Frequency Response Analysis Program

The frequency response analysis program calculates the frequency response of a system that is described in state-space form by a set of four matrices (A,B,C,D), either continuous or discrete at fixed sampling rate. It can display the frequency responses in Bode, Nichols and Nyquist plots. The systems are read from a typical systems file (*.Qdr) containing multiple systems, and the frequency responses are saved in a file with extension (*.Frq). The program includes many options. One of the options employs a variable frequency step feature for calculating smooth Nichol's and Nyquist plots which is useful in analyzing stability of systems that have very low damped resonances such as structural or slosh modes. It can also overlay multiple curves on the same plot for comparison.

The options are selected graphically using the mouse and menus. Multiple frequency responses can be calculated and saved in the same (.Frq) file for various systems and from different inputs and outputs. The user may point the cursor and read data at specific points on the locus, focus in a smaller area to view details which are otherwise not visible in a larger scale, or expand in a larger area. The program is also used to analyze the existence of limit-cycles in non-linear systems by overlaying the inverse of the describing function of the non-linearity on a Nichols or Nyquist plot.

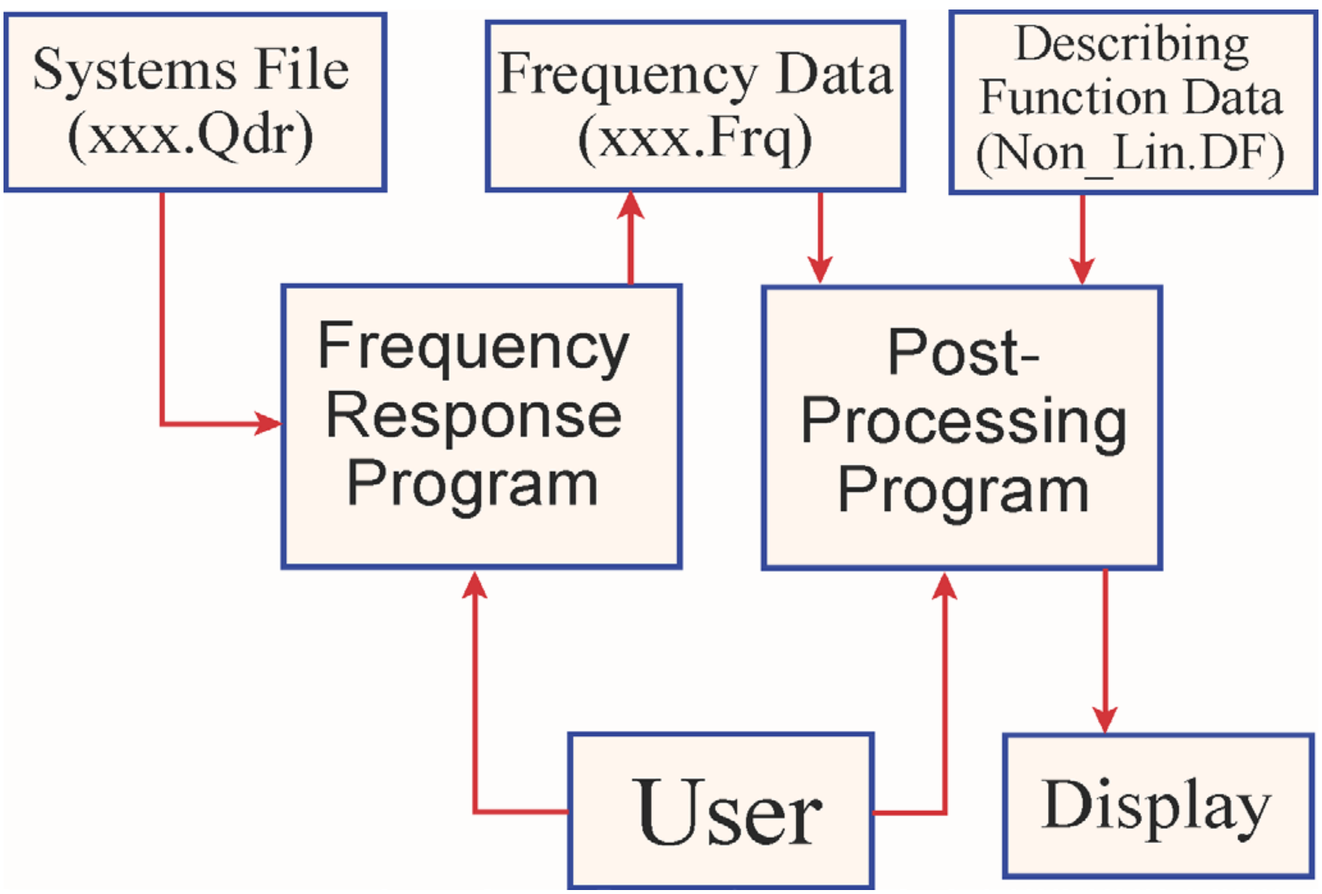


Figure 1 Frequency Response Analysis Program

Program Overview

The frequency response program consists of two parts: the frequency response calculation program, and the graphical analysis program that post-processes the frequency response data and generates different types of plots on the screen.

Frequency Response Calculation: The frequency response calculation program reads the state-space matrices from the systems file (xxx.Qdr), where the filename “xxx” typically describes the flight vehicle. It calculates the frequency responses of more than one system between specific inputs and outputs, and it saves the frequency response data in file (xxx.Frq). The input system is either continuous system (s-plane) described by a set of four state-space matrices or a discrete system (z-plane) described by four state-difference matrices. In addition to the quadruple matrices, the program also reads the sampling period (δt), and the system title. If the system is continuous (δt=0). Some of the program options are initialized from a settings table that includes some default values. For example, two algorithms are available for calculating the system’s frequency response. The user may adjust the initialization settings before proceeding with the frequency response calculation. Other parameters to adjust are: the frequency range, the number of points etc. There is also a variable frequency step (VFS) option for generating smooth Nichol's and Nyquist plots. This option is important for measuring phase and gain margins in systems that have low damped resonances, such as structural or slosh modes. The program saves the frequency response data in file xxx.Frq and calls the graphics processor to plot the frequency data on the screen.

Graphics Post-Processor: The graphics processing program reads the frequency response data from one or two (.Frq) files and plots it on the screen in three different forms. From the options menu, the user may choose to display the data in Bode, Nichol's, or Nyquist plots. The post-processing program either plots one curve from one file, or it can overlay two frequency response curves on the same plot from two separate frequency response files. The first curve from the first file, x1.frq, appears in blue. The second overlay curve from a second file x2.frq appears in red. The two sets of data from the two files must be compatible with each other for overlay in terms of frequency range. The graphics post-processor allows the user to display the frequency response data in the user desired form. The user may point the cursor on the locus and read the gain, phase, and frequency at that point. With the mouse you may also focus in a smaller area of the plot and magnify that area in order to observe more details there.

The graphics processor is also used to analyze stability of non-linear systems and evaluate the existence and size of limit-cycles by using the Describing Function (DF) method. The system’s frequency response is calculated across the non-linearity and the inverse of the DF is plotted in the same Nichols or Nyquist diagram. The user must provide the DF of the non-linearity in a separate file that has an extension (.DF). This file contains: the gain and phase of the fundamental non-linearity output calculated at different sinusoidal input amplitudes. The DF is either obtained analytically or experimentally using Simulink models. See examples for details.

Program Files

The frequency response analysis program uses three different types of files:

Standard Flixan systems file (.Qdr): This file contains the state-space systems to be analyzed. The systems are either continuous or discrete. The program only reads data from this file. The systems are generated either from a modelling program, transfer functions interconnections, or by the systems interconnection program. The systems file is associated with a certain vehicle analysis and it may contain several systems to be analyzed. The user must select one system at a time to be analyzed and specify the input/ output path. When the program completes the frequency response calculation of a system, another input/ output path or another system can be selected for analysis from the same file, and the process is repeated.

Frequency Response File (.Frq): The program generates a frequency response data file that has an extension (.Frq). This file is also used as input to the graphics post-processor. The first part of the filename is identical to the systems file. Only the extension is different. The graphics program can accept one or two frequency data files, but when two files are used the data must be compatible for overlay (same frequency range and number of points). Each frequency file may contain more than one set of frequency response data that are generated in a single process from one systems file (.Qdr).

The first line of each frequency data-set includes the number of the system’s inputs and outputs, and also some parameters which are passed to the post-processing program. The second line above the frequency data includes the system title, which is the same as the title that appears in the systems file above the system quadruples. The input and output numbers across which the system's frequency response was calculated, including short input/ output definitions are also included below the title, and the number of frequency points.

The frequency response file consists of five columns of data. The first column contains the frequencies in (rad/sec), the second and third columns consist of the real and the imaginary parts of the transfer function, the third column includes the Gain in (dB), and the fifth column is the system's phase in (degrees). The first three characters on the left side of the frequency file contain reference numbers used by the graphics post-processor to locate and read the appropriate data, and also to navigate forwards and backwards in the frequency data file.

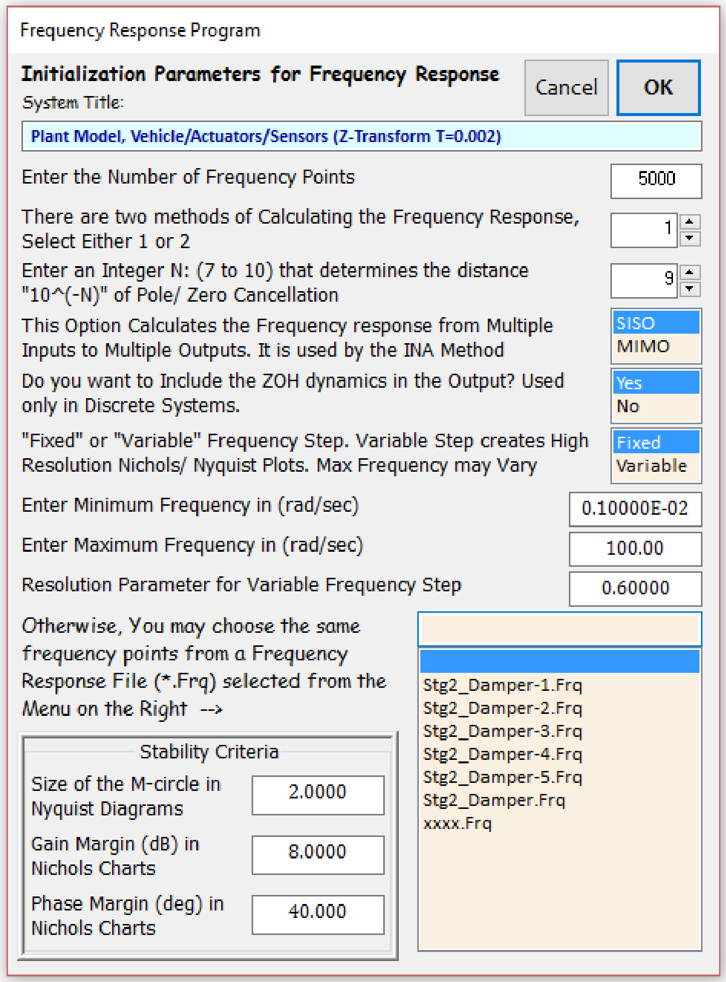
```
1  Number of Inputs= 6    Outputs= 14      M-Circle, Gain and Phase Margins:  2.00    8.00    40.0
   Plant Model, Vehicle/Actuators/Sensors (Z-Transform T=0.002)
2  Frequency Response for the following Transfer Function path
   Output(  2)-Roll Rate to FCS          / Input(  1)-Roll FCS Command (DP-TVC)  ; Decades= 5
5
    NPT= 8000 OMEGA         X             Y          GAIN(DB)    PHASE(DEGR)
      1  0.100000E-02 -0.701176E+00 -0.222287E+01  0.735022E+01 -0.107507E+03
      2  0.100340E-02 -0.705218E+00 -0.222950E+01  0.737828E+01 -0.107553E+03
      3  0.101358E-02 -0.717412E+00 -0.224932E+01  0.746175E+01 -0.107690E+03
```

The Describing Function File (.DF): This file is only included when analyzing stability of non-linear systems using the DF method. It contains the title of the non-linearity followed by the DF data. That is: in the first column we have the amplitude of the sinusoidal that is applied to the non-linearity, and in the second and third columns we have the gain and phase of the fundamental frequency coming out of the non-linearity for the corresponding input amplitude. The DF is either obtained analytically or experimentally using Simulink models.

```
User defined Non-Linearity
Amplitude  Gain       Phase (deg)
0.1000     0.5000      0.0000
0.1250     0.5300      0.0000
0.1750     0.6000      0.0000
0.2000     0.6300      0.0000
0.2250     0.6700      0.0000
0.2750     0.7500     -0.5000
0.3000     0.8000     -1.0000
0.3200     0.7800     -2.0000
```

Program Options

After selecting the systems file and then one of the systems to analyze, the program requires some options to be selected by the user and the defaults are shown in the following menu.



1. The number of points to be included in the frequency response calculation, it should be less than 20,000 points.
2. The program provides two methods for calculating the system's frequency response. Method #1 is significantly faster than method #2. We recommend using the default method #1 because it is faster. Use method # 2 as a backup.
3. The next option defines the accuracy of the frequency response calculations that you want to obtain. The program computes the poles and zeros of the system between a specified input and a specific output. When the poles and zeros are very close together, their influence on the frequency response is negligible and canceling them out simplifies the calculations. The amount of cancellation between poles and zeros depends on a number (n) that must be entered. This number must be in the range 7-10, depending on the amount of cancellation desired. The number (n) defines the cancellation distance d between poles and zeros, ($d=10^{-n}$). If there is a pole/ zero pair that are closer together than d, they will cancel out

and they will not affect the calculations. If the transfer gain between a system's input and output is very small, we recommend that you increase the value of n to 12 or 14. However, if the system to be analyzed has a very low gain or very high gain, i.e. the magnitudes of the elements of matrices B,C,D are very small (in the order of 10^{-9} or less), or very high (10^{+9} or more), we recommend that you scale this system up or down accordingly before analyzing it. This is a good practice in almost every application.

4. The next option is used for calculating a multivariable (MIMO) frequency response. That is, from multiple inputs to multiple outputs. The default selection is SISO, that is, one input/ output pair at a time. The only time that you would choose the MIMO option is when you want to allow the program to automatically compute frequency responses between multiple inputs and multiple outputs in one batch, that is, from every input to every output. This is useful when performing multivariable frequency response analysis such as "Inverse Nyquist Arrays". When you select the multivariable option the program will ask you to select the input numbers and the output numbers of the system that you want to compute the MIMO frequency response. Note that in order to apply the INA method the number of inputs must be equal to the number of outputs, so the program assumes that they are equal.
5. The next initialization option is applies only in discrete-time systems and it has to do with the option of including or not the dynamic effect of the zero-order-hold in the frequency response computations. The zero-order-hold will introduce some additional phase delay and attenuation especially at frequencies near the Nyquist frequency. The default zero-order-hold value is "include" and it must be included when performing open-loop calculations for stability analysis.
6. The user must also specify the frequency range by entering the initial and final frequencies.
7. The next option is for defining the frequency step size in the frequency response computation. When the selection is "*Fixed*", which is the default, the frequency step will be defined from the number of points and the specified frequency range. If the selection is "*Variable*", a variable frequency step (VFS) is selected. This option is useful for generating smooth and nice looking Nichols and Nyquist curves by spreading the points evenly to maintain an nearly constant resolution. The VFS option allows the frequency step between computations to vary according to gain and phase variation. When the gain and phase do not change significantly with frequency, such as at very low frequencies, the frequency step is increased. When the gain and phase begin to change more rapidly with frequency, such as at low damped resonances, the frequency step is reduced. The relative step size is adjusted by the resolution parameter that must be entered by the user. The default value is 0.6. The disadvantage of using the VFS option is that it may require several attempts to cover the specified frequency range by adjusting the resolution parameter.
8. There is a third option for defining the frequency points in the frequency calculation and that is to use the same frequency points as those defined from a previously calculated (.Frq) file. This is useful when overlaying frequency data obtained from similar systems and you want them to be in the same frequency range and include the same number of points at the same frequencies, especially when the first file is calculated using the VFS option. In order to activate this option you must select the (.Frq) file from the menu shown in the initialization menu above.

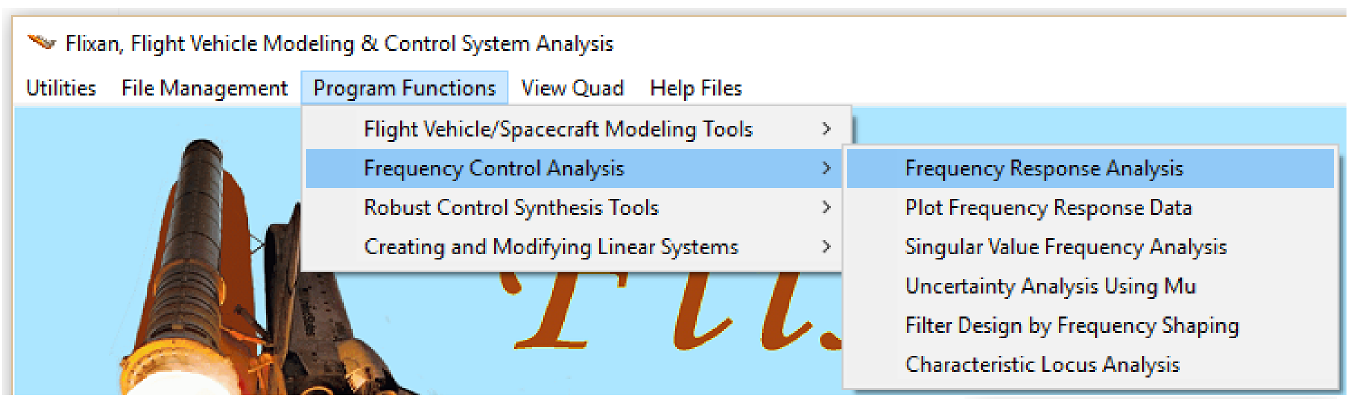
9. Finally, the initialization parameters include the user defined gain and phase margins that are used for plotting the area to avoid rectangle around the critical point. The Nichols locus must avoid this rectangle in order to satisfy the required gain and phase margins. The size of the M-circle is also defined which plays a similar role in the Nyquist diagrams.

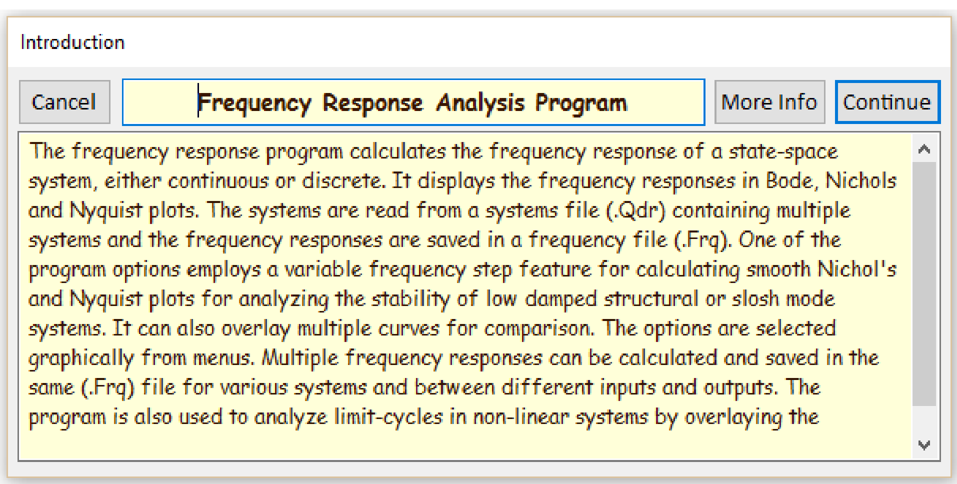
Running the Frequency Response Program

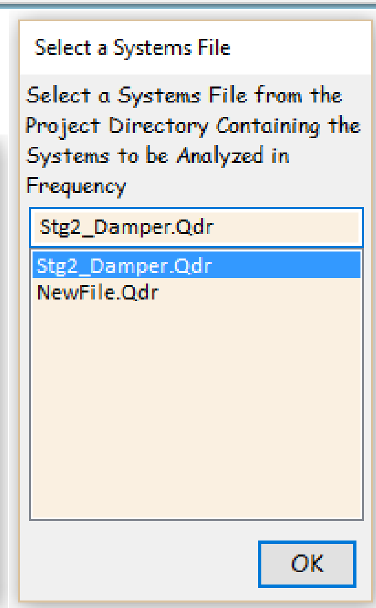
We will demonstrate the frequency response analysis program using two examples. The first example calculates responses of several systems from a systems file, and the second example demonstrates the use of the variable frequency step and overlaying two frequency response files.

Example 1 Frequency Response Calculations from Multiple Systems

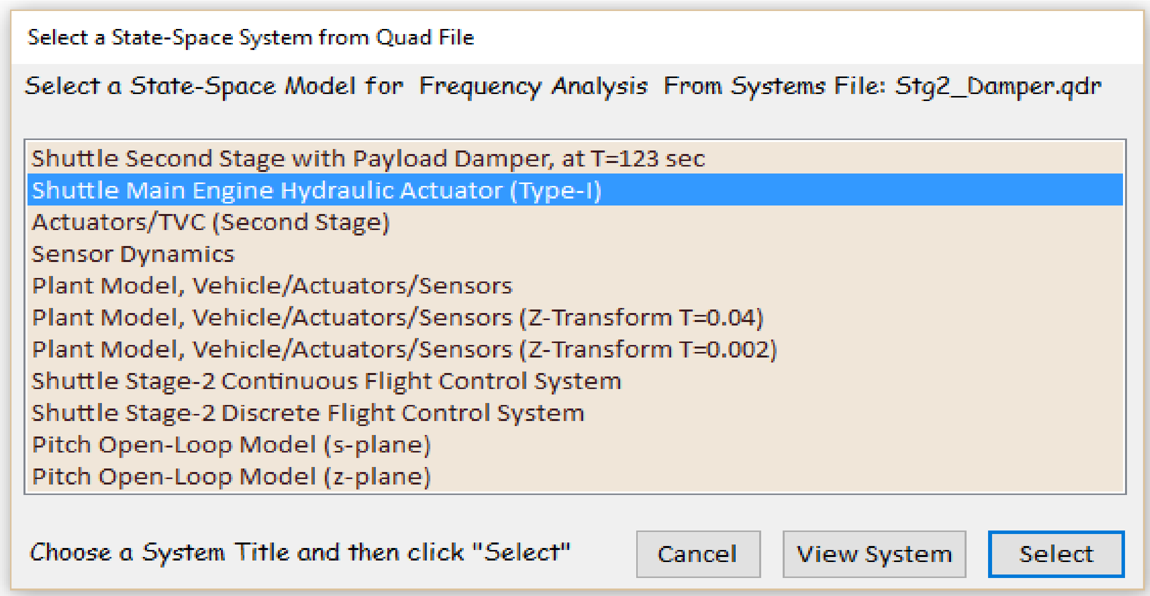
Our first example is in folder "*C:\Flixan\Frequ\Examples\Ex1*". We will calculate and plot frequency responses of systems which are located in systems file "*Stg2_Damper.Qdr*". Start the Flixan program, select the project directory and from the main menu select "*Program Functions*", "*Frequency Control Analysis*", and then "*Frequency Response Analysis*". The following is an introduction dialog and click "*Continue*". Select also the systems file from the next menu and click "OK".

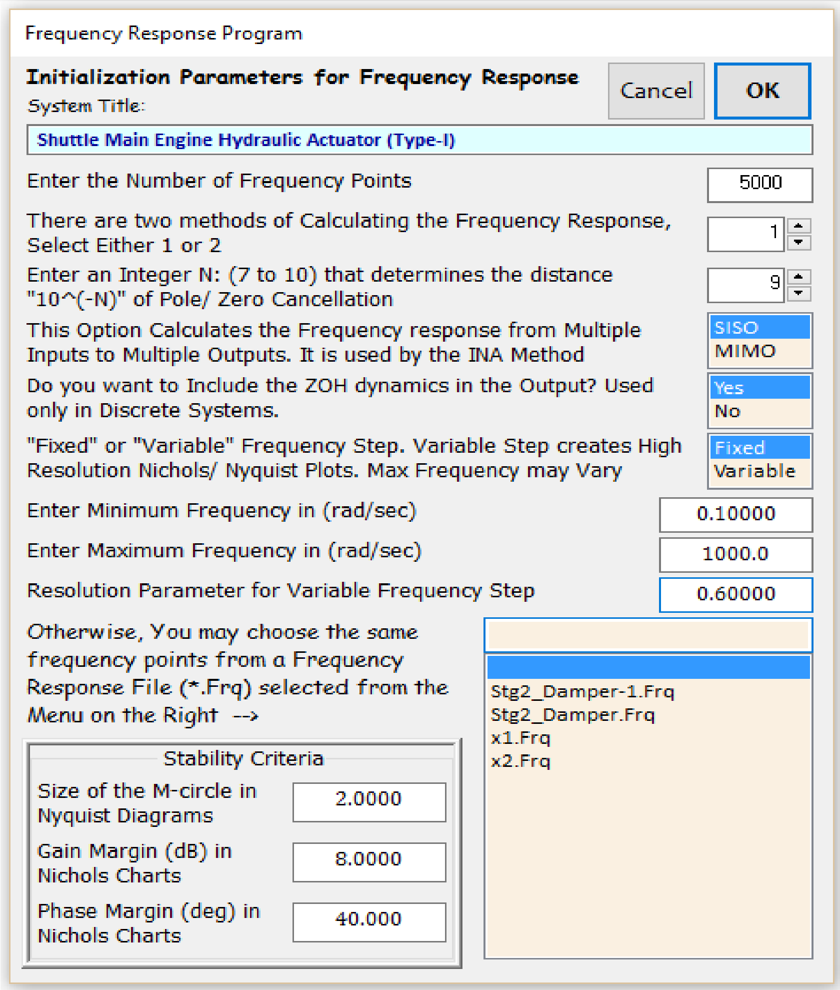




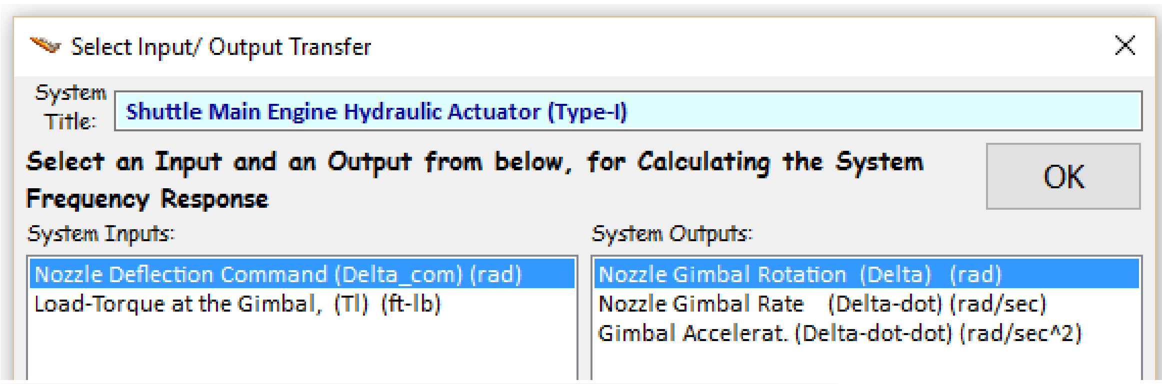


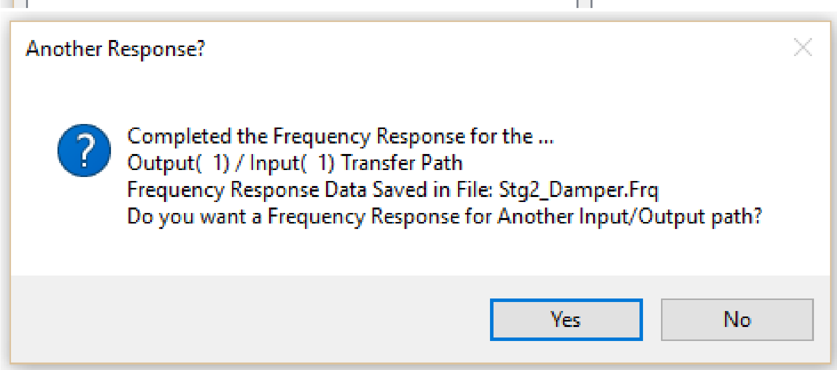
The systems file contains several systems. Using the systems selection menu below, select one of the system titles "*Shuttle Main Engine Hydraulic Actuator (Type-I)*" and click on "*Select*". Use the dialog below to set the initialization parameters for the actuator frequency response calculations, as shown, and click "OK".

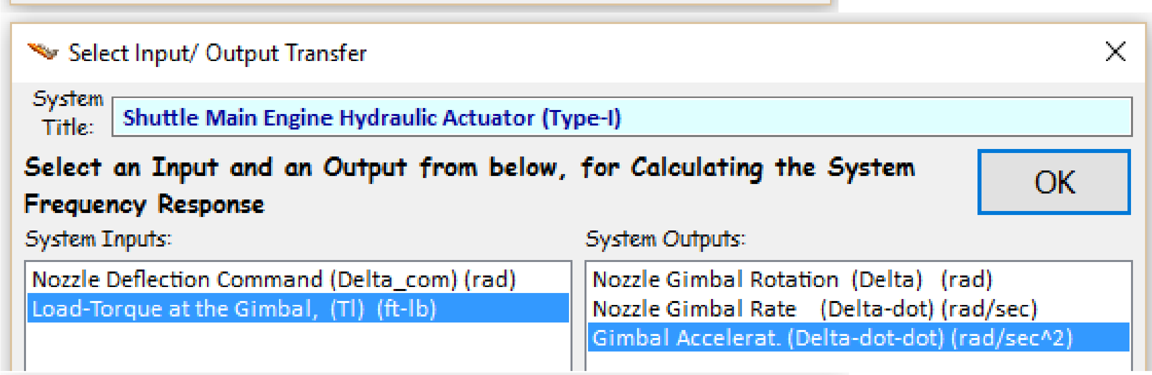


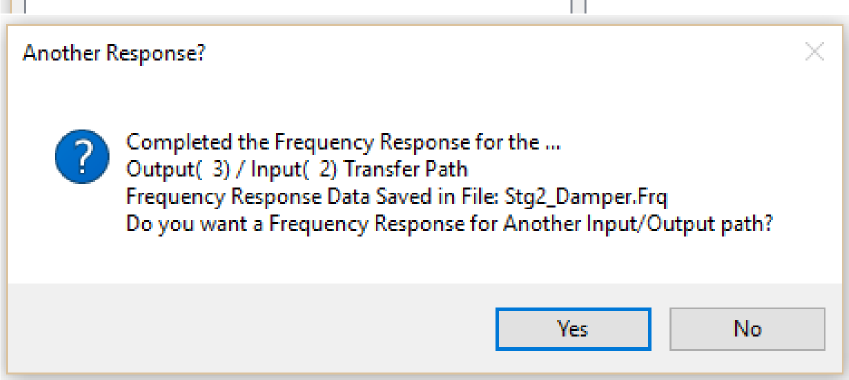


You must now define the actuator system's input and output across which the program will calculate the frequency response. Select first the "*Nozzle Command*" input and the "*Nozzle Deflection*" output, and click "OK". The program calculates the frequency response for the selected input/output pair and asks the user if he wishes to calculate another frequency response for another input/output pair. Click on "Yes" and calculate the frequency response between the "*Load-Torque*" input and the "*Nozzle Gimbal Acceleration*" output, as shown.

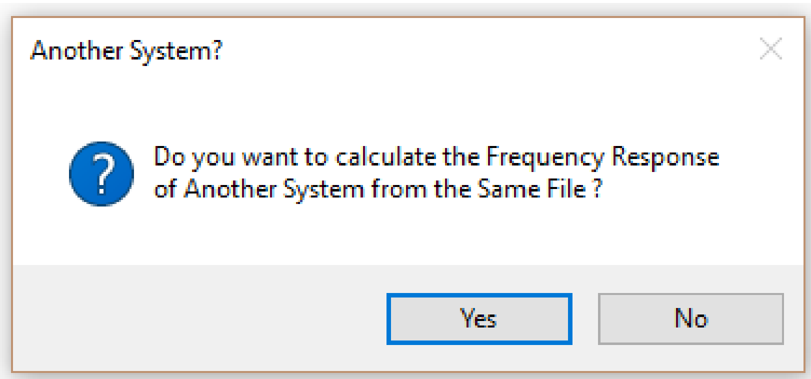


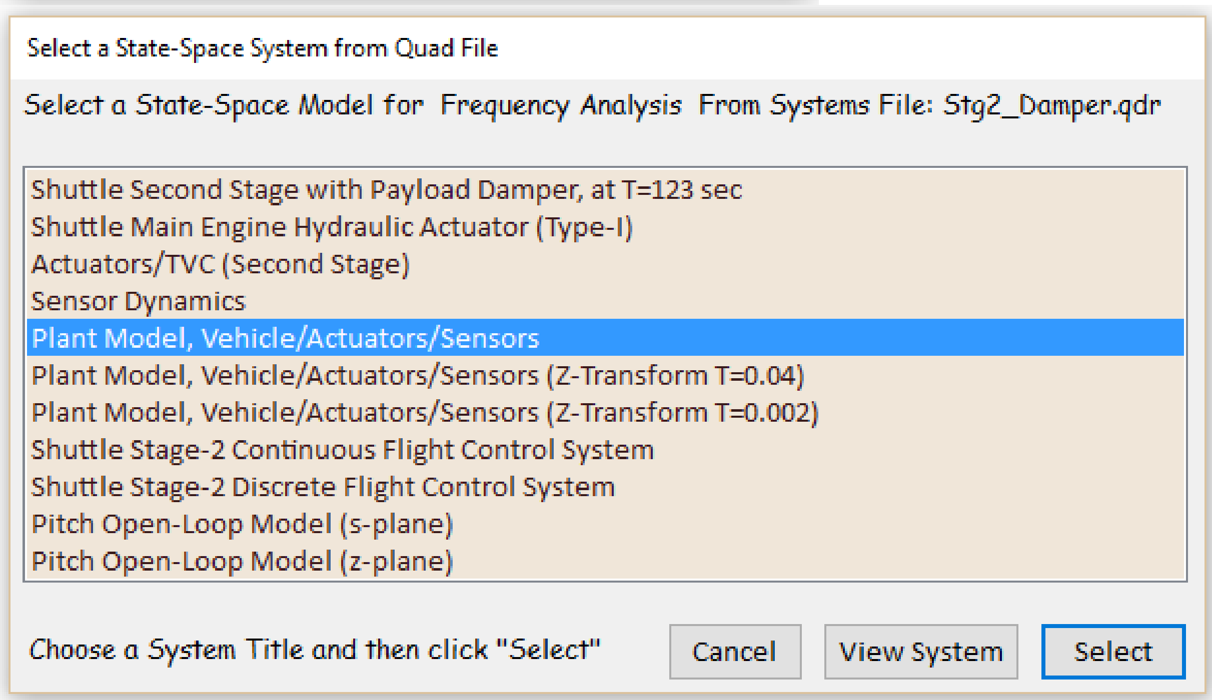




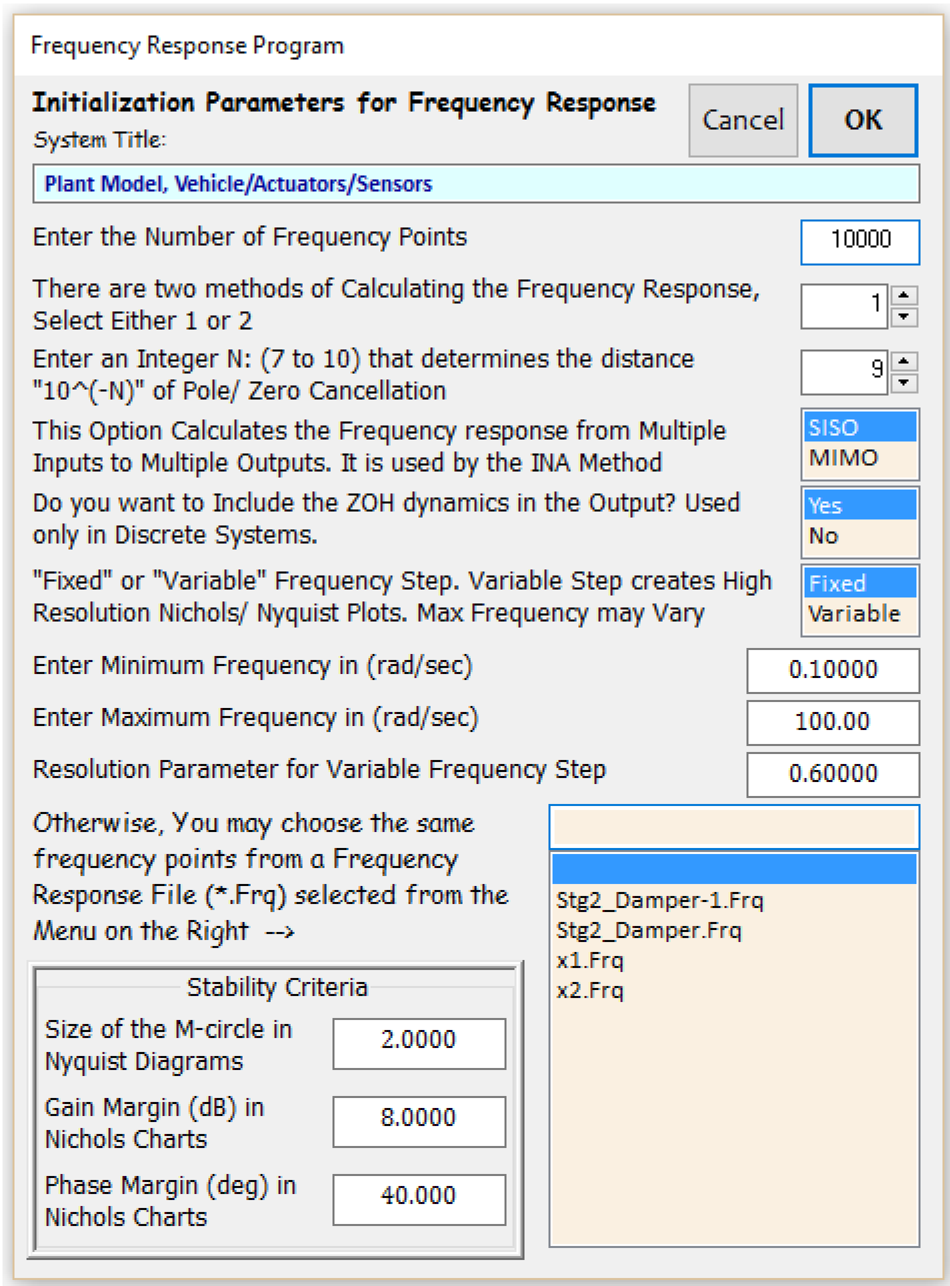


When you finish, answer "No" that you don't wish to calculate another frequency response using this actuator system. You would like, however, (answer "Yes" below) to calculate the frequency response of another system from the same file, and from the following menu select the system "*Plant Model, Vehicle/ Actuators/ Sensors*", and click "*Select*".

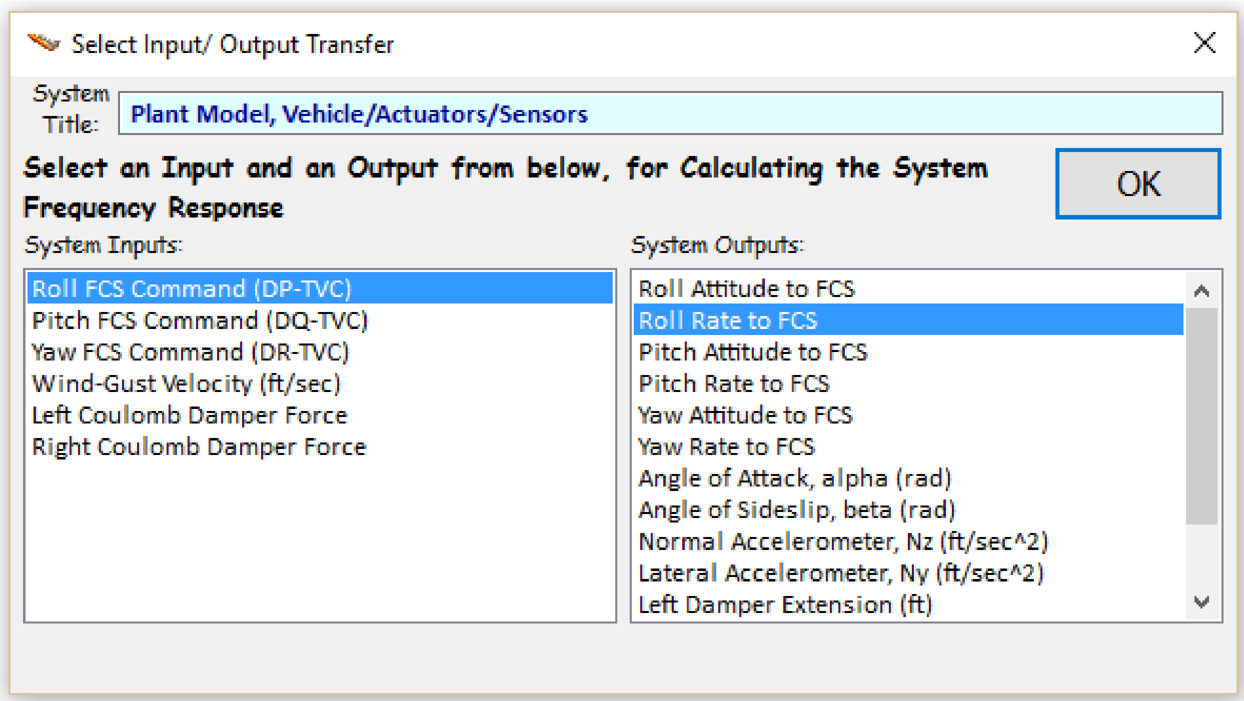


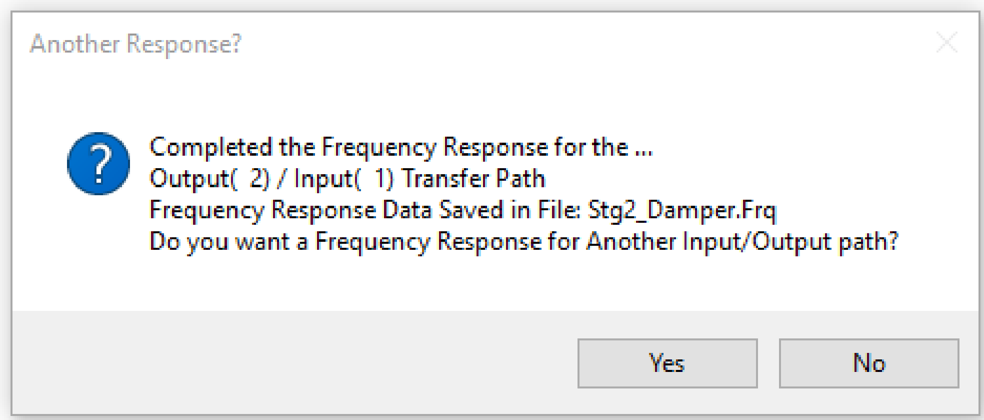


The following dialog is used to set the parameters in the calculation of the frequency response of the "Plant Model" system. We are using 10,000 points because this system has a lot of structural and slosh modes.

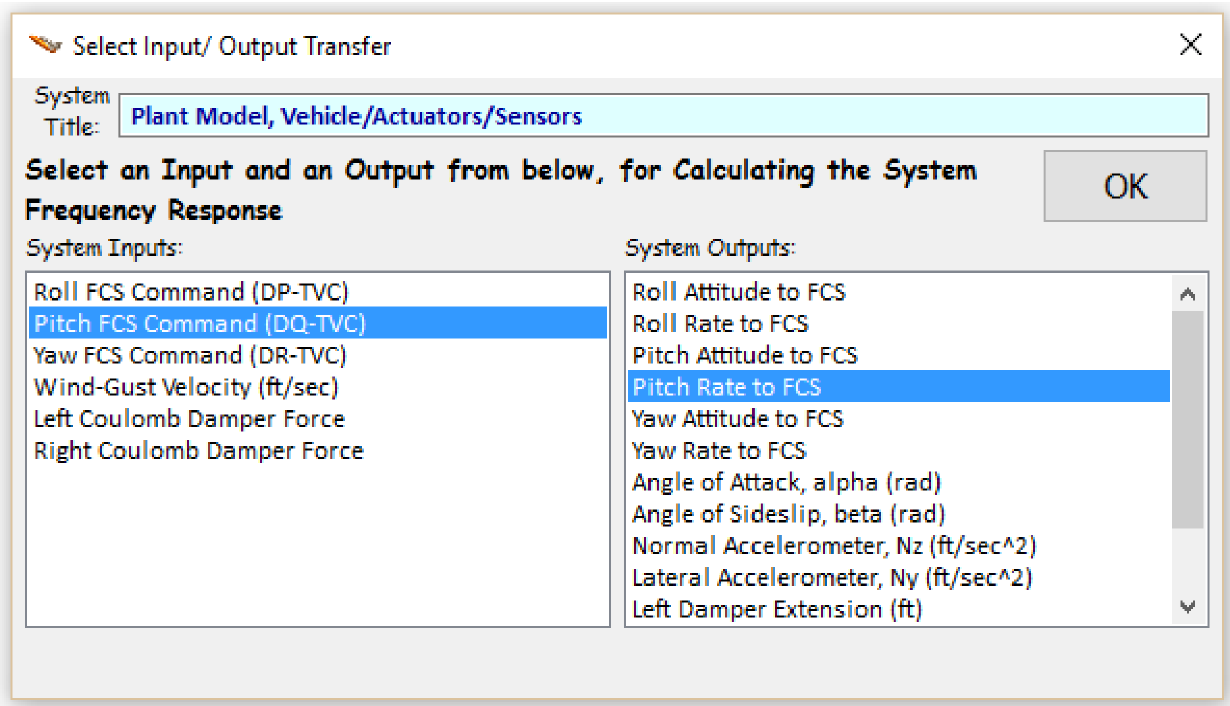


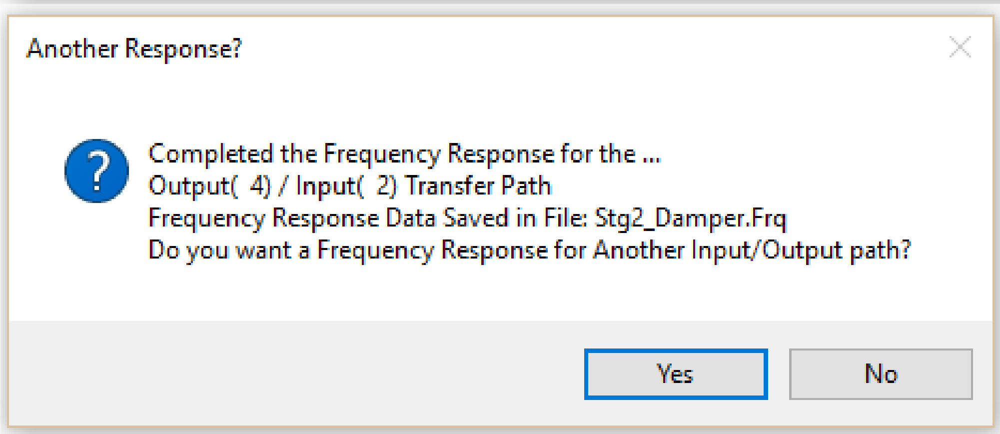
We will use this Plant Model system to calculate three frequency responses for the Roll, Pitch and Yaw axes, as shown below. In the menus below select the input and output for the roll axis. Click on “OK”, and in the next dialog click on “Yes” to select another input/output pair for the pitch axis.



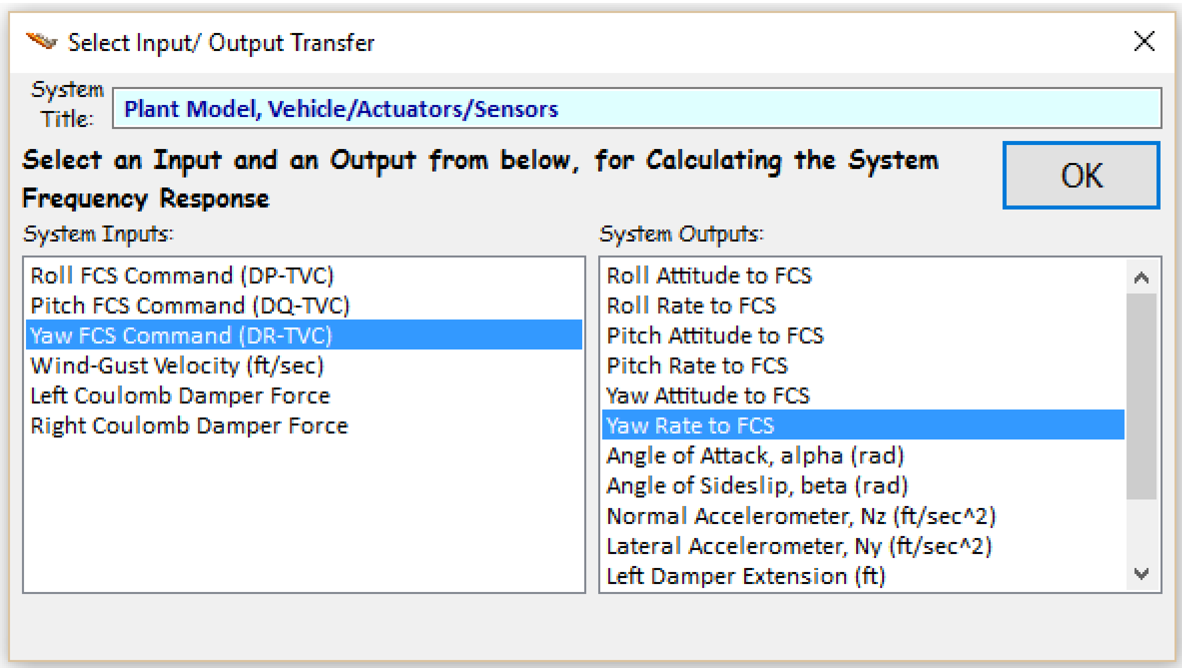


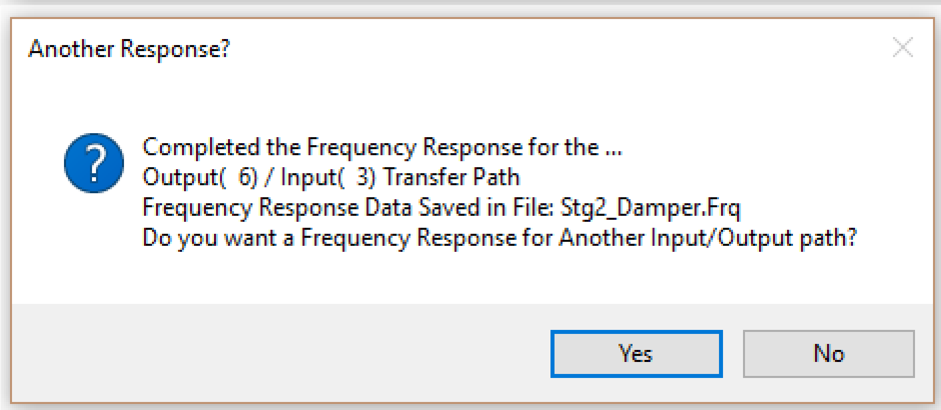
In the next dialog select the input and output for the pitch axis. Click on “OK”, and in the next dialog click on “Yes” to select another input/output pair for the yaw axis.

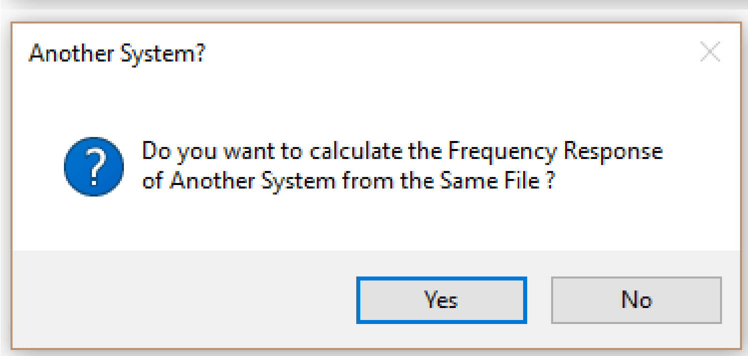


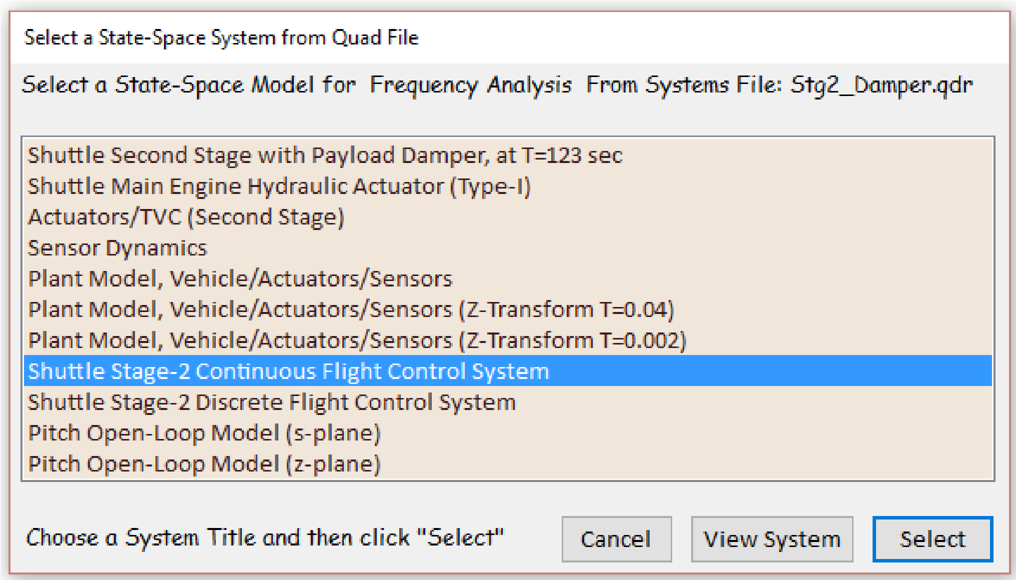


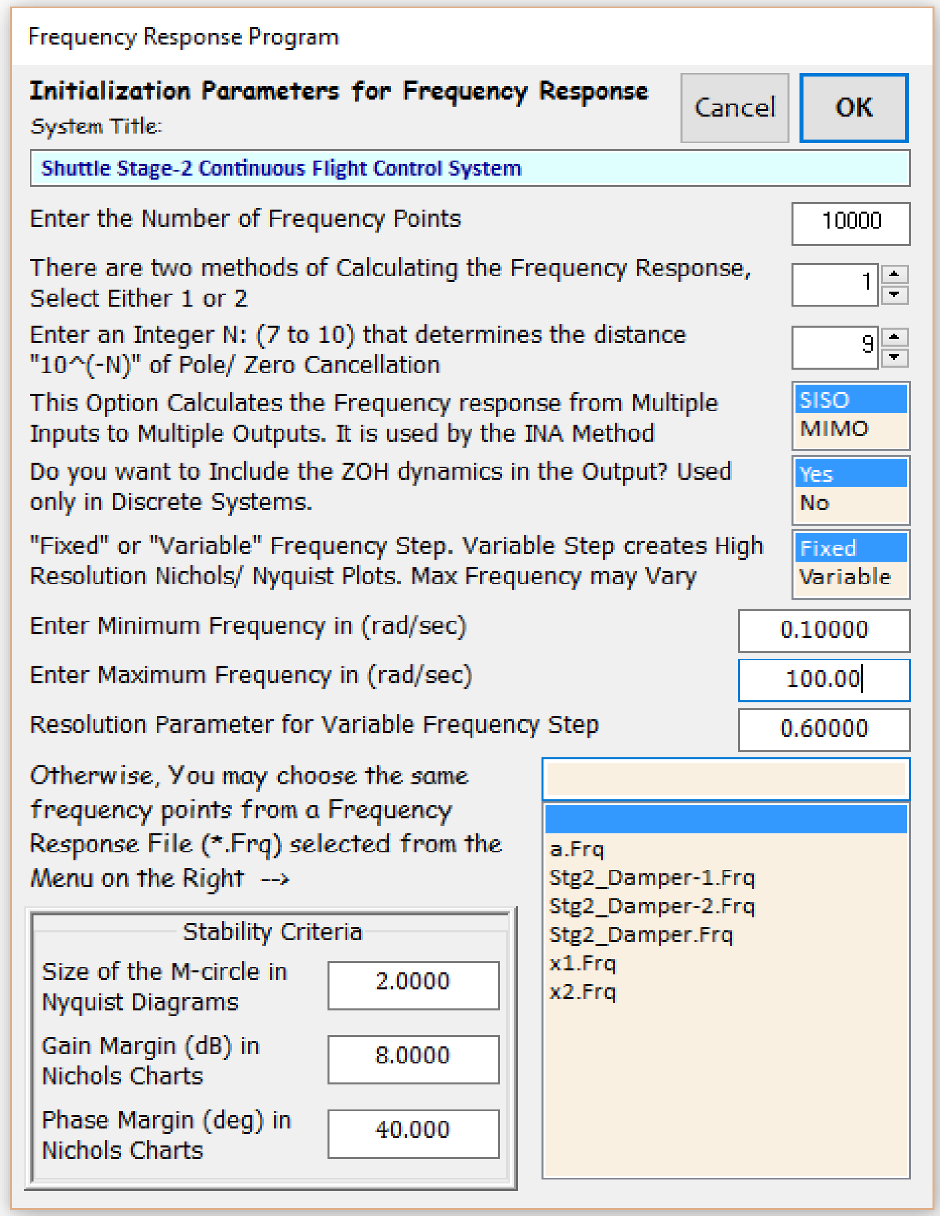
Select an input/output pair for the yaw axis and click on “No” that you don’t want to compute another frequency response using the Plant Model system. In the next dialog answer “Yes” to select another system for frequency response analysis. This time select “Shuttle Stage-2 Continuous Flight Control System” and use the next dialog to set the program parameters as before.



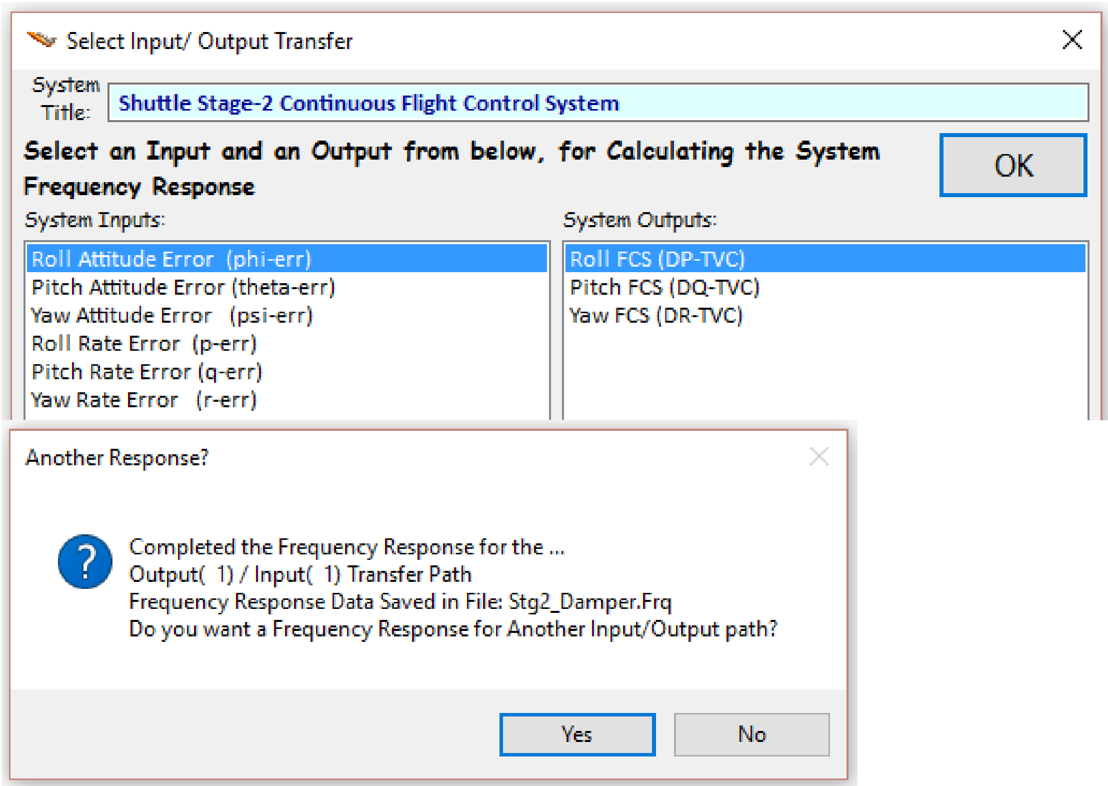


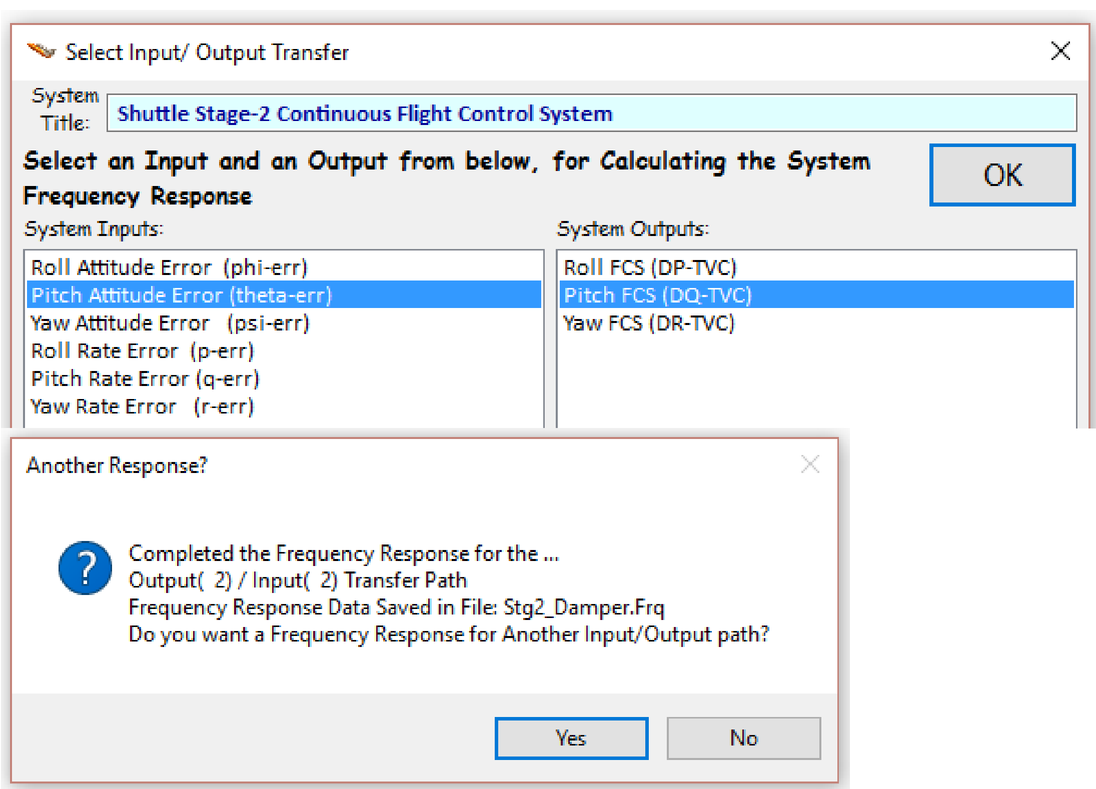


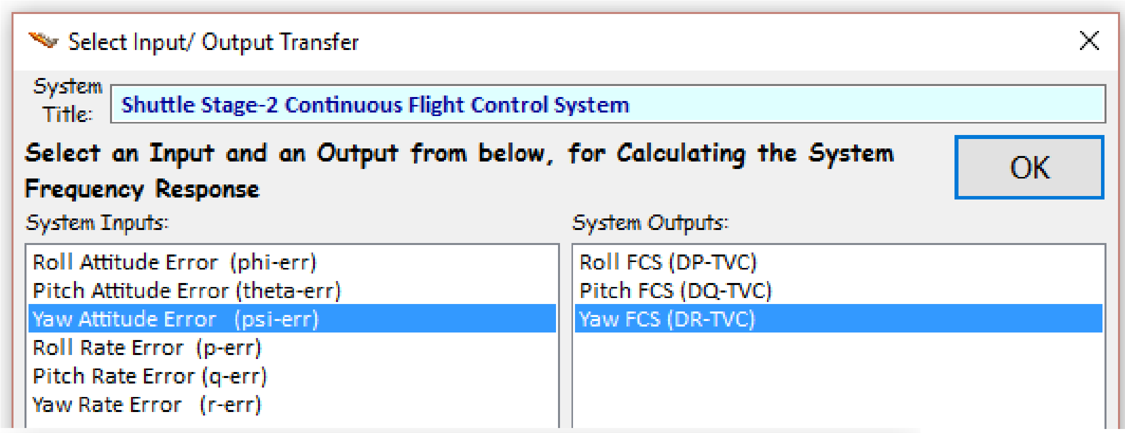

Select a State-Space System from Quad File
Select a State-Space Model for Frequency Analysis From Systems File: Stg2_Damper.qdr
Shuttle Second Stage with Payload Damper, at T=123 sec
Shuttle Main Engine Hydraulic Actuator (Type-I)
Actuators/TVC (Second Stage)
Sensor Dynamics
Plant Model, Vehicle/Actuators/Sensors
Plant Model, Vehicle/Actuators/Sensors (Z-Transform T=0.04)
Plant Model, Vehicle/Actuators/Sensors (Z-Transform T=0.002)
Shuttle Stage-2 Continuous Flight Control System
Shuttle Stage-2 Discrete Flight Control System
Pitch Open-Loop Model (s-plane)
Pitch Open-Loop Model (z-plane)
Choose a System Title and then click "Select"
Cancel
View System
Select

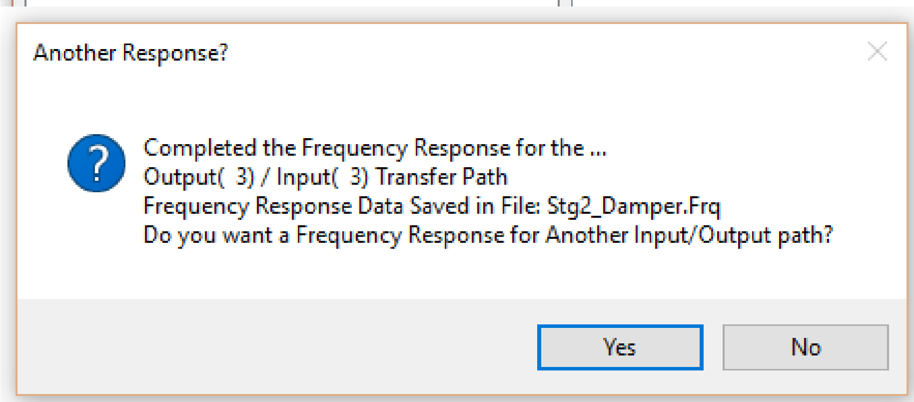

Frequency Response Program
Initialization Parameters for Frequency Response
Cancel
OK
System Title:
Shuttle Stage-2 Continuous Flight Control System
Enter the Number of Frequency Points
10000
There are two methods of Calculating the Frequency Response, Select Either 1 or 2
1
Enter an Integer N: (7 to 10) that determines the distance "10^(-N)" of Pole/ Zero Cancellation
9
This Option Calculates the Frequency response from Multiple Inputs to Multiple Outputs. It is used by the INA Method
SISO
MIMO
Do you want to Include the ZOH dynamics in the Output? Used only in Discrete Systems.
Yes
No
"Fixed" or "Variable" Frequency Step. Variable Step creates High Resolution Nichols/ Nyquist Plots. Max Frequency may Vary
Fixed
Variable
Enter Minimum Frequency in (rad/sec)
0.10000
Enter Maximum Frequency in (rad/sec)
100.00
Resolution Parameter for Variable Frequency Step
0.60000
Otherwise, You may choose the same frequency points from a Frequency Response File (*.Frq) selected from the Menu on the Right -->
a.Frq
Stg2_Damper-1.Frq
Stg2_Damper-2.Frq
Stg2_Damper.Frq
x1.Frq
x2.Frq
Stability Criteria
Size of the M-circle in Nyquist Diagrams
2.0000
Gain Margin (dB) in Nichols Charts
8.0000
Phase Margin (deg) in Nichols Charts
40.000

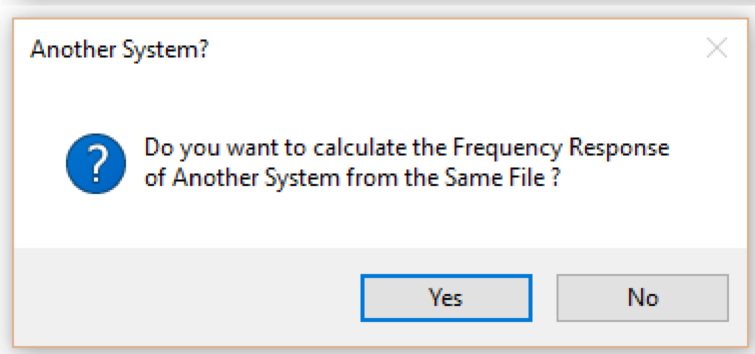
We will now analyze the discrete Shuttle flight control system and calculate the three frequency responses between roll, pitch and yaw attitude errors and the flight control system outputs.





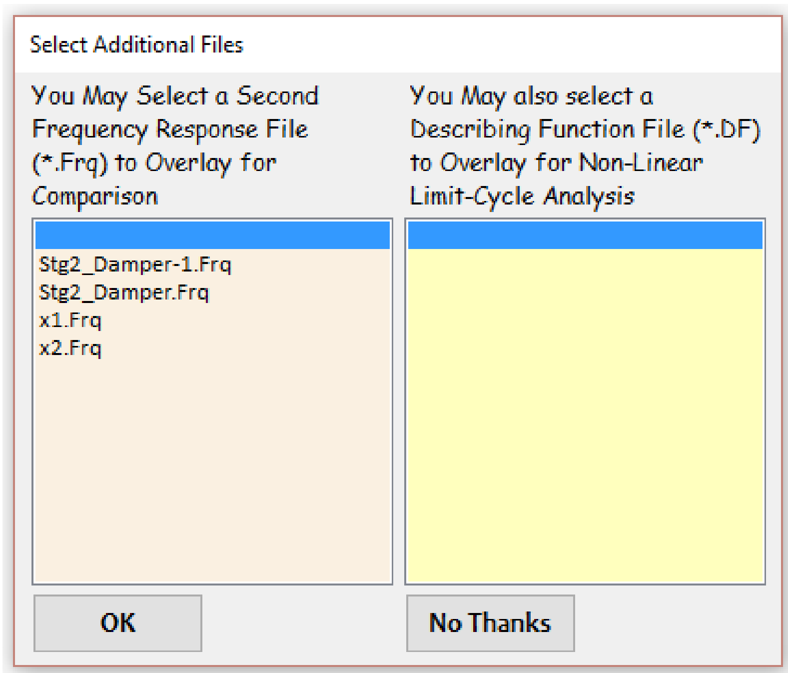




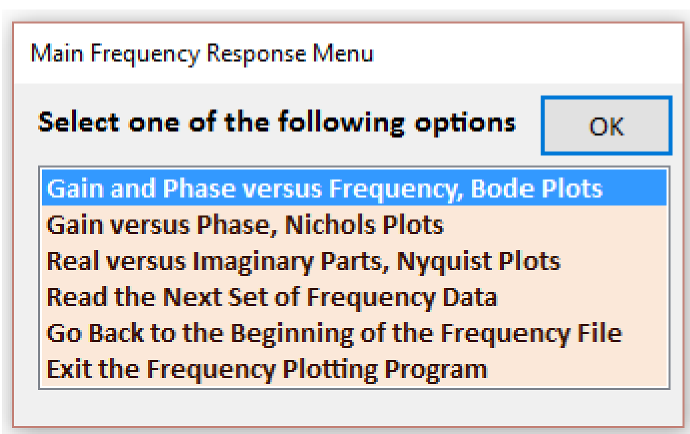


Click on “No” in both of the above dialogs and the program will plot the frequency response data which are saved in file “Stg2_Damper.Frq”.

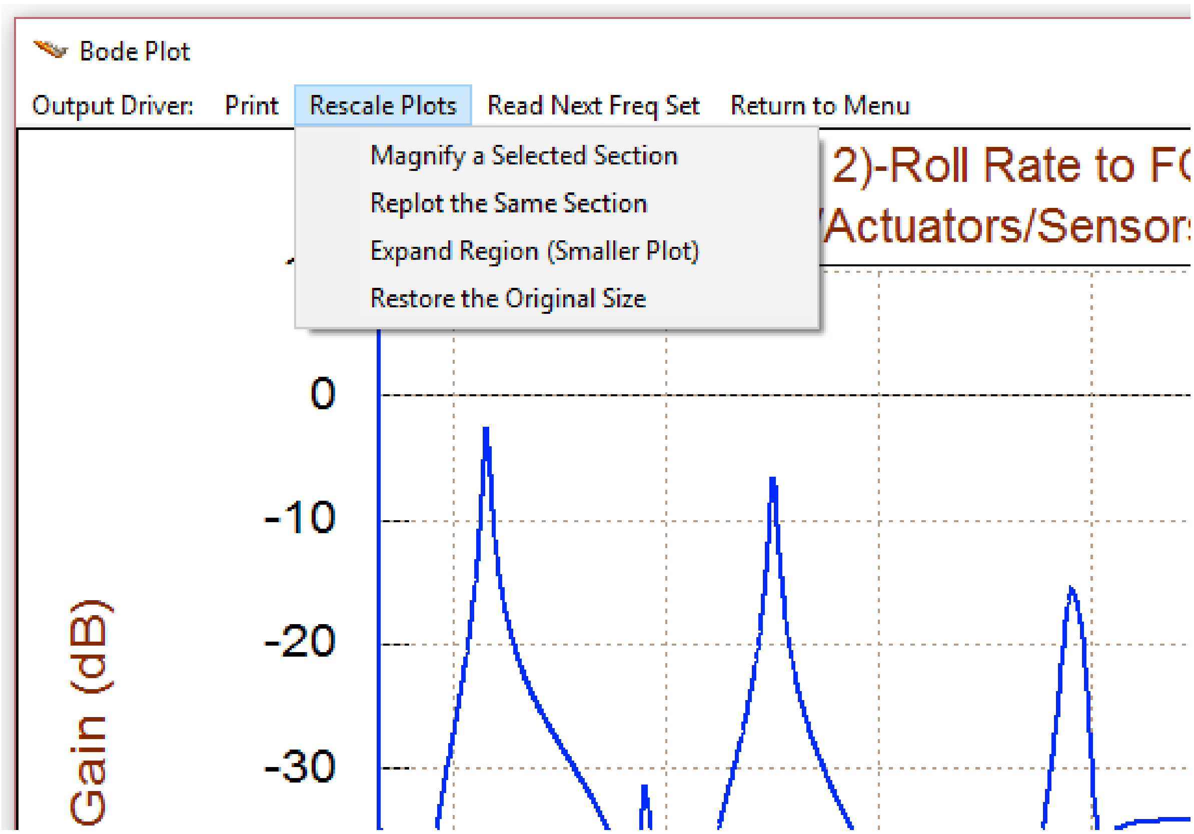
Click “OK” in the menus below without selecting any files because you don’t want to overlay any data from another previously calculated (.Frq) file or from a “Describing Function” file.



The following menu is used for plotting the frequency response data in Bode, Nichols, and Nyquist formats. The frequency response data is read from file “Stg2_Damper.Frq” which contains multiple sets of frequency data from various systems. The user can advance to the next set or go back to previous sets.

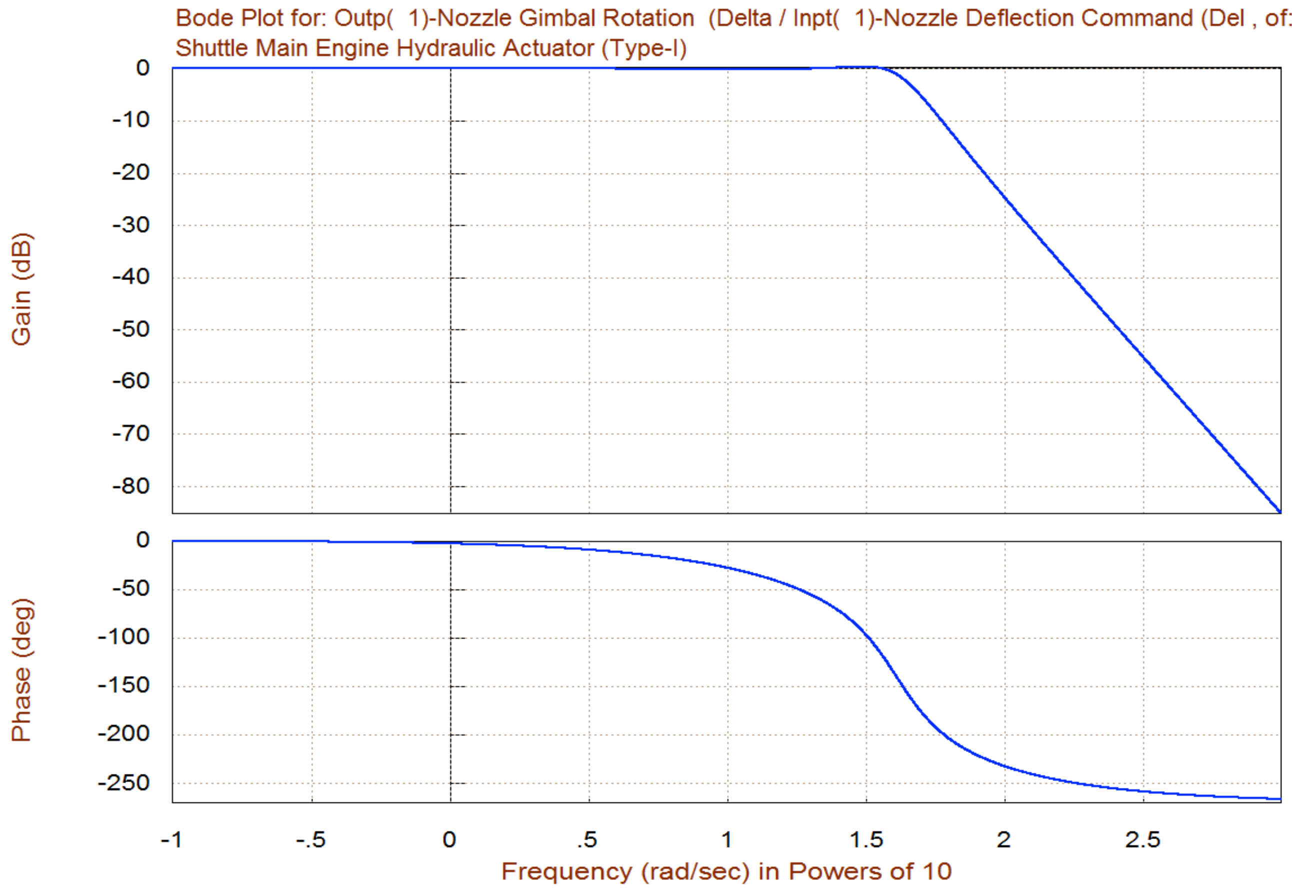


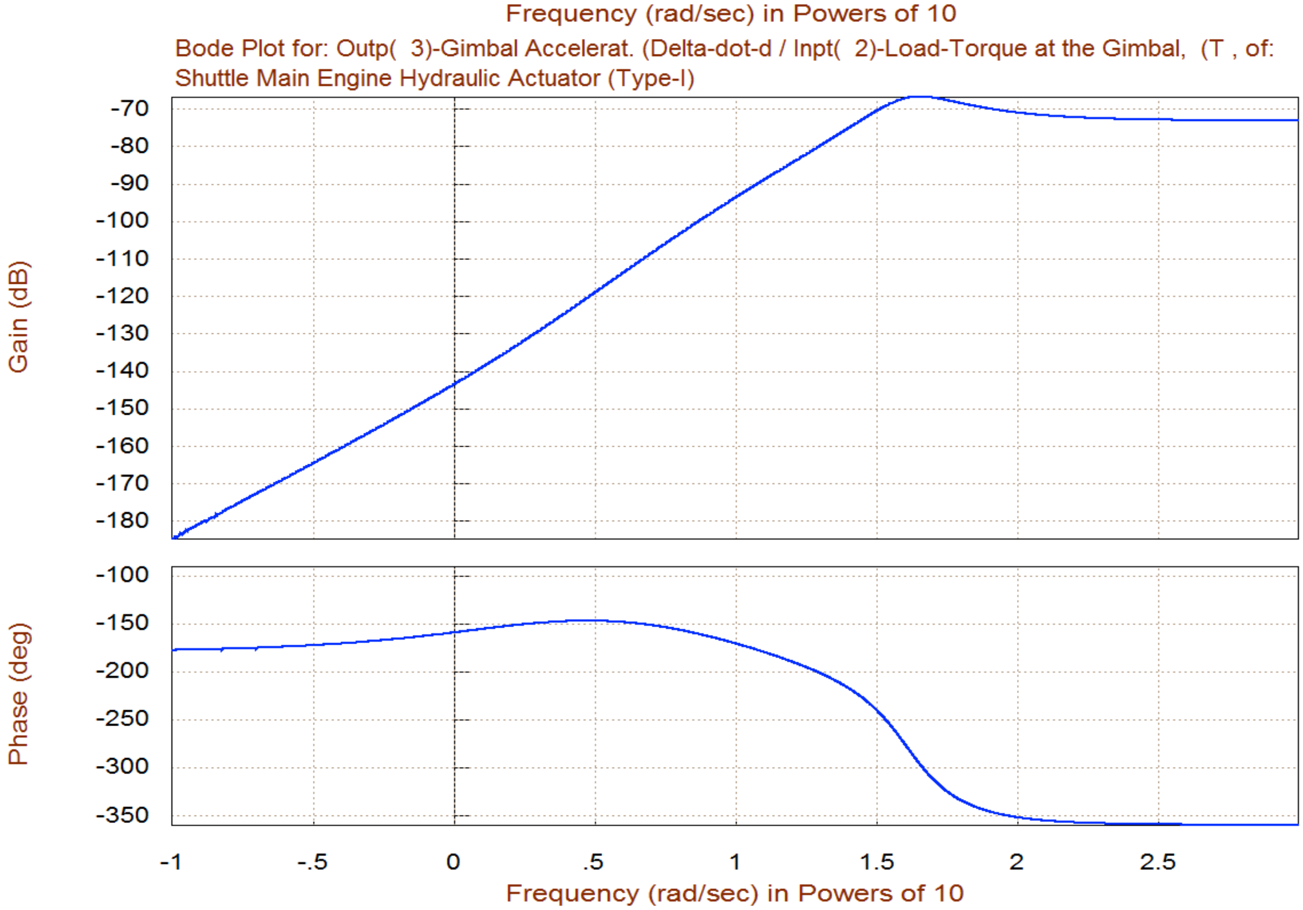
There is a menu located above each plot that is used to perform several functions. The first two options are used for selecting drivers for different output formats or for sending the plot to the printer. The "*Rescale Plots*" option has a drop-down menu used for adjusting the size of the plots, focusing in smaller areas or expanding the scales size to cover a larger area. You may also advance and plot the next set of frequency data or return to the main menu to choose a different plotting option.



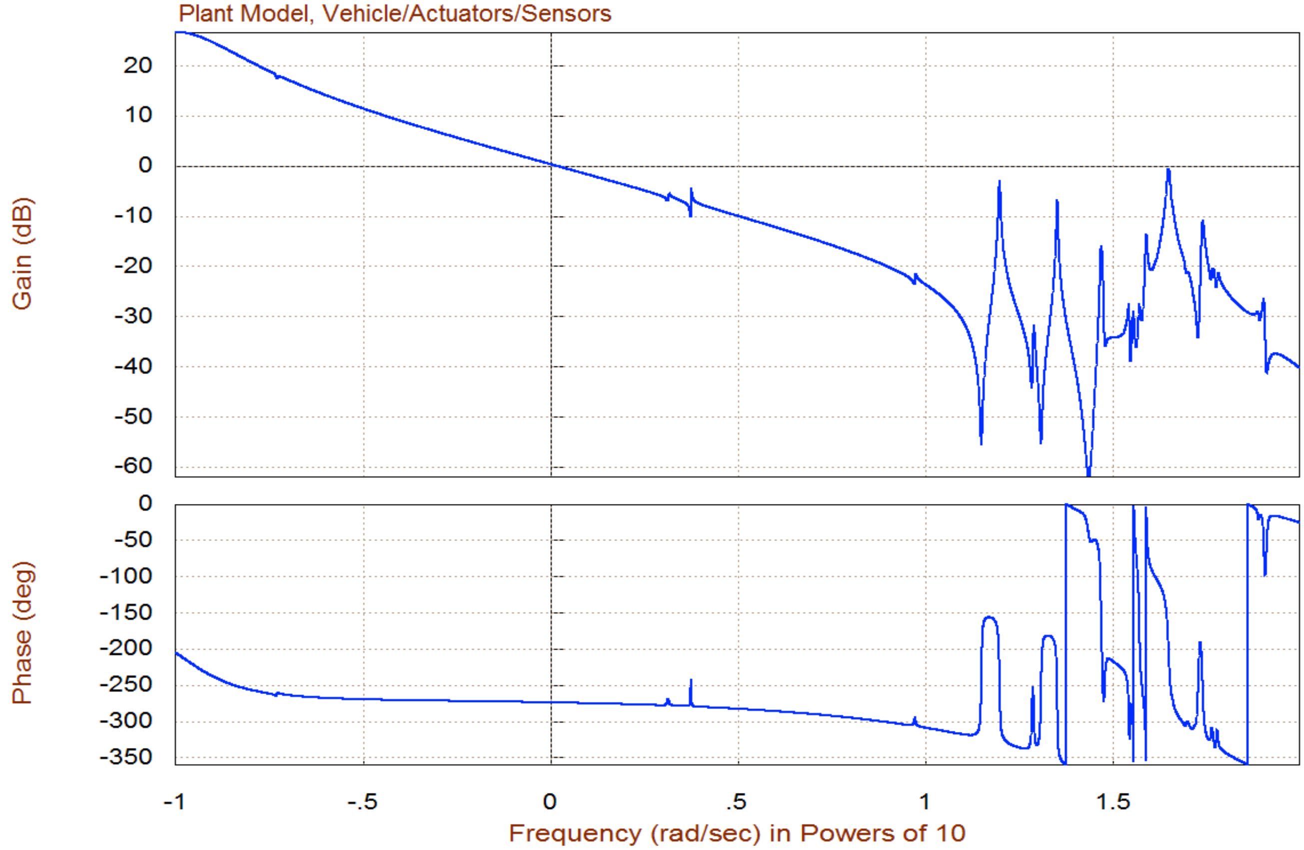
The following plots were obtained from the frequency response calculations described, beginning with two frequency responses from the actuator system, three responses from the Plant Model, and three responses from the Flight Control System. In separate plots below the regular Bode plots of the Plant Model, the regions around the flex modes are also shown expanded and in a linear scale. Those plots were obtained using the "Magnify a Selected Region" option from the menu. The peaks of the resonances are also labeled by clicking with the cursor on the actual locus.

The last three sets of plots show the frequency responses of the flight control system during second stage for the roll, pitch, and yaw axes calculated between the attitude error inputs and the control system outputs. They are presented in Bode and Nyquist plot formats.

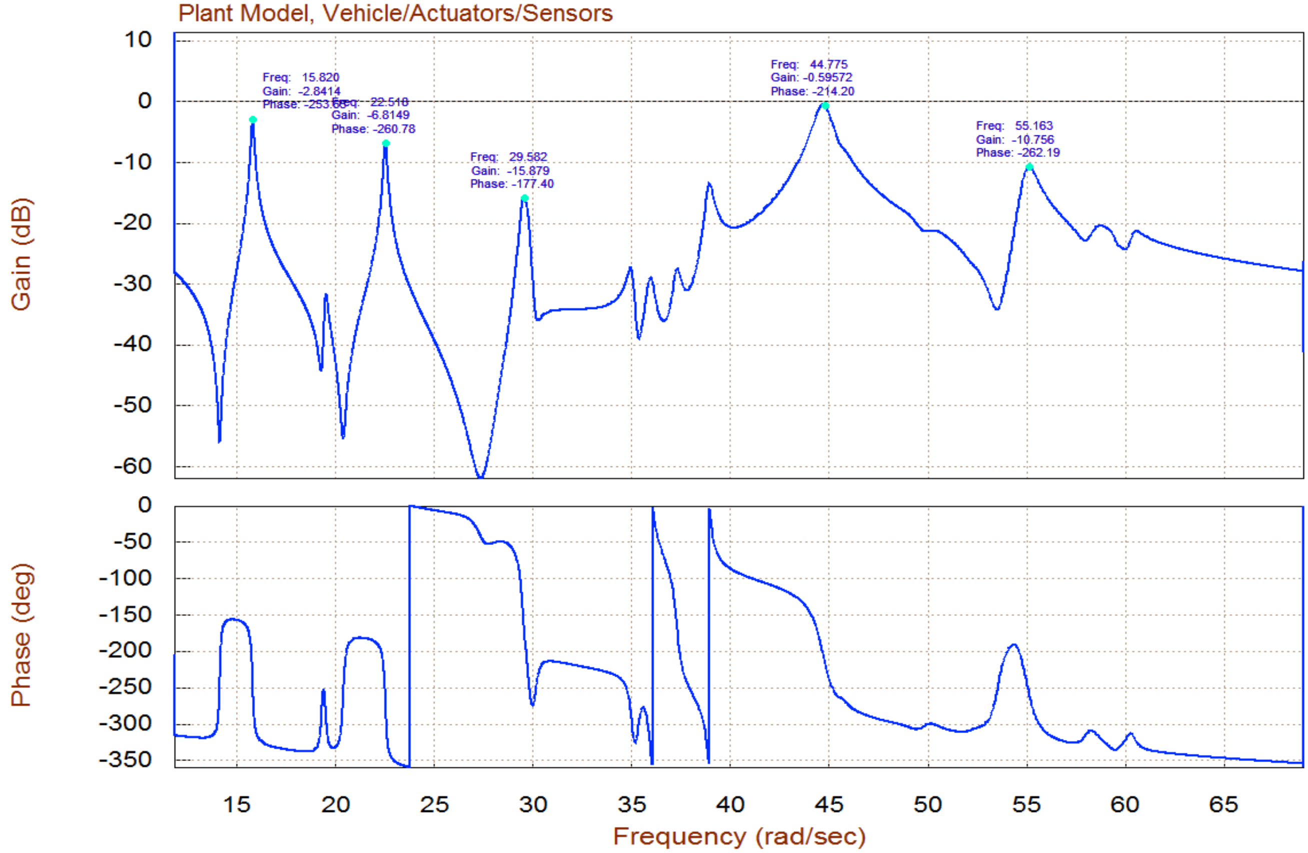

Bode Plot for: Outp(1)-Nozzle Gimbal Rotation (Delta / Inpt(1)-Nozzle Deflection Command (Del , of:
Shuttle Main Engine Hydraulic Actuator (Type-I)
Gain (dB)
Phase (deg)
Frequency (rad/sec) in Powers of 10

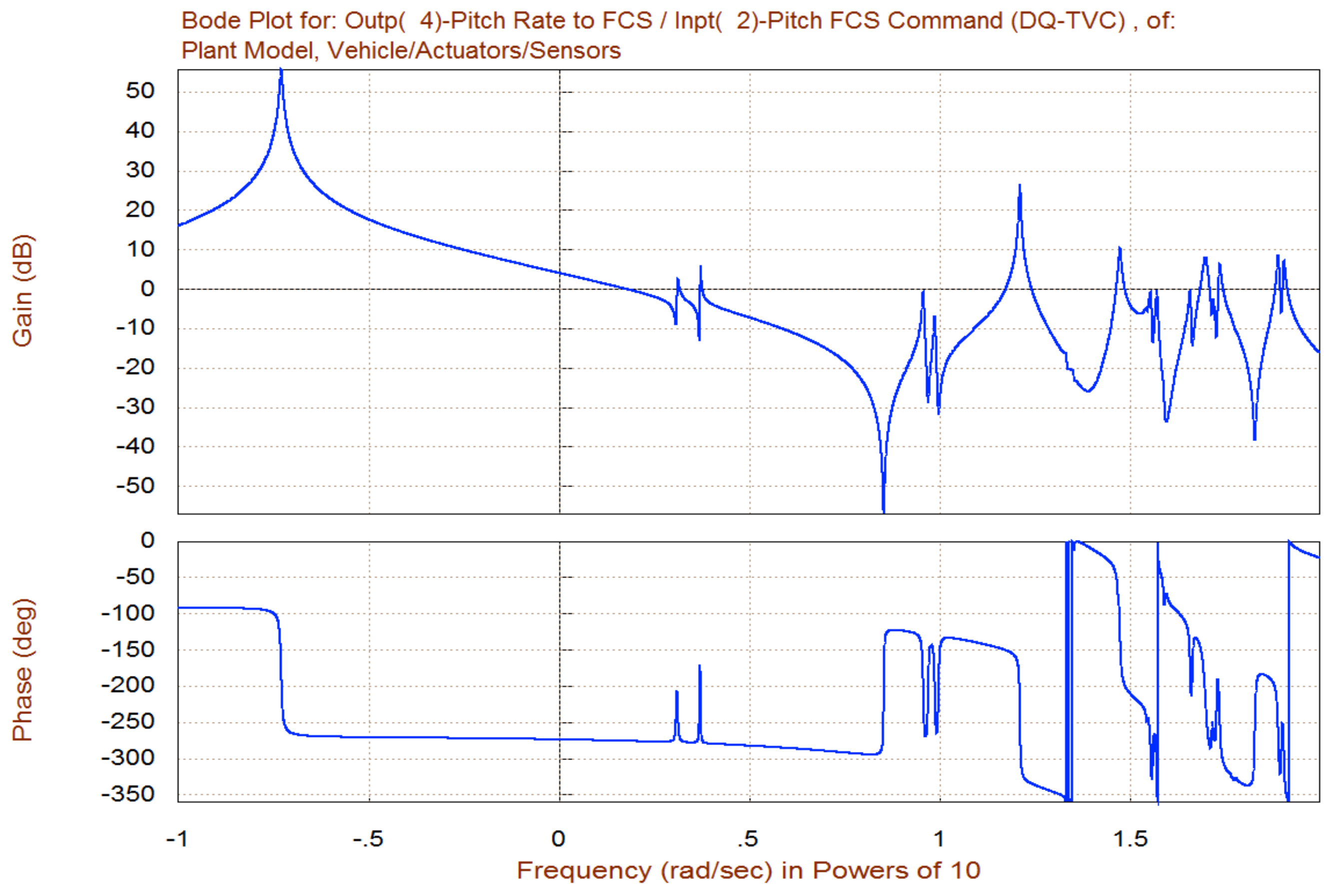

Bode Plot for: Outp(3)-Gimbal Accelerat. (Delta-dot-d / Inpt(2)-Load-Torque at the Gimbal, (T , of:
Shuttle Main Engine Hydraulic Actuator (Type-I)
Gain (dB)
Phase (deg)
Frequency (rad/sec) in Powers of 10

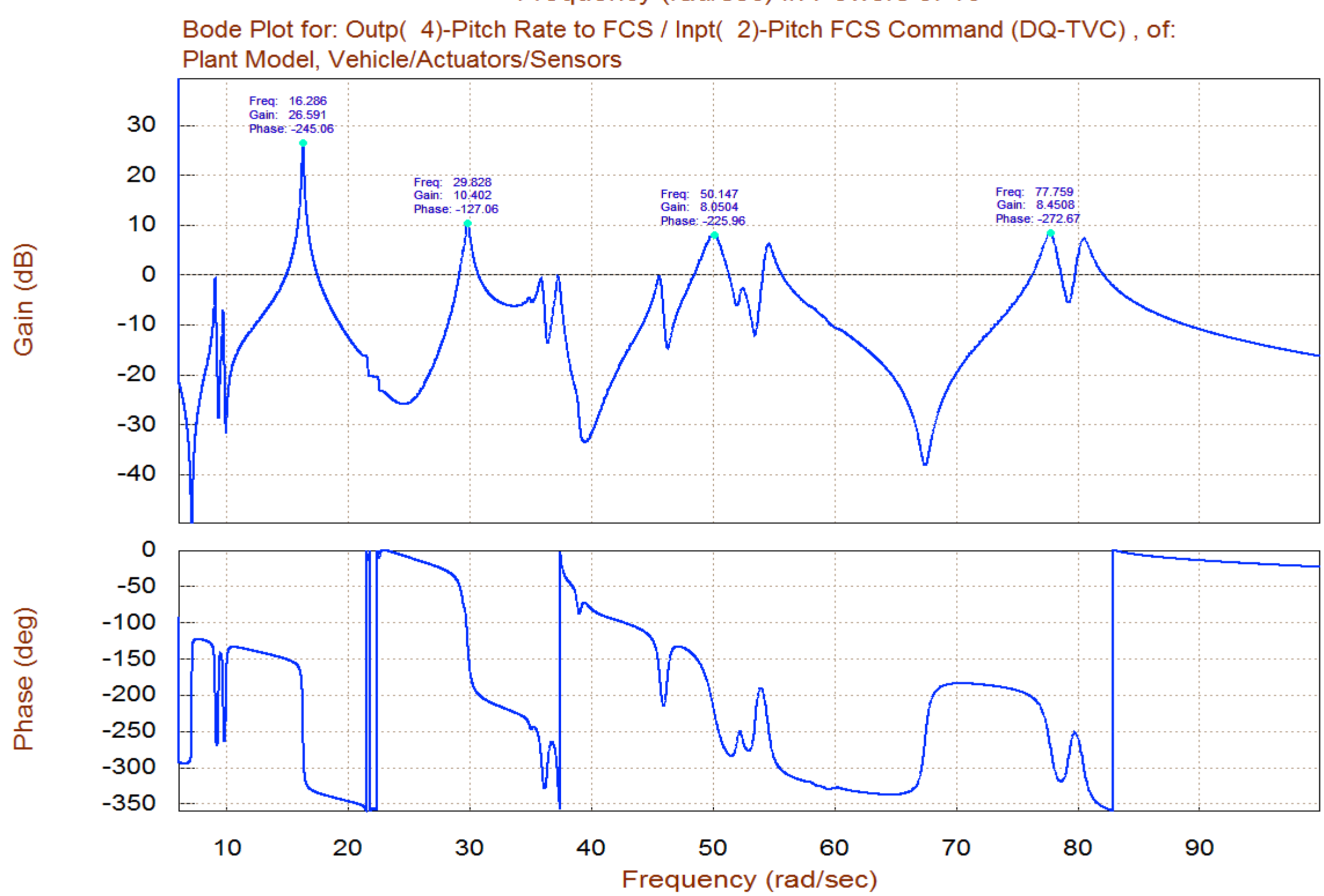
Bode Plot for: Outp(2)-Roll Rate to FCS / Inpt(1)-Roll FCS Command (DP-TVC) , of:
Plant Model, Vehicle/Actuators/Sensors



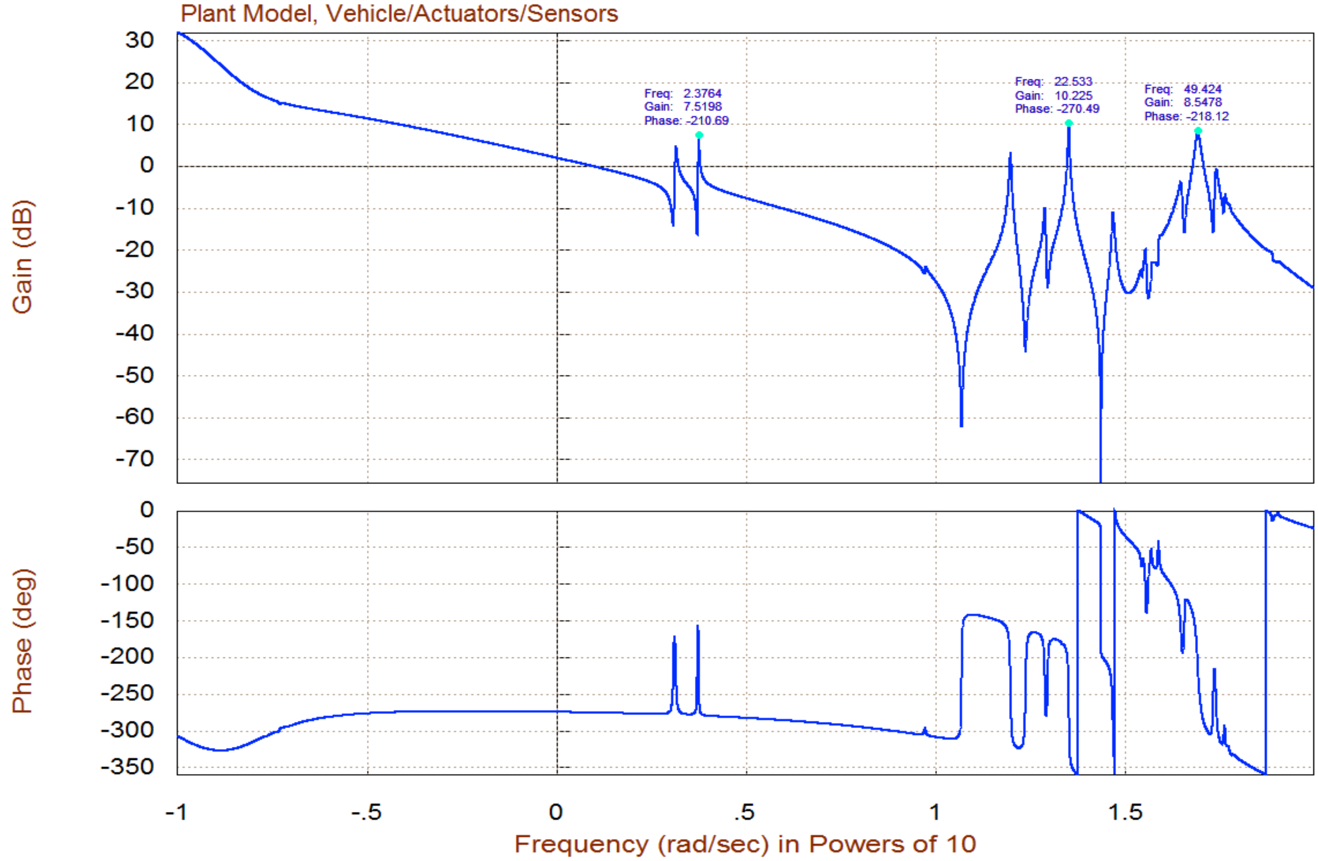
Bode Plot for: Outp(2)-Roll Rate to FCS / Inpt(1)-Roll FCS Command (DP-TVC) , of:
Plant Model, Vehicle/Actuators/Sensors



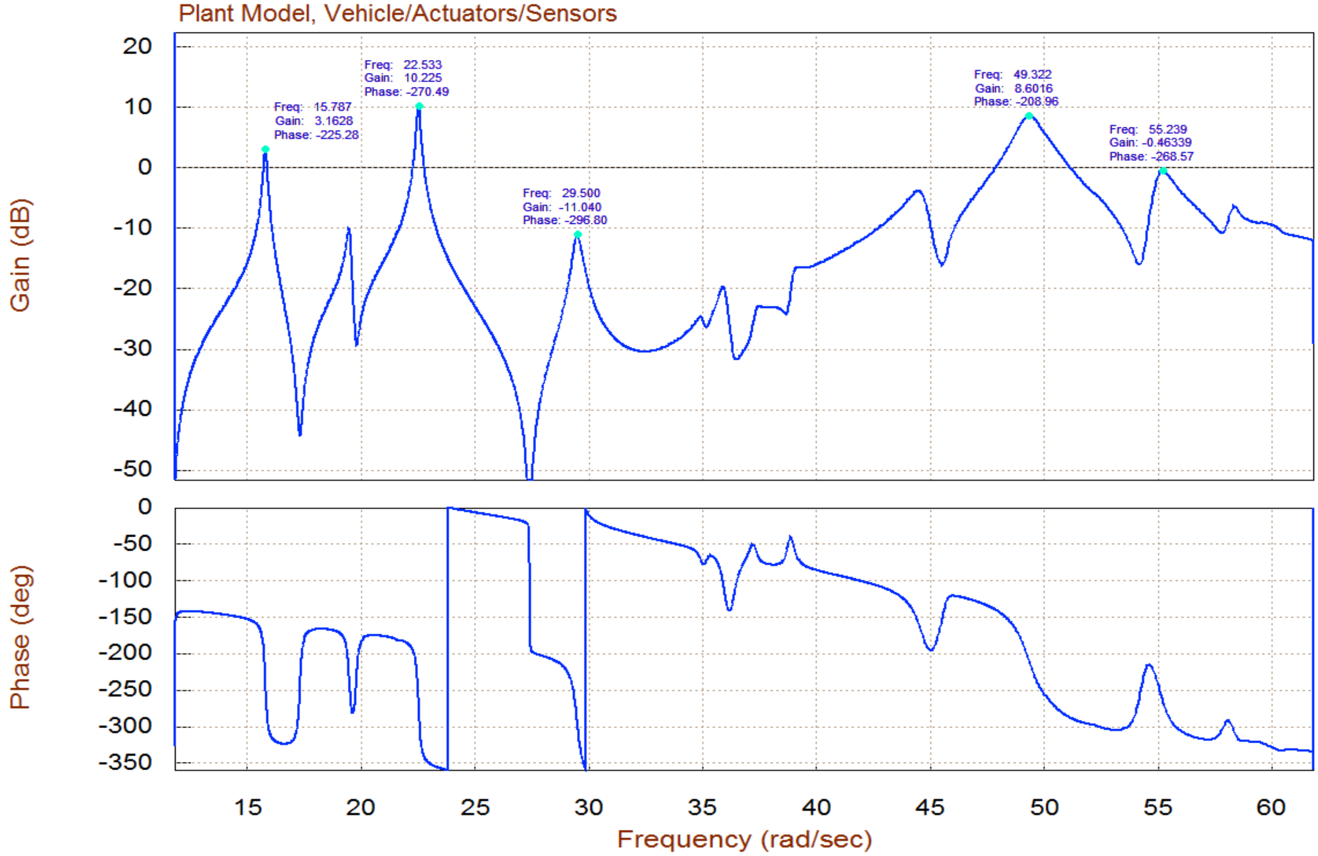

Bode Plot for: Outp(4)-Pitch Rate to FCS / Inpt(2)-Pitch FCS Command (DQ-TVC) , of:
Plant Model, Vehicle/Actuators/Sensors
Gain (dB)
Phase (deg)
Frequency (rad/sec) in Powers of 10


Bode Plot for: Outp(4)-Pitch Rate to FCS / Inpt(2)-Pitch FCS Command (DQ-TVC) , of:
Plant Model, Vehicle/Actuators/Sensors
Freq: 16.286 Gain: 26.591 Phase: -245.06
Freq: 29.828 Gain: 10.402 Phase: -127.06
Freq: 50.147 Gain: 8.0504 Phase: -225.96
Freq: 77.759 Gain: 8.4508 Phase: -272.67
Gain (dB)
Phase (deg)
Frequency (rad/sec)

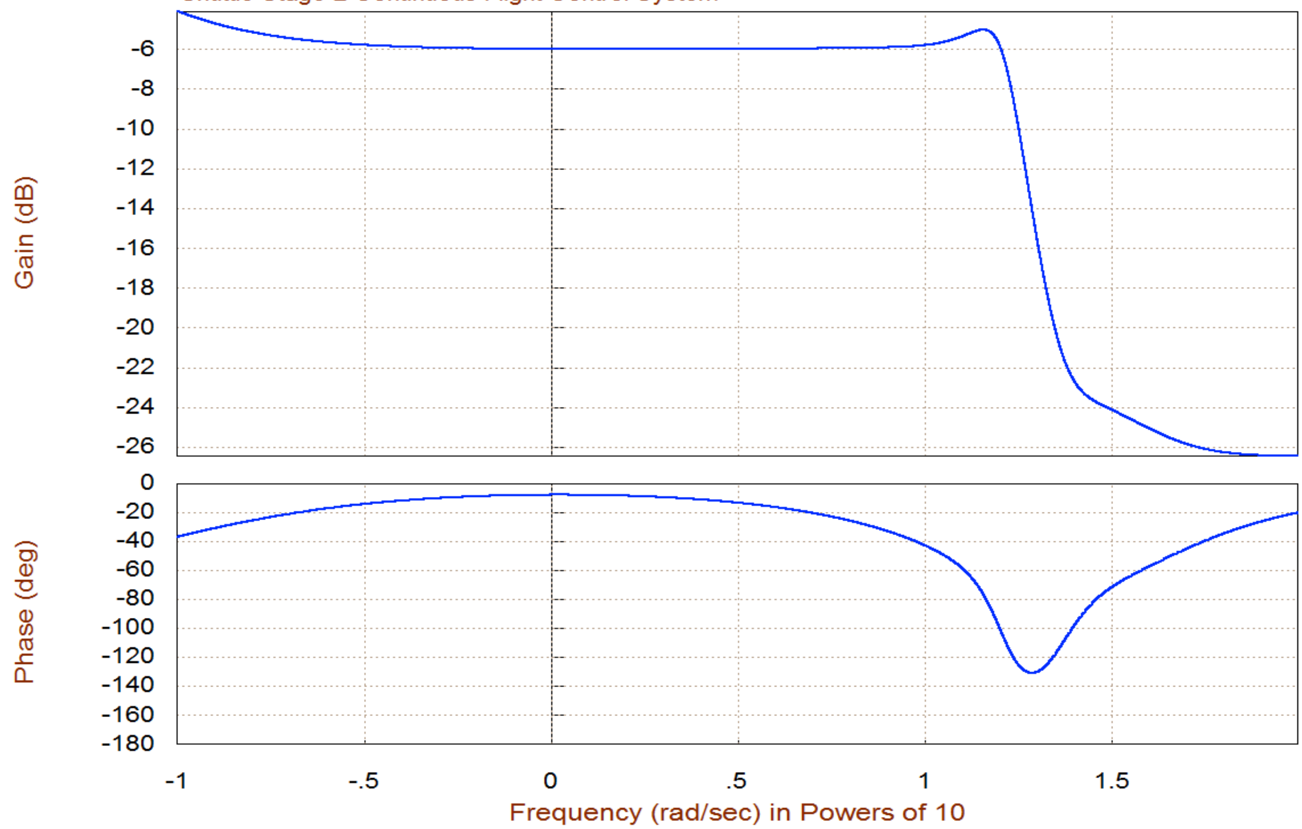
Bode Plot for: Outp(6)-Yaw Rate to FCS / Inpt(3)-Yaw FCS Command (DR-TVC) , of:
Plant Model, Vehicle/Actuators/Sensors



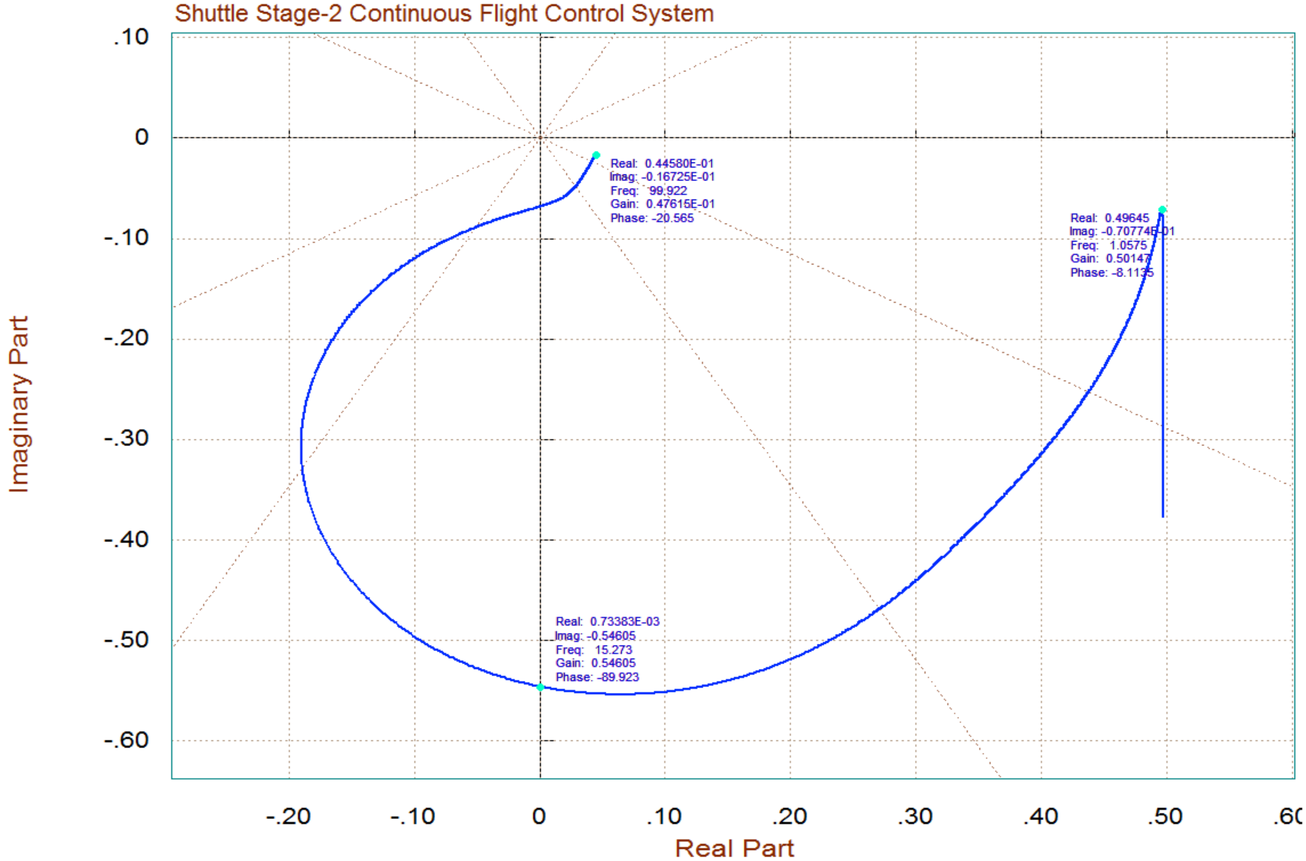
Bode Plot for: Outp(6)-Yaw Rate to FCS / Inpt(3)-Yaw FCS Command (DR-TVC) , of:
Plant Model, Vehicle/Actuators/Sensors



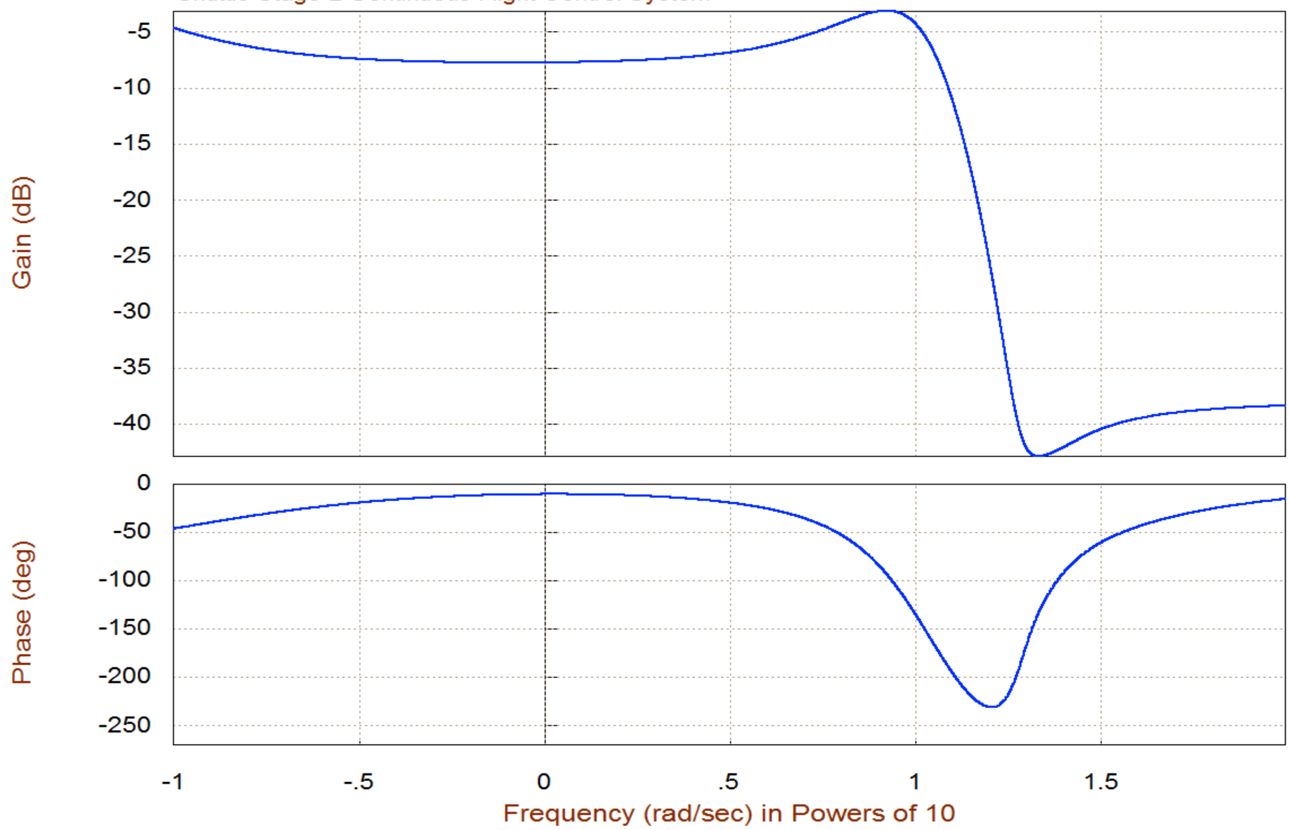
Bode Plot for: Outp(1)-Roll FCS (DP-TVC) / Inpt(1)-Roll Attitude Error (phi-err) , of:
Shuttle Stage-2 Continuous Flight Control System



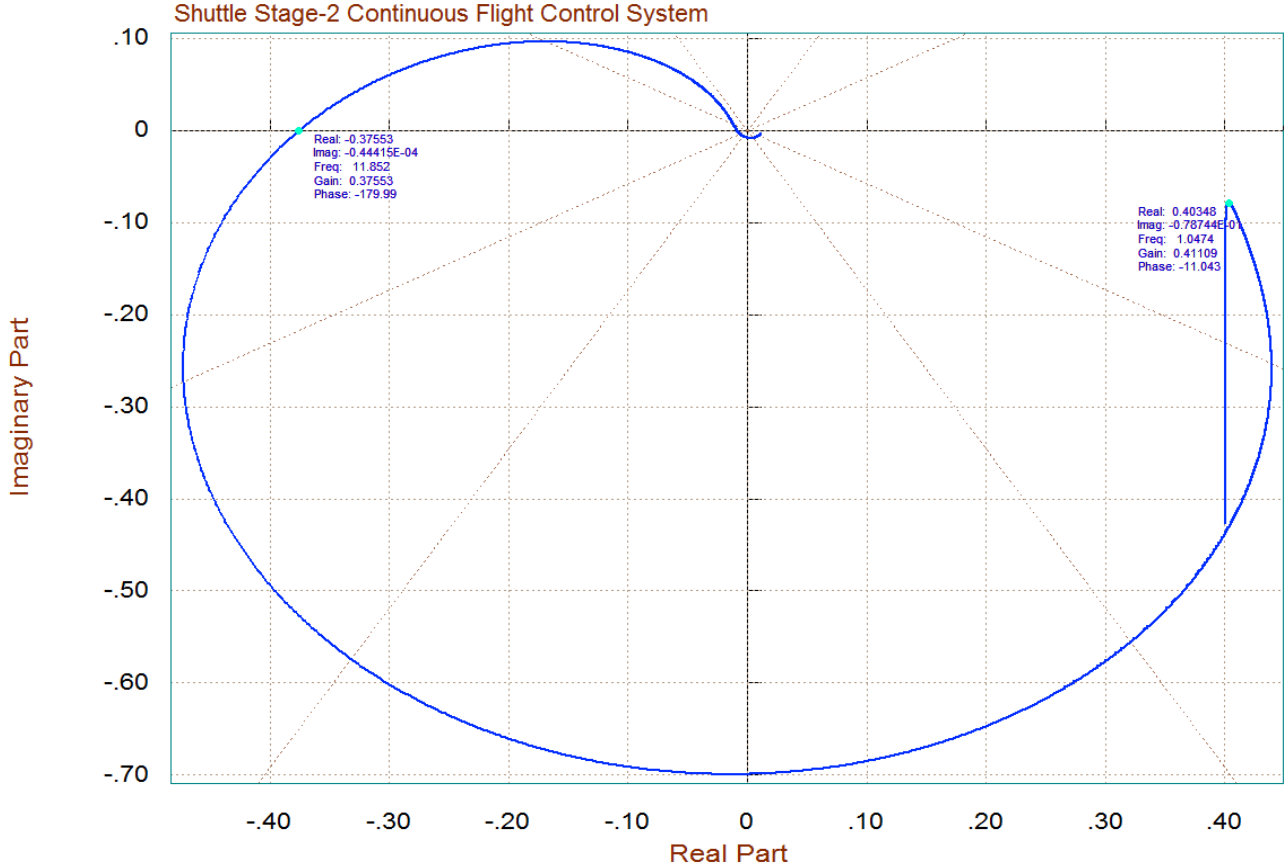
Nyquist Plot for: Outp(1)-Roll FCS (DP-TVC) / Inpt(1)-Roll Attitude Error (phi-err) , of:
Shuttle Stage-2 Continuous Flight Control System



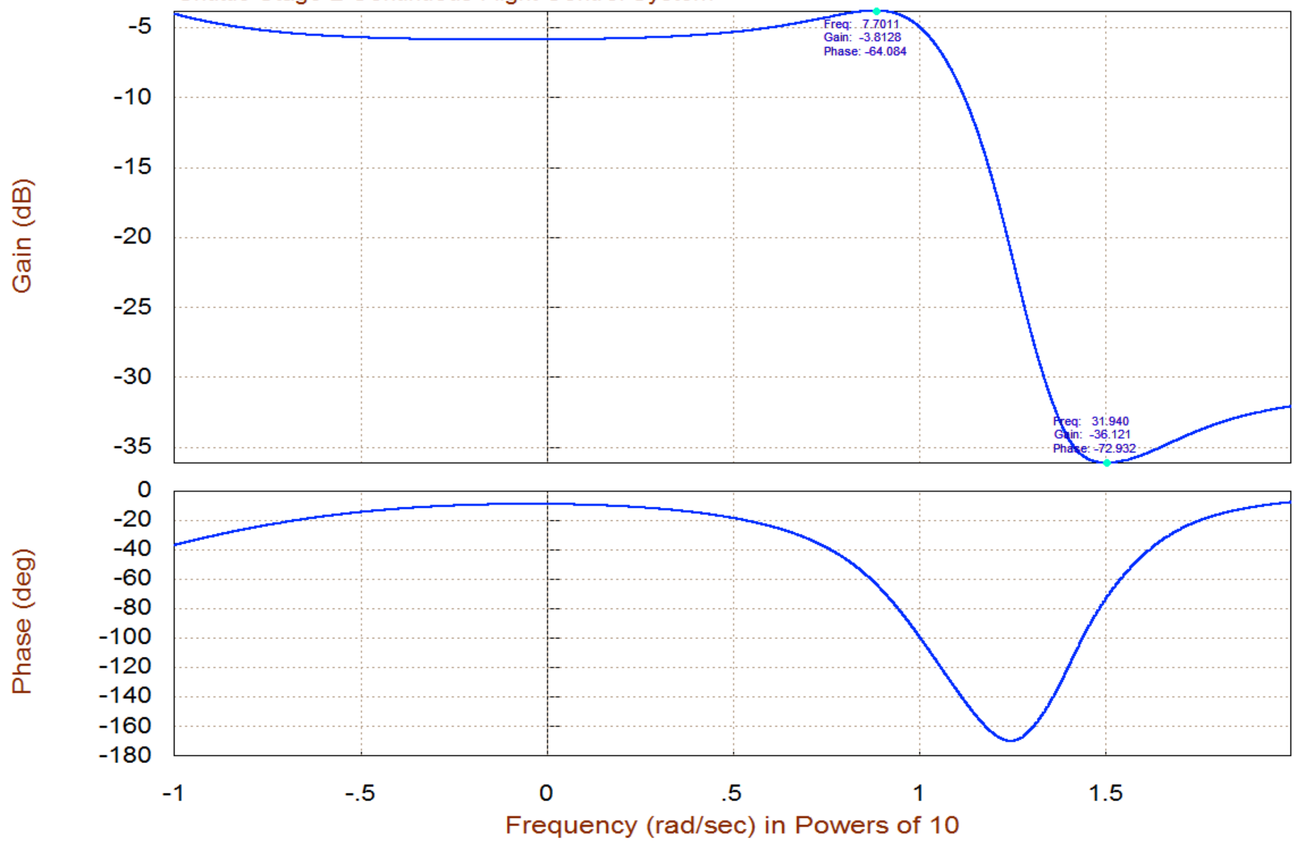
Bode Plot for: Outp(2)-Pitch FCS (DQ-TVC) / Inpt(2)-Pitch Attitude Error (theta-er , of:
Shuttle Stage-2 Continuous Flight Control System



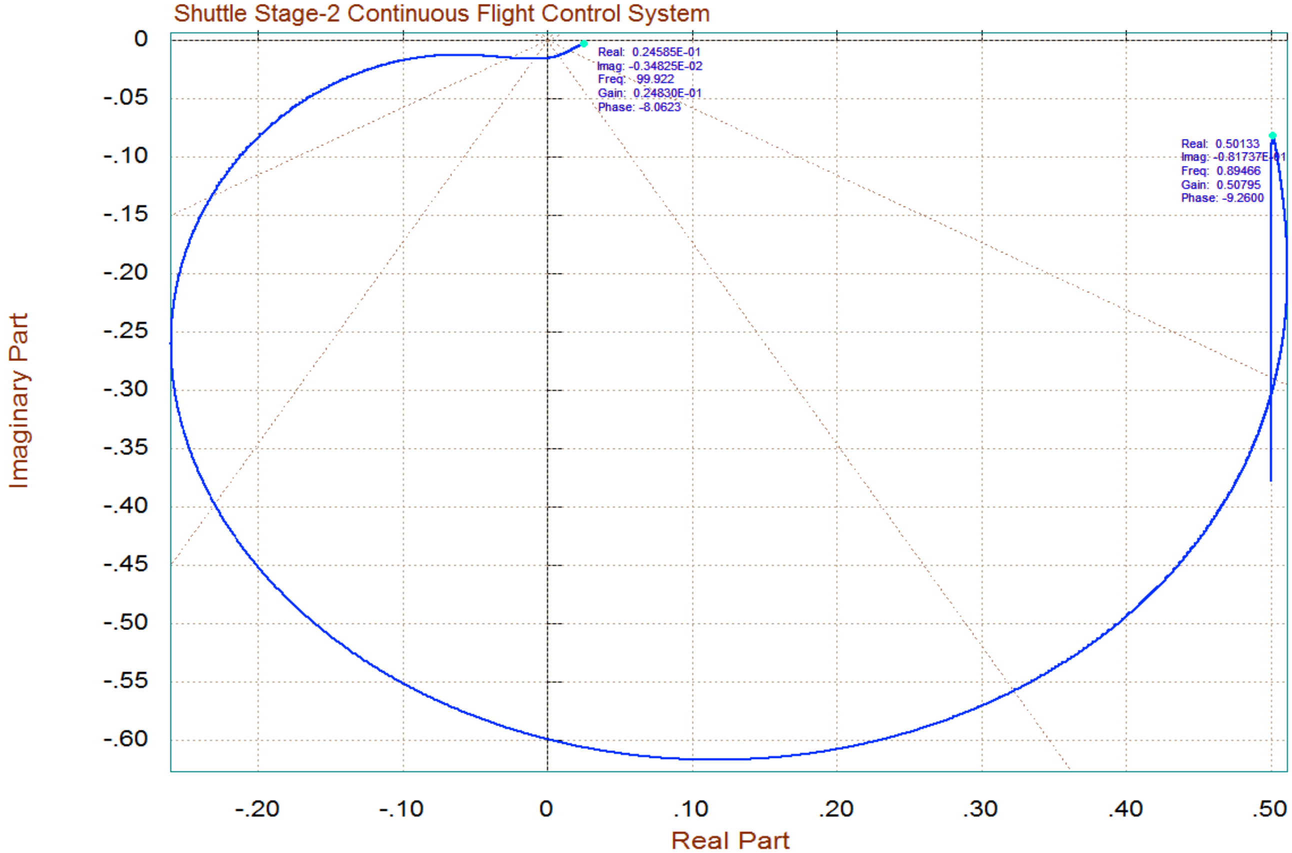
Nyquist Plot for: Outp(2)-Pitch FCS (DQ-TVC) / Inpt(2)-Pitch Attitude Error (theta-er , of:
Shuttle Stage-2 Continuous Flight Control System



Bode Plot for: Outp(3)-Yaw FCS (DR-TVC) / Inpt(3)-Yaw Attitude Error (psi-err) , of:
Shuttle Stage-2 Continuous Flight Control System

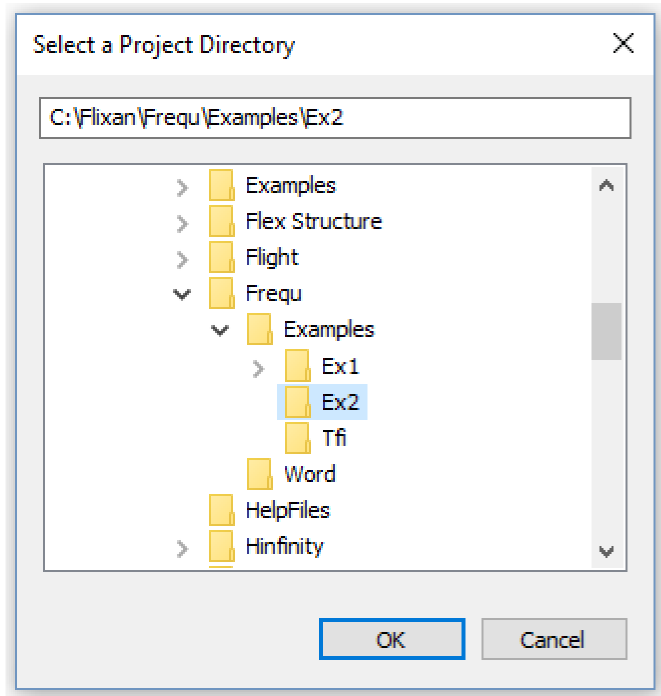


Nyquist Plot for: Outp(3)-Yaw FCS (DR-TVC) / Inpt(3)-Yaw Attitude Error (psi-err) , of:
Shuttle Stage-2 Continuous Flight Control System

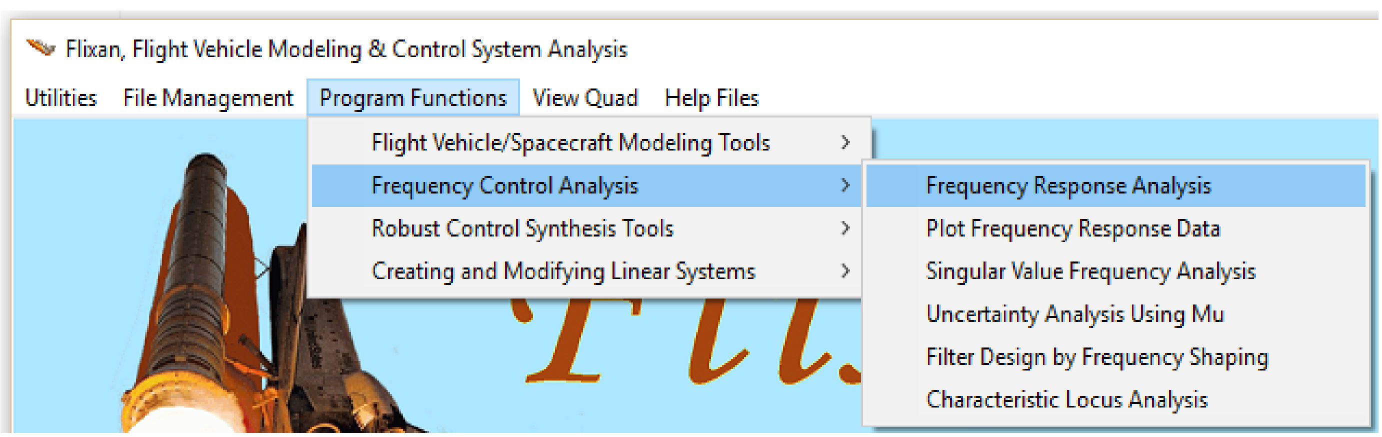


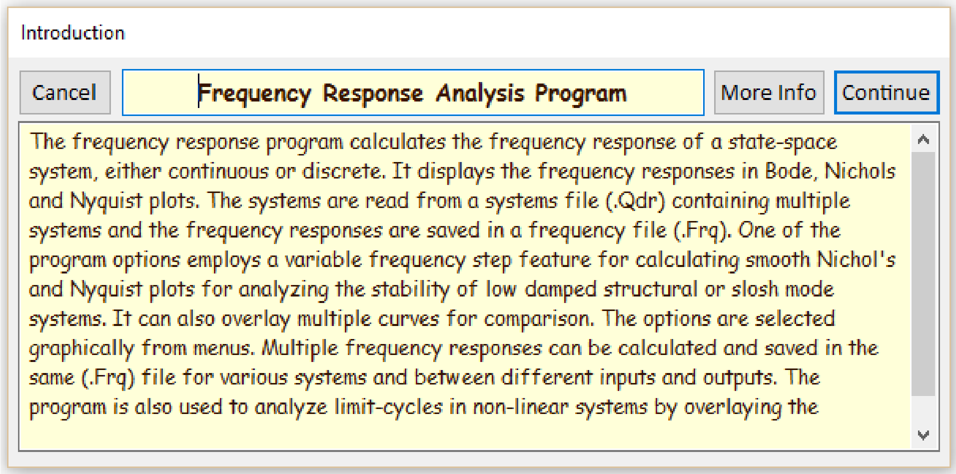
Example 2 Variable Frequency Step/ Overlay

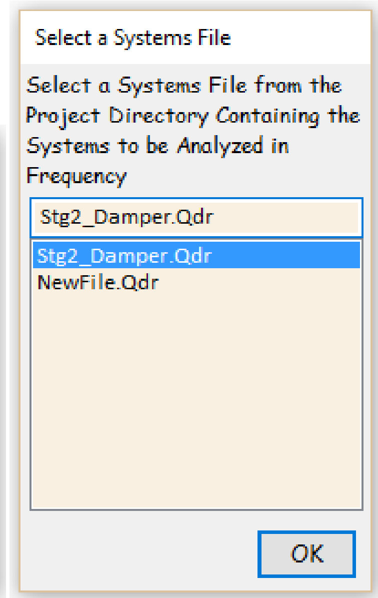
Our second example is in folder "*C:\Flixan\Frequ\Examples\Ex2*". In this example we will calculate the frequency response of two similar systems and place them in two separate (.Frq) files in order to overlay them in the same plots for comparison. The two systems represent the open-loop dynamics of the Shuttle vehicle with structural flexibility and propellant sloshing. The actuator, sensor, and control system dynamics are included in the models. The loops are opened in the pitch axis and they will be used for pitch stability analysis. The only difference between the two systems is that the first one is a continuous system that includes a continuous controller, but the second system is discrete that includes a discrete controller. The sampling period of the second system is 40 msec. We will calculate the frequency response of the continuous system and save the response in file vfs1.frq. Then we will calculate the response of the discrete system and save the response in file vfs2.frq. Then we will plot the responses from both systems together to show that the discrete system has some additional attenuation and more phase-lag at high frequencies.



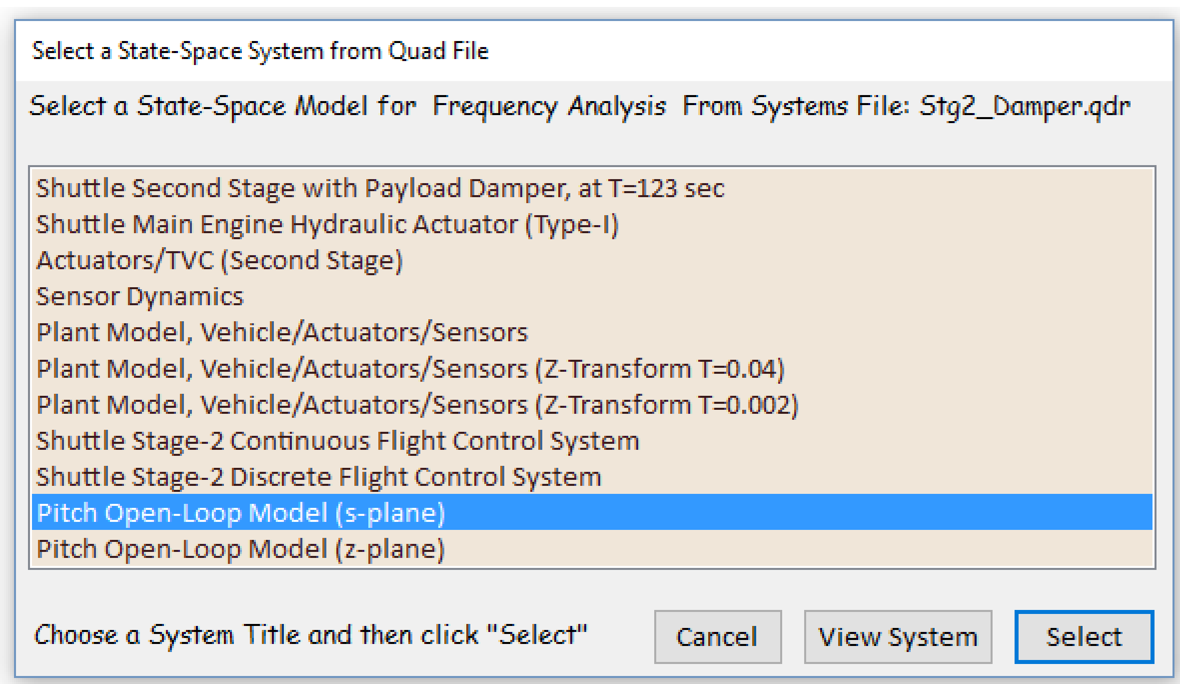
Start the Flixan program, select the project directory and from the main menu select "*Program Functions*", "*Frequency Control Analysis*", and then "*Frequency Response Analysis*". The following is an introduction dialog and click "*Continue*". Select also the systems file "Stg2_Damper.Qdr" from the next menu and click "OK".



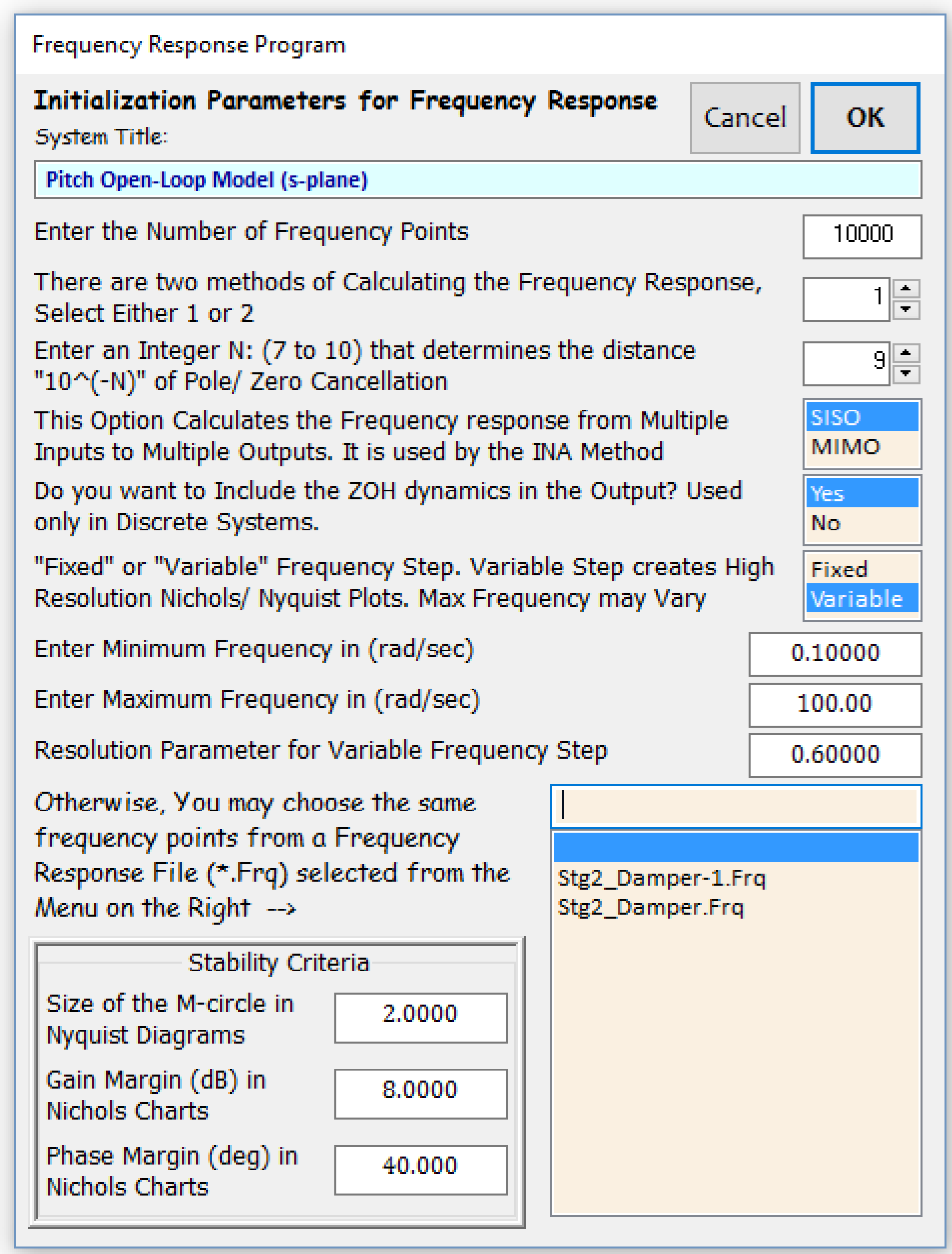




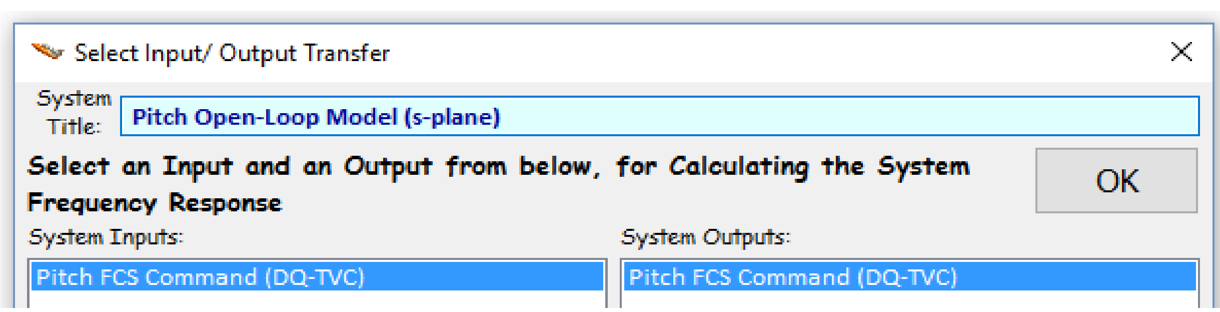
From the systems selection menu below, select the continuous system title "*Pitch Open-Loop Model (s-plane)*" and click on "*Select*".



Use the dialog below to set the initialization parameters for the frequency response calculations of the continuous system, as shown, and click "OK". Notice that the variable frequency step (VFS) option is selected and the number of points are 10,000.



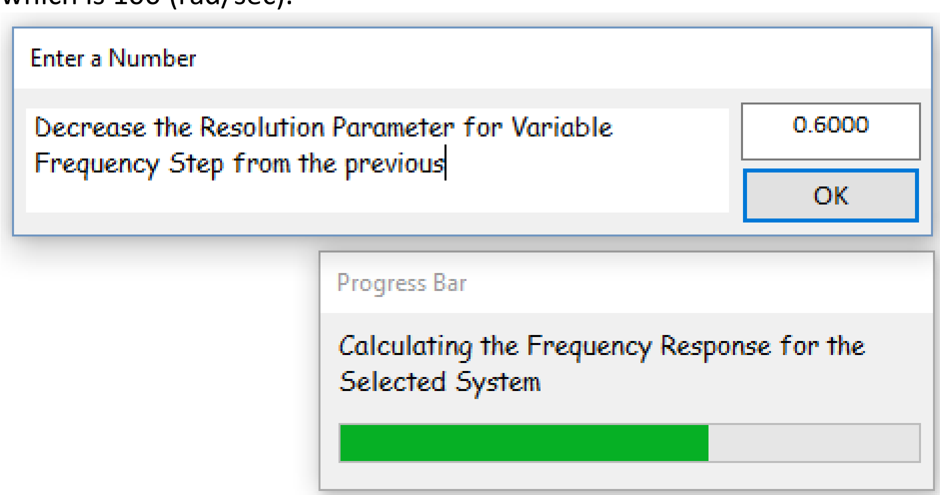
From the following input/ output selection menu, select the single input and the single output to calculate the frequency response of the system that has the loop opened at the input of the pitch actuator, and click “OK”.

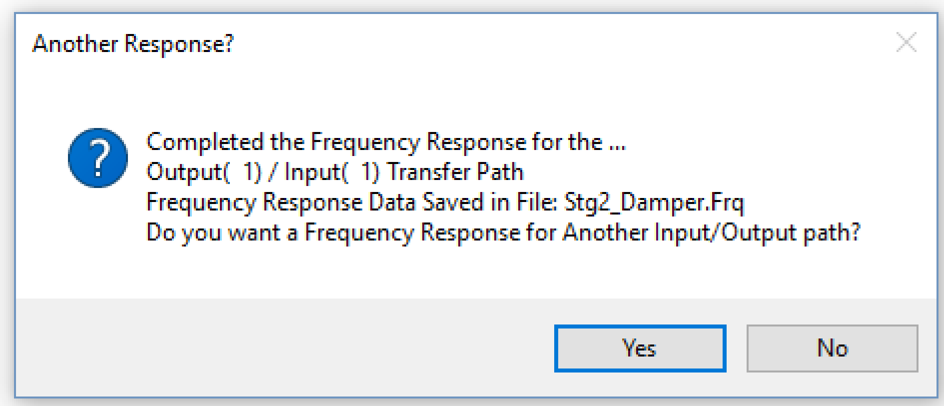


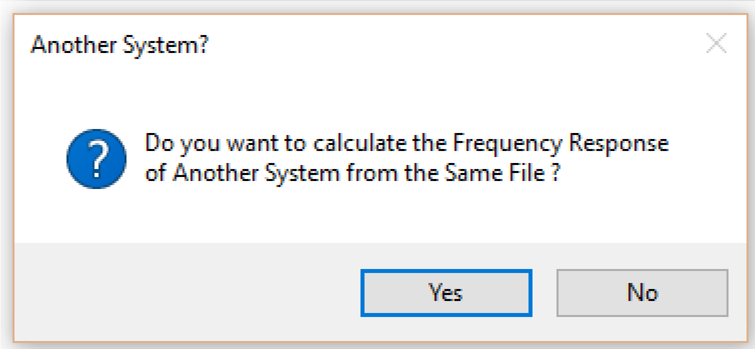
When VFS is selected the algorithm adjusts the frequency step in order to generate smooth Nichol's and Nyquist plots. This feature is useful when analyzing systems that include low damping resonances such as slosh and structural modes. The smoothness of the Nichol's and Nyquist plots depends on the number of points used. If only a few points are used the modes will appear like polygons instead of circular. The VFS algorithm is attempting to adjust the frequency step between calculations in order to keep a certain variable as close to the resolution parameter as possible. This criterion variable is a combination of three terms {δ(phase) + δ(gain) + K δ(frequency) }. That is, a change in gain, phase, and frequency which is maintained almost constant between subsequent calculations.

It is not easy to set the frequency range exactly as desired when using the VFS option and several attempts are needed in order to achieve the specified final frequency. The resolution parameter controls the spacing between the frequency points and it is typically set between (0.4 and 0.9). If the value of the resolution parameter is small there will be more points used to describe each resonance and the final frequency for a fixed number of points will be small. When a large resolution is used there will be less points used per resonance, and the final frequency will be higher than expected for the same number of points. The frequency range is, therefore, controlled by selecting the proper resolution parameter value depending on the number of modes and the number of points selected. This may require several trials of adjusting the resolution parameter in order to cover the desired frequency range.

The program will assist the user in the process of selecting the resolution parameter. For example, if you begin with a resolution value of 0.6, and if the final frequency is a lot smaller than the specified range, the program will ask you to increase the resolution parameter. If the final frequency exceeds the specified final frequency by a significant amount, the program will ask you to reduce the resolution and repeat the process. The frequency at the final point will be acceptable when it falls close to 80% of the specified final frequency, otherwise, you will be prompted to adjust the resolution parameter. In this case the resolution parameter was reduced to 0.35 in order to get near the specified final frequency which is 100 (rad/sec).

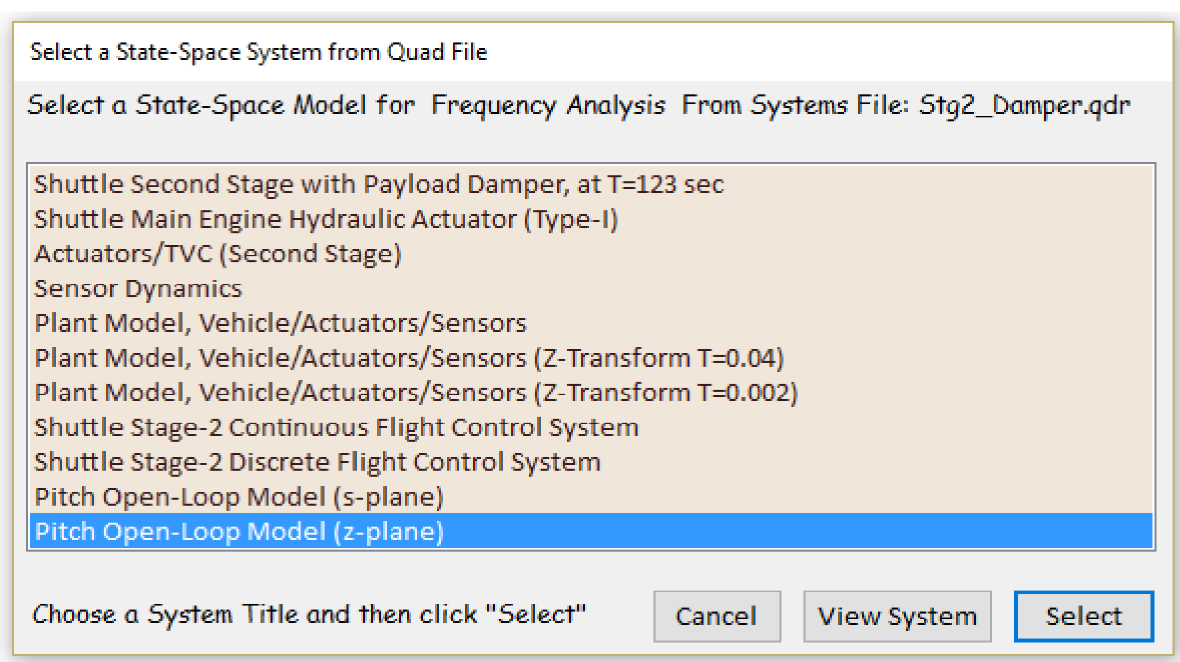




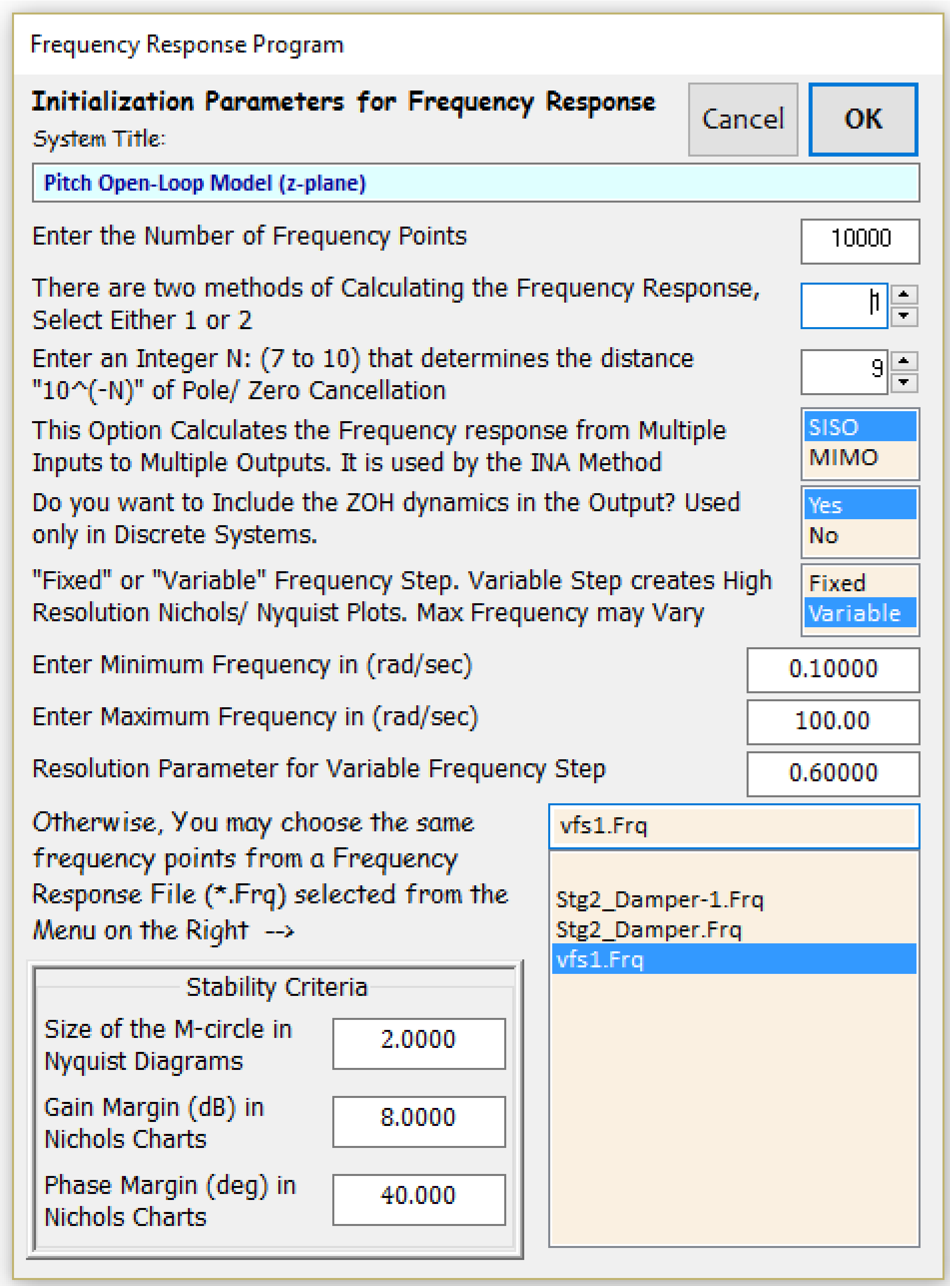


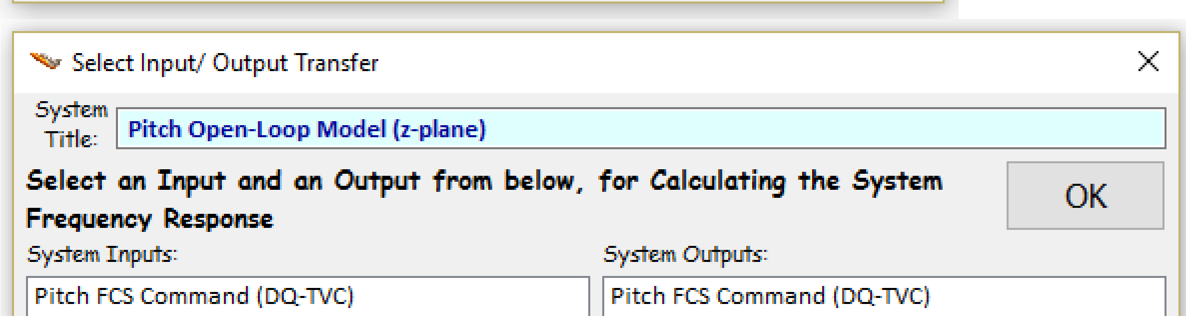
Click on “No” in both of the above dialogs and the program will plot the frequency response data which are saved in file “Stg2_Damper.Frq”. This file is also saved under a different name “vfs1.frq” to avoid being overwritten.

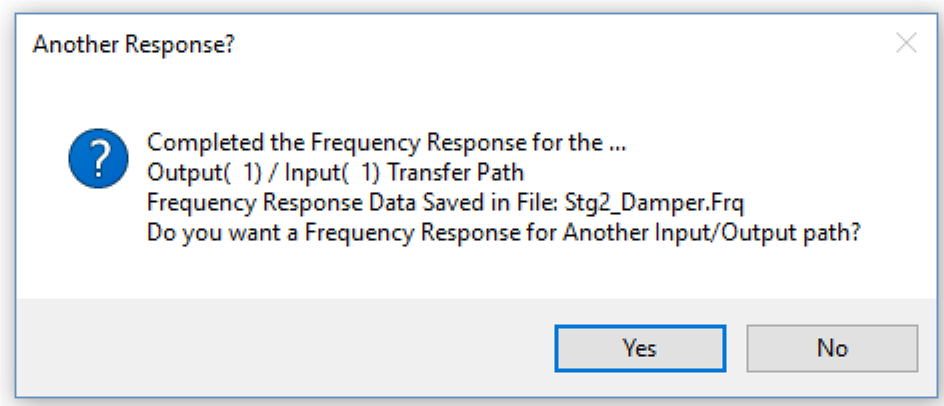
Do not plot the data yet but repeat the frequency response calculation process as before, by selecting the same systems filename “*Stg2_Damper.Qdr*” and then from the systems selection menu, select the discrete system title “*Pitch Open-Loop Model (z-plane)*” and click on “*Select*”.

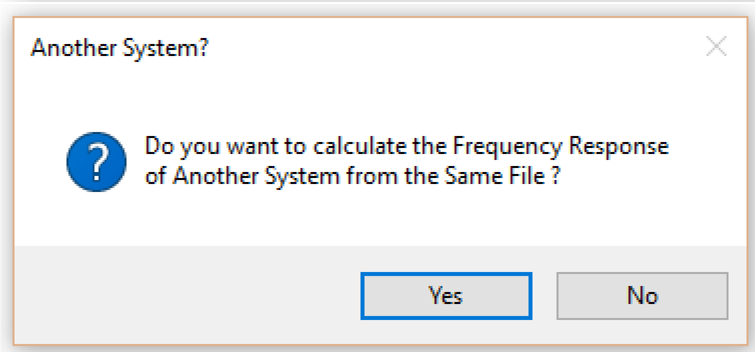


Initialize the frequency response calculations parameters for of the discrete system, as shown below, and click "OK". Notice that this time we are going to use the same frequency points that were used in the previously calculated file "vfs1.frq". Then from the following menu select the single input and the single output of that system.

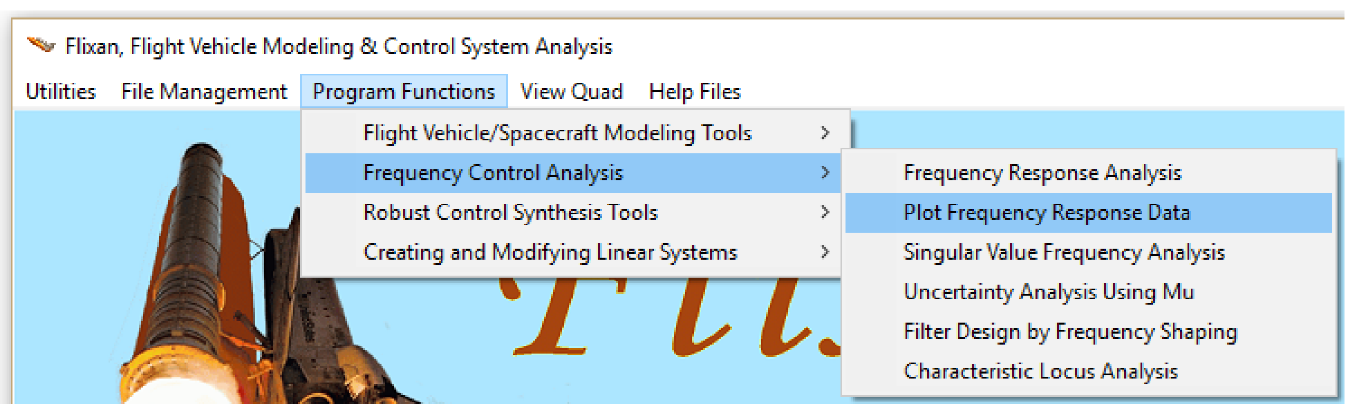




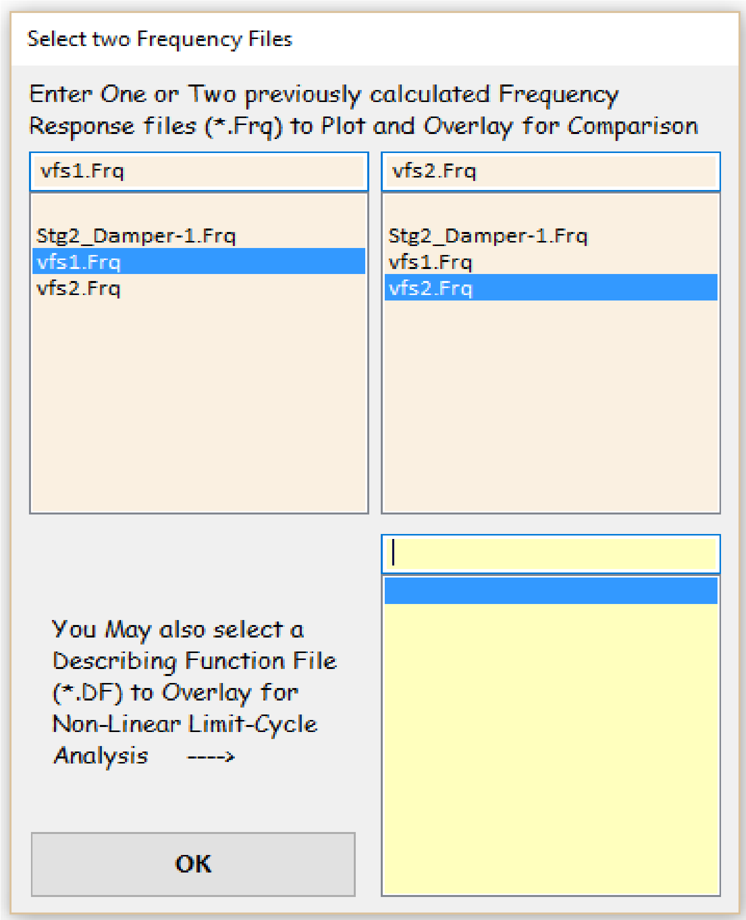


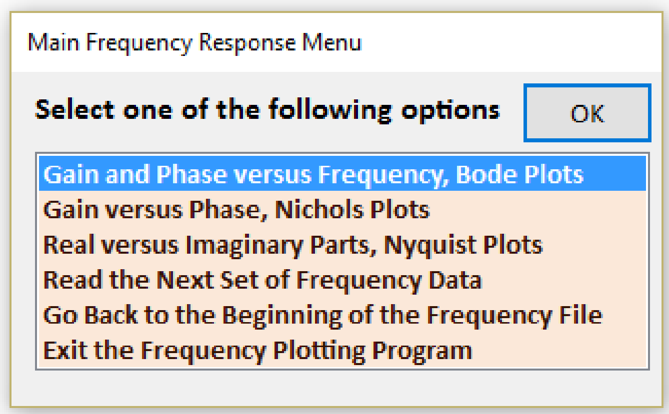


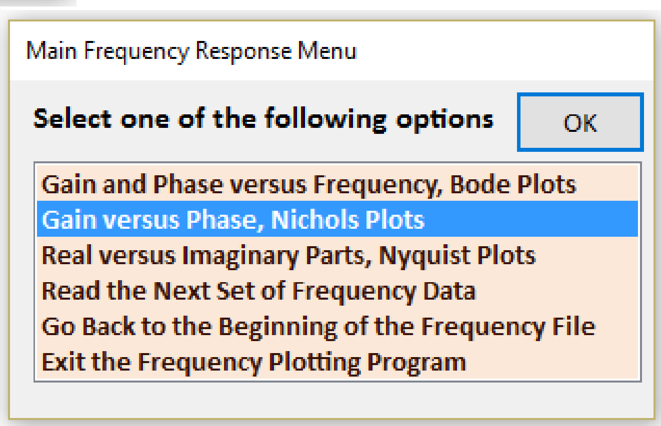
Click on "No" in both of the above dialogs and the frequency response data will be saved in file "*Stg2_Damper.Frq*". This file is also saved under a different name "*vfs2.frq*" to avoid being overwritten. You may now plot the frequency response data from the two files. Go to the Flixan main menu and select "*Program Functions*", "*Frequency Control Analysis*", and then "*Plot Frequency Response Data*".

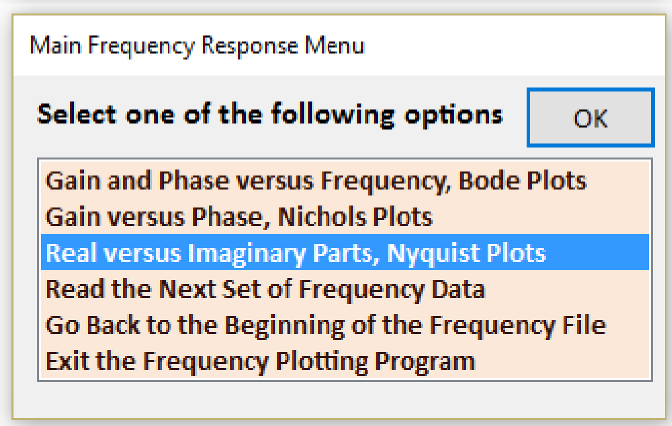


From the top two menus of the following dialog select the two files "*vfs1.frq*" and "*vfs2.frq*" containing the frequency responses of the two systems and click "OK". The third menu is used for selecting a Describing Function file which is applicable only for non-linear system analysis. Use the main menu of the post-processing program to generate Bode, Nichols, and Nyquist plots as shown.

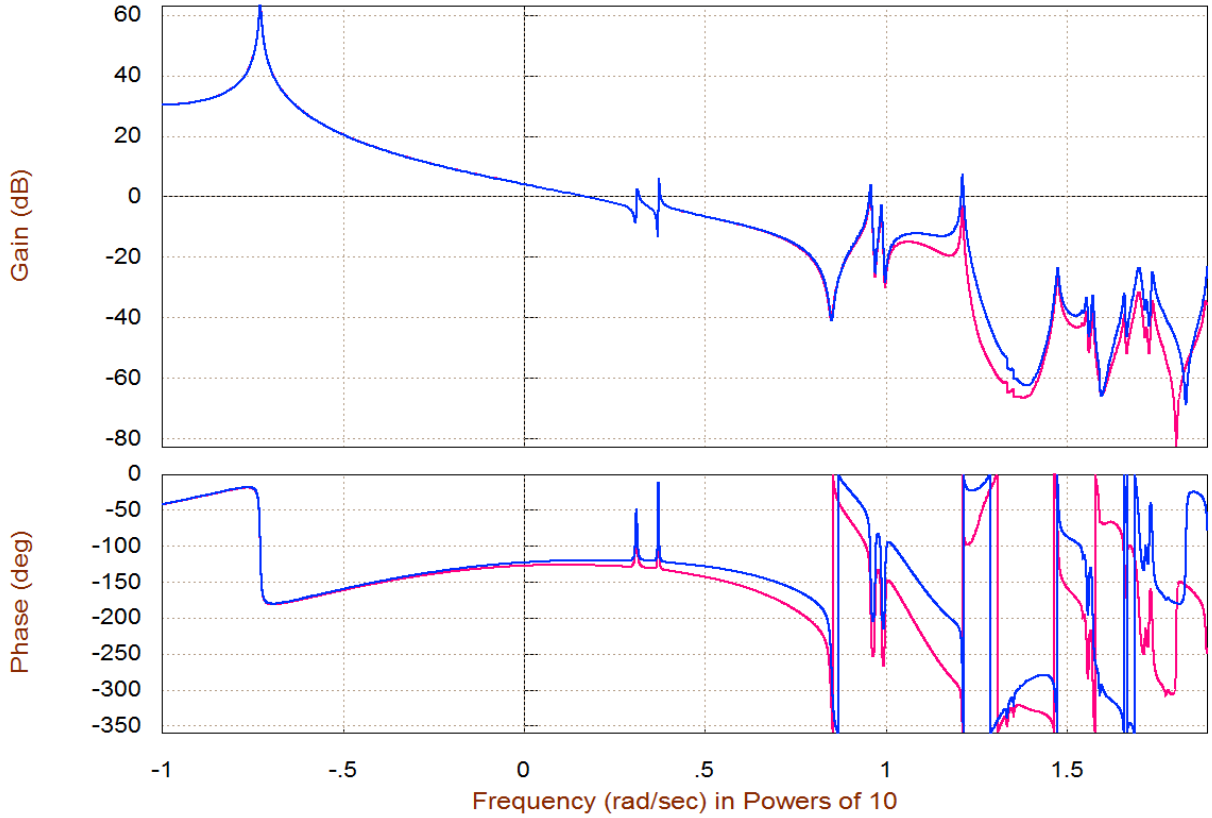

Select two Frequency Files
Enter One or Two previously calculated Frequency Response files (*.Frq) to Plot and Overlay for Comparison
vfs1.Frq
Stg2_Damper-1.Frq
vfs1.Frq
vfs2.Frq
vfs2.Frq
Stg2_Damper-1.Frq
vfs1.Frq
vfs2.Frq
You May also select a Describing Function File (*.DF) to Overlay for Non-Linear Limit-Cycle Analysis ---->
OK


Main Frequency Response Menu
Select one of the following options
OK
Gain and Phase versus Frequency, Bode Plots
Gain versus Phase, Nichols Plots
Real versus Imaginary Parts, Nyquist Plots
Read the Next Set of Frequency Data
Go Back to the Beginning of the Frequency File
Exit the Frequency Plotting Program

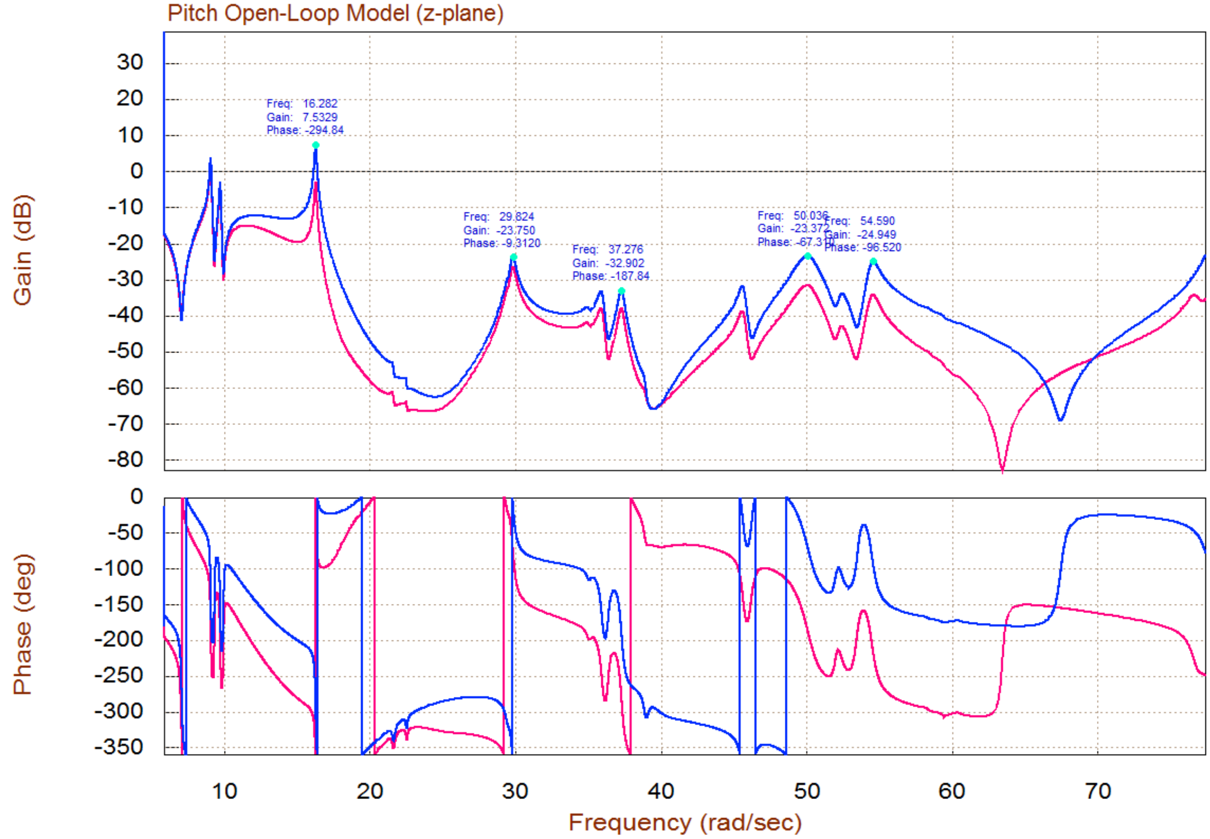

Main Frequency Response Menu
Select one of the following options
OK
Gain and Phase versus Frequency, Bode Plots
Gain versus Phase, Nichols Plots
Real versus Imaginary Parts, Nyquist Plots
Read the Next Set of Frequency Data
Go Back to the Beginning of the Frequency File
Exit the Frequency Plotting Program

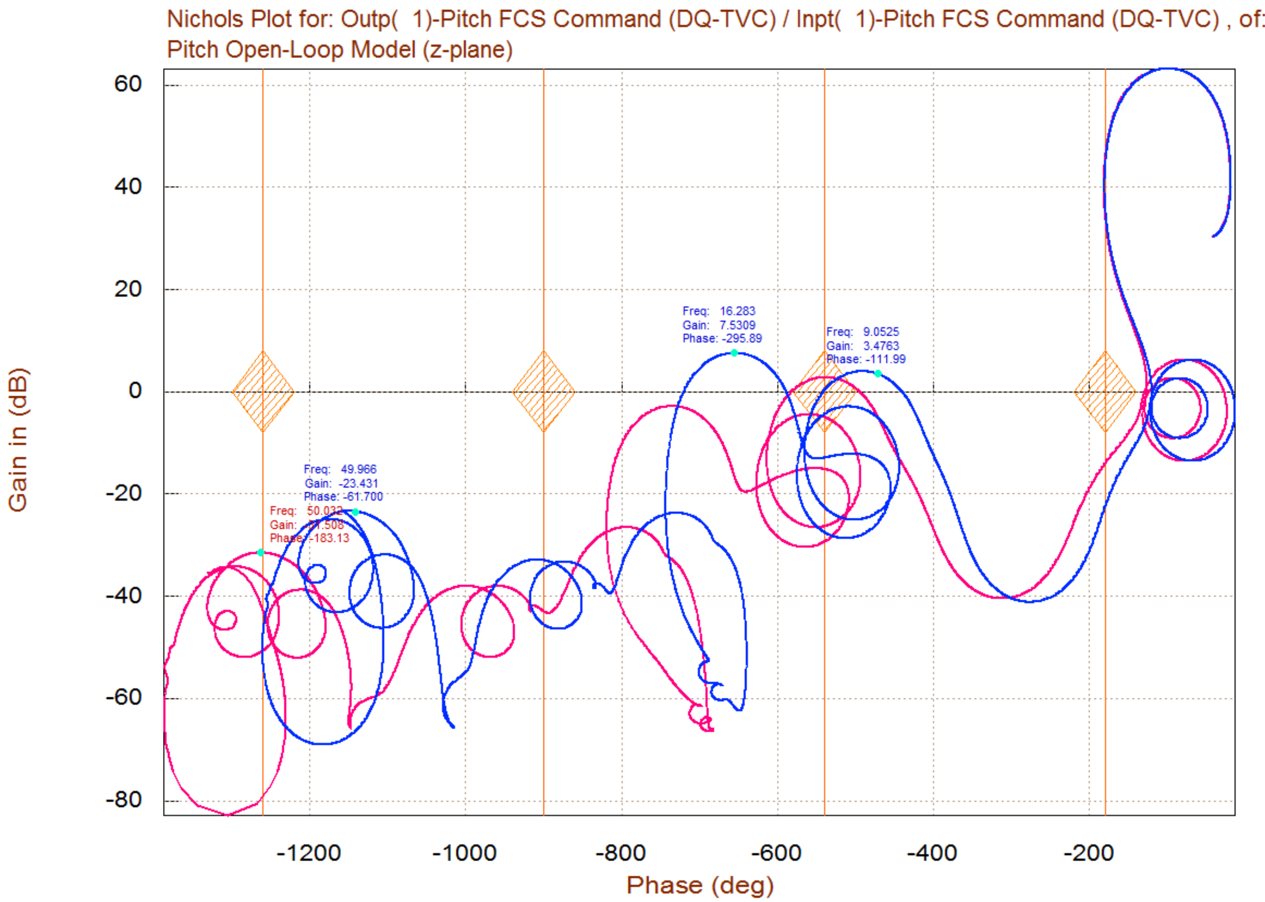

Main Frequency Response Menu
Select one of the following options
OK
Gain and Phase versus Frequency, Bode Plots
Gain versus Phase, Nichols Plots
Real versus Imaginary Parts, Nyquist Plots
Read the Next Set of Frequency Data
Go Back to the Beginning of the Frequency File
Exit the Frequency Plotting Program

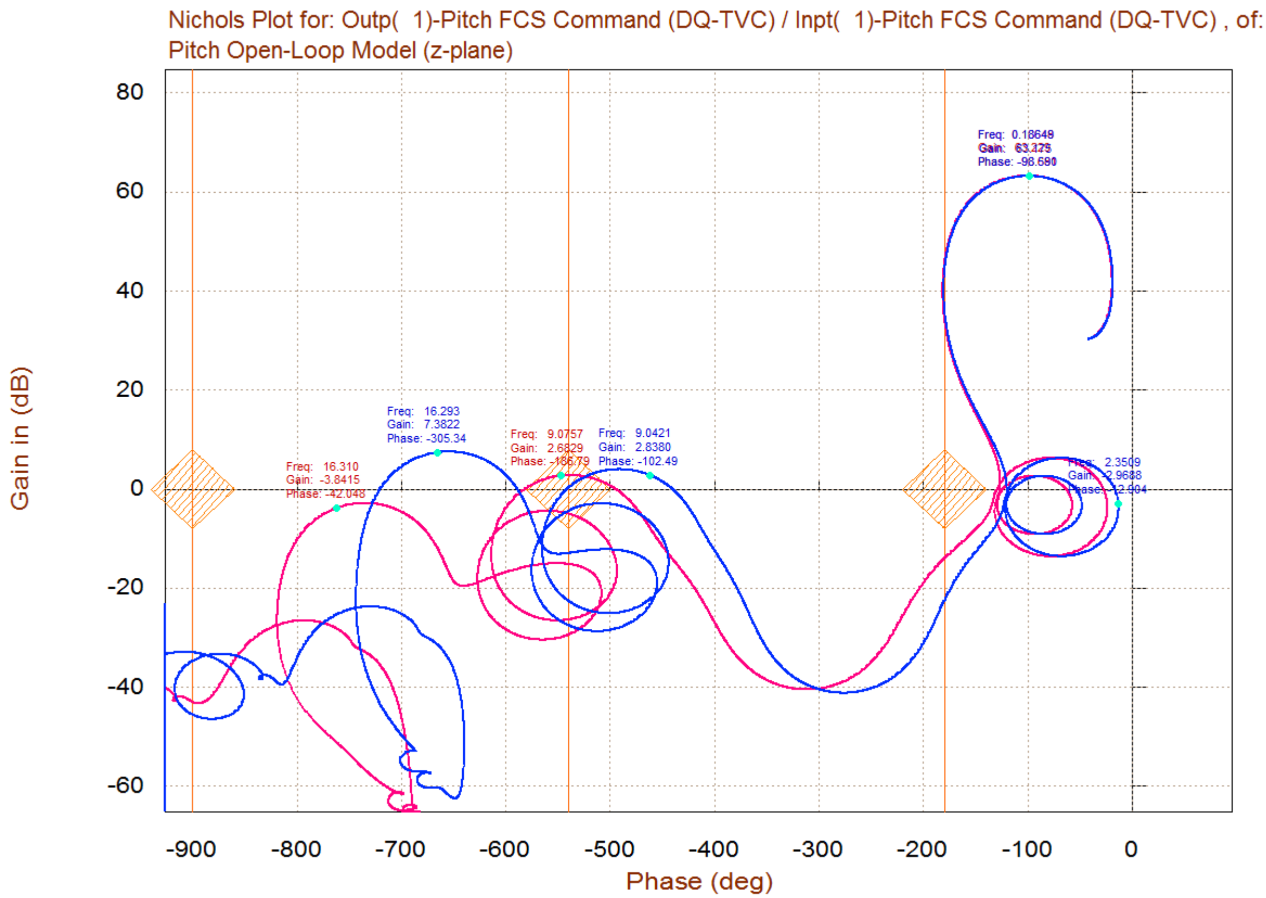
Bode Plot for: Outp(1)-Pitch FCS Command (DQ-TVC) / Inpt(1)-Pitch FCS Command (DQ-TVC) , of:
Pitch Open-Loop Model (z-plane)

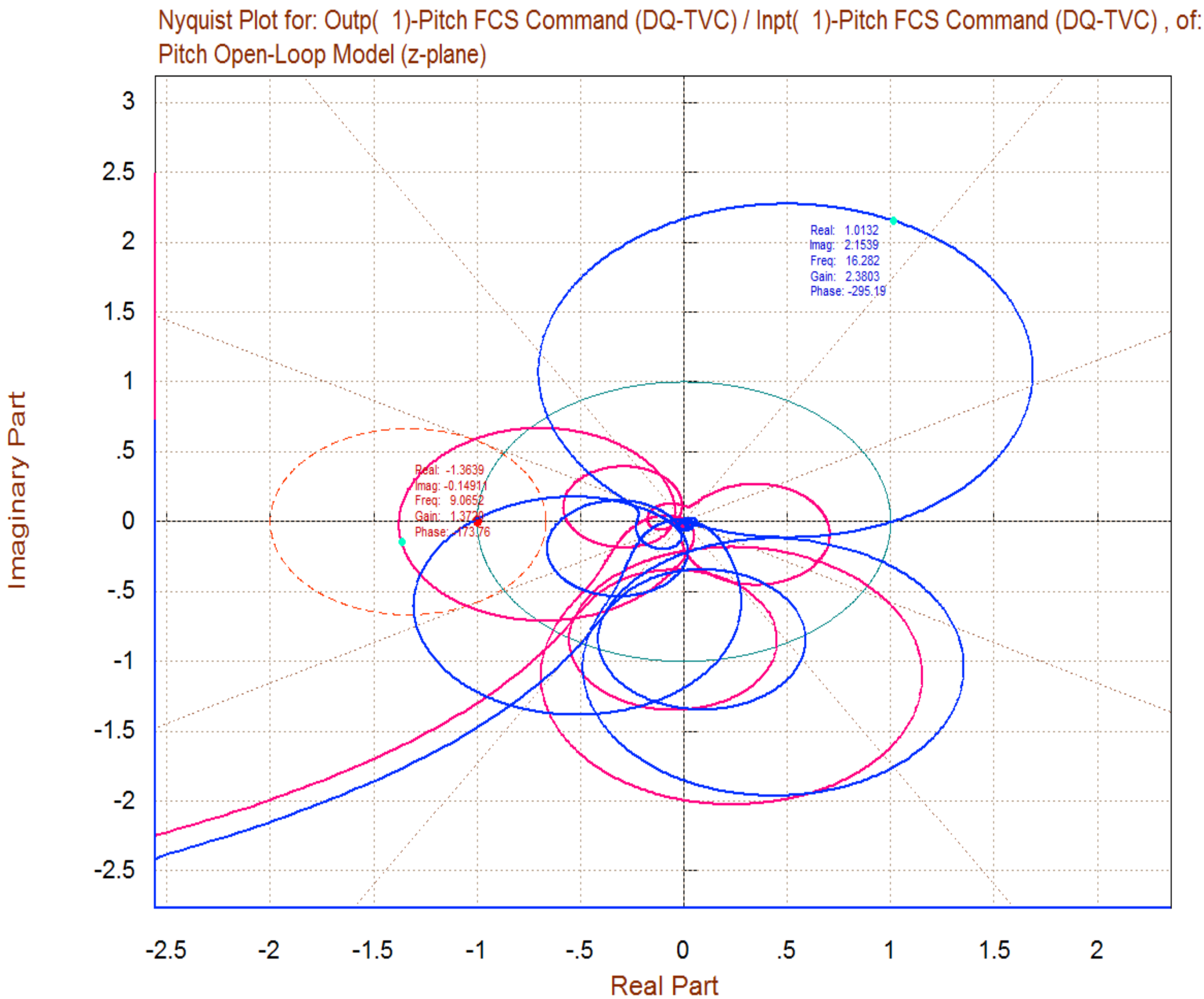


Bode Plot for: Outp(1)-Pitch FCS Command (DQ-TVC) / Inpt(1)-Pitch FCS Command (DQ-TVC) , of:
Pitch Open-Loop Model (z-plane)




Nichols Plot for: Outp(1)-Pitch FCS Command (DQ-TVC) / Inpt(1)-Pitch FCS Command (DQ-TVC) , of:
Pitch Open-Loop Model (z-plane)
Gain in (dB)
Phase (deg)
60
40
20
0
-20
-40
-60
-80
-1200
-1000
-800
-600
-400
-200
Freq: 16.283
Gain: 7.5309
Phase: -295.89
Freq: 9.0525
Gain: 3.4763
Phase: -111.99
Freq: 49.966
Gain: -23.431
Phase: -61.700


Nichols Plot for: Outp(1)-Pitch FCS Command (DQ-TVC) / Inpt(1)-Pitch FCS Command (DQ-TVC) , of:
Pitch Open-Loop Model (z-plane)
Gain in (dB)
Phase (deg)
80
60
40
20
0
-20
-40
-60
-900
-800
-700
-600
-500
-400
-300
-200
-100
0
Freq: 16.293
Gain: 7.3822
Phase: -305.34
Freq: 16.310
Gain: -3.8415
Phase: -42.048
Freq: 9.0757
Gain: 2.6829
Phase: -186.79
Freq: 9.0421
Gain: 2.8380
Phase: -102.49



Notice how the discrete system's response (magenta colour), although similar in nature, it has some additional phase-lag and attenuation which is more observable at high frequencies. It is caused mainly by the zero-order-hold. Notice also that one of the structural modes, at 9 (rad/sec) is unstable because it encircles the critical point. This is noticeable in both Nichols and Nyquist plots. If not attenuated, this structural mode will create divergent structural oscillations at 9 (rad/sec). This can be observed in closed-loop simulations.

Example 3 Describing Function Analysis

In this example we will use the Describing Function (DF) method to analyze the structural stability of the Space Shuttle payload. The Shuttle is carrying a flexible payload which is rigidly attached to the cargo bay at aft end. The front section of the payload, however, is attached to the cargo bay by means of two Coulomb Dampers which are non-linear devices, as shown in Figure 1. The purpose of the Coulomb Dampers is to attenuate oscillations and to reduce disturbances on the payload by dissipating structural energy.

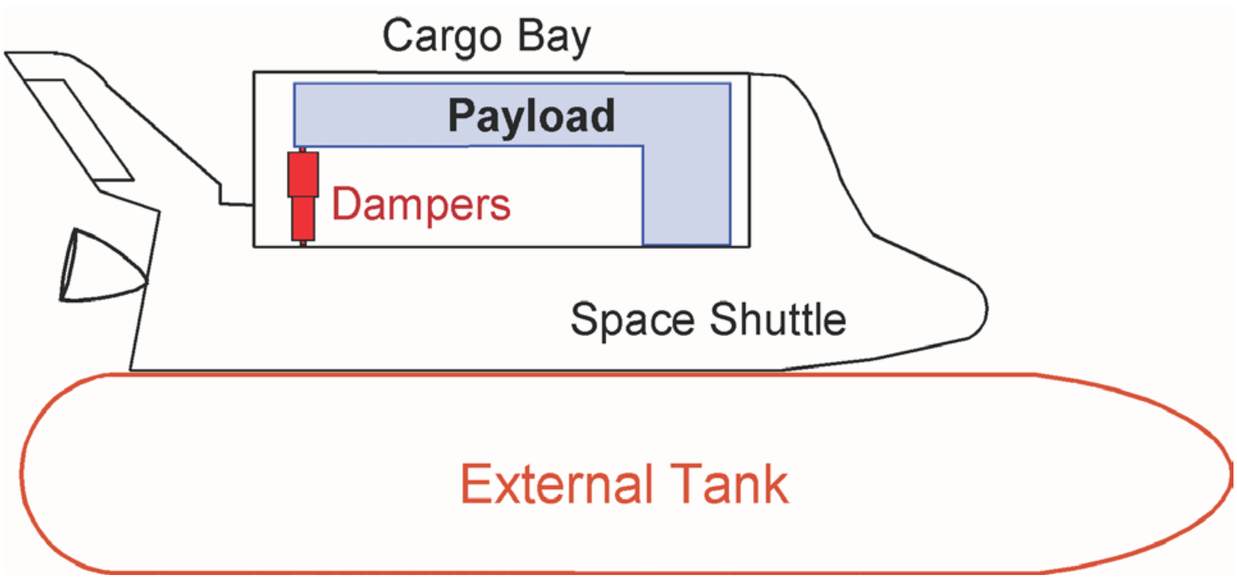


Figure 3.1 Payload is rigidly attached to the Shuttle on one side and on the other side it is attached to the Cargo Bay by two Coulomb Dampers

In this example we will analyze the dynamic interaction between the flexible structure and the non-linear Coulomb dampers by frequency domain analysis and simulation. We will use the Describing Function (DF) method to estimate the size and frequency of the limit-cycles caused by the non-linear devices, and simulations to validate the stability analysis. The files for this Shuttle second stage analysis example are located in directory "*C:\Flixan\Frequ\Examples\Ex3*". The input data file is "*Damper_DF.Inp*" and the systems file is "*Damper_DF.Qdr*" containing the vehicle state-space systems. The DF of the Coulomb damper is already calculated and saved in file "Coulomb.DF".

The systems file includes the system "*Pitch DF Analysis Model (s-plane)*". This system consists of the flexible vehicle, sensor and actuator dynamics and the flight control system with the 3 loops closed, as shown in the block diagram Figure 3.2. The mechanical loop, however, across the damper is opened for frequency response analysis. This system is used for analyzing stability in the pitch direction by exciting and measuring the structural modes symmetrically. The input is force applied across the left and right damper in (lb), and the output is the average displacement across the two dampers. A similar model is used for analyzing stability in the anti-symmetric direction by applying equal and opposite forces on the left and right dampers and measuring the differential displacements of the two dampers. This example is presented in more detail in "*Flixan\Examples\Payload Damper*".

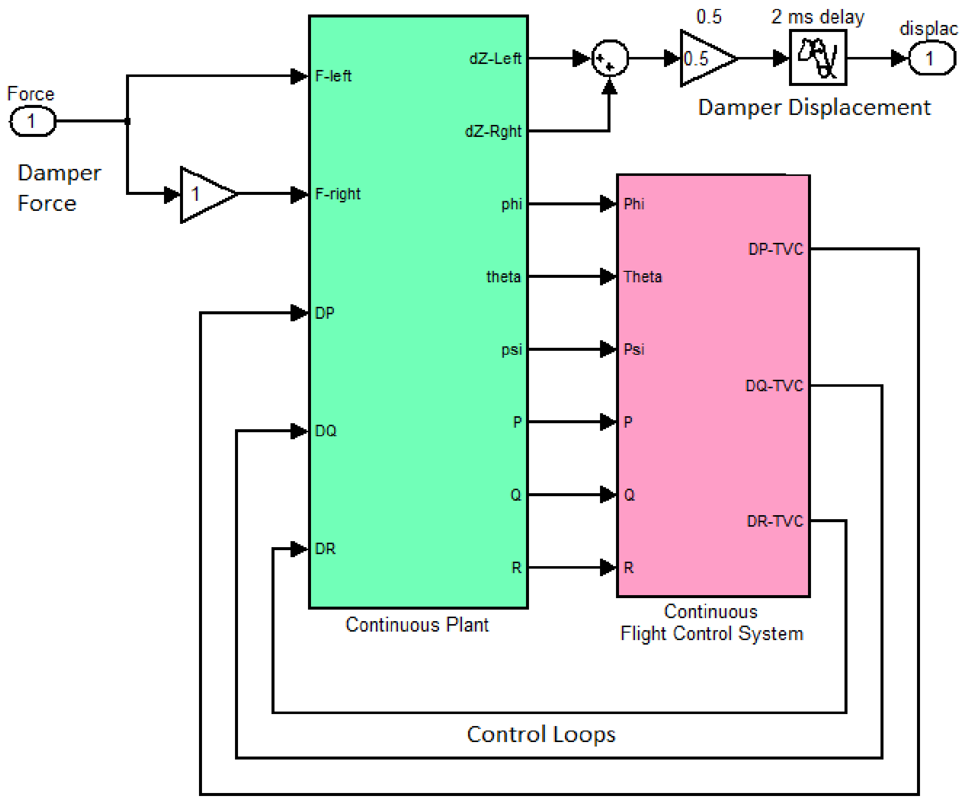
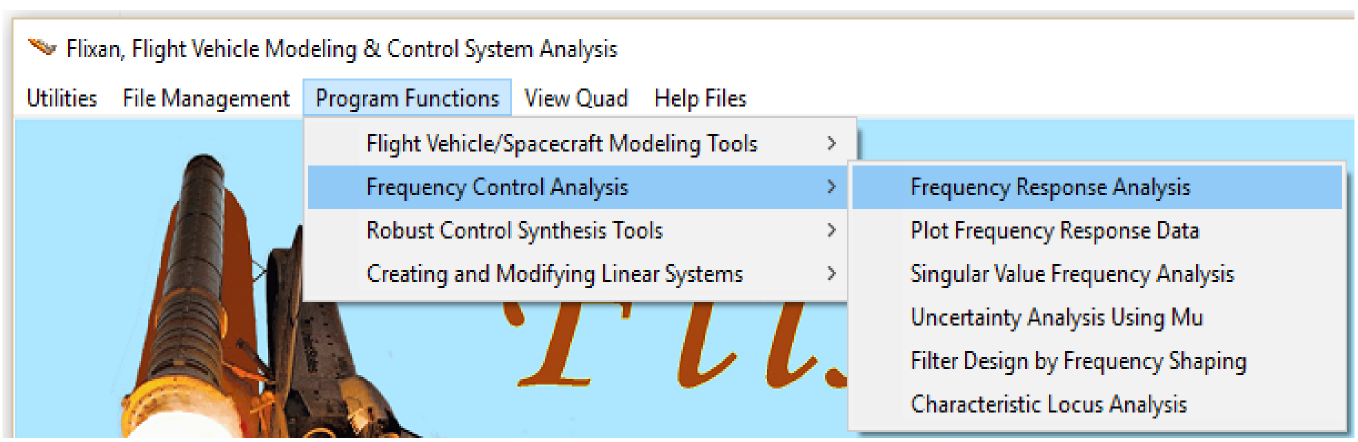
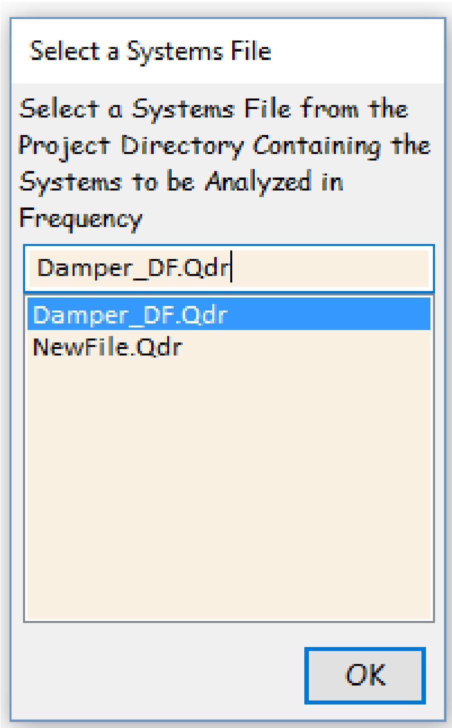


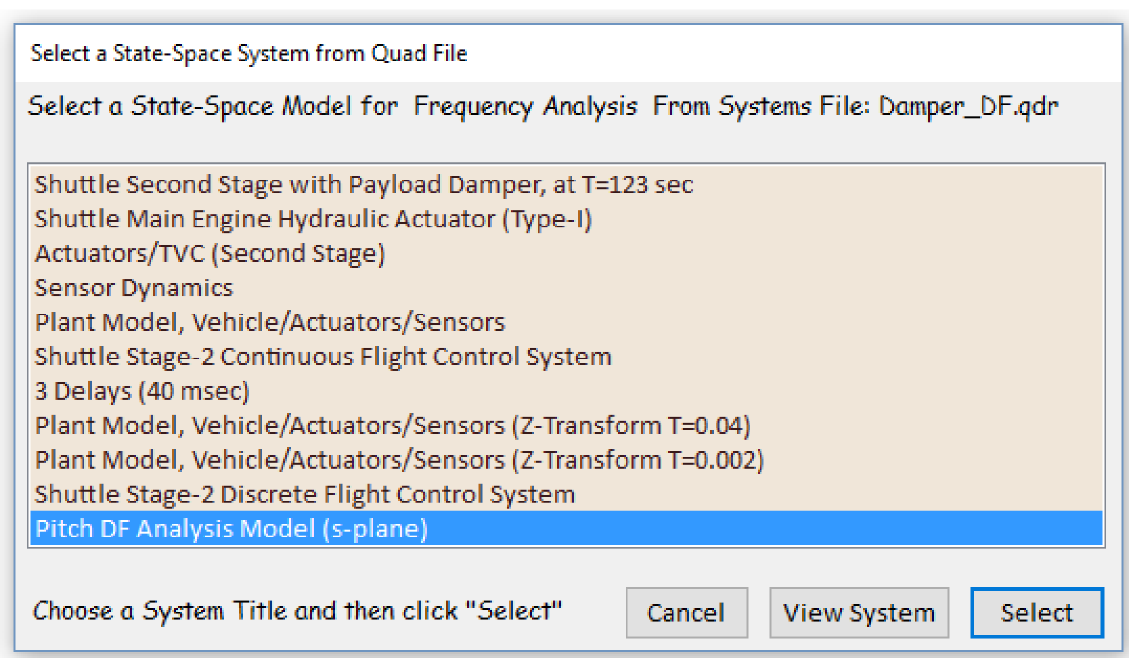
Figure 3.2 Pitch DF Analysis Model; The Flight Control Loop is Closed and the Loop is Opened across the Dampers

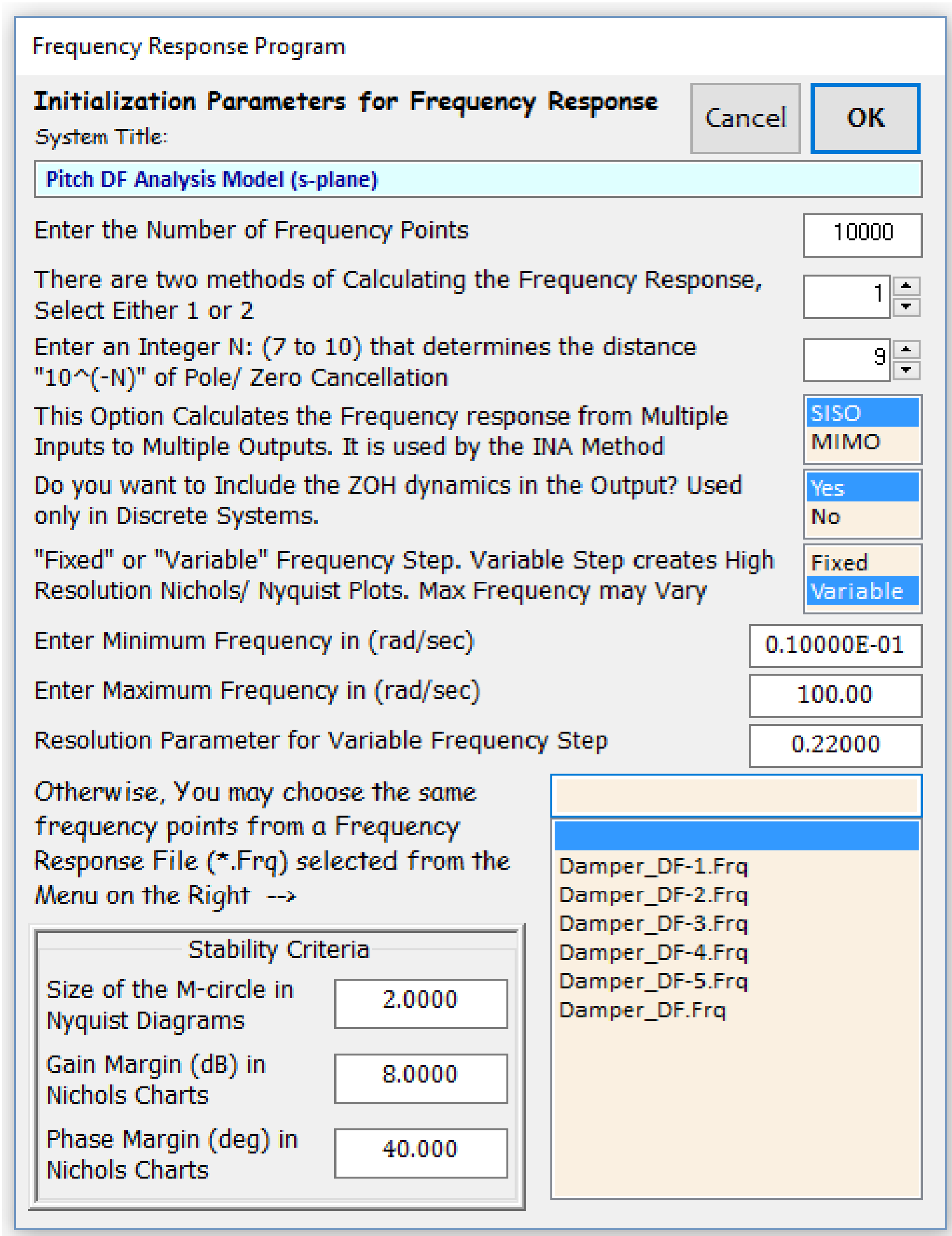
Start the Flixan program, select the project directory and from the main menu select "*Program Functions*", "*Frequency Control Analysis*", and then "*Frequency Response Analysis*". From the systems filename selection menu select the systems file "*Damper_DF.Qdr*", and click "OK".

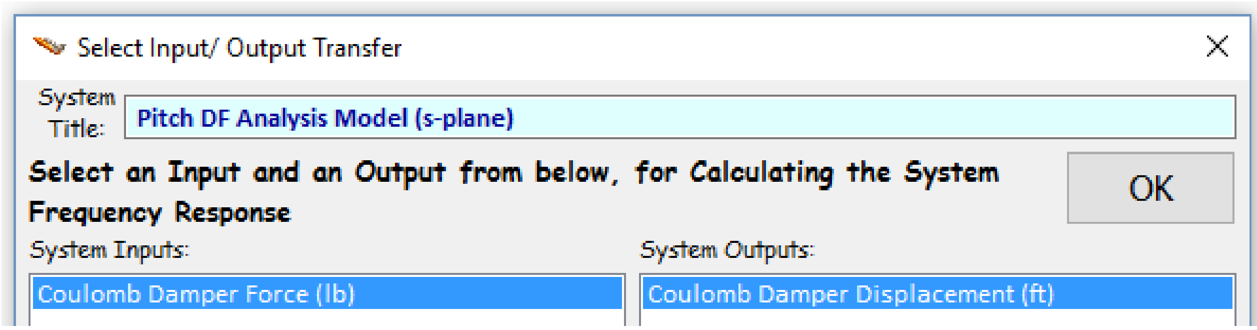


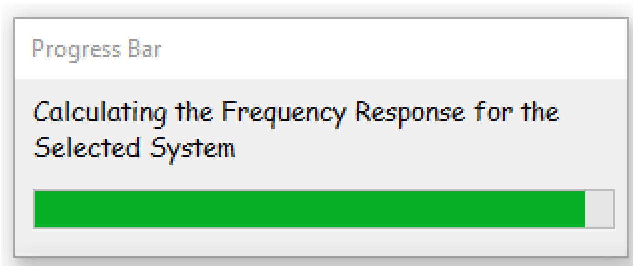


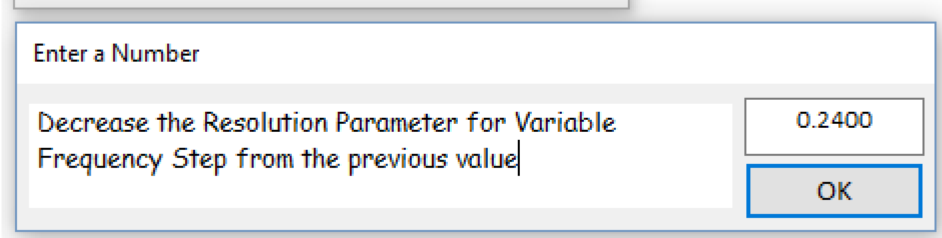
From the following menu select the system "*Pitch DF Analysis Model (s-plane)*". This system has the flight control loops closed, as already described in Figure 3.2, and the mechanical loop across the damper is opened for frequency response analysis. Click on "Select" and the parameters initialization dialog opens up, as shown below. Use the variable frequency step with 10,000 points to obtain smooth resonances. Then select the only one input and the only one output corresponding to the force on the vehicle generated by the damper and the displacement across the damper.

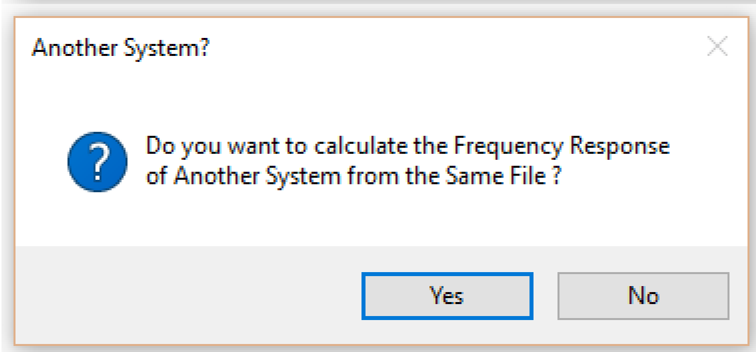



Frequency Response Program
Initialization Parameters for Frequency Response
System Title:
Cancel
OK
Pitch DF Analysis Model (s-plane)
Enter the Number of Frequency Points
10000
There are two methods of Calculating the Frequency Response, Select Either 1 or 2
1
Enter an Integer N: (7 to 10) that determines the distance "10^(-N)" of Pole/ Zero Cancellation
9
This Option Calculates the Frequency response from Multiple Inputs to Multiple Outputs. It is used by the INA Method
SISO
MIMO
Do you want to Include the ZOH dynamics in the Output? Used only in Discrete Systems.
Yes
No
"Fixed" or "Variable" Frequency Step. Variable Step creates High Resolution Nichols/ Nyquist Plots. Max Frequency may Vary
Fixed
Variable
Enter Minimum Frequency in (rad/sec)
0.10000E-01
Enter Maximum Frequency in (rad/sec)
100.00
Resolution Parameter for Variable Frequency Step
0.22000
Otherwise, You may choose the same frequency points from a Frequency Response File (*.Frq) selected from the Menu on the Right -->
Damper_DF-1.Frq
Damper_DF-2.Frq
Damper_DF-3.Frq
Damper_DF-4.Frq
Damper_DF-5.Frq
Damper_DF.Frq
Stability Criteria
Size of the M-circle in Nyquist Diagrams
2.0000
Gain Margin (dB) in Nichols Charts
8.0000
Phase Margin (deg) in Nichols Charts
40.000









Do not calculate any more frequency responses from other systems. From the following filename selection menus do not select another frequency response file, but select the non-linearity file "*Coulomb.DF*" that includes the DF data for the Coulomb damper and click on "OK". This file includes sinusoidal input amplitude in (feet) versus gain, and phase in (deg).

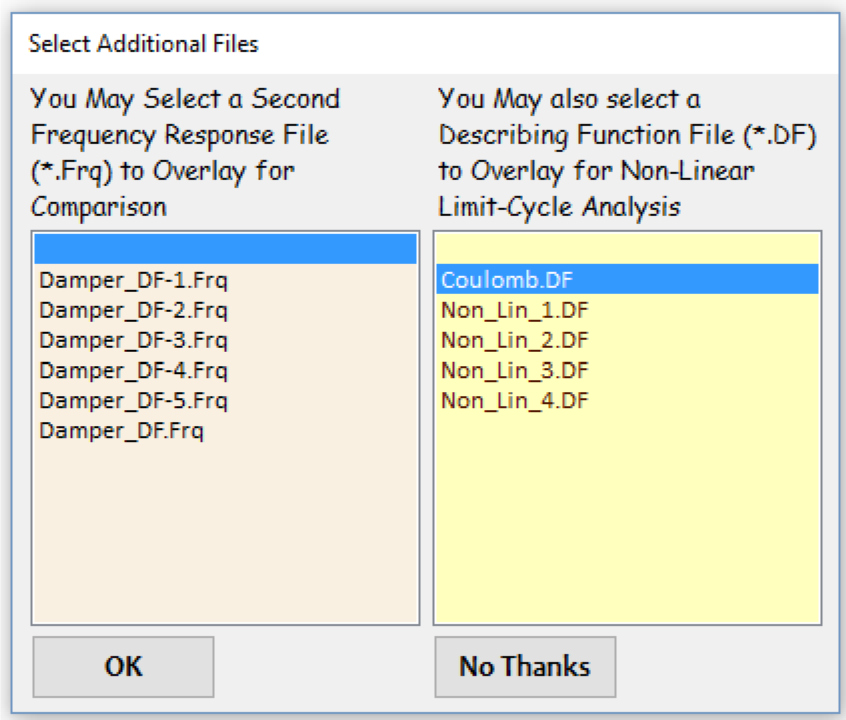


Figure 3.3a shows the Bode frequency response of the symmetric closed-loop plant $G_s(s)$ calculated across the Coulomb damper non-linearity. The excitation forces in (lb) are applied on the vehicle structure in phase at the two damper attachments in order to excite the pitch flexibility. The output of $G_s(s)$ is the average displacement at the two dampers. It shows that the structure has two big resonances at around 9 (rad/sec). The Nichols plot in Figure 3.3b shows the overlay of two loci, the symmetric system $G_s(j\omega)$ co-plotted with the inverse of the damper DF which is -1/N(a). The -1/N(a) locus is shown twice because it repeats every 360° similar to the (+) point in the classical Nichols charts. The intersections indicate that according to the Nyquist criteria there is a convergent limit cycle at 10.5 (rad/sec) indicating that the system will maintain a symmetric oscillation at 10.5 (rad/sec) at an amplitude of 0.004 (ft). There is also a convergent limit-cycle at a lower amplitude of 0.0013 (feet) which is at 9.78 (rad/sec) frequency. This intersection point is in the vertical ±180° section of the 1/N(a) describing function locus which corresponds to the device operating in the spring/ dead-zone region before breakout. The amplitude of the Coulomb damper oscillation (zero to peak) is obtained from the amplitude of the DF locus at the intersection with the plant $G_s(j\omega)$ locus. There is also a divergent (shown in green) intersection point between the two convergent limit-cycles. The loci are almost touching at a higher frequency which indicates the possibility of a limit cycle at 24 (rad/sec). However, it is of low amplitude and its existence is questionable.

The same conclusions are obtained by analyzing the Nyquist diagram in Figure 3.4. The region around the intersections between the loci is expanded in Figure 3.4b to highlight the convergent and divergent limit-cycles. The existence of the limit-cycles is confirmed by time-domain simulations. For more details read the "Shuttle Coulomb Damper" example in directory "*Flixan\Examples\Payload Damper*".

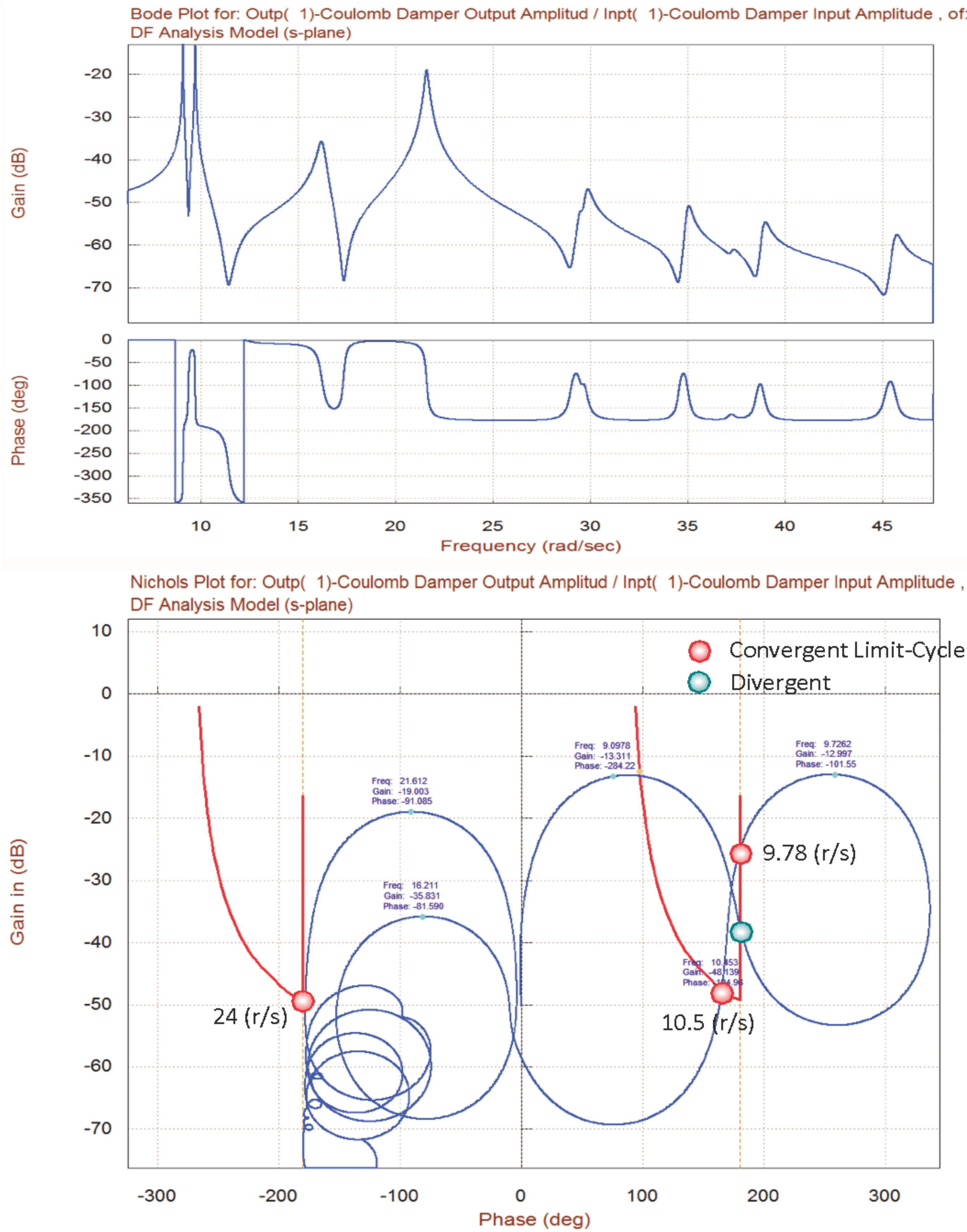


Figure 3.3 Bode and Nichols Plots across the Coulomb Damper Non-Linearity

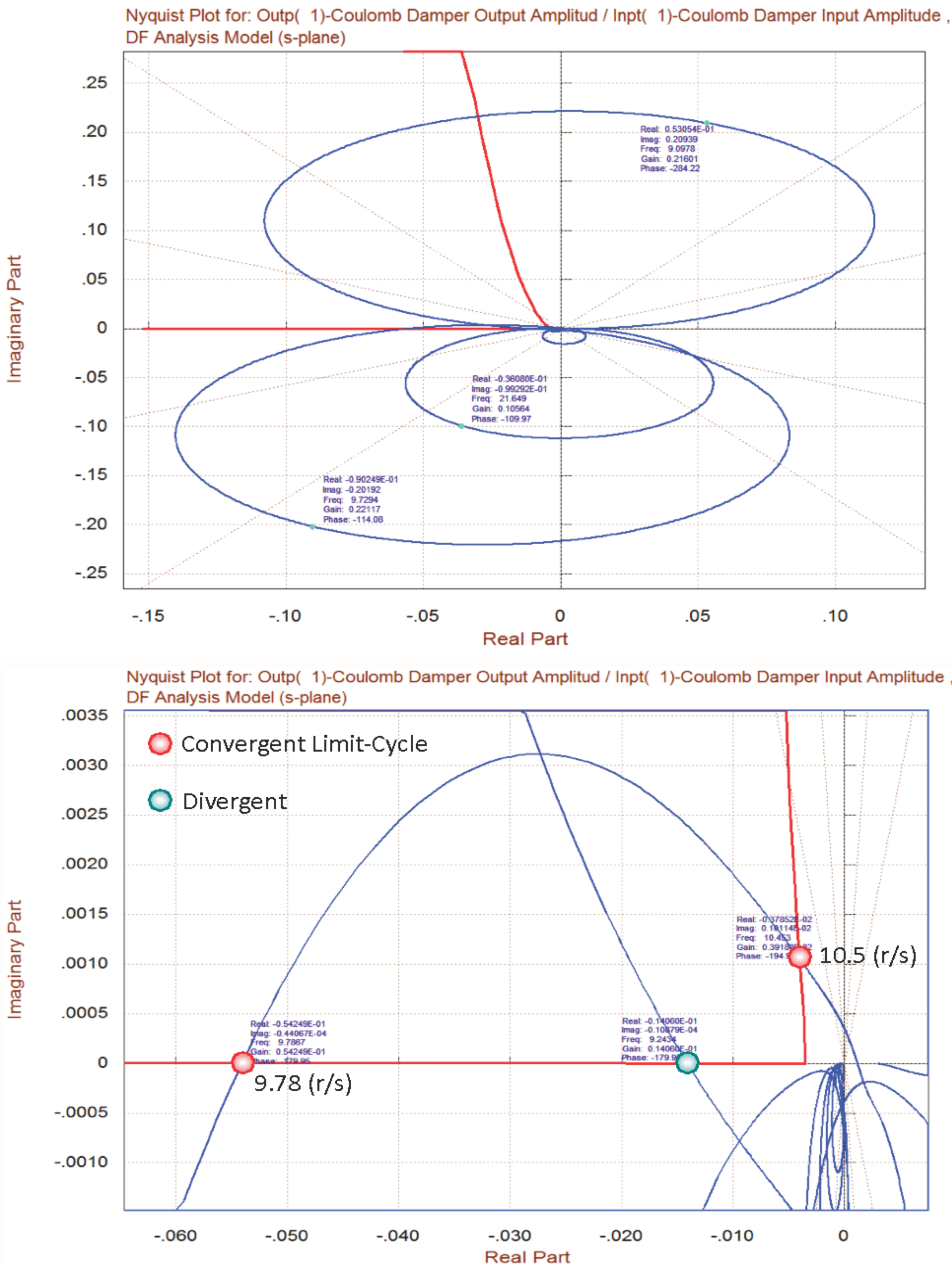


Figure 3.4 Nyquist Plot across the Coulomb Damper Non-Linearity