unstable. The Matlab μ-analysis algorithm assumes real parameter variations (not complex). Note that complex variations are too conservative in this case due to the low damped resonances and it would violate the robustness criterion.
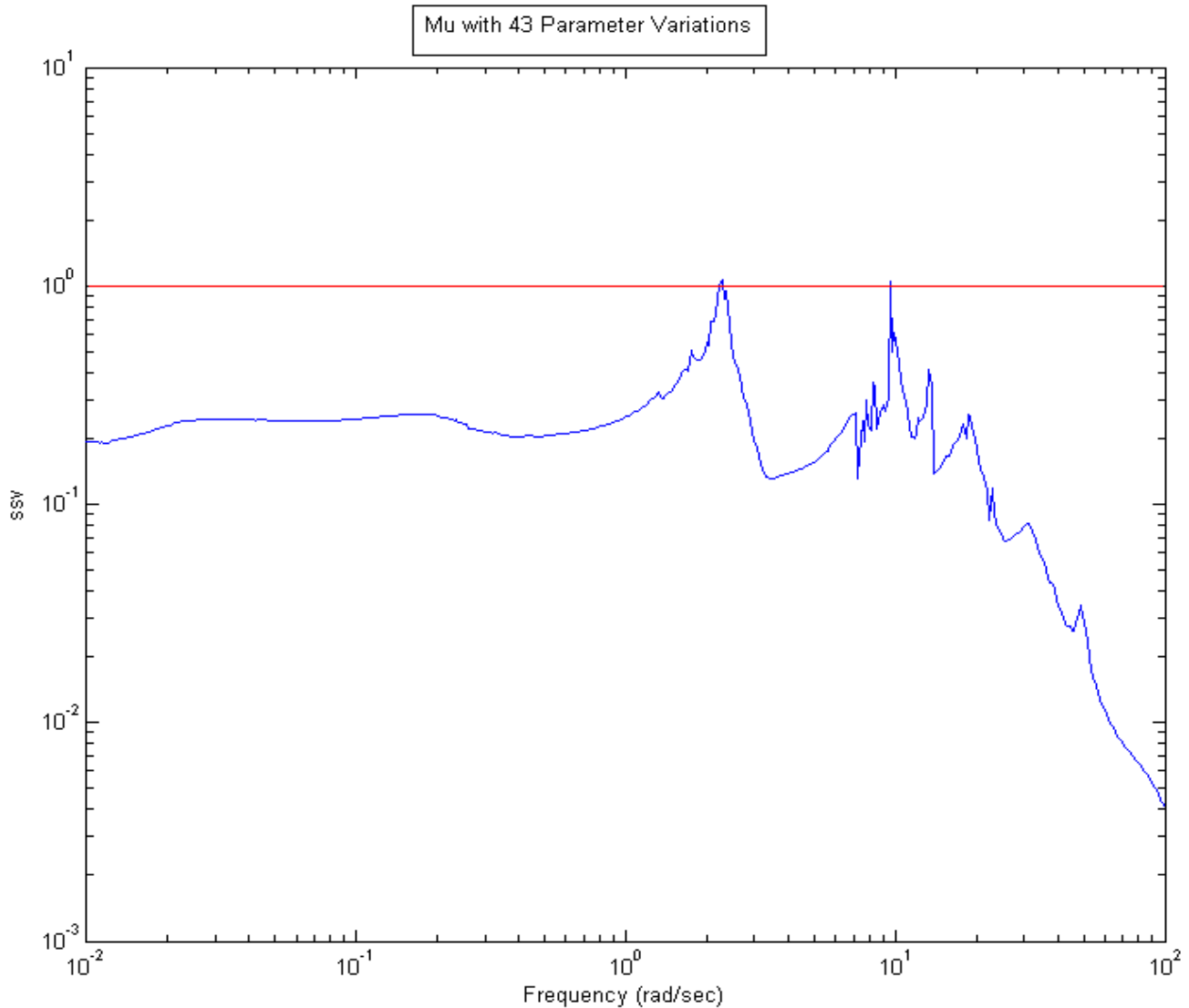
Figure 6.6, μ-analysis Shows that the Control System Satisfies the Robustness Requirements

8.7 Robustness Analysis of a Flexible Satellite with Reaction Wheels

In this example we will analyze robustness of a flexible satellite that is controlled by 3 reaction wheels. The project files are in folder "*\Flixan\Examples\Robust Analysis Param Uncertainties\ Satellite_RW*". The input data file that contains the satellite parameters is "*Satellite_RW.Inp*". Its title is "*Flex Satellite with Reaction Wheels*". This data set in addition to vehicle mass properties it contains also: the spin axis direction of the 3 reaction wheels, their initial rotational speeds in (rpm) which are zero in this case, and the moment of inertia for each wheel about its spin axis, which is 0.1 (slug-ft^2). It contains also 3 rate gyros, 3 attitude sensors, and 2 accelerometer sensors. The satellite model also contains 60 structural flexibility modes. The modes are already pre-processed and selected from a finite elements model and the selected modes are included in the input data file "*Satellite_RW.Inp*". The flex modes title is: "*Flex Satellite with Reaction Wheels (60 Flex Modes)*". The block of data for each mode contains the mode frequency and the mode shapes at the locations of the 3 reaction wheels, the 6 gyros, and the 2 accelerometers. The modal data title is also included in the satellite input data-set, under the line "*Number of Flex Modes: 60*".

The input file "*Satellite_RW.Inp*" also includes the uncertainties data-set located below the satellite input data. The Flixan program recognizes the parameter variations data-set by the label "*UNCERTAIN PARAMETER VARIATIONS FROM NOMINAL …*". The uncertainties title is "*Uncertainties for the Flex Satellite with Reaction Wheels*" and it is located below the label. It is also included in the satellite data below the line "*Parameter Uncertainties Data*". The uncertainties data consists of variations in the moments of inertia, in the RW spin axis directions, the RW momentum bias, and the rotor moment of inertia. Variations in some of the flex mode frequencies are also included. The flex mode numbers that have uncertain frequencies appear in line "*Flex Mode Uncertainties (Mode Number): 1 3 4 5 6 42 43 50*". The corresponding max frequency variations are included in the next line "*Flex Mode Frequency Variation (additive): 0.3 0.4 0.6 2.0 2.0 10.0 10.0 10.0*".

FLIGHT VEHICLE INPUT DATA ......
Flex Satellite with Reaction Wheels
! This is a simple Satellite model that uses 3 Reaction Wheels, one wheel per axis.
! It contains also 60 flex modes. The modal data are included in bottom of file.
! Title of Parameter Uncertainties data-set is also included for IFL modeling.
! The augmented spacecraft model is used to analyze robustness to structural uncertainties
!
Body Axes Output, Euler Angles

```
Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft)    :   50.0   0.0001 0.20896E+08
Moments and products of Inertias Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)          :   1045.3, 641.02,  505.05,
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)         :   -0.05,  0.1,  22.625
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)       :   0.0   29000.0   0.0001
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)         :   0.0    0.0  0.0  0.0
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                         :   0.0    0.0
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)   :   0.0 0.0 0.0 0.0, -0.0647,
External Torque and direction about (x,y,z)                                              :   Torque 0.0 1.0   0.0
Surface Reference Area (feet^2), Mean Aerodynamic Chord (ft), Wing Span in (feet)        :   0.0    1.0 1.0
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial_rho/ Partial_H} :   0.0  0.0   0.0  0.0
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:   0.0  0.0   0.0  0.0
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}:   0.0 -0.0   0.0  0.0 0.0
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}:   0.0  0.0   0.0  0.0
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:   0.0  0.0   0.0  0.0 0.0
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}:   0.0  0.0   0.0  0.0 0.0
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}:   0.0  0.0   0.0  0.0 0.0

Number of Thruster Engines, RCS Jets, No Engine Gimbaling                                 :   0


Number of Reaction Wheels                                                                 :   3
RW No:  1, Spin Direct. Unit Vect, Init Rot Speed (rpm), Rotor Mom of Inertia (slg-ft^2) :   1.0  0.0  0.0  0.0  0.1
RW No:  2, Spin Direct. Unit Vect, Init Rot Speed (rpm), Rotor Mom of Inertia (slg-ft^2) :   0.0  1.0  0.0  0.0  0.1
RW No:  3, Spin Direct. Unit Vect, Init Rot Speed (rpm), Rotor Mom of Inertia (slg-ft^2) :   0.0  0.0  1.0  0.0  0.1

Number of Gyros, (Attitude and Rate)                                                     :   6
Gyro No  1 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 6    :   Roll   Rate -0.747 0.114
Gyro No  2 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 6    :   Pitch  Rate -0.747 0.114
Gyro No  3 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 6    :   Yaw    Rate -0.747 0.114
Gyro No  4 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 6    :   Roll   Attitu -0.747 0.114
Gyro No  5 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 6    :   Pitch  Attitu -0.747 0.114
Gyro No  6 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 6    :   Yaw    Attitu -0.747 0.114

Number of Accelerometers, Along Axes: (x,y,z)                                             :   2
Acceleromet No  1 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 7   :   X-axis Accel 0.338 0.64 25
Acceleromet No  2 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 7   :   Y-axis Accel 0.338 0.64 25

Parameter Uncertainties Data
```
Uncertainties for the Flex Satellite with Reaction Wheels

```
Number of Bending Modes                                                                  :   60
```
Flex Satellite with Reaction Wheels (60 Flex Modes)
```
-----------------------------------------------------------------------------------------------------------
```

UNCERTAIN PARAMETER VARIATIONS FROM NOMINAL ......
New Uncertainties for Satellite
! This is a new set of parameter variations for a Satellite with three Reaction Wheels. The wheel
! moment of inertia about the rotor, and the spin axis direction are not known very accurately.
! Also the wheel speed is unknown. There are also uncertainties in the spacecraft moments of
! inertia, and also in the flex mode frequencies and damping coefficient (zeta).

```
Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft)    :   0.00    0.0    0.0
Moments and products of Inertias Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)          :   50.0  30.0   25.0 0.0
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)        :   0.0    0.0    0.0
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)       :   0.00   0.0    0.0
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)         :   0.0    0.0    0.0  0.0
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                         :   0.0    0.0    0.0  0.0
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)   :   0.0    0.0    0.0  0.0
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:   0.0    0.0    0.0  0.0
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}:   0.0    0.0    0.0  0.0
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}:   0.0    0.0    0.0  0.0
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:   0.0    0.0    0.0  0.0
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}:   0.0    0.0    0.0  0.0
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}:   0.0    0.0    0.0  0.0

Number of Reaction Wheels, (Variations from Nominal Wheel Parameters)                     :   3
RW No:  1, Spin Direct. Unit Vect, Init Rot Speed (rpm), Rotor Mom of Inertia (slg-ft^2) :   0.0  0.1  0.1  10.0  0.02
RW No:  2, Spin Direct. Unit Vect, Init Rot Speed (rpm), Rotor Mom of Inertia (slg-ft^2) :   0.1  0.0  0.1  10.0  0.02
RW No:  3, Spin Direct. Unit Vect, Init Rot Speed (rpm), Rotor Mom of Inertia (slg-ft^2) :   0.1  0.1  0.0  10.0  0.02

Flex Mode Uncertainties (Mode Number)                                                    :   1    3    4    5    6   42
43    50
Flex Mode Frequency (omega) Variation (additive)  (rad/sec)                              :   0.3  0.4  0.6  2.0  2.5 25
Flex Mode Damping Coefficient (zeta) Variation (additive)                                : 0.001 0. 0.001 0.0 0.001 0.
-----------------------------------------------------------------------------------------------------------
```

```
SELECTED MODAL DATA AND LOCATIONS FOR : Pre-Selected
Flex Satellite with Reaction Wheels (60 Flex Modes)
! The flex modes used in this analysis are the same modes that were selected and used in a
! previous flex spacecraft analysis. We start from Mode #7 because the first 6 rigid-body modes
! are not included in the modal data because the rigid-body dynamics are included in the vehicle.
! The Nodes correspond to vehicle locations defined in the vehicle data above.

MODE#  1/  7, Frequency (rad/sec), Damping (zeta), Generalized Mass=     3.0998      0.50000E-02   12.000
DEFINITION OF LOCATIONS (NODES)            phi along X   phi along Y   phi along Z   sigm about X  sigm about Y  sigm

                        Node ID#     Modal Data at the  3 Reaction Wheels...
Reaction Wheel #1 CG      58041     -0.20700D+00  -0.43105D-03   0.41763D-01  -0.11013D-02   0.64943D-01  -
Reaction Wheel #2 CG      58042     -0.20783D+00  -0.16859D-02  -0.31758D-01  -0.11002D-02   0.64942D-01  -
Reaction Wheel #3 CG      58043     -0.20632D+00  -0.17810D-02  -0.38780D-01  -0.11033D-02   0.64942D-01  -

                        Node ID#     Modal Data at the  6 Gyros ...
Inertial Attitude Sensors  31001     0.51466D-01   0.39036D-02   0.38773D-01  -0.11017D-02   0.64943D-01  -
Inertial Attitude Sensors  31001     0.51466D-01   0.39036D-02   0.38773D-01  -0.11017D-02   0.64943D-01  -
Inertial Attitude Sensors  31001     0.51466D-01   0.39036D-02   0.38773D-01  -0.11017D-02   0.64943D-01  -
Inertial Attitude Sensors  31001     0.51466D-01   0.39036D-02   0.38773D-01  -0.11017D-02   0.64943D-01  -
Inertial Attitude Sensors  31001     0.51466D-01   0.39036D-02   0.38773D-01  -0.11017D-02   0.64943D-01  -
Inertial Attitude Sensors  31001     0.51466D-01   0.39036D-02   0.38773D-01  -0.11017D-02   0.64943D-01  -

                        Node ID#     Modal Data at the  2 Accelerometers, along (x,y,z)...
Accelerometers             31002     0.51889D-01   0.27108D-02  -0.32279D-01
Accelerometers             31002     0.51889D-01   0.27108D-02  -0.32279D-01

                        Node ID#     Modal Data at the Disturbance Point
RCS Jet #8    (+X)         98008    -0.19474D+00   0.14764D-02   0.14237D+00  -0.11013D-02   0.64937D-01

----------------------------------------------------------------------------------------------------
```

## Generating the Uncertain Satellite State-Space Model

There are several ways of processing this input data file in Flixan and to create the satellite state-space model with the additional IFL uncertainty I/Os. It can either be processed by running the batch set which is located on the top of file "*Satellite_RW.Inp*" similar to the previous two examples, or it can be processed by running the flight vehicle modeling program just like we would normally run a typical flight vehicle input data. The uncertainties data will also be processed because they are referenced by their title in the vehicle data. Notice, that if the uncertainties title is not included in the vehicle data they will be ignored and the vehicle model will not include the additional input/ output pairs that connect to the Δ block. In this example we will use the third option which is to run the parameter uncertainties modeling program. The uncertainties are already in the input file.

Start the Flixan program and select the current project folder: "*\Flixan\Examples\Robust Analysis Param Uncertainties\Satellite_RW*". Then from the main menu select "*Program Functions*", "*Robust Control Synthesis Tools*", and "*Modeling Vehicle Parameter Uncertainties*", as shown below.
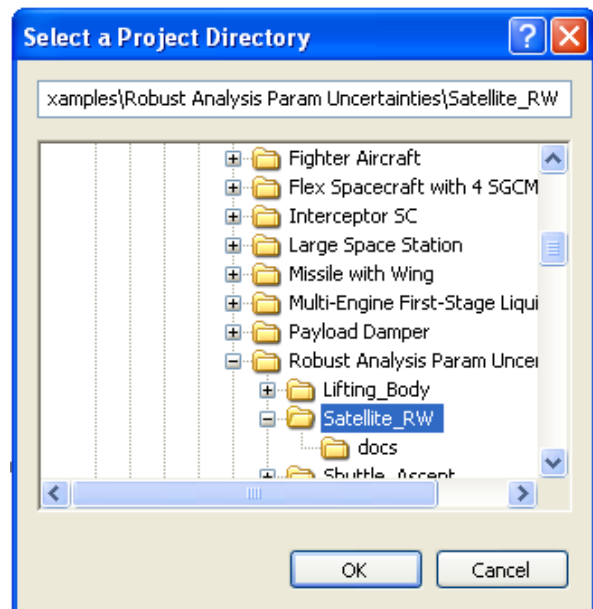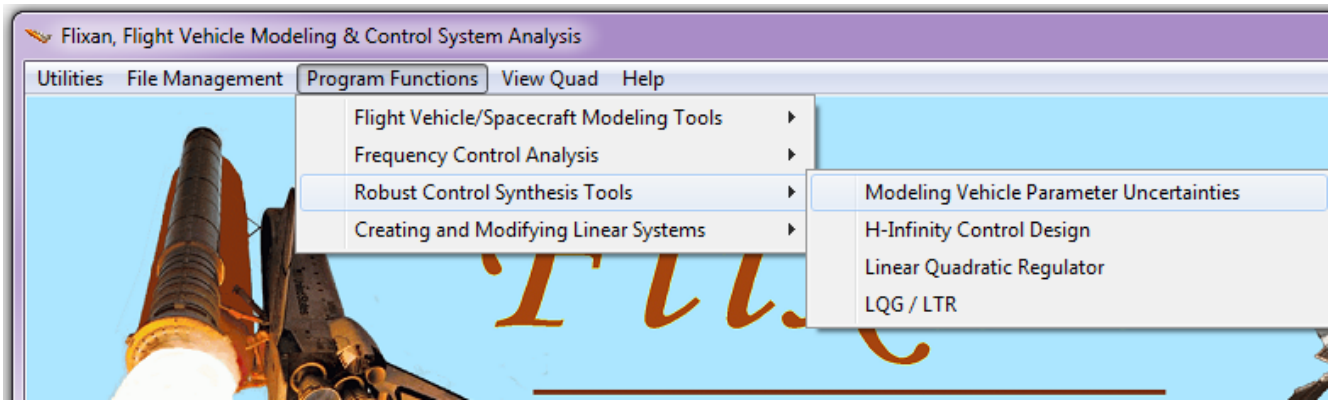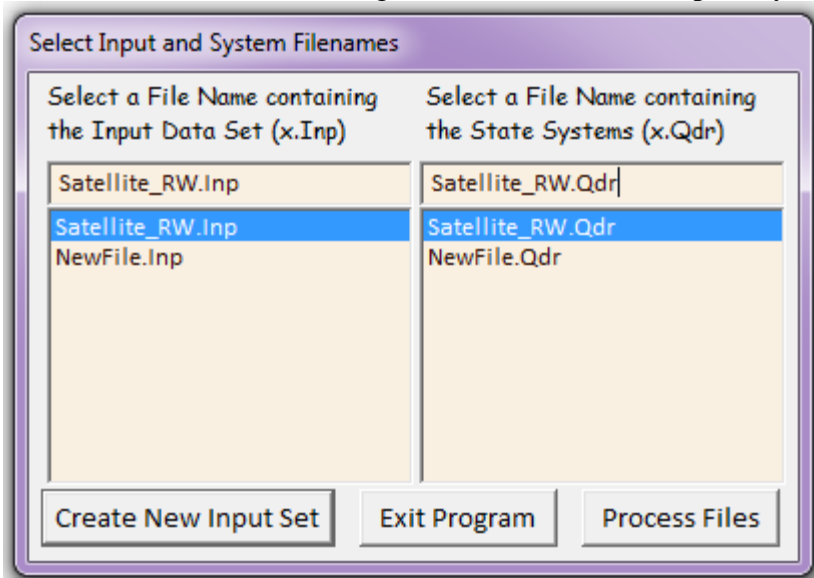
The following window presents an introductory paragraph that describes the function of the parameter uncertainties modeling program. Click on "*Continue*" to continue.
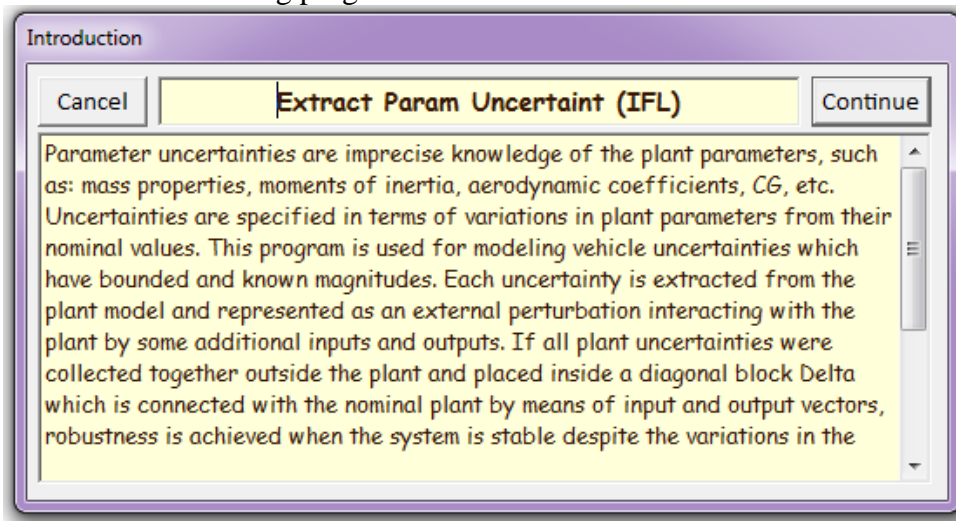
Select the already prepared satellite input data file "*Satellite_RW.Inp*" and the systems file "*Satellite_RW.Qdr*" for saving the flex satellite state-space system and click on "*Process Files*".

The following menu shows the titles of flight vehicle input data-sets which are in file "*Satellite_RW.Inp*". In this case there is only one set, we select "*Flex Satellite with Reaction Wheels*" and click on "*Run Input Set*". Similarly, the following menu shows the titles of uncertainty data-sets in the input file. Select the title "*Uncertainties for the Flex Satellite with Reaction Wheels*" and click on "*Run Input Set*".
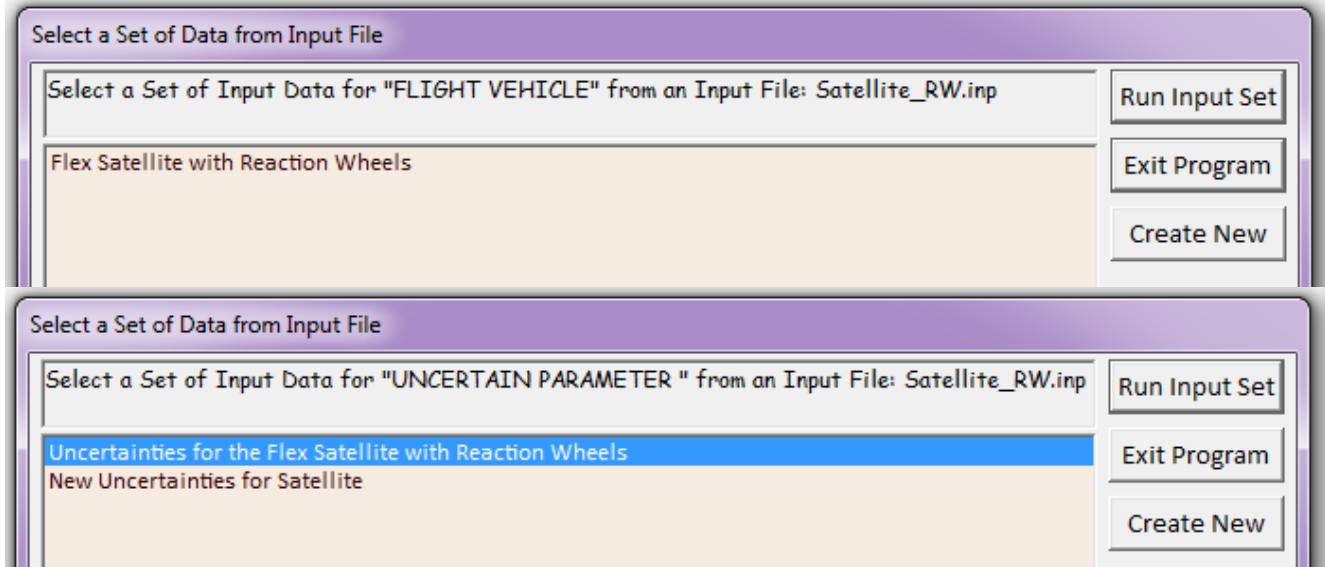
The following dialog consists of various tabs and displays for validating or modifying the uncertainties data. The uncertainties title appears on the top. Each tab consists of a group of data. In the mass properties tab below the satellite mass properties and the max variations are shown. The nominal vehicle data are in the light blue fields on the left and the max parameter variations are on the right.
The user may also check the reaction wheel data, and the flex mode variations by clicking in the appropriate tabs.

Flight Vehicle Parameter Uncertainties

Uncertainties Title

Uncertainties for the Flex Satellite with Reaction Wheels

Edit Input File	Exit
Update Data	Run
Save in File	

| Gimbaling Engines or Jets. Include Tail-Wags-Dog? | 0 | Reaction Wheels? | 3 | Structure Bending Modes | 60 |
| Rotating Control Surfaces. Include Tail-Wags-Dog? | 0 | Single Gimbal CMGs? | 0 | Fuel Sloshing Modes | 0 |

| Reaction Wheels | Single Gimbal CMGs | Double Gimbal CMG System | Fuel Slosh | Flex Modes | User Comments |
| Mass Properties | Trajectory Data | Aero Force Coeffs | Aero Moment Coeffs | Control Surfaces | Gimbal Engines/ RCS |

Vehicle Mass Properties

		Nominal Data	Param. Variations
Vehicle Mass in (slugs)		50.00000	0.000000
Moments of Inertia in (slugs-ft^2)	Ixx	1045.300	50.00000
	Iyy	641.0200	30.00000
	Izz	505.0500	25.00000
Products of Inertia in (slugs-ft^2)	Ixy	0.000000	0.000000
	Ixz	-19.00000	0.000000
	Iyz	-40.00000	0.000000
CG Location in (feet)	Xcg	-0.5000000E-01	0.000000
	Ycg	0.1000000	0.000000
	Zcg	22.62500	0.000000

Flight Vehicle Parameter Uncertainties

Uncertainties Title

Uncertainties for the Flex Satellite with Reaction Wheels

Edit Input File	Exit
Update Data	Run
Save in File	

| Gimbaling Engines or Jets. Include Tail-Wags-Dog? | 0 | Reaction Wheels? | 3 | Structure Bending Modes | 60 |
| Rotating Control Surfaces. Include Tail-Wags-Dog? | 0 | Single Gimbal CMGs? | 0 | Fuel Sloshing Modes | 0 |

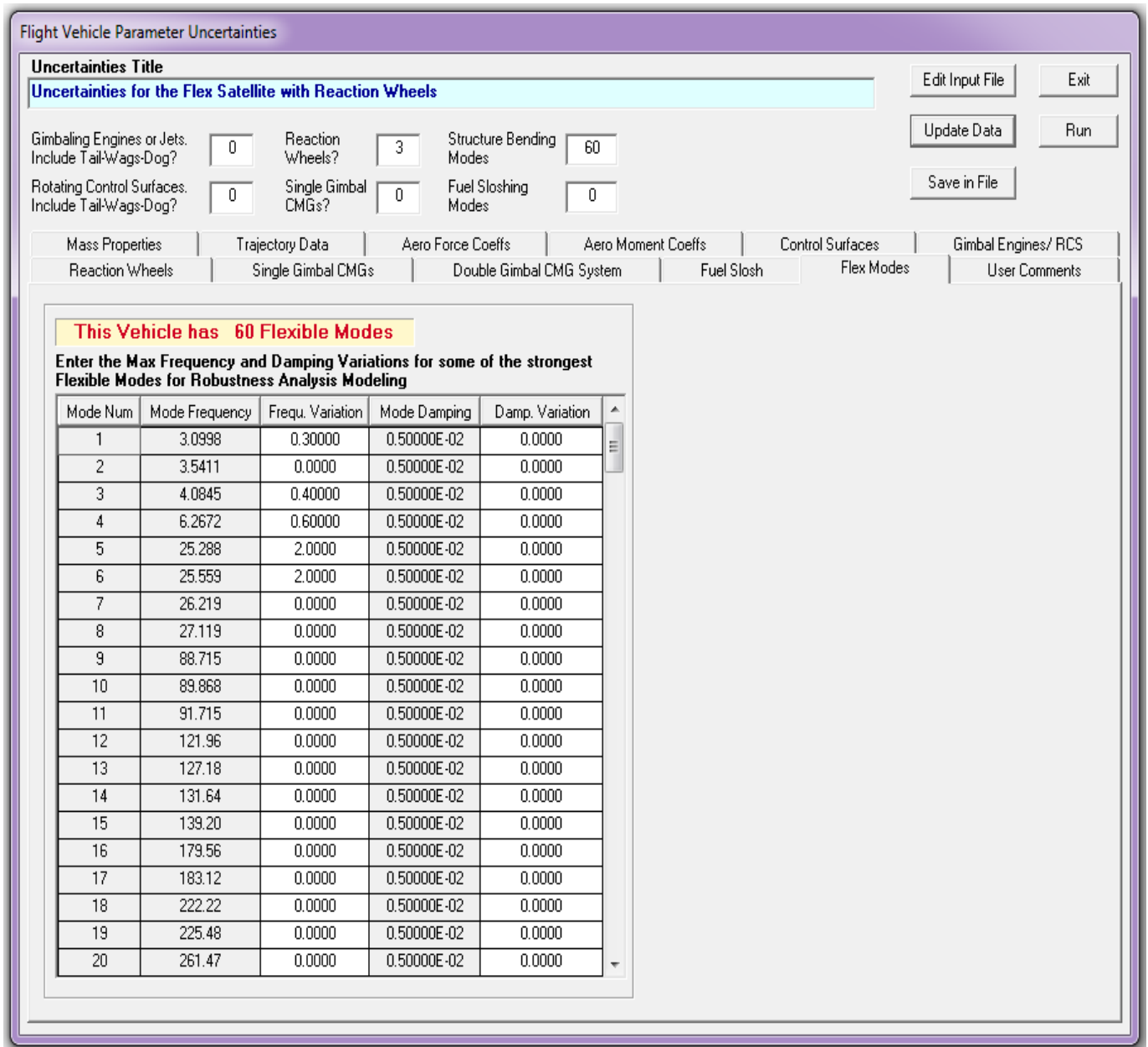| Mass Properties | Trajectory Data | Aero Force Coeffs | Aero Moment Coeffs | Control Surfaces | Gimbal Engines/ RCS |
| Reaction Wheels | Single Gimbal CMGs | Double Gimbal CMG System | Fuel Slosh | Flex Modes | User Comments |

This Vehicle has 3 Reaction/Momentum Wheels

Spin Direction, Rotor Inertia, and Initial rmp Speed for RW No: 2

RW Spin Direction Unit Vector

	Nominal Values	Enter Param. Variations
X	0.0000	0.10000
Y	1.0000	0.0000
Z	0.0000	0.10000

[Next RW]

| RW Rotor Mom.Inertia | 0.10000 | 0.20000E-01 |
| Initial RW Speed (rpm) | 0.0000 | 10.000 |

Flight Vehicle Parameter Uncertainties

Uncertainties Title

Uncertainties for the Flex Satellite with Reaction Wheels

[Edit Input File] [Exit]

[Update Data] [Run]

[Save in File]

Gimbaling Engines or Jets. Include Tail-Wags-Dog?	0	Reaction Wheels? 3	Structure Bending Modes 60
Rotating Control Surfaces. Include Tail-Wags-Dog?	0	Single Gimbal CMGs? 0	Fuel Sloshing Modes 0

Mass Properties | Trajectory Data | Aero Force Coeffs | Aero Moment Coeffs | Control Surfaces | Gimbal Engines/ RCS

Reaction Wheels | Single Gimbal CMGs | Double Gimbal CMG System | Fuel Slosh | Flex Modes | User Comments

This Vehicle has 60 Flexible Modes

Enter the Max Frequency and Damping Variations for some of the strongest Flexible Modes for Robustness Analysis Modeling

Mode Num	Mode Frequency	Frequ. Variation	Mode Damping	Damp. Variation
1	3.0998	0.30000	0.50000E-02	0.0000
2	3.5411	0.0000	0.50000E-02	0.0000
3	4.0845	0.40000	0.50000E-02	0.0000
4	6.2672	0.60000	0.50000E-02	0.0000
5	25.288	2.0000	0.50000E-02	0.0000
6	25.559	2.0000	0.50000E-02	0.0000
7	26.219	0.0000	0.50000E-02	0.0000
8	27.119	0.0000	0.50000E-02	0.0000
9	88.715	0.0000	0.50000E-02	0.0000
10	89.868	0.0000	0.50000E-02	0.0000
11	91.715	0.0000	0.50000E-02	0.0000
12	121.96	0.0000	0.50000E-02	0.0000
13	127.18	0.0000	0.50000E-02	0.0000
14	131.64	0.0000	0.50000E-02	0.0000
15	139.20	0.0000	0.50000E-02	0.0000
16	179.56	0.0000	0.50000E-02	0.0000
17	183.12	0.0000	0.50000E-02	0.0000
18	222.22	0.0000	0.50000E-02	0.0000
19	225.48	0.0000	0.50000E-02	0.0000
20	261.47	0.0000	0.50000E-02	0.0000

Click on "Run" and the program generates the flex satellite state-space model and it will save it in file "*Satellite_RW.Qdr*". This system is augmented because it includes the 28 additional inputs and the 28 additional outputs that connect with the "pulled out" uncertainty block Δ. It was also converted to Matlab state-space function m-file format "*satellite_60flx.m*" for μ-analysis in Matlab. In our μ-analysis we don't physically connect it with a Δ block but we simply calculate the μ(ω) between the 28 inputs and outputs. Notice, that the input/ output pairs connecting to the uncertainty block Δ is greater than the number of parameter variations. This is because some of the uncertainties, such as: moments of inertia, RW momentum bias, etc, are higher than rank-1 dependency coupling into more than one direction, and they appear in 2 or 3 equations.

Creating a new set of Uncertainties Data

To create a new set of parameter variations start the Flixan program as before, select the current project folder: "*Flixan\Examples\Robust Analysis Param Uncertainties\ Satellite_RW*". From the main menu select "*Program Functions*", "*Robust Control Synthesis Tools*", and "*Modeling Vehicle Parameter Uncertainties*". Select the same input data file "*Satellite_RW.Inp*" and systems file "*Satellite_RW.Qdr*", like before. From the menu that selects the title of the vehicle data select again "*Flex Satellite with Reaction Wheels*" and click on "*Run Input Set*", like before. However, now from the menu that selects an uncertainties title, do not select and run an already existing uncertainties data-set, like before, but click on "*Create New*" button instead.
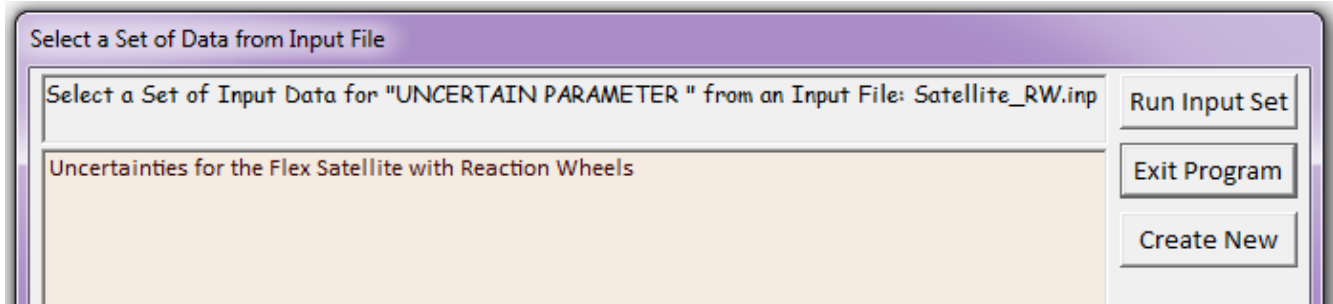
This will activate the uncertainties data preparation utility and it will display the following dialog for entering the parameter variations data in the appropriate fields. The dialog consists of several sub-dialogs selectable by tabs. On the top of the main dialog we must enter the new uncertainties title "*New Uncertainties for Satellite*" and click on "Update Data". The update button does not save the data in file but it updates the internal memory because, otherwise, it may get erased when you change tabs. The dialog also shows that the satellite has 3 reaction wheels, and 60 flex modes. Click on the "Reaction Wheels" tab and it will show the reaction wheel parameters. In the fields on the right side you may enter the parameter variations for each wheel separately. Starting from wheel #1, we enter a small amount of uncertainty in the direction of the spin axis (x, y, z), an uncertainty in the rotor inertia, and in the initial reaction wheel speed ±10 (rmp). Click on "Next RW" and repeat for the second and third RW. Click on "Update Data" before selecting the next tab which is "Flex Modes".
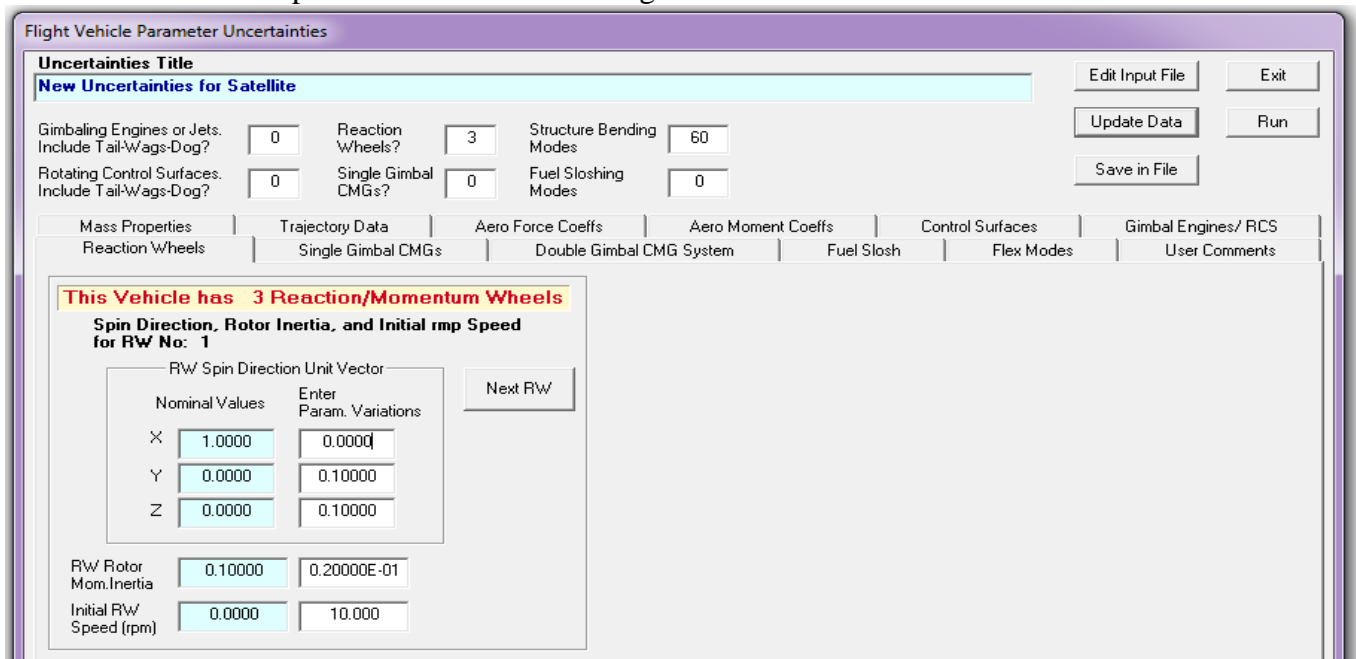
The Flex Modes tab consists of 5 columns. You may only enter data in the 3rd and 5th columns which are the frequency and damping uncertainties. The first column is the mode numbers as listed in the input data file. The 2nd column is the mode frequencies in (rad/sec). In the 3rd column you may enter the frequency variations, obviously, not in every single mode but only in the problematic modes that dominate in the flex dynamics. The 4th column shows the modal damping and you can't change it. In the 5th column you may enter the modal damping variations which are relative to the nominal damping values. Make sure that the variations are reasonable, for example, not bigger than the actual damping coefficients. Click on the "Update Data" and select the mass properties tab to enter variations in the vehicle moments of inertia.

Flight Vehicle Parameter Uncertainties

Uncertainties Title
New Uncertainties for Satellite

Edit Input File | Exit

| Gimbaling Engines or Jets. Include Tail-Wags-Dog? | 0 | Reaction Wheels? | 3 | Structure Bending Modes | 60 |
| Rotating Control Surfaces. Include Tail-Wags-Dog? | 0 | Single Gimbal CMGs? | 0 | Fuel Sloshing Modes | 0 |

Update Data | Run
Save in File

Mass Properties | Trajectory Data | Aero Force Coeffs | Aero Moment Coeffs | Control Surfaces | Gimbal Engines/ RCS
Reaction Wheels | Single Gimbal CMGs | Double Gimbal CMG System | Fuel Slosh | Flex Modes | User Comments

This Vehicle has 60 Flexible Modes

Enter the Max Frequency and Damping Variations for some of the strongest Flexible Modes for Robustness Analysis Modeling

Mode Num	Mode Frequency	Frequ. Variation	Mode Damping	Damp. Variation
1	3.0998	0.30000	0.50000E-02	0.10000E-02
2	3.5411	0.0000	0.50000E-02	0.0000
3	4.0845	0.40000	0.50000E-02	0.0000
4	6.2672	0.60000	0.50000E-02	0.10000E-02
5	25.288	2.0000	0.50000E-02	0.0000
6	25.559	2.5000	0.50000E-02	0.10000E-02
7	26.219	0.0000	0.50000E-02	0.0000
8	27.119	0.0000	0.50000E-02	0.0000
9	88.715	0.0000	0.50000E-02	0.0000
10	89.868	0.0000	0.50000E-02	0.0000
11	91.715	0.0000	0.50000E-02	0.0000
12	121.96	0.0000	0.50000E-02	0.0000
13	127.18	0.0000	0.50000E-02	0.0000
14	131.64	0.0000	0.50000E-02	0.0000
15	139.20	0.0000	0.50000E-02	0.0000
16	179.56	0.0000	0.50000E-02	0.0000
17	183.12	0.0000	0.50000E-02	0.0000
18	222.22	0.0000	0.50000E-02	0.0000
19	225.48	0.0000	0.50000E-02	0.0000
20	261.47	0.0000	0.50000E-02	0.0000

Finally you may enter some user notes for book-keeping purposes. Click on the "User Comments" tab, and in the following field comes up where you may enter a short description of the newly created parameter variations data set. This description note will be included as comments in the new uncertainty data-set in file "*Satellite_RW.Inp*". Remember again to click on the "Update Data" button after completing the note. Finally, when the parameter variations data enetring is complete in all tabs, click on "Save in File" button to save the uncertainty data in file "*Satellite_RW.Inp*", under the new title and user comments. You may finally click on "Run" to process the vehicle and uncertainties data, and to create the augmented vehicle system in file "*Satellite_RW.Qdr*".
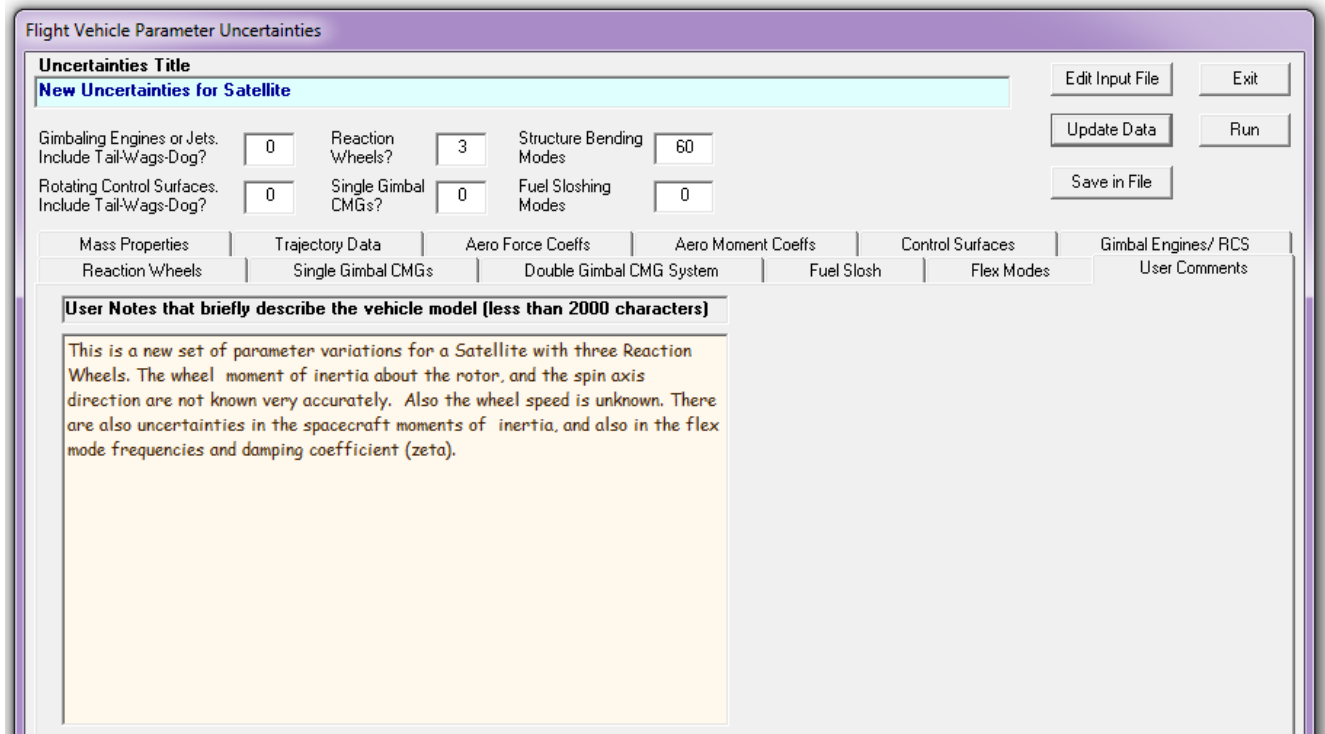
Analysis

The Matlab script file "run.m" loads the flex satellite state-space model from Matlab function "*satellite_60flx.m*". It loads also the spacecraft moments of inertia matrix which is used in the attitude control laws to provide the same ACS bandwidth in all directions. The Simulink model "*Open-Loop.mdl*", in Figure 7.1, is used to perform open-loop frequency response analysis in order to determine the ACS phase and gain margins. In this case it is configured for yaw axis analysis, with pitch and roll axes loops closed. The yaw Nichols is shown in Figure 7.2, showing also the phase and gain margins.

```
close all
r2d=180/pi; d2r=pi/180;
[Au, Bu, Cu, Du] = satellite_60flx;
npv=28;                                   % Number of Param Variation
J= [1045.3,  0,        19; ...            % Moments of Inertia Matrix
     0,        641.02,  40; ...
     19,       40,       505.05]; JI=inv(J);

label='Yaw Axis Stability, (Pitch & Roll Closed)';
[Ao,Bo,Co,Do]=linmod('Open_Loop'); sys=ss(Ao,Bo,Co,Do);
w=logspace(-2,3,12000);                   % and Frequ domain analysis
figure(11); Nichols(sys,w); title(label)
figure(12); Bode(sys,w); title(label)

w=logspace(-3,3,300);                     % and Frequ domain analysis
[Acl,Bcl,Ccl,Dcl]=linmod('Closed_Loop'); sys=ss(Acl,Bcl,Ccl,Dcl);
mu=ssv(Acl,Bcl,Ccl,Dcl,w);
figure(13); loglog(w,mu);

[Acl,Bcl,Ccl,Dcl]=linmod('Closed_Loop'); sys=ss(Acl,Bcl,Ccl,Dcl);
sysf= FRD(sys,w);
blk=[-ones(npv,1), zeros(npv,1)];
[bnd,muinfo]= mussv(sysf,blk);

ff= get(muinfo.bnds, 'frequency');
muu=get(muinfo.bnds, 'responsedata');
muu=squeeze(muu);
muu=muu(1,:);
figure(14); loglog(ff,muu)
xlabel('Frequency (rad/sec)')
ylabel('ssv')
```
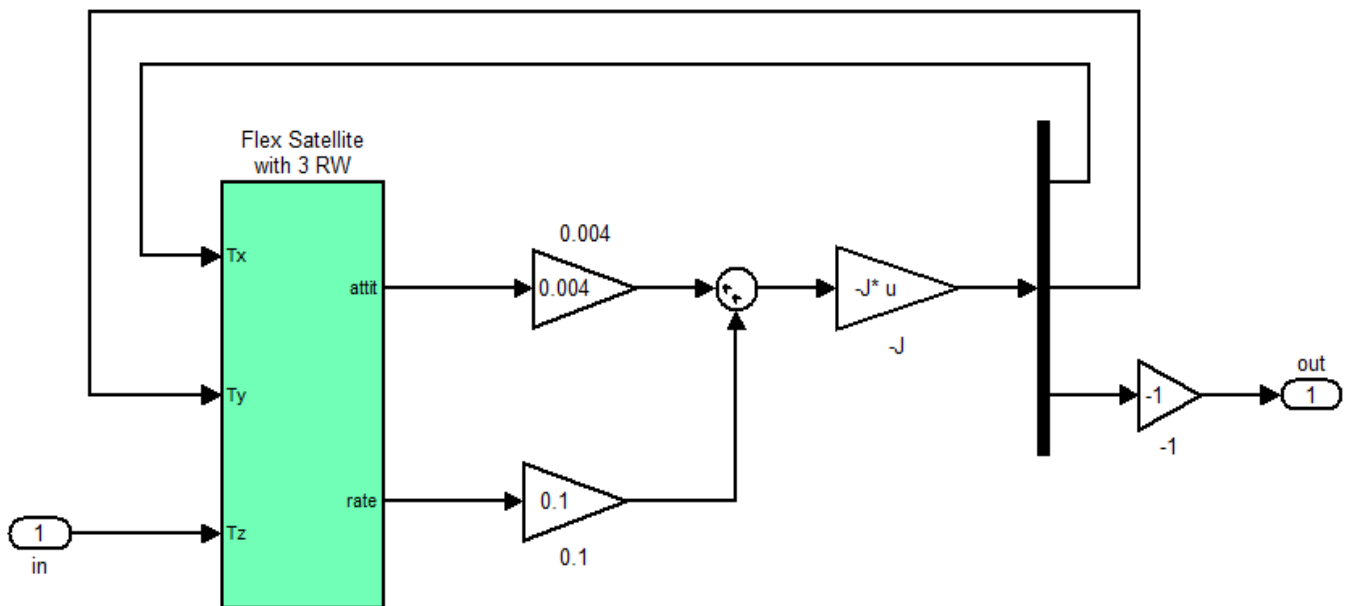

Figure 7.1 Simulink Model "Open_Loop.Mdl" used for Open-Loop Stability Analysis (configured here for yaw analysis)
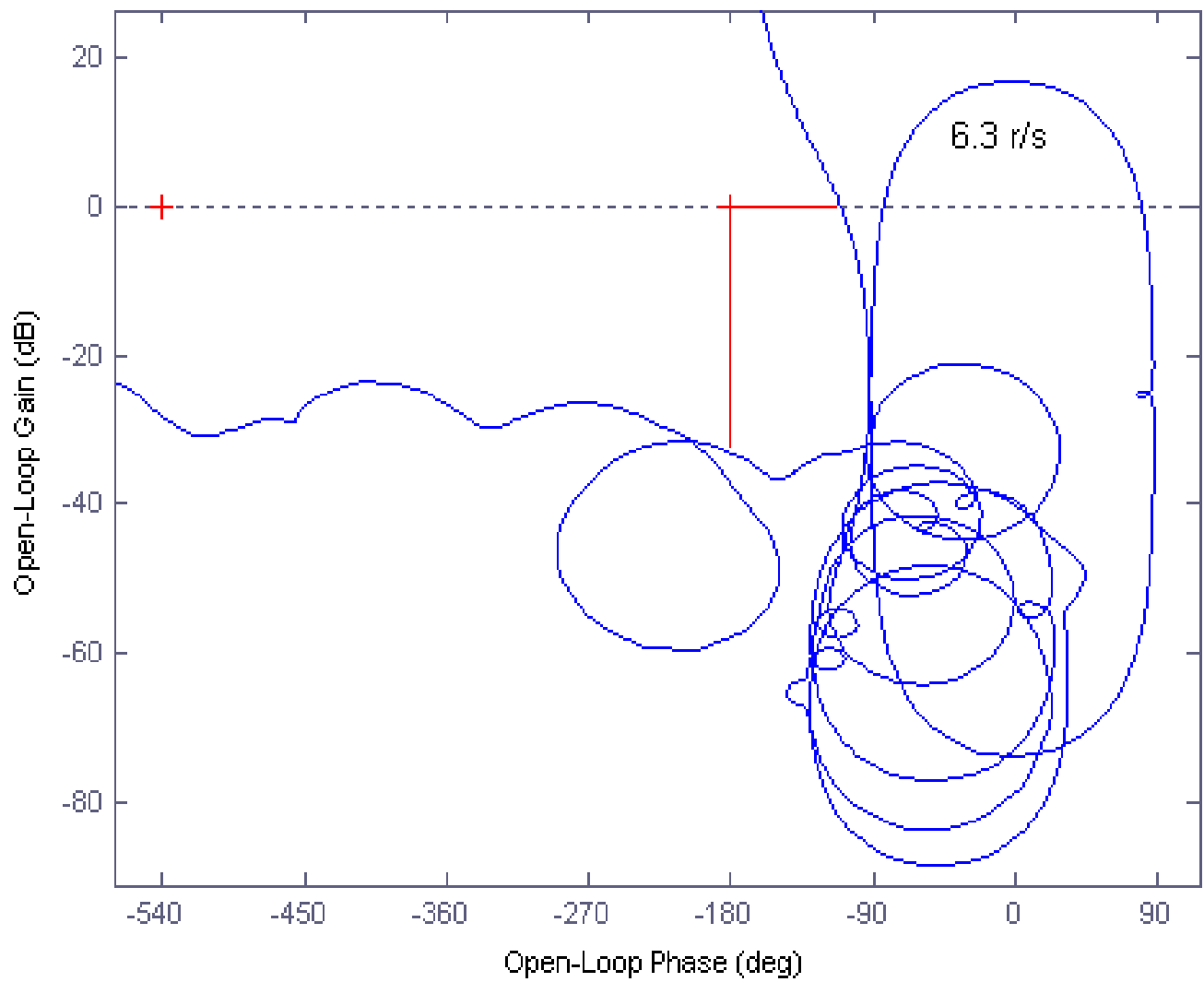
Figure 7.2 Yaw Nichols for Stability Analysis, Pitch and Roll Loops are Closed

The Simulink model in Figure 7.3 is "*Closed_Loop.mdl*" and the green satellite block includes the Flixan generated flex vehicle with uncertainties system "satellite_60flx.m". The satellite dynamics model (green block) is shown in detail in figure (4), and the controller is a simple PD control law. The file "run.m" calculates also the structural singular value (μ) frequency response of the closed-loop system with the loop opened across the parameter variations block Δ, see Figure 7.3. It calculates the (μ) using two different methods, (a) assuming that the variations in block Δ are complex with magnitudes less than 1 (this method is usually too conservative for low damped resonances), and (b) assuming that the variations in block Δ are real numbers with magnitudes varying between ± 1 (slower to compute).

Figure 7.3 Closed-Loop Simulation Model "*Closed_Loop.mdl*" uses a simple PD control law.

Flex Satellite with 3 Reaction Wheels


```
% Inputs = 32
% 1   Reaction Wheel No 1 Input Torque  (ft-lb)
% 2   Reaction Wheel No 2 Input Torque  (ft-lb)
% 3   Reaction Wheel No 3 Input Torque  (ft-lb)
% 4   Ext Torque Direct.=( 0.0000 1.0000 0.0000)

Param Variations= 28
% 5    I_xx   :    4.783 % Variation
% 6    I_xx   :    4.783 % Variation
% 7    I_yy   :    4.680 % Variation
% 8    I_yy   :    4.680 % Variation
% 9    I_yy   :    4.680 % Variation
% 10   I_zz   :    4.950 % Variation
% 11   I_zz   :    4.950 % Variation
% 12   RWinert 1:   20.000 % Variation
% 13   RW_H0  1: 0.2094E-01 Addit. Variat.
% 14   RW_H0  1: 0.2094E-01 Addit. Variat.
% 15   HDirect 1:   14.003 % Variation
% 16   HDirect 1:   14.003 % Variation
% 17   RWinert 2:   20.000 % Variation
% 18   RW_H0  2: 0.2094E-01 Addit. Variat.
% 19   RW_H0  2: 0.2094E-01 Addit. Variat.
% 20   HDirect 2:   14.003 % Variation
% 21   RWinert 3:   20.000 % Variation
% 22   RW_H0  3: 0.2094E-01 Addit. Variat.
% 23   RW_H0  3: 0.2094E-01 Addit. Variat.
% 24   HDirect 3:   14.003 % Variation
% 25   W_flex 1: 0.3000    Addit. Variat.
% 26   W_flex 2: 0.4000    Addit. Variat.
% 27   W_flex 3: 0.6000    Addit. Variat.
% 28   W_flex 4: 2.000    Addit. Variat.
% 29   W_flex 5: 2.000    Addit. Variat.
% 30   W_flex 6: 10.00    Addit. Variat.
% 31   W_flex 7: 10.00    Addit. Variat.
% 32   W_flex 8: 10.00    Addit. Variat.
```

```
% Outputs =  50
% 1    Roll Attitude  (phi-body) (radians)
% 2    Roll Rate    (p-body)  (rad/sec)
% 3    Pitch Attitude (thet-bdy) (radians)
% 4    Pitch Rate    (q-body)  (rad/sec)
% 5    Yaw  Attitude  (psi-body) (radians)
% 6    Yaw  Rate    (r-body)  (rad/sec)
% 7    Angle of attack, alfa, (radians)
% 8    Angle of sideslip, beta, (radian)
% 9    Change in Altitude, delta-h, (feet)
% 10   Forward Acceleration  (V-dot) (ft/sec)
% 11   Cross Range Velocity (Vcr) (ft/sec)
% 12   Rate-Gyro # 1, Roll Rate  (Body) (rad/sec)
% 13   Rate-Gyro # 2, Pitch Rate (Body) (rad/sec)
% 14   Rate-Gyro # 3, Yaw Rate   (Body) (rad/sec)
% 15   Gyro # 4, Roll Attitude   (Body) (radians)
% 16   Gyro # 5, Pitch Attitude  (Body) (radians)
% 17   Gyro # 6, Yaw  Attitude   (Body) (radians)
% 18   Accelerom # 1, (along X), (ft/sec^2) Translat. Accel
% 19   Accelerom # 2, (along Y), (ft/sec^2) Translat. Accel
% 20   Reaction Wheel # 1 Rate (rpm) relative to vehicle
% 21   Reaction Wheel # 2 Rate (rpm) relative to vehicle
% 22   Reaction Wheel # 3 Rate (rpm) relative to vehicle

Param Variations= 28
% 23   I_xx   :    4.783 % Variation
% 24   I_xx   :    4.783 % Variation
% 25   I_yy   :    4.680 % Variation
% 26   I_yy   :    4.680 % Variation
% 27   I_yy   :    4.680 % Variation
% 28   I_zz   :    4.950 % Variation
% 29   I_zz   :    4.950 % Variation
% 30   RWinert 1:   20.000 % Variation
% 31   RW_H0  1: 0.2094E-01 Addit. Variat.
% 32   RW_H0  1: 0.2094E-01 Addit. Variat.
% 33   HDirect 1:   14.003 % Variation
% 34   HDirect 1:   14.003 % Variation
% 35   RWinert 2:   20.000 % Variation
% 36   RW_H0  2: 0.2094E-01 Addit. Variat.
% 37   RW_H0  2: 0.2094E-01 Addit. Variat.
% 38   HDirect 2:   14.003 % Variation
% 39   RWinert 3:   20.000 % Variation
% 40   RW_H0  3: 0.2094E-01 Addit. Variat.
% 41   RW_H0  3: 0.2094E-01 Addit. Variat.
% 42   HDirect 3:   14.003 % Variation
% 43   W_flex 1: 0.3000    Addit. Variat.
% 44   W_flex 2: 0.4000    Addit. Variat.
% 45   W_flex 3: 0.6000    Addit. Variat.
% 46   W_flex 4: 2.000    Addit. Variat.
% 47   W_flex 5: 2.000    Addit. Variat.
% 48   W_flex 6: 10.00    Addit. Variat.
% 49   W_flex 7: 10.00    Addit. Variat.
% 50   W_flex 8: 10.00    Addit. Variat.
```

Figure 7.4 Spacecraft dynamics subsystem uses the uncertain state-space model "*Satellite_60flx.m*" which provides the input/ output pairs that couple with the variations block Δ

Figure 7.5 shows the µ-analysis results of the closed-loop system between the uncertainty inputs and the uncertainty outputs. The results show comparison between the two methods used to calculate the SSV: (a) using complex parameter variations, which is easier to calculate but too conservative, and (b) using real parameter variations which is slower to compute, but not as conservative as the complex variations. The complex variations being conservative violate our robustness requirement, but the (µ) with real variations satisfy the requirement of being less than one at all frequencies, with plenty of margin. It means that the system maintains stability in presence of all possible parameter variations as long as the variations lie between -1 and +1. Remember, that the individual max parameter variations were used during system creation to scale the uncertainty inputs and outputs, and the uncertainty block is, therefore, normalized assuming that its individual elements vary between -1 and +1.
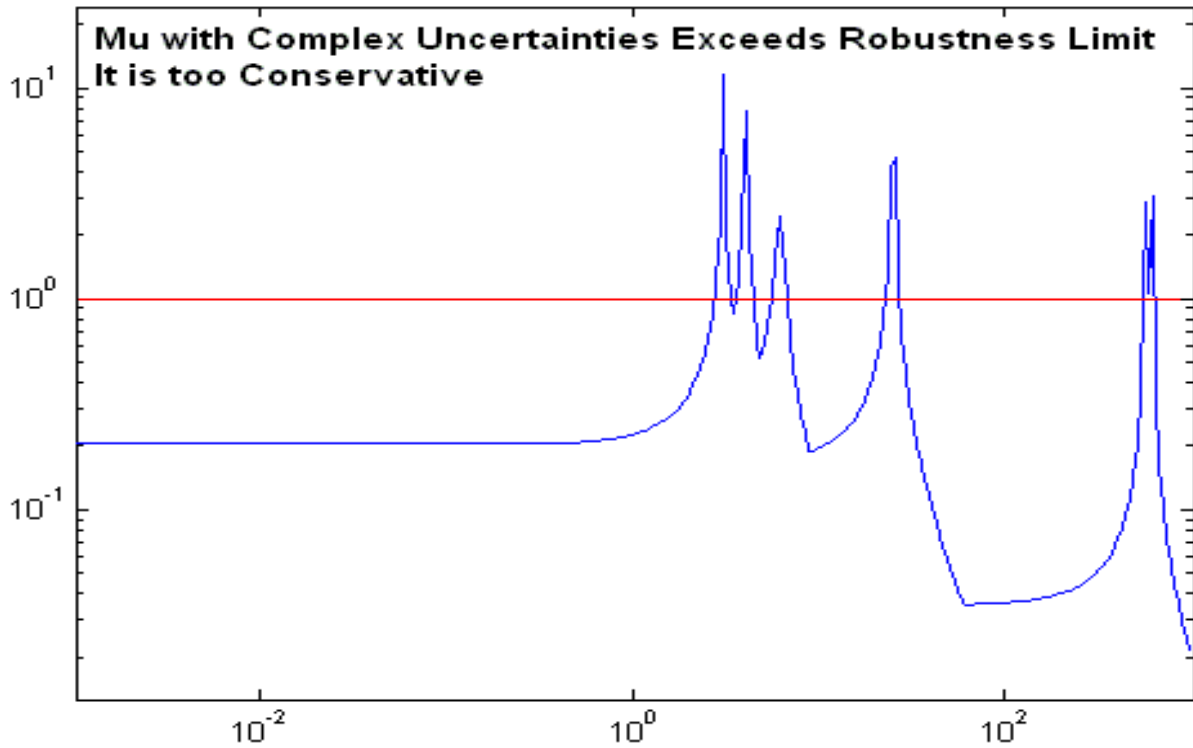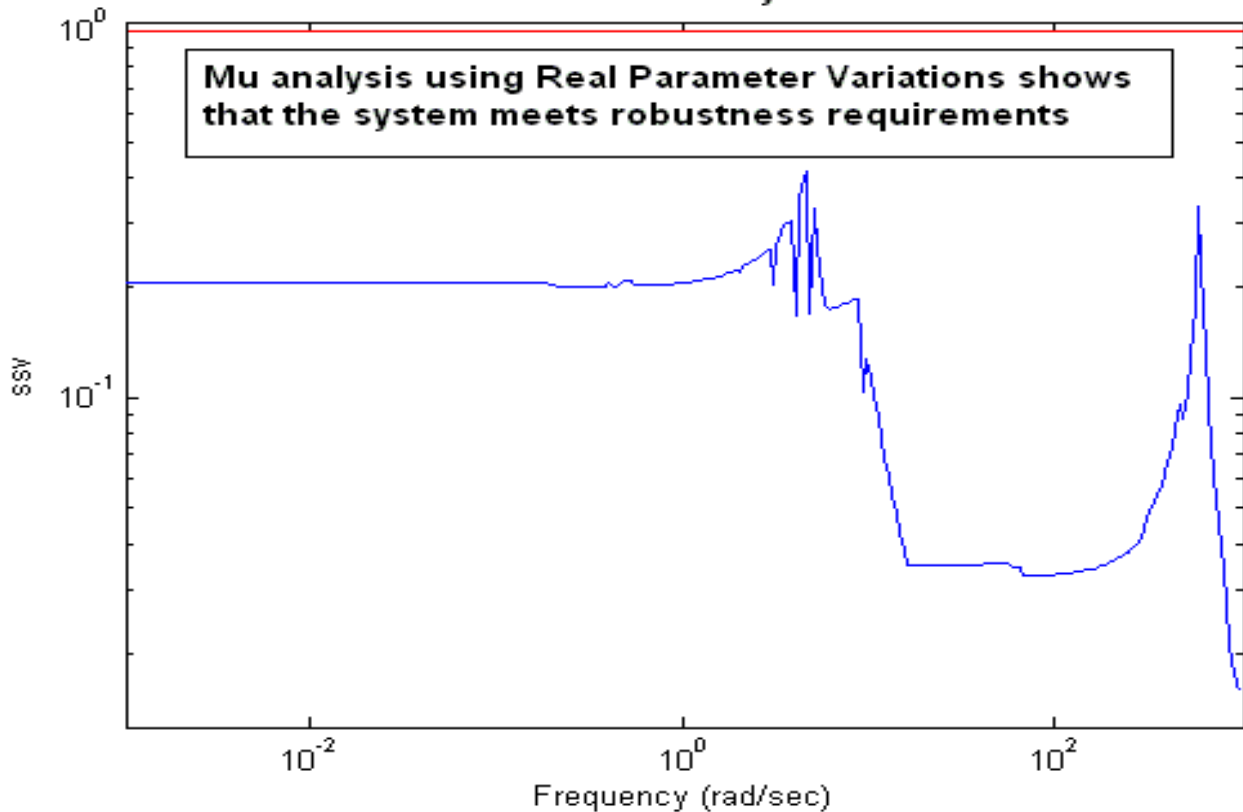
Figure 7.5 SSV of the Closed-Loop system as seen across the perturbation block Δ. The analysis using complex variations is too conservative and violates the robustness requirement, but real variations satisfies the robustness requirement

Simulation Results

The following simulation results are obtained by using the Simulink model "*Closed-Loop.mdl*". It is commanded to perform a simultaneous 20 (deg) attitude maneuver in all 3 directions. The Matlab file "pl.m" is used to plot the data. Figure 7.6 shows the attitude and rate responses in all 3 axes which are very similar. Flexibility is also visible in yaw (red).

Figure 7.7 shows the reaction wheel torques in (ft-lb) and rates in (rpm), versus time during the attitude maneuver. It also shows the response of the pitch and yaw accelerometers which are sensitive to structure flexibility.
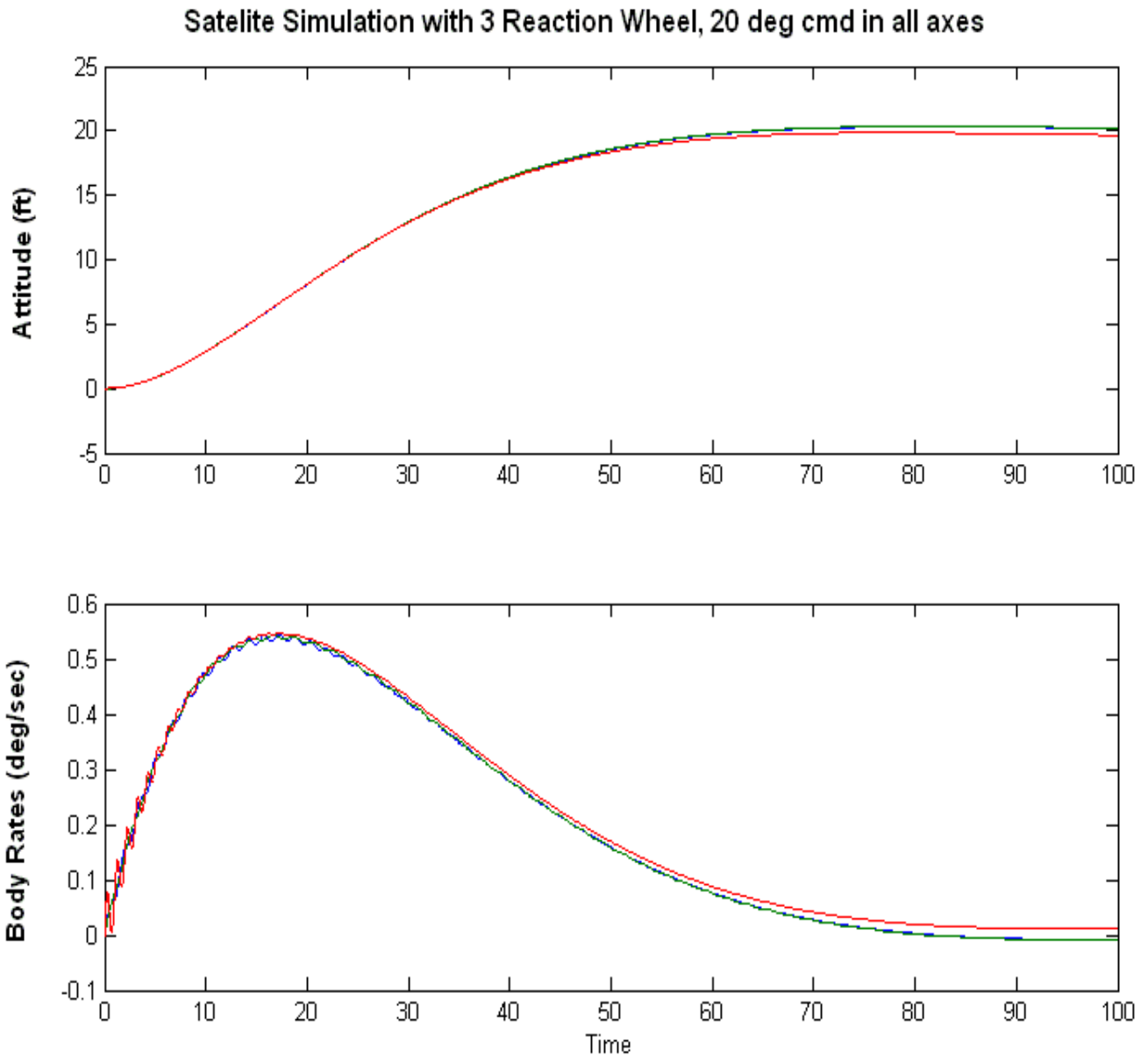
Figure 7.6 Attitude and rate response to the 20 (deg) Command in all directions. It shows that the bandwidth is the same in all directions.

Satelite Simulation with 3 Reaction Wheel, 20 deg cmd in all axes

Satelite Simulation with 3 Reaction Wheel, 20 deg cmd in all axes

Satelite Simulation with 3 Reaction Wheel, 20 deg cmd in all axes
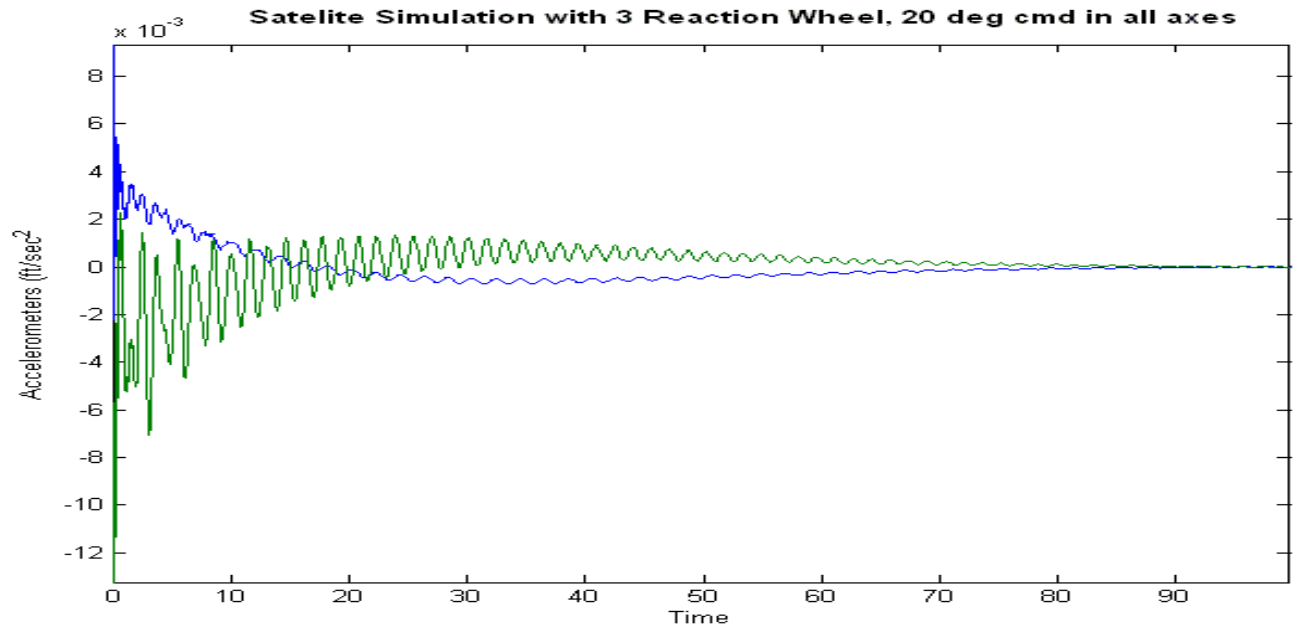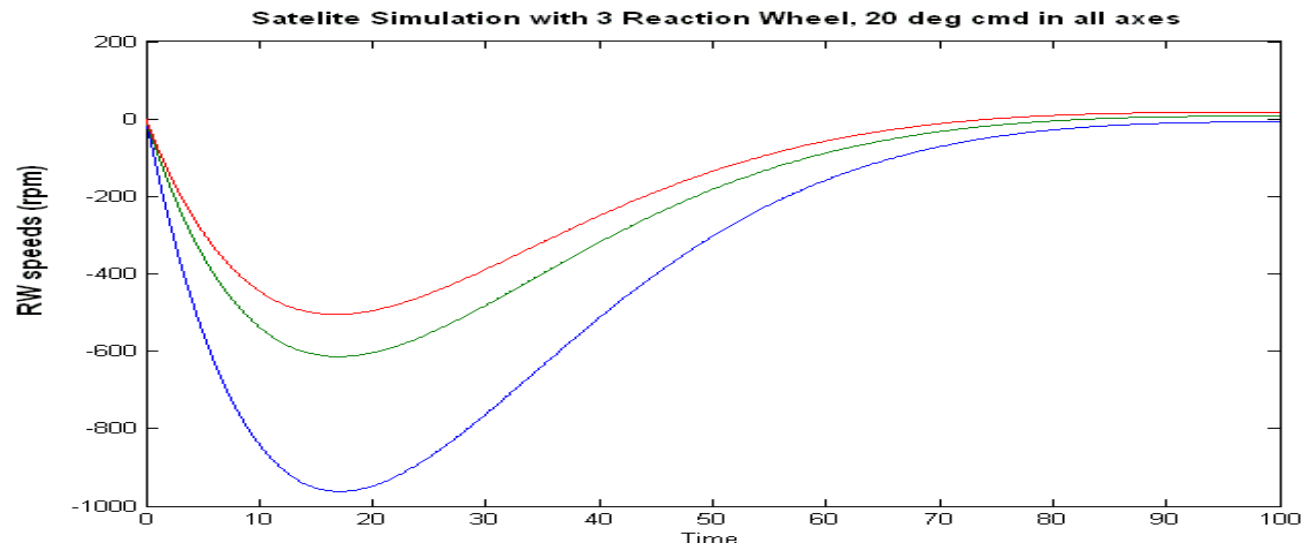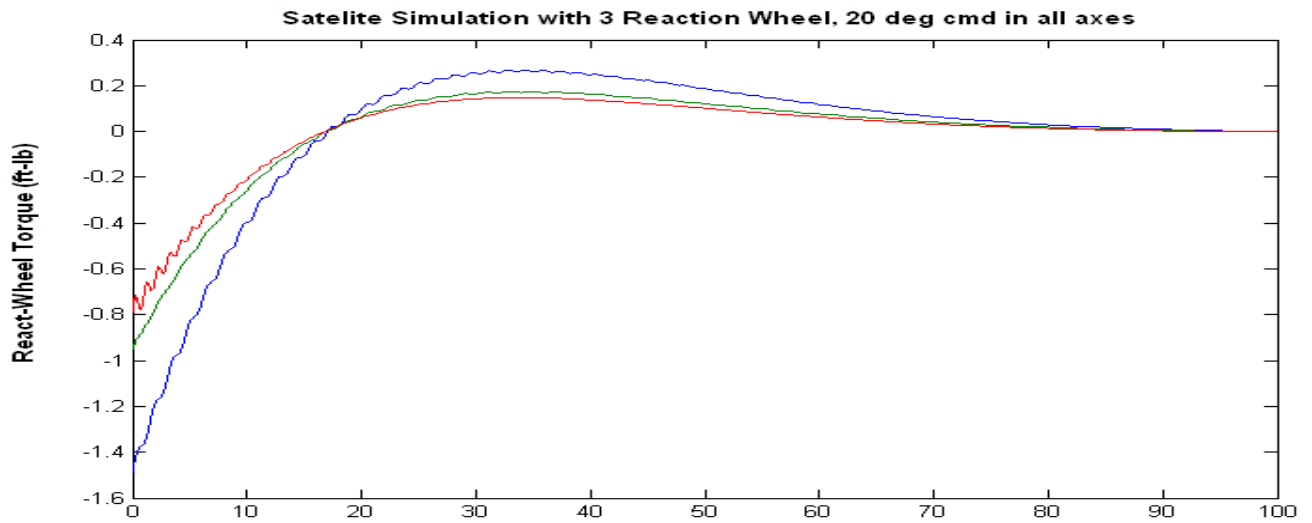
8.8 Robustness Analysis of a Flexible Satellite with Single Gimbal Control Moment Gyros

In this example we have a flexible spacecraft that is controlled by a cluster of four Single-Gimbal CMGs. The dynamics of a spacecraft with CMGs is highly non-linear and it is a function of the CMG gimbal angles. In this analysis, however, we will use a linearized model generated by the Flixan flight vehicle modeling program and also use linear control laws to analyze stability and robustness at a fixed gimbal angle positions. The analysis is performed at the initialization gimbal positions where the total system momentum is zero. The four SGCMGs have the same momentum 1200 (ft-lb-sec) and they are mounted on a four-sided pyramid configuration, as shown in Figure 2. The pyramid angle (β) is 68°. The four gimbaling directions are perpendicular to the surfaces of the pyramid and the initial gimbal angles are zero, that is, where the four momentum vectors (h_i) are parallel to the base of the pyramid, producing a total CMG momentum equal to zero. The positions shown in Figure 2 represent the CMG initialization state, and we will analyze the system stability and robustness to parameter uncertainties relative to this position. The Flight Vehicle Modeling program (FVMP) is used to create dynamic models in this initial orientation. The gimbal angles, gimbal directions, and momentum reference directions are inputs to the program. It is also possible to create linear models in other CMG orientations for analysis. The control analysis will be performed in Matlab.

Figure 8.1, shows a cluster of four SG-CMGs mounted on a rigid support structure which is isolated from the spacecraft structure by means of vibration isolation struts. Notice that, the pyramid mounting structure of Figure 8.2 is a standard arrangement used for visualization purposes. The actual mounting of the CMGs are not necessarily as shown in the pyramid, but they can be translated anywhere on the spacecraft, as long as, the gimbal direction vectors and the momentum reference directions are parallel to those shown in the pyramid model. The momentum reference directions for each CMG are defined to be the momentum directions when the gimbal angles are zero.
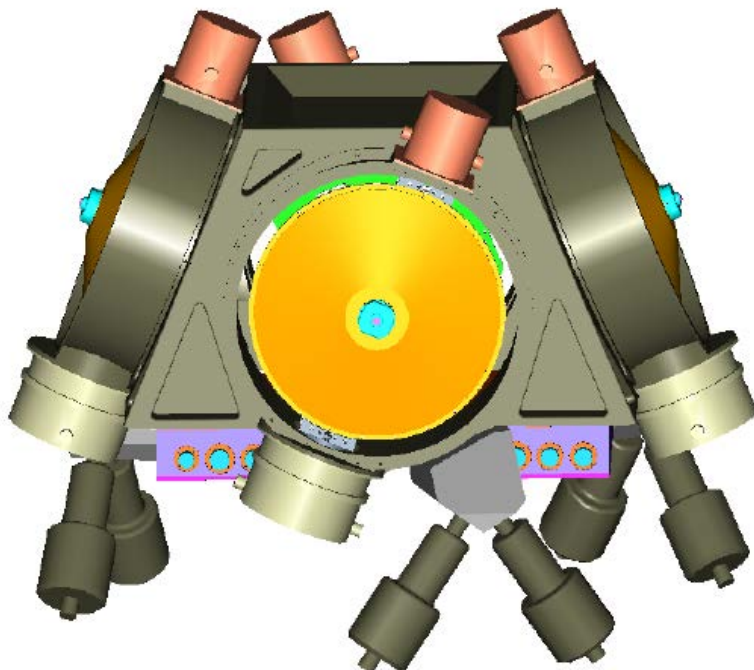
Figure 8.1 Cluster of 4 SG-CMGs

In our example, the FVMP creates two spacecraft models, a nominal model, and a similar model that has 61 additional inputs and outputs for the 61 parameter uncertainties. Dynamically, both models are the same, but the second model includes the additional 61 inputs and outputs that would connect with the parameter uncertainties that have been "pulled-out" in the normalized Δ block. We will use the nominal model to analyze stability and to prove that the nominal system is stable and then we will use the model with the uncertainties to analyze robustness to variations. We must, therefore, design the control system, close the control loop in order to stabilize the system without commands, and perform μ-analysis across the connections with the uncertainty inputs and outputs to make sure that there is no combination of the uncertainties (which may vary between ±1) that will drive it unstable.

The analysis is performed in folder "*C:\Flixan\Examples\Robust Analysis Param Uncertainties\ Satellite_SGCMG*". The spacecraft parameters are in the input file "*FlexSc_4CMG.Inp*". The file includes also a set of modal data consisting of 40 selected modes. The mode selection process is not shown here because it is fully described in other examples. The input file also includes the parameter uncertainties in a separate dataset. Its title is: "*Uncertainties for Flexible Agile Spacecraft with 4 SG-CMG*". The uncertainties are additive variations to the nominal spacecraft parameters. Notice that, not all parameters should be varied and the user must use caution in selecting which parameters to vary, because when we perturb some parameters it does not create a plant variation and causes errors. In this example we shall vary the spacecraft moments and products of inertia, the frequencies in 8 flex modes, which are the strongest. We also include uncertainty in the CMG momentum ±50 (ft-lb-sec), the momentum direction, the initial gimbal angle δ_0, the CMG moment of inertia about the gimbal axis J_g, and the pyramid surface orientation angles (β, γ). We did not include variations in the gimbal directions and the CMG inertias (J_s, J_o) because they do not create plant variations in this example.
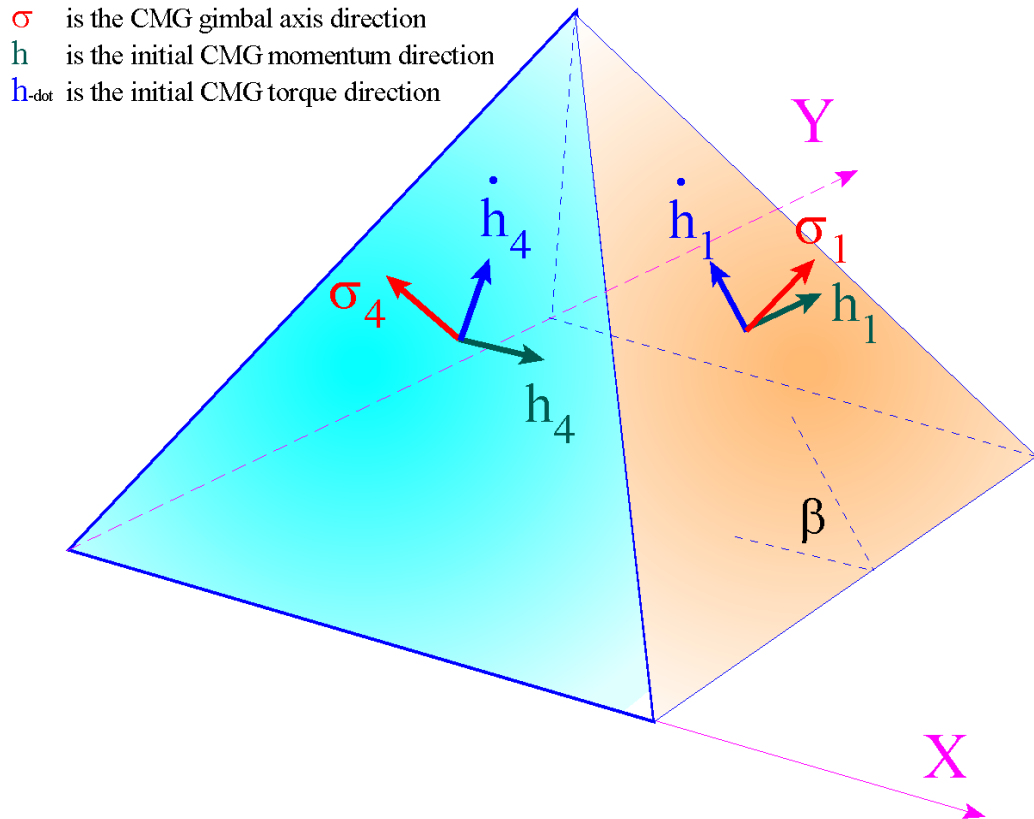
Figure 8.2 Array of four CMGs in a Pyramid Configuration

8-55

Spacecraft Model Creation

We will now process the input file "FlexSc-4CMG.Inp" that already contains the spacecraft data, the uncertainties, and the modal data.

```
FLIGHT VEHICLE INPUT DATA ......
Flexible Agile Spacecraft with 4 SG-CMG (Uncertainties)
! This is a Flex Spacecraft model that includes four 1200 (ft-lb-sec) SGCMGs
! Uncertainties in the mass properties and in the CMGs are also included
Body Axes Output, Attitude=Euler Angles

Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft)   :   202.05  32.17 0.20896E+08
Moments and products of Inertias Ixx, Iyy, Izz, -Ixy,-Ixz,-Iyz, in (lb-sec^2-ft)        :   1.41E+4  1.3E+4 0.1759E+4
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)        :   0.0  -0.20975  0.62
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)       :   0.0 25500.0 0.0 700000.
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)        :   0.0  0.0  0.0  0.0
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                         :   0.0  0.0  0.0  0.0
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)  :   0.0  0.0  0.0  0.0
External Force direction unit vect: (x,y,z), Force application point: (x,y,z)   (feet)   :   Force  1.0 0.0 0.0  -12.1
Surface Reference Area (feet^2), Mean Aerodynamic Chord (ft), Wing Span in (feet)        :   1.0    1.0        1.0
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial_rho/ Partial_H} :   0.0  -0.209  0.6201  0.0
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:  0.0  0.0  0.0  0.0  0.0
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}:  0.0  0.0  0.0  0.0  0.0
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:  0.0  0.0  0.0  0.0  0.0
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}:  0.0  0.0  0.0  0.0  0.0
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}:  0.0  0.0  0.0  0.0  0.0

Number of Single Gimbal Control Moment Gyros (SG CMG) ?                                   :   4
CMG: 1 Angular Momentum magnitude H0 (ft-lb-sec), Initial Gimbal Angle (delta) (deg)     :   1200.0      0.0
        Gimbal Direction Vector (m)                                                       :   0.927183    0.0      0.3746
        Momentum Reference Direction Vector (r)                                           :   0.0         1.0      0.0
        SGCMG Surface Orientation angles in Pyramid (beta & gamma), (Figure 2.2)          :   68.0        90.0
        SGCMG Moments of Inertia (Js, Jg, Jo) about: spin, gimbal and output axes         :   1.2         0.6      0.8
CMG: 2 Angular Momentum magnitude H0 (ft-lb-sec), Initial Gimbal Angle (delta) (deg)     :   1200.0      0.0
        Gimbal Direction Vector (m)                                                       :   0.0         0.927 0.3746
        Momentum Reference Direction Vector (r)                                           :   -1.0        0.0      0.0
        SGCMG Surface Orientation angles in Pyramid (beta & gamma), (Figure 2.2)          :   68.0        180.0
        SGCMG Moments of Inertia (Js, Jg, Jo) about: spin, gimbal and output axes         :   1.2         0.6      0.8
CMG: 3 Angular Momentum magnitude H0 (ft-lb-sec), Initial Gimbal Angle (delta) (deg)     :   1200.0      0.0
        Gimbal Direction Vector (m)                                                       :   -0.927183   0.0  0.3746
        Momentum Reference Direction Vector (r)                                           :   0.0        -1.0      0.0
        SGCMG Surface Orientation angles in Pyramid (beta & gamma), (Figure 2.2)          :   68.0        270.0
        SGCMG Moments of Inertia (Js, Jg, Jo) about: spin, gimbal and output axes         :   1.2         0.6      0.8
CMG: 4 Angular Momentum magnitude H0 (ft-lb-sec), Initial Gimbal Angle (delta) (deg)     :   1200.0      0.0
        Gimbal Direction Vector (m)                                                       :   0.0        -0.9271 0.3746
        Momentum Reference Direction Vector (r)                                           :   1.0         0.0      0.0
        SGCMG Surface Orientation angles in Pyramid (beta & gamma), (Figure 2.2)          :   68.0        0.0
        SGCMG Moments of Inertia (Js, Jg, Jo) about: spin, gimbal and output axes         :   1.2         0.6      0.8

Number of Gyros, (Attitude and Rate)                                                     :   9
Gyro No  1 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 2    :   Roll   Attit 6.53 0.0 -1.1
Gyro No  2 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 2    :   Pitch  Attit 6.53 0.0 -1.1
Gyro No  3 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 2    :   Yaw    Attit 6.53 0.0 -1.1
Gyro No  4 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 2    :   Roll   Rate  6.53 0.0 -1.1
Gyro No  5 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 2    :   Pitch  Rate  6.53 0.0 -1.1
Gyro No  6 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 2    :   Yaw    Rate  6.53 0.0 -1.1
Gyro No  7 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 1    :   Roll   Rate -2.93 0.2 -0.2
Gyro No  8 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 1    :   Pitch  Rate -2.93 0.2 -0.2
Gyro No  9 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 1    :   Yaw    Rate -2.93 0.2 -0.2

Number of Accelerometers, Along Axes: (x,y,z)                                            :   6
Acceleromet No  1 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 2   :   X-axis Accel  6.5 0.0 -1.1
Acceleromet No  2 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 2   :   Y-axis Accel  6.5 0.0 -1.1
Acceleromet No  2 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 2   :   Z-axis Accel  6.5 0.0 -1.1
Acceleromet No  4 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 4   :   X-axis Accel  5.2 0.0 -2.0
Acceleromet No  5 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 4   :   Y-axis Accel  5.2 0.0 -2.0
Acceleromet No  6 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 4   :   Z-axis Accel  5.2 0.0 -2.0

Parameter Uncertainties Data
Uncertainties for Flexible Agile Spacecraft with 4 SG-CMG

Number of Bending Modes                                                                  : 40
Flexible Agile Spacecraft with 4 SG-CMG, All Flex Modes
---------------------------------------------------------------------------------------------------------
```

Uncertainties for Flexible Agile Spacecraft with 4 SG-CMG
! This is a set of max parameter variations for the Agile Spacecraft with 4 SG-CMGs.
! We perturb the vehicle moments of inertia matrix, the momentum reference direction
! by introducing components in the zero skewed directions, we do not vary the gimbal
! direction vectors because in this case they do not create deltas. We also perturb
! the CMG surface orientation angles beta and gamma, and the CMG moment of inertia Jg
! about the gimbal axis. The other two moments of inertia, Js and Jo, do not create deltas.
! We should only perturb parameters that create plant variations, otherwise errors occur.

```
Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft)  :  0.0    0.0    0.0
Moments and products of Inertias Ixx, Iyy, Izz, -Ixy,-Ixz,-Iyz, in (lb-sec^2-ft)        : 160.0  150.0  25.0, 0.4,
0.2, 1.8
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)       :  0.0    0.0    0.0
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)      :  0.0    0.0    0.0    0.0
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)        :  0.0    0.0    0.0    0.0
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                        :  0.0    0.0    0.0    0.0
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)  :  0.0    0.0    0.0    0.0
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}: 0.0,  0.0,   0.0,   0.0,
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}: 0.0,  0.0,   0.0,   0.0,
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}: 0.0,  0.0,   0.0,   0.0,
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}: 0.0,  0.0,   0.0,   0.0,
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}: 0.0,  0.0,   0.0,   0.0,


Number of Single Gimbal Control Moment Gyros (SG CMG) Variations ?                      :   4
CMG: 1 Angular Momentum magnitude H0 (ft-lb-sec), Initial Gimbal Angle (delta) (deg)   :  50.0   1.0
       Gimbal Direction Vector (m) variation                                           :   0.0    0.0      0.0
       Momentum Reference Direction Vector (r) variation                               :   0.1    0.0      0.1
       SGCMG Surface Orientation angle in Pyramid (beta & gamma) variations,(Figure 2.2) :  1.5    1.5
       SGCMG Moments of Inertia (Js, Jg, Jo) variat. about: spin, gimbal and output axes :  0.0    0.06     0.0
CMG: 2 Angular Momentum magnitude H0 (ft-lb-sec), Initial Gimbal Angle (delta) (deg)   :  50.0   1.0
       Gimbal Direction Vector (m) variation                                           :   0.0    0.0      0.0
       Momentum Reference Direction Vector (r) variation                               :   0.0    0.1      0.1
       SGCMG Surface Orientation angle in Pyramid (beta & gamma) variations,(Figure 2.2) :  1.5    1.5
       SGCMG Moments of Inertia (Js, Jg, Jo) variat. about: spin, gimbal and output axes :  0.0    0.06     0.0
CMG: 3 Angular Momentum magnitude H0 (ft-lb-sec), Initial Gimbal Angle (delta) (deg)   :  50.0   1.0
       Gimbal Direction Vector (m) variation                                           :   0.0    0.0      0.0
       Momentum Reference Direction Vector (r) variation                               :   0.1    0.0      0.1
       SGCMG Surface Orientation angle in Pyramid (beta & gamma) variations,(Figure 2.2) :  1.5    1.5
       SGCMG Moments of Inertia (Js, Jg, Jo) variat. about: spin, gimbal and output axes :  0.0    0.06     0.0
CMG: 4 Angular Momentum magnitude H0 (ft-lb-sec), Initial Gimbal Angle (delta) (deg)   :  50.0   1.0
       Gimbal Direction Vector (m) variation                                           :   0.0    0.0      0.0
       Momentum Reference Direction Vector (r) variation                               :   0.0    0.1      0.1
       SGCMG Surface Orientation angle in Pyramid (beta & gamma) variations,(Figure 2.2) :  1.5    1.5
       SGCMG Moments of Inertia (Js, Jg, Jo) variat. about: spin, gimbal and output axes :  0.0    0.06     0.0

Flex Mode Uncertainties (Mode Number)                                                  :  1   4   6  24  26  33  34
Flex Mode Frequency Variation (additive)                                               : 0.4 0.7 1.2 3.0 3.2 3.4 4.0
```

SELECTED MODAL DATA AND LOCATIONS FOR :
Flexible Agile Spacecraft with 4 SG-CMG, All Flex Modes
! All the flex modes are selected except for the first 6 rigid-body modes. A total of 40
! structural modes.

```
MODE#  1/  7, Frequency (rad/sec), Damping (zeta), Generalized Mass=     3.1905      0.30000E-02    12.000
DEFINITION OF LOCATIONS (NODES)             phi along X   phi along Y   phi along Z   sigm about X  sigm about Y  sigm


                            Node ID#     Modal Data at the  4 Single Gimbal GMGs...
Control Moment Gyros (CMG) Loc     6     0.10717D-02  -0.16639D-01  -0.31137D-01  -0.42835D-01  -0.29132D-02  -
Control Moment Gyros (CMG) Loc     6     0.10717D-02  -0.16639D-01  -0.31137D-01  -0.42835D-01  -0.29132D-02  -
Control Moment Gyros (CMG) Loc     6     0.10717D-02  -0.16639D-01  -0.31137D-01  -0.42835D-01  -0.29132D-02  -
Control Moment Gyros (CMG) Loc     6     0.10717D-02  -0.16639D-01  -0.31137D-01  -0.42835D-01  -0.29132D-02  -
                            Node ID#     Modal Data at the  9 Gyros ...
Attitude, Rate, Accelerom Sens     2     0.45900D-02  -0.76587D-01   0.46744D-01  -0.42835D-01  -0.29132D-02  -
Attitude, Rate, Accelerom Sens     2     0.45900D-02  -0.76587D-01   0.46744D-01  -0.42835D-01  -0.29132D-02  -
Attitude, Rate, Accelerom Sens     2     0.45900D-02  -0.76587D-01   0.46744D-01  -0.42835D-01  -0.29132D-02  -
Attitude, Rate, Accelerom Sens     2     0.45900D-02  -0.76587D-01   0.46744D-01  -0.42835D-01  -0.29132D-02  -
Attitude, Rate, Accelerom Sens     2     0.45900D-02  -0.76587D-01   0.46744D-01  -0.42835D-01  -0.29132D-02  -
Attitude, Rate, Accelerom Sens     2     0.45900D-02  -0.76587D-01   0.46744D-01  -0.42835D-01  -0.29132D-02  -
Pointing Antena                    1     0.19735D-02  -0.34853D-01   0.29482D-01  -0.42912D-01  -0.26934D-02  -
Pointing Antena                    1     0.19735D-02  -0.34853D-01   0.29482D-01  -0.42912D-01  -0.26934D-02  -
Pointing Antena                    1     0.19735D-02  -0.34853D-01   0.29482D-01  -0.42912D-01  -0.26934D-02  -
                            Node ID#     Modal Data at the  6 Accelerometers, along (x,y,z)...
Attitude, Rate, Accelerom Sens     2     0.45900D-02  -0.76587D-01   0.46744D-01
Attitude, Rate, Accelerom Sens     2     0.45900D-02  -0.76587D-01   0.46744D-01
Attitude, Rate, Accelerom Sens     2     0.45900D-02  -0.76587D-01   0.46744D-01
Solar Array Hinge                  4    -0.28312D-01   0.44163D-02   0.23976D+00
Solar Array Hinge                  4    -0.28312D-01   0.44163D-02   0.23976D+00
Solar Array Hinge                  4    -0.28312D-01   0.44163D-02   0.23976D+00
                            Node ID#     Modal Data at the Disturbance Point
Reboost Engine Thruster 110 (l     5     0.16319D-02  -0.22063D-01  -0.79956D-01  -0.43084D-01  -0.27097D-02
```
--

We begin the Flixan program and select the project folder: "*Flixan\ Examples\Robust Analysis Param Uncertainties\Satellite_SGCMG*". Unlike the previous examples, this time we will create the uncertainty model using the FVMP. From the Flixan main menu select "*Program Functions*", "*Flight Vehicle/ Spacecraft Modeling Tools*", and "*Flight Vehicle State-Space Model*", as shown below.
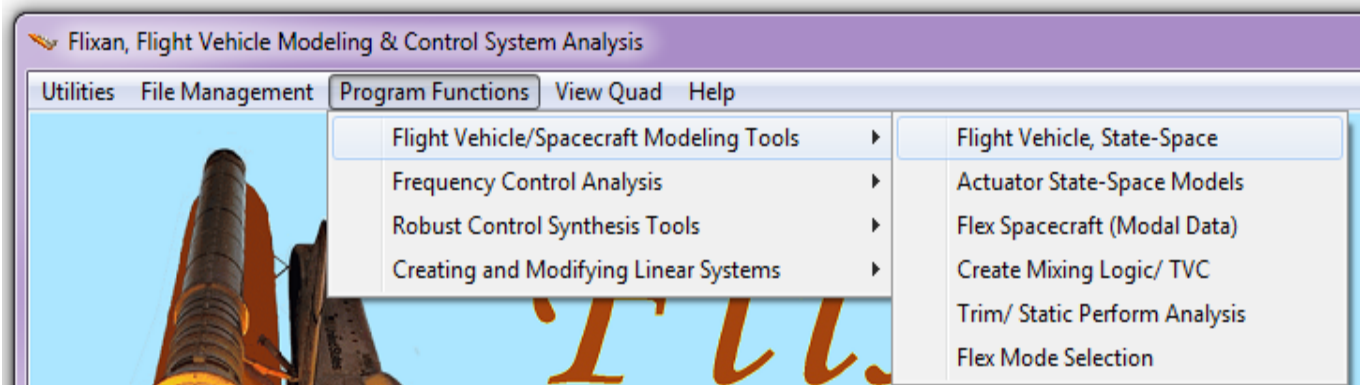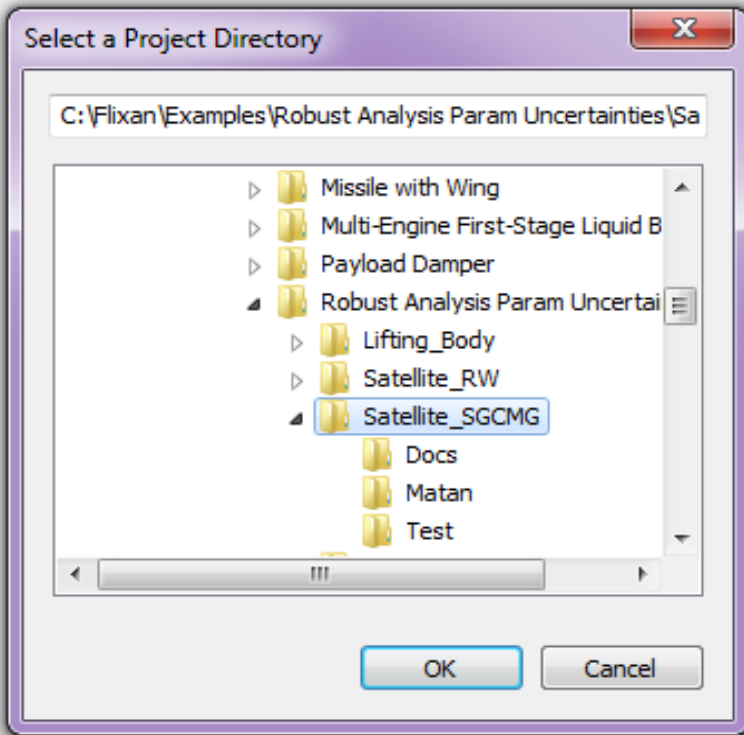
From the filename selection menu select the input data file "*FlexSc_4CMG.Inp*", and the output systems file "*FlexSc_4CMG.Qdr*", and click on "Select Files" button.
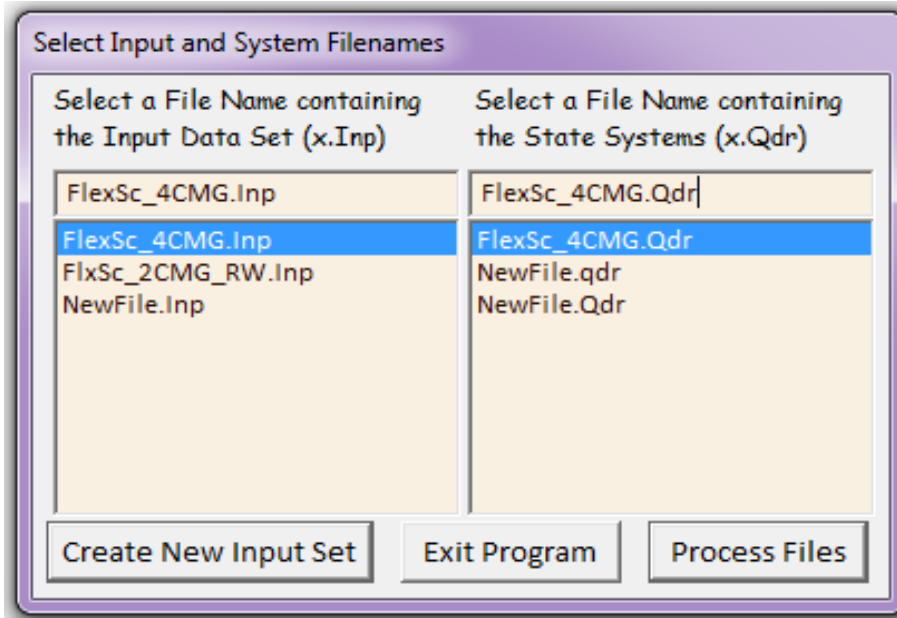
From the following menu select the spacecraft data title "*Flexible Agile Spacecraft with 4 SG-CMG (Uncertainties)*" that includes references to the uncertainties and to the modal data sets. So when we process the data via the FVMP we do not need to select the uncertainties data-set because it is included in the spacecraft data. The spacecraft data set contains the spacecraft and CMG input data for the initial CMG gimbal configuration. Click on "*Run Input Set*" to process the data.
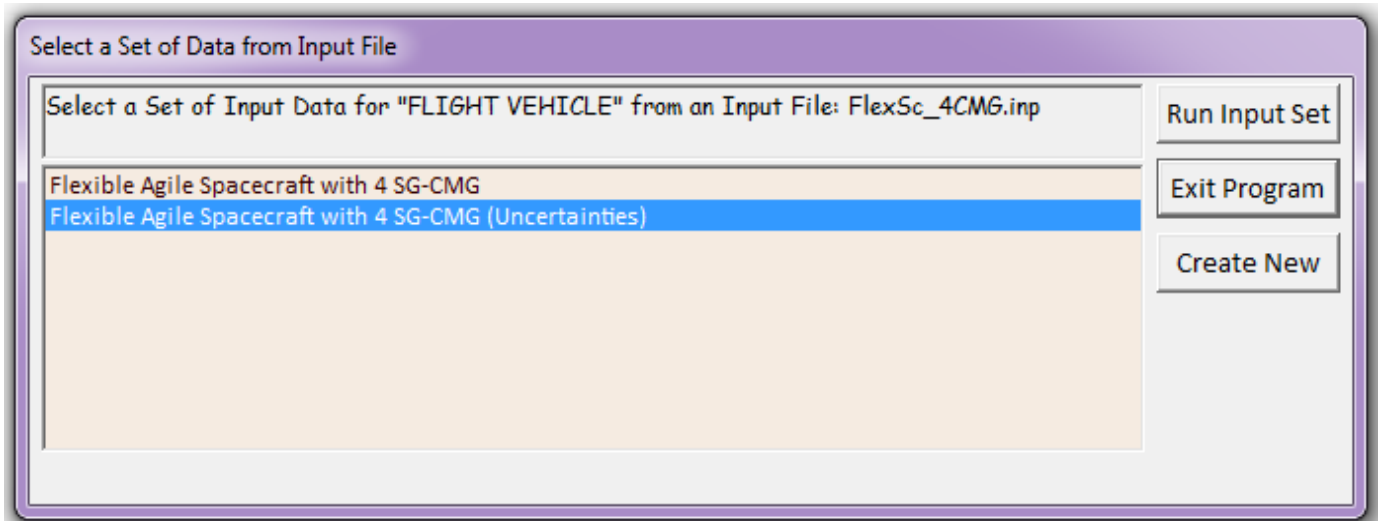
The following dialog is generated by the vehicle modeling program and it consists of several tabs for browsing and modifying the spacecraft data. Each tab consists of a group of data. The following tab shows the SG-CMG data.
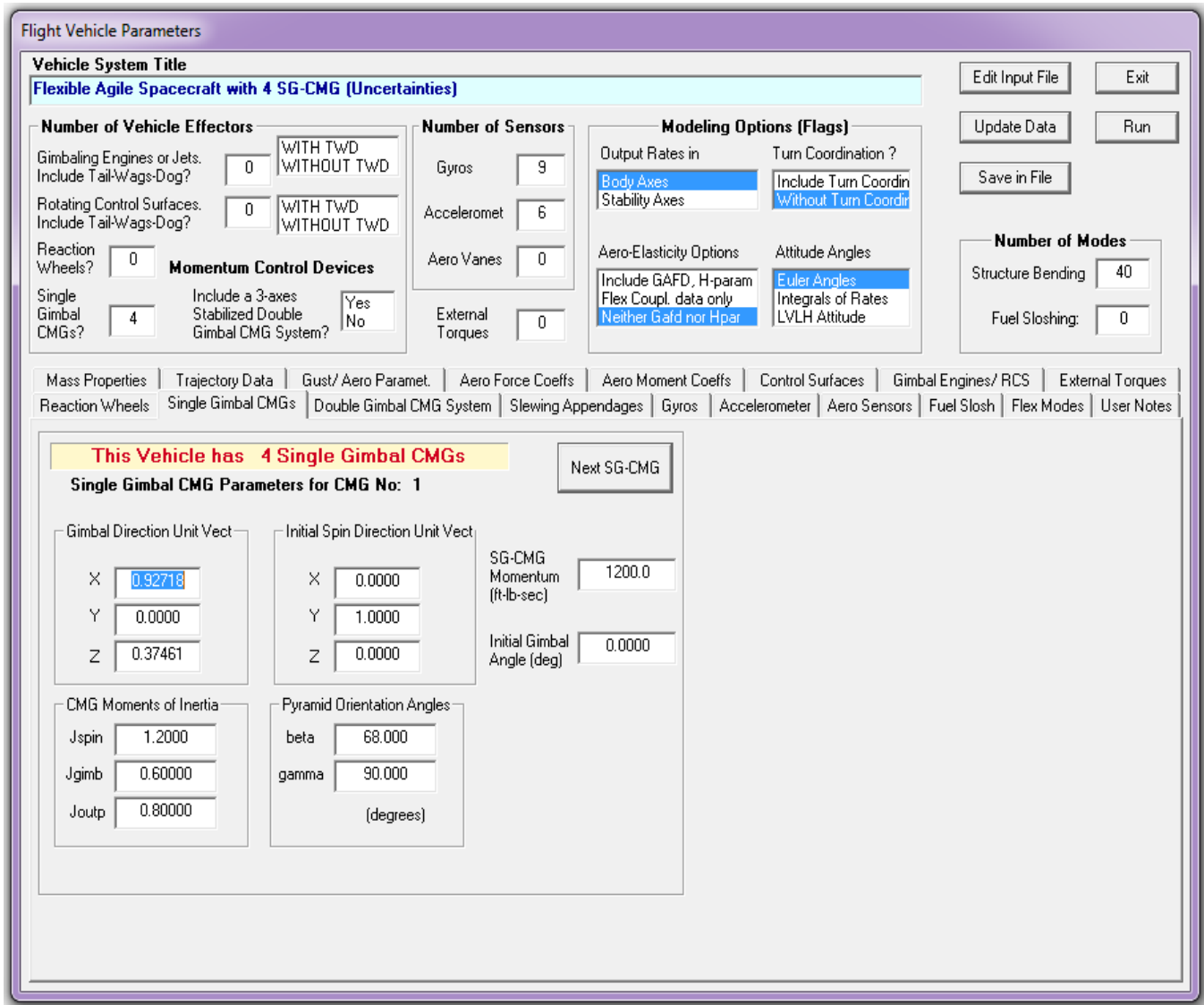
Click on "Run" and the program will create a linearized state-space model for the agile satellite and it will save it in file "*FlexSc_4CMG.Qdr*". This system is also converted as a Matlab function in m-file: "*sc_4cmg_flex_unc.m*" that will be used to perform μ-analysis in Matlab. This system includes the 61 additional inputs and the 61 additional outputs that connect to the uncertainty block Δ. In our μ-analysis we don't connect it with a Δ block but we simply compute the μ frequency response between the 61 inputs and outputs. Notice, that the input/ output pairs connecting to the uncertainty block Δ is greater than the number of parameter variations. This is because some of the uncertainties are higher than rank-1 dependency, and they couple into more than one direction since they appear in 2 equations. The products of inertia, for example, couple in two directions. Since we do not decouple the system but analyze roll, pitch, and yaw axes together, it is acceptable to treat then as two separate parameter variations although they originate from a single uncertainty.

Batch Processing

Batch processing is the easiest and fastest method for generating the spacecraft models. The input file "*FlexSc_4CMG.Inp*" includes a batch set "*Batch for Spacecraft with 4 SG CMG*" located on the top of the input data file that can be processed to generate the systems as follows. Start the Flixan program and select the folder "*Examples\Robust Analysis Param Uncertainties\Satellite_SGCMG*" as before. Then, from the Flixan main menu go to "*File Management*", "*Manage Input Files*", and "*Process/ Edit Input Data*", as shown below.
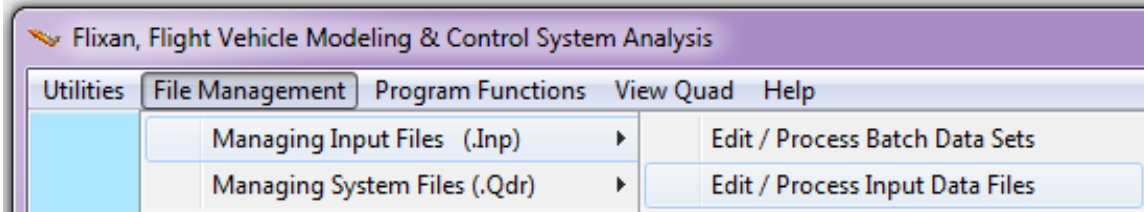
The following Menu/ Dialog comes up, and from the file selection menu on the left select the input file "*FlexSc_4CMG.Inp*". The menu on the right shows the titles of the data-sets which are included in the input file. Select the batch on the top "*Batch for Spacecraft with 4 SG CMG*", and click on "*Process Input Data*" to process the input file. The Flixan program generates the systems and saves them in systems file "*FlexSc_4CMG.Qdr*". If a previous version of the systems file already exists in folder the program will ask permission to recreate it, answer "Yes".
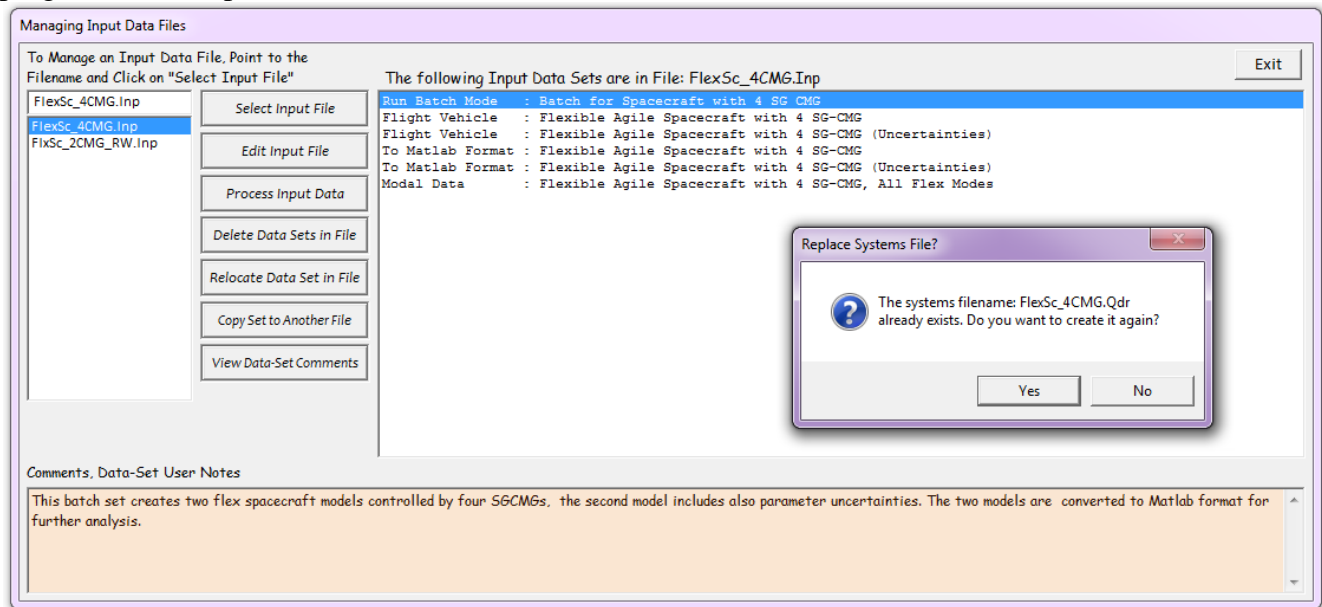
The batch performs the following operations. It creates a nominal spacecraft model with 40 flex modes, title: "*Flexible Agile Spacecraft with 4 SG-CMG*" in file "*FlexSc_4CMG.Qdr*". It also creates the uncertainty model with the 61 parameter variations, title: "*Flexible Agile Spacecraft with 4 SG-CMG (Uncertainties)*". The two state-space models are then converted to Matlab system functions, "*sc_4cmg_flex.m*" and "*sc_4cmg_flex_unc.m*", respectively that can be loaded into Matlab. Both systems contain roll, pitch, and yaw coupled vehicle dynamics. The second system, in addition to the standard inputs and outputs present in the first system, it includes also the 61 pairs of inputs and outputs that connect to the uncertainties Δ block. This is the diagonal block that contains the normalized uncertainties that vary between -1 and +1. Some of the uncertainties couple only in one axis, but some uncertainties couple in more than one axis.

Simulation Model

Now, let us take a look at the linear simulation model in file "*Lin_Flex_Sim.Mdl*", shown in Figure 8.2 that uses the nominal spacecraft system from file "*sc_4cmg_flex.m*", without the uncertainties. This simulation model is initialized using file "start.m", which also loads the two spacecraft systems into the Matlab workspace.
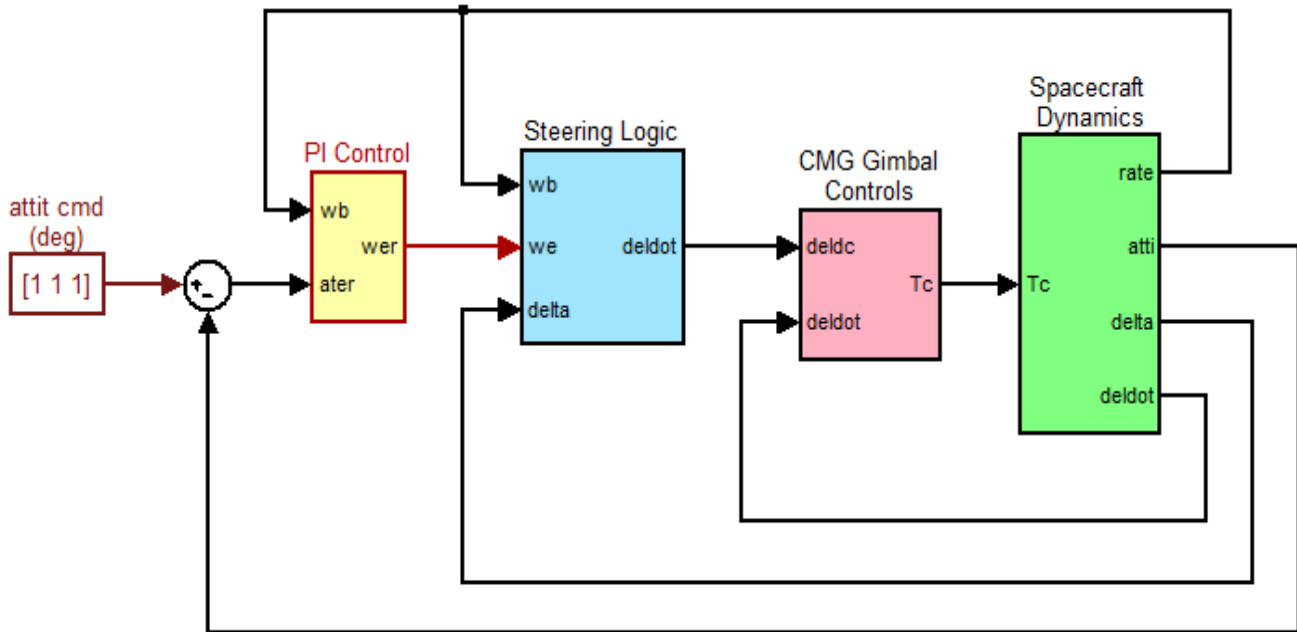
Figure 8.2 Simulink model for the Flex Agile Spacecraft with 4 SG-CMGs, in file "*Lin_Flex_Sim.mdl*"

The spacecraft dynamics (green) block consists of the state-space system "*sc_4cmg-flex.m*" (without uncertainties). The input (Tc) is a vector of four CMG gimbal torques which control the gimbal rates. The outputs are: spacecraft attitude, rates, CMG gimbal angles, and gimbal rates. The gimbal rate commands come from the CMG steering logic and the gimbal rate control system provides the gimbal torques required to control the rates. The purpose of the steering logic is to control the spacecraft rate by creating gimbal rate commands at the 4 CMG gimbals. The inputs to the steering logic are: spacecraft rates, gimbal angles, and spacecraft rate error. The attitude control system is a simple PI. The (D) part of the PID is included in the steering. In the simulation the spacecraft is commanded to perform a one degree rotation in all 3 directions. The purpose of the simulation is to demonstrate nominal system stability.
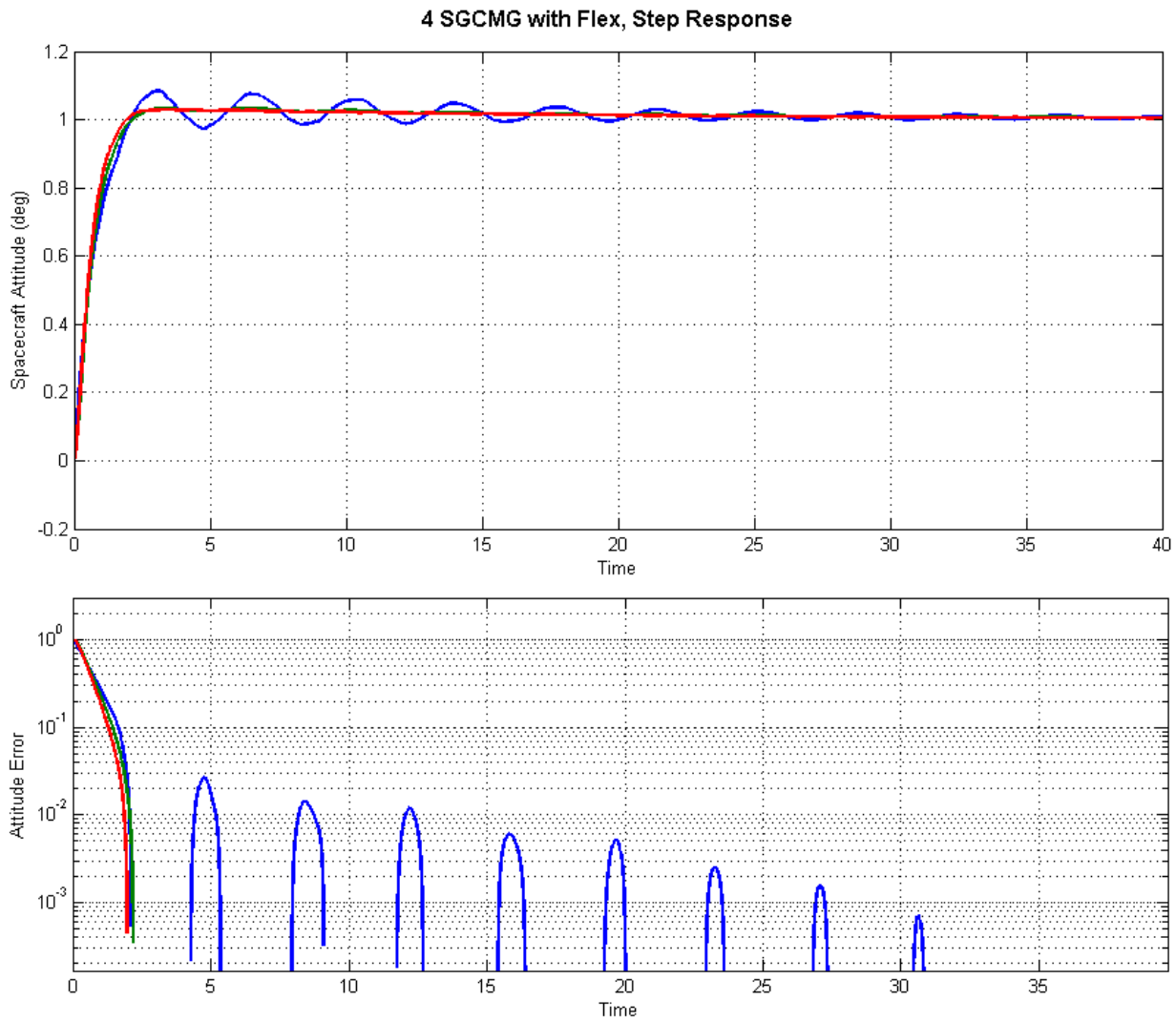
Figure 8.3a Spacecraft attitude response to one degree command in all 3 directions

Figure 8.3a shows a stable attitude response to 1° command in all 3 directions. The roll axis (blue) takes longer to settle because flexibility is stronger in roll. Figure 8.3b shows the spacecraft rate and acceleration at two separate locations with different flex mode sensitivity. Figure 8.3c shows the gimbal angles and gimbal rates. It also shows the CMG momentum. The roll momentum oscillates as the CMGs respond to the roll structural oscillations.
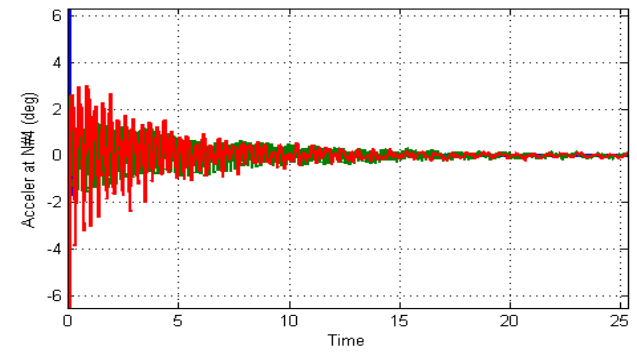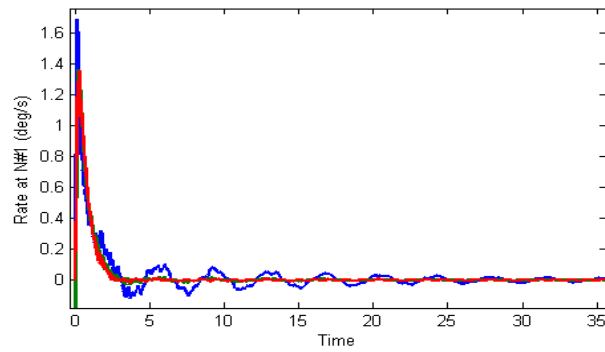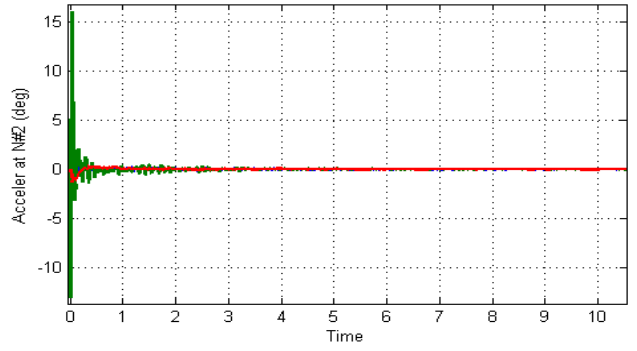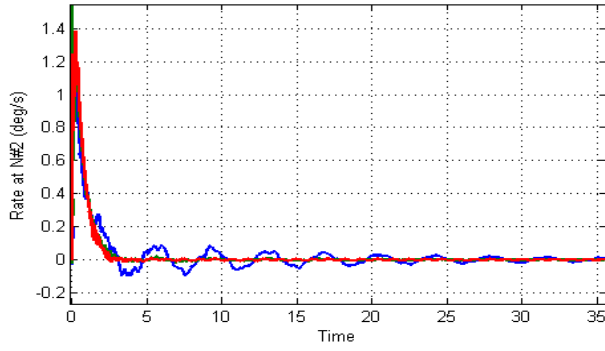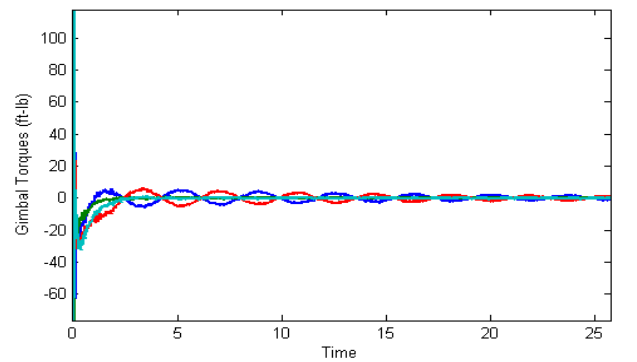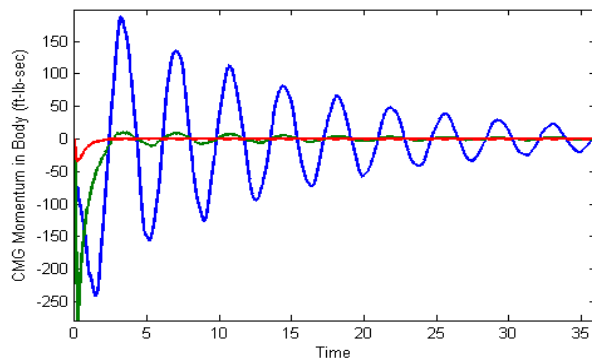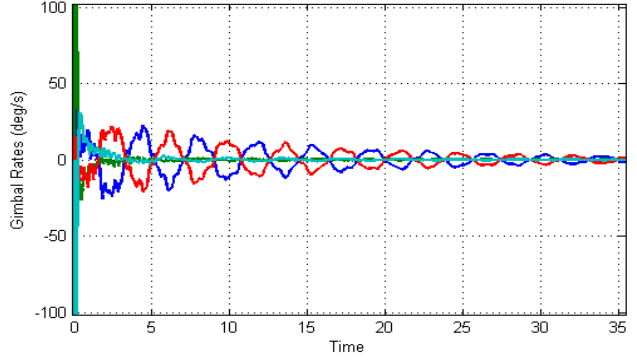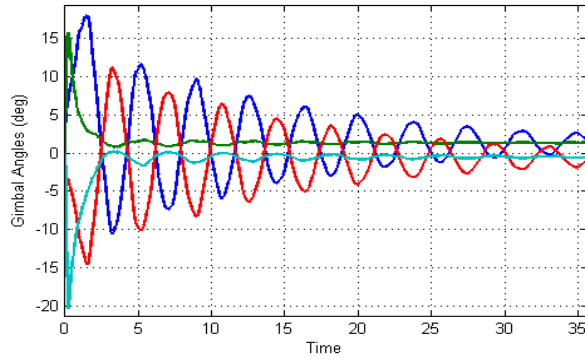
Figures (8.3b & 8.3c) Spacecraft response to the one degree command in 3 directions

Linear Stability Analysis

A similar Simulink model "*Open_Loop.mdl*" shown in Figure 8.4 is used to perform linear stability analysis and to determine the phase and gain margins in the nominal CMG configuration. It consists of the same subsystems as the simulation model having three control loops (roll, pitch, and yaw), two of which are closed and one is opened. The file "freq.m" performs the open-loop frequency response analysis. The Matlab script linearizes this system across the opened input and output using the "linmod" function. It calculates its frequency response and plots the Nichols charts, as shown in Figures (8.5a-8.5c).
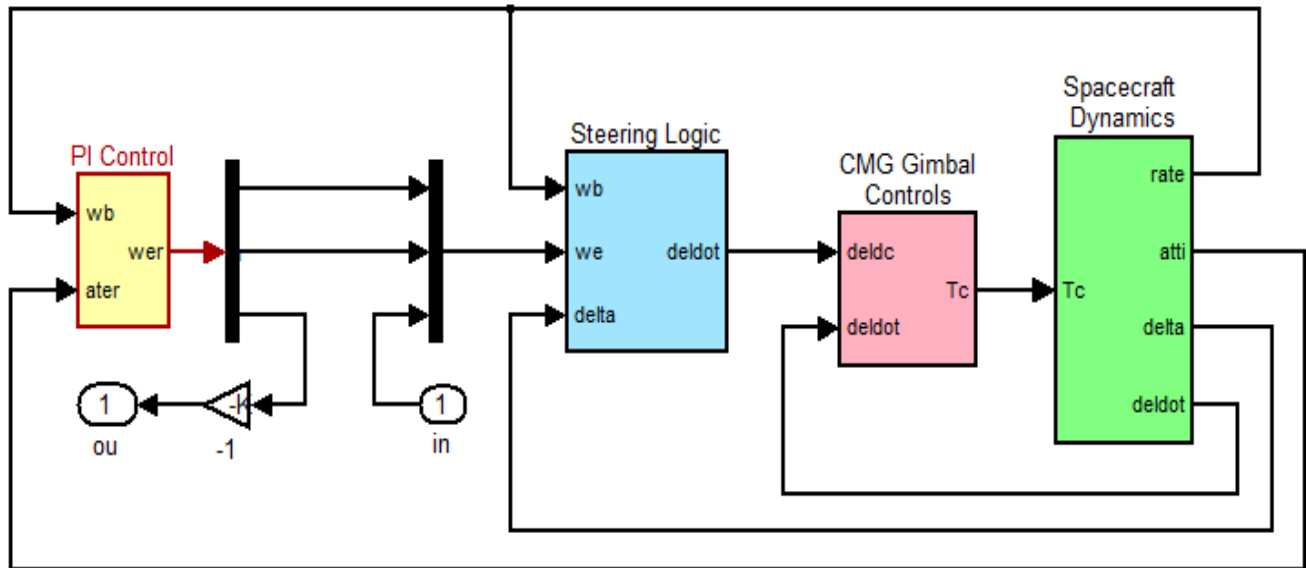
Figure 8.4 Simulink model "*Open_Loop.mdl*" used for linear stability analysis

Nominal System Stability Margins in the Yaw Axis

Nichols Chart
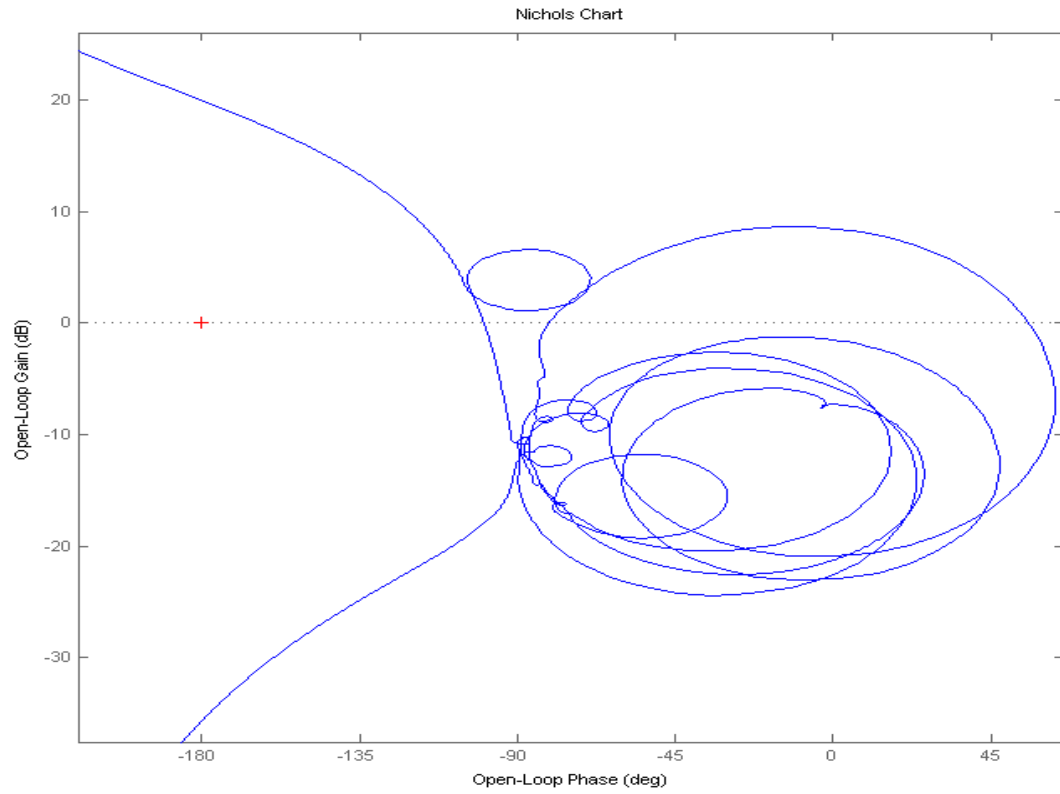
Nominal System Stability Margins in the Roll Axis

Nichols Chart

8-66
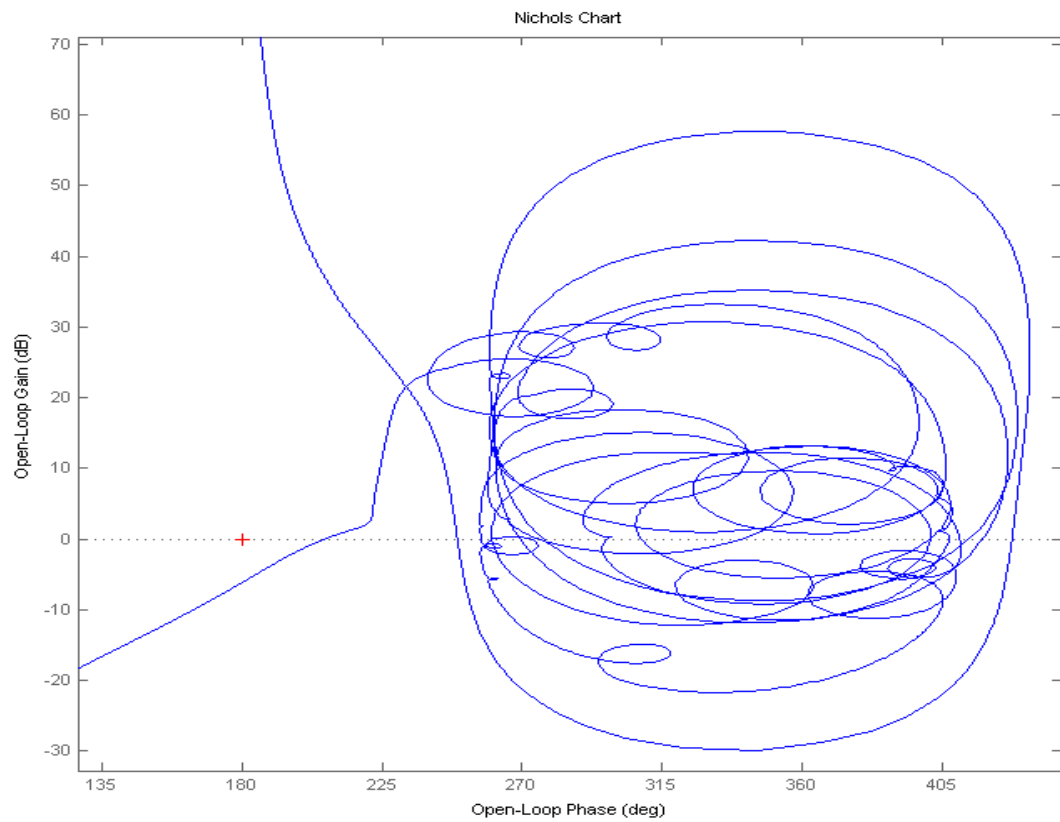
Nominal System Stability Margins in the Pitch Axis
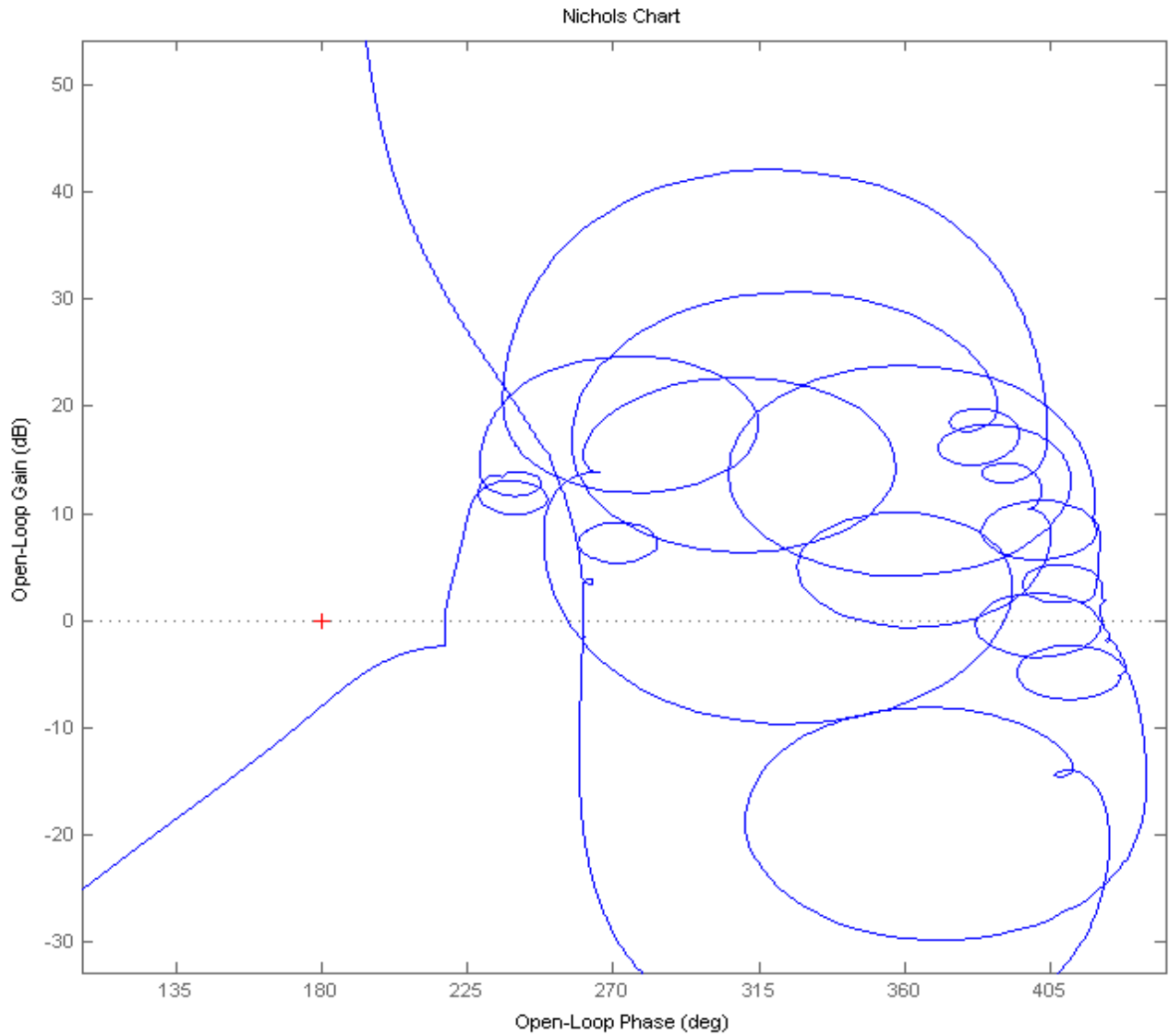
Nichols Chart

Figure 8.5c System is Nominally Stable in all 3 Axes

Robustness Analysis

Robustness analysis is performed by closing the attitude control loops (assuming that the closed-loop system is stable) and by calculating the μ-frequency response of the system across the normalized perturbations block Δ. Since the scaled diagonal elements of Δ do not vary more than ±1, the system is assumed to be robust when the SSV of the closed-loop system across the diagonal perturbations is less than one at all frequencies. The following Simulink model "Robust-Anal.mdl" is used to calculate the SSV.
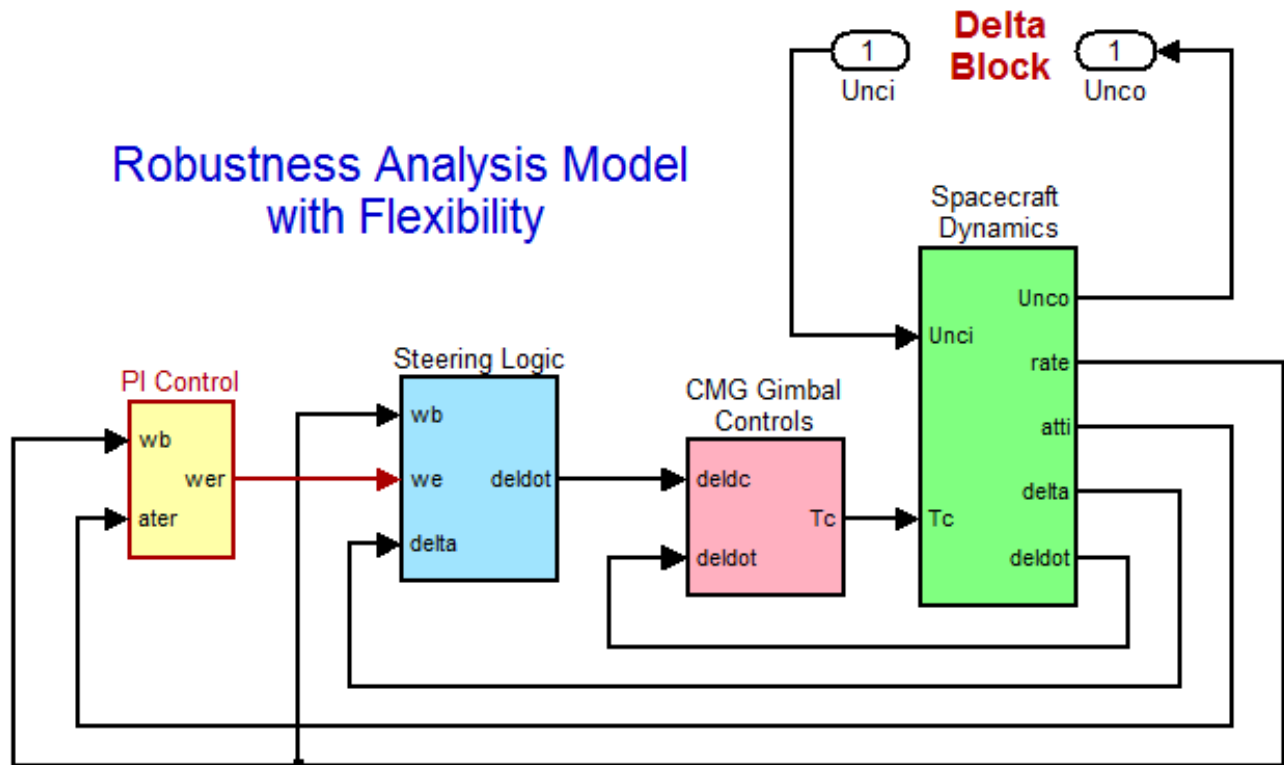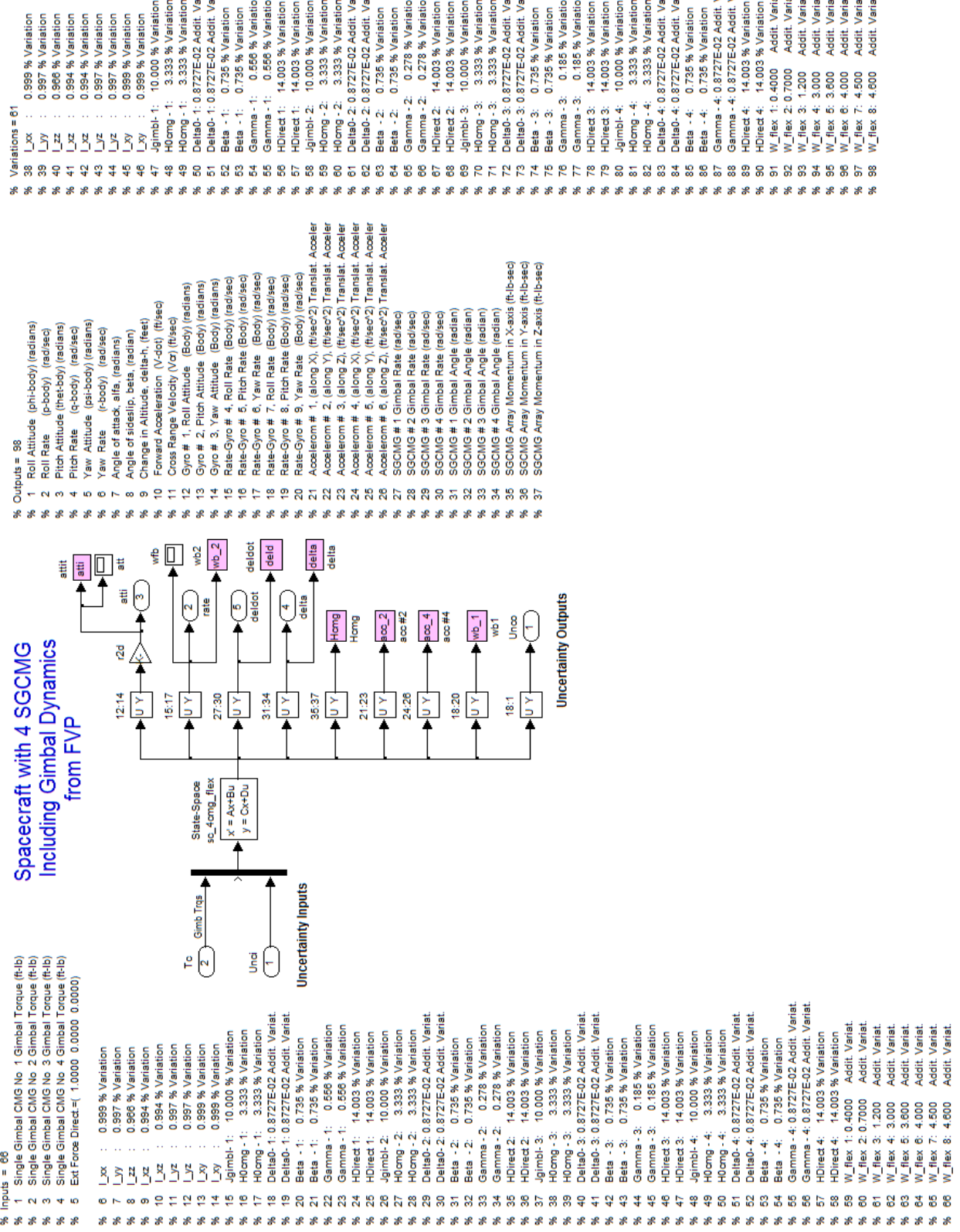

Figure 8.6 Simulink model "Robust_Anal.mdl" used to calculate system robustness to parameter variations

The green spacecraft dynamics block now includes the additional inputs and outputs that connect to the uncertainty block Δ. The control system, steering logic, and CMG gimbal control subsystems are the same as in previous models. The spacecraft subsystem is shown in detail in Figure 8.7. It uses the state-space system "*sc_4cmg_flex_unc.m*" which includes the 61 additional inputs and outputs that connect with the IFL block. The Matlab script "freq.m" also calculates the μ-frequency response of the linearized system in Figure 8.6 assuming that the parameter variations are "real" (not "complex" because complex variations are very conservative especially with flex mode variations). Figure 8.8 shows the μ-frequency response across the Δ block. It is less than one at all frequencies. We therefore conclude that the system is robust to all parameter variations.

Spacecraft with 4 SGCMG Including Gimbal Dynamics from FVP

State-Space sc_4cmg_flex x' = Ax+Bu y = Cx+Du

Tc — Gimb Trqs   Unci — Uncertainty Inputs

Outputs: attit (att), wfb (wb_2), deldot (deld), delta, Hcmg, acc #2, acc #4, wb_1, Unco — Uncertainty Outputs

```
% Inputs = 66
%   1  Single Gimbal CMG No. 1 Gimbal Torque (ft-lb)
%   2  Single Gimbal CMG No. 2 Gimbal Torque (ft-lb)
%   3  Single Gimbal CMG No. 3 Gimbal Torque (ft-lb)
%   4  Single Gimbal CMG No. 4 Gimbal Torque (ft-lb)
%   5  Ext Force Direct.=( 1.0000  0.0000  0.0000)
%   6  I_xx   :   0.999 % Variation
%   7  I_yy   :   0.997 % Variation
%   8  I_zz   :   0.966 % Variation
%   9  I_xz   :   0.994 % Variation
%  10  I_xz   :   0.984 % Variation
%  11  I_yz   :   0.997 % Variation
%  12  I_yz   :   0.999 % Variation
%  13  I_xy   :   0.999 % Variation
%  14  I_xy   :   0.999 % Variation
%  15  Jgimbl-1:  10.000 % Variation
%  16  H0omg-1:    3.333 % Variation
%  17  H0omg-1:    3.333 % Variation
%  18  Delta0-1: 0.8727E-02 Addit. Variat.
%  19  Delta0-1: 0.8727E-02 Addit. Variat.
%  20  Beta  -1:   0.735 % Variation
%  21  Beta  -1:   0.735 % Variation
%  22  Gamma -1:   0.556 % Variation
%  23  Gamma -1:   0.556 % Variation
%  24  HDirect 1: 14.003 % Variation
%  25  HDirect 1: 14.003 % Variation
%  26  Jgimbl-2:  10.000 % Variation
%  27  H0omg - 2:  3.333 % Variation
%  28  H0omg - 2:  3.333 % Variation
%  29  Delta0-2: 0.8727E-02 Addit. Variat.
%  30  Delta0-2: 0.8727E-02 Addit. Variat.
%  31  Beta  - 2:  0.735 % Variation
%  32  Beta  - 2:  0.735 % Variation
%  33  Gamma - 2:  0.278 % Variation
%  34  Gamma - 2:  0.278 % Variation
%  35  HDirect 2: 14.003 % Variation
%  36  HDirect 2: 14.003 % Variation
%  37  Jgimbl-3:  10.000 % Variation
%  38  H0omg - 3:  3.333 % Variation
%  39  H0omg - 3:  3.333 % Variation
%  40  Delta0- 3: 0.8727E-02 Addit. Variat.
%  41  Delta0- 3: 0.8727E-02 Addit. Variat.
%  42  Beta  - 3:  0.735 % Variation
%  43  Beta  - 3:  0.735 % Variation
%  44  Gamma - 3:  0.185 % Variation
%  45  Gamma - 3:  0.185 % Variation
%  46  HDirect 3: 14.003 % Variation
%  47  HDirect 3: 14.003 % Variation
%  48  Jgimbl-3:  10.000 % Variation
%  49  H0omg - 4:  3.333 % Variation
%  50  H0omg - 4:  3.333 % Variation
%  51  Delta0- 4: 0.8727E-02 Addit. Variat.
%  52  Delta0- 4: 0.8727E-02 Addit. Variat.
%  53  Beta  - 4:  0.735 % Variation
%  54  Beta  - 4:  0.735 % Variation
%  55  Gamma - 4: 0.8727E-02 Addit. Variat.
%  56  Gamma - 4: 0.8727E-02 Addit. Variat.
%  57  HDirect 4: 14.003 % Variation
%  58  HDirect 4: 14.003 % Variation
%  59  W_flex 1: 0.4000  Addit. Variat.
%  60  W_flex 2: 0.7000  Addit. Variat.
%  61  W_flex 3: 1.200   Addit. Variat.
%  62  W_flex 4: 3.000   Addit. Variat.
%  63  W_flex 5: 3.600   Addit. Variat.
%  64  W_flex 6: 4.000   Addit. Variat.
%  65  W_flex 7: 4.500   Addit. Variat.
%  66  W_flex 8: 4.600   Addit. Variat.
```

```
% Outputs = 98
%   1  Roll Attitude   (phi-body) (radians)
%   2  Roll Rate       (p-body)  (rad/sec)
%   3  Pitch Attitude  (thet-body) (radians)
%   4  Pitch Rate      (q-body)  (rad/sec)
%   5  Yaw Attitude    (psi-body) (radians)
%   6  Yaw Rate        (r-body)  (rad/sec)
%   7  Angle of attack, alfa, (radians)
%   8  Angle of sideslip, beta, (radian)
%   9  Change in Altitude, delta-h, (feet)
%  10  Forward Acceleration (V-dot) (ft/sec)
%  11  Cross Range Velocity (Vcr) (ft/sec)
%  12  Gyro # 1, Roll Attitude  (Body) (radians)
%  13  Gyro # 2, Pitch Attitude (Body) (radians)
%  14  Gyro # 3, Yaw Attitude   (Body) (radians)
%  15  Rate-Gyro # 4, Roll Rate  (Body) (rad/sec)
%  16  Rate-Gyro # 5, Pitch Rate (Body) (rad/sec)
%  17  Rate-Gyro # 6, Yaw Rate   (Body) (rad/sec)
%  18  Rate-Gyro # 7, Roll Rate  (Body) (rad/sec)
%  19  Rate-Gyro # 8, Pitch Rate (Body) (rad/sec)
%  20  Rate-Gyro # 9, Yaw Rate   (Body) (rad/sec)
%  21  Accelerom # 1, (along X), (ft/sec^2) Translat. Acceler
%  22  Accelerom # 2, (along Y), (ft/sec^2) Translat. Acceler
%  23  Accelerom # 3, (along Z), (ft/sec^2) Translat. Acceler
%  24  Accelerom # 4, (along X), (ft/sec^2) Translat. Acceler
%  25  Accelerom # 5, (along Y), (ft/sec^2) Translat. Acceler
%  26  Accelerom # 6, (along Z), (ft/sec^2) Translat. Acceler
%  27  SGCMG # 1 Gimbal Rate (rad/sec)
%  28  SGCMG # 2 Gimbal Rate (rad/sec)
%  29  SGCMG # 3 Gimbal Rate (rad/sec)
%  30  SGCMG # 4 Gimbal Rate (rad/sec)
%  31  SGCMG # 1 Gimbal Angle (radian)
%  32  SGCMG # 2 Gimbal Angle (radian)
%  33  SGCMG # 3 Gimbal Angle (radian)
%  34  SGCMG # 4 Gimbal Angle (radian)
%  35  SGCMG Array Momentum in X-axis (ft-lb-sec)
%  36  SGCMG Array Momentum in Y-axis (ft-lb-sec)
%  37  SGCMG Array Momentum in Z-axis (ft-lb-sec)
```

```
% Variations = 61
%  38  I_xx   :   0.999 % Variation
%  39  I_yy   :   0.997 % Variation
%  40  I_zz   :   0.966 % Variation
%  41  I_xz   :   0.994 % Variation
%  42  I_xz   :   0.994 % Variation
%  43  I_yz   :   0.997 % Variation
%  44  I_yz   :   0.997 % Variation
%  45  I_xy   :   0.999 % Variation
%  46  I_xy   :   0.999 % Variation
%  47  Jgimbl- 1:  10.000 % Variation
%  48  H0omg - 1:   3.333 % Variation
%  49  H0omg - 1:   3.333 % Variation
%  50  Delta0- 1: 0.8727E-02 Addit. Variat.
%  51  Delta0- 1: 0.8727E-02 Addit. Variat.
%  52  Beta  - 1:   0.735 % Variation
%  53  Beta  - 1:   0.735 % Variation
%  54  Gamma - 1:   0.556 % Variation
%  55  Gamma - 1:   0.556 % Variation
%  56  HDirect 1:  14.003 % Variation
%  57  HDirect 1:  14.003 % Variation
%  58  Jgimbl- 2:  10.000 % Variation
%  59  H0omg - 2:   3.333 % Variation
%  60  H0omg - 2:   3.333 % Variation
%  61  Delta0- 2: 0.8727E-02 Addit. Variat.
%  62  Delta0- 2: 0.8727E-02 Addit. Variat.
%  63  Beta  - 2:   0.735 % Variation
%  64  Beta  - 2:   0.735 % Variation
%  65  Gamma - 2:   0.278 % Variation
%  66  Gamma - 2:   0.278 % Variation
%  67  HDirect 2:  14.003 % Variation
%  68  HDirect 2:  14.003 % Variation
%  69  Jgimbl- 3:  10.000 % Variation
%  70  H0omg - 3:   3.333 % Variation
%  71  H0omg - 3:   3.333 % Variation
%  72  Delta0- 3: 0.8727E-02 Addit. Variat.
%  73  Delta0- 3: 0.8727E-02 Addit. Variat.
%  74  Beta  - 3:   0.735 % Variation
%  75  Beta  - 3:   0.735 % Variation
%  76  Gamma - 3:   0.185 % Variation
%  77  Gamma - 3:   0.185 % Variation
%  78  HDirect 3:  14.003 % Variation
%  79  HDirect 3:  14.003 % Variation
%  80  Jgimbl- 4:  10.000 % Variation
%  81  H0omg - 4:   3.333 % Variation
%  82  H0omg - 4:   3.333 % Variation
%  83  Delta0- 4: 0.8727E-02 Addit. Variat.
%  84  Delta0- 4: 0.8727E-02 Addit. Variat.
%  85  Beta  - 4:   0.735 % Variation
%  86  Beta  - 4:   0.735 % Variation
%  87  Gamma - 4: 0.8727E-02 Addit. Variat.
%  88  Gamma - 4: 0.8727E-02 Addit. Variat.
%  89  HDirect 4:  14.003 % Variation
%  90  HDirect 4:  14.003 % Variation
%  91  W_flex 1: 0.4000  Addit. Variat.
%  92  W_flex 2: 0.7000  Addit. Variat.
%  93  W_flex 3: 1.200   Addit. Variat.
%  94  W_flex 4: 3.000   Addit. Variat.
%  95  W_flex 5: 3.600   Addit. Variat.
%  96  W_flex 6: 4.000   Addit. Variat.
%  97  W_flex 7: 4.500   Addit. Variat.
%  98  W_flex 8: 4.600   Addit. Variat.
```

Figure 8.7 Spacecraft System from "*sc_4cmg_flex_unc.m*" that includes the uncertainty inputs and outputs.
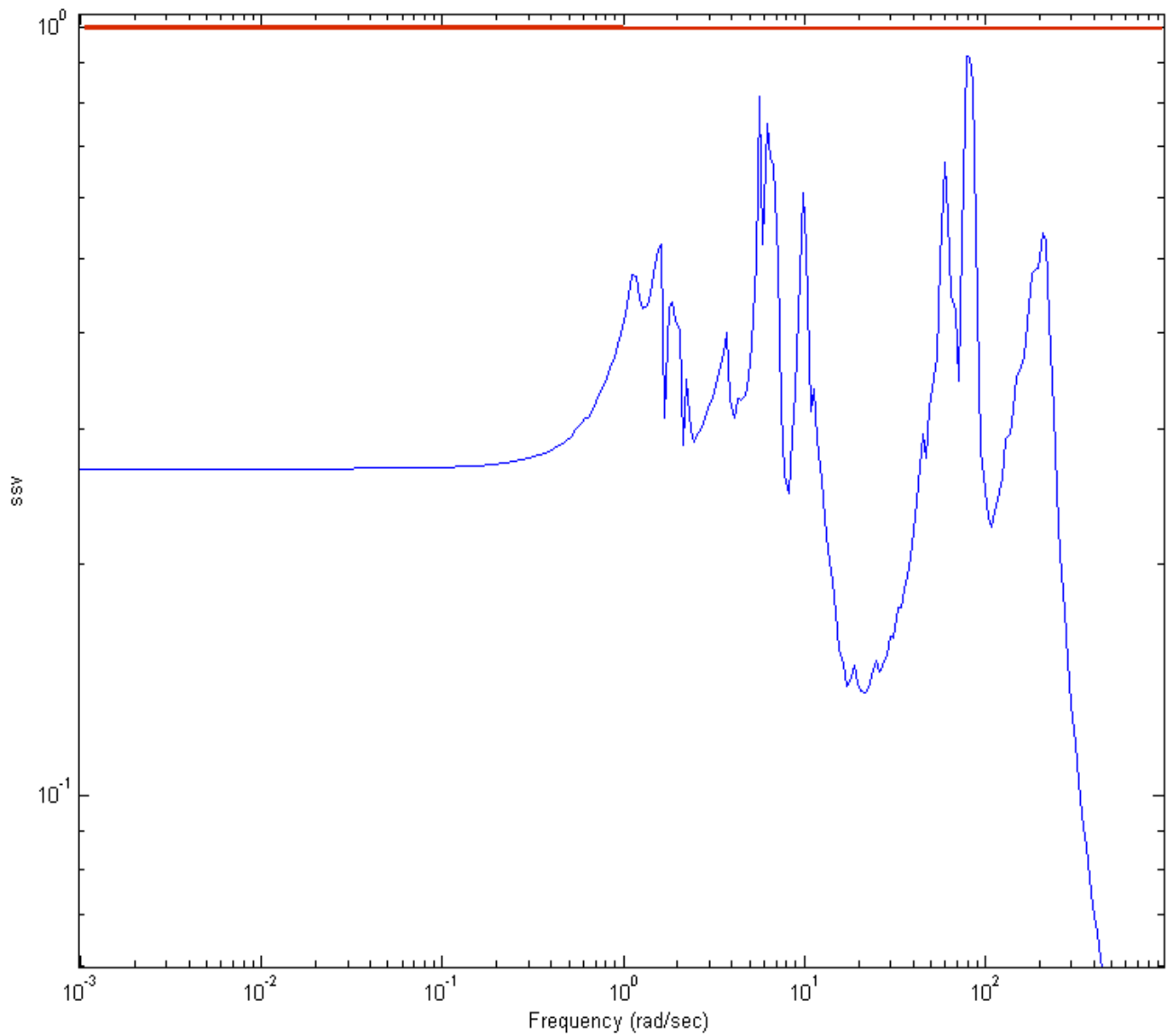
Figure 8.8 Structured Singular Value frequency response across the perturbations block Δ is less than one at all frequencies. The system, therefore, is robust to the parameter variations.