

4 Actuator Models

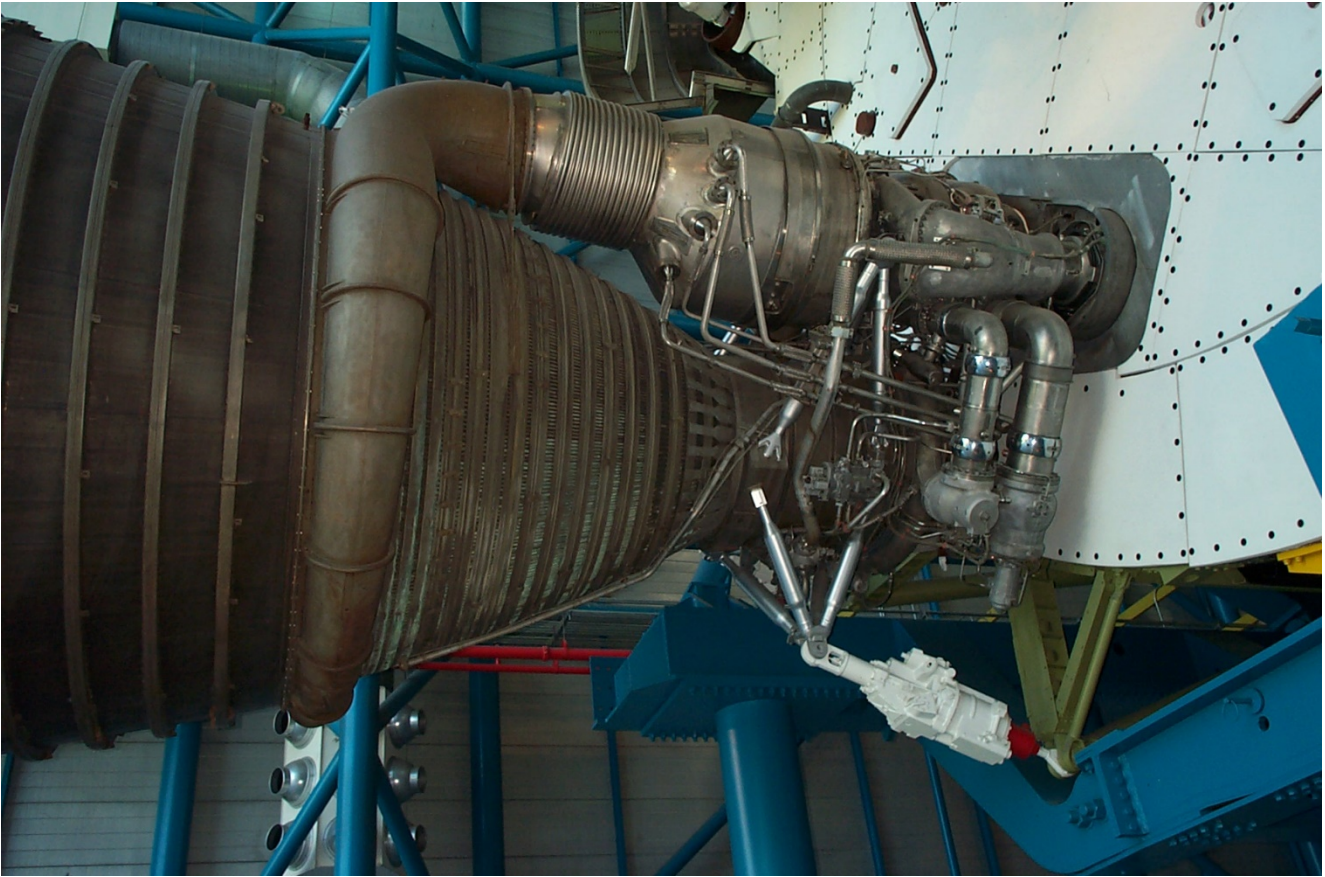
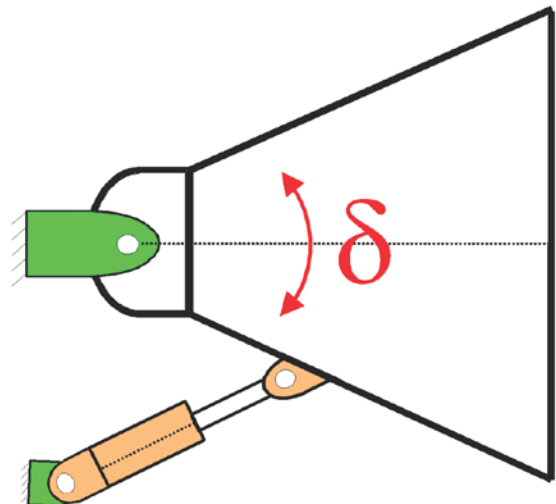


Figure 4.1 TVC engine for a Saturn-5 rocket

Flight vehicles are controlled by Thrust-Vector-Control (TVC) engines that rotate about a gimbal or by control surfaces rotating about a hinge. The purpose of the servo-actuator system is to provide the control force that is needed to rotate an engine or aerosurface in the direction required to maneuver and to stabilize the vehicle. One side of the actuator attaches to the vehicle and the other side which extends is attached to the rotating effector. Since the actuator dynamics greatly affect and couple with the vehicle motion we will dedicate this section to develop actuator models and provide some examples. In launch vehicles, a

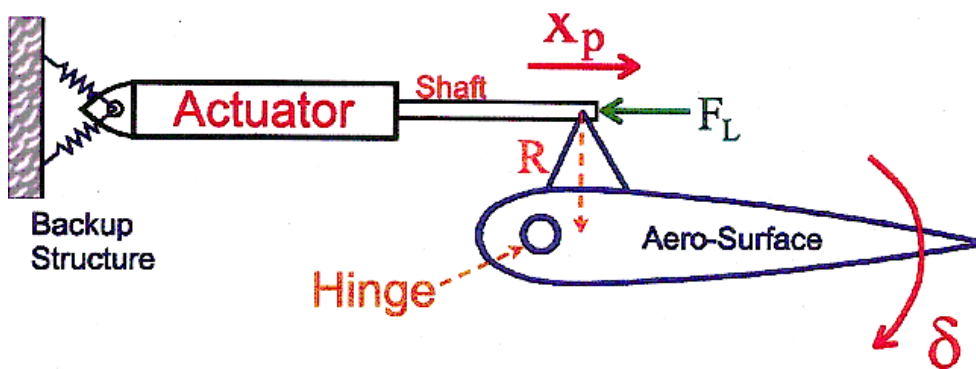


small rotation of the thrust vector angle about the gimbal (typically between $\pm 5^\circ$ and $\pm 10^\circ$ in pitch and in yaw) is sufficient to create normal and lateral control forces to stabilize and steer the vehicle, overcome the wind-gust disturbances, and to balance the aerodynamic moments. In an aircraft the control surface deflections are larger, in the order of $\pm 40^\circ$ relative to the trim position. The control surface is normally the trailing edge of a wing, tail or vertical stabilizer that rotates about a hinge line which is parallel to the

aerosurface and the actuator provides the force to rotate it against the aerodynamic loads. The rotation of the aerosurface changes the airflow around the aircraft and creates the aerodynamic forces and moments needed to trim and to control the flight vehicle. In some cases, it is the leading edge that rotates, and in some aircraft, it is the entire aerosurface that rotates and not just the edge.

Figure 4.1 shows a TVC nozzle of a Saturn-5 rocket connected to the vehicle via the gimbal. The thrust force is transmitted to the vehicle through the gimbal which allows the engine to rotate in pitch and yaw. The actuator is the white hydro-mechanical servo device that provides the force to control the deflection δ of the engine in one direction. A TVC engine is typically controlled in two directions by rotating it in pitch and yaw by two orthogonal actuators. One end of the actuator is attached to the vehicle and the other end is an extendable shaft that is attached on the pivoting nozzle via a linkage, as shown in the Saturn-5 Figure 4.1. The extension of the actuator shaft can be varied and it is controlled by a closed-loop servo system that provides hydraulic or electro-mechanical forces on the shaft to rotate and to control the nozzle deflection angle. Each actuator has a sensor that measures the shaft extension and provides feedback to the position control system. The angular rotation of the nozzle is almost proportional to the piston extension, although, not exactly because the measurement is corrupted by the bending and deformation of the support structure, as we shall see. During flight the control system calculates the required changes in the engine's pitch and yaw rotations and the actuators are commanded to either extend or to retract in order to achieve the demanded nozzle deflections and the control forces to guide and stabilize the vehicle.

Figure 4.1b shows an actuator system for an aerosurface. The shaft extension provides the force to rotate the aerosurface about the hinge line. The actuator has to counteract the external forces resisting the motion by providing a load-force F_L against it. The external forces are mainly due to aerodynamics, but also inertial due to vehicle normal acceleration and vibrations. The actuator extension x_p is used to calculate the aerosurface rotation. The measurement, however, is slightly corrupted due to deformation of the support structure, shown by the springs.



The selection of an actuator device is determined mainly by the power requirement of the load. The power is determined by the aerodynamic forces and the speed of response. A flight vehicle is easier to be controlled when its static stability is marginal. If the vehicle is passively unstable and diverges too fast then the actuator has to respond fast enough in order to catch up and prevent the instability. If, on the other hand, the vehicle is too stable it becomes less maneuverable and requires a lot of actuator power to steer it. Other factors to be considered for the selection of an actuator include the dynamic characteristics, the power sources available, the reliability of the equipment, and other physical and economic limitations. In this section we will present five actuator models which are typically used in flight control system analysis to pivot the aerosurfaces or TVC nozzles. The actuator models not only

include the shaft motion but also the rotational dynamics of the load and the structural stiffnesses that couple the two dynamic systems together. That is, stiffnesses of the support structure, shaft and load linkage. The shaft position is controlled by a local servo system inside the actuator that receives commands from flight control. The actuator models capture the response of the servo-actuator to commands and the engine or aerosurface rotation at the hinge and the interaction between the two dynamic systems via forces and gear torques.

The actuator models are not only intended to be used alone but also to be coupled with the vehicle flex models and analyze the combined vehicle/ actuator system stability and performance. This is in order to analyze the effects of the "tail-wags-dog" forces on the vehicle at the gimbal produced by the rotational acceleration of the engines and aerosurfaces and also the loading effects against the actuator forces "load-torque" dynamics generated by the vehicle motion. The combined vehicle/ actuator dynamic model must include one actuator per vehicle effector input. The dynamic coupling between vehicle and actuator dynamics must be implemented properly in order to capture the TWD and load-torque dynamic effects. It defines the dynamic interaction between actuators, vehicle dynamics and local structural flexibility at the hinges that affect the local structural stability and actuator performance at the engines or aerosurfaces. The interaction between non-linearities, such as: dead-bands, limits, and Coulomb friction with stiffnesses in the actuator support structure often causes "tail-wag-dog" type of aerosurface oscillations or limit-cycles, if the actuator control system is not properly designed. It is a good practice to keep the engine or aerosurface local bending mode above the TWD zero frequency. We shall demonstrate the dynamic coupling between the vehicle and the actuator systems with examples and will describe the associated stiffnesses. The models are implemented using the Flixan actuator modeling program located under the "*Flight Vehicle Modeling*" group. Each vehicle aerosurface or TVC engine DOF requires a separate actuator model but the actuator models are frequently the same. In order to properly couple the vehicle and actuator dynamics including the TWD/ LT effects the vehicle model must be configured to include the "tail-wags-dog" flag "WITH TWD" that will include the dynamic interaction between local flexibility, actuator dynamics and the flight control system.

The dynamic models require actuator data consisting of parameters such as: piston area, load moment of inertia, amplifier gains, friction coefficients, gear ratios, piston and backup stiffnesses, local geometry, etc. The equations of motion for the five linear actuator types are internally coded in the Flixan actuator program which is similar to the flight vehicle modeling program. It reads the actuator parameters from an input file (.Inp), processes the data, and generates different types of actuator state-space systems in a systems file (.Qdr). The actuator systems have two inputs: (1) engine deflection command δ_c in (radians) coming from the flight control system, and (2) load-torque T_L in (ft-lb), which is an external torque on the actuator generated by aero or vehicle acceleration. The actuator models have three outputs that become inputs to the vehicle dynamic model: (1) effector rotation angle (δ) in (radians), (2) effector rate in (rad/sec), and (3) acceleration in (rad/sec²). Definitions of the inputs, states, and output variables are included below the system matrices in (.Qdr) file. In Section (4.3) we will describe the five standard actuator types and demonstrate the operation of the actuator program with examples, but before we get into the actuator details let us first describe how the actuators are combined with the vehicle model.

4.1 Combining a Flexible Vehicle Model with Actuators

In Flixan the actuator systems are not included in the vehicle equations of motion but they are implemented and analyzed as separate subsystems and are eventually combined with the vehicle dynamics. This offers flexibility in selecting different types of actuators, adjusting the complexity of the model, allows you to include or remove actuator non-linearities in simulations and to analyze the actuator subsystem separately. The actuator models are not just actuator but they include the rotational dynamics of the effectors and the associated stiffnesses of the support structure, actuator/ load linkage, and the compressibility of the shaft. Factors such as, hydraulic fluid compressibility, gimbal friction, nozzle or control surface flexibility, backup structure stiffness, load inertia, non-linearities, and other parameters should be accounted correctly in the actuator model.

The rotational dynamics of the effector load about the hinge is not in the vehicle model but in the actuator. The vehicle model receives the rotational position in (rad), velocity (rad/sec), and acceleration signals from the actuators and calculates the control and reaction forces from the actuator signals. Note, the stiffnesses in the actuator model must be properly accounted depending on the flexibility definition in the vehicle model. If the flexibility of the support structure is already included in the vehicle structural model, it should obviously not be included again in the actuator subsystem. The actuator output signals are not inertial but they represent the relative motion of the effector at the hinge or gimbal, with or without local flexibility.

Figure 4.1.1 shows the interconnection between a flight vehicle and three actuators. The vehicle is controlled by three aerosurfaces (elevon, aileron, and rudder) and each aerosurface is controlled by a separate actuator. The mixing-logic matrix receives roll, pitch and yaw acceleration demands from the flight control system and converts them to elevon, aileron and rudder actuator commands. The aerosurface deflection, rate, and acceleration outputs from the actuators drive the vehicle dynamics. The vehicle, mixing logic matrix, and actuator subsystems are combined together using the Flixan® systems interconnection program or by using Matlab/Simulink®. The aerosurface deflections generate the aero forces (or in the case of a gimbaling engine the TVC forces) that control the vehicle and the accelerations generate the tail-wag-dog forces. The rates create damping forces which are not frequently included.

The actuator subsystems have a second load-torque input to receive external load torques. These are reaction torques generated by external aero forces and vehicle accelerations that close a mechanical feedback loop between vehicle and actuator, see Figure (4.1.1). The load-torques typically oppose the actuator control torques and the actuator must overcome it in order to control the position of the aerosurface or engine. The vehicle model includes load-torque outputs at each hinge. In the case of a TVC engine the vehicle model provides two load-torque outputs for pitch and yaw gimbaling that feed-back to two separate actuators.

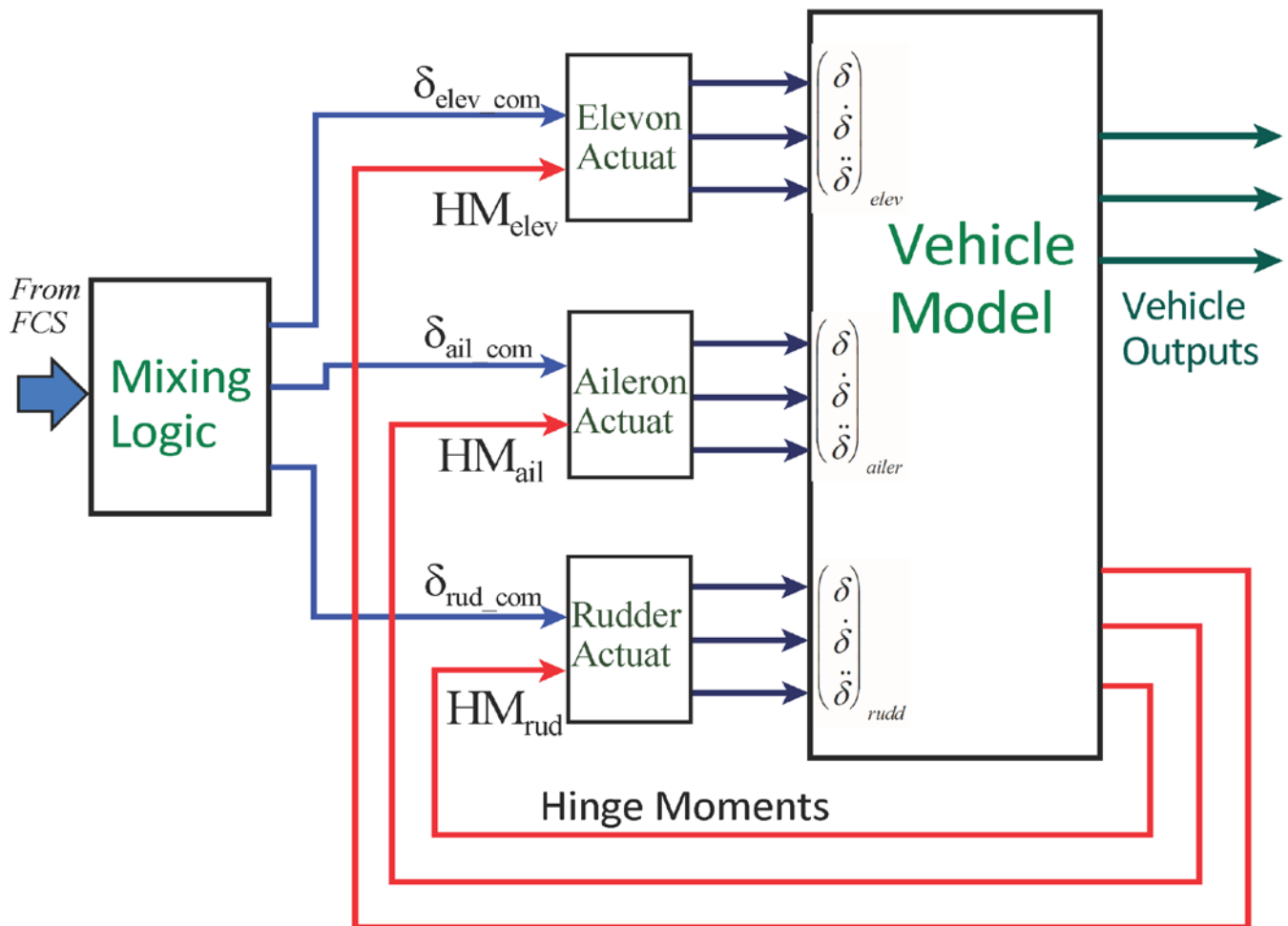


Figure (4.1.1) Vehicle and Actuator Interconnection Diagram

The analyst must be careful in selecting the stiffness coefficients in the actuator model in order to avoid including them twice, both in the actuator and in the structural models. There are three types of stiffnesses involved in modeling the servo system that controls the position of a control surface or a nozzle relative to the vehicle and they add up together in series to produce a local mode at the hinge:

- (1) The stiffness of the backup structure which is the stiffness of the support structure at the point where the actuator is attached to the vehicle.
- (2) The actuator shaft stiffness consisting of piston plus oil or electrical stiffness, and
- (3) The load linkage stiffness due to structural flexibility at the linkage and the aerosurface itself or the engine nozzle.

The three stiffnesses combine together to a combined stiffness K_T that in combination with the load moment of inertia produces a local mode. The frequency of this mode must be higher than the actuator bandwidth to avoid exciting it or destabilizing the mode. When analyzing rigid vehicle models, the backup and load stiffnesses are obviously not included in the vehicle and, therefore, all three stiffnesses must be included in the actuator model and in this case the actuator model itself captures the local resonance of the load oscillating at the pivot. The deflection δ , rate, and acceleration outputs consist of, not only rigid rotation, but also the effects due to the combined spring constant K_T from all three stiffnesses.

The position feedback measurement in the actuator control loop is the shaft extension. However, the position sensor measurement includes an error due to the shaft deformation. The measurement does not see the deformation of the backup structure and load. Therefore, all three stiffnesses should be included in the actuator model when: (a) the vehicle model is a rigid body, or (b) vehicle includes only body bending but not local flexibility at the actuator attachments, that is, the back-up structure and the actuator linkage stiffnesses are “rigid” in the finite-elements-model. When the flex vehicle model, however, includes the local structure compliances, they should not be included again in the actuator, otherwise, they are introduced twice. In this case, the actuator model must be stiff and only the shaft or piston stiffness should be included in the model. This type of modeling is important for analyzing the stability and performance of the actuator loop because the flight control system interacts strongly with the actuator dynamics, the load dynamics, and the backup structure, and they often excite each other to instability, mostly due to flexibility and non-linearities, such as, Coulomb friction, backlash, dead-bands, etc.

In the situation where the entire vehicle structure is included in the finite element model, that includes: the aero-surfaces, nozzles, and actuator linkage dynamics. In this case the backup and load stiffnesses are included in the structural model and they must not be included again in the actuator. The hinges or gimbals in the structural model are locked, and it is assumed that there is a rigid link across the two ends of the actuator. In the actuator model the backup and load stiffnesses must be set to infinity and only the actuator piston stiffness should be included. The locked hinges are released in the simulation equations by the actuator models that calculate the rigid rotations, rates, and accelerations for the vehicle equations. The flexing at the hinge due to the effector motion is calculated from the inertial coupling coefficients $h_s(k,j)$, according to equations 2.7.4. The actuator models in this case must be “stiff” because they must produce only the “rigid” component of the deflection resulting from the linear extension of the shaft. The deflection due to structural bending at the hinge is already included in the finite element model. Only the actuator shaft stiffness in series with the oil stiffness (for hydraulic actuators), or the shaft stiffness in series with the electrical stiffness (for electro-mechanical actuators), that is K_{act} instead of K_T , should be included in the “stiff” actuator model. In the stiff actuator model in Figure 4.3.1b, the backup and load stiffnesses are infinite, because it is assumed that they are included in the structural model. The feedback path via $(1/K_L)$, shown in Figure 4.3.1a becomes zero. The shaft stiffness K_{act} is much stiffer than K_T which is included in the “soft” actuator model.

4.2 Introducing a Compensator

An actuator is a closed-loop position control servo. It is an independent system that controls the position of the effector but also interacting with the flight control system. This local control system must be properly stabilized and perform as expected within the required bandwidth with a satisfactory step response. The dynamic coupling, however, between the actuator, the load and the support structure flexibility introduces a local mode that may degrade the servo system's performance, stability, and it often produces local instability. The actuator servo systems, therefore, often include first or a second order filter compensators $C(s)$ in the forward or feedback path to enhance the actuator loop stability and improve command following. The actuator modeling program provides the capability to include a first or a second order compensator which is defined in the actuator input data: (1= first order, 2= second order, 0=No compensation). The filters are unit gain, lead-lag, 1st or 2nd order transfer functions defined as follows:

$$\text{First Order } C(s) = \frac{V_{ef}(s)}{V_e(s)} = \left(\frac{\tau_n s + 1}{\tau_d s + 1} \right)$$

$$\text{Second Order } C(s) = \frac{V_{ef}(s)}{V_e(s)} = \frac{\omega_d^2 \left(s^2 + 2\zeta_n \omega_n s + \omega_n^2 \right)}{\omega_n^2 \left(s^2 + 2\zeta_d \omega_d s + \omega_d^2 \right)}$$

The above transfer functions are implemented in state-space form and are combined with the actuator dynamics. The first order compensator is expressed by the following first order differential equation, where V_e is the filter input and V_{ef} is the filter output.

$$\tau_d \dot{x} = -x + \left[1 - \frac{\tau_n}{\tau_d} \right] V_e \quad V_{ef} = x + \left(\frac{\tau_n}{\tau_d} \right) V_e$$

The second order filter is expressed by the following state-space representation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -a_2 \\ 1 & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_2 \\ b_1 \end{bmatrix} V_e \quad V_{ef} = \begin{pmatrix} \omega_d^2 \\ \omega_n^2 \end{pmatrix} \left\{ \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + V_e \right\}$$

where

$$a_1 = 2\zeta_d \omega_d; \quad a_2 = \omega_d^2; \quad b_1 = 2(\zeta_n \omega_n - \zeta_d \omega_d); \quad b_2 = \omega_n^2 - \omega_d^2$$

- The first order filter is specified in the input data by the numerator and denominator time constants (t_n and t_d).
- The second order filter is specified by the numerator and denominator damping coefficients, and by their frequencies in (rad/sec), ($\zeta_n, \omega_n, \zeta_d, \omega_d$).

4.3 Actuator Models

In this section we present equations of motion that describe the dynamics of some commonly used actuator models which are implemented in the Flixan actuator program. Examples of each model are also presented. The nomenclature used between actuator models, such as, moments of inertia, stiffnesses, etc. may be different. The actuators most commonly used in flight vehicles for positioning aerosurfaces and engine nozzles fall into three categories: (a) electro-hydraulic, (b) electro-mechanical, and (c) pneumatic. Pneumatic have limited usage due to low bandwidth and lack of fine positioning accuracy and they are not included here. We begin with a very simple generic model where most of the servo system details are simplified. It captures, however, the main functions which are: actuator shaft positioning servo, load dynamics, and stiffness of the actuator and the supporting structure. This model is used in the early phases of a flight vehicle design when the actuator details are not yet fully defined. It is simple because the servo loop dynamics is reduced to a first order transfer function driving the load dynamics. The next two models are hydraulic actuators taken from two separate sources and the last two are electro-mechanical (EMA). The parameters of these standard models are read by the program from an input data file and they can be adjusted to fit most actuator systems.

Electro-hydraulic actuators are most commonly used for launch vehicle thrust-vector-control, and have gained the widest acceptance in the aerospace industry. They have great power capability and can deliver larger torques than electrical equipment of comparable size and weight. For continuous operation they offer a minimum (equipment/power) ratio. Where intermittent operation is required, a hydraulic system can provide large amount of power from a small volume of accumulator. Their dynamic characteristics are expressed by small time constants and they develop higher peak torque per inertia ratios. The most common form of utilization of hydraulic servos in vehicle control loops consists of high-pressure supply (pump), an electro-hydraulic servo valve, a hydraulic actuator (cylinder), a feedback transducer and a servo-amplifier. The hydraulic power supplies currently used are of two main types. The first type employs a variable displacement pump whose output flow is controlled by means of a servo sensing the high-pressure side of the hydraulic system. A relief valve is also connected from the high-pressure side to the low-pressure side to minimize pressure transients above the operating pressures of the system. For normal operations the valve remains closed, opening only when the pressure exceeds a value overcoming the pre-load on the relief valve. The second type of power supply uses a fixed displacement pump with a relief valve to maintain the supply pressure within set limits, as well as to meet the normal flow requirements. In this system the relief valve is normally open so that supply pressure and valve opening maintain flow through the relief valve equal to the flow output of the fixed displacement pump. When there is a flow demand the relief-valve closes and the supply pressure is reduced. The dynamics of both power supply and relief valve exhibit a fairly flat response with small phase shift within the bandwidth of the overall servo loop, therefore, the supply pressure will be assumed to be constant at the value of zero flow demand. Electro-hydraulic valves are designed for flow or pressure control. These units are highly complex devices and exhibit high order non-linear responses. Still, in the frequency range of interest, they can be represented by a first or second order transfer functions.

The Electro-Mechanical actuators (EMA) are becoming more popular mainly because of simplicity, reliability and reduced need for maintenance in comparison with the hydraulic systems. The electro-mechanical actuator system is powered by a dc motor. The motor is driving a screw gear through a gear reduction that increases the screw torque. The spiral screw is rotating inside the shaft which surrounds the screw like a nut. When the screw mechanism rotates it converts the rotational motion from the motor to translation that extends or retracts the shaft as it pushes against the load. The end of the shaft is connected to the load via a linkage mechanism that can make it pivot about a hinge.

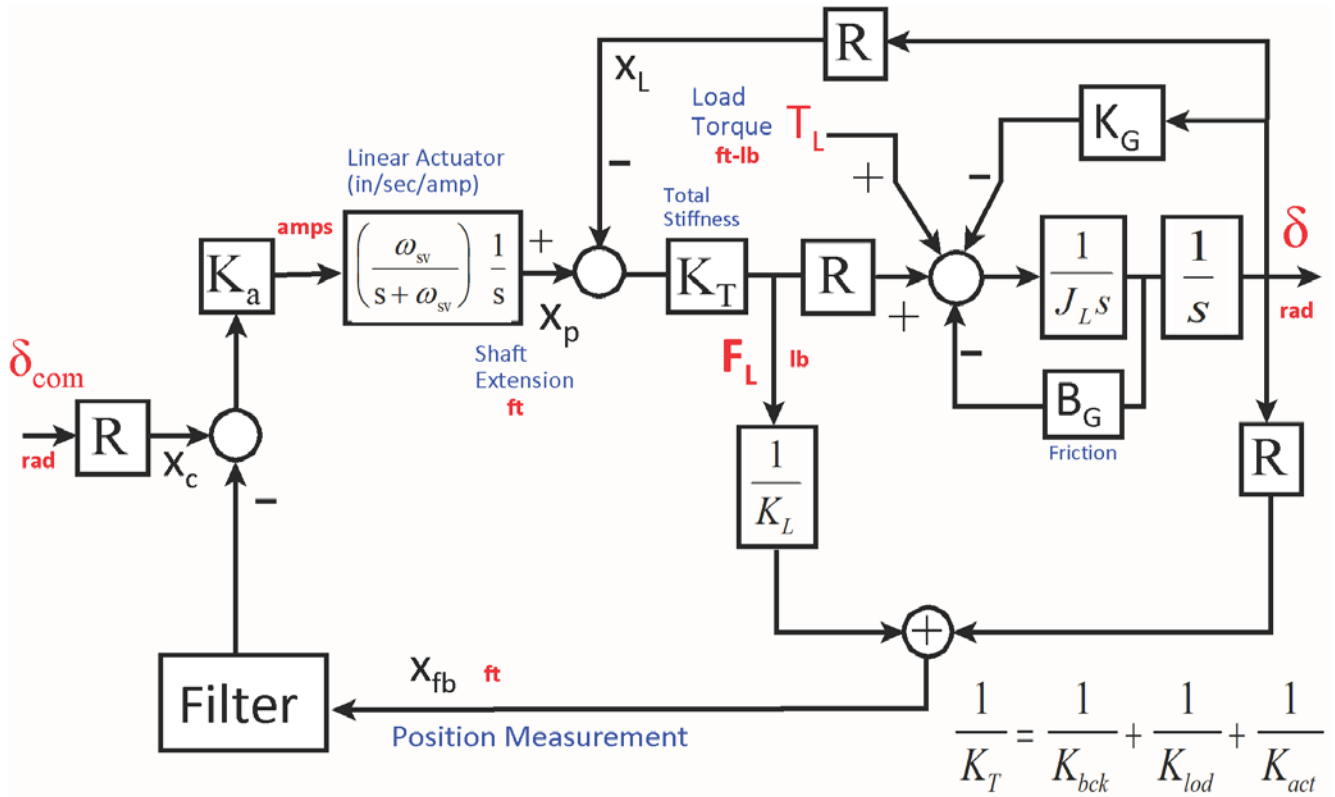
4.3.1 Simple Actuator Model

A simple actuator model is shown in figure 4.3.1. The input is gimbal deflection command δ_{com} in (rad) which is converted to shaft extension command x_c in (feet). On the left side we have the actuator device which is a linear velocity control servo that converts the position error to shaft velocity v_p which is integrated to calculate shaft position x_p . The bandwidth of the linear actuator is ω_{sv} represented by a first order transfer function. The input to the actuator device is amps coming from the feedback error amplifier that has gain K_a which is converted to shaft velocity v_p and integrated into shaft position X_p . The shaft is pushing against the total system stiffness K_T and creates a shaft force F_L . K_T consists of three type of stiffnesses combined in series:

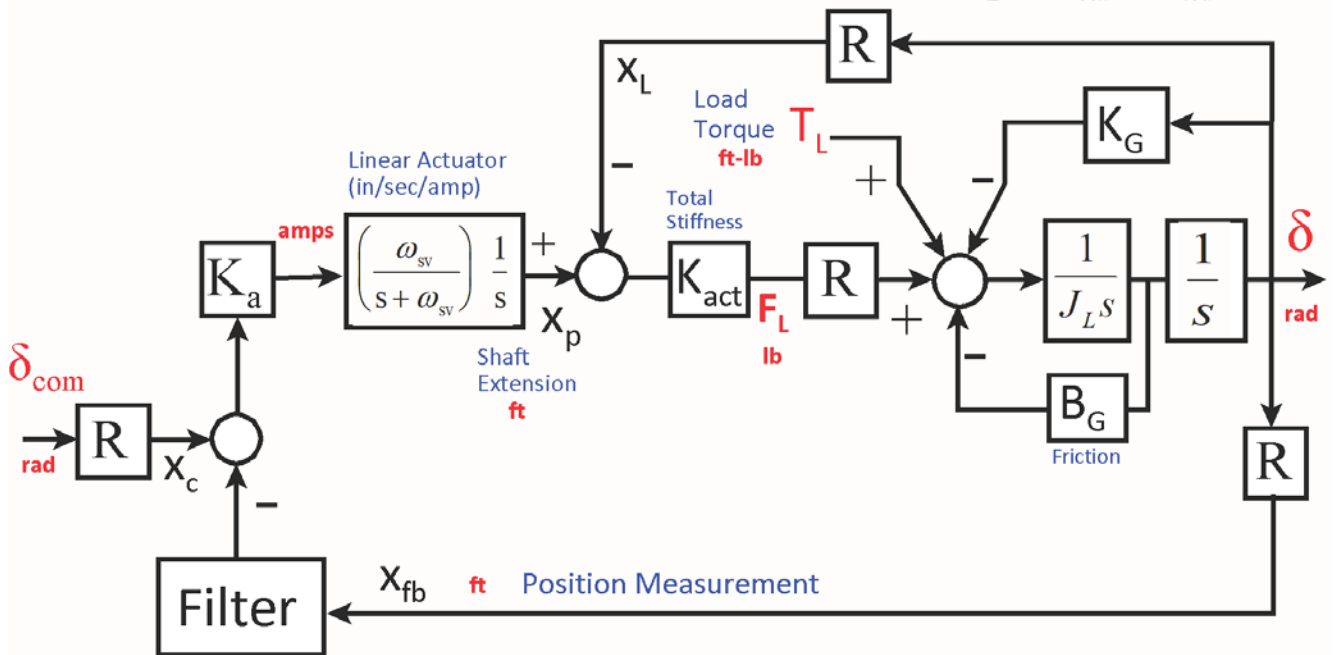
- (1) Stiffness of the backup structure K_{bck} , that is, where one side of the actuator is attached to the vehicle,
- (2) Actuator stiffness K_{act} consisting of piston plus oil (or electrical) stiffness, and
- (3) Load stiffness K_{lod} caused by the flexibility at the linkage where the shaft is attached to the load.

The shaft force is pushing against the load through the moment arm R and it produces the control torque that rotates the nozzle or aerosurface that has inertia J_L . X_p is the position of the shaft's end relative to the actuator and X_L represents the linear displacement at the nozzle attachment as a result of a load rotation angle δ , but this is not necessarily equal to X_p , because of structural bending at the attachments. In fact, it is the difference between the piston extension and linear nozzle displacement ($X_p - X_L$) that creates the force F_L pushing against the load via the combined stiffness K_T .

The control torque due to shaft extension is not the only torque driving the load. There is also an external load torque T_L that is produced by the vehicle inertial accelerations (both rigid and flex) and by the aerodynamic forces. This external to actuator load torque comes from the vehicle system outputs and creates a mechanical feedback loop. The rotations of the load ($\delta, \dot{\delta}, \ddot{\delta}$) are relative to the vehicle gimbal as a result due to shaft extension X_p plus external loading torques pushing against the combined structural stiffness K_T . The gimbal torques also include a component due to viscous damping at the gimbal B_G and a rotational stiffness at the gimbal K_G caused by pipes and other mechanical hardware connected to the nozzle. Static and Stiction friction cannot be included in the linear model but only in simulations.



Simple Actuator Model

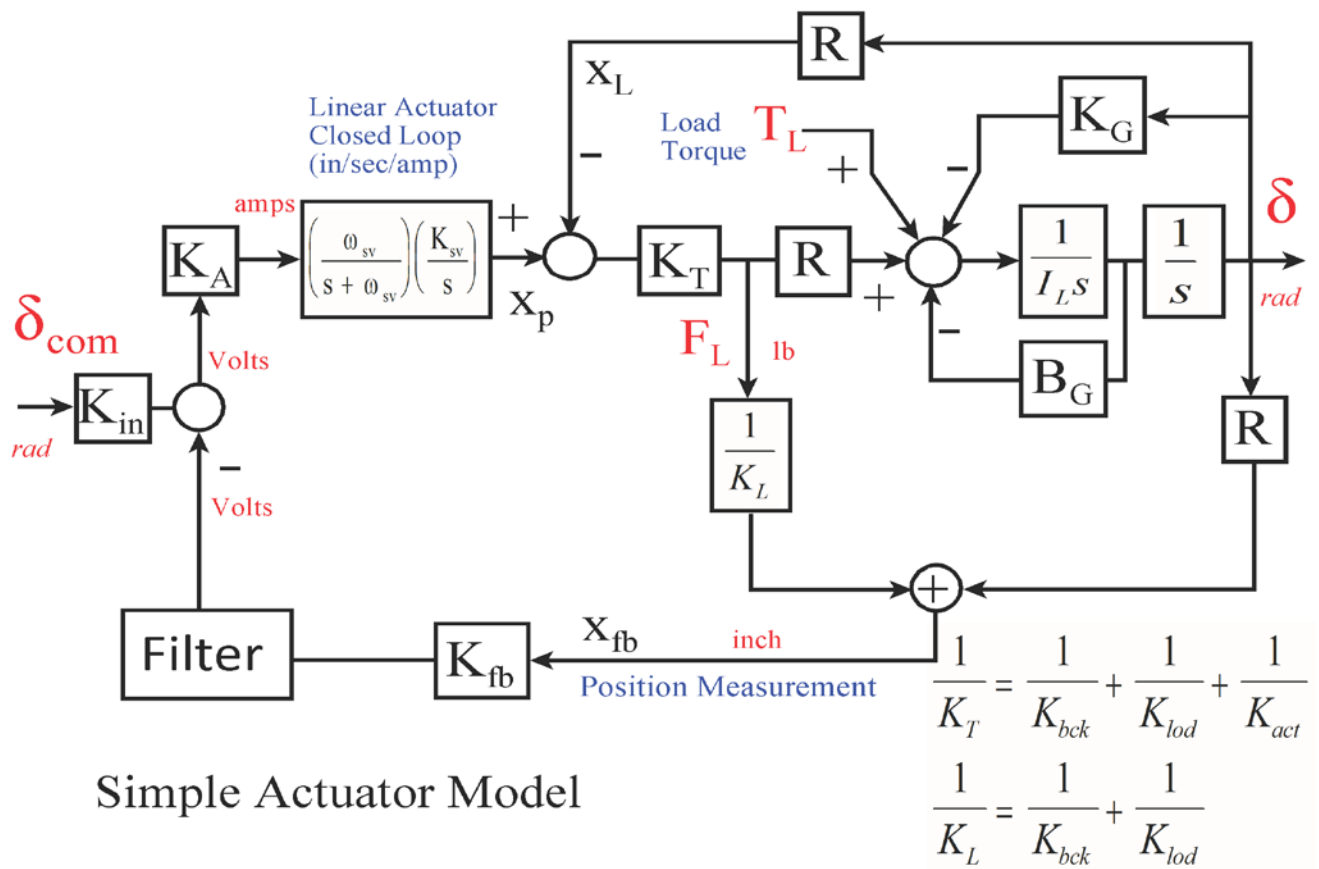


Stiff Actuator Model

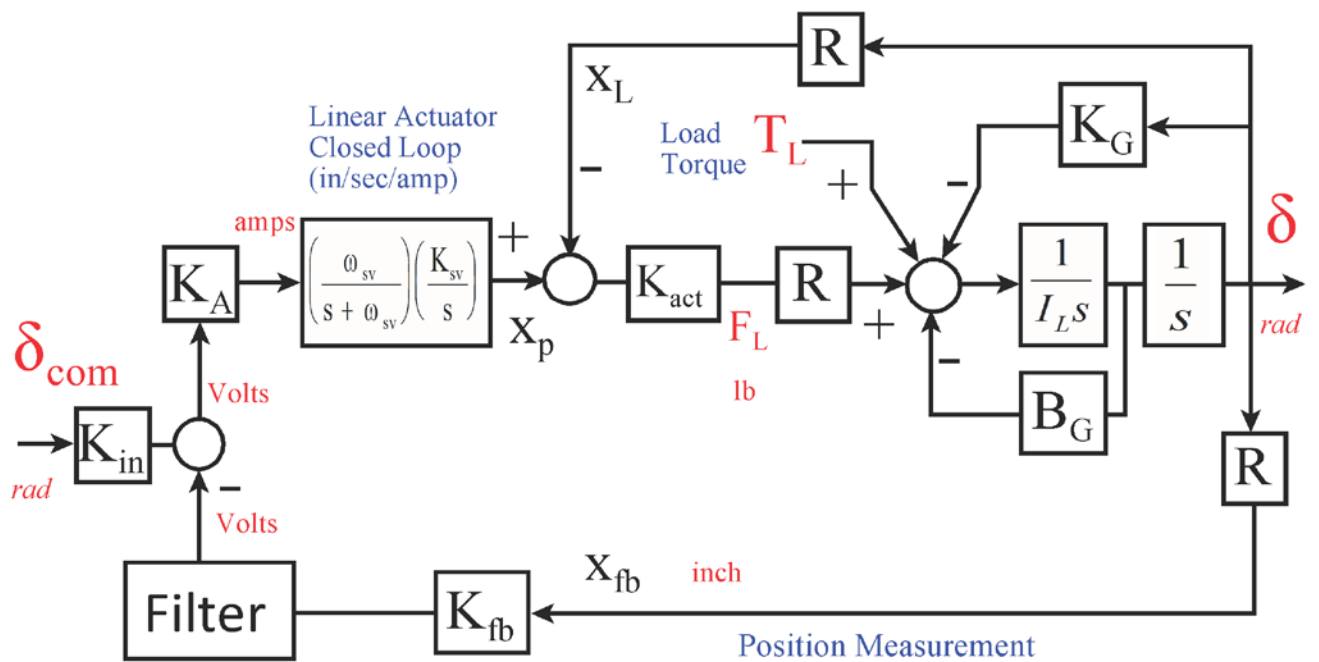
Figure 4.3.1 Simple Actuator Model that includes all 3 Stiffnesses and a “Stiff” version that includes only the Actuator Shaft Stiffness K_{act} ; Backup and Load Stiffnesses are Infinite.

The actuator position measurement X_{fb} is located inside the actuator device and it is intended to measure the actuator shaft position. If you ignore the structural stiffnesses, the effector deflection for small angles is nearly proportional to the actuator position. The sensor measures the extension of the rod relative to the actuator device, but this does not necessarily measure the total rotation of the nozzle because the vehicle support structure where the actuator is attached bends as a result of the applied force. The linkage mechanism that connects between the shaft and load also bends, and the actuator rod itself compresses under load. The actual displacement of the effector, therefore, is not only due to the rotation at the hinge but it includes a component due to structural deformations that makes it different from the measurement, and this error must be captured in the sensor model. The position sensor inside the device cannot obviously “see” the deformations at the nozzle and at the backup structure attachments, both stiffnesses combined in K_L . The measurement, however, is affected by the actuator shaft stiffness K_{act} . When it compresses, the end of the rod is not exactly where the measurement thinks that it is. In Figure (4.3.1a) the position sensor measurement signal X_{fb} consists of two components: a component $(R\delta)$ due to the rotation at the gimbal, and a component, $(K_T/K_L)(x_p - R\delta)$, where K_T is the combined stiffness from all three sources.

Let us assume that the actuator piston position is locked at zero and an external load T_L is applied to the system. The effector will deflect at an angle δ as a result of all three stiffnesses: shaft, load, and backup stiffnesses combined in K_T . The position sensor in the actuator, instead of measuring zero because the piston does not move, it is measuring a small displacement caused by the compression of the piston, plus some additional compression due to oil or electrical stiffness which are all lumped in the actuator stiffness K_{act} . The sensor does not see the bending of the backup structure and load. In the measurement we, therefore, subtract from the total flex rotation δ the displacement components due to backup and load stiffnesses (F_L/K_L) and the remaining measurement is the actuator shaft deformation due to stiffness K_{act} . In the extreme situation, when $(K_{act} = \infty)$ and $(K_T = K_L)$, the feedback measurement becomes equal to the actual shaft position ($X_{fb} = X_p$). In Figure (4.3.1b) we assume that the backup and load stiffnesses are infinite and they have been taken out of the actuator model because they are included in the vehicle structural model, thus avoiding including them twice. Figure (4.3.1b) is used in flexible models.



Simple Actuator Model



Stiff Actuator Model

Figure (4.3.1b) Another Version of Simple actuator model and a “Stiff” alternative that includes only the actuator Shaft Stiffness K_{act} . Backup and Load Stiffnesses are Infinite.

4.3.2 Hydraulic Actuator, Model (A)

The first actuator is an electro-hydraulic servo system shown in Figure (4.3.2). It comprises of a high-pressure hydraulic supply (pump), a servo amplifier, a hydraulic actuator, a servo valve, and a position transducer. The electro-hydraulic servo valve is a flow control type. There is a leakage orifice across the load piston and a differential pressure feedback loop from a load pressure transducer to the servo-valve. The hydraulic cylinder is attached to the vehicle at a support point that has a backup stiffness K_{mc} . On the other side of the cylinder the piston is attached to the load (a TVC engine). The piston has a spring constant K_{mp} . This type of electro-hydraulic actuator can be described by the following equations.

The error voltage signal V_e at the input of the servo-amplifier is: $V_e = V_c - V_f$, where V_c and V_f are the command and feedback signals respectively. The feedback transducer is integrally built within the cylinder and it measures the nozzle position by measuring the the piston displacement relative to the cylinder. It feeds it back into the servo-amplifier.

$$V_c = K_c \delta_{com} ; \quad V_f = K_f (x_p - x_c) \quad (4.3.2.2 \text{ \& } 3)$$

The error signal V_e drives a unit gain compensator $C(s)$, and the output from the compensator is:

$$V_{ef}(s) = C(s) V_e(s) \quad (4.3.2.4)$$

The servo-valve is a flow-control type coupled to an actuator having a leakage orifice across the load piston to provide damping. The output piston rod is hinged to the engine nozzle that moves about its gimbal point, see figure (4.3.2.1). We can assume that the flow from the servo-valve into one side of the cylinder is equal to the flow out of the other side. For low load pressures, the flow of hydraulic fluid (Q_v) from the servo-valve is:

$$Q_v = K_2 x_v \sqrt{P_s - P_r} - l_v P_r \quad (4.3.2.5)$$

Where:

- x_v is the valve spool displacement (ft),
- P_s is the supply pressure to the valve (lb/ft²),
- P_r is the reservoir or exhaust pressure (lb/ft²),
- K_2 is a constant,
- P_L is the differential pressure across the piston,
- $l_v P_L$ is the leakage across the spool (ft³/sec).

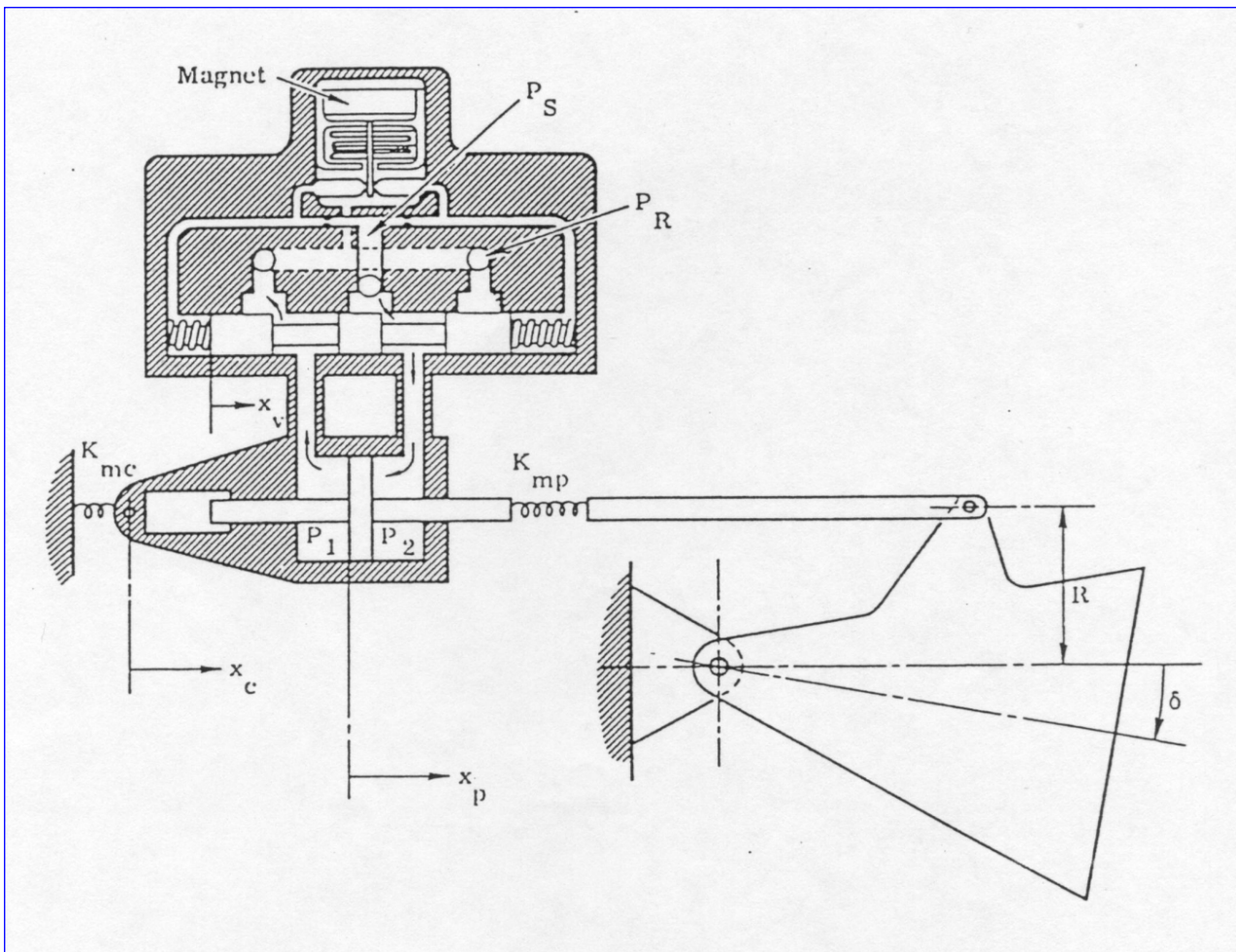


Figure (4.3.2) Schematic of a Hydraulic Servo

The total flow of hydraulic fluid, (Q_v) from the servo valve into the actuator cylinder is the sum of the piston displacement flow, leakage, and compressibility flow.

$$Q_v = A(\dot{x}_p - \dot{x}_c) + C_l P_L + \left(\frac{V_t}{4B} \right) \dot{P}_L \quad (4.3.2.6)$$

Where:

- A is the piston cross sectional area (ft^2)
- P_L is the load pressure in (lb/ft^2)
- C_l is the orifice leakage coefficient in $(\text{ft}^3/\text{sec})/(\text{lb}/\text{ft}^2)$
- δ_c is the commanded engine rotation in (radians)
- x_p is the piston rod displacement, (ft)
- x_c is the cylinder mount displacement with respect to the vehicle, in (ft)
- R is the moment arm in (ft) from the engine pivot to the piston rod, see figure (4.3.2.1).
- K_f is the feedback transducer gain (volt/ft)
- V_t is the volume of fluid under compression (ft^3)
- B is the bulk modulus of the hydraulic fluid (lb/ft^2).

The load pressure (P_L) displaces the piston rod and the engine. The displacement (x_p) with respect to the cylinder, and the displacement of the cylinder mount relative to the vehicle structure (x_c) is obtained from the following equations

$$A P_L = K_{mp} (x_p - R \delta); \quad \text{and} \quad x_c = -\frac{A P_L}{K_{mc}} \quad (4.3.2.7 \ \& \ 8)$$

Where:

- K_{mp} is the spring constant of the piston rod (lb/ft),
- K_{mc} is the spring constant of the cylinder mount (lb/ft),
- R is the moment arm of the actuator from engine gimbal (ft)
- δ is the engine rotation in radians.

From equations (4.3.2.6) to (4.3.2.8) we obtain the following equation, where Q_v represents the valve flow rate in (ft³/sec) which is proportional to the spool displacement.

$$\dot{P}_L = \frac{(-C_l P_L + Q_v - A R \dot{\delta})}{\left[\frac{V_t}{4B} + A^2 \left(\frac{1}{K_{mp}} + \frac{1}{K_{mc}} \right) \right]} \quad \text{where :} \quad (4.3.2.9), (4.3.2.10)$$

$$Q_v = K_v \left\{ K_a V_{ef} - K_{dpf} \left(\frac{\tau_c s}{\tau_c s + 1} \right) P_L \right\}$$

The second term in equation (4.3.2.10) is a dynamic pressure feedback term from the load pressure to the servo valve and it is used to control system damping, where:

- K_v is the servo-valve flow gain in (ft³/sec/ma)
- K_a is the servo-amplifier gain in (ma/volt)
- K_{dpf} is the dynamic pressure feedback gain in (ma/lb/ft²)
- τ_c is the dynamic pressure feedback time constant in (sec)
- A is the piston cross sectional area (ft²)

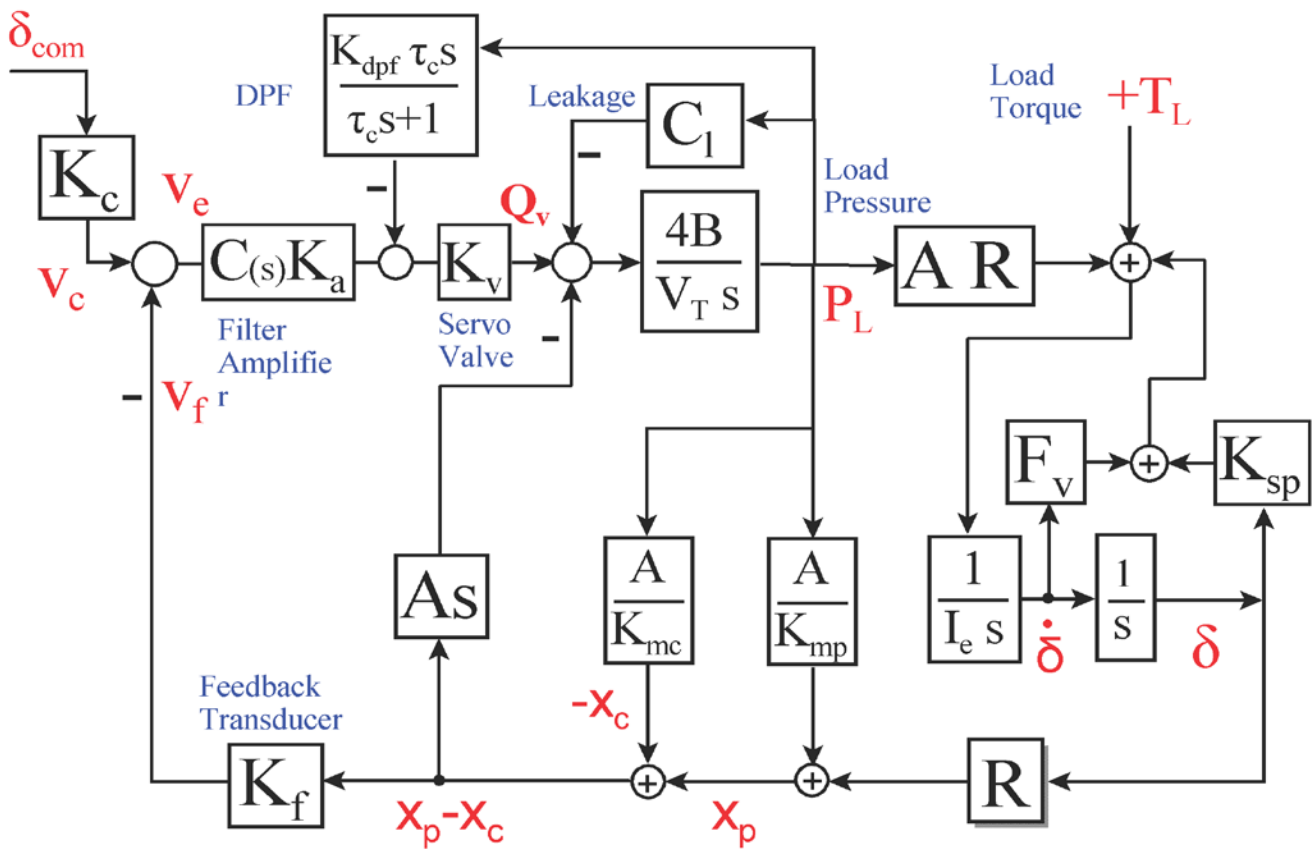
The moment equation of the engine nozzle rotation about its pivot is as following:

$$I_e \ddot{\delta} = (R A) P_L + T_L - F_v \dot{\delta} - K_{sp} \delta \quad (4.3.2.11)$$

Where:

- I_e is the engine inertia in (slug-ft²),
- F_v is the viscous friction coefficient (ft-lb)/(rad/sec),
- K_{sp} is the engine bearing spring constant (ft-lb/rad),
- δ is the engine rotation in radians.
- T_L is the load torque due to the vehicle motion (ft-lb)

The first term on the right-hand side of equation (4.3.2.11) represents the torque on the engine nozzle due to the actuator piston pressure. The second term is the external load torque created by the vehicle motion (rigid body and flexibility). The third term is a damping torque due to friction at the gimbal, and the fourth term is the torque due to spring stiffness at the pivot.



Electro-Hydraulic Actuator Model (A)

4.3.3 Hydraulic Actuator, Model (B)

This actuator model is also an electro-hydraulic type and it was used to model the Space Shuttle actuators for linear analysis. Its block diagram is shown in Figure (4.3.3) and the equations of motion are shown below. The model is relatively simple because a lot of the details are left out or combined into gains. The piston force is calculated from the following equation:

$$F_L = K_T (x_r - R \delta) \tag{4.3.3.1}$$

Where:

- F_L is the piston load force in (lb)
- K_t is the piston rod stiffness in (lb/ft)
- x_r is the actuator rod displacement in (ft)
- R is the distance between the engine pivot and the actuator rod (ft), see Figure (4.3.2)
- δ is the nozzle or surface rotation about the gimbal (radians).

The nozzle deflection measurement is calculated from a displacement sensor located in the actuator and not at the gimbal. The measurement in Equation (4.3.3.2) is corrected to account for the backup structure deformation, as it was already explained in the simple actuator model, in Section (4.3.1).

$$\delta_{fb} = K_{fb} \left(R \delta + \frac{F_L}{K_L} \right) \quad (4.3.3.2)$$

Where:

- K_{fb} is the position feedback gain in (rad/ft)
- K_L is the engine mount structural stiffness in (lb/ft)

The error signal δ_e is the difference between the commanded and the sensed engine rotation. This signal drives the unit gain compensator $C(s)$, which is optional. It may be used to improve the actuator loop stability or performance.

$$\delta_e = \delta_c - \delta_{fb} \quad \text{and} \quad \delta_{ef} = C(s) \delta_e \quad (4.3.3.3)$$

The Differential Pressure Feedback (DPF) loop is an acceleration feedback. It uses a primary pressure feedback loop and it is intended to smooth out the actuator response by increasing the system damping and to minimize the steady-state error. A sensor measures the pressure differential across the piston, the signal is shaped with a high pass filter in the electronics and it is fed back in the actuator command. The gain K_{dp} is a non-linear function that relates the rate feedback signal as a function differential force. The DPF loop can be approximated with the following first order transfer function

$$\frac{T_D(s)}{F_L(s)} = K_{dp} \frac{\tau_c s}{\tau_c s + 1} \quad (4.3.3.4)$$

Where:

- K_{dp} is the DPF loop linearization gain in (ft-lb/lb)
- τ_c is the DPF loop time constant

The actuator piston displacement is:

$$\dot{x}_r = K_{act} \left(K_{av} \delta_{ef} - T_D \right) \quad (4.3.3.5)$$

Where:

- K_{act} is the power valve and actuator gain, (ft/sec)/(ft-lb)
- K_{av} is the amplifier & torque motor gain, (ft-lb/rad)

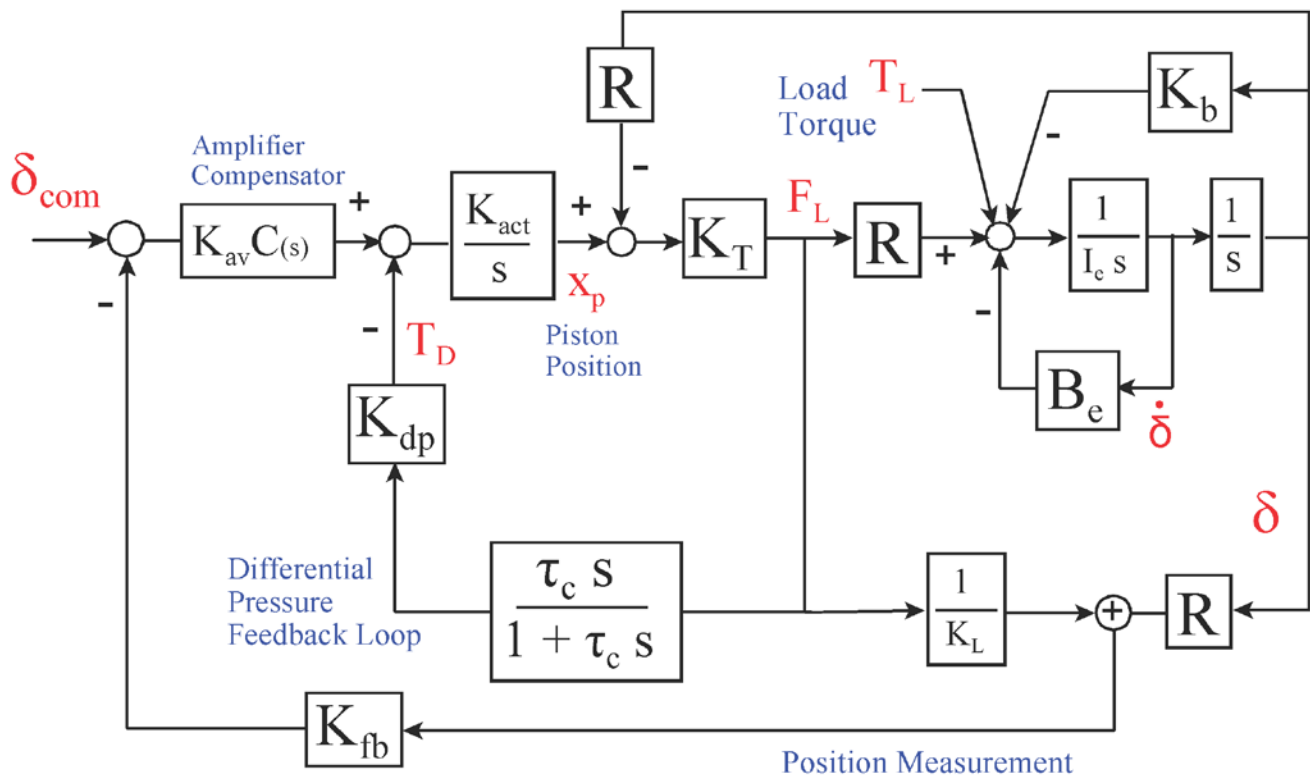
The effector deflection in (radians) is obtained from the following equation

$$I_e \ddot{\delta} = -B_e \dot{\delta} - (K_b + R^2 K_T) \delta + T_L + (R K_T) x_r \quad (4.3.3.7)$$

Where:

- B_e is the engine viscous damping in (ft-lb)/(rad/sec)
- K_b is the engine gimbal spring constant in (ft-lb/rad)
- T_L is the load torque in (ft-lb)
- K_T is the combined stiffness in (ft-lb/rad)

The first term on the right side of equation (4.3.3.7) represents the engine torque due to viscous damping, the second term is engine torque due to the combined (gimbal plus piston) stiffness, the third term is the external load torque generated by the normal acceleration of the vehicle, (rigid body plus bending). The last term represents the torque on the engine due to the actuator piston displacement (x_r) pushing against the combined system stiffness K_T , consisting of: backup structure, load linkage, and actuator piston stiffness. They combine in series by adding their inverses, as calculated in Figure (4.3.1).



Hydraulic Actuator Model (B)

Figure (4.3.3) Shuttle Hydraulic Actuator Block Diagram

4.3.4 Electro-Mechanical Actuator (EMA), Model (A)

The block diagram of an EMA servo actuator is shown in Figure (4.3.4). It consists of an electric motor that provides the torque to move the actuator rod through a system of gears that convert the rotational motion to linear motion of the rod. The actuator is attached to the load (which is either an engine nozzle or a control surface) via a stiff mounting mechanism, and the load is free to rotate about a gimbal. The actuator motor “sees” the engine load via a large gear reduction ratio (N). The motor control system uses rotation angle position feedback (Θ) to control the motor position, in combination with motor speed for damping. An optional filter C(s) may be included to compensate the error signal (V_e) and to improve performance, if necessary. The filter output drives the amplifier which provides current to the motor. The following equations describe the EMA servo system dynamics starting with the Motor Inertia Equation.

$$J_m \ddot{\Theta} = T_m - K_{md} \dot{\Theta} - \left(\frac{G_t}{N} K_m R \right) x_f \quad (4.3.4.1)$$

Where T_m is the torque generated by the electric motor in (ft-lb), the second term on the right is the torque due to viscous damping, and the third term is the torque due to the actuator displacement via the stiff mounting mechanism.

- J_m is the motor shaft inertia in (ft-lb-sec²)
- Θ is the motor rotation angle in (radians)
- R is the actuator moment arm from the engine pivot in (ft).
- K_m is the spring constant of the actuator/engine flexion attachment (lb/ft)
- N is the Gear Ratio
- G_t is the efficiency factor for the motor mechanical conversion, (typical value 1 - 1.5)
- x_f is the actuator rod displacement in (ft)
- K_{md} is the motor viscous damping coefficient (ft-lb/rad/sec).

The following equation couples the motor rotation angle to actuator displacement x_f (ft).

$$x_f = R \left(\frac{\Theta}{N} - \delta \right) \quad (4.3.4.2)$$

Where: the first right term of equation (4.3.4.2) is due to the motor shaft rotation (Θ), and the second is due to the engine rotation (δ). The engine nozzle rotation is obtained from the following equation.

$$J_e \ddot{\delta} = -F_v \dot{\delta} - K_{sp} \delta + T_L + K_m R x_f \quad (4.3.4.3)$$

Where the first term on the right hand side is the torque due to engine bearing viscous friction (F_v), the second term is due to engine bearing spring constant (K_{sp}), the third is load torque due to vehicle acceleration (coming from the vehicle model), and the last term is the torque due to the actuator motion (x_f) coupled to the engine via the flexion spring constant K_m . The position error signal V_e in (volt) is described by the following equation:

$$V_e = G_v \{G_p \delta_c - K_{pl} (\delta + B_i x_f) - K_{pm} \Theta\} \quad (4.3.4.4)$$

Where:

- J_e is the engine load inertia in (ft-lb-sec²)
- δ is the engine rotation angle in (radians)
- G_v is a position to voltage conversion constant (volt/count)
- G_p is an input angle command conversion constant (count/rad)
- K_{pl} is position feedback gain from the load (counts/rad)
- B_i is the sensor bias (rad/rad)
- K_{pm} is position feedback gain from the motor in (counts/rad)
- K_v is the rate feedback gain volts/(rad/sec)

The signal (V_i) that drives the servo-motor amplifier and the servo-amplifier transfer function are as follows, where $C(s)$ is the compensator

$$V_i = C(s)V_e - K_v \dot{\Theta} \quad \frac{V_o(s)}{V_i(s)} = K_a \left[\frac{I}{\tau_a s + I} \right] \quad (4.3.4.5) \text{ \& } (4.3.4.6)$$

The Motor Torque Equation is

$$T_m(s) = \frac{K_t}{R_m} \left(\frac{I}{\tau_m s + I} \right) (V_o - K_{emf} \dot{\Theta}) \quad (4.3.4.7)$$

Where:

- K_a is the amplifier gain in (volts/volt)
- τ_a is the amplifier time constant in seconds
- T_m is the motor torque in (ft-lb)
- K_t is the motor current to torque constant (ft-lb/amps)
- R_m is the motor winding resistance in (ohms)
- τ_m is the motor time constant in (seconds)
- K_{emf} is the motor back-emf gain (volt/rad/sec)

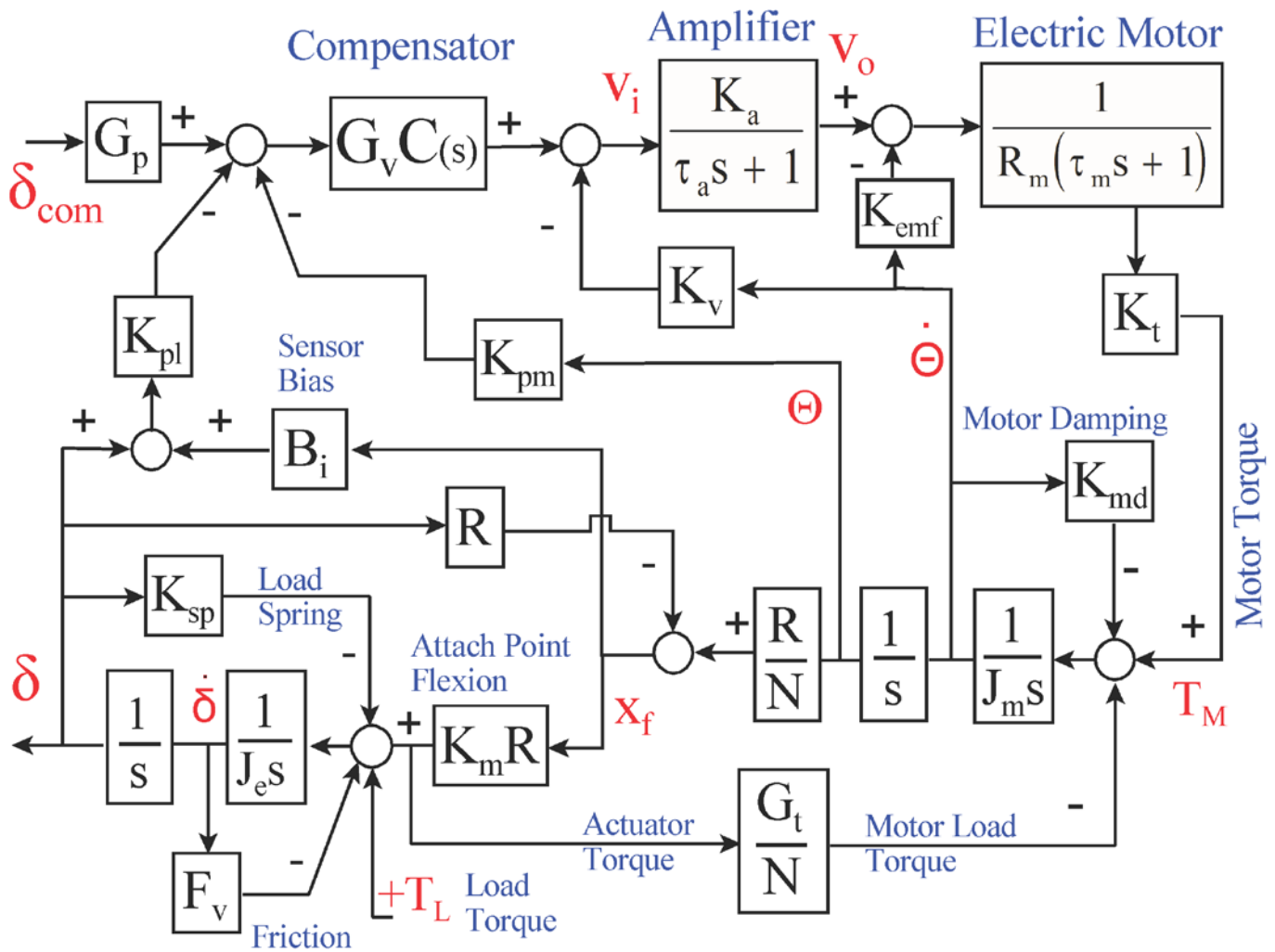


Figure (4.3.4) Electro-Mechanical Actuator Model (A) Block Diagram

4.3.5 Electro-Mechanical Actuator (EMA), Model (B)

The EMA model in Figure 4.3.5.1 is different from the previous EMA model (A). It uses a spinning motor that is controlled by dc voltage via a current amplifier and it provides the power to drive the load. The motor torque is transferred from the rotor to a spiral screw that is rotating via a gear mechanism, a small gear driving a bigger gear, and it produces a higher torque. The rotation of the screw converts the rotational motion to translational that extends or retracts the shaft. The end of the shaft is connected to an engine or aerosurface via a linkage mechanism and it is pushing against the load that rotates about a hinge in order to control the vehicle. The other side of the actuator is attached to a stiff point on the vehicle structure. The EMA system uses two gear ratios. The first gear ratio N_{gear} defines the number of motor spins for one rotation of the screw gear and the second gear ratio N_{screw} defines the number of screw rotations per unit length extension of the shaft.

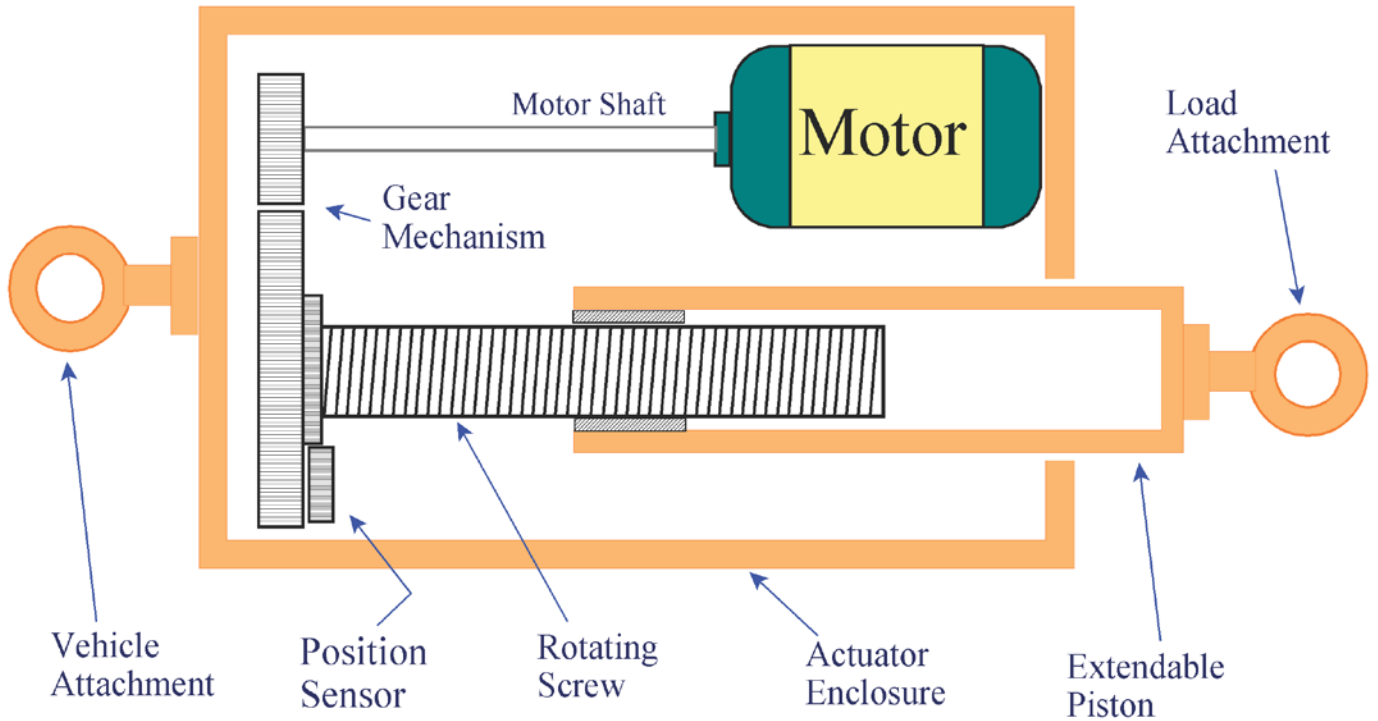


Figure (4.3.5.1) Electro-Mechanical Actuator System (B)

A current amplifier supplies the dc current to the motor armature that drives the dc motor and the current is converted to torque that spins the rotor. The voltage output of the PID controller is converted to torque T_m that spins the rotor $\dot{\mu}$.

$$i = \left(\frac{1}{Ls + R} \right) [v_{in} - K_{emf} \dot{\mu}] \quad (\text{motor current})$$

$$T_c = K_T i - K \dot{\mu} \quad (\text{electric torque})$$

$$I_m \dot{\mu} = T_c - \frac{K_{gs} \varepsilon}{N_{gear}} - K_{mfr} \dot{\varepsilon} - K_{visc} \dot{\mu} - K_c \text{sign}(\dot{\mu}) \quad (4.3.5.1)$$

$$T_m = K_{eff} K_{gs} \varepsilon \quad (\text{motor output torque})$$

$$P = R i^2 \quad (\text{dissipated power})$$

The EMA system block diagram is shown in Figure (4.3.5.2). It includes the actuator, the rotational dynamics of the load and also the mechanical stiffnesses which are coupling them together. The motor speed and the shaft extension rate are limited by the amplifier current supply. A resolver is measuring the shaft position by counting the number of screw rotations and it is used for position feedback x_{fb} . A rate sensor from the motor speed provides the necessary damping and stability characteristics in the actuator control loop. The gimbal deflection command δ_{com} is in (radians). It is converted to actuator position command in (feet) which is subtracted from the measured shaft position to become a position error commanding the PID control system. The controller generates the voltage v_i and current as a

function of error that drive the dc motor. Integral and rate feedback from the motor rate are included in the PID control system to provide the desired closed-loop response. A low-pass 1st or 2nd order compensator C(s) may be also included.

The motor dynamics Equation 4.3.5.1 is simplified by approximating the electrical characteristics and the back-emf with a first order lag of bandwidth ω_m and the motor torque gain K_v in (ft-lb/volt) which includes the current amplifier, control electronics and it converts the input voltage v_i to motor torque. The inputs to the equation are torques and the output is rotor rate ($d\mu/dt$) in (rad/sec). There are four torques that affect the motor speed: an electrical control torque that drives the motor, a reaction torque due to loading and two friction torques.

$$\dot{\mu} = \frac{1/I_m}{(s + \omega_m)} \left[K_v v_i - \frac{T_m}{N_{gear}} - K_{mfr} \dot{\varepsilon} - K_C \text{sign}(\dot{\mu}) \right] \quad (4.3.5.1)$$

The torques opposing the motor speed are:

- (a) the torque applied to the screw via the gear ratio N_{gear} but reduced in size by the same gear reduction ratio N_{gear} ,
- (b) a coulomb friction torque K_C always in the opposite direction to the rotor speed, and
- (c) a damping torque (K_{mfr}) due to the relative speed, (rotor-load) rates.

The motor drives a gear mechanism that rotates the screw at an angle (θ). The torque on the screw (T_m) generated by the motor is.

$$T_m = K_{gs} \left(\frac{\mu}{N_{gear}} - \theta \right); \quad \varepsilon = \left(\frac{\mu}{N_{gear}} - \theta \right) \quad (4.3.5.2)$$

Where:

- ω_m is the motor bandwidth in (rad/sec),
- I_m is the motor inertia (slug-ft²)
- ε is the relative position (rotor-load)
- μ is the motor rotation angle in (radians)
- θ is the screw rotation angle in (radians)
- N_{gear} is the motor to screw gear ratio
- N_{screw} is the shaft screw ratio (ft/rad)
- T_m is the torque on the screw
- K_{gs} is the gear stiffness in (ft-lb/rad).
- K_{mfr} is the motor gear damping friction, (ft-lb/rad/sec) due to relative speed
- K_C is the motor Coulomb Friction (ft-lb)

The direction of the screw rotation causes the shaft to either extend or to retract. Equation (4.3.5.3) calculates the screw position angle θ as a function of the supplied torque T_m and the load force F_L . The shaft position x_p is linearly related to the screw angle by the screw gear ratio.

$$\begin{aligned} J_a \ddot{\theta} &= K_{eff} T_m - N_{screw} F_L - K_{dmp} \dot{\theta} - K_C \text{sign}(\theta) \\ F_L &= K_T [x_p - x_L] \quad x_L = R\delta \\ x_p &= N_{screw} \theta \end{aligned} \quad (4.3.5.3)$$

Where:

- N_{screw} is the screw gear ratio in (ft/radian). It represents the length of the piston extension when the screw mechanism rotates one radian.
- F_L is the force across the actuator shaft in (lb) due to the load.
- J_a is the combined inertia of the gears plus the screw
- R is the moment arm, shaft to hinge in (ft)
- K_{dmp} is the shaft friction (ft-lb/rad/sec)
- x_p is the piston extension and it is related to the spiral shaft rotation (θ) by the screw gear ratio

Equation (4.3.5.4) calculates the load angular acceleration as a function of the applied torques.

$$J_e \ddot{\delta} = F_L R - B_e \dot{\delta} - T_{stat} \text{sign}(\dot{\delta}) - K_g \delta + T_{Ext_Load} \quad (4.3.5.4)$$

Where:

- J_e is the load moment of inertia of the engine or the control surface about the hinge
- δ is the load deflection in (rad)
- K_g is the stiffness at the hinge (ft-lb/rad)
- B_e is the Viscous Friction (ft-lb/rad/sec)
- T_{static} is the Static Friction in (ft-lb)
- T_{ext_load} is the external torque due to aerodynamics and vehicle acceleration forces.

The variable ($x_L=R\delta$) represents the distance traveled at the end of the shaft (the point where it is attached to the load) when it rotates at an angle δ . It is not only the piston extension x_p that creates the force F_L against the load, but it is the difference (x_p-x_L) that is pushing against the total system stiffness K_T . The rotation angle δ represents the combined rotation of the load at the gimbal consisting of two components: a component due to the piston extension x_p , plus a component due to structural bending K_T . Equation (4.3.5.5) calculates the resolver feedback measurement x_{fb} .

$$x_{fb} = x_p \frac{K_T}{K_L} + x_L \left(1 - \frac{K_T}{K_L} \right) = x_p \frac{K_T}{K_L} + \frac{F_L}{K_{act}} \quad (4.3.5.5)$$

It represents what the position sensor measures inside the actuator. It consists of two components, a component due to actual shaft position (x_p), and a component due to compression inside the actuator caused by the actuator internal stiffness (K_{act}). The system has a total stiffness (K_T). Part of the total stiffness is due to backup structure and load stiffnesses (K_L), and the other part is due to the shaft and electrical compression (K_{act}) inside the actuator. The sensor measures shaft rotations due to compression (K_{act}) but it doesn't see the deformation due to (K_L). The stiffnesses are related by the following equation

$$\frac{1}{K_T} = \frac{1}{K_L} + \frac{1}{K_{act}} \quad (4.3.5.6)$$

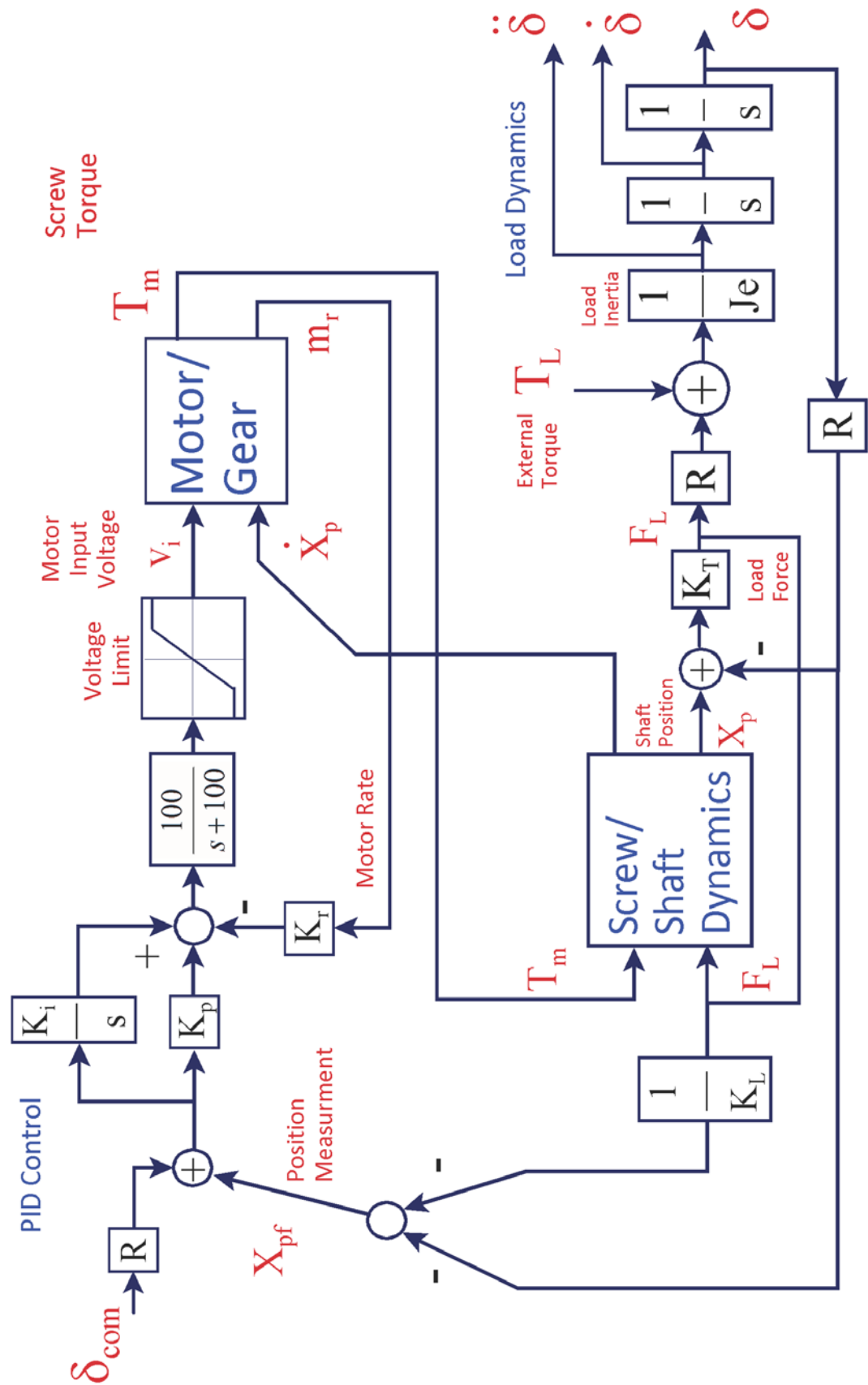
Consider the two extreme situations. First, when the load is very light, there is no friction, and the load does not resist the motion of the shaft (x_p). The shaft force (F_L) is reduced to zero. The extension due to (δ), ($x_L = x_p$), and the feedback position measurement at the output of the summing junction becomes $x_{fb} = x_p$. This is exactly what you would expect for the position sensor to measure. Let us now assume that there is infinite stiffness in the actuator, and K_{act} is infinite. The total stiffness now reduces to backup plus load flexure ($K_T = K_L$). Assume also that the shaft is locked at zero position ($x_p=0$). If we apply an external load torque to the load (T_L), the piston force is: ($F_L = -T_L/R$). The feedback sensor measurement x_{fb} will be zero as you would expect because the sensor cannot see the deformation due to (K_L) outside the actuator box.

$$x_{fb} = x_L - \frac{K_T}{K_L} x_L = 0$$

In actuality, however, when you are holding (x_p) to zero and are applying an external load (T_L) the sensor at the shaft will measure the small compression of the motor and shaft. The sensor measurement x_{fb} will be:

$$x_{fb} = x_L \left(1 - \frac{K_T}{K_L} \right) = x_L \frac{K_T}{K_{act}} = \frac{T_L/R}{K_{act}}$$

This is exactly what you would expect from the sensor to see; an extension measurement due to the external load acting against (K_{act}) alone.



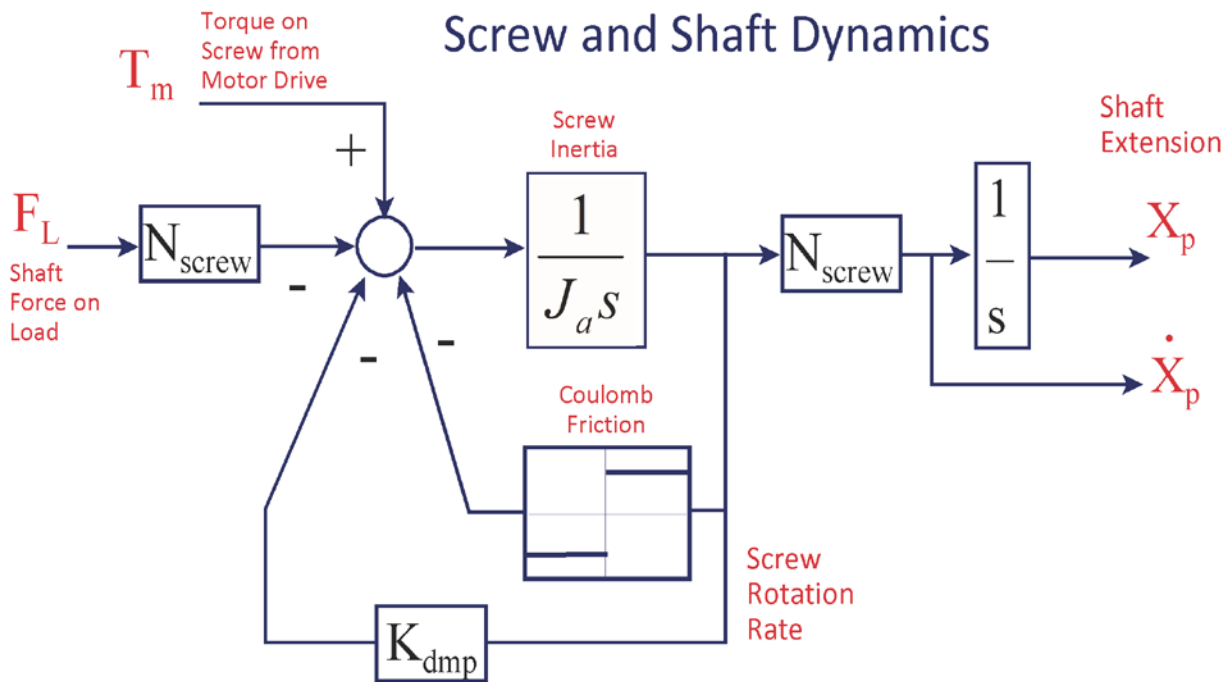
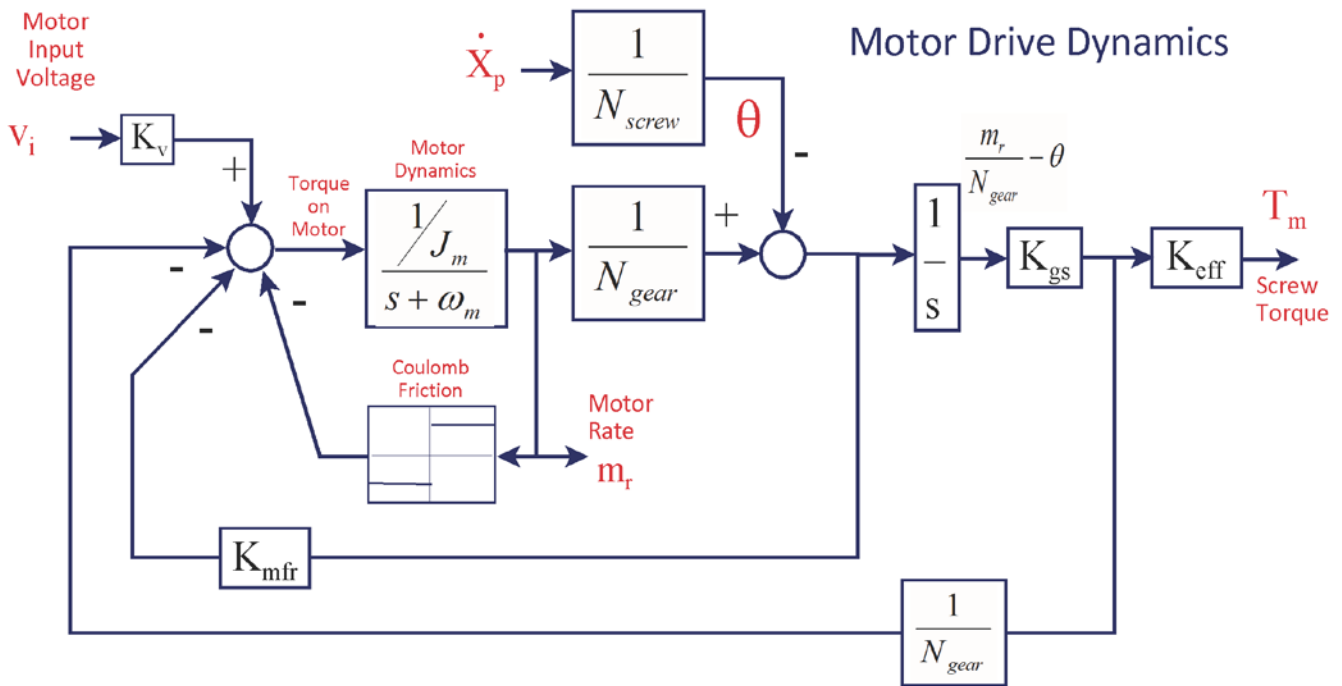
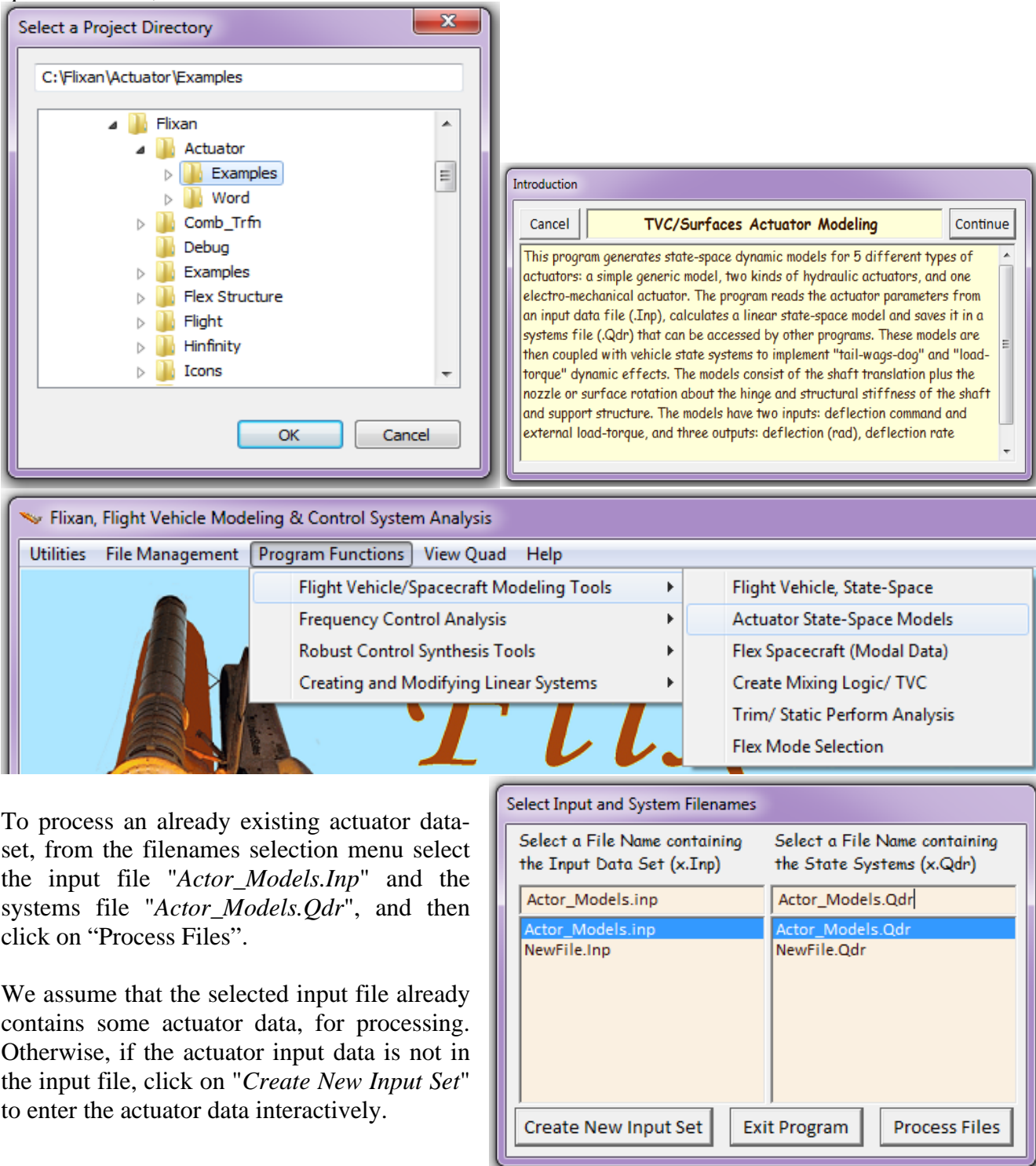


Figure 4.3.5.2 Electro-Mechanical Actuator (B) Block Diagram with Subsystems

4.4 Running the Actuator Modeling Program

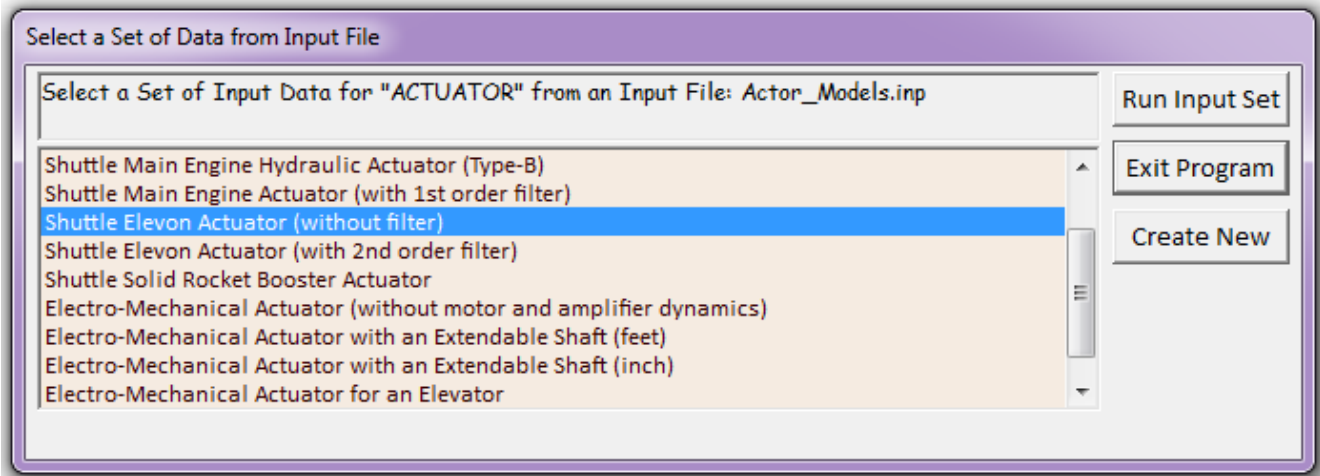
The actuator modeling program includes the five different types of actuator models described. It runs either interactively or in batch mode, similar to the vehicle modeling program. To run it interactively, start the Flixan program, select the project directory which in this example it is "C:\Flixan\Actuator\Examples" and contains the input data and the system files (.Inp) and (.Qdr). Then, from the Flixan main menu select "Program Functions", "Flight Vehicle/Spacecraft Modeling Tools", and "Actuator State-Space Models", as shown below.



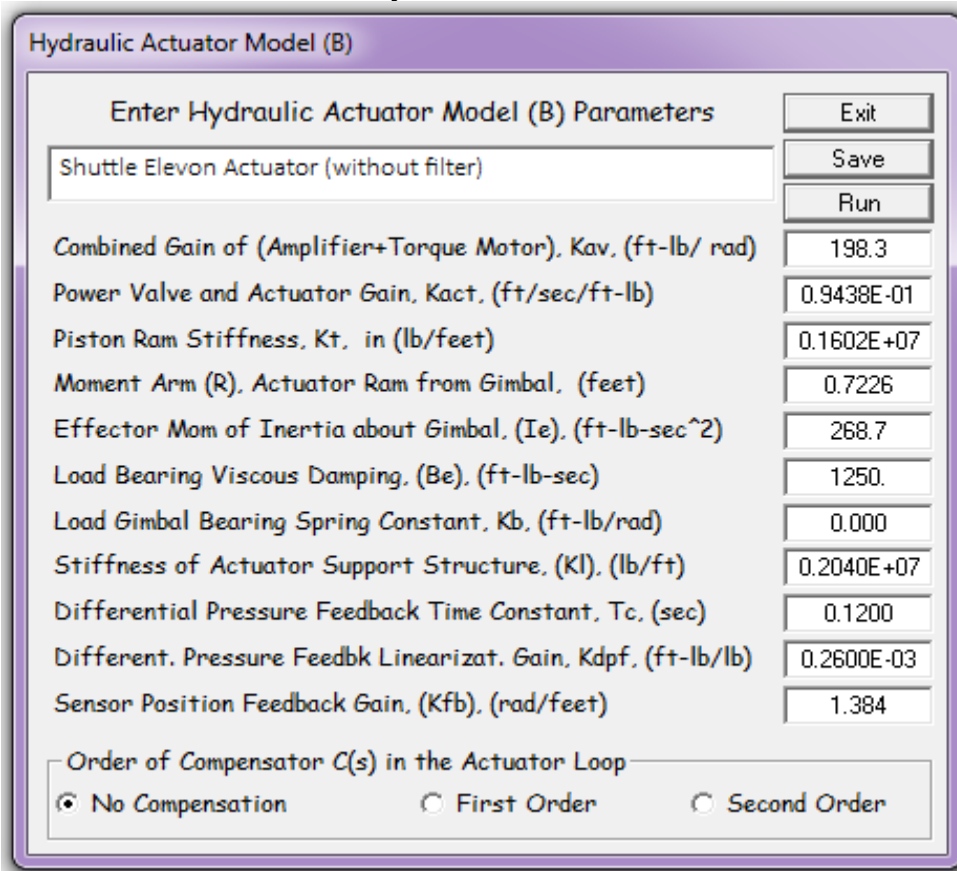
To process an already existing actuator data-set, from the filenames selection menu select the input file "Actor_Models.Inp" and the systems file "Actor_Models.Qdr", and then click on "Process Files".

We assume that the selected input file already contains some actuator data, for processing. Otherwise, if the actuator input data is not in the input file, click on "Create New Input Set" to enter the actuator data interactively.

The following menu shows the titles of the actuator data-sets which are included in the input file "Actor_Models.Inp". Select the title "Shuttle Elevon Actuator (without filter)" and click on "Run Input Set" to process the actuator data. You may also click on "Create New" to create a new set or "Exit the Program".



The program reads the input data and opens the following dialog that shows the actuator parameters to be processed. The user can modify some of the parameters using this dialog and save the new data-set under a different title. Click on "Run" and the actuator program will process the data-set and create the actuator state-space system in file "Actor_Models.Qdr" using the same title "Shuttle Elevon Actuator (without filter)". The title of the system is the same as the title in the input data. The comment lines from the data-set are also transferred to the system.



The following data-set is the actuator data that we just processed from file "Actor_Models.Inp". The first line: "ACTUATOR INPUT DATA ... HYDRAULIC TYPE B" is the actuator id line which identifies the type of data. The Flxan program recognizes it, and also the actuator type which is (Hydraulic type B in this case), and uses the proper utility to process the data. The second line is the actuator title: "Shuttle Elevon Actuator (without filter)". The green lines below the title beginning with (!) are comments that further describe the actuator.

```

ACTUATOR INPUT DATA ..... HYDRAULIC TYPE B
Shuttle Elevon Actuator (without filter)
! Hydraulic Actuator Type (B) for the Space Shuttle Elevon,
! Without a Filter Compensator C(s)
Symbol      Parameter Description      (Units)      Value
C(s)      Order of Compensat:(0,1,2), Coefficients      (---)      0
Kav      Total Gain of Amplifier + Torque Motor      (ft-lb/rad)      198.329
Kact      Actuator Gain .....(ft/s/ft-lb)      0.09438
Kt      Piston Ram Stiffness ..... (lb/ft)      0.16020E+07
R      Moment Arm of Actuator Ram from Gimbal      (feet)      0.7226
Ie      Elevon Inertia about Gimbal .....(ft-lb-s^2)      268.67
Be      Elevon Viscous Damping .....(ft-lb-sec)      1250.0
Kb      Elevon Gimbal Bearing Spring Constant      (ft-lb/rad)      0.0
Kl      Elevon Mount Structural Stiffness ..... (lb/ft)      0.204E+07
Tc      Different. Pressure Feedbk Time Constant      (seconds)      0.12
Kdcpf      Differ Pressure Feedbk Linearizat. Gain      (ft-lb/lb)      0.26E-03
Kfb      Position Feedback Gain ..... (rad/feet)      1.38396

```

The following state-space system is created by the program in file "Actor_Models.Qdr" and corresponds to the above actuator data. It is identified with the same title and uses the same comments as the input data.

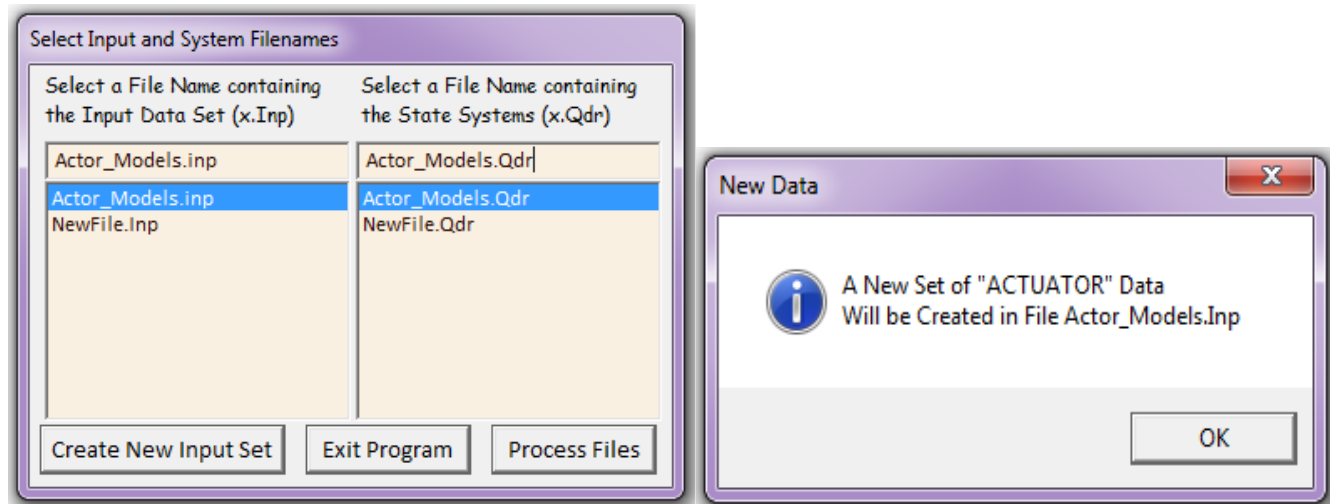
```

STATE-SPACE SYSTEM ...
Shuttle Elevon Actuator (without filter)
! Hydraulic Actuator Type (B) for the Space Shuttle Elevon, Without Filter
Number of Inputs, States, Outputs, Sample Time dT (for discrete)= 2 4 3 0.0000
Matrices: (A,B,C,D)
Matrix A      Size = 4 X 4
      1-Column      2-Column      3-Column      4-Column
1-Row -0.476444909333E+02 -0.847340446418E+04 -0.167404640751E+04 -0.300977352000E+03
2-Row -0.943800000000E-01 -0.203433315667E+02 -0.401912611041E+01 0.000000000000E+00
3-Row 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.100000000000E+01
4-Row 0.000000000000E+00 0.430865076116E+04 -0.311343104001E+04 -0.465254773514E+01
-----
Matrix B      Size = 4 X 2
      1-Column      2-Column
1-Row 0.779654257565E+04 0.000000000000E+00
2-Row 0.187182910200E+02 0.000000000000E+00
3-Row 0.000000000000E+00 0.000000000000E+00
4-Row 0.000000000000E+00 0.372203818811E-02
-----
Matrix C      Size = 3 X 4
      1-Column      2-Column      3-Column      4-Column
1-Row 0.000000000000E+00 0.000000000000E+00 0.100000000000E+01 0.000000000000E+00
2-Row 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.100000000000E+01
3-Row 0.000000000000E+00 0.430865076116E+04 -0.311343104001E+04 -0.465254773514E+01
-----
Matrix D      Size = 3 X 2
      1-Column      2-Column
1-Row 0.000000000000E+00 0.000000000000E+00
2-Row 0.000000000000E+00 0.000000000000E+00
3-Row 0.000000000000E+00 0.372203818811E-02

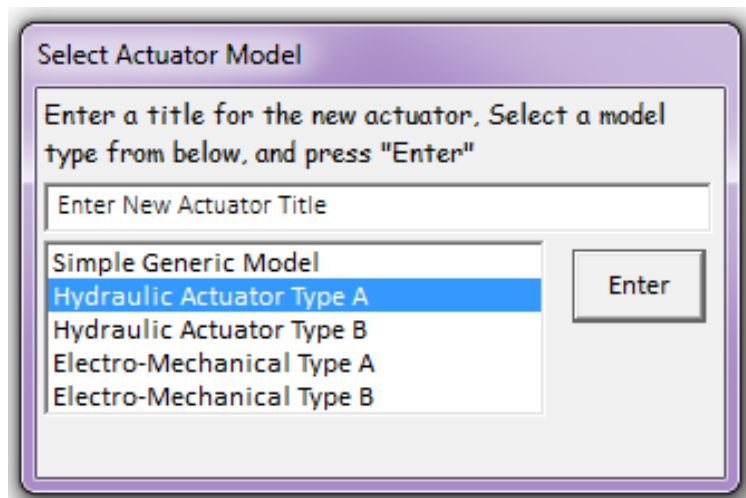
```

Creating a new set of actuator data in file

When the actuator data are not in the input data file, they can be entered interactively, saved, and then processed, as follows. Begin the actuator program and use the filenames selection menu to select the input and system files, as before. This time click on "Create New Input Set" instead of processing an already existing dataset.



In the next dialog, click OK to confirm a new actuator. From the following menu you must select the type of actuator that you want to create. There are 5 existing types to choose from. In this case we select the "Hydraulic Actuator Type A", and click on "Enter".



The program presents the following data entry dialog for entering the parameters of the selected actuator type. Start by entering the title "Missile Actuator-A", which should be less than 100 characters long and different from any other titles already saved in the input file. Enter also the actuator parameters in the corresponding fields. After entering the data, do not run it yet, but click on "Save", and the program will save the new actuator dataset in the input file "Actor-Models.Inp". Then you may click on "Run" to process it, and to create the new actuator state-space system in file "Actor-Models.Qdr". The reader can find more actuator modeling examples in file "\\Flixan\Actuator\Examples\Actor-Models.Inp".

Hydraulic Actuator Model (A)

Enter Hydraulic Actuator Model (A) Parameters

Missile Actuator-A

Input Command Gain, (Kc), (Volt/Rad)	46.12
Gain of Servo-Amplifier, Ka, (ma/volt)	20.40
Power Valve Flow Gain, Kv, (ft ³ /sec/ma)	0.5960E-04
Different Pressure Feedback Time Const. Tc, (sec)	0.1000
Different. Pressure Feedback Gain, Kdpf, (ma/lb/ft ²)	0.1000E-04
Volume of Compressed Fluid, (Vt), (feet ³)	0.1160E-02
Bulk Modulus of Hydraulic Fluid, (BM), (lb/ft ²)	0.2280E+08
Stiffness of Piston plus Mount Backup, Km, (lb/ft)	0.1330E+08
Leakage Coeffic. (Orifice+Spool), (ft ³ /sec/lb/ft ²)	0.2000E-10
Moment Arm, Actuator Rod from Gimbal, (R) in (ft)	0.3000
Piston Cross-Sectional Area, (A), (feet ²)	0.1500E-01
Load Inertia about Gimbal, (Ie), (ft-lb-sec ²)	0.5600
Gimbal Bearing Viscous Damping, (Fv), (ft-lb-sec)	410.0
Engine Gimbal Bearing Spring Constant, Ksp, (ft-lb/rad)	0.1110E+06
Actuator Position Feedback Gain, Kf, (volts/ft)	132.0

Order of Compensator C(s) in the Forward Loop

No Compensation
 First Order
 Second Order

Exit
Save
Run

Compensator Options

As we already discussed the actuator is an inner local loop that must be stable with an acceptable response to commands within the required bandwidth and step response requirements. The dynamic coupling between actuator, vehicle, and support structure flexibility introduces local resonances and TWD dynamics that may degrade the servo system's performance and stability, and the actuator servo systems loop often includes a compensators in order to improve the actuator loop stability and improve the command following. The actuator modeling program provides the capability to include a first or a second order compensator which is defined in the actuator input data. After entering the actuator data interactively using the data entry dialog the user may decide to include either: a first order or a second order compensator, or no compensator (default), using the 3-option buttons located at the bottom of the dialog. If you select to include a compensator a new dialog comes up where the user enters the transfer function coefficients. The compensator coefficients are saved in the input file data-set together with the actuator parameters. In fact, the first line in the input data defines the compensator transfer function $C(s)$ (if any) which is inserted in the actuator position control loop as shown in the actuator block diagrams.

For a first order compensator the input data line consists of 3 numbers: “**Order of Compensator: (0,1,2), Coefficients: 1, τ_n , τ_d** ” where: τ_n and τ_d are the numerator and denominator time constants of the compensator transfer function:

$$C(s) = \frac{\tau_n s + 1}{\tau_d s + 1}$$

For a second order compensator the input data line consists of 5 numbers: “**Order of Compensator: (0,1,2), Coefficients: 2, ζ_n , ω_n , ζ_d , ω_d** ” where ζ_n , and ω_n are the numerator damping coefficient and frequency in (rad/sec), and ζ_d , and ω_d are the denominator damping and frequency in (rad/sec) of the compensator transfer function.

$$C(s) = \frac{\omega_d^2 \left(s^2 + 2\zeta_n \omega_n s + \omega_n^2 \right)}{\omega_n^2 \left(s^2 + 2\zeta_d \omega_d s + \omega_d^2 \right)}$$