Overhead Crane Position Control

In this example the plant consists of two masses m_1 and m_2 which are connected with a rope and the mass m_1 is suspended from m_2 by the rope, as shown in Figure 1. They represent a simple model of an overhead crane where the mass m_2 can only move along they y direction as a result of the control force F_c which is applied on m_2 along they y direction. Equations 1 are describing the motion of the two masses.

$$m_1\ddot{y}_1 = m_1\, g\sin\theta + F_d$$

$$m_2\ddot{y}_2 = F_c - m_1\, g\sin\theta \qquad \sin\theta = \frac{x_2 - x_1}{l} \qquad (1)$$

Where:

g is the acceleration due to gravity
θ is the angle of the string from vertical
L is the length of the pendulum

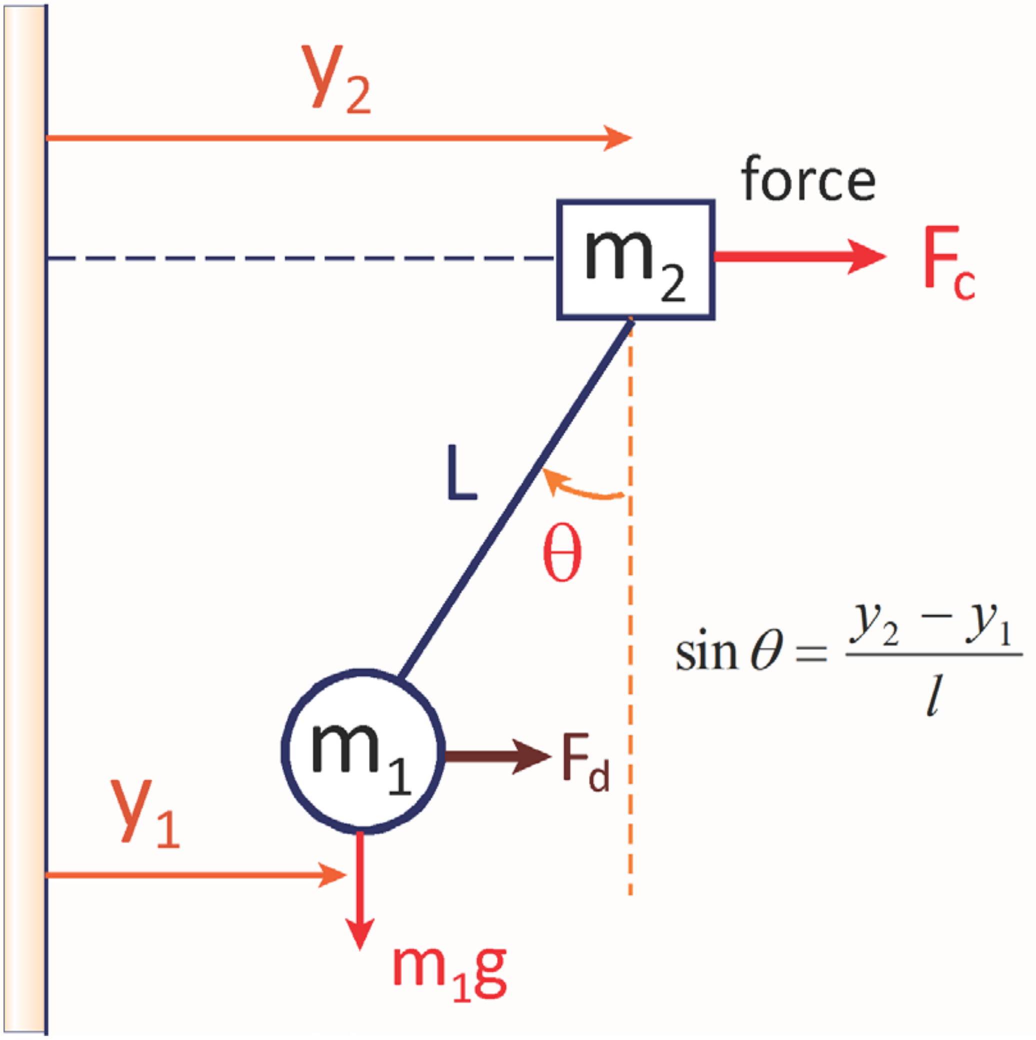


Figure 1 Simple Overhead Crane Plant Model

There is also a disturbance force F_d that is acting on the bottom mass m_1 and it is along the y_1 direction. The design requirement for this plant is to control the position y_1 of the bottom mass m_1 by applying a control force F_c on the top mass m_2 which affects the position of both masses (y_1 & y_2). We should be able to command the position of m_1 by moving m_2 and we should also be able to maintain the m_1 position constant when the disturbance force F_d is applied on m_1. From equations 1 we can write the plant equations in state-space form, assuming that g/l =1 and m_1=m_2

$$\underline{\dot{x}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \underline{x} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} u_c \\ w_d \end{pmatrix} + G\,w \tag{2}$$

Where:

$\underline{x(t)}$ is the state vector, $\underline{x}$= [y_1, y_2, $\dot{y}_1, \dot{y}_2$]
$u_c(t)$ is the control force F_c
$w_d(t)$ is the disturbance force F_d
w(t) is the process noise vector

The output vector in equation 3 consists of only two deterministic measurements: the position y_1 of the mass m_1 and the pendulum angle θ of the rope from vertical. Where: $\underline{v}$ is a zero mean white measurement noise vector.

$$\underline{z} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{bmatrix} \underline{x} + \underline{v} \tag{3}$$

Design Approaches

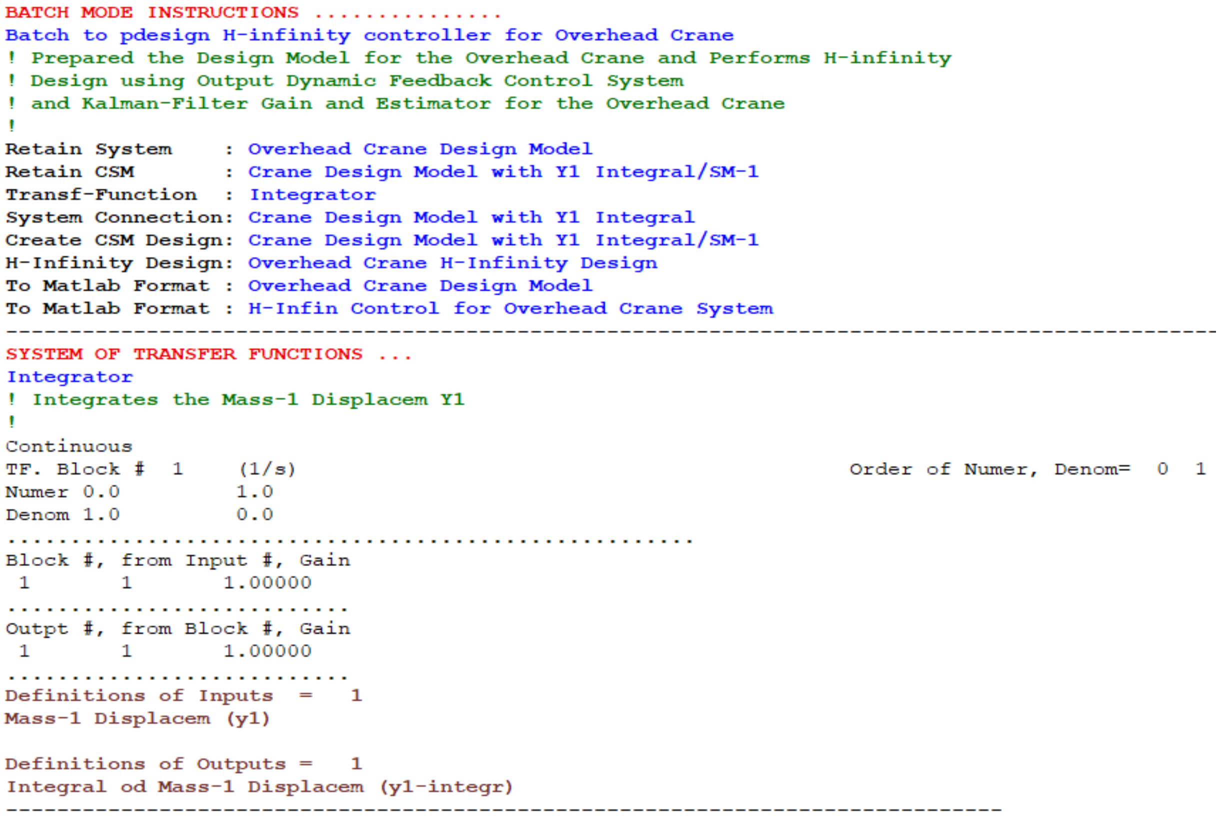
In this example we will design control systems that use the two output measurements, the m_1 position y_1 and the pendulum angle θ to calculate the required control force on m_2 in order to control the position of the bottom mass y_1. The LQR and H-infinity control design methods guarantee stable solutions but since we want to control the position y_1 of the bottom mass m_1 under the influence of disturbance forces we will introduce one additional state in the design model (y_1-integral) and we will design a state-feedback controller for the augmented 5-state plant. However, since most of the states are not measurable, we cannot directly apply state-feedback and must design an estimator for the four-state vector, $\underline{x}$= [y_1, y_2, $\dot{y}_1, \dot{y}_2$] and apply the state-feedback from the estimated states plus the y_1-integral which is known and it does not need to be estimated. We will demonstrate three approaches of finding solutions to this problem: (a) an H-infinity output feedback dynamic controller, (b) an H-infinity state-feedback gain controller in combination with a Kalman-Filter estimator, and (c) an LQG state-feedback/ estimator approach. We will use the Flixan program to create the dynamic models for control design and analysis and produce three separate control systems for each design method. We will then analyze performance of each system in Matlab using simulation models and will examine stability using Bode and Nichols plots.

1. H-Infinity Design Using Output Feedback

This H-infinity design creates a dynamic controller that closes the loop between the 2 plant outputs y_1 and θ, and the control force that drives m_1. We will use the plant model "*Overhead Crane Design Model*" to create the Design Model and then the Synthesis Model for the H-Infinity algorithm. This system has two inputs: the control force F_c on m_2, and the disturbance force F_d on m_1. It also has three outputs: (y_1, θ, and y_1-dot). The states are the two mass positions and velocities. We will augment this system by adding one more state, the integral of m_1 position (y_1-integral). This additional state is needed in order for the system to maintain y_1 position in the presence of steady disturbance forces F_d. The augmented system title is "*Crane Design Model with Y1 Integral*" and it has one additional state and output, the integral of m1 position, y_1-integral. We will then use this augmented design model to create the H-infinity SM, a 9-matrices system which is the input to the H-infinity program. The creation of the Synthesis Model is an interactive process where the user defines the control and disturbance inputs, and the measurement and performance criteria outputs. The user must also define the input and output scaling gains which is an iterative process.

1.1 The Flixan Files

The files for this H-infinity design are in directory: "*\Flixan\Control Analysis\Hinfinity\Examples\Crane Hinf\1-Output Fbk Hinf*". The input file is "*Crane Hinf1.Inp*" and it contains the Flixan datasets that generate the plant models and calculate the dynamic H-infinity controller. It begins with a batch set that can be used to process the entire file. The batch retains the previously created plant system and the control synthesis model in the systems file "*Crane Hinf1.Qdr*".



```
BATCH MODE INSTRUCTIONS ...............
Batch to pdesign H-infinity controller for Overhead Crane
! Prepared the Design Model for the Overhead Crane and Performs H-infinity
! Design using Output Dynamic Feedback Control System
! and Kalman-Filter Gain and Estimator for the Overhead Crane
!
Retain System     : Overhead Crane Design Model
Retain CSM        : Crane Design Model with Y1 Integral/SM-1
Transf-Function   : Integrator
System Connection: Crane Design Model with Y1 Integral
Create CSM Design: Crane Design Model with Y1 Integral/SM-1
H-Infinity Design: Overhead Crane H-Infinity Design
To Matlab Format : Overhead Crane Design Model
To Matlab Format : H-Infin Control for Overhead Crane System
-----------------------------------------------------------------------------------------
SYSTEM OF TRANSFER FUNCTIONS ...
Integrator
! Integrates the Mass-1 Displacem Y1
!
Continuous
TF. Block #  1     (1/s)                                      Order of Numer, Denom=  0  1
Numer 0.0          1.0
Denom 1.0          0.0
......................................................
Block #, from Input #, Gain
 1       1       1.00000
...........................
Outpt #, from Block #, Gain
 1       1       1.00000
...........................
Definitions of Inputs  =   1
Mass-1 Displacem (y1)

Definitions of Outputs =   1
Integral od Mass-1 Displacem (y1-integr)
-------------------------------------------------------------------------------
```

```
INTERCONNECTION OF SYSTEMS .....
Crane Design Model with Y1 Integral
! Creates an Augmented plant for control Design by including the integral
! of mass-1 displacement in the states and output.
!
Titles of Systems to be Combined
Title 1 Overhead Crane Design Model
Title 2 Integrator
SYSTEM INPUTS TO SUBSYSTEM  1                                                         Plant(s)
System Input  1 to Subsystem  1, Input  1, Gain= 1.0                                  Control Force
System Input  2 to Subsystem  1, Input  2, Gain= 1.0                                  Disturb Force
.................................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                                      Plant Outputs
System Output  1 from Subsystem  1, Output  1, Gain= 1.0                              y1 displacem
System Output  2 from Subsystem  1, Output  2, Gain= 1.0                              theta
System Output  3 from Subsystem  1, Output  3, Gain= 1.0                              y1-dot
.................................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                                                      Integrator
System Output  4 from Subsystem  2, Output  1, Gain= 1.0                              y1 integral
.................................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2                                               Plant Outp to Control Input
Subsystem  1, Output  1 to Subsystem  2, Input  1, Gain= 1.0                          y1 displacem
.................................................................................
Definitions of Inputs  =   2
Control Force on m2    (Fc)
Disturb Force on m1    (Fd)

Definitions of States  =   5
Bottom Mass Position, y1
Top Mass Position, y2
Bottom Mass Velocity, y1-dot
Top Mass Velocity, y2-dot
Bot Mass-1 Position Integral, y1-int

Definitions of Outputs =   4
Mass-1 Displacement       (y1)
Pendulum Angle            (theta)
Bottom Mass Velocity,     (y1-dot)
Bot Mass1 Position-Integr (y1-int)
--------------------------------------------------------------------------------------------------------
```

```
CREATE A SYNTHESIS MODEL FOR H-INFINITY CONTROL DESIGN
Crane Design Model with Y1 Integral/SM-1
Crane Design Model with Y1 Integral
Number of Uncertainty I/O Pairs    :    0
Number of Disturbance Inputs       :    1
Disturbance Input Numbers          :    2
Number of Control Inputs           :    1
Control Input Numbers              :    1
Number of Performance Outputs      :    4
Perform Optimization Output Numbrs:     1    2    3    4
Number of Commanded Outputs        :    1
Command Regulated Output Numbers   :    1
Number of Measurement Outputs      :    3    2
Measurement Output Numbers         :    1    2    4
Disturbance Input  & Command Gains:  0.00018   0.0007    0.0012    0.0008    0.0033
Performance Output & Control Gains:  0.00015   0.5       0.2       0.0001    0.00015   0.0007
--------------------------------------------------------------------------------------------------------
```

```
H-INFINITY CONTROL DESIGN ......
Overhead Crane H-Infinity Design
Synthesis Model for Control Design in file (.Qdr) :        Crane Design Model with Y1 Integral/SM-1
Peak Value of the Sensitivity Function Gamma (dB) :        70.0
Dynamic Output Feedback via an Estimator for      :        Overhead Crane System
--------------------------------------------------------------------------------------------------------
```

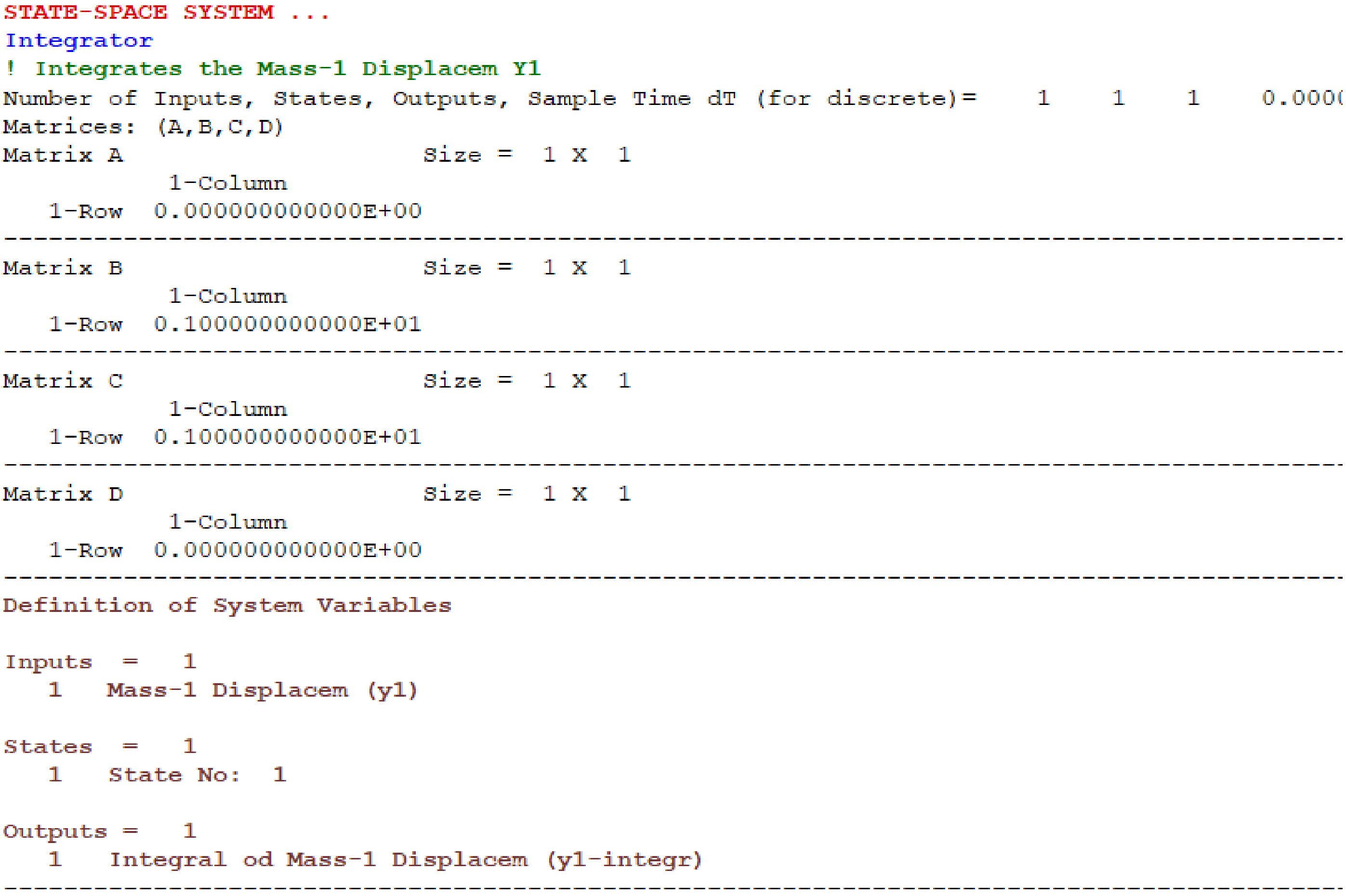
```
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Overhead Crane Design Model
System
Crane
-------------------------------------------------------------------------------------------
```

```
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
H-Infin Control for Overhead Crane System
System
Control
-------------------------------------------------------------------------------------------
```

The input file includes the H-infinity design dataset, title: “*Overhead Crane H-Infinity Design*” that creates the control state-space system by reading and processing the SM, title: “*Crane Design Model with Y1 Integral/SM-2*”, which is already created and saved in the systems file. The control system is saved in the systems file under the title “*H-Infinity Control for Overhead Crane System*”. There is an H-infinity model creation dataset in the input file that can create the 9-matrices SM via the batch set, but the SM is first created interactively as we shall see in the section 1.6. It includes two titles. The first title “*Crane Design Model with Y1 Integral/SM-1*” is the name of the SM that will be created. The second title “*Crane Design Model with Y1 Integral*” is the system that it will be created from. The dataset creates the SM by processing the instructions that follow and it includes the scaling gains. The crane and controller systems are converted to Matlab m-functions “crane.m” and “control.m” and are loaded into Matlab by executing the script “init.m”.

```
% Init.m
r2d=180/pi;
[Ad,Bd,Cd,Dd]= crane;           % Load the Plant Model
[Ac,Bc,Cc,Dc]= control;         % Load H-infinity Control System
```

1.2 Systems File “Crane Hinf1.Qdr”



```
STATE-SPACE SYSTEM ...
Integrator
! Integrates the Mass-1 Displacem Y1
Number of Inputs, States, Outputs, Sample Time dT (for discrete)=    1    1    1    0.000
Matrices: (A,B,C,D)
Matrix A                    Size =  1 X  1
           1-Column
   1-Row  0.000000000000E+00
------------------------------------------------------------------------------------------
Matrix B                    Size =  1 X  1
           1-Column
   1-Row  0.100000000000E+01
------------------------------------------------------------------------------------------
Matrix C                    Size =  1 X  1
           1-Column
   1-Row  0.100000000000E+01
------------------------------------------------------------------------------------------
Matrix D                    Size =  1 X  1
           1-Column
   1-Row  0.000000000000E+00
------------------------------------------------------------------------------------------
Definition of System Variables

Inputs  =   1
   1   Mass-1 Displacem (y1)

States  =   1
   1   State No:  1

Outputs =   1
   1   Integral od Mass-1 Displacem (y1-integr)
------------------------------------------------------------------------------------------
```

```
STATE-SPACE SYSTEM ...
Overhead Crane Design Model
! A dynamic model of two masses representing an overhead crane. The
! two inputs are control and disturbance forces, the outputs are
! mass position y1, velocity y1-dot, and pendulum angle theta
!
Number of Inputs, States, Outputs, Sample Time dT (for discrete)=    2    4    3    0.0000
Matrices: (A,B,C,D)
Matrix A                    Size =  4 X  4
           1-Column            2-Column            3-Column            4-Column
   1-ROW  0.0                 0.0                 1.0                 0.0
   2-ROW  0.0                 0.0                 0.0                 1.0
   3-ROW -1.0                 1.0                 0.0                 0.0
   4-ROW  1.0                -1.0                 0.0                 0.0
-----------------------------------------------------------------------------------------------
Matrix B                    Size =  4 X  2
           1-Column            2-Column
   1-ROW  0.0                 0.0
   2-ROW  0.0                 0.0
   3-ROW  0.0                 1.0
   4-ROW  1.0                 0.0
-----------------------------------------------------------------------------------------------
Matrix C                    Size =  3 X  4
          1-COLUMN             2-COLUMN            3-COLUMN            4-COLUMN
   1-ROW  1.0                 0.0                 0.0                 0.0
   2-ROW -1.0                 1.0                 0.0                 0.0
   3-ROW  0.0                 0.0                 1.0                 0.0
-----------------------------------------------------------------------------------------------
Matrix D                    Size =  3 X  2
          1-COLUMN
   1-ROW  0.0                 0.0
   2-ROW  0.0                 0.0
   3-ROW  0.0                 0.0
-----------------------------------------------------------------------------------------------
Definition of System Variables

Inputs  =   2
   1   Control Force on Top Mass m2
   2   Disturb Force on Bot Mass m1

States  =   4
   1   Bottom Mass Position, y1
   2   Top Mass Position, y2
   3   Bottom Mass Velocity, y1-dot
   4   Top Mass Velocity, y2-dot

Outputs =   3
   1   Bottom Mass Position, y1
   2   Pendulum Angle, theta
   3   Bottom Mass Velocity, y1-dot
------------------------------------------------------------------------
```

```
SYNTHESIS MODEL FOR H-INFINITY CONTROL
Crane Design Model with Y1 Integral/SM-1
Number of: States (x), Uncertainty Inp/Outputs from Plant Variations (dP)=   5   0   0
Number of: Extern Disturbance Inputs (Wi), Control Inputs (Uc)          =   1   1
Number of: Output Criteria (Zo), Regulated Outputs (Zr), Measurements (y)=   4   1   3
Synthes Model Matrices: A, B1,B2,C1,C2, D11,D12,D21,D22, Sample Time (dT)=   0.0000
Matrix A                Size =  5 X  5
          1-Column            2-Column            3-Column            4-Column            5-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   3-Row -0.100000000000E+01  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.100000000000E+01 -0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Matrix B1               Size =  5 X  5
          1-Column            2-Column            3-Column            4-Column            5-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Matrix B2               Size =  5 X  1
          1-Column
   1-Row  0.000000000000E+00
   2-Row  0.000000000000E+00
   3-Row  0.000000000000E+00
   4-Row  0.100000000000E+01
   5-Row  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Matrix C1               Size =  6 X  5
          1-Column            2-Column            3-Column            4-Column            5-Column
   1-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row -0.100000000000E+01  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
   5-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Matrix C2               Size =  3 X  5
          1-Column            2-Column            3-Column            4-Column            5-Column
   1-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row -0.100000000000E+01  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
------------------------------------------------------------------------------------------------------------------------
Matrix D11              Size =  6 X  5
          1-Column            2-Column            3-Column            4-Column            5-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00 -0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
-----------------------------------------------------------------------------------------------------------------------
Matrix D12              Size =  6 X  1
          1-Column
   1-Row  0.000000000000E+00
   2-Row  0.000000000000E+00
   3-Row  0.000000000000E+00
   4-Row  0.000000000000E+00
   5-Row  0.000000000000E+00
   6-Row  0.100000000000E+01
-----------------------------------------------------------------------------------------------------------------------
Matrix D21              Size =  3 X  5
          1-Column            2-Column            3-Column            4-Column            5-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
-----------------------------------------------------------------------------------------------------------------------
Matrix D22              Size =  3 X  1
          1-Column
   1-Row  0.000000000000E+00
   2-Row  0.000000000000E+00
   3-Row  0.000000000000E+00
-----------------------------------------------------------------------------------------------------------------------
```

```
Definition of Synthesis Model Variables                          Max Scaling Factors

States (x) ..........   =   5
  1     Bottom Mass Position, y1
  2     Top Mass Position, y2
  3     Bottom Mass Velocity, y1-dot
  4     Top Mass Velocity, y2-dot
  5     Bot Mass-1 Position Integral, y1-int

Excitation Inputs (w)   =   5
  1     Disturb Force on m1    (Fd)                              *  0.00018
  2     Commd for Outpt: Mass-1 Displacement       (y1)          *  0.0007
  3     Noise at Output: Mass-1 Displacement       (y1)          *  0.0012
  4     Noise at Output: Pendulum Angle            (theta)       *  0.0008
  5     Noise at Output: Bot Mass1 Position-Integr (y1-int)      *  0.0033

Control Inputs (u) ... =   1
  1     Control: Control Force on m2   (Fc)                      *  1.0000

Performance Outputs (z)=   6
  1     Mass-1 Displacement        (y1)                          /  0.00015
  2     Pendulum Angle             (theta)                       /  0.5
  3     Bottom Mass Velocity,      (y1-dot)                      /  0.2
  4     Bot Mass1 Position-Integr (y1-int)                       /  0.0001
  5     Track Error: Mass-1 Displacement        (y1)             /  0.00015
  6     Contrl Criter. Control Force on m2    (Fc)               /  0.0007

Measurement Outputs (y)=   3
  1     Measurm: Mass-1 Displacement        (y1)                 /  1.0000
  2     Measurm: Pendulum Angle             (theta)              /  1.0000
  3     Measurm: Bot Mass1 Position-Integr (y1-int)              /  1.0000
-----------------------------------------------------------------------------------------
STATE-SPACE SYSTEM ...
Crane Design Model with Y1 Integral
! Creates an Augmented Plant for control Design by including the integral of mass-1 displacement
! in the states and output.
Number of Inputs, States, Outputs, Sample Time dT (for discrete)=    2    5    4    0.0000
Matrices: (A,B,C,D)
Matrix A                  Size =  5 X  5
          1-Column            2-Column            3-Column            4-Column            5-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   3-Row -0.100000000000E+01  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.100000000000E+01 -0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
---------------------------------------------------------------------------------------------------------------
Matrix B                  Size =  5 X  2
          1-Column            2-Column
   1-Row  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.100000000000E+01
   4-Row  0.100000000000E+01  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00
---------------------------------------------------------------------------------------------------------------
Matrix C                  Size =  4 X  5
          1-Column            2-Column            3-Column            4-Column            5-Column
   1-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row -0.100000000000E+01  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
---------------------------------------------------------------------------------------------------------------
Matrix D                  Size =  4 X  2
          1-Column            2-Column
   1-Row  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00
---------------------------------------------------------------------------------------------------------------
```

```
Definition of System Variables

Inputs  =   2
   1   Control Force on m2   (Fc)
   2   Disturb Force on m1   (Fd)

States  =   5
   1   Bottom Mass Position, y1
   2   Top Mass Position, y2
   3   Bottom Mass Velocity, y1-dot
   4   Top Mass Velocity, y2-dot
   5   Bot Mass-1 Position Integral, y1-int

Outputs =   4
   1   Mass-1 Displacement       (y1)
   2   Pendulum Angle            (theta)
   3   Bottom Mass Velocity,     (y1-dot)
   4   Bot Mass1 Position-Integr (y1-int)
-------------------------------------------------------------------------
STATE-SPACE SYSTEM ...
H-Infin Control for Overhead Crane System
Number of Inputs, States, Outputs, Sample Time dT (for discrete)=    3    5    1    0.0000
Matrices: (A,B,C,D)
Matrix A                    Size =  5 X  5
           1-Column            2-Column            3-Column            4-Column            5-Column
   1-Row -0.384352971951E+00  0.771084837111E-01  0.100000000000E+01  0.000000000000E+00 -0.791504459031E-01
   2-Row -0.200089080082E+00 -0.728849709528E-01  0.000000000000E+00  0.100000000000E+01 -0.786815482796E-01
   3-Row -0.109061054575E+01  0.101841772222E+01  0.649874209788E-06  0.895218380198E-07 -0.100797487155E-01
   4-Row -0.719767575907E+01 -0.996660288899E+01 -0.135387964904E+02 -0.423614181498E+01 -0.701167525298E+01
   5-Row  0.393446166733E+00  0.797858612500E-02  0.000000000000E+00  0.000000000000E+00 -0.333095908327E+00
-------------------------------------------------------------------------------------------------------------
Matrix B                    Size =  5 X  3
           1-Column            2-Column            3-Column
   1-Row  0.307244488239E+00 -0.771084837111E-01  0.791504459031E-01
   2-Row  0.272974051035E+00  0.728849709528E-01  0.786815482796E-01
   3-Row  0.721937550216E-01 -0.184173967377E-01  0.100802102974E-01
   4-Row  0.644292606089E-01 -0.584524320586E-02  0.116729513961E-01
   5-Row  0.598575247142E+00 -0.797858612500E-02  0.333095908327E+00
-------------------------------------------------------------------------------------------------------------
Matrix C                    Size =  1 X  5
           1-Column            2-Column            3-Column            4-Column            5-Column
   1-Row -0.812740125525E+01 -0.897244813220E+01 -0.135387964904E+02 -0.423614181498E+01 -0.700000230158E+01
-------------------------------------------------------------------------------------------------------------
Matrix D                    Size =  1 X  3
           1-Column            2-Column            3-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
-------------------------------------------------------------------------------------------------------------
Definition of System Variables

Inputs  =   3
   1   Measurm: Mass-1 Displacement       (y1)
   2   Measurm: Pendulum Angle            (theta)
   3   Measurm: Bot Mass1 Position-Integr (y1-int)

States  =   5
   1   Bottom Mass Position, y1
   2   Top Mass Position, y2
   3   Bottom Mass Velocity, y1-dot
   4   Top Mass Velocity, y2-dot
   5   Bot Mass-1 Position Integral, y1-int

Outputs =   1
   1   Control: Control Force on m2   (Fc)
-------------------------------------------------------------------------------------------------------------
```

1.3 Control Analysis

The Simulink model “*Open_Loop.Slx*” in Figure 1.1, is used to analyze control stability. It includes the two Flixan generated systems: the crane model and the control system in open-loop. The loop is opened at the control force input to the top mass m_2. The disturbance force input is not used for stability analysis. The two outputs of the crane model (y_1 and θ) are inputs to the control system. The control system was designed based on the augmented design system that includes y1-integral, and a y1-integrator is included to drive the controller third input. The script file “frequ.m” below, calculates the frequency responses and plots the Bode and Nichols plots shown in Figure 1.2. Notice that the system has a big resonance at 1.41 (rad/sec) which is at the pendulum frequency. The control system counteracts the natural pendulum mode by introducing an anti-resonance at the same frequency since it is designed around the plant model.

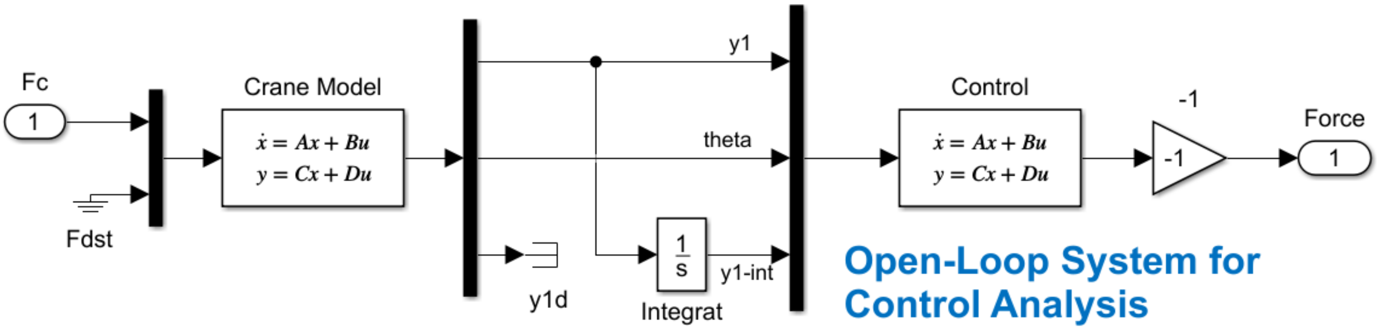
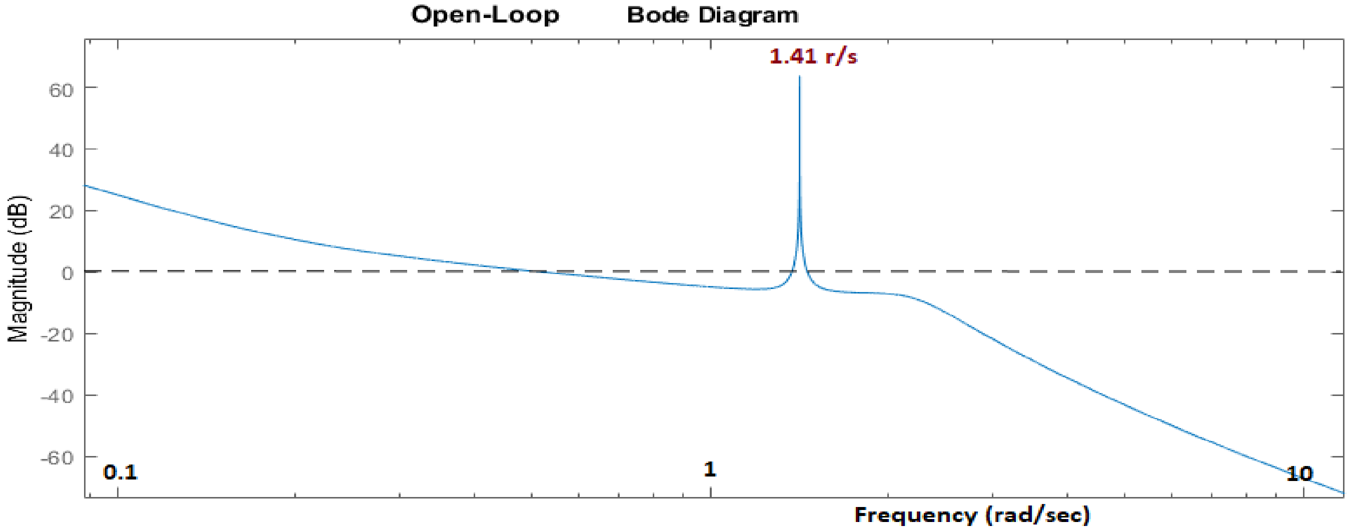


Figure 1.1 Frequency Response Analysis Model “Open_Loop.Slx”

```
% Frequency Response Analysis
init
[A1,B1,C1,D1]= linmod('Open_Loop'); % Stabil Anal Open-Loop Simulink model
sys= ss(A1,B1,C1,D1);               % Create SS System
w1=logspace(-2, 2, 10000);          % Define Frequ Range
figure(1); nichols(sys,sys1,w1)     % Nichols Plot
figure(2); bode(sys,w1)             % Bode Plot
```



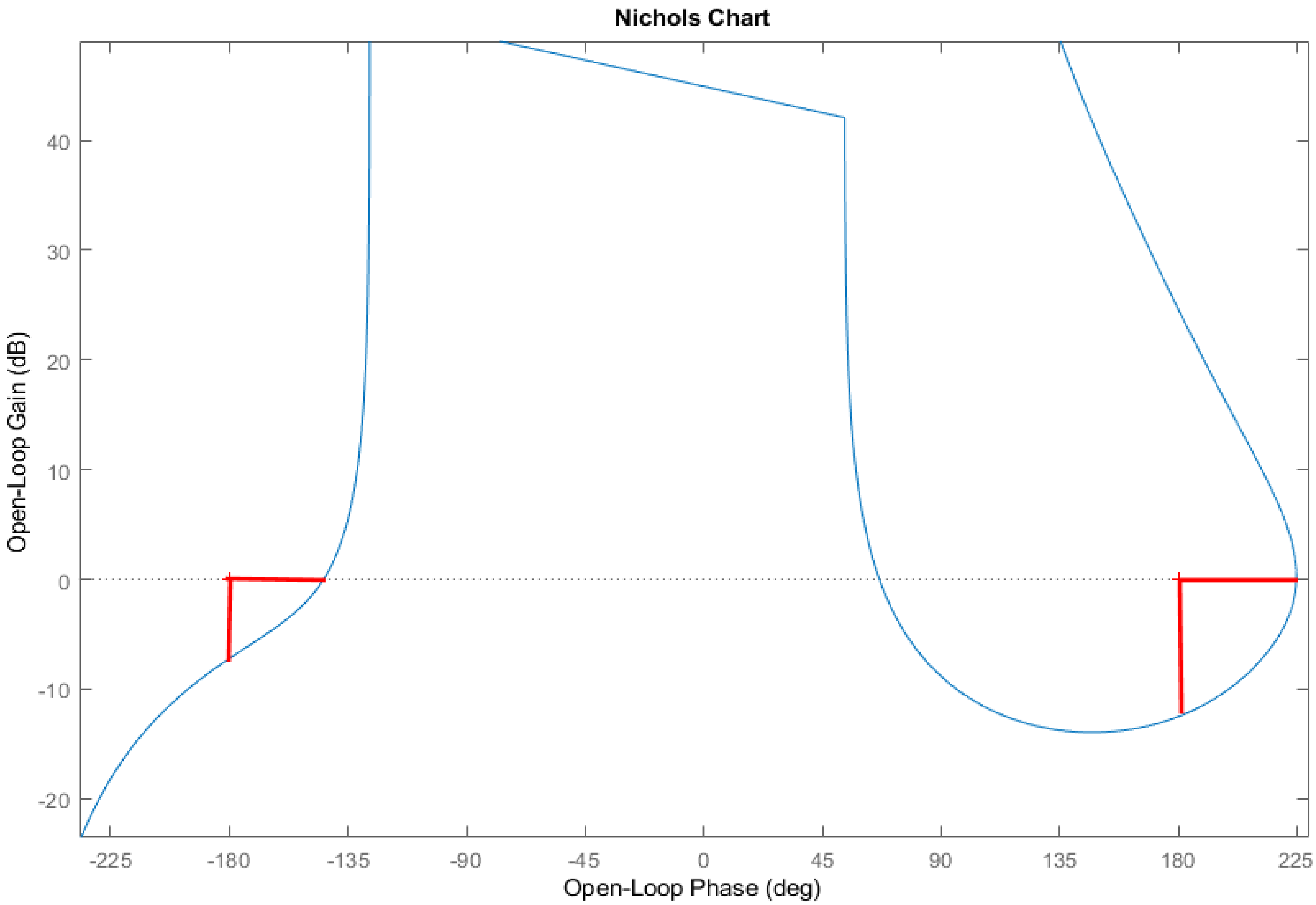


Figure 1.2 Frequency Response Stability Analysis Results Showing Phase and Gain Margins

```
% Frequency Response Analysis
init
[A1,B1,C1,D1]= linmod('Open_Loop'); % Stabil Anal Open-Loop Simulink model
sys= ss(A1,B1,C1,D1);               % Create SS System
w1=logspace(-2, 2, 10000);          % Define Frequ Range
figure(1); nichols(sys,sys1,w1)     % Nichols Plot
figure(2); bode(sys,w1)             % Bode Plot
```

The script file “frequ.m” calculates the open-loop system frequency response and plots the Bode and Nichols plots in Figure 1.2.

1.4 Simulation Models

Figure 1.3 shows two Simulink models that analyze the H-infinity control system performance. The first model “*Crane_Sim-1*” is used to test the controller state-feedback gain Cc directly from the four states: $\underline{x}$= [y_1, y_2, $\dot{y}_1, \dot{y}_2$] without a state estimator. The second model “*Crane_Sim-2*” includes the entire dynamic controller from the two plant outputs (y_1, θ) plus y_1-integral.

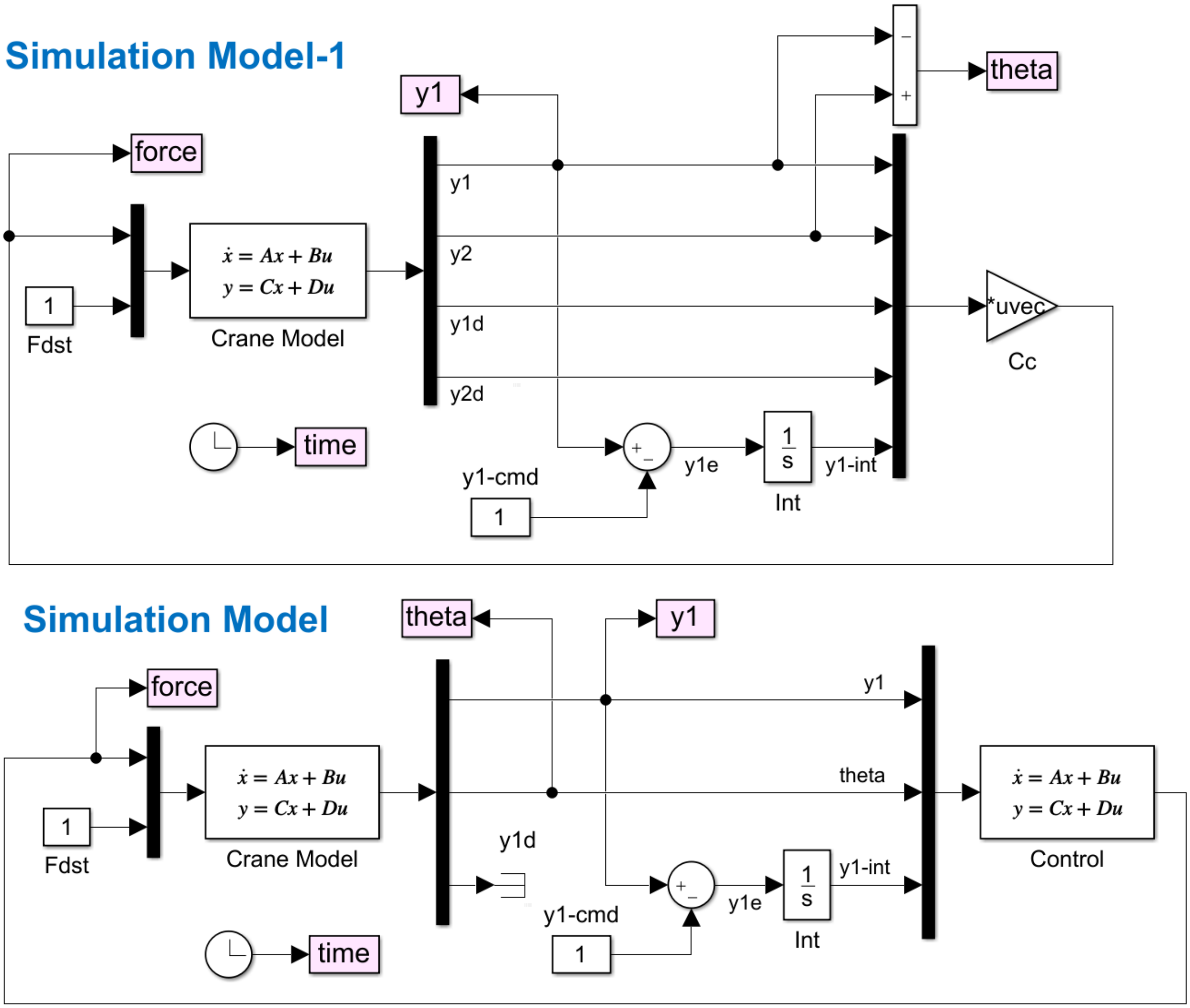


Figure 1.3 Two Simulation Models "Crane_Sim1" and "Crane_Sim2". The first one uses State-Feedback via matrix Cc. The second one uses Output Feedback via the H-Infinity Control System

1.5 Simulation Results

Figure 1.4 shows the system's response to m_1 displacement command y_{1cmd}. The pendulum swings under the influence of the control force, the force reverses and the pendulum angle returns to zero. In the meantime m1 catches up to the command.

Figure 1.5 shows the system's response to a steady disturbance force F_{dst}=1 on the bottom mass m_1. Mass1 moves in the direction of force under the influence of the disturbance force and the pendulum angle θ swings negative and oscillates. A negative control force -1 is applied on the top mass m_2 to counteract the disturbance force. The bottom mass m_1 returns to its original position under the influence of the control force Fc=-1. The top mass m_2 moved to the left and the pendulum angle θ stabilized at -1.

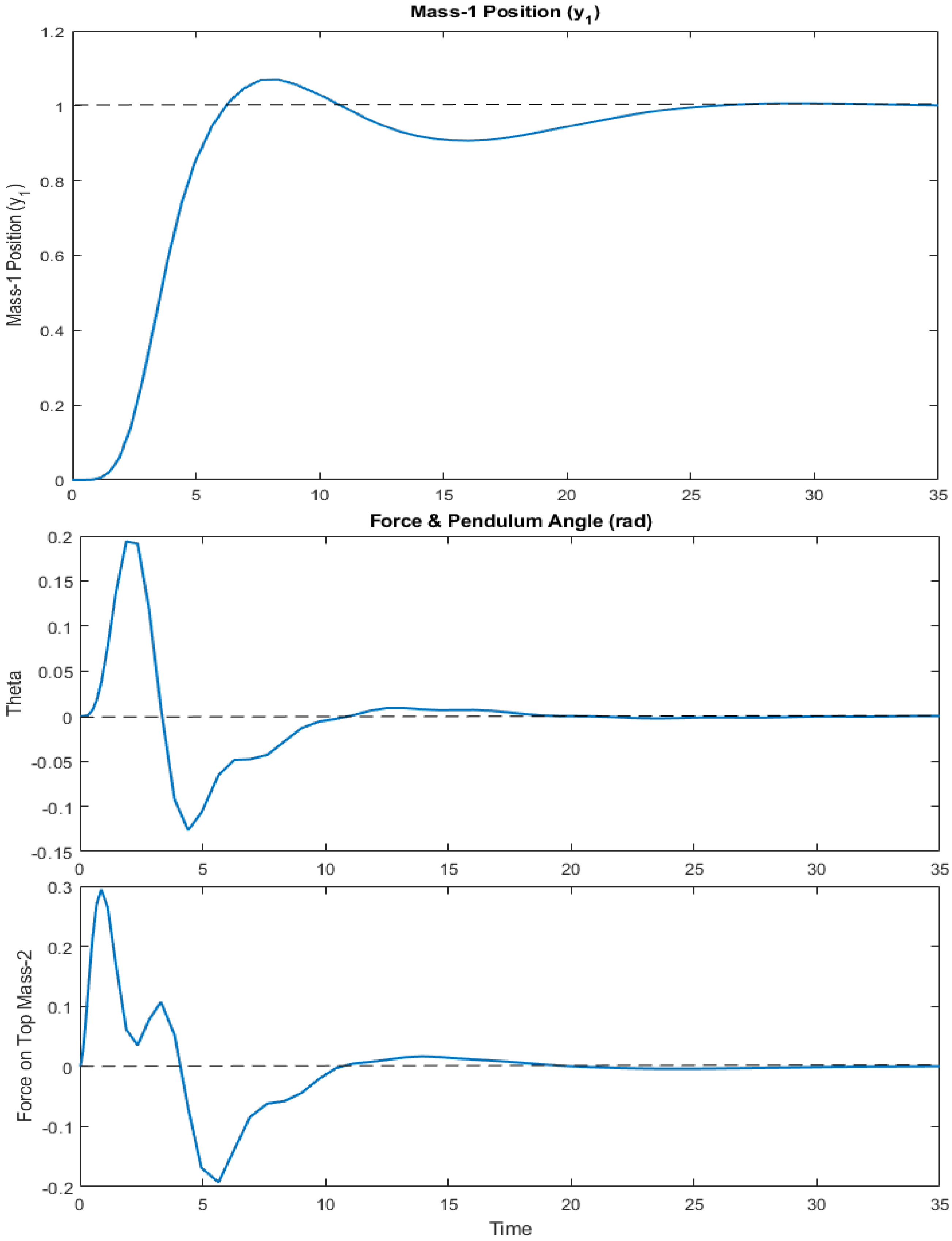


Figure 1.4 System's Response to m1 Position Command

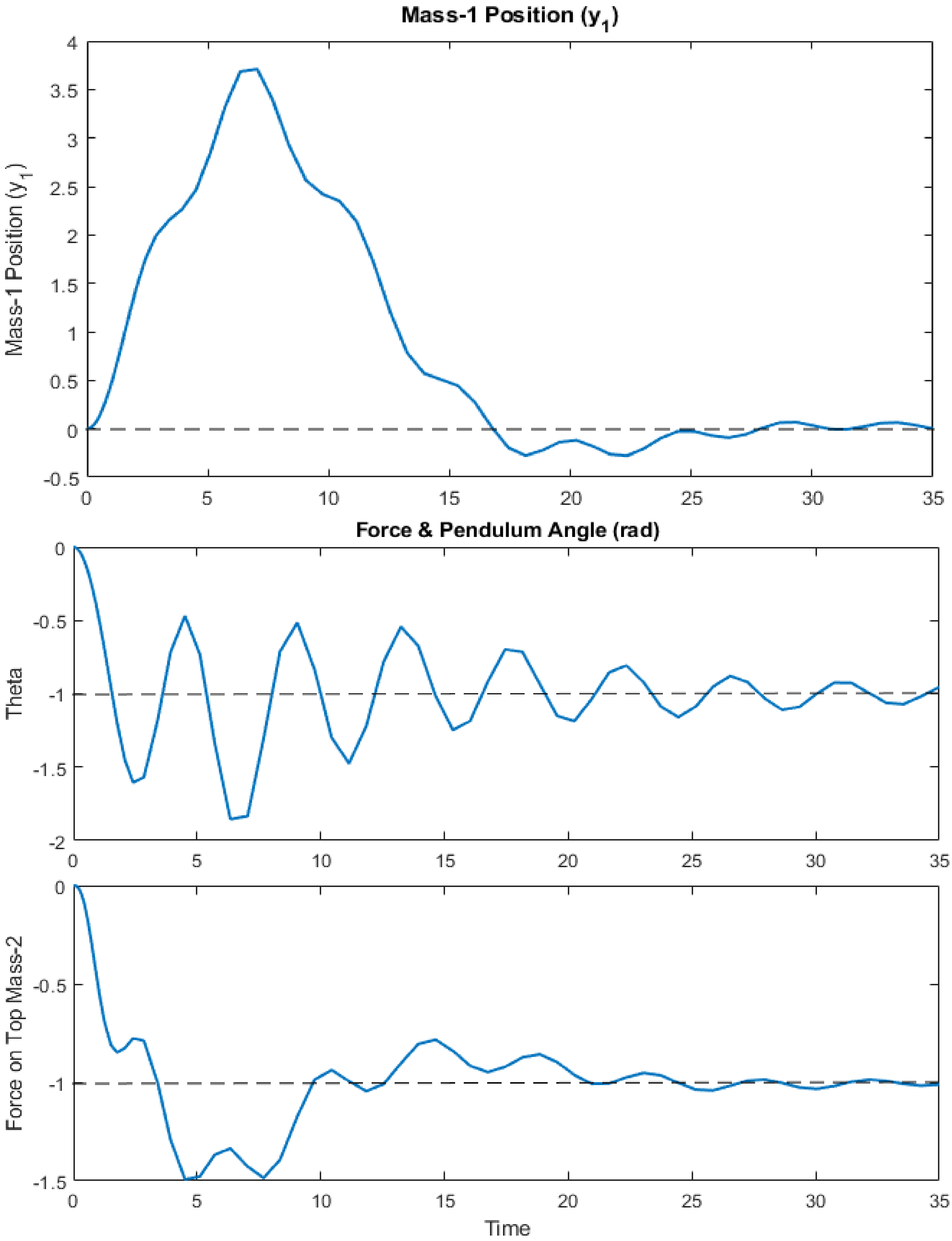
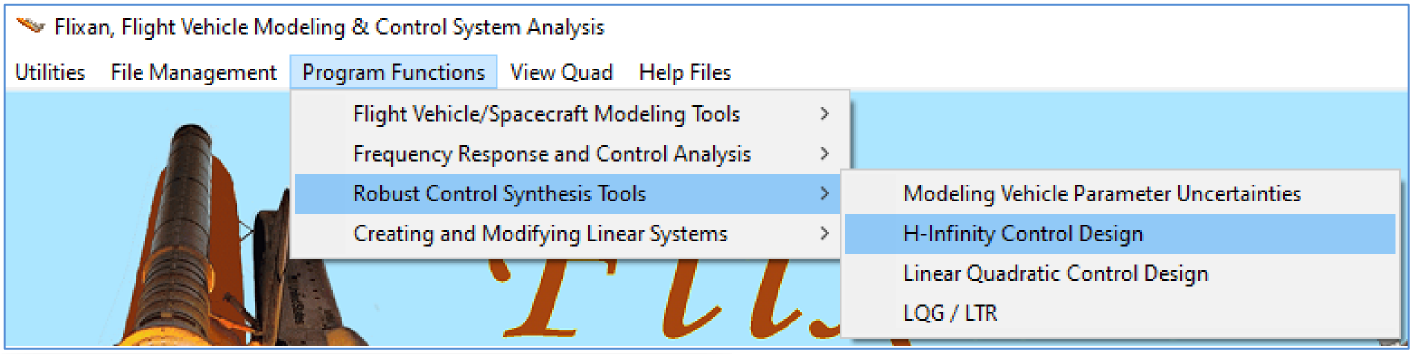
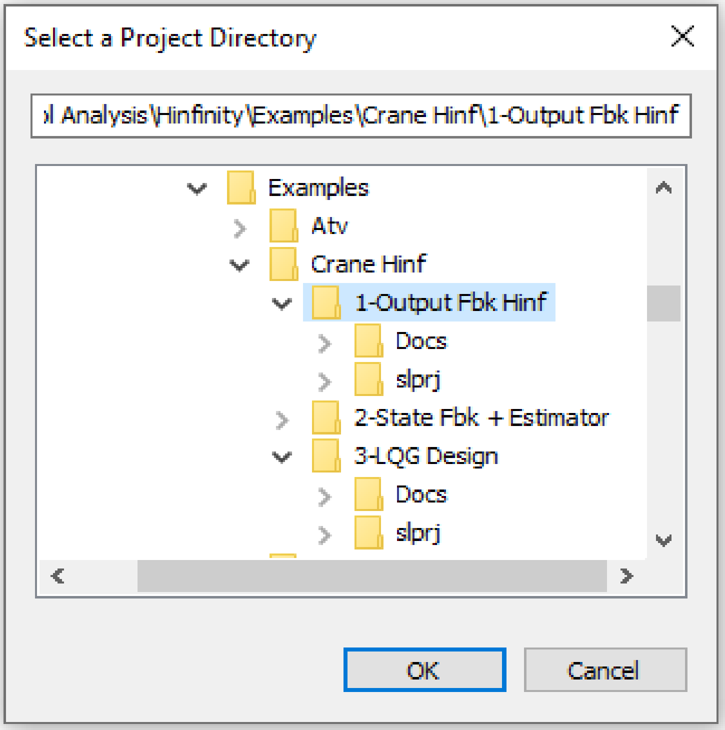


Figure 1.5 System's Response to an External Disturbance Force on m1

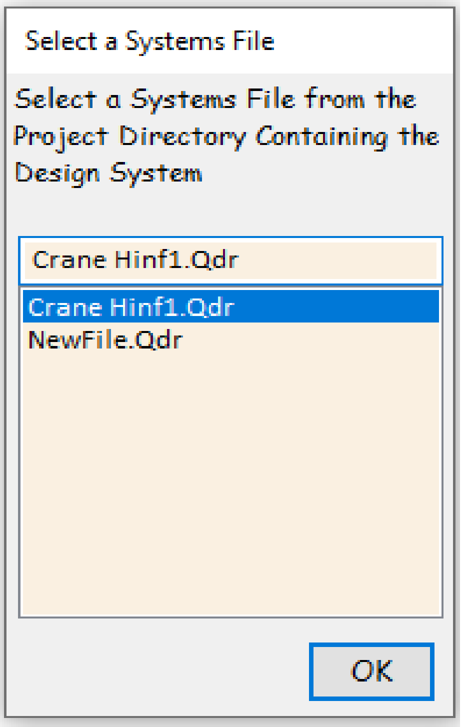
1.6 Running the H-Infinity Program Interactively

The input and systems files in this example are already set up to be processed in batch mode but in this section, we will run the H-infinity program interactively. We will first create the Synthesis Model from the design system “*Crane Design Model with Y1 Integral*” which is already saved in the systems file “*Crane Hinf1.Qdr*”. The Synthesis Model is a 9-matrices system that is used by the optimization algorithm to create the control system. The SM defines which inputs are controls and disturbances and which outputs are measurements and optimization criteria. It also includes gains which are used to define the control requirements trade-off between bandwidth, stability and performance in the H-infinity optimization. We will design the control system interactively using the SM, design the control system and save it in the systems file. We begin by running the H-infinity design program and selecting the first option as shown below. Use the menus to select the systems filename and the augmented design system.




Select a Project Directory

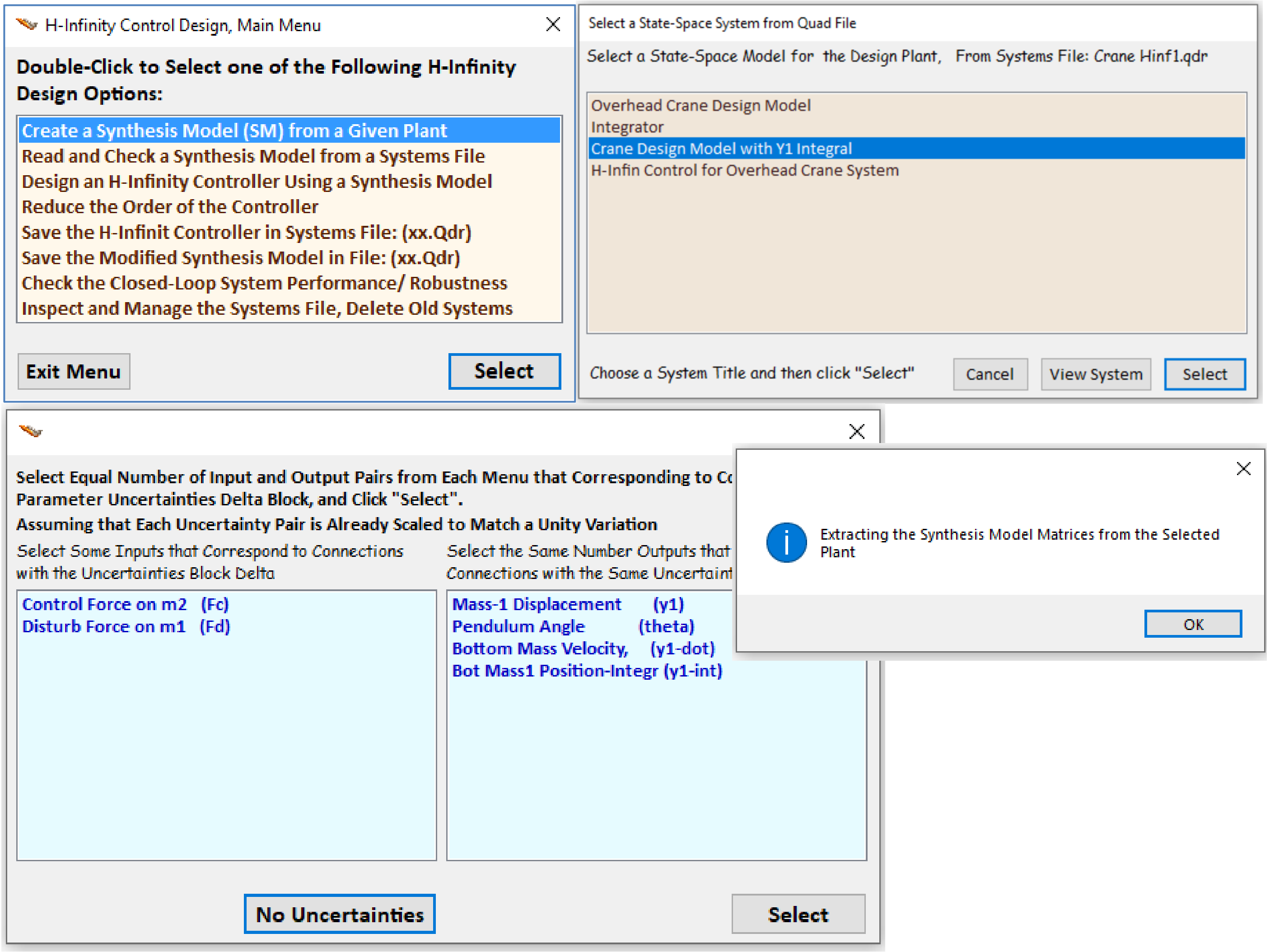
ıl Analysis\Hinfinity\Examples\Crane Hinf\1-Output Fbk Hinf


Select a Systems File

Select a Systems File from the Project Directory Containing the Design System

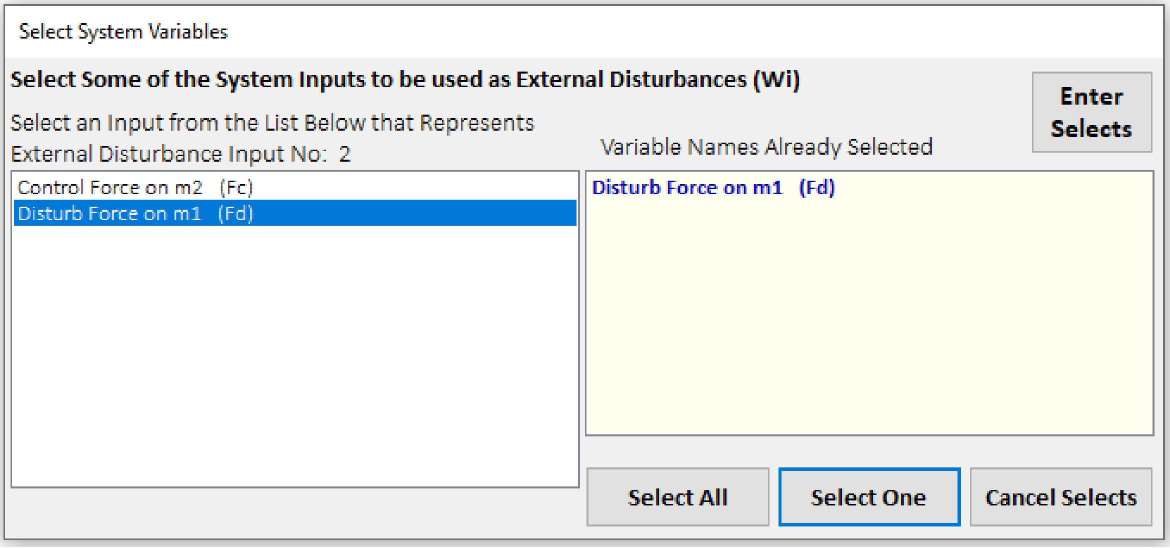
Crane Hinf1.Qdr

The SM will be created from the augmented system “Crane Design Model with Y1 Integral”, by selecting some inputs and outputs using menus and placing them into groups. The first dialog is for selecting parameter variation pairs that connect with uncertainties. In this case we don’t have any, so click on “No Uncertainties”.

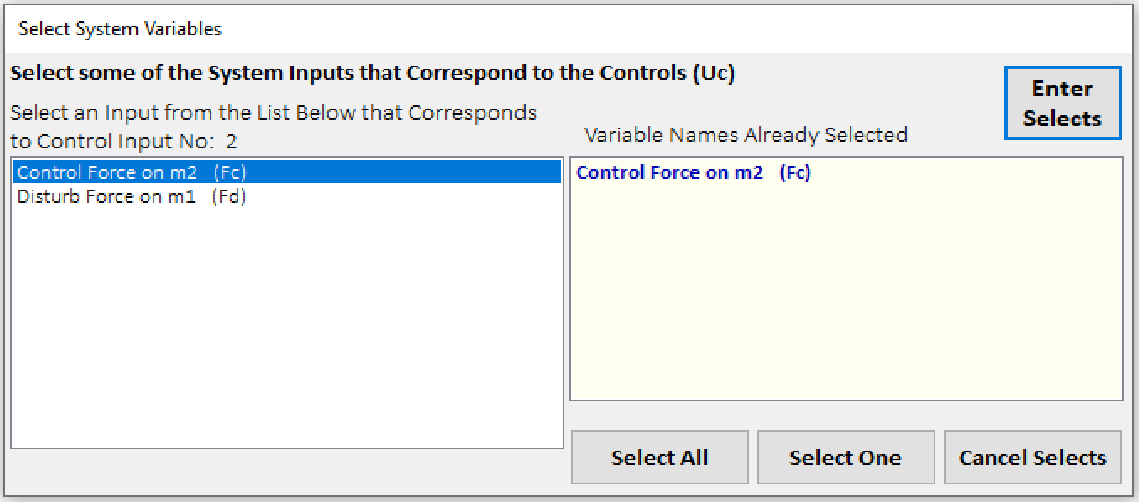


Extracting the Synthesis Model Matrices from the Selected Plant

The next menu is for defining external disturbance inputs. The system has two inputs. Select the second input which is the disturbance force on m1, and click on "Enter Selects" to continue.



The next menu is for selecting the control inputs u_c. There is only one control input in the design system which is the control force om m2. Select it and then click on “Enter Selects” to continue.



Select System Variables

Select some of the System Inputs that Correspond to the Controls (Uc)

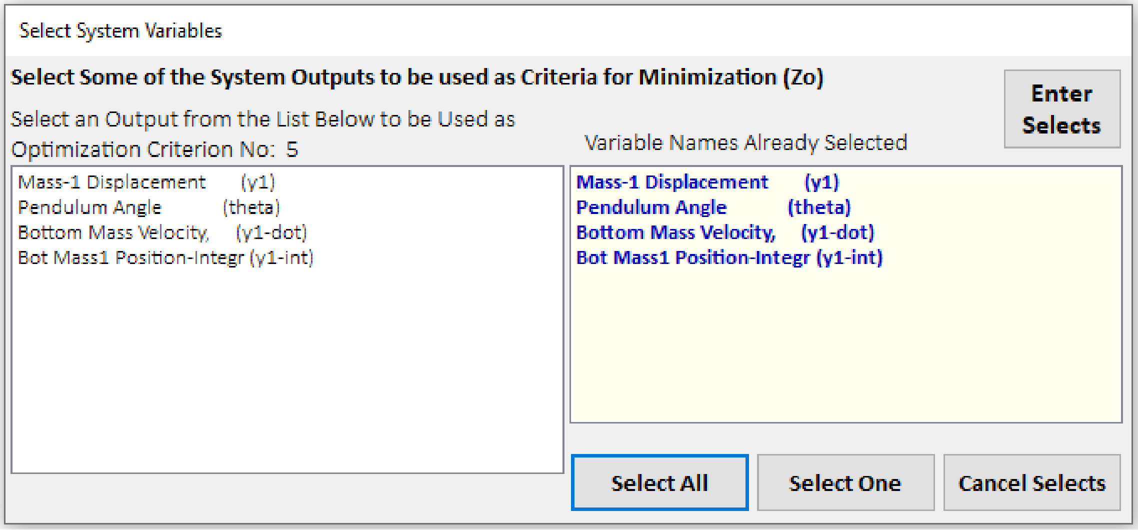
Select an Input from the List Below that Corresponds to Control Input No: 2

Control Force on m2 (Fc)
Disturb Force on m1 (Fd)

Variable Names Already Selected

Control Force on m2 (Fc)

The next menu selects the variables to be used in the performance optimization criterion. The design system has 4 outputs and we will include all 4. Select one at a time, or click on “Select All”, and then click on “Enter Selects” to continue. The next menu is for selecting outputs which are regulated by commands. Select the mass1 displacement (y1) and click on “Enter Selects”.



Select System Variables

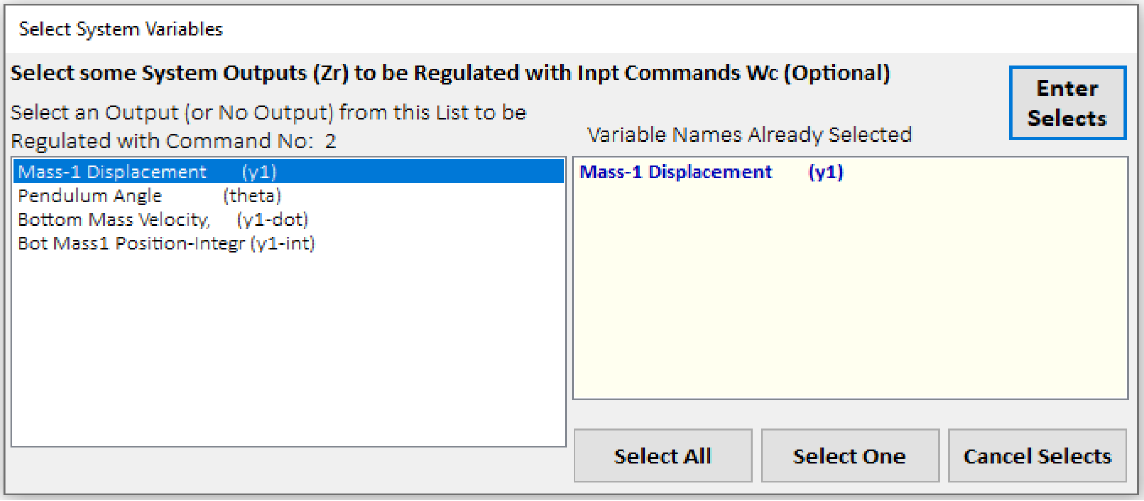
Select Some of the System Outputs to be used as Criteria for Minimization (Zo)

Select an Output from the List Below to be Used as Optimization Criterion No: 5

Mass-1 Displacement (y1)
Pendulum Angle (theta)
Bottom Mass Velocity, (y1-dot)
Bot Mass1 Position-Integr (y1-int)

Variable Names Already Selected

Mass-1 Displacement (y1)
Pendulum Angle (theta)
Bottom Mass Velocity, (y1-dot)
Bot Mass1 Position-Integr (y1-int)



Select System Variables

Select some System Outputs (Zr) to be Regulated with Inpt Commands Wc (Optional)

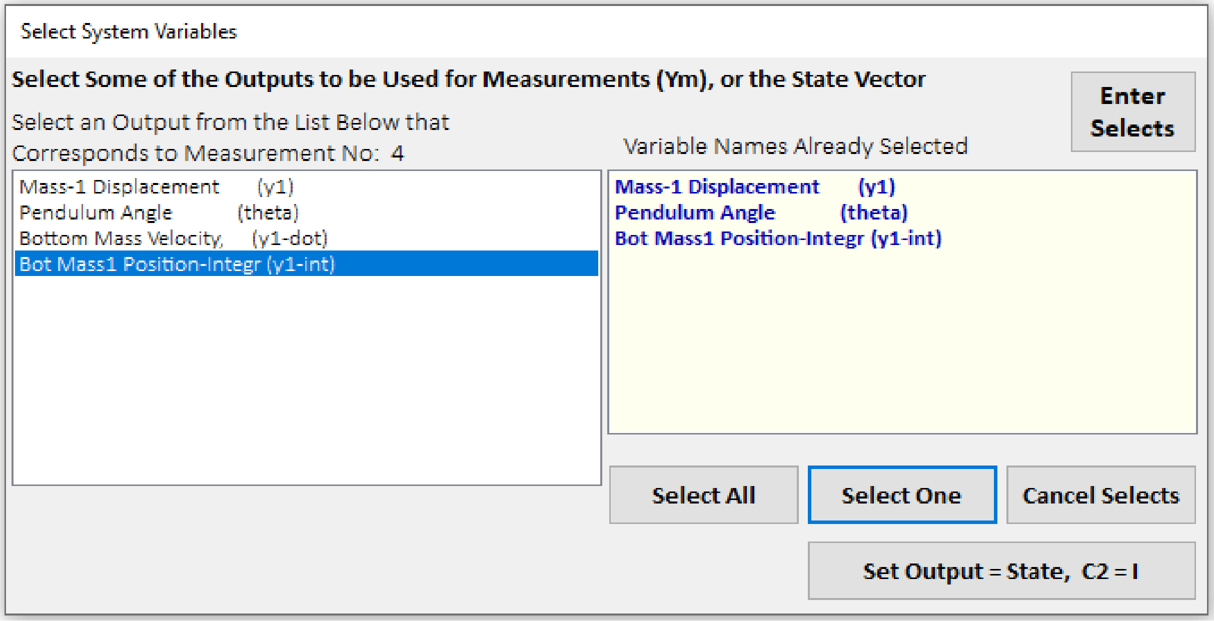
Select an Output (or No Output) from this List to be Regulated with Command No: 2

Mass-1 Displacement (y1)
Pendulum Angle (theta)
Bottom Mass Velocity, (y1-dot)
Bot Mass1 Position-Integr (y1-int)

Variable Names Already Selected

Mass-1 Displacement (y1)

The next menu is used for selecting the output measurements. Select the m_1 position (y_1), the pendulum angle θ, and the integral of m_1 position since it is also measurable. Click on “Enter Selects” to continue.



Select System Variables

Select Some of the Outputs to be Used for Measurements (Ym), or the State Vector

Select an Output from the List Below that
Corresponds to Measurement No: 4

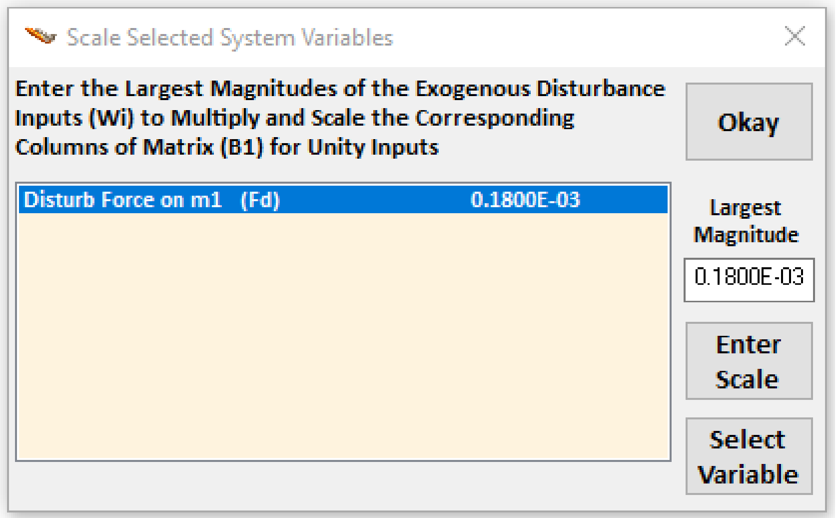
Mass-1 Displacement (y1)
Pendulum Angle (theta)
Bottom Mass Velocity, (y1-dot)
Bot Mass1 Position-Integr (y1-int)

Variable Names Already Selected

Mass-1 Displacement (y1)
Pendulum Angle (theta)
Bot Mass1 Position-Integr (y1-int)

We are finished defining the input and output variables. We must now enter the gains that will be used to scale them. The trade-off between bandwidth and performance versus sensitivity and stability is adjusted in the optimization by those gains. Initially we don’t know what gains will produce the desired performance versus stability, so we begin to scale the disturbance inputs by setting their gains equal to the magnitudes of the maximum expected disturbances, and for the output performance criteria we set their gains equal to the maximum acceptable magnitude at each output. The control input is also included in the criteria outputs and it is scaled by the maximum control magnitude. The measurements noise is also included in the disturbances vector and we must set the scaling gains equal to the maximum noise magnitude at each measurement. In this example we have to estimate the state-vector and we must enter the expected noise at each measurement.

In the dialog below enter the gain that will scale the disturbance force on m_1. Double-click on the input or click on “Select Variable”, enter the scaling gain that defines the disturbance magnitude, and click on “Enter Scale” to accept it. Click “Okay” to go to the next dialog.



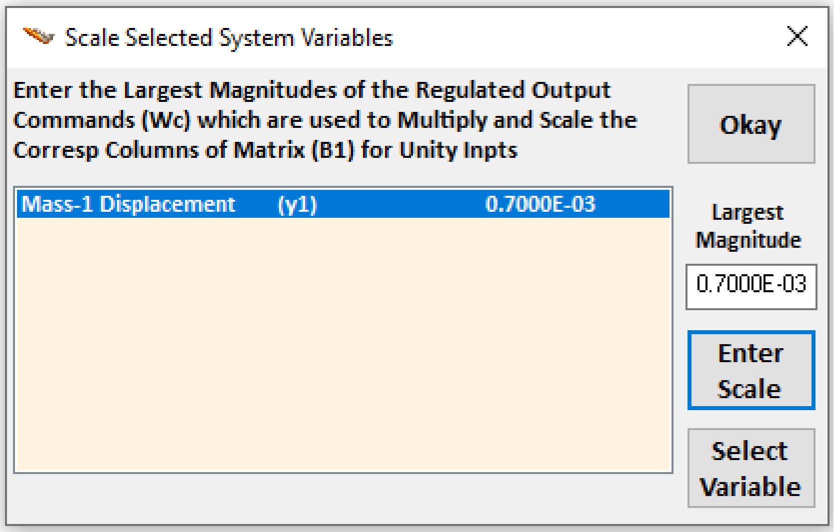
Scale Selected System Variables

Enter the Largest Magnitudes of the Exogenous Disturbance Inputs (Wi) to Multiply and Scale the Corresponding Columns of Matrix (B1) for Unity Inputs

Disturb Force on m1 (Fd)	0.1800E-03

Largest Magnitude: 0.1800E-03

In the next dialog below enter the largest magnitude of the expected input that commands the regulated output (y_1). The biggest magnitude of y_1_cmd, and click "Okay". In the next dialog you must enter the noise magnitude at the 3 measurements. Select one at a time. Enter the noise magnitude and click on "Enter Scale". The value appears in the display next to the variable label. When you finish click "Okay" to go to the next dialog.

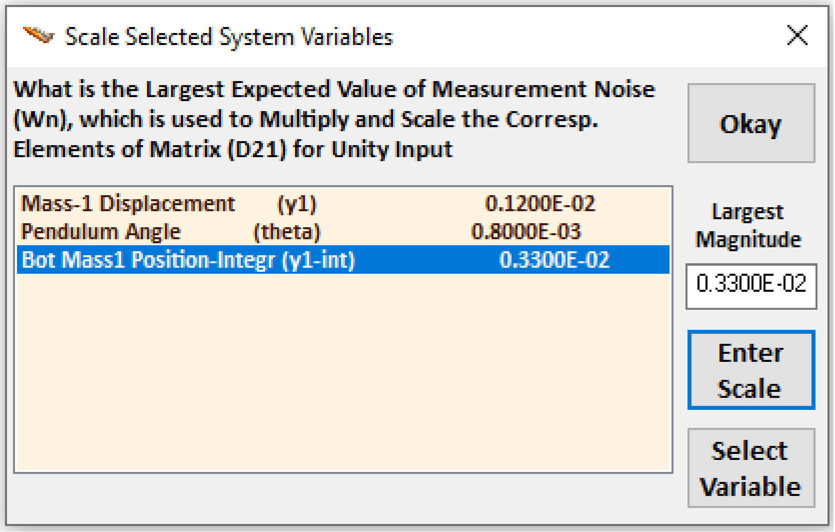


Scale Selected System Variables

Enter the Largest Magnitudes of the Regulated Output Commands (Wc) which are used to Multiply and Scale the Corresp Columns of Matrix (B1) for Unity Inpts

Mass-1 Displacement (y1) 0.7000E-03

Largest Magnitude: 0.7000E-03

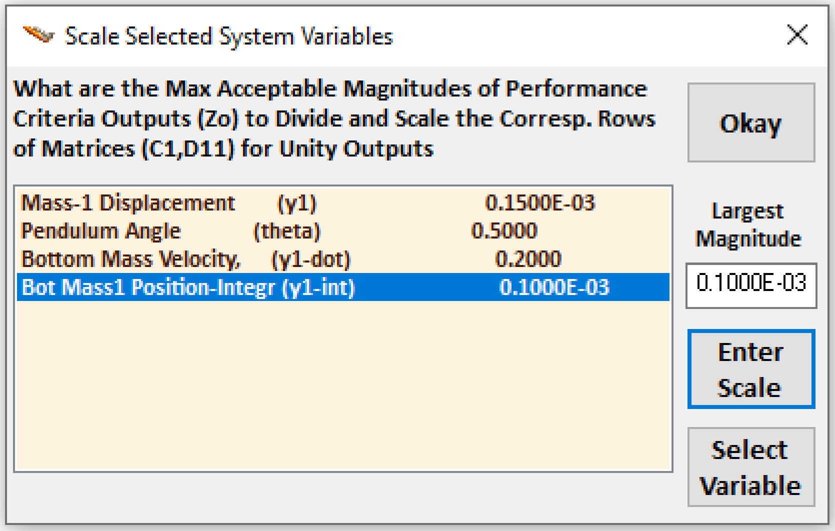


Scale Selected System Variables

What is the Largest Expected Value of Measurement Noise (Wn), which is used to Multiply and Scale the Corresp. Elements of Matrix (D21) for Unity Input

Mass-1 Displacement (y1) 0.1200E-02
Pendulum Angle (theta) 0.8000E-03
Bot Mass1 Position-Integr (y1-int) 0.3300E-02

Largest Magnitude: 0.3300E-02



Scale Selected System Variables

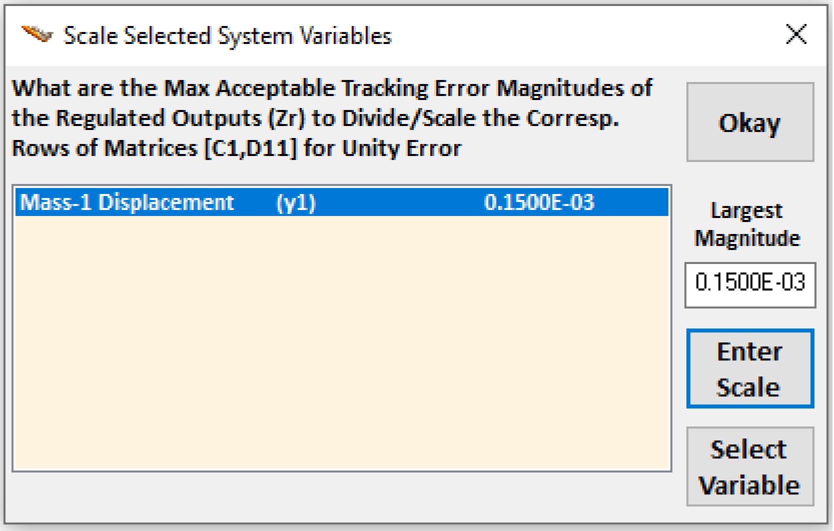
What are the Max Acceptable Magnitudes of Performance Criteria Outputs (Zo) to Divide and Scale the Corresp. Rows of Matrices (C1,D11) for Unity Outputs

Mass-1 Displacement (y1) 0.1500E-03
Pendulum Angle (theta) 0.5000
Bottom Mass Velocity, (y1-dot) 0.2000
Bot Mass1 Position-Integr (y1-int) 0.1000E-03

Largest Magnitude: 0.1000E-03

The fourth dialog is for defining the gains at the performance optimization criteria outputs. That is, the maximum acceptable magnitudes at the 4 performance outputs: (y1, θ, y_1-dot, y_1-integral). Reducing the gain value at a performance output produces better performance and smaller transient in the corresponding variable. Select one variable at a time, enter the gain and click on "Enter Scale". When you finish click on "Okay" to go to the next dialog.

The next dialog is for entering the gain that defines the max acceptable magnitude of the regulated output error (z_{re}). That is the magnitude of the error: y1-output minus y1-command.

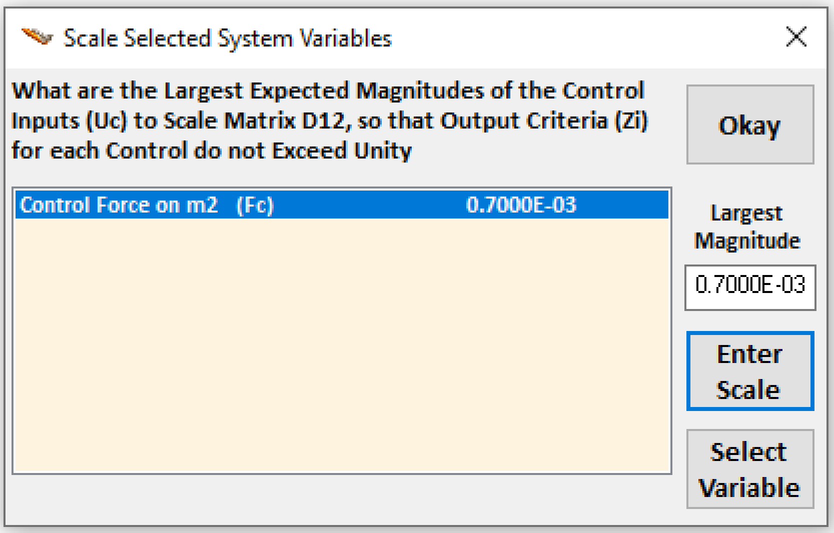


What are the Max Acceptable Tracking Error Magnitudes of the Regulated Outputs (Zr) to Divide/Scale the Corresp. Rows of Matrices [C1,D11] for Unity Error

Mass-1 Displacement (y1) 0.1500E-03

Largest Magnitude: 0.1500E-03

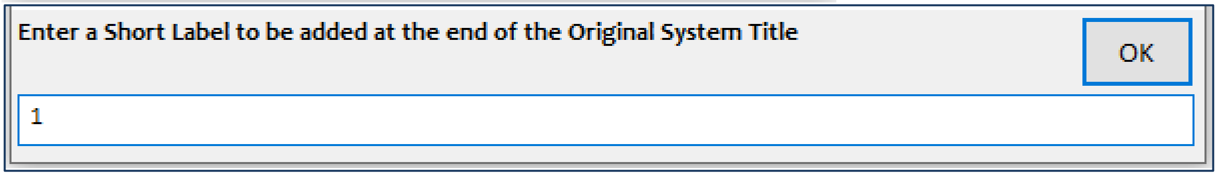
The last dialog is for entering the maximum control magnitude because the control is also included in the optimization criteria. In this example we only have the control force Fc. Finally, enter a short label to appear at the end of the Synthesis Model title.



What are the Largest Expected Magnitudes of the Control Inputs (Uc) to Scale Matrix D12, so that Output Criteria (Zi) for each Control do not Exceed Unity

Control Force on m2 (Fc) 0.7000E-03

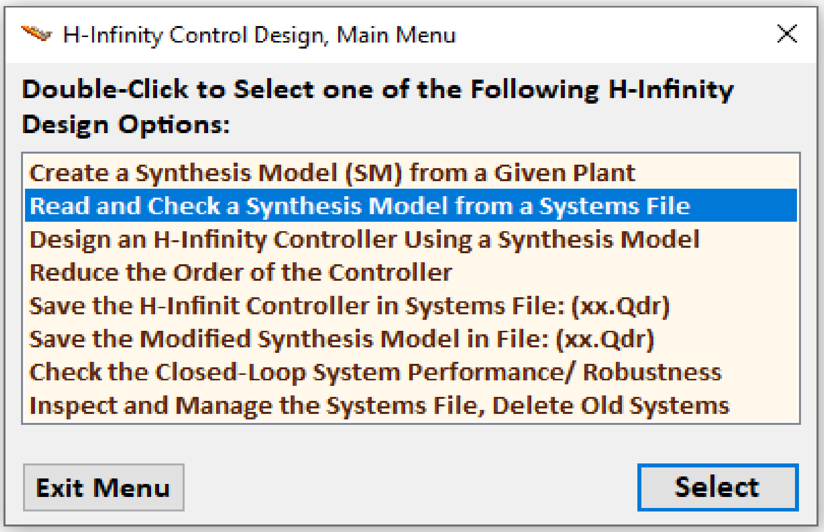
Largest Magnitude: 0.7000E-03

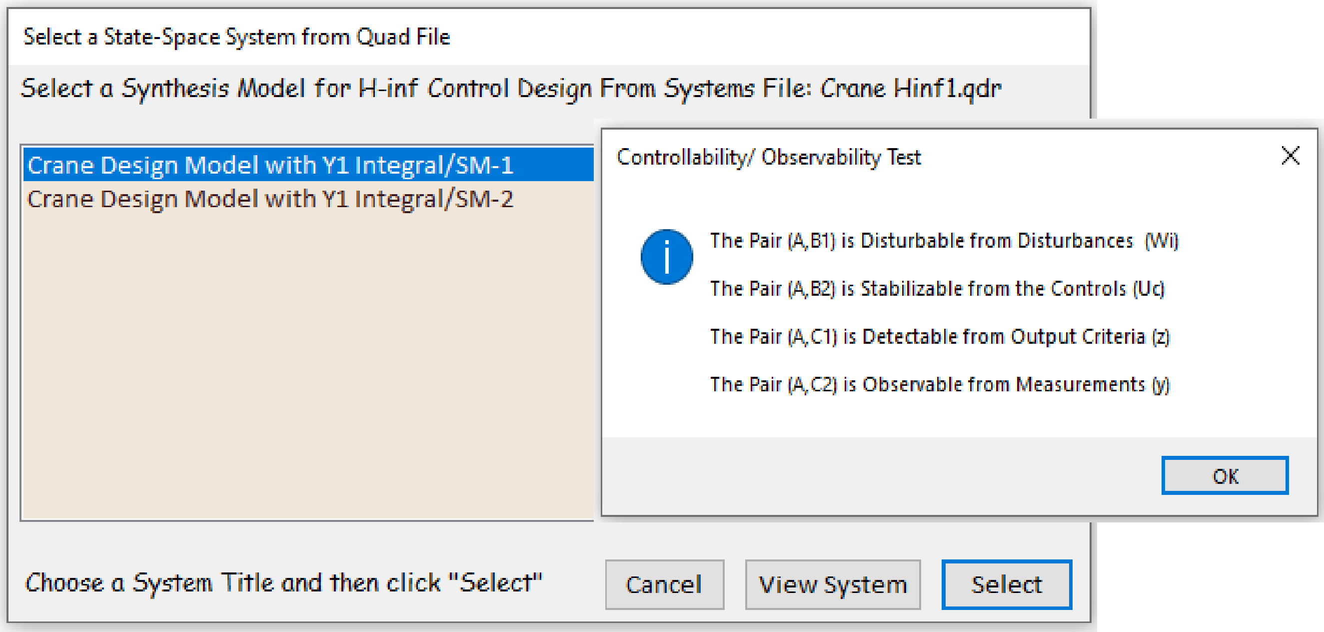


Enter a Short Label to be added at the end of the Original System Title

1

We are back in the main menu and now ready to begin the control design. Select the 2nd option to view and check the SM. Select the SM from the systems file and click on "Select". They are both identical because we just created the second one. Make sure the SM is controllable, disturbable from the disturbance inputs, observable from the measurements, and detectable from the performance criteria.





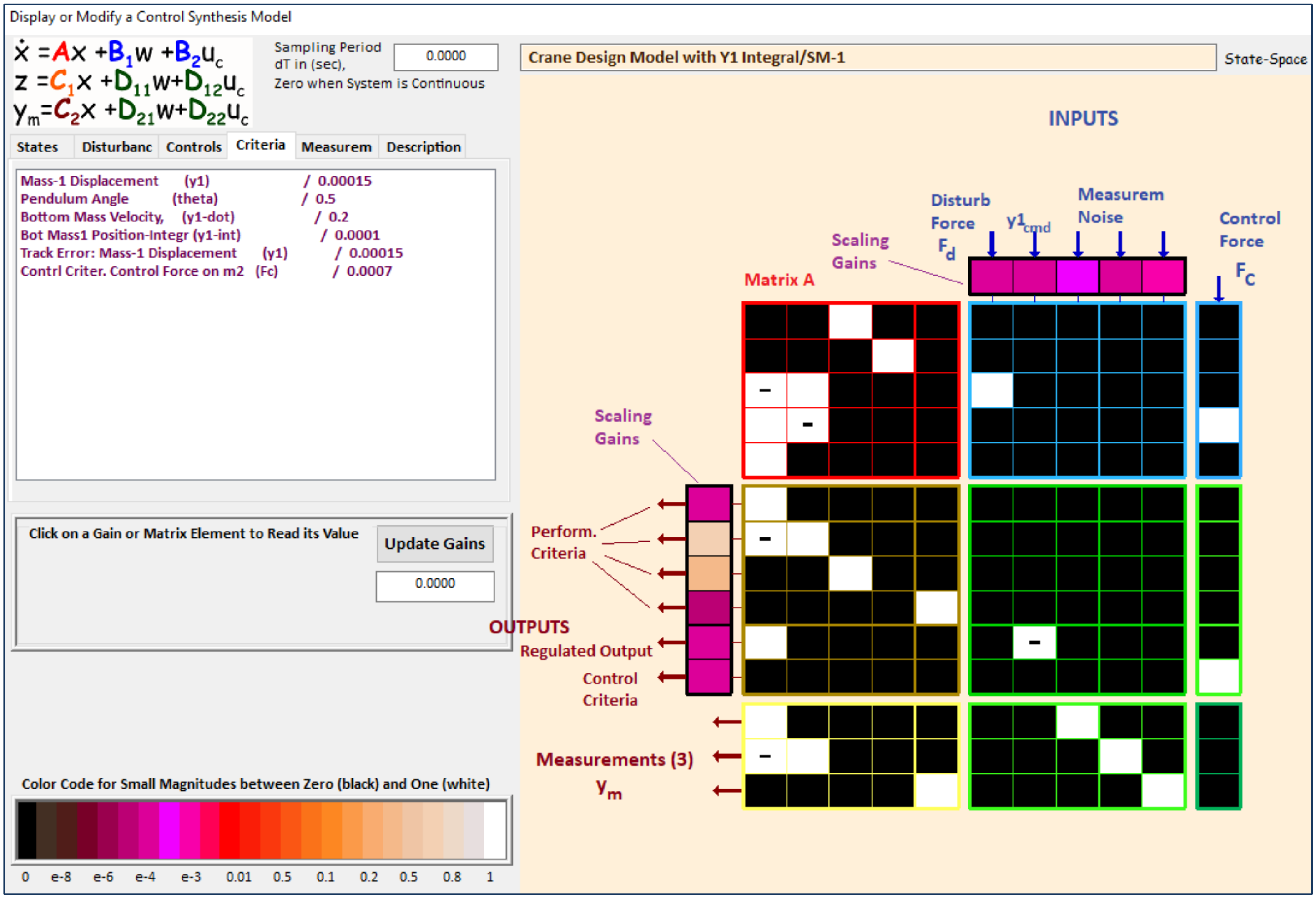
The Pair (A,B1) is Disturbable from Disturbances (Wi)

The Pair (A,B2) is Stabilizable from the Controls (Uc)

The Pair (A,C1) is Detectable from Output Criteria (z)

The Pair (A,C2) is Observable from Measurements (y)

The program confirms that the SM meets the expected requirements and displays the SM matrices graphically in system's form in the next dialog. The 9 SM matrices are color coded and also the scaling gains are included that scale the disturbances and the criteria. The A-matrix has 5 states. There are 5 disturbance inputs which are: 1 external force Fd, the command for 1 regulated output, and 3 measurements noise inputs. There is one control input Fc. We also have 6 performance criteria which are: y_1, θ, y_1_dot, y_1_integral, y_1_error, and Fc. The 3 measurements from matrix C2 are: y_1, θ, and y_1_integral.



Display or Modify a Control Synthesis Model

$\dot{x} = Ax + B_1 w + B_2 u_c$

$z = C_1 x + D_{11} w + D_{12} u_c$

$y_m = C_2 x + D_{21} w + D_{22} u_c$

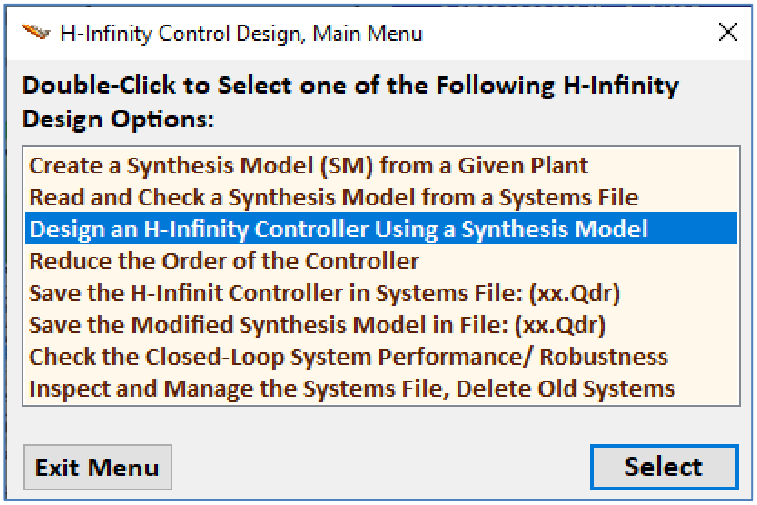
Sampling Period dT in (sec), Zero when System is Continuous: 0.0000

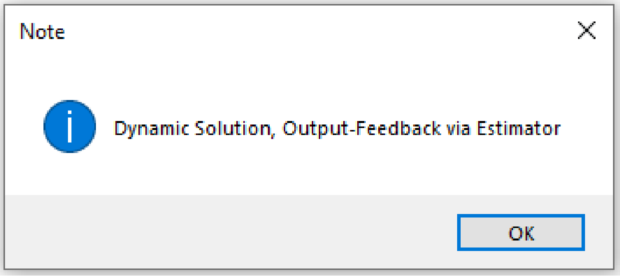
Crane Design Model with Y1 Integral/SM-1

Mass-1 Displacement (y1) / 0.00015
Pendulum Angle (theta) / 0.5
Bottom Mass Velocity, (y1-dot) / 0.2
Bot Mass1 Position-Integr (y1-int) / 0.0001
Track Error: Mass-1 Displacement (y1) / 0.00015
Contrl Criter. Control Force on m2 (Fc) / 0.0007

Click on a Gain or Matrix Element to Read its Value: 0.0000

Select the third option from the main menu to design the H-infinity controller, and click "Select". The program confirms that the solution is Output Feedback Controller which includes an Estimator.

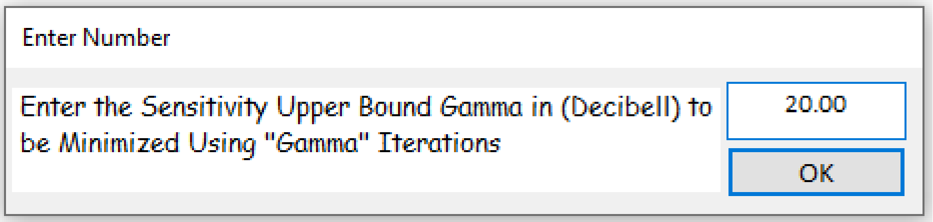




Note

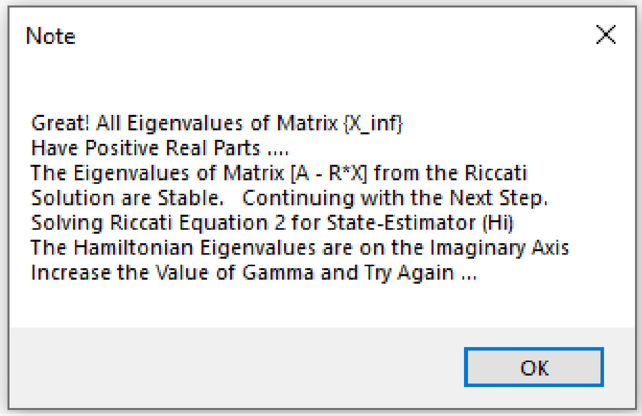
Dynamic Solution, Output-Feedback via Estimator

Now we begin the iterative process of minimizing the upper bound γ of the infinity norm of the sensitivity transfer function between the disturbance inputs and the output criteria vectors. We begin with an arbitrarily small γ upper bound and try to find the smallest γ that will not violate the algorithm requirements. After a few iterations we find that γ=70 (dB) works and we click on "No" meaning that we do not want to try another value but to accept the current controller.

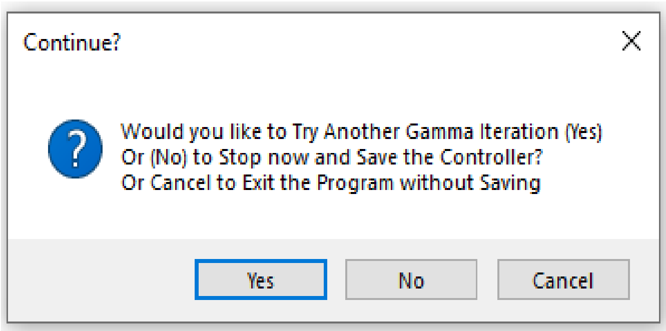


Enter the Sensitivity Upper Bound Gamma in (Decibell) to be Minimized Using "Gamma" Iterations

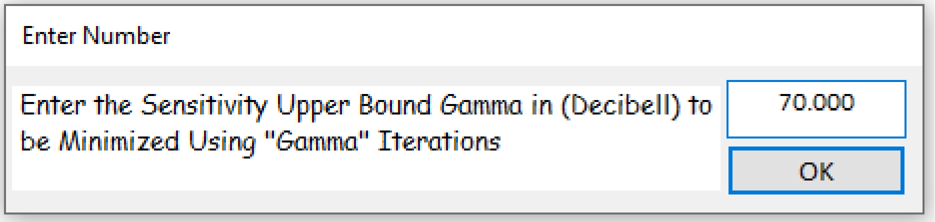
20.00



Great! All Eigenvalues of Matrix {X_inf}
Have Positive Real Parts ....
The Eigenvalues of Matrix [A - R*X] from the Riccati
Solution are Stable. Continuing with the Next Step.
Solving Riccati Equation 2 for State-Estimator (Hi)
The Hamiltonian Eigenvalues are on the Imaginary Axis
Increase the Value of Gamma and Try Again ...

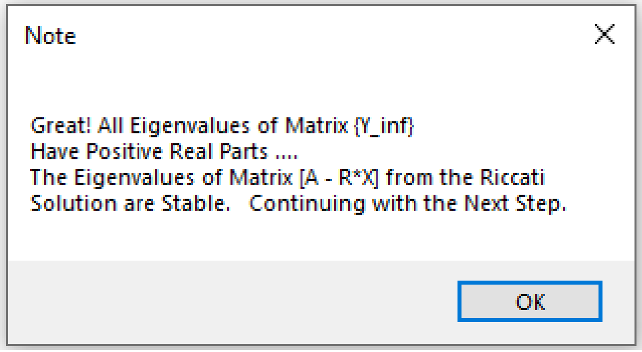


Would you like to Try Another Gamma Iteration (Yes)
Or (No) to Stop now and Save the Controller?
Or Cancel to Exit the Program without Saving

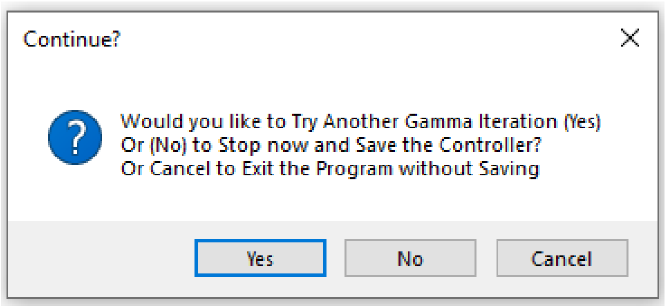


Enter the Sensitivity Upper Bound Gamma in (Decibell) to be Minimized Using "Gamma" Iterations

70.000



Great! All Eigenvalues of Matrix {Y_inf}
Have Positive Real Parts ....
The Eigenvalues of Matrix [A - R*X] from the Riccati
Solution are Stable. Continuing with the Next Step.



Would you like to Try Another Gamma Iteration (Yes)
Or (No) to Stop now and Save the Controller?
Or Cancel to Exit the Program without Saving

Figure 1.6 shows the closed-loop poles of the system. Notice that there is a complex pair of poles which are near the pendulum mode and there are two more complex pairs. The control system is finally saved in the systems file under the title "*H-Infin Control for Overhead Crane System*".

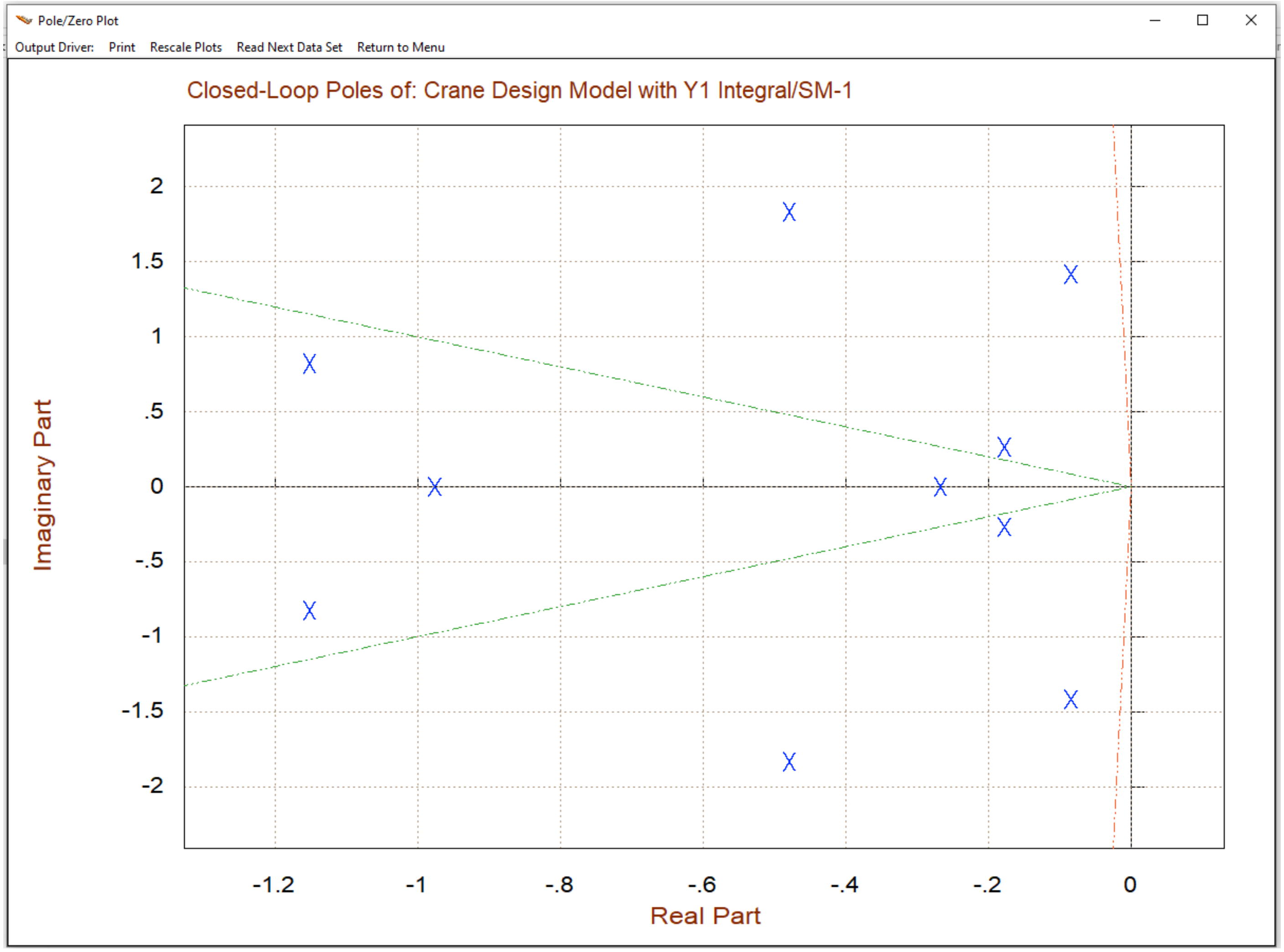
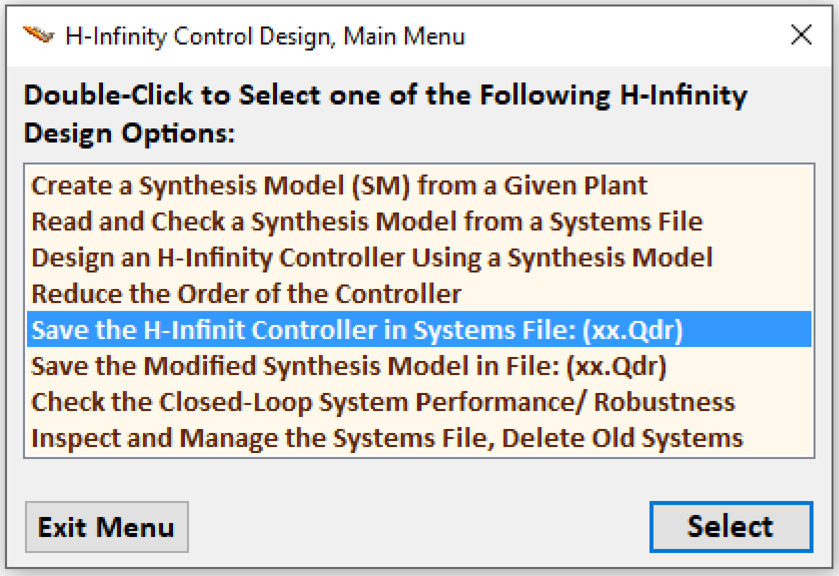


Figure 1.6 Closed-Loop Poles



Double-Click to Select one of the Following H-Infinity Design Options:

- Create a Synthesis Model (SM) from a Given Plant
- Read and Check a Synthesis Model from a Systems File
- Design an H-Infinity Controller Using a Synthesis Model
- Reduce the Order of the Controller
- Save the H-Infinit Controller in Systems File: (xx.Qdr)
- Save the Modified Synthesis Model in File: (xx.Qdr)
- Check the Closed-Loop System Performance/ Robustness
- Inspect and Manage the Systems File, Delete Old Systems

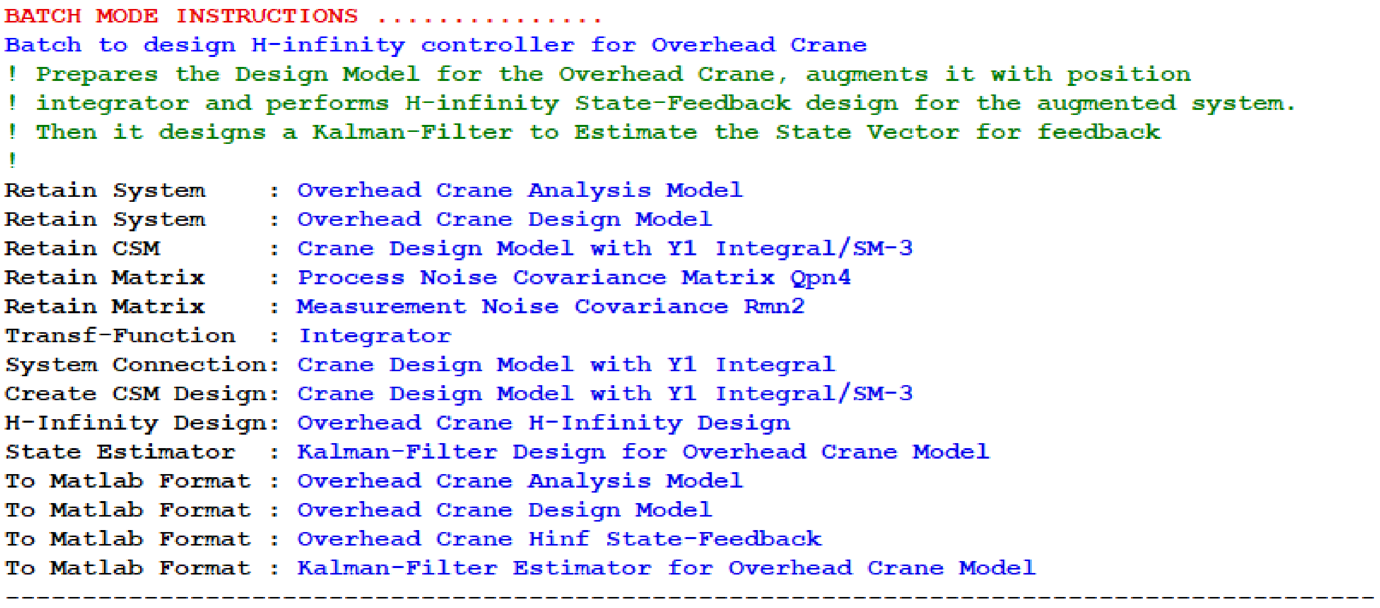
2. State-Feedback H-Infinity Design with State Estimator

This is also an H-infinity design that uses the crane design model including the y_1-integral state. It is separated in two parts: (a) the state-feedback design, and (b) the estimator design. The control design model has two inputs: the control and disturbance forces (F_c, F_d) and it has 5 outputs, the entire state vector consisting of: bottom mass-1 position and velocity (y_1, $\dot{y}_1$), top mass-2 position and velocity (y_2, $\dot{y}_2$), and y_1-integral. The control design model will be used to create the H-infinity Synthesis Model and design the state-feedback gain Kc. There is a second vehicle model in the systems file. It is called the analysis model, similar to the design model but it does not include the y_1-integral and its outputs are only y_1 and the pendulum angle θ. This model will be used to design the state estimator that will estimate the four states: (y_1, y_2, $\dot{y}_1, \dot{y}_2$) from the two measurements (y_1 and θ). The 5th state (y_1-integral) is not included in the estimator because it is directly measurable. In addition to the plant model the Kalman-Filter estimator requires the (4x4) process noise matrix Q_{pn} and the measurement noise matrix R_{mn}. They are covariance matrices that define the amount of noise corrupting the 4-states and the 2 measurements.

2.1 The Flixan Files

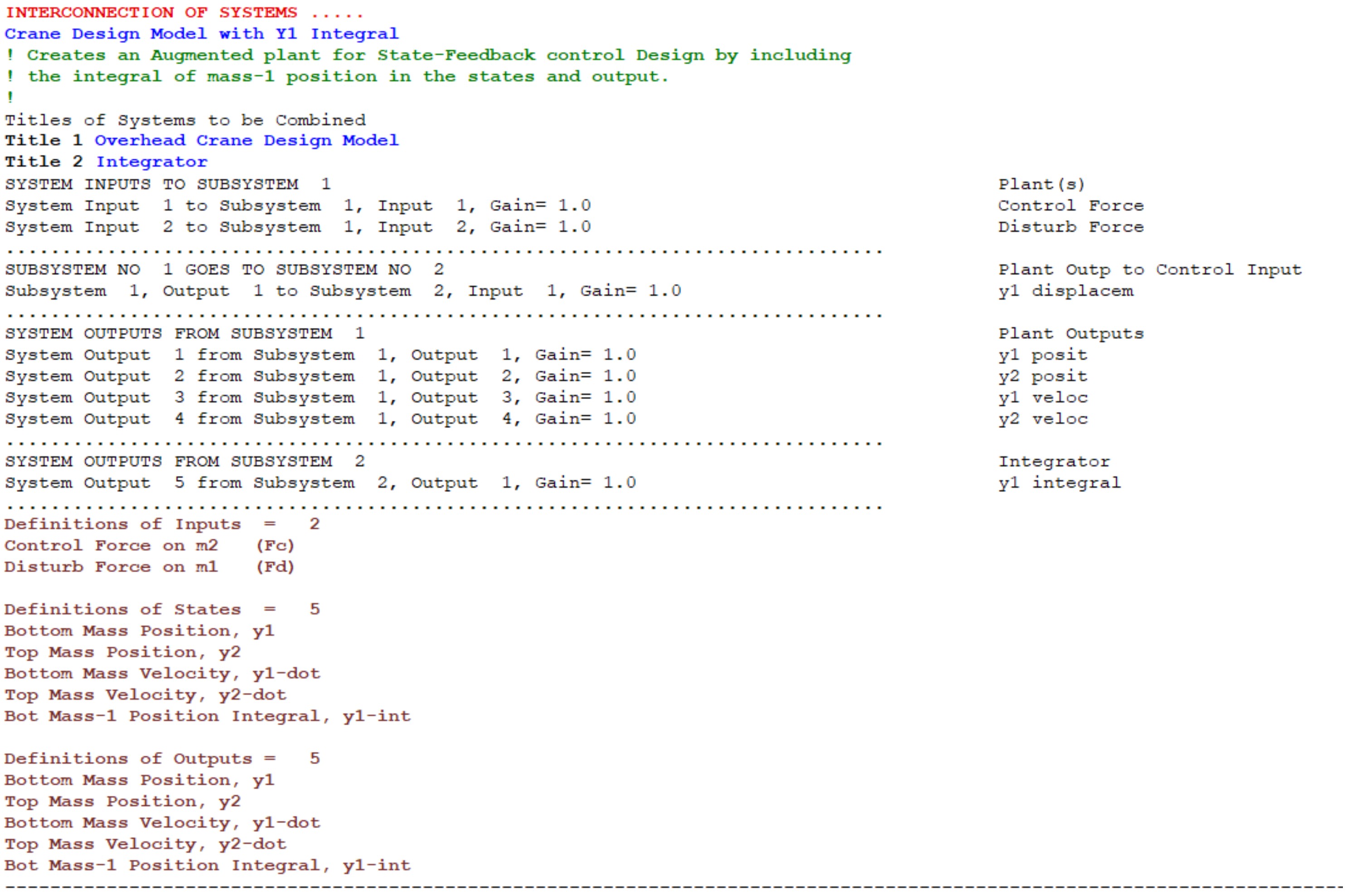
The files for the LQG design are in directory: "*\Flixan\Control Analysis\Hinfinity\Examples\Crane Hinf\2-State Fbk + Estimator*". The input file is "*Crane Hinf2.Inp*" and it contains the Flixan datasets that generate the plant models, the synthesis model, calculates the H-infinity controller, and the Kalman-Filter. It begins with a batch set that can be used to process the entire file. The batch preserves the two plant systems: "*Overhead Crane Design Model*" and the "*Overhead Crane Analysis Model*". It also retains the KF covariance matrices Qpn4 and Rmn2 and the control Synthesis Model in the systems file "*Crane Hinf2.Qdr*". The state-feedback controller Kc and the Kalman-Filter matrix Kf are saved in the systems file, and also in Matlab format for further analysis.

Input File

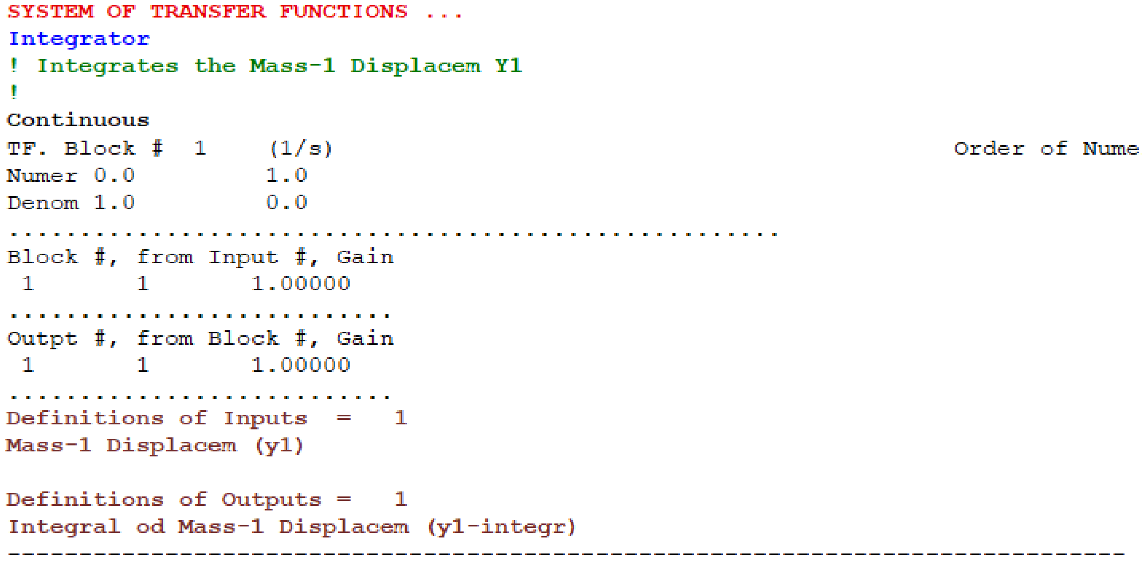


```
BATCH MODE INSTRUCTIONS ...............
Batch to design H-infinity controller for Overhead Crane
! Prepares the Design Model for the Overhead Crane, augments it with position
! integrator and performs H-infinity State-Feedback design for the augmented system.
! Then it designs a Kalman-Filter to Estimate the State Vector for feedback
!
Retain System     : Overhead Crane Analysis Model
Retain System     : Overhead Crane Design Model
Retain CSM        : Crane Design Model with Y1 Integral/SM-3
Retain Matrix     : Process Noise Covariance Matrix Qpn4
Retain Matrix     : Measurement Noise Covariance Rmn2
Transf-Function   : Integrator
System Connection: Crane Design Model with Y1 Integral
Create CSM Design: Crane Design Model with Y1 Integral/SM-3
H-Infinity Design: Overhead Crane H-Infinity Design
State Estimator   : Kalman-Filter Design for Overhead Crane Model
To Matlab Format : Overhead Crane Analysis Model
To Matlab Format : Overhead Crane Design Model
To Matlab Format : Overhead Crane Hinf State-Feedback
To Matlab Format : Kalman-Filter Estimator for Overhead Crane Model
------------------------------------------------------------------------------------------
```

The following interconnection dataset combines the 4-state design model with the y1-integrator to create the augmented 5-state design model that will be used to create the H-infinity Synthesis Model.



```
INTERCONNECTION OF SYSTEMS .....
Crane Design Model with Y1 Integral
! Creates an Augmented plant for State-Feedback control Design by including
! the integral of mass-1 position in the states and output.
!
Titles of Systems to be Combined
Title 1 Overhead Crane Design Model
Title 2 Integrator
SYSTEM INPUTS TO SUBSYSTEM  1                                                         Plant(s)
System Input  1 to Subsystem  1, Input  1, Gain= 1.0                                  Control Force
System Input  2 to Subsystem  1, Input  2, Gain= 1.0                                  Disturb Force
..............................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2                                               Plant Outp to Control Input
Subsystem  1, Output  1 to Subsystem  2, Input  1, Gain= 1.0                          y1 displacem
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                                      Plant Outputs
System Output  1 from Subsystem  1, Output  1, Gain= 1.0                              y1 posit
System Output  2 from Subsystem  1, Output  2, Gain= 1.0                              y2 posit
System Output  3 from Subsystem  1, Output  3, Gain= 1.0                              y1 veloc
System Output  4 from Subsystem  1, Output  4, Gain= 1.0                              y2 veloc
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                                                      Integrator
System Output  5 from Subsystem  2, Output  1, Gain= 1.0                              y1 integral
..............................................................................
Definitions of Inputs  =   2
Control Force on m2   (Fc)
Disturb Force on m1   (Fd)

Definitions of States  =   5
Bottom Mass Position, y1
Top Mass Position, y2
Bottom Mass Velocity, y1-dot
Top Mass Velocity, y2-dot
Bot Mass-1 Position Integral, y1-int

Definitions of Outputs =   5
Bottom Mass Position, y1
Top Mass Position, y2
Bottom Mass Velocity, y1-dot
Top Mass Velocity, y2-dot
Bot Mass-1 Position Integral, y1-int
------------------------------------------------------------------------------------------------------------------------
```



```
SYSTEM OF TRANSFER FUNCTIONS ...
Integrator
! Integrates the Mass-1 Displacem Y1
!
Continuous
TF. Block #  1    (1/s)                                 Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
......................................................
Block #, from Input #, Gain
 1       1       1.00000
...........................
Outpt #, from Block #, Gain
 1       1       1.00000
...........................
Definitions of Inputs  =   1
Mass-1 Displacem (y1)

Definitions of Outputs =   1
Integral od Mass-1 Displacem (y1-integr)
------------------------------------------------------------------------------
```

The following dataset is used to create the SM from the 5-state “Crane Design Model with Y1 Integral”. It defines which inputs are controls and which disturbances. Also, which outputs are measurements and which are optimization criteria. It also includes the scaling gains. The SM title is “*Crane Design Model with Y1 Integral/SM-3*”.

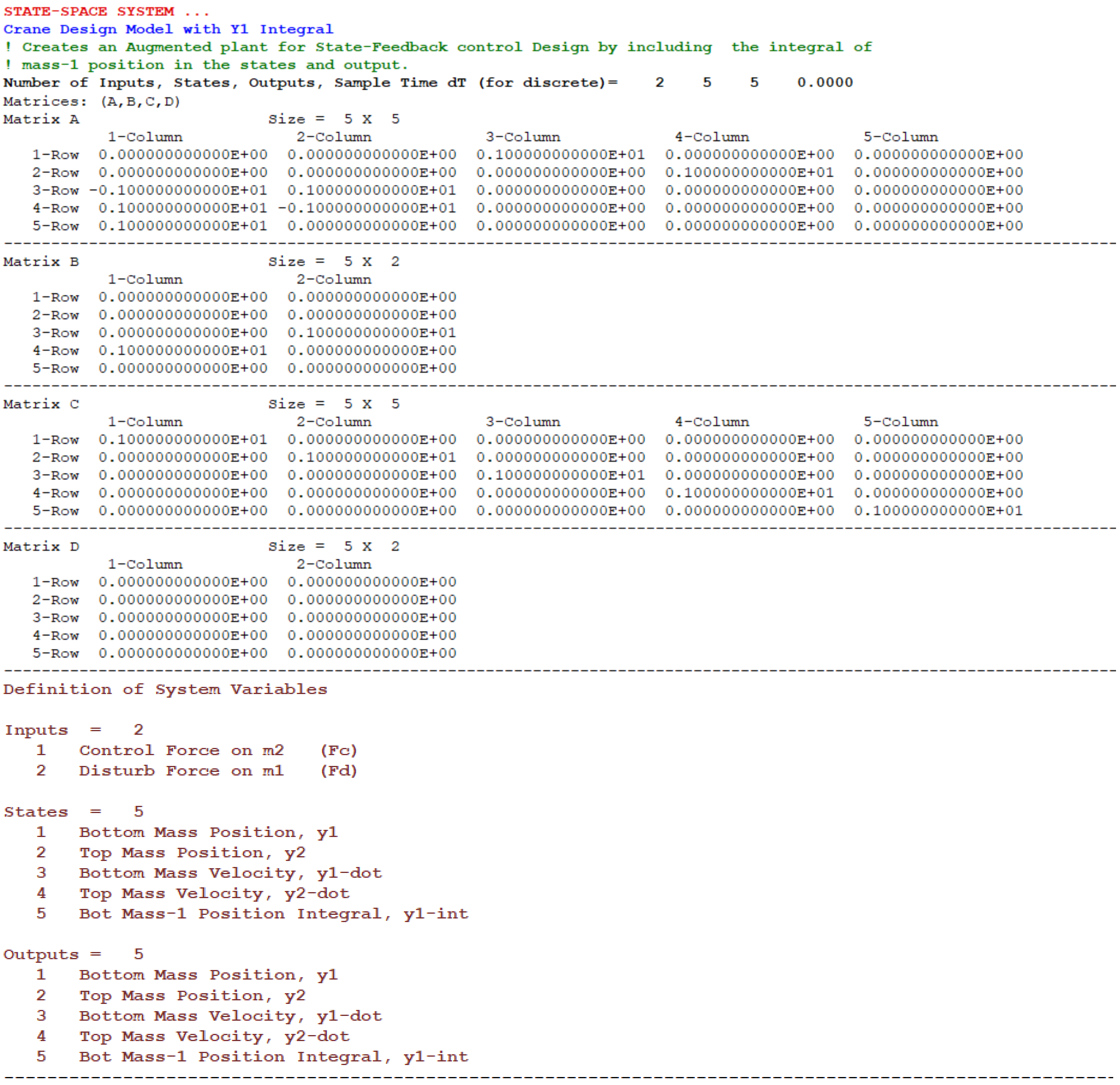
```
CREATE A SYNTHESIS MODEL FOR H-INFINITY CONTROL DESIGN
Crane Design Model with Y1 Integral/SM-3
Crane Design Model with Y1 Integral
! This dataset creates the 9-matrices Syntesis Model from the Design Crane system
! that includes y1-integral and it saves it in the Systems file: Crane_Hinf2.Qdr
!
Number of Uncertainty I/O Pairs    :    0
Number of Disturbance Inputs       :    1
Disturbance Input Numbers          :    2
Number of Control Inputs           :    1
Control Input Numbers              :    1
Number of Performance Outputs      :    3
Perform Optimization Output Numbrs:     1    3    5
Number of Commanded Outputs        :    1
Command Regulated Output Numbers   :    1
Number of Measurement Outputs      :    5    3
Measurement Output Numbers         :    1    2    3    4    5
Disturbance Input  & Command Gains:  0.001     0.008     0.00      0.00      0.00      0.00      0.00
Performance Output & Control Gains:  0.001     0.01      0.002     0.001     0.002
----------------------------------------------------------------------------------------------------------------
H-INFINITY CONTROL DESIGN ......
Overhead Crane H-Infinity Design
Synthesis Model for Control Design in file (.Qdr) :        Crane Design Model with Y1 Integral/SM-3
Peak Value of the Sensitivity Function Gamma (dB) :        5.0
State-Feedback Control Solution via Gain Kqhinf   :Kc      Overhead Crane Hinf State-Feedback
----------------------------------------------------------------------------------------------------------------
KALMAN-BUCY FILTER STATE ESTIMATOR DESIGN
Kalman-Filter Design for Overhead Crane Model
! State Observer for the Original 4-state Crane Model, Estimating
! Positions and Velocities of the two masses from the plant output
!
Plant Model Used to Design the Kalman-Filter from:         Overhead Crane Analysis Model
Input Process Noise Matrix (G) is the  Identity
Process Noise Covariance Qpn is Matrix Qpn4                Process Noise Covariance Matrix Qpn4
Measurement Noise Covariance is Matrix Rmn2                Measurement Noise Covariance Rmn2
Kalman-Filter Estimator is Gain Matrix Kf                  Kalman-Filter Estimator for Overhead Crane Model
----------------------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Overhead Crane Analysis Model
System
Crane
-----------------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Overhead Crane Design Model
System
Design
-----------------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Overhead Crane Hinf State-Feedback
Matrix Kc
-----------------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Kalman-Filter Estimator for Overhead Crane Model
Matrix Kf
-----------------------------------------------------------------------------------------------------------
```

The H-infinity control design dataset "*Overhead Crane H-Infinity Design*" reads the 9-matrices SM "*Crane Design Model with Y1 Integral/SM-3*" and it creates the state-feedback matrix Kc. Its title is "*Overhead Crane Hinf State-Feedback*" and it is saved in the systems file. The upper value of gamma is set to γ=5 (dB). The Kalman-Filter estimator dataset calculates the Kalman-Filter gain Kf. It uses the 4-state plant model: "*Overhead Crane Analysis Model*" which does not include the y_1-integral. It also reads the noise covariance matrices Qpn4 and Rmn2 which are located in the systems file "*Crane Hinf2.Qdr*". Kf will be used in the observer simulation to estimate the 4 states from the outputs y_1 and θ. The analysis and design plant models "Crane" and "Design" and the gain matrices Kc and Kf are exported into Matlab by the conversion datasets which are included at the bottom of the Flixan input file. They are converted to m-functions and "mat" matrices that can be loaded into Matlab by running the script file "init.m".

2.2 Systems File

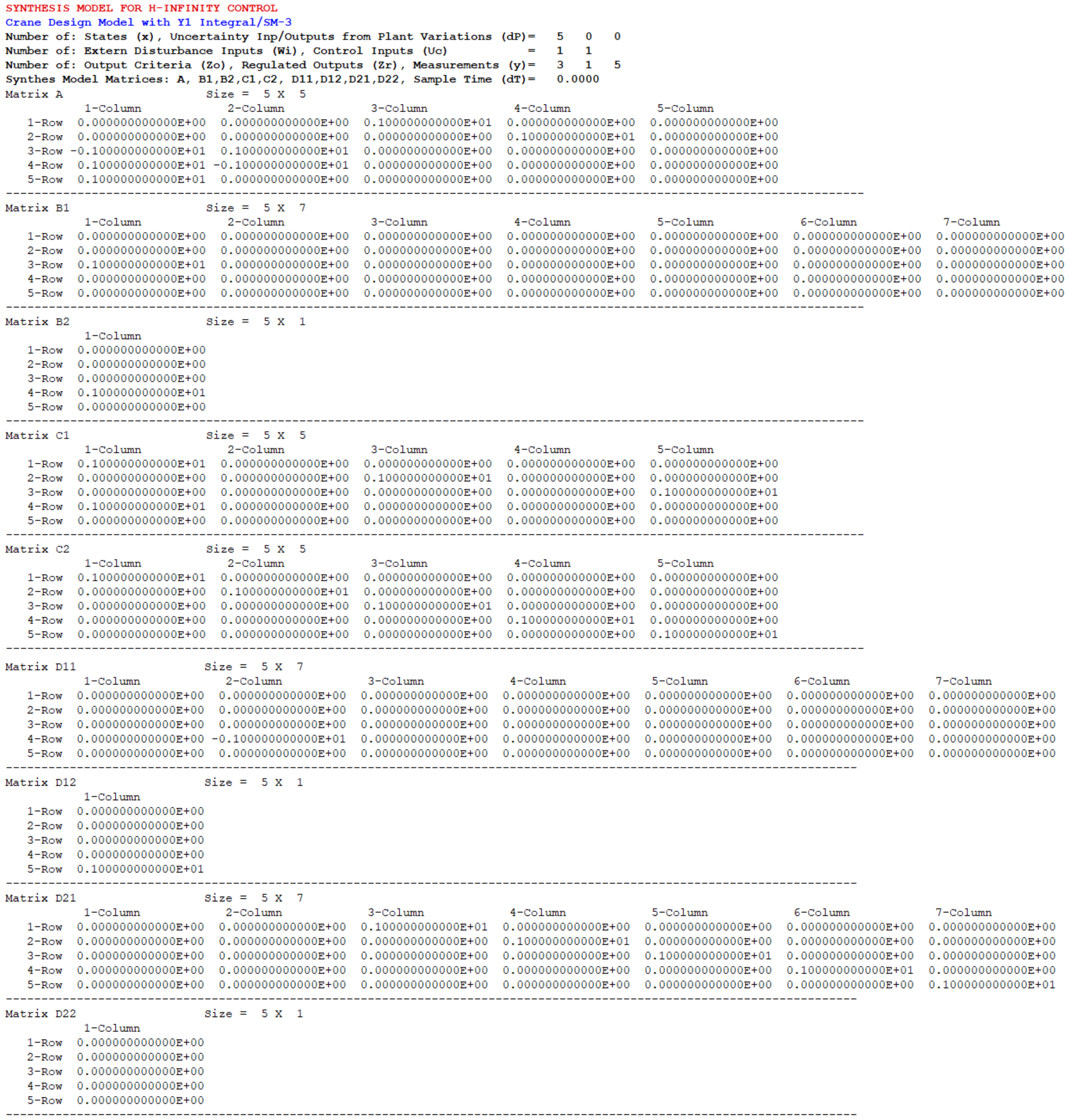
```
STATE-SPACE SYSTEM ...
Overhead Crane Analysis Model
! A dynamic model of two masses representing an overhead crane.
! The two Inputs are the Control and Disturbance Forces.
! The two Outputs are mass-1 position y1, and pendulum angle theta
!
Number of Inputs, States, Outputs, Sample Time dT (for discrete)=    2    4    2    0.0000
Matrices: (A,B,C,D)
Matrix A                    Size =  4 X  4
           1-Column            2-Column            3-Column            4-Column
   1-ROW  0.0                 0.0                 1.0                 0.0
   2-ROW  0.0                 0.0                 0.0                 1.0
   3-ROW -1.0                 1.0                 0.0                 0.0
   4-ROW  1.0                -1.0                 0.0                 0.0
-----------------------------------------------------------------------------------------------------
Matrix B                    Size =  4 X  2
           1-Column            2-Column
   1-ROW  0.0                 0.0
   2-ROW  0.0                 0.0
   3-ROW  0.0                 1.0
   4-ROW  1.0                 0.0
-----------------------------------------------------------------------------------------------------
Matrix C                    Size =  2 X  4
          1-COLUMN             2-COLUMN            3-COLUMN            4-COLUMN
   1-ROW  1.0                 0.0                 0.0                 0.0
   2-ROW -1.0                 1.0                 0.0                 0.0
-----------------------------------------------------------------------------------------------------
Matrix D                    Size =  2 X  2
           1-Column            2-Column
   1-ROW  0.0                 0.0
   2-ROW  0.0                 0.0
-----------------------------------------------------------------------------------------------------
Definition of System Variables

Inputs   =    2
   1    Control Force on Top Mass m2
   2    Disturb Force on Bot Mass m1

States   =    4
   1    Bottom Mass Position, y1
   2    Top Mass Position, y2
   3    Bottom Mass Velocity, y1-dot
   4    Top Mass Velocity, y2-dot

Outputs =    2
   1    Bottom Mass Position, y1
   2    Pendulum Angle, theta
-----------------------------------------------------------------------------------------------------
```

```
STATE-SPACE SYSTEM ...
Overhead Crane Design Model
! A dynamic model of two masses representing an overhead crane.
! The input is the control force, the outputs is the state vector
!
Number of Inputs, States, Outputs, Sample Time dT (for discrete)=    2    4    4    0.0000
Matrices: (A,B,C,D)
Matrix A                    Size =  4 X  4
           1-Column             2-Column             3-Column             4-Column
   1-ROW  0.0                 0.0                  1.0                  0.0
   2-ROW  0.0                 0.0                  0.0                  1.0
   3-ROW -1.0                 1.0                  0.0                  0.0
   4-ROW  1.0                -1.0                  0.0                  0.0
----------------------------------------------------------------------------------------------------------------
Matrix B                    Size =  4 X  2
           1-Column             2-Column
   1-ROW  0.0                 0.0
   2-ROW  0.0                 0.0
   3-ROW  0.0                 1.0
   4-ROW  1.0                 0.0
----------------------------------------------------------------------------------------------------------------
Matrix C                    Size =  4 X  4
          1-COLUMN              2-COLUMN             3-COLUMN             4-COLUMN
   1-ROW  1.0                 0.0                  0.0                  0.0
   2-ROW  0.0                 1.0                  0.0                  0.0
   3-ROW  0.0                 0.0                  1.0                  0.0
   4-ROW  0.0                 0.0                  0.0                  1.0
----------------------------------------------------------------------------------------------------------------
Matrix D                    Size =  4 X  2
          1-COLUMN
   1-ROW  0.0                 0.0
   2-ROW  0.0                 0.0
   3-ROW  0.0                 0.0
   4-ROW  0.0                 0.0
----------------------------------------------------------------------------------------------------------------
Definition of System Variables

Inputs  =   2
   1   Control Force on Top Mass m2
   2   Disturb Force on Bot Mass m1

States  =   4
   1   Bottom Mass Position, y1
   2   Top Mass Position, y2
   3   Bottom Mass Velocity, y1-dot
   4   Top Mass Velocity, y2-dot

Outputs =   4
   1   Bottom Mass Position, y1
   2   Top Mass Position, y2
   3   Bottom Mass Velocity, y1-dot
   4   Top Mass Velocity, y2-dot
-----------------------------------------------------------------------------------------------------------------
GAIN MATRIX FOR ...
Process Noise Covariance Matrix Qpn4
! Noise Intensity at the four states
Matrix Qpn4                Size =  4 X  4
           1-Column            2-Column            3-Column            4-Column
   1-Row  0.1                0.0                 0.0                 0.0
   2-Row  0.0                0.1                 0.0                 0.0
   3-Row  0.0                0.0                 0.1                 0.0
   4-Row  0.0                0.0                 0.0                 0.1
-----------------------------------------------------------------------------------------------------------------
GAIN MATRIX FOR ...
Measurement Noise Covariance Rmn2
! Noise Intensity at the 2 measurements
Matrix Rmn2                Size =  2 X  2
           1-Column             2-Column
   1-ROW    0.03               0.0
   2-ROW    0.0                0.03
-----------------------------------------------------------------------------------------------------------------
```



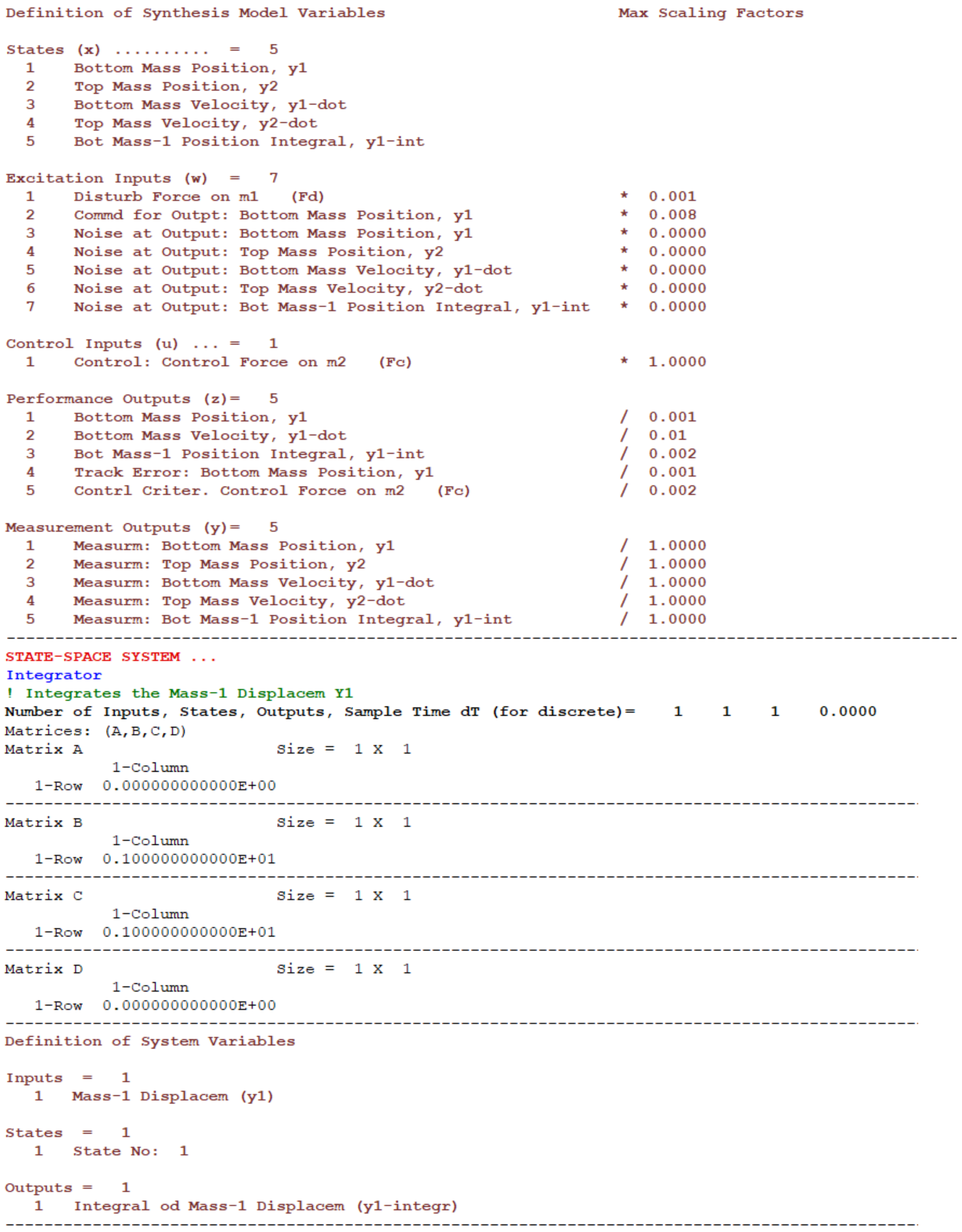
```
STATE-SPACE SYSTEM ...
Crane Design Model with Y1 Integral
! Creates an Augmented plant for State-Feedback control Design by including  the integral of
! mass-1 position in the states and output.
Number of Inputs, States, Outputs, Sample Time dT (for discrete)=    2    5    5    0.0000
Matrices: (A,B,C,D)
Matrix A                    Size =  5 X  5
           1-Column             2-Column             3-Column             4-Column             5-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   3-Row -0.100000000000E+01  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.100000000000E+01 -0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
-----------------------------------------------------------------------------------------------------------------------
Matrix B                    Size =  5 X  2
           1-Column             2-Column
   1-Row  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.100000000000E+01
   4-Row  0.100000000000E+01  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00
-----------------------------------------------------------------------------------------------------------------------
Matrix C                    Size =  5 X  5
           1-Column             2-Column             3-Column             4-Column             5-Column
   1-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
-----------------------------------------------------------------------------------------------------------------------
Matrix D                    Size =  5 X  2
           1-Column             2-Column
   1-Row  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00
-----------------------------------------------------------------------------------------------------------------------
Definition of System Variables

Inputs   =   2
   1   Control Force on m2   (Fc)
   2   Disturb Force on m1   (Fd)

States   =   5
   1   Bottom Mass Position, y1
   2   Top Mass Position, y2
   3   Bottom Mass Velocity, y1-dot
   4   Top Mass Velocity, y2-dot
   5   Bot Mass-1 Position Integral, y1-int

Outputs =    5
   1   Bottom Mass Position, y1
   2   Top Mass Position, y2
   3   Bottom Mass Velocity, y1-dot
   4   Top Mass Velocity, y2-dot
   5   Bot Mass-1 Position Integral, y1-int
-----------------------------------------------------------------------------------------------------------------------
```

The above augmented system is very similar to the original design plant but it includes one additional state, the y1-integral. The system output is equal to the 5-state vector. This system will be used to create the SM and the state-feedback gain Kc.



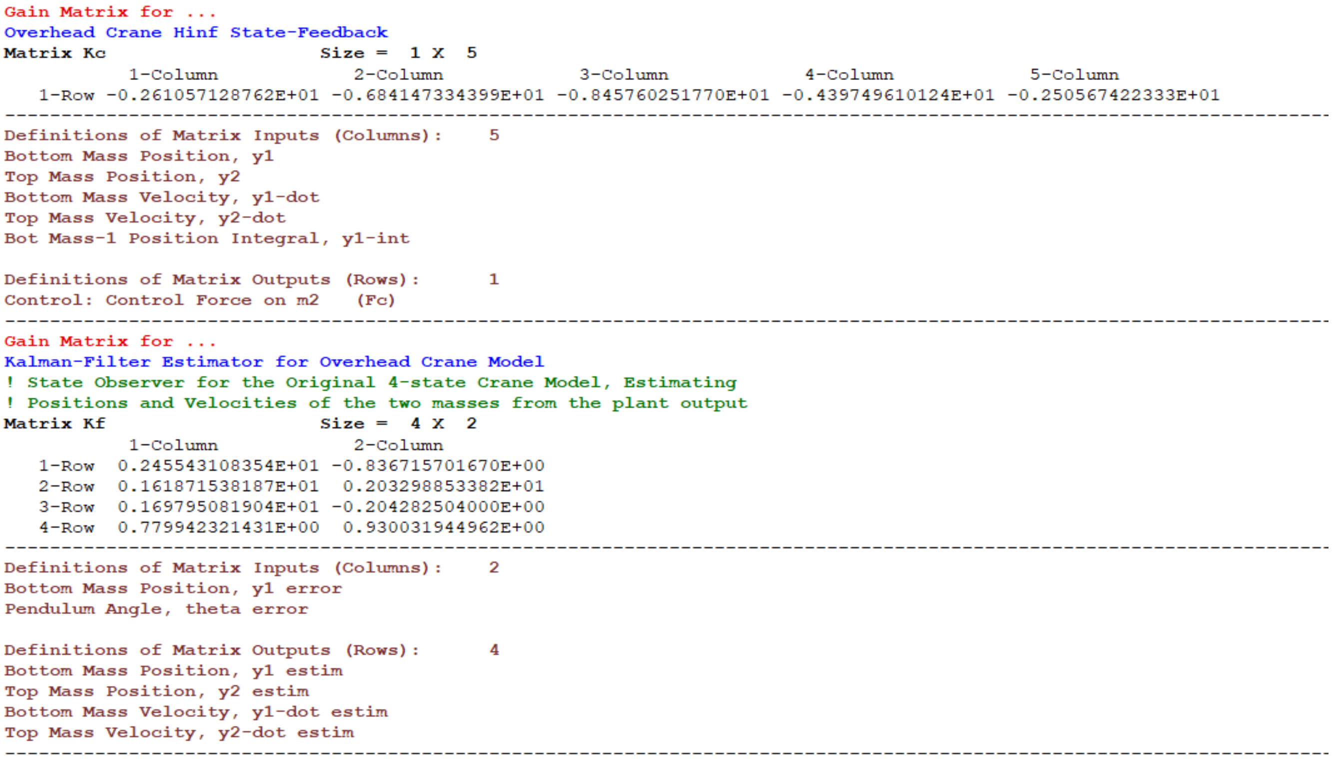
```
SYNTHESIS MODEL FOR H-INFINITY CONTROL
Crane Design Model with Y1 Integral/SM-3
Number of: States (x), Uncertainty Inp/Outputs from Plant Variations (dP)=   5   0   0
Number of: Extern Disturbance Inputs (Wi), Control Inputs (Uc)           =   1   1
Number of: Output Criteria (Zo), Regulated Outputs (Zr), Measurements (y)=   3   1   5
Synthes Model Matrices: A, B1,B2,C1,C2, D11,D12,D21,D22, Sample Time (dT)=   0.0000
Matrix A                    Size =  5 X  5
           1-Column             2-Column             3-Column             4-Column             5-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   3-Row -0.100000000000E+01  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.100000000000E+01 -0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Matrix B1                   Size =  5 X  7
           1-Column             2-Column             3-Column             4-Column             5-Column             6-Column             7-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Matrix B2                   Size =  5 X  1
           1-Column
   1-Row  0.000000000000E+00
   2-Row  0.000000000000E+00
   3-Row  0.000000000000E+00
   4-Row  0.100000000000E+01
   5-Row  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Matrix C1                   Size =  5 X  5
           1-Column             2-Column             3-Column             4-Column             5-Column
   1-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
   4-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Matrix C2                   Size =  5 X  5
           1-Column             2-Column             3-Column             4-Column             5-Column
   1-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
------------------------------------------------------------------------------------------------------------------------
Matrix D11                  Size =  5 X  7
           1-Column             2-Column             3-Column             4-Column             5-Column             6-Column             7-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00 -0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Matrix D12                  Size =  5 X  1
           1-Column
   1-Row  0.000000000000E+00
   2-Row  0.000000000000E+00
   3-Row  0.000000000000E+00
   4-Row  0.000000000000E+00
   5-Row  0.100000000000E+01
------------------------------------------------------------------------------------------------------------------------
Matrix D21                  Size =  5 X  7
           1-Column             2-Column             3-Column             4-Column             5-Column             6-Column             7-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
------------------------------------------------------------------------------------------------------------------------
Matrix D22                  Size =  5 X  1
           1-Column
   1-Row  0.000000000000E+00
   2-Row  0.000000000000E+00
   3-Row  0.000000000000E+00
   4-Row  0.000000000000E+00
   5-Row  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
```

This is the Synthesis Model used by the H-infinity algorithm to create the control system or just state-feedback gain in this case. It consists of 9 matrices plus the scaling gains. It is derived from the system "*Crane Design Model with Y1 Integral*" through an interactive process that will be described later.

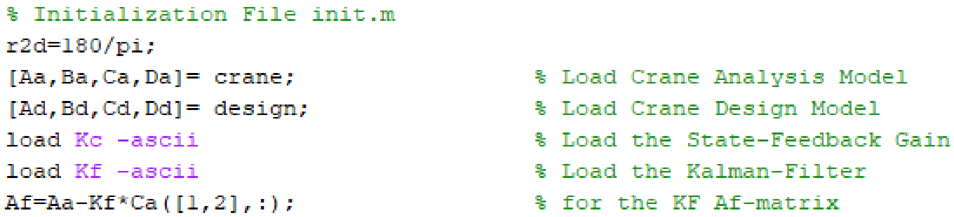


```
Definition of Synthesis Model Variables                          Max Scaling Factors

States (x) ..........  =   5
  1    Bottom Mass Position, y1
  2    Top Mass Position, y2
  3    Bottom Mass Velocity, y1-dot
  4    Top Mass Velocity, y2-dot
  5    Bot Mass-1 Position Integral, y1-int

Excitation Inputs (w)  =   7
  1    Disturb Force on m1   (Fd)                                 *  0.001
  2    Commd for Outpt: Bottom Mass Position, y1                  *  0.008
  3    Noise at Output: Bottom Mass Position, y1                  *  0.0000
  4    Noise at Output: Top Mass Position, y2                     *  0.0000
  5    Noise at Output: Bottom Mass Velocity, y1-dot              *  0.0000
  6    Noise at Output: Top Mass Velocity, y2-dot                 *  0.0000
  7    Noise at Output: Bot Mass-1 Position Integral, y1-int      *  0.0000

Control Inputs (u) ... =   1
  1    Control: Control Force on m2   (Fc)                        *  1.0000

Performance Outputs (z)=   5
  1    Bottom Mass Position, y1                                   /  0.001
  2    Bottom Mass Velocity, y1-dot                               /  0.01
  3    Bot Mass-1 Position Integral, y1-int                       /  0.002
  4    Track Error: Bottom Mass Position, y1                      /  0.001
  5    Contrl Criter. Control Force on m2   (Fc)                  /  0.002

Measurement Outputs (y)=   5
  1    Measurm: Bottom Mass Position, y1                          /  1.0000
  2    Measurm: Top Mass Position, y2                             /  1.0000
  3    Measurm: Bottom Mass Velocity, y1-dot                      /  1.0000
  4    Measurm: Top Mass Velocity, y2-dot                         /  1.0000
  5    Measurm: Bot Mass-1 Position Integral, y1-int              /  1.0000
-----------------------------------------------------------------------------------------------
STATE-SPACE SYSTEM ...
Integrator
! Integrates the Mass-1 Displacem Y1
Number of Inputs, States, Outputs, Sample Time dT (for discrete)=    1    1    1    0.0000
Matrices: (A,B,C,D)
Matrix A                    Size =  1 X  1
           1-Column
   1-Row  0.000000000000E+00
-------------------------------------------------------------------------------------------
Matrix B                    Size =  1 X  1
           1-Column
   1-Row  0.100000000000E+01
-------------------------------------------------------------------------------------------
Matrix C                    Size =  1 X  1
           1-Column
   1-Row  0.100000000000E+01
-------------------------------------------------------------------------------------------
Matrix D                    Size =  1 X  1
           1-Column
   1-Row  0.000000000000E+00
-------------------------------------------------------------------------------------------
Definition of System Variables

Inputs  =   1
   1   Mass-1 Displacem (y1)

States  =   1
   1   State No:  1

Outputs =   1
   1   Integral od Mass-1 Displacem (y1-integr)
-------------------------------------------------------------------------------------------
```

Notice, the measurement noise is set to zero in the Synthesis Model scaling gains. This is because in this case the output vector is equal to the state vector and we are solving the state-feedback H-infinity problem. This is how we tell the program that we are solving the state-feedback problem rather than the output feedback. If the measurement noise was not set to zero, the program would solve the output feedback problem and it would include a dynamic controller with an estimator. Even though all 5 states are measurable, the introduction of noise would require an estimator.



```
Gain Matrix for ...
Overhead Crane Hinf State-Feedback
Matrix Kc                  Size =  1 X  5
           1-Column            2-Column            3-Column            4-Column            5-Column
   1-Row -0.261057128762E+01 -0.684147334399E+01 -0.845760251770E+01 -0.439749610124E+01 -0.250567422333E+01
--------------------------------------------------------------------------------------------------------------------
Definitions of Matrix Inputs (Columns):    5
Bottom Mass Position, y1
Top Mass Position, y2
Bottom Mass Velocity, y1-dot
Top Mass Velocity, y2-dot
Bot Mass-1 Position Integral, y1-int

Definitions of Matrix Outputs (Rows):      1
Control: Control Force on m2    (Fc)
--------------------------------------------------------------------------------------------------------------------
Gain Matrix for ...
Kalman-Filter Estimator for Overhead Crane Model
! State Observer for the Original 4-state Crane Model, Estimating
! Positions and Velocities of the two masses from the plant output
Matrix Kf                  Size =  4 X  2
           1-Column            2-Column
   1-Row  0.245543108354E+01 -0.836715701670E+00
   2-Row  0.161871538187E+01  0.203298853382E+01
   3-Row  0.169795081904E+01 -0.204282504000E+00
   4-Row  0.779942321431E+00  0.930031944962E+00
--------------------------------------------------------------------------------------------------------------------
Definitions of Matrix Inputs (Columns):    2
Bottom Mass Position, y1 error
Pendulum Angle, theta error

Definitions of Matrix Outputs (Rows):      4
Bottom Mass Position, y1 estim
Top Mass Position, y2 estim
Bottom Mass Velocity, y1-dot estim
Top Mass Velocity, y2-dot estim
--------------------------------------------------------------------------------------------------------------------
```

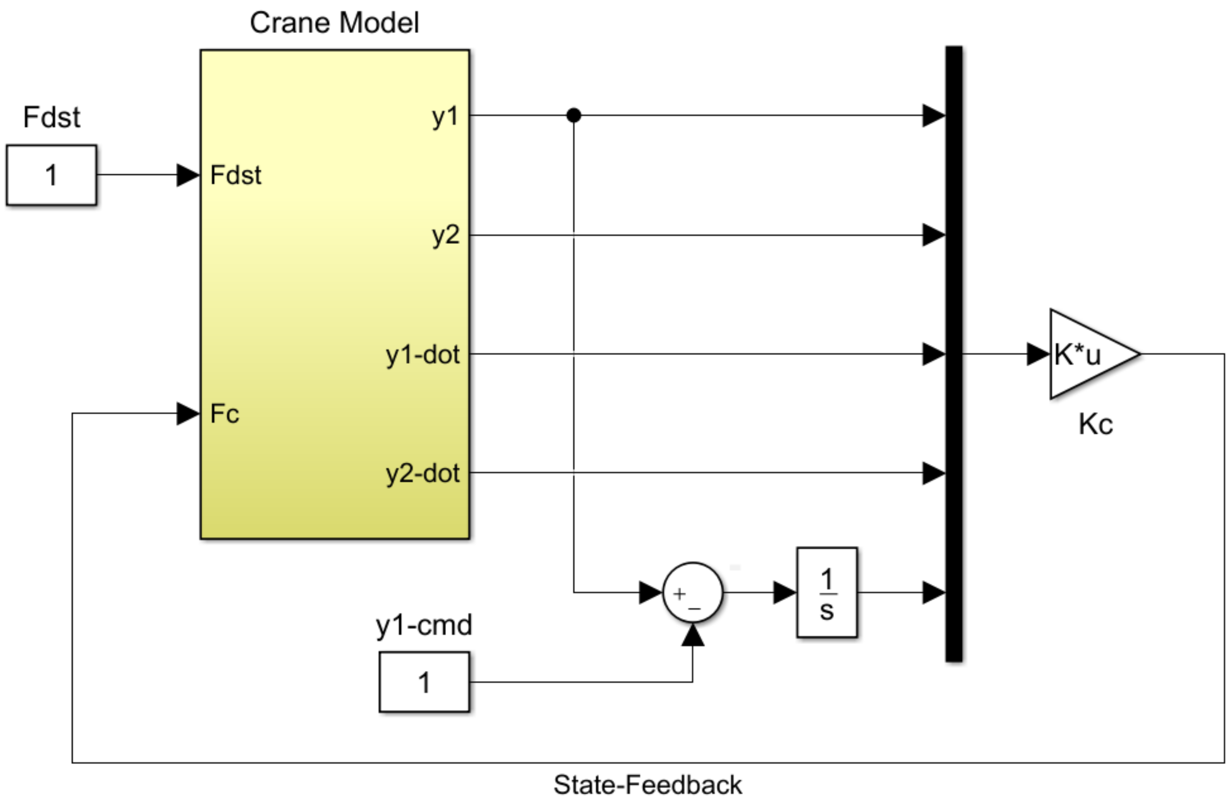


```
% Initialization File init.m
r2d=180/pi;
[Aa,Ba,Ca,Da]= crane;                % Load Crane Analysis Model
[Ad,Bd,Cd,Dd]= design;               % Load Crane Design Model
load Kc -ascii                       % Load the State-Feedback Gain
load Kf -ascii                       % Load the Kalman-Filter
Af=Aa-Kf*Ca([1,2],:);                % for the KF Af-matrix
```

2.3 Simulation Models

We will use two simulation models, shown in Figure 2.1, to design and analyze the H-infinity control system. The first model "*Crane_Sim_1.mdl*" is used to test the state-feedback gain Kc, that is directly from the four states: x= [y_1, y_2, $\dot{y}_1, \dot{y}_2$] which of course they are not measurable. It is intended to check out the control design and to adjust the weight matrices Qc2 and Rc as needed for good performance. It uses the 4-state design model with the 4-state outputs. The y1-integrator is introduced to provide the 5th state-feedback needed by the control gain Kc.

The second simulation "*Crane_Sim_2.mdl*" uses the analysis model with the two outputs (y_1 and θ). The 2 outputs are inputs to the Kalman-Filter which estimates the 4-state vector. The estimated 4-state vector together with y1-integral become inputs to the 5-state control gain Kc which calculates the control force Fc and closes the control loop. The state estimator is shown in detail in Figure 2.2. In addition to the plant outputs the estimator requires knowledge of the two forces: control Fc and disturbance Fd. Control force is understandable and easily attainable but the disturbance force is not easy to measure. That's why we use the integrator so that we don't have to know the disturbance force.



Closed-Loop System via Estimator

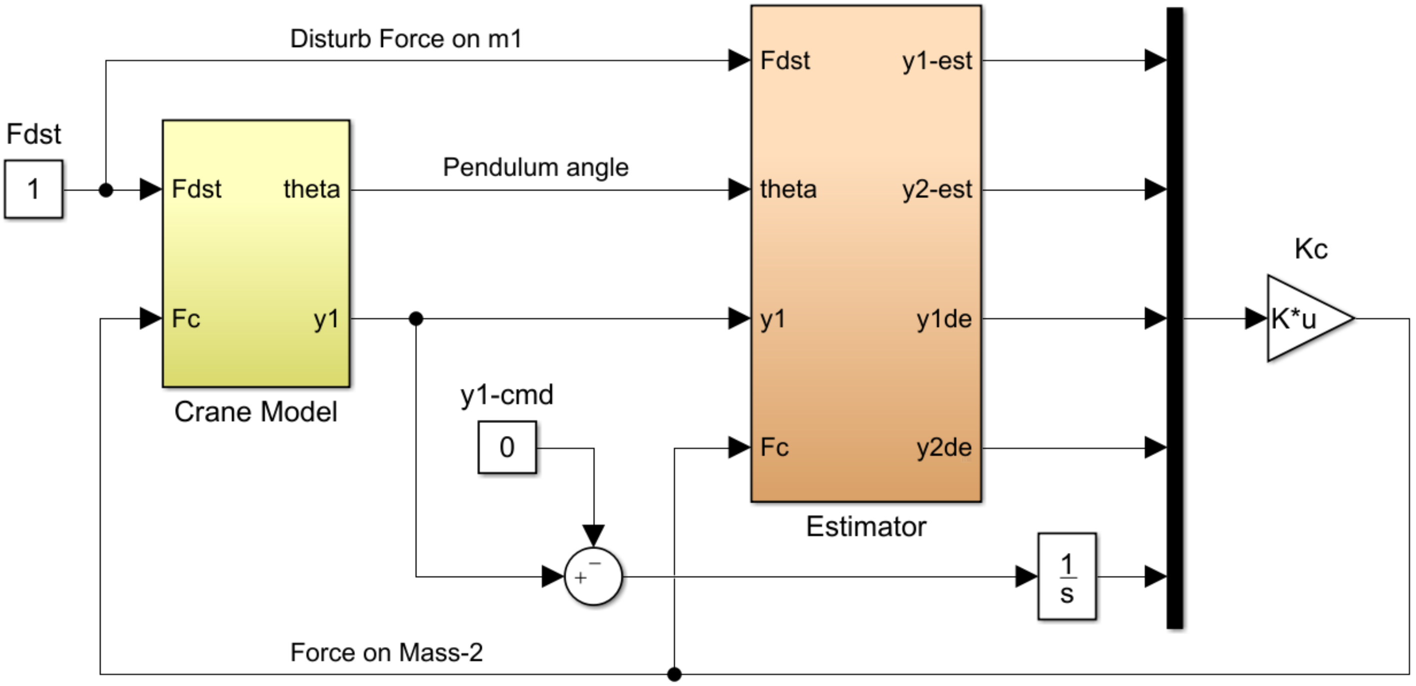


Figure 2.1 Closed-Loop Simulation Models “Crane_Sim_1” and “Crane_Sim_2”

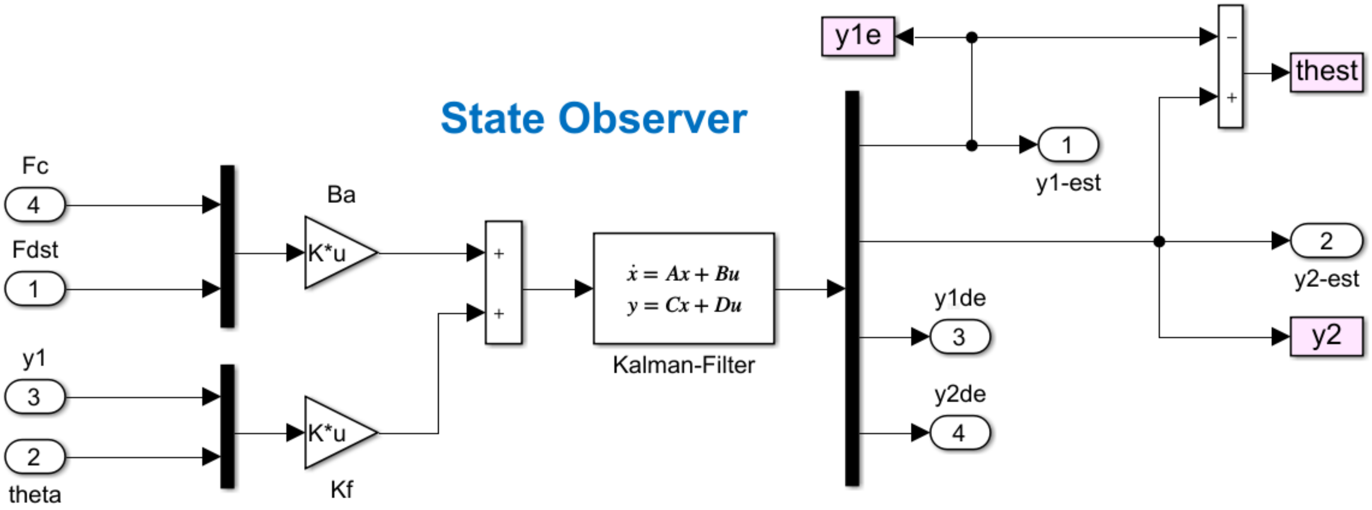


Figure 2.2 Kalman-Filter State Estimator

2.4 Simulation Results

Figure 2.3 shows the response of the second simulation model to a displacement command on m_1 equal to y_{1comd} = 1 unit. The m_2 mass moves towards the right under positive force almost to the half way point causing the pendulum angle to swing positive. It slows down near the half-way point waiting for the pendulum angle to swing negative. Then it applies negative force to stop the oscillation when m_1 is almost near the target position. The estimated and actual pendulum angles are identical.

Figure 2.4 shows the response of the second model to a unit steady disturbance force Fd=1 on m_1. It begins to move towards the right under the influence of the external force and the pendulum swings negative. The control system responds and applies a negative force on m_2 which moves to the left and is pulling the rope to counteract the positive disturbance. The control force stabilizes at Fc=-Fd=-1 opposing the disturbance and the pendulum angle stabilizes at a negative value θ=-1 while pulling the rope steadily against the disturbance force. The m_1 position eventually returns to its initial value y_{1cmd}=0 under the influence of the integrator and the steady pulling.

2.5 Frequency Response Analysis

Two Simulink models, shown in Figure 2.5, are used for frequency response analysis and to determine the system stability. A simple state-feedback model “Open_Loop_1” and one that uses the estimator “Open_Loop_2”. They both have the loop opened at the control force input. Figure 2.6 shows the Bode and Nichols plots and the phase and gain margins. Notice that the system has big resonance at 1.41 (rad/sec) which is at the pendulum frequency. The control system is designed around the plant model and counteracts the natural pendulum mode by introducing an anti-resonance.

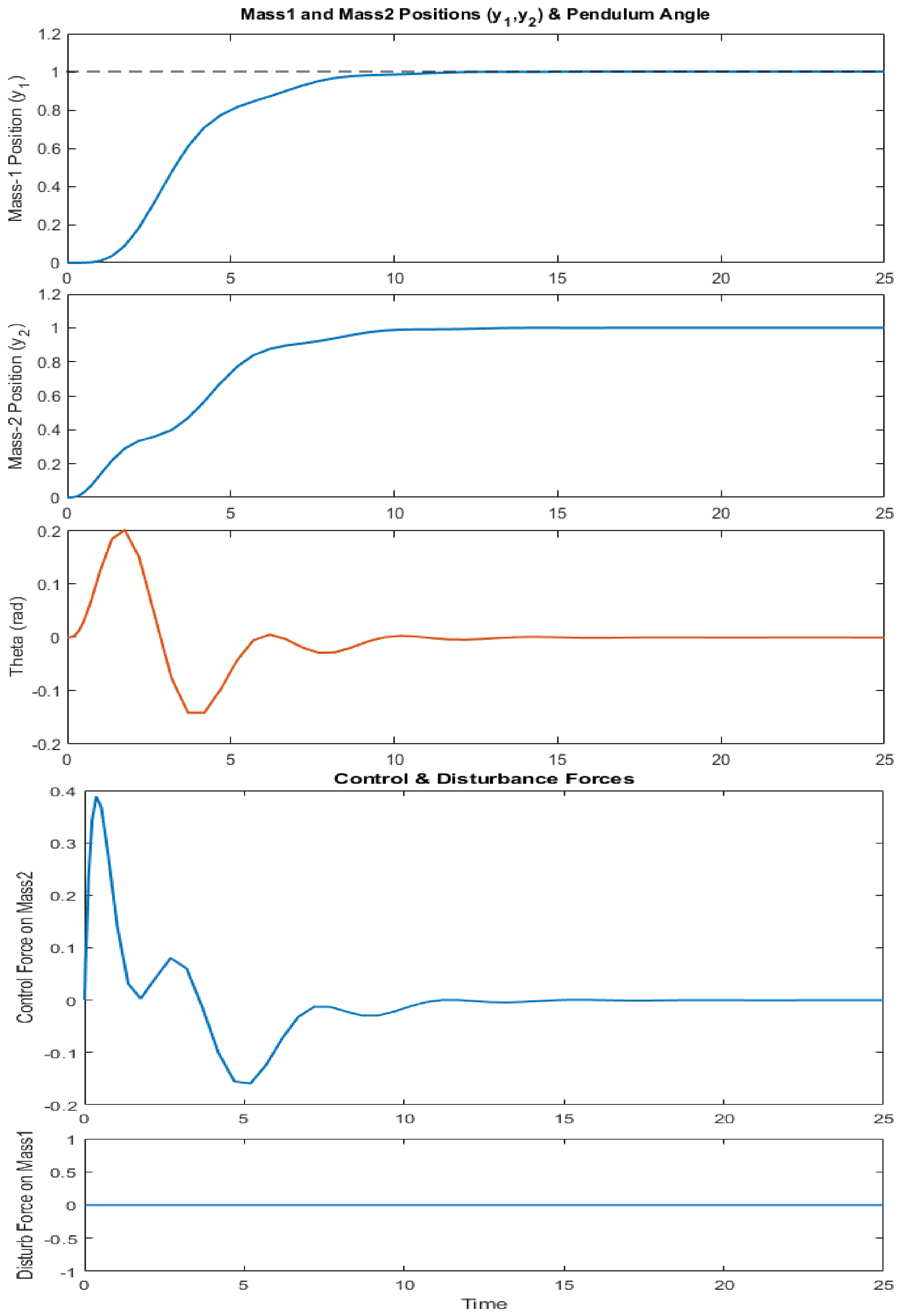


Figure 2.3 System's Response to y1-command

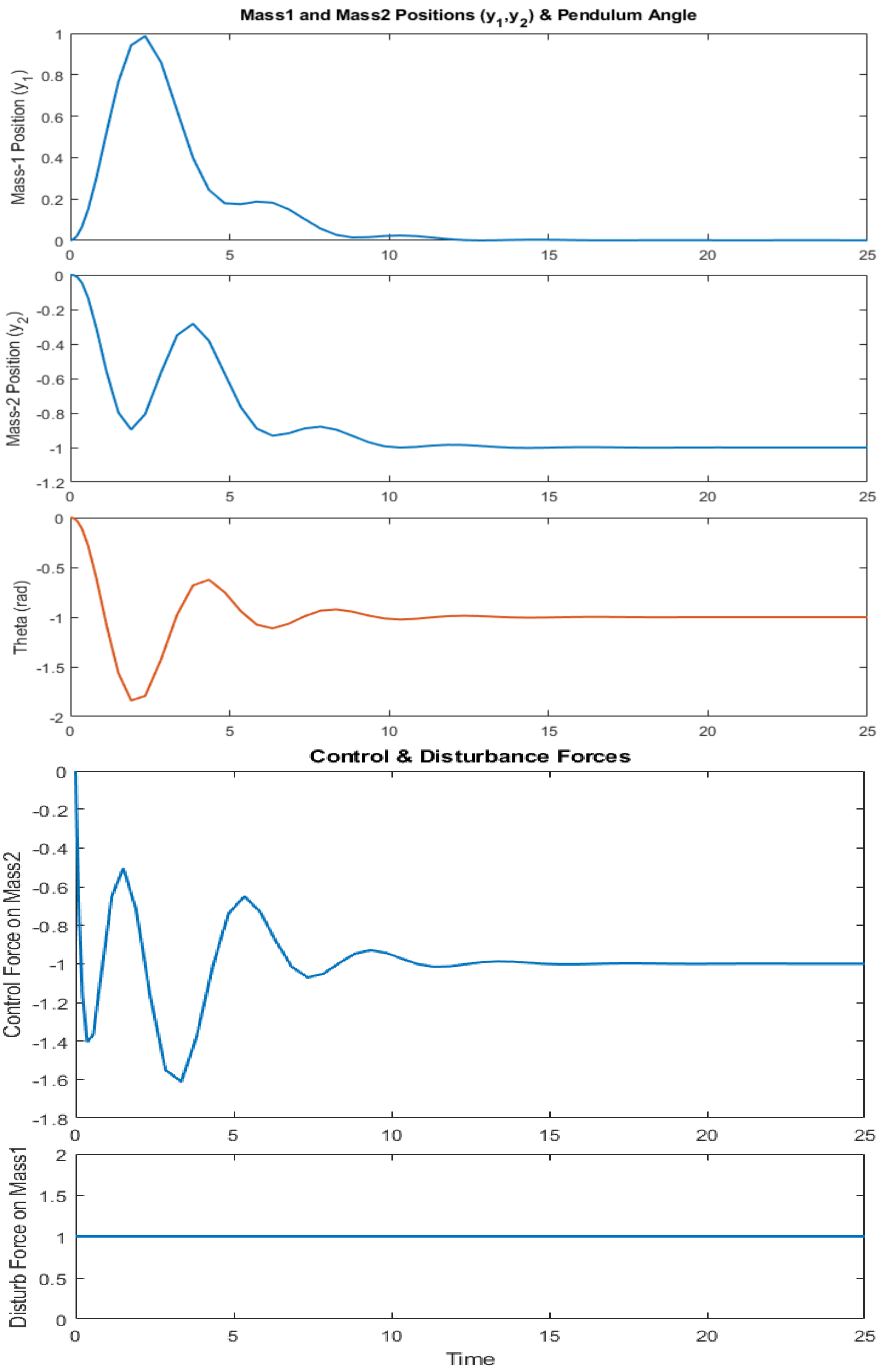


Figure 2.4 System's Response to Disturbance Force on m1

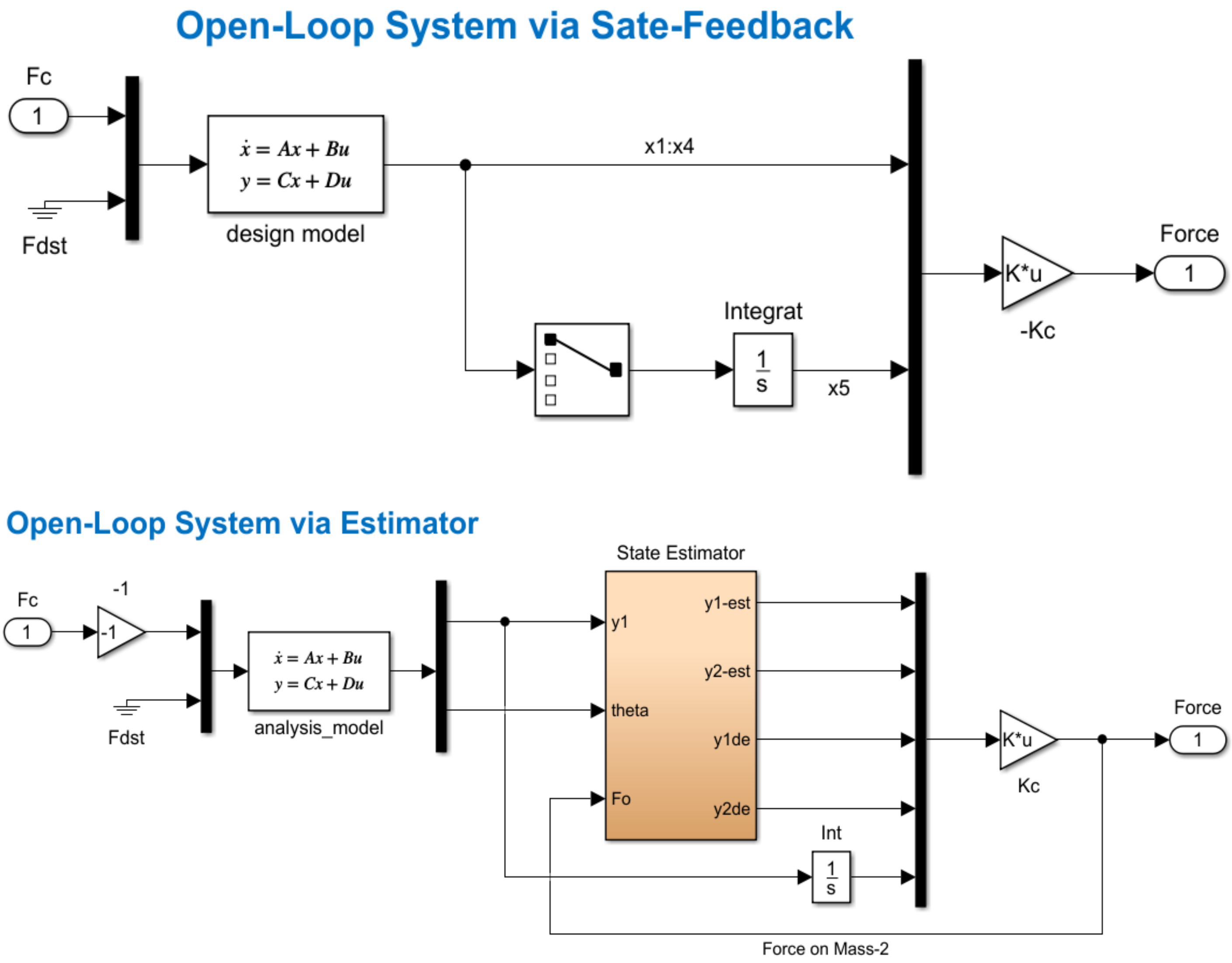


Figure 2.5 Stability Analysis Models "Open_Loop_1" and "Open_Loop_2"

The following Matlab script "frequ.m" calculates the frequency response from the Simulink model "*Open_Loop_2*" and creates the Bode and Nichols plots.

```
% Frequency Response Analysis frequ.m
init
[A1,B1,C1,D1]= linmod('Open_Loop_2');  % Linearize Open-Loop Simulink model
sys= ss(A1,B1,C1,D1);                  % Create SS System
wl=logspace(-2, 2, 10000);             % Define Frequ Range
figure(1); nichols(sys,wl)             % Nichols Plot
figure(2); bode(sys,wl)                % Bode Plot
sys1=sys;
```

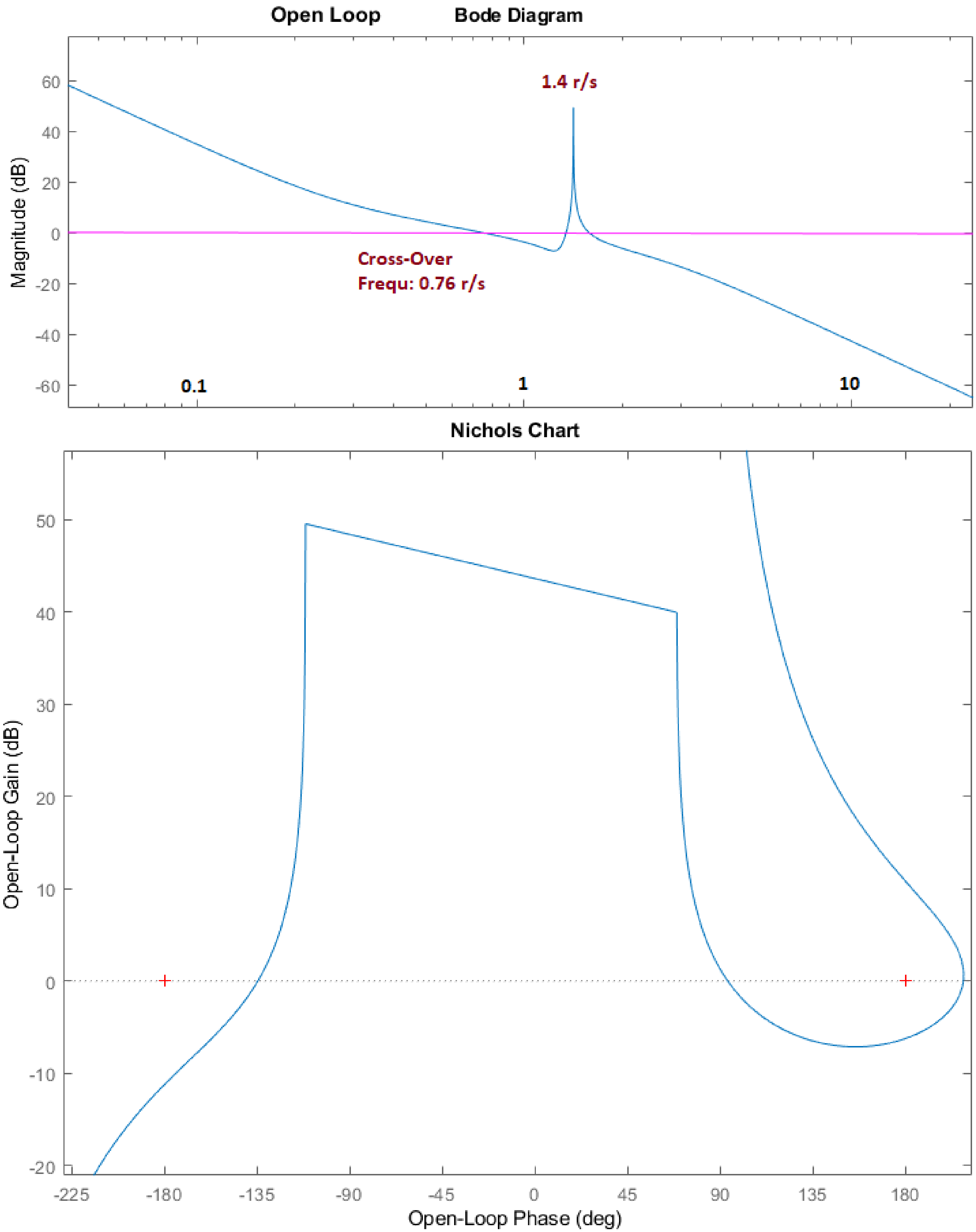
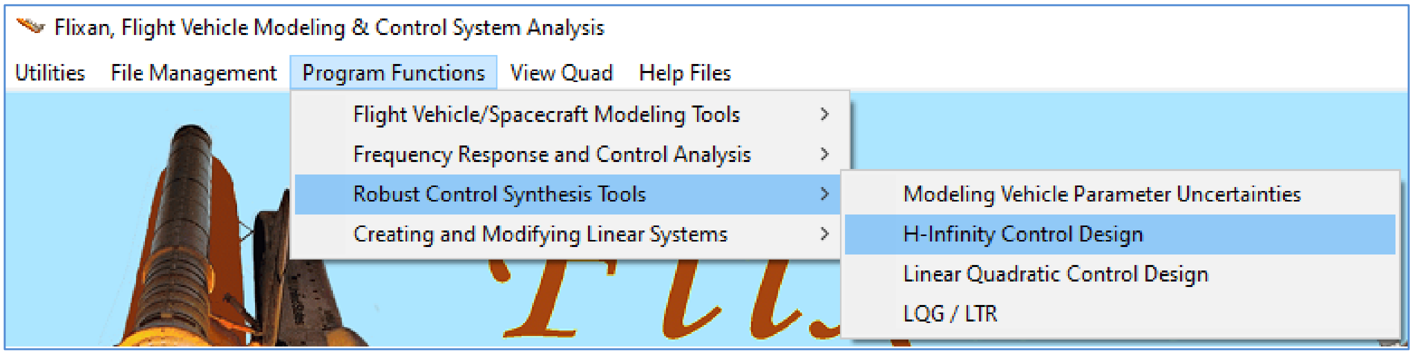
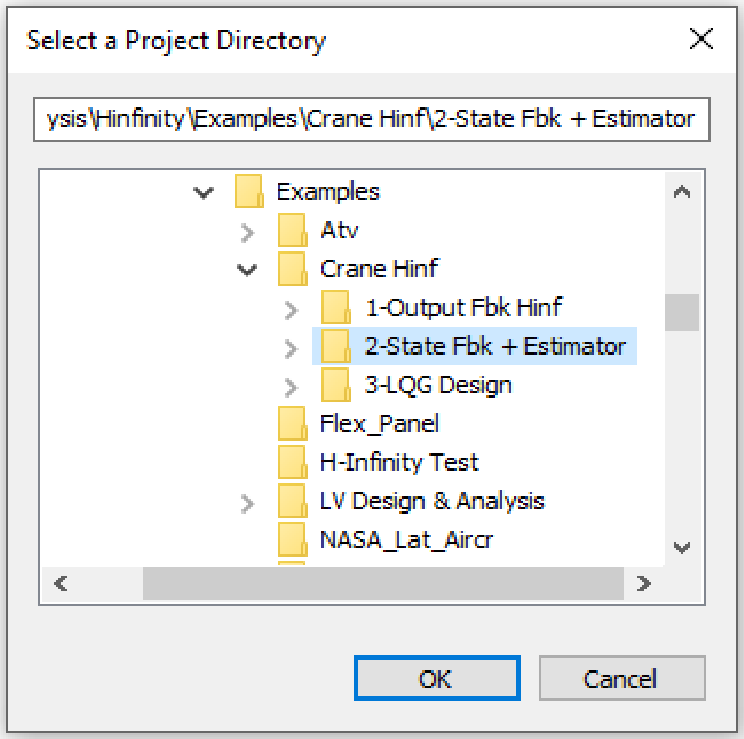


Figure 2.6 Open-Loop Bode and Nichols Plots for Analyzing Stability Using Model “Open_Loop_2”

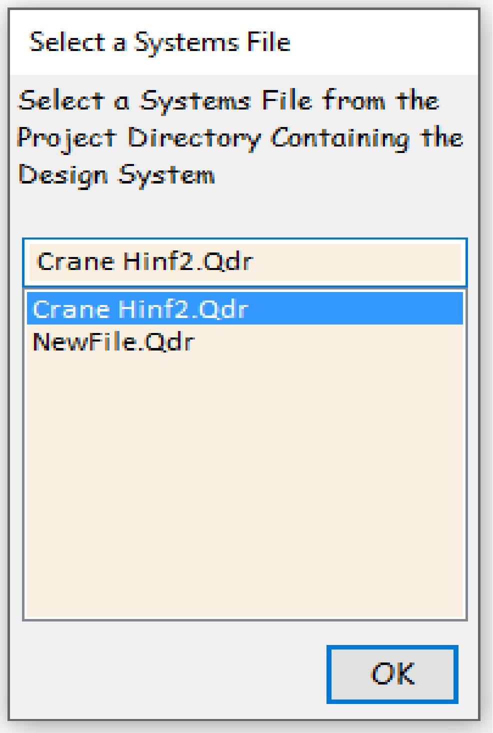
2.6 Running the H-Infinity Program Interactively

The input and systems files in this example are already prepared to be processed in batch mode but in this section, we will run the H-infinity program interactively. We will first create the 9-matrix Synthesis Model from the design system “*Crane Design Model with Y1 Integral*” which is already saved in the systems file “*Crane Hinf2.Qdr*”. The SM defines which inputs are controls and disturbances and which outputs are measurements and optimization criteria. It also includes the scaling gains. This time the measurements vector is equal to the 5 SM states which are the augmented design system states, and the measurement noise is zero. The controller derived from this SM will be a state-feedback gain Kc that will be saved in the systems file. We begin by running the H-infinity design program and from the main menu selecting the first option, as shown. Use the following menu to select the systems filename.





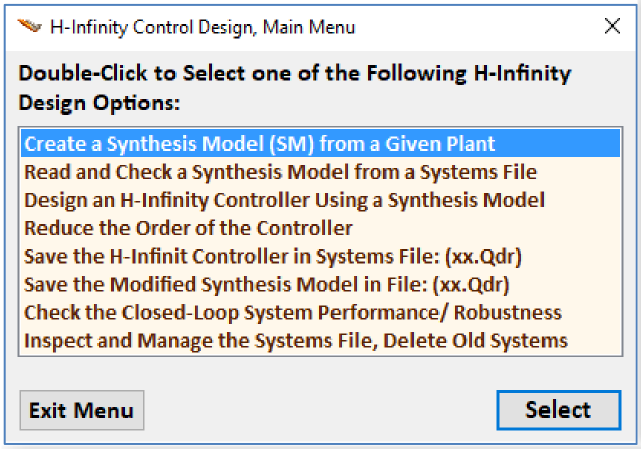
ysis\Hinfinity\Examples\Crane Hinf\2-State Fbk + Estimator



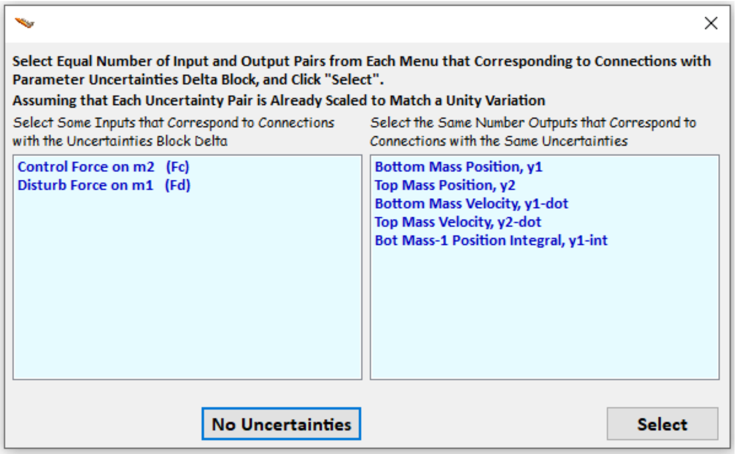
Select a Systems File from the Project Directory Containing the Design System

Crane Hinf2.Qdr

Select also the design system “Crane Design Model with Y1 Integral” that will used to create the SM by selecting some inputs and outputs using menus and placing them into groups. The first dialog is for selecting parameter variation pairs that connect with uncertainties. In this case we don’t have any, so click on “No Uncertainties”.

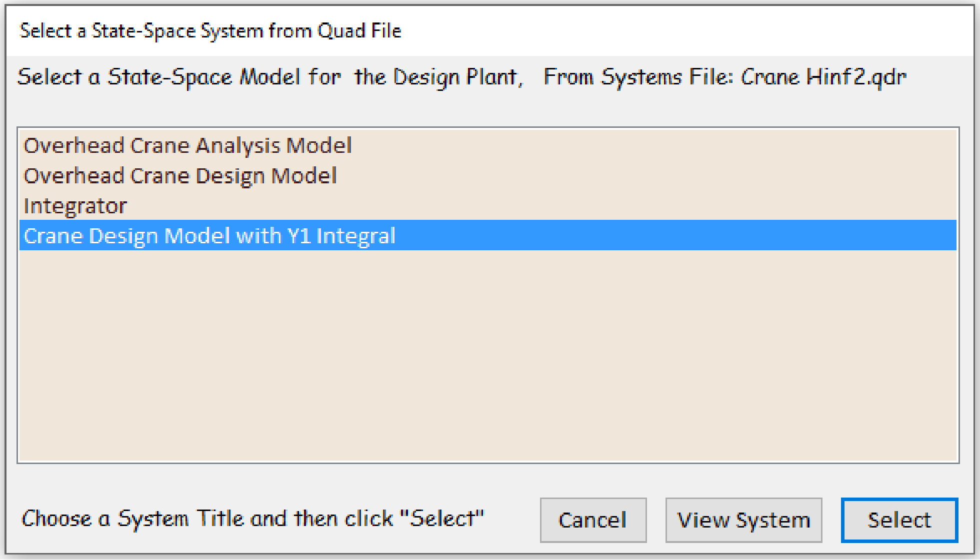

H-Infinity Control Design, Main Menu

Double-Click to Select one of the Following H-Infinity Design Options:


Select Equal Number of Input and Output Pairs from Each Menu that Corresponding to Connections with Parameter Uncertainties Delta Block, and Click "Select".
Assuming that Each Uncertainty Pair is Already Scaled to Match a Unity Variation

Select Some Inputs that Correspond to Connections with the Uncertainties Block Delta

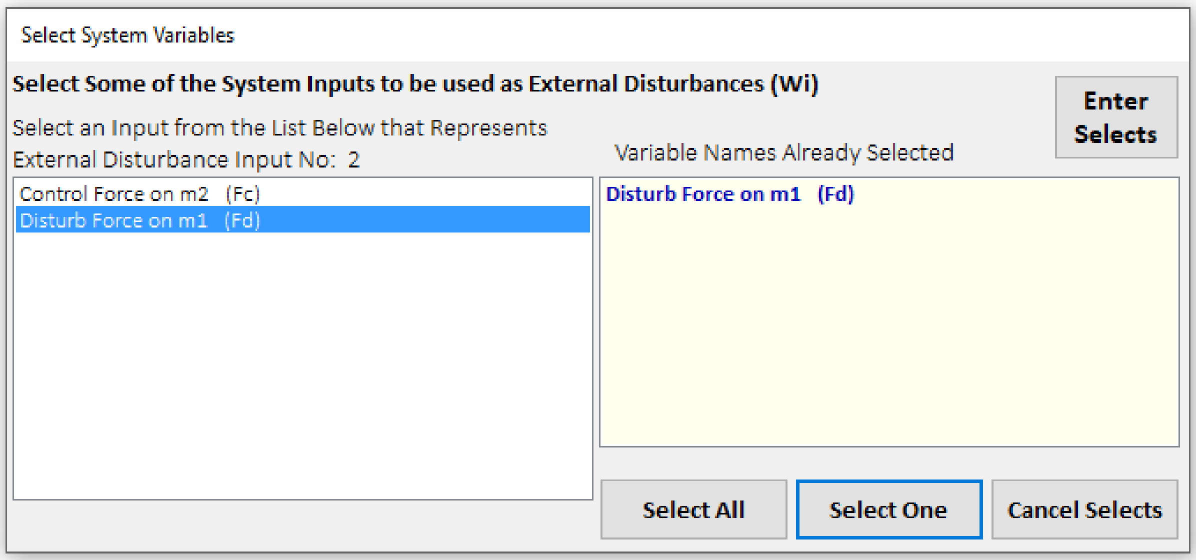
Select the Same Number Outputs that Correspond to Connections with the Same Uncertainties


Select a State-Space System from Quad File

Select a State-Space Model for the Design Plant, From Systems File: Crane Hinf2.qdr

Choose a System Title and then click "Select"

The next menu is for defining external disturbance inputs. The system has two inputs. Select the second input which is the disturbance force on m1, and click on "Enter Selects" to continue.


Select System Variables

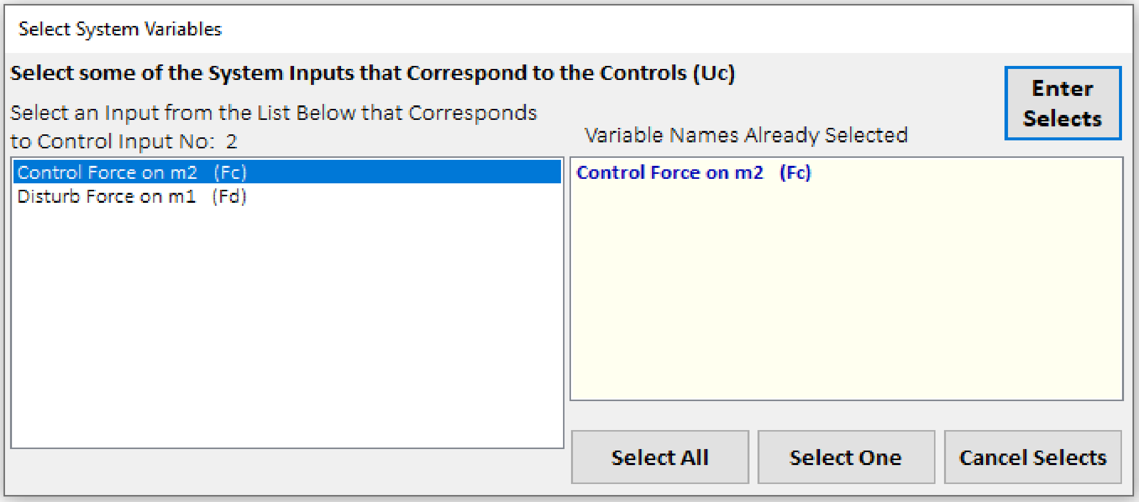
Select Some of the System Inputs to be used as External Disturbances (Wi)

Select an Input from the List Below that Represents External Disturbance Input No: 2

Variable Names Already Selected

Disturb Force on m1 (Fd)

The next menu is for selecting the control inputs u_c. There is only one control input in the design system which is the control force om m2. Select it and then click on “Enter Selects” to continue.



Select System Variables

Select some of the System Inputs that Correspond to the Controls (Uc)

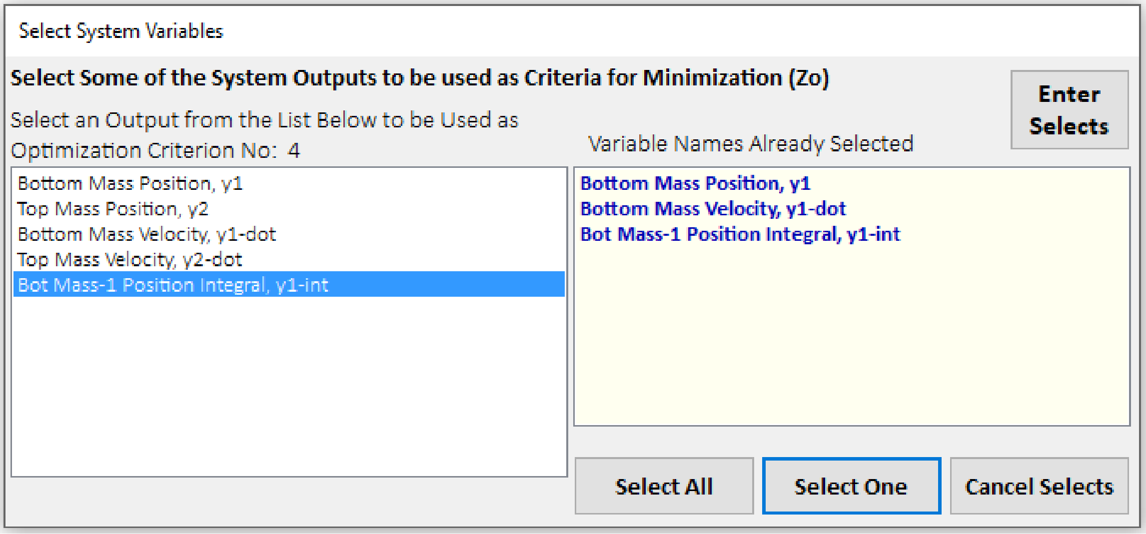
Select an Input from the List Below that Corresponds to Control Input No: 2

Control Force on m2 (Fc)
Disturb Force on m1 (Fd)

Variable Names Already Selected

Control Force on m2 (Fc)

The next menu selects the variables to be used in the performance optimization criterion. The design system has 5 outputs which are also states. We will select 3: y1 position, y1-dot, and y1-integral. Select one at a time, and then click on “Enter Selects” to continue. The next menu is for selecting outputs which are regulated by commands. Select the mass1 displacement (y1) and click on “Enter Selects”.



Select System Variables

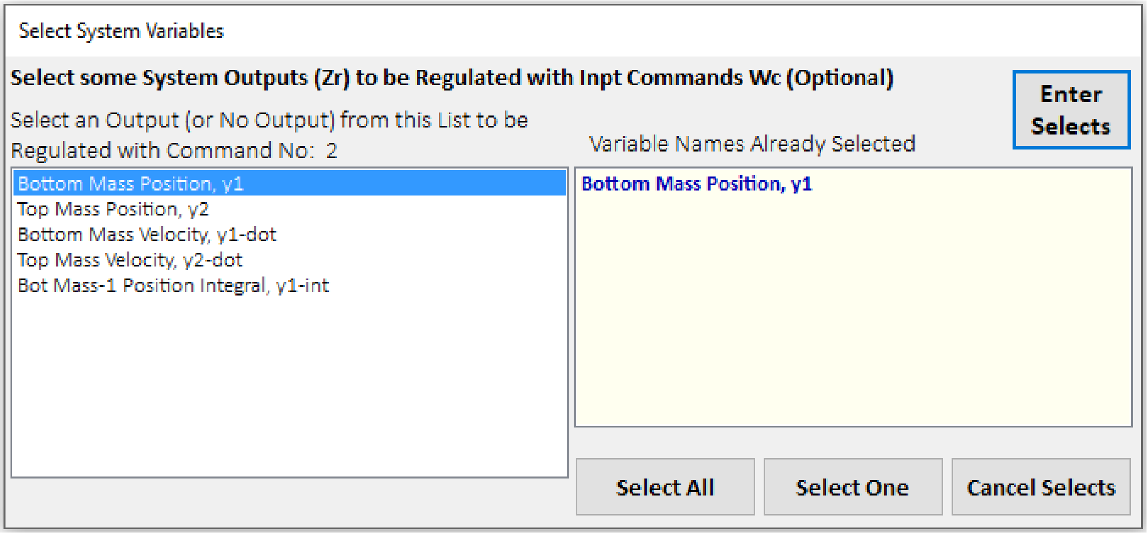
Select Some of the System Outputs to be used as Criteria for Minimization (Zo)

Select an Output from the List Below to be Used as Optimization Criterion No: 4

Bottom Mass Position, y1
Top Mass Position, y2
Bottom Mass Velocity, y1-dot
Top Mass Velocity, y2-dot
Bot Mass-1 Position Integral, y1-int

Variable Names Already Selected

Bottom Mass Position, y1
Bottom Mass Velocity, y1-dot
Bot Mass-1 Position Integral, y1-int



Select System Variables

Select some System Outputs (Zr) to be Regulated with Inpt Commands Wc (Optional)

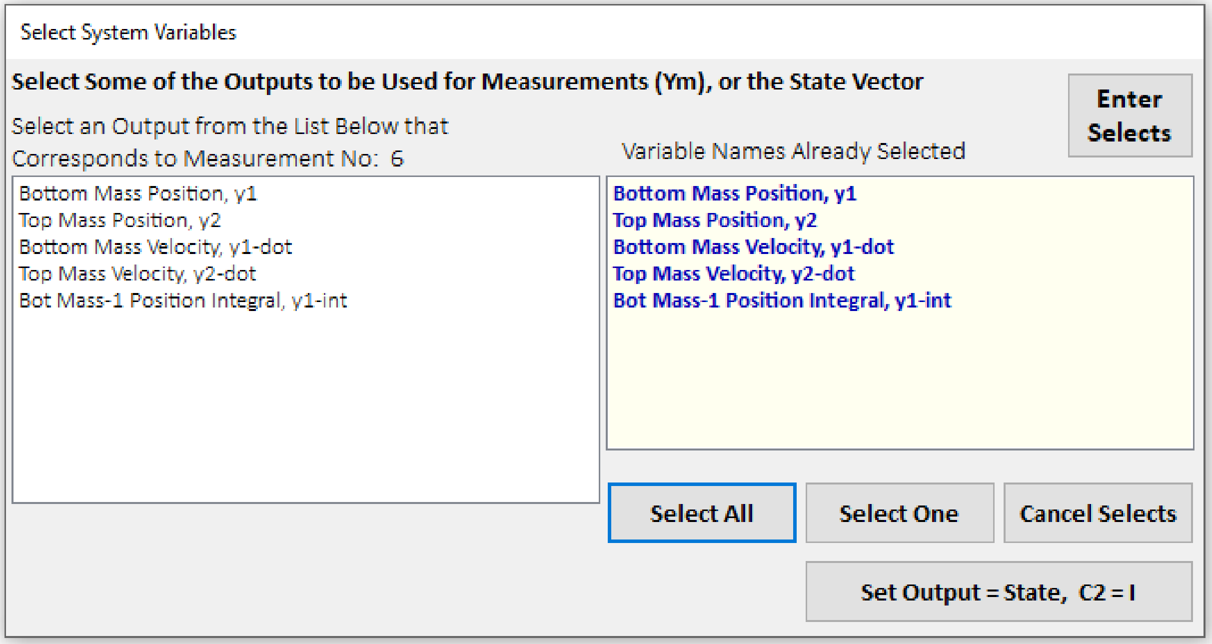
Select an Output (or No Output) from this List to be Regulated with Command No: 2

Bottom Mass Position, y1
Top Mass Position, y2
Bottom Mass Velocity, y1-dot
Top Mass Velocity, y2-dot
Bot Mass-1 Position Integral, y1-int

Variable Names Already Selected

Bottom Mass Position, y1

The next menu is used for selecting the output measurements. Click on “Select All” to select all the outputs which is also the state-vector, and then click on “Enter Selects” to continue.


Select System Variables

Select Some of the Outputs to be Used for Measurements (Ym), or the State Vector

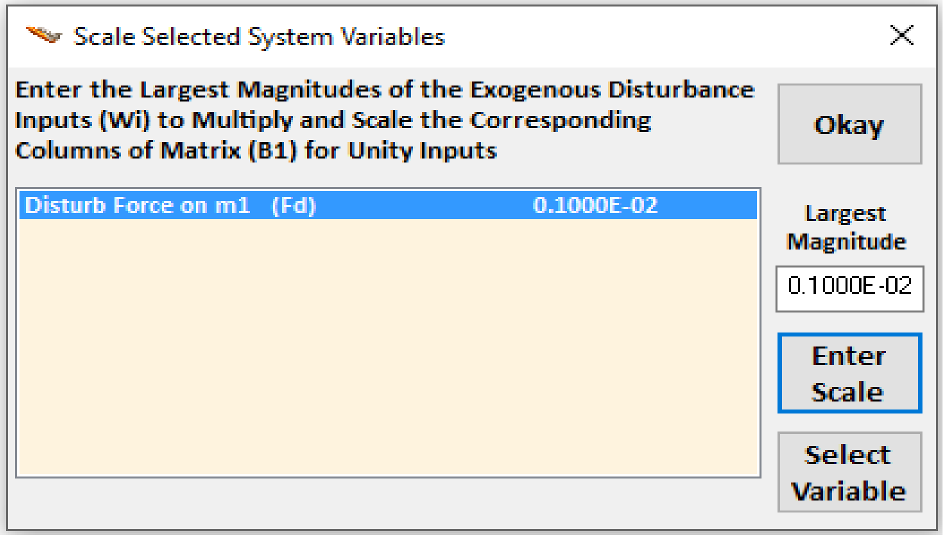
Select an Output from the List Below that Corresponds to Measurement No: 6

Bottom Mass Position, y1
Top Mass Position, y2
Bottom Mass Velocity, y1-dot
Top Mass Velocity, y2-dot
Bot Mass-1 Position Integral, y1-int

Variable Names Already Selected

Bottom Mass Position, y1
Top Mass Position, y2
Bottom Mass Velocity, y1-dot
Top Mass Velocity, y2-dot
Bot Mass-1 Position Integral, y1-int

We finished defining the input and output variables. We must now enter the gains that will be used to scale them. The gains are used to adjust the trade-off between bandwidth and performance versus sensitivity and stability in the optimization. We begin to scale the disturbance inputs by setting their gains equal to the magnitudes of the maximum expected disturbances, and for the output performance criteria we set their gains equal to the maximum acceptable magnitude at each output. The control input is also included in the criteria outputs and it is scaled by the magnitude of maximum control force. The gains for the measurements noise are set to zero in this case because it’s a state-feedback. We will design the estimator later. In the dialog below enter the gain that will scale the disturbance force on m_1. Double-click on the input or click on “Select Variable”, enter the scaling gain that defines the disturbance magnitude, and click on “Enter Scale” to accept it. Click “Okay” to go to the next dialog.

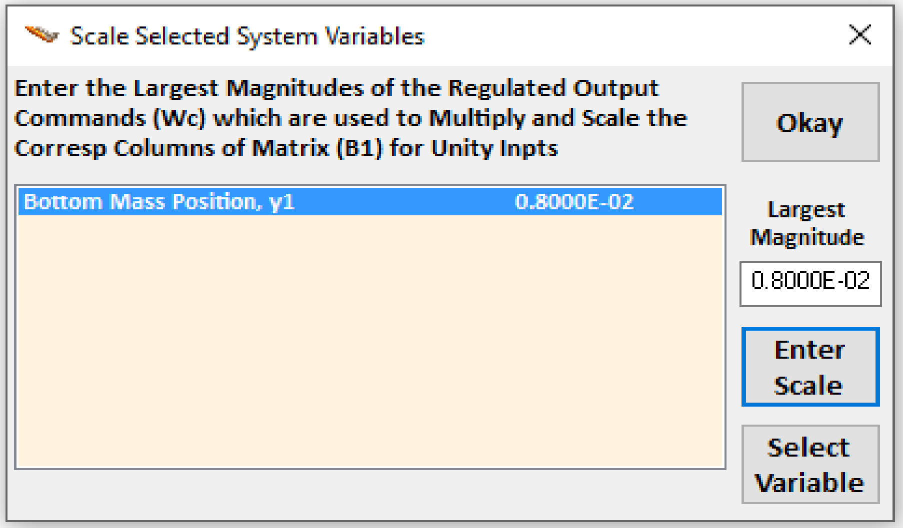

Scale Selected System Variables

Enter the Largest Magnitudes of the Exogenous Disturbance Inputs (Wi) to Multiply and Scale the Corresponding Columns of Matrix (B1) for Unity Inputs

Disturb Force on m1 (Fd) 0.1000E-02

Largest Magnitude: 0.1000E-02

In the next dialog below enter the largest expected magnitude at the input that commands the output (y_1). That is, the biggest magnitude of y_1_cmd, and click “Okay”.



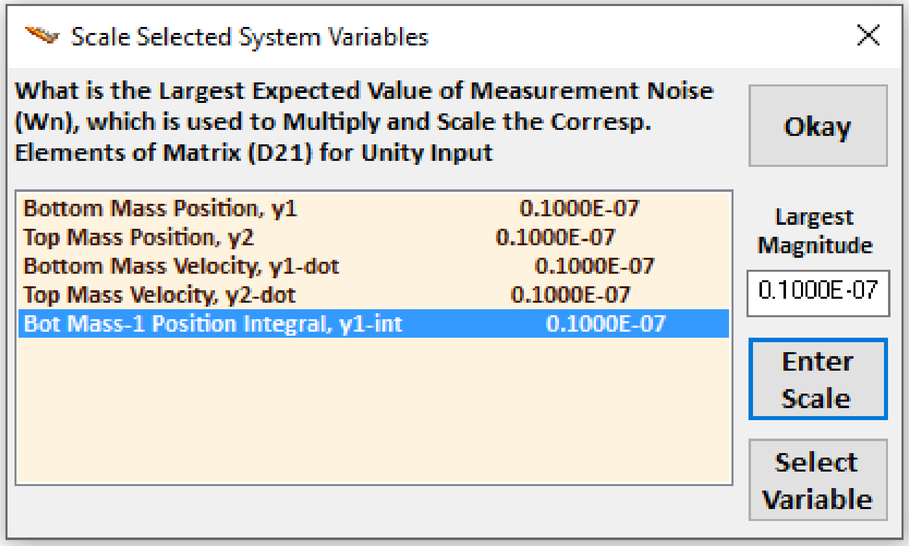
Scale Selected System Variables

Enter the Largest Magnitudes of the Regulated Output Commands (Wc) which are used to Multiply and Scale the Corresp Columns of Matrix (B1) for Unity Inpts

Bottom Mass Position, y1 0.8000E-02

Largest Magnitude: 0.8000E-02

In the next dialog you must enter the noise magnitude at the 5 measurements. Select one at a time and enter a very small number in all of them and then click on "Enter Scale". When you finish click "Okay" to go to the next dialog.

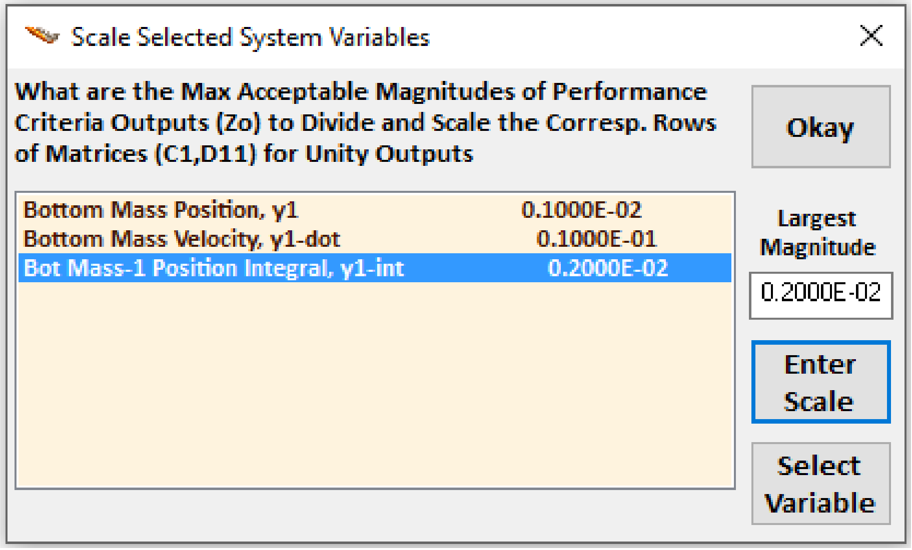


Scale Selected System Variables

What is the Largest Expected Value of Measurement Noise (Wn), which is used to Multiply and Scale the Corresp. Elements of Matrix (D21) for Unity Input

Bottom Mass Position, y1	0.1000E-07
Top Mass Position, y2	0.1000E-07
Bottom Mass Velocity, y1-dot	0.1000E-07
Top Mass Velocity, y2-dot	0.1000E-07
Bot Mass-1 Position Integral, y1-int	0.1000E-07

Largest Magnitude: 0.1000E-07



Scale Selected System Variables

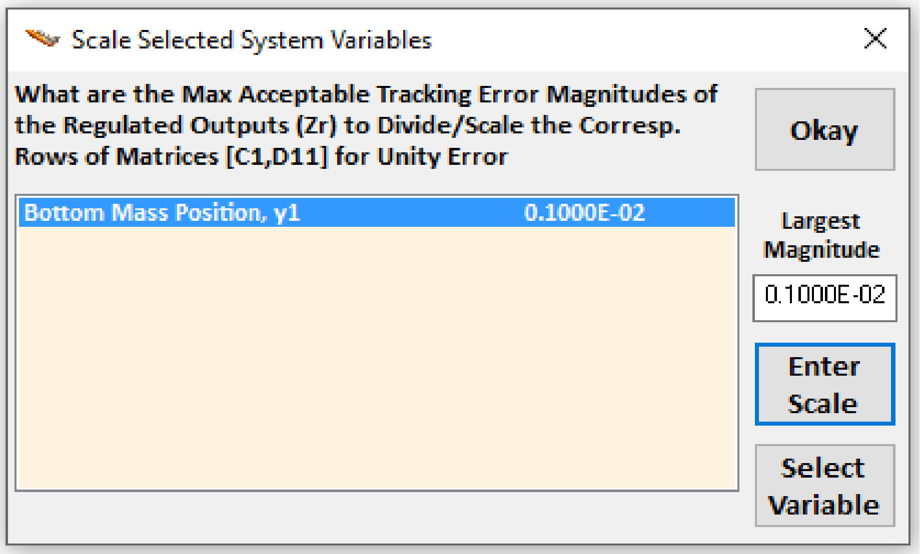
What are the Max Acceptable Magnitudes of Performance Criteria Outputs (Zo) to Divide and Scale the Corresp. Rows of Matrices (C1,D11) for Unity Outputs

Bottom Mass Position, y1	0.1000E-02
Bottom Mass Velocity, y1-dot	0.1000E-01
Bot Mass-1 Position Integral, y1-int	0.2000E-02

Largest Magnitude: 0.2000E-02

The next dialog is for defining the gains at the performance optimization criteria outputs. That is, the maximum acceptable magnitudes at the 3 performance outputs: ($y1$, y_1-dot, and y_1-integral). Reducing the gain value at a performance output produces better performance and smaller transient in the corresponding variable. Select one variable at a time, enter the gain and click on "Enter Scale". When you finish click on "Okay" to go to the next dialog.

The next dialog is for entering the gain that defines the max acceptable magnitude of the regulated output error (z_{re}). That is the magnitude of the error: y1-output minus y1-command.

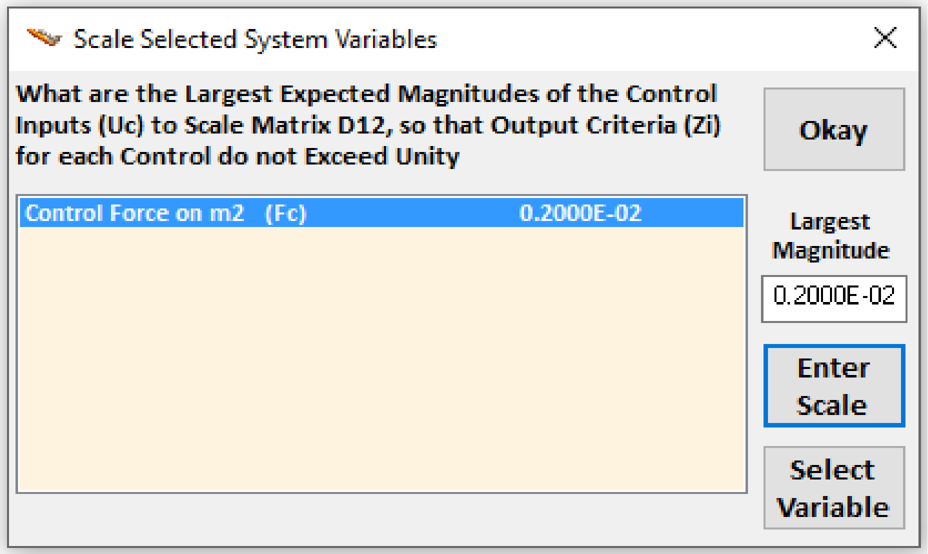


What are the Max Acceptable Tracking Error Magnitudes of the Regulated Outputs (Zr) to Divide/Scale the Corresp. Rows of Matrices [C1,D11] for Unity Error

Bottom Mass Position, y1 0.1000E-02

Largest Magnitude: 0.1000E-02

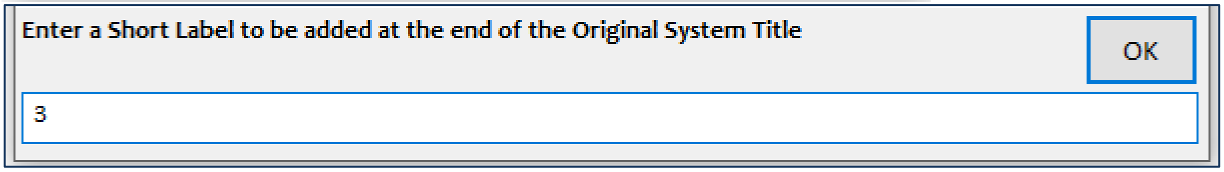
The last dialog is for entering the maximum control magnitude because the control is also included in the optimization criteria. In this example we only have the control force Fc. Finally, enter a short label to appear at the end of the Synthesis Model title.



What are the Largest Expected Magnitudes of the Control Inputs (Uc) to Scale Matrix D12, so that Output Criteria (Zi) for each Control do not Exceed Unity

Control Force on m2 (Fc) 0.2000E-02

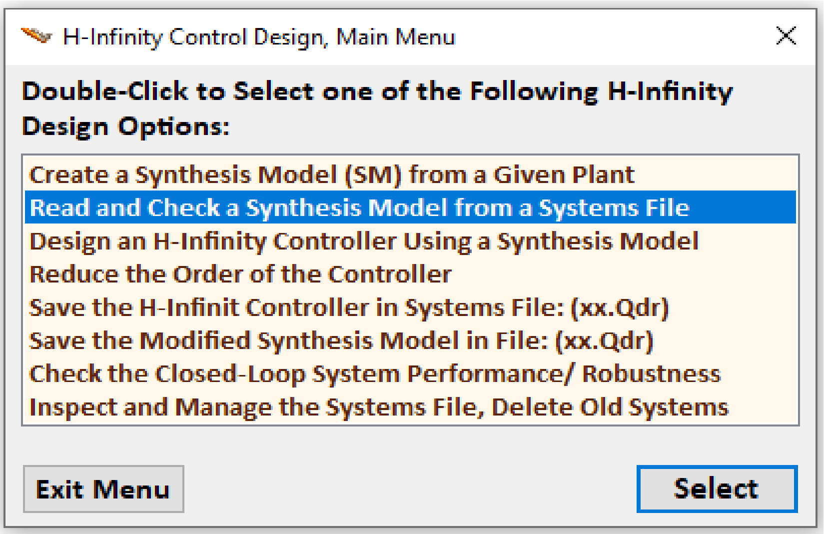
Largest Magnitude: 0.2000E-02

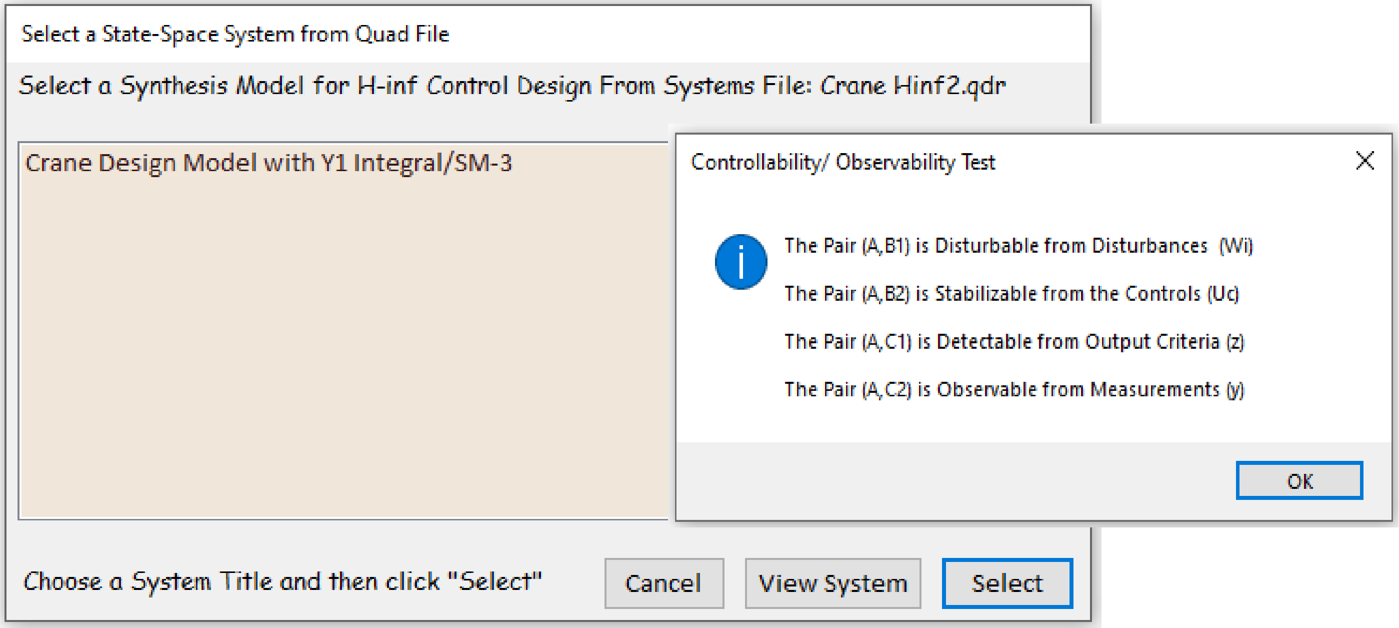


Enter a Short Label to be added at the end of the Original System Title

3

We now return to the main menu and begin the control design. Select the 2nd option to check the SM. Select the only one SM from the systems file and click on “Select”. Make sure the SM is controllable, disturbable from the disturbance inputs, observable from the measurements, and detectable from the performance criteria.





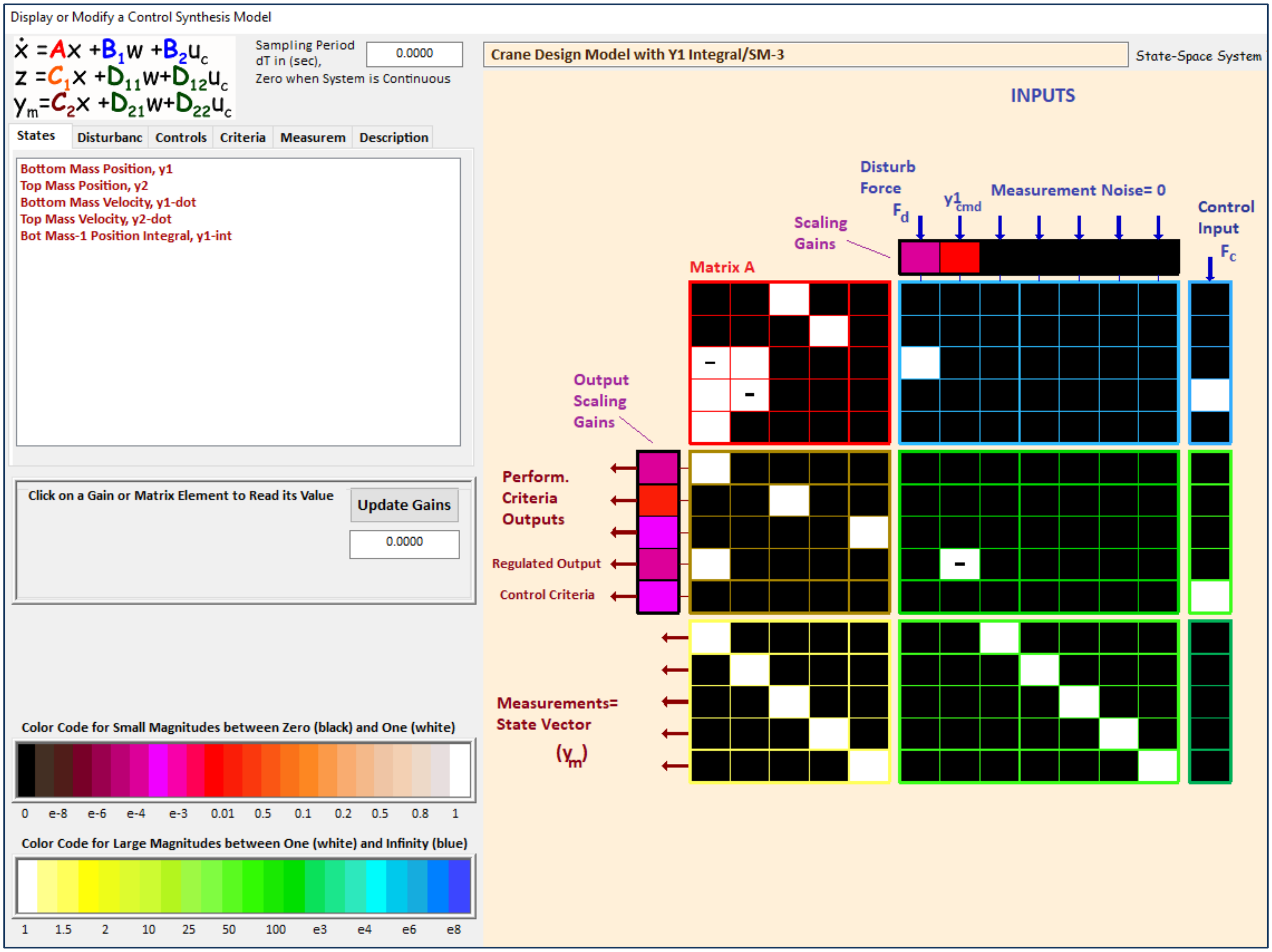
The Pair (A,B1) is Disturbable from Disturbances (Wi)

The Pair (A,B2) is Stabilizable from the Controls (Uc)

The Pair (A,C1) is Detectable from Output Criteria (z)

The Pair (A,C2) is Observable from Measurements (y)

The program confirms that the SM meets the expected requirements and it presents the SM matrices graphically in system’s form, shown in the next dialog. The 9 SM matrices are color coded and also the scaling gains are included that scale the disturbances and the criteria. The A-matrix has 5 states. There are 7 disturbance inputs which are: 1 external force Fd, the y_{1_command} for 1 regulated output y_{1_error}, and 5 measurements noise inputs that have zero (black) gains. There is one control input Fc. We also have 5 performance criteria which are: y_1, y_1_dot, y_1_integral, y_1_error, and Fc. The 5 measurements from matrix C2 are equal to the states: (y_1, y_2, $\dot{y}_1, \dot{y}_2$ and y_1_integral).



Display or Modify a Control Synthesis Model

$$\dot{x} = Ax + B_1 w + B_2 u_c$$
$$z = C_1 x + D_{11} w + D_{12} u_c$$
$$y_m = C_2 x + D_{21} w + D_{22} u_c$$

Sampling Period dT in (sec), Zero when System is Continuous: 0.0000

Crane Design Model with Y1 Integral/SM-3

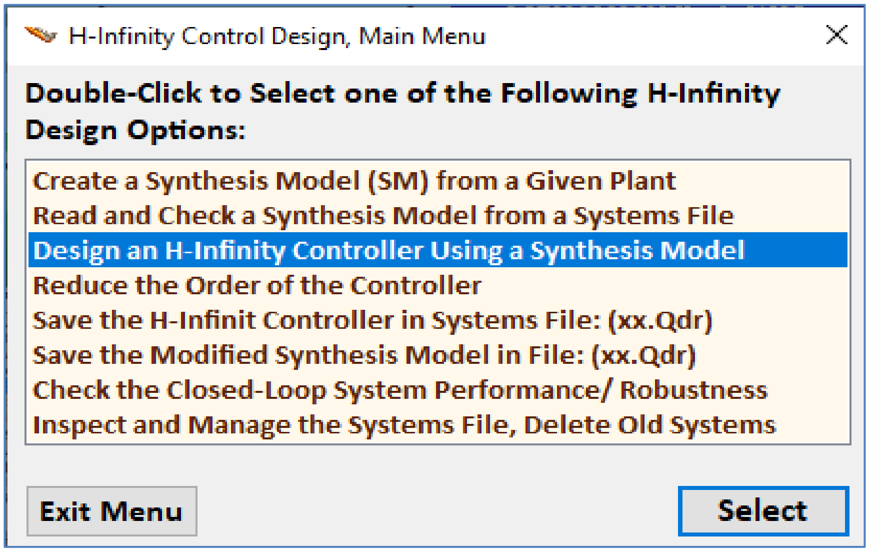
State-Space System

States:

Bottom Mass Position, y1
Top Mass Position, y2
Bottom Mass Velocity, y1-dot
Top Mass Velocity, y2-dot
Bot Mass-1 Position Integral, y1-int

Click on a Gain or Matrix Element to Read its Value: 0.0000

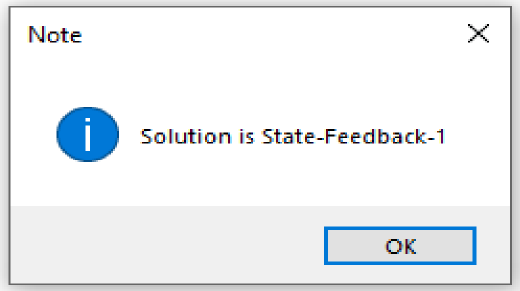
Select the third option from the main menu to design the H-infinity controller, and click “Select”. The program confirms that the solution is State-Feedback Controller and click “OK”.



H-Infinity Control Design, Main Menu

Double-Click to Select one of the Following H-Infinity Design Options:

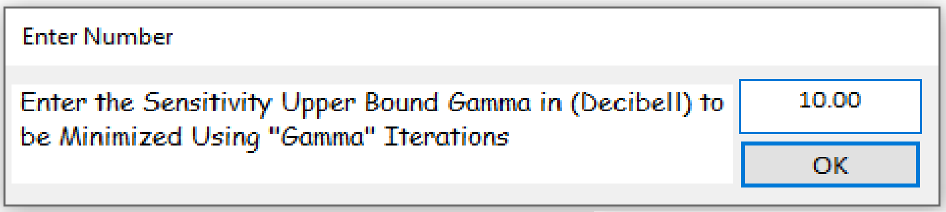
- Create a Synthesis Model (SM) from a Given Plant
- Read and Check a Synthesis Model from a Systems File
- Design an H-Infinity Controller Using a Synthesis Model
- Reduce the Order of the Controller
- Save the H-Infinit Controller in Systems File: (xx.Qdr)
- Save the Modified Synthesis Model in File: (xx.Qdr)
- Check the Closed-Loop System Performance/ Robustness
- Inspect and Manage the Systems File, Delete Old Systems



Note

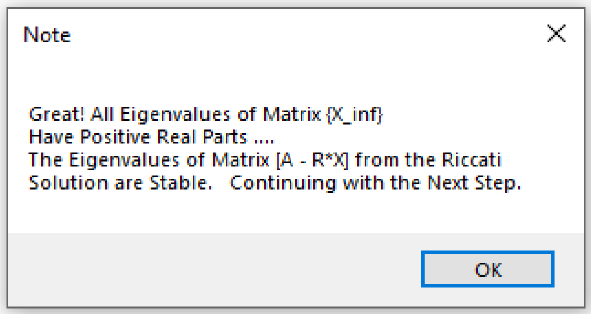
Solution is State-Feedback-1

Now we begin the iterative process of minimizing the upper bound γ of the infinity norm of the sensitivity transfer function between the disturbance inputs and the output criteria vectors. We begin with an arbitrary γ upper bound and try to find the smallest γ that will not violate the algorithm requirements. After a few iterations we end up with γ=5 (dB) and click on "No" meaning that we do not want to try another value but to accept the current controller.

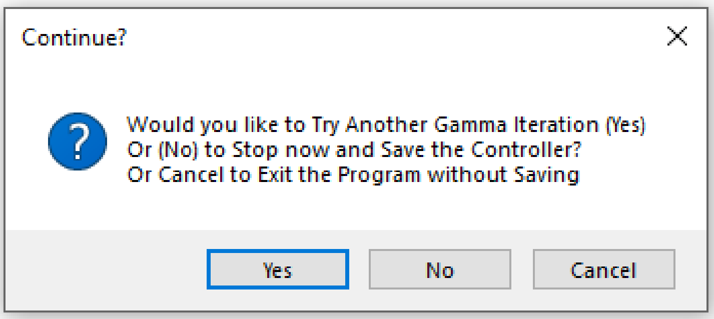


Enter the Sensitivity Upper Bound Gamma in (Decibell) to be Minimized Using "Gamma" Iterations

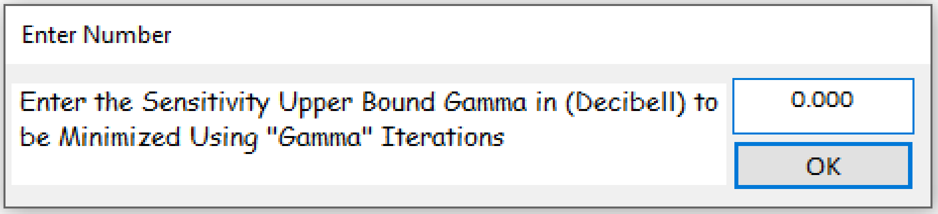
10.00



Great! All Eigenvalues of Matrix {X_inf}
Have Positive Real Parts ....
The Eigenvalues of Matrix [A - R*X] from the Riccati
Solution are Stable. Continuing with the Next Step.

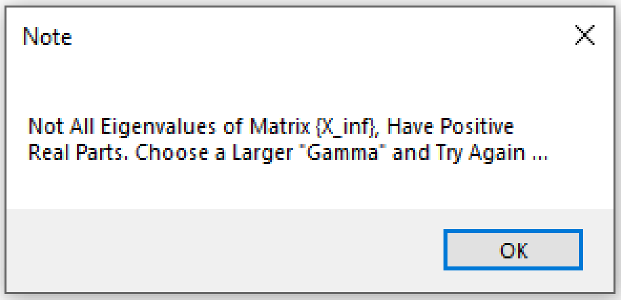


Would you like to Try Another Gamma Iteration (Yes)
Or (No) to Stop now and Save the Controller?
Or Cancel to Exit the Program without Saving

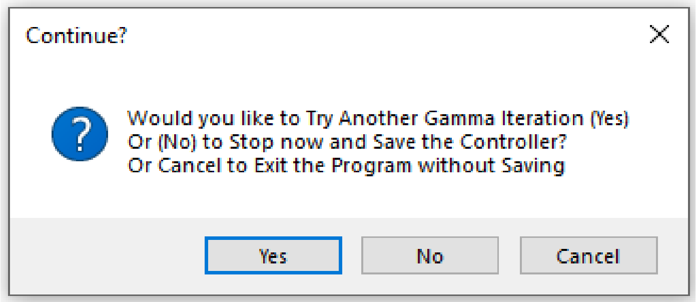


Enter the Sensitivity Upper Bound Gamma in (Decibell) to be Minimized Using "Gamma" Iterations

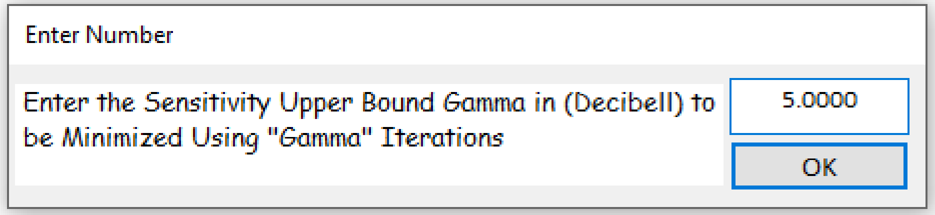
0.000



Not All Eigenvalues of Matrix {X_inf}, Have Positive
Real Parts. Choose a Larger "Gamma" and Try Again ...

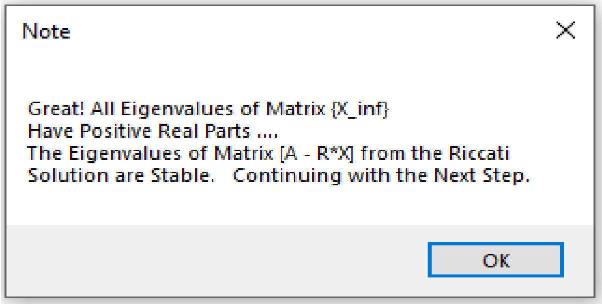


Would you like to Try Another Gamma Iteration (Yes)
Or (No) to Stop now and Save the Controller?
Or Cancel to Exit the Program without Saving

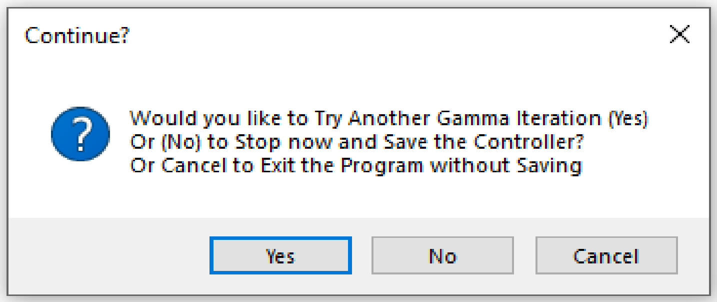


Enter the Sensitivity Upper Bound Gamma in (Decibell) to be Minimized Using "Gamma" Iterations

5.0000



Great! All Eigenvalues of Matrix {X_inf}
Have Positive Real Parts ....
The Eigenvalues of Matrix [A - R*X] from the Riccati
Solution are Stable. Continuing with the Next Step.



Would you like to Try Another Gamma Iteration (Yes)
Or (No) to Stop now and Save the Controller?
Or Cancel to Exit the Program without Saving

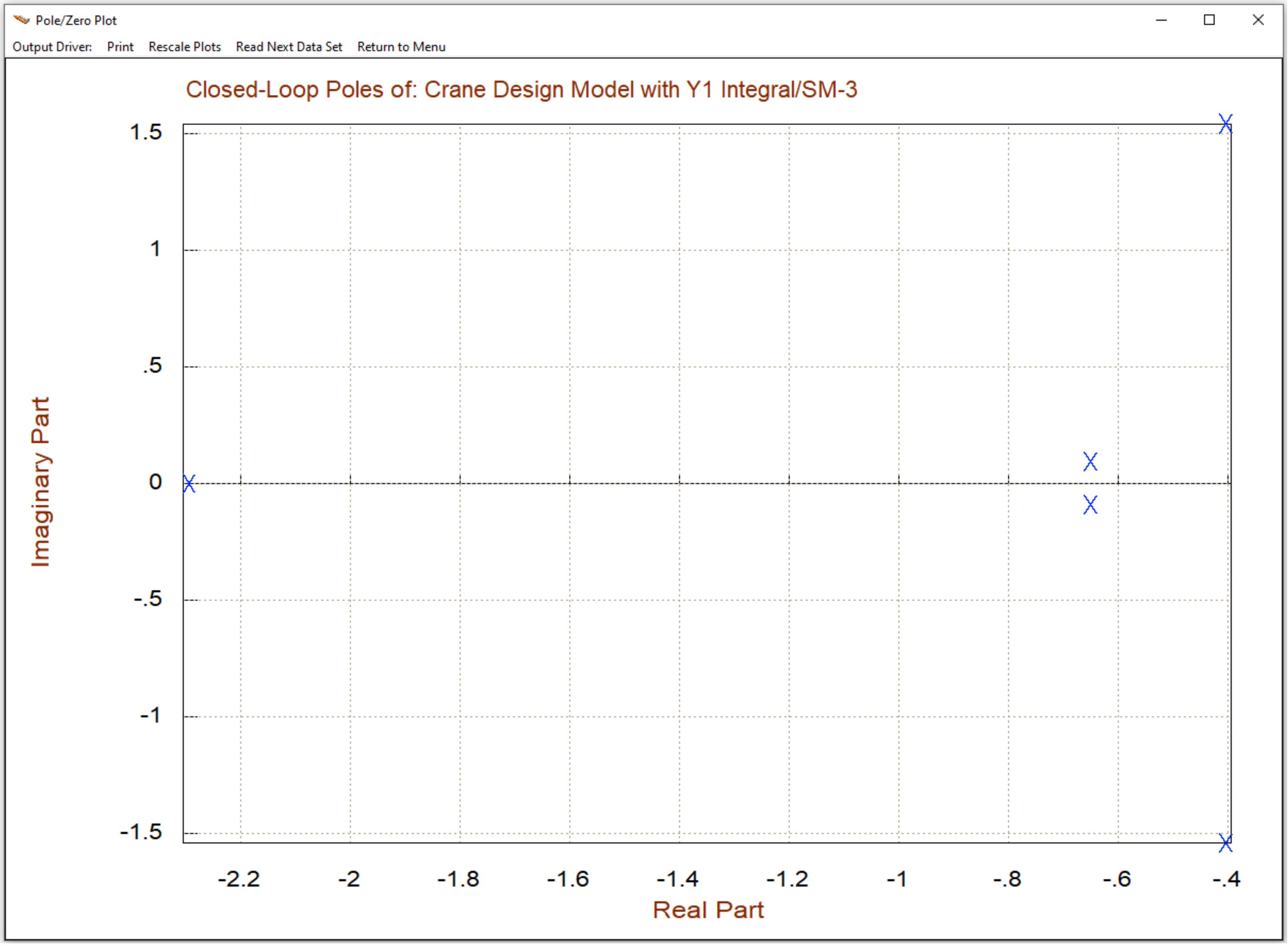
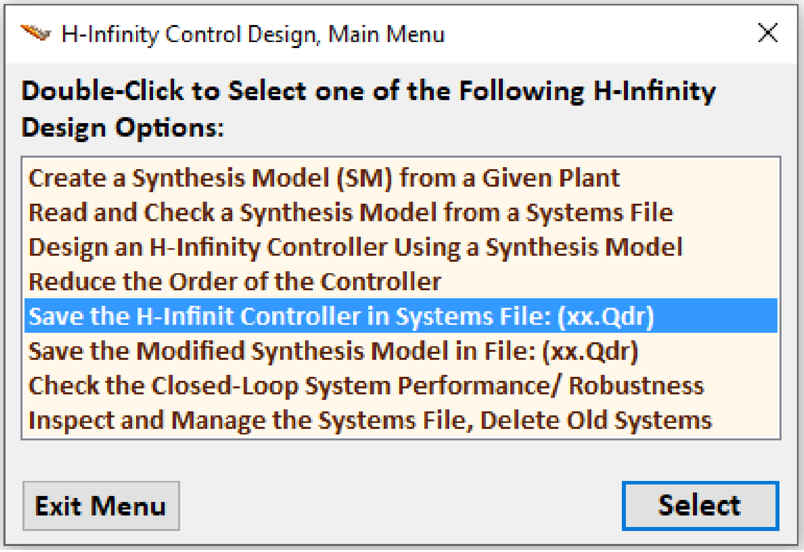


Figure 2.7 Closed-Loop System Poles

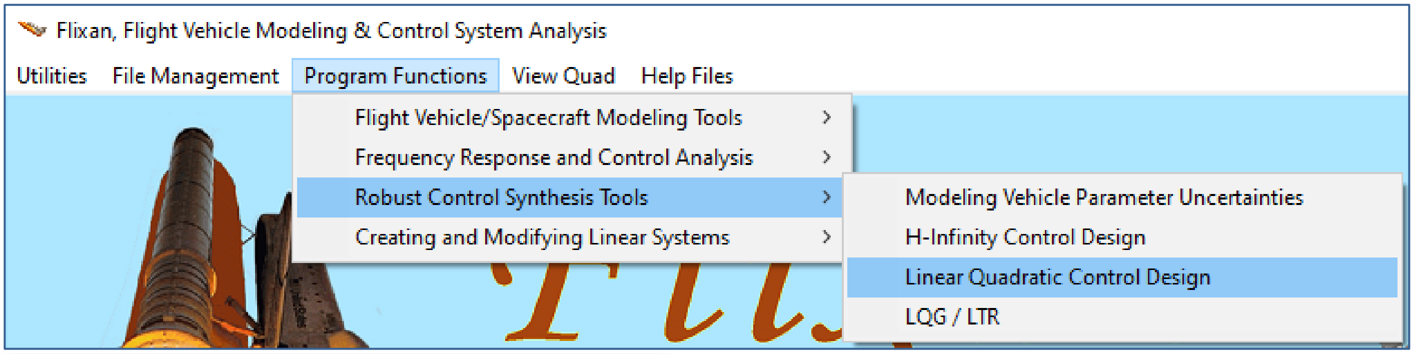
Figure 2.7 shows the closed-loop poles of the system. Notice that there is a complex pair of poles which are near the pendulum mode frequency. We return to the main menu and save the state-feedback gain Kc under the title “*Overhead Crane Hinf State-Feedback*”.

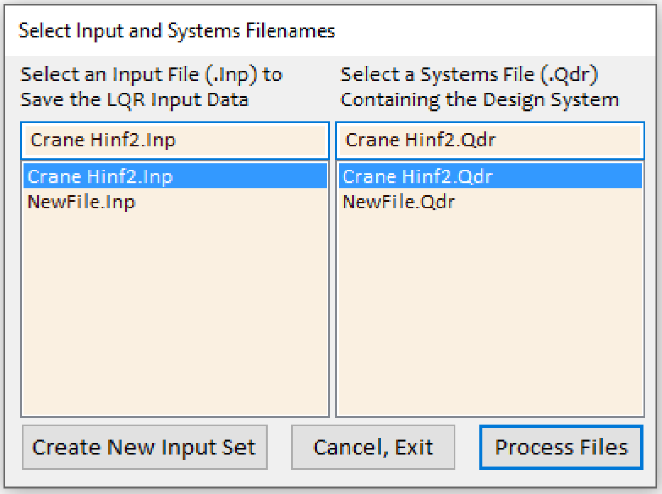


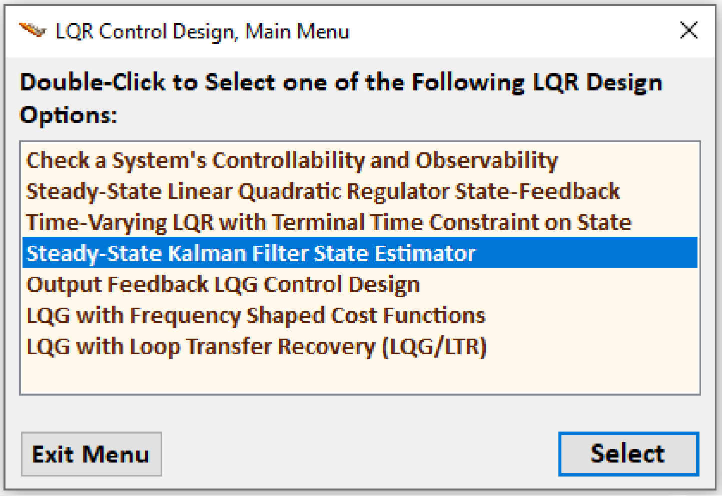
Double-Click to Select one of the Following H-Infinity Design Options:

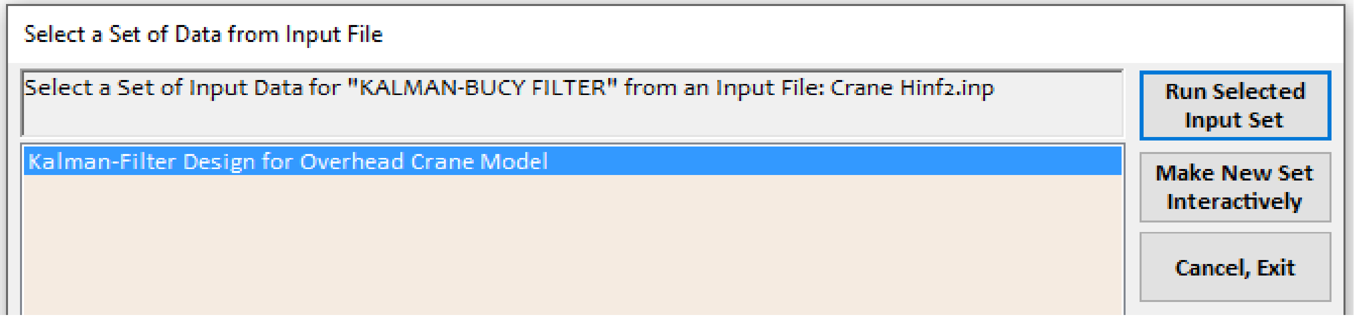
2.7 Kalman-Filter

The second step in the design is to create the Kalman-Filter gain interactively. The KF dataset is already in the systems file but we will recreate it from scratch using the LQR program. Start the LQR program, select the project files, and from the main menu select "*Steady-State Kalman-Filter Estimator*". The next menu shows that there is already a KF dataset in file. If you run it, it will reprocess the dataset that's already in file. Select "Make New Set Interactively" to create a new KF dataset instead.

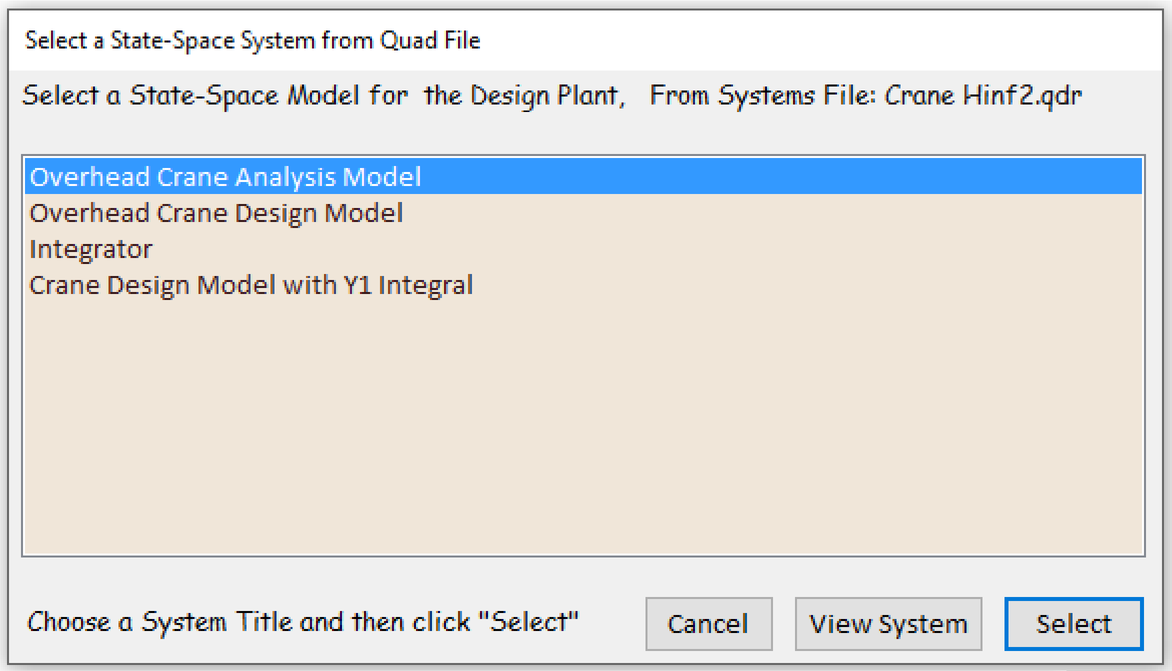








From the menu below select the system from which the KF gain will be designed. Select the original Overhead Crane Design Model, not the one with the y1-integral, and enter a title for the new KF dataset that will contain instructions for batch processing and will be saved in the input file.

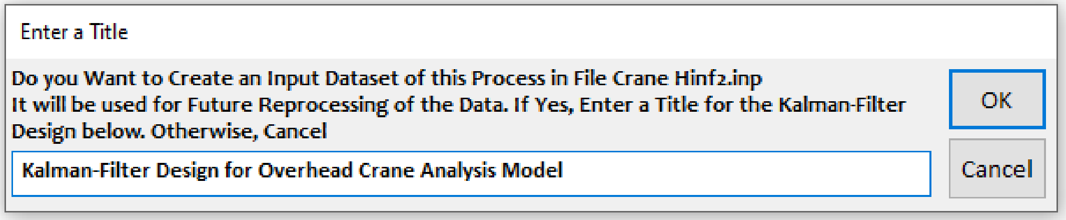


Select a State-Space System from Quad File

Select a State-Space Model for the Design Plant, From Systems File: Crane Hinf2.qdr

- Overhead Crane Analysis Model
- Overhead Crane Design Model
- Integrator
- Crane Design Model with Y1 Integral

Choose a System Title and then click "Select"

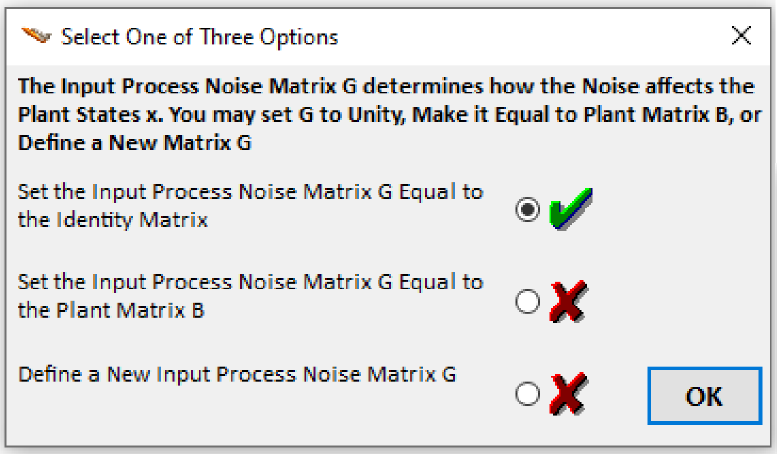


Enter a Title

Do you Want to Create an Input Dataset of this Process in File Crane Hinf2.inp
It will be used for Future Reprocessing of the Data. If Yes, Enter a Title for the Kalman-Filter Design below. Otherwise, Cancel

Kalman-Filter Design for Overhead Crane Analysis Model

We must now select the process noise intensity matrix G. The matrix through which noise enters the system. You can either choose the identity matrix to affect each of the 4 states directly, or the input matrix B, or enter a new noise matrix G. In this case we choose the identity matrix. Great, the system is disturbable from G, as it should be. We must also choose the (4x4) process noise covariance matrix that defines how much noise corrupts each state, and the (2x2) measurement noise covariance matrix that defines the noise at the 2 measurements.



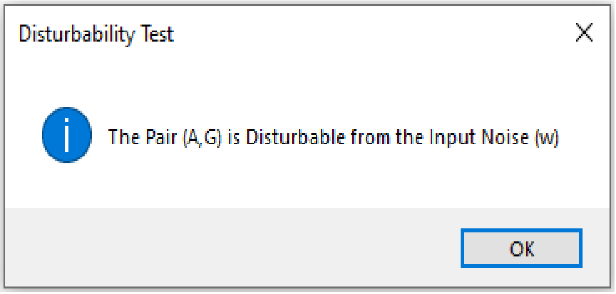
Select One of Three Options

The Input Process Noise Matrix G determines how the Noise affects the Plant States x. You may set G to Unity, Make it Equal to Plant Matrix B, or Define a New Matrix G

Set the Input Process Noise Matrix G Equal to the Identity Matrix ● ✓

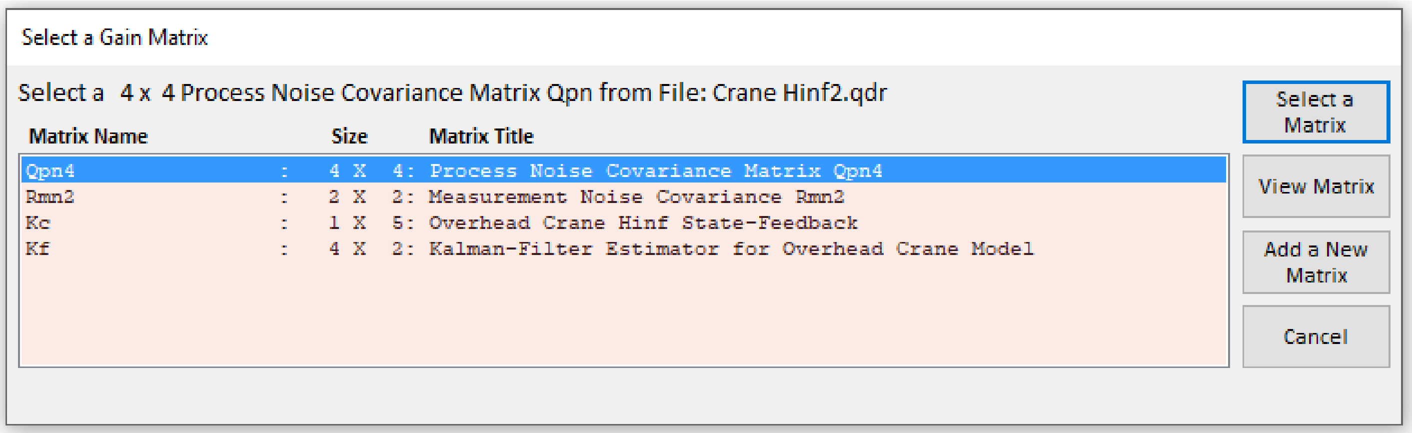
Set the Input Process Noise Matrix G Equal to the Plant Matrix B ○ ✗

Define a New Input Process Noise Matrix G ○ ✗



Disturbability Test

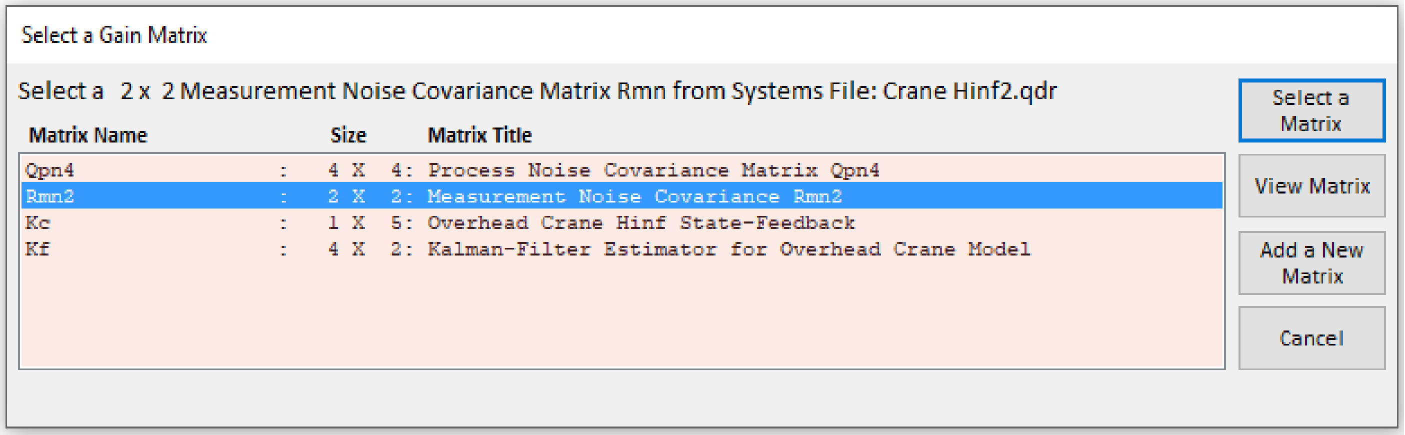
The Pair (A,G) is Disturbable from the Input Noise (w)



Select a 4 x 4 Process Noise Covariance Matrix Qpn from File: Crane Hinf2.qdr

Matrix Name	Size	Matrix Title

```
Qpn4                 :   4 X   4: Process Noise Covariance Matrix Qpn4
Rmn2                 :   2 X   2: Measurement Noise Covariance Rmn2
Kc                   :   1 X   5: Overhead Crane Hinf State-Feedback
Kf                   :   4 X   2: Kalman-Filter Estimator for Overhead Crane Model
```

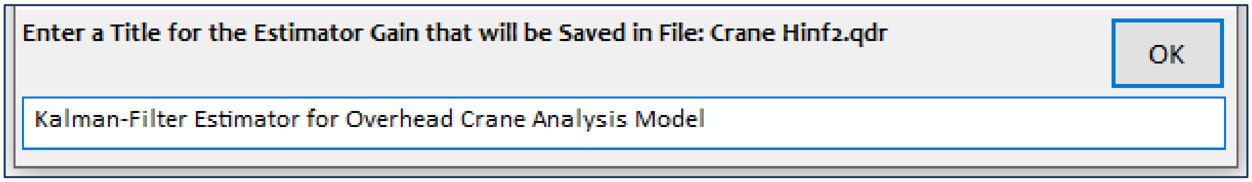


Select a 2 x 2 Measurement Noise Covariance Matrix Rmn from Systems File: Crane Hinf2.qdr

Matrix Name	Size	Matrix Title

```
Qpn4                 :   4 X   4: Process Noise Covariance Matrix Qpn4
Rmn2                 :   2 X   2: Measurement Noise Covariance Rmn2
Kc                   :   1 X   5: Overhead Crane Hinf State-Feedback
Kf                   :   4 X   2: Kalman-Filter Estimator for Overhead Crane Model
```

We must finally enter a title for the Kalman-Filter gain. The title of the KF gain will be saved in the systems file together with the Kf matrix.



Enter a Title for the Estimator Gain that will be Saved in File: Crane Hinf2.qdr

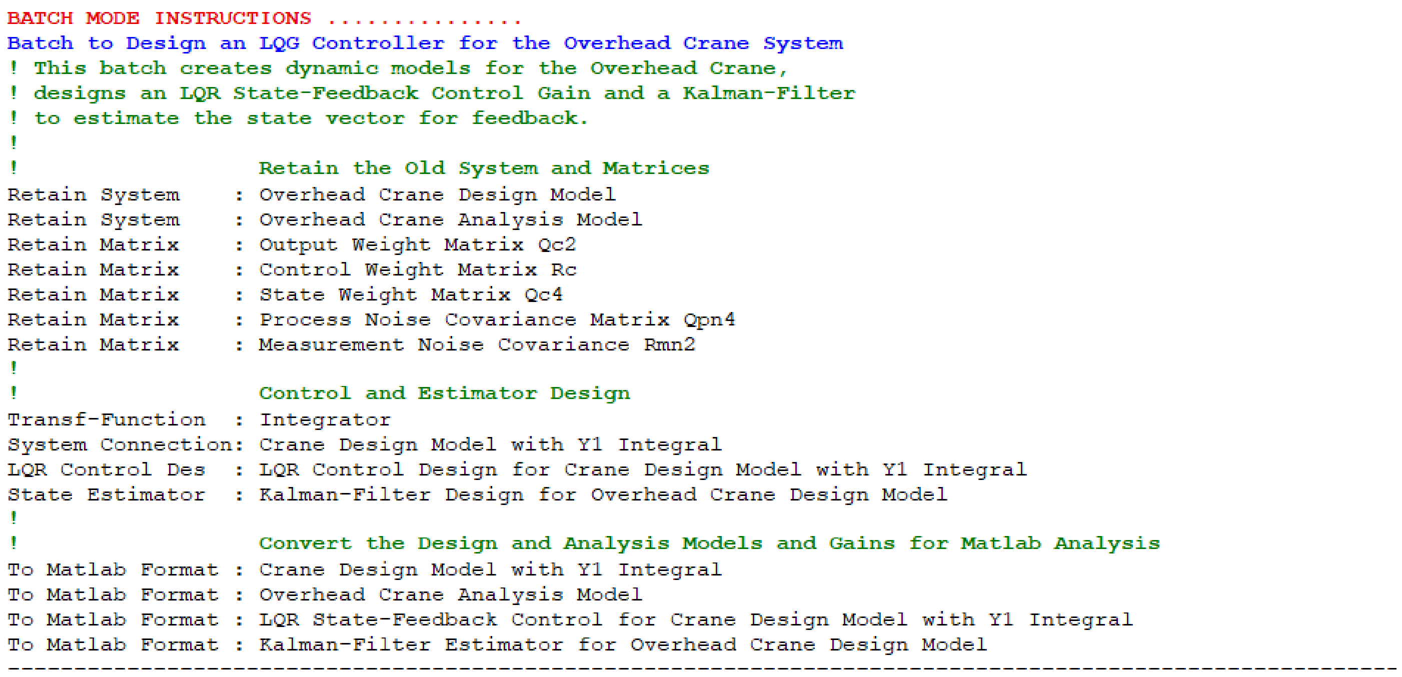
Kalman-Filter Estimator for Overhead Crane Analysis Model

3. LQG Design

The LQG design consists of the 4-state plant model augmented with a 5th state, the y_1-integral, to improve tracking in the presence of disturbance forces. The LQG is a two-step design. We will first design an LQR state-feedback control gain that calculates the control force from a linear combination of the 5 states, assuming they are all measurable. We will test the controller performance using a simple state-feedback simulation model and adjust the Q_c and R_c penalty matrices as needed for good performance. Then we will design the Kalman-Filter observer using the original 4-states plant. The Kalman-Filter estimates the four-state vector: $\underline{x}= [y_1, y_2, \dot{y}_1, \dot{y}_2]$ from the two measurements (y_1, θ). We will use a second simulation to analyze the estimator design where the KF estimated 4 states plus y_1-integral replace the state feedback. The process noise Q_{pn} and the measurement noise R_{mn} covariance matrices will be adjusted as needed for good performance and stability margins.

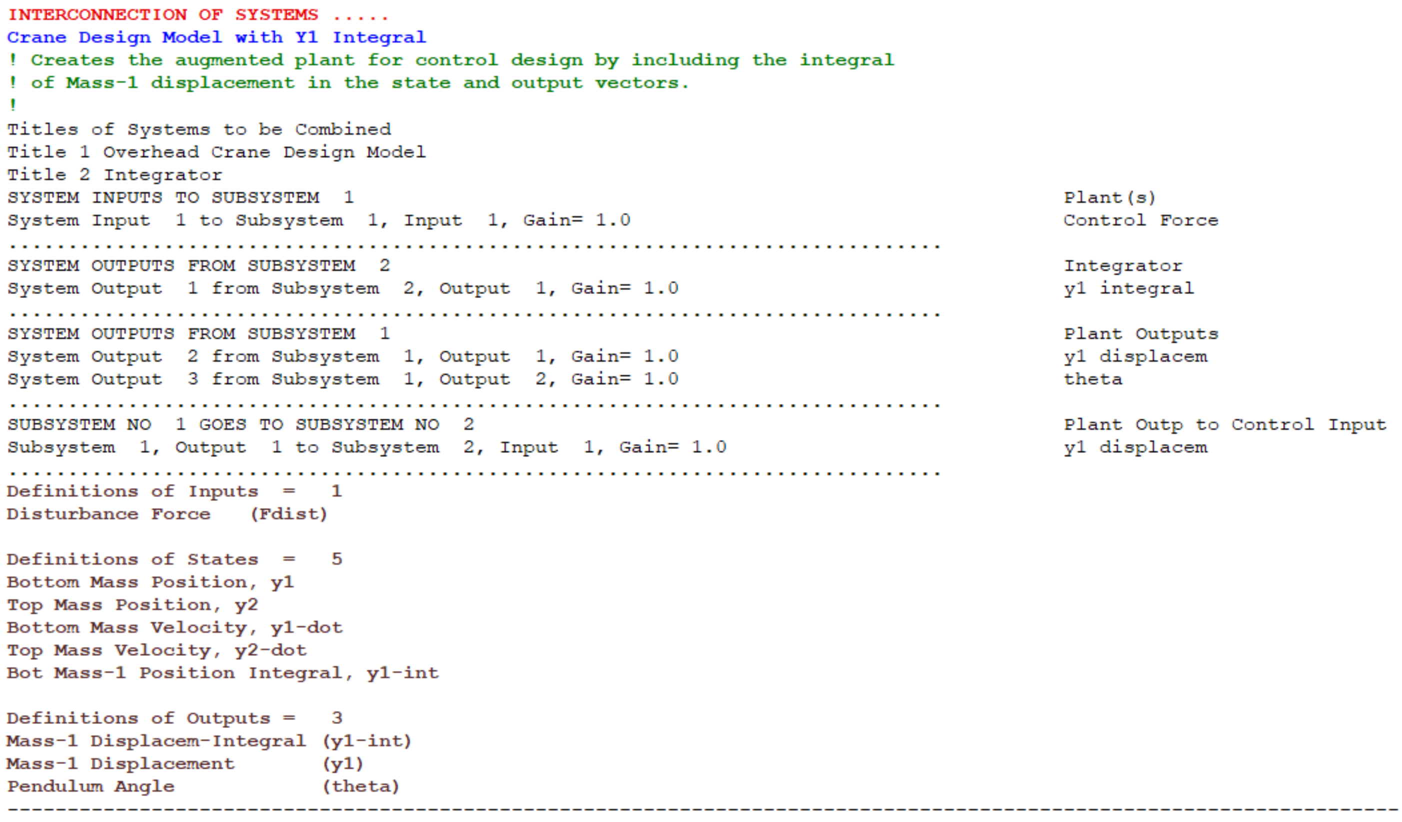
3.1 Flixan Files

The files for the LQG design are in directory: "*\Flixan\Control Analysis\Hinfinity\Examples\Crane Hinf\3-LQG Design*". The input file is "*Crane_LQG.Inp*" and it contains the Flixan datasets that generate the plant models, calculate the LQR controller, and the Kalman-Filter gains. It begins with a batch set that can be used to process the entire file. The batch preserves the systems and LQG weight matrices used in the design. There are two plant models in the systems file which are similar, the "*Overhead Crane Design Model*" and the "*Overhead Crane Analysis Model*".

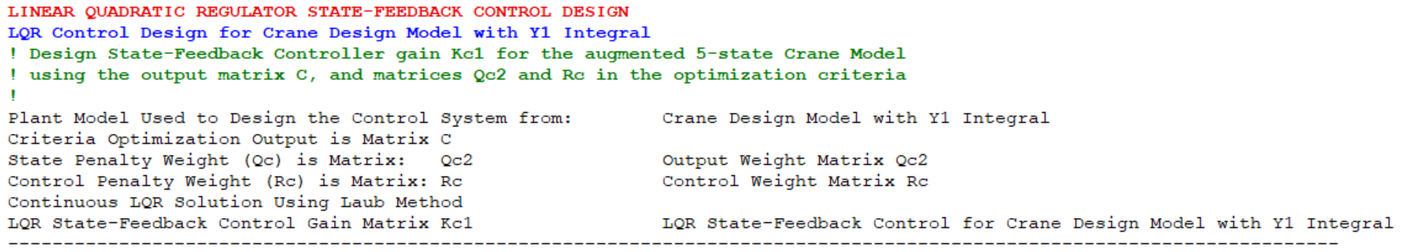


```
BATCH MODE INSTRUCTIONS ...............
Batch to Design an LQG Controller for the Overhead Crane System
! This batch creates dynamic models for the Overhead Crane,
! designs an LQR State-Feedback Control Gain and a Kalman-Filter
! to estimate the state vector for feedback.
!
!                  Retain the Old System and Matrices
Retain System    : Overhead Crane Design Model
Retain System    : Overhead Crane Analysis Model
Retain Matrix    : Output Weight Matrix Qc2
Retain Matrix    : Control Weight Matrix Rc
Retain Matrix    : State Weight Matrix Qc4
Retain Matrix    : Process Noise Covariance Matrix Qpn4
Retain Matrix    : Measurement Noise Covariance Rmn2
!
!                  Control and Estimator Design
Transf-Function  : Integrator
System Connection: Crane Design Model with Y1 Integral
LQR Control Des  : LQR Control Design for Crane Design Model with Y1 Integral
State Estimator  : Kalman-Filter Design for Overhead Crane Design Model
!
!                  Convert the Design and Analysis Models and Gains for Matlab Analysis
To Matlab Format : Crane Design Model with Y1 Integral
To Matlab Format : Overhead Crane Analysis Model
To Matlab Format : LQR State-Feedback Control for Crane Design Model with Y1 Integral
To Matlab Format : Kalman-Filter Estimator for Overhead Crane Design Model
-------------------------------------------------------------------------------------------
```

The design system is augmented by including the y_1-integral state. The following dataset creates the augmented 5-state design model "*Crane Design Model with Y1 Integral*" that is used to design the state-feedback controller gain.



```
INTERCONNECTION OF SYSTEMS .....
Crane Design Model with Y1 Integral
! Creates the augmented plant for control design by including the integral
! of Mass-1 displacement in the state and output vectors.
!
Titles of Systems to be Combined
Title 1 Overhead Crane Design Model
Title 2 Integrator
SYSTEM INPUTS TO SUBSYSTEM  1                                                             Plant(s)
System Input  1 to Subsystem  1, Input  1, Gain= 1.0                                      Control Force
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                                                          Integrator
System Output  1 from Subsystem  2, Output  1, Gain= 1.0                                  y1 integral
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                                          Plant Outputs
System Output  2 from Subsystem  1, Output  1, Gain= 1.0                                  y1 displacem
System Output  3 from Subsystem  1, Output  2, Gain= 1.0                                  theta
..............................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2                                                   Plant Outp to Control Input
Subsystem  1, Output  1 to Subsystem  2, Input  1, Gain= 1.0                              y1 displacem
..............................................................................
Definitions of Inputs  =   1
Disturbance Force   (Fdist)

Definitions of States  =   5
Bottom Mass Position, y1
Top Mass Position, y2
Bottom Mass Velocity, y1-dot
Top Mass Velocity, y2-dot
Bot Mass-1 Position Integral, y1-int

Definitions of Outputs =   3
Mass-1 Displacem-Integral (y1-int)
Mass-1 Displacement       (y1)
Pendulum Angle            (theta)
-------------------------------------------------------------------------------------------------------------------
```



```
LINEAR QUADRATIC REGULATOR STATE-FEEDBACK CONTROL DESIGN
LQR Control Design for Crane Design Model with Y1 Integral
! Design State-Feedback Controller gain Kc1 for the augmented 5-state Crane Model
! using the output matrix C, and matrices Qc2 and Rc in the optimization criteria
!
Plant Model Used to Design the Control System from:       Crane Design Model with Y1 Integral
Criteria Optimization Output is Matrix C
State Penalty Weight (Qc) is Matrix:   Qc2                Output Weight Matrix Qc2
Control Penalty Weight (Rc) is Matrix: Rc                 Control Weight Matrix Rc
Continuous LQR Solution Using Laub Method
LQR State-Feedback Control Gain Matrix Kc1                LQR State-Feedback Control for Crane Design Model with Y1 Integral
-------------------------------------------------------------------------------------------------------------------
```

The LQR Control Design dataset is using the system "*Crane Design Model with Y1 Integral*" to design the 5-state feedback gain Kc1. The output matrix C defines the 3 output variables (y_1-integral, y_1, and θ) to be optimized by LQR via the weight matrix Qc2 which penalizes the 3 variables. The control force is penalized via the scalar Rc. A satisfactory trade-off between speed of convergence and control force usage is achieved by adjusting the two matrices. The state-feedback controller gain Kc1 is saved in the systems file "*Crane_LQG.Qdr*" and its title is "*LQR State-Feedback Control for Crane Design Model with Y1 Integral*".

Similarly, the Kalman-Filter estimator dataset below calculates the Kalman-Filter gain Kf1. It uses the original 4-state plant model: "*Overhead Crane Design Model*" which does not include the y_1-integral. It also reads the noise covariance matrices Qpn4 and Rmn2 which are located in the systems file "*Crane-LQG.Qdr*". Kf1 is used in the observer simulation to estimate the 4 states from the outputs y_1 and θ. The two plant models and the gain matrices Kc1 and Kf1 are exported into Matlab by the conversion datasets which are included at the bottom of the Flixan input file. They are converted to m-functions and "mat" matrices that can be loaded into Matlab by running the script file "init.m".

```
KALMAN-BUCY FILTER STATE ESTIMATOR DESIGN
Kalman-Filter Design for Overhead Crane Design Model
! State Observer for the Original 4-state Crane Model, Estimating
! Positions and Velocities of the two masses from the 2 plant outputs using
! Process Noise and Measurement Noise Covariance Matrices Qpn4 & Rmn2
!
Plant Model Used to Design the Kalman-Filter from:          Overhead Crane Design Model
Input Process Noise Matrix (G) is the  Identity
Process Noise Covariance Qpn is Matrix Qpn4                 Process Noise Covariance Matrix Qpn4
Measurement Noise Covariance is Matrix Rmn2                 Measurement Noise Covariance Rmn2
Kalman-Filter Estimator is Gain Matrix Kf1                  Kalman-Filter Estimator for Overhead Crane Design Model
------------------------------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........       (Title, System/Matrix, m-filename)
Overhead Crane Analysis Model
System
Analysis_Plant
------------------------------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........       (Title, System/Matrix, m-filename)
Crane Design Model with Y1 Integral
System
Design_Plant_Int
------------------------------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........       (Title, System/Matrix, m-filename)
LQR State-Feedback Control for Crane Design Model with Y1 Integral
Matrix Kc1
------------------------------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........       (Title, System/Matrix, m-filename)
Kalman-Filter Estimator for Overhead Crane Design Model
Matrix Kf1
------------------------------------------------------------------------------------------------------------------------
```

```
% Initialization Script
r2d=180/pi;
[Ao,Bo,Co,Do]= design_plant_int;                % Load Design Model
[Aa,Ba,Ca,Da]= analysis_plant;                  % Load Analysis Model
load Kc1 -ascii                                 % State-Feedback Gain
load Kf1 -ascii                                 % Kalman-Filter Gain
Af=Aa-Kf1*Ca([1,3],:);                          % Kalman-Filter Af Matrix
```

3.2 Simulation Models

Figure 3.2 shows a simulation model "*Crane_Sim-1.mdl*" that is used to test the state-feedback gain Kc1 directly from the four states: $\underline{x}$= [y_1, y_2, $\dot{y}_1, \dot{y}_2$] which of course they are not measurable. It is intended to check the control design and to adjust the weight matrices Qc2 and Rc as needed for good performance. Figure 3.1 shows the system's response to y_1 displacement command. It is logically what a person would do naturally using common sense. First, move the top mass as fast as possible half way towards the intended position and then pause for a short period while waiting for the bottom mass to swing over towards the target. The motion of the bottom mass-1 is somewhat delayed until the pendulum angle θ is sufficiently big to exert a side force. When the bottom mass swings over to the extreme opposite side of the pendulum angle at -θ, which is close to the intended position, the top mass-2 is moved as fast as possible above the target position to prevent it from oscillating further. This is essentially what the LQR control system does in Figure 3.1 but it also takes into consideration the limited bandwidth of the control system. This action requires knowledge of the pendulum frequency which is captured in the design plant and subsequently in the control design in order to actively dampen the pendulum oscillations. Notice the "hick-up" in the y_2 response as it waits for the bottom mass-1 to swing over on the opposite side of the pendulum.

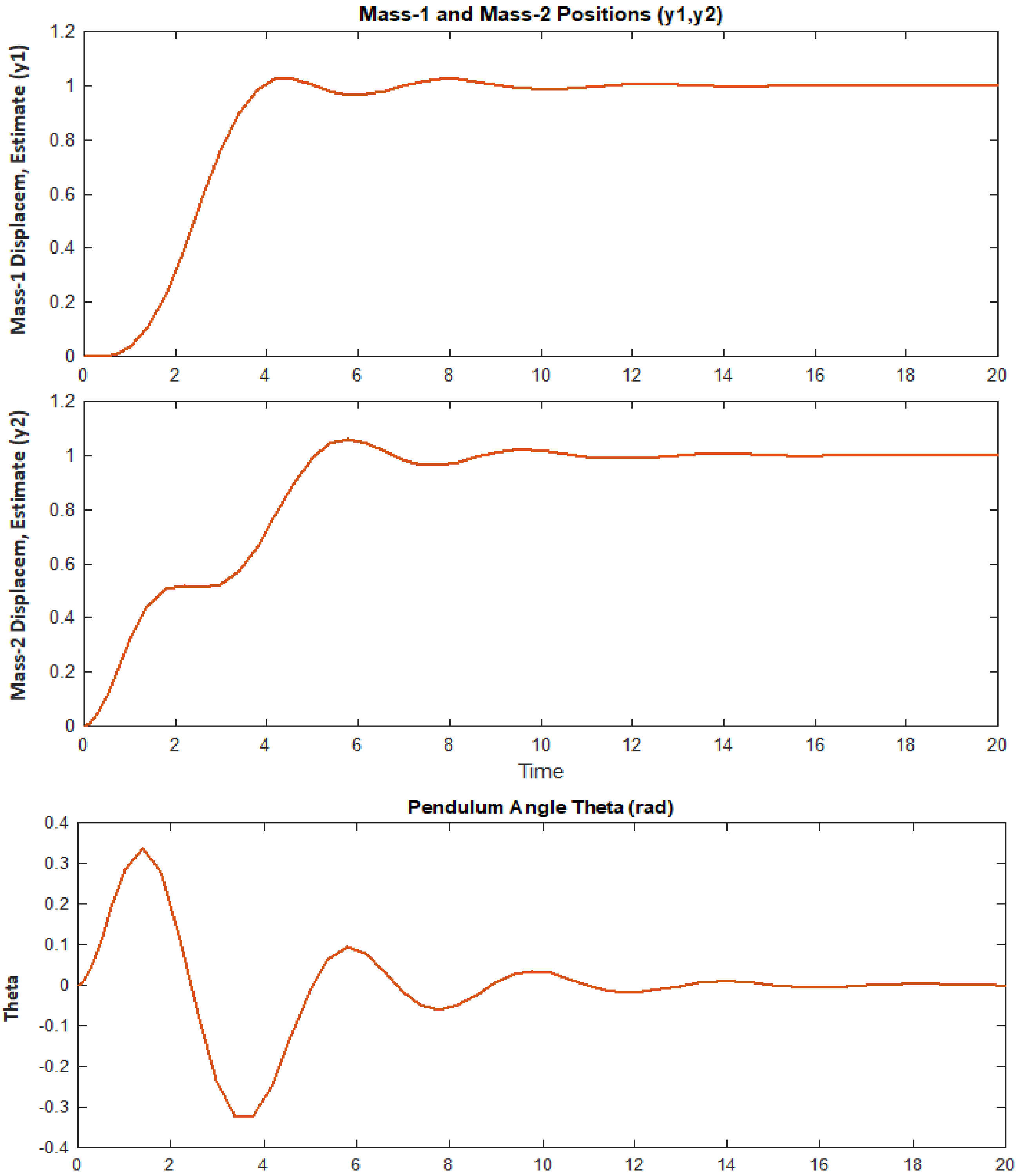


Figure 3.1 System's Response to a Displacement Command y1-command

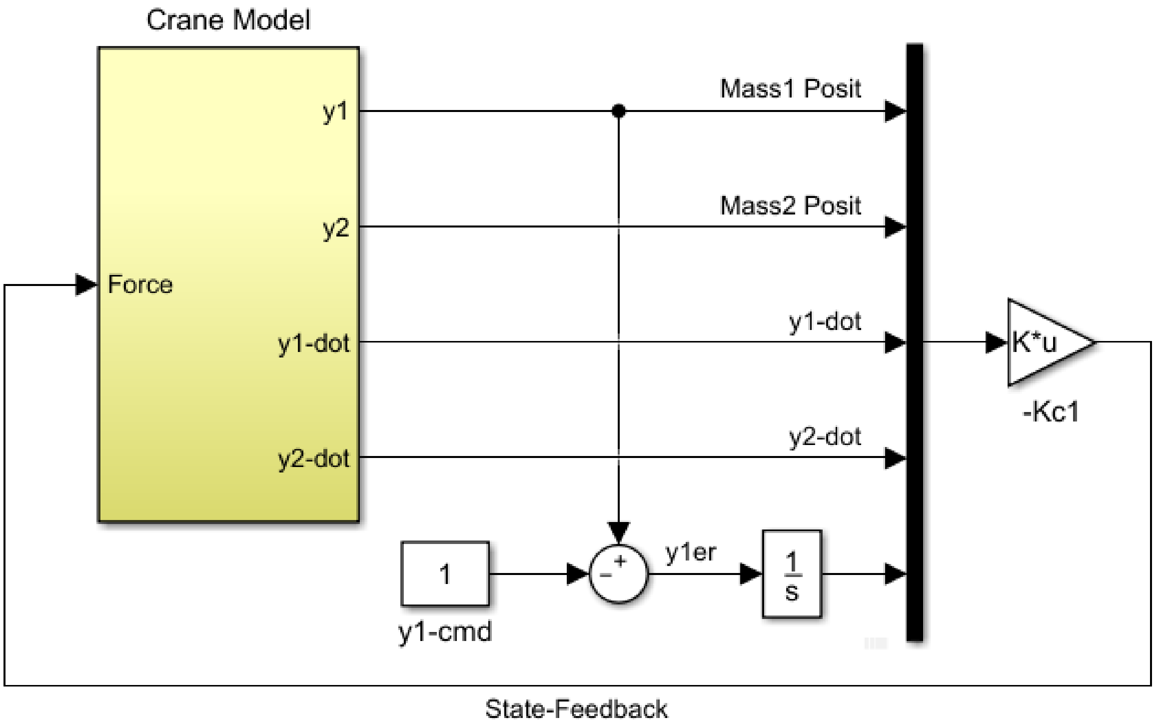


Figure 3.2 State-Feedback Simulation model "*Crane_Sim-1.mdl*"

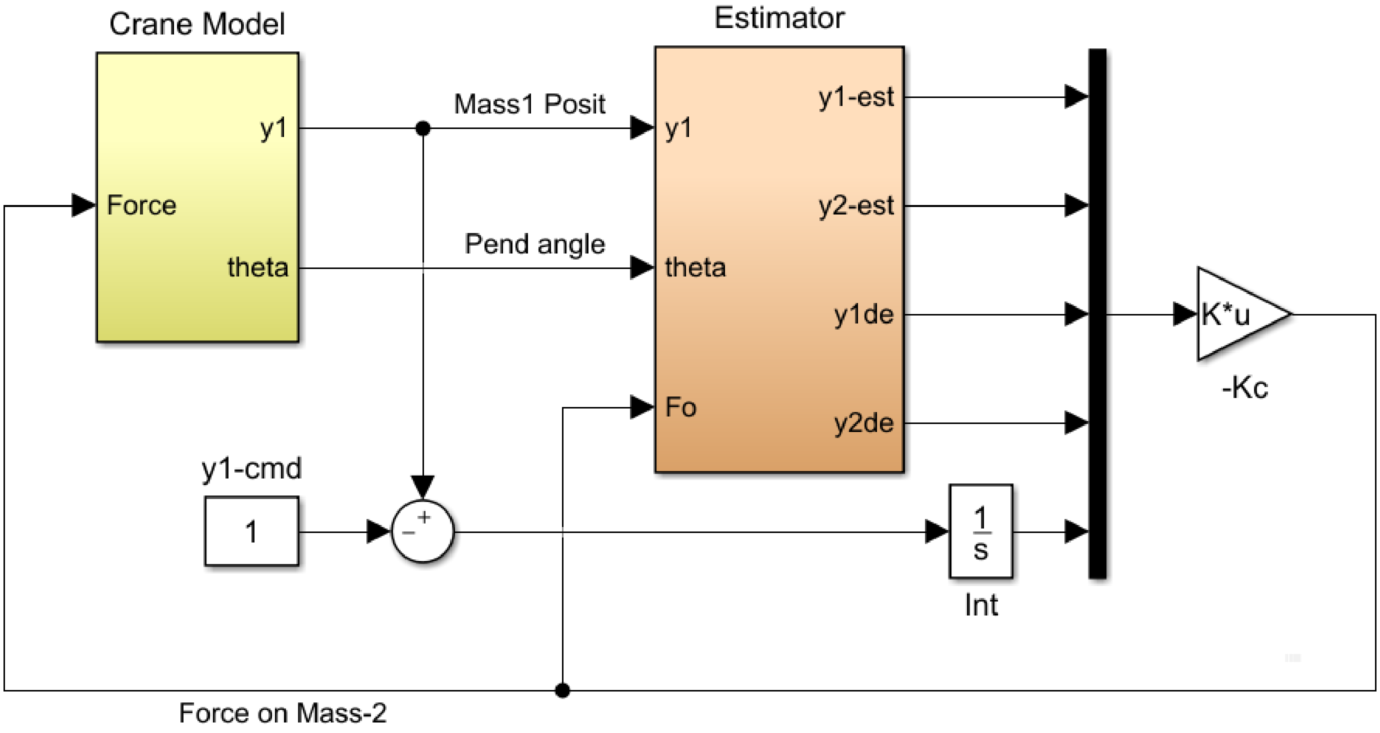


Figure 3.3 Closed-Loop Simulation Model "*Crane_Sim-2.mdl*" with State-Estimator

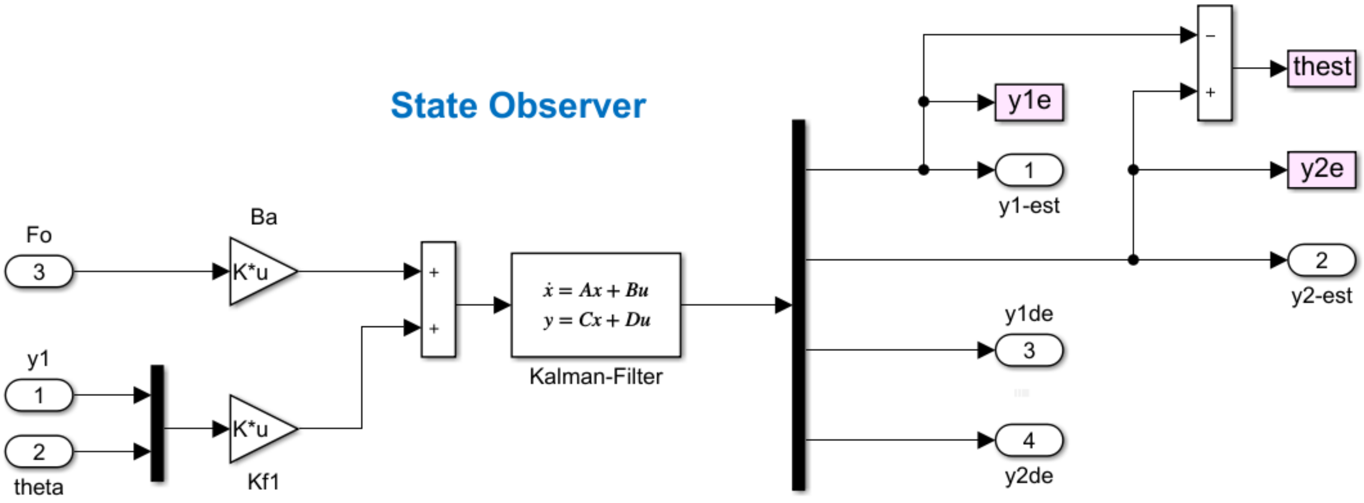


Figure 3.4 Kalman-Filter State Estimator

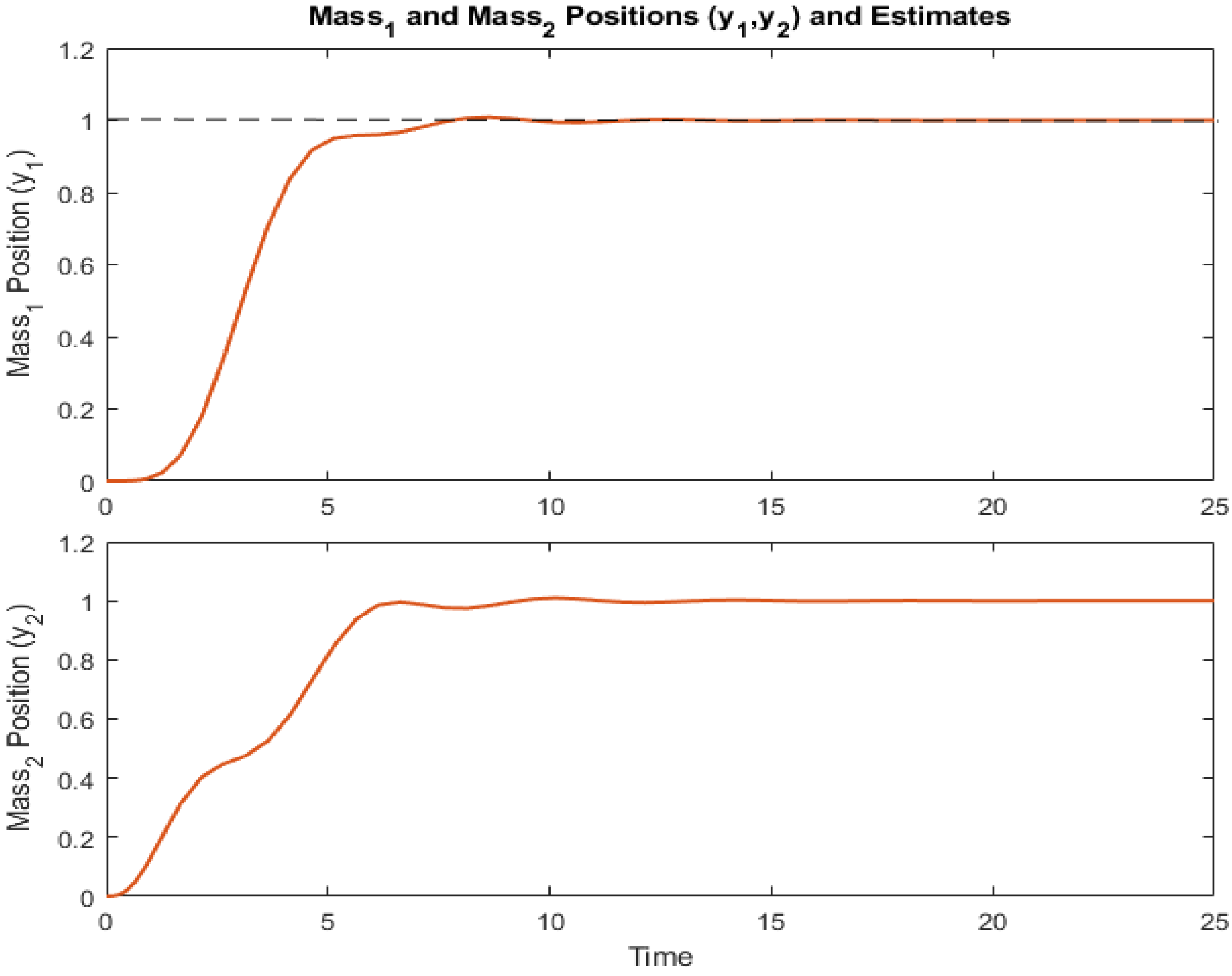


Figure 3.5a Position Response of Simulation Model "Crane_Sim-2" to y_1 Displacement Command

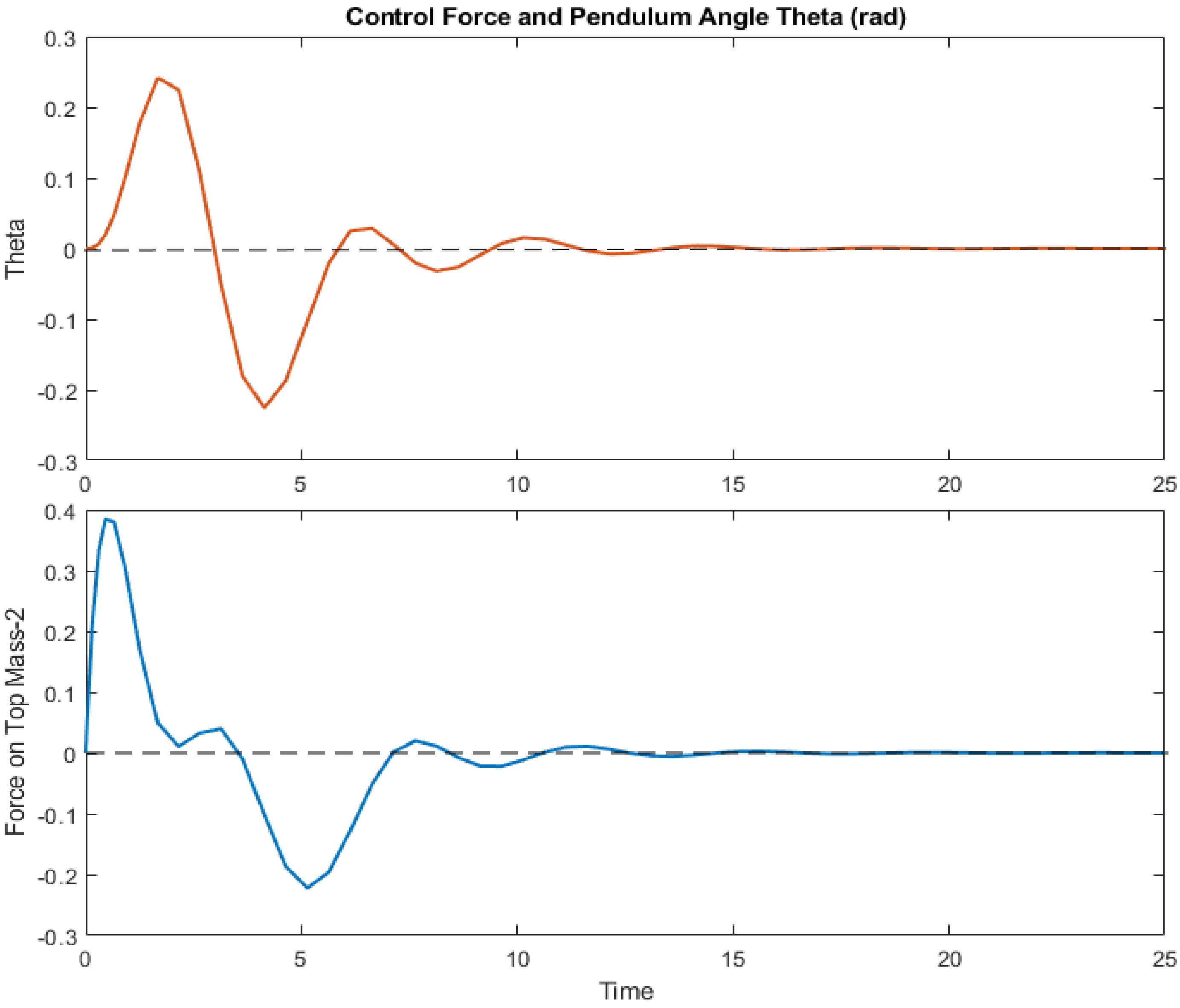


Figure 3.5b Theta and Control Force Response of "Crane_Sim-2l" Model to y1 Command

The simulation model "Crane_Sim-2.mdl" in Figure 3.3 shows the control system including the KF estimator which estimates the 4 states required for feedback via the control gain Kc1. The inputs to the estimator are the two measurements: y_1, and θ, and also the control force. The y_1-itegral is not included in the KF because it is directly measured. Figure 3.5 shows the response of the output-feedback system which is similar to the state-feedback. The hick-up on the y_2 displacement at the half-way point is not as intense and the oscillation is damping faster.

3.3 Frequency Response Analysis

Frequency response analysis is used to analyze the control system's stability by measuring the gain and phase margins. The Simulink model "*Open_Loop.Mdl*" in Figure 3.6, has the loop opened at the control force input and it is used to calculate the frequency response. Figure 3.7 shows the Bode and Nichols plots and the stability margins. Notice that the system has big resonance at 1.42 (rad/sec) which is at the pendulum frequency.

The control system counteracts the natural pendulum frequency by introducing an anti-resonance because it is designed using the plant model. Figure 3.7 shows the phase and gain margins before and after the resonance and they have acceptable stability.

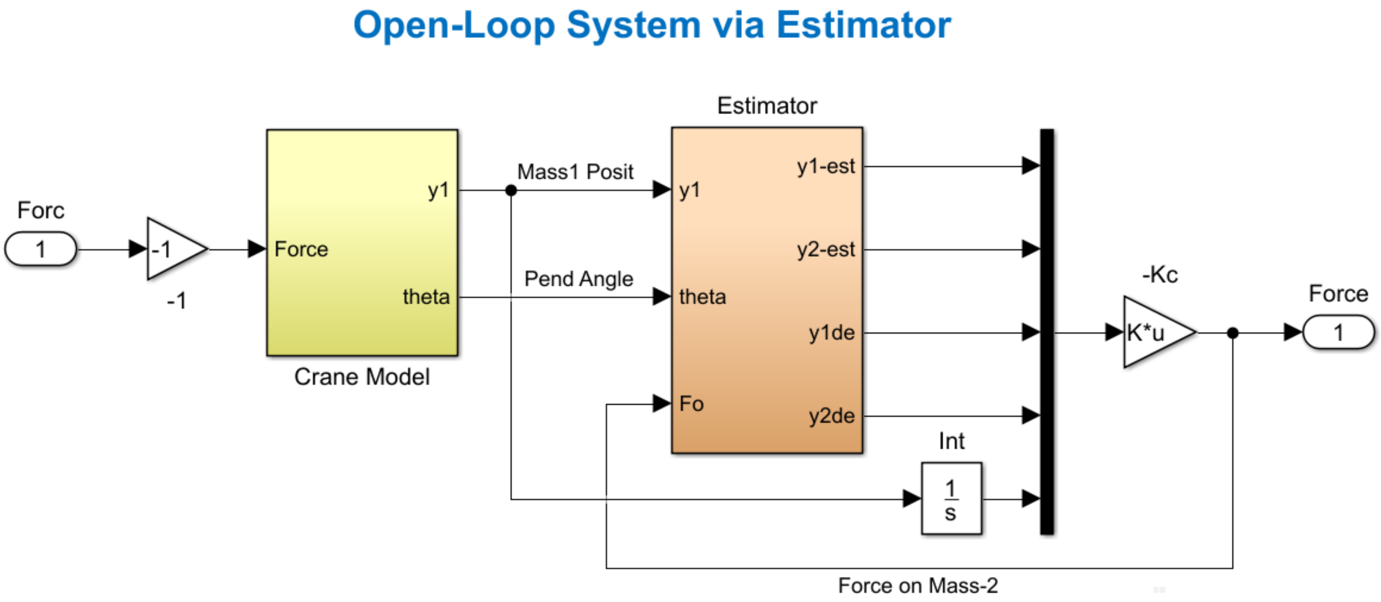


Figure 3.6 Stability Analysis Model "Open_Loop"

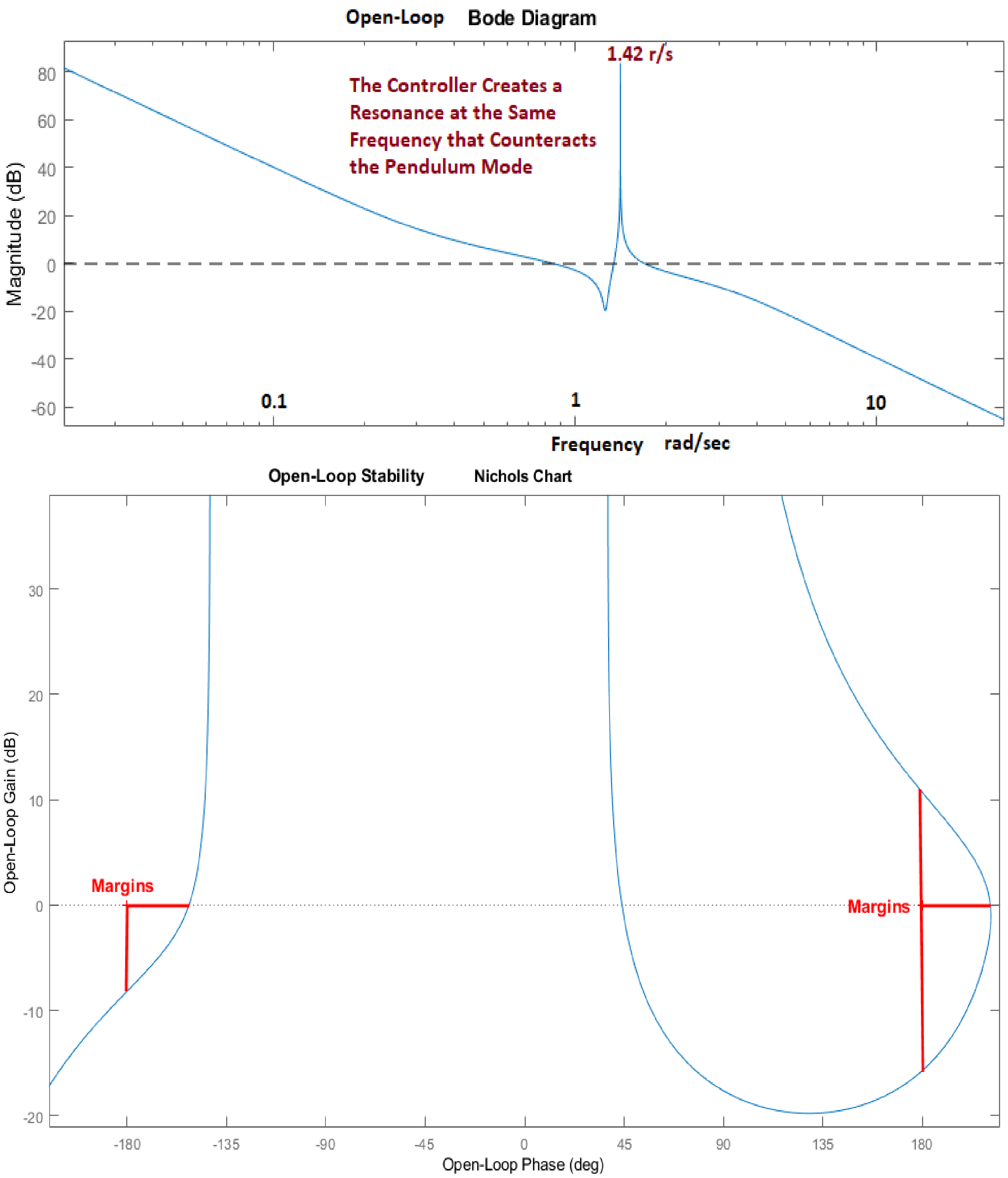
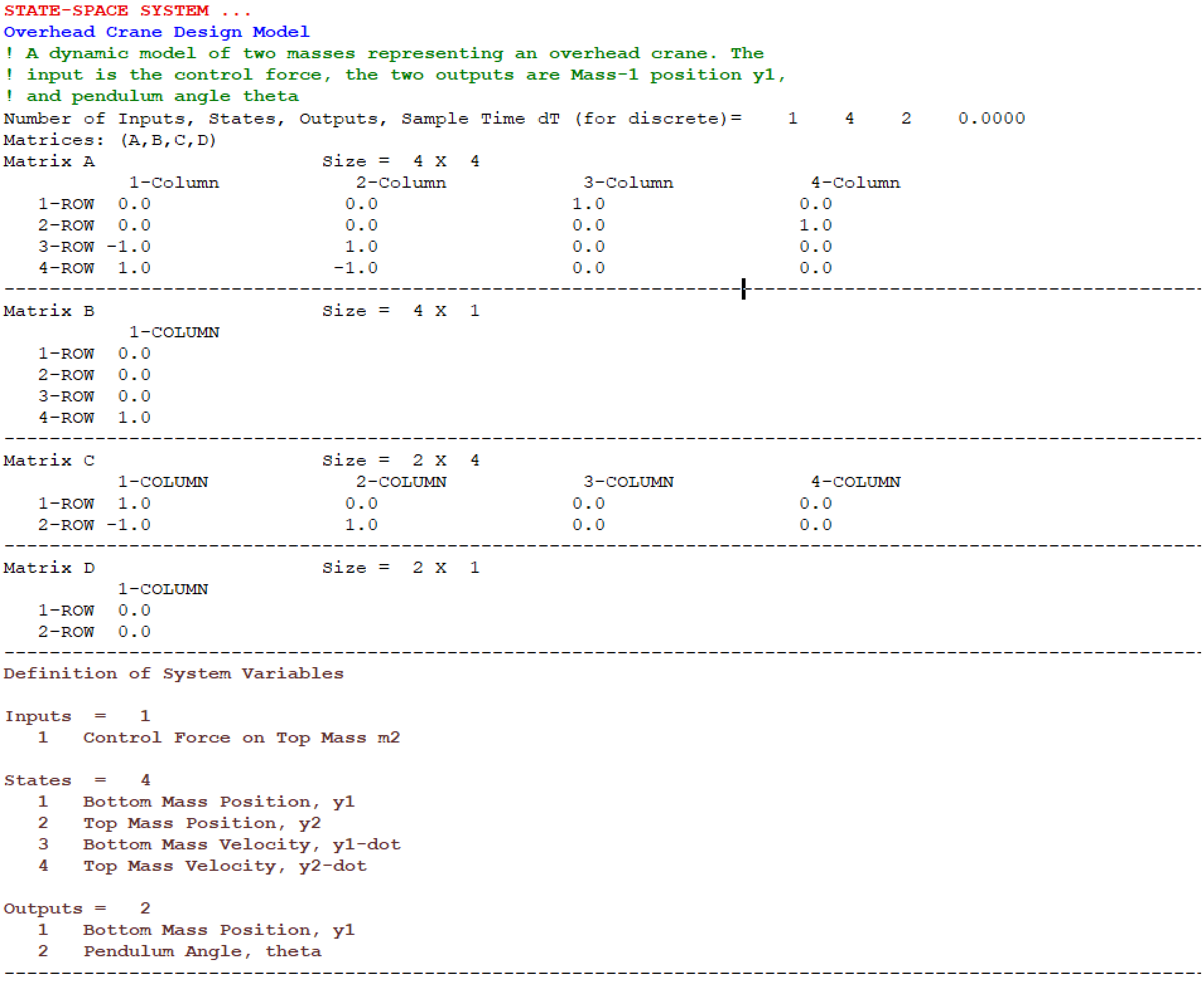
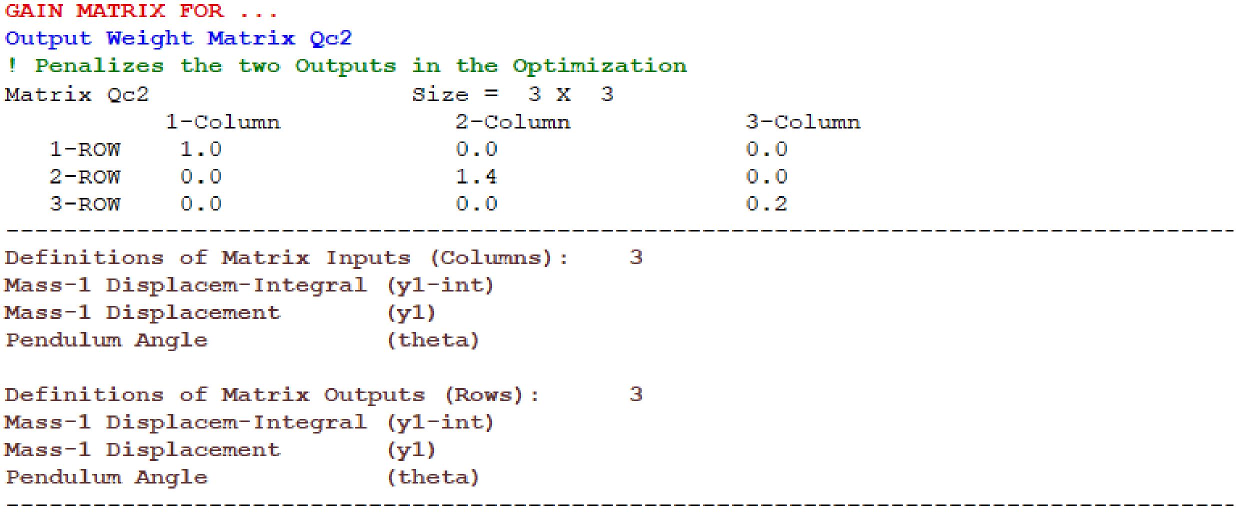


Figure 3.7 Open-Loop Frequency Response Analysis, Bode and Nichols Plots

3.4 Systems File “Crane_LQG.Qdr”



```
STATE-SPACE SYSTEM ...
Overhead Crane Design Model
! A dynamic model of two masses representing an overhead crane. The
! input is the control force, the two outputs are Mass-1 position y1,
! and pendulum angle theta
Number of Inputs, States, Outputs, Sample Time dT (for discrete)=    1    4    2    0.0000
Matrices: (A,B,C,D)
Matrix A                    Size =  4 X  4
           1-Column                2-Column                3-Column                4-Column
   1-ROW   0.0                     0.0                     1.0                     0.0
   2-ROW   0.0                     0.0                     0.0                     1.0
   3-ROW  -1.0                     1.0                     0.0                     0.0
   4-ROW   1.0                    -1.0                     0.0                     0.0
------------------------------------------------------------------------------------------------------------
Matrix B                    Size =  4 X  1
           1-COLUMN
   1-ROW   0.0
   2-ROW   0.0
   3-ROW   0.0
   4-ROW   1.0
------------------------------------------------------------------------------------------------------------
Matrix C                    Size =  2 X  4
          1-COLUMN                 2-COLUMN                3-COLUMN                4-COLUMN
   1-ROW   1.0                     0.0                     0.0                     0.0
   2-ROW  -1.0                     1.0                     0.0                     0.0
------------------------------------------------------------------------------------------------------------
Matrix D                    Size =  2 X  1
          1-COLUMN
   1-ROW   0.0
   2-ROW   0.0
------------------------------------------------------------------------------------------------------------
Definition of System Variables

Inputs  =   1
   1    Control Force on Top Mass m2

States  =   4
   1    Bottom Mass Position, y1
   2    Top Mass Position, y2
   3    Bottom Mass Velocity, y1-dot
   4    Top Mass Velocity, y2-dot

Outputs =   2
   1    Bottom Mass Position, y1
   2    Pendulum Angle, theta
------------------------------------------------------------------------------------------------------------
```



```
GAIN MATRIX FOR ...
Output Weight Matrix Qc2
! Penalizes the two Outputs in the Optimization
Matrix Qc2                    Size =  3 X  3
           1-Column                2-Column                3-Column
   1-ROW    1.0                    0.0                     0.0
   2-ROW    0.0                    1.4                     0.0
   3-ROW    0.0                    0.0                     0.2
-------------------------------------------------------------------------------------------
Definitions of Matrix Inputs (Columns):     3
Mass-1 Displacem-Integral  (y1-int)
Mass-1 Displacement        (y1)
Pendulum Angle             (theta)

Definitions of Matrix Outputs (Rows):       3
Mass-1 Displacem-Integral  (y1-int)
Mass-1 Displacement        (y1)
Pendulum Angle             (theta)
-------------------------------------------------------------------------------------------
```

```
GAIN MATRIX FOR ...
State Weight Matrix Qc4
! Penalizes the four States individually in the Optimization
Matrix Qc4                  Size =  4 X  4
           1-Column            2-Column            3-Column            4-Column
   1-ROW   .10000000E+01       .00000000E+00       .00000000E+00       .00000000E+00
   2-ROW   .00000000E+00       .10000000E+01       .00000000E+00       .00000000E+00
   3-ROW   .00000000E+00       .00000000E+00       .10000000E+01       .00000000E+00
   4-ROW   .00000000E+00       .00000000E+00       .00000000E+00       .10000000E+01
-----------------------------------------------------------------------------------------------------
Definitions of Matrix Inputs (Columns):     4
Bottom Mass Position (y1)
Top Mass Position    (y2)
Bottom Mass Velocity (y1-dot)
Top Mass Velocity    (y2-dot)

Definitions of Matrix Outputs (Rows):      4
Bottom Mass Position
Top Mass Position
Bottom Mass Velocity
Top Mass Velocity
-----------------------------------------------------------------------------------------------------
GAIN MATRIX FOR ...
Control Weight Matrix Rc
! Penalizes the Control Force (F)
Matrix Rc                   Size =  1 X  1
           1-COLUMN
   1-ROW   0.5
-----------------------------------------------------------------------------------------------------
GAIN MATRIX FOR ...
Process Noise Covariance Matrix Qpn4
! Noise Intensity at the 4 states
Matrix Qpn4                 Size =  4 X  4
           1-Column            2-Column            3-Column            4-Column
   1-Row  0.1                 0.0                 0.0                 0.0
   2-Row  0.0                 0.1                 0.0                 0.0
   3-Row  0.0                 0.0                 0.1                 0.0
   4-Row  0.0                 0.0                 0.0                 0.1
-----------------------------------------------------------------------------------------------------
Gain Matrix for ...
LQR State-Feedback Control for Crane Design Model with Y1 Integral
! State-Feedback Controller gain Kc1 for the augmented 5-state Crane Model
! using the output matrix C, and matrices Qc2 and Rc in the optimization criteria
Matrix Kc1                  Size =  1 X  5
           1-Column            2-Column            3-Column            4-Column            5-Column
   1-Row  0.121524129997E+01  0.338666257656E+01  0.389487298736E+01  0.260256126789E+01  0.14142135623 7E+01
-----------------------------------------------------------------------------------------------------
Definitions of Matrix Inputs (Columns):     5
Bottom Mass Position, y1
Top Mass Position, y2
Bottom Mass Velocity, y1-dot
Top Mass Velocity, y2-dot
Bot Mass-1 Position Integral, y1-int

Definitions of Matrix Outputs (Rows):      1
Control Force   (Fc)
-----------------------------------------------------------------------------------------------------
Gain Matrix for ...
Kalman-Filter Estimator for Overhead Crane Design Model
! State Observer for the Original 4-state Crane Model, Estimating
! Positions and Velocities of the two masses from the 2 plant outputs using
! Process Noise and Measurement Noise Covariance Matrices Qpn4 & Rmn2
Matrix Kf1                  Size =  4 X  2
           1-Column            2-Column
   1-Row  0.205874704209E+01 -0.662514394792E+00
   2-Row  0.139623264730E+01  0.157715990309E+01
   3-Row  0.133868235331E+01 -0.990584865904E-01
   4-Row  0.589974827283E+00  0.628474655367E+00
-----------------------------------------------------------------------------------------
Definitions of Matrix Inputs (Columns):     2
Bottom Mass Position, y1 error
Pendulum Angle, theta error

Definitions of Matrix Outputs (Rows):      4
Bottom Mass Position, y1 estim
Top Mass Position, y2 estim
Bottom Mass Velocity, y1-dot estim
Top Mass Velocity, y2-dot estim
-----------------------------------------------------------------------------------------
```

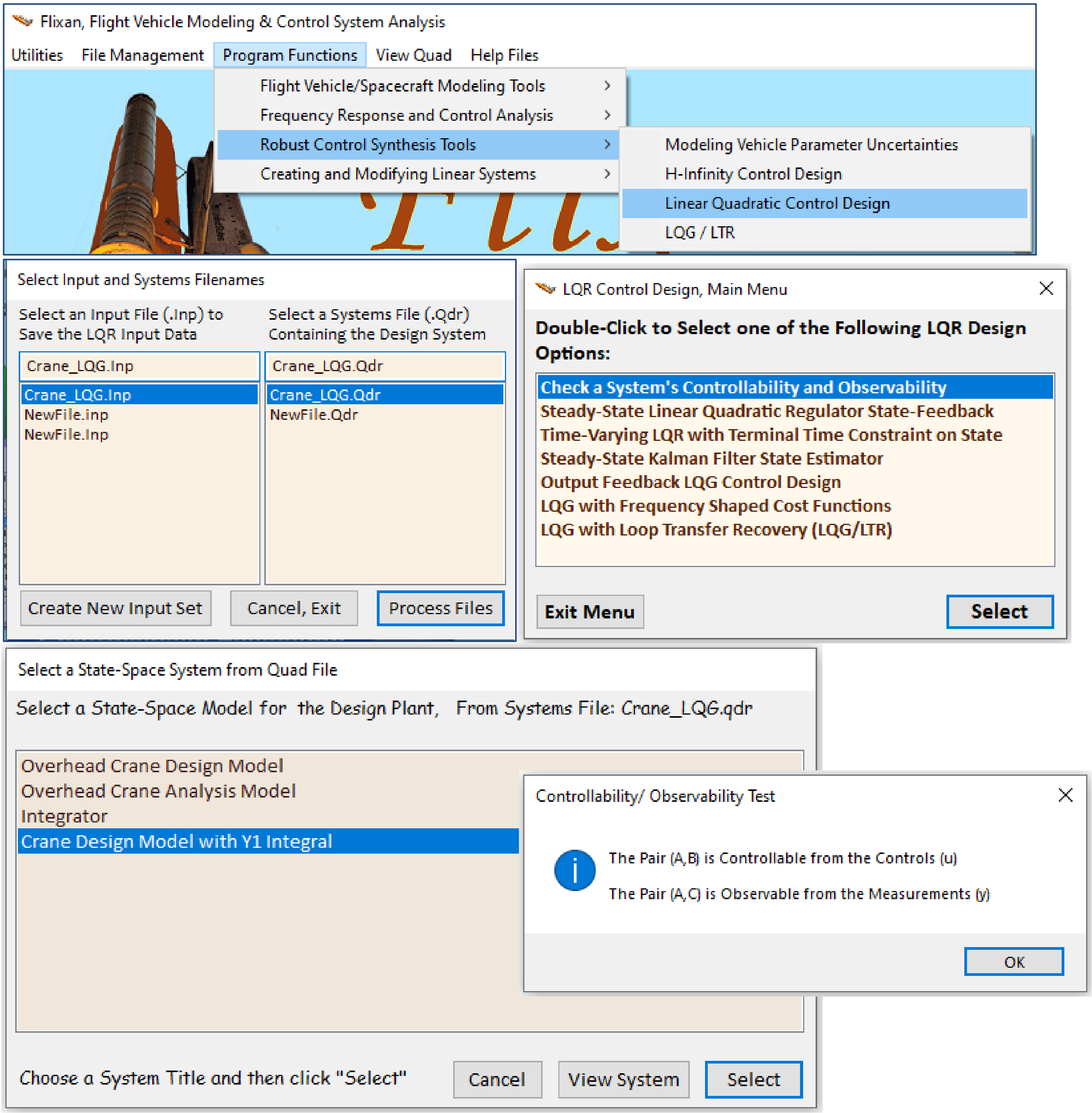
```
STATE-SPACE SYSTEM ...
Overhead Crane Analysis Model
! A dynamic model of two masses representing an overhead crane.
! The input is the control force, and the three outputs are Mass-1
! position y1, Mass-2 position y2, and pendulum angle theta.
!
Number of Inputs, States, Outputs, Sample Time dT (for discrete)=    1    4    3    0.0
Matrices: (A,B,C,D)
Matrix A                    Size =  4 X  4
           1-Column            2-Column            3-Column            4-Column
   1-ROW  0.0                 0.0                 1.0                 0.0
   2-ROW  0.0                 0.0                 0.0                 1.0
   3-ROW -1.0                 1.0                 0.0                 0.0
   4-ROW  1.0                -1.0                 0.0                 0.0
------------------------------------------------------------------------------------------------
Matrix B                    Size =  4 X  1
           1-COLUMN
   1-ROW  0.0
   2-ROW  0.0
   3-ROW  0.0
   4-ROW  1.0
------------------------------------------------------------------------------------------------
Matrix C                    Size =  3 X  4
          1-COLUMN             2-COLUMN            3-COLUMN            4-COLUMN
   1-row  1.0                 0.0                 0.0                 0.0
   2-row  0.0                 1.0                 0.0                 0.0
   3-row -1.0                 1.0                 0.0                 0.0
------------------------------------------------------------------------------------------------
Matrix D                    Size =  3 x  1
          1-COLUMN
   1-row  0.0
   2-row  0.0
   3-row  0.0
------------------------------------------------------------------------------------------------
Definition of System Variables

Inputs  =   1
   1   Force F on the Top Mass m2

States  =   4
   1   Bottom Mass Position, y1
   2   Top Mass Position, y2
   3   Bottom Mass Velocity, y1-dot
   4   Top Mass Velocity, y2-dot

Outputs =   3
   1   Bottom Mass Position, (y1)
   2   Top     Mass Position, (y2)
   3   Pendulum Angle,       theta
--------------------------------------------------------------------------------------
```

```
STATE-SPACE SYSTEM ...
Crane Design Model with Y1 Integral
! Creates the augmented plant for control design by including the integral of Mass-1 displacement
! in the state and output vectors.
Number of Inputs, States, Outputs, Sample Time dT (for discrete)=    1    5    3    0.0000
Matrices: (A,B,C,D)
Matrix A                    Size =  5 X  5
           1-Column             2-Column             3-Column             4-Column             5-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   3-Row -0.100000000000E+01  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.100000000000E+01 -0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Matrix B                    Size =  5 X  1
           1-Column
   1-Row  0.000000000000E+00
   2-Row  0.000000000000E+00
   3-Row  0.000000000000E+00
   4-Row  0.100000000000E+01
   5-Row  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Matrix C                    Size =  3 X  5
           1-Column             2-Column             3-Column             4-Column             5-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
   2-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row -0.100000000000E+01  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Matrix D                    Size =  3 X  1
           1-Column
   1-Row  0.000000000000E+00
   2-Row  0.000000000000E+00
   3-Row  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Definition of System Variables

Inputs  =   1
   1   Disturbance Force   (Fdist)

States  =   5
   1   Bottom Mass Position, y1
   2   Top Mass Position, y2
   3   Bottom Mass Velocity, y1-dot
   4   Top Mass Velocity, y2-dot
   5   Bot Mass-1 Position Integral, y1-int

Outputs =   3
   1   Mass-1 Displacem-Integral  (y1-int)
   2   Mass-1 Displacement        (y1)
   3   Pendulum Angle             (theta)
-----------------------------------------------------------------------------------------
```

3.5 Running the LQR Program Interactively

The input and systems files in this example are already prepared and they can be processed in batch mode but in this section, we will run the LQG process interactively. The program will read the augmented 5-states plant model G(s), the output weighting matrix Q_c that penalizes the 3 criteria variables which are defined by the output matric C, and the control weighting matrix R_c that penalizes the control force. We will solve the LQR problem interactively, calculate the 5-state feedback gain matrix Kc1, and save it in the systems file. We will also calculate the KF estimator that will estimate the 4 states from the two measurements using the original 4-state plant and not the augmented 5-states plant. We will use the process noise intensity matrix Q_{pn}, and the measurement noise covariance matrix R_{mn}, solve the KF observer problem, calculate the Kalman-Filter gain matric K_{f1} and save it in the same systems file.

The Linear Quadratic Control design program is selected from the Flixan main menu by going to “*Program Functions*”, “*Robust Control Synthesis Tools*”, and then “*Linear Quadratic Control Design*”, as shown below. Select the input and the systems filenames, and click on “*Process Files*”. Select the first option from the LQR menu to check the system’s Observability and Controllability and make sure that it can be used.

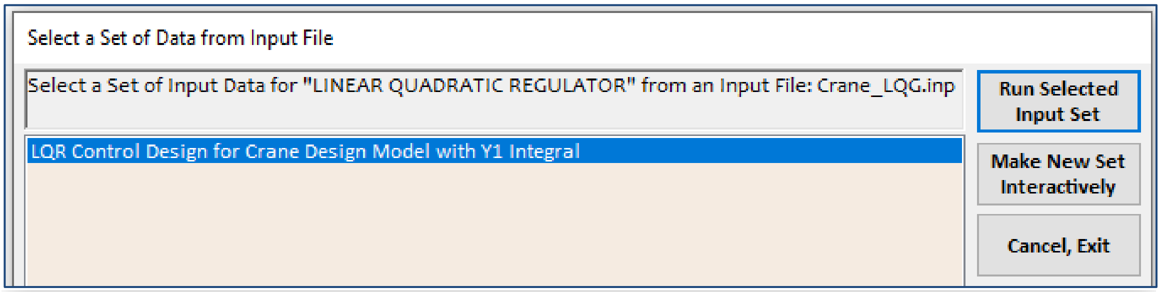


The Pair (A,B) is Controllable from the Controls (u)

The Pair (A,C) is Observable from the Measurements (y)

Then chose the second option from the LQR menu “*Steady-State LQR State-Feedback Design*” and click on “*Select*”.

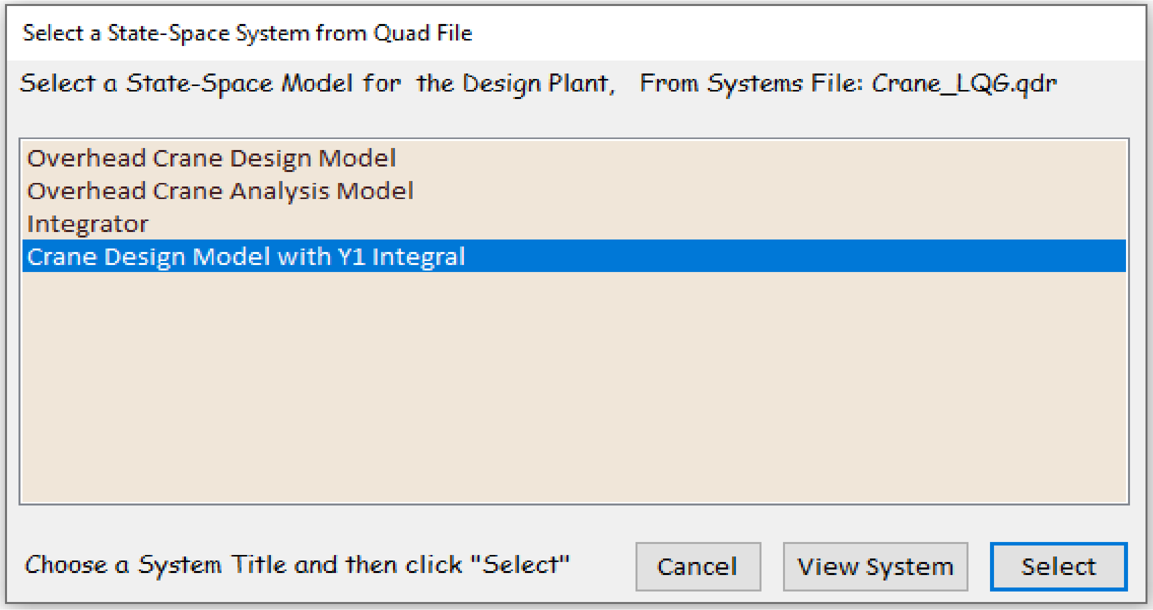
The next menu lists the titles of LQR datasets that already exist in the input file. They contain LQR instructions for batch processing. There is 1 LQR dataset but we won't use it this time because we will create a new one interactively. So, click on "*Make a New Set Interactively*". The program now wants to read the plant model G(s). Select the 5-state system that includes the y1-integrator and click "Select".



Select a Set of Data from Input File

Select a Set of Input Data for "LINEAR QUADRATIC REGULATOR" from an Input File: Crane_LQG.inp

LQR Control Design for Crane Design Model with Y1 Integral



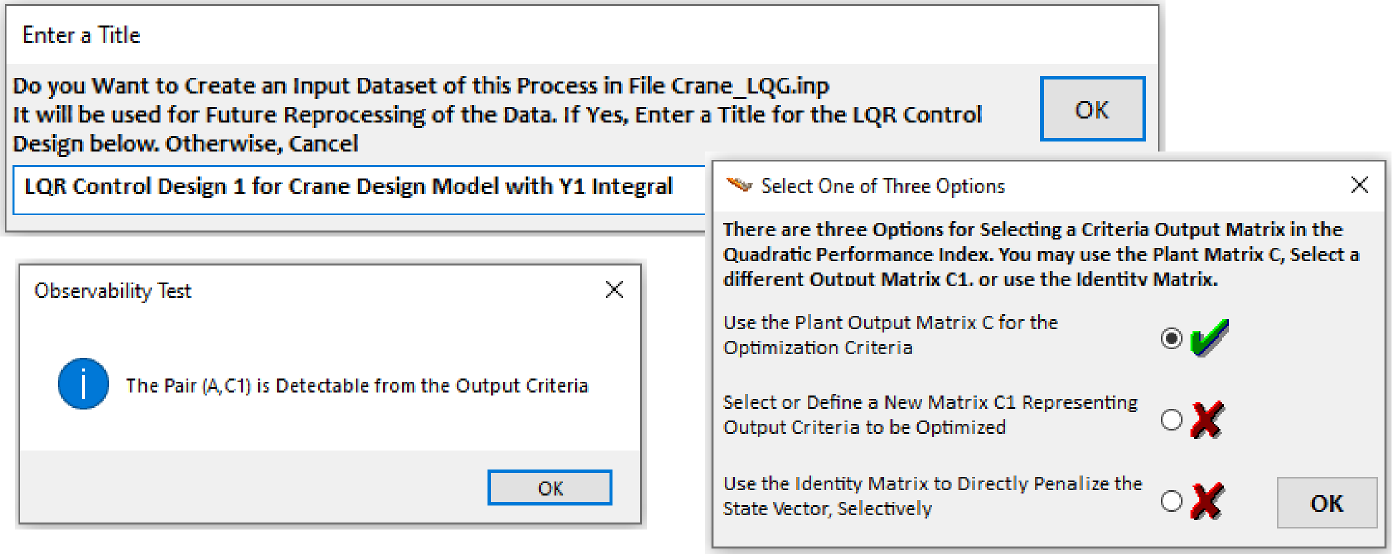
Select a State-Space System from Quad File

Select a State-Space Model for the Design Plant, From Systems File: Crane_LQG.qdr

Overhead Crane Design Model
Overhead Crane Analysis Model
Integrator
Crane Design Model with Y1 Integral

Choose a System Title and then click "Select"

The new LQR design dataset, like all datasets, requires a title. Enter the new LQR title in the following dialog and click "OK". It can be used to reprocess this operation in the future when you run the program in batch mode. The next step is to define the output criteria to be optimized. They are defined by the output matrix. You can either use the existing output matrix C, or the identity matrix, or to define a new set of output criteria by picking a different matrix C_1. In this case we will chose the C matrix which outputs: (y_1, y_1-integr, and θ) and the program checks the observability okay from C.



Enter a Title

**Do you Want to Create an Input Dataset of this Process in File Crane_LQG.inp
It will be used for Future Reprocessing of the Data. If Yes, Enter a Title for the LQR Control Design below. Otherwise, Cancel**

LQR Control Design 1 for Crane Design Model with Y1 Integral

Select One of Three Options

There are three Options for Selecting a Criteria Output Matrix in the Quadratic Performance Index. You may use the Plant Matrix C, Select a different Output Matrix C1, or use the Identity Matrix.

● Use the Plant Output Matrix C for the Optimization Criteria ✓

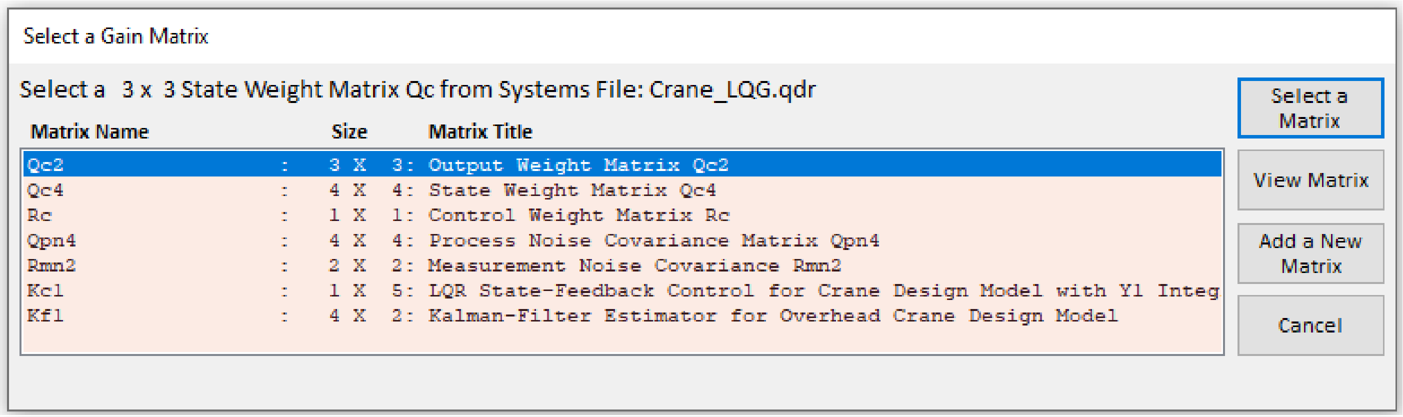
○ Select or Define a New Matrix C1 Representing Output Criteria to be Optimized ✗

○ Use the Identity Matrix to Directly Penalize the State Vector, Selectively ✗

Observability Test

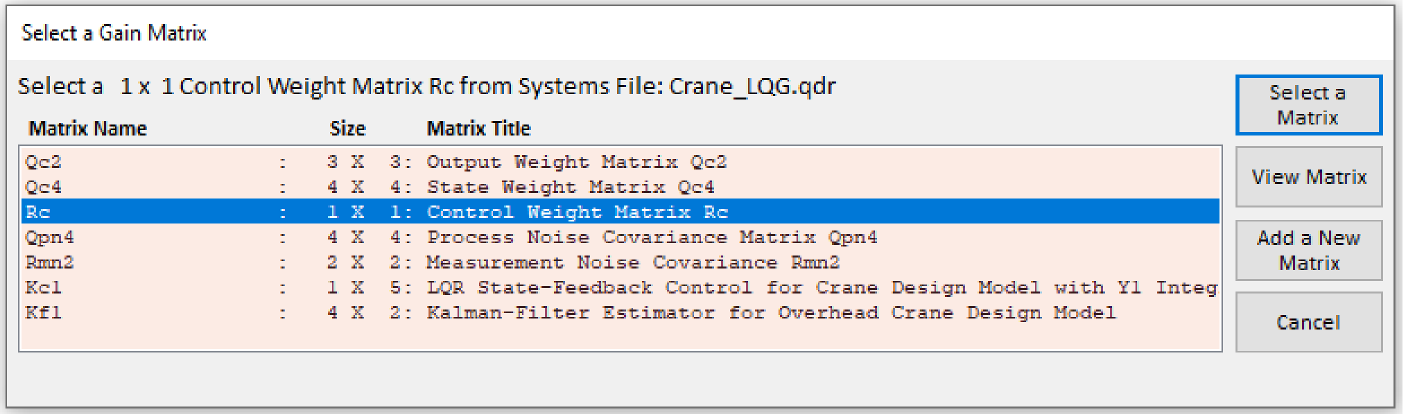
The Pair (A,C1) is Detectable from the Output Criteria

We must now select the two weight matrices Qc and Rc which are already saved in the systems file. The (3x3) matrix Qc that penalizes the 3 criteria outputs which are defined by the output matrix C, and the scalar Rc that penalizes the control input force on m_2.



Select a 3 x 3 State Weight Matrix Qc from Systems File: Crane_LQG.qdr

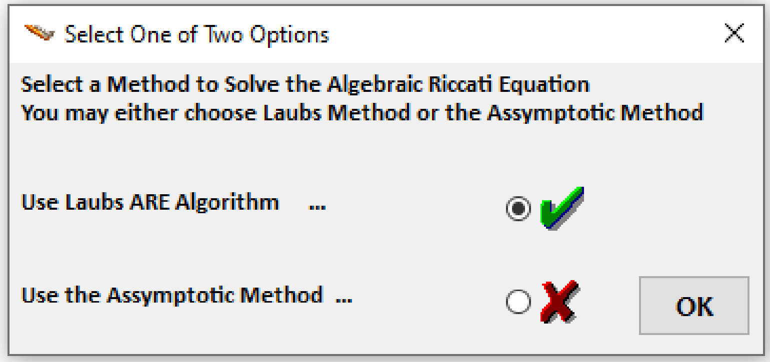
```
Matrix Name              Size     Matrix Title
Qc2                  :   3 X  3: Output Weight Matrix Qc2
Qc4                  :   4 X  4: State Weight Matrix Qc4
Rc                   :   1 X  1: Control Weight Matrix Rc
Qpn4                 :   4 X  4: Process Noise Covariance Matrix Qpn4
Rmn2                 :   2 X  2: Measurement Noise Covariance Rmn2
Kc1                  :   1 X  5: LQR State-Feedback Control for Crane Design Model with Y1 Integ
Kf1                  :   4 X  2: Kalman-Filter Estimator for Overhead Crane Design Model
```



Select a 1 x 1 Control Weight Matrix Rc from Systems File: Crane_LQG.qdr

```
Matrix Name              Size     Matrix Title
Qc2                  :   3 X  3: Output Weight Matrix Qc2
Qc4                  :   4 X  4: State Weight Matrix Qc4
Rc                   :   1 X  1: Control Weight Matrix Rc
Qpn4                 :   4 X  4: Process Noise Covariance Matrix Qpn4
Rmn2                 :   2 X  2: Measurement Noise Covariance Rmn2
Kc1                  :   1 X  5: LQR State-Feedback Control for Crane Design Model with Y1 Integ
Kf1                  :   4 X  2: Kalman-Filter Estimator for Overhead Crane Design Model
```

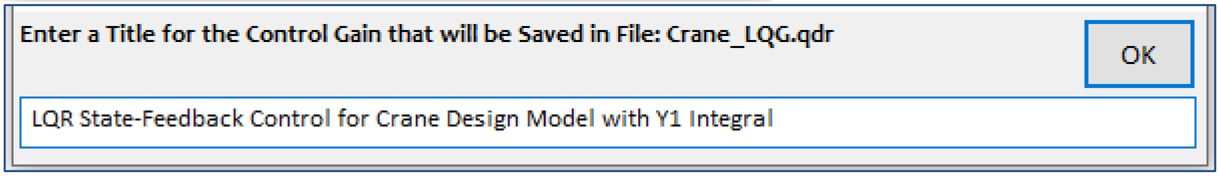
We must finally select the algorithm that will be used to solve the asymptotic Riccati equation. The program has 2 options. Laub's algorithm is chosen in this case. We must also define a title for the state-feedback gain Kc1 that will be saved in the systems file. The new LQR dataset will also be saved in the input file and it is identical to the old one. Click "OK" to return to the main LQR menu.



Select a Method to Solve the Algebraic Riccati Equation
You may either choose Laubs Method or the Assymptotic Method

Use Laubs ARE Algorithm ... ●

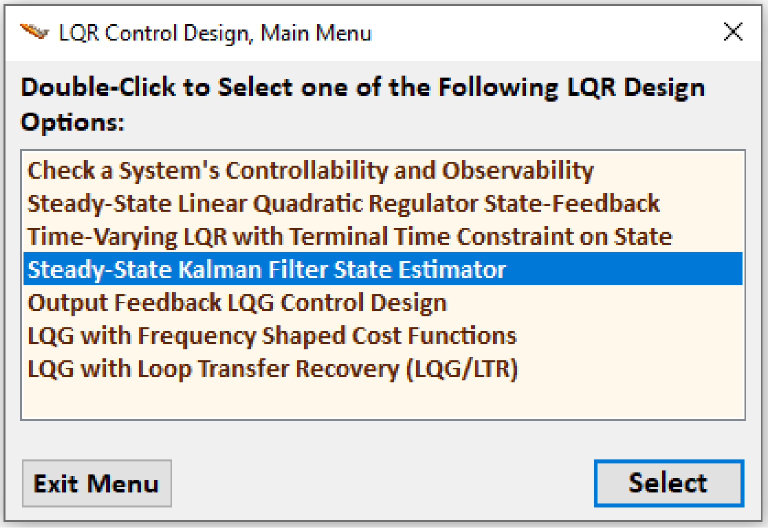
Use the Assymptotic Method ... ○



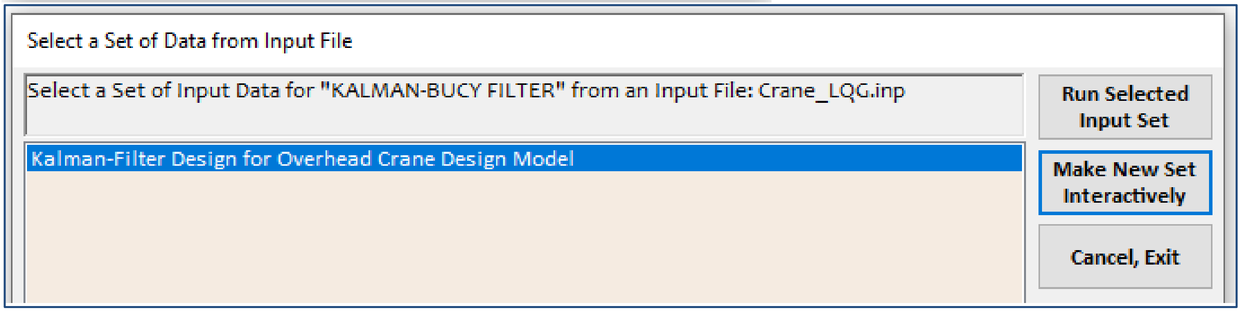
Enter a Title for the Control Gain that will be Saved in File: Crane_LQG.qdr

LQR State-Feedback Control for Crane Design Model with Y1 Integral

We will now solve the Kalman-Filter design interactively. The KF dataset is already in the systems file and ready to be processed but we will recreate it from scratch. From the LQR main menu select "*Steady-State Kalman-Filter Estimator*". The next menu shows that there is already a KF dataset in file. If you run it, it will just reprocess the one that's already there. Select "*Make New Set Interactively*" to create a new set instead.


LQR Control Design, Main Menu

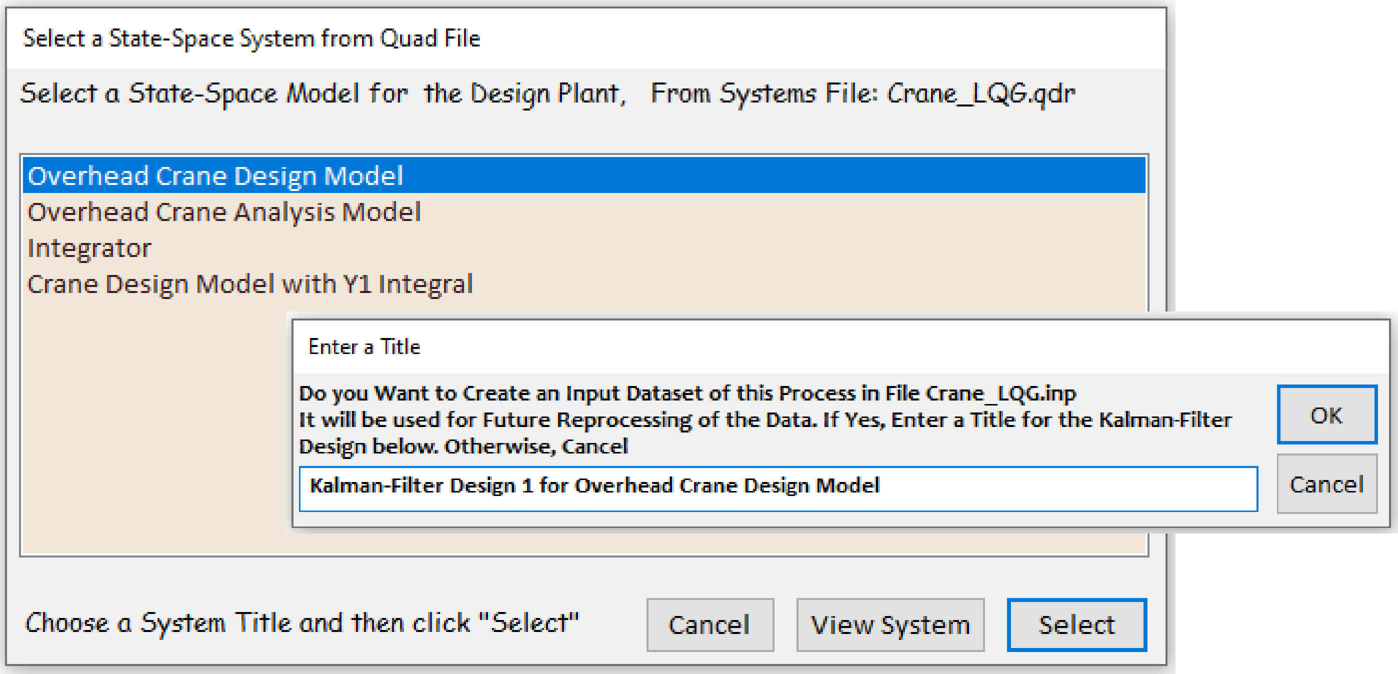
Double-Click to Select one of the Following LQR Design Options:


Select a Set of Data from Input File

Select a Set of Input Data for "KALMAN-BUCY FILTER" from an Input File: Crane_LQG.inp

Kalman-Filter Design for Overhead Crane Design Model

From the menu below select the system from which the KF gain will be designed. Pick the original "Overhead Crane Design Model", not the one with the y1-integral, and enter a title for the new KF dataset that will include instructions for batch re-processing and it will be saved in the input file.


Select a State-Space System from Quad File

Select a State-Space Model for the Design Plant, From Systems File: Crane_LQG.qdr

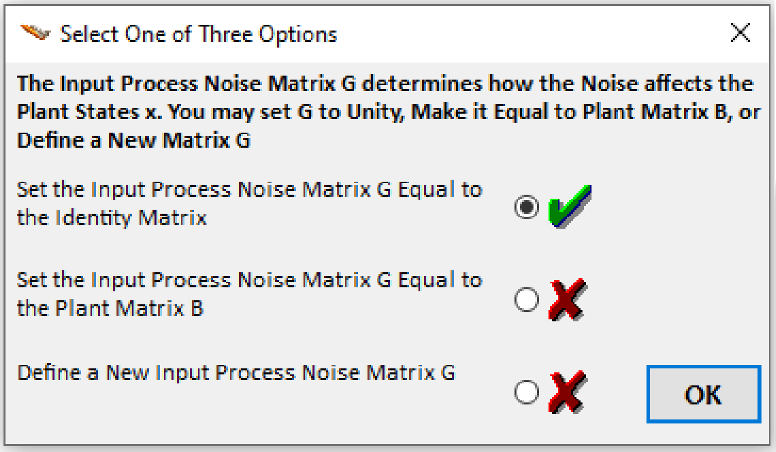
Enter a Title

Do you Want to Create an Input Dataset of this Process in File Crane_LQG.inp
It will be used for Future Reprocessing of the Data. If Yes, Enter a Title for the Kalman-Filter Design below. Otherwise, Cancel

Kalman-Filter Design 1 for Overhead Crane Design Model

Choose a System Title and then click "Select"

We must now select the process noise intensity matrix G. That's the matrix through which noise enters the system. You can either choose the identity matrix to affect each of the 4 states directly, or the input matrix B, or enter a new noise matrix G. In this case we choose the identity matrix, and the system is disturbable through G. We must also choose the (4x4) process noise covariance matrix that defines how much noise corrupts each state individually, and the (2x2) measurement noise covariance matrix that defines the noise at the 2 measurements.



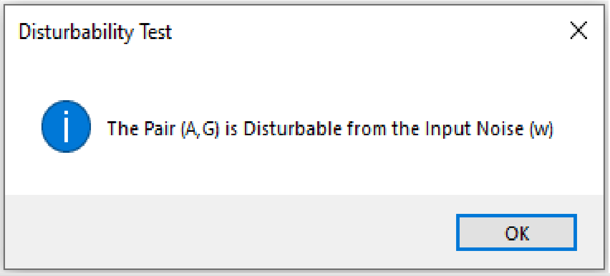
Select One of Three Options

The Input Process Noise Matrix G determines how the Noise affects the Plant States x. You may set G to Unity, Make it Equal to Plant Matrix B, or Define a New Matrix G

Set the Input Process Noise Matrix G Equal to the Identity Matrix ● ✓

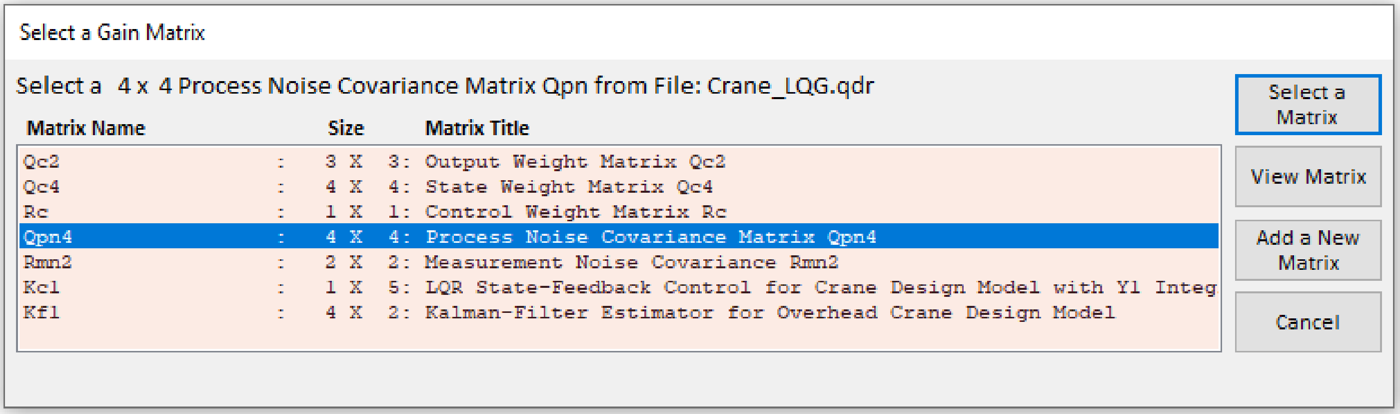
Set the Input Process Noise Matrix G Equal to the Plant Matrix B ○ ✗

Define a New Input Process Noise Matrix G ○ ✗



Disturbability Test

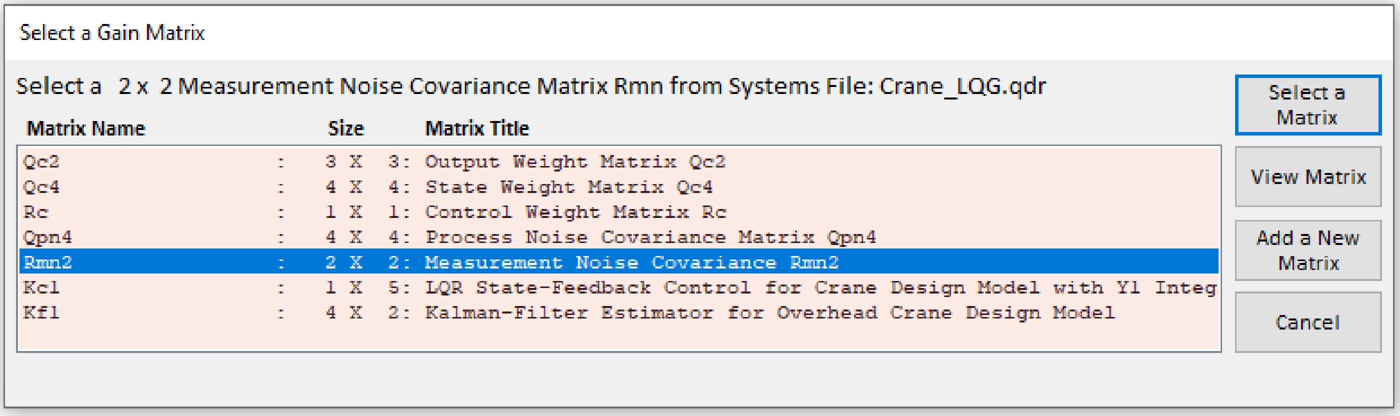
The Pair (A,G) is Disturbable from the Input Noise (w)



Select a Gain Matrix

Select a 4 x 4 Process Noise Covariance Matrix Qpn from File: Crane_LQG.qdr

```
Matrix Name                 Size    Matrix Title
Qc2                    :    3 X   3: Output Weight Matrix Qc2
Qc4                    :    4 X   4: State Weight Matrix Qc4
Rc                     :    1 X   1: Control Weight Matrix Rc
Qpn4                   :    4 X   4: Process Noise Covariance Matrix Qpn4
Rmn2                   :    2 X   2: Measurement Noise Covariance Rmn2
Kc1                    :    1 X   5: LQR State-Feedback Control for Crane Design Model with Y1 Integ
Kf1                    :    4 X   2: Kalman-Filter Estimator for Overhead Crane Design Model
```

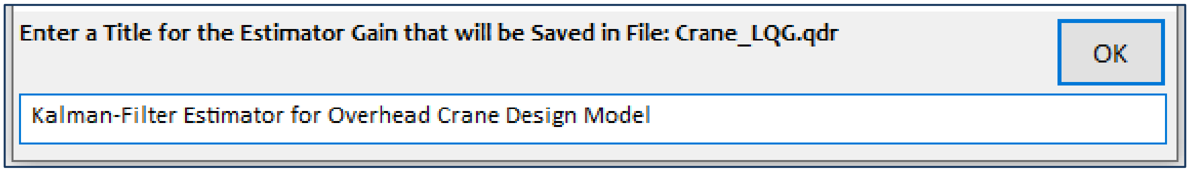


Select a Gain Matrix

Select a 2 x 2 Measurement Noise Covariance Matrix Rmn from Systems File: Crane_LQG.qdr

```
Matrix Name                 Size    Matrix Title
Qc2                    :    3 X   3: Output Weight Matrix Qc2
Qc4                    :    4 X   4: State Weight Matrix Qc4
Rc                     :    1 X   1: Control Weight Matrix Rc
Qpn4                   :    4 X   4: Process Noise Covariance Matrix Qpn4
Rmn2                   :    2 X   2: Measurement Noise Covariance Rmn2
Kc1                    :    1 X   5: LQR State-Feedback Control for Crane Design Model with Y1 Integ
Kf1                    :    4 X   2: Kalman-Filter Estimator for Overhead Crane Design Model
```

We must finally enter a title for the Kalman-Filter gain. The title of the KF gain will appear in the systems file together with the Kf1 matrix.



Enter a Title for the Estimator Gain that will be Saved in File: Crane_LQG.qdr

Kalman-Filter Estimator for Overhead Crane Design Model

Control of an Inverted Pendulum

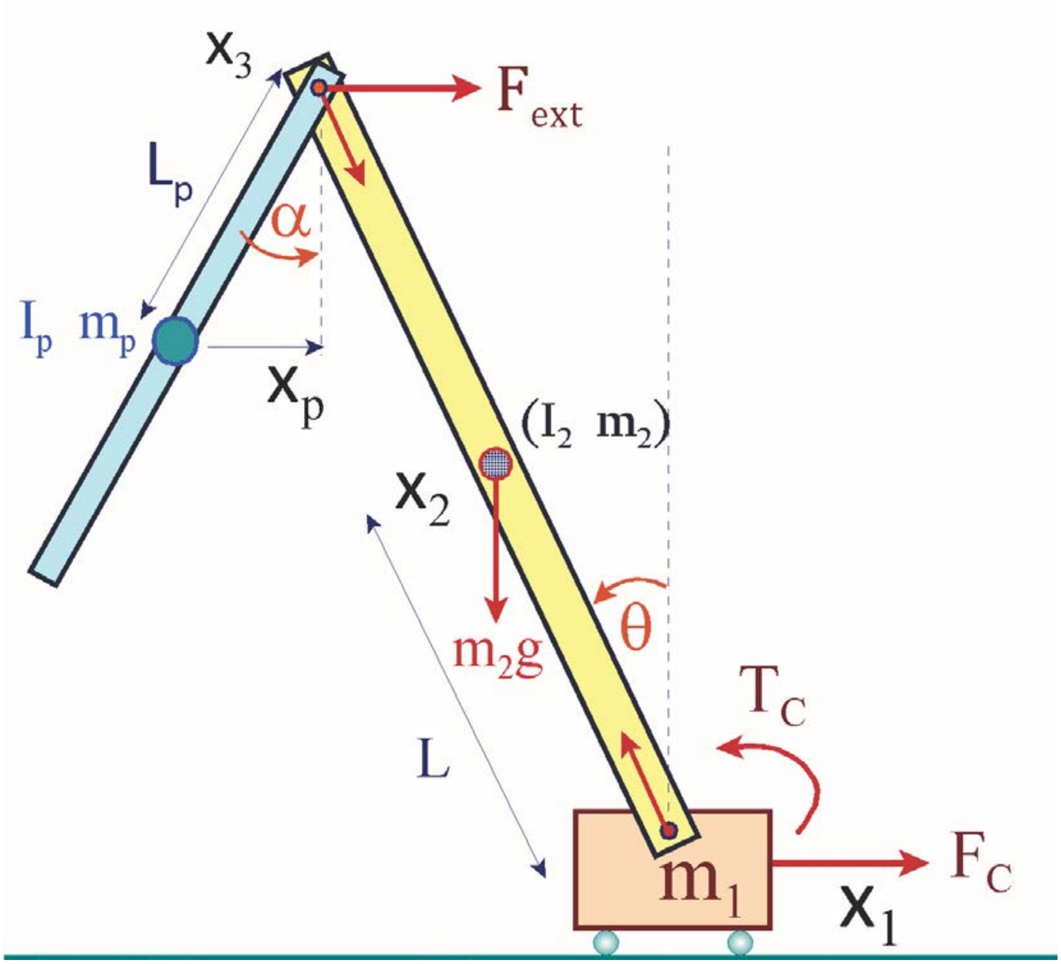


Figure 1 Inverted Pendulum System

The system in this control design example is an inverted pendulum rod of mass m_2 which is supported by a cart of mass m_1 and the cart can move in the x-direction, as shown in Figure-1. The length of the rod is 2L and it has a moment of inertia I_2 about its center of mass. The position of the cart x_1 can be moved by a control force Fc which is applied in the x direction. The vertical bar is attached to the cart via a hinge and it can rotate relative to local vertical at an angle θ. There is also a servo-motor that can apply a torque Tc at the hinge. To make the problem a little more interesting, there is also a compound pendulum bar of mass m_p and inertia I_p that is suspended from the top of the rod x_3 and it can oscillate freely at an angle α relative to vertical. The problem is to design a multivariable control system that will balance the inverted rod vertically and also control the top bar position point x3 by commanding its x-position. In addition, the control system should be able to dampen the oscillations of the pendulum mass m_p and should maintain the top bar x_3 position in the presence of a steadily applied external force F_{ext} which is applied at x_3 along x. This system is obviously open-loop unstable and we will use the H-infinity method to stabilize it. This problem is similar to controlling a launch vehicle with a sloshing propellant tank.

1. Non-Linear Equations of Motion

The x and y position and velocity at the center of the bar x_2 are:

$$x_2 = x_1 - L\sin\theta \qquad y_2 = L\cos\theta$$
$$\dot{x}_2 = \dot{x}_1 - L\,\dot{\theta}\cos\theta \qquad \dot{y}_2 = -L\,\dot{\theta}\sin\theta$$

The x and y position and velocity at the top of the rod point x_3 are:

$$x_3 = x_1 - 2L\sin\theta \qquad y_3 = 2L\cos\theta$$
$$\dot{x}_3 = \dot{x}_1 - 2L\,\dot{\theta}\cos\theta \qquad \dot{y}_3 = -2L\,\dot{\theta}\sin\theta$$

The x and y position and velocity at the pendulum mass x_p are:

$$x_{\mathrm{p}} = x_3 + l_p\sin\alpha = \ x_1 - 2L\sin\theta + l_p\sin\alpha$$
$$y_{\mathrm{p}} = y_3 - l_p\cos\alpha = \ 2L\cos\theta - l_p\cos\alpha$$
$$\dot{x}_{\mathrm{p}} = \dot{x}_3 + l_p\,\dot{a}\cos\alpha = \dot{x}_1 - 2L\,\dot{\theta}\cos\theta + l_p\,\dot{a}\cos\alpha$$
$$\dot{y}_{\mathrm{p}} = \dot{\mathrm{y}}_3 + l_p\,\dot{\alpha}\sin\alpha = l_p\,\dot{\alpha}\sin\alpha - 2L\,\dot{\theta}\sin\theta$$

The combined potential energy of masses m_2 and m_p relative to m_1 is:

$$V = (m_2 + 2m_p)gL\cos\theta - m_p g l_p\cos\alpha$$

The combined kinetic energy of the system is:

$$T = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}I_2\dot{\theta}^2 + \frac{1}{2}I_p\dot{\alpha}^2 + \frac{1}{2}m_2(\dot{x}_2^2 + \dot{y}_2^2) + \frac{1}{2}m_p\left(\dot{x}_p^2 + \dot{y}_p^2\right)$$

$$T = \frac{1}{2}(m_1 + m_2)\dot{x}_1^2 + \frac{1}{2}\left(I_2 + m_2L^2\right)\dot{\theta}^2 - m_2\dot{x}_1\dot{\theta}L\cos\theta$$

$$+\frac{m_p}{2}\left\{\dot{x}_1^2 + 4L^2\dot{\theta}^2 + l_p^2\dot{\alpha}^2 - 4\dot{x}_1L\dot{\theta}\cos\theta + 2\dot{x}_1l_p\dot{\alpha}\cos\alpha - 4Ll_p\dot{\theta}\dot{\alpha}\cos(\theta - \alpha)\right\}$$

$$T = \frac{1}{2}\left(m_1 + m_2 + m_p\right)\dot{x}_1^2 + \frac{1}{2}\left(I_2 + m_2L^2 + 4m_pL^2\right)\dot{\theta}^2 + \frac{1}{2}I_p\dot{\alpha}^2$$
$$- \left(m_2 + 2m_p\right)L\dot{x}_1\dot{\theta}\cos\theta + m_pl_p\dot{x}_1\dot{\alpha}\cos\alpha - 2m_pLl_p\dot{\theta}\dot{\alpha}\cos(\theta - \alpha)$$

$$T = \frac{1}{2}M_o\dot{x}_1^2 + \frac{1}{2}I_o\dot{\theta}^2 + \frac{1}{2}I_p\dot{\alpha}^2 - m_3L\dot{x}_1\dot{\theta}\cos\theta + m_pl_p\dot{x}_1\dot{\alpha}\cos\alpha - I_x\dot{\theta}\dot{\alpha}\cos(\theta - \alpha)$$

The following is the Euler Lagrange equation where q_j are the Generalized Coordinates, and Q_j are the Generalized Force components in the q_j directions

$$\frac{d}{dt}\left(\frac{\delta T}{\delta \dot{q}_j}\right) - \left(\frac{\delta T}{\delta q_j}\right) + \left(\frac{\delta V}{\delta q_j}\right) = \boldsymbol{Q_j}$$

X1 Equation

$$\frac{d}{dt}\left(\frac{\delta T}{\delta \dot{x}_1}\right) = M_o \ddot{x}_1 - m_3 L \ddot{\theta} \cos\theta + m_3 L \dot{\theta}^2 \sin\theta$$

$$\left(\frac{\delta T}{\delta x_1}\right) = \left(\frac{\delta V}{\delta x_1}\right) = 0$$

$$M_o \ddot{x}_1 = m_3 L \ddot{\theta} \cos\theta - m_3 L \dot{\theta}^2 \sin\theta + F_C + F_{ext}$$

θ Equation

$$\frac{d}{dt}\left(\frac{\delta T}{\delta \dot{\theta}}\right) = I_o \ddot{\theta} - m_3 \ddot{x}_1 L \cos\theta + \cancel{m_3 L \dot{x}_1 \dot{\theta} \sin\theta} - I_x \ddot{\alpha} \cos(\theta - \alpha) + I_x \dot{\alpha}\left(\dot{\theta} - \dot{\alpha}\right) \sin(\theta - \alpha)$$

$$\left(\frac{\delta T}{\delta \theta}\right) = \cancel{m_3 L \dot{x}_1 \dot{\theta} \sin\theta} + \cancel{I_x \dot{\alpha} \dot{\theta} \sin(\theta - \alpha)}$$

$$\left(\frac{\delta V}{\delta \theta}\right) = -\left(m_2 + 2m_p\right) g \mathrm{L} \sin\theta$$

$$I_o \ddot{\theta} = m_3 \ddot{x}_1 L \cos\theta + I_x \ddot{\alpha} \cos(\theta - \alpha) + I_x \dot{\alpha}^2 \sin(\theta - \alpha) + \left(m_2 + 2m_p\right) g L \sin\theta + T_C - 2 L F_{ext} \cos\theta$$

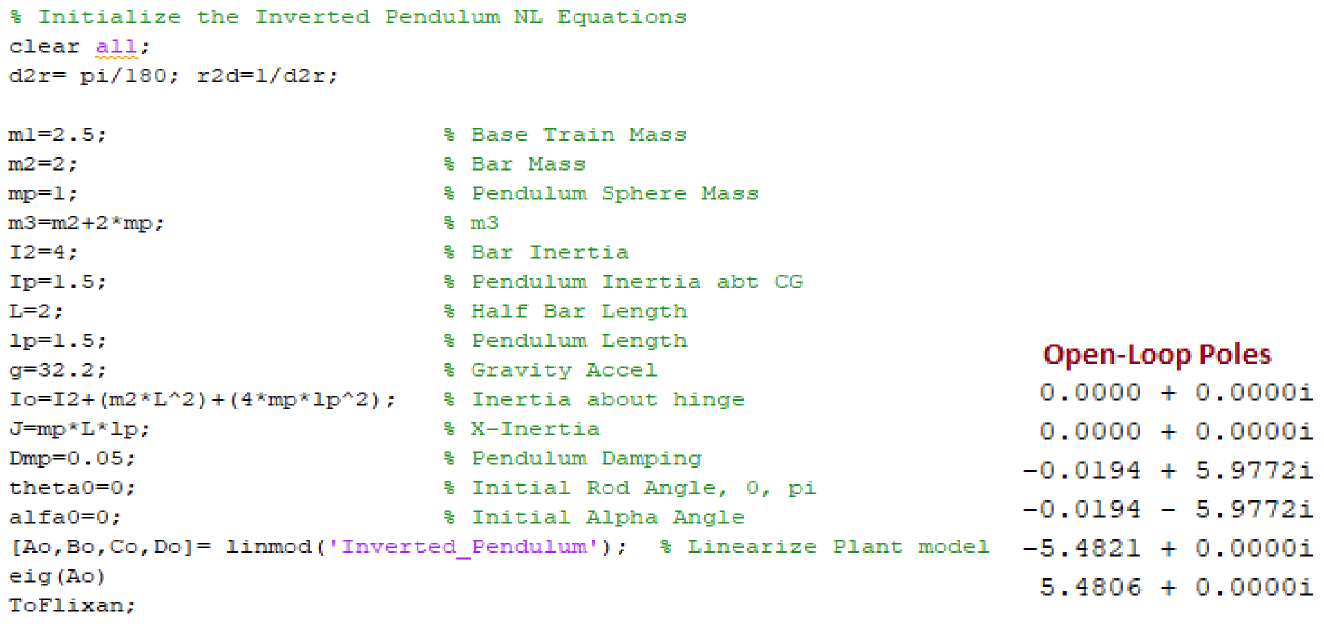
α Equation

$$\frac{d}{dt}\left(\frac{\delta T}{\delta \dot{\alpha}}\right) = I_p \ddot{\alpha} + m_p l_p \ddot{x}_1 \cos\alpha - \cancel{m_p l_p \dot{x}_1 \dot{\alpha} \sin\alpha} - I_x \ddot{\theta} \cos(\theta - \alpha) + I_x \dot{\theta}\left(\dot{\theta} - \dot{\alpha}\right) \sin(\theta - \alpha)$$

$$\left(\frac{\delta T}{\delta \alpha}\right) = -\cancel{I_x \dot{\alpha} \dot{\theta} \sin(\theta - \alpha)} - \cancel{m_p l_p \dot{x}_1 \dot{\alpha} \sin\alpha}$$

$$\left(\frac{\delta V}{\delta \alpha}\right) = m_p g l_p \sin\alpha$$

$$I_p \ddot{\alpha} = -m_p l_p \ddot{x}_1 \cos\alpha + I_x \ddot{\theta} \cos(\theta - \alpha) - I_x \dot{\theta}^2 \sin(\theta - \alpha) - m_p g l_p \sin\alpha - D\dot{\alpha}$$



```
% Initialize the Inverted Pendulum NL Equations
clear all;
d2r= pi/180; r2d=1/d2r;

m1=2.5;                     % Base Train Mass
m2=2;                       % Bar Mass
mp=1;                       % Pendulum Sphere Mass
m3=m2+2*mp;                 % m3
I2=4;                       % Bar Inertia
Ip=1.5;                     % Pendulum Inertia abt CG
L=2;                        % Half Bar Length
lp=1.5;                     % Pendulum Length
g=32.2;                     % Gravity Accel
Io=I2+(m2*L^2)+(4*mp*lp^2); % Inertia about hinge
J=mp*L*lp;                  % X-Inertia
Dmp=0.05;                   % Pendulum Damping
theta0=0;                   % Initial Rod Angle, 0, pi
alfa0=0;                    % Initial Alpha Angle
[Ao,Bo,Co,Do]= linmod('Inverted_Pendulum');  % Linearize Plant model
eig(Ao)
ToFlixan;
```

Open-Loop Poles

```
 0.0000 + 0.0000i
 0.0000 + 0.0000i
-0.0194 + 5.9772i
-0.0194 - 5.9772i
-5.4821 + 0.0000i
 5.4806 + 0.0000i
```

Figure 2 Initialization Script and Open-Loop System Eigenvalues

Dynamic Modeling

The non-linear equations are implemented in a Simulink model "*Inverted Pendulum*" which is shown in Figures (3 &4) and it is located in this folder "*Flixan\Control Analysis\Hinfinity\Examples\Inverted Pendulum\1-Inverted Pend Model*". The model parameters are initialized by running the script "init.m", in Fig.2. It is initialized at an angle θ_0=0, linearized, and the eigenvalues have an unstable pole because of the up-side-down rod position. It also has a low-damped oscillatory complex pair at 6 (rad/sec) which is the pendulum mode. The cart modes are at zero in this initial state.

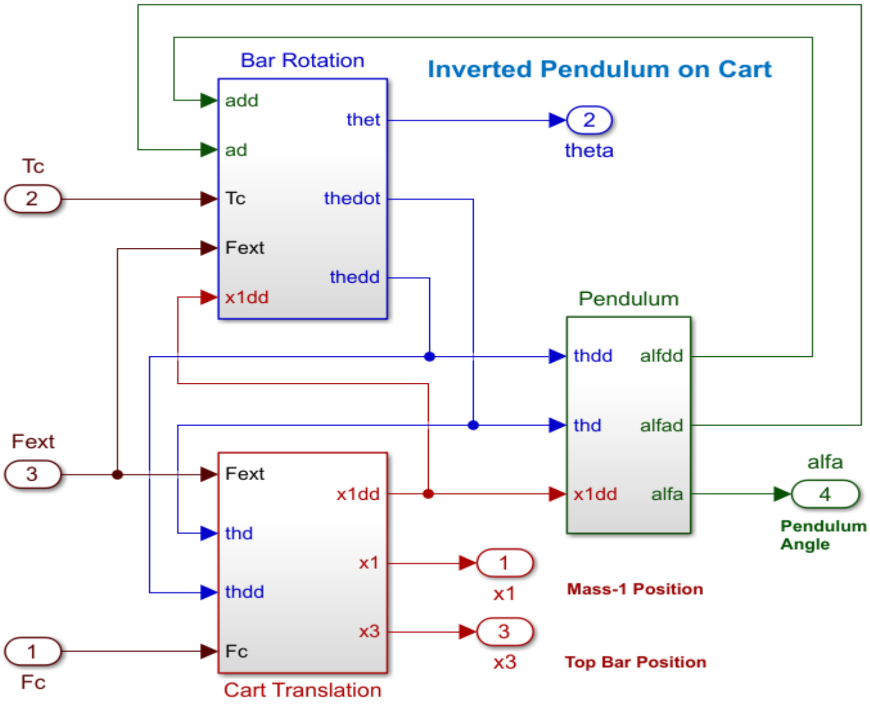


Figure 3 Simulink Model "Inverted_Pendulum" which Includes the Non-Linear Equations and Consists of 3 Subsystems

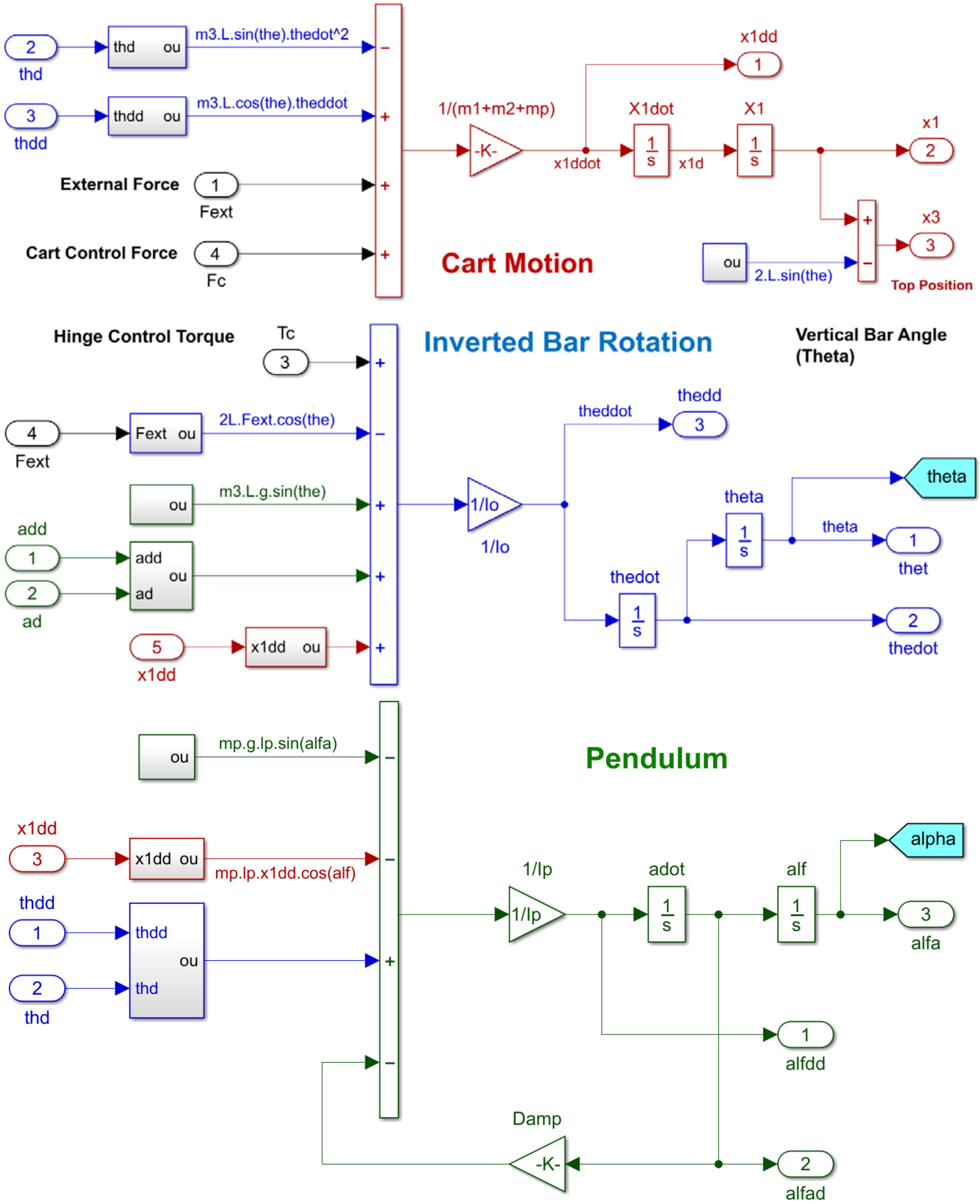
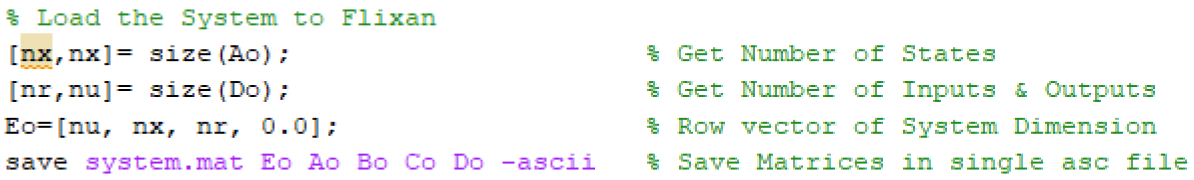


Figure 4 The Three 2nd Order Subsystems of the Inverted Pendulum Model: Cart, Inverted Bar, and Pendulum

The Matlab generated linearized state-space system (A_0, B_0, C_0, D_0] is saved in file “system.mat” in order to be imported into the Flixan files. The script “ToFlixan.m” saves the system dimensions and the four matrices in file “system.mat”. This file will be moved into the analysis folder and read by Flixan program.

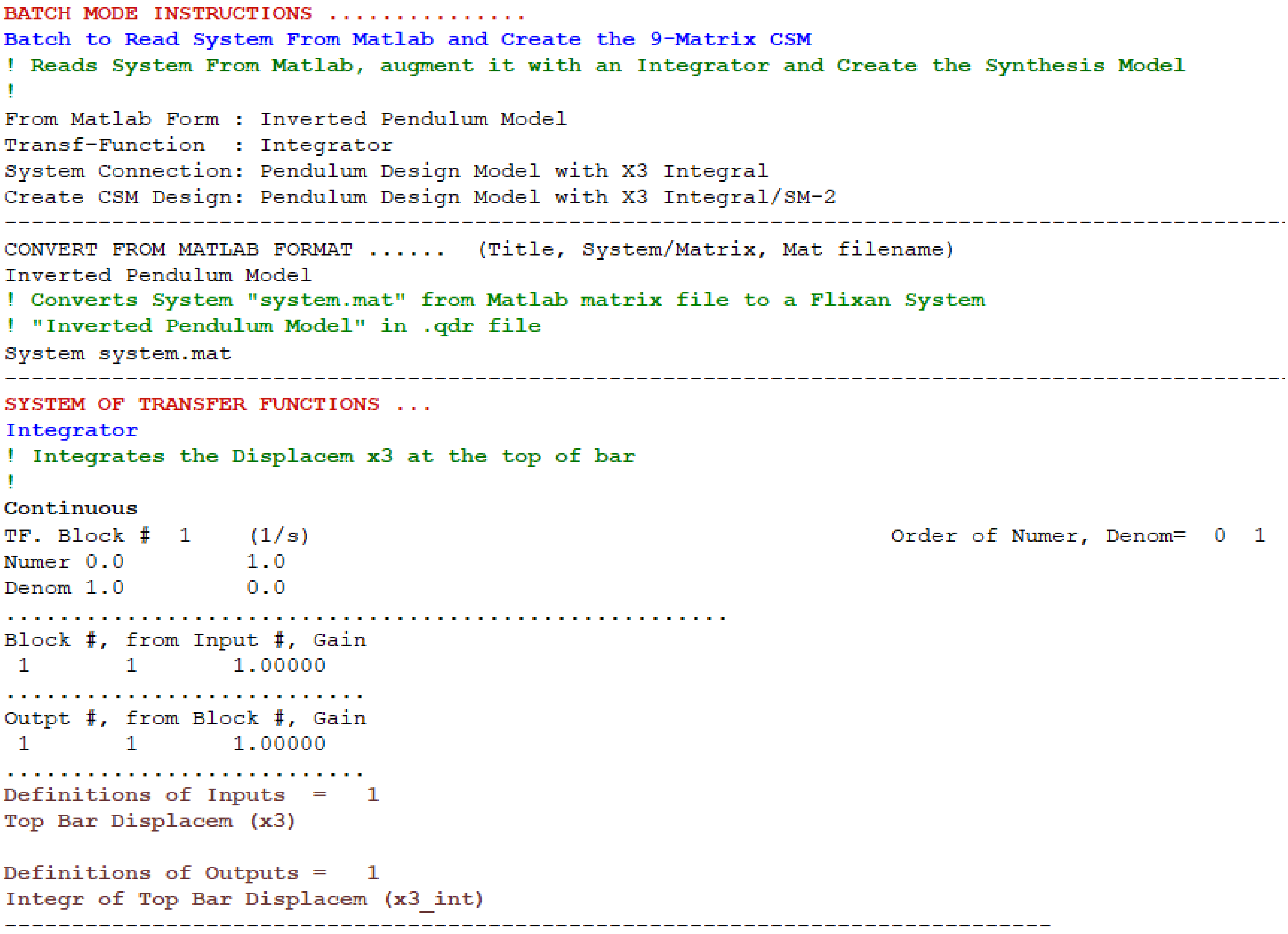


```
% Load the System to Flixan
[nx,nx]= size(Ao);                    % Get Number of States
[nr,nu]= size(Do);                    % Get Number of Inputs & Outputs
Eo=[nu, nx, nr, 0.0];                 % Row vector of System Dimension
save system.mat Eo Ao Bo Co Do -ascii % Save Matrices in single asc file
```

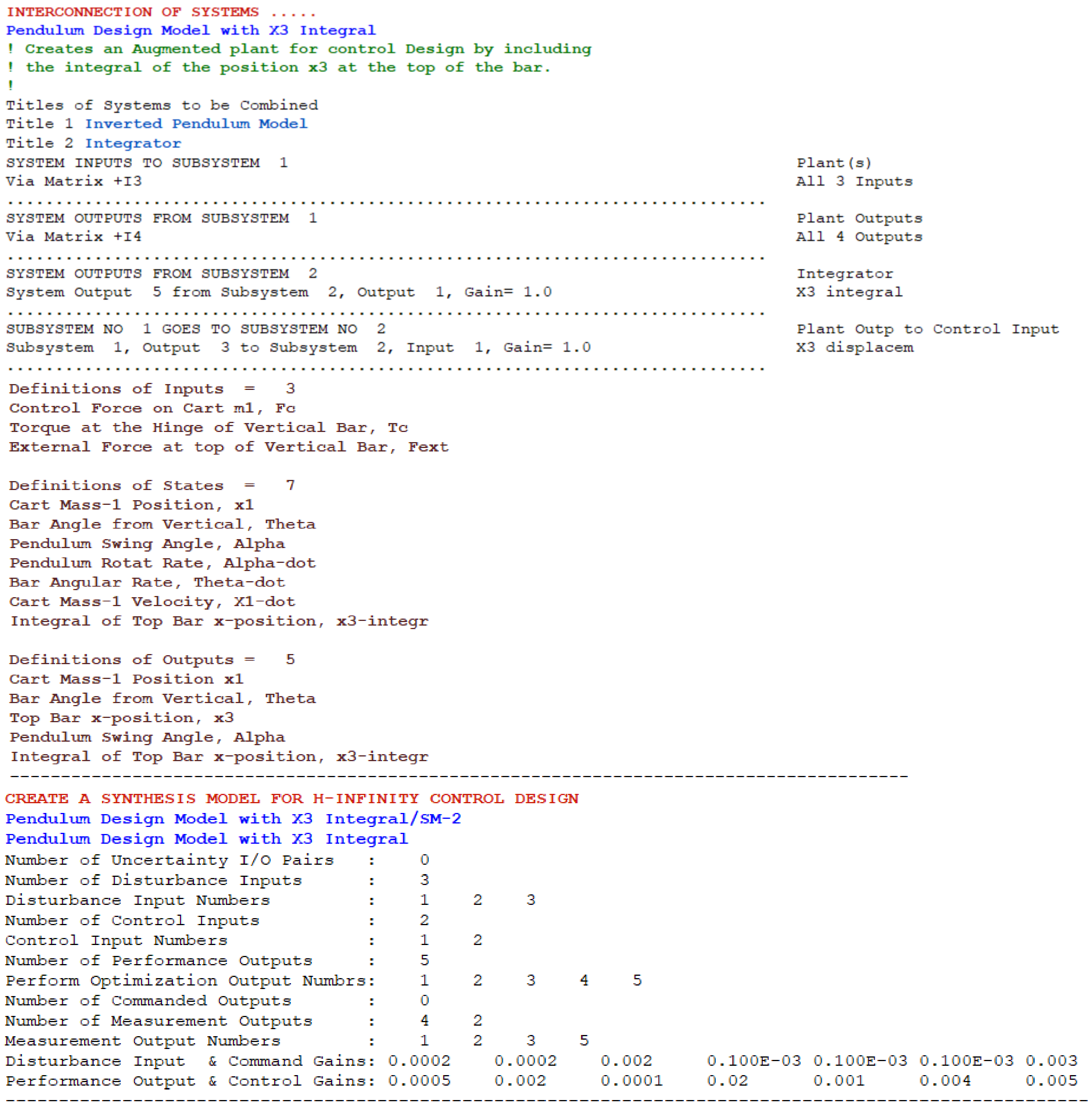
2. Creating the Synthesis Model for H-Infinity Design

The linear design and analysis, is performed in folder “*Flixan\Control Analysis\ Hinfinity\ Examples\ Inverted Pendulum\2-Control Design & Analysis*”. This folder has two input files. The first one “*Create-CSM.Inp*” is used to create the SM. It reads the linear system from file “system.mat” and creates the design model “*Pendulum Design Model with X3 Integral*” by adding one more state, x3-integral and also defines the input output variables. The design or plant model is used to create the synthesis model which is needed for the H-infinity control design which is the next step.

File: Create_CSM.Inp



```
BATCH MODE INSTRUCTIONS ...............
Batch to Read System From Matlab and Create the 9-Matrix CSM
! Reads System From Matlab, augment it with an Integrator and Create the Synthesis Model
!
From Matlab Form : Inverted Pendulum Model
Transf-Function  : Integrator
System Connection: Pendulum Design Model with X3 Integral
Create CSM Design: Pendulum Design Model with X3 Integral/SM-2
-----------------------------------------------------------------------------------------------
CONVERT FROM MATLAB FORMAT ......  (Title, System/Matrix, Mat filename)
Inverted Pendulum Model
! Converts System "system.mat" from Matlab matrix file to a Flixan System
! "Inverted Pendulum Model" in .qdr file
System system.mat
-----------------------------------------------------------------------------------------------
SYSTEM OF TRANSFER FUNCTIONS ...
Integrator
! Integrates the Displacem x3 at the top of bar
!
Continuous
TF. Block #  1    (1/s)                                      Order of Numer, Denom=  0  1
Numer 0.0        1.0
Denom 1.0        0.0
.......................................................
Block #, from Input #, Gain
 1       1       1.00000
...........................
Outpt #, from Block #, Gain
 1       1       1.00000
...........................
Definitions of Inputs  =   1
Top Bar Displacem (x3)

Definitions of Outputs =   1
Integr of Top Bar Displacem (x3_int)
-----------------------------------------------------------------------------
```

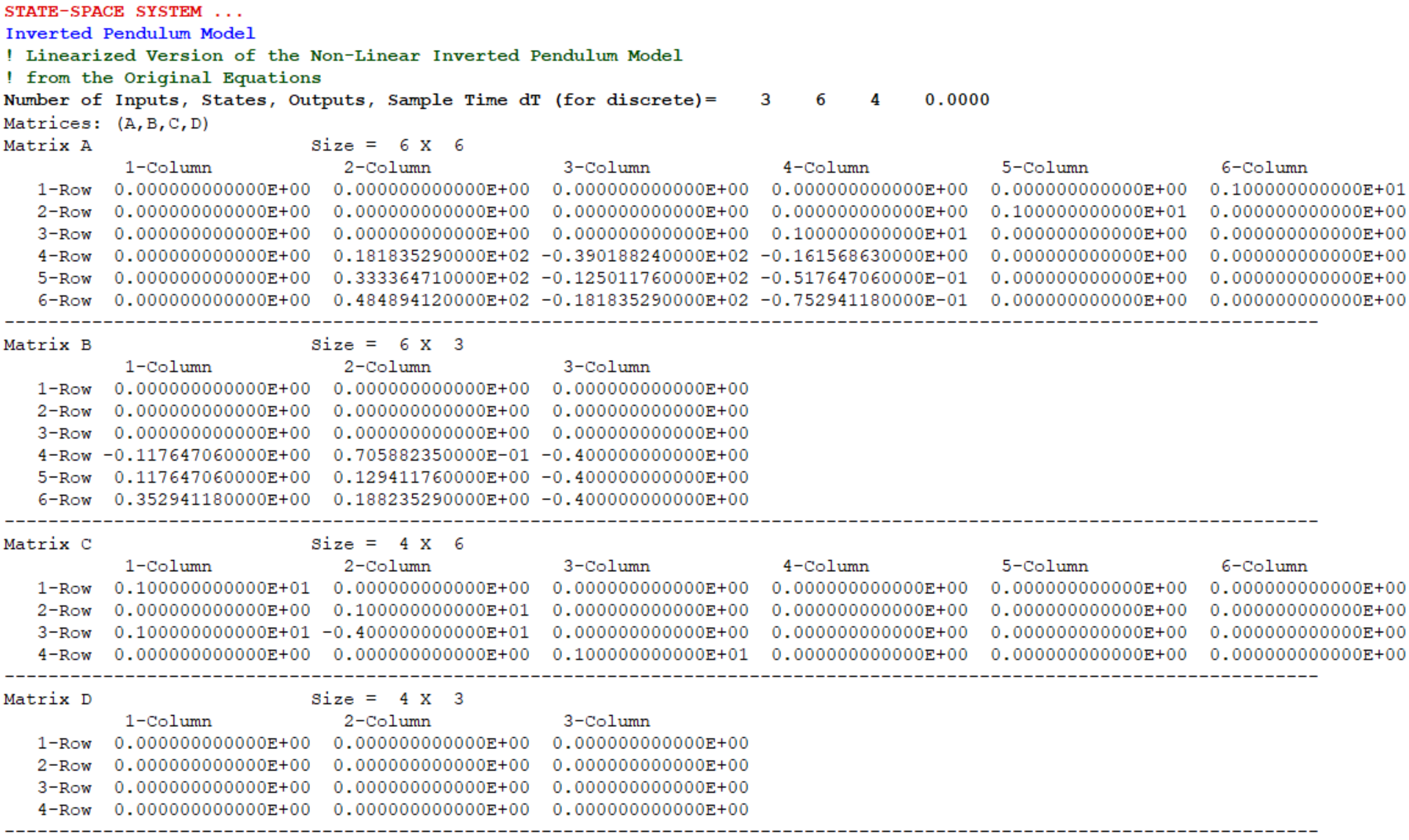


```
INTERCONNECTION OF SYSTEMS .....
Pendulum Design Model with X3 Integral
! Creates an Augmented plant for control Design by including
! the integral of the position x3 at the top of the bar.
!
Titles of Systems to be Combined
Title 1 Inverted Pendulum Model
Title 2 Integrator
SYSTEM INPUTS TO SUBSYSTEM  1                                          Plant(s)
Via Matrix +I3                                                         All 3 Inputs
...........................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                       Plant Outputs
Via Matrix +I4                                                         All 4 Outputs
...........................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                                       Integrator
System Output  5 from Subsystem  2, Output  1, Gain= 1.0               X3 integral
...........................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2                                Plant Outp to Control Input
Subsystem  1, Output  3 to Subsystem  2, Input  1, Gain= 1.0           X3 displacem
...........................................................................
 Definitions of Inputs  =   3
 Control Force on Cart m1, Fc
 Torque at the Hinge of Vertical Bar, Tc
 External Force at top of Vertical Bar, Fext

 Definitions of States  =   7
 Cart Mass-1 Position, x1
 Bar Angle from Vertical, Theta
 Pendulum Swing Angle, Alpha
 Pendulum Rotat Rate, Alpha-dot
 Bar Angular Rate, Theta-dot
 Cart Mass-1 Velocity, X1-dot
 Integral of Top Bar x-position, x3-integr

 Definitions of Outputs =   5
 Cart Mass-1 Position x1
 Bar Angle from Vertical, Theta
 Top Bar x-position, x3
 Pendulum Swing Angle, Alpha
 Integral of Top Bar x-position, x3-integr
 ------------------------------------------------------------------------------------------
CREATE A SYNTHESIS MODEL FOR H-INFINITY CONTROL DESIGN
Pendulum Design Model with X3 Integral/SM-2
Pendulum Design Model with X3 Integral
Number of Uncertainty I/O Pairs    :    0
Number of Disturbance Inputs       :    3
Disturbance Input Numbers          :    1    2    3
Number of Control Inputs           :    2
Control Input Numbers              :    1    2
Number of Performance Outputs      :    5
Perform Optimization Output Numbrs:     1    2    3    4    5
Number of Commanded Outputs        :    0
Number of Measurement Outputs      :    4    2
Measurement Output Numbers         :    1    2    3    5
Disturbance Input  & Command Gains: 0.0002    0.0002    0.002     0.100E-03 0.100E-03 0.100E-03 0.003
Performance Output & Control Gains: 0.0005    0.002     0.0001    0.02      0.001     0.004     0.005
----------------------------------------------------------------------------------------------------------
```

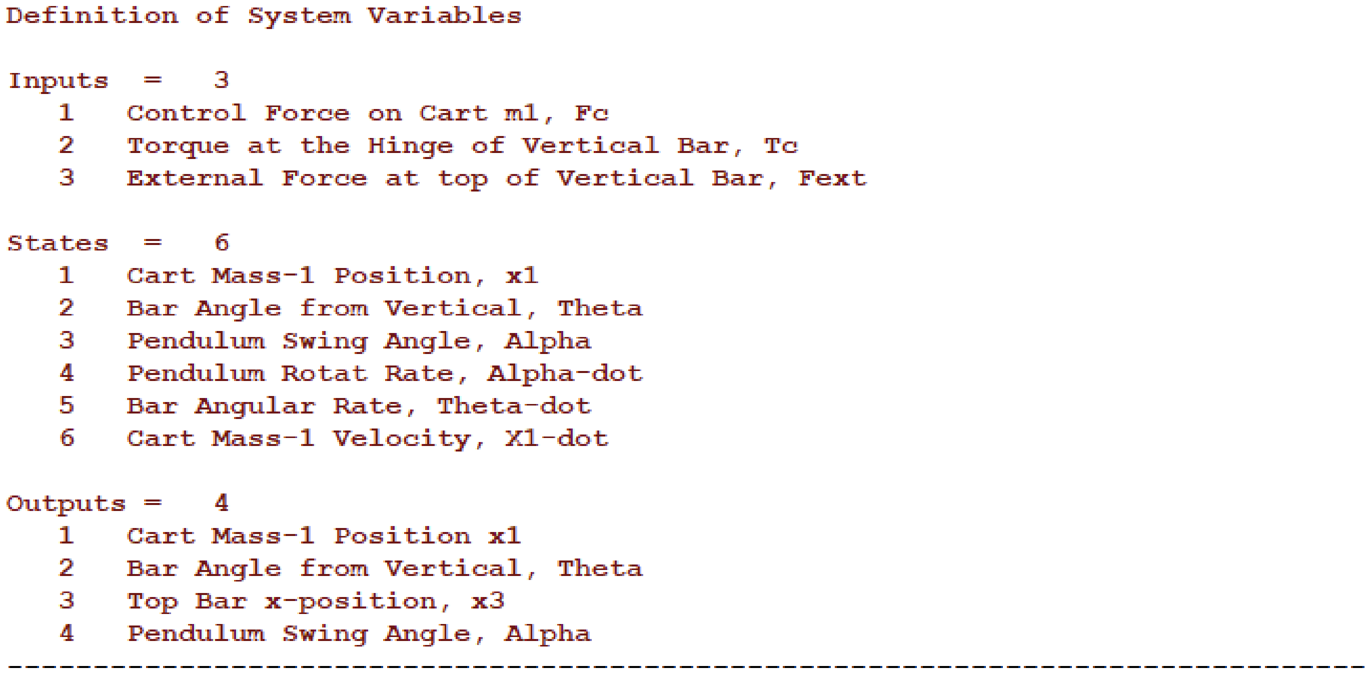
This input file can be processed by running the batch set which reads the linearized system's (A,B,C,D) matrices from the Matlab file "system.mat" and saves it in file "Create_CSM.Qdr" under the title "*Inverted Pendulum Model*". From this system it creates an augmented system "*Pendulum Design Model with X3 Integral*" by adding the x3-integral in the states and in the outputs. The x3-integral variable is used to provide better x3 position control under the influence of a steady disturbance force Fext. Then it creates the Synthesis Model "*Pendulum Design Model with X3 Integral/SM-2*" by processing the already prepared SM creation dataset which defines the inputs, outputs and the scaling gains.

File: Create_CSM.Qdr

The following is the systems file "*Create_CSM.Qdr*" that includes the linearized system, the augmented design system that includes the x3 integral, and the Synthesis Model.

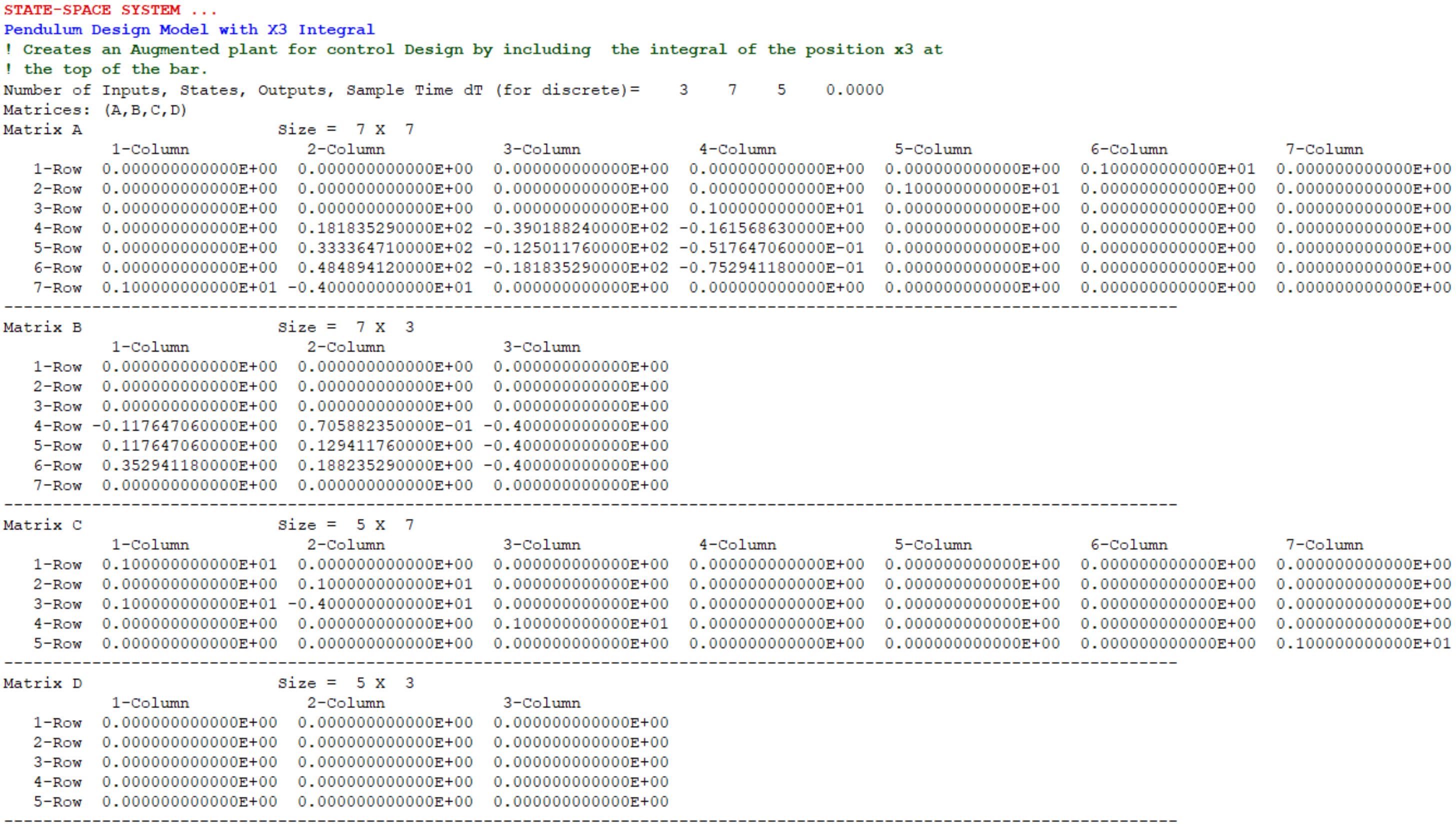


```
STATE-SPACE SYSTEM ...
Inverted Pendulum Model
! Linearized Version of the Non-Linear Inverted Pendulum Model
! from the Original Equations
Number of Inputs, States, Outputs, Sample Time dT (for discrete)=    3    6    4    0.0000
Matrices: (A,B,C,D)
Matrix A                    Size =  6 X  6
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.181835290000E+02 -0.390188240000E+02 -0.161568630000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.333364710000E+02 -0.125011760000E+02 -0.517647060000E-01  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.484894120000E+02 -0.181835290000E+02 -0.752941180000E-01  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Matrix B                    Size =  6 X  3
           1-Column            2-Column            3-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row -0.117647060000E+00  0.705882350000E-01 -0.400000000000E+00
   5-Row  0.117647060000E+00  0.129411760000E+00 -0.400000000000E+00
   6-Row  0.352941180000E+00  0.188235290000E+00 -0.400000000000E+00
------------------------------------------------------------------------------------------------------------------------
Matrix C                    Size =  4 X  6
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.100000000000E+01 -0.400000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Matrix D                    Size =  4 X  3
           1-Column            2-Column            3-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
```

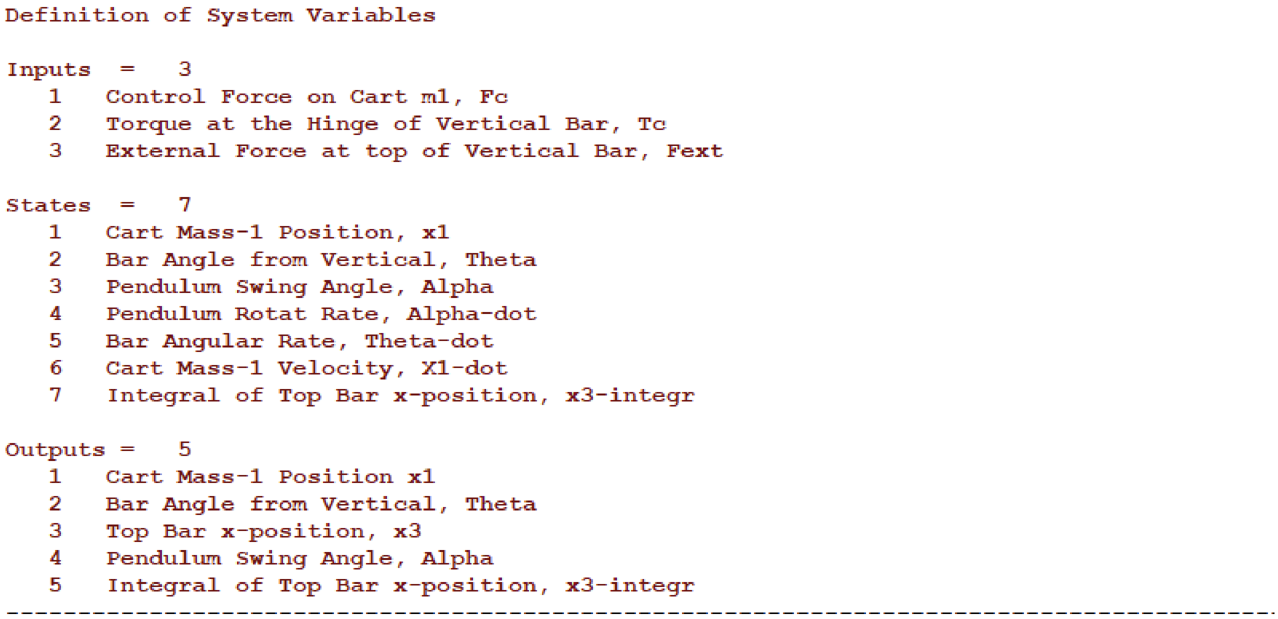


```
Definition of System Variables

Inputs   =   3
   1   Control Force on Cart m1, Fc
   2   Torque at the Hinge of Vertical Bar, Tc
   3   External Force at top of Vertical Bar, Fext

States   =   6
   1   Cart Mass-1 Position, x1
   2   Bar Angle from Vertical, Theta
   3   Pendulum Swing Angle, Alpha
   4   Pendulum Rotat Rate, Alpha-dot
   5   Bar Angular Rate, Theta-dot
   6   Cart Mass-1 Velocity, X1-dot

Outputs =   4
   1   Cart Mass-1 Position x1
   2   Bar Angle from Vertical, Theta
   3   Top Bar x-position, x3
   4   Pendulum Swing Angle, Alpha
--------------------------------------------------------------------------------
```

The following is the design model “*Pendulum Design Model with X3 Integral*” which includes x3-integral in the states and output vectors. The state variables are defined below.



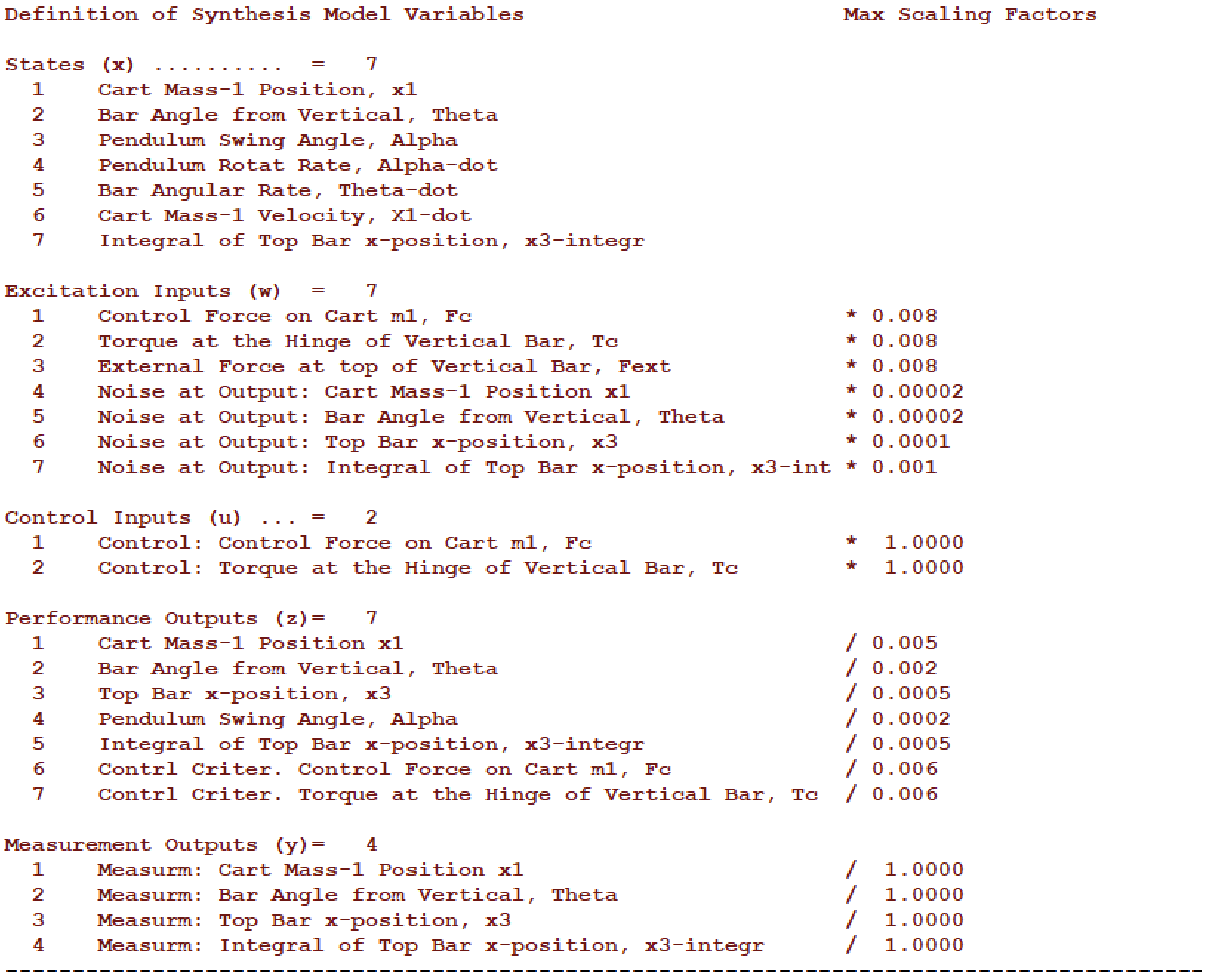
```
STATE-SPACE SYSTEM ...
Pendulum Design Model with X3 Integral
! Creates an Augmented plant for control Design by including  the integral of the position x3 at
! the top of the bar.
Number of Inputs, States, Outputs, Sample Time dT (for discrete)=    3    7    5    0.0000
Matrices: (A,B,C,D)
Matrix A                    Size =  7 X  7
           1-Column             2-Column             3-Column             4-Column             5-Column             6-Column             7-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.181835290000E+02 -0.390188240000E+02 -0.161568630000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.333364710000E+02 -0.125011760000E+02 -0.517647060000E-01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.484894120000E+02 -0.181835290000E+02 -0.752941180000E-01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.100000000000E+01 -0.400000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
-------------------------------------------------------------------------------------------------------------------------
Matrix B                    Size =  7 X  3
           1-Column             2-Column             3-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row -0.117647060000E+00  0.705882350000E-01 -0.400000000000E+00
   5-Row  0.117647060000E+00  0.129411760000E+00 -0.400000000000E+00
   6-Row  0.352941180000E+00  0.188235290000E+00 -0.400000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
-------------------------------------------------------------------------------------------------------------------------
Matrix C                    Size =  5 X  7
           1-Column             2-Column             3-Column             4-Column             5-Column             6-Column             7-Column
   1-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.100000000000E+01 -0.400000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
-------------------------------------------------------------------------------------------------------------------------
Matrix D                    Size =  5 X  3
           1-Column             2-Column             3-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
-------------------------------------------------------------------------------------------------------------------------
```



```
Definition of System Variables

Inputs  =   3
   1   Control Force on Cart m1, Fc
   2   Torque at the Hinge of Vertical Bar, Tc
   3   External Force at top of Vertical Bar, Fext

States  =   7
   1   Cart Mass-1 Position, x1
   2   Bar Angle from Vertical, Theta
   3   Pendulum Swing Angle, Alpha
   4   Pendulum Rotat Rate, Alpha-dot
   5   Bar Angular Rate, Theta-dot
   6   Cart Mass-1 Velocity, X1-dot
   7   Integral of Top Bar x-position, x3-integr

Outputs =   5
   1   Cart Mass-1 Position x1
   2   Bar Angle from Vertical, Theta
   3   Top Bar x-position, x3
   4   Pendulum Swing Angle, Alpha
   5   Integral of Top Bar x-position, x3-integr
-------------------------------------------------------------------------------------------
```

The Synthesis Model is shown below and consists of 9 matrices. It also includes the definitions of variables at the bottom with the performance parameter scaling gains. The SM is copied in a separate systems file “IP_Design.Qdr” where together with its pair “IP_Design.Inp” we will perform the H-infinity control design in separate files to avoid data mixup.

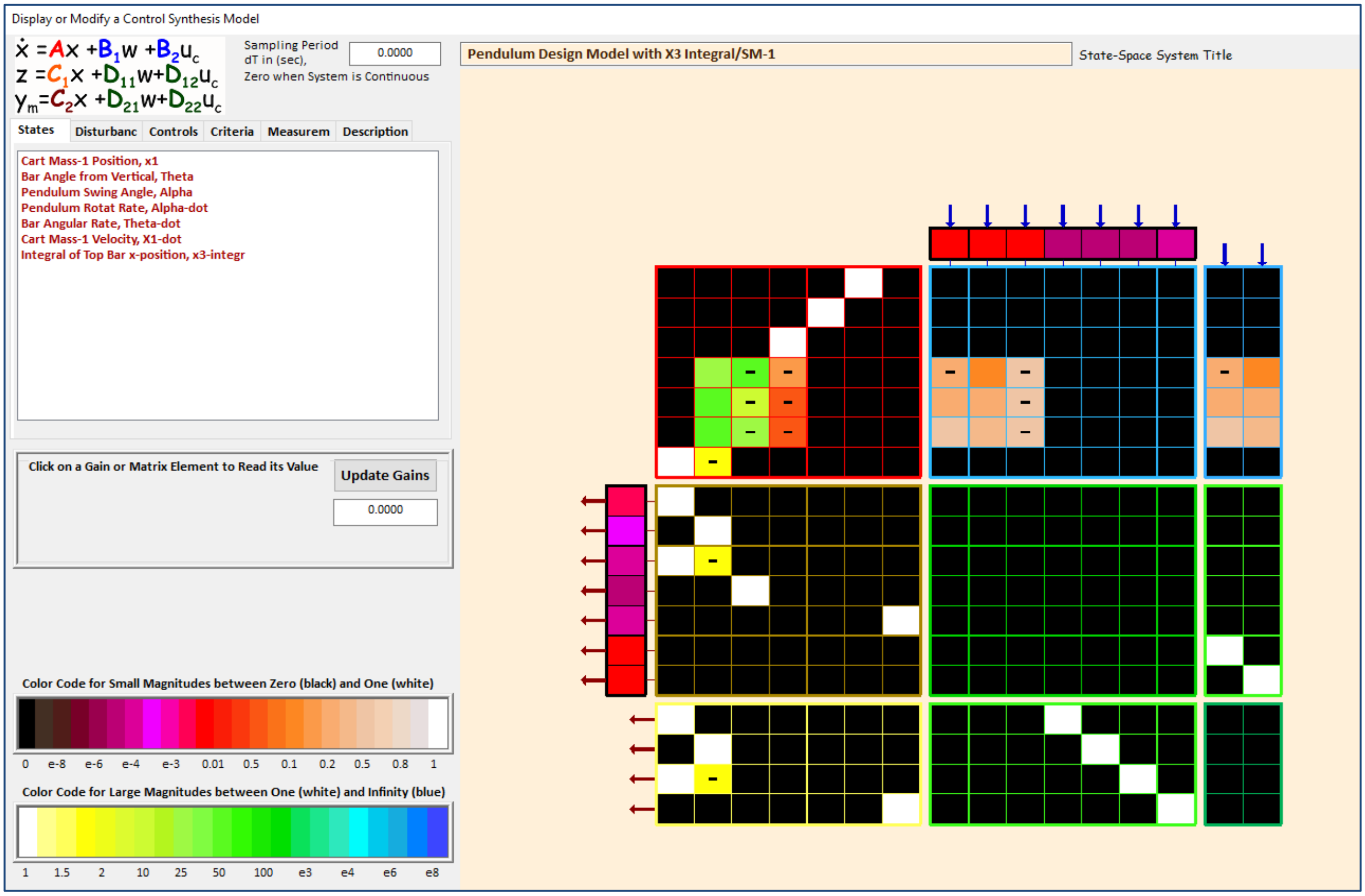
```
SYNTHESIS MODEL FOR H-INFINITY CONTROL DESIGN, EXTRACTED FROM SYSTEM ...
Pendulum Design Model with X3 Integral/SM-2
Number of: States (x), Uncertainty Inp/Outputs from Plant Variations (dP)=   7    0    0
Number of: Extern Disturbance Inputs (Wi), Control Inputs (Uc)           =   3    2
Number of: Output Criteria (Zo), Regulated Outputs (Zr), Measurements (y)=   5    0    4
Synthes Model Matrices: A, B1,B2,C1,C2, D11,D12,D21,D22, Sample Time (dT)=   0.0000
Matrix A                    Size =  7 X  7
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column            7-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.181835290000E+02 -0.390188240000E+02 -0.161568630000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.333364710000E+02 -0.125011760000E+02 -0.517647060000E-01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.484894120000E+02 -0.181835290000E+02 -0.752941180000E-01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.100000000000E+01 -0.400000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Matrix B1                   Size =  7 X  7
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column            7-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row -0.117647060000E+00  0.705882350000E-01 -0.400000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.117647060000E+00  0.129411760000E+00 -0.400000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.352941180000E+00  0.188235290000E+00 -0.400000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Matrix B2                   Size =  7 X  2
           1-Column            2-Column
   1-Row  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00
   4-Row -0.117647060000E+00  0.705882350000E-01
   5-Row  0.117647060000E+00  0.129411760000E+00
   6-Row  0.352941180000E+00  0.188235290000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Matrix C1                   Size =  7 X  7
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column            7-Column
   1-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.100000000000E+01 -0.400000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Matrix C2                   Size =  4 X  7
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column            7-Column
   1-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.100000000000E+01 -0.400000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
------------------------------------------------------------------------------------------------------------------------
Matrix D11                  Size =  7 X  7
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column            7-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Matrix D12                  Size =  7 X  2
           1-Column            2-Column
   1-Row  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00
   6-Row  0.100000000000E+01  0.000000000000E+00
   7-Row  0.000000000000E+00  0.100000000000E+01
------------------------------------------------------------------------------------------------------------------------
Matrix D21                  Size =  4 X  7
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column            7-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
------------------------------------------------------------------------------------------------------------------------
Matrix D22                  Size =  4 X  2
           1-Column            2-Column
   1-Row  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
```



```
Definition of Synthesis Model Variables                          Max Scaling Factors

States (x) .........  =   7
  1    Cart Mass-1 Position, x1
  2    Bar Angle from Vertical, Theta
  3    Pendulum Swing Angle, Alpha
  4    Pendulum Rotat Rate, Alpha-dot
  5    Bar Angular Rate, Theta-dot
  6    Cart Mass-1 Velocity, X1-dot
  7    Integral of Top Bar x-position, x3-integr

Excitation Inputs (w)  =   7
  1    Control Force on Cart m1, Fc                             * 0.008
  2    Torque at the Hinge of Vertical Bar, Tc                  * 0.008
  3    External Force at top of Vertical Bar, Fext              * 0.008
  4    Noise at Output: Cart Mass-1 Position x1                 * 0.00002
  5    Noise at Output: Bar Angle from Vertical, Theta          * 0.00002
  6    Noise at Output: Top Bar x-position, x3                  * 0.0001
  7    Noise at Output: Integral of Top Bar x-position, x3-int * 0.001

Control Inputs (u) ... =   2
  1    Control: Control Force on Cart m1, Fc                    *  1.0000
  2    Control: Torque at the Hinge of Vertical Bar, Tc         *  1.0000

Performance Outputs (z)=   7
  1    Cart Mass-1 Position x1                                  / 0.005
  2    Bar Angle from Vertical, Theta                           / 0.002
  3    Top Bar x-position, x3                                   / 0.0005
  4    Pendulum Swing Angle, Alpha                              / 0.0002
  5    Integral of Top Bar x-position, x3-integr                / 0.0005
  6    Contrl Criter. Control Force on Cart m1, Fc              / 0.006
  7    Contrl Criter. Torque at the Hinge of Vertical Bar, Tc   / 0.006

Measurement Outputs (y)=   4
  1    Measurm: Cart Mass-1 Position x1                         /  1.0000
  2    Measurm: Bar Angle from Vertical, Theta                  /  1.0000
  3    Measurm: Top Bar x-position, x3                          /  1.0000
  4    Measurm: Integral of Top Bar x-position, x3-integr       /  1.0000
-------------------------------------------------------------------------------------------
```

3. Control Design Using File: IP_Design.Inp

The SM is shown graphically in Figure 5. It consists of 2 controls and 4 measurements and it is set up to synthesize an output feedback dynamic controller. It includes an external force F_{ext} which is applied at the top of the bar and the system must be able to counteract that disturbance while balancing the rod and maintaining the x_3 position. It also includes disturbances at the two controls and noise at the 4 measurements. The performance criteria consist of: the top bar position error x_3-err and its integral, the m_1 position x_1-err because we want to prevent it from drifting, the vertical bar angle θ because we want to keep it vertical, and the pendulum swing angle α because we want to use the controls to dampen the α-oscillations. The two controls are also included in the performance criteria. The H-infinity program will process the SM and create the control system. Figure 5 also shows the closed-loop system eigenvalues which are all stable.



Display or Modify a Control Synthesis Model

$\dot{x} = Ax + B_1 w + B_2 u_c$

$z = C_1 x + D_{11} w + D_{12} u_c$

$y_m = C_2 x + D_{21} w + D_{22} u_c$

Sampling Period dT in (sec), Zero when System is Continuous: 0.0000

Pendulum Design Model with X3 Integral/SM-1 — State-Space System Title

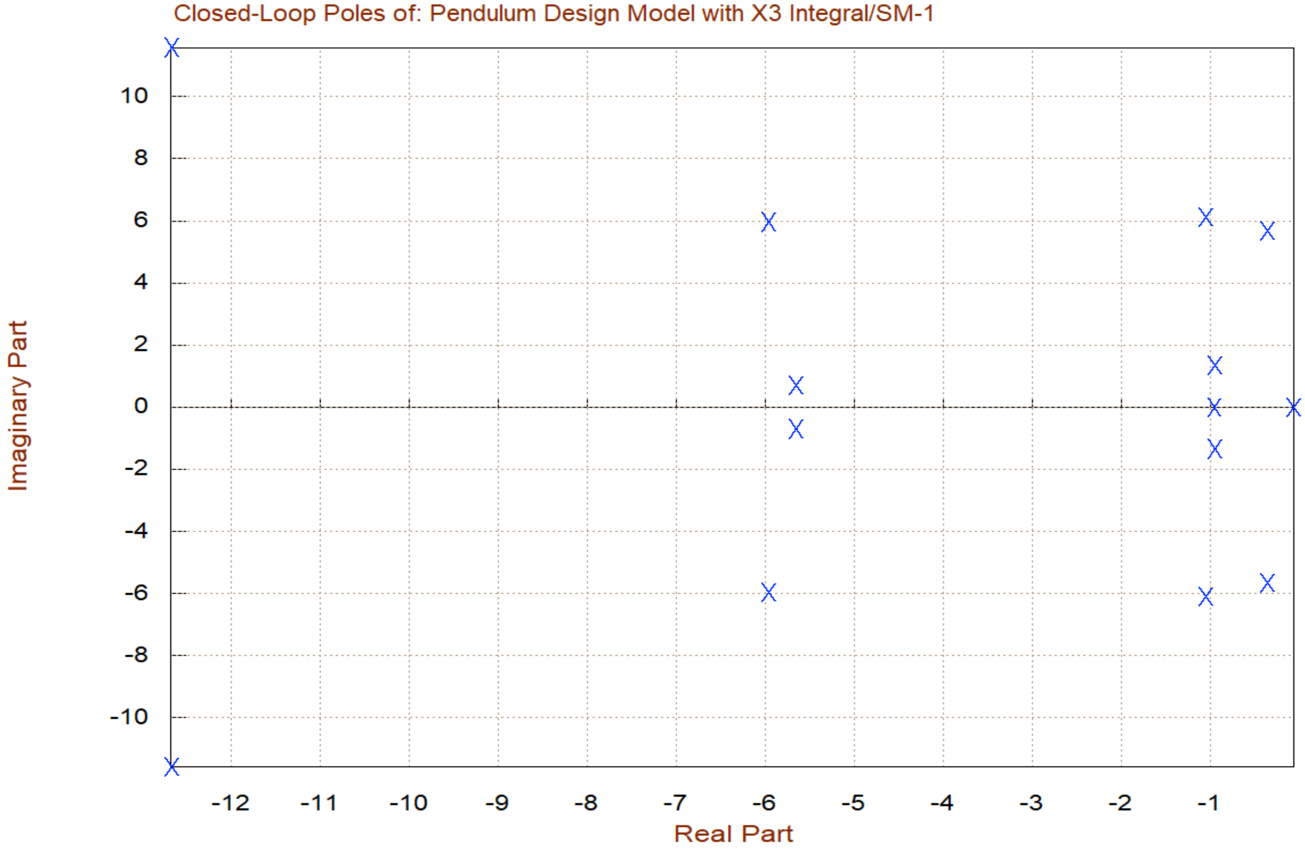
Cart Mass-1 Position, x1
Bar Angle from Vertical, Theta
Pendulum Swing Angle, Alpha
Pendulum Rotat Rate, Alpha-dot
Bar Angular Rate, Theta-dot
Cart Mass-1 Velocity, X1-dot
Integral of Top Bar x-position, x3-integr

Click on a Gain or Matrix Element to Read its Value

0.0000

Color Code for Small Magnitudes between Zero (black) and One (white)

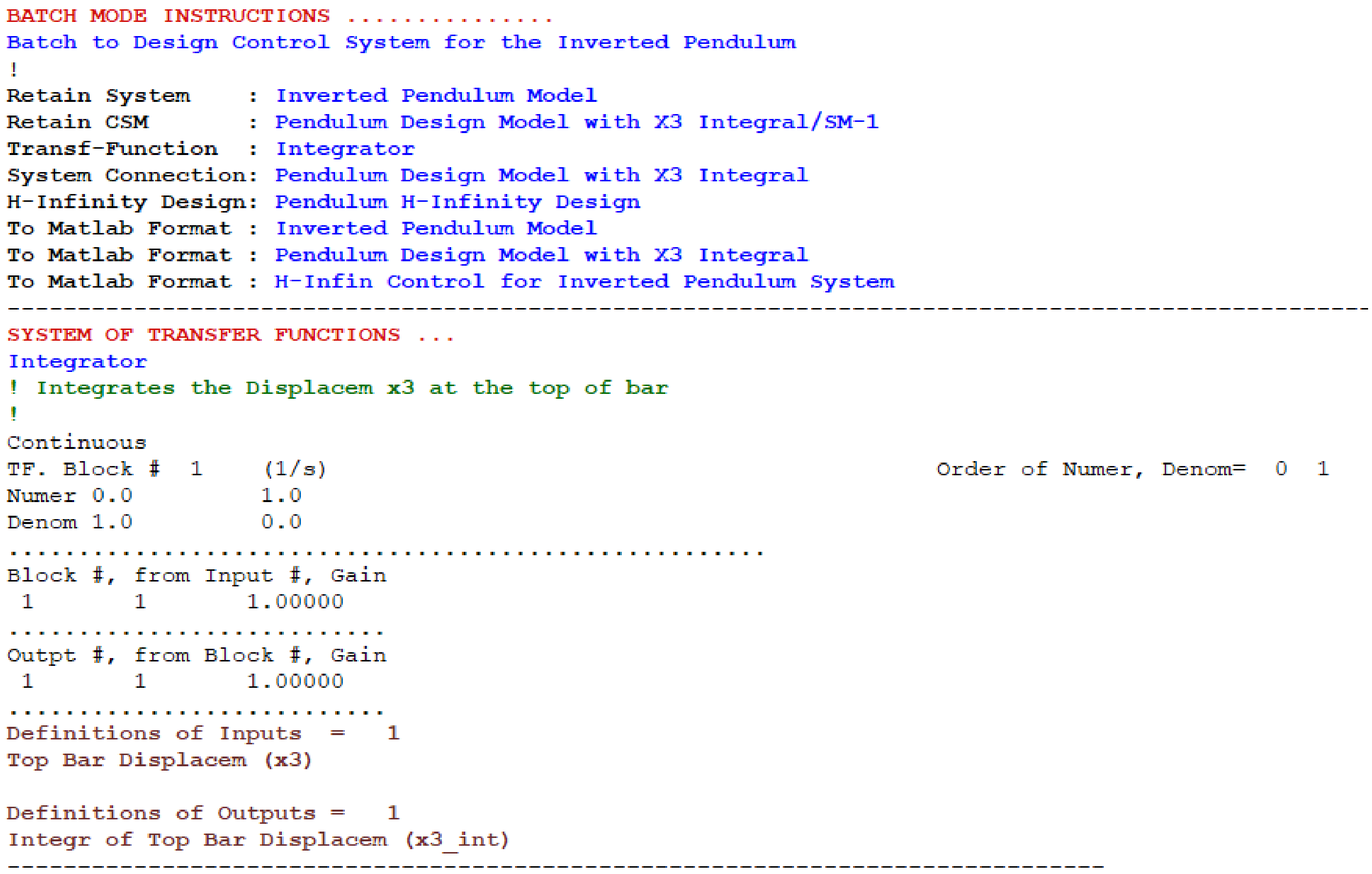
Color Code for Large Magnitudes between One (white) and Infinity (blue)



Closed-Loop Poles of: Pendulum Design Model with X3 Integral/SM-1

Figure 5 Synthesis Model and the Closed-Loop System Eigenvalues

The H-infinity control design is implemented in files “*IP_Design.Inp*” and “*IP_Design.Qdr*” located in folder “*Flixan\Control Analysis\Hinfinity\Examples\Inverted Pendulum\2-Control Design & Analysis*”, and it can be processed by running the batch set. The program preserves the original inverted pendulum system and the SM which are already in the systems file and it processes the H-infinity design dataset “*Pendulum H-Infinity Design*” which is set up to create an output feedback dynamic controller with the peak sensitivity gamma adjusted to γ=36 (dB). The control system is saved in the systems file under the title “*H-Infinity Control for Inverted Pendulum System*”. The plant and controller systems are also converted to Matlab functions “pendulum.m” and “control.m” that can be loaded into Matlab for the control analysis.



```
BATCH MODE INSTRUCTIONS ...............
Batch to Design Control System for the Inverted Pendulum
!
Retain System    : Inverted Pendulum Model
Retain CSM       : Pendulum Design Model with X3 Integral/SM-1
Transf-Function  : Integrator
System Connection: Pendulum Design Model with X3 Integral
H-Infinity Design: Pendulum H-Infinity Design
To Matlab Format : Inverted Pendulum Model
To Matlab Format : Pendulum Design Model with X3 Integral
To Matlab Format : H-Infin Control for Inverted Pendulum System
--------------------------------------------------------------------------------------------------
SYSTEM OF TRANSFER FUNCTIONS ...
Integrator
! Integrates the Displacem x3 at the top of bar
!
Continuous
TF. Block #  1    (1/s)                                           Order of Numer, Denom=   0  1
Numer 0.0         1.0
Denom 1.0         0.0
......................................................
Block #, from Input #, Gain
 1       1        1.00000
...........................
Outpt #, from Block #, Gain
 1       1        1.00000
...........................
Definitions of Inputs  =   1
Top Bar Displacem (x3)

Definitions of Outputs =   1
Integr of Top Bar Displacem (x3_int)
-------------------------------------------------------------------------------
```

```
INTERCONNECTION OF SYSTEMS .....
Pendulum Design Model with X3 Integral
! Creates an Augmented plant for control Design by including
! the integral of the position x3 at the top of the bar.
!
Titles of Systems to be Combined
Title 1 Inverted Pendulum Model
Title 2 Integrator
SYSTEM INPUTS TO SUBSYSTEM  1                                               Plant(s)
Via Matrix +I3                                                              All 3 Inputs
.............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                            Plant Outputs
Via Matrix +I4                                                              All 4 Outputs
.............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                                            Integrator
System Output  5 from Subsystem  2, Output  1, Gain= 1.0                    X3 integral
.............................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2                                     Plant Outp to Control Input
Subsystem  1, Output  3 to Subsystem  2, Input  1, Gain= 1.0                X3 displacem
.............................................................................
Definitions of Inputs  =   3
Control Force on Cart m1, Fc
Torque at the Hinge of Vertical Bar, Tc
External Force at top of Vertical Bar, Fext

Definitions of States  =   7
Cart Mass-1 Position, x1
Bar Angle from Vertical, Theta
Pendulum Mass mp X-Displacem from Hinge, Xp
Cart Mass-1 Velocity, x1-dot
Bar Angular Rate, Theta-dot
Pendulum Mass mp Horiz Velocity, Xp-dot
Integral of Top Bar x-position, x3-integr

Definitions of Outputs =   5
Cart Mass-1 Position x1
Bar Angle from Vertical, Theta
X-Position at Top of Bar (Pend. Suspension), x3
Pendulum Swing Angle, Alpha
Integral of Top Bar x-position, x3-integr
-----------------------------------------------------------------------------------------------------
H-INFINITY CONTROL DESIGN ......
Pendulum H-Infinity Design
Synthesis Model for Control Design in file (.Qdr) :        Pendulum Design Model with X3 Integral/SM-1
Peak Value of the Sensitivity Function Gamma (dB) :        30.0
Dynamic Output Feedback via an Estimator for      :        Inverted Pendulum System
-----------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT .......      (Title, System/Matrix, m-filename)
Inverted Pendulum Model
System
pendulum
-----------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT .......      (Title, System/Matrix, m-filename)
Pendulum Design Model with X3 Integral
System
pend_int
-----------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT .......      (Title, System/Matrix, m-filename)
H-Infin Control for Inverted Pendulum System
System
control
-----------------------------------------------------------------------------------------------------
```

4. Control Analysis

The initialization file “init2.m” loads the linearized plant model and the control system into Matlab for control analysis and simulation. The file “frequ.m” is used to calculate the Bode and Nichols plots.

```
% Init2.m
r2d=180/pi;
[Ad,Bd,Cd,Dd]= pendulum;        % Load the Plant Model
[Ag,Bg,Cg,Dg]= aug_pend;        % Load Augmented Plant Model
[Ac,Bc,Cc,Dc]= control;         % Load H-infinity Control System
```

```
% Frequency Response Analysis
init2
[Al,Bl,Cl,Dl]= linmod('Open_Loop'); % Stabil Anal Open-Loop Simulink model
sys= ss(Al,Bl,Cl,Dl);               % Create SS System
wl=logspace(-2, 2, 20000);          % Define Frequ Range
figure(1); nichols(sys,sys,wl)      % Nichols Plot
figure(2); bode(sys,wl)             % Bode Plot
```

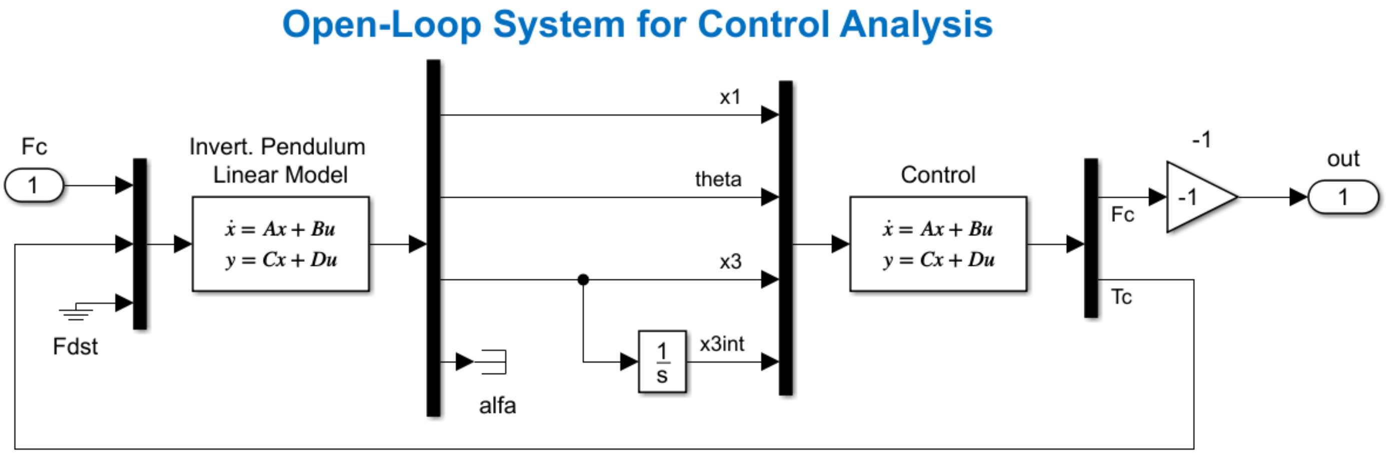


Figure 6 Stability Analysis Model “Open_Loop.slx” Shown with the Cart Force Loop Opened and the Hinge Torque Loop Closed.

The Simulink model “Open_Loop.slx” in Figure-6 is used to analyze the control system stability by creating Bode and Nichols plots for the two control loops. One loop is opened at a time with the other loop closed. The inputs to the control system are the four measurements that were defined in the SM. That is, the cart position x_1, the angle θ of the bar from vertical, the horizontal location at the top of the bar x_3, and the integral of x_3. The control system outputs are the control force Fc on m_1 and the control torque Tc at the hinge between the bar and m_1. The phase and gain margins of the two loops are shown in the two Nichols plots in Figure 7. They were created separately from the Open-Loop model in Figure 6 by opening one loop at a time. The pendulum mode at 6 (rad/s) is phase-stabilized. High gain at the pendulum resonance provide active attenuation for that mode.

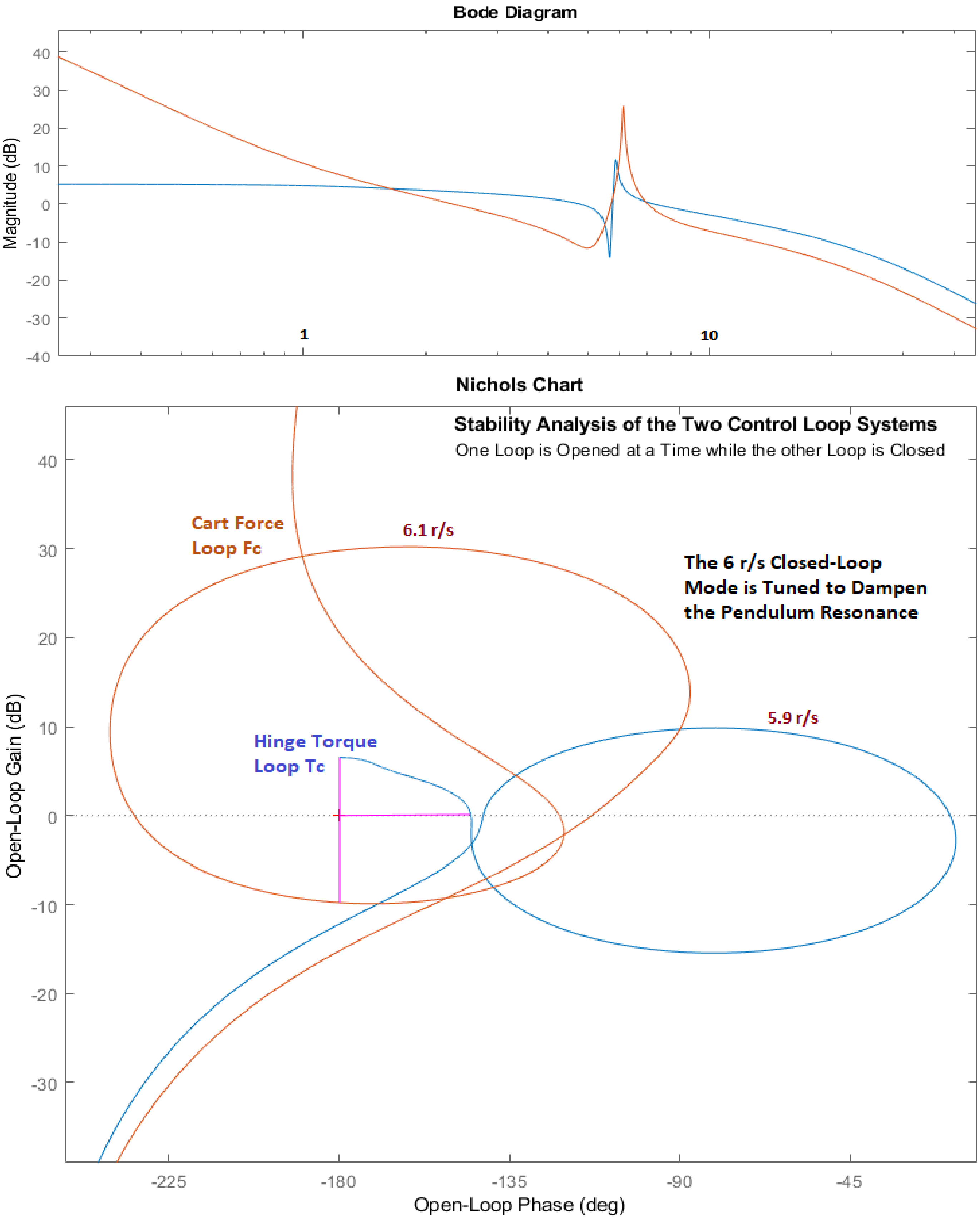


Figure 7 Nichols Plots Showing Stability Margins in the Two Control Loops

5. Simulation

The simulation model "*Linear_Sim.slx*" in Figure 8, includes the same linear model and controller as Figure 6 and it is used to analyze the linear system response to position commands and disturbances. We will not show the results here because they are similar to the non-linear model and we would rather move to the non-linear analysis folder "…*Examples**Inverted Pendulum**3-Non-Linear Closed-Loop Sim*" and run the Simulink model "*Closed_Loop_Sim.slx*" which is shown in Figure-9 and includes the non-linear equations that were implemented in "*Inverted_Pendulum.slx*" in Figure-3. It is initialized by the script "init3.m".

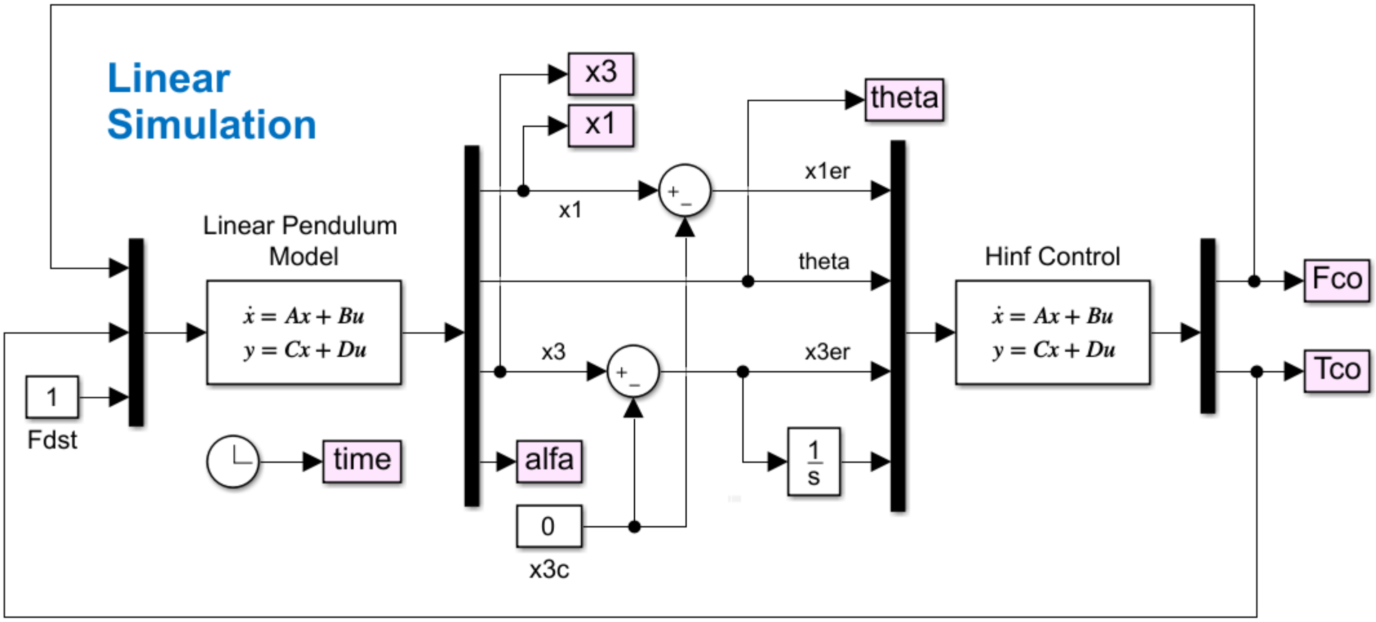


Figure 8 Linear Simulation Model "Linear_Sim.slx"

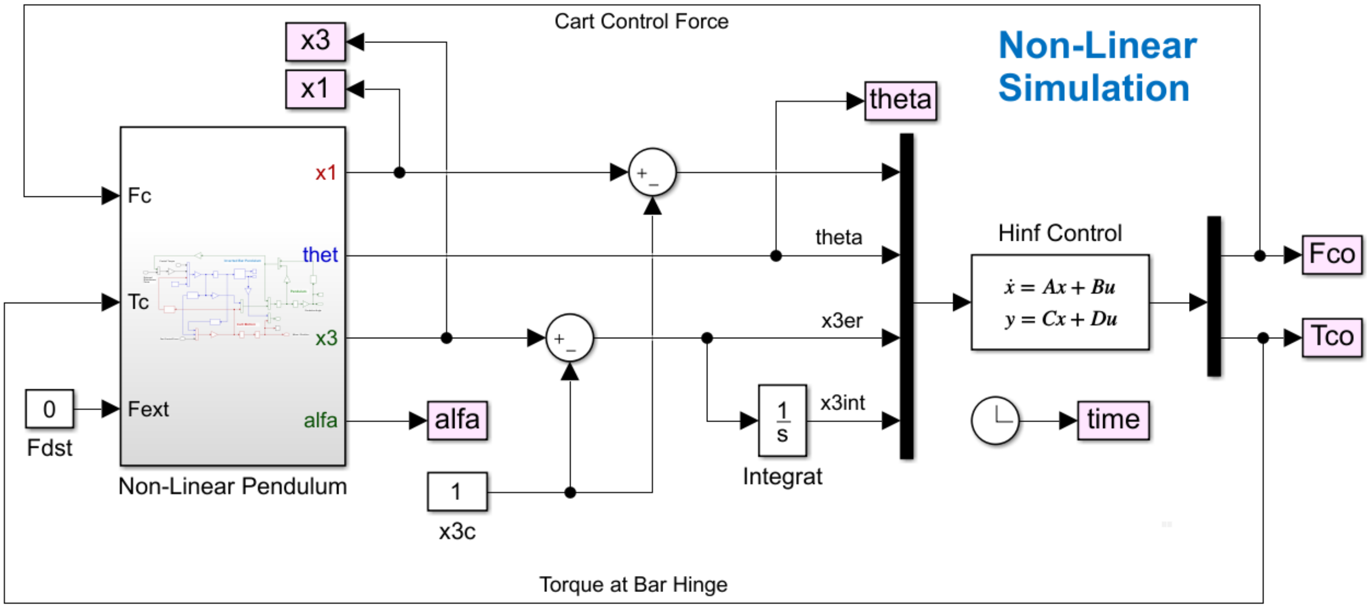
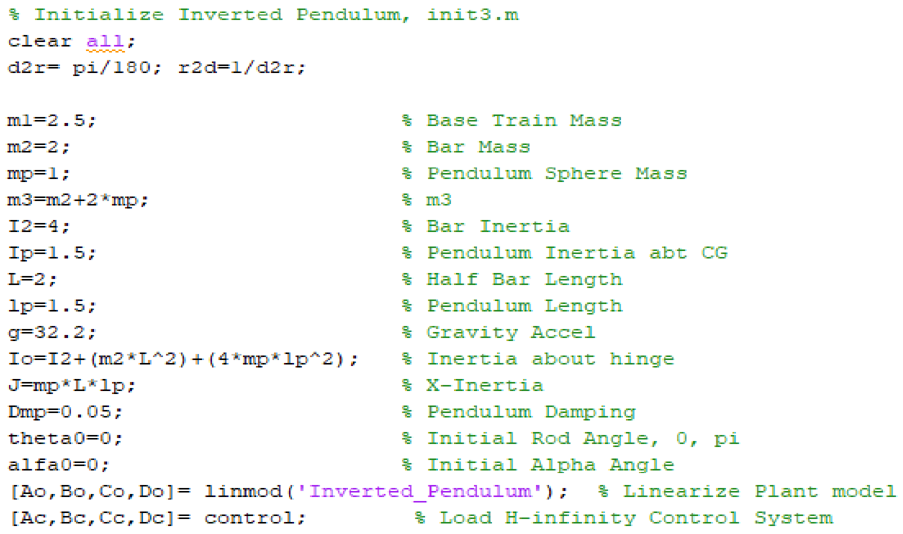
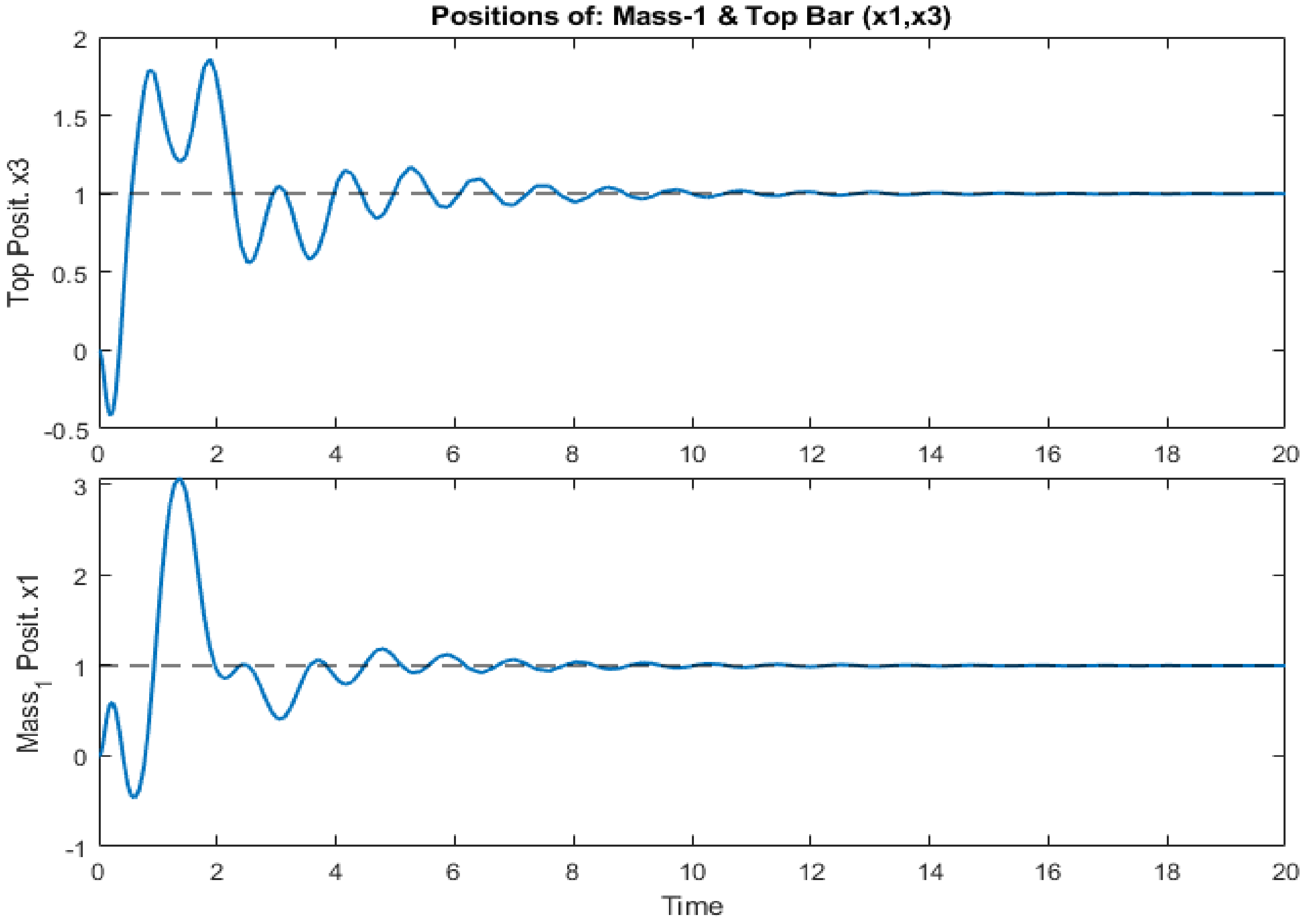


Figure 9 Non-Linear Simulation Model "Closed_Loop_Sim.slx"



```
% Initialize Inverted Pendulum, init3.m
clear all;
d2r= pi/180; r2d=1/d2r;

m1=2.5;                        % Base Train Mass
m2=2;                          % Bar Mass
mp=1;                          % Pendulum Sphere Mass
m3=m2+2*mp;                    % m3
I2=4;                          % Bar Inertia
Ip=1.5;                        % Pendulum Inertia abt CG
L=2;                           % Half Bar Length
lp=1.5;                        % Pendulum Length
g=32.2;                        % Gravity Accel
Io=I2+(m2*L^2)+(4*mp*lp^2);    % Inertia about hinge
J=mp*L*lp;                     % X-Inertia
Dmp=0.05;                      % Pendulum Damping
theta0=0;                      % Initial Rod Angle, 0, pi
alfa0=0;                       % Initial Alpha Angle
[Ao,Bo,Co,Do]= linmod('Inverted_Pendulum');  % Linearize Plant model
[Ac,Bc,Cc,Dc]= control;         % Load H-infinity Control System
```

Response to Position Command: The non-linear simulation model "*Closed_Loop_Sim*" is initialized from file "*init3.m*" which loads the control system from the previous step. We will first examine the closed-loop system's response to a step command in the x_3 position. That is, command the position at the top of the bar which also includes the x_3-integral trim function. In Figure-9 we are commanding both x_3 and x_1 with the same displacement input because we want to keep the bar vertical. Notice that the bar angle θ stabilizes vertical at zero and the pendulum angle α dampens much faster than its original low ζ. Both controls converge at zero because the bar stabilizes vertical. You can also see how the control force and torque are actively counteracting the pendulum oscillations.



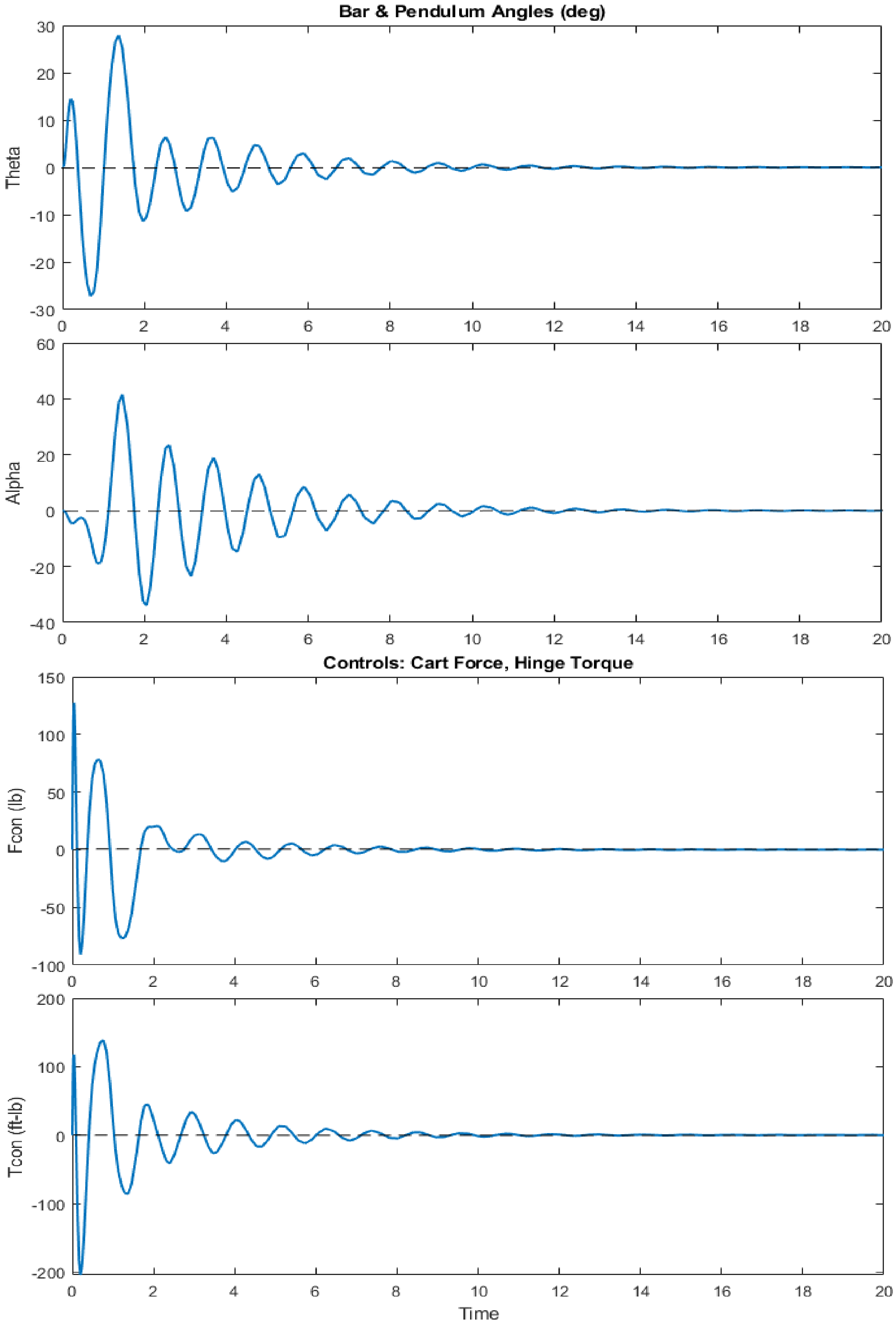


Figure 10 System Response to a Position Change Unit Step-Command in x3

Response to Disturbance Force: It is also interesting to analyze the system's response to an external disturbance force of 10 (lbf) which is applied at the top of the bar, which is x3 the pendulum attach point. The position initially succumbs to the force and moves towards the force direction but eventually the x3-integrator kicks-in and brings it back towards its initial position. The cart x1 initially follows the motion at the top in order to balance the rod but eventually it moves left towards negative values and the rod angle stabilizes at θ= -3.8° against the external force Fext and it applies a positive torque. It is interesting that the control force settles at 10 (lbf) which is exactly opposite to the disturbance force. The negative steady-state moments due to disturbance Fext plus the moments due to pendulum weights are also equal and opposite to the 57 (ft-lb) positive control torque reacting against the disturbance.

To summarize, the control system is able to: (a) to balance the inverted pendulum rod which has the second pendulum attached to it, (b) to command the top position x3 and to maintain its position under the influence of an external force Fext applied at the top of the bar, and (c) to actively dampen the lightly damped oscillations of the attached pendulum.

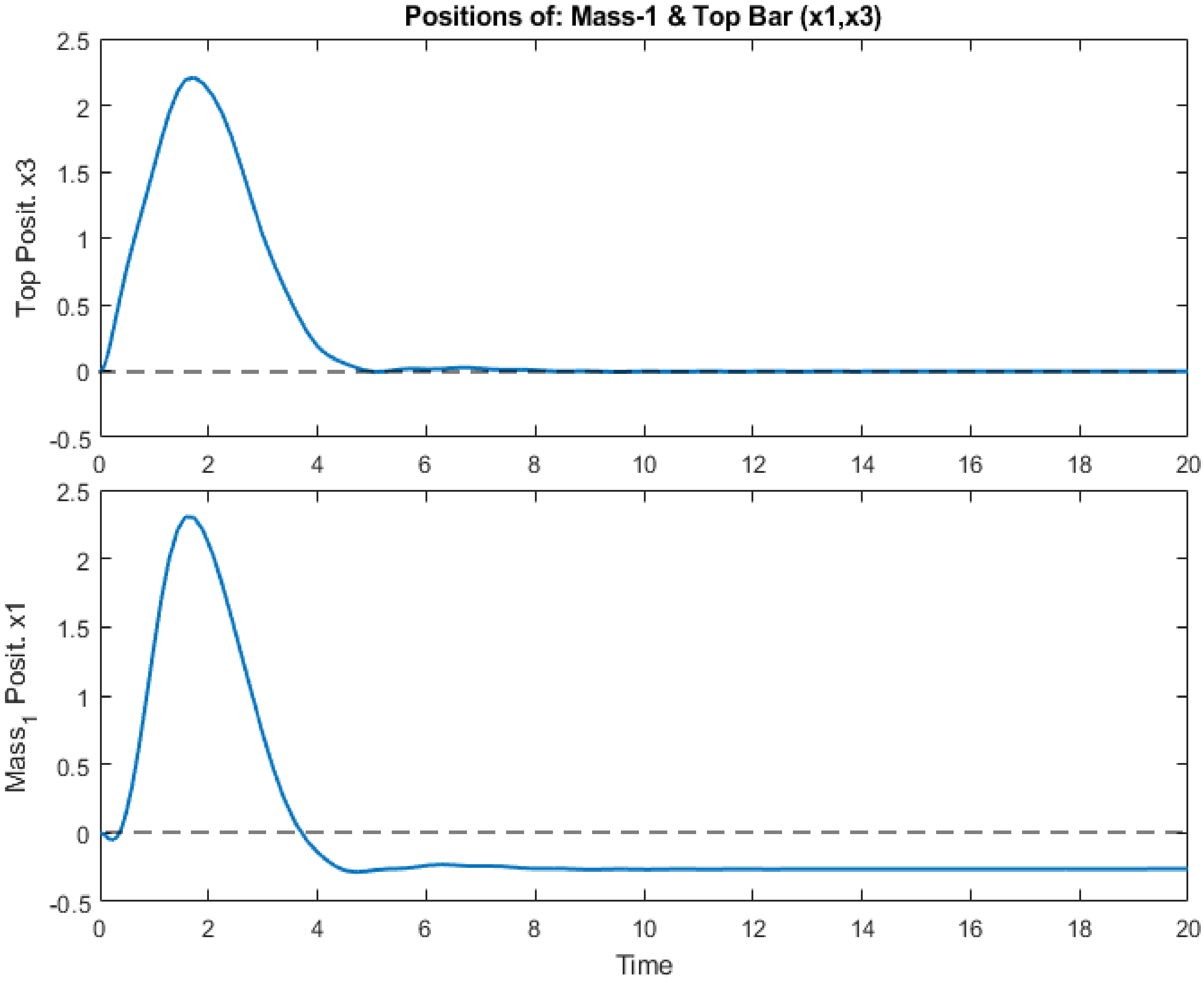
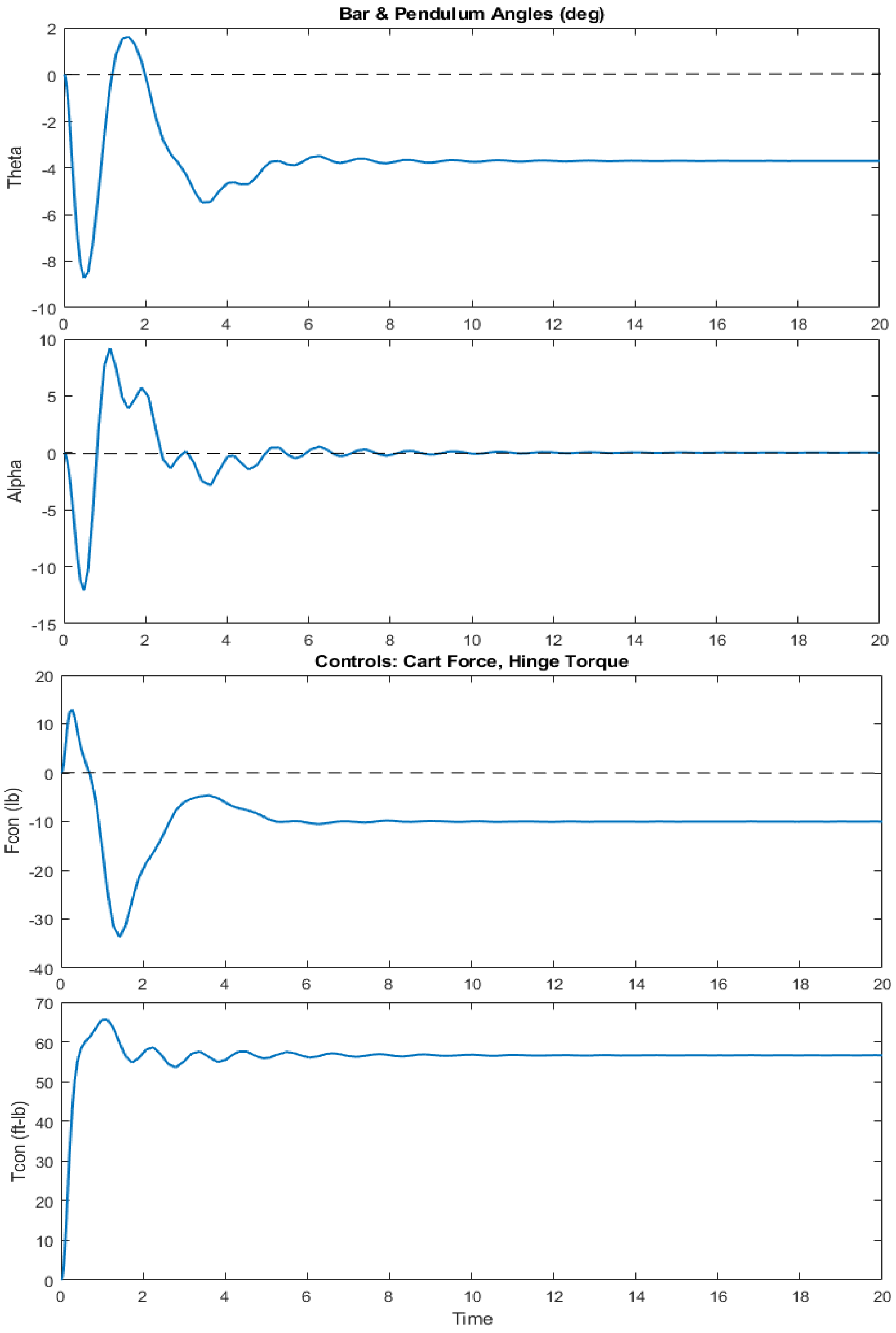


Figure 11 System Response to 10 (lbf) External Disturbance Step Applied at the Top of the Rod


Bar & Pendulum Angles (deg)
Theta
2
0
-2
-4
-6
-8
-10
Alpha
10
5
0
-5
-10
-15
Controls: Cart Force, Hinge Torque
Fcon (lb)
20
10
0
-10
-20
-30
-40
Tcon (ft-lb)
70
60
50
40
30
20
10
0
0
2
4
6
8
10
12
14
16
18
20
Time

Space Shuttle Control Design at Max-Q

This is an H-infinity design demo and we will use the Space Shuttle vehicle during first stage at maximum dynamic pressure to design the flight control system (FCS) at a fixed Max-Q flight condition. Separate pitch and lateral rigid-body models will be used to derive the control gains for the pitch and lateral FCS. The control system gains will be analyzed further using more detailed vehicle models that include slosh and flexibility. Some of the flex modes are unstable and bending filters are included in order to stabilize them. The design and analysis will be separated in sections, beginning with rigid-body pitch and lateral axes, adding slosh, then flexibility, and finally analyzing the coupled pitch and lateral system in the z-domain. The analysis includes stability margins, sensitivity to gusts, and time-slice simulations that show the control system's response to gusts and to attitude commands. Note that, at High-Q the FCS does not receive any significant attitude commands from guidance because the emphasis in this time period of 15-20 sec is to reduce the normal and lateral aerodynamic loads against the structure. This is done by the load-relief system which decreases the loads by applying feedback from the N_Y and N_Z accelerometers. This action, however, degrades the capability of the FCS to efficiently track commands from guidance and its response to attitude commands is slower than at lower Q-pressure. The roll and yaw axes are strongly coupled in this vehicle mainly due to the cross-product of inertia I_{xz} and the huge vertical stabilizer in the back of the Shuttle that couples the roll and yaw axes aerodynamically ($Cl\beta$). Roll and yaw dynamics must, therefore, be analyzed together and cross-coupling gains are derived to compensate for this coupling.

In Section-1 we will design a simple lateral FCS using feedback from the roll and yaw attitudes, (p & r) rates, angle of sideslip beta, and β-integral. In Section-2 we will modify the lateral control system to include a disturbance filter instead of β-integral. The purpose of the disturbance filter is to actively attenuate aero-loading due to gust disturbances coming at a fixed frequency of 1 (rad/sec), rather than at steady-state. Typical aerodynamic disturbances are random, cyclic and their average frequency occurs at around 1 (rad/sec). We, therefore, like to have a dip in the sensitivity transfer-function at that frequency, and this is achieved by including a β-filter in the design process. In section-3 we repeat the H-infinity design in the pitch axis. An α-filter is introduced in the synthesis model that will reduce vehicle sensitivity to aero-loading at 1 (rad/sec). The angles of attack and sideslip, however, are not directly measurable and we must design (α & β) estimators using the accelerometer signals. In Section-4 we solve the lateral problem by designing a different type of β-estimator from the lateral measurements using an H-infinity output-feedback dynamic controller. In Section-5 we use the pitch and lateral state-feedback control gains derived from previous sections to analyze stability of the coupled vehicle system including propellant sloshing and structural flexibility. Two slosh modes and 25 flex modes are included in the vehicle model. Some of the modes are unstable and low-pass and notch filters are included to stabilize them. The filters degrade the original rigid-body margins and some of the gains also needed slight adjustment in order to restore the stability margins without affecting much of the performance. Finally, the vehicle model and the FCS are discretized and the control system is analyzed in the z-domain.

1. Preliminary Lateral H-Infinity Design

In this section we will design a simple state-feedback control law for the roll and yaw axes of the Space Shuttle vehicle. We will create a lateral rigid vehicle design model consisting of states: roll and yaw attitudes (φ, ψ), the body rates (p, r) and angle of sideslip (β). We will augment the design model by adding one more state: (β-integral) and will design a (2x6) H-Infinity state-feedback control law. The TVC matrix is combined with the vehicle model and the design plant has two control inputs: roll and yaw controls (DP & DR). We will then use the augmented vehicle model to create interactively using Flixan the synthesis model (SM) which consists of 9 matrices and some adjustable gains. The gains are experimentally adjusted to trade-off control system bandwidth versus stability and sensitivity to disturbances. A typical SM is shown in Figure-1 and it has 3 sets of inputs and outputs.

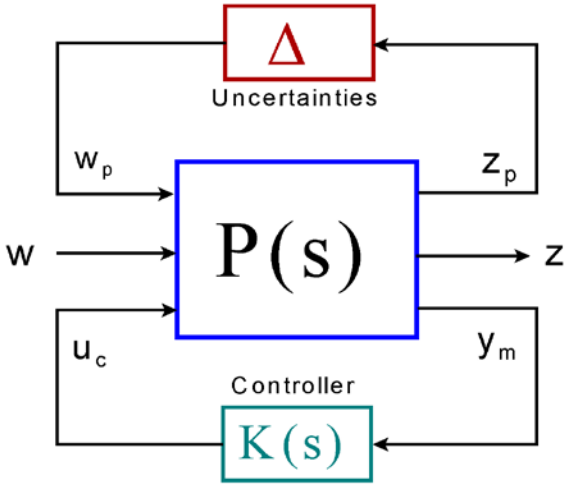
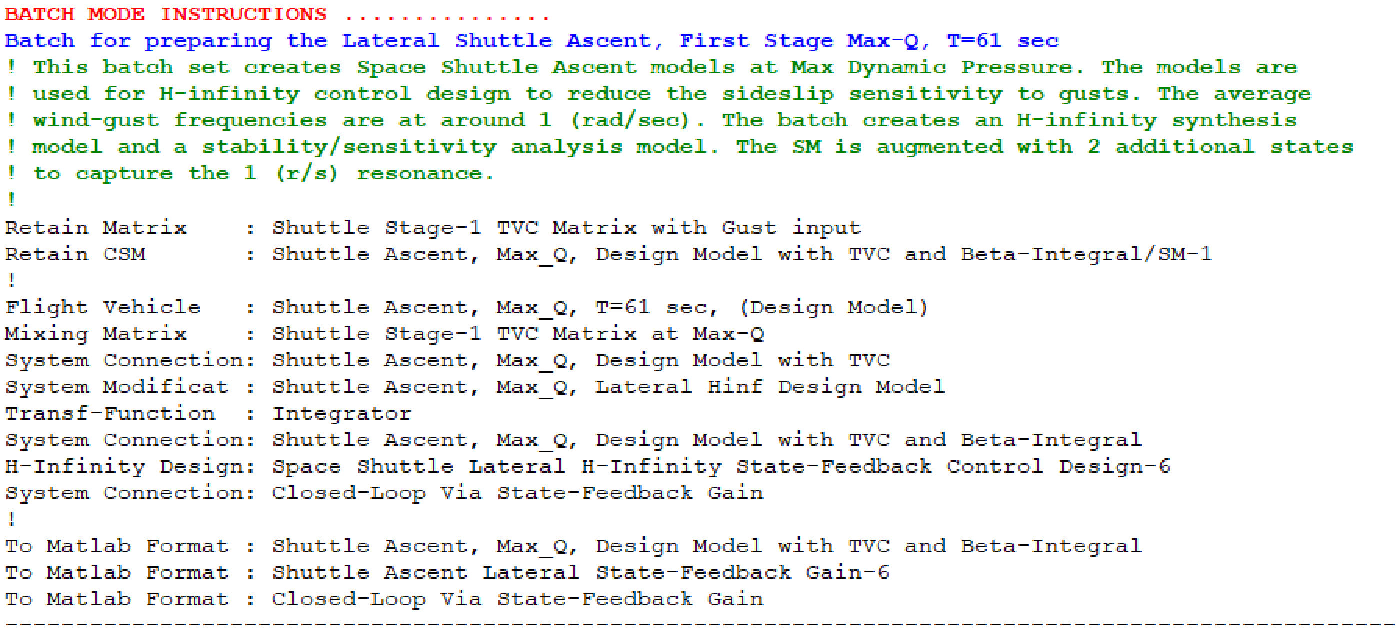


Figure 1 Synthesis Model

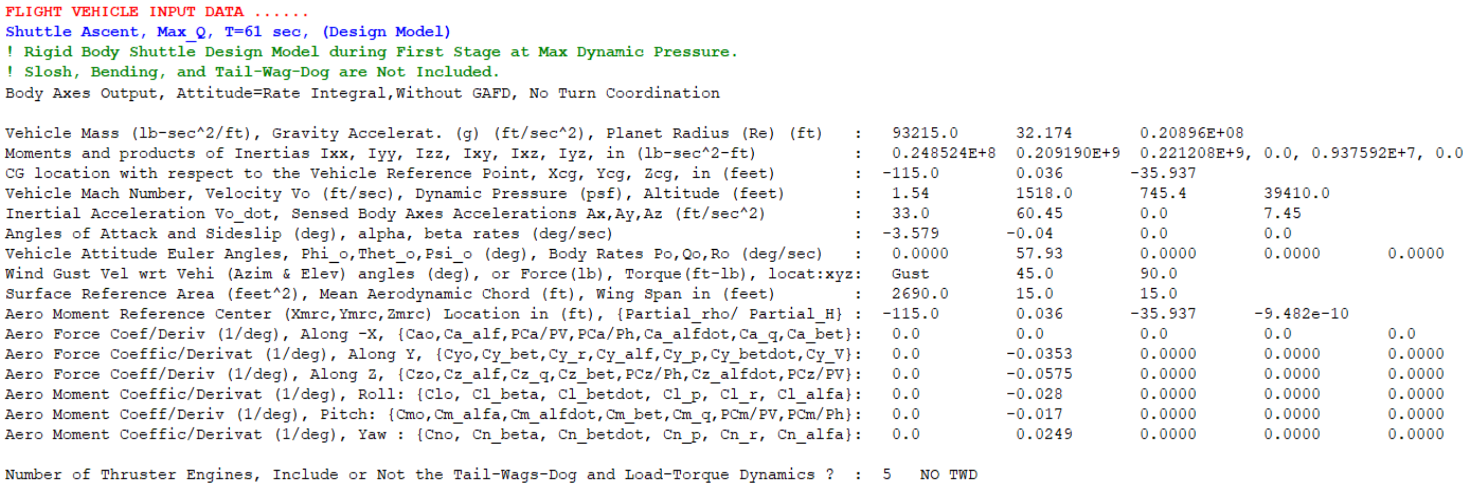
The first set (w_p and z_p) define the fictitious inputs and outputs that connect to plant uncertainties which are extracted to a block Δ. In this example we don't have them because we did not include any parameter uncertainties in the vehicle model. The second input/ output set (w and z) are the external disturbances and performance criteria outputs. In this case the disturbance w is the wind-gust velocity (V_{gust}) and the criterion z is the sideslip angle β which is affected by gusts and it is considered to be a load indicator. The last I/O set (u_c and y_m) are the inputs and measurements used for control. They connect to the control system K(s) which in this case is just a state-feedback.

The input file in this example is "*Lateral_MaxQ1.Inp*" and it is located in this directory: "*Flixan\Control Analysis\Hinfinity\Examples\Shuttle Ascent Hinfinity Design\1-Lateral Design w Beta-Integral*". It includes the vehicle dataset "*Shuttle Ascent, Max_Q, T=61 sec, (Design Model)"* which generates the Shuttle vehicle system at Max-Q. There is a dataset that generates the TVC matrix "Shuttle Stage-1 TVC Matrix at Max-Q" which converts the roll, pitch and yaw FCS demands to 5 pitch and 5 yaw gimbal deflection commands. The TVC and the vehicle model are combined together to system "*Shuttle Ascent, Max_Q, Design Model with TVC*", and then only the lateral states, inputs and outputs are extracted and retained in the subsystem "*Shuttle Ascent, Max_Q, Lateral Hinf Design Model*". The pitch variables are ignored for now. The lateral subsystem is augmented by including a 6th state, the β-integrator, and creating the lateral design plant "*Shuttle Ascent, Max_Q, Design Model with TVC and Beta-Integral*" which will be used in Section 1.2 to create the SM. A batch set is included that can perform the entire modeling and design process in batch mode, but for now we will describe the interactive process in detail.

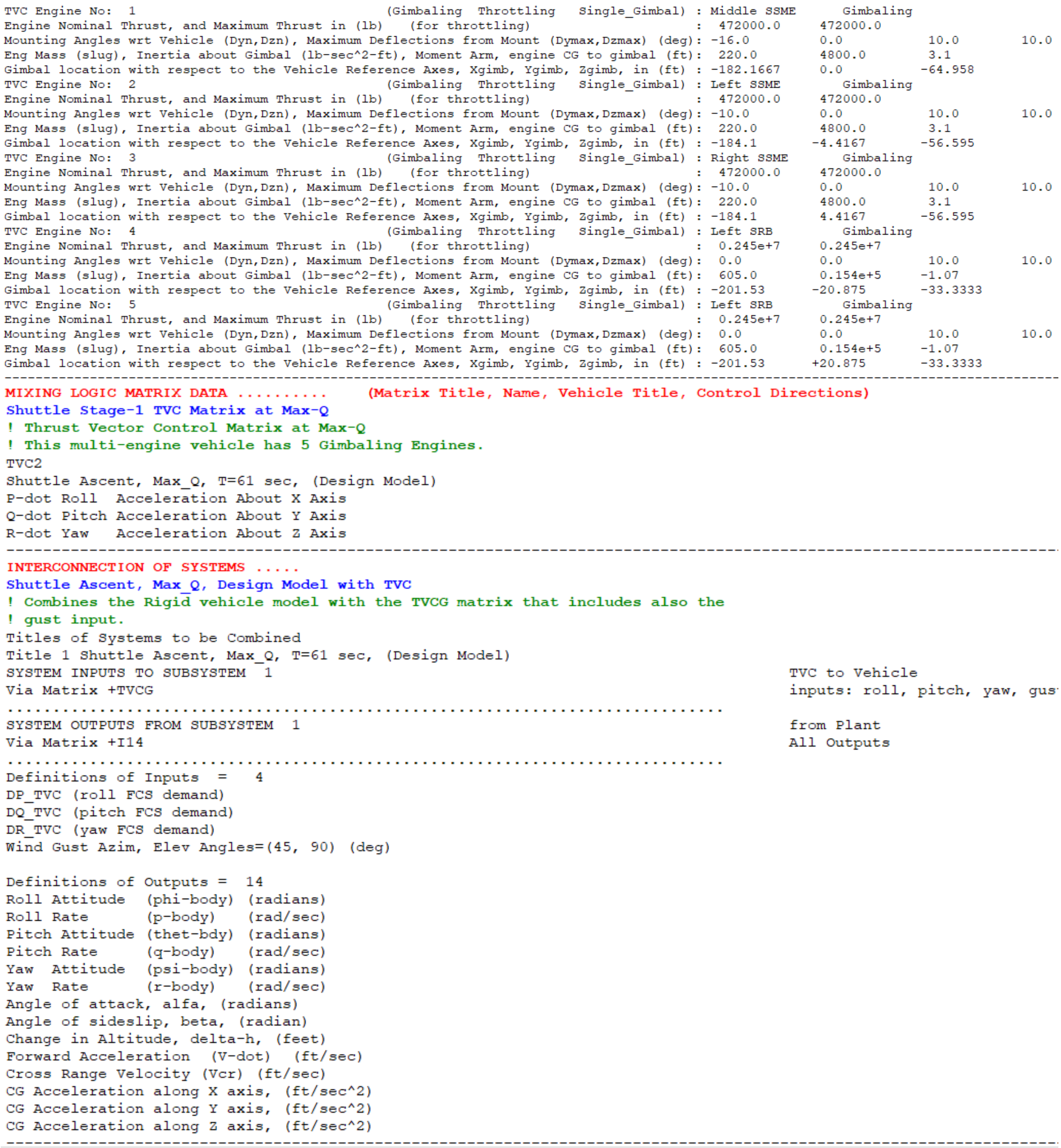
1.1 Input File: Lateral_MaxQ1.Inp



```
BATCH MODE INSTRUCTIONS ...............
Batch for preparing the Lateral Shuttle Ascent, First Stage Max-Q, T=61 sec
! This batch set creates Space Shuttle Ascent models at Max Dynamic Pressure. The models are
! used for H-infinity control design to reduce the sideslip sensitivity to gusts. The average
! wind-gust frequencies are at around 1 (rad/sec). The batch creates an H-infinity synthesis
! model and a stability/sensitivity analysis model. The SM is augmented with 2 additional states
! to capture the 1 (r/s) resonance.
!
Retain Matrix     : Shuttle Stage-1 TVC Matrix with Gust input
Retain CSM        : Shuttle Ascent, Max_Q, Design Model with TVC and Beta-Integral/SM-1
!
Flight Vehicle    : Shuttle Ascent, Max_Q, T=61 sec, (Design Model)
Mixing Matrix     : Shuttle Stage-1 TVC Matrix at Max-Q
System Connection: Shuttle Ascent, Max_Q, Design Model with TVC
System Modificat : Shuttle Ascent, Max_Q, Lateral Hinf Design Model
Transf-Function   : Integrator
System Connection: Shuttle Ascent, Max_Q, Design Model with TVC and Beta-Integral
H-Infinity Design: Space Shuttle Lateral H-Infinity State-Feedback Control Design-6
System Connection: Closed-Loop Via State-Feedback Gain
!
To Matlab Format : Shuttle Ascent, Max_Q, Design Model with TVC and Beta-Integral
To Matlab Format : Shuttle Ascent Lateral State-Feedback Gain-6
To Matlab Format : Closed-Loop Via State-Feedback Gain
----------------------------------------------------------------------------------------------------
```

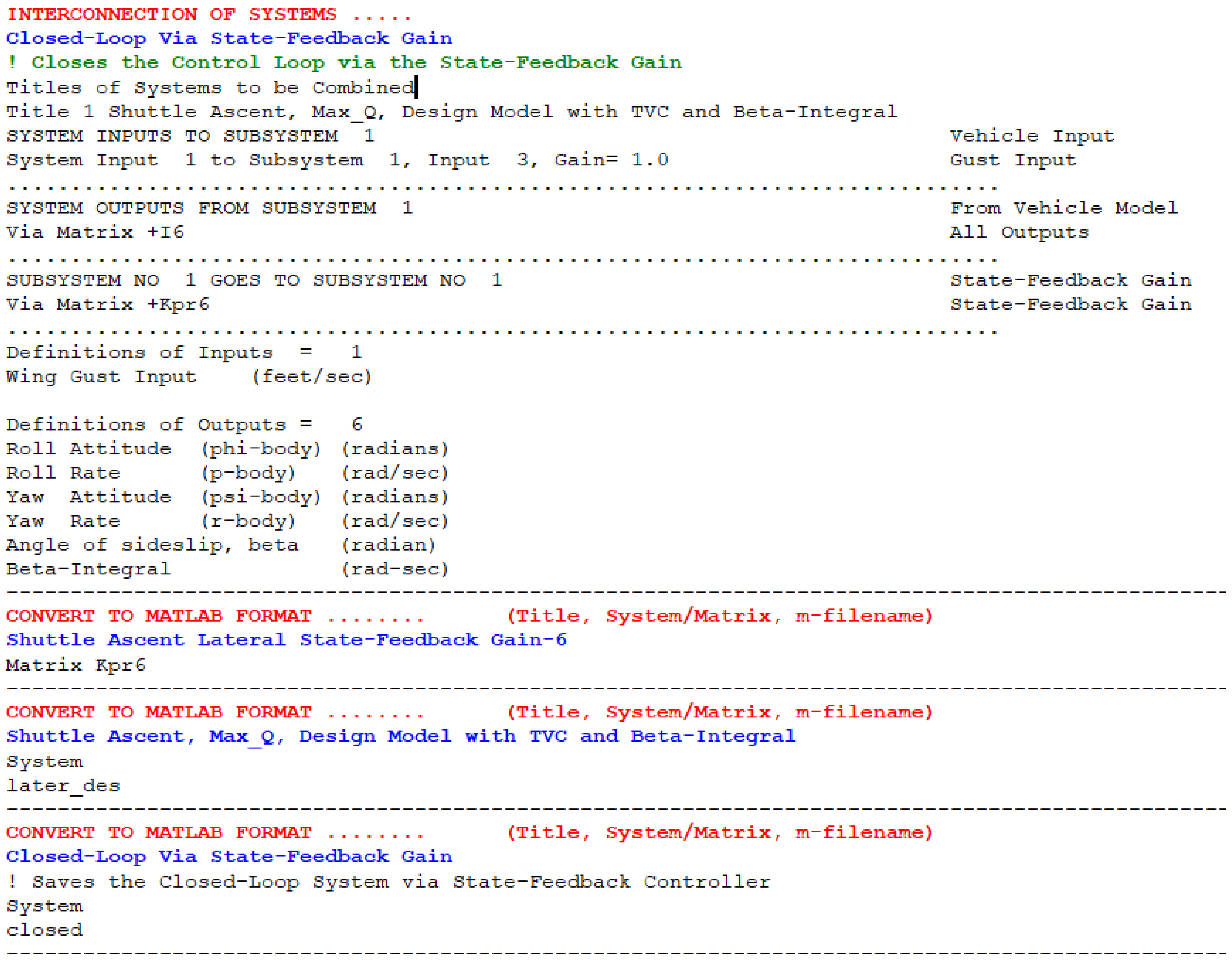


```
FLIGHT VEHICLE INPUT DATA ......
Shuttle Ascent, Max_Q, T=61 sec, (Design Model)
! Rigid Body Shuttle Design Model during First Stage at Max Dynamic Pressure.
! Slosh, Bending, and Tail-Wag-Dog are Not Included.
Body Axes Output, Attitude=Rate Integral,Without GAFD, No Turn Coordination

Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Planet Radius (Re) (ft)   :   93215.0      32.174       0.20896E+08
Moments and products of Inertias Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)          :   0.248524E+8  0.209190E+9  0.221208E+9, 0.0, 0.937592E+7, 0.0
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)        :  -115.0        0.036       -35.937
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)       :   1.54         1518.0       745.4        39410.0
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)         :   33.0         60.45        0.0          7.45
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                         :  -3.579       -0.04         0.0          0.0
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)   :   0.0000       57.93        0.0000       0.0000       0.0000
Wind Gust Vel wrt Vehi (Azim & Elev) angles (deg), or Force(lb), Torque(ft-lb), locat:xyz:   Gust         45.0         90.0
Surface Reference Area (feet^2), Mean Aerodynamic Chord (ft), Wing Span in (feet)        :   2690.0       15.0         15.0
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial_rho/ Partial_H} :  -115.0        0.036       -35.937      -9.482e-10
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:   0.0          0.0          0.0          0.0          0.0
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}:   0.0         -0.0353       0.0000       0.0000       0.0000
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}:   0.0         -0.0575       0.0000       0.0000       0.0000
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:   0.0         -0.028        0.0000       0.0000       0.0000
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}:   0.0         -0.017        0.0000       0.0000       0.0000
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}:   0.0          0.0249       0.0000       0.0000       0.0000

Number of Thruster Engines, Include or Not the Tail-Wags-Dog and Load-Torque Dynamics ?  :  5   NO TWD
```



```
TVC Engine No:  1                              (Gimbaling  Throttling   Single_Gimbal) : Middle SSME      Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)                   :  472000.0     472000.0
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): -16.0        0.0          10.0         10.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  220.0       4800.0       3.1
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -182.1667    0.0          -64.958
TVC Engine No:  2                              (Gimbaling  Throttling   Single_Gimbal) : Left SSME        Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)                   :  472000.0     472000.0
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): -10.0        0.0          10.0         10.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  220.0       4800.0       3.1
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -184.1      -4.4167       -56.595
TVC Engine No:  3                              (Gimbaling  Throttling   Single_Gimbal) : Right SSME       Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)                   :  472000.0     472000.0
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): -10.0        0.0          10.0         10.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  220.0       4800.0       3.1
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -184.1       4.4167       -56.595
TVC Engine No:  4                              (Gimbaling  Throttling   Single_Gimbal) : Left SRB         Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)                   :  0.245e+7     0.245e+7
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg):  0.0         0.0          10.0         10.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  605.0       0.154e+5     -1.07
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -201.53     -20.875       -33.3333
TVC Engine No:  5                              (Gimbaling  Throttling   Single_Gimbal) : Left SRB         Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)                   :  0.245e+7     0.245e+7
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg):  0.0         0.0          10.0         10.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  605.0       0.154e+5     -1.07
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -201.53     +20.875       -33.3333
----------------------------------------------------------------------------------------------------------------------------------
MIXING LOGIC MATRIX DATA ..........    (Matrix Title, Name, Vehicle Title, Control Directions)
Shuttle Stage-1 TVC Matrix at Max-Q
! Thrust Vector Control Matrix at Max-Q
! This multi-engine vehicle has 5 Gimbaling Engines.
TVC2
Shuttle Ascent, Max_Q, T=61 sec, (Design Model)
P-dot Roll  Acceleration About X Axis
Q-dot Pitch Acceleration About Y Axis
R-dot Yaw   Acceleration About Z Axis
----------------------------------------------------------------------------------------------------
INTERCONNECTION OF SYSTEMS .....
Shuttle Ascent, Max_Q, Design Model with TVC
! Combines the Rigid vehicle model with the TVCG matrix that includes also the
! gust input.
Titles of Systems to be Combined
Title 1 Shuttle Ascent, Max_Q, T=61 sec, (Design Model)
SYSTEM INPUTS TO SUBSYSTEM  1                                            TVC to Vehicle
Via Matrix +TVCG                                                         inputs: roll, pitch, yaw, gus
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                         from Plant
Via Matrix +I14                                                          All Outputs
..............................................................................
Definitions of Inputs  =   4
DP_TVC (roll FCS demand)
DQ_TVC (pitch FCS demand)
DR_TVC (yaw FCS demand)
Wind Gust Azim, Elev Angles=(45, 90) (deg)

Definitions of Outputs =  14
Roll Attitude  (phi-body) (radians)
Roll Rate      (p-body)   (rad/sec)
Pitch Attitude (thet-bdy) (radians)
Pitch Rate     (q-body)   (rad/sec)
Yaw  Attitude  (psi-body) (radians)
Yaw  Rate      (r-body)   (rad/sec)
Angle of attack, alfa, (radians)
Angle of sideslip, beta, (radian)
Change in Altitude, delta-h, (feet)
Forward Acceleration  (V-dot)  (ft/sec)
Cross Range Velocity (Vcr) (ft/sec)
CG Acceleration along X axis, (ft/sec^2)
CG Acceleration along Y axis, (ft/sec^2)
CG Acceleration along Z axis, (ft/sec^2)
----------------------------------------------------------------------------------------------------
```

```
SYSTEM OF TRANSFER FUNCTIONS ...
Integrator
Continuous
TF. Block #  1  (1/s)                                    Order of Numer, Denom=  0  1
Numer 0.0        1.0
Denom 1.0        0.0
....................................................
Block #, from Input #, Gain
  1  1    1.0
..........................
Outpt #, from Block #, Gain
  1  1    1.0
....................................................
------------------------------------------------------------------------------------------
CREATE A NEW SYSTEM FROM AN OLD SYSTEM... (Titles of the New and Old Systems)
Shuttle Ascent, Max_Q, Lateral Hinf Design Model
Shuttle Ascent, Max_Q, Design Model with TVC
TRUNCATE OR REORDER THE SYSTEM INPUTS, STATES, AND OUTPUTS
Extract Inputs :    1  3  4
Extract States :    1  2  5  6  8
Extract Outputs:    1  2  5  6  8
------------------------------------------------------------------------------------------
INTERCONNECTION OF SYSTEMS .....
Shuttle Ascent, Max_Q, Design Model with TVC and Beta-Integral
! Same as Above Including Beta Integral
!
Titles of Systems to be Combined
Title 1 Shuttle Ascent, Max_Q, Lateral Hinf Design Model
Title 2 Integrator
SYSTEM INPUTS TO SUBSYSTEM  1                                              TVC to Vehicle
Via Matrix +I3                                                             Inputs: roll, yaw, gust
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1
Via Matrix +I5                                                             All Outputs
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                                           From Integrator
System Output  6 from Subsystem  2, Output  1, Gain= 1.0                   Beta-Integral
..............................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2                                    Beta to Integrator
Subsystem  1, Output  5 to Subsystem  2, Input  1, Gain= 1.0               Beta
..............................................................................
Definitions of Inputs  =   3
DP_TVC (roll FCS demand)
DR_TVC (yaw FCS demand)
Wind-Gust Azim, Elev Angles=(45,90) (deg)

Definitions of Outputs =   6
Roll Attitude  (phi-body) (radians)
Roll Rate      (p-body)   (rad/sec)
Yaw  Attitude  (psi-body) (radians)
Yaw  Rate      (r-body)   (rad/sec)
Angle of sideslip, beta   (radian)
Beta-Integral             (rad-sec)
------------------------------------------------------------------------------------------
H-INFINITY CONTROL DESIGN ......
Space Shuttle Lateral H-Infinity State-Feedback Control Design-6
Synthesis Model for Control Design in file (.Qdr) :        Shuttle Ascent, Max_Q, Design Model with TVC and Beta-Integral/
Peak Value of the Sensitivity Function Gamma (dB) :        30.0
State-Feedback Control Solution via Gain          :Kpr6    Shuttle Ascent Lateral State-Feedback Gain-6
------------------------------------------------------------------------------------------
```



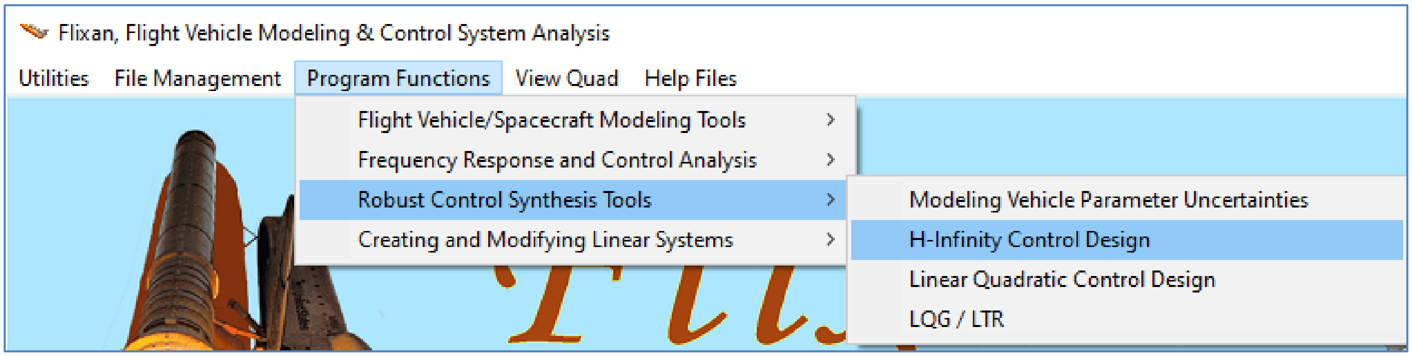
```
INTERCONNECTION OF SYSTEMS .....
Closed-Loop Via State-Feedback Gain
! Closes the Control Loop via the State-Feedback Gain
Titles of Systems to be Combined
Title 1 Shuttle Ascent, Max_Q, Design Model with TVC and Beta-Integral
SYSTEM INPUTS TO SUBSYSTEM  1                                         Vehicle Input
System Input  1 to Subsystem  1, Input  3, Gain= 1.0                  Gust Input
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                      From Vehicle Model
Via Matrix +I6                                                        All Outputs
..............................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  1                               State-Feedback Gain
Via Matrix +Kpr6                                                      State-Feedback Gain
..............................................................................
Definitions of Inputs  =   1
Wing Gust Input    (feet/sec)

Definitions of Outputs =   6
Roll Attitude  (phi-body) (radians)
Roll Rate      (p-body)   (rad/sec)
Yaw  Attitude  (psi-body) (radians)
Yaw  Rate      (r-body)   (rad/sec)
Angle of sideslip, beta   (radian)
Beta-Integral             (rad-sec)
-----------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Shuttle Ascent Lateral State-Feedback Gain-6
Matrix Kpr6
-----------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Shuttle Ascent, Max_Q, Design Model with TVC and Beta-Integral
System
later_des
-----------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Closed-Loop Via State-Feedback Gain
! Saves the Closed-Loop System via State-Feedback Controller
System
closed
-----------------------------------------------------------------------------------------------
```

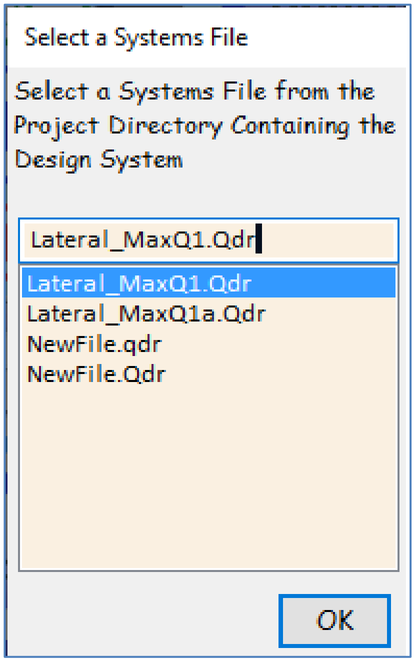
1.2 Creating the Synthesis Model

The systems and matrices of this example are saved in file: “Lateral_MaxQ1.Qdr”. The SM consists of 9 matrices and it will also be saved in the same systems file. It will be created from the design plant “*Shuttle Ascent, Max_Q, Design Model with TVC and Beta-Integral*” which is an (A, B, C, D) system by extracting some inputs and outputs and placing them into groups as we will describe in the following interactive process. The SM includes the plant dynamics and also defines the trade-off requirements that the H-infinity algorithm must optimize. The designer must define which inputs are disturbances and which inputs are controls. Also, which outputs are criteria to be optimized and which ones are measurements. In this example the measurements are the state-vector and we won’t need an estimator. We define the trade-off between bandwidth and performance versus sensitivity and stability in the optimization algorithm by adjusting some gains which are “knobs” that scale the disturbance inputs and some gains that scale the criteria outputs. Obviously, initially we don’t know what gains will produce the desired performance versus stability, so we begin to scale the disturbance inputs by entering the magnitudes of the maximum expected disturbances in the input gains, and for the output gains we enter the maximum allowed magnitude at each performance criterion. The controls are also included in the criteria outputs and we must scale them by the maximum amount of control allowed. In contrast, the measurement noise is also included in the disturbances vector and we must enter the maximum value of noise at each measurement. Fortunately, in this example the state-vector is measurable and we are not estimating it. Our measurement noise is zero and we enter very small values for measurement noise.

Those gains are used to properly scale the disturbance inputs and criteria outputs in the optimization. Typically, several iterations are needed to converge to the desired trade-off of performance versus robustness. A simple, preliminary simulation model is used to evaluate the design. If we find that we are using too much control, we must reduce the corresponding control gain in the performance criteria output and repeat the design. If a regulated output such as vehicle attitude doesn't converge to its commanded value fast, the gain of the corresponding attitude criterion must be reduced.

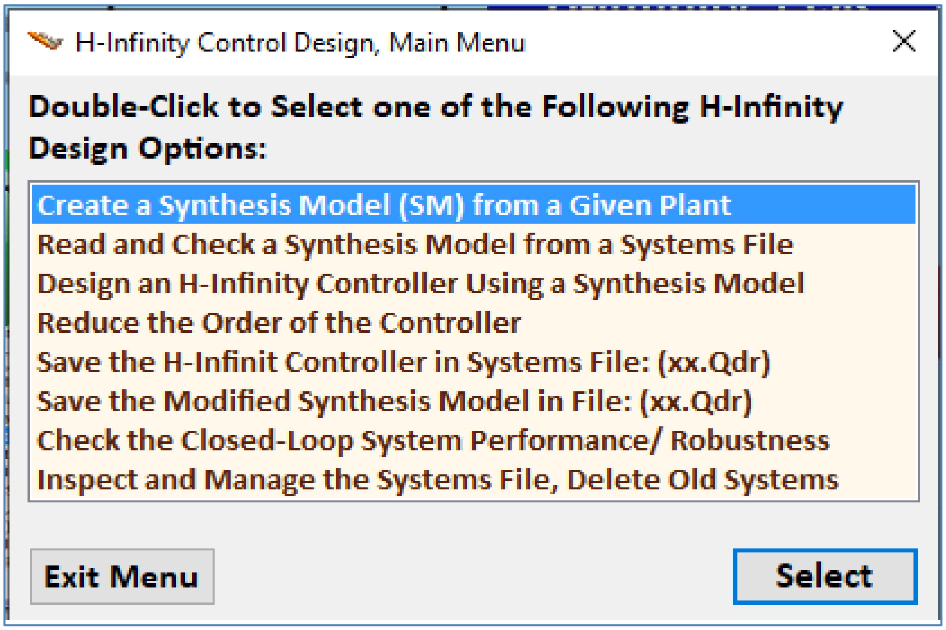
To create the Synthesis Model, we begin by running the H-Infinity Control Design program from the Flixan main menu and then select the systems file that contains the design vehicle plant and where the SM will be saved. Then from the H-Infinity main menu select "Create a Synthesis Model (SM)" to create the SM from the vehicle design model.




Select a Systems File

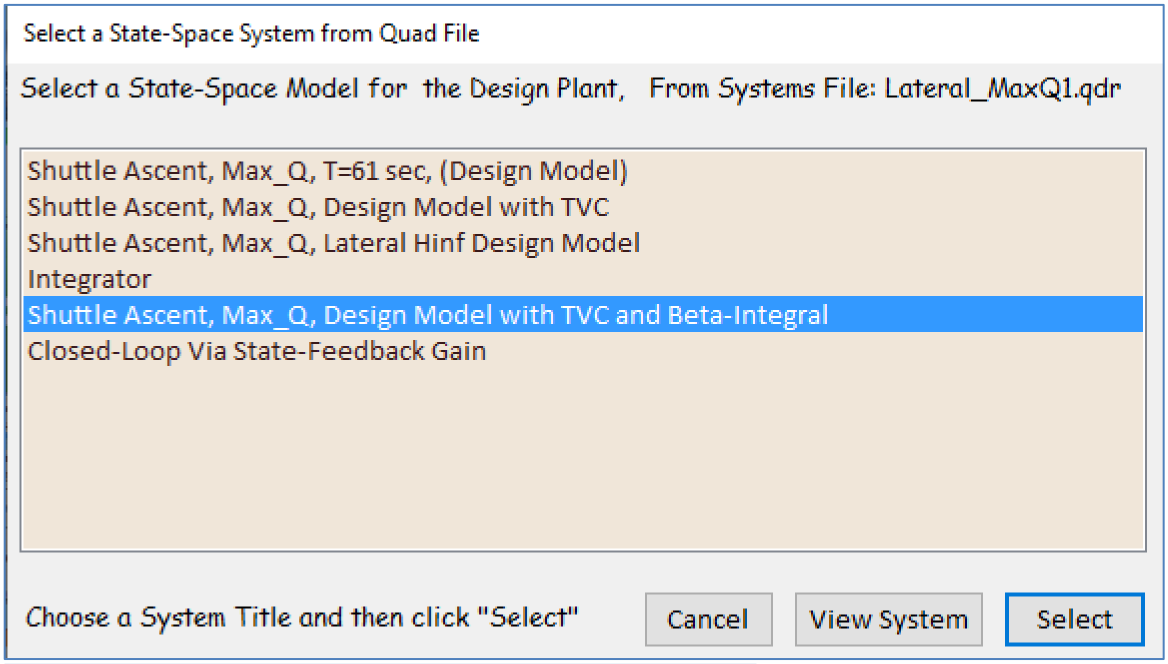
Select a Systems File from the Project Directory Containing the Design System

Lateral_MaxQ1.Qdr


H-Infinity Control Design, Main Menu

Double-Click to Select one of the Following H-Infinity Design Options:

The following menu shows the titles of the systems which are included in the systems file. Select the vehicle design plant that includes β-integral and click on "Select". The H-infinity SM will be created from this system.

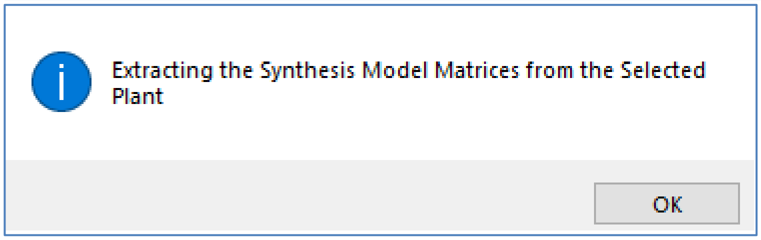


Select a State-Space System from Quad File

Select a State-Space Model for the Design Plant, From Systems File: Lateral_MaxQ1.qdr

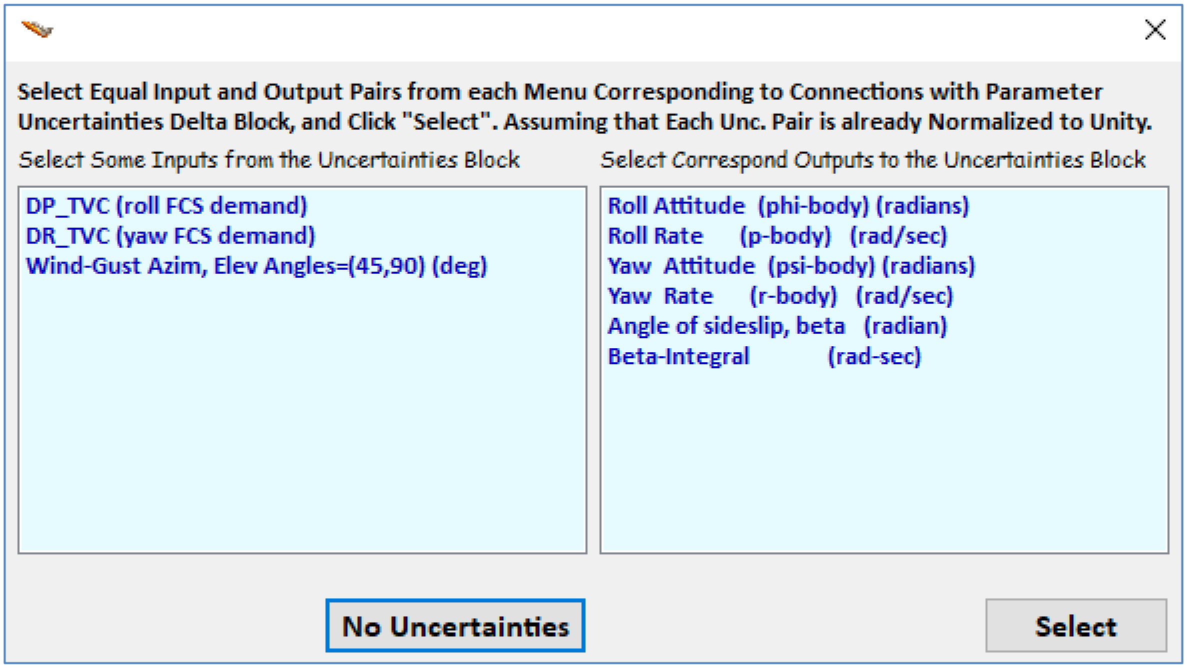
Shuttle Ascent, Max_Q, T=61 sec, (Design Model)
Shuttle Ascent, Max_Q, Design Model with TVC
Shuttle Ascent, Max_Q, Lateral Hinf Design Model
Integrator
Shuttle Ascent, Max_Q, Design Model with TVC and Beta-Integral
Closed-Loop Via State-Feedback Gain

Choose a System Title and then click "Select"



Extracting the Synthesis Model Matrices from the Selected Plant

The first menu is used for defining parameter uncertainties. That is, pre-scaled inputs and outputs that connect to the uncertainties Δ block. In this example we have not defined any uncertain parameters and have not created any uncertainty inputs and outputs. We therefore, click on “No Uncertainties” to proceed.



Select Equal Input and Output Pairs from each Menu Corresponding to Connections with Parameter Uncertainties Delta Block, and Click "Select". Assuming that Each Unc. Pair is already Normalized to Unity.

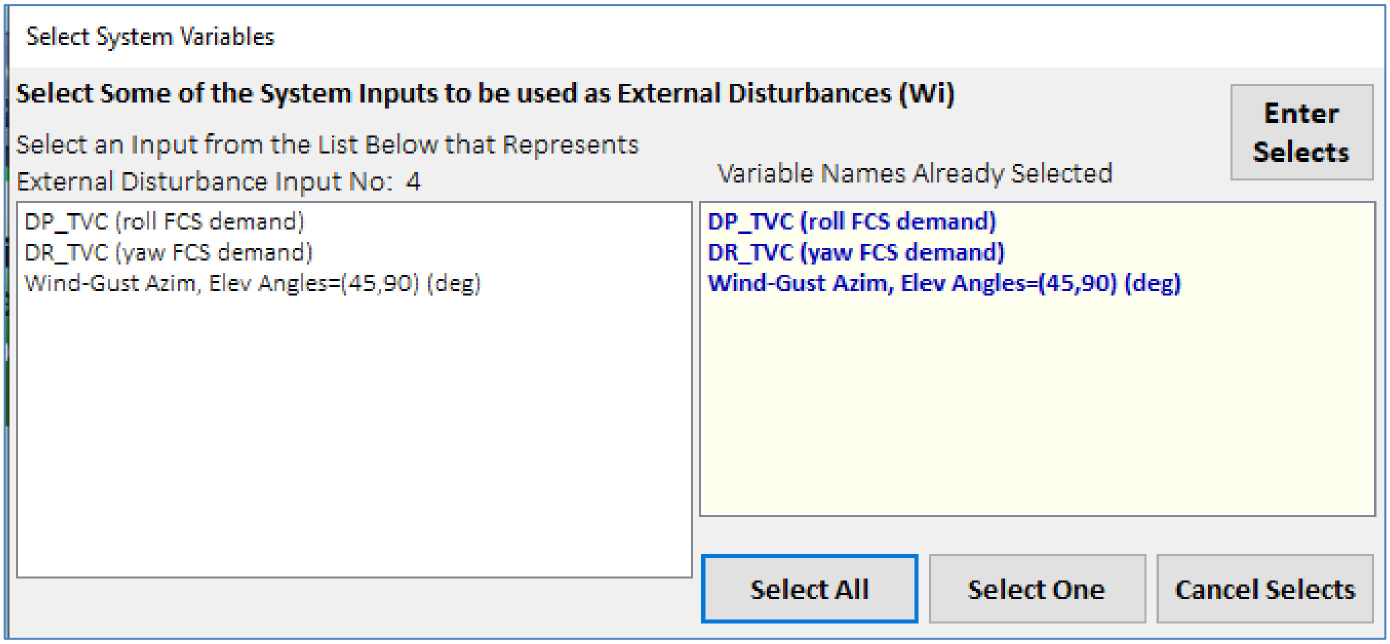
Select Some Inputs from the Uncertainties Block

DP_TVC (roll FCS demand)
DR_TVC (yaw FCS demand)
Wind-Gust Azim, Elev Angles=(45,90) (deg)

Select Correspond Outputs to the Uncertainties Block

Roll Attitude (phi-body) (radians)
Roll Rate (p-body) (rad/sec)
Yaw Attitude (psi-body) (radians)
Yaw Rate (r-body) (rad/sec)
Angle of sideslip, beta (radian)
Beta-Integral (rad-sec)

The next menu is for defining external disturbance inputs. The design model has 3 inputs and all 3 are considered as disturbances. It is mainly the wind-gust but the two controls are also subject to disturbances. So, all 3 are selected and click on “Enter Selects” to continue.



Select System Variables

Select Some of the System Inputs to be used as External Disturbances (Wi)

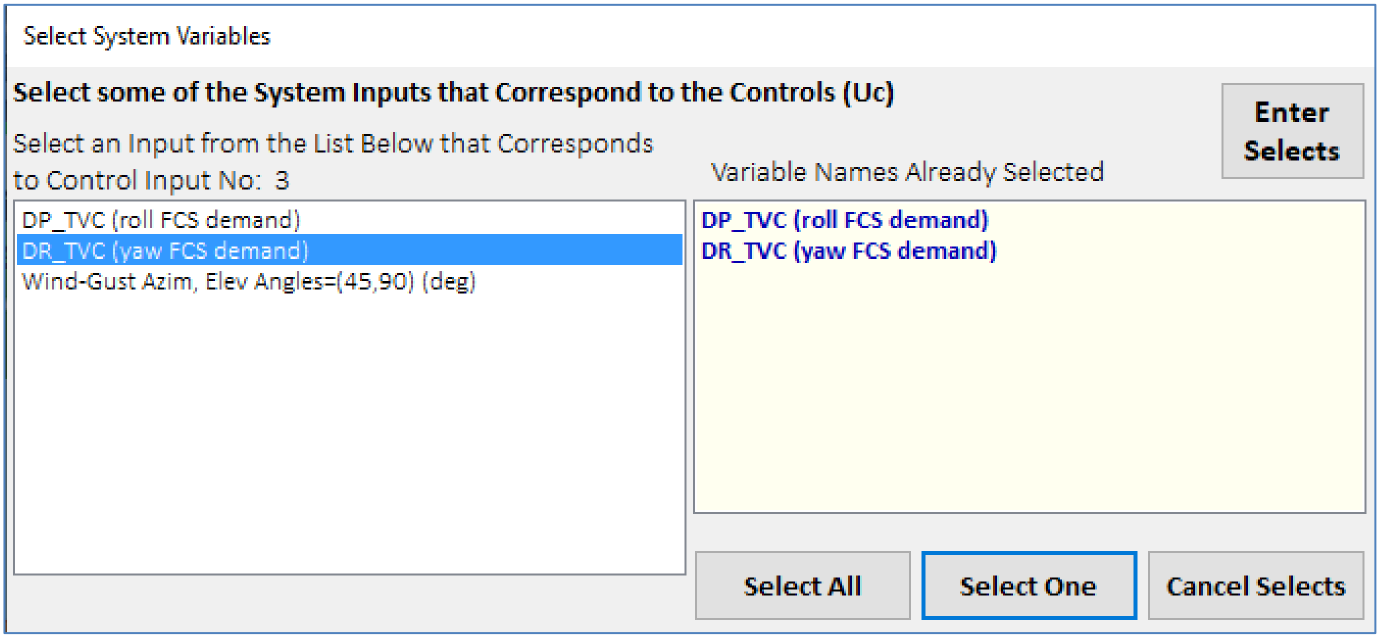
Select an Input from the List Below that Represents External Disturbance Input No: 4

DP_TVC (roll FCS demand)
DR_TVC (yaw FCS demand)
Wind-Gust Azim, Elev Angles=(45,90) (deg)

Variable Names Already Selected

DP_TVC (roll FCS demand)
DR_TVC (yaw FCS demand)
Wind-Gust Azim, Elev Angles=(45,90) (deg)

The next menu is for selecting the control inputs. There are two control inputs, the roll and yaw demands, which are the inputs to the TVC matrix since the design model already includes the TVC. Select one at a time and then click on “Enter Selects” to continue.



Select System Variables

Select some of the System Inputs that Correspond to the Controls (Uc)

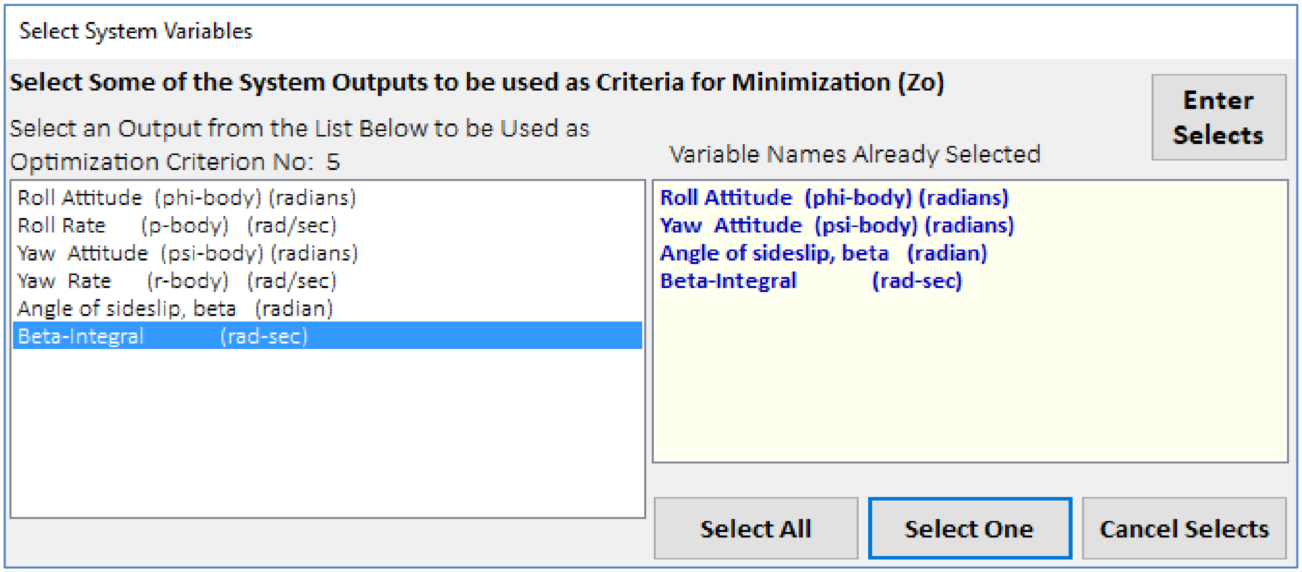
Select an Input from the List Below that Corresponds to Control Input No: 3

DP_TVC (roll FCS demand)
DR_TVC (yaw FCS demand)
Wind-Gust Azim, Elev Angles=(45,90) (deg)

Variable Names Already Selected

DP_TVC (roll FCS demand)
DR_TVC (yaw FCS demand)

The design system output consists of the entire state vector of 6 variables. We will optimize only four of those state variables, the two attitudes, beta, and β-integral. Select one at a time and then click on “Enter Selects” to continue. In the next menu do not select anything. Skip this option and click on “Enter Selects”. The next menu is for selecting the output measurements. Select all of them which is the entire state vector or click on “Set Output= State” and then click on “Enter Selects” to continue.


Select System Variables

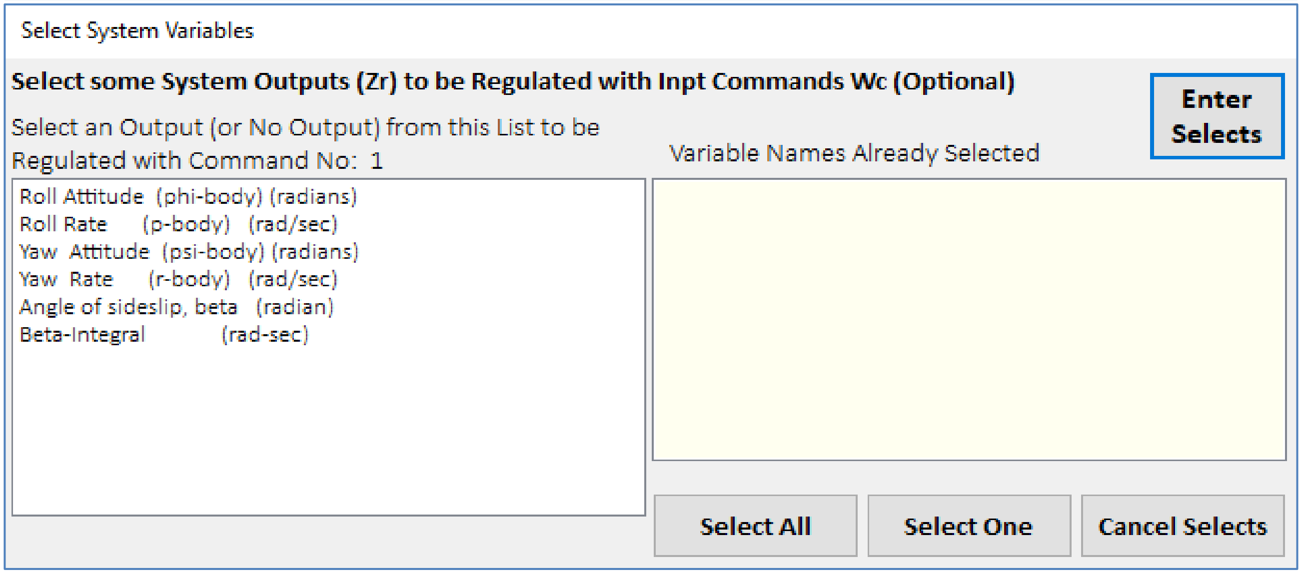
Select Some of the System Outputs to be used as Criteria for Minimization (Zo)

Select an Output from the List Below to be Used as
Optimization Criterion No: 5

Roll Attitude (phi-body) (radians)
Roll Rate (p-body) (rad/sec)
Yaw Attitude (psi-body) (radians)
Yaw Rate (r-body) (rad/sec)
Angle of sideslip, beta (radian)
Beta-Integral (rad-sec)

Variable Names Already Selected

Roll Attitude (phi-body) (radians)
Yaw Attitude (psi-body) (radians)
Angle of sideslip, beta (radian)
Beta-Integral (rad-sec)

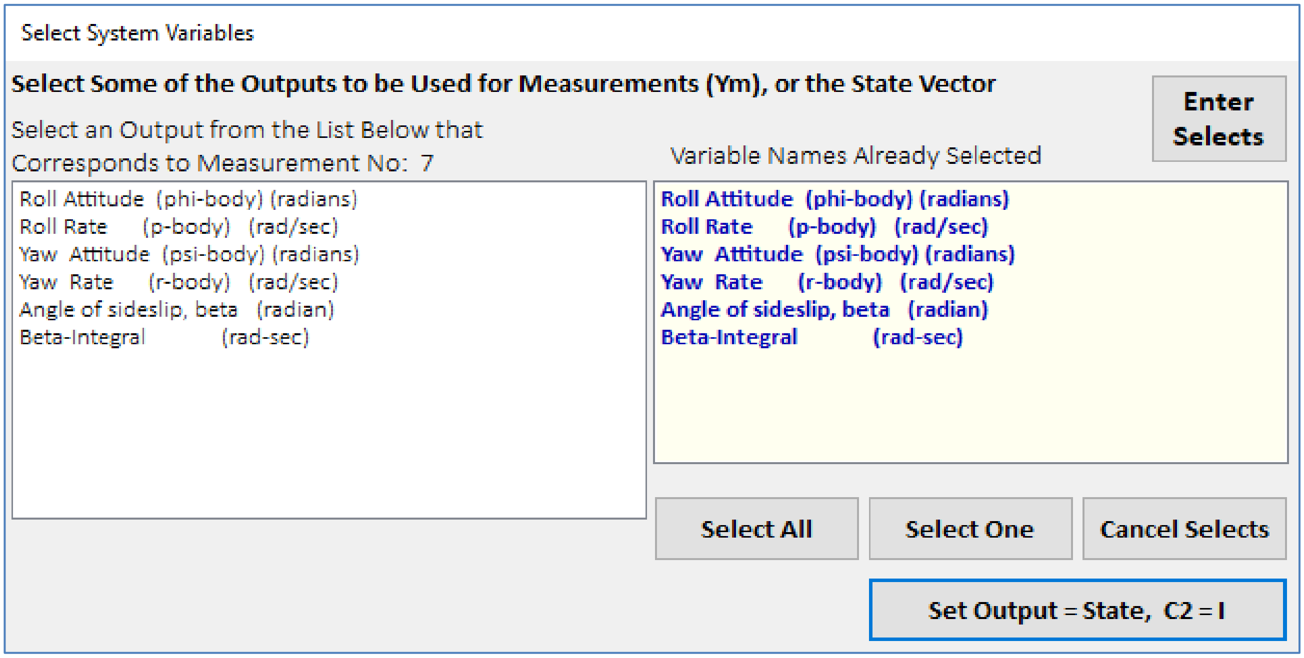

Select System Variables

Select some System Outputs (Zr) to be Regulated with Inpt Commands Wc (Optional)

Select an Output (or No Output) from this List to be
Regulated with Command No: 1

Roll Attitude (phi-body) (radians)
Roll Rate (p-body) (rad/sec)
Yaw Attitude (psi-body) (radians)
Yaw Rate (r-body) (rad/sec)
Angle of sideslip, beta (radian)
Beta-Integral (rad-sec)

Variable Names Already Selected


Select System Variables

Select Some of the Outputs to be Used for Measurements (Ym), or the State Vector

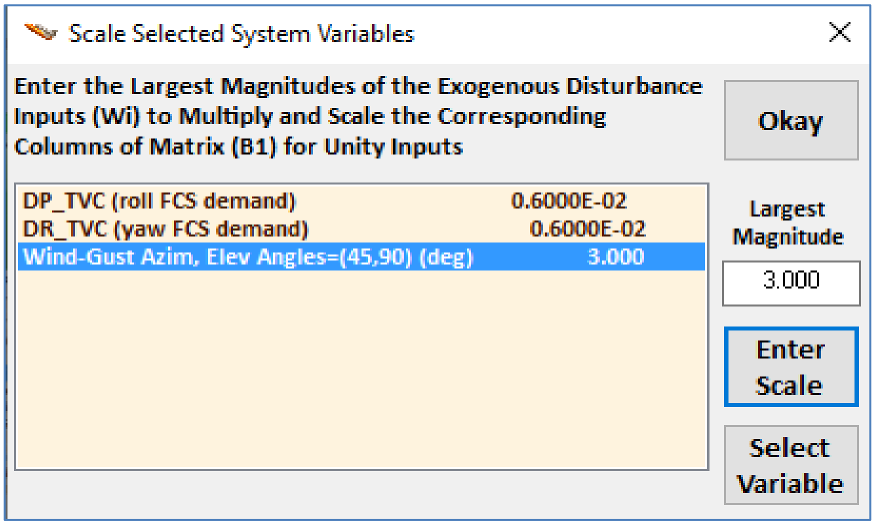
Select an Output from the List Below that
Corresponds to Measurement No: 7

Roll Attitude (phi-body) (radians)
Roll Rate (p-body) (rad/sec)
Yaw Attitude (psi-body) (radians)
Yaw Rate (r-body) (rad/sec)
Angle of sideslip, beta (radian)
Beta-Integral (rad-sec)

Variable Names Already Selected

Roll Attitude (phi-body) (radians)
Roll Rate (p-body) (rad/sec)
Yaw Attitude (psi-body) (radians)
Yaw Rate (r-body) (rad/sec)
Angle of sideslip, beta (radian)
Beta-Integral (rad-sec)

We have now finished defining the input and output variables. The next step is to define the gains that will be used to scale them. Those gains may be changed in the next design iteration. In the dialog below enter the gains that will scale the disturbance inputs. That is, the maximum expected disturbance at each input. Highlight the input, click on "Select Variable" and click on "Enter Scale" to accept it, one at a time. The value appears in the display next to the variable label. When you finish click on "Okay" to go to the next dialog.



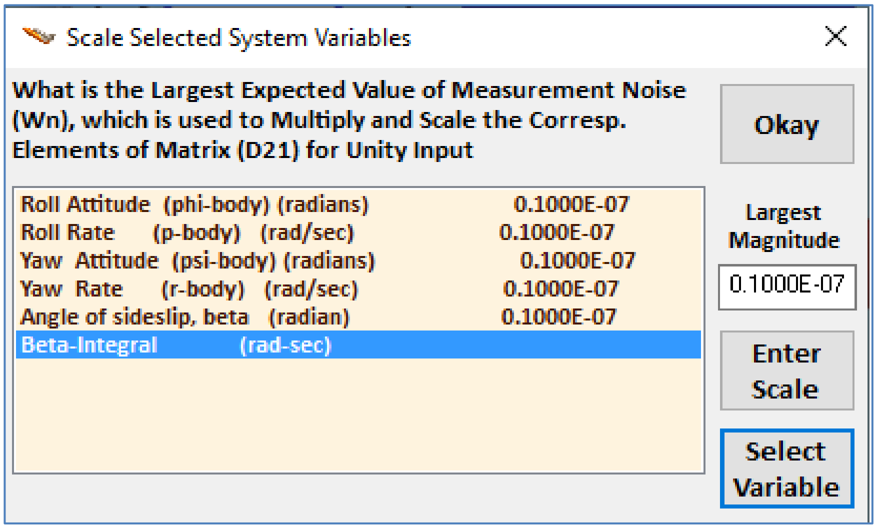
Scale Selected System Variables

Enter the Largest Magnitudes of the Exogenous Disturbance Inputs (Wi) to Multiply and Scale the Corresponding Columns of Matrix (B1) for Unity Inputs

DP_TVC (roll FCS demand) 0.6000E-02
DR_TVC (yaw FCS demand) 0.6000E-02
Wind-Gust Azim, Elev Angles=(45,90) (deg) 3.000

Largest Magnitude: 3.000

This dialog is for entering the measurement noise. In this example the measurement is the entire state-vector and the program knows that, but we do not want to build a state estimator. If the state-vector measurements were noisy then we would need one, even though we are measuring the entire state. So, we tell the program that we don't want the estimator by inserting zero noise or a very small noise magnitude in each variable. The program requires a confirmation that you don't want the estimator, so you enter "Yes" to calculate a state-feedback control gain and not a dynamic controller.

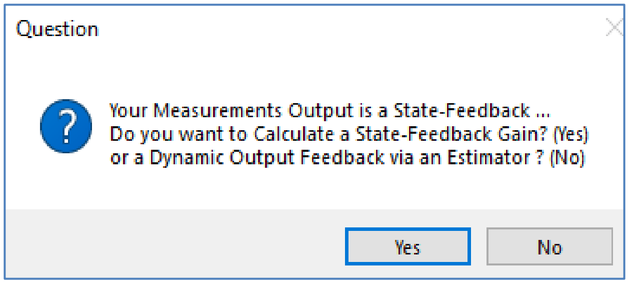


Scale Selected System Variables

What is the Largest Expected Value of Measurement Noise (Wn), which is used to Multiply and Scale the Corresp. Elements of Matrix (D21) for Unity Input

Roll Attitude (phi-body) (radians) 0.1000E-07
Roll Rate (p-body) (rad/sec) 0.1000E-07
Yaw Attitude (psi-body) (radians) 0.1000E-07
Yaw Rate (r-body) (rad/sec) 0.1000E-07
Angle of sideslip, beta (radian) 0.1000E-07
Beta-Integral (rad-sec)

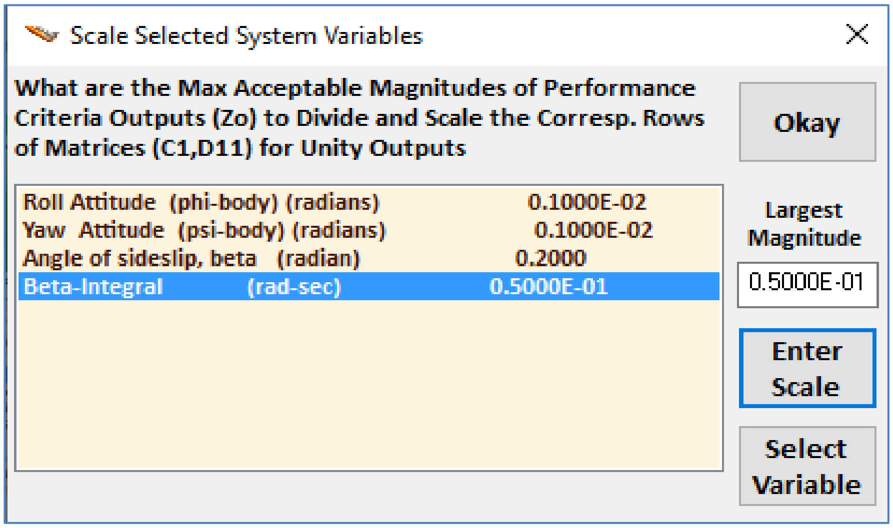
Largest Magnitude: 0.1000E-07



Question

Your Measurements Output is a State-Feedback ...
Do you want to Calculate a State-Feedback Gain? (Yes)
or a Dynamic Output Feedback via an Estimator ? (No)

We must now define the gains for the performance optimization criteria. That is, the maximum acceptable magnitude at the criteria outputs defined, which are: the maximum roll and yaw attitude errors, maximum beta transient magnitude and its integral. Reducing the gain value for a specific performance output results into better performance and smaller transient for that variable. Select one variable at a time, enter the gain and click on “enter scale” to accept it. When you finish click on “Okay” to go to the next dialog.

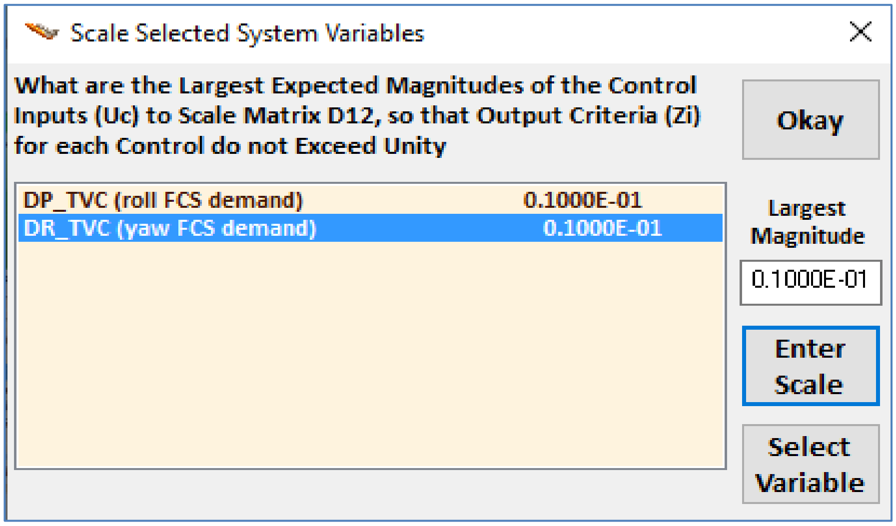


What are the Max Acceptable Magnitudes of Performance Criteria Outputs (Zo) to Divide and Scale the Corresp. Rows of Matrices (C1,D11) for Unity Outputs

Roll Attitude (phi-body) (radians)	0.1000E-02
Yaw Attitude (psi-body) (radians)	0.1000E-02
Angle of sideslip, beta (radian)	0.2000
Beta-Integral (rad-sec)	0.5000E-01

Largest Magnitude: 0.5000E-01

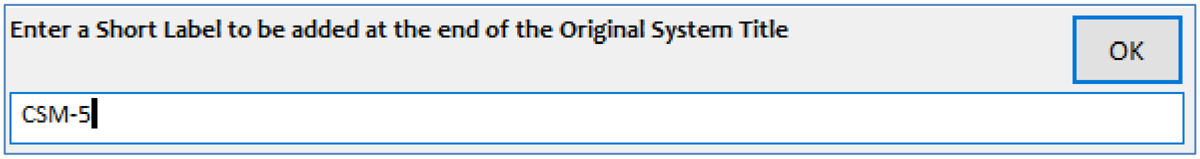
The controls are also included in the optimization criteria. Between the two criteria we define the dividing line of the trade-off between performance and control bandwidth. In this example we have two controls, roll and yaw control demands. If we increase the gain in one of them, let’s say the roll control, we are telling the mathematic algorithm to allow more control in the roll axis which means bigger bandwidth in roll and the system will be faster in roll. Enter the two gains as before and click on “Okay” to proceed. Finally enter a short label that will appear at the end of the Synthesis Model title in the systems file.



What are the Largest Expected Magnitudes of the Control Inputs (Uc) to Scale Matrix D12, so that Output Criteria (Zi) for each Control do not Exceed Unity

DP_TVC (roll FCS demand)	0.1000E-01
DR_TVC (yaw FCS demand)	0.1000E-01

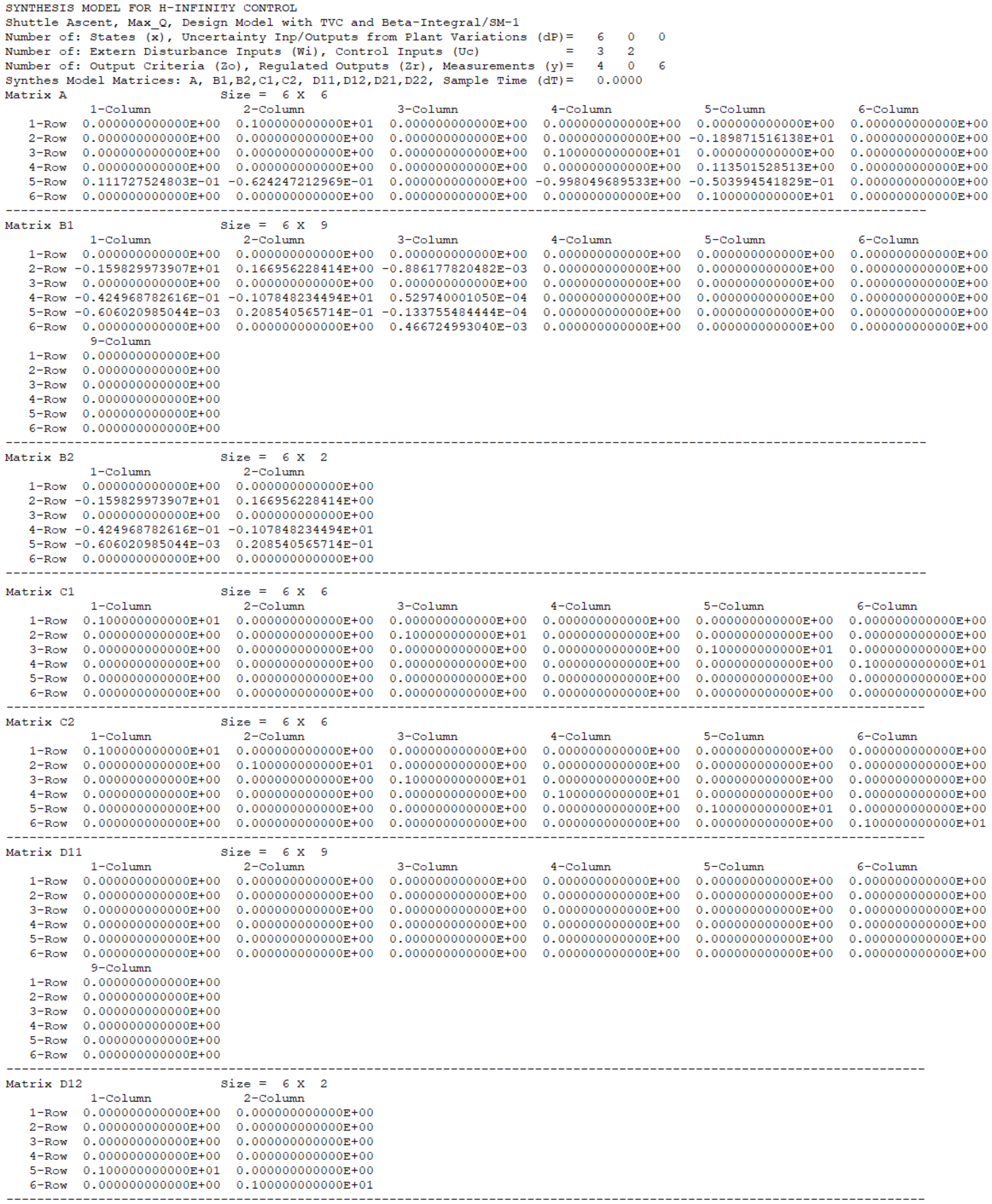
Largest Magnitude: 0.1000E-01



Enter a Short Label to be added at the end of the Original System Title

CSM-5

The following is the H-Infinity SM which is saved in file “Lateral_MaxQ1.Qdr” and it will be used to design the state-feedback controller



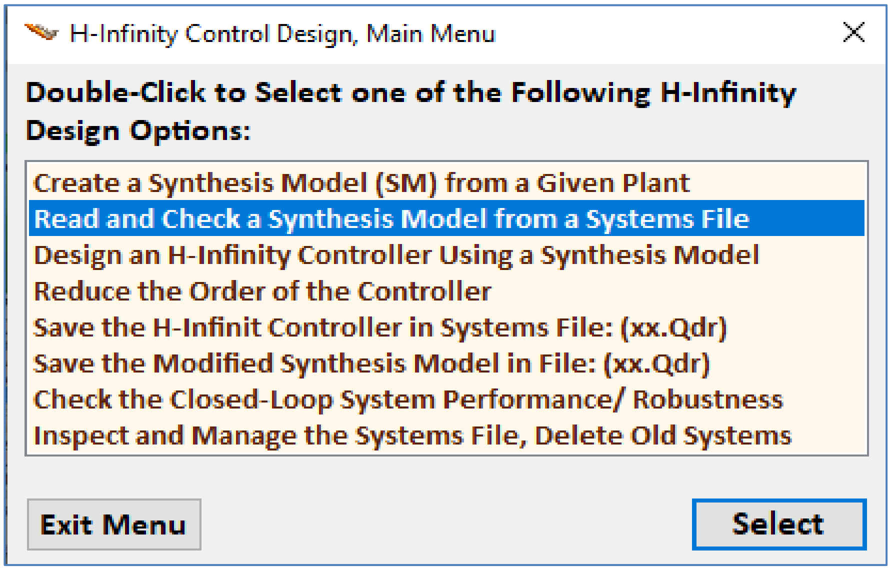
```
SYNTHESIS MODEL FOR H-INFINITY CONTROL
Shuttle Ascent, Max_Q, Design Model with TVC and Beta-Integral/SM-1
Number of: States (x), Uncertainty Inp/Outputs from Plant Variations (dP)=   6   0   0
Number of: Extern Disturbance Inputs (Wi), Control Inputs (Uc)           =   3   2
Number of: Output Criteria (Zo), Regulated Outputs (Zr), Measurements (y)=   4   0   6
Synthes Model Matrices: A, B1,B2,C1,C2, D11,D12,D21,D22, Sample Time (dT)=   0.0000
Matrix A                 Size =  6 X  6
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00 -0.189871516138E+01  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.113501528513E+00  0.000000000000E+00
   5-Row  0.111727524803E-01 -0.624247212969E-01  0.000000000000E+00 -0.998049689533E+00 -0.503994541829E-01  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
--------------------------------------------------------------------------------------------------------------------------
Matrix B1                Size =  6 X  9
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row -0.159829973907E+01  0.166956228414E+00 -0.886177820482E-03  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row -0.424968782616E-01 -0.107848234494E+01  0.529740001050E-04  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row -0.606020985044E-03  0.208540565714E-01 -0.133755484444E-04  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.466724993040E-03  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
           9-Column
   1-Row  0.000000000000E+00
   2-Row  0.000000000000E+00
   3-Row  0.000000000000E+00
   4-Row  0.000000000000E+00
   5-Row  0.000000000000E+00
   6-Row  0.000000000000E+00
--------------------------------------------------------------------------------------------------------------------------
Matrix B2                Size =  6 X  2
           1-Column            2-Column
   1-Row  0.000000000000E+00  0.000000000000E+00
   2-Row -0.159829973907E+01  0.166956228414E+00
   3-Row  0.000000000000E+00  0.000000000000E+00
   4-Row -0.424968782616E-01 -0.107848234494E+01
   5-Row -0.606020985044E-03  0.208540565714E-01
   6-Row  0.000000000000E+00  0.000000000000E+00
--------------------------------------------------------------------------------------------------------------------------
Matrix C1                Size =  6 X  6
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
--------------------------------------------------------------------------------------------------------------------------
Matrix C2                Size =  6 X  6
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
--------------------------------------------------------------------------------------------------------------------------
Matrix D11               Size =  6 X  9
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
           9-Column
   1-Row  0.000000000000E+00
   2-Row  0.000000000000E+00
   3-Row  0.000000000000E+00
   4-Row  0.000000000000E+00
   5-Row  0.000000000000E+00
   6-Row  0.000000000000E+00
--------------------------------------------------------------------------------------------------------------------------
Matrix D12               Size =  6 X  2
           1-Column            2-Column
   1-Row  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00
   5-Row  0.100000000000E+01  0.000000000000E+00
   6-Row  0.000000000000E+00  0.100000000000E+01
--------------------------------------------------------------------------------------------------------------------------
```

```
Matrix D21                 Size =  6 X  9
          1-Column            2-Column            3-Column            4-Column            5-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.466724993040E-03  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
          9-Column
   1-Row  0.000000000000E+00
   2-Row  0.000000000000E+00
   3-Row  0.000000000000E+00
   4-Row  0.000000000000E+00
   5-Row  0.000000000000E+00
   6-Row  0.100000000000E+01
----------------------------------------------------------------------------------------------------------
Matrix D22                 Size =  6 X  2
          1-Column            2-Column
   1-Row  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00
----------------------------------------------------------------------------------------------------------
Definition of Synthesis Model Variables                        Max Scaling Factors

States (x) .........  =   6
  1    Roll Attitude  (phi-body) (radians)
  2    Roll Rate      (p-body)   (rad/sec)
  3    Yaw  Attitude  (psi-body) (radians)
  4    Yaw  Rate      (r-body)   (rad/sec)
  5    Angle of sideslip, beta   (radian)
  6    Beta-Integral             (rad-sec)

 Excitation Inputs (w)  =   9
  1    DP_TVC (roll FCS demand)                              * 0.002
  2    DR_TVC (yaw FCS demand)                               * 0.002
  3    Wind-Gust Azim, Elev Angles=(45,90) (deg)             * 3.0
  4    Noise at Output: Roll Attitude  (phi-body) (radians)  * 0.10000E-07
  5    Noise at Output: Roll Rate      (p-body)   (rad/sec)  * 0.10000E-07
  6    Noise at Output: Yaw  Attitude  (psi-body) (radians)  * 0.10000E-07
  7    Noise at Output: Yaw  Rate      (r-body)   (rad/sec)  * 0.10000E-07
  8    Noise at Output: Angle of sideslip, beta   (radian)   * 0.10000E-07
  9    Noise at Output: Beta-Integral             (rad-sec)  * 0.10000E-07

 Control Inputs (u) ... =   2
  1    Control: DP_TVC (roll FCS demand)                     *  1.0000
  2    Control: DR_TVC (yaw FCS demand)                      *  1.0000

 Performance Outputs (z)=   6
  1    Roll Attitude  (phi-body) (radians)                   / 0.0001
  2    Yaw  Attitude  (psi-body) (radians)                   / 0.0003
  3    Angle of sideslip, beta   (radian)                    / 0.03
  4    Beta-Integral             (rad-sec)                   / 0.03
  5    Contrl Criter. DP_TVC (roll FCS demand)               / 0.0005
  6    Contrl Criter. DR_TVC (yaw FCS demand)                / 0.0008

 Measurement Outputs (y)=   6
  1    Measurm: Roll Attitude  (phi-body) (radians)          /  1.0000
  2    Measurm: Roll Rate      (p-body)   (rad/sec)          /  1.0000
  3    Measurm: Yaw  Attitude  (psi-body) (radians)          /  1.0000
  4    Measurm: Yaw  Rate      (r-body)   (rad/sec)          /  1.0000
  5    Measurm: Angle of sideslip, beta   (radian)           /  1.0000
  6    Measurm: Beta-Integral             (rad-sec)          /  1.0000
 -------------------------------------------------------------------------------------------
```

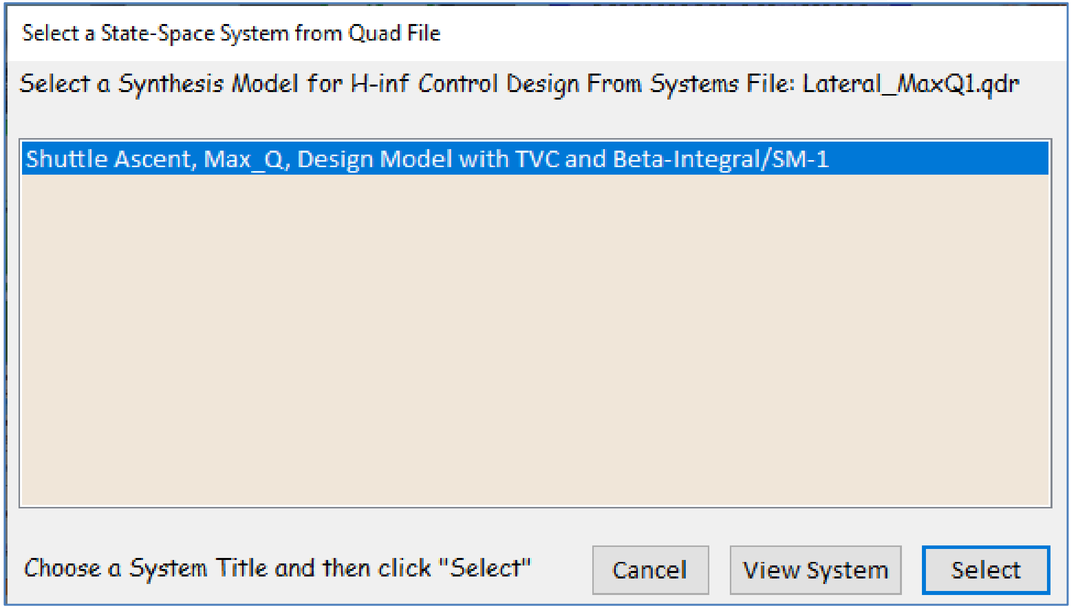
The scaling gains are included on the side of the corresponding variables to be scaled.

1.3 Designing the H-Infinity Controller Interactively

We will now use the above SM to design the H-infinity controller. We go back to the H-infinity design program and from the main menu select the second option to read the SM which is already in file. From the next menu select the title of the only SM which is in the systems file, and click “Select”.


H-Infinity Control Design, Main Menu

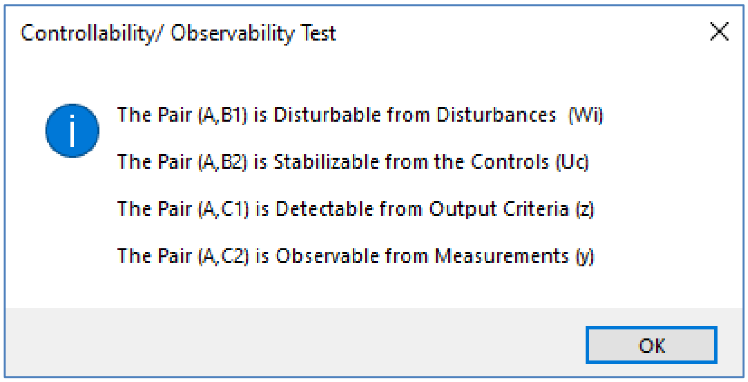
Double-Click to Select one of the Following H-Infinity Design Options:


Select a State-Space System from Quad File

Select a Synthesis Model for H-inf Control Design From Systems File: Lateral_MaxQ1.qdr

Shuttle Ascent, Max_Q, Design Model with TVC and Beta-Integral/SM-1

Choose a System Title and then click "Select"


Controllability/ Observability Test

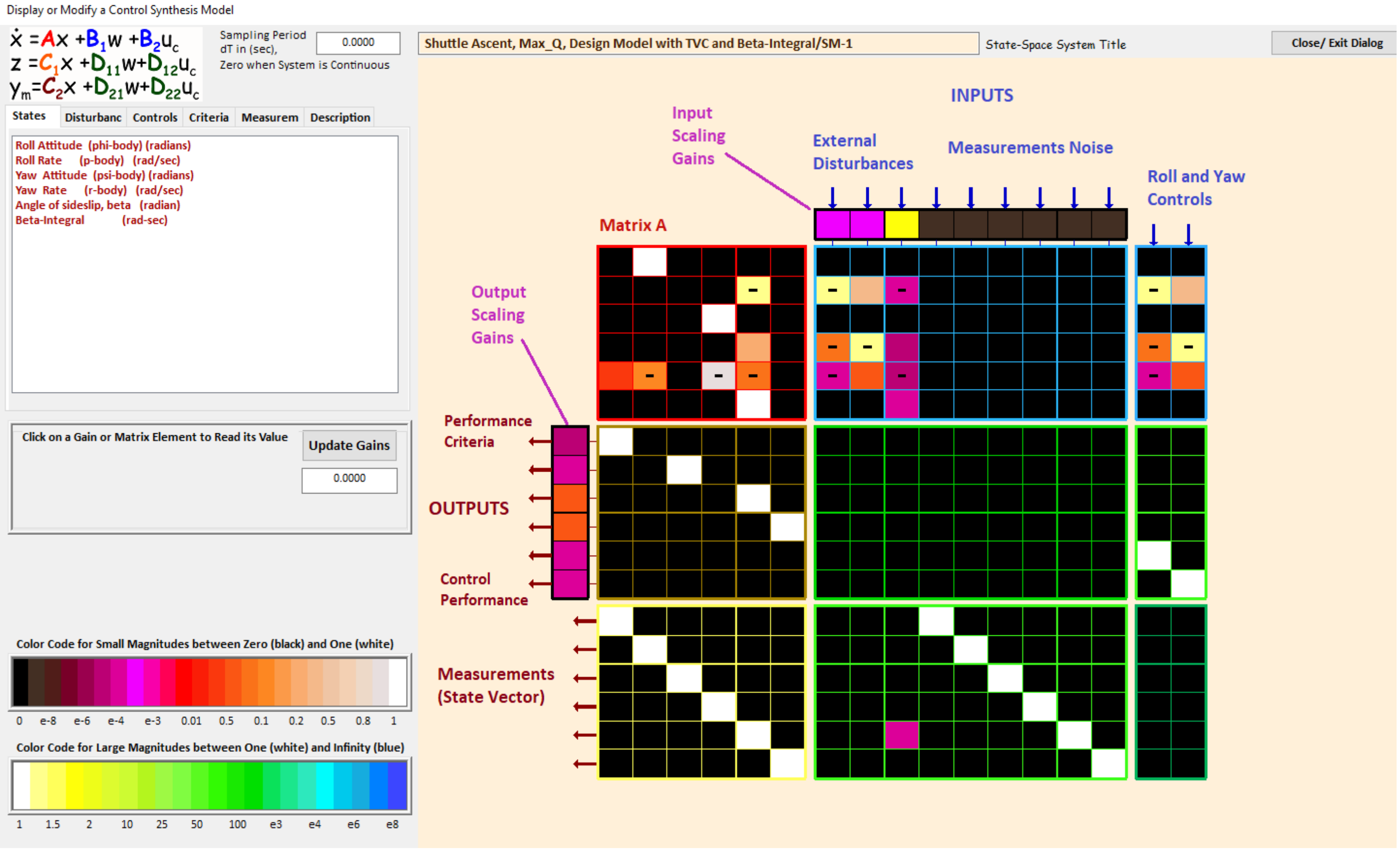
The Pair (A,B1) is Disturbable from Disturbances (Wi)

The Pair (A,B2) is Stabilizable from the Controls (Uc)

The Pair (A,C1) is Detectable from Output Criteria (z)

The Pair (A,C2) is Observable from Measurements (y)

The program confirms that the SM meets the expected observability and controllability requirements and displays the SM matrices graphically in system's form in the dialog below. The 9 SM matrices appear color coded and also the scaling gains that scale the disturbances and the criteria. The A-matrix consists of 6 states. There are 3 external disturbances, 6 measurements noise inputs which are set to almost zero (dark brown), 4 performance criteria, and 2 control utilization criteria. C2 is the identity matrix which means the 6 outputs are equal to the state vector. Also, there are 2 control inputs for roll and yaw.



Display or Modify a Control Synthesis Model

$$\dot{x} = Ax + B_1 w + B_2 u_c$$
$$z = C_1 x + D_{11} w + D_{12} u_c$$
$$y_m = C_2 x + D_{21} w + D_{22} u_c$$

Sampling Period dT in (sec), Zero when System is Continuous: 0.0000

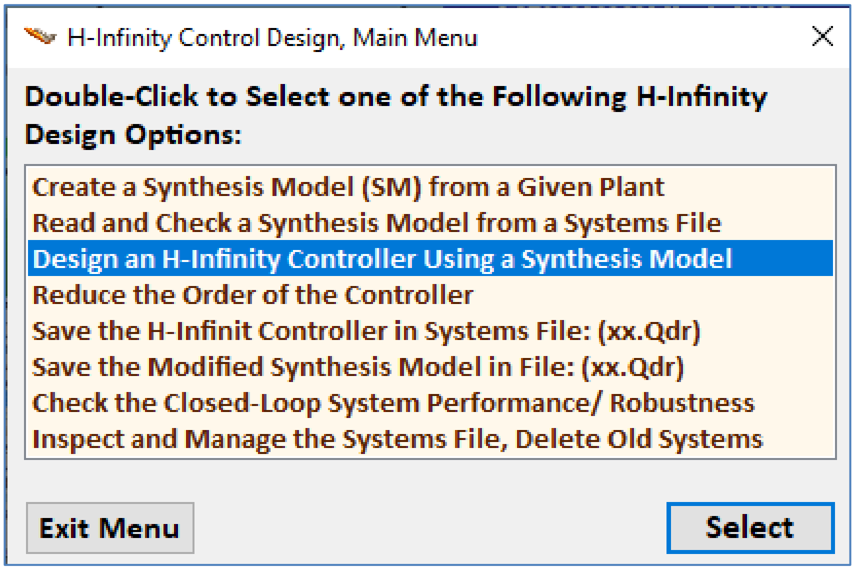
State-Space System Title: Shuttle Ascent, Max_Q, Design Model with TVC and Beta-Integral/SM-1

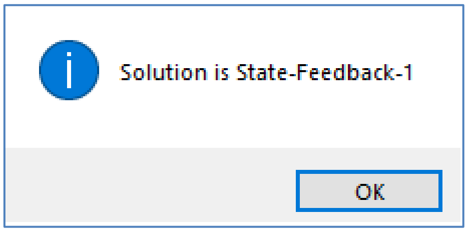
States:

Roll Attitude (phi-body) (radians)
Roll Rate (p-body) (rad/sec)
Yaw Attitude (psi-body) (radians)
Yaw Rate (r-body) (rad/sec)
Angle of sideslip, beta (radian)
Beta-Integral (rad-sec)

Click on a Gain or Matrix Element to Read its Value: 0.0000

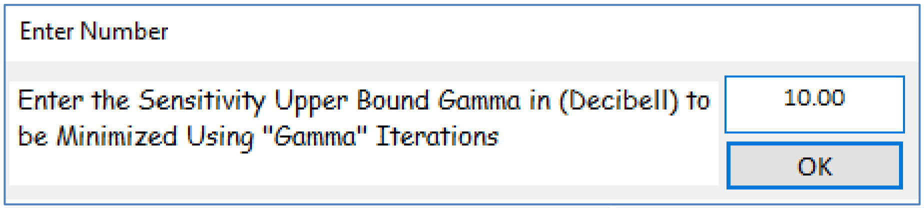
Select the third option from the main menu to design the H-infinity controller, and click "Select". The program confirms that the solution is a state-feedback gain rather than dynamic.





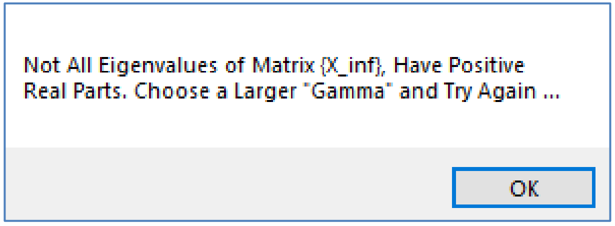
Solution is State-Feedback-1

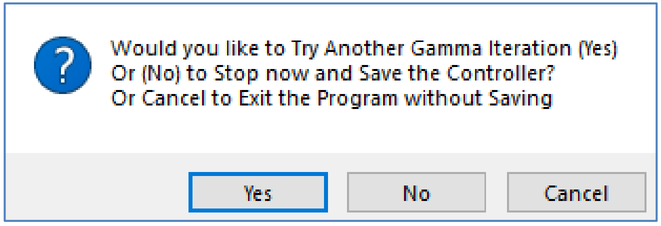
Now we begin the iterative process of trying to minimize the upper bound γ of the infinity norm of the sensitivity transfer function between the 3 disturbance inputs and the 6 output criteria (4-performance & 2-control). We begin with an arbitrary γ upper bound and try to find the smallest γ that will not violate the algorithm requirements. We must enter γ in decibels. We first enter $\gamma=10$ which is too low and click on “Yes” in the next dialog to try a bigger value. Next time we enter $\gamma=20$ which is also low and click on “Yes” again to try an even bigger value. After 2-3 iterations we find that $\gamma=30$ works and we click on “No” meaning that we do not want to try another value but to accept the current controller.

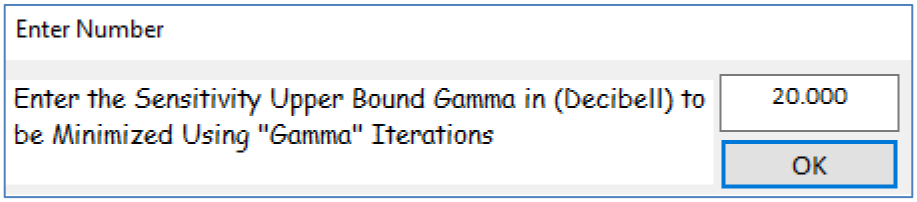

Enter Number

Enter the Sensitivity Upper Bound Gamma in (Decibell) to be Minimized Using "Gamma" Iterations

10.00

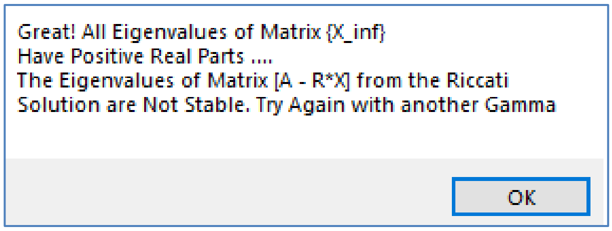

Not All Eigenvalues of Matrix {X_inf}, Have Positive Real Parts. Choose a Larger "Gamma" and Try Again ...

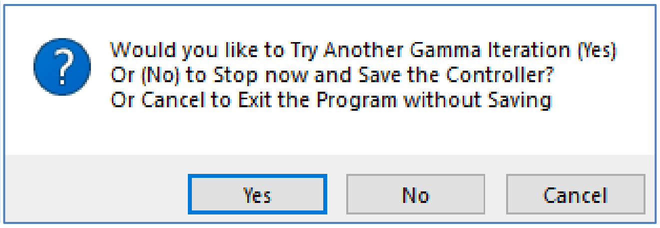

Would you like to Try Another Gamma Iteration (Yes)
Or (No) to Stop now and Save the Controller?
Or Cancel to Exit the Program without Saving


Enter Number

Enter the Sensitivity Upper Bound Gamma in (Decibell) to be Minimized Using "Gamma" Iterations

20.000


Great! All Eigenvalues of Matrix {X_inf}
Have Positive Real Parts ....
The Eigenvalues of Matrix [A - R*X] from the Riccati
Solution are Not Stable. Try Again with another Gamma


Would you like to Try Another Gamma Iteration (Yes)
Or (No) to Stop now and Save the Controller?
Or Cancel to Exit the Program without Saving

The next Figure-2 shows the eigenvalues of the system with the control loop closed as in Figure-1 between the inputs (w) and the outputs (z). They are all stable, es expected. We return to the H-infinity main menu, and at this point we can save the controller gain by clicking on “*Save the H-infinity Controller in Systems File (x.Qdr)*”.

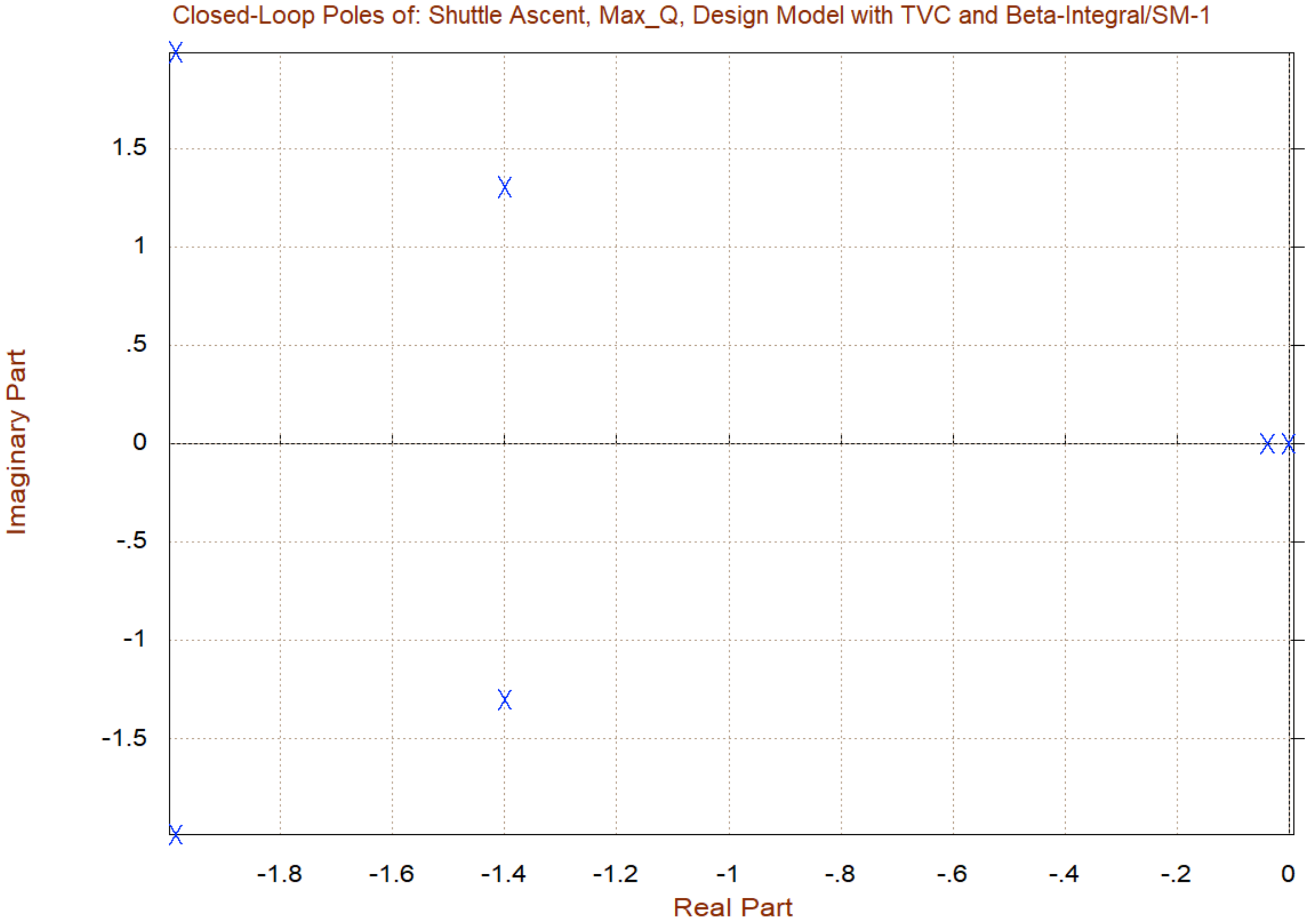
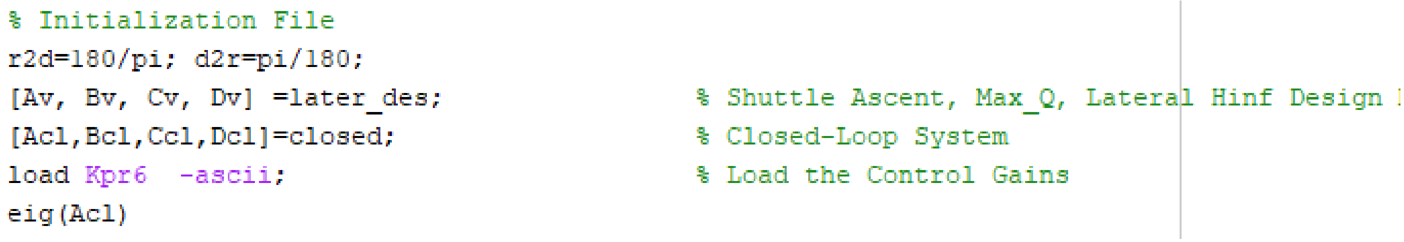


Figure 2 Closed-Loop System Poles

1.4 Control Analysis

We will now analyze the control system stability and sensitivity to wind-gust disturbances in Matlab. The analysis files are in directory "*Flixan\Control Analysis\Hinfinity\Examples\Shuttle Ascent Hinfinity Design\1-Lateral Design w Beta-Integral*". There is an initialization file "*init.m*" that loads the lateral design model "*Shuttle Ascent, Max_Q, Design Model with TVC and Beta-Integral*" from file "*later_des.m*". It also loads the closed-loop system from file "closed.m" and the H-infinity derived state-feedback matrix from file "Kpr6.mat".



```
% Initialization File
r2d=180/pi; d2r=pi/180;
[Av, Bv, Cv, Dv] =later_des;                       % Shuttle Ascent, Max_Q, Lateral Hinf Design
[Acl,Bcl,Ccl,Dcl]=closed;                          % Closed-Loop System
load Kpr6  -ascii;                                 % Load the Control Gains
eig(Acl)
```

There is also an open-loop Simulink model "*Open_Loop.slx*" used for stability analysis, shown in Figure-3 configured for open yaw loop frequency response analysis with the roll loop closed.

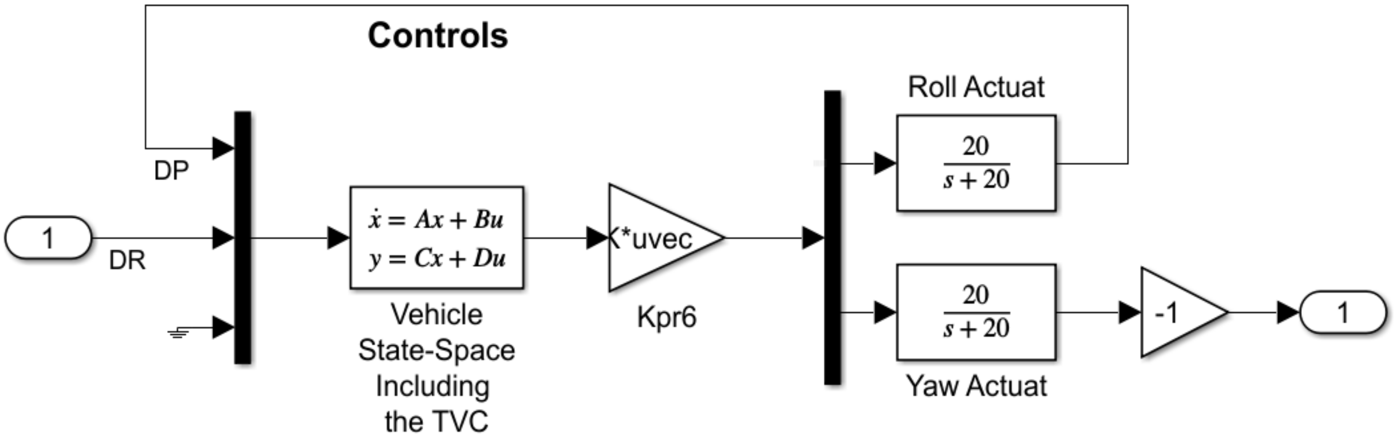
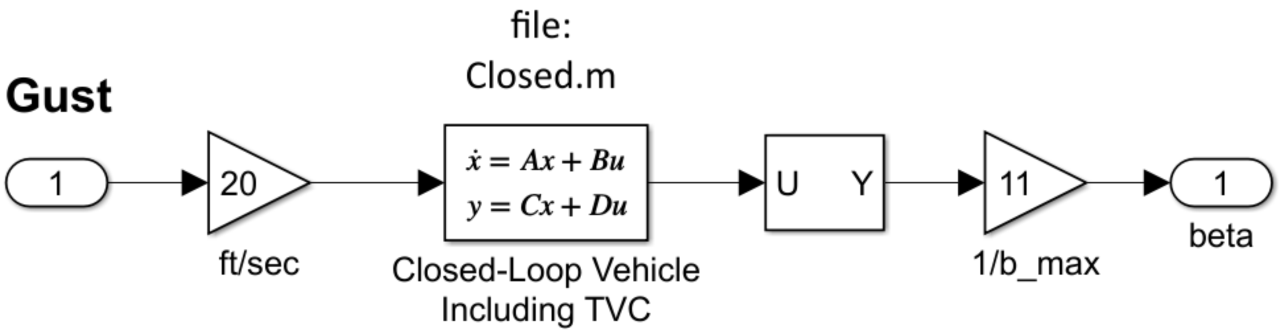
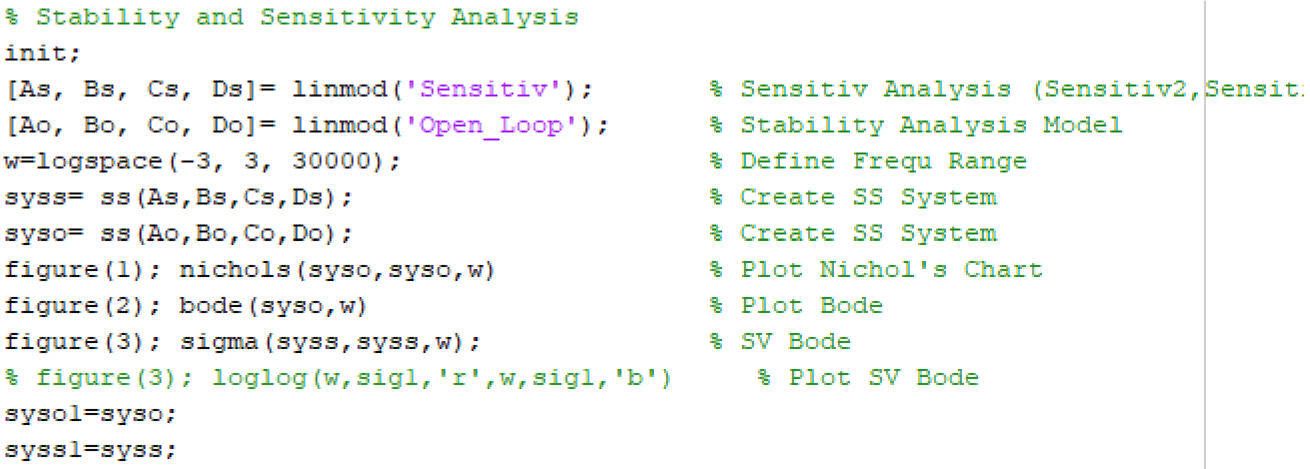


Figure 3 Open-Loop Stability Analysis Model "Open_Loop.slx"

Sensitivity to gusts is analyzed using the closed-loop model "*Sensitiv.slx*" which includes the closed-loop system "*Closed-Loop Via State-Feedback Gain*". Its input is scaled by multiplying it with the maximum expected wind-gust velocity which is 20 (feet/sec) and its output is scaled by dividing it with the maximum allowed β angle, which is 5° or 0.09 (rad). Therefore, the peak of the scaled sensitivity transfer function should be less than 1. The script file "freq.m" shown below calculates the yaw Nichols and Bode plots (Fig.4) and the Sigma plot of the sensitivity function. In Fig.5 the disturbance attenuation at low frequencies is very good because of the β-integral gain in the state-feedback. The overall gain is less than 1 or 0 (dB).





```
% Stability and Sensitivity Analysis
init;
[As, Bs, Cs, Ds]= linmod('Sensitiv');          % Sensitiv Analysis (Sensitiv2,Sensit
[Ao, Bo, Co, Do]= linmod('Open_Loop');         % Stability Analysis Model
w=logspace(-3, 3, 30000);                      % Define Frequ Range
syss= ss(As,Bs,Cs,Ds);                         % Create SS System
syso= ss(Ao,Bo,Co,Do);                         % Create SS System
figure(1); nichols(syso,syso,w)                % Plot Nichol's Chart
figure(2); bode(syso,w)                        % Plot Bode
figure(3); sigma(syss,syss,w);                 % SV Bode
% figure(3); loglog(w,sig1,'r',w,sig1,'b')       % Plot SV Bode
syso1=syso;
syss1=syss;
```

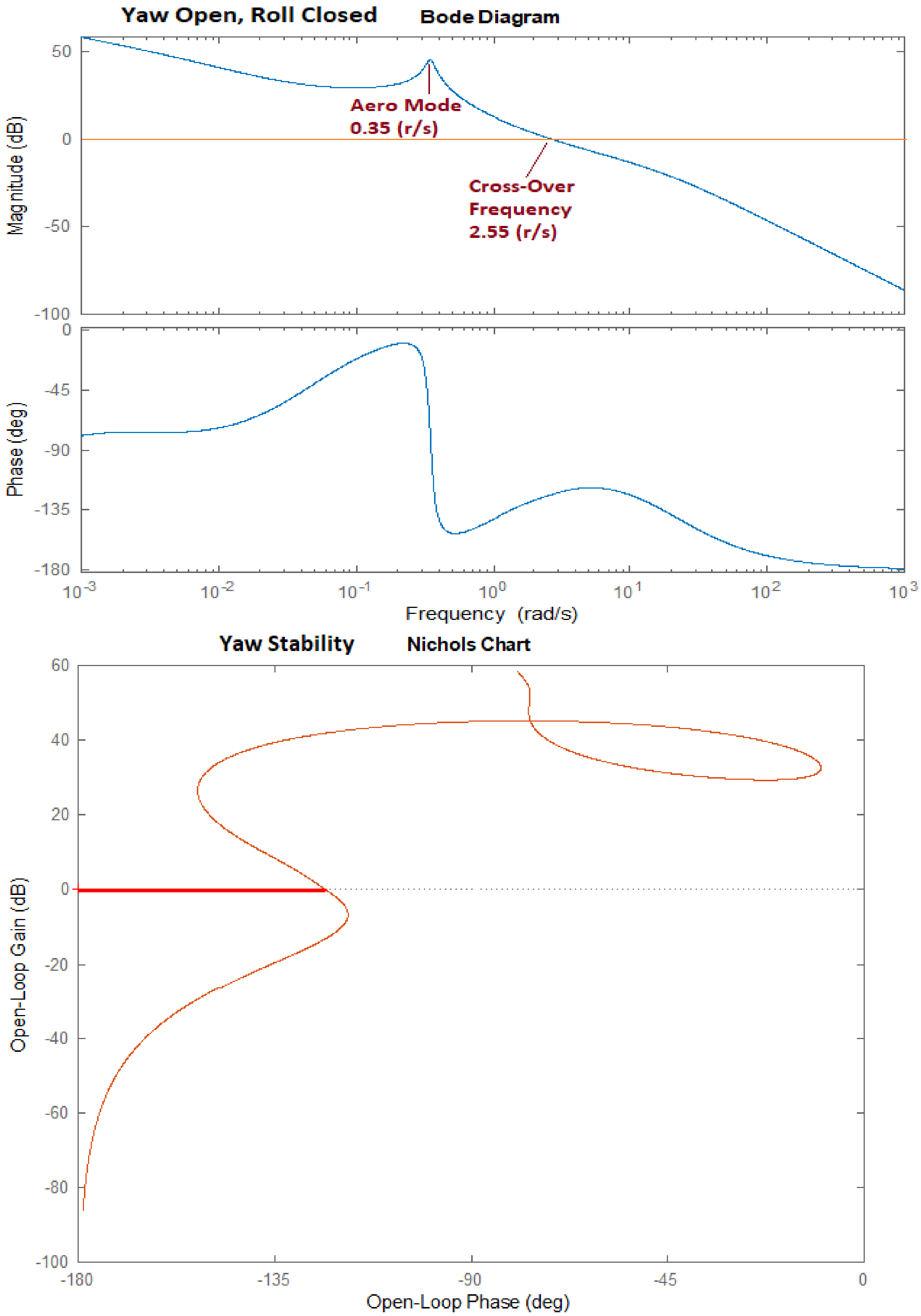


Figure 4 Yaw Axis Bode and Nichols Plots Showing Stability Margin

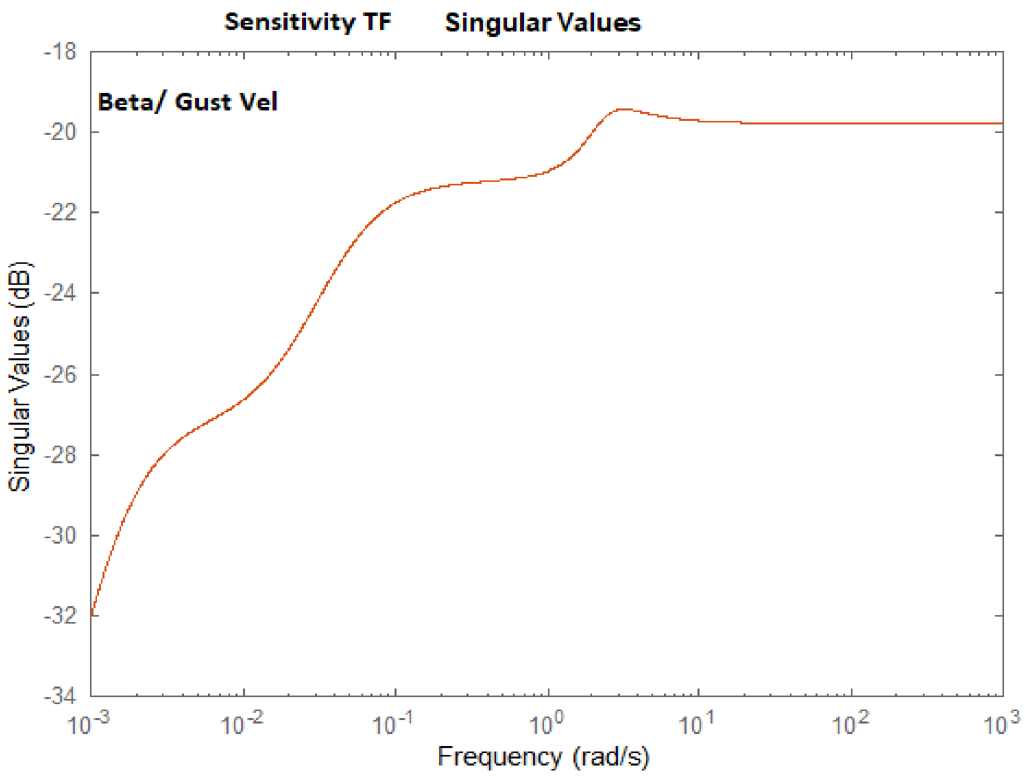


Figure 5 Sensitivity Response, Gust Velocity to Sideslip β

The simulation model in file “Closed_Loop.slx”, shown in Fig.6, is used to calculate the system responses to gusts and to attitude commands. Figure-7 shows the system response to unit attitude commands.

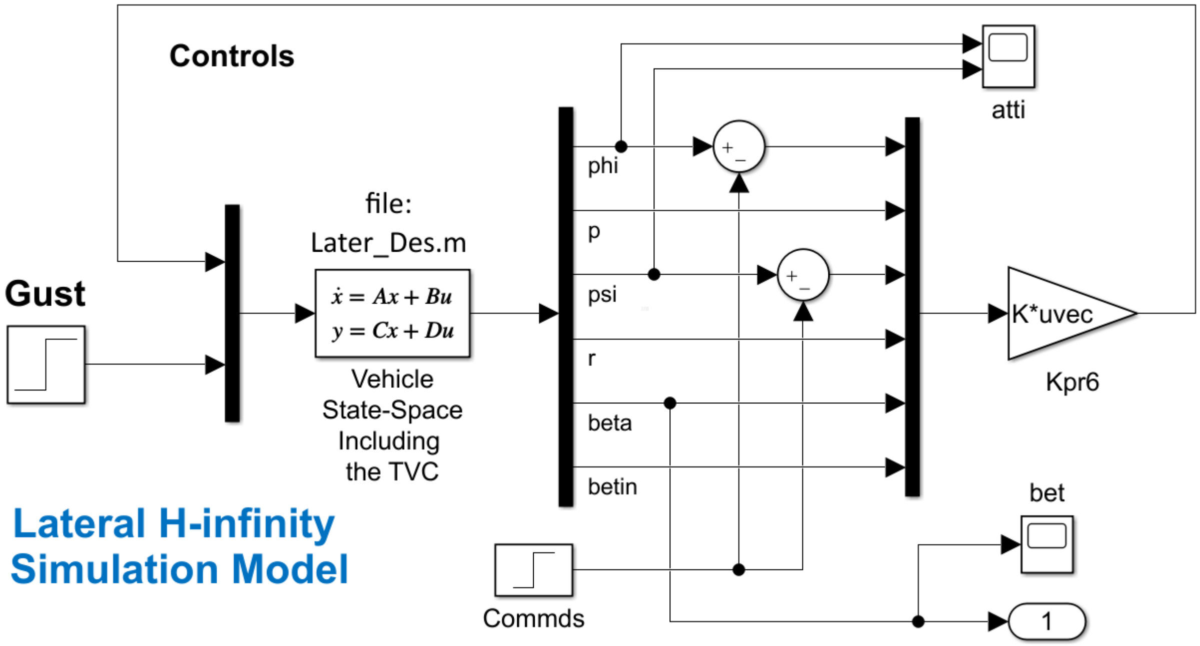


Figure 6 Simulation Model “Closed_Loop.slx”

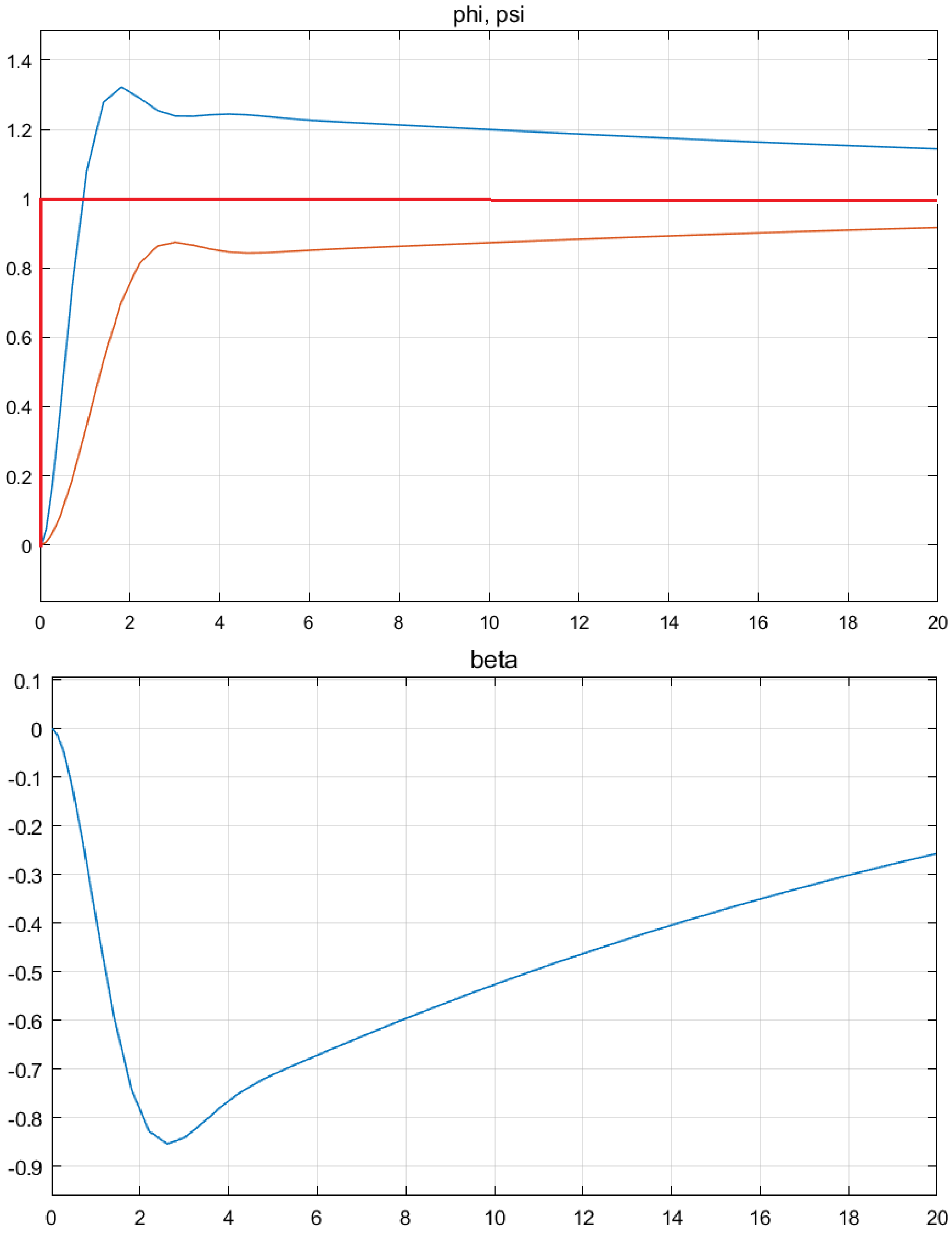


Figure 7 Closed-Loop System's Response to Attitude Commands in Roll and Yaw

2. Lateral H-Infinity Design with Gust Filter

In this section we will present a different lateral design approach. It is still a state-feedback controller but we will replace the β-integrator gain with two additional gains from the states of a second order filter that is inserted in the vehicle model and is replacing the integrator. The vehicle is at Max-Q condition which lasts only 15-20 sec and the disturbances are mostly cyclic random gusts rather than steady-state shear. We assume that the gust disturbance has a sharp peak at 1 (rad/sec). We, therefore, like our control system sensitivity function to have a dip at the disturbance frequency. The filter produces a resonance that is tuned at the 1 (rad/sec) disturbance frequency. We set up the H-infinity synthesis model so that the resonance is excited by beta and the optimization algorithm will produce a state-feedback gain matrix that will inhibit the excitation of that resonance. It means that when the vehicle is excited by a cyclic wind-gust at 1 (rad/sec), the control system will respond to the oscillation by turning the vehicle nose towards the wind and therefore reducing somewhat the sideslip angle in steady-state. This of course is only possible when the TVC system can respond at that frequency.

The work files for Section-2 are in directory: "*Flixan\Control Analysis\Hinfinity\Examples\Shuttle Ascent Hinfinity Design\2-Lateral Design w Gust-Filter*". The Flixan input file containing the vehicle data is "*Lateral-MaxQ2.Inp*". In this analysis we will develop two vehicle models, a simple rigid model for control design and a more complex model for stability/ performance analysis. The second model includes also propellant sloshing and tail-wags-dog dynamics. The simple model is augmented by combining it with the beta-filter and the TVC to produce the design model. The design model is an (A, B, C, D) system and we use it to create the Synthesis Model which is a 9-matrix system (A, B1, B2, C1, C2, D11, D12, D21, D22) by selecting the control and disturbance inputs, the control measurements and performance criteria outputs, as it was shown in the previous section. The SM is already saved in file: "*Lateral_MaxQ2.Qdr*" and it will be used by the H-infinity program to calculate the state-feedback matrix Kpr7. Then, we shall load the systems, control gains and TVC matrices into Matlab and analyze control system stability, command following and performance to gusts in various complexity levels. We will also create a β-estimator from the accelerometer measurement because β is not directly measurable.

2.1 Input File

The input file contains the datasets which perform various Flixan functions. There is a batch set at the top of the file that can process the entire file in batch mode. The file includes two Shuttle vehicle models: a simple model "*Shuttle Ascent, Max_Q, T=61 sec, (Design Model)*" that is used for developing the synthesis model, and a more complex model that includes propellant sloshing, accelerometers and TWD dynamics. Its title is "*Shuttle Ascent, Max_Q, T=61 sec, Rigid-Body/ Slosh/ TWD/ Accelerometer*". The simple vehicle model is combined with the TVC matrix and reduced by truncating the longitudinal variables and retaining only the lateral inputs, states and outputs. The title of the reduced lateral system is "*Shuttle Ascent, Max_Q, Lateral Hinf Design Model*". A similar reduced lateral model is extracted from the complex vehicle model that includes pitch and lateral dynamics. Its title is "*Shuttle Ascent, Max_Q, Lateral Analysis Model with Slosh & TVC*" and it will be used for control analysis. A TVC matrix is also created using the mixing-logic program.

```
BATCH MODE INSTRUCTIONS ...............
Batch for preparing the Lateral Shuttle Ascent, First Stage Max-Q, T=61 sec
! This batch set creates Space Shuttle Ascent models at Max Dynamic Pressure. The models are
! used for H-infinity control design to reduce the sideslip sensitivity to gusts. The average
! frequency of the wind-gust is around 1 (rad/sec). The batch creates an H-infinity synthesis
! model that includes a beta filter and is augmented with 2 additional states to include the
! 1 (r/s) resonance. A state-feedback H-infinity controller is created. A second vehicle model
! is created that includes slosh, TWD, and accelerometers for further analysis.
!
Retain Matrix     : Shuttle Stage-1 TVC Matrix with Gust input
Retain CSM        : Shuttle Ascent, Max_Q, Design Model with TVC and Beta Filter/SM-2
!
Flight Vehicle    : Shuttle Ascent, Max_Q, T=61 sec, (Design Model)
Flight Vehicle    : Shuttle Ascent, Max_Q, T=61 sec, Rigid-Body/ Slosh/ TWD/ Acceleromet
System Connection: Shuttle Ascent, Max_Q, Design Model with TVC
System Modificat  : Shuttle Ascent, Max_Q, Lateral Hinf Design Model
System Modificat  : Shuttle Ascent, Max_Q, Lateral Analysis Model with Slosh & TVC
Mixing Matrix     : Shuttle Stage-1 TVC Matrix at Max-Q
Transf-Function   : Integrator
Transf-Function   : Beta Filter
System Connection: Shuttle Ascent, Max_Q, Design Model with TVC and Beta Filter
H-Infinity Design: Space Shuttle Lateral H-Infinity State-Feedback Control Design-7
System Connection: Closed-Loop Via State-Feedback Gain
!
To Matlab Format : Shuttle Ascent, Max_Q, Design Model with TVC and Beta Filter
To Matlab Format : Shuttle Ascent, Max_Q, Lateral Analysis Model with Slosh & TVC
To Matlab Format : Shuttle Ascent Lateral State-Feedback Gain-7
To Matlab Format : Closed-Loop Via State-Feedback Gain
To Matlab Format : Beta Filter
To Matlab Format : Shuttle Stage-1 TVC Matrix with Gust input
------------------------------------------------------------------------------------------
```

```
FLIGHT VEHICLE INPUT DATA ......
Shuttle Ascent, Max_Q, T=61 sec, (Design Model)
! Rigid Body Shuttle Design Model during First Stage at Max Dynamic Pressure.
! Slosh, Bending, and Tail-Wag-Dog are Not Included.
Body Axes Output, Attitude=Rate Integral,Without GAFD, No Turn Coordination

Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Planet Radius (Re) (ft)   :   93215.0      32.174       0.20896E+08
Moments and products of Inertias Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)          :   0.248524E+8  0.209190E+9  0.221208E+9, 0.0, 0.937592E+7,
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)        :  -115.0        0.036       -35.937
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)       :   1.54         1518.0       745.4        39410.0
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)         :   33.0         60.45        0.0          7.45
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                         :  -3.579       -0.04         0.0          0.0
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)   :   0.0000       57.93        0.0000       0.0000       0.00
Wind Gust Vel wrt Vehi (Azim & Elev) angles (deg), or Force(lb), Torque(ft-lb), locat:xyz:   Gust         45.0         90.0
Surface Reference Area (feet^2), Mean Aerodynamic Chord (ft), Wing Span in (feet)        :   2690.0       15.0         15.0
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial_rho/ Partial_H} :  -115.0        0.036       -35.937      -9.482e-10
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:   0.0          0.0          0.0          0.0          0.0
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}:   0.0         -0.0353       0.0000       0.0000       0.00
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}:   0.0         -0.0575       0.0000       0.0000       0.00
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:   0.0         -0.028        0.0000       0.0000       0.00
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}:   0.0         -0.017        0.0000       0.0000       0.00
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}:   0.0          0.0249       0.0000       0.0000       0.00

Number of Thruster Engines, Include or Not the Tail-Wags-Dog and Load-Torque Dynamics ?  :  5   NO TWD

TVC Engine No:  1                          (Gimbaling  Throttling   Single_Gimbal) : Middle SSME     Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)               :  472000.0    472000.0
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): -16.0       0.0          10.0        10.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  220.0       4800.0       3.1
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -182.1667    0.0         -64.958
TVC Engine No:  2                          (Gimbaling  Throttling   Single_Gimbal) : Left SSME       Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)               :  472000.0    472000.0
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): -10.0       0.0          10.0        10.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  220.0       4800.0       3.1
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -184.1      -4.4167      -56.595
TVC Engine No:  3                          (Gimbaling  Throttling   Single_Gimbal) : Right SSME      Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)               :  472000.0    472000.0
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): -10.0       0.0          10.0        10.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  220.0       4800.0       3.1
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -184.1       4.4167      -56.595
TVC Engine No:  4                          (Gimbaling  Throttling   Single_Gimbal) : Left SRB        Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)               :  0.245e+7    0.245e+7
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg):  0.0        0.0          10.0        10.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  605.0       0.154e+5    -1.07
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -201.53     -20.875      -33.3333
TVC Engine No:  5                          (Gimbaling  Throttling   Single_Gimbal) : Left SRB        Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)               :  0.245e+7    0.245e+7
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg):  0.0        0.0          10.0        10.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  605.0       0.154e+5    -1.07
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -201.53     +20.875      -33.3333
----------------------------------------------------------------------------------------------------------------------------
```

The simple vehicle model does not include tail-wag-dog (TWD) dynamics. It has 5 TVC engines: 3 Shuttle Main Engines of 472,000 (lb) thrust each, and 2 Solid Rocket Boosters of 2.45 million pounds thrust each. The second vehicle model includes TWD. It also includes roll, pitch and yaw rate gyros, NY and NZ accelerometers, and two slosh modes for the LOX and LH2 tanks.

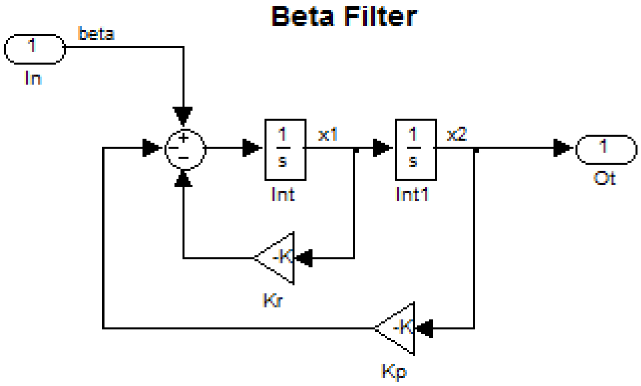
```
FLIGHT VEHICLE INPUT DATA ......
Shuttle Ascent, Max_Q, T=61 sec, Rigid-Body/ Slosh/ TWD/ Acceleromet
! Rigid Body Shuttle model during First Stage at Max Dynamic pressure.
! Slosh and Tail-Wag-Dog is Included.
Body Axes Output, Attitude=Rate Integral,Without GAFD, No Turn Coordination

Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Planet Radius (Re) (ft)   :  93215.0      32.174       0.20896E+08
Moments and products of Inertias Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)          :  0.248524E+8  0.209190E+9  0.221208E+9, 0.0, 0.937592E+7,
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)        : -115.0        0.036       -35.937
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)       :  1.54         1518.0       745.4        39410.0
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)         :  33.0         60.45        0.0          7.45
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                         : -3.579       -0.04         0.0          0.0
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)   :  0.0000       57.93        0.0000       0.0000       0.000
Wind Gust Vel wrt Vehi (Azim & Elev) angles (deg), or Force(lb), Torque(ft-lb), locat:xyz:  Gust         45.0         90.0
Surface Reference Area (feet^2), Mean Aerodynamic Chord (ft), Wing Span in (feet)        :  2690.0       15.0         15.0
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial_rho/ Partial_H} : -115.0        0.036       -35.937       -9.482e-10
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:  0.0          0.0          0.0          0.0          0.0
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}:  0.0         -0.0353       0.0000       0.0000       0.000
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}:  0.0         -0.0575       0.0000       0.0000       0.000
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:  0.0         -0.028        0.0000       0.0000       0.000
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}:  0.0         -0.017        0.0000       0.0000       0.000
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}:  0.0          0.0249       0.0000       0.0000       0.000

Number of Thruster Engines, Include or Not the Tail-Wags-Dog and Load-Torque Dynamics ?  :  5   WITH TWD
TVC Engine No:  1                               (Gimbaling  Throttling   Single_Gimbal) : Middle SSME      Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)                    :  472000.0     472000.0
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): -16.0         0.0          10.0         10.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  220.0        4800.0       3.1
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -182.1667     0.0         -64.958
TVC Engine No:  2                               (Gimbaling  Throttling   Single_Gimbal) : Left SSME        Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)                    :  472000.0     472000.0
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): -10.0         0.0          10.0         10.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  220.0        4800.0       3.1
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -184.1       -4.4167      -56.595
TVC Engine No:  3                               (Gimbaling  Throttling   Single_Gimbal) : Right SSME       Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)                    :  472000.0     472000.0
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): -10.0         0.0          10.0         10.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  220.0        4800.0       3.1
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -184.1        4.4167      -56.595
TVC Engine No:  4                               (Gimbaling  Throttling   Single_Gimbal) : Left SRB         Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)                    :  0.245e+7     0.245e+7
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg):  0.0          0.0          10.0         10.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  605.0        0.154e+5    -1.07
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -201.53      -20.875      -33.3333
TVC Engine No:  5                               (Gimbaling  Throttling   Single_Gimbal) : Left SRB         Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)                    :  0.245e+7     0.245e+7
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg):  0.0          0.0          10.0         10.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  605.0        0.154e+5    -1.07
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -201.53      +20.875      -33.3333

Number of Gyros, (Attitude and Rate)                                                     :  3
Gyro No  1 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Location in (feet) :  Roll   Rate           -93.625     0.00    -61.5417
Gyro No  2 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Location in (feet) :  Pitch  Rate           -93.625     0.00    -61.5417
Gyro No  3 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Location in (feet) :  Yaw    Rate           -93.625     0.00    -61.5417

Number of Accelerometers, Along Axis: (x,y,z)                                            :  2
Acceleromet No  1 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Location (ft) :  Y-axis Accelerat.     -93.625     0.00    -61.5417
Acceleromet No  2 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Location (ft) :  Z-axis Accelerat.     -93.625     0.00    -61.5417

Number of Slosh Modes                                                                    :  2
LOX Slosh Mass (slug), Frequ Wy,Wz 1g (rad/s), Damp (zeta-y-z), Locat {Xsl,Ysl,Zsl} (ft) :  4100.0  2.33  2.33  0.00164 0.00164 -57.96   0.0  -34.0
LH2 Slosh Mass (slug), Frequ Wy,Wz 1g (rad/s), Damp (zeta-y-z), Locat {Xsl,Ysl,Zsl} (ft) :  512.5  2.066  2.066 0.00244 0.00244 -115.02  0.0  -34.29
----------------------------------------------------------------------------------------------------------------------------------------------
```

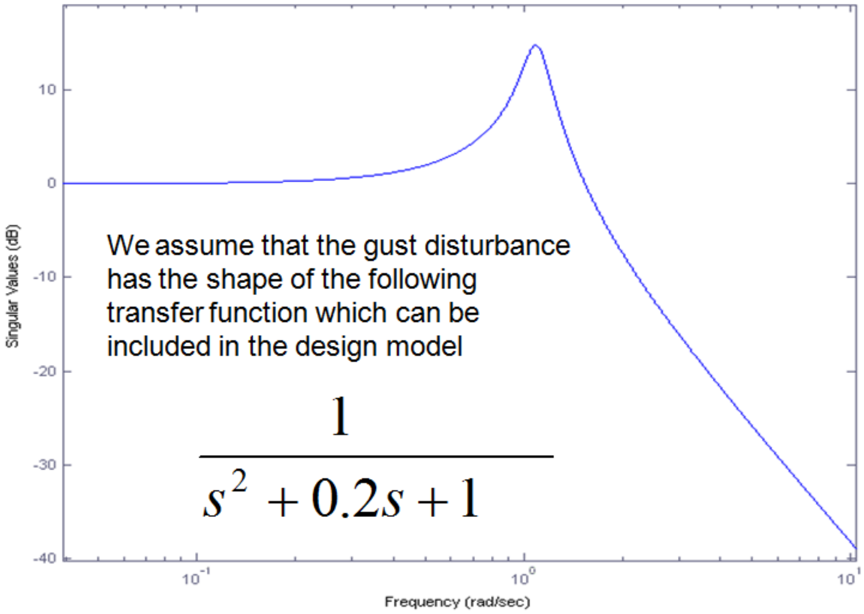
```
INTERCONNECTION OF SYSTEMS .....
Shuttle Ascent, Max_Q, Design Model with TVC
! Combines the Rigid vehicle model with the TVCG matrix that includes also the
! gust input.
Titles of Systems to be Combined
Title 1 Shuttle Ascent, Max_Q, T=61 sec, (Design Model)
SYSTEM INPUTS TO SUBSYSTEM  1                                                        TVC to Vehicle
Via Matrix +TVCG                                                                     inputs: roll, pitch, yaw,
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                                     from Plant
Via Matrix +I14                                                                      All Outputs
..............................................................................
Definitions of Inputs  =   4
DP_TVC (roll FCS demand)
DQ_TVC (pitch FCS demand)
DR_TVC (yaw FCS demand)
Wind Gust Azim, Elev Angles=(45, 90) (deg)

Definitions of Outputs =  14
Roll Attitude  (phi-body) (radians)
Roll Rate      (p-body)   (rad/sec)
Pitch Attitude (thet-bdy) (radians)
Pitch Rate     (q-body)   (rad/sec)
Yaw  Attitude  (psi-body) (radians)
Yaw  Rate      (r-body)   (rad/sec)
Angle of attack, alfa, (radians)
Angle of sideslip, beta, (radian)
Change in Altitude, delta-h, (feet)
Forward Acceleration  (V-dot)  (ft/sec)
Cross Range Velocity (Vcr) (ft/sec)
CG Acceleration along X axis, (ft/sec^2)
CG Acceleration along Y axis, (ft/sec^2)
CG Acceleration along Z axis, (ft/sec^2)
----------------------------------------------------------------------------------------------------------------
```

This Interconnection set combines the first vehicle model with the TVC matrix and also allows for a Wind-Gust input. The inputs to this model are: Roll, Pitch, Yaw Acceleration Demands and Wind-Gust.

Note, a fourth column was added to the TVC matrix for the gust (TVCG)

```
SYSTEM OF TRANSFER FUNCTIONS ...
Integrator
Continuous
TF. Block #  1  (1/s)                                            Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
.....................................................
Block #, from Input #, Gain
  1  1    1.0
...........................
Outpt #, from Block #, Gain
  1  1    1.0
.....................................................
--------------------------------------------------------------------------------------------
SYSTEM OF TRANSFER FUNCTIONS ...
Beta Filter
Continuous
TF. Block #  1  ( 1/s )                                          Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
TF. Block #  2  ( 1/s )                                          Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
.....................................................
Block #, from Input #, Gain
1        1     1.0
...........................
Block #, from Block #, Gain
2        1     1.0
1        1    -0.3
1        2    -1.0
...........................
Outpt #, from Block #, Gain
1        1     1.0
2        2     1.0
...........................
Definitions of Inputs  =   1
Beta In

Definitions of Outputs =   2
Filter State x1
Filter State x2
--------------------------------------------------------------------------------------------
```





We assume that the gust disturbance has the shape of the following transfer function which can be included in the design model

$$\frac{1}{s^2 + 0.2s + 1}$$

The beta-filter is a 2nd order transfer-function that is excited by β and produces two additional states x_1 and x_2 in the design model. It is implemented by the transfer-function dataset above.

The following Mixing Logic dataset generates the (10x3) TVC2 matrix that converts the 3 (roll, pitch, yaw) control demands to gimbal deflection commands. The other two sets create lateral design and analysis vehicle models by extracting the lateral variables from the corresponding coupled vehicle models.

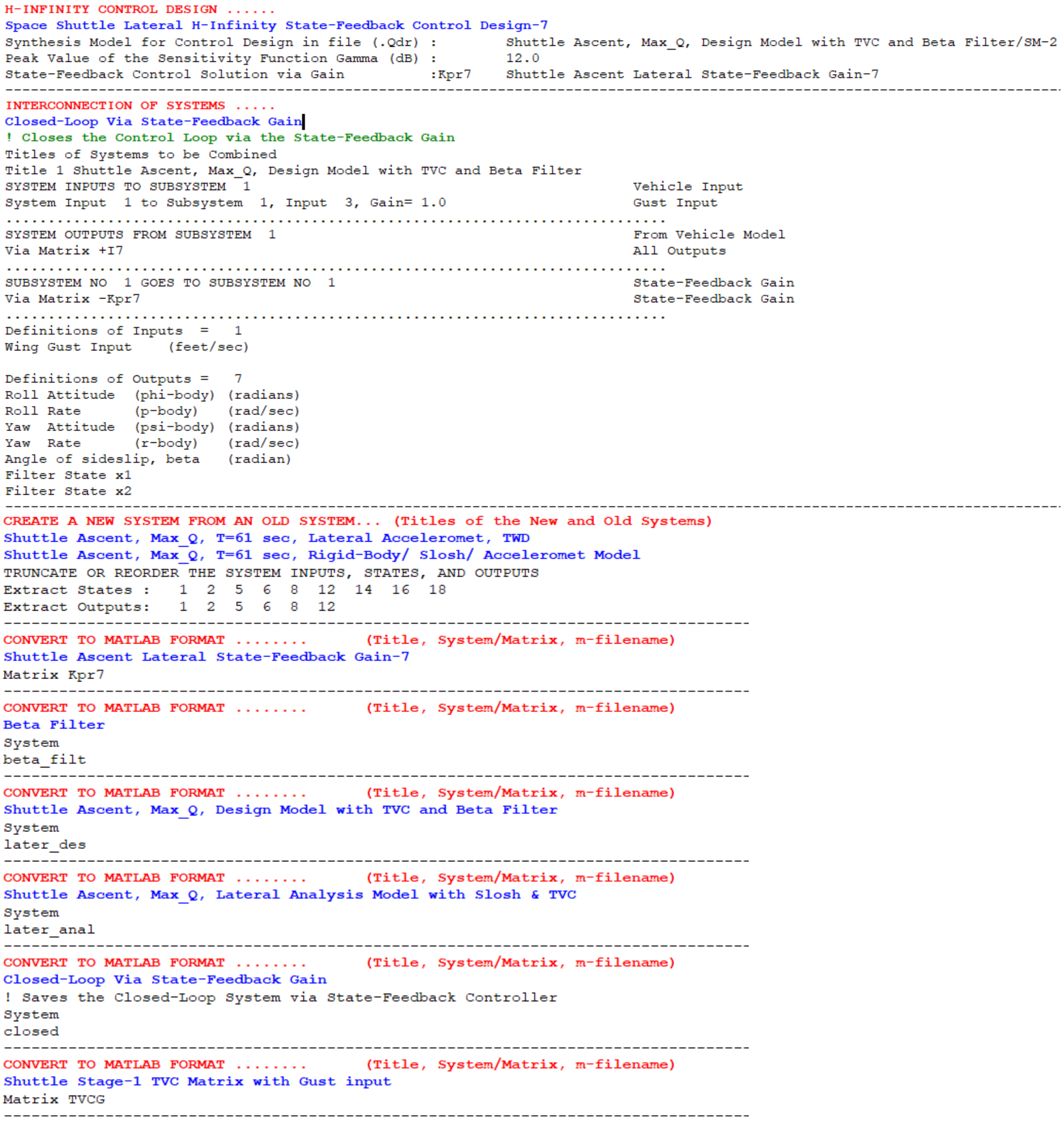
```
MIXING LOGIC MATRIX DATA ..........     (Matrix Title, Name, Vehicle Title, Control Directions)
Shuttle Stage-1 TVC Matrix at Max-Q
! Thrust Vector Control Matrix at Max-Q
! This multi-engine vehicle has 5 Gimbaling Engines.
TVC2
Shuttle Ascent, Max_Q, T=61 sec, (Design Model)
P-dot Roll  Acceleration About X Axis
Q-dot Pitch Acceleration About Y Axis
R-dot Yaw   Acceleration About Z Axis
-----------------------------------------------------------------------------------------------
CREATE A NEW SYSTEM FROM AN OLD SYSTEM... (Titles of the New and Old Systems)
Shuttle Ascent, Max_Q, Lateral Hinf Design Model
Shuttle Ascent, Max_Q, Design Model with TVC
TRUNCATE OR REORDER THE SYSTEM INPUTS, STATES, AND OUTPUTS
Extract Inputs :   1  3  4
Extract States :   1  2  5  6  8
Extract Outputs:   1  2  5  6  8
-----------------------------------------------------------------------------------------------
CREATE A NEW SYSTEM FROM AN OLD SYSTEM... (Titles of the New and Old Systems)
Shuttle Ascent, Max_Q, Lateral Analysis Model with Slosh & TVC
Shuttle Ascent, Max_Q, T=61 sec, Rigid-Body/ Slosh/ TWD/ Acceleromet
TRUNCATE OR REORDER THE SYSTEM INPUTS, STATES, AND OUTPUTS
Extract Inputs :   1  2  3  4  5  11  12  13  14  15  21
Extract States :   1  2  5  6  8  12  14  16  18
Extract Outputs:   1  2  5  6  8  12
-----------------------------------------------------------------------------------------------
```

The following interconnection dataset combines the lateral design model with the beta-filter to produce the augmented design model that includes the beta-filter. They both include the TVC matrix.

```
INTERCONNECTION OF SYSTEMS .....
Shuttle Ascent, Max_Q, Design Model with TVC and Beta Filter
! Combines the Lateral Design Vehicle Model with the Beta Filter
!
Titles of Systems to be Combined
Title 1 Shuttle Ascent, Max_Q, Lateral Hinf Design Model
Title 2 Beta Filter
SYSTEM INPUTS TO SUBSYSTEM  1                                             Design Vehicle Inputs
System Input  1 to Subsystem  1, Input  1, Gain= 1.00000                  Roll Demand DP_tvc
System Input  2 to Subsystem  1, Input  2, Gain= 1.00000                  Yaw  Demand DR_tvc
System Input  3 to Subsystem  1, Input  3, Gain= 1.00000                  Wind-Gust
.............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                          from Vehicle
System Output  1 from Subsystem  1, Output  1, Gain= 1.0                  Roll Attitude
System Output  2 from Subsystem  1, Output  2, Gain= 1.0                  Roll Rate
System Output  3 from Subsystem  1, Output  3, Gain= 1.0                  Yaw  Attitude
System Output  4 from Subsystem  1, Output  4, Gain= 1.0                  Yaw  Rate
System Output  5 from Subsystem  1, Output  5, Gain= 1.0                  Sideslip Beta
.............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                                          from Vehicle
System Output  6 from Subsystem  2, Output  1, Gain= 1.0                  Filter State x1
System Output  7 from Subsystem  2, Output  2, Gain= 1.0                  Filter State x1
.............................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2                                   Vehicle to Filter
Subsystem  1, Output  5 to Subsystem  2, Input  1, Gain=   1.0000         Beta to Filter
......................................................................
Definitions of Inputs  =   3
DP_TVC (roll FCS demand)
DR_TVC (yaw FCS demand)
Wind Gust Azim, Elev Angles=(45, 90) (deg)

Definitions of Outputs =   7
Roll Attitude   (phi-body) (radians)
Roll Rate       (p-body)   (rad/sec)
Yaw  Attitude   (psi-body) (radians)
Yaw  Rate       (r-body)   (rad/sec)
Angle of sideslip, beta, (radian)
Filter State x1
Filter State x2
-------------------------------------------------------------------------------------------------------
```

The next set performs the H-infinity design in batch. It uses the SM "*Shuttle Ascent, Max_Q, Design Model with TVC and Beta Filter/SM-2*" which is already saved in the systems file. The γ value is set to 12 (dB), as determined from previous interactive processing. The state-feedback matrix is saved in the systems file. Its name is "Kpr7" and its title is "*Shuttle Ascent Lateral State-Feedback Gain-7*".

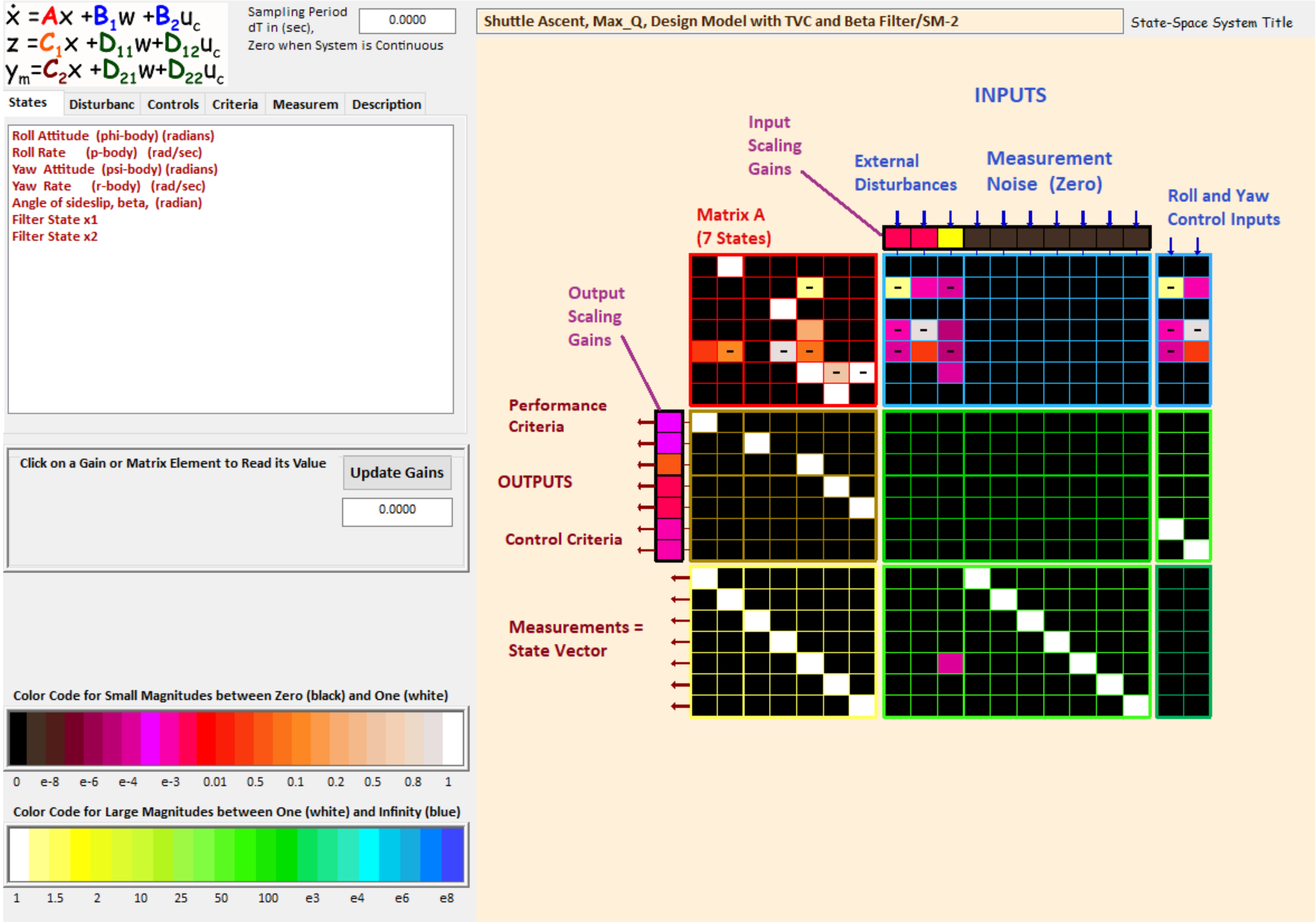
The closed-loop system is obtained by closing the control loop around the augmented design model using the H-infinity derived state-feedback matrix Kpr7. Finally, the design and analysis models, the TVC matrix, beta-filter, the control gain matrix Kpr7, and the closed-loop system are exported and loaded into Matlab.



```
H-INFINITY CONTROL DESIGN ......
Space Shuttle Lateral H-Infinity State-Feedback Control Design-7
Synthesis Model for Control Design in file (.Qdr) :        Shuttle Ascent, Max_Q, Design Model with TVC and Beta Filter/SM-2
Peak Value of the Sensitivity Function Gamma (dB) :        12.0
State-Feedback Control Solution via Gain          :Kpr7    Shuttle Ascent Lateral State-Feedback Gain-7
--------------------------------------------------------------------------------------------------------------------------
INTERCONNECTION OF SYSTEMS .....
Closed-Loop Via State-Feedback Gain
! Closes the Control Loop via the State-Feedback Gain
Titles of Systems to be Combined
Title 1 Shuttle Ascent, Max_Q, Design Model with TVC and Beta Filter
SYSTEM INPUTS TO SUBSYSTEM  1                                             Vehicle Input
System Input  1 to Subsystem  1, Input  3, Gain= 1.0                      Gust Input
..........................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                          From Vehicle Model
Via Matrix +I7                                                            All Outputs
..........................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  1                                   State-Feedback Gain
Via Matrix -Kpr7                                                          State-Feedback Gain
..........................................................................
Definitions of Inputs  =   1
Wing Gust Input    (feet/sec)

Definitions of Outputs =   7
Roll Attitude  (phi-body)  (radians)
Roll Rate      (p-body)    (rad/sec)
Yaw  Attitude  (psi-body)  (radians)
Yaw  Rate      (r-body)    (rad/sec)
Angle of sideslip, beta    (radian)
Filter State x1
Filter State x2
--------------------------------------------------------------------------------------------------------------------------
CREATE A NEW SYSTEM FROM AN OLD SYSTEM... (Titles of the New and Old Systems)
Shuttle Ascent, Max_Q, T=61 sec, Lateral Acceleromet, TWD
Shuttle Ascent, Max_Q, T=61 sec, Rigid-Body/ Slosh/ Acceleromet Model
TRUNCATE OR REORDER THE SYSTEM INPUTS, STATES, AND OUTPUTS
Extract States :   1  2  5  6  8  12  14  16  18
Extract Outputs:   1  2  5  6  8  12
--------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Shuttle Ascent Lateral State-Feedback Gain-7
Matrix Kpr7
--------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Beta Filter
System
beta_filt
--------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Shuttle Ascent, Max_Q, Design Model with TVC and Beta Filter
System
later_des
--------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Shuttle Ascent, Max_Q, Lateral Analysis Model with Slosh & TVC
System
later_anal
--------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Closed-Loop Via State-Feedback Gain
! Saves the Closed-Loop System via State-Feedback Controller
System
closed
--------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Shuttle Stage-1 TVC Matrix with Gust input
Matrix TVCG
--------------------------------------------------------------------------------
```

2.2 H-Infinity Synthesis Model

Figure-8 shows the lateral control synthesis model in systems form, color coded, as it appears when running the H-infinity program interactively. The approximate color code values are shown at the lower left corner. The A-matrix consists of 7 states, 5 from the original vehicle and 2 from the α-filter. There are 3 external disturbances (w), 2 control inputs (u_c) for roll and yaw control, 7 measurements (y_m) which are equal to the 7 states (C2 is the identity matrix), 5 performance criteria (z), and 2 control evaluation criteria. There are also 7 measurement noise inputs which are set to almost zero (dark brown) because they don't play a role here when designing a state-feedback, only when we include a state estimator.



$$\dot{x} = Ax + B_1 w + B_2 u_c$$
$$z = C_1 x + D_{11} w + D_{12} u_c$$
$$y_m = C_2 x + D_{21} w + D_{22} u_c$$

Sampling Period dT in (sec), Zero when System is Continuous: 0.0000

Shuttle Ascent, Max_Q, Design Model with TVC and Beta Filter/SM-2 — State-Space System Title

States:
Roll Attitude (phi-body) (radians)
Roll Rate (p-body) (rad/sec)
Yaw Attitude (psi-body) (radians)
Yaw Rate (r-body) (rad/sec)
Angle of sideslip, beta, (radian)
Filter State x1
Filter State x2

Click on a Gain or Matrix Element to Read its Value: 0.0000

Color Code for Small Magnitudes between Zero (black) and One (white)

Color Code for Large Magnitudes between One (white) and Infinity (blue)

Figure 8 H-Infinity Synthesis Model

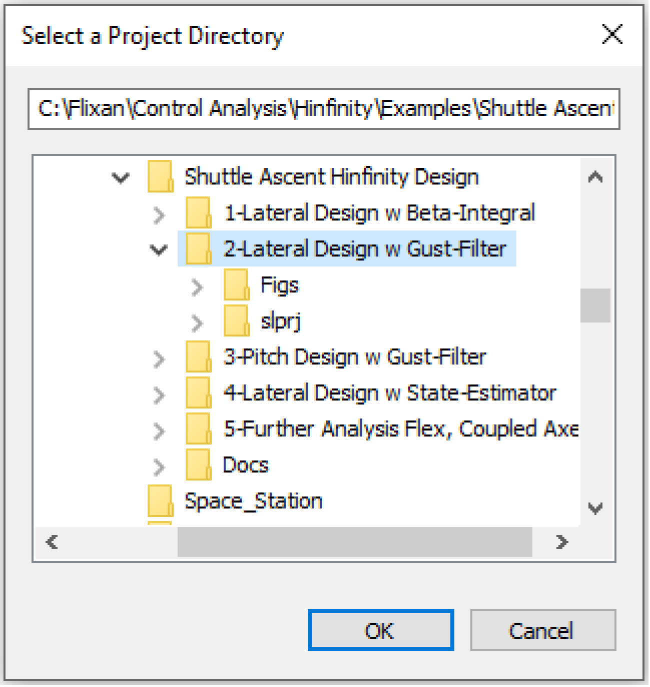
The SM is shown in detail below as copied from the systems file "Lateral_MaxQ2.Inp". Description of the variables are shown at the bottom, together with the scaling gains.

```
SYNTHESIS MODEL FOR H-INFINITY CONTROL
Shuttle Ascent, Max_Q, Design Model with TVC and Beta Filter/SM-2
Number of: States (x), Uncertainty Inp/Outputs from Plant Variations (dP)=   7   0   0
Number of: Extern Disturbance Inputs (Wi), Control Inputs (Uc)          =   3   2
Number of: Output Criteria (Zo), Regulated Outputs (Zr), Measurements (y)=   5   0   7
Synthes Model Matrices: A, B1,B2,C1,C2, D11,D12,D21,D22, Sample Time (dT)=   0.0000
Matrix A                    Size =  7 X  7
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00 -0.189871516138E+01  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.113501528513E+00  0.000000000000E+00
   5-Row  0.111727524803E-01 -0.624247212969E-01  0.000000000000E+00 -0.998049689533E+00 -0.503994541829E-01  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01 -0.280000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
------------------------------------------------------------------------------------------------------------------------
Matrix B1                   Size =  7 X 10
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row -0.100900317547E+01  0.288640277257E-02 -0.886177820482E-03  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row -0.317315938752E-02 -0.981762414631E+00  0.529740001050E-04  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row -0.814302307770E-03  0.179457917000E-01 -0.133755484444E-04  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.466724993040E-03  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
           9-Column           10-Column
   1-Row  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Matrix B2                   Size =  7 X  2
           1-Column            2-Column
   1-Row  0.000000000000E+00  0.000000000000E+00
   2-Row -0.100900317547E+01  0.288640277257E-02
   3-Row  0.000000000000E+00  0.000000000000E+00
   4-Row -0.317315938752E-02 -0.981762414631E+00
   5-Row -0.814302307770E-03  0.179457917000E-01
   6-Row  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Matrix C1                   Size =  7 X  7
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Matrix C2                   Size =  7 X  7
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Matrix D11                  Size =  7 X 10
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
           9-Column           10-Column
   1-Row  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
```

```
Matrix D12                  Size =  7 X  2
           1-Column            2-Column
   1-Row  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00
   6-Row  0.100000000000E+01  0.000000000000E+00
   7-Row  0.000000000000E+00  0.100000000000E+01
------------------------------------------------------------------------------------------------------------------------
Matrix D21                  Size =  7 X 10
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.466724993040E-03  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
           9-Column           10-Column
   1-Row  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00
   6-Row  0.100000000000E+01  0.000000000000E+00
   7-Row  0.000000000000E+00  0.100000000000E+01
------------------------------------------------------------------------------------------------------------------------
Matrix D22                  Size =  7 X  2
           1-Column            2-Column
   1-Row  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Definition of Synthesis Model Variables                              Max Scaling Factors

States (x) ..........  =   7
  1     Roll Attitude   (phi-body) (radians)
  2     Roll Rate       (p-body)   (rad/sec)
  3     Yaw  Attitude   (psi-body) (radians)
  4     Yaw  Rate       (r-body)   (rad/sec)
  5     Angle of sideslip, beta,   (radian)
  6     Filter State x1
  7     Filter State x2

Excitation Inputs (w)  =  10
  1     DP_TVC (roll FCS demand)                                     * 0.005
  2     DR_TVC (yaw FCS demand)                                      * 0.005
  3     Wind Gust Azim, Elev Angles=(45, 90) (deg)                   * 3.000
  4     Noise at Output: Roll Attitude     (phi-body) (radians)     * 0.10000E-07
  5     Noise at Output: Roll Rate         (p-body)   (rad/sec)     * 0.10000E-07
  6     Noise at Output: Yaw  Attitude     (psi-body) (radians)     * 0.10000E-07
  7     Noise at Output: Yaw  Rate         (r-body)   (rad/sec)     * 0.10000E-07
  8     Noise at Output: Angle of sideslip, beta,     (radian)      * 0.10000E-07
  9     Noise at Output: Filter State x1                             * 0.10000E-07
 10     Noise at Output: Filter State x2                             * 0.10000E-07

Control Inputs (u)  ... =   2
  1     Control: DP_TVC (roll FCS demand)                            *  1.0000
  2     Control: DR_TVC (yaw FCS demand)                             *  1.0000

Performance Outputs (z)=   7
  1     Roll Attitude   (phi-body) (radians)                         / 0.002
  2     Yaw  Attitude   (psi-body) (radians)                         / 0.0015
  3     Angle of sideslip, beta,   (radian)                          / 0.04
  4     Filter State x1                                              / 0.0045
  5     Filter State x2                                              / 0.0045
  6     Contrl Criter. DP_TVC (roll FCS demand)                      / 0.003
  7     Contrl Criter. DR_TVC (yaw FCS demand)                       / 0.003

Measurement Outputs (y)=   7
  1     Measurm: Roll Attitude    (phi-body) (radians)               /  1.0000
  2     Measurm: Roll Rate        (p-body)   (rad/sec)               /  1.0000
  3     Measurm: Yaw  Attitude    (psi-body) (radians)               /  1.0000
  4     Measurm: Yaw  Rate        (r-body)   (rad/sec)               /  1.0000
  5     Measurm: Angle of sideslip, beta,    (radian)                /  1.0000
  6     Measurm: Filter State x1                                     /  1.0000
  7     Measurm: Filter State x2                                     /  1.0000
---------------------------------------------------------------------------------------------
```

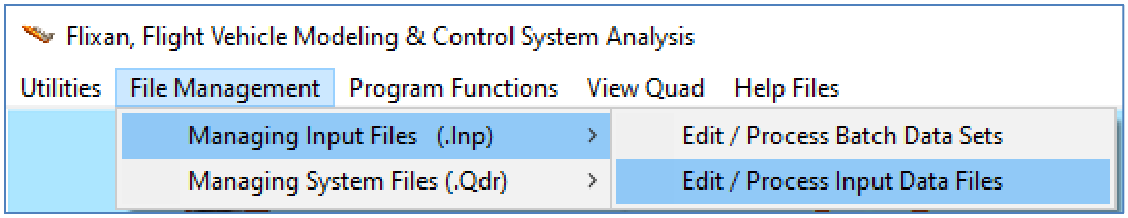
2.3 Processing the Input Data File

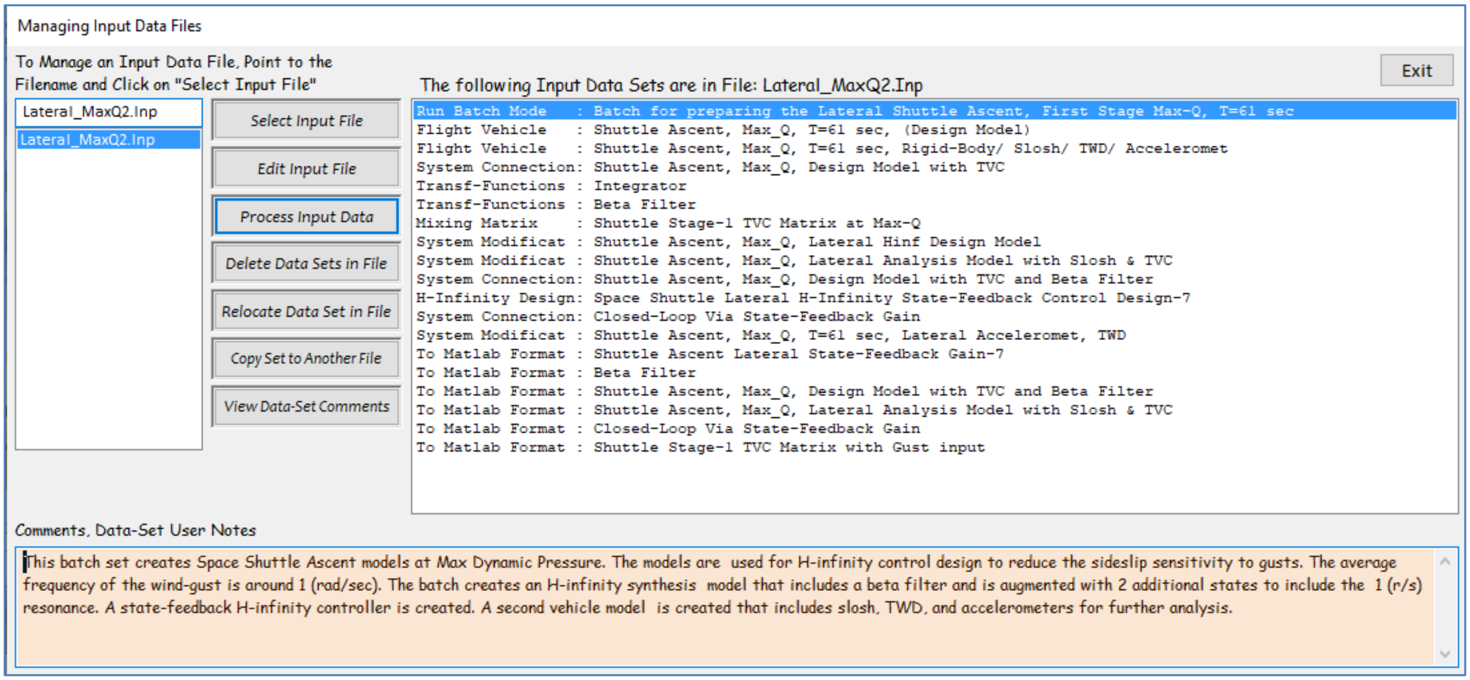
The complete modeling and control design are included in the input file “Lateral_MaxQ2.Inp”. This time we will not run it interactively but we will process the entire file in batch mode by running the batch set which is located at the top of the file. Start the Flixan program and select the project directory: “Flixan\Control Analysis\Hinfinity\Examples\Shuttle Ascent Hinfinity Design\2-Lateral Design w Gust-Filter”. From the main menu select “File Management”, “Managing Input Files”, and then “Edit/ Process Input Data Files”, as shown below.



C:\Flixan\Control Analysis\Hinfinity\Examples\Shuttle Ascen

The following dialog comes up that includes two menus. The menu on the left side lists the input data files in the project directory. There is only one. Highlight it and click on “Select Input File” button. The menu on the right shows the datasets which are in the input file. Select the batch set which is at the top of the list and click on “Process Input Data”.



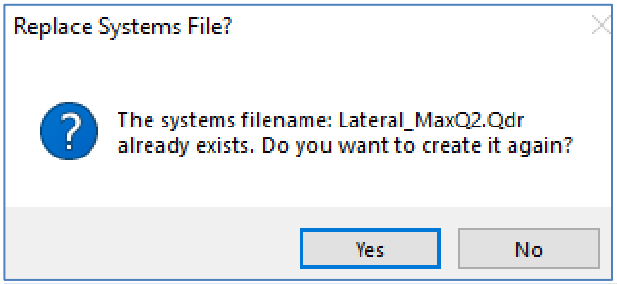


The following Input Data Sets are in File: Lateral_MaxQ2.Inp

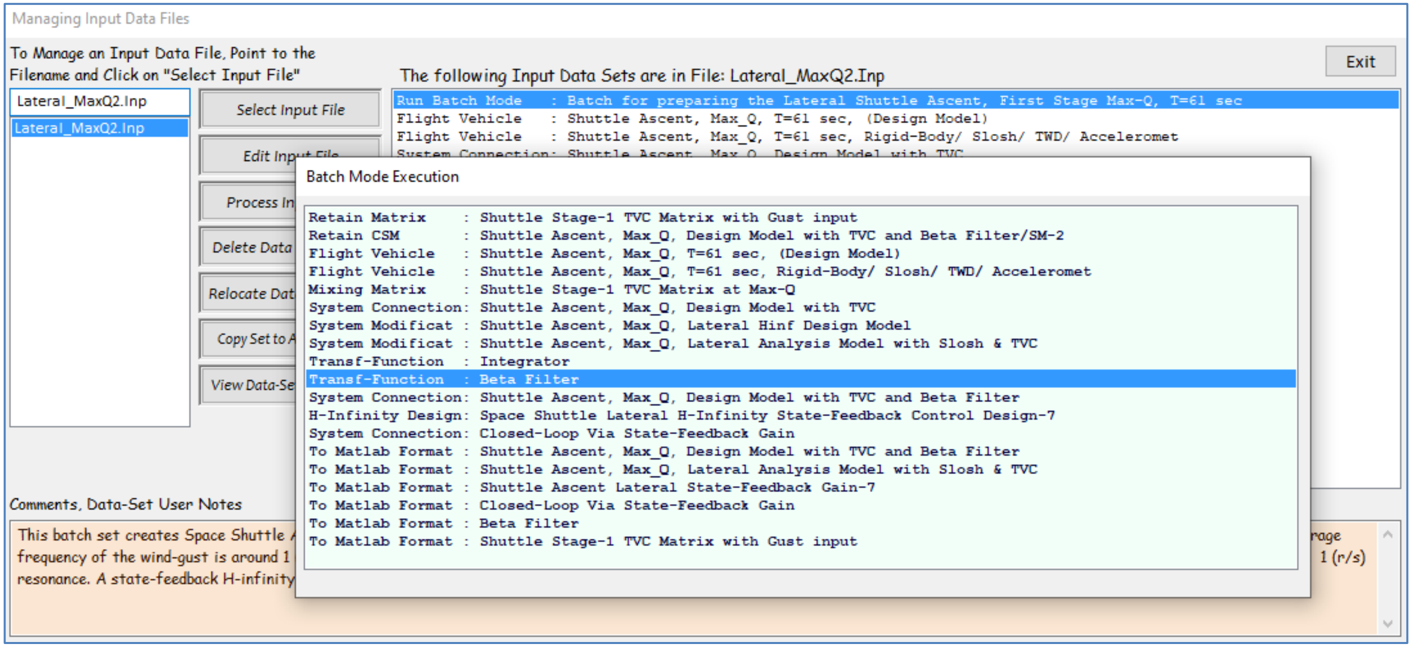
```
Run Batch Mode    : Batch for preparing the Lateral Shuttle Ascent, First Stage Max-Q, T=61 sec
Flight Vehicle    : Shuttle Ascent, Max_Q, T=61 sec, (Design Model)
Flight Vehicle    : Shuttle Ascent, Max_Q, T=61 sec, Rigid-Body/ Slosh/ TWD/ Acceleromet
System Connection: Shuttle Ascent, Max_Q, Design Model with TVC
Transf-Functions : Integrator
Transf-Functions : Beta Filter
Mixing Matrix     : Shuttle Stage-1 TVC Matrix at Max-Q
System Modificat : Shuttle Ascent, Max_Q, Lateral Hinf Design Model
System Modificat : Shuttle Ascent, Max_Q, Lateral Analysis Model with Slosh & TVC
System Connection: Shuttle Ascent, Max_Q, Design Model with TVC and Beta Filter
H-Infinity Design: Space Shuttle Lateral H-Infinity State-Feedback Control Design-7
System Connection: Closed-Loop Via State-Feedback Gain
System Modificat : Shuttle Ascent, Max_Q, T=61 sec, Lateral Acceleromet, TWD
To Matlab Format : Shuttle Ascent Lateral State-Feedback Gain-7
To Matlab Format : Beta Filter
To Matlab Format : Shuttle Ascent, Max_Q, Design Model with TVC and Beta Filter
To Matlab Format : Shuttle Ascent, Max_Q, Lateral Analysis Model with Slosh & TVC
To Matlab Format : Closed-Loop Via State-Feedback Gain
To Matlab Format : Shuttle Stage-1 TVC Matrix with Gust input
```

This batch set creates Space Shuttle Ascent models at Max Dynamic Pressure. The models are used for H-infinity control design to reduce the sideslip sensitivity to gusts. The average frequency of the wind-gust is around 1 (rad/sec). The batch creates an H-infinity synthesis model that includes a beta filter and is augmented with 2 additional states to include the 1 (r/s) resonance. A state-feedback H-infinity controller is created. A second vehicle model is created that includes slosh, TWD, and accelerometers for further analysis.

In the following question, answer “Yes”, which is okay to delete the old systems file and recreate it. The batch executes and creates the systems and matrices that can now be loaded into Matlab.



The systems filename: Lateral_MaxQ2.Qdr already exists. Do you want to create it again?



To Manage an Input Data File, Point to the Filename and Click on "Select Input File"

Lateral_MaxQ2.Inp
Lateral_MaxQ2.Inp

The following Input Data Sets are in File: Lateral_MaxQ2.Inp

```
Run Batch Mode    : Batch for preparing the Lateral Shuttle Ascent, First Stage Max-Q, T=61 sec
Flight Vehicle    : Shuttle Ascent, Max_Q, T=61 sec, (Design Model)
Flight Vehicle    : Shuttle Ascent, Max_Q, T=61 sec, Rigid-Body/ Slosh/ TWD/ Acceleromet
System Connection: Shuttle Ascent, Max_Q, Design Model with TVC
```

```
Retain Matrix     : Shuttle Stage-1 TVC Matrix with Gust input
Retain CSM        : Shuttle Ascent, Max_Q, Design Model with TVC and Beta Filter/SM-2
Flight Vehicle    : Shuttle Ascent, Max_Q, T=61 sec, (Design Model)
Flight Vehicle    : Shuttle Ascent, Max_Q, T=61 sec, Rigid-Body/ Slosh/ TWD/ Acceleromet
Mixing Matrix     : Shuttle Stage-1 TVC Matrix at Max-Q
System Connection: Shuttle Ascent, Max_Q, Design Model with TVC
System Modificat  : Shuttle Ascent, Max_Q, Lateral Hinf Design Model
System Modificat  : Shuttle Ascent, Max_Q, Lateral Analysis Model with Slosh & TVC
Transf-Function   : Integrator
Transf-Function   : Beta Filter
System Connection: Shuttle Ascent, Max_Q, Design Model with TVC and Beta Filter
H-Infinity Design: Space Shuttle Lateral H-Infinity State-Feedback Control Design-7
System Connection: Closed-Loop Via State-Feedback Gain
To Matlab Format  : Shuttle Ascent, Max_Q, Design Model with TVC and Beta Filter
To Matlab Format  : Shuttle Ascent, Max_Q, Lateral Analysis Model with Slosh & TVC
To Matlab Format  : Shuttle Ascent Lateral State-Feedback Gain-7
To Matlab Format  : Closed-Loop Via State-Feedback Gain
To Matlab Format  : Beta Filter
To Matlab Format  : Shuttle Stage-1 TVC Matrix with Gust input
```

Comments, Data-Set User Notes

This batch set creates Space Shuttle A
frequency of the wind-gust is around 1
resonance. A state-feedback H-infinity

rage
1 (r/s)

2.4 Control Analysis

The Matlab analysis begins by running the initialization file “init.m” which loads the systems, control gains and TVC matrices into Matlab. The design model includes the β-filter. The β-filter is also loaded separately in state-space form. The analysis model includes slosh and TWD dynamics but no β-filter. It was included in the design plant for the controller calculation, but now the β-filter has to be included in the control system. Some vehicle parameters that will be used later for estimating β are also loaded into Matlab. We also check the eigenvalues of the closed-loop system to make sure that it’s stable. We will demonstrate two versions of the control design. We will first analyze the system using β-feedback directly. Then we will replace the direct β-measurement with an estimate from the NY accelerometer because β is not directly measurable.

```
% Initialization File
r2d=180/pi; d2r=pi/180;
[Av, Bv, Cv, Dv] =later_des;                % Shuttle Ascent, Max_Q, Lateral Hinf Design Model
[An, Bn, Cn, Dn] =later_anal;               % Shuttle Ascent, Lateral Analysis Model
[Acl,Bcl,Ccl,Dcl]=closed;                   % Closed-Loop System
[Af, Bf, Cf, Df] =beta_filt;                % Beta Filter
load Kpr7  -ascii;                          % Load the Control Gains
load TVCG  -ascii;                          % Load the TVC Matrix with Gust

% Alpha/Beta Estimator Parameters
Mass=93215; Sref= 2690; Qbar=745.4;
Cyb=-0.0353; Cza=-0.0574;
Thr=[470000, 470000, 470000, 0.245e7, 0.245e7]';
eig(Acl)
```

2.4.1 Control Analysis Using Direct Beta Measurement

We will first analyze roll and yaw stability and the control system sensitivity to gust disturbances which is a frequency domain analysis. We will use the analysis model "*Shuttle Ascent, Max_Q, Lateral Analysis Model with Slosh & TVC*" which was loaded from file "*later_anal.m*" and it includes slosh, TWD and the accelerometers. Stability is calculated using the open-loop analysis model "*Open_Loop1.slx*", shown in Figure-9. The filter states x_1 and x_2 are excited by the sideslip angle β. The state-vector feedback in addition to the vehicle 5 states, it also includes the two filter states x_1 and x_2. Yaw stability is analyzed by opening the yaw loop and closing the roll loop, as shown below.

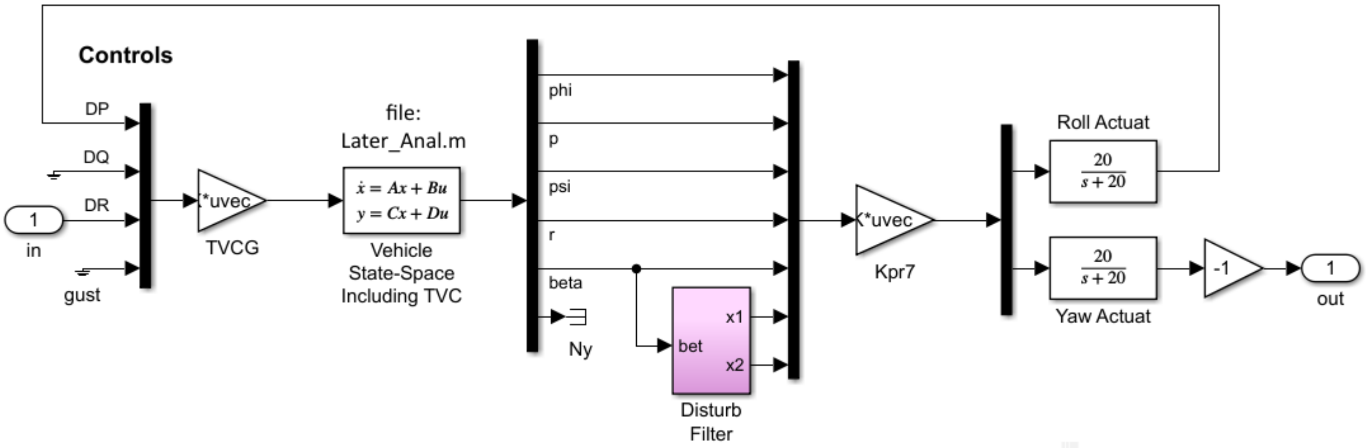


Figure 9 Open-Loop Stability Analysis Model "Open_Loop1.slx" Configured for Yaw Analysis (Roll Loop Closed)

Similarly, the sensitivity to disturbances is analyzed in the frequency domain using the closed-loop sensitivity analysis model "Sensitiv3.slx" shown in Figure-10, by calculating the SV sigma-plot from gust input to β output. The subsystems of this model are the same as in Fig-9. It is scaled by multiplying the gust input by the largest expected wind-gust velocity and by dividing the β-output by the maximum allowed β-angle. The system satisfies sensitivity requirement when the Sensitivity TF magnitude is less than one at all frequencies.

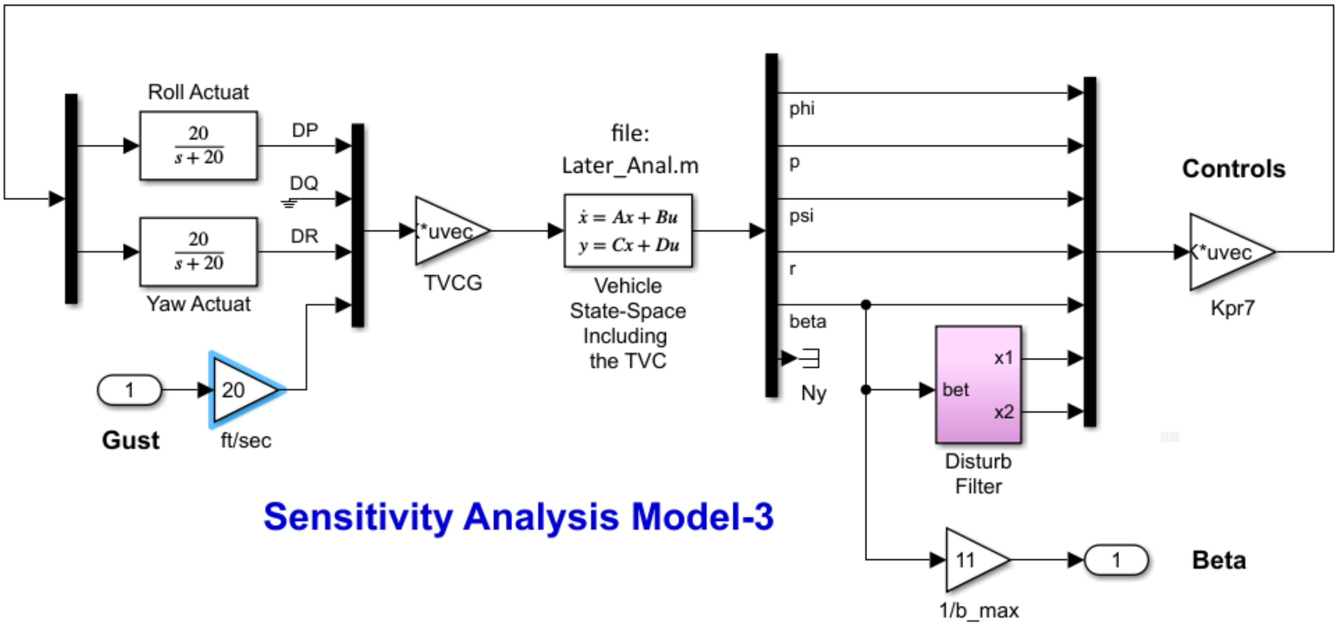


Figure 10 Sensitivity Analysis Model "Sensitiv3.slx"

Figure 11 shows the open yaw loop Bode and Nichols plots (roll closed) calculated using the Simulink model "*Open_Loop1.slx*". They are calculated by running the script file "frequ.m". Figure 12 shows Bode and Nichols plots for the open roll loop with the yaw loop closed. The margins are great at this point since we don't have bending modes. Slosh is phase-stable.

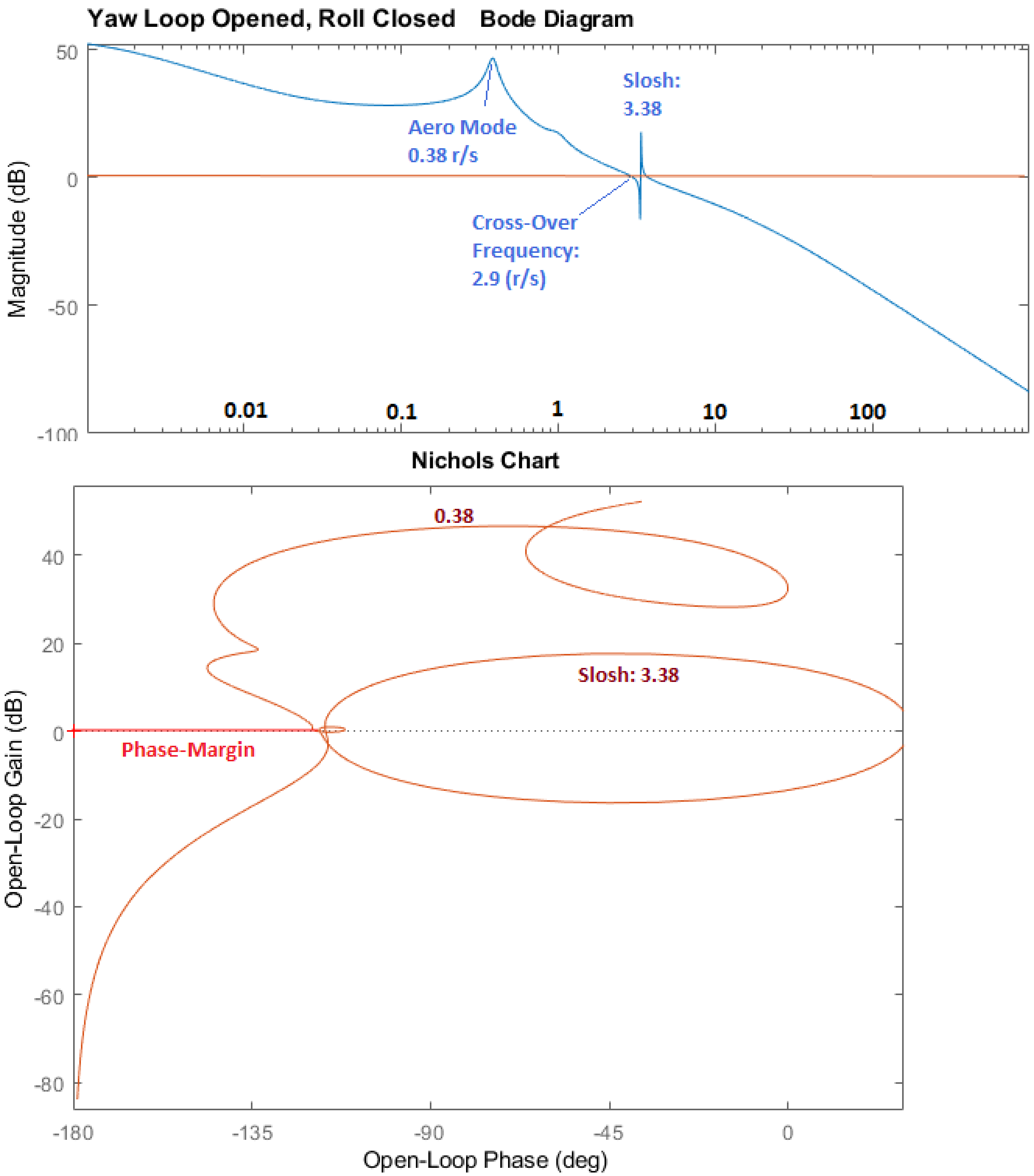
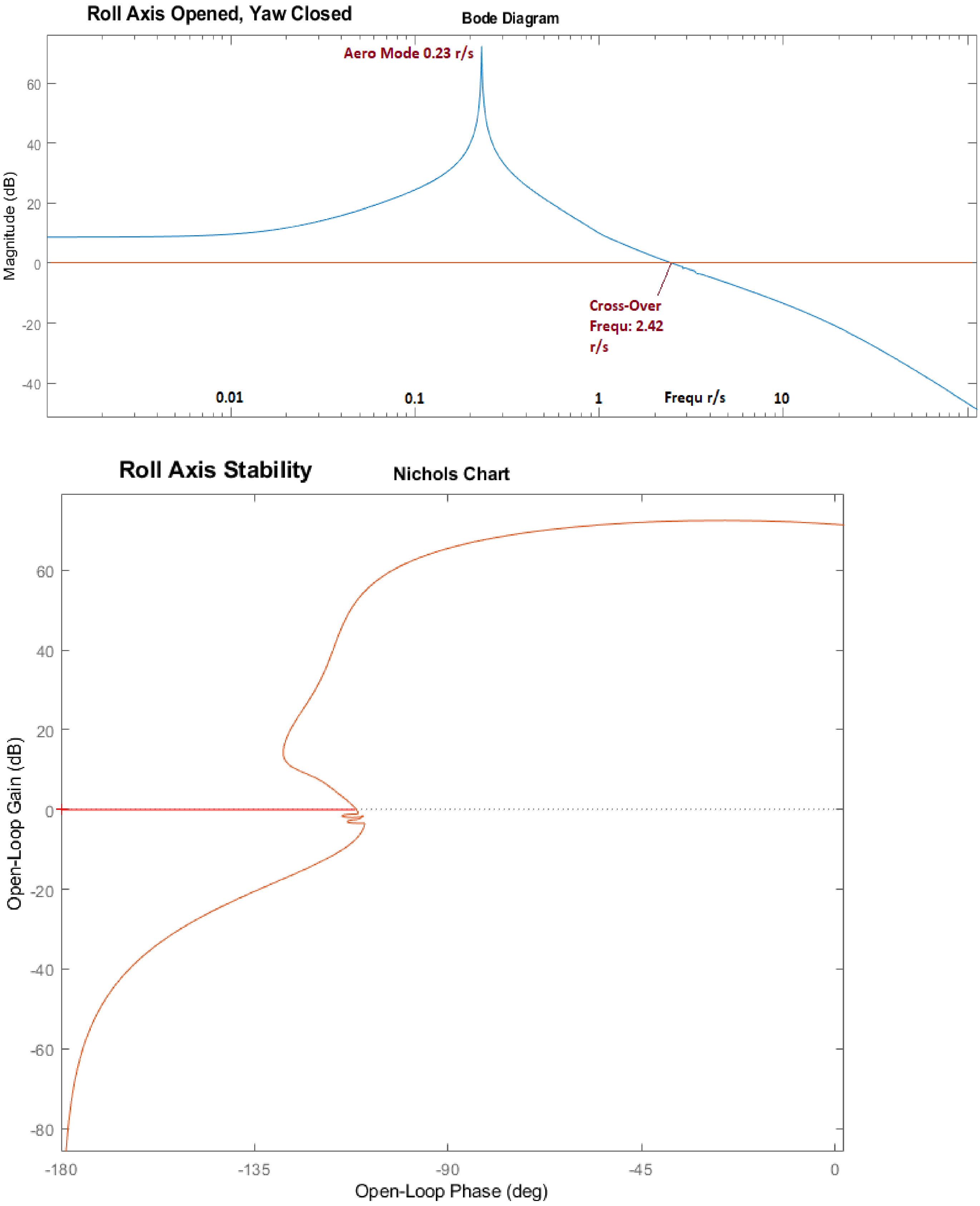
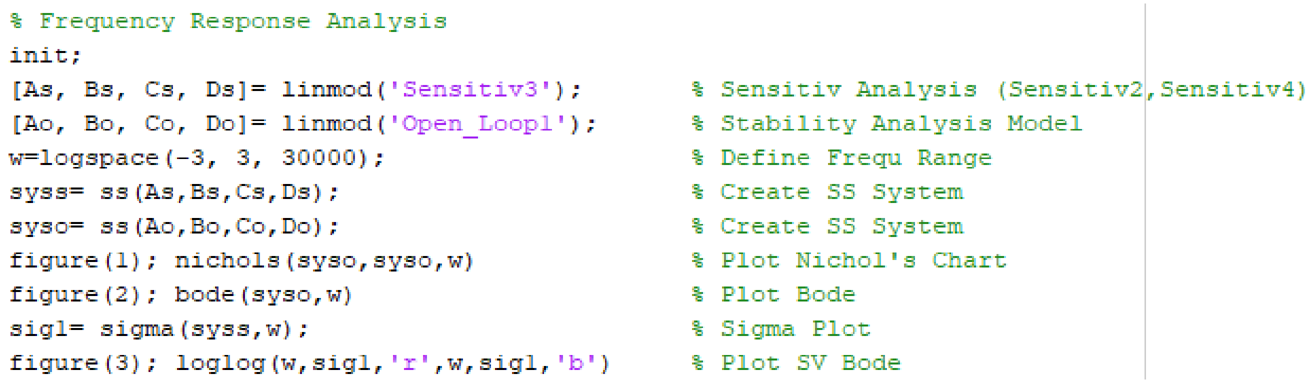


Figure 11 Yaw Axis Stability


Roll Axis Opened, Yaw Closed

Roll Axis Stability

Figure 12 Roll Axis Stability



```
% Frequency Response Analysis
init;
[As, Bs, Cs, Ds]= linmod('Sensitiv3');       % Sensitiv Analysis (Sensitiv2,Sensitiv4)
[Ao, Bo, Co, Do]= linmod('Open_Loop1');      % Stability Analysis Model
w=logspace(-3, 3, 30000);                    % Define Frequ Range
syss= ss(As,Bs,Cs,Ds);                       % Create SS System
syso= ss(Ao,Bo,Co,Do);                       % Create SS System
figure(1); nichols(syso,syso,w)              % Plot Nichol's Chart
figure(2); bode(syso,w)                      % Plot Bode
sig1= sigma(syss,w);                         % Sigma Plot
figure(3); loglog(w,sig1,'r',w,sig1,'b')     % Plot SV Bode
```

The script file “freq.m” uses the Simulink models “Open_Loop1.slxc” and “Sensitiv3.slx” to calculate the Bode, Nichols and SV (Sigma) plots. The sensitivity function has a significant dip at 1 (rad/sec). This would reduce the beta amplitude response at the disturbance frequency.

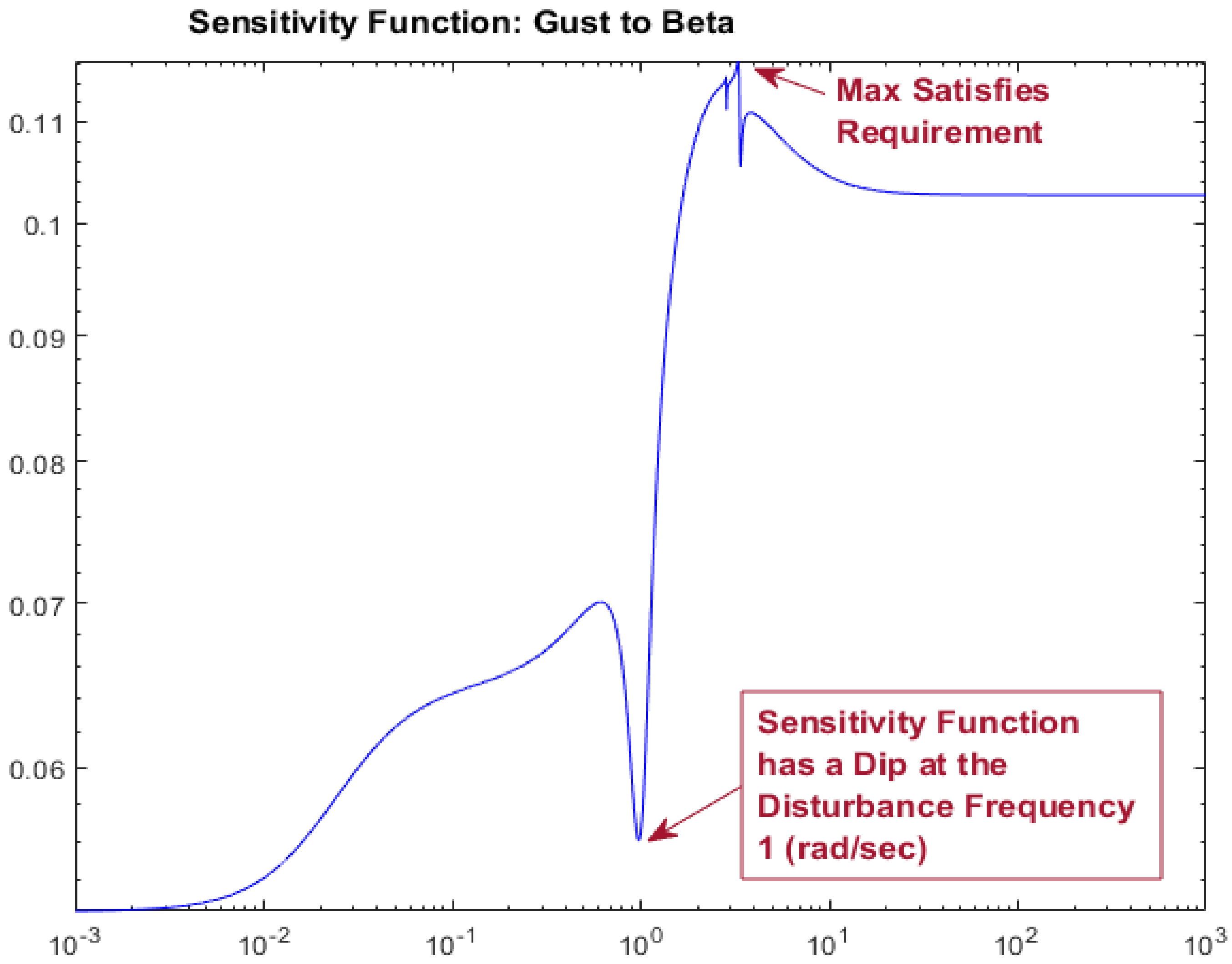


Figure 13 Sigma Plot of the Sensitivity Function Between Gust Disturbance and Beta Output

Simulation Using Direct Beta Measurement

A couple of simulation models were created to test the H-infinity controller using direct feedback from beta including the β-filter. They are shown in Figure 14 below. The first one in file "Closed_Loop1.slx" uses the design model from file "Later_Des.m" which includes the β-filter. The second one in file "Closed_Loop2.slx" uses the analysis model from file "Later_Anal.m" which does not include the β-filter but it includes slosh and TWD. The β-filter is included in a separate block. The H-infinity derived (2x7) state-feedback gain Kpr7 closes the control loop from the augmented 7-states vector via the roll and yaw actuator models. Figure 15 shows the attitude and beta responses to unit step (ϕ & ψ) attitude commands as calculated by the second simulation model. The two attitude responses converge towards the commanded values but they are slow because of the Max-Q and load-relief situation.

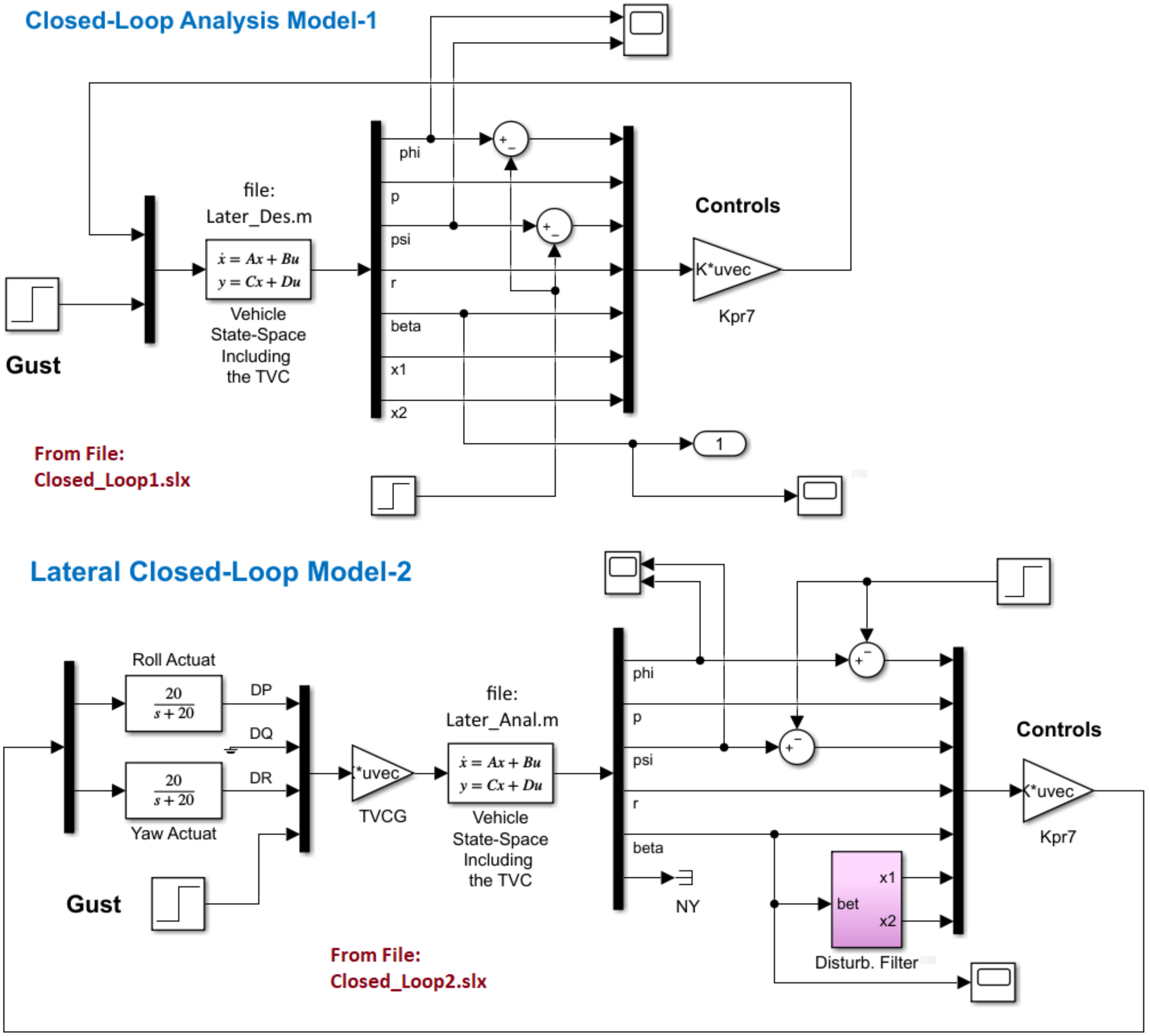


Figure 14 Closed-Loop Time-Slice Simulation Models

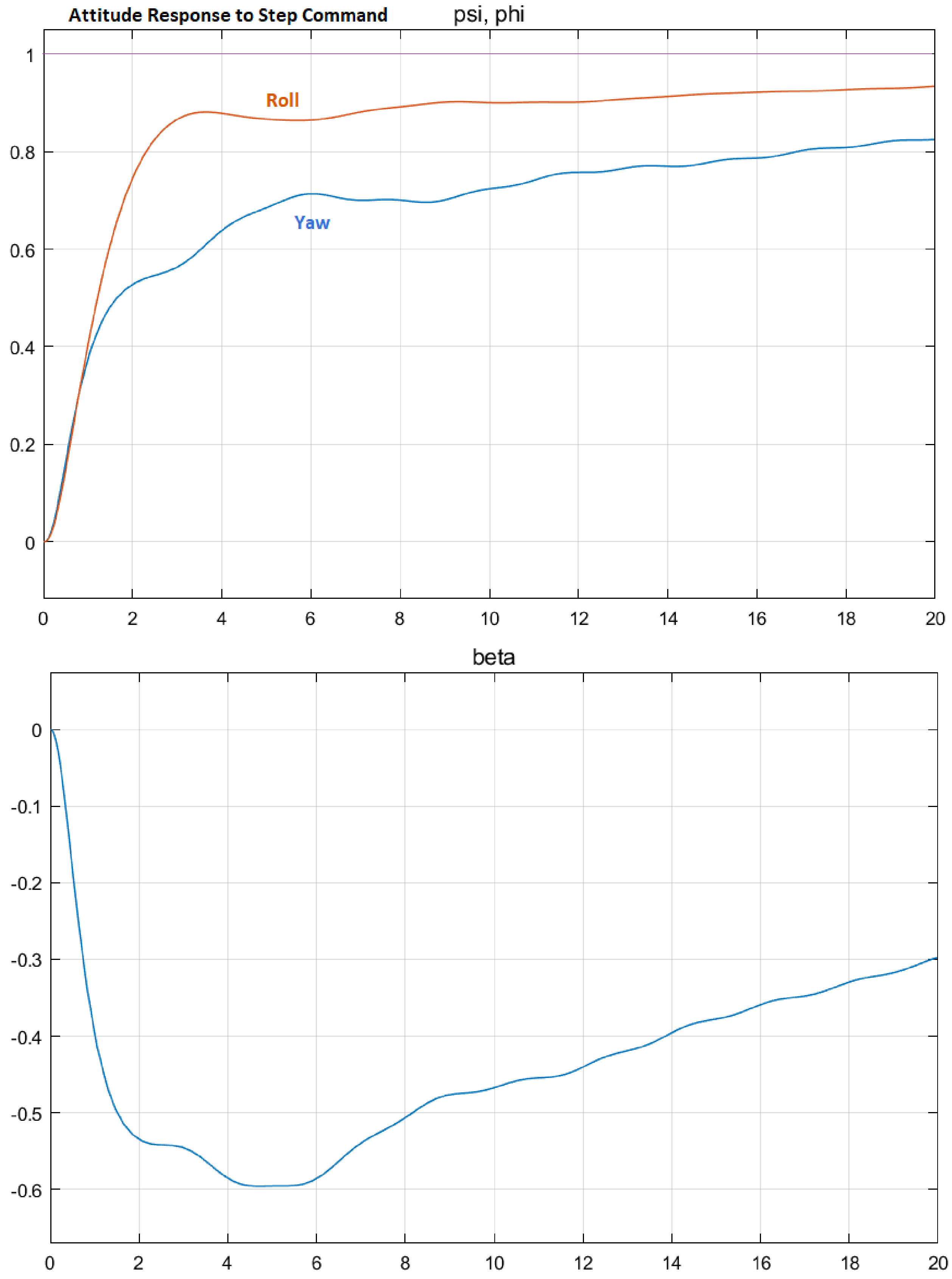


Figure 15 Vehicle Response to Simultaneously Applied Unit Step Roll and Yaw Attitude Commands

2.4.2 Control Analysis Using Feedback from a Beta Estimator

The Shuttle vehicle does not have an air-data probe to directly measure the angles of attack and sideslip (α & β). We will, therefore, design estimators that will calculate (α & β) estimates from the (N_Y & N_Z) accelerometer measurements and other variables. The β-estimator block is shown in Figure-14. The inputs are N_Y acceleration, yaw gimbal deflections (δz_i) and filtered roll & yaw rates. The output is β-estimate in (rad).

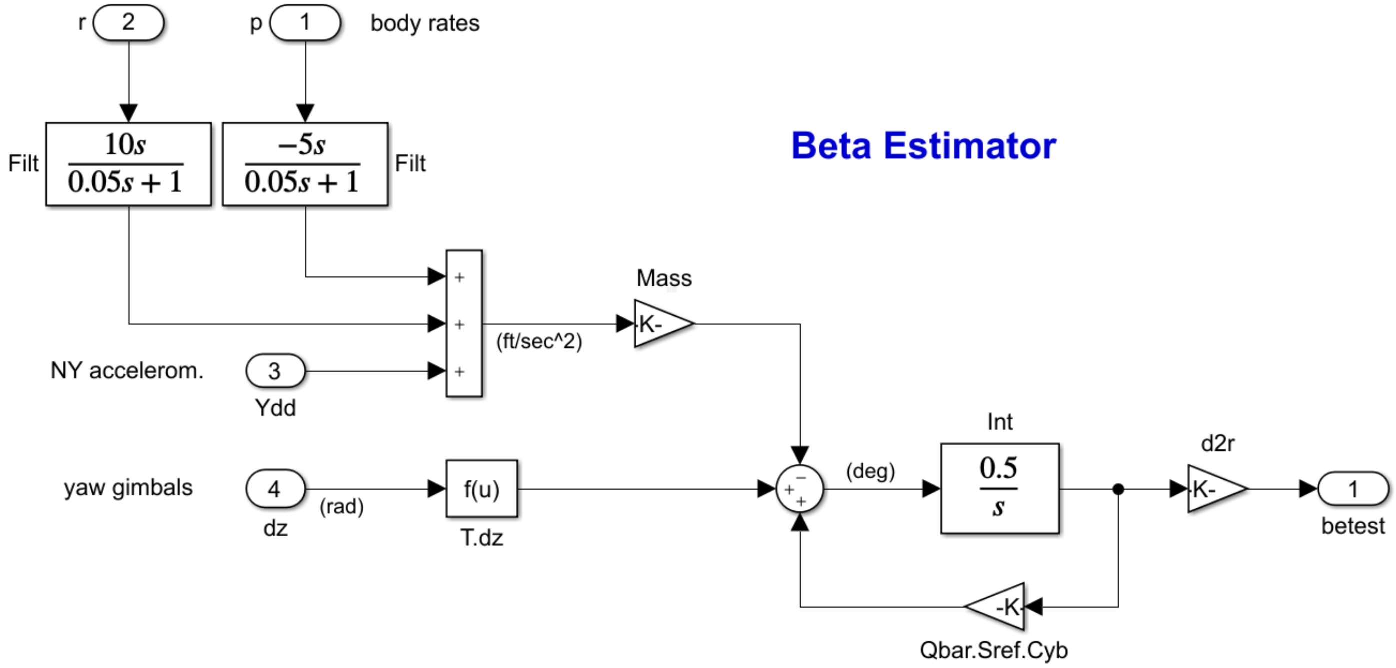


Figure 16 Beta-Estimator

The Simulink model "*Open_Loop2.slx*" in Figure-17 is used to analyze roll and yaw stability. It is similar to Figure-9 and includes the same analysis system but beta is no longer used and instead the estimated beta is used for state-feedback. Also, the β-filter states x_1 and x_2 are no longer excited by the sideslip angle, but the estimated β.

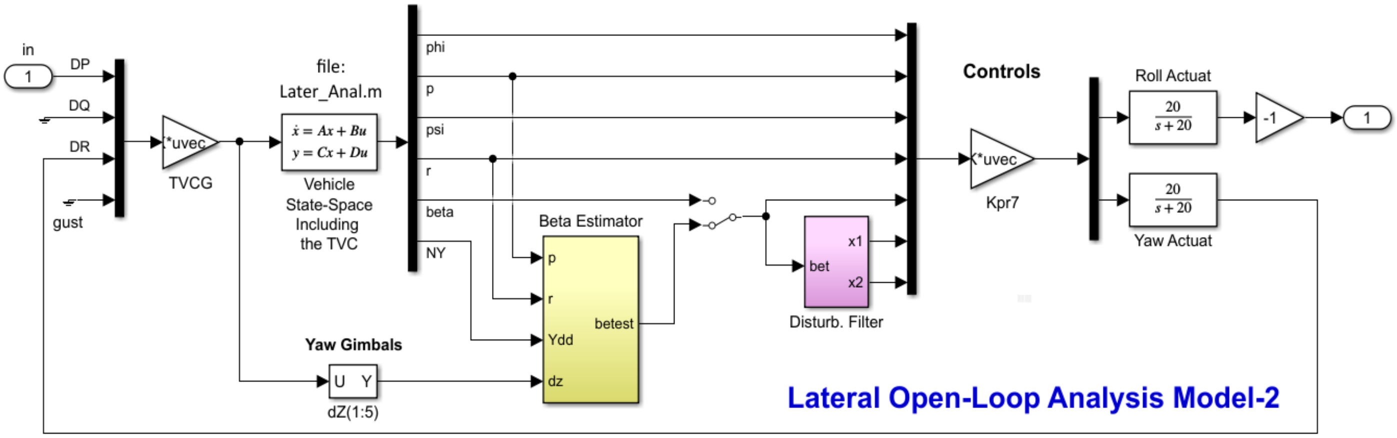


Figure 17 Open-Loop Stability Analysis Model "Open_Loop2.slx" Configured for Roll Analysis (Yaw Closed)

Similarly, the sensitivity analysis model which is shown in Figure-18, it is implemented in the Simulink model “Sensitiv4.slx” and it includes the β-estimator. Otherwise, it is similar to Figure-10. Its gust input is scaled by the largest wind-gust velocity and the β-output is scaled by the maximum allowed β-angle. The output is the actual beta, not the estimate. It is only used for evaluation not for control.

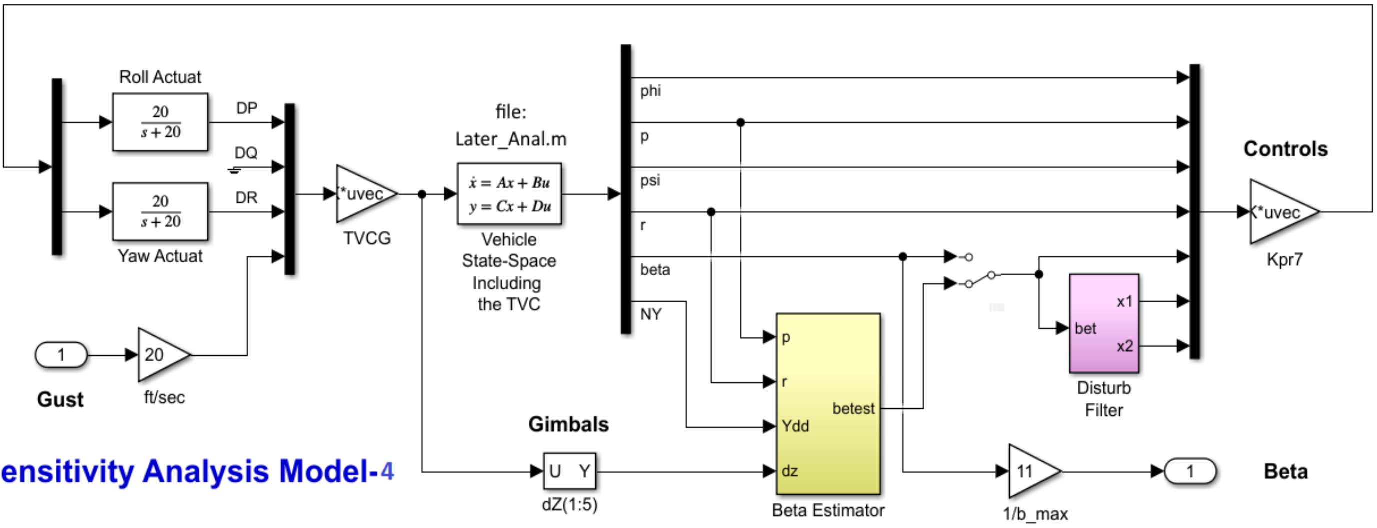
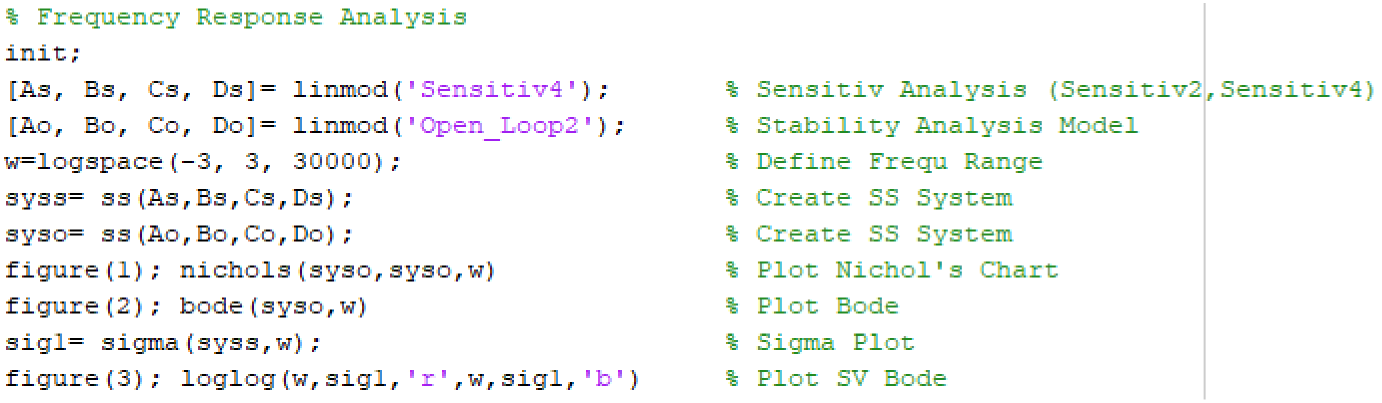


Figure 18 Sensitivity Analysis Model “Sensitiv4.slx”

The script file “freq.m” uses the above two Simulink models to calculate the Open-Loop Bode and Nichols plots and also the sensitivity Sigma plot β(jω)/Vgust. Figure 19 shows the open yaw loop Bode and Nichols plots (roll loop closed) calculated using the Simulink model “*Open_Loop2.slx*”. Figure 20 shows the Bode and Nichols plots for the open roll loop with the yaw loop closed. The margins are good at this point since we don’t have bending modes. Slosh is phase-stable.



```
% Frequency Response Analysis
init;
[As, Bs, Cs, Ds]= linmod('Sensitiv4');         % Sensitiv Analysis (Sensitiv2,Sensitiv4)
[Ao, Bo, Co, Do]= linmod('Open_Loop2');        % Stability Analysis Model
w=logspace(-3, 3, 30000);                      % Define Frequ Range
syss= ss(As,Bs,Cs,Ds);                         % Create SS System
syso= ss(Ao,Bo,Co,Do);                         % Create SS System
figure(1); nichols(syso,syso,w)                % Plot Nichol's Chart
figure(2); bode(syso,w)                        % Plot Bode
sigl= sigma(syss,w);                           % Sigma Plot
figure(3); loglog(w,sigl,'r',w,sigl,'b')       % Plot SV Bode
```

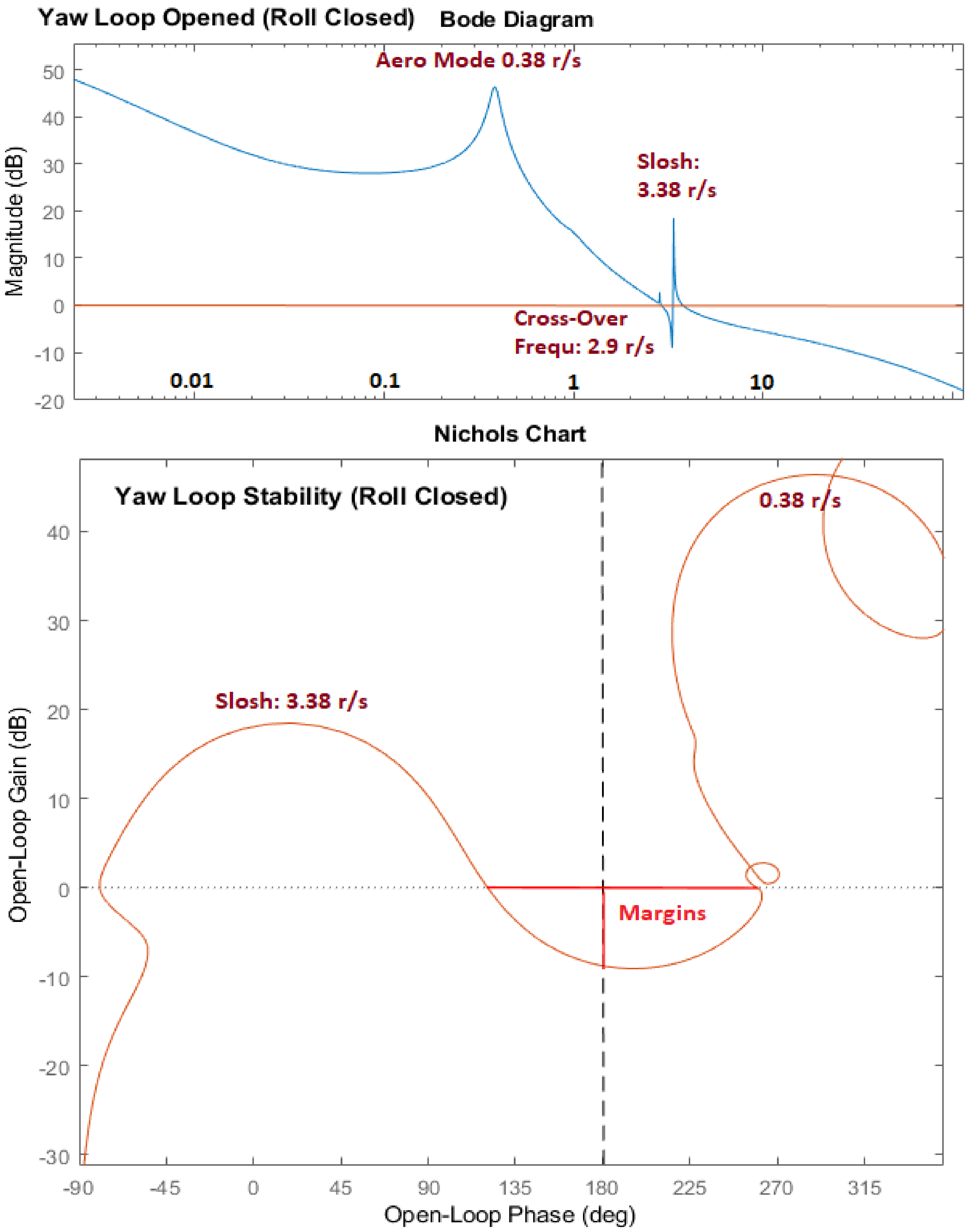


Figure 19 Yaw Axis Stability

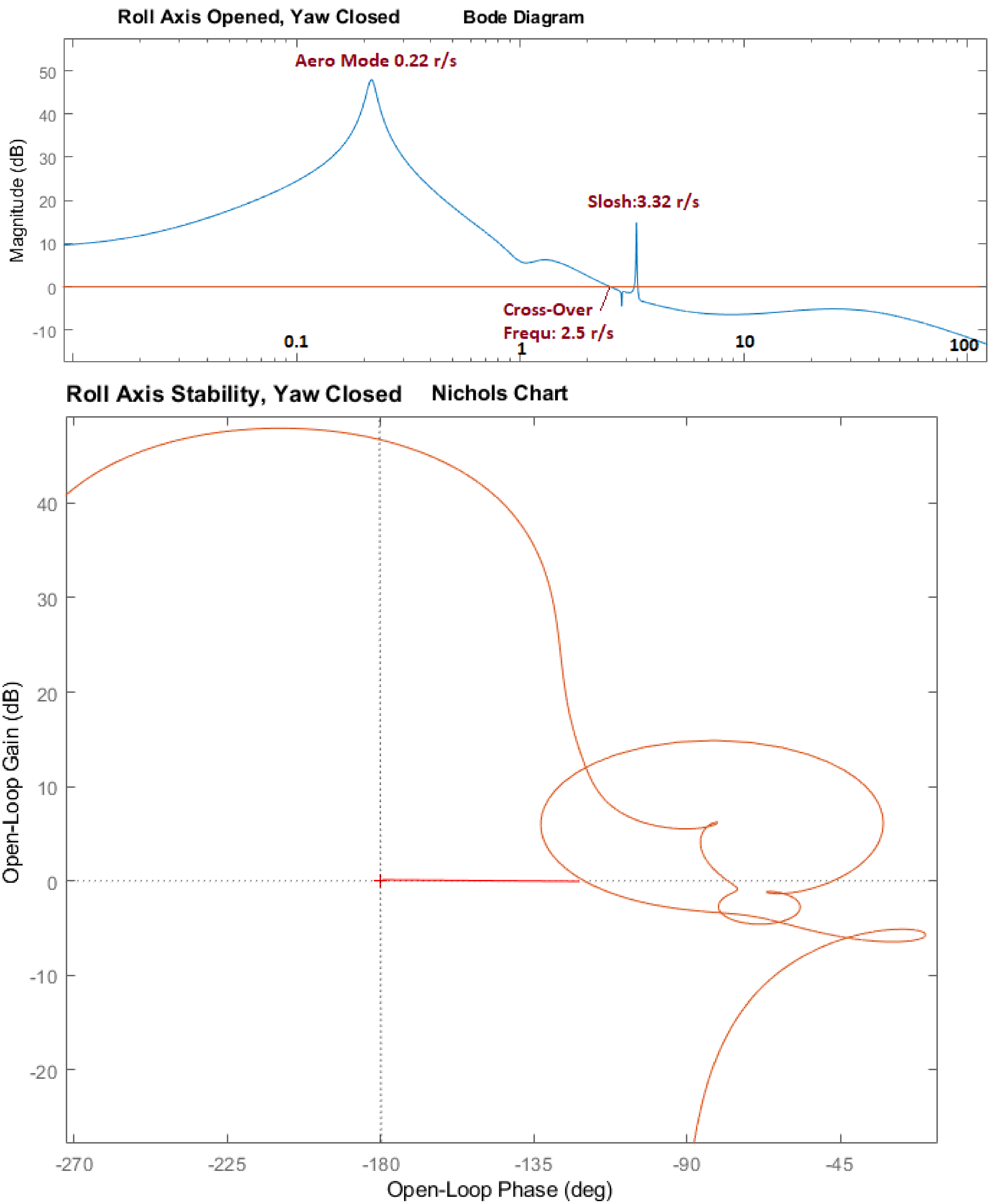


Figure 20 Roll Axis Stability

The sensitivity response in Figure-21 has a peak magnitude less than one, as expected to meet the sensitivity to gust requirement. It also has the dip at the average disturbance frequency which is accomplished by the β-filter, which reduces the beta amplitude response at the disturbance frequency.

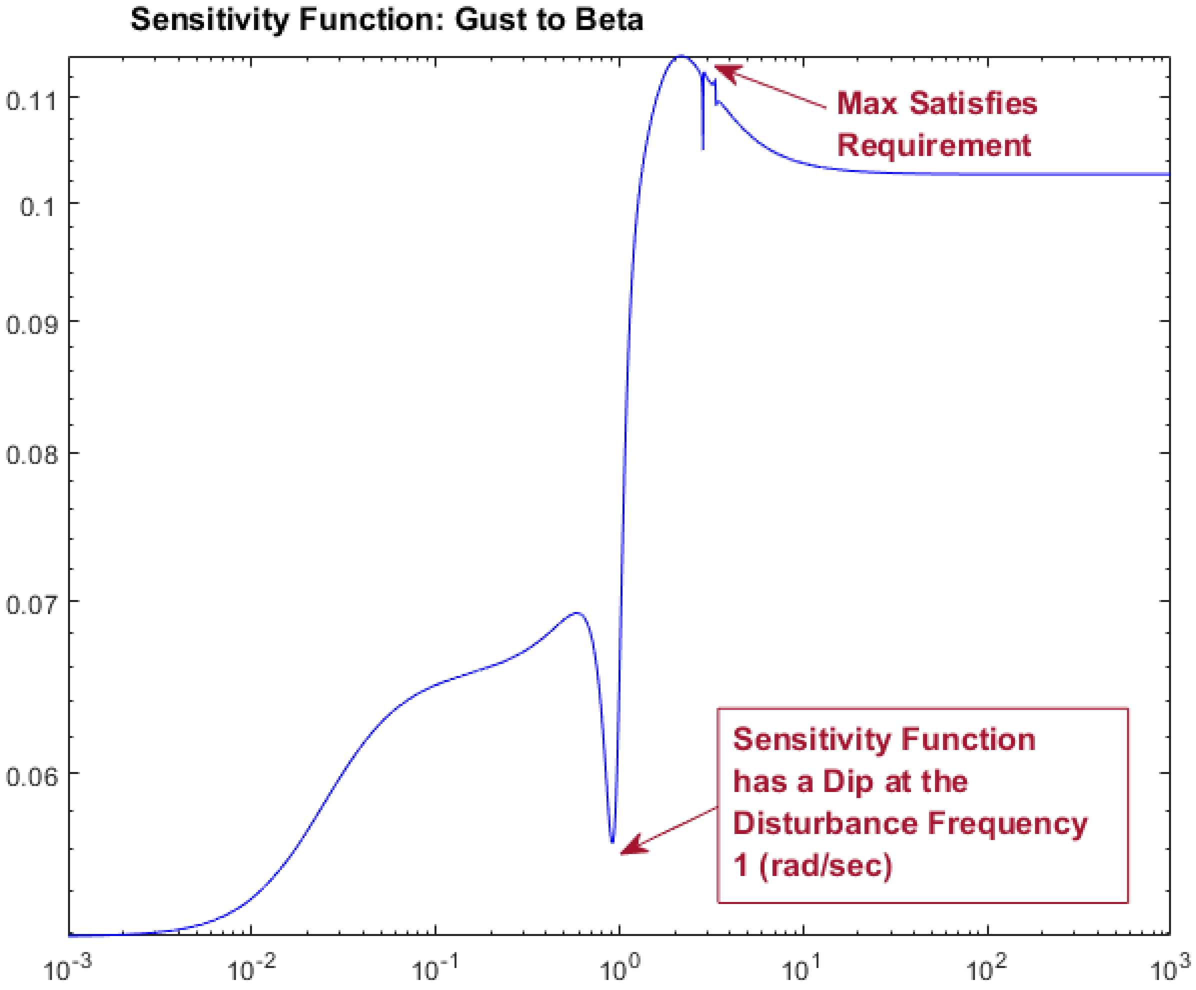


Figure 21 Sigma Plot of the Sensitivity Function Between Gust Disturbance and Beta Output

Lateral Simulation Using the Beta Estimator

Figure-22 shows the Simulink model "Closed_Loop3.slx" used for the lateral simulation. It is similar to "Closed_Loop2.slx" in Figure-14 but it includes the β-estimator in the state-feedback instead of a direct β-measurement. The estimator also drives the β-filter. The H-infinity derived (2x7) state-feedback gain Kpr7 closes the control loop from the 7-states vector via the roll and yaw actuator models.

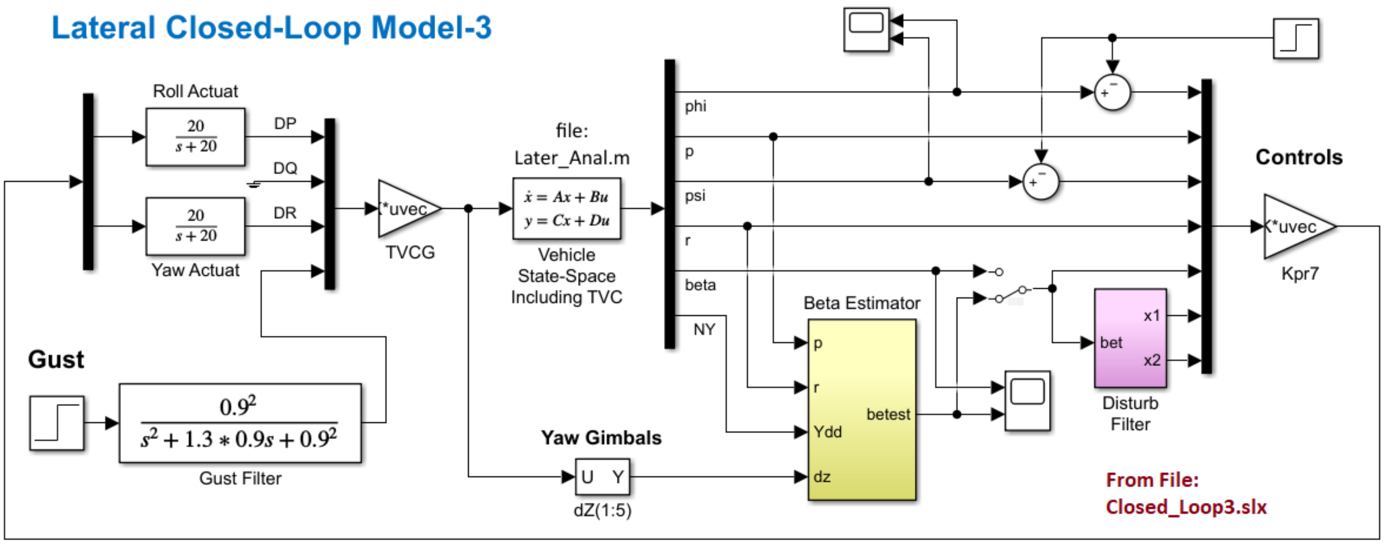


Figure 22 Closed-Loop Simulation Model "Closed_Loop3.slx" With Beta-Estimator and Filter

Figure 23 shows the vehicle roll and yaw attitude responses to unit step (ϕ & ψ) attitude commands. The two attitude responses converge towards the commanded values but they are slow because of the Max-Q and load-relief limitations.

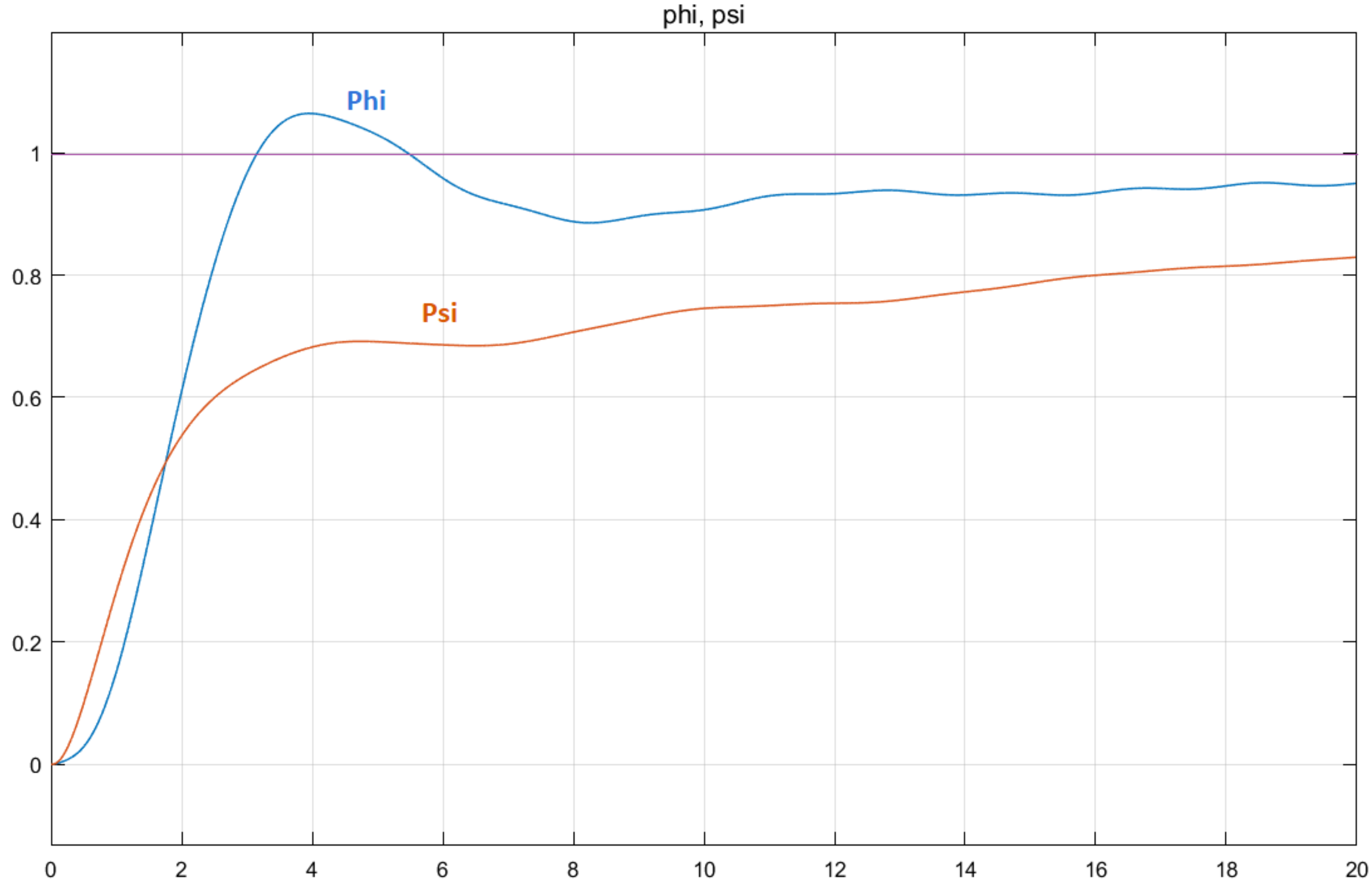


Figure 23 Vehicle Response to Simultaneously Applied Unit Step Roll and Yaw Attitude Commands

3. Pitch Axis H-Infinity Design with Gust Filter

The pitch design is similar to Section-2 and even simpler because we only have one pitch control (DQ) which is implemented via the TVC logic. The H-Infinity control law is a (1x5) state-feedback controller from the 3 vehicle states: (θ, q, α) and a 2nd order angle of attack filter: (x_1, x_2) which is intended improve sensitivity to aero disturbances at 1 (rad/sec). We will set up the H-infinity synthesis model so that the α-filter mode is excited by alpha and the optimization algorithm will produce a state-feedback gain matrix that will inhibit the excitation of the filter resonance and, therefore, provide a dip in the sensitivity function at that frequency. It means that when the vehicle is excited by cyclic disturbances at 1 (rad/sec), the control system will respond by turning the vehicle nose towards the wind reducing the angle of attack.

The work files for Section-3 are in directory: "*Flixan\Control Analysis\Hinfinity\Examples\ Shuttle Ascent Hinfinity Design\3-Pitch Design w Gust-Filter*". The Flixan input file containing the vehicle data is "*Pitch-MaxQ3.Inp*". Similar to Section-2, we will develop two vehicle models, a simple rigid model for control design and a more complex model for stability/ performance analysis. The second model includes propellant sloshing and tail-wags-dog dynamics. The simple model is augmented by combining it with the α-filter and the TVC to produce the design model. The design model is used to create the Synthesis Model, which is a 9-matrix system, via an interactive input/ output selection process already described. The SM is permanently saved in file: "*Pitch_MaxQ3.Qdr*" and it will be used by the H-infinity program to calculate the state-feedback matrix Kqhinf. Then, we shall load the systems, control gains and TVC matrices into Matlab and analyze control system stability, command following and performance to gusts in various complexity levels. We will also create an α-estimator from the Nz accelerometer measurement because α is not directly measurable.

3.1 Input File

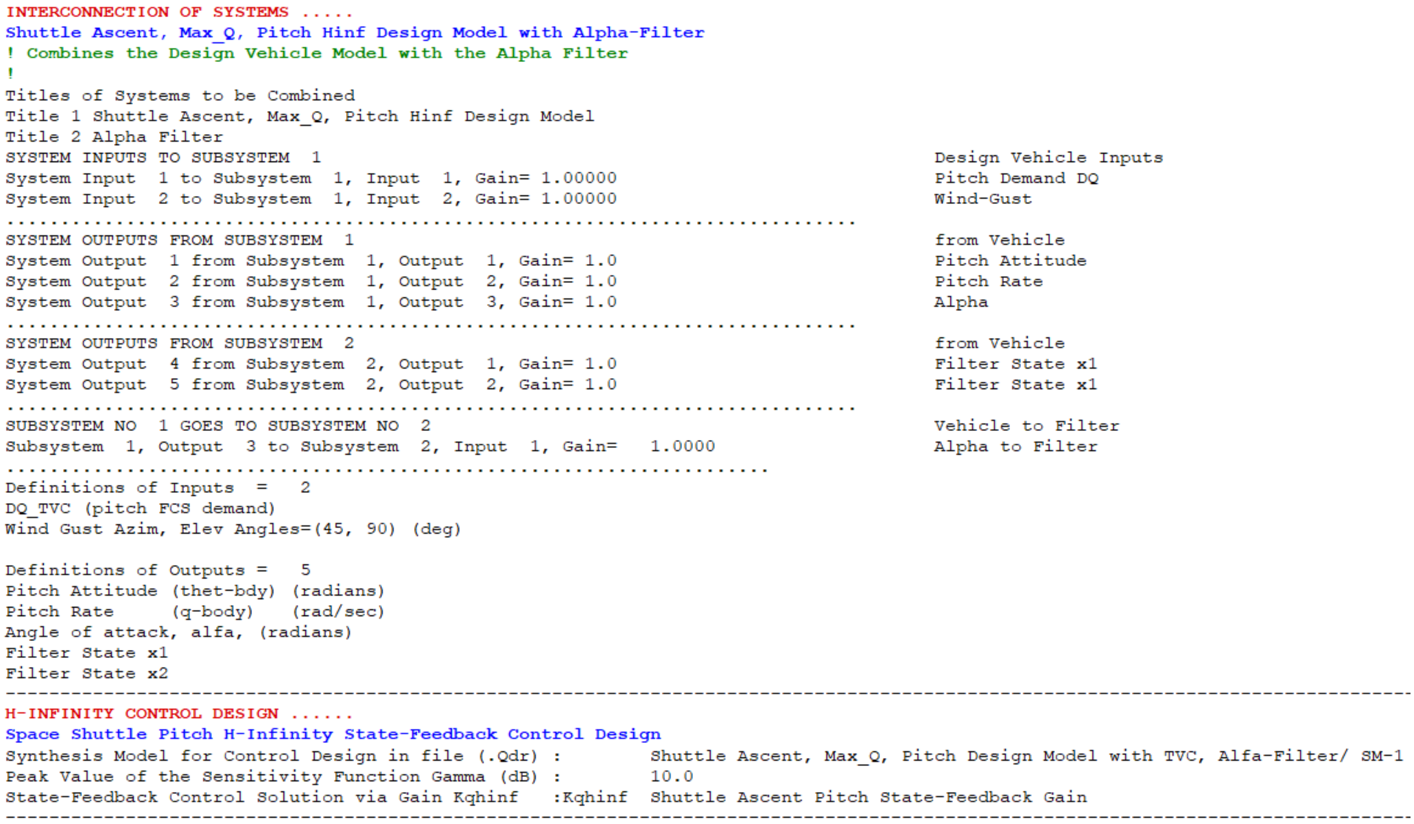
The input file "*Pitch_MaxQ3.Inp*" contains the datasets that perform various Flixan functions. There is a batch set at the top of the file that processes the entire file in batch mode. The file includes two Shuttle vehicle models: a simple model "*Shuttle Ascent, Max_Q, T=61 sec, (Design Model)*" that is used for developing the synthesis model, and a more complex model that includes propellant sloshing, accelerometers and TWD dynamics and it is used for analysis. Its title is "*Shuttle Ascent, Max_Q, T=61 sec, Rigid-Body/ Slosh Analysis Model*". The simple vehicle model is combined with the TVC matrix and is reduced by truncating the lateral variables and retaining only the pitch inputs, states and outputs. The title of the reduced pitch system is "*Shuttle Ascent, Max_Q, Pitch Hinf Design Model*". It is also combined with the α-filter to create the design plant "*Shuttle Ascent, Max_Q, Pitch Hinf Design Model with Alpha-Fi*lter" which is also used to create the pitch SM "*Shuttle Ascent, Max_Q, Pitch Design Model with TVC, Alfa-Filter/ SM-1*".

A similar reduced pitch model is extracted from the second vehicle model that includes pitch and lateral dynamics. Its title is "*Shuttle Ascent, Max_Q, Pitch Analysis with Slosh & TVC*" and it will be used for control analysis. The TVC matrix is also created using the mixing-logic program.

```
BATCH MODE INSTRUCTIONS ...............
Batch for preparing the Pitch Shuttle Ascent Models at First Stage Max-Q, T=61 sec
! This batch set creates Pitch Shuttle Ascent models at Max dynamic pressure. The models are
! used for H-infinity control design to reduce alpha sensitivity to gusts. The average gust
! frequencies are 1 (rad/sec). Two vehicle models are created, a rigid-body for control design,
! and an analysis model that includes slosh and TWD dynamics. The design model is used to create
! the H-infinity synthesis model interactively using the H-infinity program. The SM is augmented
! with 2 additional states to capture a resonance at the 1 (r/s) disturbance. The SM is used
! by the program to create the state-feedback controller. It is saved in the systems file and
! permanently retained. Flex modes are not included.
!
!................ Preserve Older Systems ......................................
Retain Matrix    : Shuttle Stage-1 TVC Matrix with Gust input
Retain CSM       : Shuttle Ascent, Max_Q, Pitch Design Model with TVC, Alfa-Filter/ SM-1
!
! ..............  Control Design ...............................................
Flight Vehicle   : Shuttle Ascent, Max_Q, T=61 sec, (Design Model)
Mixing Matrix    : Shuttle Stage-1 TVC Matrix at Max-Q
Transf-Function  : Alpha Filter
System Connection: Shuttle Ascent, Max_Q, Design Model with TVC
System Modificat : Shuttle Ascent, Max_Q, Pitch Hinf Design Model
System Connection: Shuttle Ascent, Max_Q, Pitch Hinf Design Model with Alpha-Filter
H-Infinity Design: Space Shuttle Pitch H-Infinity State-Feedback Control Design
!
!...............  Analysis Models ..............................................
Flight Vehicle   : Shuttle Ascent, Max_Q, T=61 sec, Rigid-Body/ Slosh Analysis Model
System Connection: Shuttle Ascent, Max_Q, Analysis Model with Slosh & TVC
System Modificat : Shuttle Ascent, Max_Q, Pitch Analysis with Slosh & TVC
!
!...............  Send Systems to Matlab .........................................
To Matlab Format : Alpha Filter
To Matlab Format : Shuttle Stage-1 TVC Matrix at Max-Q
To Matlab Format : Shuttle Ascent Pitch State-Feedback Gain
To Matlab Format : Shuttle Ascent, Max_Q, Pitch Hinf Design Model with Alpha-Filter
To Matlab Format : Shuttle Ascent, Max_Q, Pitch Analysis with Slosh & TVC
----------------------------------------------------------------------------------------------------------
FLIGHT VEHICLE INPUT DATA ......
Shuttle Ascent, Max_Q, T=61 sec, (Design Model)
! Rigid Body Shuttle Design Model during First Stage at Max Dynamic Pressure.
! Slosh, Bending, and Tail-Wag-Dog are Not Included.
Body Axes Output, Attitude=Rate Integral,Without GAFD, No Turn Coordination

Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Planet Radius (Re) (ft)    :  93215.0      32.174       0.20896E+08
Moments and products of Inertias Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)           :  0.248524E+8  0.209190E+9  0.221208E+9, 0.0, 0.937592E+7,
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)         : -115.0        0.036       -35.937
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)        :  1.54         1518.0       745.4        39410.0
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)          :  33.0         60.45        0.0          7.45
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                          : -3.579       -0.04         0.0          0.0
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)    :  0.0000       57.93        0.0000       0.0000       0.000
Wind Gust Vel wrt Vehi (Azim & Elev) angles (deg), or Force(lb), Torque(ft-lb), locat:xyz:  Gust         45.0         90.0
Surface Reference Area (feet^2), Mean Aerodynamic Chord (ft), Wing Span in (feet)         :  2690.0       15.0         15.0
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial_rho/ Partial_H} : -115.0        0.036       -35.937      -9.482e-10
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:  0.0          0.0          0.0          0.0          0.0
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}:  0.0         -0.0353       0.0000       0.0000       0.000
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}:  0.0         -0.0575       0.0000       0.0000       0.000
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:  0.0         -0.028        0.0000       0.0000       0.000
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}:  0.0         -0.017        0.0000       0.0000       0.000
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}:  0.0          0.0249       0.0000       0.0000       0.000

Number of Thruster Engines, Include or Not the Tail-Wags-Dog and Load-Torque Dynamics ?  :  5   NO TWD

TVC Engine No:  1                            (Gimbaling  Throttling   Single_Gimbal) : Middle SSME      Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                 :  472000.0     472000.0
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): -16.0        0.0          10.0         10.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  220.0        4800.0       3.1
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -182.1667    0.0         -64.958
TVC Engine No:  2                            (Gimbaling  Throttling   Single_Gimbal) : Left SSME        Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                 :  472000.0     472000.0
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): -10.0        0.0          10.0         10.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  220.0        4800.0       3.1
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -184.1      -4.4167      -56.595
TVC Engine No:  3                            (Gimbaling  Throttling   Single_Gimbal) : Right SSME       Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                 :  472000.0     472000.0
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): -10.0        0.0          10.0         10.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  220.0        4800.0       3.1
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -184.1       4.4167      -56.595
TVC Engine No:  4                            (Gimbaling  Throttling   Single_Gimbal) : Left SRB         Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                 :  0.245e+7     0.245e+7
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg):  0.0         0.0          10.0         10.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  605.0        0.154e+5    -1.07
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -201.53     -20.875      -33.3333
TVC Engine No:  5                            (Gimbaling  Throttling   Single_Gimbal) : Left SRB         Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                 :  0.245e+7     0.245e+7
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg):  0.0         0.0          10.0         10.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  605.0        0.154e+5    -1.07
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -201.53     +20.875      -33.3333
----------------------------------------------------------------------------------------------------------------------------------
```

```
MIXING LOGIC MATRIX DATA ..........    (Matrix Title, Name, Vehicle Title, Control Directions)
Shuttle Stage-1 TVC Matrix at Max-Q
! Thrust Vector Control Matrix at Max-Q
! This multi-engine vehicle has 5 Gimbaling Engines.
TVC
Shuttle Ascent, Max_Q, T=61 sec, (Design Model)
P-dot Roll  Acceleration About X Axis
Q-dot Pitch Acceleration About Y Axis
R-dot Yaw   Acceleration About Z Axis
---------------------------------------------------------------------------------------------------------
SYSTEM OF TRANSFER FUNCTIONS ...
Wind Gust Shaping
Continuous
TF. Block #  1  0.9^2/(s^2 +1.17s +0.9^2)                    Order of Numer, Denom=  0  2
Numer 0.0         0.0        0.81
Denom 1.0         1.17       0.81
......................................................
Block #, from Input #, Gain
  1  1    1.0
...........................
Outpt #, from Block #, Gain
  1  1    1.0
...........................
---------------------------------------------------------------------------------------------------------
SYSTEM OF TRANSFER FUNCTIONS ...
Alpha Filter
Continuous
TF. Block #  1  ( 1/s )                              Order of Numer, Denom=  0  1
Numer 0.0        1.0
Denom 1.0        0.0
TF. Block #  2  ( 1/s )                              Order of Numer, Denom=  0  1
Numer 0.0        1.0
Denom 1.0        0.0
...................................................
Block #, from Input #, Gain
1         1      1.0
..........................
Block #, from Block #, Gain
2         1      1.0
1         1     -0.3
1         2     -1.0
..........................
Outpt #, from Block #, Gain
1         1      1.0
2         2      1.0
..........................
Definitions of Inputs  =   1
Alpha In

Definitions of Outputs =   2
Filter State x1
Filter State x2
---------------------------------------------------------------------------------------------------------
INTERCONNECTION OF SYSTEMS .....
Shuttle Ascent, Max_Q, Design Model with TVC
! Combines the Rigid vehicle model with the TVCG matrix that includes also the
! gust input.
Titles of Systems to be Combined
Title 1 Shuttle Ascent, Max_Q, T=61 sec, (Design Model)
SYSTEM INPUTS TO SUBSYSTEM  1                                     TVC to Vehicle
Via Matrix +TVCG                                                  inputs: roll, pitch, yaw, gust
..........................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                  from Plant
Via Matrix +I14                                                   All Outputs
..........................................................................
Definitions of Inputs  =    4
DP_TVC (roll FCS demand)
DQ_TVC (pitch FCS demand)
DR_TVC (yaw FCS demand)
Wind Gust Azim, Elev Angles=(45, 90) (deg)

Definitions of Outputs =  14
Roll Attitude   (phi-body) (radians)
Roll Rate       (p-body)   (rad/sec)
Pitch Attitude  (thet-bdy) (radians)
Pitch Rate      (q-body)   (rad/sec)
Yaw  Attitude   (psi-body) (radians)
Yaw  Rate       (r-body)   (rad/sec)
Angle of attack, alfa, (radians)
Angle of sideslip, beta, (radian)
Change in Altitude, delta-h, (feet)
Forward Acceleration  (V-dot)  (ft/sec)
Cross Range Velocity (Vcr) (ft/sec)
CG Acceleration along X axis, (ft/sec^2)
CG Acceleration along Y axis, (ft/sec^2)
CG Acceleration along Z axis, (ft/sec^2)
-----------------------------------------------------------------------------------------------------------------
```

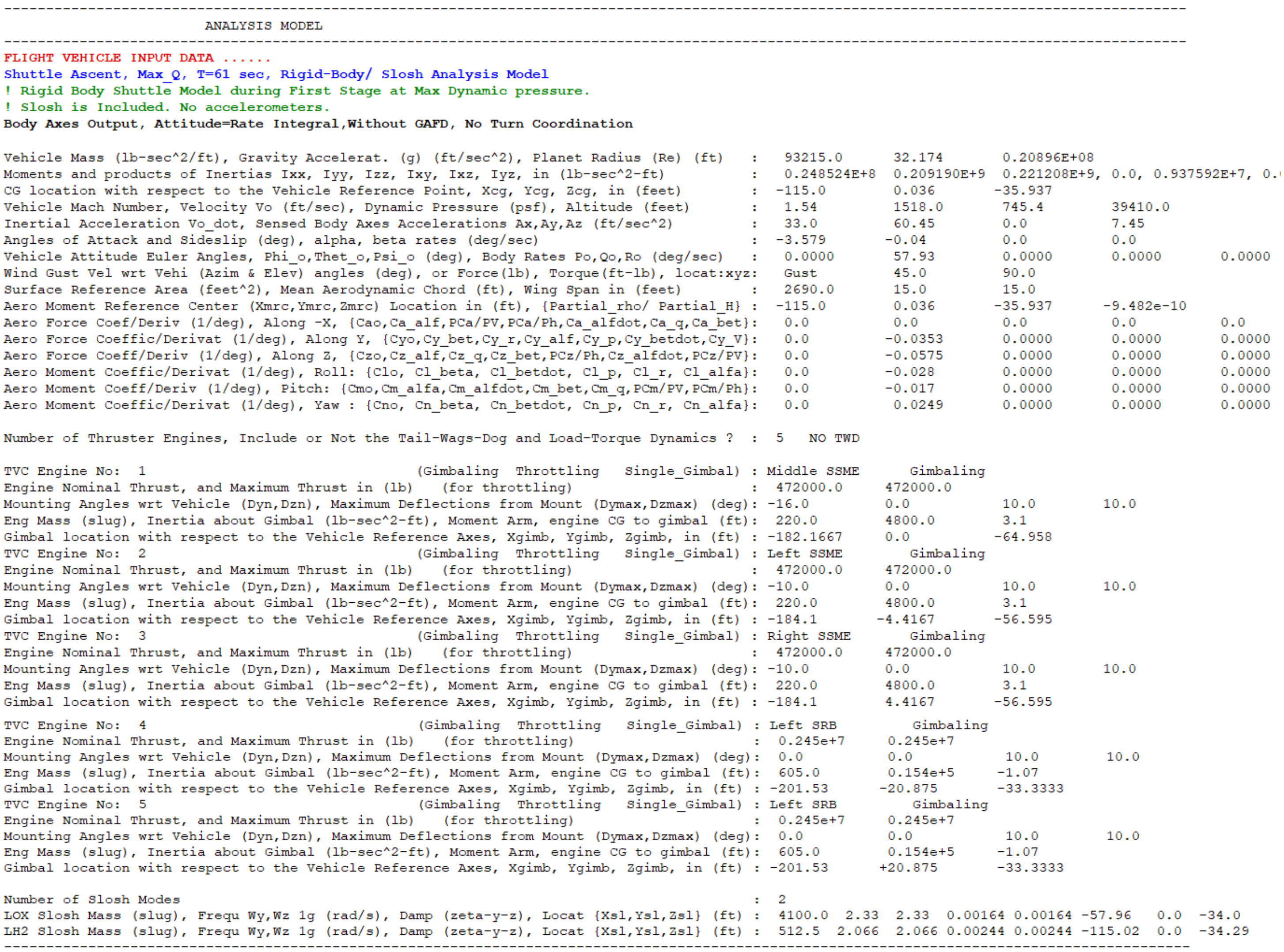
The following interconnection dataset combines the pitch design model with the alpha-filter to produce the augmented design model that includes the alpha-filter. They both include the TVC matrix.



```
INTERCONNECTION OF SYSTEMS .....
Shuttle Ascent, Max_Q, Pitch Hinf Design Model with Alpha-Filter
! Combines the Design Vehicle Model with the Alpha Filter
!
Titles of Systems to be Combined
Title 1 Shuttle Ascent, Max_Q, Pitch Hinf Design Model
Title 2 Alpha Filter
SYSTEM INPUTS TO SUBSYSTEM  1                                                         Design Vehicle Inputs
System Input  1 to Subsystem  1, Input  1, Gain= 1.00000                              Pitch Demand DQ
System Input  2 to Subsystem  1, Input  2, Gain= 1.00000                              Wind-Gust
...............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                                      from Vehicle
System Output  1 from Subsystem  1, Output  1, Gain= 1.0                              Pitch Attitude
System Output  2 from Subsystem  1, Output  2, Gain= 1.0                              Pitch Rate
System Output  3 from Subsystem  1, Output  3, Gain= 1.0                              Alpha
...............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                                                      from Vehicle
System Output  4 from Subsystem  2, Output  1, Gain= 1.0                              Filter State x1
System Output  5 from Subsystem  2, Output  2, Gain= 1.0                              Filter State x1
...............................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2                                               Vehicle to Filter
Subsystem  1, Output  3 to Subsystem  2, Input  1, Gain=   1.0000                     Alpha to Filter
.......................................................................
Definitions of Inputs  =   2
DQ_TVC (pitch FCS demand)
Wind Gust Azim, Elev Angles=(45, 90) (deg)

Definitions of Outputs =   5
Pitch Attitude (thet-bdy) (radians)
Pitch Rate     (q-body)   (rad/sec)
Angle of attack, alfa, (radians)
Filter State x1
Filter State x2
-----------------------------------------------------------------------------------------------------------------------------------
H-INFINITY CONTROL DESIGN ......
Space Shuttle Pitch H-Infinity State-Feedback Control Design
Synthesis Model for Control Design in file (.Qdr) :         Shuttle Ascent, Max_Q, Pitch Design Model with TVC, Alfa-Filter/ SM-1
Peak Value of the Sensitivity Function Gamma (dB) :         10.0
State-Feedback Control Solution via Gain Kqhinf    :Kqhinf  Shuttle Ascent Pitch State-Feedback Gain
-----------------------------------------------------------------------------------------------------------------------------------
```

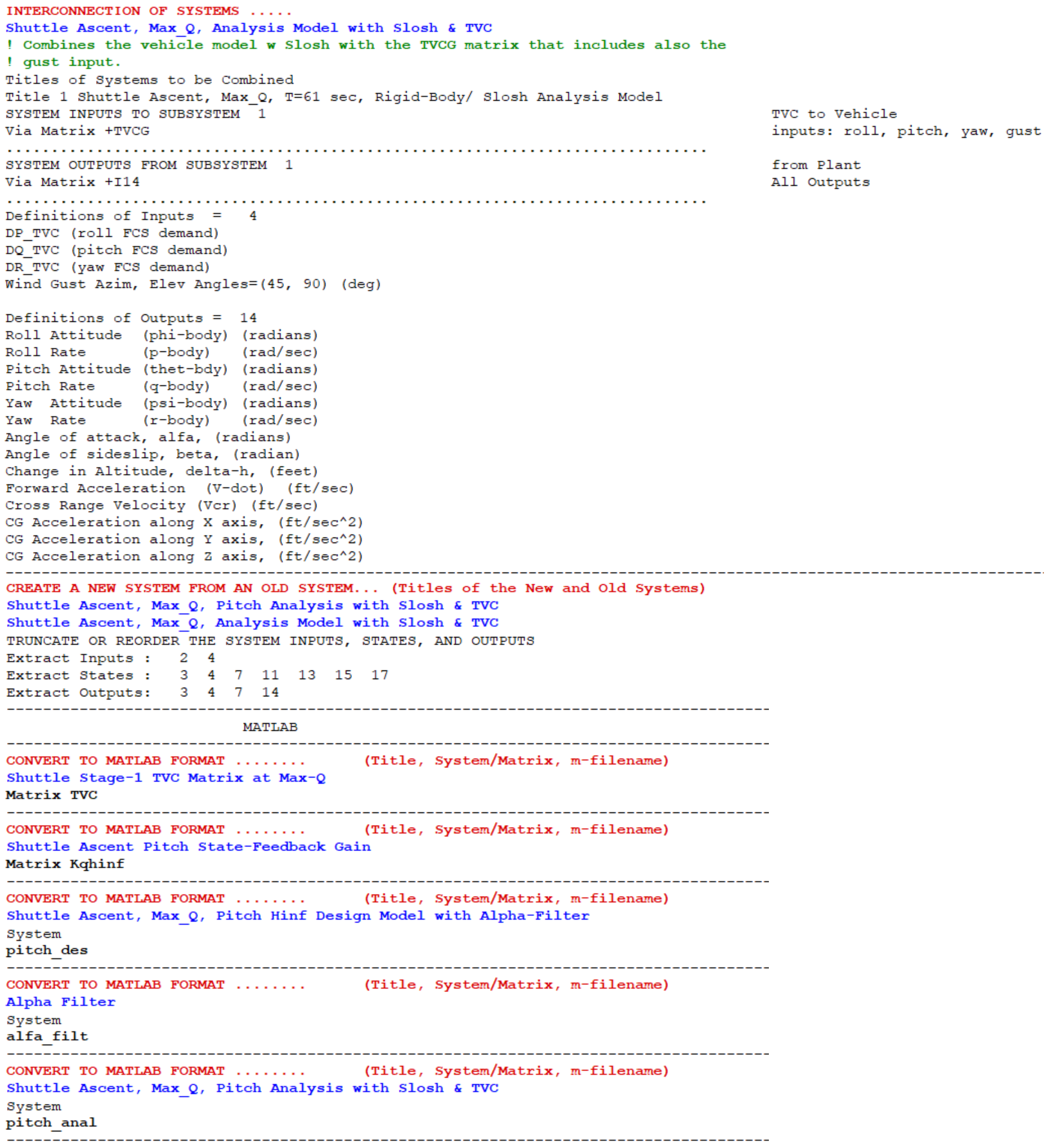
The next H-infinity design set performs the control design in batch. It uses the SM "*Shuttle Ascent, Max_Q, Pitch Design Model with TVC, Alfa-Filter/SM-1*" which is already saved in the systems file. The γ value is preset to 10 (dB), as determined from previous interactive processing. The state-feedback matrix is saved in the systems file. Its name is "Kqhinf" and its title is "*Shuttle Ascent Pitch State-Feedback Gain*".

The next is a vehicle dataset that creates the analysis vehicle model "*Shuttle Ascent, Max_Q, T=61 sec, Rigid-Body/ Slosh Analysis Model*" that includes slosh for the LOX and the LH2 tanks and TWD dynamics. It also includes rate gyros and accelerometer sensors.



```
--------------------------------------------------------------------------------------------------------------------------------------------
                        ANALYSIS MODEL
--------------------------------------------------------------------------------------------------------------------------------------------
FLIGHT VEHICLE INPUT DATA ......
Shuttle Ascent, Max_Q, T=61 sec, Rigid-Body/ Slosh Analysis Model
! Rigid Body Shuttle Model during First Stage at Max Dynamic pressure.
! Slosh is Included. No accelerometers.
Body Axes Output, Attitude=Rate Integral,Without GAFD, No Turn Coordination

Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Planet Radius (Re) (ft)   :   93215.0      32.174       0.20896E+08
Moments and products of Inertias Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)          :   0.248524E+8  0.209190E+9  0.221208E+9, 0.0, 0.937592E+7, 0.
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)        :  -115.0        0.036       -35.937
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)       :   1.54         1518.0       745.4        39410.0
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)         :   33.0         60.45        0.0          7.45
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                         :  -3.579       -0.04         0.0          0.0
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)   :   0.0000       57.93        0.0000       0.0000       0.0000
Wind Gust Vel wrt Vehi (Azim & Elev) angles (deg), or Force(lb), Torque(ft-lb), locat:xyz:   Gust         45.0         90.0
Surface Reference Area (feet^2), Mean Aerodynamic Chord (ft), Wing Span in (feet)        :   2690.0       15.0         15.0
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial_rho/ Partial_H} :  -115.0        0.036       -35.937      -9.482e-10
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:   0.0          0.0          0.0          0.0          0.0
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}:   0.0         -0.0353       0.0000       0.0000       0.0000
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}:   0.0         -0.0575       0.0000       0.0000       0.0000
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:   0.0         -0.028        0.0000       0.0000       0.0000
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}:   0.0         -0.017        0.0000       0.0000       0.0000
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}:   0.0          0.0249       0.0000       0.0000       0.0000

Number of Thruster Engines, Include or Not the Tail-Wags-Dog and Load-Torque Dynamics ?  :  5   NO TWD

TVC Engine No:  1                                (Gimbaling  Throttling   Single_Gimbal) : Middle SSME      Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                     :  472000.0     472000.0
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): -16.0         0.0          10.0         10.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  220.0        4800.0        3.1
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -182.1667     0.0         -64.958
TVC Engine No:  2                                (Gimbaling  Throttling   Single_Gimbal) : Left SSME        Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                     :  472000.0     472000.0
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): -10.0         0.0          10.0         10.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  220.0        4800.0        3.1
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -184.1       -4.4167       -56.595
TVC Engine No:  3                                (Gimbaling  Throttling   Single_Gimbal) : Right SSME       Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                     :  472000.0     472000.0
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): -10.0         0.0          10.0         10.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  220.0        4800.0        3.1
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -184.1        4.4167       -56.595

TVC Engine No:  4                                (Gimbaling  Throttling   Single_Gimbal) : Left SRB         Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                     :  0.245e+7     0.245e+7
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg):  0.0          0.0          10.0         10.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  605.0        0.154e+5     -1.07
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -201.53      -20.875       -33.3333
TVC Engine No:  5                                (Gimbaling  Throttling   Single_Gimbal) : Left SRB         Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                     :  0.245e+7     0.245e+7
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg):  0.0          0.0          10.0         10.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  605.0        0.154e+5     -1.07
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -201.53      +20.875       -33.3333

Number of Slosh Modes                                                                    :  2
LOX Slosh Mass (slug), Frequ Wy,Wz 1g (rad/s), Damp (zeta-y-z), Locat {Xsl,Ysl,Zsl} (ft) :  4100.0  2.33  2.33  0.00164 0.00164 -57.96   0.0  -34.0
LH2 Slosh Mass (slug), Frequ Wy,Wz 1g (rad/s), Damp (zeta-y-z), Locat {Xsl,Ysl,Zsl} (ft) :  512.5  2.066  2.066 0.00244 0.00244 -115.02  0.0  -34.29
--------------------------------------------------------------------------------------------------------------------------------------------
```

The next two sets combine the analysis vehicle model, which includes both: pitch and lateral variables, with the TVC matrix and retain only the pitch variables. The lateral variables are ignored in this section. They will be analyzed together in the last section with flex modes.

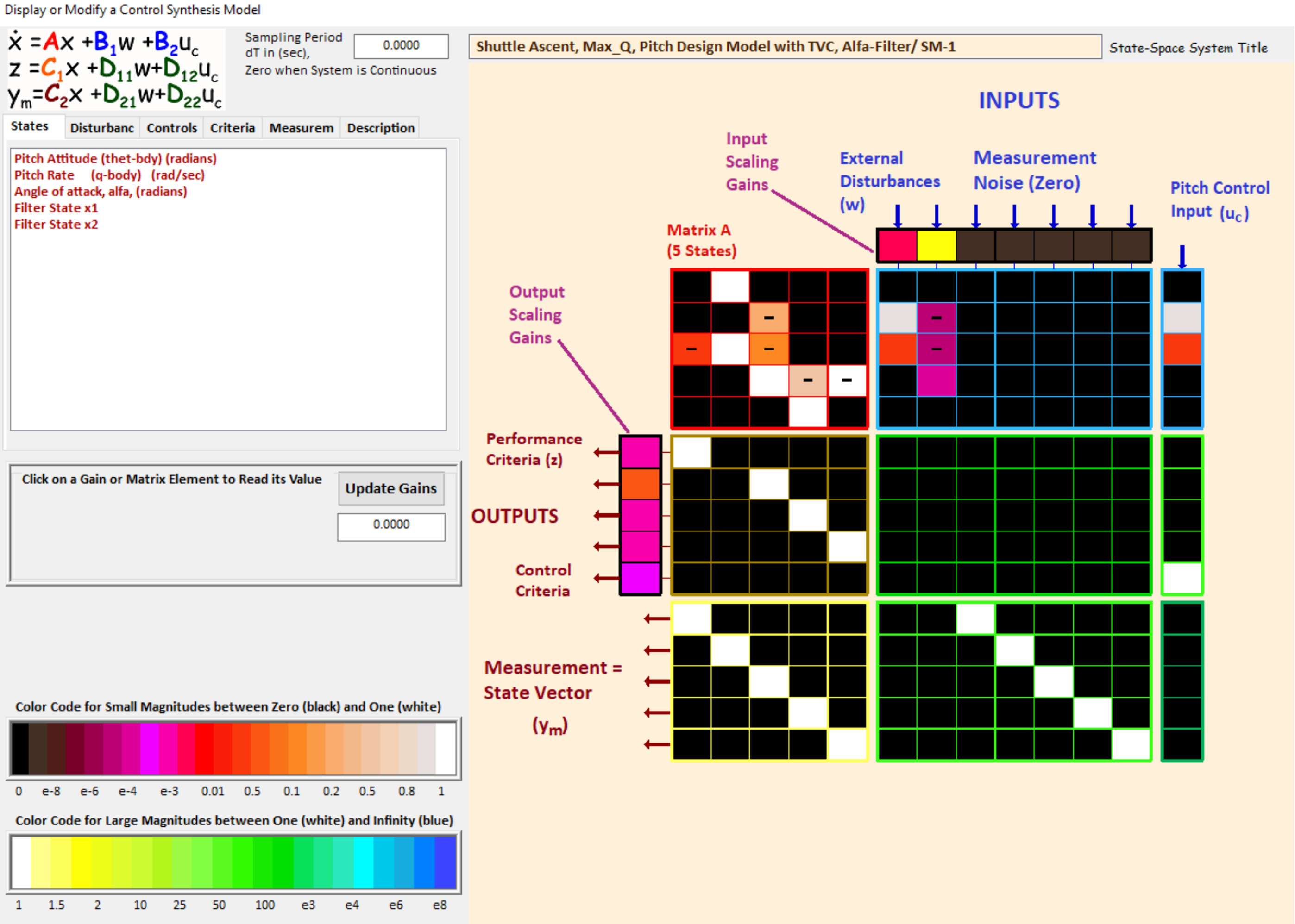


```
INTERCONNECTION OF SYSTEMS .....
Shuttle Ascent, Max_Q, Analysis Model with Slosh & TVC
! Combines the vehicle model w Slosh with the TVCG matrix that includes also the
! gust input.
Titles of Systems to be Combined
Title 1 Shuttle Ascent, Max_Q, T=61 sec, Rigid-Body/ Slosh Analysis Model
SYSTEM INPUTS TO SUBSYSTEM  1                                                      TVC to Vehicle
Via Matrix +TVCG                                                                   inputs: roll, pitch, yaw, gust
.............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                                   from Plant
Via Matrix +I14                                                                    All Outputs
.............................................................................
Definitions of Inputs  =   4
DP_TVC (roll FCS demand)
DQ_TVC (pitch FCS demand)
DR_TVC (yaw FCS demand)
Wind Gust Azim, Elev Angles=(45, 90) (deg)

Definitions of Outputs =  14
Roll Attitude  (phi-body) (radians)
Roll Rate      (p-body)   (rad/sec)
Pitch Attitude (thet-bdy) (radians)
Pitch Rate     (q-body)   (rad/sec)
Yaw  Attitude  (psi-body) (radians)
Yaw  Rate      (r-body)   (rad/sec)
Angle of attack, alfa, (radians)
Angle of sideslip, beta, (radian)
Change in Altitude, delta-h, (feet)
Forward Acceleration  (V-dot)  (ft/sec)
Cross Range Velocity (Vcr) (ft/sec)
CG Acceleration along X axis, (ft/sec^2)
CG Acceleration along Y axis, (ft/sec^2)
CG Acceleration along Z axis, (ft/sec^2)
----------------------------------------------------------------------------------------------------------------------
CREATE A NEW SYSTEM FROM AN OLD SYSTEM... (Titles of the New and Old Systems)
Shuttle Ascent, Max_Q, Pitch Analysis with Slosh & TVC
Shuttle Ascent, Max_Q, Analysis Model with Slosh & TVC
TRUNCATE OR REORDER THE SYSTEM INPUTS, STATES, AND OUTPUTS
Extract Inputs :   2  4
Extract States :   3  4  7  11  13  15  17
Extract Outputs:   3  4  7  14
-------------------------------------------------------------------------------------
                          MATLAB
-------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Shuttle Stage-1 TVC Matrix at Max-Q
Matrix TVC
-------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Shuttle Ascent Pitch State-Feedback Gain
Matrix Kqhinf
-------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Shuttle Ascent, Max_Q, Pitch Hinf Design Model with Alpha-Filter
System
pitch_des
-------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Alpha Filter
System
alfa_filt
-------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Shuttle Ascent, Max_Q, Pitch Analysis with Slosh & TVC
System
pitch_anal
-------------------------------------------------------------------------------------
```

Finally, the design and analysis models, the TVC matrix, alpha-filter, and the control gain matrix Kqhinf are exported and loaded into Matlab for further analysis.

3.2 H-Infinity Synthesis Model

Figure-24 shows the pitch control synthesis model in systems form, color coded, as it appears when running the H-infinity program interactively. The approximate color code values are shown at the lower left corner. The A-matrix consists of 5 states, 3 from the original vehicle and 2 from the α-filter. There are 2 external disturbances (w), 1 control (u_c), 5 measurements (y_m) which are equal to the 5 states (C2 is the identity matrix), 4 performance criteria (z), and 1 control criterion. There are also 5 measurement noise inputs which are set to almost zero (dark brown) because they don't play a role when designing a state-feedback, only when we include a state estimator.



Display or Modify a Control Synthesis Model

$$\dot{x} = Ax + B_1 w + B_2 u_c$$
$$z = C_1 x + D_{11} w + D_{12} u_c$$
$$y_m = C_2 x + D_{21} w + D_{22} u_c$$

Sampling Period dT in (sec), Zero when System is Continuous: 0.0000

State-Space System Title: Shuttle Ascent, Max_Q, Pitch Design Model with TVC, Alfa-Filter/ SM-1

States

Pitch Attitude (thet-bdy) (radians)
Pitch Rate (q-body) (rad/sec)
Angle of attack, alfa, (radians)
Filter State x1
Filter State x2

Click on a Gain or Matrix Element to Read its Value: 0.0000

Figure 24 Pitch Synthesis Model in Systems Form with Scaling Gains

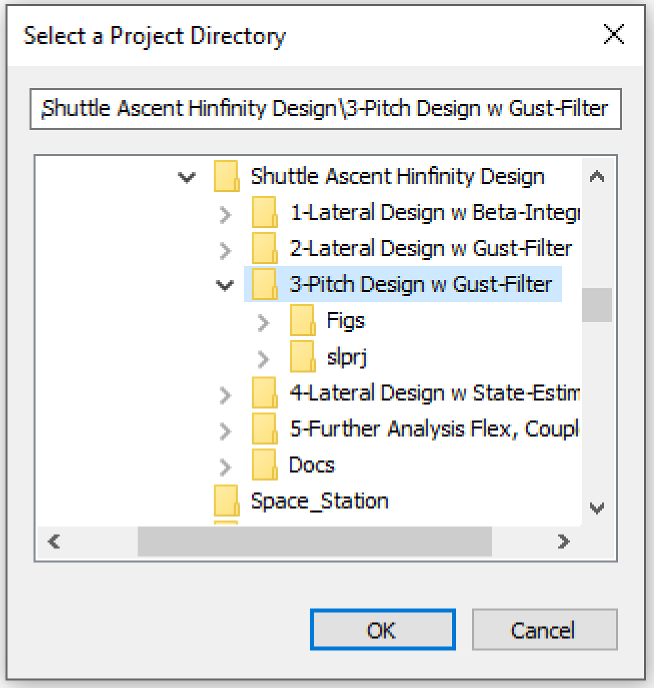
The SM is permanently saved in the systems file "Pitch_MaxQ3.Inp" and it shown in detail below. Description of the variables are shown at the bottom, together with the scaling gains.

```
SYNTHESIS MODEL FOR H-INFINITY CONTROL
Shuttle Ascent, Max_Q, Pitch Design Model with TVC, Alfa-Filter/ SM-1
Number of: States (x), Uncertainty Inp/Outputs from Plant Variations (dP)=   5   0   0
Number of: Extern Disturbance Inputs (Wi), Control Inputs (Uc)            =   2   1
Number of: Output Criteria (Zo), Regulated Outputs (Zr), Measurements (y)=   4   0   5
Synthes Model Matrices: A, B1,B2,C1,C2, D11,D12,D21,D22, Sample Time (dT)=   0.0000
Matrix A                Size =  5 X  5
          1-Column           2-Column           3-Column           4-Column           5-Column
   1-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00 -0.140043700534E+00  0.000000000000E+00  0.000000000000E+00
   3-Row -0.184943171585E-01  0.999999986911E+00 -0.689606149584E-01  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01 -0.300000000000E+00 -0.100000000000E+01
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
-----------------------------------------------------------------------------------------------------------
Matrix B1               Size =  5 X  7
          1-Column           2-Column           3-Column           4-Column           5-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.998581332818E+00 -0.651090951754E-04  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.171330380602E-01 -0.216623128103E-04  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.464919842357E-03  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
-----------------------------------------------------------------------------------------------------------
Matrix B2               Size =  5 X  1
          1-Column
   1-Row  0.000000000000E+00
   2-Row  0.998581332818E+00
   3-Row  0.171330380602E-01
   4-Row  0.000000000000E+00
   5-Row  0.000000000000E+00
-----------------------------------------------------------------------------------------------------------
Matrix C1               Size =  5 X  5
          1-Column           2-Column           3-Column           4-Column           5-Column
   1-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
-----------------------------------------------------------------------------------------------------------
Matrix C2               Size =  5 X  5
          1-Column           2-Column           3-Column           4-Column           5-Column
   1-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
-----------------------------------------------------------------------------------------------------------
Matrix D11              Size =  5 X  7
          1-Column           2-Column           3-Column           4-Column           5-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
-----------------------------------------------------------------------------------------------------------
Matrix D12              Size =  5 X  1
          1-Column
   1-Row  0.000000000000E+00
   2-Row  0.000000000000E+00
   3-Row  0.000000000000E+00
   4-Row  0.000000000000E+00
   5-Row  0.100000000000E+01
-----------------------------------------------------------------------------------------------------------
Matrix D21              Size =  5 X  7
          1-Column           2-Column           3-Column           4-Column           5-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
-----------------------------------------------------------------------------------------------------------
Matrix D22              Size =  5 X  1
          1-Column
   1-Row  0.000000000000E+00
   2-Row  0.000000000000E+00
   3-Row  0.000000000000E+00
   4-Row  0.000000000000E+00
   5-Row  0.000000000000E+00
-----------------------------------------------------------------------------------------------------------
```

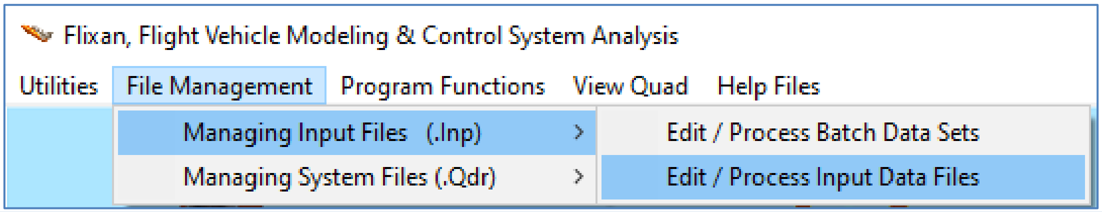
```
States (x) ..........  =   5
   1     Pitch Attitude (thet-bdy) (radians)
   2     Pitch Rate      (q-body)   (rad/sec)
   3     Angle of attack, alfa, (radians)
   4     Filter State x1
   5     Filter State x2

Excitation Inputs (w)  =   7
   1     DQ_TVC (pitch FCS demand)                              * 0.005
   2     Wind Gust Azim, Elev Angles=(45, 90) (deg)             * 3.000
   3     Noise at Output: Pitch Attitude (thet-bdy) (radians)   * 0.10000E-07
   4     Noise at Output: Pitch Rate      (q-body)   (rad/sec)  * 0.10000E-07
   5     Noise at Output: Angle of attack, alfa, (radians)      * 0.10000E-07
   6     Noise at Output: Filter State x1                       * 0.10000E-07
   7     Noise at Output: Filter State x2                       * 0.10000E-07

Control Inputs (u) ... =   1
   1     Control: DQ_TVC (pitch FCS demand)                     *  1.0000

Performance Outputs (z)=   5
   1     Pitch Attitude (thet-bdy) (radians)                    / 0.004
   2     Angle of attack, alfa, (radians)                       / 0.04
   3     Filter State x1                                        / 0.003
   4     Filter State x2                                        / 0.003
   5     Contrl Criter. DQ_TVC (pitch FCS demand)               / 0.002

Measurement Outputs (y)=   5
   1     Measurm: Pitch Attitude (thet-bdy) (radians)           /  1.0000
   2     Measurm: Pitch Rate      (q-body)   (rad/sec)          /  1.0000
   3     Measurm: Angle of attack, alfa, (radians)              /  1.0000
   4     Measurm: Filter State x1                               /  1.0000
   5     Measurm: Filter State x2                               /  1.0000
```

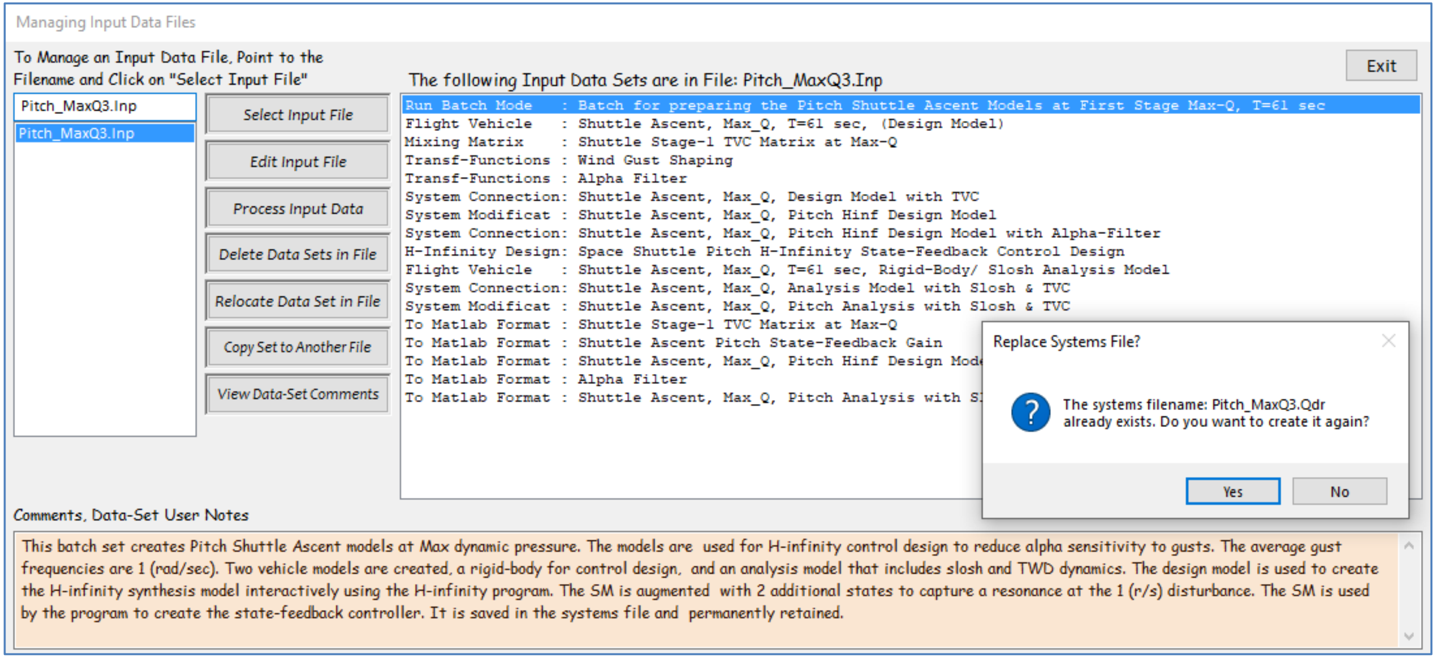
3.3 Processing the Input Data File

The datasets of the entire pitch modeling and control design are included in the input file “Pitch-MaxQ3.Inp”. We will now process this file in batch mode by running the batch set which is located at the top of the file. Start the Flixan program and select the project directory: “*Flixan\ Control Analysis\ Hinfinity\ Examples\ Shuttle Ascent Hinfinity Design\3-Pitch Design w Gust-Filter*”. From the main menu select “*File Management*”, “*Managing Input Files*”, and then “*Edit/ Process Input Data Files*”, as shown below. The following dialog comes up that includes two menus. The menu on the left side lists the input data files in the project directory. There is only one. Highlight it and click on “*Select Input File*” button. The menu on the right shows the datasets which are in the input file. Select the batch set which is at the top of the list and click on “*Process Input Data*”.


Select a Project Directory

Shuttle Ascent Hinfinity Design\3-Pitch Design w Gust-Filter





The following Input Data Sets are in File: Pitch_MaxQ3.Inp

```
Run Batch Mode    : Batch for preparing the Pitch Shuttle Ascent Models at First Stage Max-Q, T=61 sec
Flight Vehicle    : Shuttle Ascent, Max_Q, T=61 sec, (Design Model)
Mixing Matrix     : Shuttle Stage-1 TVC Matrix at Max-Q
Transf-Functions  : Wind Gust Shaping
Transf-Functions  : Alpha Filter
System Connection: Shuttle Ascent, Max_Q, Design Model with TVC
System Modificat  : Shuttle Ascent, Max_Q, Pitch Hinf Design Model
System Connection: Shuttle Ascent, Max_Q, Pitch Hinf Design Model with Alpha-Filter
H-Infinity Design: Space Shuttle Pitch H-Infinity State-Feedback Control Design
Flight Vehicle    : Shuttle Ascent, Max_Q, T=61 sec, Rigid-Body/ Slosh Analysis Model
System Connection: Shuttle Ascent, Max_Q, Analysis Model with Slosh & TVC
System Modificat  : Shuttle Ascent, Max_Q, Pitch Analysis with Slosh & TVC
To Matlab Format  : Shuttle Stage-1 TVC Matrix at Max-Q
To Matlab Format  : Shuttle Ascent Pitch State-Feedback Gain
To Matlab Format  : Shuttle Ascent, Max_Q, Pitch Hinf Design Mod
To Matlab Format  : Alpha Filter
To Matlab Format  : Shuttle Ascent, Max_Q, Pitch Analysis with Sl
```

Replace Systems File?

The systems filename: Pitch_MaxQ3.Qdr already exists. Do you want to create it again?

Comments, Data-Set User Notes

This batch set creates Pitch Shuttle Ascent models at Max dynamic pressure. The models are used for H-infinity control design to reduce alpha sensitivity to gusts. The average gust frequencies are 1 (rad/sec). Two vehicle models are created, a rigid-body for control design, and an analysis model that includes slosh and TWD dynamics. The design model is used to create the H-infinity synthesis model interactively using the H-infinity program. The SM is augmented with 2 additional states to capture a resonance at the 1 (r/s) disturbance. The SM is used by the program to create the state-feedback controller. It is saved in the systems file and permanently retained.

In the following question, answer "Yes", which is okay to delete the old systems file and recreate it. The batch executes and creates the systems and matrices that can now be loaded into Matlab.

3.4 Control Analysis

The Matlab analysis begins by running the initialization file "init.m" which loads the systems, control gains and TVC matrices into Matlab. The design model includes the α-filter. The β-filter is also loaded separately in state-space form. The analysis model includes slosh and TWD dynamics but no β-filter. It was included in the design plant for the controller calculation, but now the β-filter has to be included in the control system. Some vehicle parameters that will be used later for estimating β are also loaded into Matlab. We also check the eigenvalues of the closed-loop system to make sure that it's stable. We will demonstrate two versions of the control design. We will first analyze the system using α-feedback directly. Then we will replace the direct α-measurement with an estimate from the N_Z accelerometer because α is not directly measurable.

```
% Initialization File
r2d=180/pi; d2r=pi/180;
[Ad, Bd, Cd, Dd]= pitch_des;                    % Pitch design sustem
[An, Bn, Cn, Dn]= pitch_anal;                   % Simple analysis system
[Af, Bf, Cf, Df]= alfa_filt;                    % Alpha Filter
load TVC.mat -ascii                             % TVC Matrix
load Kqhinf.mat -ascii                          % Control Gain Matrix

%... Beta Estimator parameters
Mass=93215; Sref= 2690; Qbar=745.4; Cza=-0.0574;
Thr=[470000, 470000, 470000, 0.245e7, 0.245e7]';
```

Stability and Sensitivity Analysis Models

We will now analyze pitch axis stability in the frequency domain. We will use the analysis system: "*Shuttle Ascent, Max_Q, Pitch Analysis with Slosh & TVC*" which was loaded from file "*pitch_anal.m*" and it includes slosh, TWD and the accelerometers. We will first analyze stability using direct α-measurement for feedback and also to drive the α-filter using the open-loop analysis model "*Anal2.slx*", shown in Figure-25. The filter states x_1 and x_2 are directly excited by the angle of attack α. The state-vector in addition to the 3 vehicle states, it also includes the two filter states x_1 and x_2. A low-pass filter is included which represents the actuator. Pitch stability is analyzed by opening the control loop as shown below. We will also analyze pitch stability using the model "*Anal3.slx*", shown in Figure-26 which does not have a direct α-measurement but it uses an α-estimator to estimate α from the N_Z accelerometer, Figure-27.

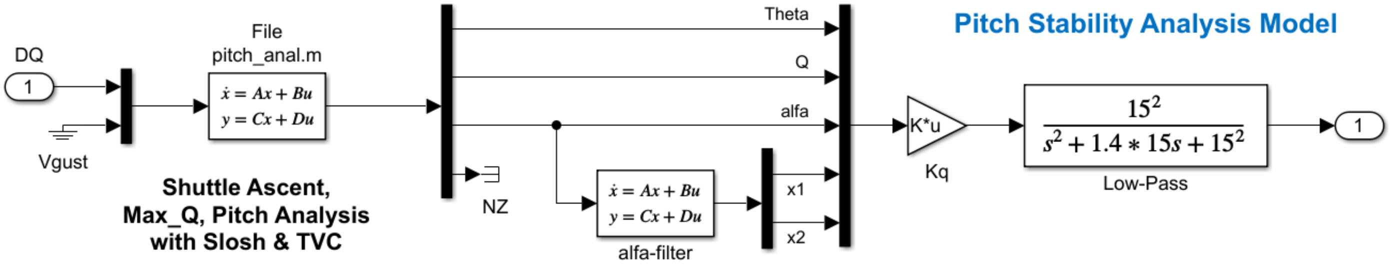


Figure 25 Open-Loop Pitch Stability Analysis Model from file "Anal2.slx"

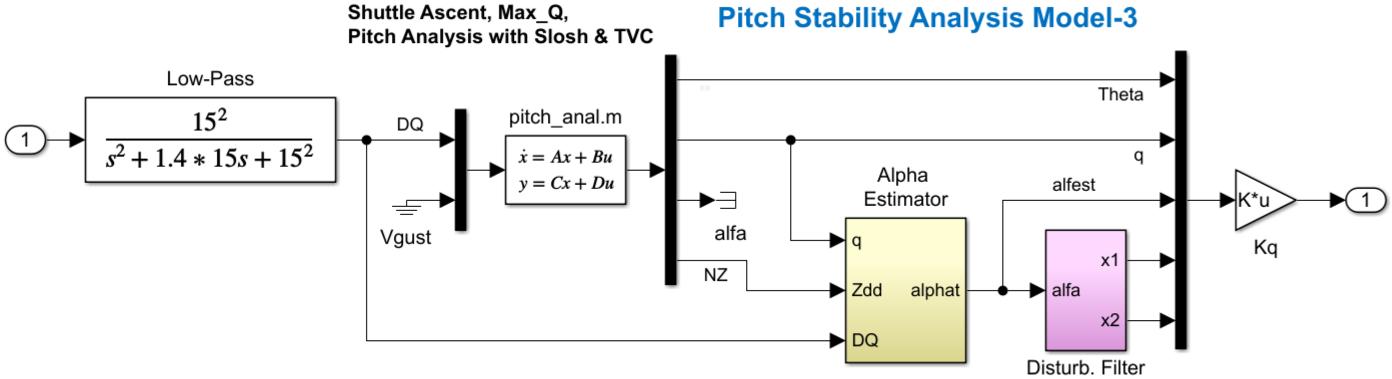


Figure 26 Open-Loop Pitch Stability Analysis Model from file "Anal3.slx" which includes the α-estimator

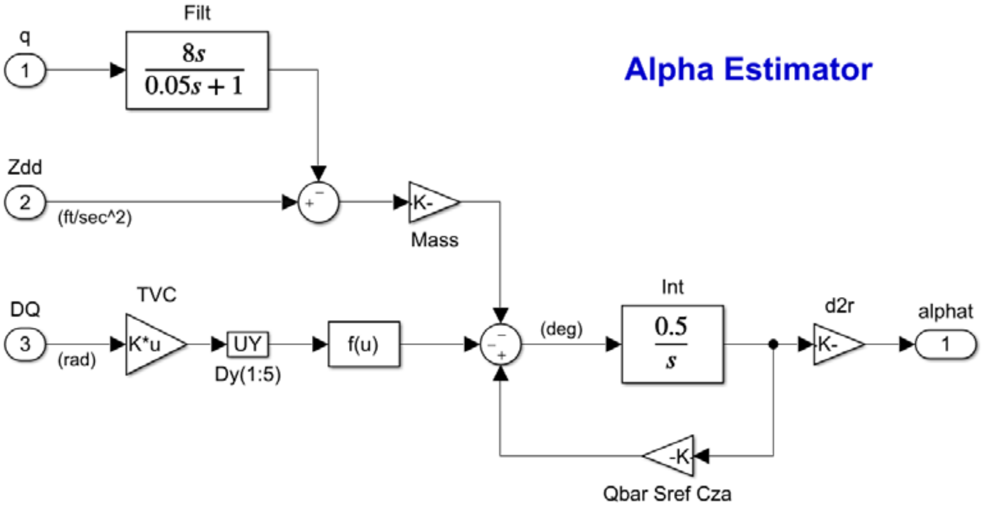


Figure 27 Alpha-Estimator

Figure-28 is a closed-loop system used for analyzing sensitivity to gusts in the frequency domain using Singular Value (Sigma) plots. It is implemented in the Simulink model "Sensit3.slx" and it includes the α–estimator. The gust input is scaled by the largest wind-gust velocity and the α–output is scaled by the maximum allowed α-angle. The output is the actual alpha, not the estimate. The magnitude of the Sensitivity Function (α/Vgust) is expected to be less than one at all frequencies.

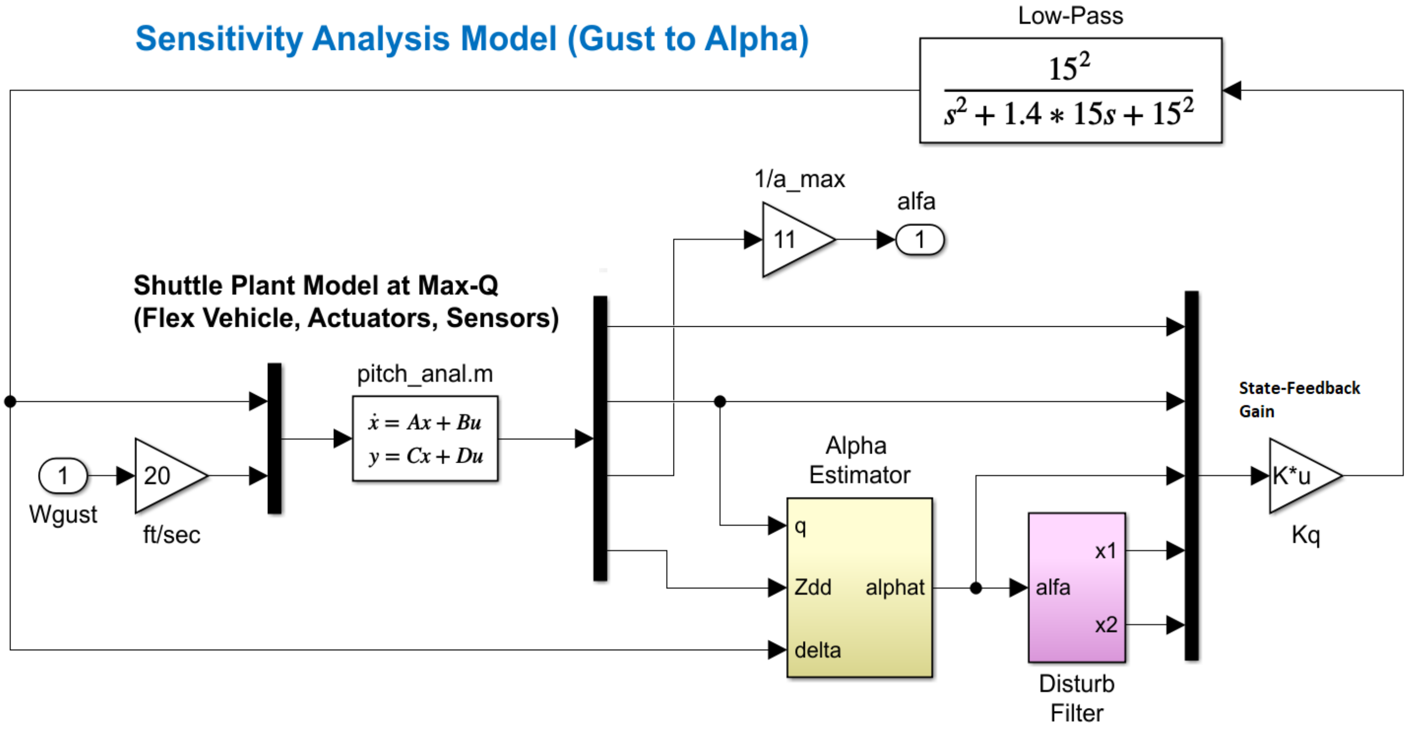
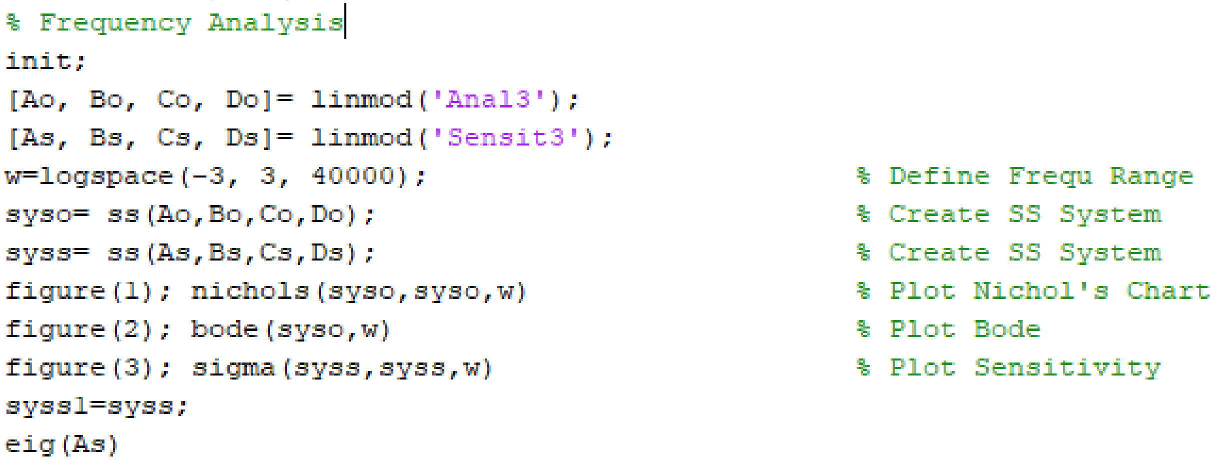


Figure 28 Sensitivity Analysis Model "Sensit3.slx"

The following Matlab script file "freq.m" calculates the Bode, Nichols and Sigma plots. Figure-29 shows the pitch axis stability margins, the two slosh modes at 3.37 (rad/sec) and the aero mode at 0.46 (rad/sec).



```
% Frequency Analysis
init;
[Ao, Bo, Co, Do]= linmod('Anal3');
[As, Bs, Cs, Ds]= linmod('Sensit3');
w=logspace(-3, 3, 40000);                      % Define Frequ Range
syso= ss(Ao,Bo,Co,Do);                         % Create SS System
syss= ss(As,Bs,Cs,Ds);                         % Create SS System
figure(1); nichols(syso,syso,w)                % Plot Nichol's Chart
figure(2); bode(syso,w)                        % Plot Bode
figure(3); sigma(syss,syss,w)                  % Plot Sensitivity
syssl=syss;
eig(As)
```

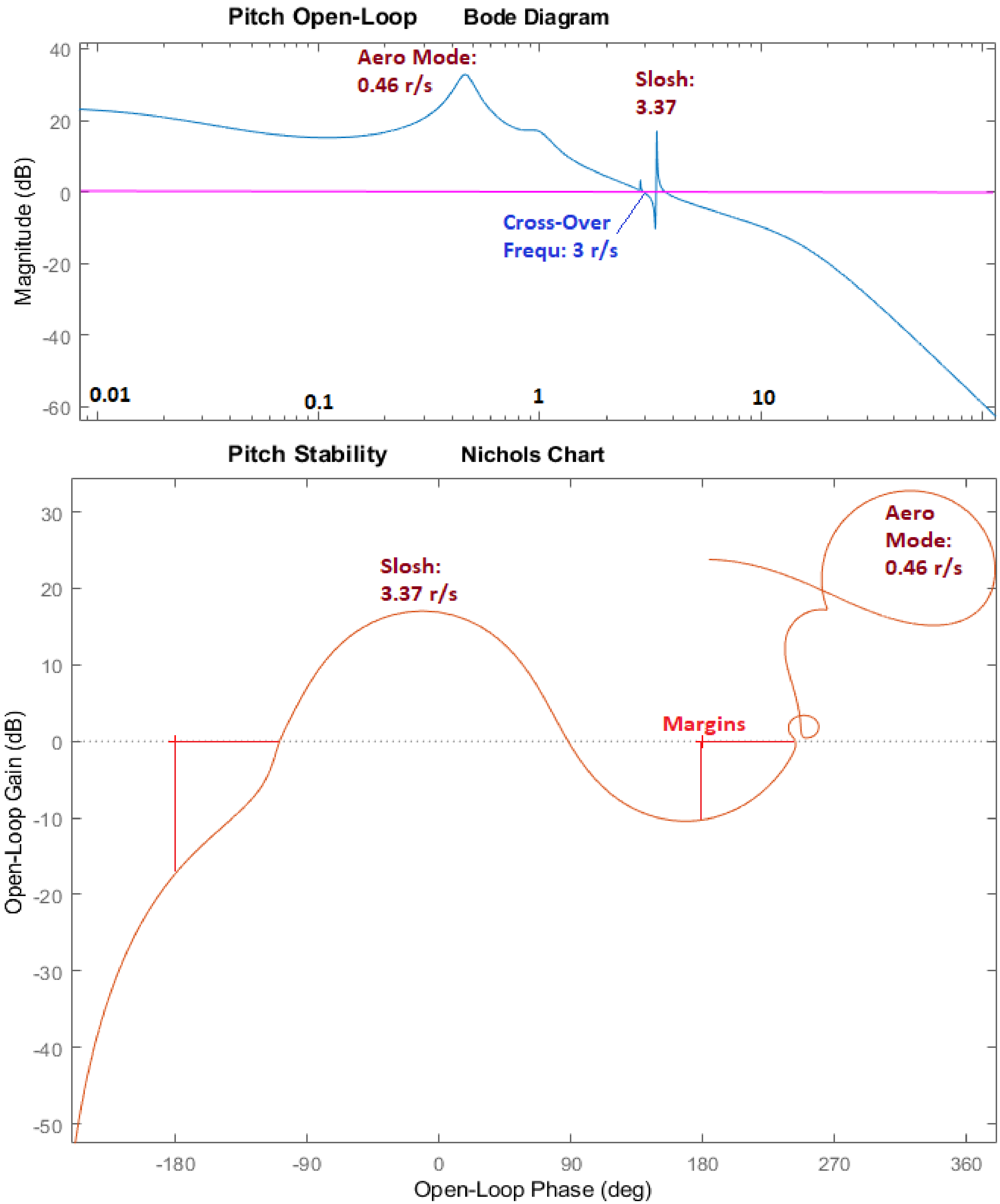


Figure 29 Pitch Axis Stability

The sensitivity response in Figure-30 has a peak magnitude less than one, as expected to meet the sensitivity to gust requirement. It also has the dip at the average disturbance frequency which is accomplished by the α-filter, which reduces the alpha response at the disturbance frequency.

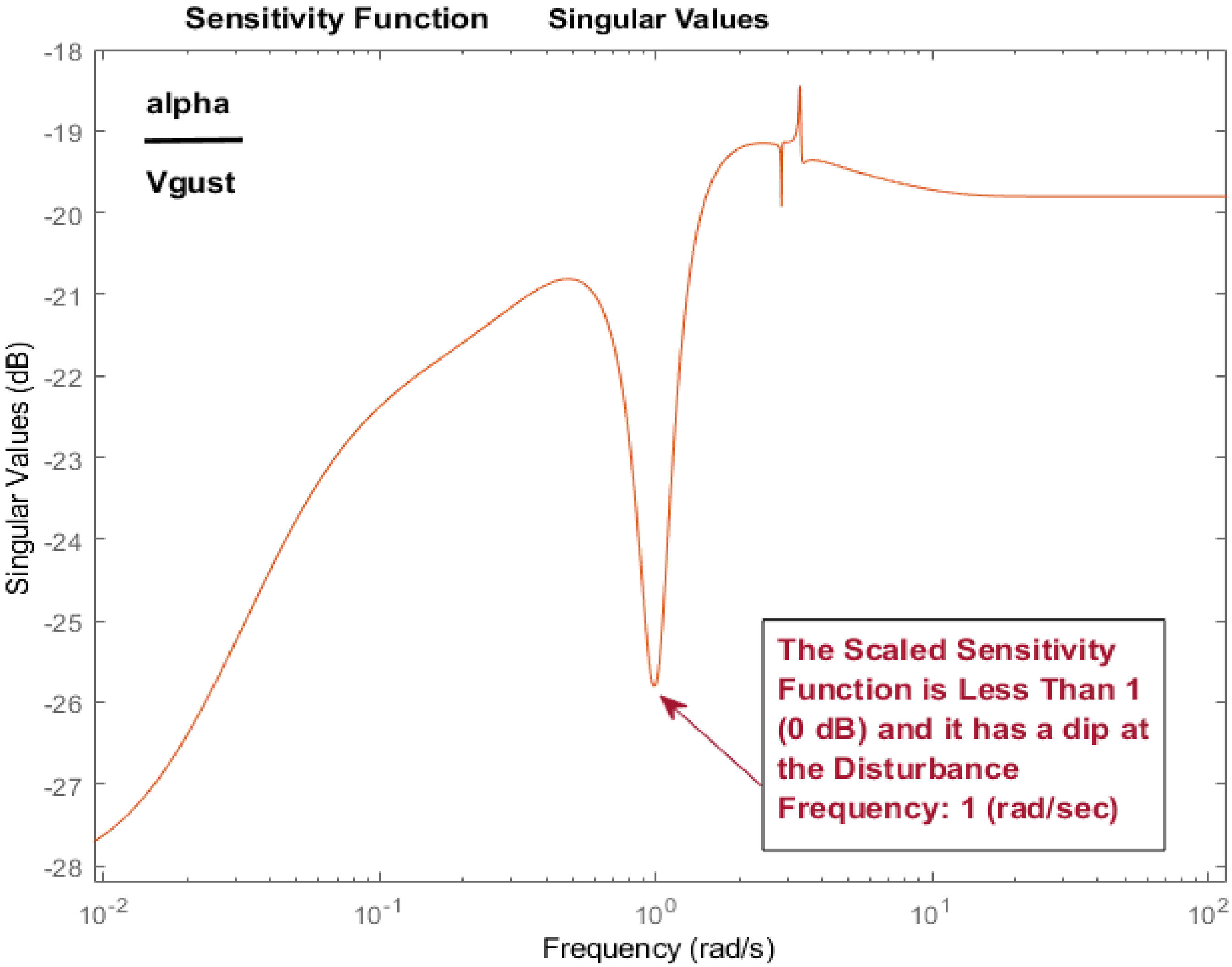


Figure 30 Sigma Plot of the Sensitivity Function Between Gust Disturbance and Alpha Output

Pitch Simulation Models, With and Without Alpha Estimator

We have a couple of closed-loop pitch axis simulation models in files "Sim2.slx" and "Sim3.slx" in Figures (31 & 32). They both use the analysis system "*Shuttle Ascent, Max_Q, Pitch Analysis with Slosh & TVC*". The first one uses direct α-measurement for feedback and the second one has the α-estimator for feedback and to drive the α-filter. The H-infinity derived state-feedback gain Kqhinf is (1x5) matrix and closes the control loop from the 5-states vector via the actuator system. Figure 33 shows the system response to a unit step θ-command. The attitude converges towards the commanded value but its response is slow because of Max-Q and the load-relief limitations. The α-estimate is tracking the real α but it has an oscillation because the N_Z accelerometer picks up the slosh disturbances from the external tank.

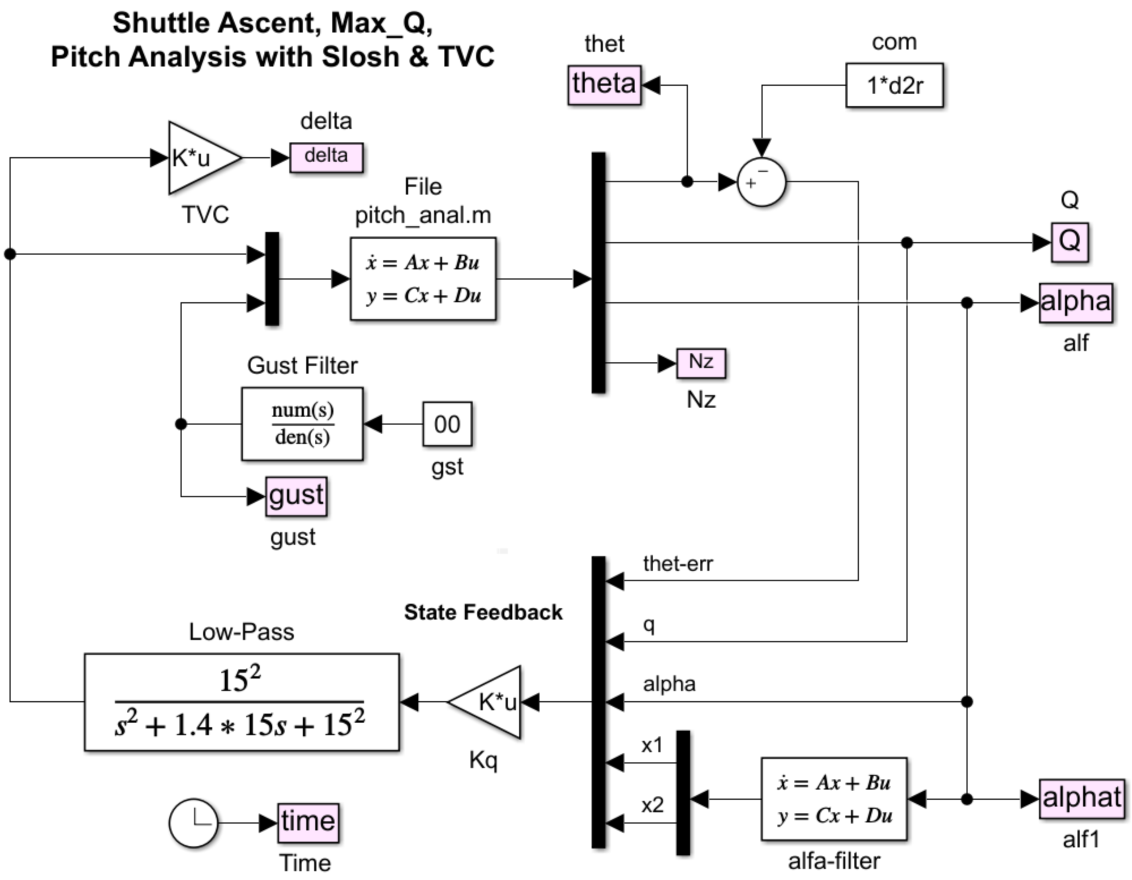


Figure 31 Simulation Model "Sim2.slx" Using Direct Alpha Measurement

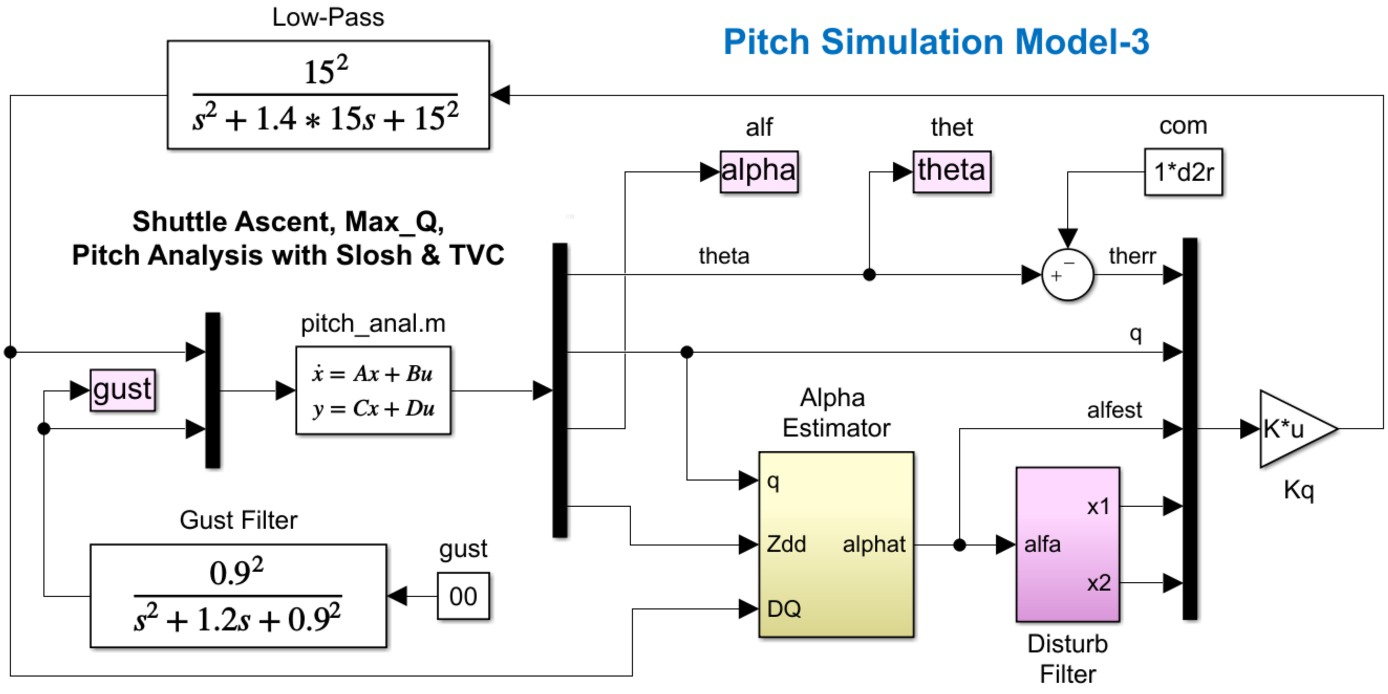


Figure 32 Simulation Model "Sim3.slx" Using the Alpha Estimator

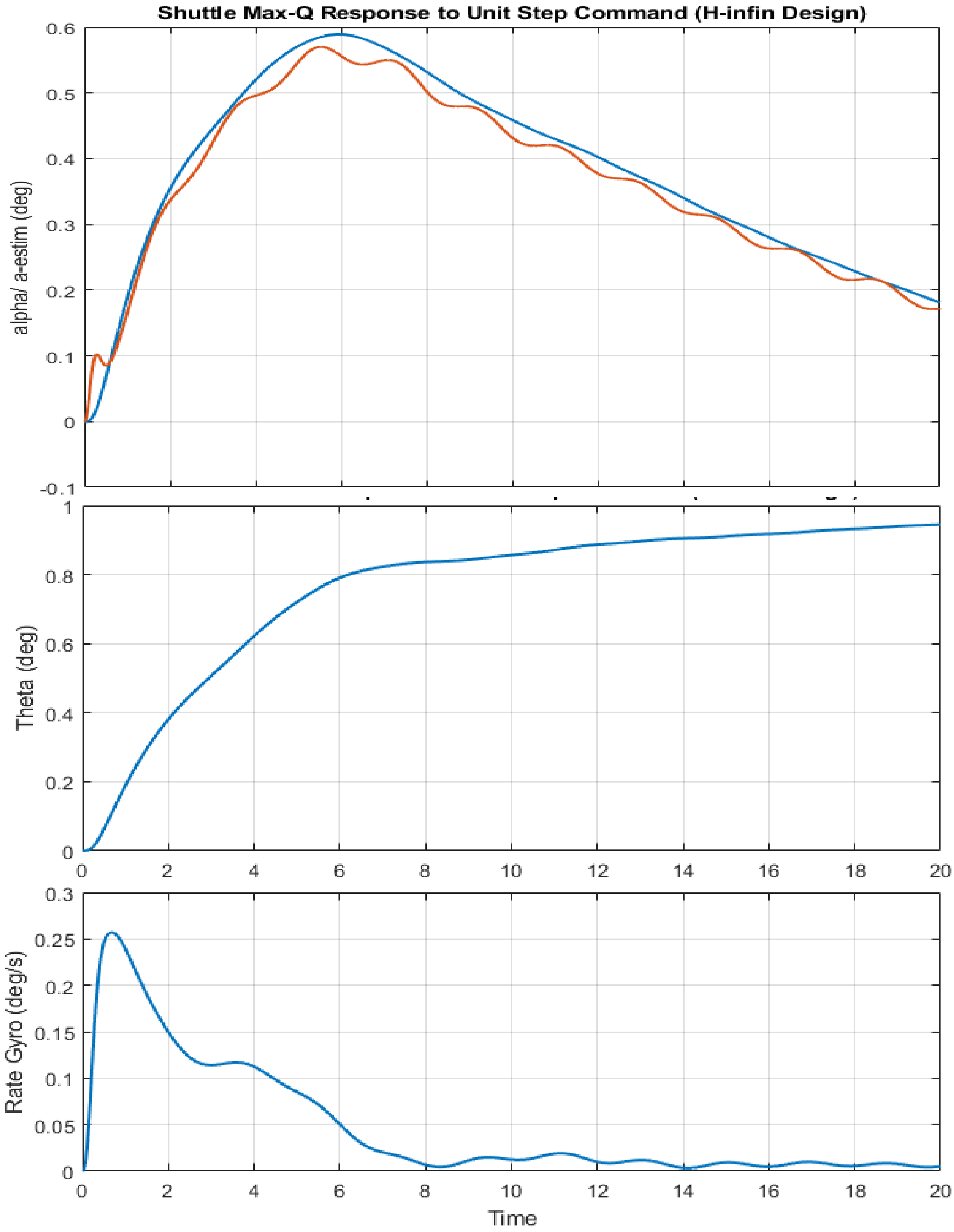


Figure 33 System Response to Unit Step Pitch Attitude Command

4. Lateral H-Infinity Design with State-Estimation

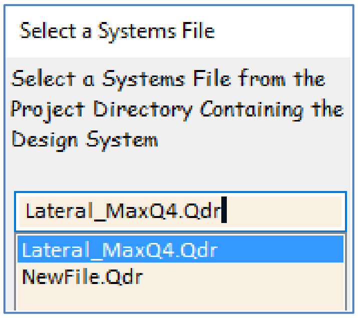
In our previous H-infinity designs we assumed a perfect measurement of the state vector and the controllers were state-feedback gains. In this example the H-infinity controller is a dynamical system that includes a state estimator that estimates the state-vector from the measurements which are only attitudes, rates, and accelerometer. The angle of sideslip β is not measurable. Similar to Section 2, in this design we will include the β-filter in the SM to provide additional attenuation at the 1 (rad/sec) disturbance frequency. Finally, we will simplify the control law to use direct measurements for the attitudes and rates, instead of their estimates, and use the estimator only for the remaining 3 states.

The input file in this example is "*Lateral_MaxQ4.Inp*" and it is located in this directory: "*Flixan\Control Analysis\Hinfinity\Examples\Shuttle Ascent Hinfinity Design\ 4-Lateral Design w State-Estimator*". It includes two Shuttle vehicle datasets at Max-Q. The design vehicle system: "*Shuttle Ascent, Max_Q, T=61 sec, (Design Model)",* and the analysis system: "*Shuttle Ascent, Max_Q, T=61 sec, Rigid-Body/ Slosh/ TWD/ Acceleromet*" which includes propellant sloshing, accelerometers, and TWD dynamics. There is a dataset for generating the TVC matrix "*Shuttle Stage-1 TVC Matrix at Max-Q*", the 2nd order NY disturbance filer "NY Filter". The lateral system is extracted in "*Shuttle Ascent, Max_Q, Lateral Hinf Design Model*", and an interconnection set combines the lateral design vehicle, the TVC, and the NY filter into the design plant: "*Shuttle Ascent, Max_Q, Lateral Design Model with TVC and NY Filter*" which is also used to create the Synthesis Model for the H-infinity design program. A lateral analysis model is also created by extracting the lateral variables from the analysis model that includes slosh and TWD and ignoring the pitch dynamics. The title of the lateral analysis system is "*Shuttle Ascent, Max_Q, Lateral Analysis Model with Slosh & TWD*". A batch set is included at the top of the file that can perform the entire modeling and design process in batch mode, but let us describe the interactive process of creating the SM in detail.

4.1 Creating the Synthesis Model

The systems and matrices of this example are saved in file: "Lateral_MaxQ4.Qdr". The SM consists of 9 matrices and it will be saved in the same systems file. It must be created interactively from the design plant "*Shuttle Ascent, Max_Q, Lateral Design Model with TVC and NY Filter*" which is an (A, B, C, D) system, by extracting some inputs and outputs and placing them into groups as we will describe in the following process. The SM matrices consist of the plant dynamics and also defines the trade-off requirements that the H-infinity algorithm must optimize. The designer must define which inputs are disturbances and which inputs are controls. Also, which outputs are criteria to be optimized and which ones are measurements. In this example some of the measurements are also in the state-vector but the NY output is not a state and the state β plus two other states (x_1 & x_2) have to be estimated. We must, therefore, include the estimator in the design. We define the trade-off between bandwidth and performance versus sensitivity and stability in the optimization algorithm by adjusting some gains which are "knobs" that scale the disturbance inputs and some gains that scale the criteria outputs. Obviously, initially we don't know what gains will produce the desired performance versus stability, so we begin to scale the disturbance inputs by entering the magnitudes of the maximum expected disturbances in the input gains, and for the output gains we enter the maximum allowed magnitude at each performance criterion. The controls are also included in the criteria outputs and we must scale them by the maximum amount of control allowed. In contrast, the measurement noise is also included in the disturbances vector and we must enter the maximum value of noise at each measurement. In this example we must inject measurement noise because we will include a state estimator. But we will introduce very little noise on the attitude and rate measurements and more noise on the accelerometer and filter states.

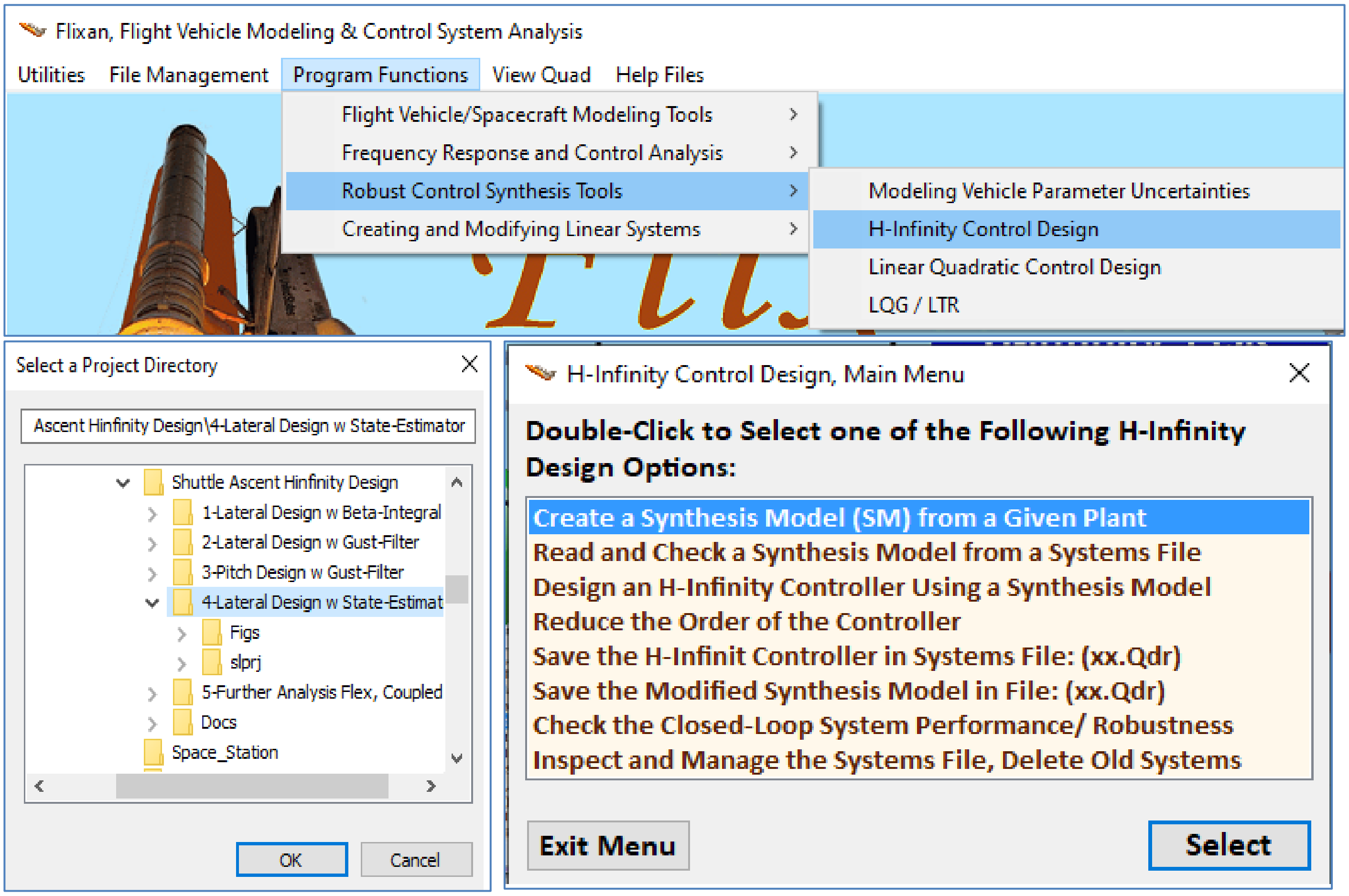
Typically, several gain iterations are needed to converge to the desired trade-off of performance versus robustness. A simple, preliminary simulation model is used to evaluate the design. If we find that we are using too much control, we must reduce the corresponding control gain in the performance criteria output and repeat the design. If a regulated output such as vehicle attitude doesn't converge to the commanded value fast, the gain that corresponds to the attitude criterion must be reduced.


Select a Systems File

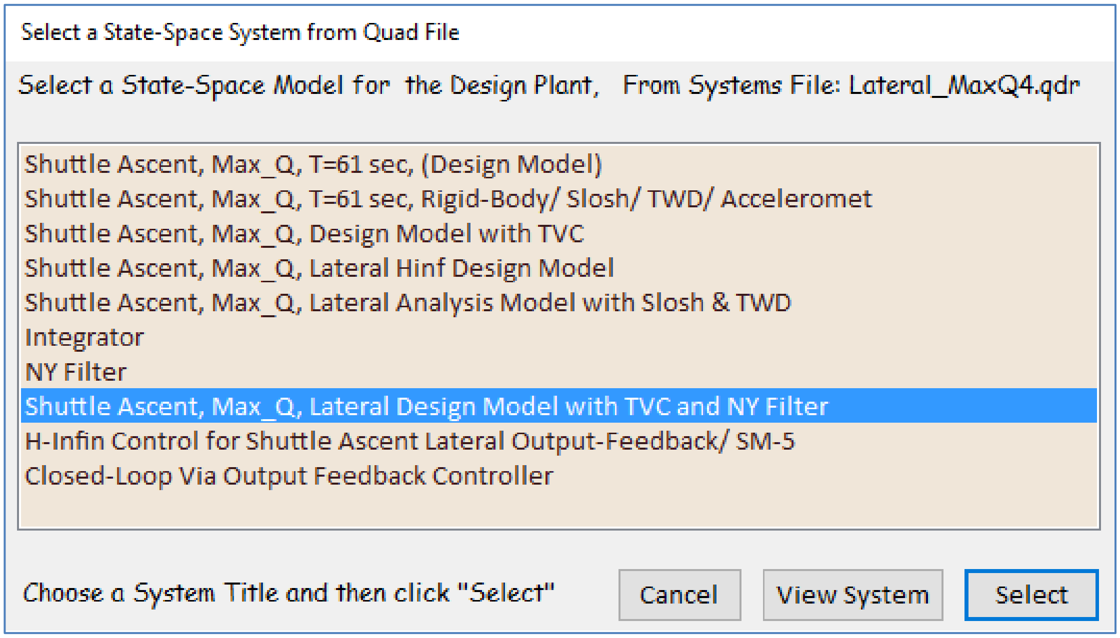
Select a Systems File from the Project Directory Containing the Design System

Lateral_MaxQ4.Qdr

To create the Synthesis Model, we begin by running the H-Infinity Control Design program from the Flixan main menu and then select the systems file that contains the design vehicle plant and where the SM will be saved. Then from the H-Infinity main menu select "Create a Synthesis Model (SM)" to create the SM from the vehicle design model.



The following menu shows the titles of the systems which are included in the systems file. Select the design plant "*Shuttle Ascent, Max_Q, Lateral Design Model with TVC and NY Filter*" that includes Ny-filter and click on "Select". The H-infinity SM will be created from this system.

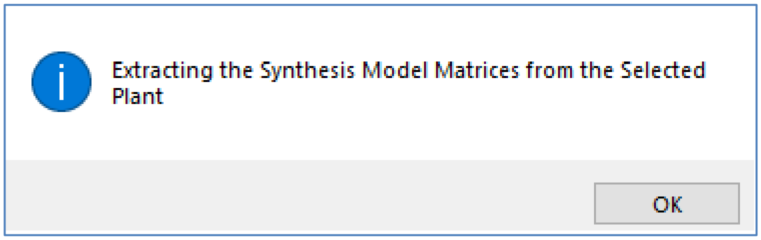


Select a State-Space System from Quad File

Select a State-Space Model for the Design Plant, From Systems File: Lateral_MaxQ4.qdr

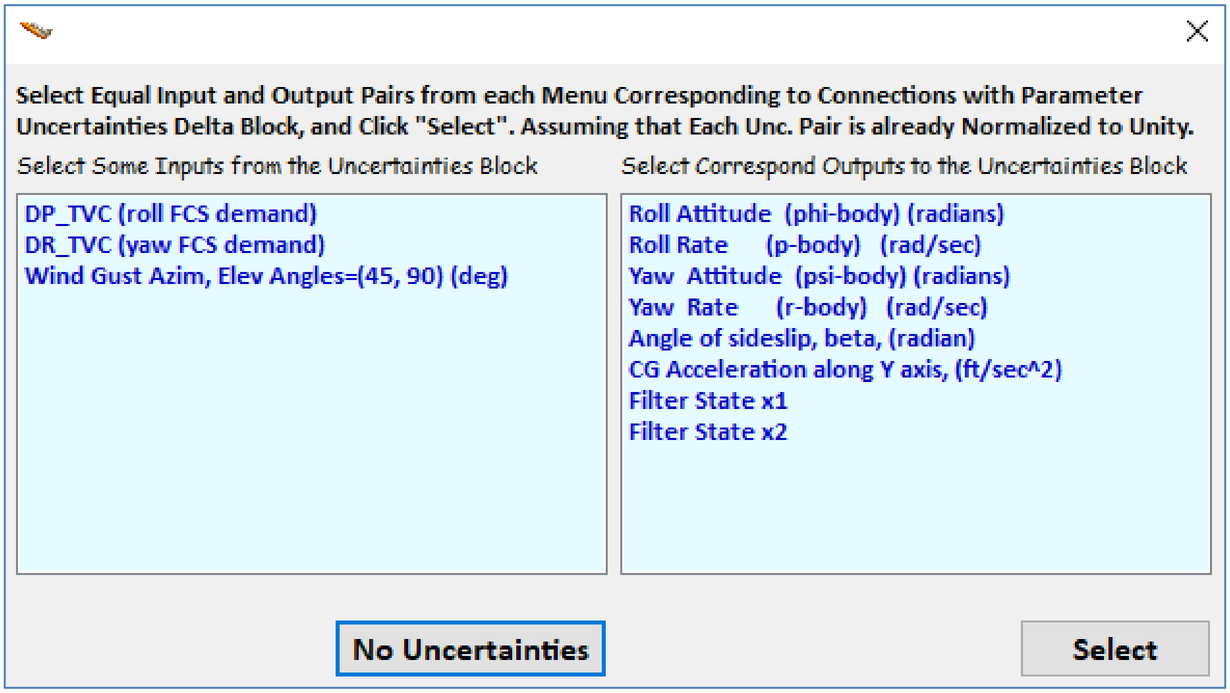
- Shuttle Ascent, Max_Q, T=61 sec, (Design Model)
- Shuttle Ascent, Max_Q, T=61 sec, Rigid-Body/ Slosh/ TWD/ Acceleromet
- Shuttle Ascent, Max_Q, Design Model with TVC
- Shuttle Ascent, Max_Q, Lateral Hinf Design Model
- Shuttle Ascent, Max_Q, Lateral Analysis Model with Slosh & TWD
- Integrator
- NY Filter
- Shuttle Ascent, Max_Q, Lateral Design Model with TVC and NY Filter
- H-Infin Control for Shuttle Ascent Lateral Output-Feedback/ SM-5
- Closed-Loop Via Output Feedback Controller

Choose a System Title and then click "Select"



Extracting the Synthesis Model Matrices from the Selected Plant

The first menu is for defining parameter uncertainties. That is, pre-scaled inputs and outputs that connect to the uncertainties Δ block. In this example we have not defined any uncertain parameters in the vehicle model with uncertainty inputs and outputs. We therefore, click on “No Uncertainties” to continue.



Select Equal Input and Output Pairs from each Menu Corresponding to Connections with Parameter Uncertainties Delta Block, and Click "Select". Assuming that Each Unc. Pair is already Normalized to Unity.

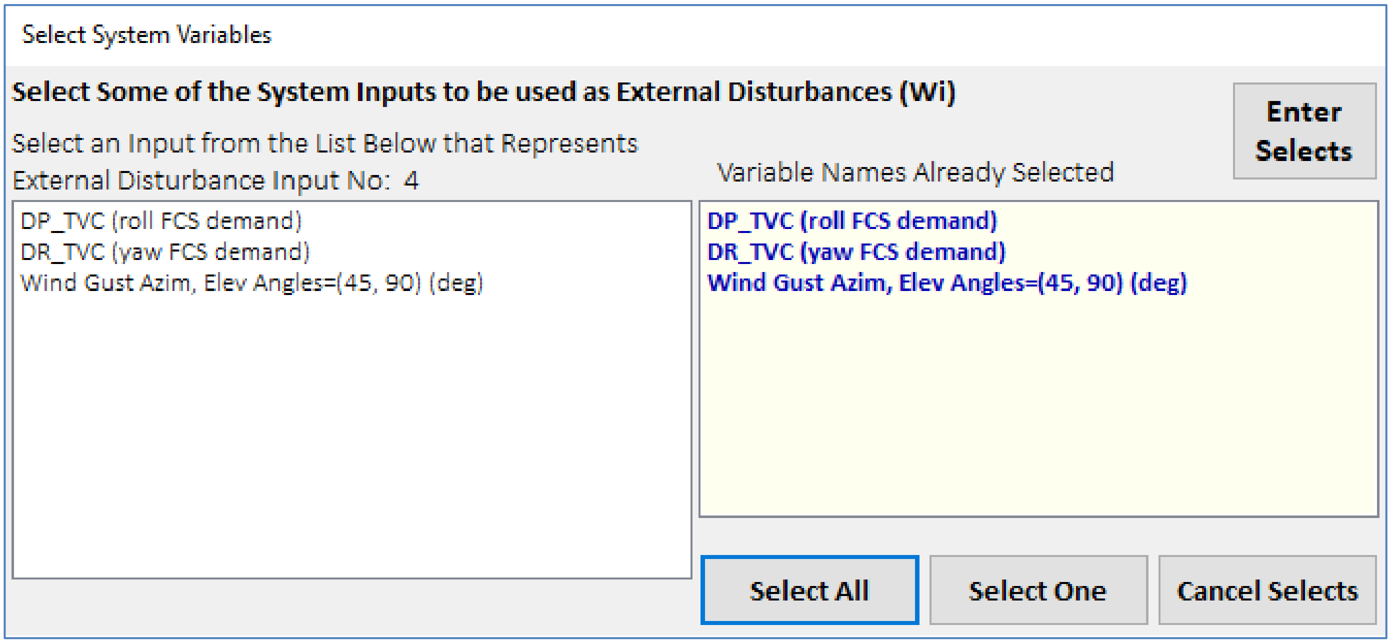
Select Some Inputs from the Uncertainties Block

- DP_TVC (roll FCS demand)
- DR_TVC (yaw FCS demand)
- Wind Gust Azim, Elev Angles=(45, 90) (deg)

Select Correspond Outputs to the Uncertainties Block

- Roll Attitude (phi-body) (radians)
- Roll Rate (p-body) (rad/sec)
- Yaw Attitude (psi-body) (radians)
- Yaw Rate (r-body) (rad/sec)
- Angle of sideslip, beta, (radian)
- CG Acceleration along Y axis, (ft/sec^2)
- Filter State x1
- Filter State x2

The next menu is for defining external disturbance inputs. The design model has 3 inputs and all 3 are considered as disturbance sources. The biggest disturbance is the wind-gust but the two controls are also subjected to disturbances. So, all 3 are selected and click on “Enter Selects” to continue.



Select System Variables

Select Some of the System Inputs to be used as External Disturbances (Wi)

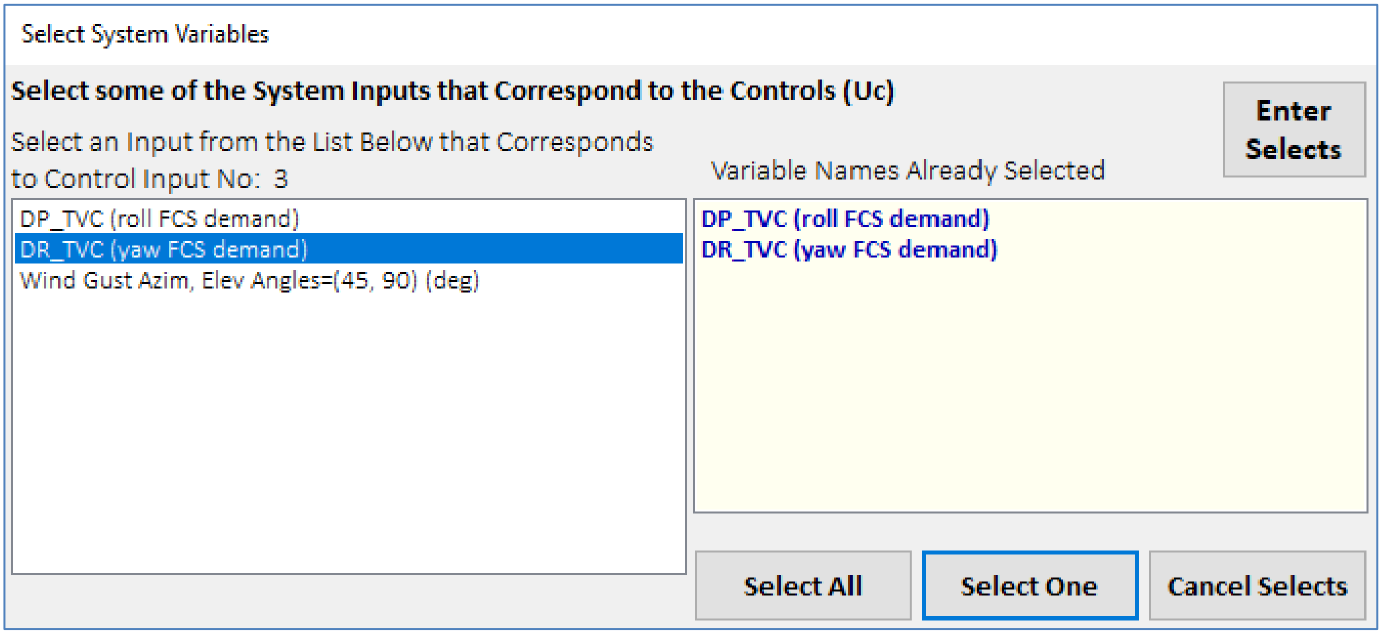
Select an Input from the List Below that Represents
External Disturbance Input No: 4

DP_TVC (roll FCS demand)
DR_TVC (yaw FCS demand)
Wind Gust Azim, Elev Angles=(45, 90) (deg)

Variable Names Already Selected

DP_TVC (roll FCS demand)
DR_TVC (yaw FCS demand)
Wind Gust Azim, Elev Angles=(45, 90) (deg)

The next menu is used for selecting the control inputs. There are two control inputs, the roll and yaw demands, which are the inputs to the TVC matrix since the design model already includes the TVC. Select them one at a time and then click on “Enter Selects” to continue.



Select System Variables

Select some of the System Inputs that Correspond to the Controls (Uc)

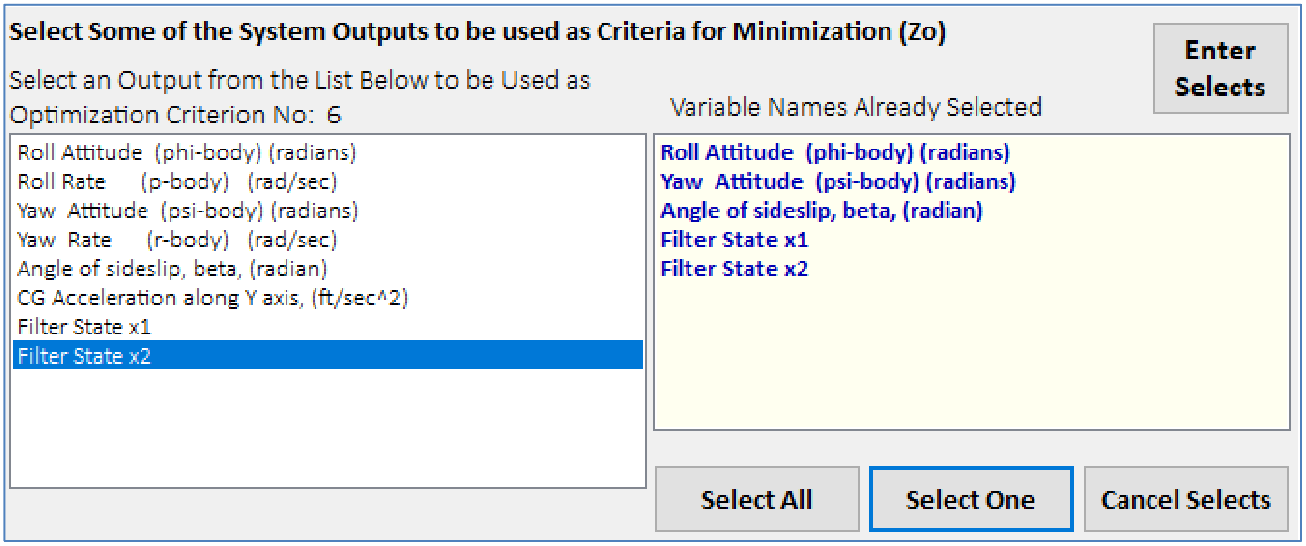
Select an Input from the List Below that Corresponds
to Control Input No: 3

DP_TVC (roll FCS demand)
DR_TVC (yaw FCS demand)
Wind Gust Azim, Elev Angles=(45, 90) (deg)

Variable Names Already Selected

DP_TVC (roll FCS demand)
DR_TVC (yaw FCS demand)

The design system output consists of attitudes, rates, beta, Ny-acceleration, and the two filter states. In this case we select only the outputs that we would like to minimize in the presence of disturbances, and that is, the two attitudes, the sideslip β, and the two filter states x_1 and x_2 that will produce the dip at the disturbance frequency. Select one at a time and then click on “Enter Selects” to continue. In the next menu do not select anything. Skip this option and click on “Enter Selects”.



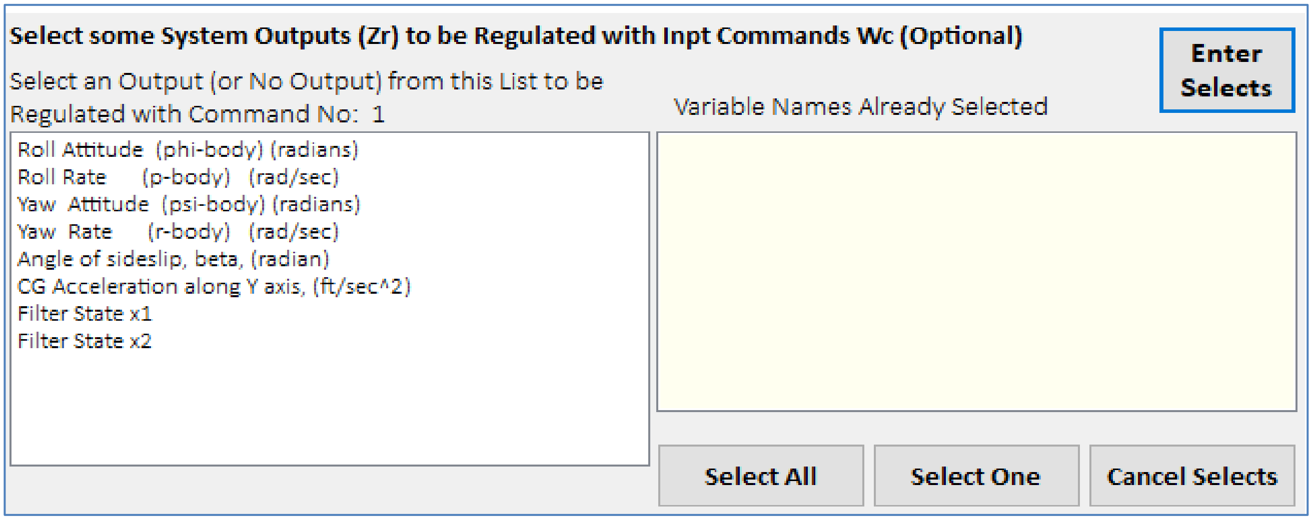
Select Some of the System Outputs to be used as Criteria for Minimization (Zo)

Select an Output from the List Below to be Used as
Optimization Criterion No: 6

Roll Attitude (phi-body) (radians)
Roll Rate (p-body) (rad/sec)
Yaw Attitude (psi-body) (radians)
Yaw Rate (r-body) (rad/sec)
Angle of sideslip, beta, (radian)
CG Acceleration along Y axis, (ft/sec^2)
Filter State x1
Filter State x2

Variable Names Already Selected

Roll Attitude (phi-body) (radians)
Yaw Attitude (psi-body) (radians)
Angle of sideslip, beta, (radian)
Filter State x1
Filter State x2



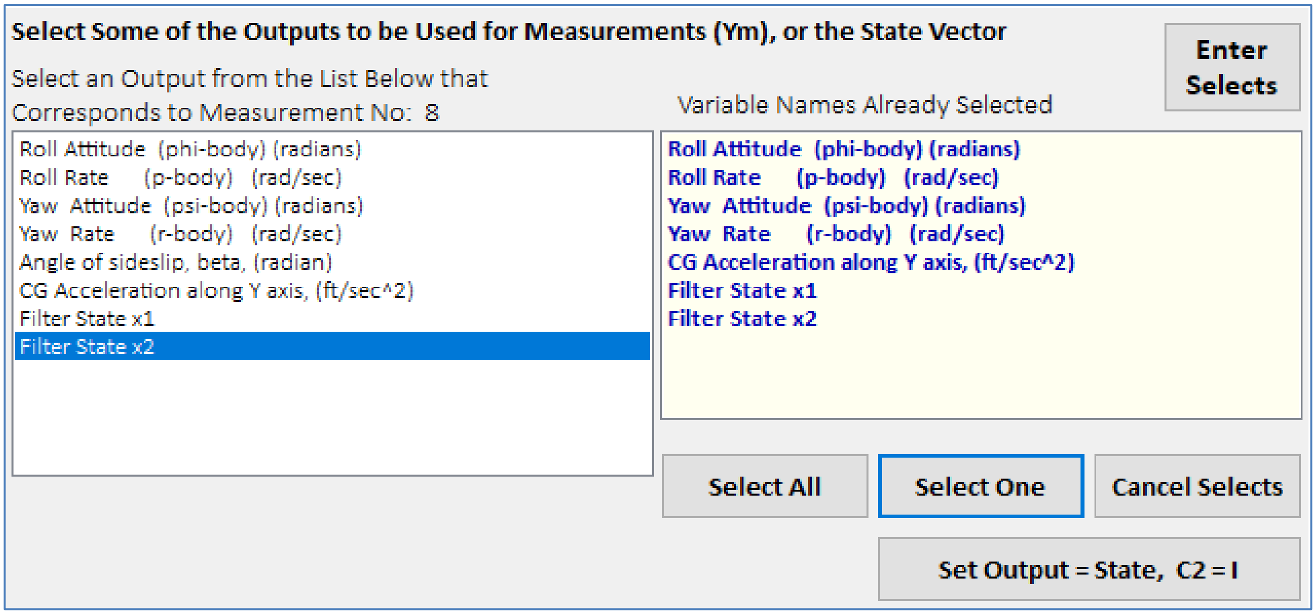
Select some System Outputs (Zr) to be Regulated with Inpt Commands Wc (Optional)

Select an Output (or No Output) from this List to be
Regulated with Command No: 1

Roll Attitude (phi-body) (radians)
Roll Rate (p-body) (rad/sec)
Yaw Attitude (psi-body) (radians)
Yaw Rate (r-body) (rad/sec)
Angle of sideslip, beta, (radian)
CG Acceleration along Y axis, (ft/sec^2)
Filter State x1
Filter State x2

Variable Names Already Selected

The next menu is for selecting the output measurements. Select the two attitudes, the two body rates, the Ny-accelerometer, the two filter states, and then click on “Enter Selects” to continue.



Select Some of the Outputs to be Used for Measurements (Ym), or the State Vector

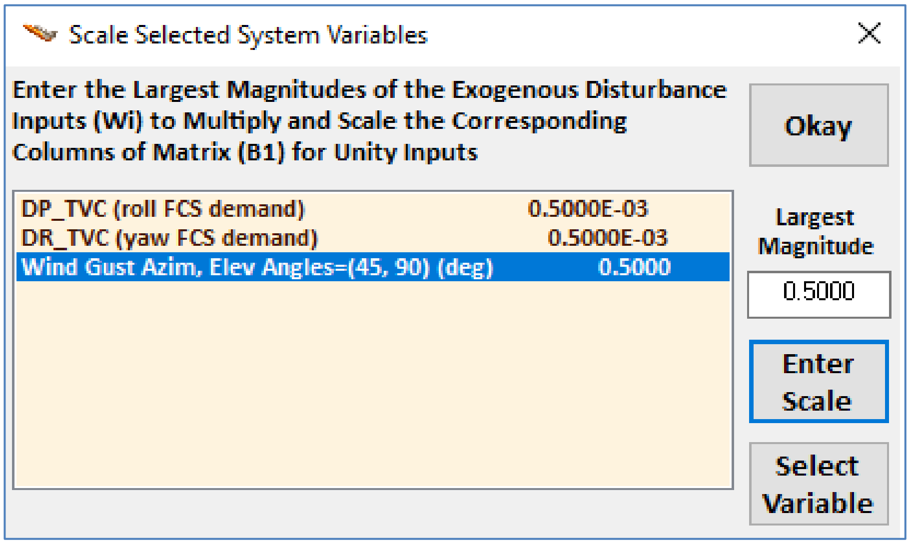
Select an Output from the List Below that
Corresponds to Measurement No: 8

Roll Attitude (phi-body) (radians)
Roll Rate (p-body) (rad/sec)
Yaw Attitude (psi-body) (radians)
Yaw Rate (r-body) (rad/sec)
Angle of sideslip, beta, (radian)
CG Acceleration along Y axis, (ft/sec^2)
Filter State x1
Filter State x2

Variable Names Already Selected

Roll Attitude (phi-body) (radians)
Roll Rate (p-body) (rad/sec)
Yaw Attitude (psi-body) (radians)
Yaw Rate (r-body) (rad/sec)
CG Acceleration along Y axis, (ft/sec^2)
Filter State x1
Filter State x2

We have now finished defining the input and output variables. The next step is to define the gains that will be used to scale them. Those gains may be changed for the next design iteration. In the dialog below enter the gains that will scale the disturbance inputs. That is, the maximum expected disturbance at each input. Highlight one of the inputs, click on “Select Variable”, enter the gain value, and click on “Enter Scale” to accept it, one at a time. The value appears in the display next to the variable label. When you finish click on “Okay” to go to the next dialog.

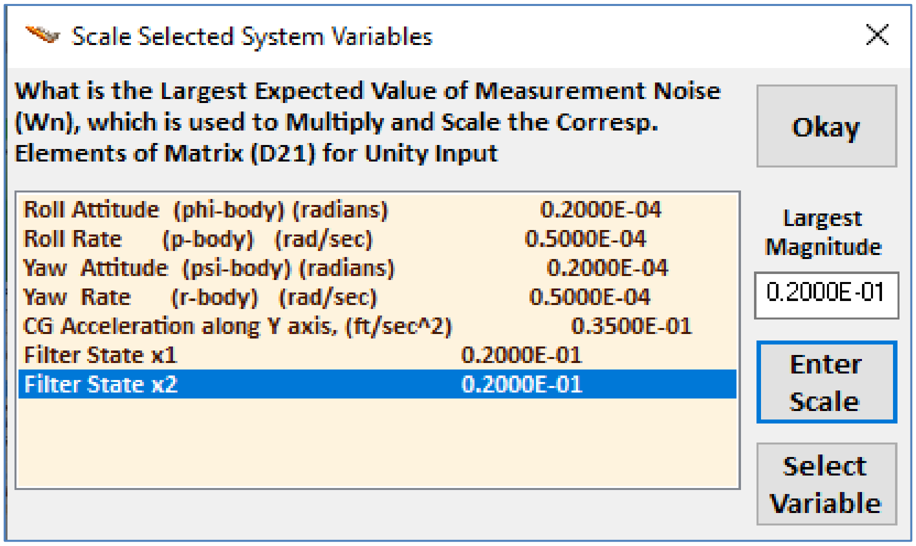


Enter the Largest Magnitudes of the Exogenous Disturbance Inputs (Wi) to Multiply and Scale the Corresponding Columns of Matrix (B1) for Unity Inputs

DP_TVC (roll FCS demand)	0.5000E-03
DR_TVC (yaw FCS demand)	0.5000E-03
Wind Gust Azim, Elev Angles=(45, 90) (deg)	0.5000

Largest Magnitude: 0.5000

This dialog is for entering the measurement noise. In this case we must include measurement noise because it is used in the estimator design. The noise value in the attitudes and rates are small because they are measurable. The noise injected in the accelerometer measurement and in the filter states is larger.

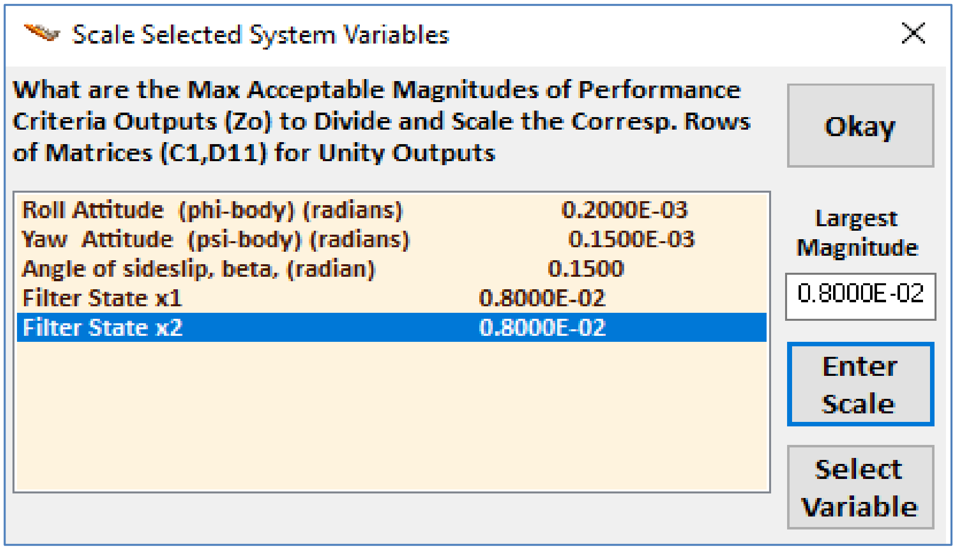


What is the Largest Expected Value of Measurement Noise (Wn), which is used to Multiply and Scale the Corresp. Elements of Matrix (D21) for Unity Input

Roll Attitude (phi-body) (radians)	0.2000E-04
Roll Rate (p-body) (rad/sec)	0.5000E-04
Yaw Attitude (psi-body) (radians)	0.2000E-04
Yaw Rate (r-body) (rad/sec)	0.5000E-04
CG Acceleration along Y axis, (ft/sec^2)	0.3500E-01
Filter State x1	0.2000E-01
Filter State x2	0.2000E-01

Largest Magnitude: 0.2000E-01

We must now define the gains for the performance optimization criteria (z). That is, the maximum acceptable magnitude at the selected criteria outputs, which are: the maximum roll and yaw attitude errors, maximum beta transient magnitude and maximum values of the filter states x1 & x2. Reducing the gain value of a specific performance output results into better performance and smaller transient for that variable. Select one variable at a time, enter the gain and click on “enter scale” to accept it. When you finish click on “Okay” to go to the next dialog.

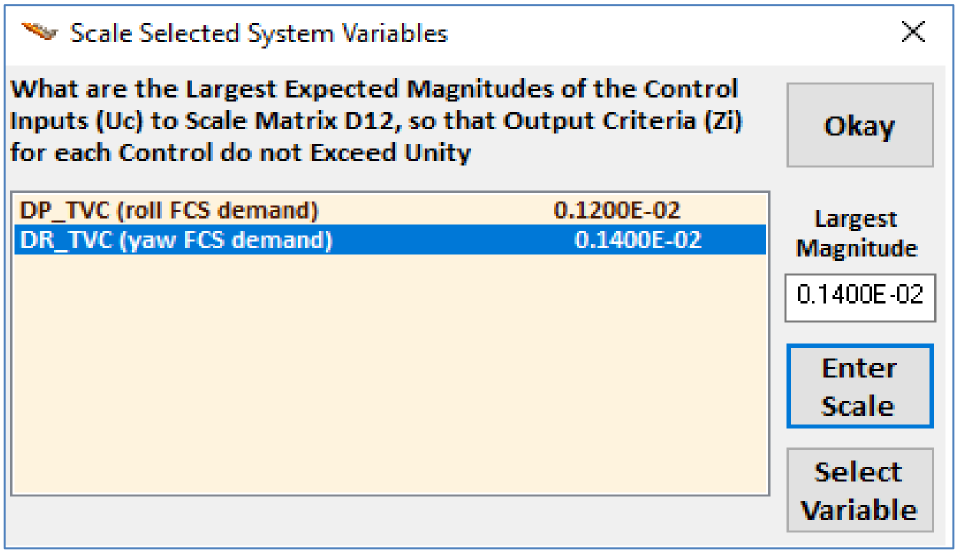


What are the Max Acceptable Magnitudes of Performance Criteria Outputs (Zo) to Divide and Scale the Corresp. Rows of Matrices (C1,D11) for Unity Outputs

Roll Attitude (phi-body) (radians)	0.2000E-03
Yaw Attitude (psi-body) (radians)	0.1500E-03
Angle of sideslip, beta, (radian)	0.1500
Filter State x1	0.8000E-02
Filter State x2	0.8000E-02

Largest Magnitude: 0.8000E-02

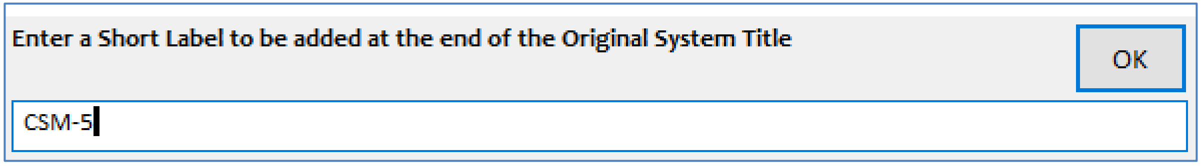
The controls are also included in the optimization criteria (z). Between the two criteria we define the trade-off between performance and control bandwidth. In this example we have two controls, roll and yaw control demands. If we increase the gain in one of them, let's say the roll control, we are telling the mathematic algorithm to allow more control in the roll axis which means bigger bandwidth in roll and the system will be faster in roll. Enter the two gains as before and click on "Okay" to proceed. Finally enter a short label that will appear at the end of the Synthesis Model title which is saved in the systems file.



What are the Largest Expected Magnitudes of the Control Inputs (Uc) to Scale Matrix D12, so that Output Criteria (Zi) for each Control do not Exceed Unity

DP_TVC (roll FCS demand)	0.1200E-02
DR_TVC (yaw FCS demand)	0.1400E-02

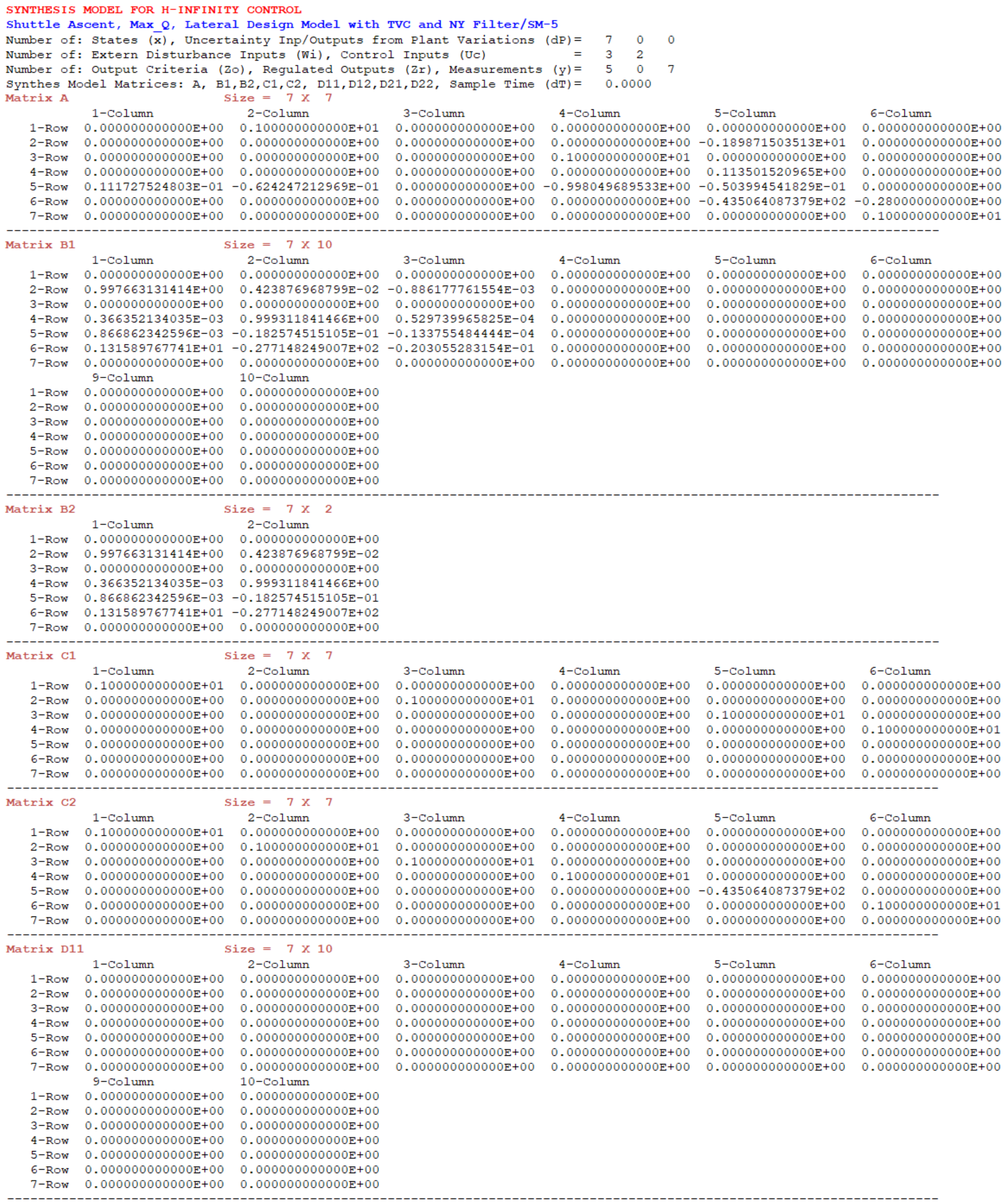
Largest Magnitude: 0.1400E-02



Enter a Short Label to be added at the end of the Original System Title

CSM-5

H-Infinity Synthesis Model in file “Lateral_MaxQ4.Qdr”

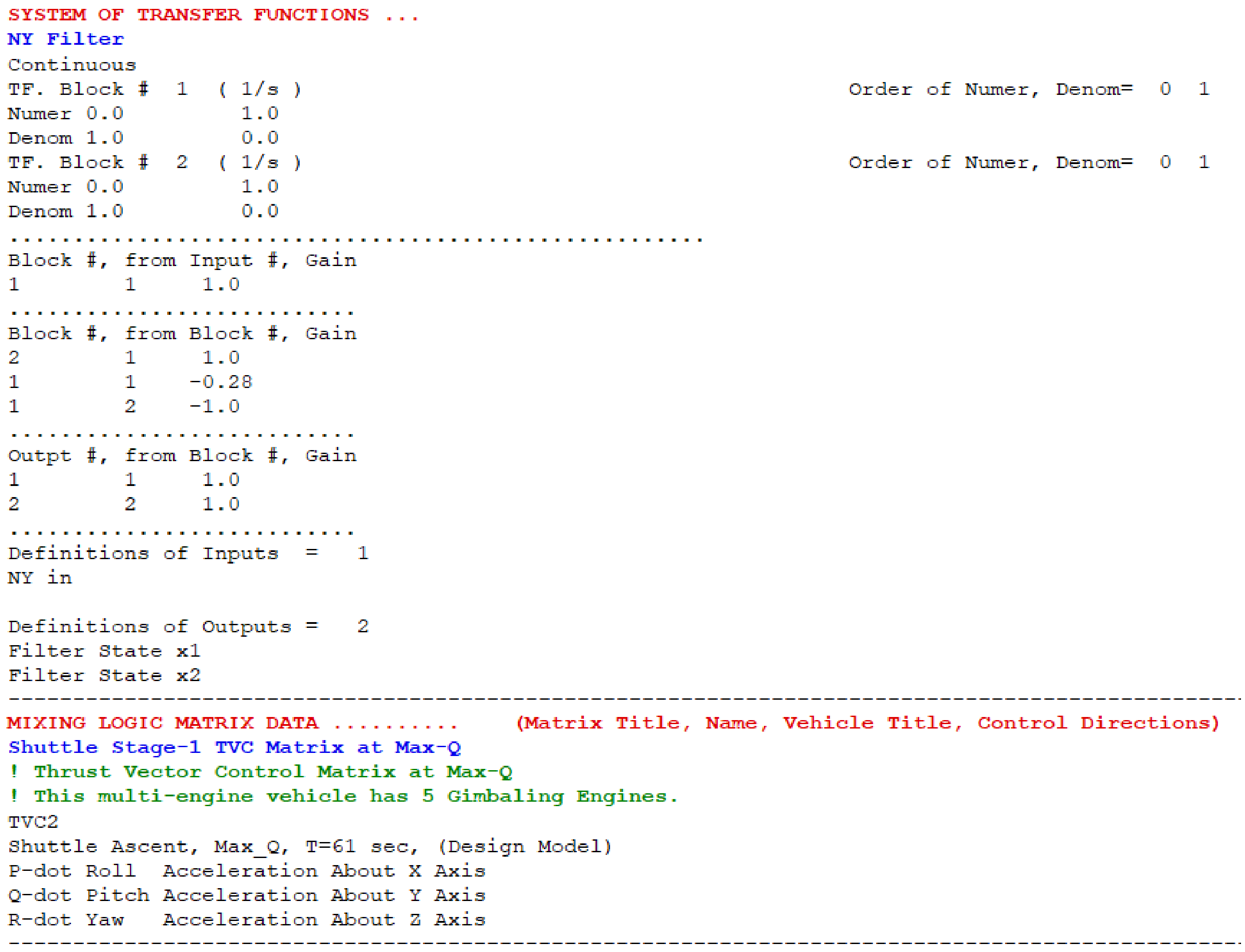


```
SYNTHESIS MODEL FOR H-INFINITY CONTROL
Shuttle Ascent, Max_Q, Lateral Design Model with TVC and NY Filter/SM-5
Number of: States (x), Uncertainty Inp/Outputs from Plant Variations (dP)=   7   0   0
Number of: Extern Disturbance Inputs (Wi), Control Inputs (Uc)           =   3   2
Number of: Output Criteria (Zo), Regulated Outputs (Zr), Measurements (y)=   5   0   7
Synthes Model Matrices: A, B1,B2,C1,C2, D11,D12,D21,D22, Sample Time (dT)=   0.0000
Matrix A                 Size =  7 X  7
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00 -0.189871503513E+01  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.113501520965E+00  0.000000000000E+00
   5-Row  0.111727524803E-01 -0.624247212969E-01  0.000000000000E+00 -0.998049689533E+00 -0.503994541829E-01  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00 -0.435064087379E+02 -0.280000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
--------------------------------------------------------------------------------------------------------------------------
Matrix B1                Size =  7 X 10
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.997663131414E+00  0.423876968799E-02 -0.886177761554E-03  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.366352134035E-03  0.999311841466E+00  0.529739965825E-04  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.866862342596E-03 -0.182574515105E-01 -0.133755484444E-04  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.131589767741E+01 -0.277148249007E+02 -0.203055283154E-01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
           9-Column           10-Column
   1-Row  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00
--------------------------------------------------------------------------------------------------------------------------
Matrix B2                Size =  7 X  2
           1-Column            2-Column
   1-Row  0.000000000000E+00  0.000000000000E+00
   2-Row  0.997663131414E+00  0.423876968799E-02
   3-Row  0.000000000000E+00  0.000000000000E+00
   4-Row  0.366352134035E-03  0.999311841466E+00
   5-Row  0.866862342596E-03 -0.182574515105E-01
   6-Row  0.131589767741E+01 -0.277148249007E+02
   7-Row  0.000000000000E+00  0.000000000000E+00
--------------------------------------------------------------------------------------------------------------------------
Matrix C1                Size =  7 X  7
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
--------------------------------------------------------------------------------------------------------------------------
Matrix C2                Size =  7 X  7
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00 -0.435064087379E+02  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
--------------------------------------------------------------------------------------------------------------------------
Matrix D11               Size =  7 X 10
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
           9-Column           10-Column
   1-Row  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00
--------------------------------------------------------------------------------------------------------------------------
```

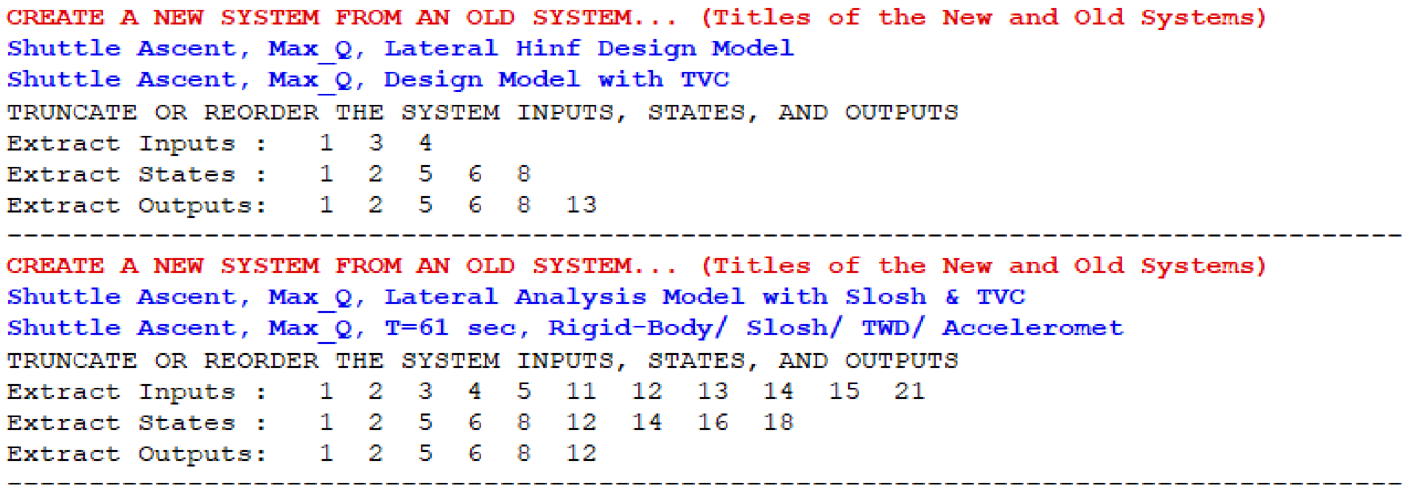
```
Matrix D12                  Size =  7 X  2
           1-Column            2-Column
   1-Row  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00
   6-Row  0.100000000000E+01  0.000000000000E+00
   7-Row  0.000000000000E+00  0.100000000000E+01
------------------------------------------------------------------------------------------------------------------------
Matrix D21                  Size =  7 X 10
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.131589767741E+01 -0.277148249007E+02 -0.203055283154E-01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
           9-Column            10-Column
   1-Row  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00
   6-Row  0.100000000000E+01  0.000000000000E+00
   7-Row  0.000000000000E+00  0.100000000000E+01
------------------------------------------------------------------------------------------------------------------------
Matrix D22                  Size =  7 X  2
           1-Column            2-Column
   1-Row  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00
   5-Row  0.131589767741E+01 -0.277148249007E+02
   6-Row  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Definition of Synthesis Model Variables                        Max Scaling Factors

States (x) ..........  =   7
  1    Roll Attitude  (phi-body)  (radians)
  2    Roll Rate      (p-body)    (rad/sec)
  3    Yaw  Attitude  (psi-body)  (radians)
  4    Yaw  Rate      (r-body)    (rad/sec)
  5    Angle of sideslip, beta, (radian)
  6    Filter State x1
  7    Filter State x2

Excitation Inputs (w)  =  10
  1    DP_TVC (roll FCS demand)                                 *  0.0005
  2    DR_TVC (yaw FCS demand)                                  *  0.0005
  3    Wind Gust Azim, Elev Angles=(45, 90) (deg)               *  0.5
  4    Noise at Output: Roll Attitude   (phi-body) (radians)    *  0.00002
  5    Noise at Output: Roll Rate       (p-body)   (rad/sec)    *  0.00005
  6    Noise at Output: Yaw  Attitude   (psi-body) (radians)    *  0.00002
  7    Noise at Output: Yaw  Rate       (r-body)   (rad/sec)    *  0.00005
  8    Noise at Output: CG Acceleration along Y axis, (ft/sec^ *  0.035
  9    Noise at Output: Filter State x1                         *  0.02
 10    Noise at Output: Filter State x2                         *  0.02

Control Inputs (u) ... =   2
  1    Control: DP_TVC (roll FCS demand)                        *  1.0000
  2    Control: DR_TVC (yaw FCS demand)                         *  1.0000

Performance Outputs (z)=   7
  1    Roll Attitude  (phi-body) (radians)                      /  0.0002
  2    Yaw  Attitude  (psi-body) (radians)                      /  0.00015
  3    Angle of sideslip, beta, (radian)                        /  0.15
  4    Filter State x1                                          /  0.008
  5    Filter State x2                                          /  0.008
  6    Contrl Criter. DP_TVC (roll FCS demand)                  /  0.002
  7    Contrl Criter. DR_TVC (yaw FCS demand)                   /  0.0025

Measurement Outputs (y)=   7
  1    Measurm: Roll Attitude  (phi-body) (radians)             /  1.0000
  2    Measurm: Roll Rate      (p-body)   (rad/sec)             /  1.0000
  3    Measurm: Yaw  Attitude  (psi-body) (radians)             /  1.0000
  4    Measurm: Yaw  Rate      (r-body)   (rad/sec)             /  1.0000
  5    Measurm: CG Acceleration along Y axis, (ft/sec^2)        /  1.0000
  6    Measurm: Filter State x1                                 /  1.0000
  7    Measurm: Filter State x2                                 /  1.0000
```

4.2 Input File: Lateral_MaxQ1.Inp

```
BATCH MODE INSTRUCTIONS ...............
Batch for preparing the Lateral Shuttle Ascent, First Stage Max-Q, T=61 sec
! This batch set creates two Space Shuttle ascent models at Max Dynamic Pressure.
! A rigid model for control design and a model that includes slosh, accelerom and TWD for
! analysis. The design model includes a filter and it is used for H-infinity control design
! to reduce the sideslip sensitivity to gusts. A Dynamic H-infinity controller is created
! from output measurements that includes state estimator.
!
Retain Matrix     : Shuttle Stage-1 TVC Matrix with Gust input
Retain CSM        : Shuttle Ascent, Max_Q, Lateral Design Model with TVC and NY Filter/SM-5
!
Flight Vehicle    : Shuttle Ascent, Max_Q, T=61 sec, (Design Model)
Flight Vehicle    : Shuttle Ascent, Max_Q, T=61 sec, Rigid-Body/ Slosh/ TWD/ Acceleromet
System Connection: Shuttle Ascent, Max_Q, Design Model with TVC
System Modificat : Shuttle Ascent, Max_Q, Lateral Hinf Design Model
System Modificat : Shuttle Ascent, Max_Q, Lateral Analysis Model with Slosh & TVC
Mixing Matrix     : Shuttle Stage-1 TVC Matrix at Max-Q
Transf-Function   : Integrator
Transf-Function   : NY Filter
System Connection: Shuttle Ascent, Max_Q, Lateral Design Model with TVC and NY Filter
H-Infinity Design: Space Shuttle Lateral H-Infinity Output-Feedback Control Design-5
System Connection: Closed-Loop Via Output Feedback Controller
!
To Matlab Format : Shuttle Ascent, Max_Q, Lateral Design Model with TVC and NY Filter
To Matlab Format : Shuttle Ascent, Max_Q, Lateral Analysis Model with Slosh & TVC
To Matlab Format : H-Infin Control for Shuttle Ascent Lateral Output-Feedback/ SM-5
To Matlab Format : Closed-Loop Via Output Feedback Controller
To Matlab Format : NY Filter
To Matlab Format : Shuttle Stage-1 TVC Matrix with Gust input
---------------------------------------------------------------------------------------------------
INTERCONNECTION OF SYSTEMS .....
Shuttle Ascent, Max_Q, Design Model with TVC
! Combines the Rigid vehicle model with the TVCG matrix that includes also the
! gust input.
Titles of Systems to be Combined
Title 1 Shuttle Ascent, Max_Q, T=61 sec, (Design Model)
SYSTEM INPUTS TO SUBSYSTEM  1                                                        TVC to Vehicle
Via Matrix +TVCG                                                                     inputs: roll, pitch,
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                                     from Plant
Via Matrix +I14                                                                      All Outputs
..............................................................................
Definitions of Inputs  =   4
DP_TVC (roll FCS demand)
DQ_TVC (pitch FCS demand)
DR_TVC (yaw FCS demand)
Wind Gust Azim, Elev Angles=(45, 90) (deg)

Definitions of Outputs =  14
Roll Attitude  (phi-body) (radians)
Roll Rate      (p-body)   (rad/sec)
Pitch Attitude (thet-bdy) (radians)
Pitch Rate     (q-body)   (rad/sec)
Yaw  Attitude  (psi-body) (radians)
Yaw  Rate      (r-body)   (rad/sec)
Angle of attack, alfa, (radians)
Angle of sideslip, beta, (radian)
Change in Altitude, delta-h, (feet)
Forward Acceleration  (V-dot)   (ft/sec)
Cross Range Velocity (Vcr) (ft/sec)
CG Acceleration along X axis, (ft/sec^2)
CG Acceleration along Y axis, (ft/sec^2)
CG Acceleration along Z axis, (ft/sec^2)
----------------------------------------------------------------------------------------------------------
```



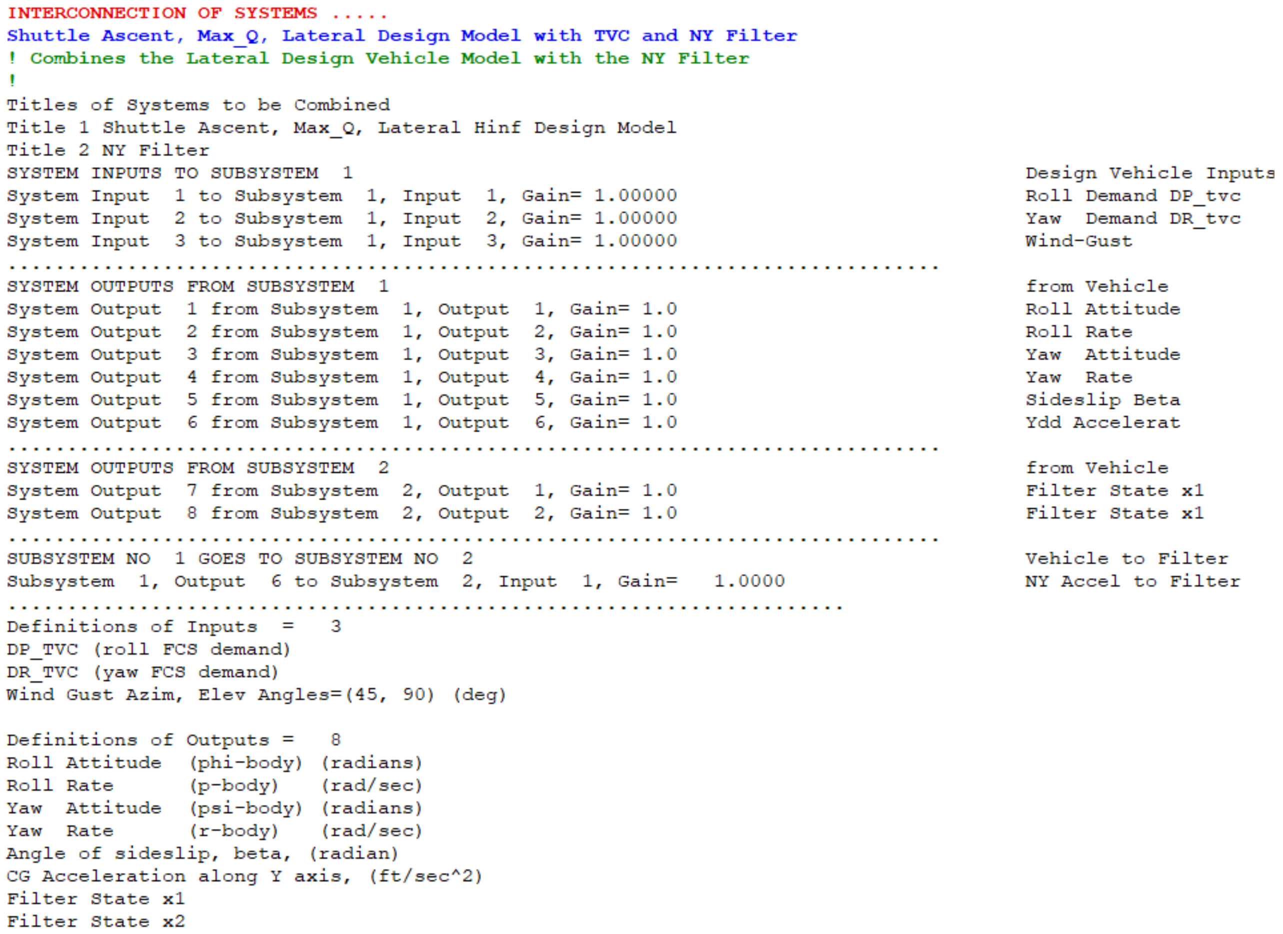
```
SYSTEM OF TRANSFER FUNCTIONS ...
NY Filter
Continuous
TF. Block #   1   ( 1/s )                                       Order of Numer, Denom=   0   1
Numer 0.0          1.0
Denom 1.0          0.0
TF. Block #   2   ( 1/s )                                       Order of Numer, Denom=   0   1
Numer 0.0          1.0
Denom 1.0          0.0
.......................................................
Block #, from Input #, Gain
1         1      1.0
...........................
Block #, from Block #, Gain
2         1      1.0
1         1     -0.28
1         2     -1.0
...........................
Outpt #, from Block #, Gain
1         1      1.0
2         2      1.0
...........................
Definitions of Inputs  =   1
NY in

Definitions of Outputs =   2
Filter State x1
Filter State x2
-----------------------------------------------------------------------------------------------
MIXING LOGIC MATRIX DATA ..........    (Matrix Title, Name, Vehicle Title, Control Directions)
Shuttle Stage-1 TVC Matrix at Max-Q
! Thrust Vector Control Matrix at Max-Q
! This multi-engine vehicle has 5 Gimbaling Engines.
TVC2
Shuttle Ascent, Max_Q, T=61 sec, (Design Model)
P-dot Roll   Acceleration About X Axis
Q-dot Pitch Acceleration About Y Axis
R-dot Yaw    Acceleration About Z Axis
-----------------------------------------------------------------------------------------------
```



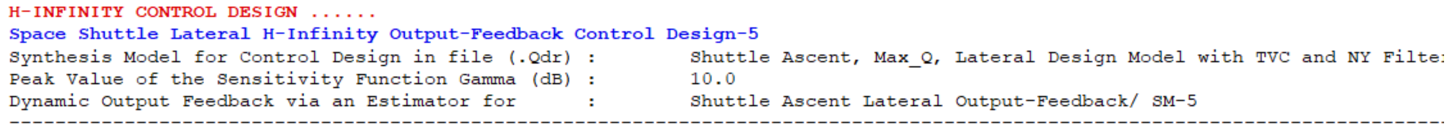
```
CREATE A NEW SYSTEM FROM AN OLD SYSTEM... (Titles of the New and Old Systems)
Shuttle Ascent, Max_Q, Lateral Hinf Design Model
Shuttle Ascent, Max_Q, Design Model with TVC
TRUNCATE OR REORDER THE SYSTEM INPUTS, STATES, AND OUTPUTS
Extract Inputs :    1   3   4
Extract States :    1   2   5   6   8
Extract Outputs:    1   2   5   6   8   13
-----------------------------------------------------------------------------------
CREATE A NEW SYSTEM FROM AN OLD SYSTEM... (Titles of the New and Old Systems)
Shuttle Ascent, Max_Q, Lateral Analysis Model with Slosh & TVC
Shuttle Ascent, Max_Q, T=61 sec, Rigid-Body/ Slosh/ TWD/ Acceleromet
TRUNCATE OR REORDER THE SYSTEM INPUTS, STATES, AND OUTPUTS
Extract Inputs :    1   2   3   4   5   11   12   13   14   15   21
Extract States :    1   2   5   6   8   12   14   16   18
Extract Outputs:    1   2   5   6   8   12
-----------------------------------------------------------------------------------
```

The above two sets extract and retain the lateral subsystem variables from the fully coupled pitch/ lateral systems. The first one is for the design model and the second one is for the analysis model that includes slosh and TWD.

The systems interconnection dataset below combines the lateral vehicle model with the disturbance filter. Note that the 2nd order filter is excited directly from the Ny-accelerometer this time and not by β. The outputs include Ny-acceleration. This new system will be used to create the Synthesis Model and it consists of both: vehicle and filter states.



```
INTERCONNECTION OF SYSTEMS .....
Shuttle Ascent, Max_Q, Lateral Design Model with TVC and NY Filter
! Combines the Lateral Design Vehicle Model with the NY Filter
!
Titles of Systems to be Combined
Title 1 Shuttle Ascent, Max_Q, Lateral Hinf Design Model
Title 2 NY Filter
SYSTEM INPUTS TO SUBSYSTEM  1                                              Design Vehicle Inputs
System Input  1 to Subsystem  1, Input  1, Gain= 1.00000                   Roll Demand DP_tvc
System Input  2 to Subsystem  1, Input  2, Gain= 1.00000                   Yaw  Demand DR_tvc
System Input  3 to Subsystem  1, Input  3, Gain= 1.00000                   Wind-Gust
.............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                           from Vehicle
System Output  1 from Subsystem  1, Output  1, Gain= 1.0                   Roll Attitude
System Output  2 from Subsystem  1, Output  2, Gain= 1.0                   Roll Rate
System Output  3 from Subsystem  1, Output  3, Gain= 1.0                   Yaw  Attitude
System Output  4 from Subsystem  1, Output  4, Gain= 1.0                   Yaw  Rate
System Output  5 from Subsystem  1, Output  5, Gain= 1.0                   Sideslip Beta
System Output  6 from Subsystem  1, Output  6, Gain= 1.0                   Ydd Accelerat
.............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                                           from Vehicle
System Output  7 from Subsystem  2, Output  1, Gain= 1.0                   Filter State x1
System Output  8 from Subsystem  2, Output  2, Gain= 1.0                   Filter State x1
.............................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2                                    Vehicle to Filter
Subsystem  1, Output  6 to Subsystem  2, Input  1, Gain=   1.0000          NY Accel to Filter
.....................................................................
Definitions of Inputs  =   3
DP_TVC (roll FCS demand)
DR_TVC (yaw FCS demand)
Wind Gust Azim, Elev Angles=(45, 90) (deg)

Definitions of Outputs =   8
Roll Attitude  (phi-body) (radians)
Roll Rate      (p-body)   (rad/sec)
Yaw  Attitude  (psi-body) (radians)
Yaw  Rate      (r-body)   (rad/sec)
Angle of sideslip, beta, (radian)
CG Acceleration along Y axis, (ft/sec^2)
Filter State x1
Filter State x2
```



```
H-INFINITY CONTROL DESIGN ......
Space Shuttle Lateral H-Infinity Output-Feedback Control Design-5
Synthesis Model for Control Design in file (.Qdr)  :       Shuttle Ascent, Max_Q, Lateral Design Model with TVC and NY Filte
Peak Value of the Sensitivity Function Gamma (dB)  :       10.0
Dynamic Output Feedback via an Estimator for       :       Shuttle Ascent Lateral Output-Feedback/ SM-5
------------------------------------------------------------------------------------------------------------------------
```

The above H-infinity dataset is processed by Flixan in batch mode. It uses the SM: "*Shuttle Ascent, Max_Q, Lateral Design Model with TVC and NY Filter/SM-5*" that is already saved in the systems file "*Lateral_MaxQ4.Qdr*" to create the dynamic output feedback H-infinity controller: "*H-Infin Control for Shuttle Ascent Lateral Output-Feedback/ SM-5*" which is also saved in the same systems file. The H-infinity upper bound is preset to γ=10 (dB).

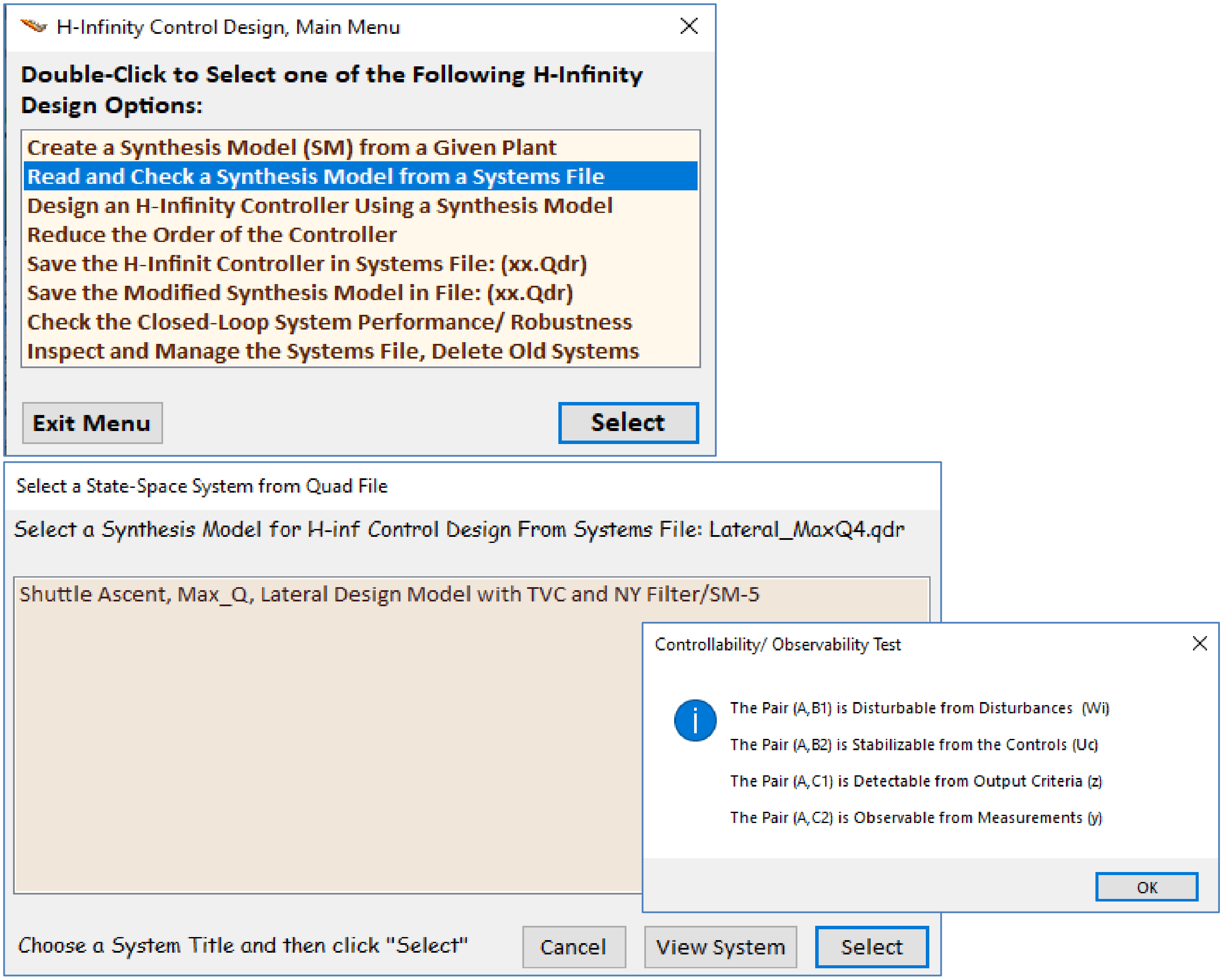
The following interconnection set combines the design plant "Shuttle Ascent, Max_Q, Lateral Design Model with TVC and NY Filter" with the H-infinity controller in closed-loop form. The input is wind-gust velocity in the pre-defined direction relative to the vehicle axes. It is used to calculate sensitivity to gusts.

The following Matlab conversion sets create systems and matrices of the control system, the Ny-disturbance filter, the vehicle design and analysis models, the TVC matrix, and the closed-loop system, that will be loaded into Matlab.

```
INTERCONNECTION OF SYSTEMS .....
Closed-Loop Via Output Feedback Controller
! Closes the Control Loop via the Output-Feedback Controller/Estimator
Titles of Systems to be Combined
Title 1 Shuttle Ascent, Max_Q, Lateral Design Model with TVC and NY Filter
Title 2 H-Infin Control for Shuttle Ascent Lateral Output-Feedback/ SM-5
SYSTEM INPUTS TO SUBSYSTEM  1                                             Vehicle Input
System Input  1 to Subsystem  1, Input  3, Gain= 1.0                      Gust Input
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                          From Vehicle Model
Via Matrix +I8                                                            All Outputs
..............................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2                                   Vehicle to Control Input
Subsystem  1, Output  1 to Subsystem  2, Input  1, Gain=   1.0000         Roll Attitude
Subsystem  1, Output  2 to Subsystem  2, Input  2, Gain=   1.0000         Roll Rate
Subsystem  1, Output  3 to Subsystem  2, Input  3, Gain=   1.0000         Yaw  Attitude
Subsystem  1, Output  4 to Subsystem  2, Input  4, Gain=   1.0000         Yaw  Rate
Subsystem  1, Output  6 to Subsystem  2, Input  5, Gain=   1.0000         Y-CG Acceleration
Subsystem  1, Output  7 to Subsystem  2, Input  6, Gain=   1.0000         Filter State x1
Subsystem  1, Output  8 to Subsystem  2, Input  7, Gain=   1.0000         Filter State x2
..............................................................................
SUBSYSTEM NO  2 GOES TO SUBSYSTEM NO  1                                   Controller to Vehicle
Subsystem  2, Output  1 to Subsystem  1, Input  1, Gain=   1.0000         DP_tvc
Subsystem  2, Output  2 to Subsystem  1, Input  2, Gain=   1.0000         DR_tvc
..............................................................................
Definitions of Inputs  =   1
Wing Gust Input    (feet/sec)

Definitions of Outputs =   8
Roll Attitude  (phi-body) (radians)
Roll Rate      (p-body)   (rad/sec)
Yaw  Attitude  (psi-body) (radians)
Yaw  Rate      (r-body)   (rad/sec)
Angle of sideslip, beta   (radian)
Y-CG Acceleration (Ydd)   (ft/sec^2)
Filter State x1
Filter State x2
-----------------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
H-Infin Control for Shuttle Ascent Lateral Output-Feedback/ SM-5
System
Hinf_cntrl
-----------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
NY Filter
System
NY_filt
-----------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Shuttle Ascent, Max_Q, Lateral Design Model with TVC and NY Filter
System
later_des
-----------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Shuttle Ascent, Max_Q, Lateral Analysis Model with Slosh & TVC
System
later_anal
-----------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Closed-Loop Via Output Feedback Controller
! Saves the Closed-Loop System via State-Feedback Controller
System
closed
-----------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Shuttle Stage-1 TVC Matrix with Gust input
Matrix TVCG
-----------------------------------------------------------------------------
```

4.3 Designing the H-Infinity Controller Interactively

We will now use the SM to design the H-infinity controller. The input file has a batch set that can run the H-infinity program in batch mode but this time we will run it interactively. We go back to the H-infinity design program and from the main menu select the second option to read the SM which is already in file. From the next menu select the title of the only SM which is in the systems file, and click “Select”.



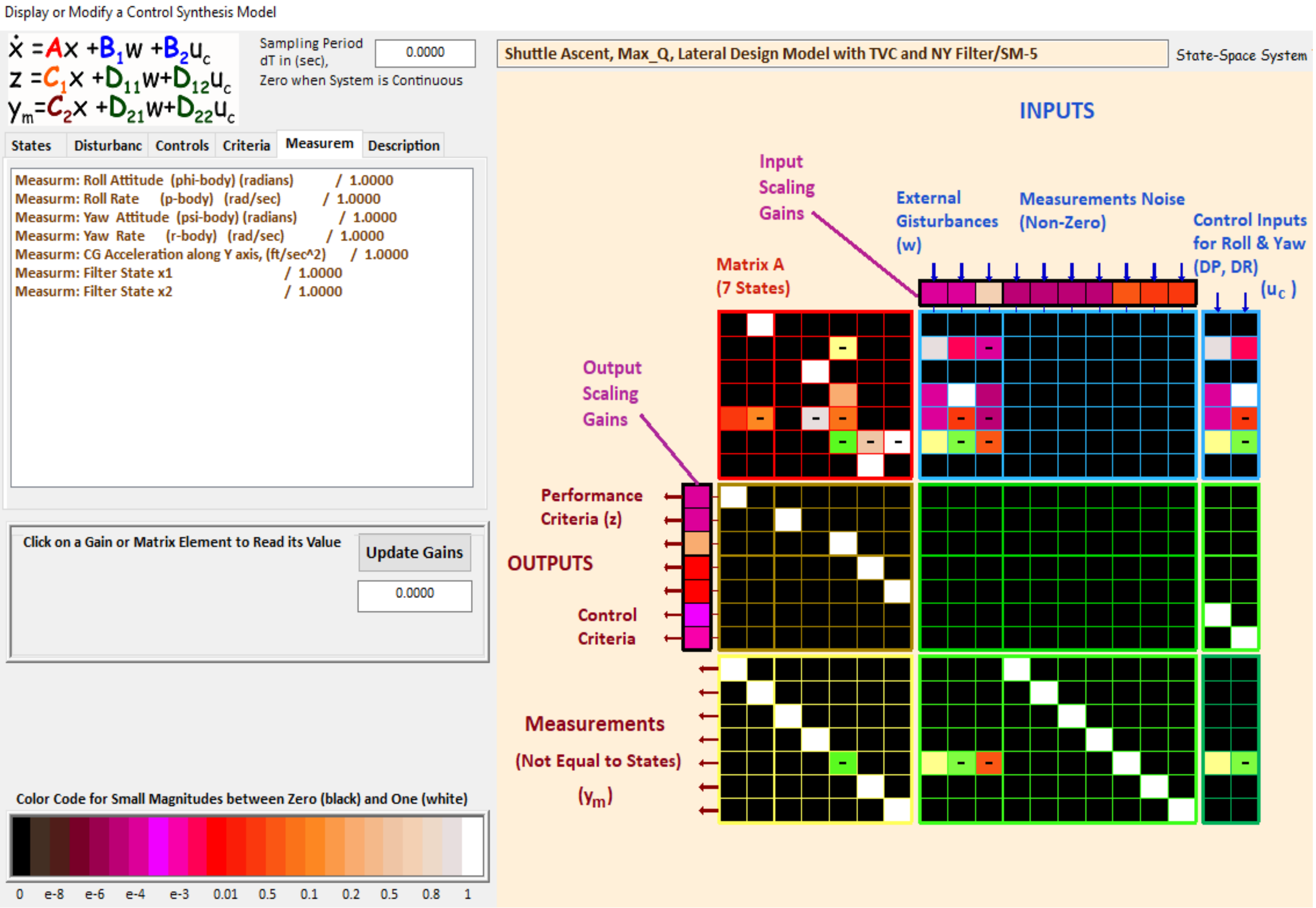
The Pair (A,B1) is Disturbable from Disturbances (Wi)

The Pair (A,B2) is Stabilizable from the Controls (Uc)

The Pair (A,C1) is Detectable from Output Criteria (z)

The Pair (A,C2) is Observable from Measurements (y)

The program confirms that the SM meets the expected observability and controllability requirements and displays the SM matrices graphically in system’s form in the dialog below. The 9 SM matrices appear color coded including the scaling gains that scale the disturbances and the criteria. The A-matrix consists of 7 states. The A-matrix consists of 7 states, 5 from the original vehicle and 2 from the Ny-filter. There are 3 external disturbances (w), 2 control inputs (u_c) for roll and yaw control, 7 measurements (y_m), 5 performance criteria (z), and 2 control evaluation criteria. Some of the measurements are equal to the states but not all. There are also 7 measurement noise inputs which are not zero this time because measurement noise plays a role here in the estimator design.


Display or Modify a Control Synthesis Model

$$\dot{x} = Ax + B_1 w + B_2 u_c$$
$$z = C_1 x + D_{11} w + D_{12} u_c$$
$$y_m = C_2 x + D_{21} w + D_{22} u_c$$

Sampling Period dT in (sec), Zero when System is Continuous: 0.0000

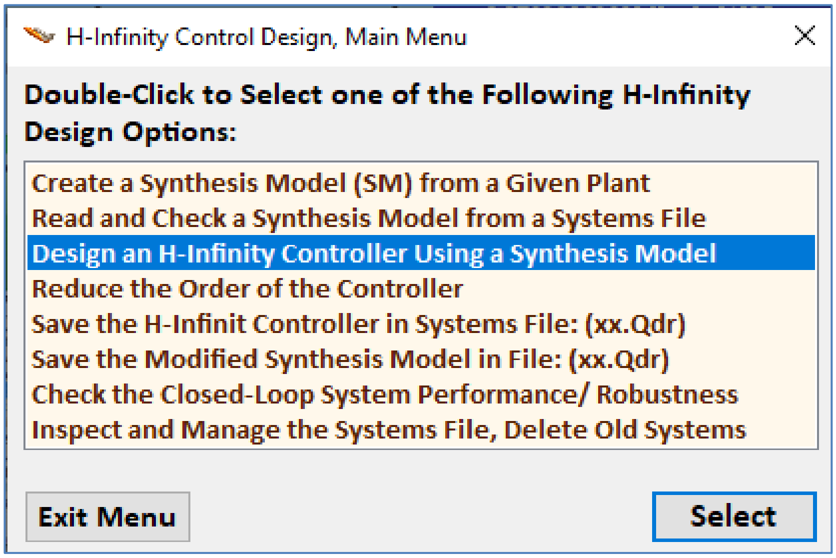
Shuttle Ascent, Max_Q, Lateral Design Model with TVC and NY Filter/SM-5

```
Measurm: Roll Attitude  (phi-body) (radians)      / 1.0000
Measurm: Roll Rate     (p-body)  (rad/sec)       / 1.0000
Measurm: Yaw  Attitude  (psi-body) (radians)      / 1.0000
Measurm: Yaw  Rate     (r-body)  (rad/sec)       / 1.0000
Measurm: CG Acceleration along Y axis, (ft/sec^2)    / 1.0000
Measurm: Filter State x1                         / 1.0000
Measurm: Filter State x2                         / 1.0000
```

Click on a Gain or Matrix Element to Read its Value: 0.0000

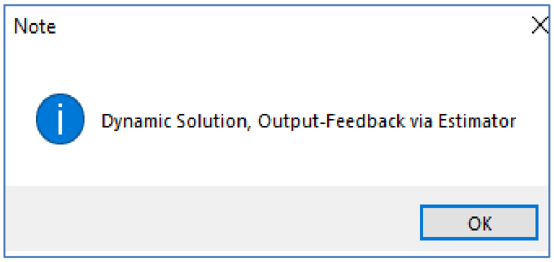
Color Code for Small Magnitudes between Zero (black) and One (white)

Select the third option from the main menu to design the H-infinity controller, and click "Select". The program notifies you that the solution will be a dynamic output feedback controller.


H-Infinity Control Design, Main Menu

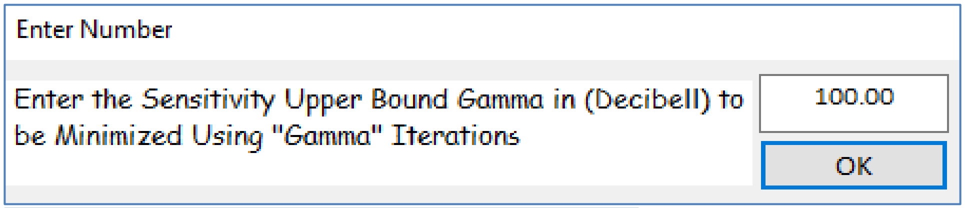
Double-Click to Select one of the Following H-Infinity Design Options:

- Create a Synthesis Model (SM) from a Given Plant
- Read and Check a Synthesis Model from a Systems File
- Design an H-Infinity Controller Using a Synthesis Model
- Reduce the Order of the Controller
- Save the H-Infinit Controller in Systems File: (xx.Qdr)
- Save the Modified Synthesis Model in File: (xx.Qdr)
- Check the Closed-Loop System Performance/ Robustness
- Inspect and Manage the Systems File, Delete Old Systems


Note

Dynamic Solution, Output-Feedback via Estimator

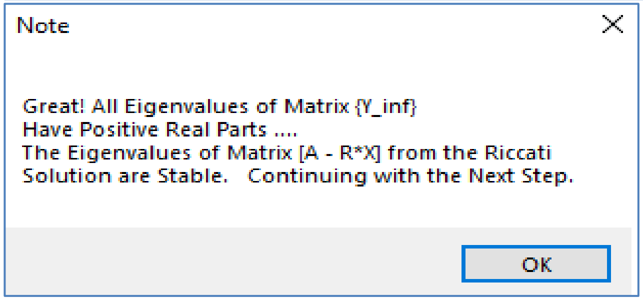
Now we begin the iterative process of minimizing the upper bound γ of the infinity norm of the sensitivity transfer function between the 3 disturbance inputs and the 7 output criteria (5-performance & 2-control). We begin with an arbitrary γ upper bound and try to find the smallest γ that will not violate the algorithm requirements. We must enter γ in decibels.



Enter Number

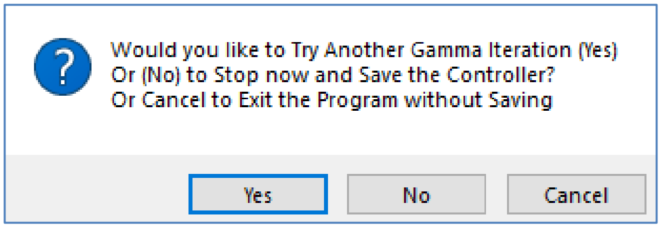
Enter the Sensitivity Upper Bound Gamma in (Decibell) to be Minimized Using "Gamma" Iterations

100.00

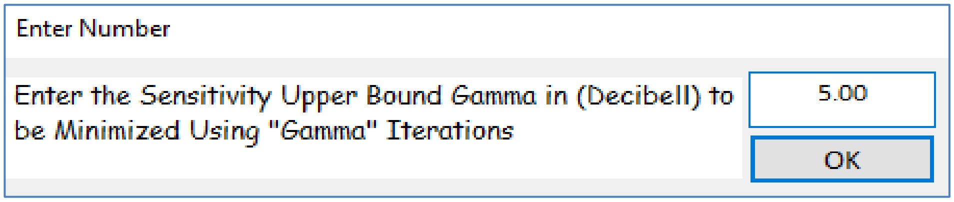


Note

Great! All Eigenvalues of Matrix {Y_inf}
Have Positive Real Parts ....
The Eigenvalues of Matrix [A - R*X] from the Riccati
Solution are Stable. Continuing with the Next Step.



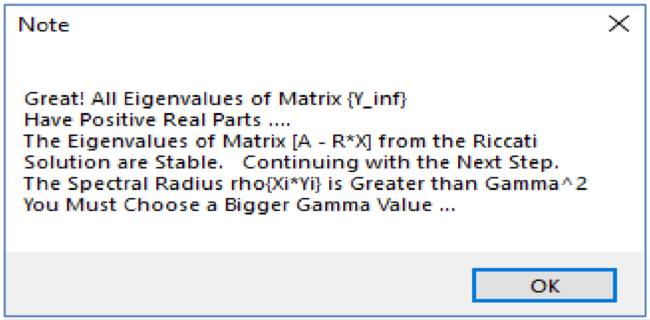
Would you like to Try Another Gamma Iteration (Yes)
Or (No) to Stop now and Save the Controller?
Or Cancel to Exit the Program without Saving



Enter Number

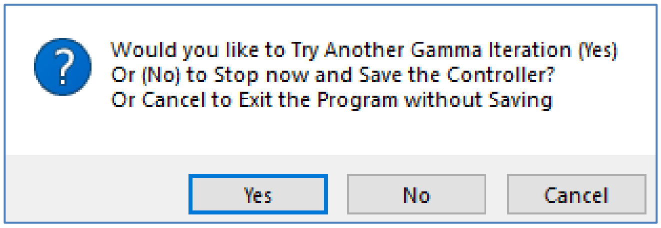
Enter the Sensitivity Upper Bound Gamma in (Decibell) to be Minimized Using "Gamma" Iterations

5.00

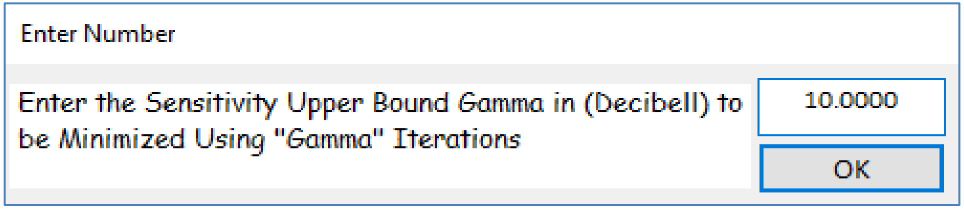


Note

Great! All Eigenvalues of Matrix {Y_inf}
Have Positive Real Parts ....
The Eigenvalues of Matrix [A - R*X] from the Riccati
Solution are Stable. Continuing with the Next Step.
The Spectral Radius rho{Xi*Yi} is Greater than Gamma^2
You Must Choose a Bigger Gamma Value ...



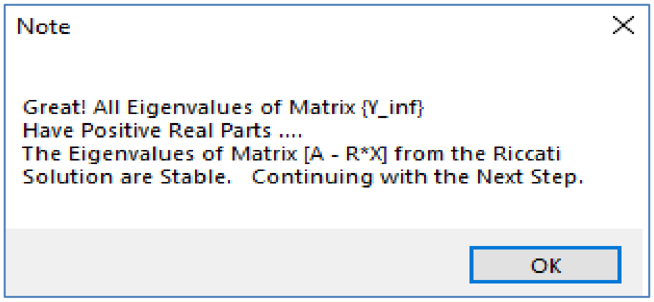
Would you like to Try Another Gamma Iteration (Yes)
Or (No) to Stop now and Save the Controller?
Or Cancel to Exit the Program without Saving



Enter Number

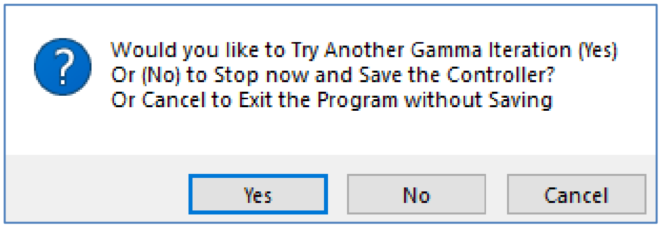
Enter the Sensitivity Upper Bound Gamma in (Decibell) to be Minimized Using "Gamma" Iterations

10.0000



Note

Great! All Eigenvalues of Matrix {Y_inf}
Have Positive Real Parts ....
The Eigenvalues of Matrix [A - R*X] from the Riccati
Solution are Stable. Continuing with the Next Step.



Would you like to Try Another Gamma Iteration (Yes)
Or (No) to Stop now and Save the Controller?
Or Cancel to Exit the Program without Saving

We arbitrarily enter γ=100 (dB) first, which is acceptable by the algorithm but high. Click on “Yes” in the dialog question to try a smaller number. Try γ=5 (dB) next time but that is too low. There is a note that tells you to try a bigger gamma. Click on “Yes” in the dialog question to enter a bigger number. Try γ=10 (dB) next time and the note says that it is acceptable. In the question about trying another (γ), click on “No” this time meaning that you don’t want to try another value but to accept the current controller with γ=10. Figure-34 shows the eigenvalues of the system with the control loop closed as in Figure-1. They are all stable, as expected. We exit this figure, return to the H-infinity main menu, and we can save the controller as a system by clicking on “*Save the H-infinity Controller in Systems File (x.Qdr)*”.

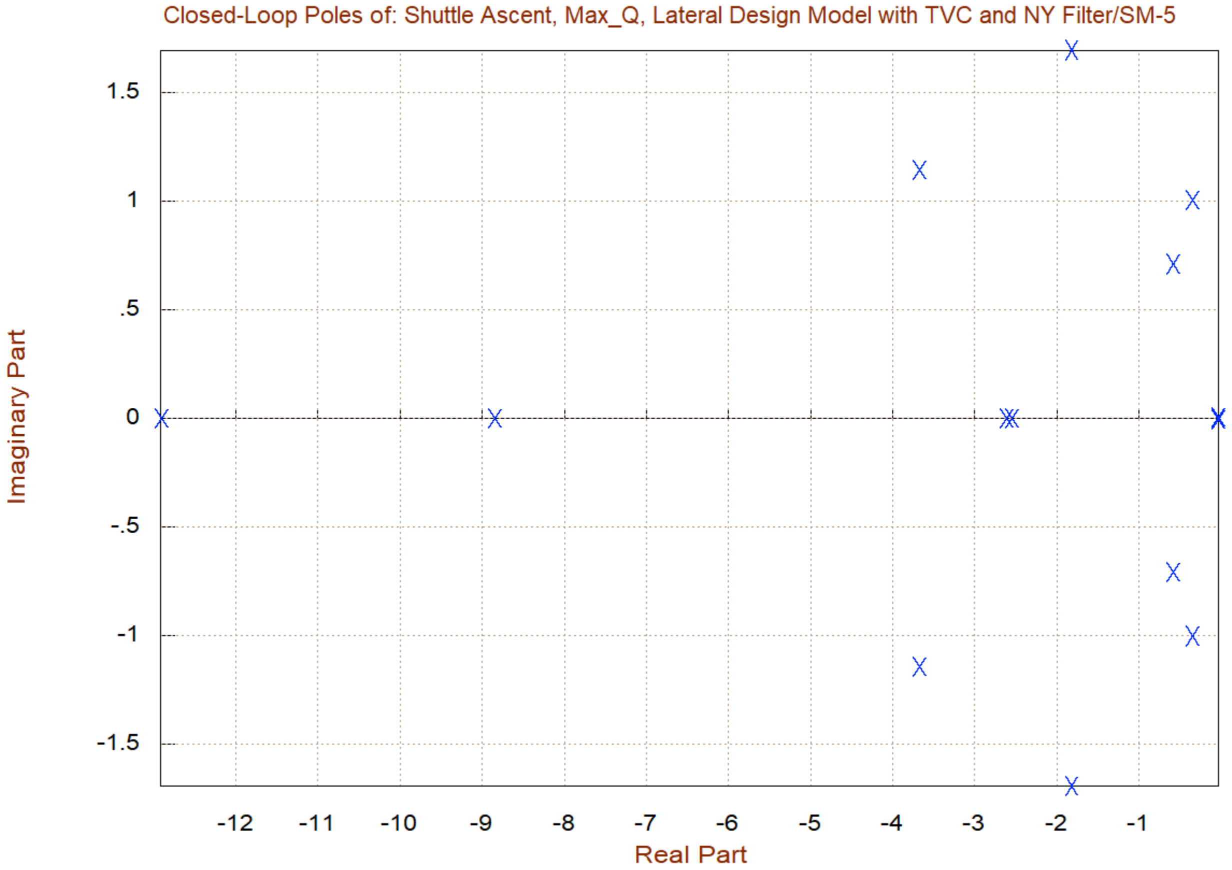
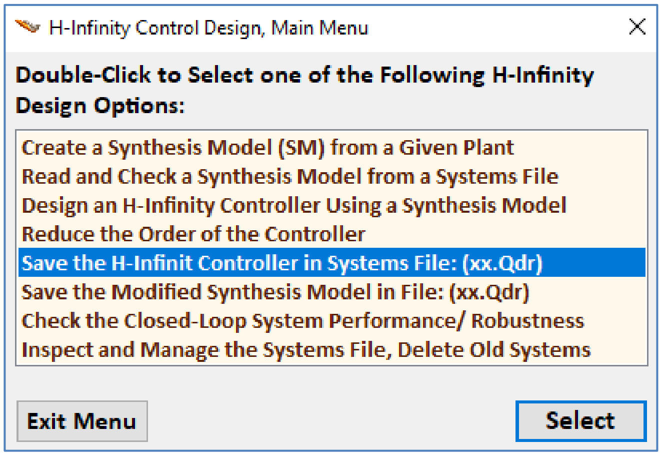
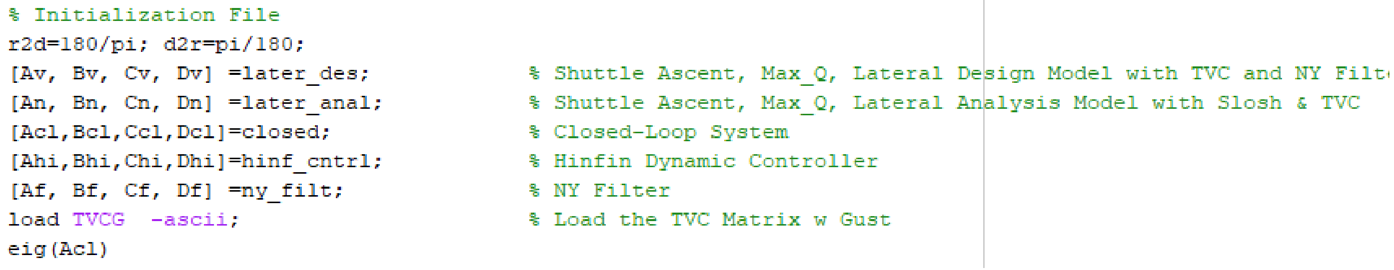


Figure 34 Closed-Loop System Poles



4.4 Control Analysis

The Matlab analysis begins by running the initialization file “init.m” which loads the systems into Matlab. That is, the design and analysis systems, the closed-loop system, the H-infinity controller, the disturbance filter, and the TVC matrix. All systems are in state-space form, including the Ny filter. The design model includes the Ny-filter and the filter input is directly excited by the Ny-accelerometer, not β. The analysis model includes slosh, accelerometer, and TWD dynamics but not the Ny-filter. It was included in the design plant for the controller calculation but now the Ny-filter must go in the control system. We also check the eigenvalues of the closed-loop system to make sure that it’s stable. We will demonstrate two versions of the control design. We will first analyze the system using the dynamic controller from all outputs. Then we will try another version of the same controller, by replacing some of the estimated states in the feedback loop with the actual measured states instead of the estimated.



```
% Initialization File
r2d=180/pi; d2r=pi/180;
[Av, Bv, Cv, Dv] =later_des;          % Shuttle Ascent, Max_Q, Lateral Design Model with TVC and NY Filt
[An, Bn, Cn, Dn] =later_anal;         % Shuttle Ascent, Max_Q, Lateral Analysis Model with Slosh & TVC
[Acl,Bcl,Ccl,Dcl]=closed;             % Closed-Loop System
[Ahi,Bhi,Chi,Dhi]=hinf_cntrl;         % Hinfin Dynamic Controller
[Af, Bf, Cf, Df] =ny_filt;            % NY Filter
load TVCG  -ascii;                    % Load the TVC Matrix w Gust
eig(Acl)
```

4.4.1 Analysis Using the Full Estimation Controller

Two Simulink models “*Open_Loop1.slx*” and “*Open_Loop2.slx*” have been created to analyze roll and yaw open-loop stability. They are shown in Figures (35 & 36). The first one uses the lateral design system from file “later_des.m” which includes the Ny-filter. The second model uses the analysis system from file “later_anal.m” which includes slosh and TWD but the Ny-filter is in a separate block. The controller system from file “hinf_cntrl.m” is a separate block. The vehicle output measurements: (ϕ, p, ψ, r, Ny-acceleration, and the two filter states x_1 and x_2) are the feeding into the controller inputs. Yaw stability is calculated by opening the yaw loop (DR), closing the roll loop (DP), as shown in figures (35 & 36). The Bode and Nichols plots are calculated across the opened loop and shown in Figures (40 & 41).

The Simulink models “Sensitiv1.slx” and “Sensitiv2.slx” in Figures (37 & 38) are used to analyze the system’s sensitivity to wind-gusts in the frequency domain using Singular Value (Sigma) plots. The first one uses the closed-loop system from file “closed.m” and the second model uses the analysis system from file “later_anal.m” which includes slosh and TWD but the Ny-filter is in a separate block. The gust input is scaled by the largest wind-gust velocity and the β–output is scaled by the maximum allowed β-angle. The output is the actual beta angle, not the estimate. The Sensitivity Function (β/Vgust) is shown in Figure 39. Its magnitude is expected to be less than one at all frequencies.

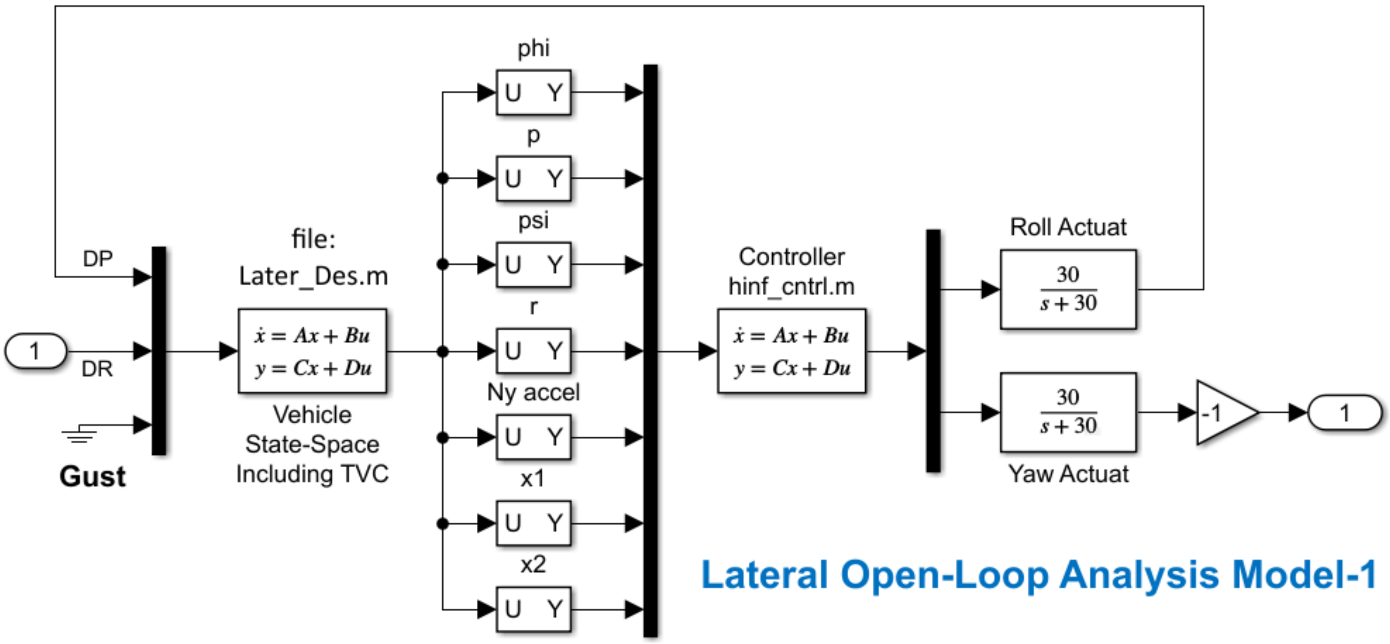


Figure 35 Open-Loop Stability Analysis Model "Open_Loop1.slx"

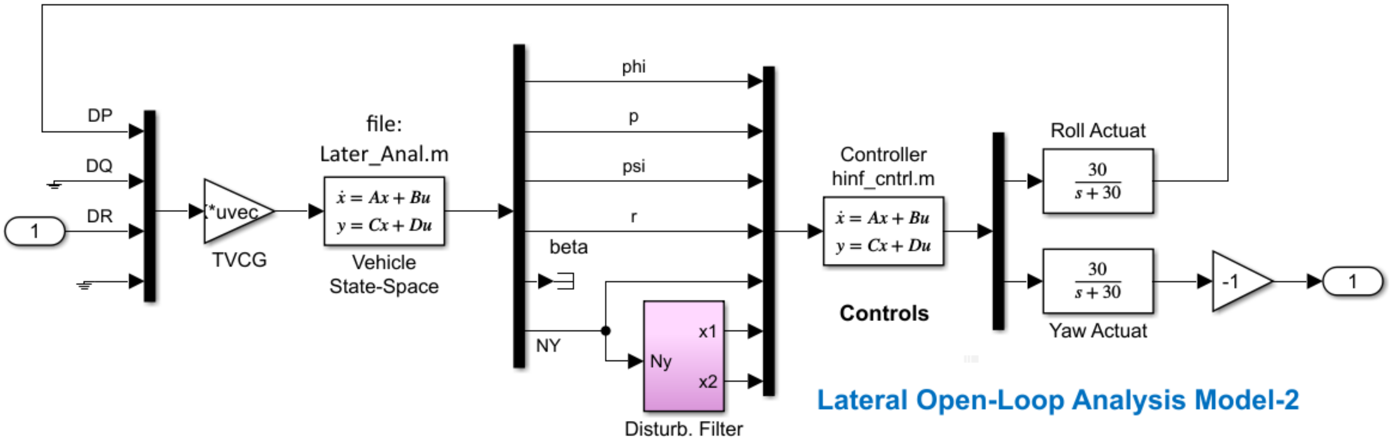


Figure 36 Open-Loop Stability Analysis Model "Open_Loop2.slx"

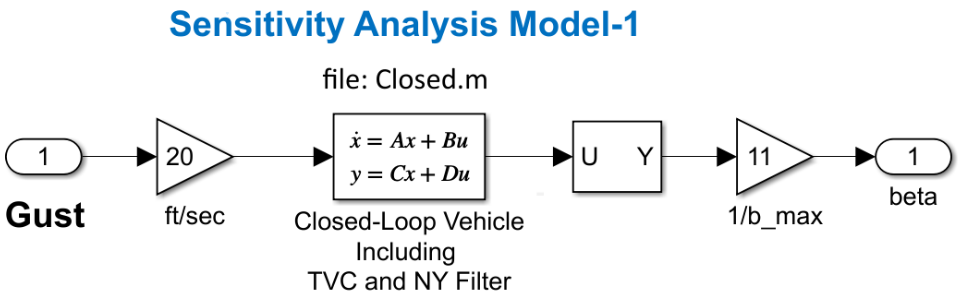


Figure 37 Scaled Sensitivity Analysis Model "Sensitiv1.slx"

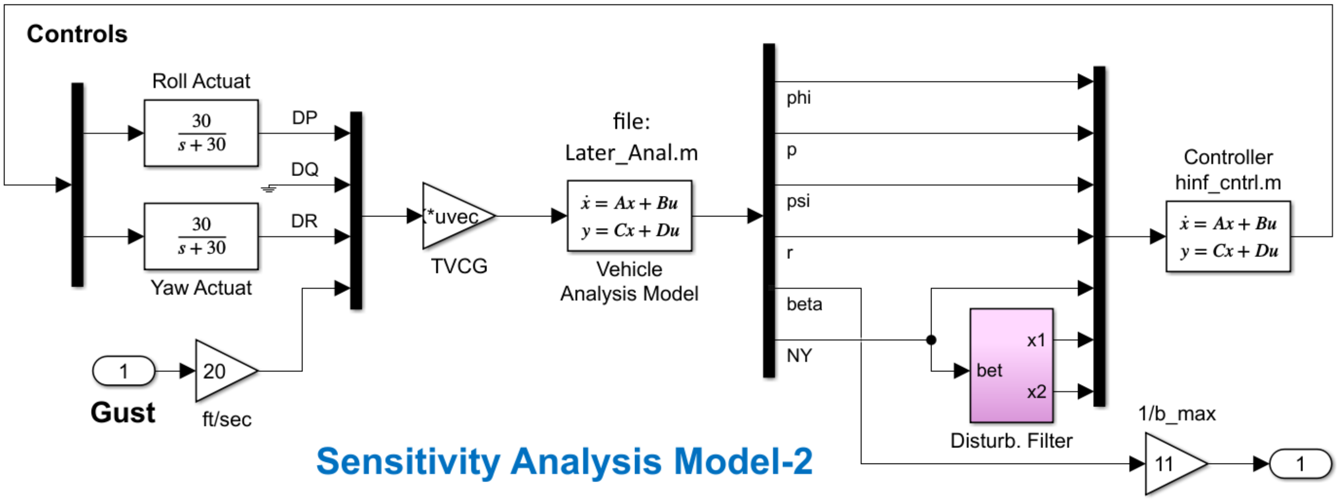
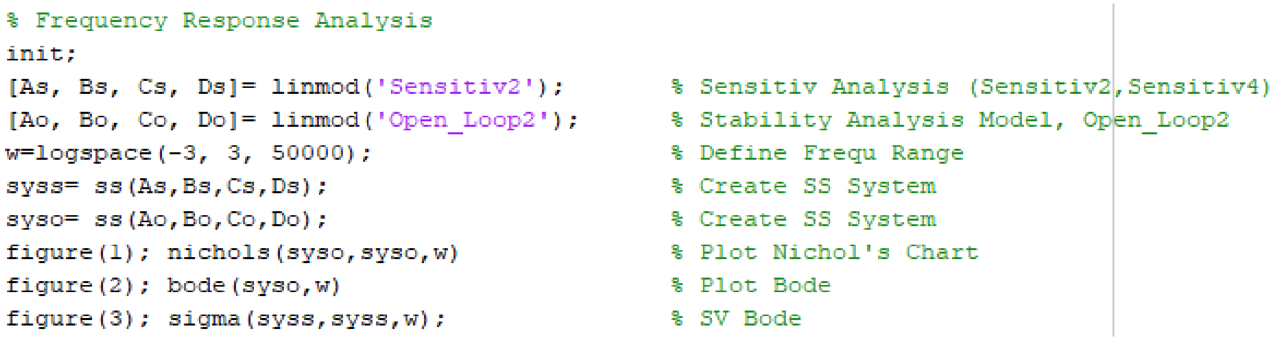


Figure 38 Scaled Sensitivity Analysis Model "Sensitiv2.slx"

The following Matlab script calculates and plots the Bode and Nichols plots of the open-loop system and the Singular Values frequency response of the scaled sensitivity analysis system.



```
% Frequency Response Analysis
init;
[As, Bs, Cs, Ds]= linmod('Sensitiv2');        % Sensitiv Analysis (Sensitiv2,Sensitiv4)
[Ao, Bo, Co, Do]= linmod('Open_Loop2');        % Stability Analysis Model, Open_Loop2
w=logspace(-3, 3, 50000);                     % Define Frequ Range
syss= ss(As,Bs,Cs,Ds);                        % Create SS System
syso= ss(Ao,Bo,Co,Do);                        % Create SS System
figure(1); nichols(syso,syso,w)               % Plot Nichol's Chart
figure(2); bode(syso,w)                       % Plot Bode
figure(3); sigma(syss,syss,w);                % SV Bode
```

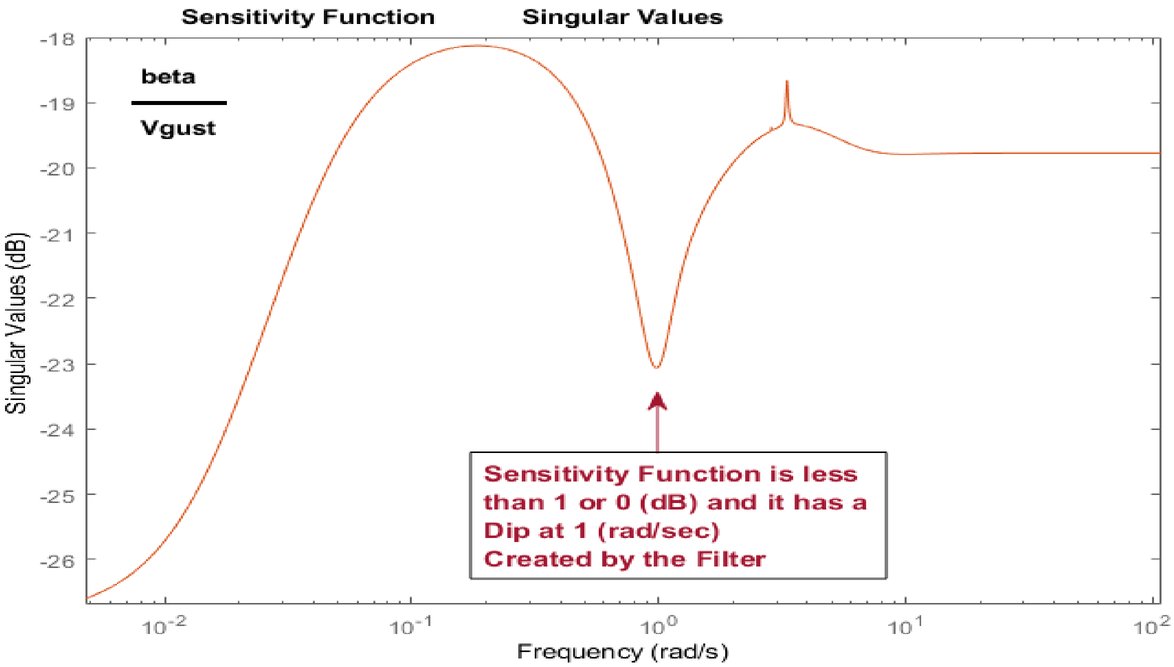


Figure 39 Scaled Sensitivity Function Sigma Plot from Gust to Beta Angle. It is less than 1 at all frequencies and it has a dip at the disturbance frequency

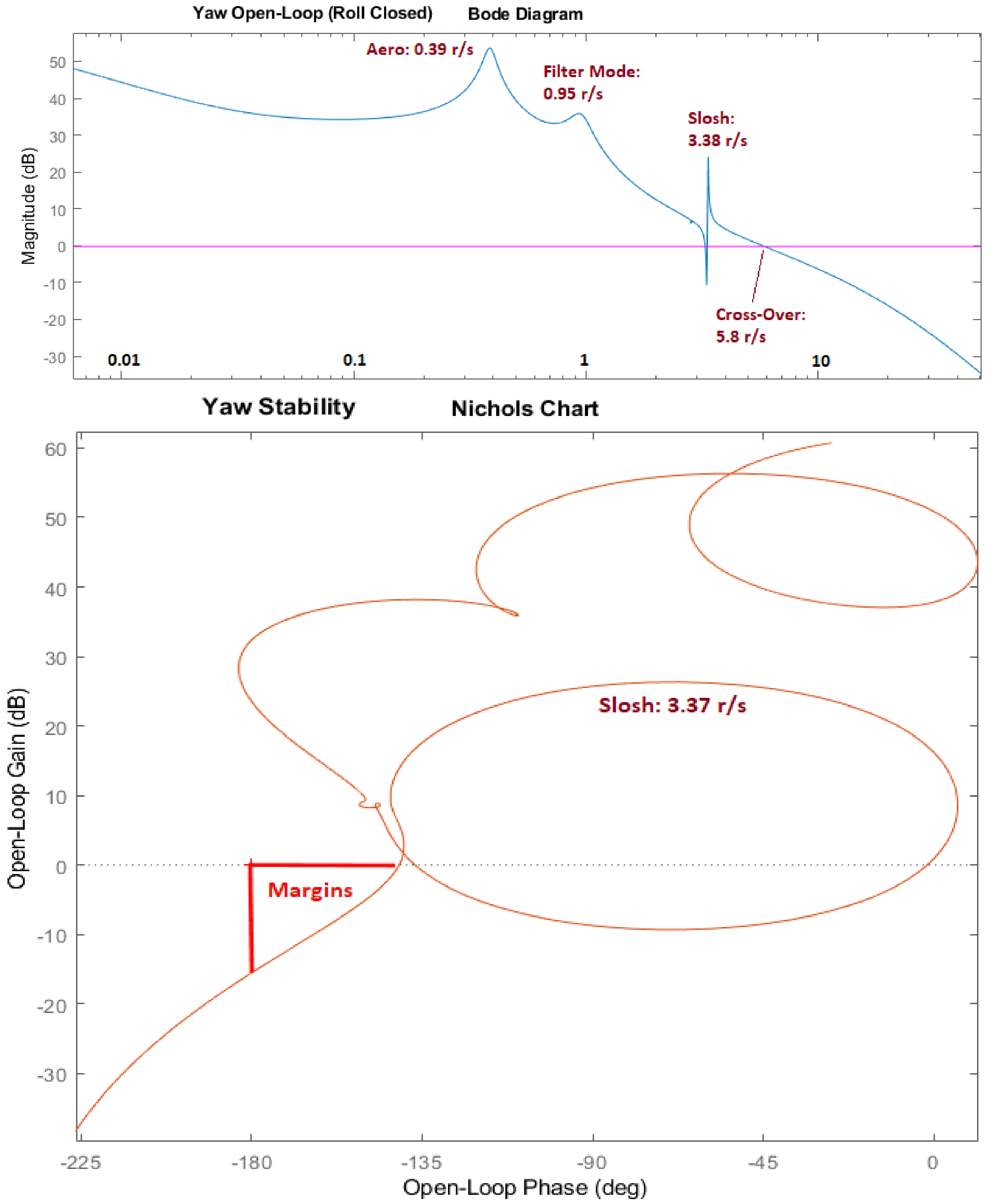


Figure 40 Yaw Stability with Roll Loop Closed. The Filter produces a Mode near the Disturbance Frequency 1 (rad/sec)

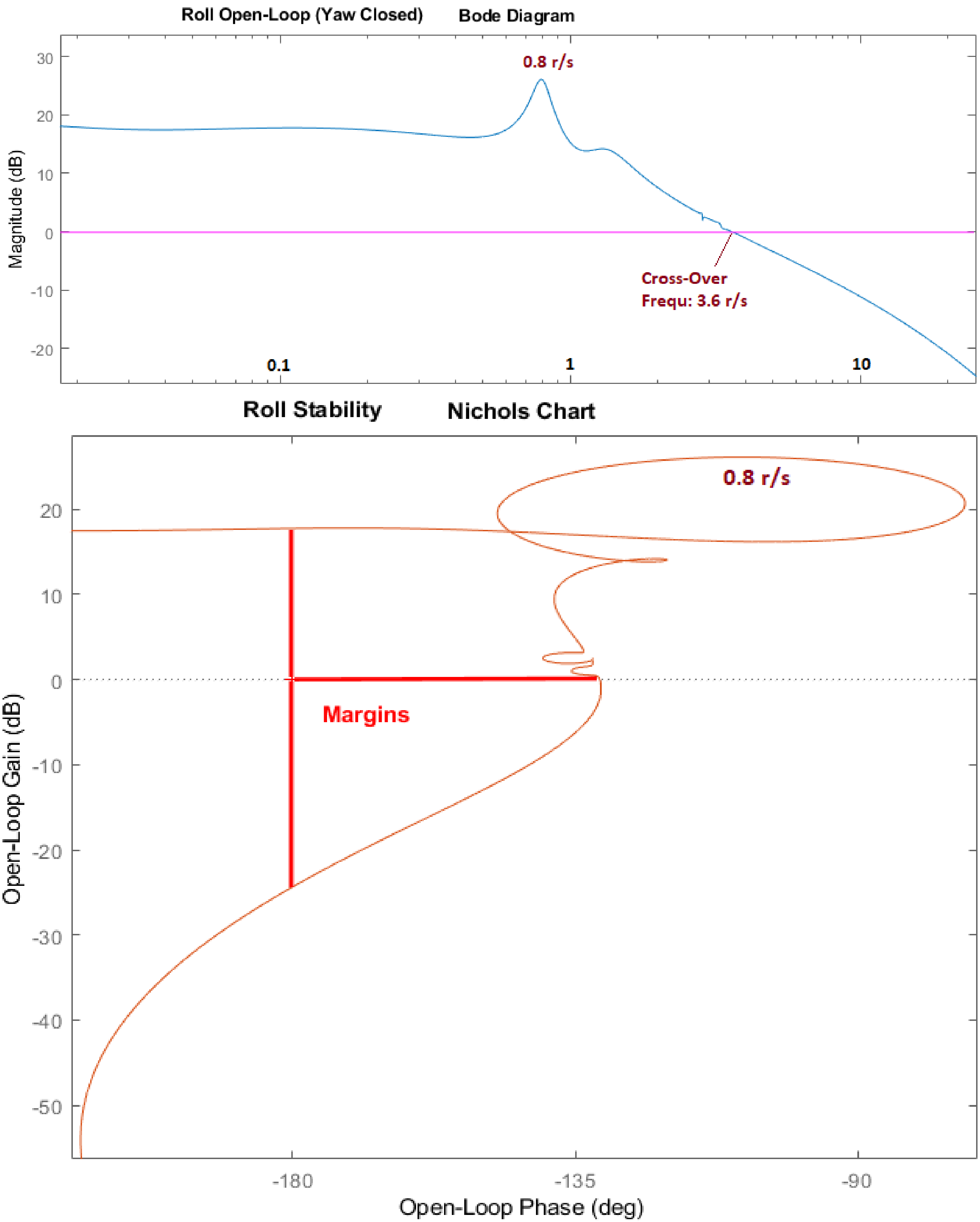


Figure 41 Roll Stability with Yaw Loop Closed

Figure 42 shows the two simulation models "*Closed_Loop1.slx*" and "*Closed_Loop2.slx*" which are used to calculate the system responses to attitude commands and to wind-gust disturbances. They are similar in structure to the previous models described. The vehicle responses to unit step attitude commands are shown in Figure 43. They are obtained from the second model which includes slosh and TWD.

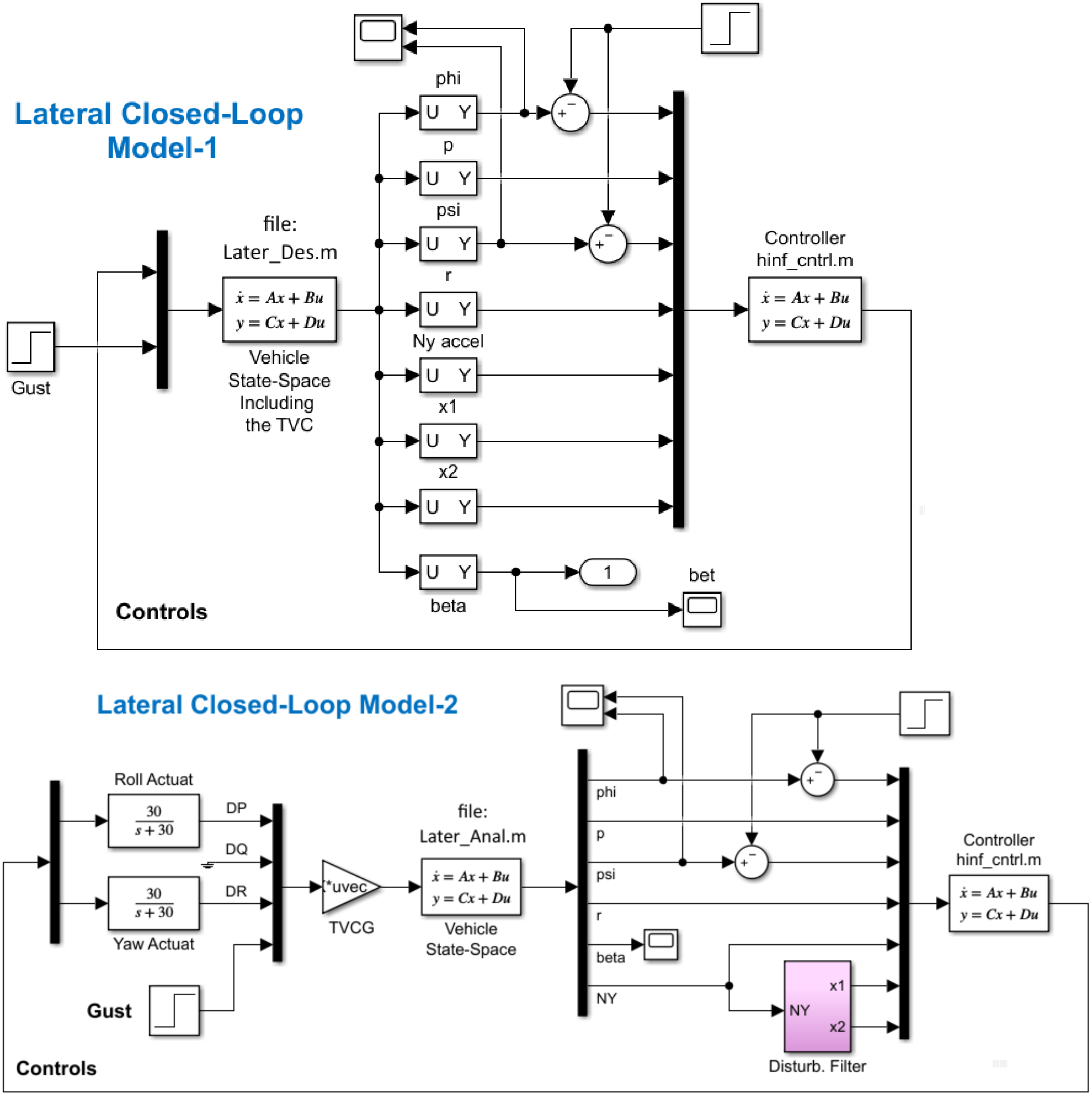


Figure 42 Closed-Loop Simulation Models "Closed_Loop1.slx" and "Closed_Loop2.slx"

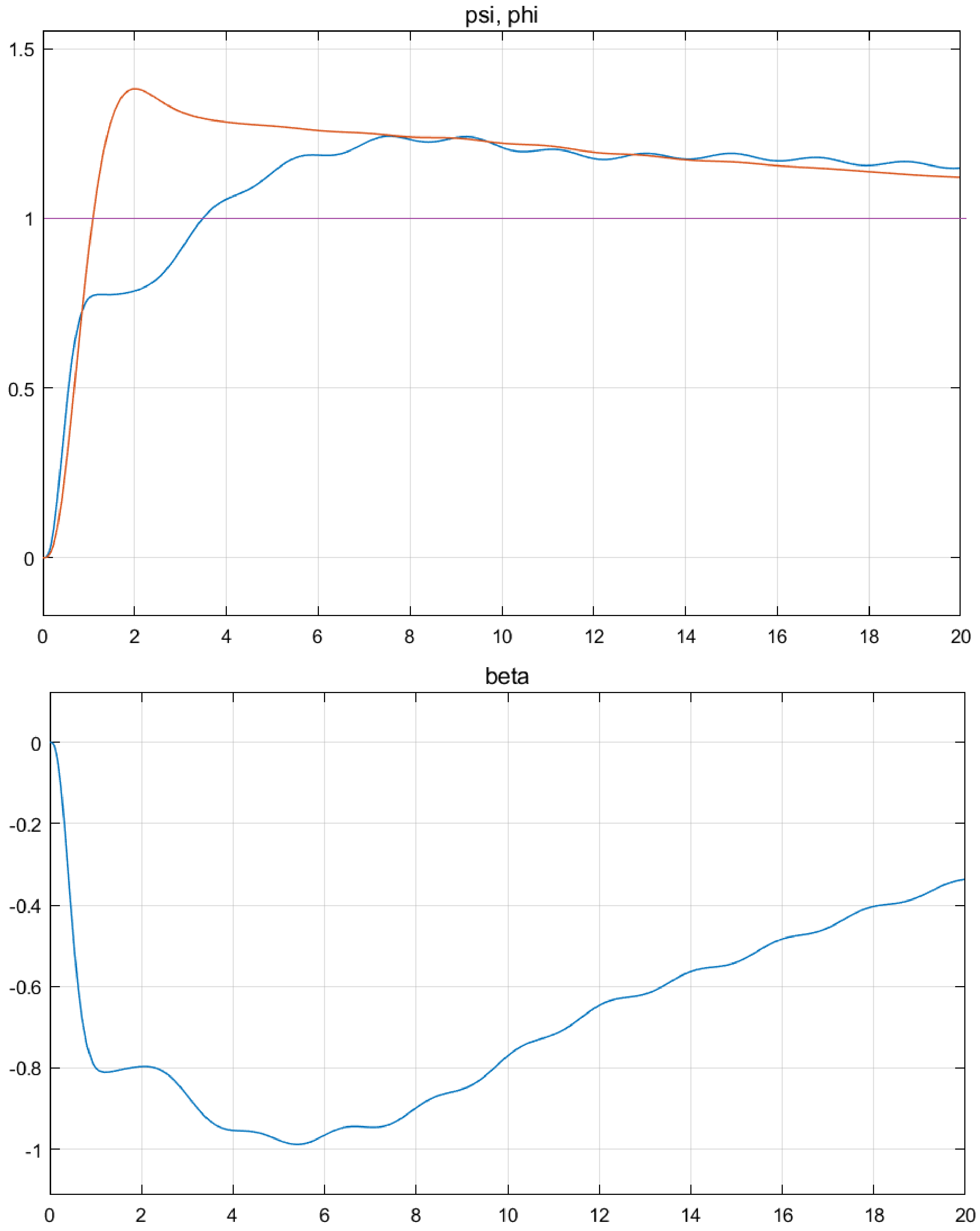


Figure 43 Vehicle Responses to Unit Step Attitude Commands

4.4.2 Analysis Using Controller with Partial Estimation

The Simulink model "*Open_Loop3.slx*" in Figure 44 is similar to "*Open_Loop2.slx*" in Figure 36 and used for analyzing stability but it is slightly modified. It is taking advantage of the fact that 4 of the vehicle system outputs are also states and instead of feeding back estimates of the states we may feed-back the exact states. The controller state-estimator has been separated from the feedback gain C_{hi}. The first four inputs of C_{hi} are the exactly measured states (body rates and attitudes) and the last three inputs are the estimated states of: β, x_1 & x_2. The sensitivity analysis model "Sensitiv3.slx" in Figure 45 and the simulation model "Closed_Loop3.slx" in Figure 46 have a similar structure and are using the partial estimator.

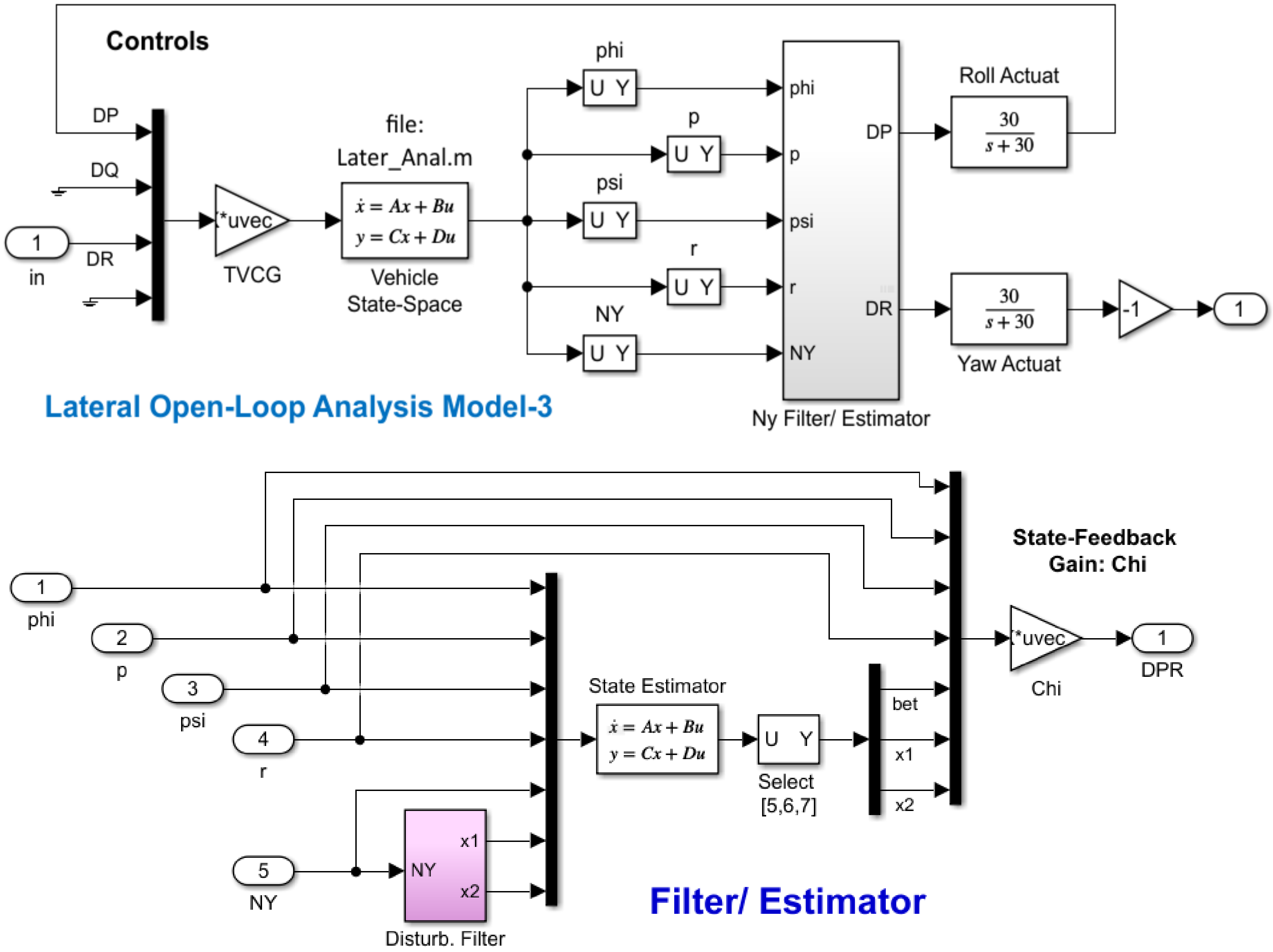


Figure 44 Stability Analysis Model "Open_Loop3.slx" that uses a Partial Estimator

The script file "freq.m" calculates the Bode and Nichols plots of the open-loop system for the yaw and roll axes, shown in Figures (47 & 48). It also analyzes the scaled model "Sensitiv3.slx" sensitivity to wind-gusts in the frequency domain by plotting the Singular Value plots. It is shown in Figure 49 and it is less than 1 at all frequencies with a dip at the expected disturbance frequency. Figure 50 shows the attitude response to unit step commands.

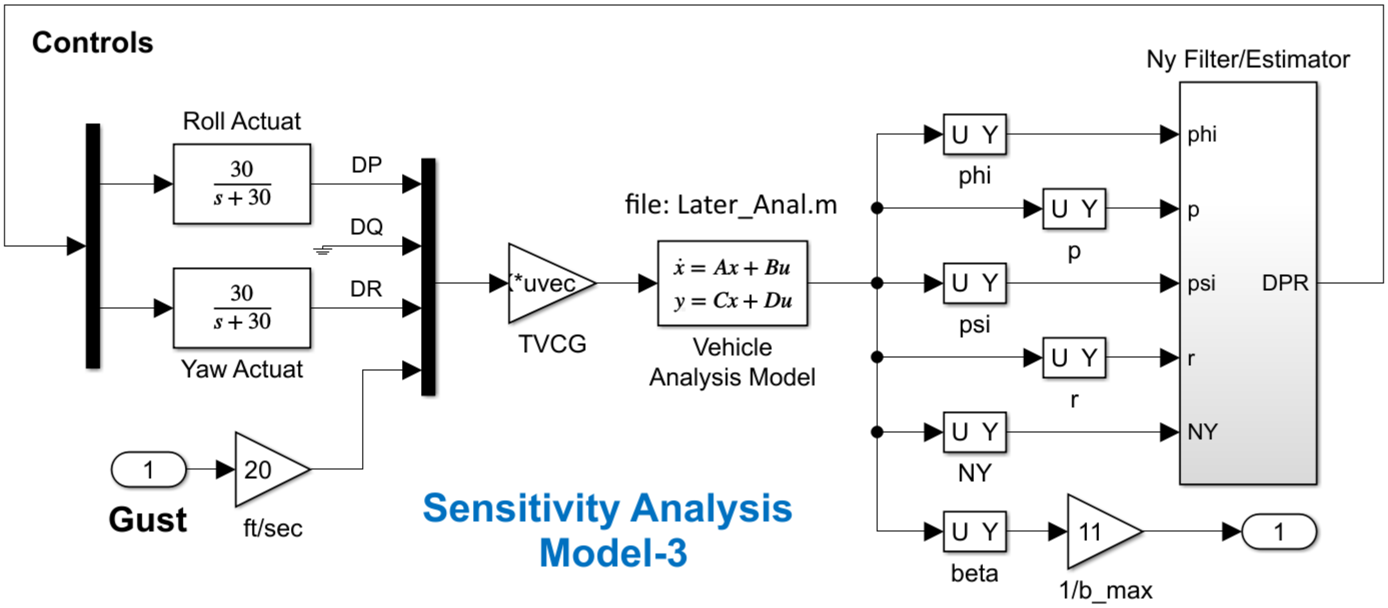


Figure 45 Sensitivity Analysis Model "Sensitiv3.slx" with Partial Estimator

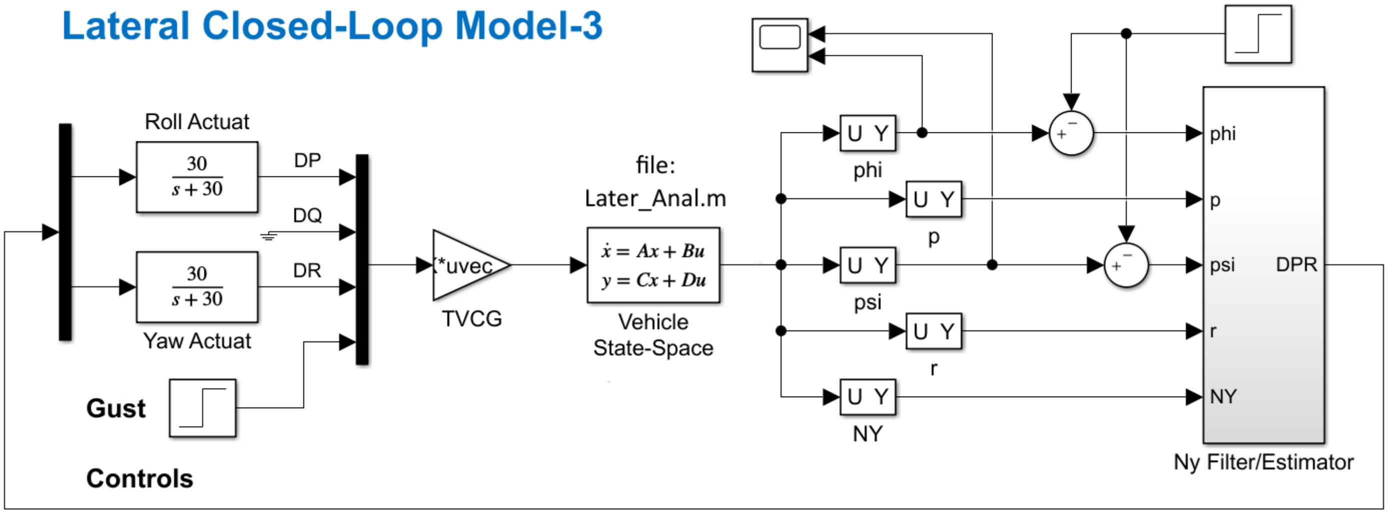
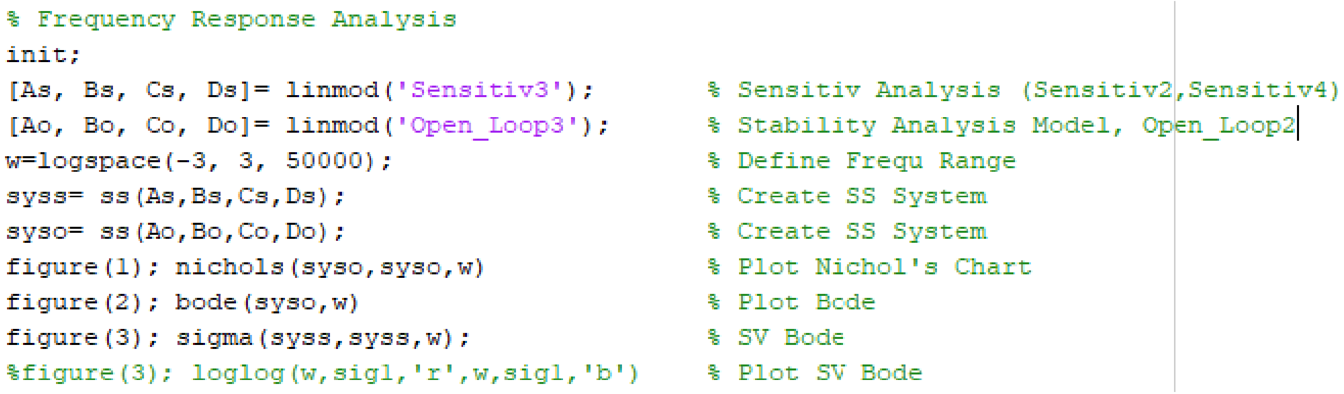


Figure 46 Closed-Loop Simulation Model "Closed_Loop3.slx" with Partial Estimator



```
% Frequency Response Analysis
init;
[As, Bs, Cs, Ds]= linmod('Sensitiv3');        % Sensitiv Analysis (Sensitiv2,Sensitiv4)
[Ao, Bo, Co, Do]= linmod('Open_Loop3');       % Stability Analysis Model, Open_Loop2
w=logspace(-3, 3, 50000);                     % Define Frequ Range
syss= ss(As,Bs,Cs,Ds);                        % Create SS System
syso= ss(Ao,Bo,Co,Do);                        % Create SS System
figure(1); nichols(syso,syso,w)               % Plot Nichol's Chart
figure(2); bode(syso,w)                       % Plot Bode
figure(3); sigma(syss,syss,w);                % SV Bode
%figure(3); loglog(w,sig1,'r',w,sig1,'b')     % Plot SV Bode
```

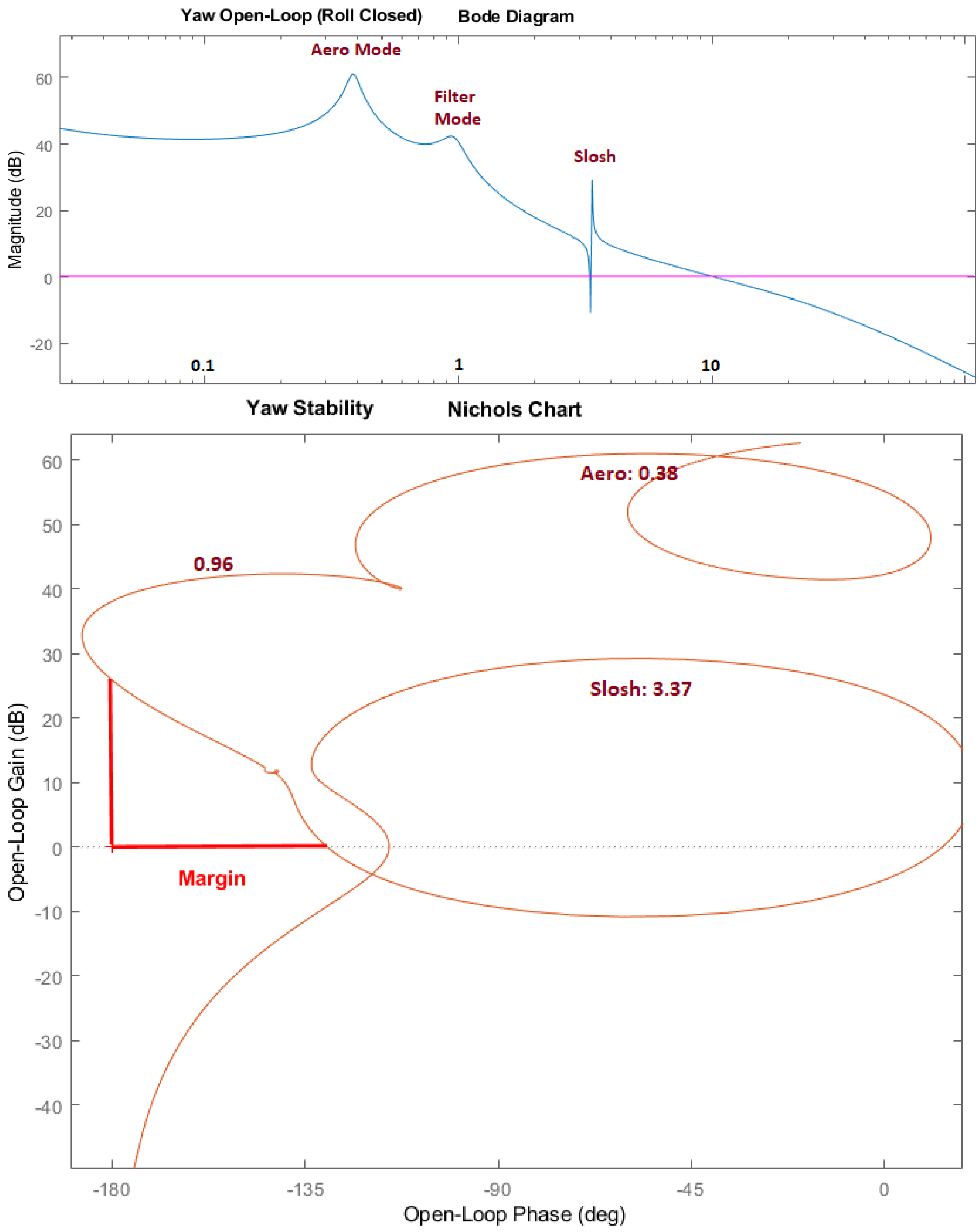


Figure 47 Yaw Axis Stability, Roll Axis Closed

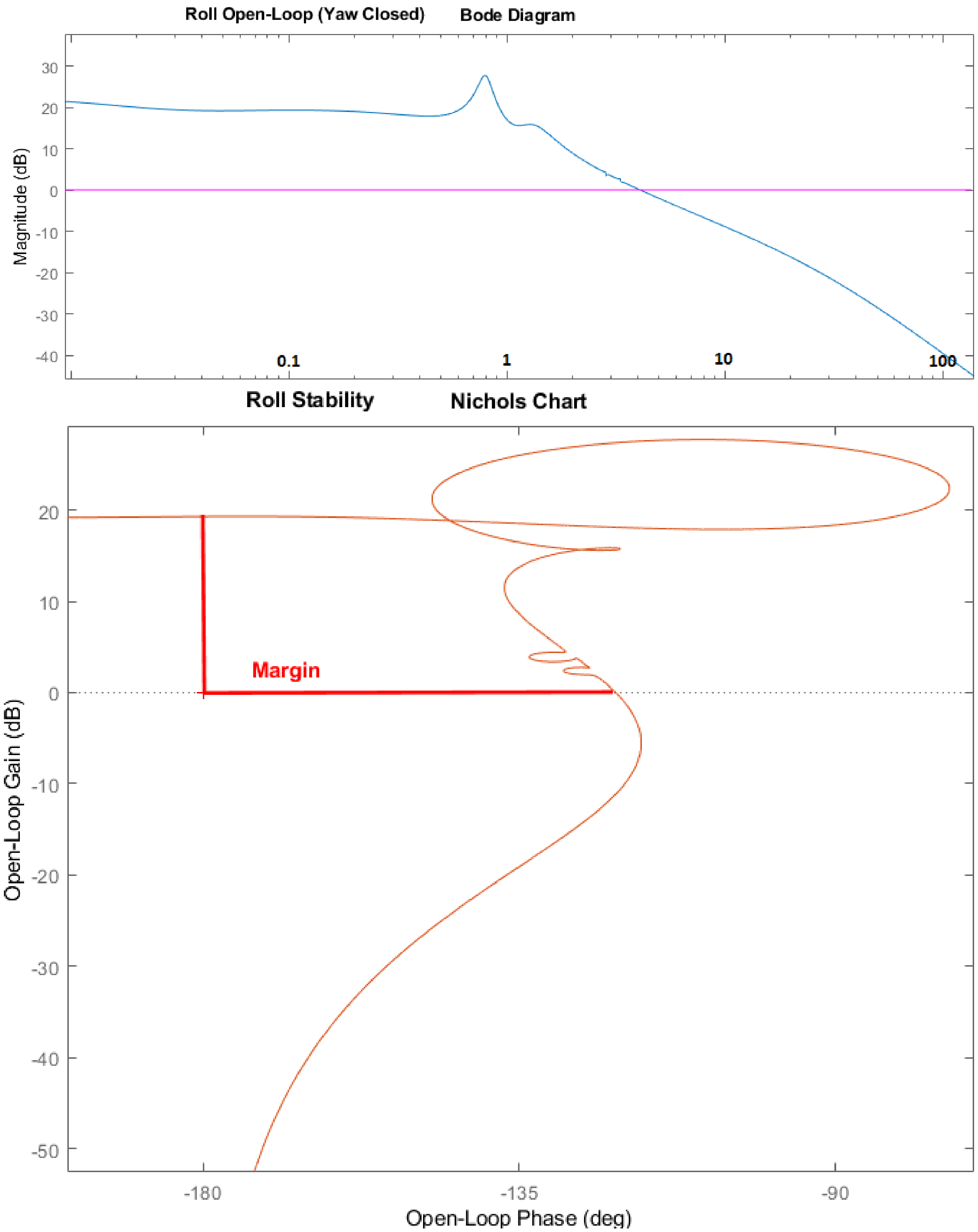


Figure 48 Roll Axis Stability, Yaw Axis Closed

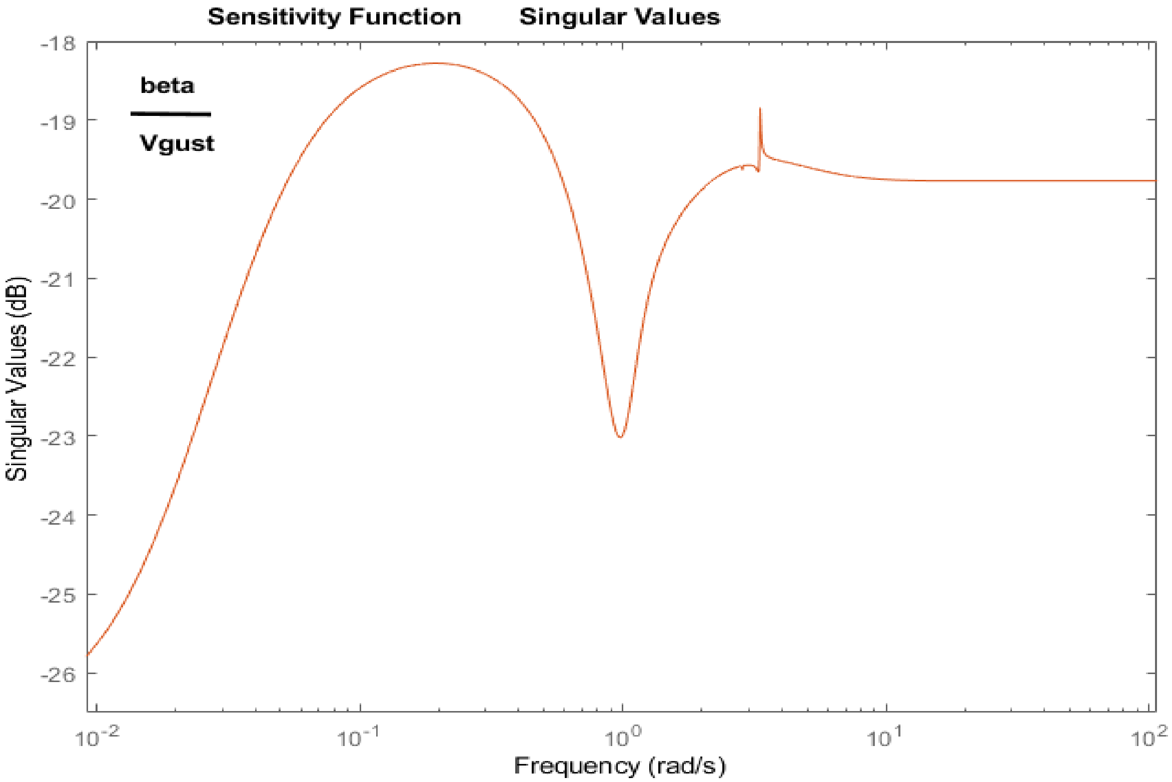


Figure 49 Scaled Sensitivity Function Sigma Plot from Gust to Beta Angle. It is less than 1 at all frequencies and it has a dip at the disturbance frequency

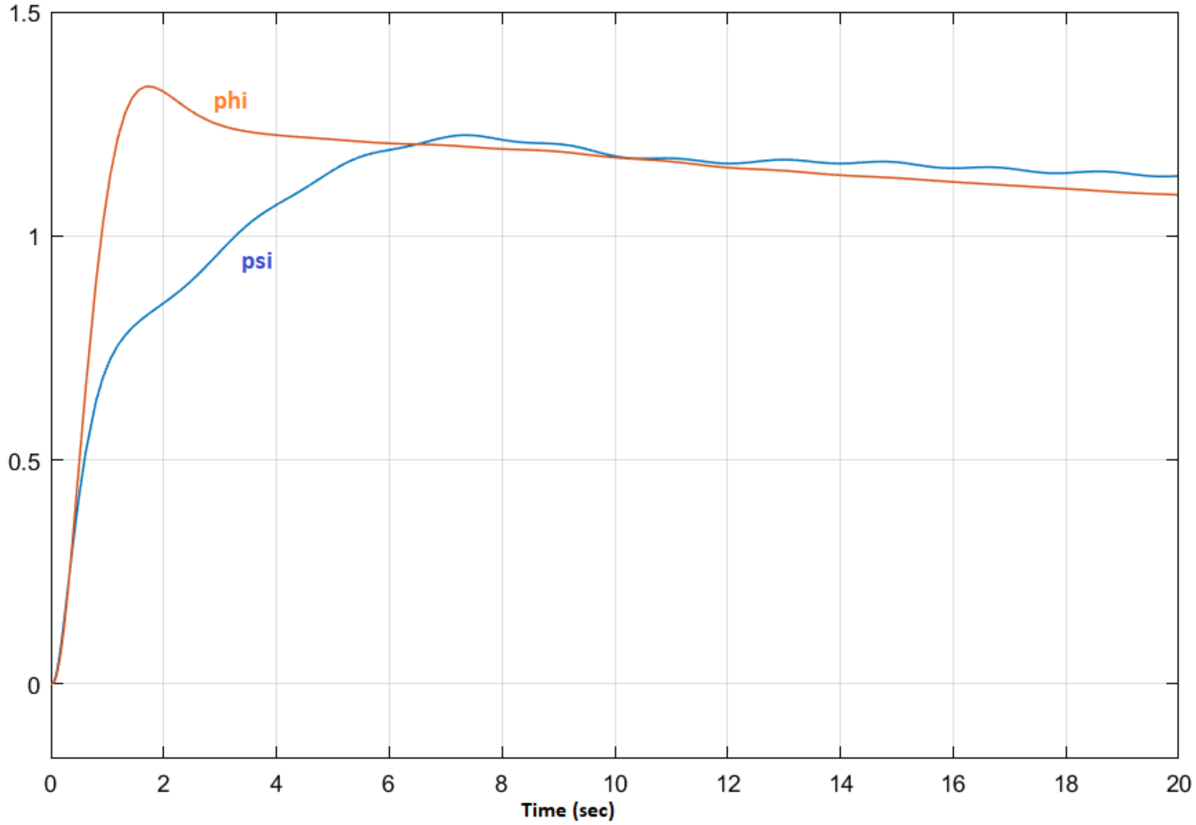


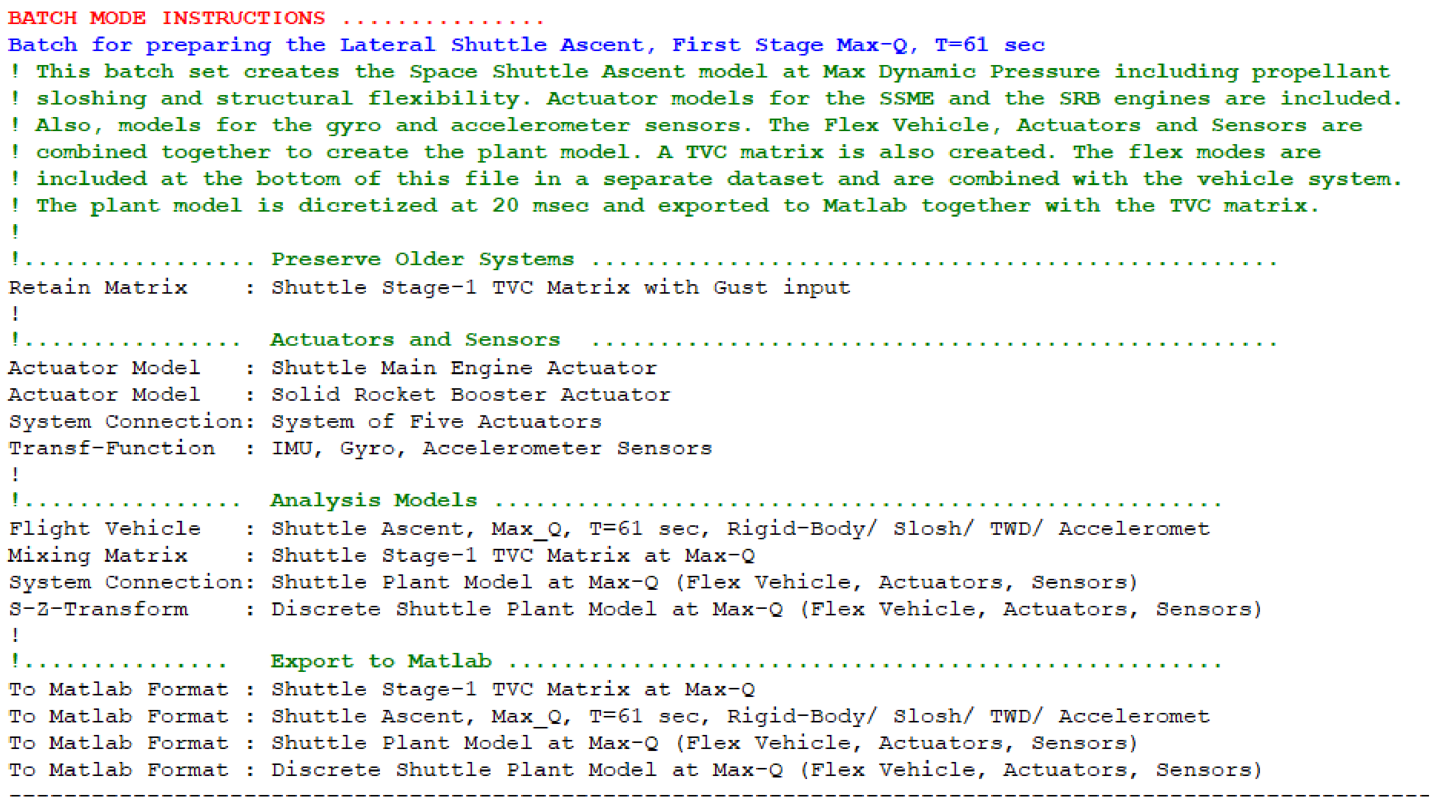
Figure 50 Vehicle Attitude Responses to Unit Step Attitude Commands

5. Coupled Axes Analysis with Slosh and Flexibility

To complete the design, we must now analyze the control system gains derived from Sections 2 and 3 with the coupled pitch and lateral vehicle system including slosh and structural flexibility. We will analyze stability in each axis, sensitivity to wind-gust disturbances and calculate the control system's response to attitude commands, as before. Detailed actuator and sensor models will be included in the plant system. The analysis will be repeated in the z-domain after discretizing and combining the plant and the control system. The analysis files for this section are in this directory: *"Flixan\Control Analysis\Hinfinity\ Examples\ Shuttle Ascent Hinfinity Design\5-Further Analysis Flex, Coupled Axes*". The Flixan input datasets are in file: "*Coupled_MaxQ.Inp*" and the systems created are saved in file: "*Coupled_MaxQ.Qdr*".

5.1 The Input File

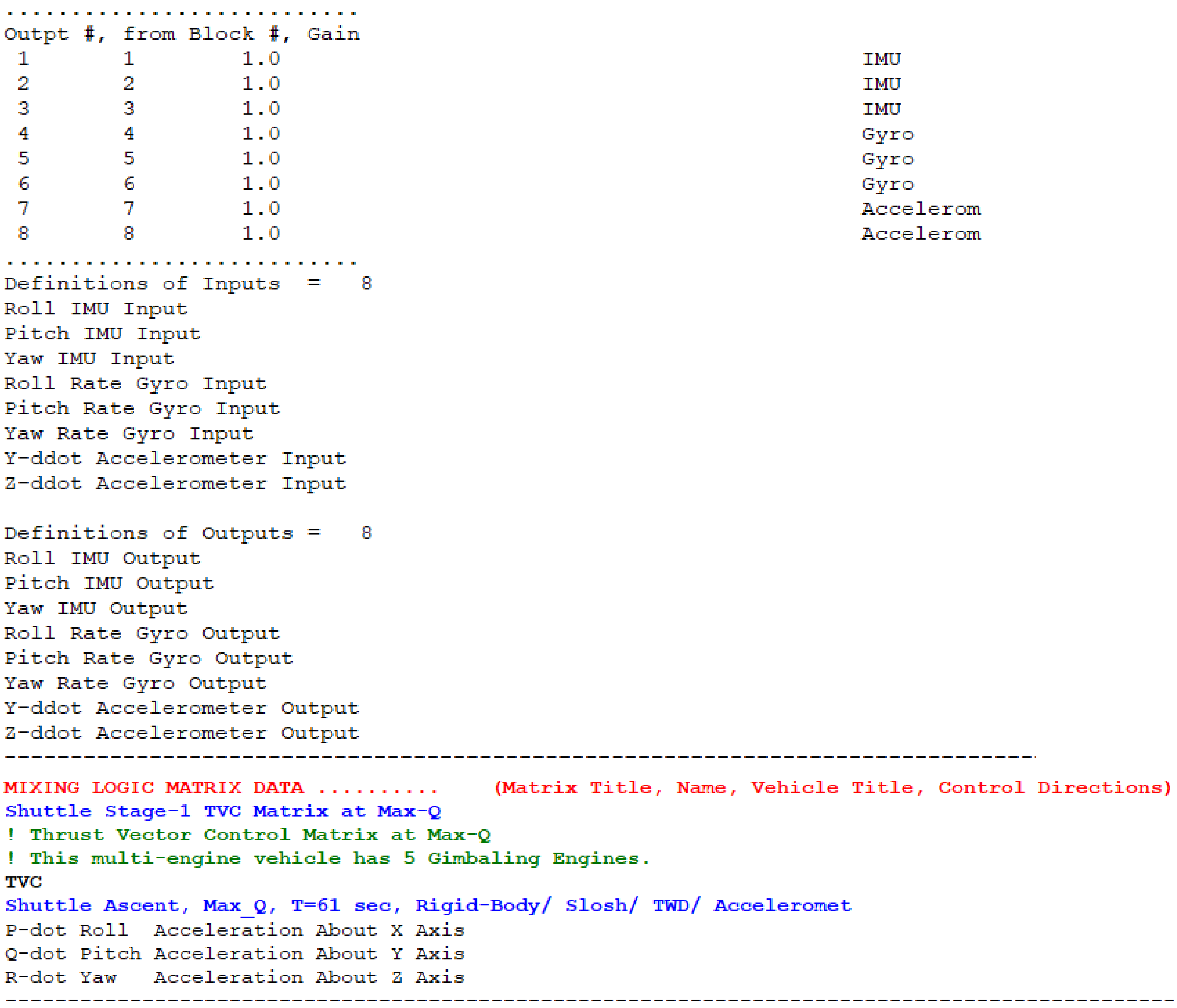
The input file "*Coupled_MaxQ.Inp*" begins with a batch dataset for processing the entire file in batch mode. It includes the vehicle model "*Shuttle Ascent, Max_Q, T=61 sec, Rigid-Body/ Slosh/ TWD/ Accelerometer*" which is combined with the modal data set "*Flex Modes for Shuttle Ascent, Max_Q*" located at the bottom of the file. The modes have already been preselected and scaled and ready to be combined with the vehicle data. There is a mixing logic dataset "*Shuttle Stage-1 TVC Matrix at Max-Q*" that creates the TVC matrix from the vehicle data. There are two actuator models, "*Shuttle Main Engine Actuator*" for the 3 SSMEs, and "*Solid Rocket Booster Actuator*" for the 2 SRB engines. The 5 engines are combined together in a single system "System of Five Actuators" which is included twice in the plant model for the pitch and lateral gimbal deflections. A separate transfer function system is created "*IMU, Gyro, Accelerometer Sensors*" which includes the sensor dynamics and is also included in the plant model.



```
BATCH MODE INSTRUCTIONS ...............
Batch for preparing the Lateral Shuttle Ascent, First Stage Max-Q, T=61 sec
! This batch set creates the Space Shuttle Ascent model at Max Dynamic Pressure including propellant
! sloshing and structural flexibility. Actuator models for the SSME and the SRB engines are included.
! Also, models for the gyro and accelerometer sensors. The Flex Vehicle, Actuators and Sensors are
! combined together to create the plant model. A TVC matrix is also created. The flex modes are
! included at the bottom of this file in a separate dataset and are combined with the vehicle system.
! The plant model is dicretized at 20 msec and exported to Matlab together with the TVC matrix.
!
!................ Preserve Older Systems ...............................................
Retain Matrix     : Shuttle Stage-1 TVC Matrix with Gust input
!
!...............  Actuators and Sensors  ...............................................
Actuator Model    : Shuttle Main Engine Actuator
Actuator Model    : Solid Rocket Booster Actuator
System Connection: System of Five Actuators
Transf-Function   : IMU, Gyro, Accelerometer Sensors
!
!...............  Analysis Models ......................................................
Flight Vehicle    : Shuttle Ascent, Max_Q, T=61 sec, Rigid-Body/ Slosh/ TWD/ Acceleromet
Mixing Matrix     : Shuttle Stage-1 TVC Matrix at Max-Q
System Connection: Shuttle Plant Model at Max-Q (Flex Vehicle, Actuators, Sensors)
S-Z-Transform     : Discrete Shuttle Plant Model at Max-Q (Flex Vehicle, Actuators, Sensors)
!
!..............   Export to Matlab .....................................................
To Matlab Format : Shuttle Stage-1 TVC Matrix at Max-Q
To Matlab Format : Shuttle Ascent, Max_Q, T=61 sec, Rigid-Body/ Slosh/ TWD/ Acceleromet
To Matlab Format : Shuttle Plant Model at Max-Q (Flex Vehicle, Actuators, Sensors)
To Matlab Format : Discrete Shuttle Plant Model at Max-Q (Flex Vehicle, Actuators, Sensors)
-----------------------------------------------------------------------------------------------------
```

```
ACTUATOR INPUT DATA ............ HYDRAULIC TYPE B
Shuttle Main Engine Actuator
! Shuttle Main Engine Actuator Without Compensator
! Using the Hydraulic Actuator Model Type (B)
Symbol       Parameter Description                  (Units)      Value
C(s)    Order of Compensat:(0,1,2), Coefficients     (---)       0
Kav     Total Gain of Amplifier + Torque Motor   (ft-lb/rad)  0.91000
Kact    Power Valve and Actuator Gain .........(ft/s/ft-lb)   53.950
Kt      Piston Ram Stiffness ..................     (lb/ft)    0.20280E+07
R       Moment Arm of Actuator Ram from Gimbal     (feet)      2.4800
Ie      Engine Inertia about Gimbal ...........(ft-lb-s^2)    4516.0
Be      Engine Viscous Damping ................(ft-lb-sec)    16500.
Kb      Engine Gimbal Bearing Spring Constant   (ft-lb/rad)   27160.0
Kl      Engine Mount Structural Stiffness ......   (lb/ft)    0.27960E+07
Tc      Different. Pressure Feedbk Time Constant  (seconds)  0.12660
Kdpf    Differ Pressure Feedbk Linearizat. Gain   (ft-lb/lb)  0.40400E-06
Kfb     Position Feedback Gain ................. (rad/feet)  0.4
---------------------------------------------------------------------------
ACTUATOR INPUT DATA ............ HYDRAULIC TYPE B
Solid Rocket Booster Actuator
Symbol       Parameter Description                  (Units)      Value
C(s)    Order of Compensat:(0,1,2), Coefficients     (---)       0
Kav     Total Gain of Amplifier + Torque Motor   (ft-lb/rad)   1.8152
Kact    Power Valve and Actuator Gain .........(ft/s/ft-lb)   75.4
Kt      Piston Ram Stiffness ..................     (lb/ft)    0.45000E+07
R       Moment Arm of Actuator Ram from Gimbal     (feet)      5.8
Ie      Engine Inertia about Gimbal ...........(ft-lb-s^2)    11000.0
Be      Engine Viscous Damping ................(ft-lb-sec)    17166.0
Kb      Engine Gimbal Bearing Spring Constant   (ft-lb/rad)   0.19166E+06
Kl      Engine Mount Structural Stiffness ......   (lb/ft)    0.60000E+07
Tc      Different. Pressure Feedbk Time Constant  (seconds)  0.1252
Kdpf    Differ Pressure Feedbk Linearizat. Gain   (ft-lb/lb)  0.40000E-06
Kfb     Position Feedback Gain ................. (rad/feet)  0.172
----------------------------------------------------------------------------
INTERCONNECTION OF SYSTEMS .....
System of Five Actuators
! A Combination of 5 actuators in parallel, 3 Space Shuttle Main Engine actuators,
! and 2 Solid Rocket Booster actuatots
!
Titles of Systems to be Combined
Title 1 Shuttle Main Engine Actuator
Title 2 Shuttle Main Engine Actuator
Title 3 Shuttle Main Engine Actuator
Title 4 Solid Rocket Booster Actuator
Title 5 Solid Rocket Booster Actuator
SYSTEM INPUTS TO SUBSYSTEM  1                                                       SSME Engine # 1
System Input  1 to Subsystem  1, Input  1, Gain= 1.00000                            Command Input
System Input  6 to Subsystem  1, Input  2, Gain= 1.00000                            Load-Torque
.......................................................................
SYSTEM INPUTS TO SUBSYSTEM  2                                                       SSME Engine # 2
System Input  2 to Subsystem  2, Input  1, Gain= 1.00000                            Command Input
System Input  7 to Subsystem  2, Input  2, Gain= 1.00000                            Load-Torque
.......................................................................
SYSTEM INPUTS TO SUBSYSTEM  3                                                       SSME Engine # 3
System Input  3 to Subsystem  3, Input  1, Gain= 1.00000                            Command Input
System Input  8 to Subsystem  3, Input  2, Gain= 1.00000                            Load-Torque
.......................................................................
SYSTEM INPUTS TO SUBSYSTEM  4                                                       SRB Engine # 4
System Input  4 to Subsystem  4, Input  1, Gain= 1.00000                            Command Input
System Input  9 to Subsystem  4, Input  2, Gain= 1.00000                            Load-Torque
.......................................................................
SYSTEM INPUTS TO SUBSYSTEM  5                                                       SRB Engine # 5
System Input  5 to Subsystem  5, Input  1, Gain= 1.00000                            Command Input
System Input 10 to Subsystem  5, Input  2, Gain= 1.00000                            Load-Torque
.......................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                                    SSME Engine # 1
System Output  1 from Subsystem  1, Output  1, Gain= 1.00000                        Deflection
System Output  6 from Subsystem  1, Output  3, Gain= 1.00000                        Acceleration
.......................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                                                    SSME Engine # 2
System Output  2 from Subsystem  2, Output  1, Gain= 1.00000                        Deflection
System Output  7 from Subsystem  2, Output  3, Gain= 1.00000                        Acceleration
.......................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  3                                                    SSME Engine # 3
System Output  3 from Subsystem  3, Output  1, Gain= 1.00000                        Deflection
System Output  8 from Subsystem  3, Output  3, Gain= 1.00000                        Acceleration
.......................................................................
```

```
SYSTEM OUTPUTS FROM SUBSYSTEM  4                                                SRB Engine # 4
System Output  4 from Subsystem  4, Output  1, Gain= 1.00000                    Deflection
System Output  9 from Subsystem  4, Output  3, Gain= 1.00000                    Acceleration
.............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  5                                                SRB Engine # 5
System Output  5 from Subsystem  5, Output  1, Gain= 1.00000                    Deflection
System Output 10 from Subsystem  5, Output  3, Gain= 1.00000                    Acceleration
.............................................................................
Definitions of Inputs  =  10
SSME Engine # 1 delta command (rad)
SSME Engine # 2 delta command (rad)
SSME Engine # 3 delta command (rad)
SRB  Engine # 4 delta command (rad)
SRB  Engine # 5 delta command (rad)
SSME Engine # 1 Load-Torque (ft-lb)
SSME Engine # 2 Load-Torque (ft-lb)
SSME Engine # 3 Load-Torque (ft-lb)
SRB  Engine # 4 Load-Torque (ft-lb)
SRB  Engine # 5 Load-Torque (ft-lb)

Definitions of Outputs =  10
SSME Engine # 1 deflection (rad)
SSME Engine # 2 deflection (rad)
SSME Engine # 3 deflection (rad)
SRB  Engine # 4 deflection (rad)
SRB  Engine # 5 deflection (rad)
SSME Engine # 1 accelerat. (rad/sec^2)
SSME Engine # 2 accelerat. (rad/sec^2)
SSME Engine # 3 accelerat. (rad/sec^2)
SRB  Engine # 4 accelerat. (rad/sec^2)
SRB  Engine # 5 accelerat. (rad/sec^2)
-----------------------------------------------------------------------------------------------------
SYSTEM OF TRANSFER FUNCTIONS ...
IMU, Gyro, Accelerometer Sensors
! Flight Control Sensor Dynamics (Low-Pass Filters)
Continuous
TF. Block #  1 IMU-roll                               Order of Numer, Denom=  1  1
Numer 0.0        7.0
Denom 1.0        7.0
TF. Block #  2 IMU-pitch                              Order of Numer, Denom=  1  1
Numer 0.0        7.0
Denom 1.0        7.0
TF. Block #  3 IMU-yaw                                Order of Numer, Denom=  1  1
Numer 0.0        7.0
Denom 1.0        7.0
TF. Block #  4 Rate Gyro Roll                         Order of Numer, Denom=  1  1
Numer 0.0        60.0
Denom 1.0        60.0
TF. Block #  5 Rate Gyro Pitch                        Order of Numer, Denom=  1  1
Numer 0.0        60.0
Denom 1.0        60.0
TF. Block #  6 Rate Gyro Yaw                          Order of Numer, Denom=  1  1
Numer 0.0        60.0
Denom 1.0        60.0
TF. Block #  7 Acceleromet-Ydd                        Order of Numer, Denom=  2  2
Numer 0.0        0.0        225.0
Denom 1.0        20.5       225.0
TF. Block #  8 Acceleromet-Zdd                        Order of Numer, Denom=  2  2
Numer 0.0        0.0        225.0
Denom 1.0        20.5       225.0
...........................................................
Block #, from Input #, Gain
 1       1       1.0                                  IMU
 2       2       1.0                                  IMU
 3       3       1.0                                  IMU
 4       4       1.0                                  Gyro
 5       5       1.0                                  Gyro
 6       6       1.0                                  Gyro
 7       7       1.0                                  Accelerom
 8       8       1.0                                  Accelerom
.............................
```



```
...........................
Outpt #, from Block #, Gain
 1        1        1.0                                                IMU
 2        2        1.0                                                IMU
 3        3        1.0                                                IMU
 4        4        1.0                                                Gyro
 5        5        1.0                                                Gyro
 6        6        1.0                                                Gyro
 7        7        1.0                                                Accelerom
 8        8        1.0                                                Accelerom
...........................
Definitions of Inputs  =   8
Roll IMU Input
Pitch IMU Input
Yaw IMU Input
Roll Rate Gyro Input
Pitch Rate Gyro Input
Yaw Rate Gyro Input
Y-ddot Accelerometer Input
Z-ddot Accelerometer Input

Definitions of Outputs =   8
Roll IMU Output
Pitch IMU Output
Yaw IMU Output
Roll Rate Gyro Output
Pitch Rate Gyro Output
Yaw Rate Gyro Output
Y-ddot Accelerometer Output
Z-ddot Accelerometer Output
---------------------------------------------------------------------------------
MIXING LOGIC MATRIX DATA ..........    (Matrix Title, Name, Vehicle Title, Control Directions)
Shuttle Stage-1 TVC Matrix at Max-Q
! Thrust Vector Control Matrix at Max-Q
! This multi-engine vehicle has 5 Gimbaling Engines.
TVC
Shuttle Ascent, Max_Q, T=61 sec, Rigid-Body/ Slosh/ TWD/ Acceleromet
P-dot Roll  Acceleration About X Axis
Q-dot Pitch Acceleration About Y Axis
R-dot Yaw   Acceleration About Z Axis
-------------------------------------------------------------------------------------------
```

The following is the Shuttle vehicle dataset at Max-Q, which is 754 (psf). It has 5 engines, 3 rate gyros (roll, pitch, yaw) rates, 2 accelerometers along (N_Y and N_Z), 2 propellant sloshing modes for the LOX and LH2 tanks, and 25 bending modes. The plant model *"Shuttle Plant Model at Max-Q (Flex Vehicle, Actuators, Sensors)"* consists of 4 systems combined: the vehicle model, the sensors, the 5 pitch actuators, and the 5 yaw actuators. The system of five actuators is used twice for pitch and yaw actuators.

```
FLIGHT VEHICLE INPUT DATA ......
Shuttle Ascent, Max_Q, T=61 sec, Rigid-Body/ Slosh/ TWD/ Acceleromet
! Shuttle Vehicle Model during First Stage at Max Dynamic pressure.
! Slosh, Structural Flexibility and Tail-Wag-Dog is Included.
Body Axes Output, Attitude=Rate Integral, Without GAFD, No Turn Coordination

Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Planet Radius (Re) (ft)    :  93215.0      32.174       0.20896E+08
Moments and products of Inertias Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)           :  0.248524E+8  0.209190E+9  0.221208E+9, 0.0, 0.937592E+7, 0.0
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)         : -115.0        0.036       -35.937
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)        :  1.54         1518.0       745.4        39410.0
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)          :  33.0         60.45        0.0          7.45
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                          : -3.579       -0.04         0.0          0.0
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)    :  0.0000       57.93        0.0000       0.0000       0.0000
Wind Gust Vel wrt Vehi (Azim & Elev) angles (deg), or Force(lb), Torque(ft-lb), locat:xyz:  Gust         45.0         90.0
Surface Reference Area (feet^2), Mean Aerodynamic Chord (ft), Wing Span in (feet)         :  2690.0       15.0         15.0
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial_rho/ Partial_H} : -115.0        0.036       -35.937      -9.482e-10
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:  0.0          0.0          0.0          0.0          0.0
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}:  0.0         -0.0353       0.0000       0.0000       0.0000
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}:  0.0         -0.0575       0.0000       0.0000       0.0000
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:  0.0         -0.028        0.0000       0.0000       0.0000
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}:  0.0         -0.017        0.0000       0.0000       0.0000
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}:  0.0          0.0249       0.0000       0.0000       0.0000

Number of Thruster Engines, Include or Not the Tail-Wags-Dog and Load-Torque Dynamics ?  :  5   WITH TWD

 TVC Engine No:  1                                (Gimbaling  Throttling   Single_Gimbal) : Middle SSME      Gimbaling
 Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)                    :  472000.0     472000.0
 Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): -16.0         0.0          10.0         10.0
 Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  220.0        4800.0       3.1
 Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -182.1667     0.0         -64.958
 TVC Engine No:  2                                (Gimbaling  Throttling   Single_Gimbal) : Left SSME        Gimbaling
 Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)                    :  472000.0     472000.0
 Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): -10.0         0.0          10.0         10.0
 Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  220.0        4800.0       3.1
 Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -184.1       -4.4167      -56.595
 TVC Engine No:  3                                (Gimbaling  Throttling   Single_Gimbal) : Right SSME       Gimbaling
 Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)                    :  472000.0     472000.0
 Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): -10.0         0.0          10.0         10.0
 Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  220.0        4800.0       3.1
 Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -184.1        4.4167      -56.595
 TVC Engine No:  4                                (Gimbaling  Throttling   Single_Gimbal) : Left SRB         Gimbaling
 Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)                    :  0.245e+7     0.245e+7
 Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg):  0.0          0.0          10.0         10.0
 Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  605.0        0.154e+5    -1.07
 Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -201.53      -20.875      -33.3333
 TVC Engine No:  5                                (Gimbaling  Throttling   Single_Gimbal) : Left SRB         Gimbaling
 Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)                    :  0.245e+7     0.245e+7
 Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg):  0.0          0.0          10.0         10.0
 Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  605.0        0.154e+5    -1.07
 Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -201.53      +20.875      -33.3333

 Number of Gyros, (Attitude and Rate)                                                     :  3
 Gyro No  1 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Location in (feet) :  Roll   Rate          -93.625    0.00    -61.5417
 Gyro No  2 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Location in (feet) :  Pitch  Rate          -93.625    0.00    -61.5417
 Gyro No  3 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Location in (feet) :  Yaw    Rate          -93.625    0.00    -61.5417

 Number of Accelerometers, Along Axis: (x,y,z)                                            :  2
 Acceleromet No  1 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Location (ft) :  Y-axis Accelerat.    -93.625    0.00    -61.5417
 Acceleromet No  2 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Location (ft) :  Z-axis Accelerat.    -93.625    0.00    -61.5417

 Number of Slosh Modes                                                                    :  2
 LOX Slosh Mass (slug), Frequ Wy,Wz 1g (rad/s), Damp (zeta-y-z), Locat {Xsl,Ysl,Zsl} (ft) :  4100.0  2.33  2.33  0.00164 0.00164 -57.96   0.0  -34.0
 LH2 Slosh Mass (slug), Frequ Wy,Wz 1g (rad/s), Damp (zeta-y-z), Locat {Xsl,Ysl,Zsl} (ft) :  512.5  2.066  2.066 0.00244 0.00244 -115.02  0.0  -34.2

 Number of Bending Modes                                                                  : 25
 Flex Modes for Shuttle Ascent, Max_Q
-----------------------------------------------------------------------------------------------------------------------------
```

```
INTERCONNECTION OF SYSTEMS .....
Shuttle Plant Model at Max-Q (Flex Vehicle, Actuators, Sensors)
! Combine the above vehicle model with the Actuators and Sensors to create a fully coupled
! pitch and lateral plant model. This system will be combined with the TVC and the
! Flight Control System to create the plant/ simulation model. Tail-Wag-Dog and
! Load-Torque feedback interconnections are included.
!
Titles of Systems to be Combined
Title 1 Shuttle Ascent, Max_Q, T=61 sec, Rigid-Body/ Slosh/ TWD/ Acceleromet
Title 2 System of Five Actuators
Title 3 System of Five Actuators
Title 4 IMU, Gyro, Accelerometer Sensors
SYSTEM INPUTS TO SUBSYSTEM  1                                         Wind to Vehicle
System Input 11 to Subsystem  1, Input 21, Gain= 1.0                  Wind-Gust
.................................................................
SYSTEM INPUTS TO SUBSYSTEM  2                                         Pitch Deflection Commands
System Input  1 to Subsystem  2, Input  1, Gain= 1.0                  dy1-com
System Input  2 to Subsystem  2, Input  2, Gain= 1.0                  dy2-com
System Input  3 to Subsystem  2, Input  3, Gain= 1.0                  dy3-com
System Input  4 to Subsystem  2, Input  4, Gain= 1.0                  dy4-com
System Input  5 to Subsystem  2, Input  5, Gain= 1.0                  dy5-com
.................................................................
SYSTEM INPUTS TO SUBSYSTEM  3                                         Yaw Deflection Commands
System Input  6 to Subsystem  3, Input  1, Gain= 1.0                  dz1-com
System Input  7 to Subsystem  3, Input  2, Gain= 1.0                  dz2-com
System Input  8 to Subsystem  3, Input  3, Gain= 1.0                  dz3-com
System Input  9 to Subsystem  3, Input  4, Gain= 1.0                  dz4-com
System Input 10 to Subsystem  3, Input  5, Gain= 1.0                  dz5-com
.................................................................
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SYSTEM OUTPUTS FROM SUBSYSTEM  4                                                    Sensors
System Output  1 from Subsystem  4, Output  1, Gain= 1.0                            roll  attitude
System Output  2 from Subsystem  4, Output  2, Gain= 1.0                            pitch attitude
System Output  3 from Subsystem  4, Output  3, Gain= 1.0                            yaw   attitude
System Output  4 from Subsystem  4, Output  4, Gain= 1.0                            roll  rate
System Output  5 from Subsystem  4, Output  5, Gain= 1.0                            pitch rate
System Output  6 from Subsystem  4, Output  6, Gain= 1.0                            yaw   rate
System Output  7 from Subsystem  4, Output  7, Gain= 1.0                            Ny Accelerat
System Output  8 from Subsystem  4, Output  8, Gain= 1.0                            Nz Accelerat
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                                    Vehicle Model
System Output  9 from Subsystem  1, Output  7, Gain= 1.00000                        alpha (rad)
System Output 10 from Subsystem  1, Output  8, Gain= 1.00000                        beta  (rad)
System Output 11 from Subsystem  1, Output 17, Gain= 1.00000                        TLY-1 (ft-lb)
System Output 12 from Subsystem  1, Output 18, Gain= 1.00000                        TLY-2 (ft-lb)
System Output 13 from Subsystem  1, Output 19, Gain= 1.00000                        TLY-3 (ft-lb)
System Output 14 from Subsystem  1, Output 20, Gain= 1.00000                        TLY-4 (ft-lb)
System Output 15 from Subsystem  1, Output 21, Gain= 1.00000                        TLY-5 (ft-lb)
System Output 16 from Subsystem  1, Output 22, Gain= 1.00000                        TLZ-1 (ft-lb)
System Output 17 from Subsystem  1, Output 23, Gain= 1.00000                        TLZ-2 (ft-lb)
System Output 18 from Subsystem  1, Output 24, Gain= 1.00000                        TLZ-3 (ft-lb)
System Output 19 from Subsystem  1, Output 25, Gain= 1.00000                        TLZ-4 (ft-lb)
System Output 20 from Subsystem  1, Output 26, Gain= 1.00000                        TLZ-5 (ft-lb)
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                                                    Pitch Actuator Model
System Output 21 from Subsystem  2, Output  1, Gain= 1.0                            dy-1
System Output 22 from Subsystem  2, Output  2, Gain= 1.0                            dy-2
System Output 23 from Subsystem  2, Output  3, Gain= 1.0                            dy-3
System Output 24 from Subsystem  2, Output  4, Gain= 1.0                            dy-4
System Output 25 from Subsystem  2, Output  5, Gain= 1.0                            dy-5
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  3                                                    Yaw Actuator Model
System Output 26 from Subsystem  3, Output  1, Gain= 1.0                            dz-1
System Output 27 from Subsystem  3, Output  2, Gain= 1.0                            dz-2
System Output 28 from Subsystem  3, Output  3, Gain= 1.0                            dz-3
System Output 29 from Subsystem  3, Output  4, Gain= 1.0                            dz-4
System Output 30 from Subsystem  3, Output  5, Gain= 1.0                            dz-5
..............................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  4                                             Vehicle to Sensors
Subsystem  1, Output  1 to Subsystem  4, Input  1, Gain= 1.0                        IMU-roll
Subsystem  1, Output  3 to Subsystem  4, Input  2, Gain= 1.0                        IMU-ptch
Subsystem  1, Output  5 to Subsystem  4, Input  3, Gain= 1.0                        IMU-yaw
Subsystem  1, Output 12 to Subsystem  4, Input  4, Gain= 1.0                        Rate-Gyro-Roll
Subsystem  1, Output 13 to Subsystem  4, Input  5, Gain= 1.0                        Rate-Gyro-Ptch
Subsystem  1, Output 14 to Subsystem  4, Input  6, Gain= 1.0                        Rate-Gyro-Yaw
Subsystem  1, Output 15 to Subsystem  4, Input  7, Gain= 1.0                        Accelerom-Yddot
Subsystem  1, Output 16 to Subsystem  4, Input  8, Gain= 1.0                        Accelerom-Zddot
..............................................................................
SUBSYSTEM NO  2 GOES TO SUBSYSTEM NO  1                                             Pitch Actuators to Vehicle
Subsystem  2, Output  1 to Subsystem  1, Input  1, Gain= 1.0                        dy-1
Subsystem  2, Output  2 to Subsystem  1, Input  2, Gain= 1.0                        dy-2
Subsystem  2, Output  3 to Subsystem  1, Input  3, Gain= 1.0                        dy-3
Subsystem  2, Output  4 to Subsystem  1, Input  4, Gain= 1.0                        dy-4
Subsystem  2, Output  5 to Subsystem  1, Input  5, Gain= 1.0                        dy-5
Subsystem  2, Output  6 to Subsystem  1, Input  6, Gain= 1.0                        dy-ddot-1
Subsystem  2, Output  7 to Subsystem  1, Input  7, Gain= 1.0                        dy-ddot-2
Subsystem  2, Output  8 to Subsystem  1, Input  8, Gain= 1.0                        dy-ddot-3
Subsystem  2, Output  9 to Subsystem  1, Input  9, Gain= 1.0                        dy-ddot-4
Subsystem  2, Output 10 to Subsystem  1, Input 10, Gain= 1.0                        dy-ddot-5
..............................................................................
SUBSYSTEM NO  3 GOES TO SUBSYSTEM NO  1                                             Yaw Actuators to Vehicle
Subsystem  3, Output  1 to Subsystem  1, Input 11, Gain= 1.0                        dz-1
Subsystem  3, Output  2 to Subsystem  1, Input 12, Gain= 1.0                        dz-2
Subsystem  3, Output  3 to Subsystem  1, Input 13, Gain= 1.0                        dz-3
Subsystem  3, Output  4 to Subsystem  1, Input 14, Gain= 1.0                        dz-4
Subsystem  3, Output  5 to Subsystem  1, Input 15, Gain= 1.0                        dz-5
Subsystem  3, Output  6 to Subsystem  1, Input 16, Gain= 1.0                        dz-ddot-1
Subsystem  3, Output  7 to Subsystem  1, Input 17, Gain= 1.0                        dz-ddot-2
Subsystem  3, Output  8 to Subsystem  1, Input 18, Gain= 1.0                        dz-ddot-3
Subsystem  3, Output  9 to Subsystem  1, Input 19, Gain= 1.0                        dz-ddot-4
Subsystem  3, Output 10 to Subsystem  1, Input 20, Gain= 1.0                        dz-ddot-5
..............................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2                                             Pitch Load-Torque Feedback
Subsystem  1, Output 17 to Subsystem  2, Input  6, Gain= 1.0                        TLY-1
Subsystem  1, Output 18 to Subsystem  2, Input  7, Gain= 1.0                        TLY-2
Subsystem  1, Output 19 to Subsystem  2, Input  8, Gain= 1.0                        TLY-3
Subsystem  1, Output 20 to Subsystem  2, Input  9, Gain= 1.0                        TLY-4
Subsystem  1, Output 21 to Subsystem  2, Input 10, Gain= 1.0                        TLY-5
..............................................................................
```
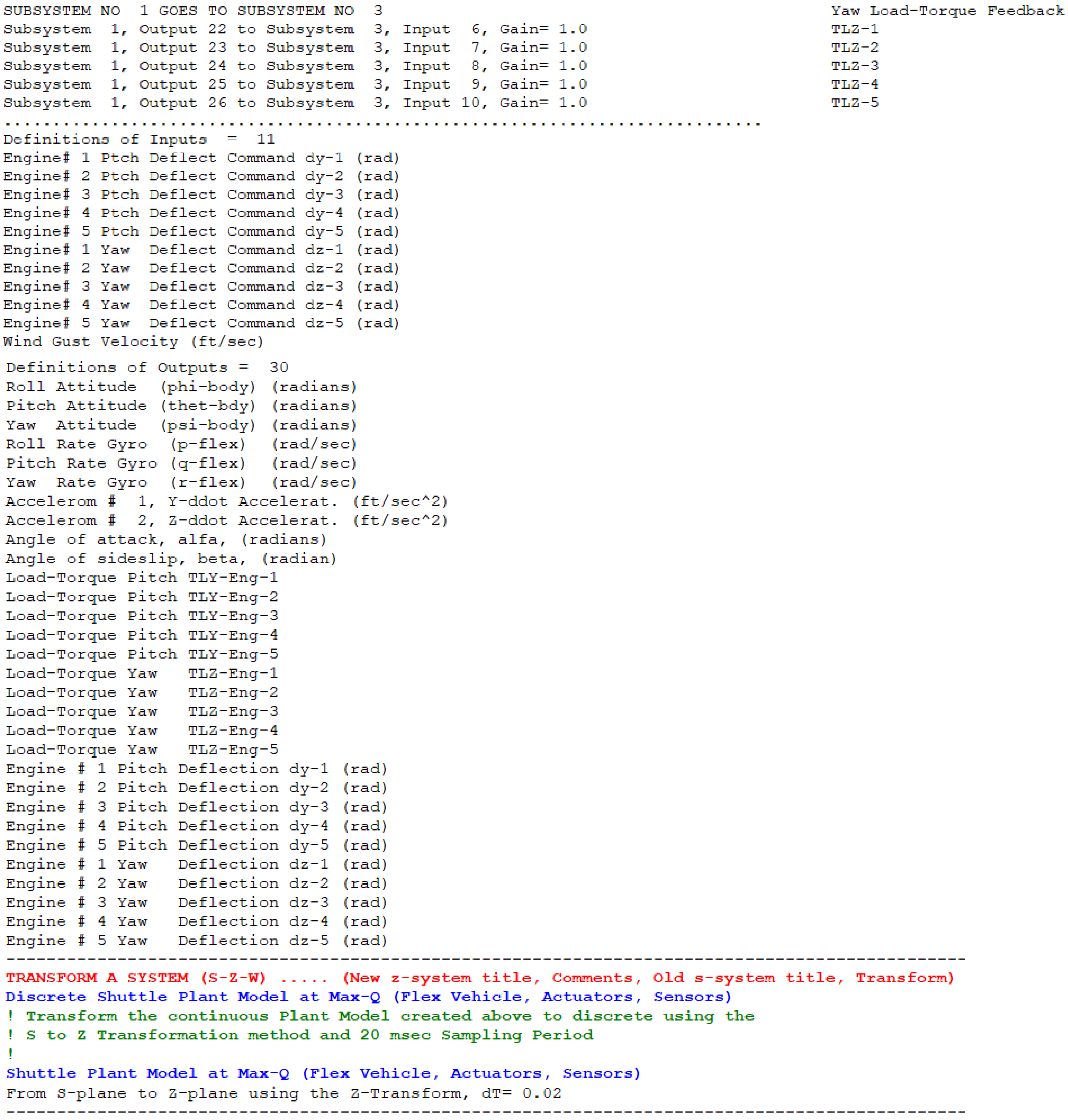



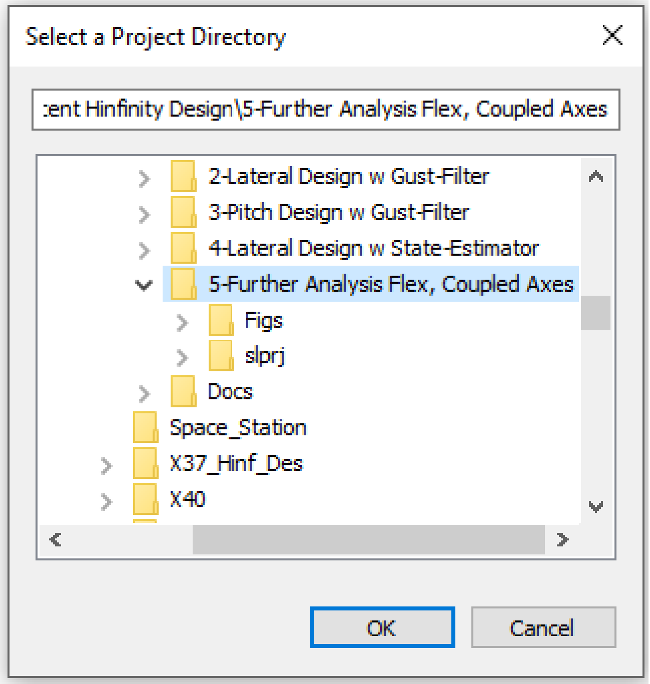
```
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  3                                    Yaw Load-Torque Feedback
Subsystem  1, Output 22 to Subsystem  3, Input  6, Gain= 1.0               TLZ-1
Subsystem  1, Output 23 to Subsystem  3, Input  7, Gain= 1.0               TLZ-2
Subsystem  1, Output 24 to Subsystem  3, Input  8, Gain= 1.0               TLZ-3
Subsystem  1, Output 25 to Subsystem  3, Input  9, Gain= 1.0               TLZ-4
Subsystem  1, Output 26 to Subsystem  3, Input 10, Gain= 1.0               TLZ-5
............................................................................
Definitions of Inputs  =  11
Engine# 1 Ptch Deflect Command dy-1 (rad)
Engine# 2 Ptch Deflect Command dy-2 (rad)
Engine# 3 Ptch Deflect Command dy-3 (rad)
Engine# 4 Ptch Deflect Command dy-4 (rad)
Engine# 5 Ptch Deflect Command dy-5 (rad)
Engine# 1 Yaw  Deflect Command dz-1 (rad)
Engine# 2 Yaw  Deflect Command dz-2 (rad)
Engine# 3 Yaw  Deflect Command dz-3 (rad)
Engine# 4 Yaw  Deflect Command dz-4 (rad)
Engine# 5 Yaw  Deflect Command dz-5 (rad)
Wind Gust Velocity (ft/sec)
Definitions of Outputs =  30
Roll Attitude  (phi-body) (radians)
Pitch Attitude (thet-bdy) (radians)
Yaw  Attitude  (psi-body) (radians)
Roll Rate Gyro  (p-flex)  (rad/sec)
Pitch Rate Gyro (q-flex)  (rad/sec)
Yaw  Rate Gyro  (r-flex)  (rad/sec)
Accelerom #  1, Y-ddot Accelerat. (ft/sec^2)
Accelerom #  2, Z-ddot Accelerat. (ft/sec^2)
Angle of attack, alfa, (radians)
Angle of sideslip, beta, (radian)
Load-Torque Pitch TLY-Eng-1
Load-Torque Pitch TLY-Eng-2
Load-Torque Pitch TLY-Eng-3
Load-Torque Pitch TLY-Eng-4
Load-Torque Pitch TLY-Eng-5
Load-Torque Yaw   TLZ-Eng-1
Load-Torque Yaw   TLZ-Eng-2
Load-Torque Yaw   TLZ-Eng-3
Load-Torque Yaw   TLZ-Eng-4
Load-Torque Yaw   TLZ-Eng-5
Engine # 1 Pitch Deflection dy-1 (rad)
Engine # 2 Pitch Deflection dy-2 (rad)
Engine # 3 Pitch Deflection dy-3 (rad)
Engine # 4 Pitch Deflection dy-4 (rad)
Engine # 5 Pitch Deflection dy-5 (rad)
Engine # 1 Yaw   Deflection dz-1 (rad)
Engine # 2 Yaw   Deflection dz-2 (rad)
Engine # 3 Yaw   Deflection dz-3 (rad)
Engine # 4 Yaw   Deflection dz-4 (rad)
Engine # 5 Yaw   Deflection dz-5 (rad)
----------------------------------------------------------------------------------------------
TRANSFORM A SYSTEM (S-Z-W) ..... (New z-system title, Comments, Old s-system title, Transform)
Discrete Shuttle Plant Model at Max-Q (Flex Vehicle, Actuators, Sensors)
! Transform the continuous Plant Model created above to discrete using the
! S to Z Transformation method and 20 msec Sampling Period
!
Shuttle Plant Model at Max-Q (Flex Vehicle, Actuators, Sensors)
From S-plane to Z-plane using the Z-Transform, dT= 0.02
----------------------------------------------------------------------------------------------
```

The above s to z transformation set converts the continuous plant model to a discrete system title “*Discrete Shuttle Plant Model at Max-Q (Flex Vehicle, Actuators, Sensors)”* sampled at 20 msec. The next four sets convert the vehicle, plant models and the TVC matrix to files that can be loaded into Matlab. The last dataset “Flex Modes for Shuttle Ascent, Max_Q” is the modal data that will be processed by Flixan together with the vehicle data to create the flexible vehicle state-space system. It contains 29 modes and the first 2 are shown below. Each mode frame contains the mode frequency, damping, modal mass, and the mode shapes and slopes at important vehicle locations (nodes).

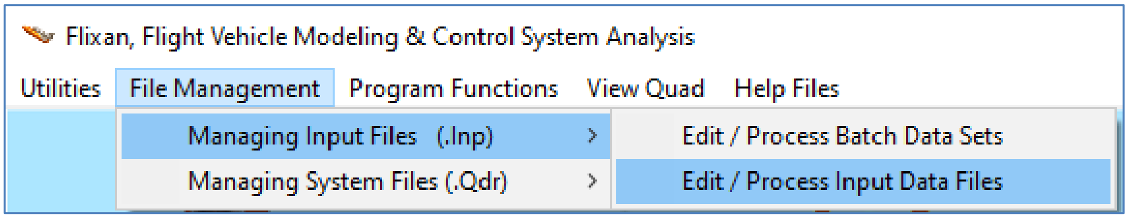
```
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Shuttle Stage-1 TVC Matrix at Max-Q
Matrix TVC
------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Shuttle Ascent, Max_Q, T=61 sec, Rigid-Body/ Slosh/ TWD/ Acceleromet
System
vehicle
------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Shuttle Plant Model at Max-Q (Flex Vehicle, Actuators, Sensors)
System
plant_s
------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Discrete Shuttle Plant Model at Max-Q (Flex Vehicle, Actuators, Sensors)
System
plant_z
------------------------------------------------------------------------------
SELECTED MODAL DATA AND LOCATIONS FOR SHUTTLE ASCENT AT MAX-Q
Flex Modes for Shuttle Ascent, Max_Q
! The following set of 29 flex modes include both symmetric (pitch) and anti-symmetric (lateral)
! modes. They were selected using the mode-selection process.

MODE # 1/  7, Frequency (rad/sec), Damping (zeta), Generalized Mass=    12.151       0.50000E-02   0.12000E+06
DEFINITION OF LOCATIONS (NODES)            phi along X   phi along Y   phi along Z   sigm about X  sigm about Y  sigm about Z

                              Node ID#     Modal Data at the  5 Engines, (x,y,z)...
Shuttle Main Engine  # 1          2088    0.61387D+00  -0.61734D-01  -0.15612D+01    0.42653D-02  -0.13384D-01    0.11436D-02
Shuttle Main Engine  # 2 (left    2254    0.50962D+00  -0.91117D-01  -0.15971D+01    0.29002D-02  -0.13111D-01  -0.75217D-03
Shuttle Main Engine  # 3 (righ    2091    0.50867D+00  -0.92870D-01  -0.15757D+01    0.44489D-02  -0.23288D-01    0.15293D-02
Solid Rocket Booster # 4 (left    1367   -0.44066D-01    0.13004D+00  -0.78224D+00  -0.17006D+00  -0.25292D-01  -0.64782D-03
Solid Rocket Booster # 5 (righ     367   -0.31986D-01  -0.13981D+00  -0.78234D+00    0.17183D+00  -0.25481D-01    0.11537D-02

                              Node ID#     Modal Data at the  3 Gyros ...
Booster Gyro (right)               372   -0.18730D-01    0.10486D+01    0.88193D+00    0.19468D+00  -0.99742D-03    0.14856D-02
Booster Gyro (right)               372   -0.18730D-01    0.10486D+01    0.88193D+00    0.19468D+00  -0.99742D-03    0.14856D-02
Booster Gyro (right)               372   -0.18730D-01    0.10486D+01    0.88193D+00    0.19468D+00  -0.99742D-03    0.14856D-02

                              Node ID#     Modal Data at the  2 Accelerometers, along (x,y,z)...
IMU/ Accelerometer (right)        2322    0.00000D+00    0.39853D-01  -0.23900D+00
IMU/ Accelerometer (right)        2322    0.00000D+00    0.39853D-01  -0.23900D+00

                              Node ID#     Modal Data at the  2 Slosh Masses...
LOX Slosh Mass                     238    0.00000D+00  -0.83451D-02  -0.68063D+00    0.00000D+00    0.00000D+00    0.00000D+00
LH2 Slosh Mass                     239    0.00000D+00    0.93311D-02  -0.37481D+00    0.00000D+00    0.00000D+00    0.00000D+00

                              Node ID#     Modal Data at the Disturbance Point
LOX Slosh Mass                     238    0.00000D+00  -0.83451D-02  -0.68063D+00    0.00000D+00    0.00000D+00    0.00000D+00

MODE # 2/  8, Frequency (rad/sec), Damping (zeta), Generalized Mass=    12.778       0.50000E-02   0.12000E+06
DEFINITION OF LOCATIONS (NODES)            phi along X   phi along Y   phi along Z   sigm about X  sigm about Y  sigm about Z

                              Node ID#     Modal Data at the  5 Engines, (x,y,z)...
Shuttle Main Engine  # 1          2088    0.91664D-02    0.18723D+01  -0.16686D-01    0.95388D-01  -0.38999D-02  -0.15823D-01
Shuttle Main Engine  # 2 (left    2254    0.14771D+00    0.93867D+00  -0.35851D+00    0.82810D-01    0.19854D-01    0.33026D-01
Shuttle Main Engine  # 3 (righ    2091   -0.14020D+00    0.94628D+00    0.35008D+00    0.76428D-01    0.12912D-01    0.25309D-01
Solid Rocket Booster # 4 (left    1367    0.20309D+00    0.26632D+00  -0.28703D+00  -0.91805D-01  -0.11800D-01  -0.11624D-01
Solid Rocket Booster # 5 (righ     367   -0.20439D+00    0.25900D+00    0.25654D+00  -0.87006D-01    0.10924D-01  -0.11574D-01

                              Node ID#     Modal Data at the  3 Gyros ...
Booster Gyro (right)               372   -0.18763D+00  -0.98496D+00  -0.37473D+00  -0.97034D-01  -0.61992D-02    0.38462D-02
Booster Gyro (right)               372   -0.18763D+00  -0.98496D+00  -0.37473D+00  -0.97034D-01  -0.61992D-02    0.38462D-02
Booster Gyro (right)               372   -0.18763D+00  -0.98496D+00  -0.37473D+00  -0.97034D-01  -0.61992D-02    0.38462D-02

                              Node ID#     Modal Data at the  2 Accelerometers, along (x,y,z)...
IMU/ Accelerometer (right)        2322    0.00000D+00    0.27026D+01    0.12267D+00
IMU/ Accelerometer (right)        2322    0.00000D+00    0.27026D+01    0.12267D+00

                              Node ID#     Modal Data at the  2 Slosh Masses...
LOX Slosh Mass                     238    0.00000D+00    0.25308D+00  -0.11050D-01    0.00000D+00    0.00000D+00    0.00000D+00
LH2 Slosh Mass                     239    0.00000D+00  -0.25045D+00  -0.49152D-02    0.00000D+00    0.00000D+00    0.00000D+00

                              Node ID#     Modal Data at the Disturbance Point
LOX Slosh Mass                     238    0.00000D+00    0.25308D+00  -0.11050D-01    0.00000D+00    0.00000D+00    0.00000D+00
```

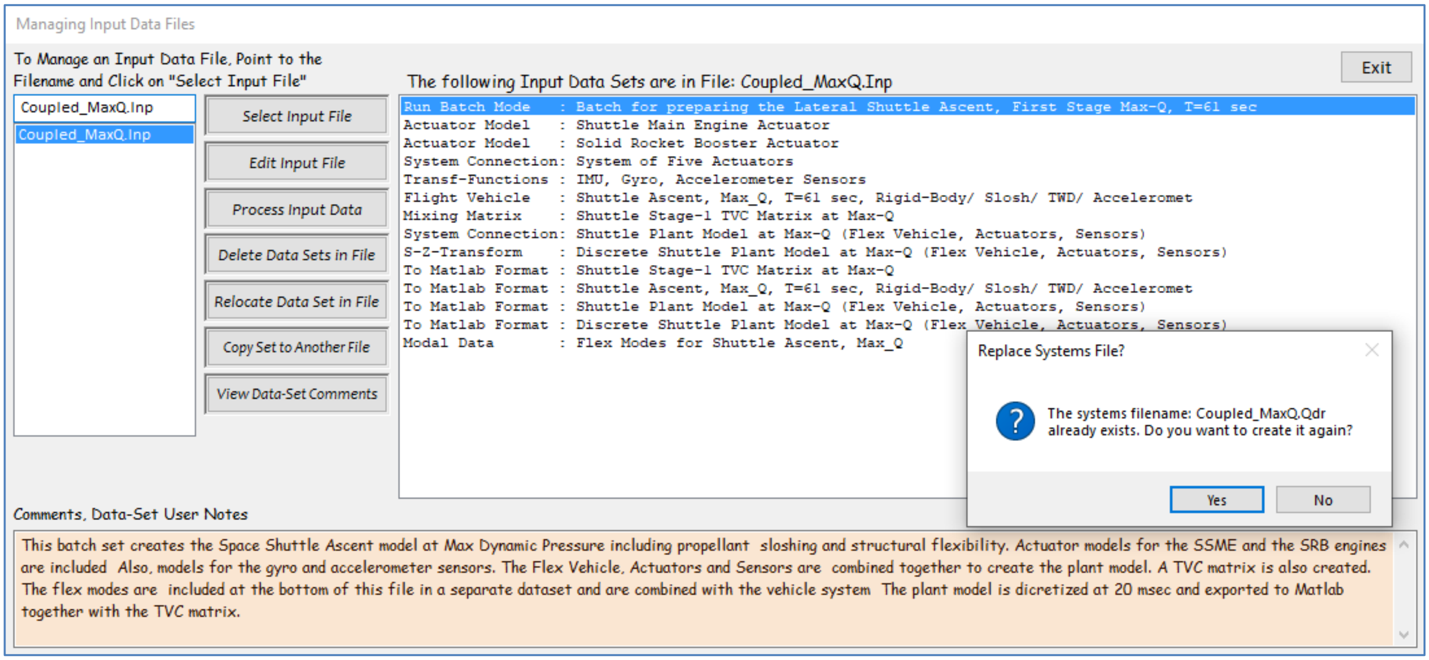
5.2 Processing the Input Data File in Batch

We can process the input file interactively in batch mode by running the batch dataset located at the top of the file. Start the Flixan program and select the project directory: "*Flixan\Control Analysis\Hinfinity\Examples\ Shuttle Ascent Hinfinity Design\5-Further Analysis Flex, Coupled Axes*". From the main menu select "File Management", "Managing Input Files", and then "Edit/ Process Input Data Files", as shown below.



The following dialog comes up that includes two menus. The menu on the left side lists the input data files in the project directory. There is only one. Highlight it and click on "Select Input File" button. The menu on the right shows the datasets which are in the input file. Select the batch set which is at the top of the list and click on "Process Input Data".





To Manage an Input Data File, Point to the Filename and Click on "Select Input File"

The following Input Data Sets are in File: Coupled_MaxQ.Inp

```
Run Batch Mode    : Batch for preparing the Lateral Shuttle Ascent, First Stage Max-Q, T=61 sec
Actuator Model    : Shuttle Main Engine Actuator
Actuator Model    : Solid Rocket Booster Actuator
System Connection: System of Five Actuators
Transf-Functions : IMU, Gyro, Accelerometer Sensors
Flight Vehicle    : Shuttle Ascent, Max_Q, T=61 sec, Rigid-Body/ Slosh/ TWD/ Acceleromet
Mixing Matrix     : Shuttle Stage-1 TVC Matrix at Max-Q
System Connection: Shuttle Plant Model at Max-Q (Flex Vehicle, Actuators, Sensors)
S-Z-Transform     : Discrete Shuttle Plant Model at Max-Q (Flex Vehicle, Actuators, Sensors)
To Matlab Format : Shuttle Stage-1 TVC Matrix at Max-Q
To Matlab Format : Shuttle Ascent, Max_Q, T=61 sec, Rigid-Body/ Slosh/ TWD/ Acceleromet
To Matlab Format : Shuttle Plant Model at Max-Q (Flex Vehicle, Actuators, Sensors)
To Matlab Format : Discrete Shuttle Plant Model at Max-Q (Flex Vehicle, Actuators, Sensors)
Modal Data        : Flex Modes for Shuttle Ascent, Max_Q
```

The systems filename: Coupled_MaxQ.Qdr already exists. Do you want to create it again?

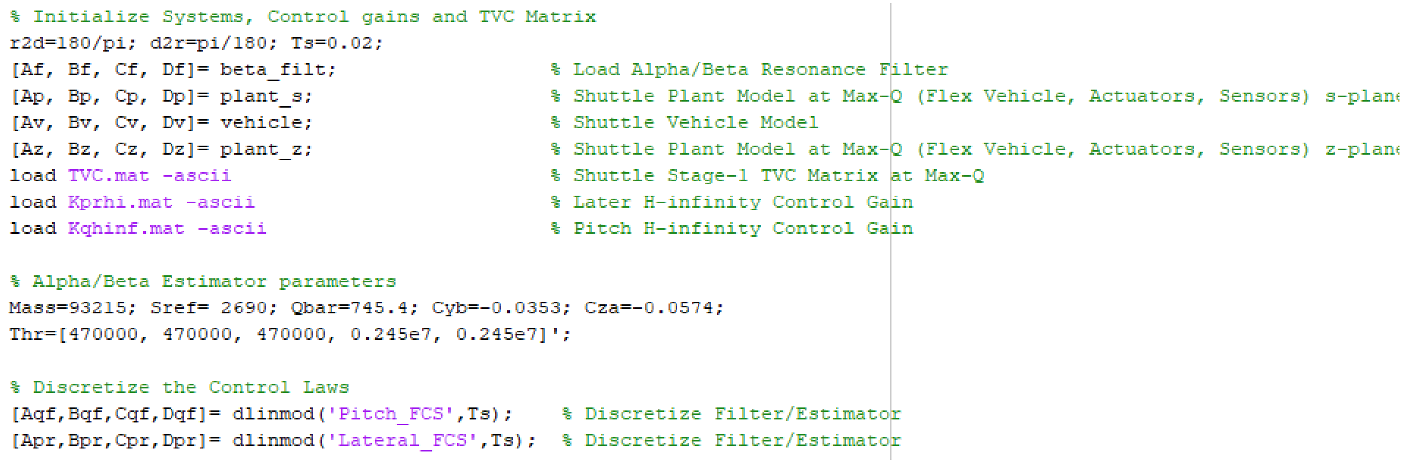
Comments, Data-Set User Notes

This batch set creates the Space Shuttle Ascent model at Max Dynamic Pressure including propellant sloshing and structural flexibility. Actuator models for the SSME and the SRB engines are included Also, models for the gyro and accelerometer sensors. The Flex Vehicle, Actuators and Sensors are combined together to create the plant model. A TVC matrix is also created. The flex modes are included at the bottom of this file in a separate dataset and are combined with the vehicle system The plant model is dicretized at 20 msec and exported to Matlab together with the TVC matrix.

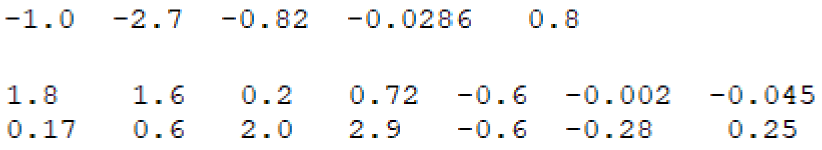
In the following question, answer "Yes", which is okay to delete the old systems file and recreate it. The batch executes and creates the systems and matrices that can now be loaded into Matlab.

5.2 Loading the Files into Matlab

The Matlab script file “init.m” loads the continuous and discrete systems, the disturbance filter and the TVC matrix into Matlab. It also loads the pitch and lateral control gains Kqhinf.mat and Kprhi.mat that were calculated in previous Sections 2 & 3. It also loads some vehicle parameters which are needed for the α & β estimators. Finally, it discretizes the pitch and lateral flight control systems in order to be combined with the discretized plant.



```
% Initialize Systems, Control gains and TVC Matrix
r2d=180/pi; d2r=pi/180; Ts=0.02;
[Af, Bf, Cf, Df]= beta_filt;                 % Load Alpha/Beta Resonance Filter
[Ap, Bp, Cp, Dp]= plant_s;                   % Shuttle Plant Model at Max-Q (Flex Vehicle, Actuators, Sensors) s-plan
[Av, Bv, Cv, Dv]= vehicle;                   % Shuttle Vehicle Model
[Az, Bz, Cz, Dz]= plant_z;                   % Shuttle Plant Model at Max-Q (Flex Vehicle, Actuators, Sensors) z-plan
load TVC.mat -ascii                          % Shuttle Stage-1 TVC Matrix at Max-Q
load Kprhi.mat -ascii                        % Later H-infinity Control Gain
load Kqhinf.mat -ascii                       % Pitch H-infinity Control Gain

% Alpha/Beta Estimator parameters
Mass=93215; Sref= 2690; Qbar=745.4; Cyb=-0.0353; Cza=-0.0574;
Thr=[470000, 470000, 470000, 0.245e7, 0.245e7]';

% Discretize the Control Laws
[Aqf,Bqf,Cqf,Dqf]= dlinmod('Pitch_FCS',Ts);    % Discretize Filter/Estimator
[Apr,Bpr,Cpr,Dpr]= dlinmod('Lateral_FCS',Ts);  % Discretize Filter/Estimator
```



```
 -1.0   -2.7   -0.82   -0.0286   0.8

 1.8     1.6    0.2    0.72   -0.6   -0.002   -0.045
 0.17    0.6    2.0    2.9    -0.6   -0.28     0.25
```

Figure 51 Pitch and Lateral State-Feedback Control Gains: Kqhinf, Kprhi

The pitch and lateral control systems consist mainly of the state-feedback gain matrices and the α & β estimators. However, considerable amount of filtering has been included in order to attenuate the bending modes and to prevent them from becoming unstable. Filters are also included in the estimator’s accelerometer measurements. Figures 52 and 53 show the pitch and lateral flight control systems.

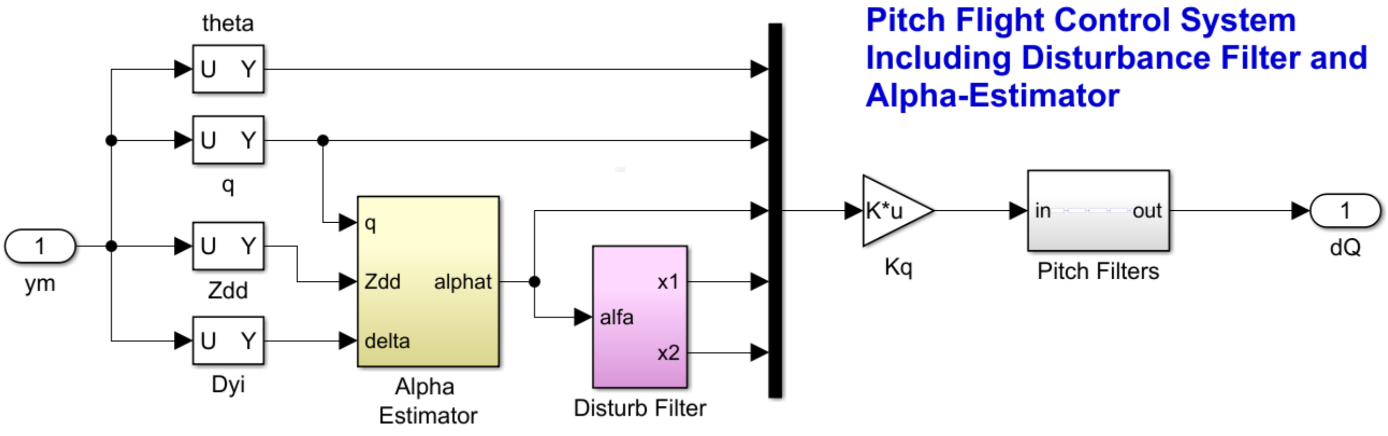


Figure 52 Pitch FCS with Estimator, α-filter, and Bending Filters

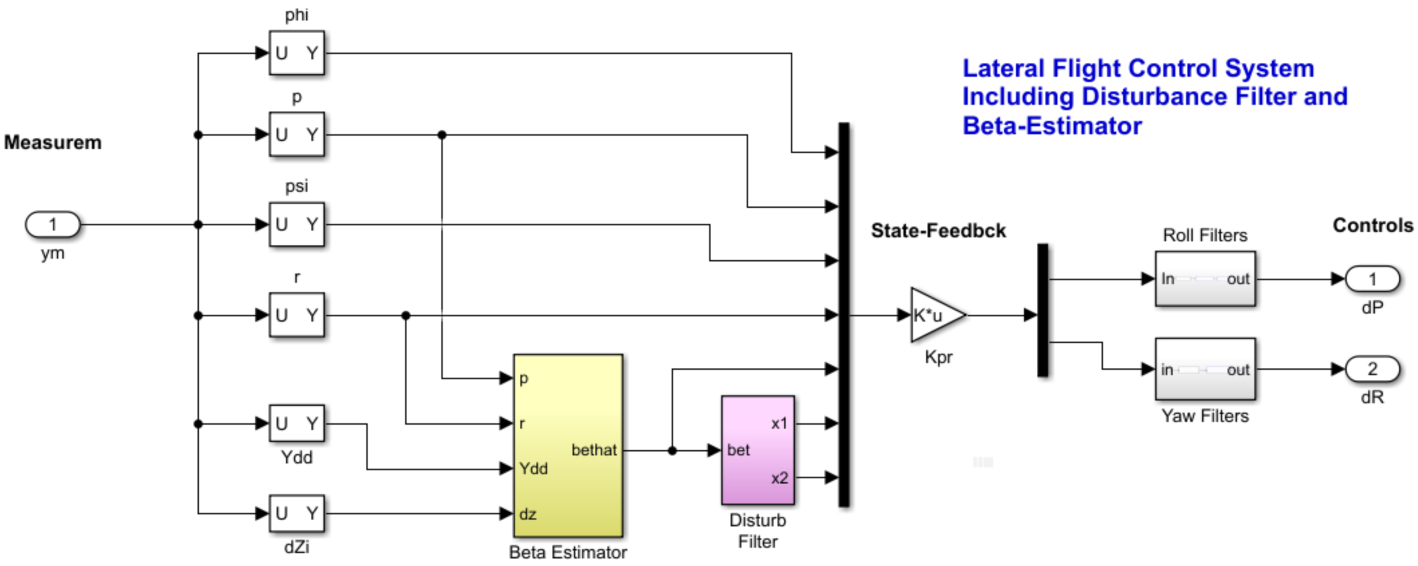
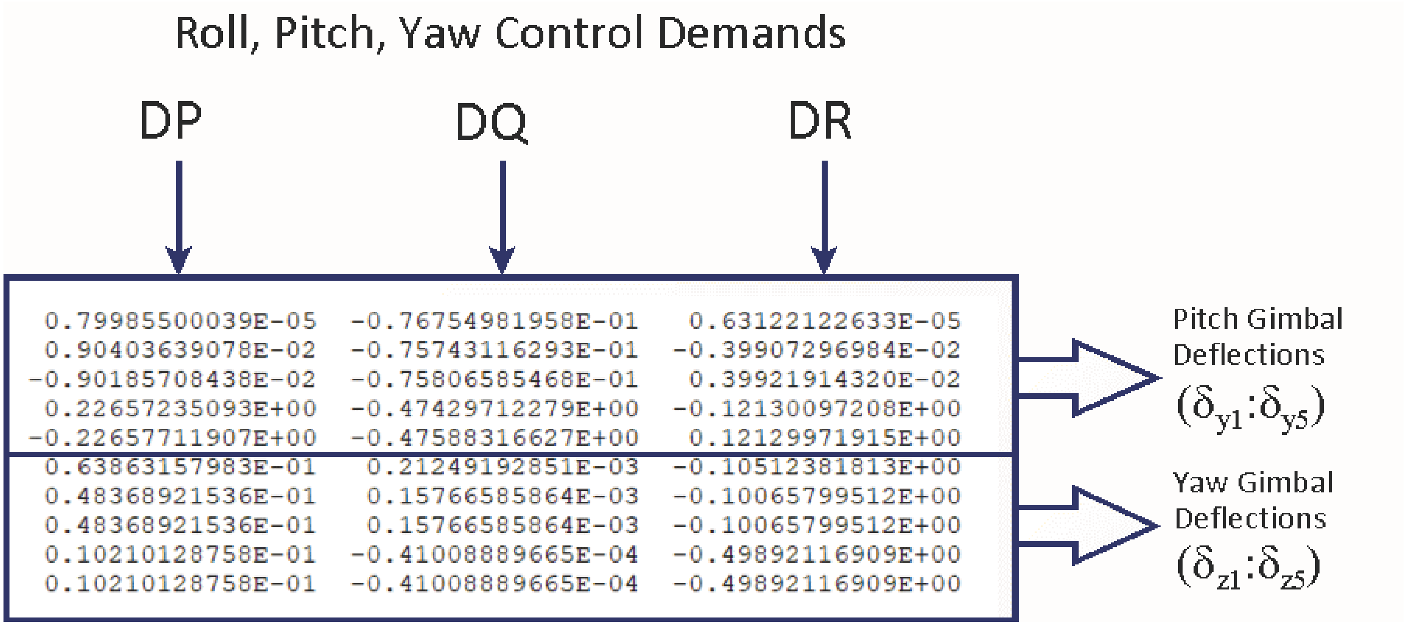


Figure 53 Roll & Yaw FCS with Estimator, β-filter, and Bending Filters

The TVC matrix converts the roll, pitch and yaw FCS demands to gimbal deflections that command the 10 actuators (5-pitch and 5-yaw commands). It is calculated by the Flixan Mixing-Logic algorithm based on TVC thrust, max gimbal deflection, and geometry relative to vehicle CG.



```
  0.79985500039E-05  -0.76754981958E-01   0.63122122633E-05
  0.90403639078E-02  -0.75743116293E-01  -0.39907296984E-02
 -0.90185708438E-02  -0.75806585468E-01   0.39921914320E-02
  0.22657235093E+00  -0.47429712279E+00  -0.12130097208E+00
 -0.22657711907E+00  -0.47588316627E+00   0.12129971915E+00
  0.63863157983E-01   0.21249192851E-03  -0.10512381813E+00
  0.48368921536E-01   0.15766585864E-03  -0.10065799512E+00
  0.48368921536E-01   0.15766585864E-03  -0.10065799512E+00
  0.10210128758E-01  -0.41008889665E-04  -0.49892116909E+00
  0.10210128758E-01  -0.41008889665E-04  -0.49892116909E+00
```

Figure 54 The TVC Converts the Roll, Pitch and Yaw Demands to Gimbal Deflection Commands

5.3 Control Analysis

Figure 55 is the open-loop model "Open_Loop.slx" used for analyzing stability, one loop at a time with the other two loops closed, shown in the pitch stability configuration in Fig.55. The entire plant model consisting of flex vehicle, actuators and sensors is included in the state-space system. Figure 56 is the closed-loop model "Sensitiv.slx" and contains the same elements. It is used for analyzing the system's sensitivity to wind-gusts in the frequency domain using Singular Value (Sigma) plots. The direction of the gust excites both pitch and yaw. The input is gust velocity scaled by the largest wind-gust velocity 30 (ft/sec) and the output consists of both (α & β) angles, divided by the maximum allowed angles 4°.

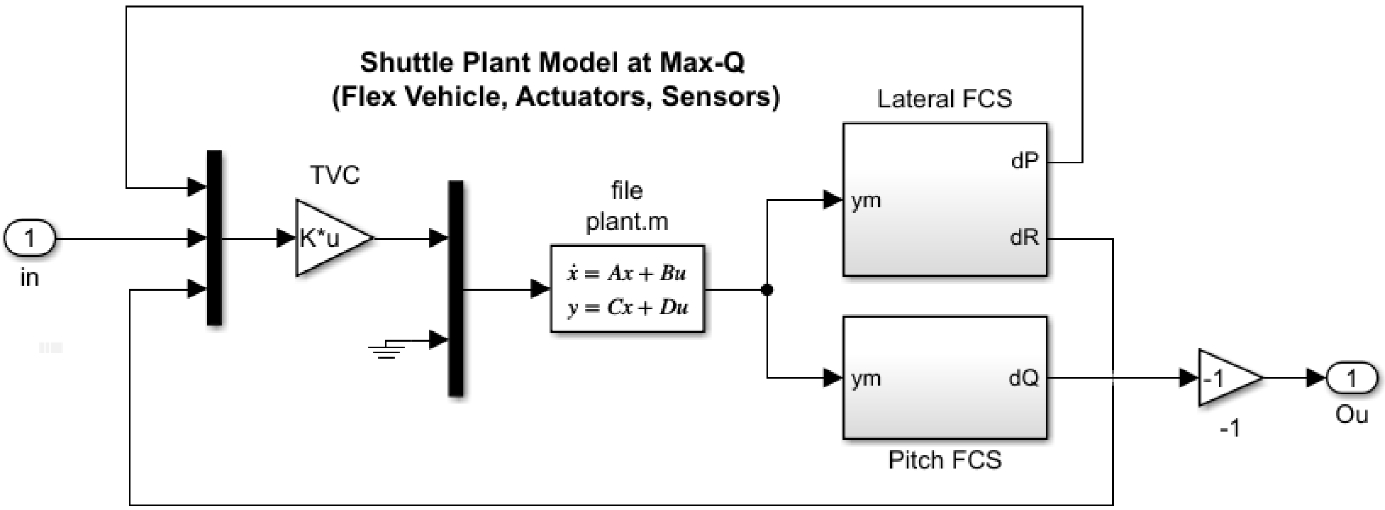


Figure 55 Coupled, Pitch and Lateral "Open_Loop.slx" Model Used for Stability Analysis

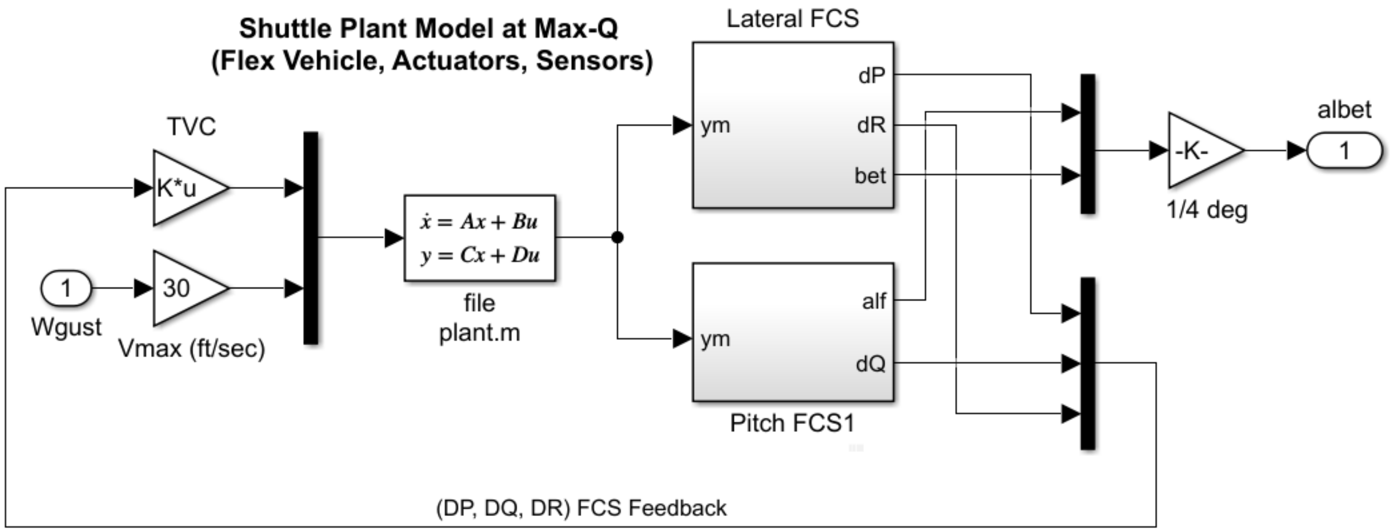


Figure 56 Scaled Sensitivity Analysis Model "Sensitiv.slx"

The file "freq.m" below uses the open-loop model in Fig.55 to calculate the Bode and Nichols plots for pitch, yaw and roll stability analysis, and the Sensitivity model in Fig.56 to analyze sensitivity to disturbances using SV plot. The Sensitivity Function: (α,β)/Vgust is shown in Figure 60. Its magnitude is less than one at all frequencies and it has a dip at 1 (rad/sec), as expected.

```
% Frequency Response Analysis File
init;
[Ao, Bo, Co, Do]= linmod('Open_Loop');         % Stability Analysis
[As, Bs, Cs, Ds]= linmod('Sensitiv');          % Sensitiv Analysis
w=logspace(-3, 3, 30000);                      % Define Frequ Range
syso= ss(Ao,Bo,Co,Do);                         % Create SS System
syss= ss(As,Bs,Cs,Ds);                         % Create SS System
figure(1); nichols(syso,syso,w)                % Plot Nichol's Chart
figure(2); bode(syso,w)                        % Plot Bode
sigl=sigma(syss,w);                            % SV Bode
figure(3); loglog(w,sigl,'r',w,sigl,'b')       % Plot SV Bode
```

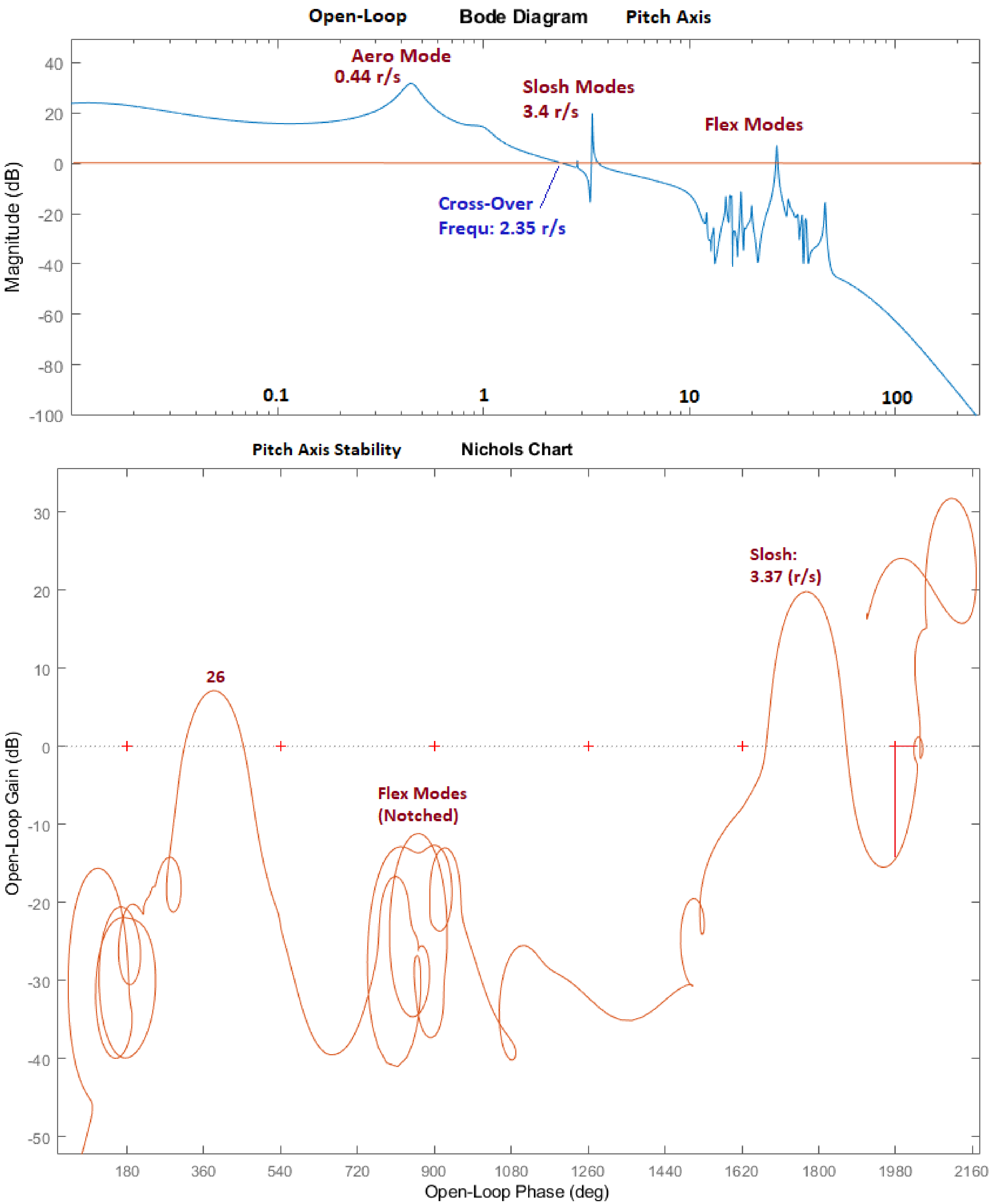


Figure 57 Pitch Axis Stability Analysis

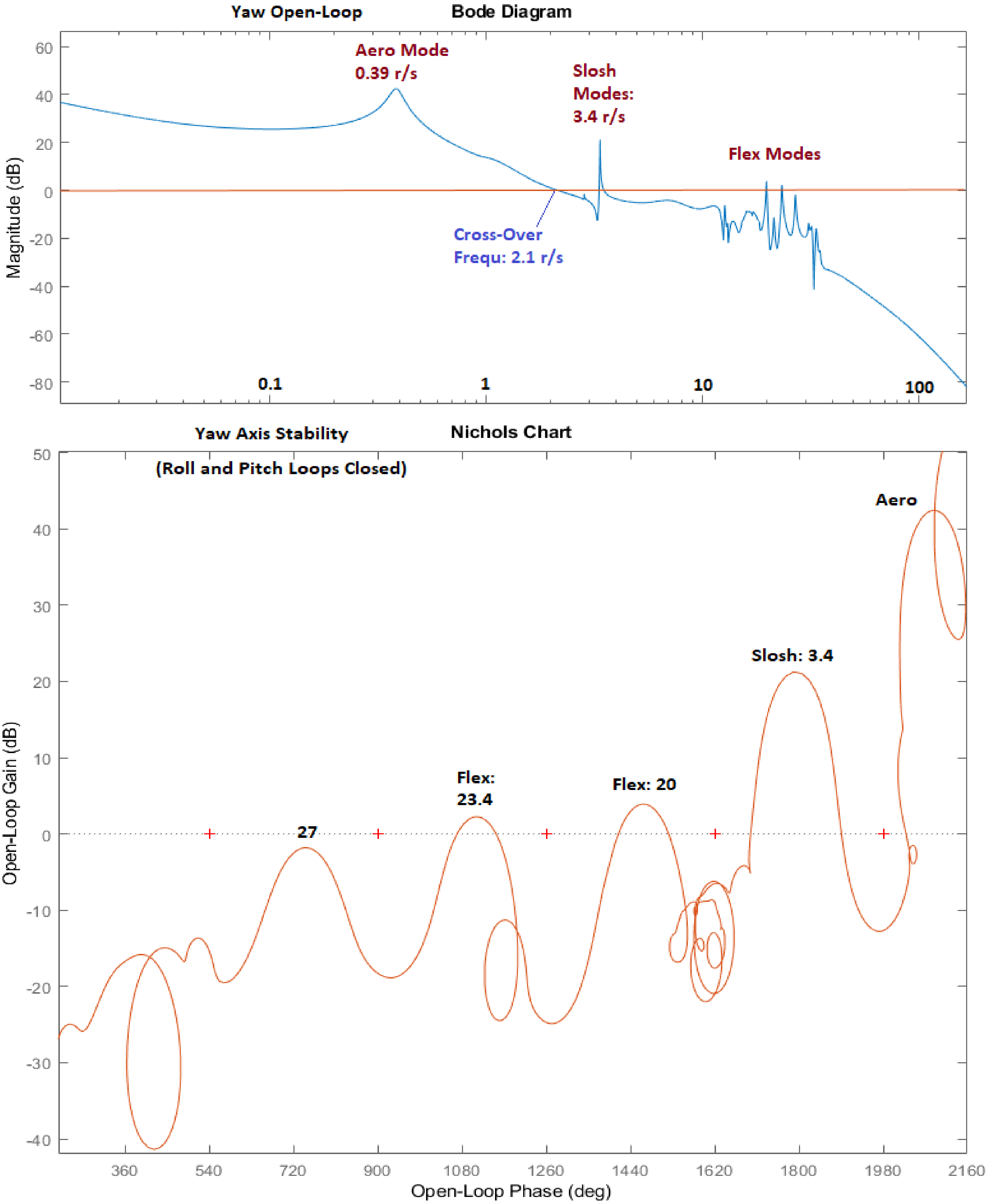
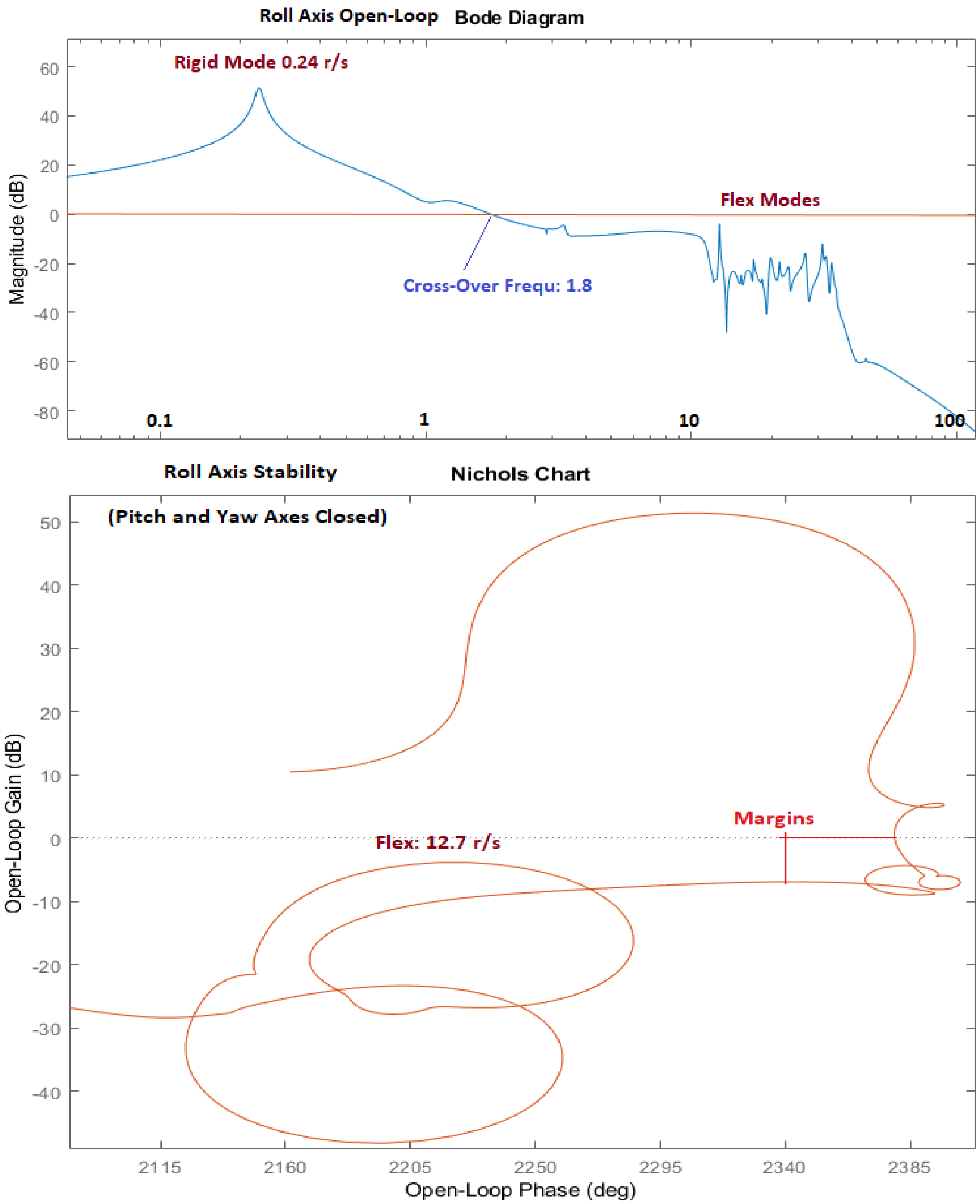


Figure 58 Yaw Axis Stability Analysis


Roll Axis Open-Loop Bode Diagram

Roll Axis Stability Nichols Chart

Figure 59 Roll Axis Stability Analysis

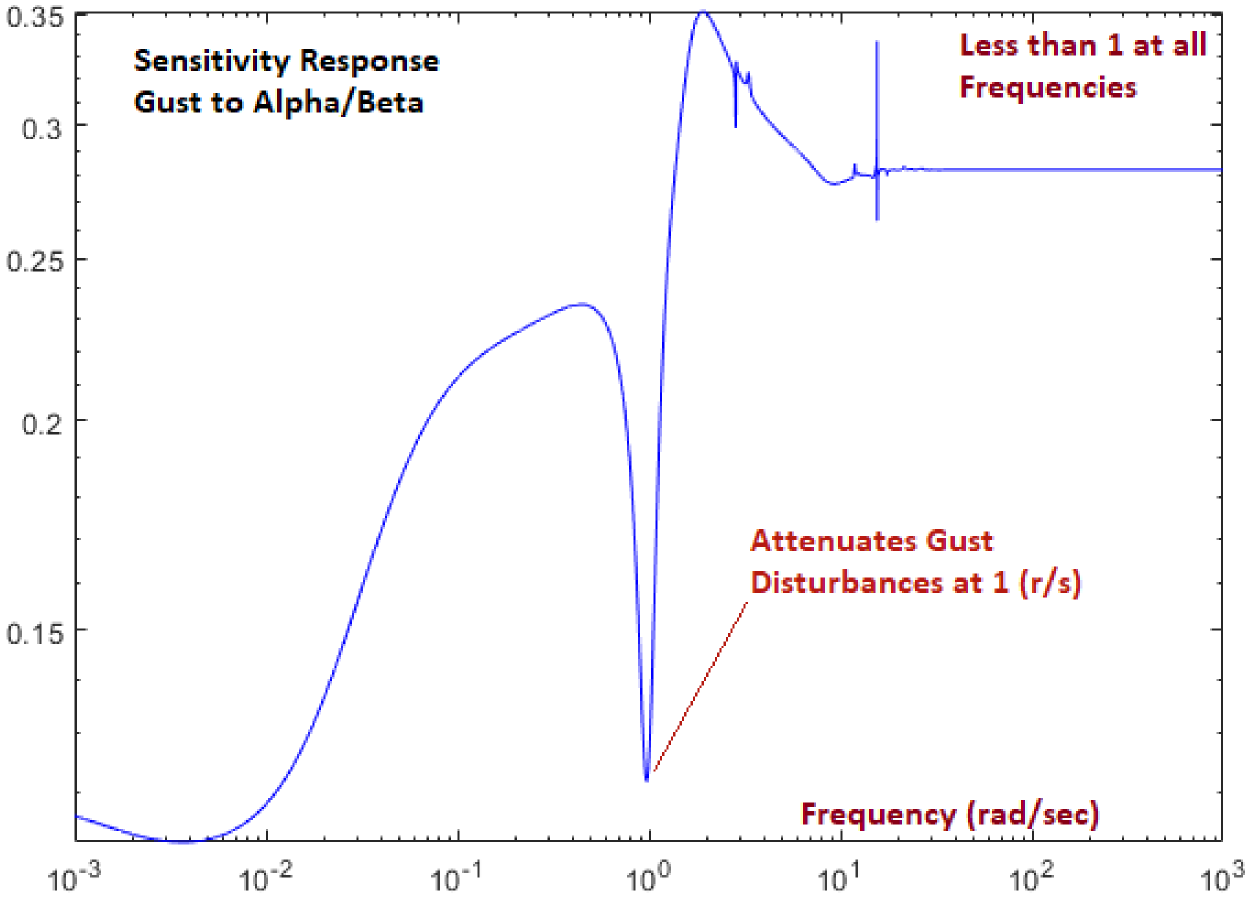


Figure 60 Sigma Plot of the Sensitivity Function Between Gust Disturbance and α & β Output

5.4 Simulation Models

We have two Simulink models to perform time-slice simulations at Max-Q. A continuous model "Sim_Flex_s.slx" in Figure 61 that uses the continuous plant "*Shuttle Plant Model at Max-Q (Flex Vehicle, Actuators, Sensors)",* and a discrete model "Sim_Flex_z.slx" in Figure 62, discretized at dT=0.02 sec, that includes the z-transformed plant "*Discrete Shuttle Plant Model at Max-Q (Flex Vehicle, Actuators, Sensors)*". The discrete simulation also includes the z-transformed pitch and lateral flight control systems which include the estimators and filters.

The first set of Figures 63 show the response of the continuous simulation "Sim_Flex_s.slx" to a wind-gust pulse of 30 (feet/sec) max velocity which lasts about 9 sec. The gust produces 0.5° of α and β transients and 0.5° of negative roll, and the engines deflect to counteract the transient and stabilize the vehicle.

The second set of Figures 64 show the response of the discrete simulation "Sim_Flex_z.slx" to unit step attitude commands in pitch and yaw. Not in roll. Pitch and yaw slowly converge to the commanded values. There is a strong roll transient due to the aerodynamic coupling that slowly decays towards zero. Flexibility is visible in the accelerometers

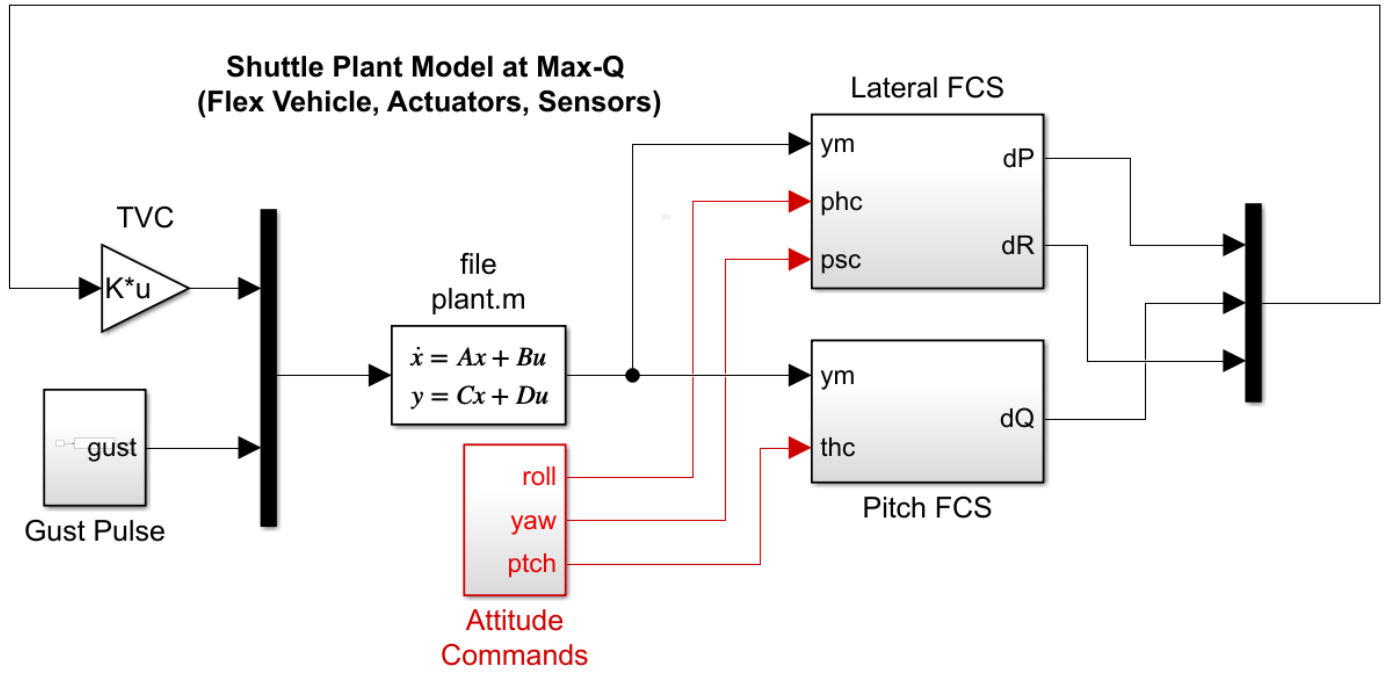


Figure 61 Closed-Loop Shuttle Ascent Simulation Model "Sim_Flex_s.slx" (S-plane)

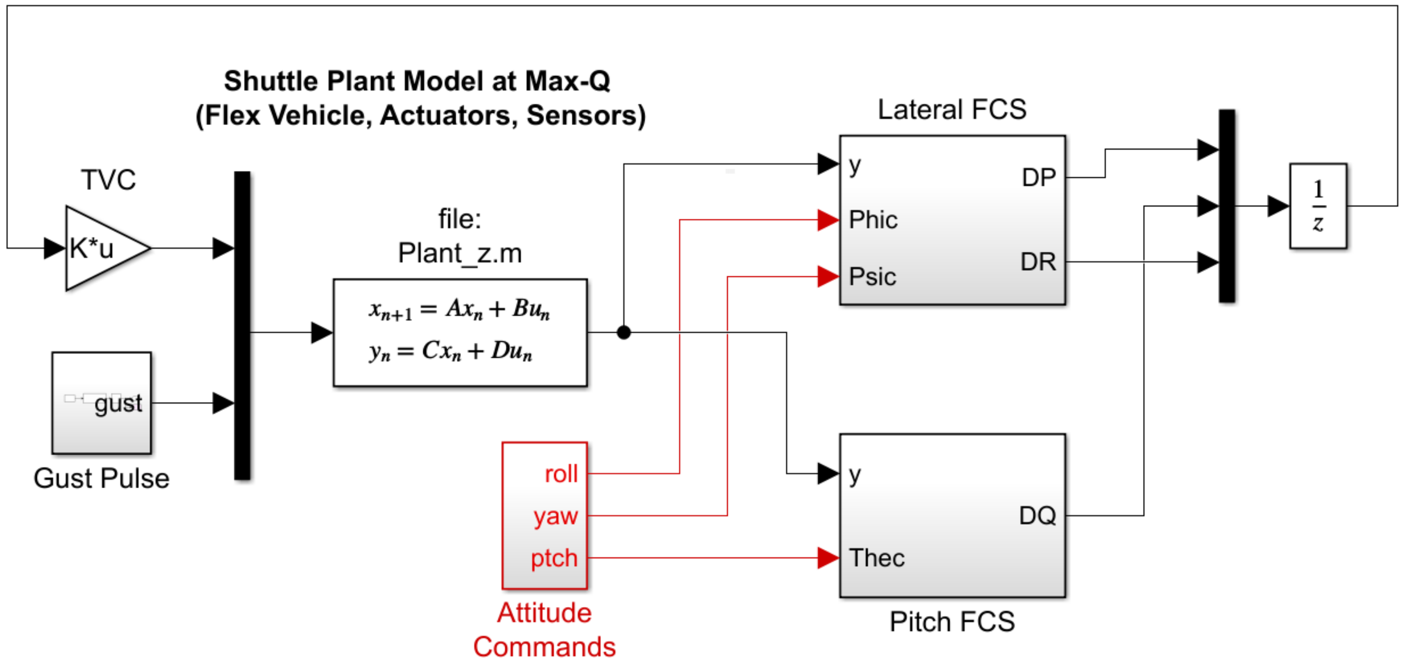
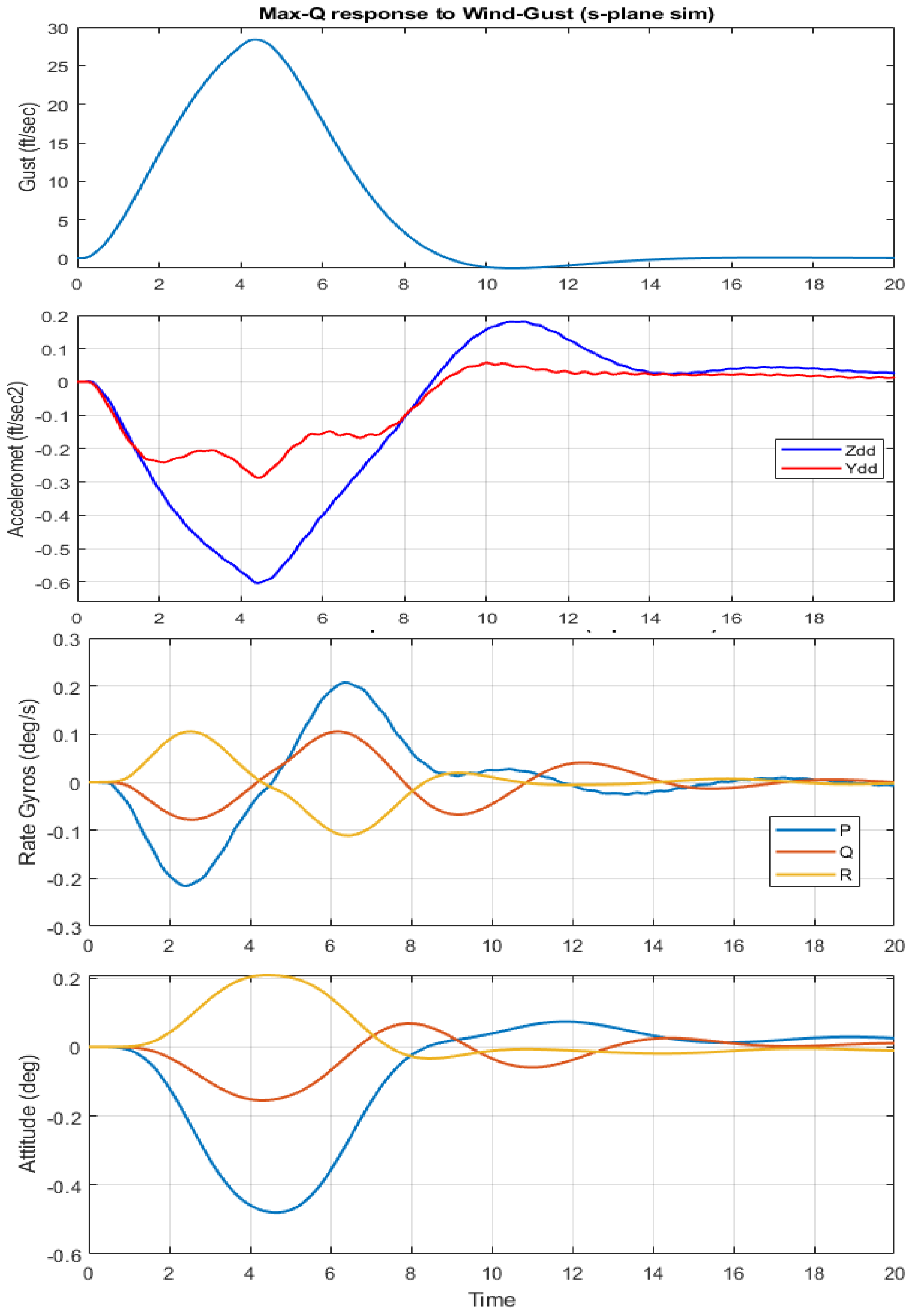


Figure 62 Closed-Loop Shuttle Ascent Simulation Model "Sim_Flex_z.slx" (Z-plane)



Max-Q response to Wind-Gust (s-plane sim)
Gust (ft/sec)
30
25
20
15
10
5
0
0
2
4
6
8
10
12
14
16
18
20
Acceleromet (ft/sec2)
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
Zdd
Ydd
0
2
4
6
8
10
12
14
16
18
Rate Gyros (deg/s)
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
P
Q
R
0
2
4
6
8
10
12
14
16
18
20
Attitude (deg)
0.2
0
-0.2
-0.4
-0.6
0
2
4
6
8
10
12
14
16
18
20
Time

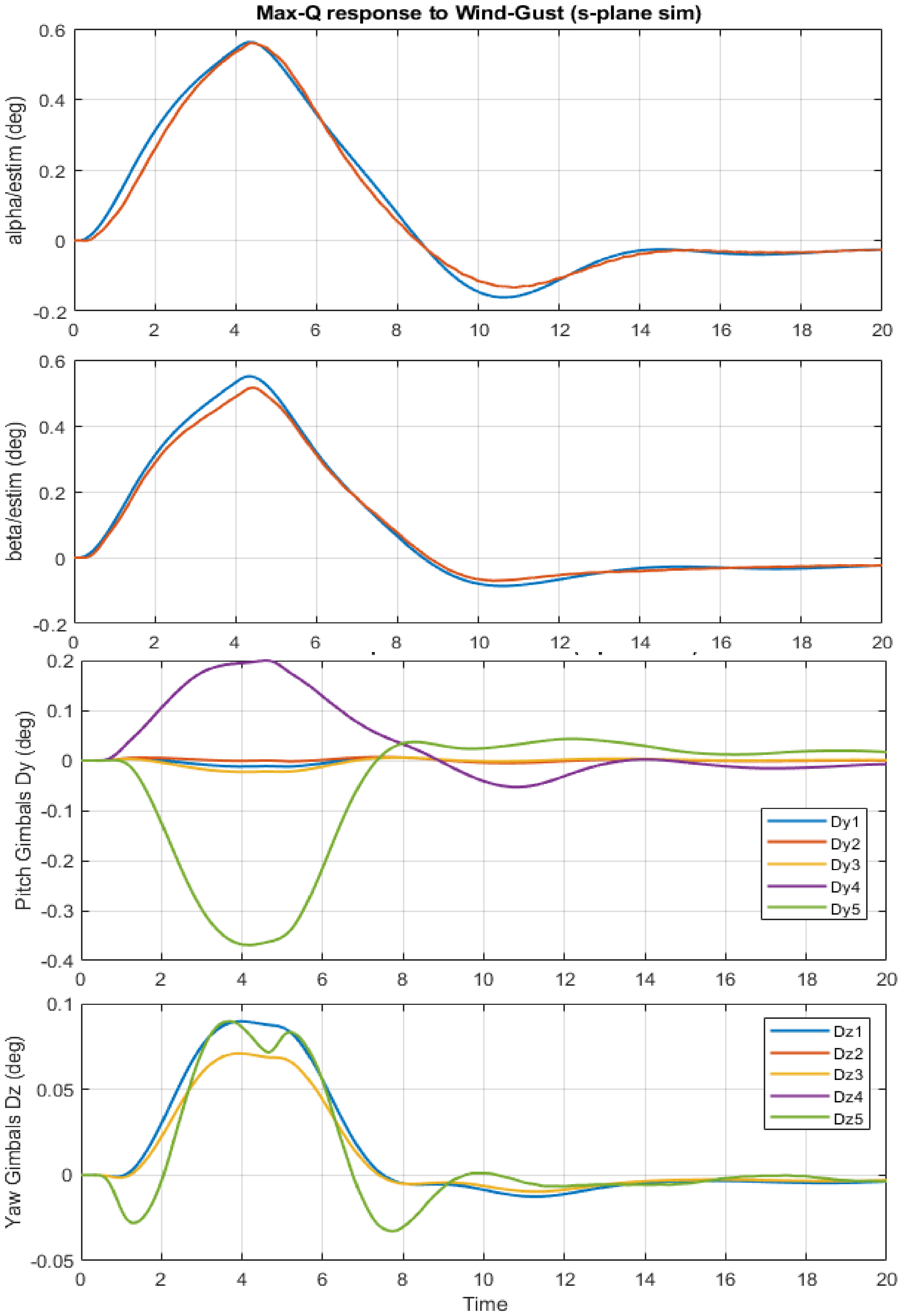
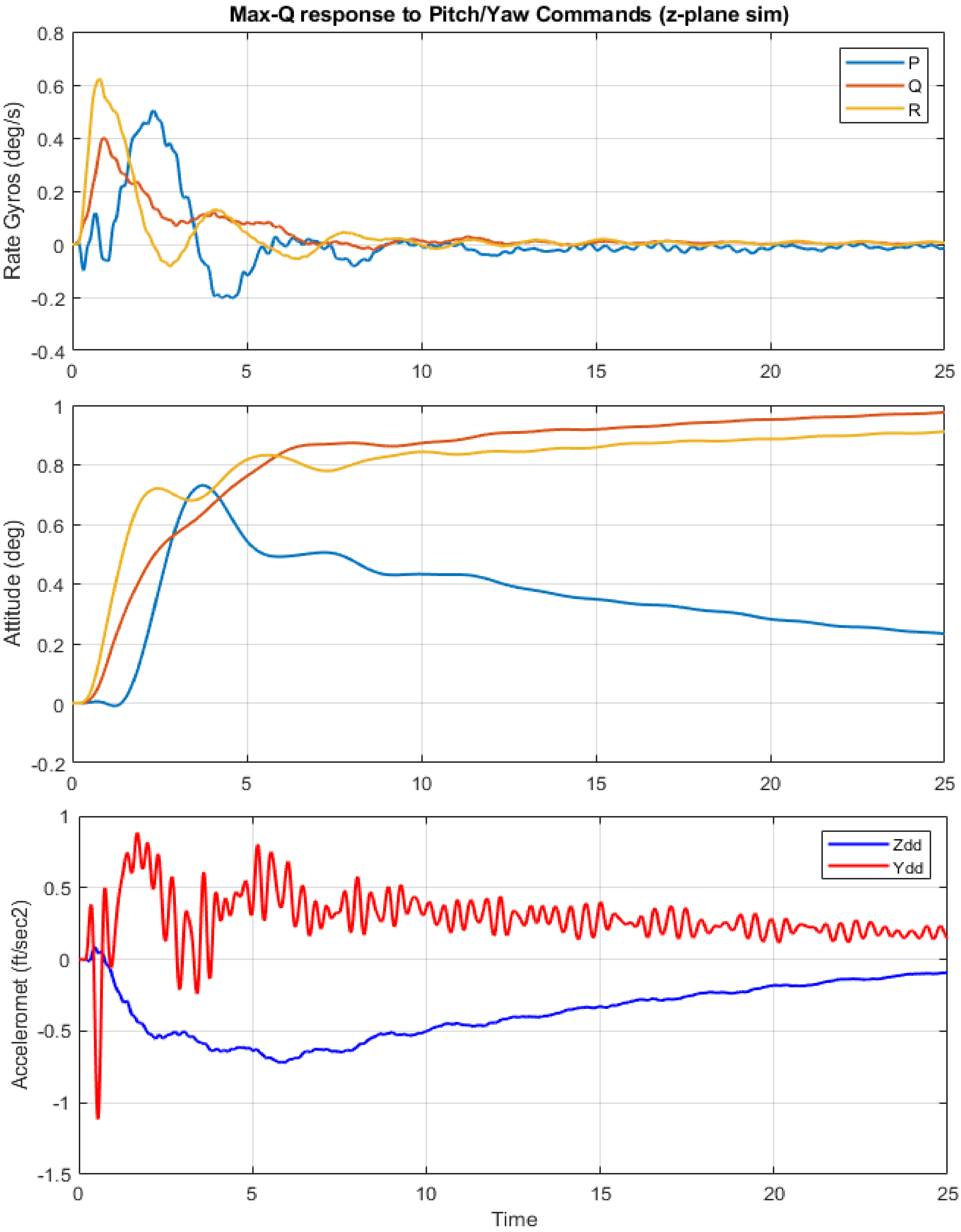


Figure 63 Continuous System “Sim_Flex_s.slx” Response to Wind-Gust Pulse


Max-Q response to Pitch/Yaw Commands (z-plane sim)
Rate Gyros (deg/s)
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
5
10
15
20
25
P
Q
R
Attitude (deg)
1
0.8
0.6
0.4
0.2
0
-0.2
0
5
10
15
20
25
Acceleromet (ft/sec2)
1
0.5
0
-0.5
-1
-1.5
0
5
10
15
20
25
Time
Zdd
Ydd

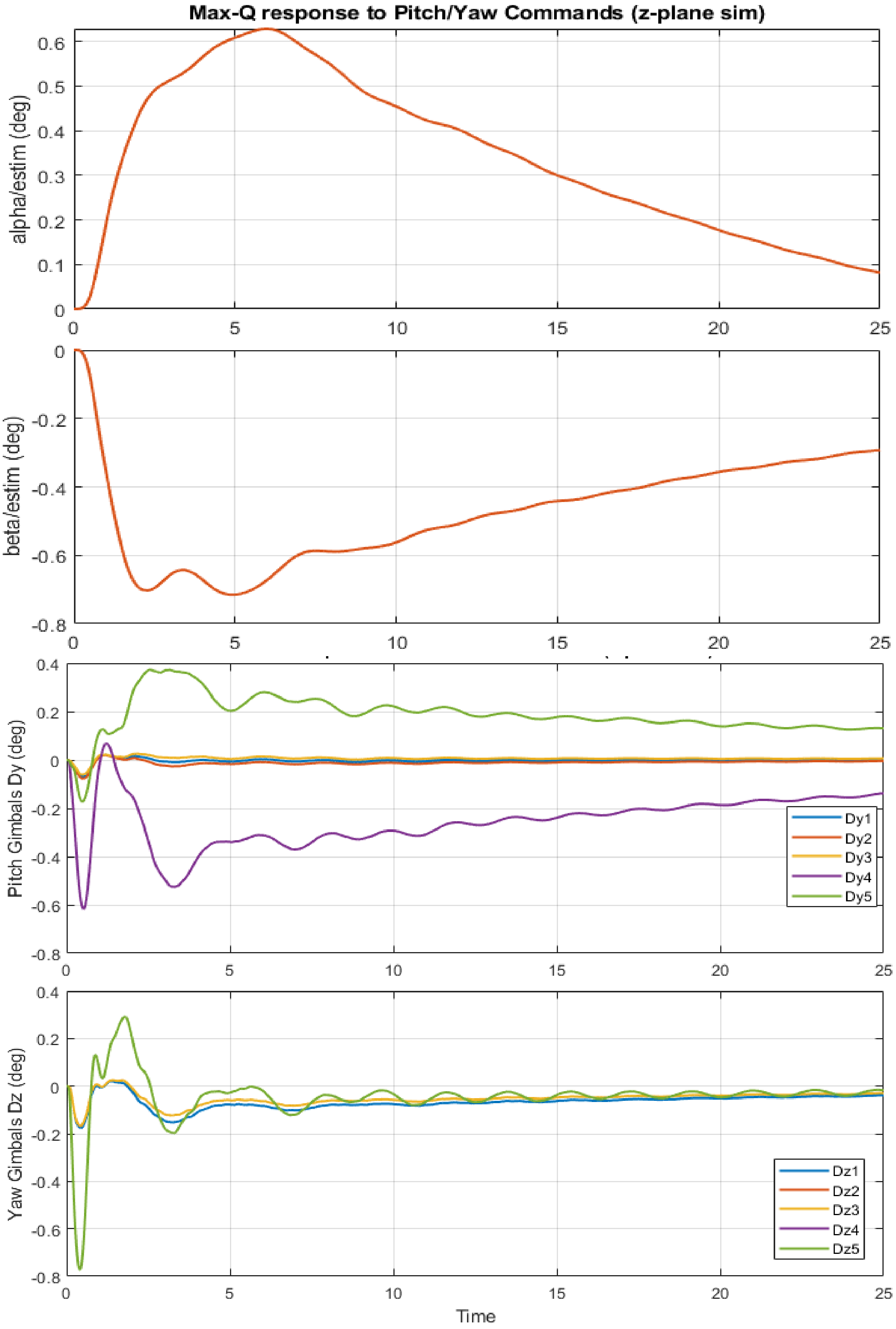


Figure 64 Discrete System Response to Pitch and Yaw Attitude Commands

Launch Vehicle Control Design

In this example we will model a launch vehicle during first stage at high dynamic pressure and use the H-infinity method to design the flight control system (FCS) at a fixed time-slice. This vehicle is cylindrical which simplifies the design because pitch and yaw are almost identical. The propulsion consists of 9 engines producing a total thrust of 230,000 (lbf) but only 8 of the engines are gimbaling in pitch and yaw. The center engine is fixed and it does not gimbal or throttle. In Section-1 we begin with a rigid body model and design a preliminary control system at a fixed flight condition, t=85 sec, a little before Max-Q. We will analyze only the pitch axis because the yaw axis is identical and the roll axis is trivial and it will be analyzed in Section-2. In this high-Q condition the control design includes several performance requirements. Our primary design goal is to reduce the aerodynamic loading on the vehicle structure caused due to cyclic aerodynamic disturbances in the frequency range between 0.6 and 0.9 (rad/sec). When the vehicle ascends through atmospheric layers of varying wind velocities, they produce aero disturbances in this frequency range. We therefore want to desensitize the vehicle in this frequency range by creating a dip in the sensitivity function. In essence, the vehicle anticipates and reacts against the cyclic disturbance by turning its nose towards the wind. This is not a typical load-relief system which reacts against a wind-shear in the steady-state but in this case it anticipates the cyclic disturbance by oscillating the vehicle attitude towards the cyclic wind and thus reducing the alpha oscillations and also the aerodynamic loading against the structure. Sensitivity reduction is accomplished by the introduction of a 4^{th} order alpha-filter. The H-infinity design model is augmented by including the alpha-filter to further improve sensitivity at the desired frequency range. The H-infinity algorithm creates a stabilizing controller and the filter is included in the FCS. In addition to cyclic disturbances, we also want to improve the control system robustness against uncertainties or variations of some internal vehicle parameters, such as aerodynamic coefficients and mass properties. They are internal parameters where the maximum amount of variation from nominal values are known. The internal variations are treated as external disturbances and performance criteria in the H-infinity optimization model and the algorithm produces a control system that is robust to those internal variations, as well as to the external disturbances described.

A design however is not complete until it is tested with all the details. In Section-2 we will further analyze the vehicle system using detailed models that include roll, pitch and yaw coupled axes, propellant sloshing, structural modes and actuator models including tail-wags-dog and load-torque feedback. Some of the modes are unstable and low-pass and notch filters are included in order to stabilize them. The analysis includes stability margins using Bode and Nichols charts, sensitivity to gusts and robust performance, which is the ability to satisfy good performance and robustness to parameter uncertainties and requires Structured Singular Values analysis. Time-slice simulations are also included to demonstrate the control system's response to gusts and to attitude commands. In Sections 1 & 2 we present a simpler state-feedback design where the angle of attack is measurable. In Sections 3 & 4 we use an estimator to replace the alpha feedback with Nz accelerometer feedback.

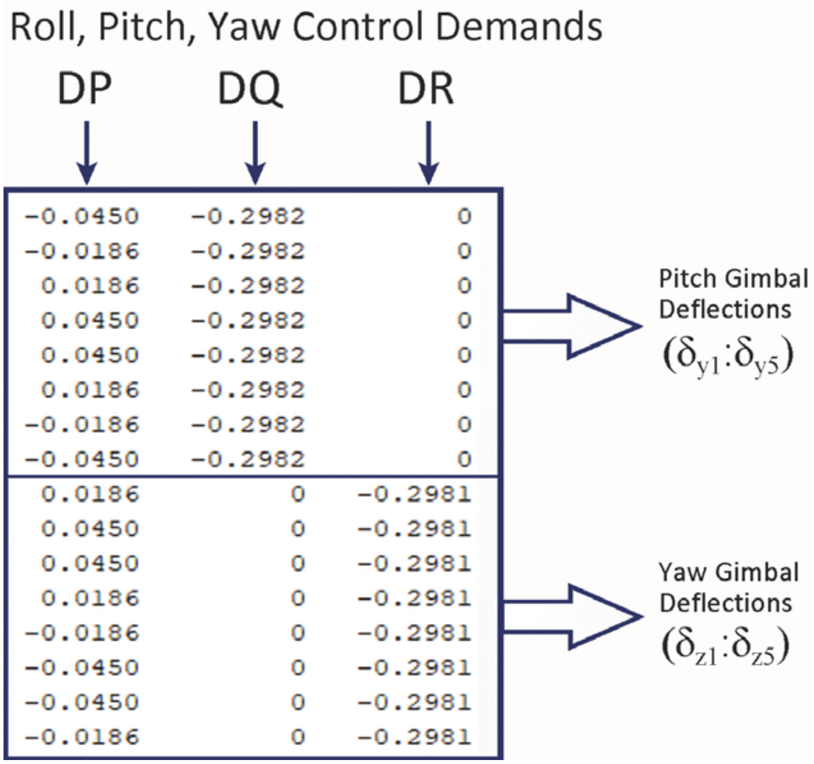
1.0 Rigid Body H-Infinity Control Design

In this section we will design the state-feedback control law for the launch vehicle pitch axes. The yaw axis is identical by symmetry. The input file in this example is "*LV_HighQ.inp*" located in directory "*Flixan\Control Analysis\Hinfinity\Examples\Launch Vehicle Design & Analysis\1-LV Gust Robust Design, Uncs, State-Feedback*". The vehicle model is rigid. Its title is "*First Stage Analysis Model, T=85.0 sec*" and it does not include TWD dynamics. We will create several models from this vehicle starting with the TVC.

1.1 The Thrust Vector Control Matrix

Figure 1 TVC Engines

The TVC consists of 9 engines. The central engine is fixed and only the 8 peripheral engines are gimbaling in pitch and yaw to produce the required roll, pitch and yaw torques required to maneuver the vehicle. The TVC matrix, in Figure 2, converts the roll, pitch and yaw acceleration demands coming from the flight control system (FCS) to pitch and yaw gimbal deflection commands for the 8 engines. The TVC matrix diagonalizes the plant between the (roll, pitch, yaw) acceleration demands and the vehicle acceleration outputs. The gimbal deflections produce acceleration outputs that approximate the acceleration demands. It is created by the Flixan program based on vehicle geometry and thrusts by processing the mixing logic dataset "*Mixing Logic for First Stage Analysis Model, T=85.0 sec*" and the vehicle data.



DP	DQ	DR
-0.0450	-0.2982	0
-0.0186	-0.2982	0
0.0186	-0.2982	0
0.0450	-0.2982	0
0.0450	-0.2982	0
0.0186	-0.2982	0
-0.0186	-0.2982	0
-0.0450	-0.2982	0
0.0186	0	-0.2981
0.0450	0	-0.2981
0.0450	0	-0.2981
0.0186	0	-0.2981
-0.0186	0	-0.2981
-0.0450	0	-0.2981
-0.0450	0	-0.2981
-0.0186	0	-0.2981

Figure 2 TVC Matrix Converts the Control Acceleration Demands to Gimbal Deflections

1.2 Creating the Design System and the Control Synthesis Model for H-Infinity Design

The H-infinity algorithm requires a Synthesis Model (SM) which is used as reference in order to generate the control system. We do this in steps. We must first combine the vehicle system with the TVC and then extract the pitch variables to create the vehicle system: "*Pitch Design Model with TVC*" which includes the TVC and consists of states: pitch attitude θ, body rate q, and angle of attack α. The inputs and outputs include 6 additional inputs and outputs that connect with the uncertainty block Δ which represent internal parameter uncertainties. Then, the design model is augmented by including 5 additional states: a 4th order filter "Alpha Filter" that amplifies the angle of attack α at the disturbance frequency range 0.6 to 0.9 (rad/sec) and a "Theta Integrator" which introduces θ-integral in the optimization to improve attitude tracking. The title of the augmented design model is "*Pitch H-inf Design Model with TVC and Alpha-Filter*" and it will be used to create the Synthesis Model (SM) which is presented to the H-infinity algorithm.

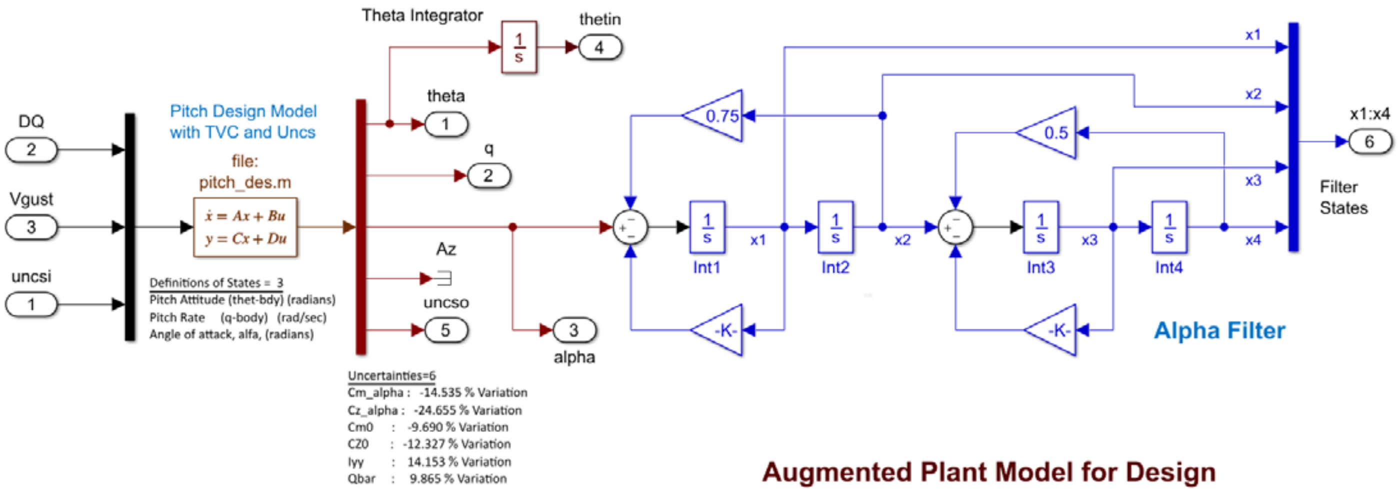
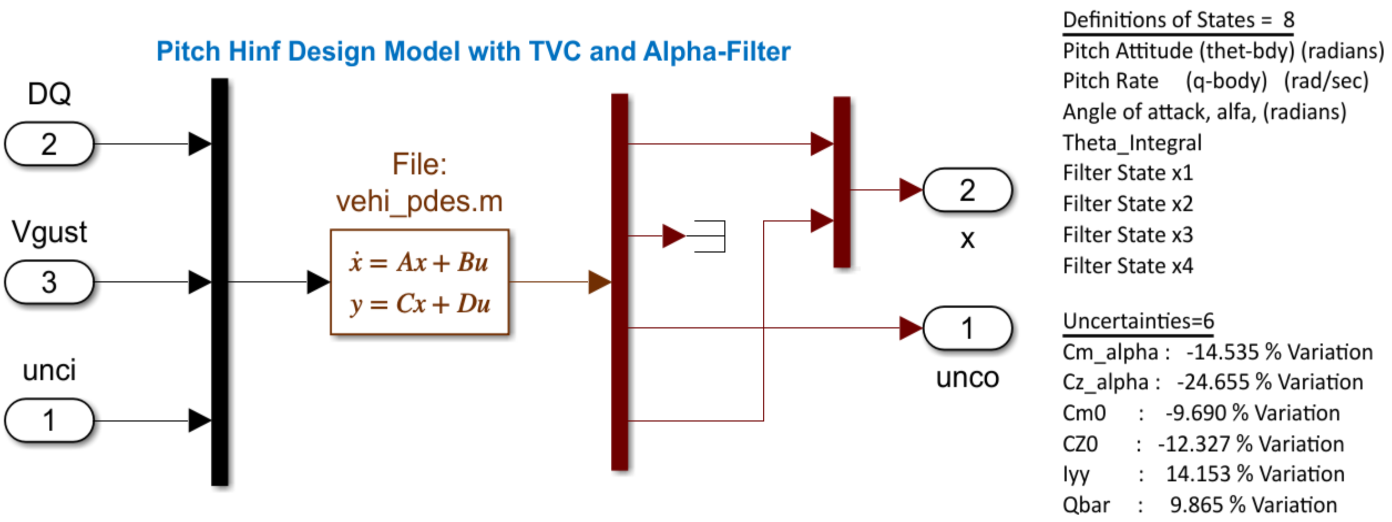


Figure 3 The Design Model is Augmented by Including a 4th Order Filter and a Theta Integrator



Definitions of States = 8
Pitch Attitude (thet-bdy) (radians)
Pitch Rate (q-body) (rad/sec)
Angle of attack, alfa, (radians)
Theta_Integral
Filter State x1
Filter State x2
Filter State x3
Filter State x4

Uncertainties=6
Cm_alpha : -14.535 % Variation
Cz_alpha : -24.655 % Variation
Cm0 : -9.690 % Variation
CZ0 : -12.327 % Variation
Iyy : 14.153 % Variation
Qbar : 9.865 % Variation

Figure 4 The Augmented Design System "Pitch H-inf Design Model with TVC and Alpha-Filter" is saved in State-Space form. It Includes the Pitch Design model with TVC, the 4th Order Notch Filter and the Theta Integrator

The SM consists of 9 matrices and the performance adjustment gains. It is created from the augmented design system interactively by selecting the necessary inputs and outputs and entering the performance gains. It is then saved in the systems file "*LV_HighQ.Qdr*", and its title is "*Pitch H-inf Design Model with TVC and Alpha-Filter/SM-2*". Figure-5 shows the SM system P(s) in block diagram form which has 3 sets of input/ output vector pairs. The first set (w_p and z_p) are fictitious inputs and outputs that connect to the plant uncertainties block Δ which is extracted from the plant. The B_1 and C_1 matrices of P(s) have already been scaled to match with uncertainties Δ that vary between ±1. The inputs (w_p) are treated like disturbances and the outputs (z_p) are included in the optimization criteria. The second input/output set (w and z) are the actual external disturbance inputs and performance criteria outputs. In this case the disturbance w is the wind-gust velocity (V_{gust}) and the criterion z is the angle of attack dispersion relative to the velocity vector which is affected by gusts and it is an aerodynamic load indicator. The last I/O set (u_c and y_m) are the control inputs and measurements that connect with the control system K(s) which in this case is (1x8) state-feedback gain matrix (Kqhi).

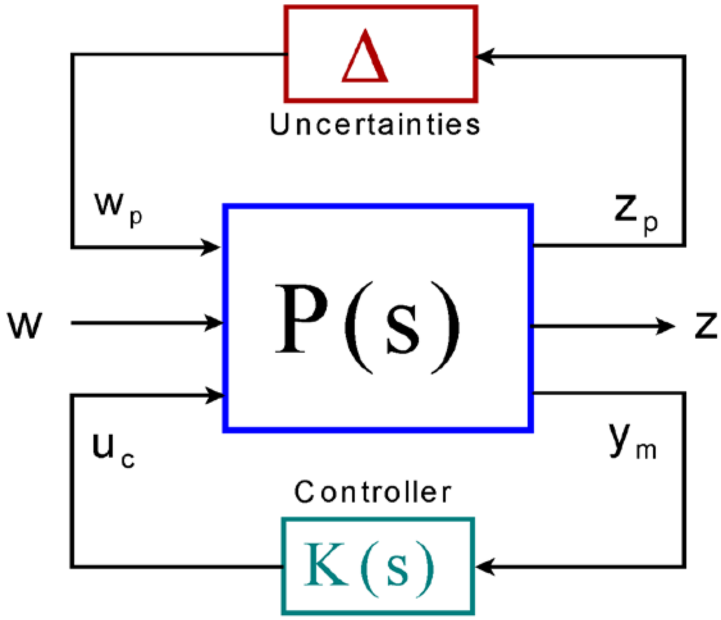
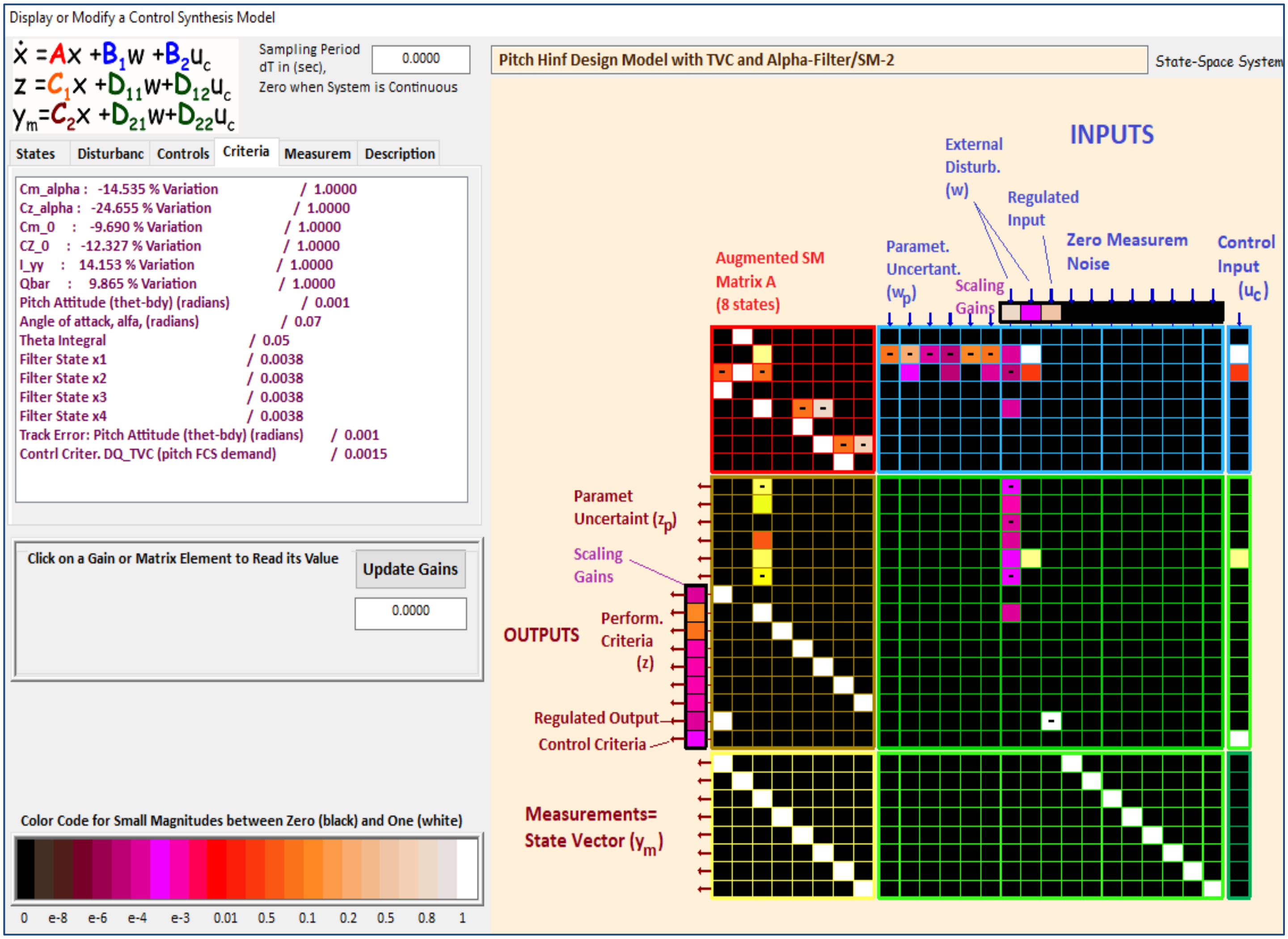


Figure 5 Synthesis Model



$$\dot{x} = Ax + B_1 w + B_2 u_c$$
$$z = C_1 x + D_{11} w + D_{12} u_c$$
$$y_m = C_2 x + D_{21} w + D_{22} u_c$$

Sampling Period dT in (sec), Zero when System is Continuous: 0.0000

Pitch Hinf Design Model with TVC and Alpha-Filter/SM-2

Cm_alpha : -14.535 % Variation / 1.0000
Cz_alpha : -24.655 % Variation / 1.0000
Cm_0 : -9.690 % Variation / 1.0000
CZ_0 : -12.327 % Variation / 1.0000
I_yy : 14.153 % Variation / 1.0000
Qbar : 9.865 % Variation / 1.0000
Pitch Attitude (thet-bdy) (radians) / 0.001
Angle of attack, alfa, (radians) / 0.07
Theta Integral / 0.05
Filter State x1 / 0.0038
Filter State x2 / 0.0038
Filter State x3 / 0.0038
Filter State x4 / 0.0038
Track Error: Pitch Attitude (thet-bdy) (radians) / 0.001
Contrl Criter. DQ_TVC (pitch FCS demand) / 0.0015

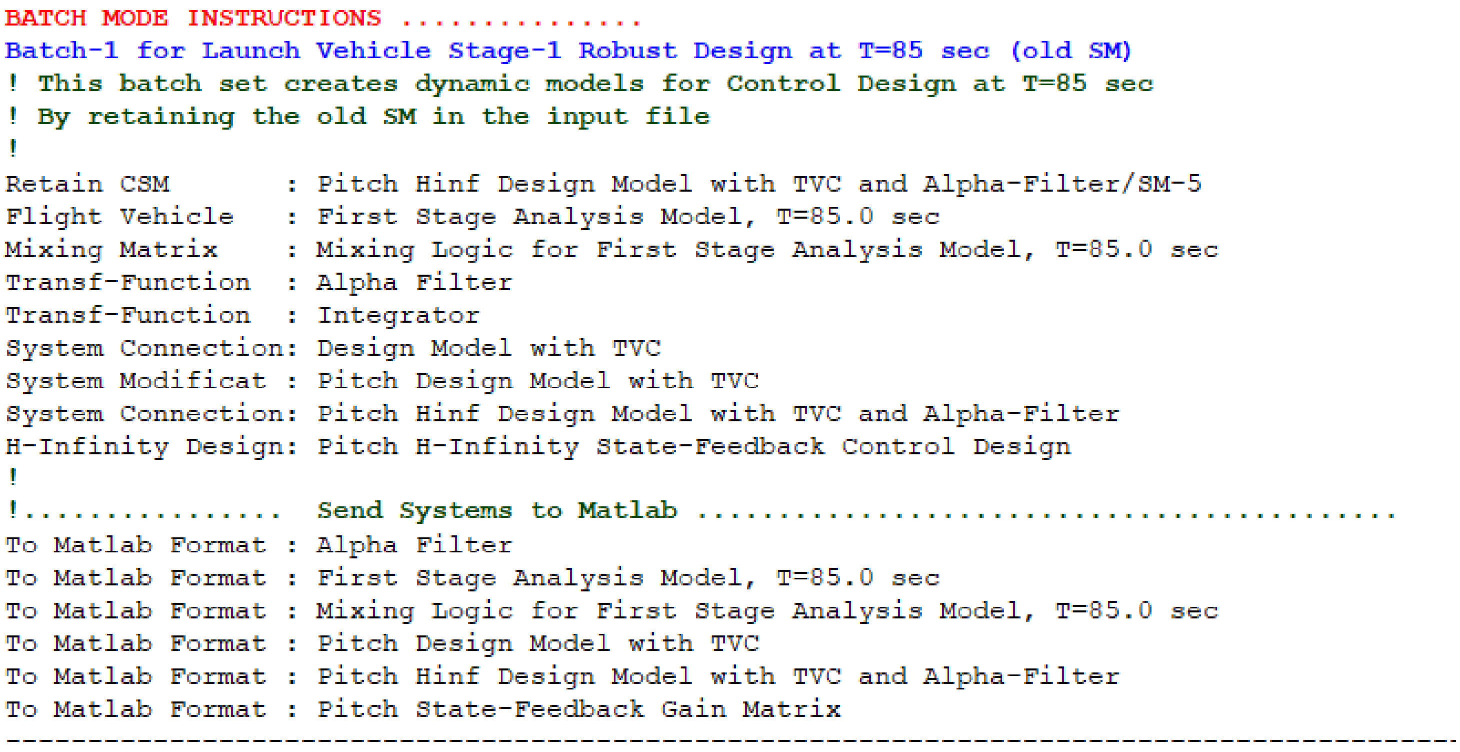
Click on a Gain or Matrix Element to Read its Value: 0.0000

Figure 6 The Synthesis Model in More Detail Systems Form

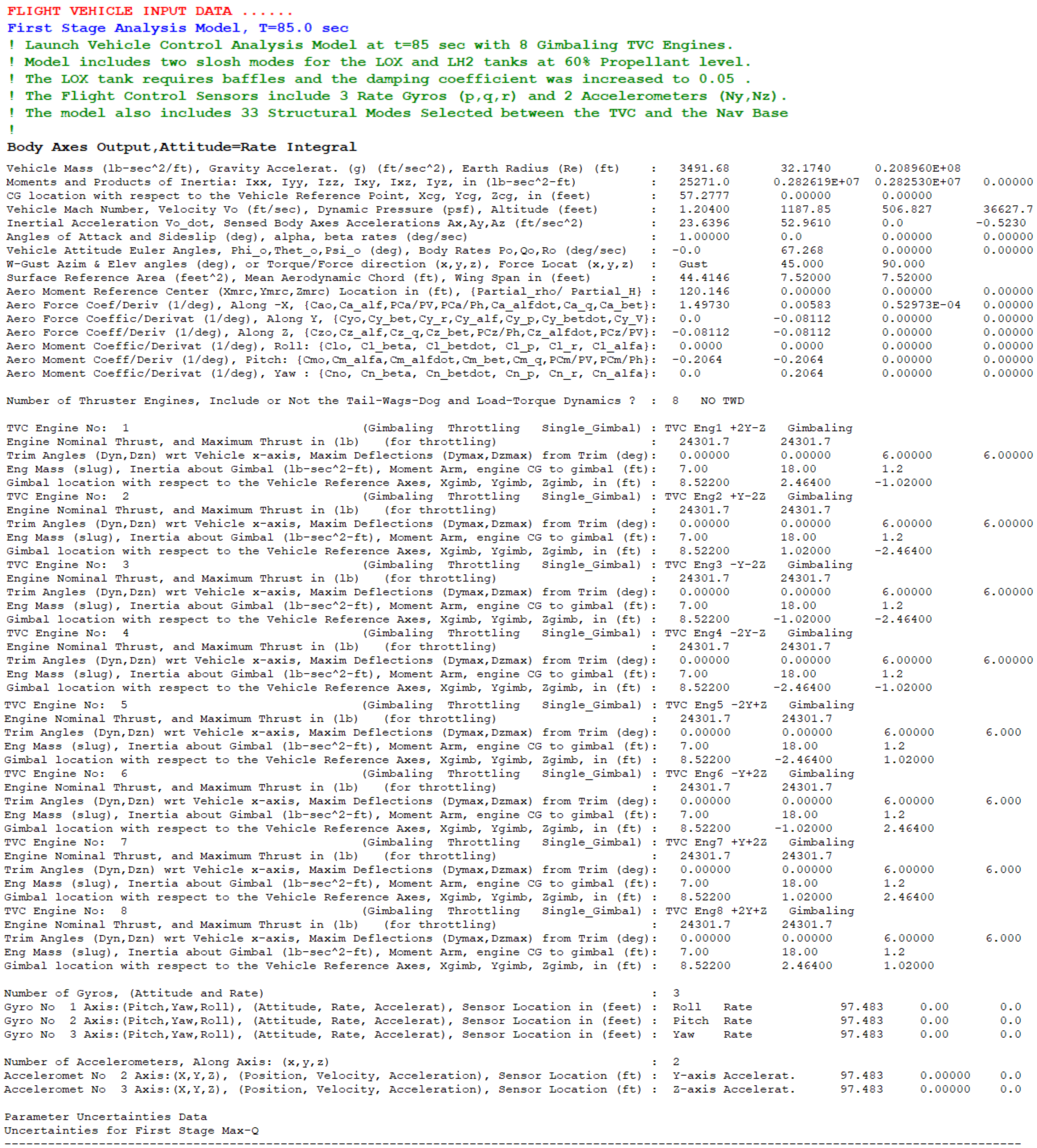
The SM is shown in a more detail systems form in Figure-6. It has 8 states and consists of 9 matrices and the adjustable gains are shown at the w-inputs and z-outputs. The w-inputs consist of: the gust velocity, control input noise, and a command for a regulated output. The z-outputs consist of: 7 outputs to be optimized, a regulated output error, and one control input. The gains are adjusted for a satisfactory trade-off between control system bandwidth versus stability and sensitivity to disturbances (both internal and external).

1.3 Input File

The input file "*LV_HighQ.Inp*" contains the datasets used by the Flixan program to create the vehicle systems and matrices. A batch set at the top of the file, title: "*Batch-1 for Launch Vehicle Stage-1 Robust Design at T=85 sec (old SM)*" is used to process the entire file in batch mode. There is a vehicle dataset "*First Stage Analysis Model, T=85.0 sec*" that generates the rigid vehicle system and includes parameter uncertainties. A mixing logic dataset "*Mixing Logic for First Stage Analysis Model, T=85.0 sec*" that uses the vehicle data to create the TVC matrix. The theta integrator and alpha-filters are implemented in transfer-function form and saved in state-space system "*Alpha Filter*". The TVC matrix is then combined with the vehicle system to create the full 3-axes design model "*Design Model with TVC*". Since we are only interested in the pitch axis, we extract the pitch subsystem and save it in "*Pitch Design Model with TVC*". This system is then augmented by including the alpha-filter and the theta-integrator, as it is shown in Figure-3, and its title is "*Pitch H-inf Design Model with TVC and Alpha-Filter*", also shown compact in Figure-4. From this system we then create the Synthesis Model: "*Pitch H-inf Design Model with TVC and Alpha-Filter/SM-2*" by means of an interactive process by selecting inputs, outputs and gains, as it will be described in Section 1.4. This batch-1 includes a statement "*Retain CSM*" that permanently preserves the SM in the systems file "*LV_HighQ.Qdr*" and it does not delete it. There is a second batch in file "*LV_HighQ.Inp*" that does not retain the SM but it creates a new SM from the design system and it will be described in Section 1.5.

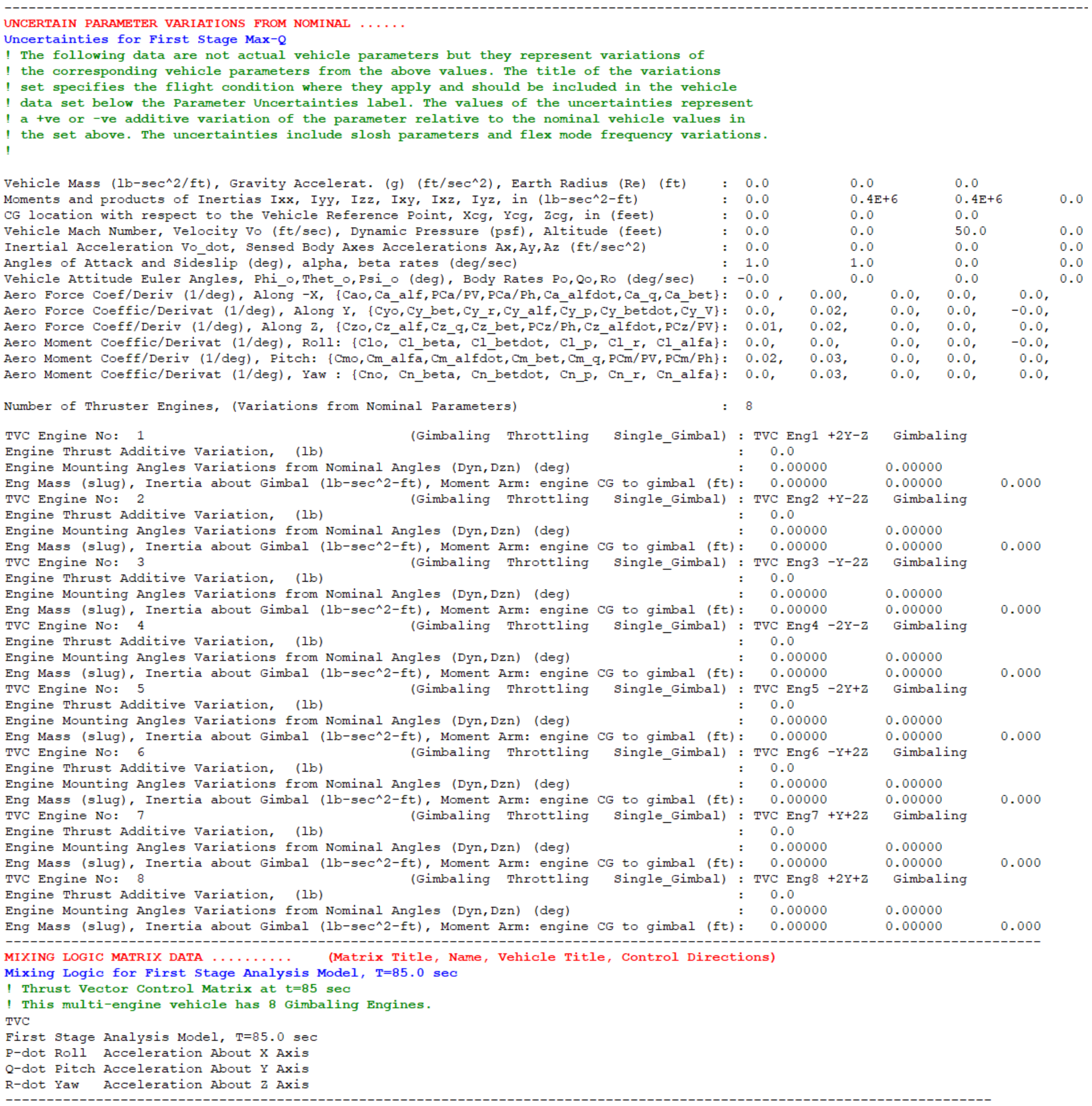


```
BATCH MODE INSTRUCTIONS ...............
Batch-1 for Launch Vehicle Stage-1 Robust Design at T=85 sec (old SM)
! This batch set creates dynamic models for Control Design at T=85 sec
! By retaining the old SM in the input file
!
Retain CSM        : Pitch Hinf Design Model with TVC and Alpha-Filter/SM-5
Flight Vehicle    : First Stage Analysis Model, T=85.0 sec
Mixing Matrix     : Mixing Logic for First Stage Analysis Model, T=85.0 sec
Transf-Function   : Alpha Filter
Transf-Function   : Integrator
System Connection: Design Model with TVC
System Modificat : Pitch Design Model with TVC
System Connection: Pitch Hinf Design Model with TVC and Alpha-Filter
H-Infinity Design: Pitch H-Infinity State-Feedback Control Design
!
!................  Send Systems to Matlab ....................................
To Matlab Format : Alpha Filter
To Matlab Format : First Stage Analysis Model, T=85.0 sec
To Matlab Format : Mixing Logic for First Stage Analysis Model, T=85.0 sec
To Matlab Format : Pitch Design Model with TVC
To Matlab Format : Pitch Hinf Design Model with TVC and Alpha-Filter
To Matlab Format : Pitch State-Feedback Gain Matrix
-----------------------------------------------------------------------------------------
```



```
FLIGHT VEHICLE INPUT DATA ......
First Stage Analysis Model, T=85.0 sec
! Launch Vehicle Control Analysis Model at t=85 sec with 8 Gimbaling TVC Engines.
! Model includes two slosh modes for the LOX and LH2 tanks at 60% Propellant level.
! The LOX tank requires baffles and the damping coefficient was increased to 0.05 .
! The Flight Control Sensors include 3 Rate Gyros (p,q,r) and 2 Accelerometers (Ny,Nz).
! The model also includes 33 Structural Modes Selected between the TVC and the Nav Base
!
Body Axes Output,Attitude=Rate Integral
Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft)    :  3491.68      32.1740      0.208960E+08
Moments and Products of Inertia: Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)          :  25271.0     0.282619E+07  0.282530E+07   0.00000
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)        :  57.2777      0.00000      0.00000
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)       :  1.20400      1187.85      506.827       36627.7
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)         :  23.6396      52.9610      0.0          -0.5230
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                         :  1.00000      0.0          0.00000       0.00000
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)   : -0.0          67.268       0.00000       0.00000
W-Gust Azim & Elev angles (deg), or Torque/Force direction (x,y,z), Force Locat (x,y,z)  :  Gust         45.000       90.000
Surface Reference Area (feet^2), Mean Aerodynamic Chord (ft), Wing Span in (feet)        :  44.4146      7.52000      7.52000
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial_rho/ Partial_H} :  120.146      0.00000      0.00000       0.00000
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:  1.49730      0.00583      0.52973E-04   0.00000
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}:  0.0         -0.08112      0.00000       0.00000
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}: -0.08112     -0.08112      0.00000       0.00000
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:  0.0000       0.0000       0.00000       0.00000
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}: -0.2064      -0.2064       0.00000       0.00000
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}:  0.0          0.2064       0.00000       0.00000

Number of Thruster Engines, Include or Not the Tail-Wags-Dog and Load-Torque Dynamics ?  :  8   NO TWD

TVC Engine No:  1                             (Gimbaling  Throttling   Single_Gimbal) : TVC Eng1 +2Y-Z   Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                    :  24301.7      24301.7
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000       6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  7.00         18.00        1.2
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  8.52200      2.46400     -1.02000
TVC Engine No:  2                             (Gimbaling  Throttling   Single_Gimbal) : TVC Eng2 +Y-2Z   Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                    :  24301.7      24301.7
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000       6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  7.00         18.00        1.2
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  8.52200      1.02000     -2.46400
TVC Engine No:  3                             (Gimbaling  Throttling   Single_Gimbal) : TVC Eng3 -Y-2Z   Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                    :  24301.7      24301.7
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000       6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  7.00         18.00        1.2
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  8.52200     -1.02000     -2.46400
TVC Engine No:  4                             (Gimbaling  Throttling   Single_Gimbal) : TVC Eng4 -2Y-Z   Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                    :  24301.7      24301.7
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000       6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  7.00         18.00        1.2
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  8.52200     -2.46400     -1.02000
TVC Engine No:  5                             (Gimbaling  Throttling   Single_Gimbal) : TVC Eng5 -2Y+Z   Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                    :  24301.7      24301.7
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000       6.000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  7.00         18.00        1.2
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  8.52200     -2.46400      1.02000
TVC Engine No:  6                             (Gimbaling  Throttling   Single_Gimbal) : TVC Eng6 -Y+2Z   Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                    :  24301.7      24301.7
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000       6.000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  7.00         18.00        1.2
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  8.52200     -1.02000      2.46400
TVC Engine No:  7                             (Gimbaling  Throttling   Single_Gimbal) : TVC Eng7 +Y+2Z   Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                    :  24301.7      24301.7
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000       6.000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  7.00         18.00        1.2
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  8.52200      1.02000      2.46400
TVC Engine No:  8                             (Gimbaling  Throttling   Single_Gimbal) : TVC Eng8 +2Y+Z   Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                    :  24301.7      24301.7
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000       6.000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  7.00         18.00        1.2
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  8.52200      2.46400      1.02000

Number of Gyros, (Attitude and Rate)                                                    :  3
Gyro No  1 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Location in (feet) :  Roll   Rate          97.483      0.00        0.0
Gyro No  2 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Location in (feet) :  Pitch  Rate          97.483      0.00        0.0
Gyro No  3 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Location in (feet) :  Yaw    Rate          97.483      0.00        0.0

Number of Accelerometers, Along Axis: (x,y,z)                                           :  2
Acceleromet No  2 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Location (ft) :  Y-axis Accelerat.     97.483      0.00000     0.0
Acceleromet No  3 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Location (ft) :  Z-axis Accelerat.     97.483      0.00000     0.0

Parameter Uncertainties Data
Uncertainties for First Stage Max-Q
--------------------------------------------------------------------------------------------------------------------------------------
```

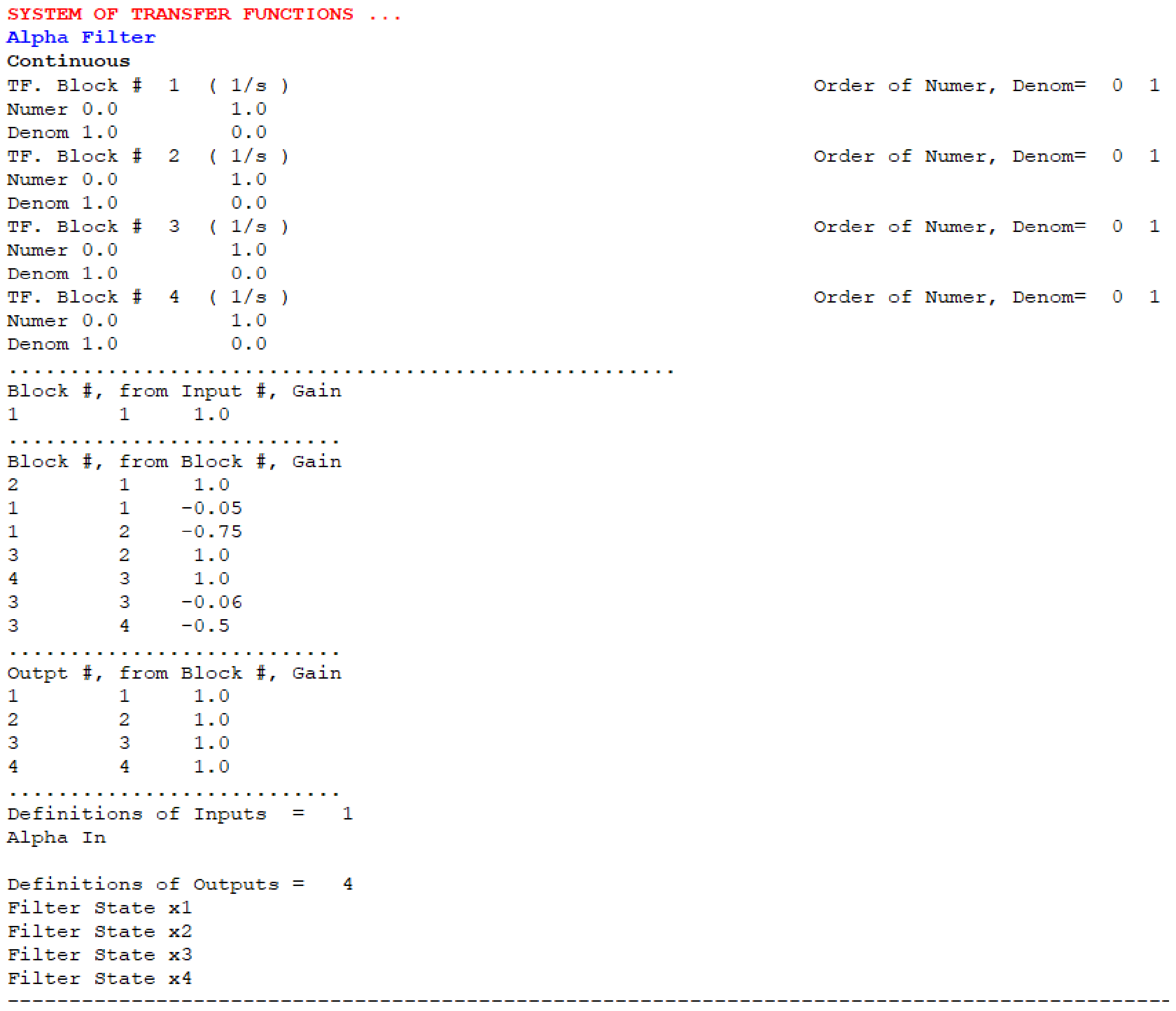
The uncertainties dataset below includes the amount of worst possible variations in some of the vehicle parameters. Each variation corresponds to a parameter in the vehicle data. The variations of the non-changing parameters are obviously set to zero. In this case we have 11 parameter variations, and 6 of them affect the pitch axis. The dynamic pressure variation affects both pitch and yaw.



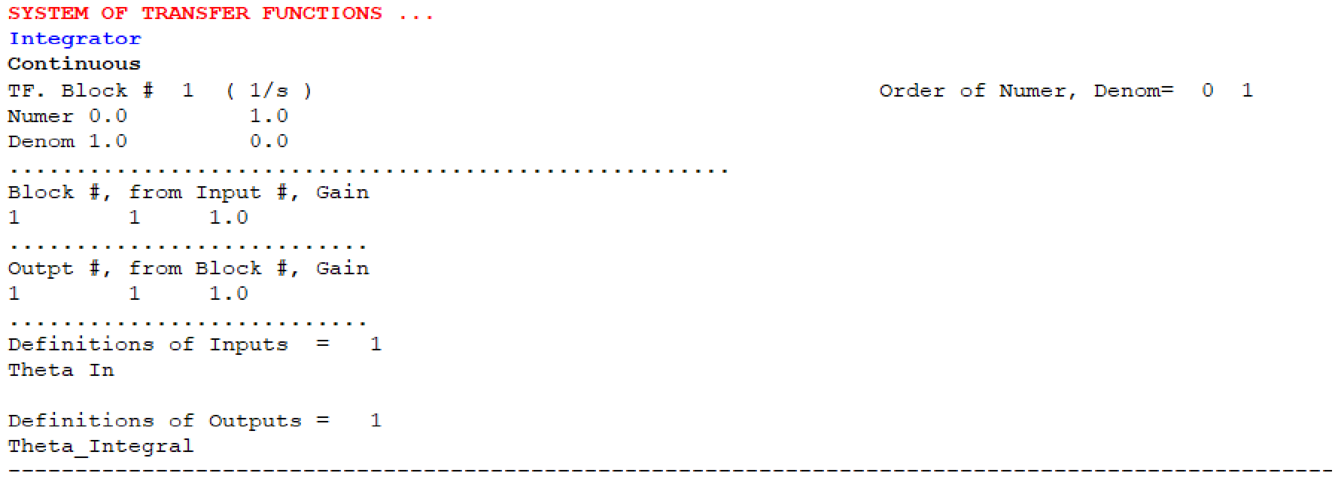
```
---------------------------------------------------------------------------------------------------------------------------------------
UNCERTAIN PARAMETER VARIATIONS FROM NOMINAL ......
Uncertainties for First Stage Max-Q
! The following data are not actual vehicle parameters but they represent variations of
! the corresponding vehicle parameters from the above values. The title of the variations
! set specifies the flight condition where they apply and should be included in the vehicle
! data set below the Parameter Uncertainties label. The values of the uncertainties represent
! a +ve or -ve additive variation of the parameter relative to the nominal vehicle values in
! the set above. The uncertainties include slosh parameters and flex mode frequency variations.
!

Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft)    :  0.0          0.0          0.0
Moments and products of Inertias Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)          :  0.0          0.4E+6       0.4E+6       0.0
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)         :  0.0          0.0          0.0
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)        :  0.0          0.0          50.0         0.0
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)          :  0.0          0.0          0.0          0.0
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                          :  1.0          1.0          0.0          0.0
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)    : -0.0          0.0          0.0          0.0
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:  0.0 ,    0.00,     0.0,   0.0,     0.0,
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}:  0.0,     0.02,     0.0,   0.0,    -0.0,
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}:  0.01,    0.02,     0.0,   0.0,     0.0,
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:  0.0,     0.0,      0.0,   0.0,    -0.0,
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}:  0.02,    0.03,     0.0,   0.0,     0.0,
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}:  0.0,     0.03,     0.0,   0.0,     0.0,

Number of Thruster Engines, (Variations from Nominal Parameters)                          :  8

TVC Engine No:  1                               (Gimbaling  Throttling   Single_Gimbal) : TVC Eng1 +2Y-Z   Gimbaling
Engine Thrust Additive Variation,  (lb)                                                  :   0.0
Engine Mounting Angles Variations from Nominal Angles (Dyn,Dzn) (deg)                    :   0.00000       0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm: engine CG to gimbal (ft):   0.00000       0.00000       0.000
TVC Engine No:  2                               (Gimbaling  Throttling   Single_Gimbal) : TVC Eng2 +Y-2Z   Gimbaling
Engine Thrust Additive Variation,  (lb)                                                  :   0.0
Engine Mounting Angles Variations from Nominal Angles (Dyn,Dzn) (deg)                    :   0.00000       0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm: engine CG to gimbal (ft):   0.00000       0.00000       0.000
TVC Engine No:  3                               (Gimbaling  Throttling   Single_Gimbal) : TVC Eng3 -Y-2Z   Gimbaling
Engine Thrust Additive Variation,  (lb)                                                  :   0.0
Engine Mounting Angles Variations from Nominal Angles (Dyn,Dzn) (deg)                    :   0.00000       0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm: engine CG to gimbal (ft):   0.00000       0.00000       0.000
TVC Engine No:  4                               (Gimbaling  Throttling   Single_Gimbal) : TVC Eng4 -2Y-Z   Gimbaling
Engine Thrust Additive Variation,  (lb)                                                  :   0.0
Engine Mounting Angles Variations from Nominal Angles (Dyn,Dzn) (deg)                    :   0.00000       0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm: engine CG to gimbal (ft):   0.00000       0.00000       0.000
TVC Engine No:  5                               (Gimbaling  Throttling   Single_Gimbal) : TVC Eng5 -2Y+Z   Gimbaling
Engine Thrust Additive Variation,  (lb)                                                  :   0.0
Engine Mounting Angles Variations from Nominal Angles (Dyn,Dzn) (deg)                    :   0.00000       0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm: engine CG to gimbal (ft):   0.00000       0.00000       0.000
TVC Engine No:  6                               (Gimbaling  Throttling   Single_Gimbal) : TVC Eng6 -Y+2Z   Gimbaling
Engine Thrust Additive Variation,  (lb)                                                  :   0.0
Engine Mounting Angles Variations from Nominal Angles (Dyn,Dzn) (deg)                    :   0.00000       0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm: engine CG to gimbal (ft):   0.00000       0.00000       0.000
TVC Engine No:  7                               (Gimbaling  Throttling   Single_Gimbal) : TVC Eng7 +Y+2Z   Gimbaling
Engine Thrust Additive Variation,  (lb)                                                  :   0.0
Engine Mounting Angles Variations from Nominal Angles (Dyn,Dzn) (deg)                    :   0.00000       0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm: engine CG to gimbal (ft):   0.00000       0.00000       0.000
TVC Engine No:  8                               (Gimbaling  Throttling   Single_Gimbal) : TVC Eng8 +2Y+Z   Gimbaling
Engine Thrust Additive Variation,  (lb)                                                  :   0.0
Engine Mounting Angles Variations from Nominal Angles (Dyn,Dzn) (deg)                    :   0.00000       0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm: engine CG to gimbal (ft):   0.00000       0.00000       0.000
---------------------------------------------------------------------------------------------------------------------------------
MIXING LOGIC MATRIX DATA ..........    (Matrix Title, Name, Vehicle Title, Control Directions)
Mixing Logic for First Stage Analysis Model, T=85.0 sec
! Thrust Vector Control Matrix at t=85 sec
! This multi-engine vehicle has 8 Gimbaling Engines.
TVC
First Stage Analysis Model, T=85.0 sec
P-dot Roll  Acceleration About X Axis
Q-dot Pitch Acceleration About Y Axis
R-dot Yaw   Acceleration About Z Axis
---------------------------------------------------------------------------------------------------------------------------
```

The mixing logic dataset above uses the vehicle data to generate the TVC matrix. The TVC matrix is defined to accept 3 rotational acceleration demands (roll, pitch and yaw), see Figure-2. The outputs are 8 pitch and 8 yaw gimbal deflection commands that go to the actuators.

The alpha-filter consists of two resonances which are excited from the angle of attack. They amplify the frequency range between 0.6 to 0.9 (rad/sec) where the wind disturbances occur. The resonances are optimized by the algorithm which creates a control logic that attempts to attenuate excitations in that frequency range. The integrator is used to create θ-integral which is also included in the optimization.



```
SYSTEM OF TRANSFER FUNCTIONS ...
Alpha Filter
Continuous
TF. Block #  1  ( 1/s )                                          Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
TF. Block #  2  ( 1/s )                                          Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
TF. Block #  3  ( 1/s )                                          Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
TF. Block #  4  ( 1/s )                                          Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
......................................................
Block #, from Input #, Gain
1        1     1.0
...........................
Block #, from Block #, Gain
2        1     1.0
1        1    -0.05
1        2    -0.75
3        2     1.0
4        3     1.0
3        3    -0.06
3        4    -0.5
...........................
Outpt #, from Block #, Gain
1        1     1.0
2        2     1.0
3        3     1.0
4        4     1.0
...........................
Definitions of Inputs  =   1
Alpha In

Definitions of Outputs =   4
Filter State x1
Filter State x2
Filter State x3
Filter State x4
----------------------------------------------------------------------------------------------
```

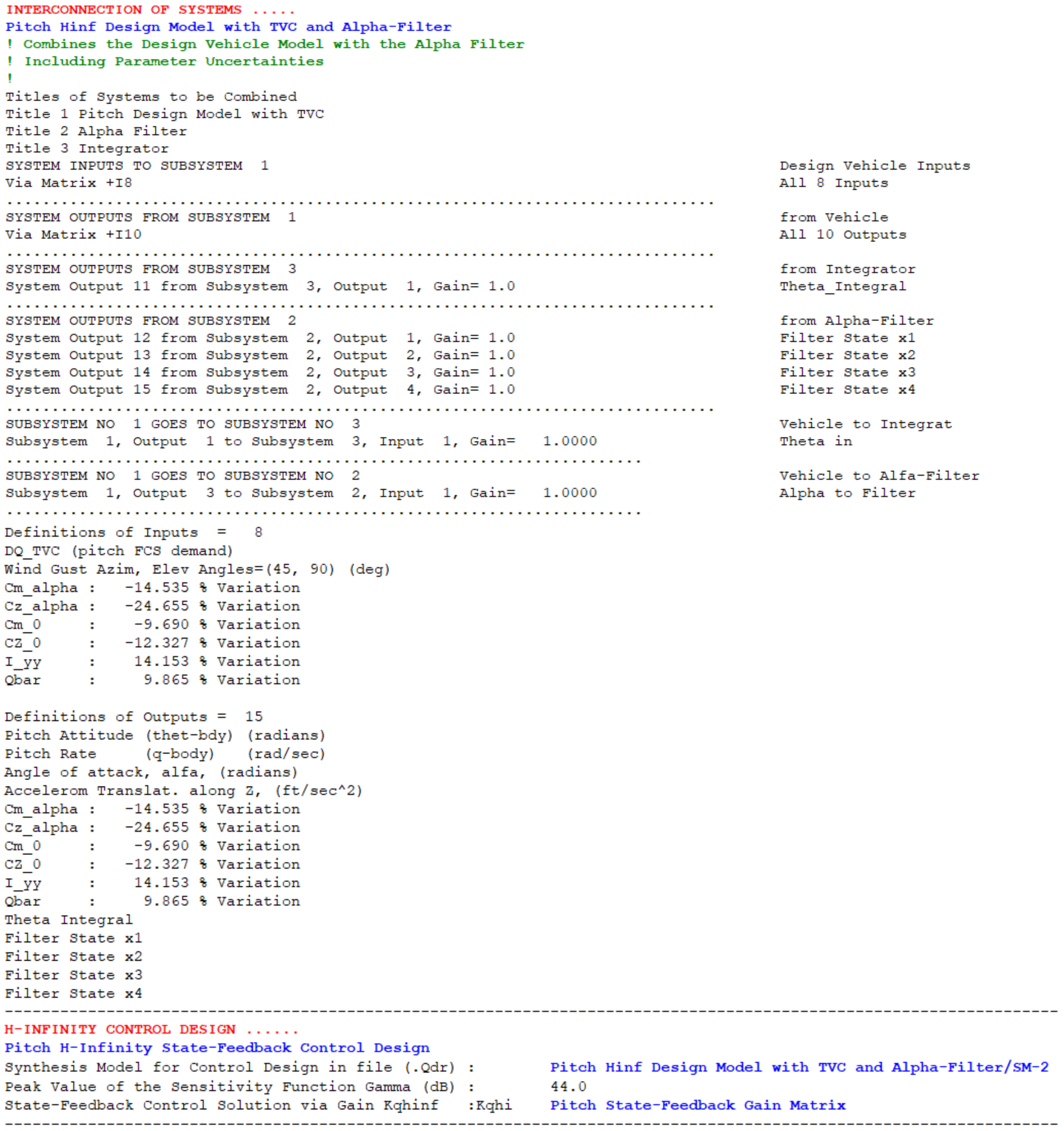


```
SYSTEM OF TRANSFER FUNCTIONS ...
Integrator
Continuous
TF. Block #  1  ( 1/s )                                          Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
......................................................
Block #, from Input #, Gain
1        1     1.0
...........................
Outpt #, from Block #, Gain
1        1     1.0
...........................
Definitions of Inputs  =   1
Theta In

Definitions of Outputs =   1
Theta_Integral
----------------------------------------------------------------------------------------------------
```

The datasets below combine the vehicle system with the TVC matrix to create “*Design Model with TVC*” and then create a reduced system “*Pitch Design Model with TVC*” by extracting only the pitch variables.

```
INTERCONNECTION OF SYSTEMS .....
Design Model with TVC
! Combines the Rigid Vehicle model with the TVC matrix.
! The Inpus include Gust plus Uncertainties.
! The Outputs include Uncertainties.
!
Titles of Systems to be Combined
Title 1 First Stage Analysis Model, T=85.0 sec
SYSTEM INPUTS TO SUBSYSTEM  1                                                         TVC to Vehicle
Via Matrix +TVC                                                                       inputs: roll, pitch, yaw,
Via Matrix +I11                                                                       Gust & 10 Uncertainties
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                                      from Plant
Via Matrix +I23                                                                       All Outputs
..............................................................................
Definitions of Inputs  =  14
DP_TVC (roll FCS demand)
DQ_TVC (pitch FCS demand)
DR_TVC (yaw FCS demand)
Wind Gust Azim, Elev Angles=(45, 90) (deg)
Cm_alpha :    -14.535 % Variation
Cn_beta  :     14.535 % Variation
Cz_alpha :    -24.655 % Variation
Cy_beta  :    -24.655 % Variation
Cm_0     :     -9.690 % Variation
CZ_0     :    -12.327 % Variation
I_yy     :     14.153 % Variation
I_zz     :     14.158 % Variation
Qbar     :      9.865 % Variation
Qbar     :      9.865 % Variation

Definitions of Outputs =  23
Roll Attitude  (phi-body) (radians)
Roll Rate      (p-body)   (rad/sec)
Pitch Attitude (thet-bdy) (radians)
Pitch Rate     (q-body)   (rad/sec)
Yaw  Attitude  (psi-body) (radians)
Yaw  Rate      (r-body)   (rad/sec)
Angle of attack, alfa, (radians)
Angle of sideslip, beta, (radian)
Change in Altitude, delta-h, (feet)
Forward Acceleration  (V-dot)  (ft/sec)
Cross Range Velocity (Vcr) (ft/sec)
Accelerom  along Y, (ft/sec^2) Translat.
Accelerom  along Z, (ft/sec^2) Translat.
Cm_alpha :    -14.535 % Variation
Cn_beta  :     14.535 % Variation
Cz_alpha :    -24.655 % Variation
Cy_beta  :    -24.655 % Variation
Cm_0     :     -9.690 % Variation
CZ_0     :    -12.327 % Variation
I_yy     :     14.153 % Variation
I_zz     :     14.158 % Variation
Qbar     :      9.865 % Variation
Qbar     :      9.865 % Variation
------------------------------------------------------------------------------------
CREATE A NEW SYSTEM FROM AN OLD SYSTEM... (Titles of the New and Old Systems)
Pitch Design Model with TVC
Design Model with TVC
TRUNCATE OR REORDER THE SYSTEM INPUTS, STATES, AND OUTPUTS
Extract Inputs :    2   4   5    7    9   10   11   13
Extract States :    3   4   7
Extract Outputs:    3   4   7  13   14   16   18   19   20   22
-------------------------------------------------------------------------------------------------
```

The dataset below combines the pitch design model with the alpha-filter and the theta-integrator to create the augmented pitch design model that is used to create the 9-matrices SM.



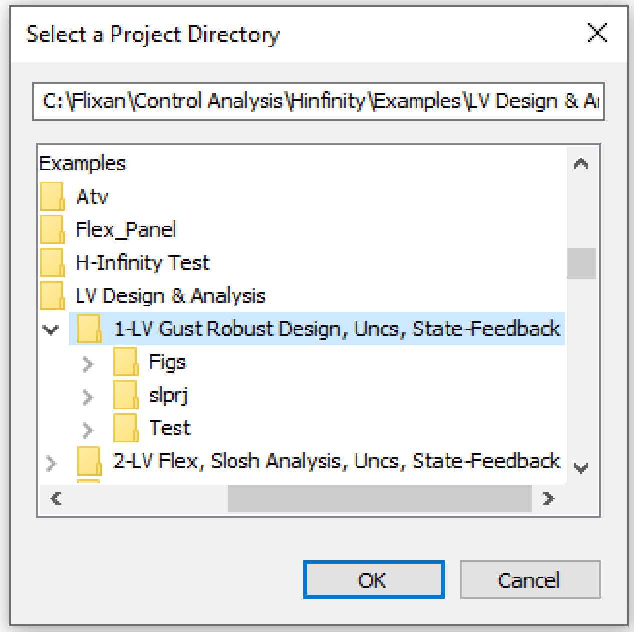
```
INTERCONNECTION OF SYSTEMS .....
Pitch Hinf Design Model with TVC and Alpha-Filter
! Combines the Design Vehicle Model with the Alpha Filter
! Including Parameter Uncertainties
!
Titles of Systems to be Combined
Title 1 Pitch Design Model with TVC
Title 2 Alpha Filter
Title 3 Integrator
SYSTEM INPUTS TO SUBSYSTEM  1                                              Design Vehicle Inputs
Via Matrix +I8                                                             All 8 Inputs
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                           from Vehicle
Via Matrix +I10                                                            All 10 Outputs
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  3                                           from Integrator
System Output 11 from Subsystem  3, Output  1, Gain= 1.0                   Theta_Integral
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                                           from Alpha-Filter
System Output 12 from Subsystem  2, Output  1, Gain= 1.0                   Filter State x1
System Output 13 from Subsystem  2, Output  2, Gain= 1.0                   Filter State x2
System Output 14 from Subsystem  2, Output  3, Gain= 1.0                   Filter State x3
System Output 15 from Subsystem  2, Output  4, Gain= 1.0                   Filter State x4
..............................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  3                                    Vehicle to Integrat
Subsystem  1, Output  1 to Subsystem  3, Input  1, Gain=   1.0000          Theta in
......................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2                                    Vehicle to Alfa-Filter
Subsystem  1, Output  3 to Subsystem  2, Input  1, Gain=   1.0000          Alpha to Filter
......................................................................
Definitions of Inputs  =   8
DQ_TVC (pitch FCS demand)
Wind Gust Azim, Elev Angles=(45, 90) (deg)
Cm_alpha :   -14.535 % Variation
Cz_alpha :   -24.655 % Variation
Cm_0     :    -9.690 % Variation
CZ_0     :   -12.327 % Variation
I_yy     :    14.153 % Variation
Qbar     :     9.865 % Variation

Definitions of Outputs =  15
Pitch Attitude (thet-bdy) (radians)
Pitch Rate     (q-body)   (rad/sec)
Angle of attack, alfa, (radians)
Accelerom Translat. along Z, (ft/sec^2)
Cm_alpha :   -14.535 % Variation
Cz_alpha :   -24.655 % Variation
Cm_0     :    -9.690 % Variation
CZ_0     :   -12.327 % Variation
I_yy     :    14.153 % Variation
Qbar     :     9.865 % Variation
Theta Integral
Filter State x1
Filter State x2
Filter State x3
Filter State x4
-------------------------------------------------------------------------------------------------------------------
H-INFINITY CONTROL DESIGN ......
Pitch H-Infinity State-Feedback Control Design
Synthesis Model for Control Design in file (.Qdr)  :      Pitch Hinf Design Model with TVC and Alpha-Filter/SM-2
Peak Value of the Sensitivity Function Gamma (dB)  :      44.0
State-Feedback Control Solution via Gain Kqhinf    :Kqhi    Pitch State-Feedback Gain Matrix
-------------------------------------------------------------------------------------------------------------------
```

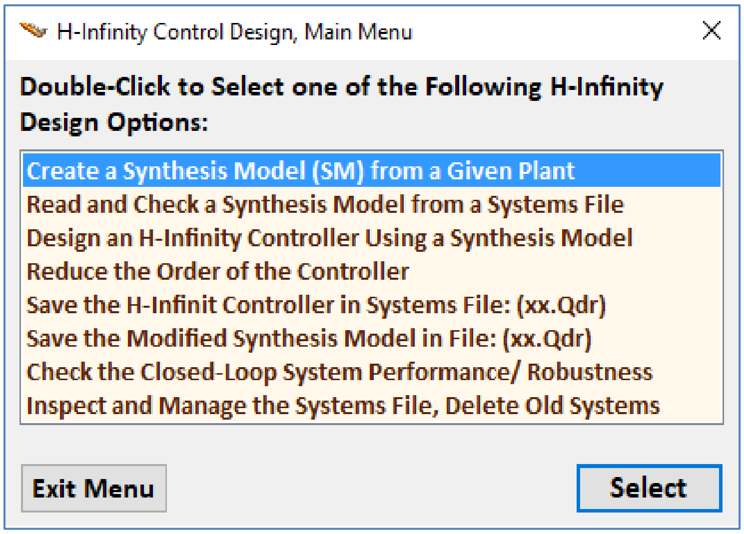
The H-Infinity design dataset reads the SM "*Pitch Hinf Design Model with TVC and Alpha-Filter/SM-2*" from the systems file: "LV_HighQ.Qdr" and creates the (1x8) state-feedback matrix "Kqhi" which is also saved in the systems file under "*Pitch State-Feedback Gain Matrix*". The upper bound gamma parameter is set to γ=44 (dB). Values smaller than γ=40 (dB) violate the algorithm criteria. The following datasets convert the Flixan generated systems and matrices from file "LV_HighQ.Qdr" to m-files and mat-files that can be loaded into Matlab for control analysis.

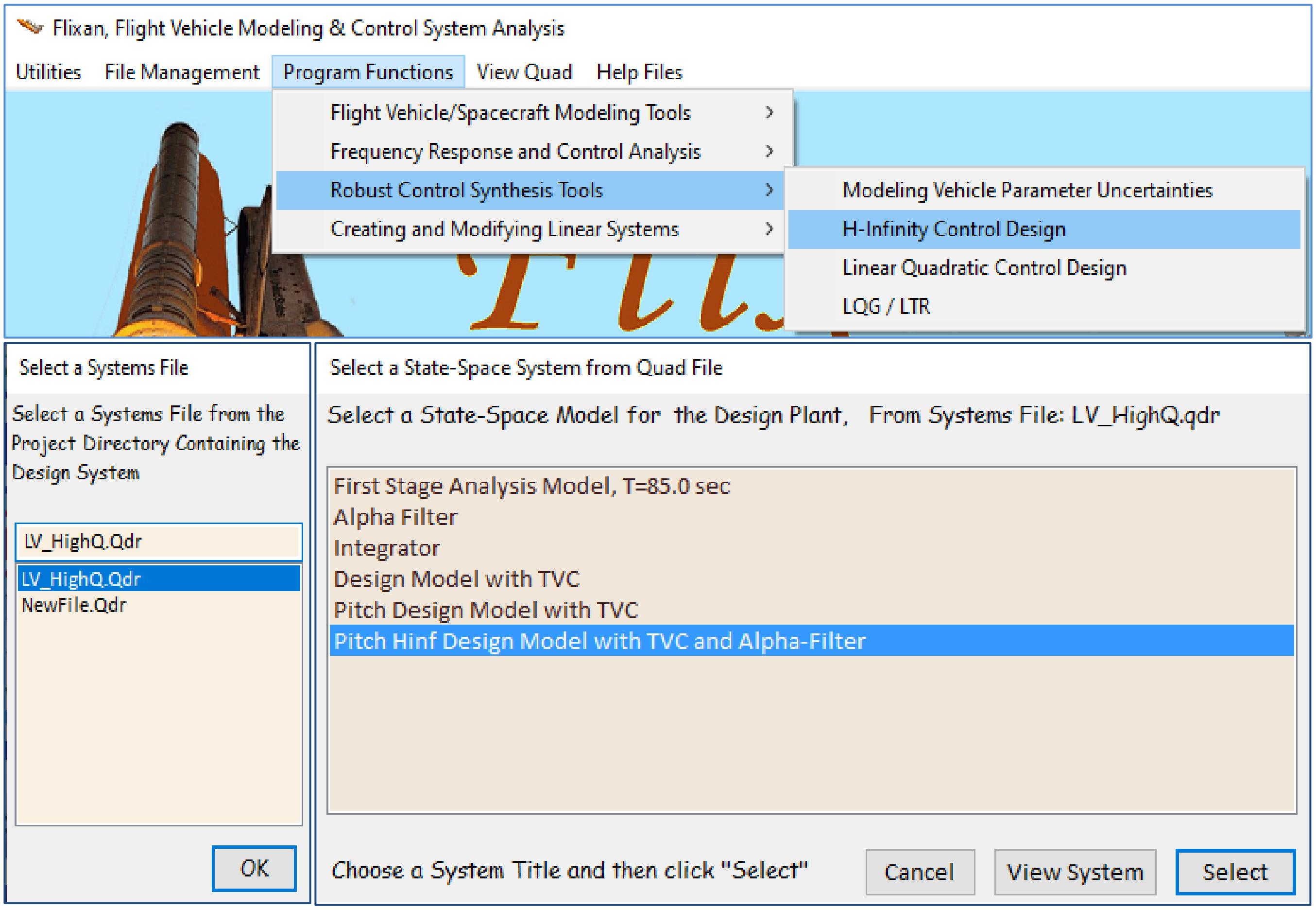
```
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Alpha Filter
System
alfa_filt
-------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Mixing Logic for First Stage Analysis Model, T=85.0 sec
Matrix TVC
-------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
First Stage Analysis Model, T=85.0 sec
System
vehicle.m
-------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Pitch Design Model with TVC
System
pitch_des.m
-------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Pitch Hinf Design Model with TVC and Alpha-Filter
System
vehi_pdes.m
-------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Pitch State-Feedback Gain Matrix
Matrix Kqhi
-------------------------------------------------------------------------------------
```

1.4 Creating the Synthesis Model Interactively

We will now show how to create the SM that will also be saved in the systems file. The SM consists of 9 matrices and includes the plant dynamics and optimization parameters. It redefines the system inputs into controls and disturbances and redefines the outputs into measurements and optimization criteria. It also includes gains which represent performance requirements that must be optimized by the H-infinity algorithm. The designer must define which inputs are disturbances and which ones are controls. Also, which outputs are criteria to be optimized and which ones are used as measurements. In this example the measurements are equal to the state-vector and we won't need an estimator. We begin by running the H-infinity design program and selecting the first option to create the SM, as shown below. Use the next two menus to select the systems filename and the augmented design system which includes the alpha-filter.


C:\Flixan\Control Analysis\Hinfinity\Examples\LV Design & A





Select a Systems File from the Project Directory Containing the Design System

LV_HighQ.Qdr

LV_HighQ.Qdr
NewFile.Qdr

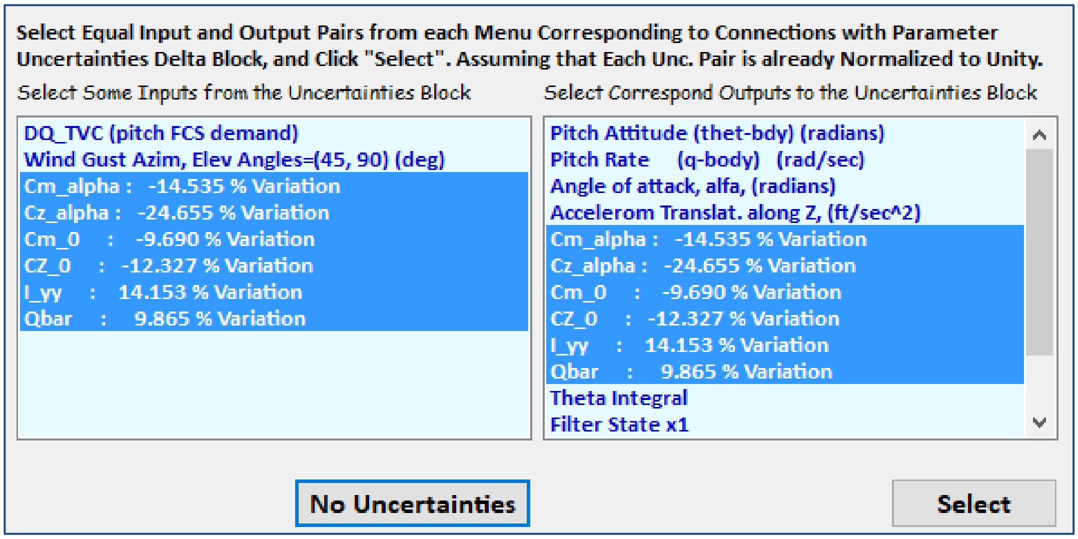
Select a State-Space Model for the Design Plant, From Systems File: LV_HighQ.qdr

First Stage Analysis Model, T=85.0 sec
Alpha Filter
Integrator
Design Model with TVC
Pitch Design Model with TVC
Pitch Hinf Design Model with TVC and Alpha-Filter

Choose a System Title and then click "Select"

The SM will be created from the design plant “*Pitch Hinf Design Model with TVC and Alpha-Filter*”, which is an (A, B, C, D) system, by selecting some inputs and outputs using menus and placing them into groups. We begin with the first dialog below which selects the inputs and outputs that connect with the uncertainty block Δ, if any. In this case we have 6 uncertainty connections. Use the left and right menus to select the 6 parameter variation pairs that connect with the uncertainty block.

Note, they must be equal number of i/o pairs. They won’t be connected to any block in this case but they will be treated like disturbance inputs and criteria outputs as described in the IFL method. Click on “Select”.



Select Equal Input and Output Pairs from each Menu Corresponding to Connections with Parameter Uncertainties Delta Block, and Click "Select". Assuming that Each Unc. Pair is already Normalized to Unity.

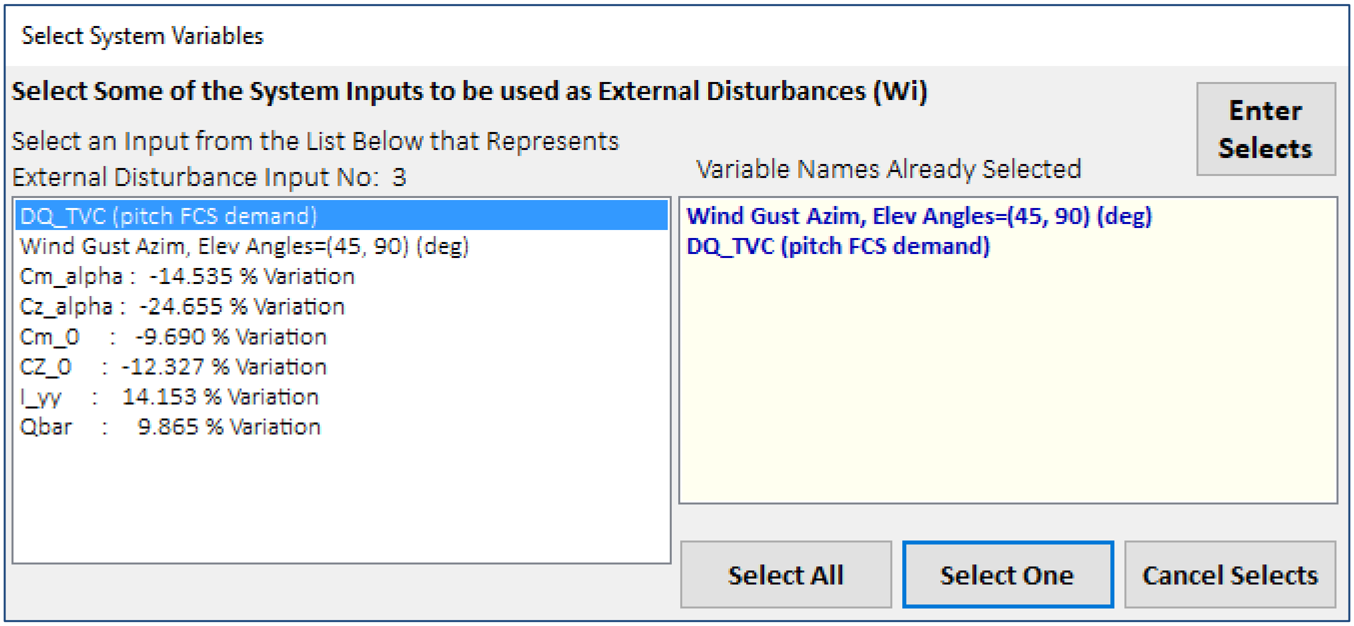
Select Some Inputs from the Uncertainties Block

DQ_TVC (pitch FCS demand)
Wind Gust Azim, Elev Angles=(45, 90) (deg)
Cm_alpha : -14.535 % Variation
Cz_alpha : -24.655 % Variation
Cm_0 : -9.690 % Variation
CZ_0 : -12.327 % Variation
I_yy : 14.153 % Variation
Qbar : 9.865 % Variation

Select Correspond Outputs to the Uncertainties Block

Pitch Attitude (thet-bdy) (radians)
Pitch Rate (q-body) (rad/sec)
Angle of attack, alfa, (radians)
Accelerom Translat. along Z, (ft/sec^2)
Cm_alpha : -14.535 % Variation
Cz_alpha : -24.655 % Variation
Cm_0 : -9.690 % Variation
CZ_0 : -12.327 % Variation
I_yy : 14.153 % Variation
Qbar : 9.865 % Variation
Theta Integral
Filter State x1

The next menu is for defining external disturbance inputs. The first two inputs are selected as disturbances. That is, the wind-gust velocity, and noise to be added at the control input DQ_tvc. Select one at a time, and click on “Enter Selects” to continue.



Select System Variables

Select Some of the System Inputs to be used as External Disturbances (Wi)

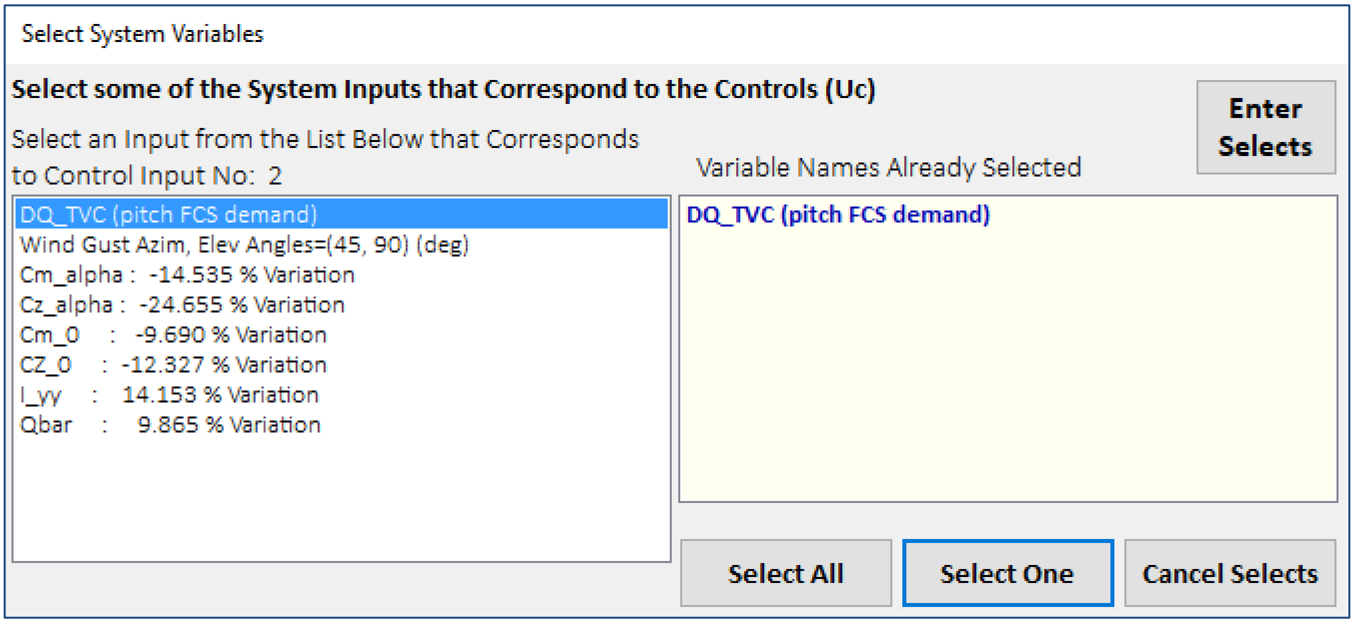
Select an Input from the List Below that Represents External Disturbance Input No: 3

DQ_TVC (pitch FCS demand)
Wind Gust Azim, Elev Angles=(45, 90) (deg)
Cm_alpha : -14.535 % Variation
Cz_alpha : -24.655 % Variation
Cm_0 : -9.690 % Variation
CZ_0 : -12.327 % Variation
I_yy : 14.153 % Variation
Qbar : 9.865 % Variation

Variable Names Already Selected

Wind Gust Azim, Elev Angles=(45, 90) (deg)
DQ_TVC (pitch FCS demand)

The next menu is used for selecting the control inputs. There is only one control input in the design system, the pitch demand to the TVC. It is actually the second input to the TVC matrix but the TVC is already included in the plant. Select the DQ_tvc input and then click on “Enter Selects” to continue.



Select System Variables

Select some of the System Inputs that Correspond to the Controls (Uc)

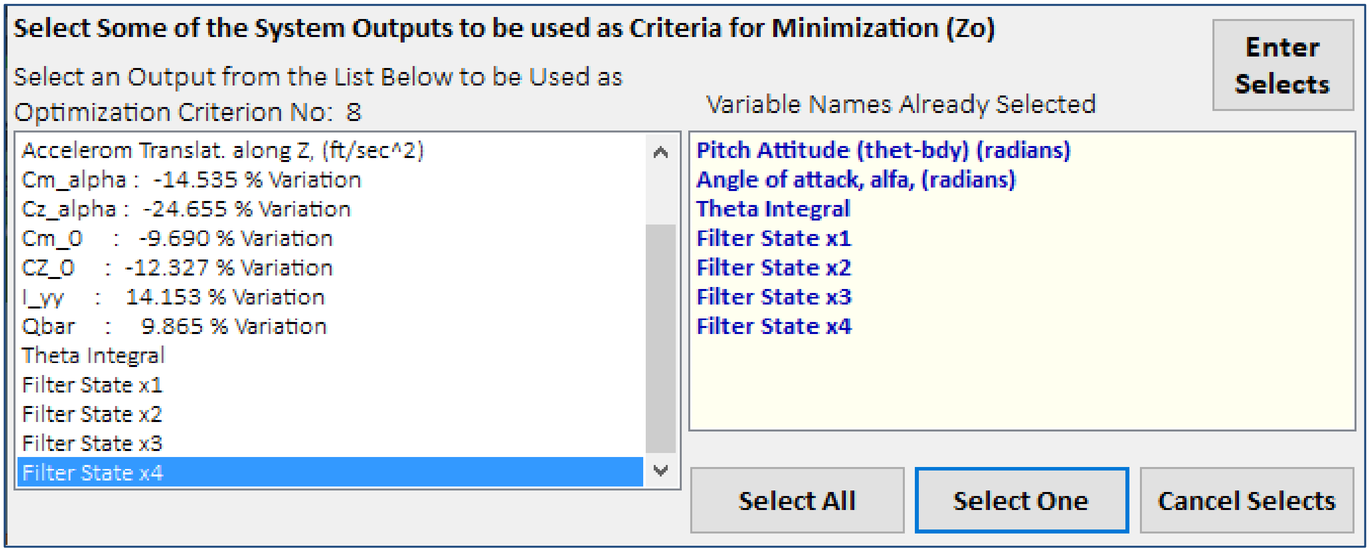
Select an Input from the List Below that Corresponds to Control Input No: 2

DQ_TVC (pitch FCS demand)
Wind Gust Azim, Elev Angles=(45, 90) (deg)
Cm_alpha : -14.535 % Variation
Cz_alpha : -24.655 % Variation
Cm_0 : -9.690 % Variation
CZ_0 : -12.327 % Variation
I_yy : 14.153 % Variation
Qbar : 9.865 % Variation

Variable Names Already Selected

DQ_TVC (pitch FCS demand)

The design system has several outputs. We will include only 7 to be optimized which are also state variables. That is, the pitch attitude θ, the angle of attack α, θ-integral, and the four filter states, x1 to x4. Select one at a time and then click on “Enter Selects” to continue.


Select Some of the System Outputs to be used as Criteria for Minimization (Zo)

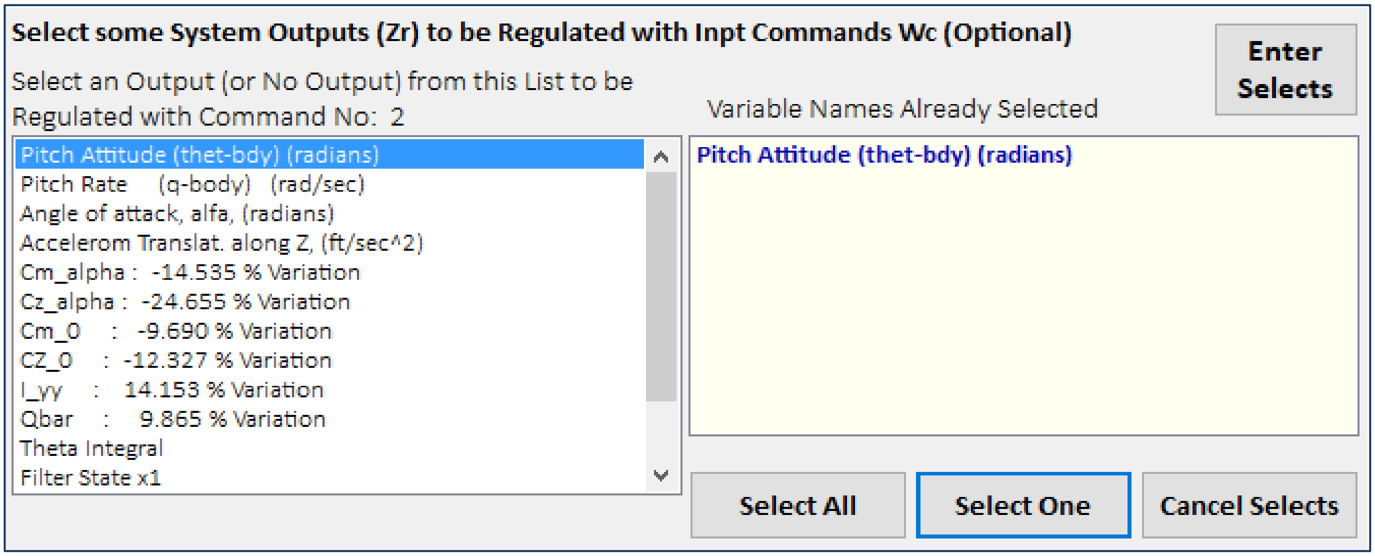
Select an Output from the List Below to be Used as
Optimization Criterion No: 8

Accelerom Translat. along Z, (ft/sec^2)
Cm_alpha : -14.535 % Variation
Cz_alpha : -24.655 % Variation
Cm_0 : -9.690 % Variation
CZ_0 : -12.327 % Variation
I_yy : 14.153 % Variation
Qbar : 9.865 % Variation
Theta Integral
Filter State x1
Filter State x2
Filter State x3
Filter State x4

Variable Names Already Selected

Pitch Attitude (thet-bdy) (radians)
Angle of attack, alfa, (radians)
Theta Integral
Filter State x1
Filter State x2
Filter State x3
Filter State x4

In the next menu select one output to be commanded. We only have one output which is regulated by command, the pitch attitude. Select it and click on “Enter Selects”. The next menu is used for selecting the output measurements. Select the ones which are also states in order to create a state-feedback gain (no estimator will be used in this case), then click on “Enter Selects” to continue.

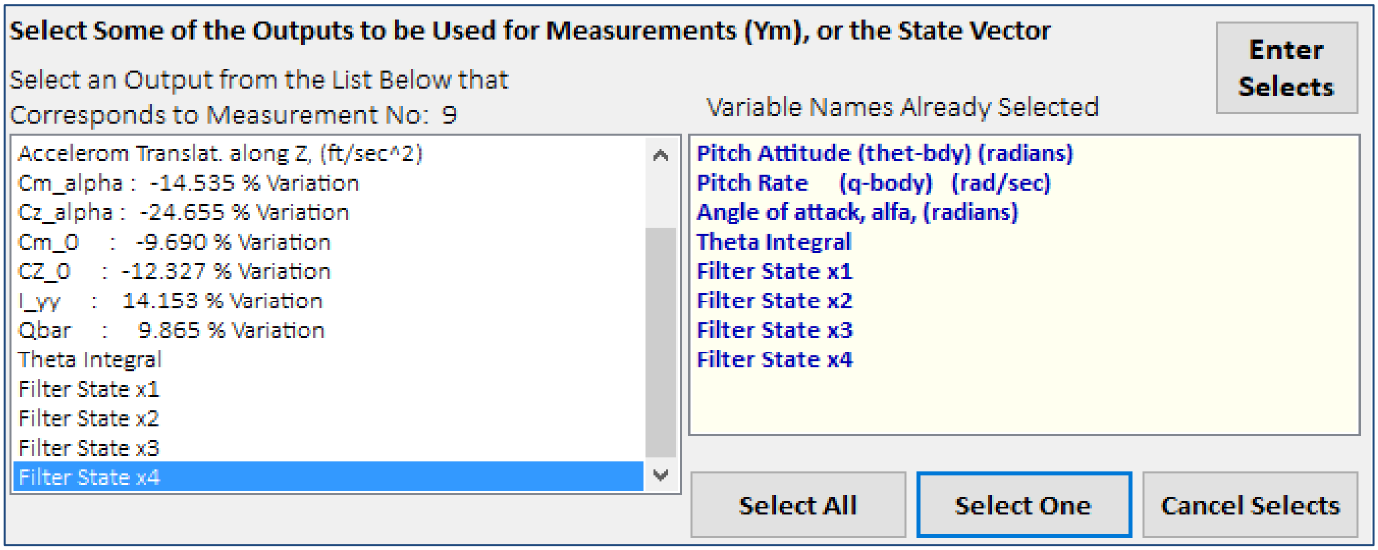

Select some System Outputs (Zr) to be Regulated with Inpt Commands Wc (Optional)

Select an Output (or No Output) from this List to be
Regulated with Command No: 2

Pitch Attitude (thet-bdy) (radians)
Pitch Rate (q-body) (rad/sec)
Angle of attack, alfa, (radians)
Accelerom Translat. along Z, (ft/sec^2)
Cm_alpha : -14.535 % Variation
Cz_alpha : -24.655 % Variation
Cm_0 : -9.690 % Variation
CZ_0 : -12.327 % Variation
I_yy : 14.153 % Variation
Qbar : 9.865 % Variation
Theta Integral
Filter State x1

Variable Names Already Selected

Pitch Attitude (thet-bdy) (radians)


Select Some of the Outputs to be Used for Measurements (Ym), or the State Vector

Select an Output from the List Below that
Corresponds to Measurement No: 9

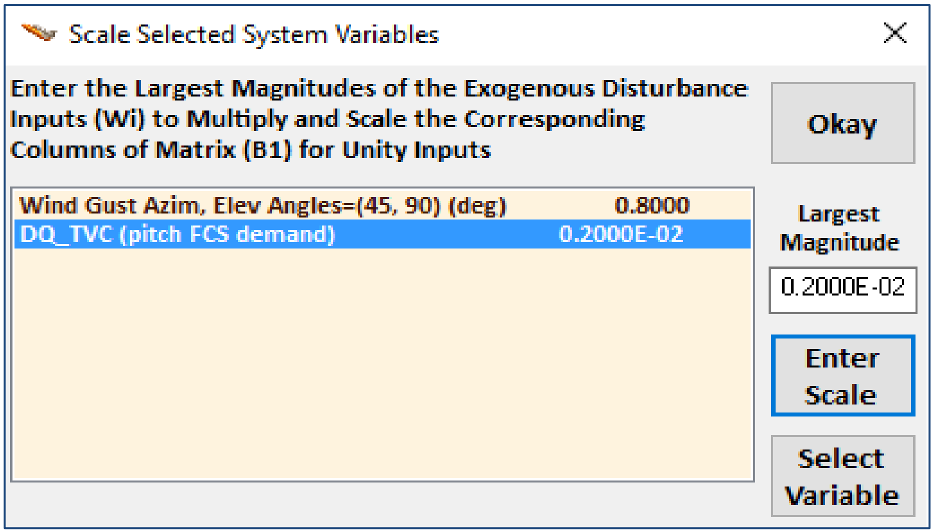
Accelerom Translat. along Z, (ft/sec^2)
Cm_alpha : -14.535 % Variation
Cz_alpha : -24.655 % Variation
Cm_0 : -9.690 % Variation
CZ_0 : -12.327 % Variation
I_yy : 14.153 % Variation
Qbar : 9.865 % Variation
Theta Integral
Filter State x1
Filter State x2
Filter State x3
Filter State x4

Variable Names Already Selected

Pitch Attitude (thet-bdy) (radians)
Pitch Rate (q-body) (rad/sec)
Angle of attack, alfa, (radians)
Theta Integral
Filter State x1
Filter State x2
Filter State x3
Filter State x4

We have now finished defining the input and output variables of the SM. We must now enter the gains that will be used to scale them in the optimization. The trade-off between bandwidth and performance versus sensitivity and stability are defined in the optimization algorithm by adjusting those gains which are like "knobs" that scale the disturbance inputs and the criteria outputs and they can be changed in the next design iteration when not satisfied with the result. Initially we don't know what gains will produce the desired performance versus stability, so we begin to scale the disturbance inputs by entering the magnitudes of the maximum expected disturbances in the input gains and for the output gains we enter the maximum magnitude permitted at each performance criterion. The controls are also included in the criteria outputs and we must scale them by entering the maximum amount of control allowed.

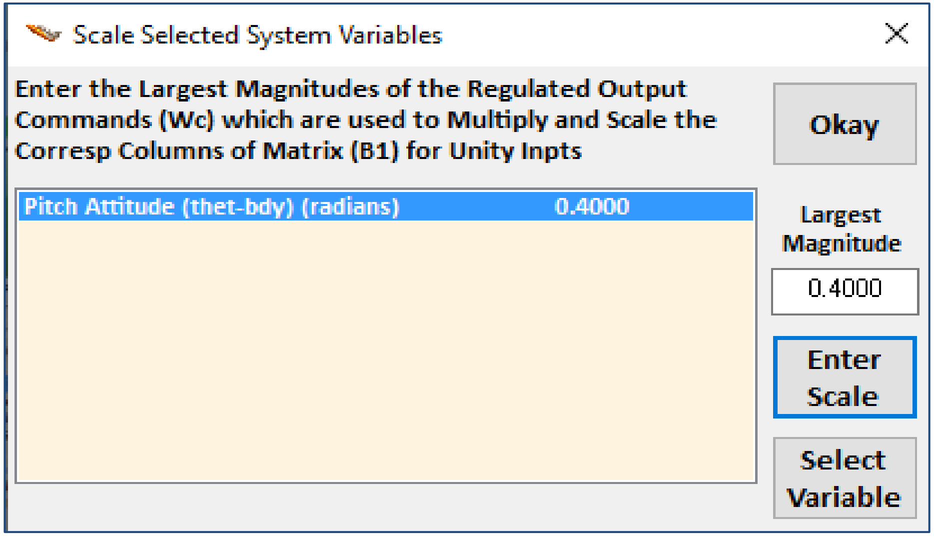
The measurement noise is also included in the disturbances vector and we must enter the maximum value of noise at each measurement. Fortunately, in this example the state-vector is measurable and since we are not estimating it, the measurement noise is set to zero or a very small value. In the dialog below enter the gains that will scale the disturbance inputs. That is, the maximum expected disturbance at each input. Highlight the input, click on "Select Variable", enter value, and click on "Enter Scale" to accept it, one at a time. The value appears in the display next to the variable label. When you finish click on "Okay" to go to the next dialog. In the two dialogs below enter the magnitudes of the largest disturbances expected and the magnitude of the control input and click "Okay".



Enter the Largest Magnitudes of the Exogenous Disturbance Inputs (Wi) to Multiply and Scale the Corresponding Columns of Matrix (B1) for Unity Inputs

Wind Gust Azim, Elev Angles=(45, 90) (deg) 0.8000
DQ_TVC (pitch FCS demand) 0.2000E-02

Largest Magnitude: 0.2000E-02

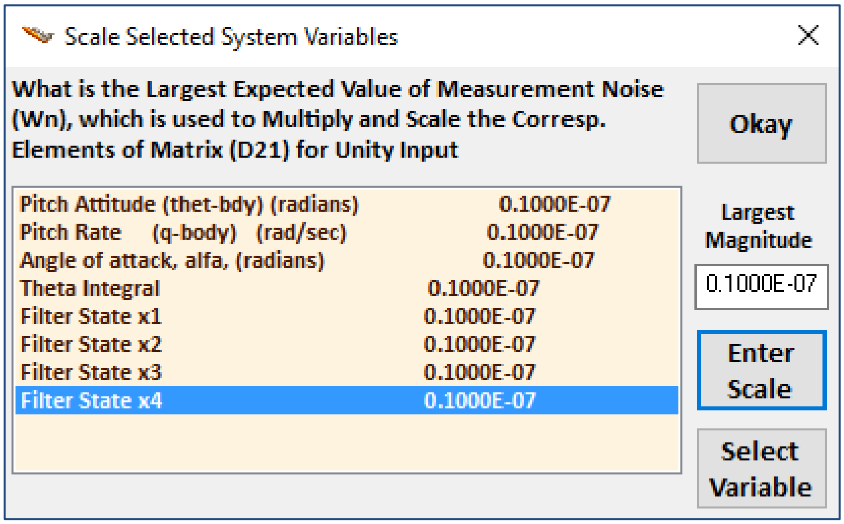


Enter the Largest Magnitudes of the Regulated Output Commands (Wc) which are used to Multiply and Scale the Corresp Columns of Matrix (B1) for Unity Inpts

Pitch Attitude (thet-bdy) (radians) 0.4000

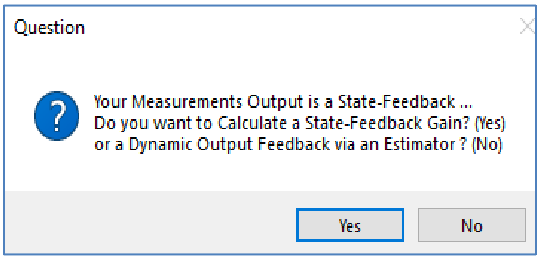
Largest Magnitude: 0.4000

The next dialog is for entering noise at the measurements. In this example the measurement is the entire state-vector and the program knows that, but we do not want to build a state estimator. If the state-vector measurements were noisy then we would need an estimator, even though we are measuring the entire state. So, we tell the program that we don’t want the estimator by inserting zero noise or a very small noise magnitude in each variable. The program requires a confirmation that you don’t want the estimator, so you enter “Yes” to calculate a state-feedback control gain and not a dynamic controller



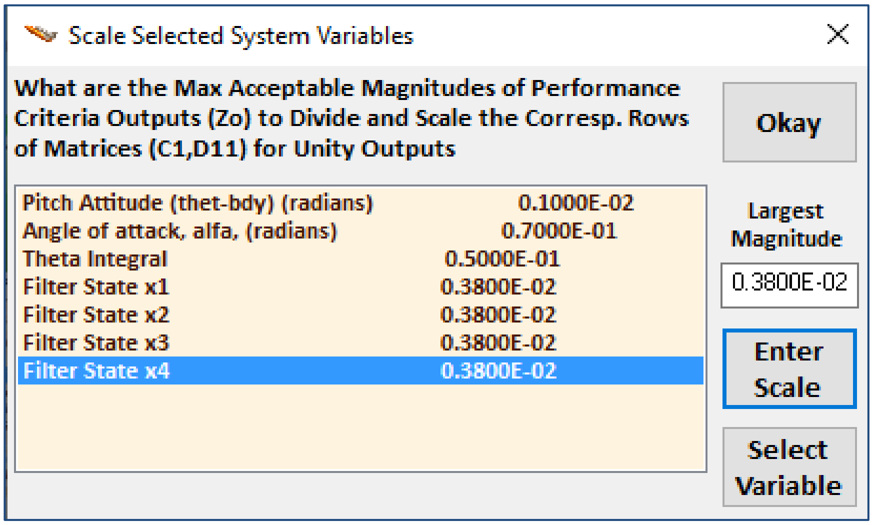
What is the Largest Expected Value of Measurement Noise (Wn), which is used to Multiply and Scale the Corresp. Elements of Matrix (D21) for Unity Input

Pitch Attitude (thet-bdy) (radians)	0.1000E-07
Pitch Rate (q-body) (rad/sec)	0.1000E-07
Angle of attack, alfa, (radians)	0.1000E-07
Theta Integral	0.1000E-07
Filter State x1	0.1000E-07
Filter State x2	0.1000E-07
Filter State x3	0.1000E-07
Filter State x4	0.1000E-07



Your Measurements Output is a State-Feedback ...
Do you want to Calculate a State-Feedback Gain? (Yes)
or a Dynamic Output Feedback via an Estimator ? (No)

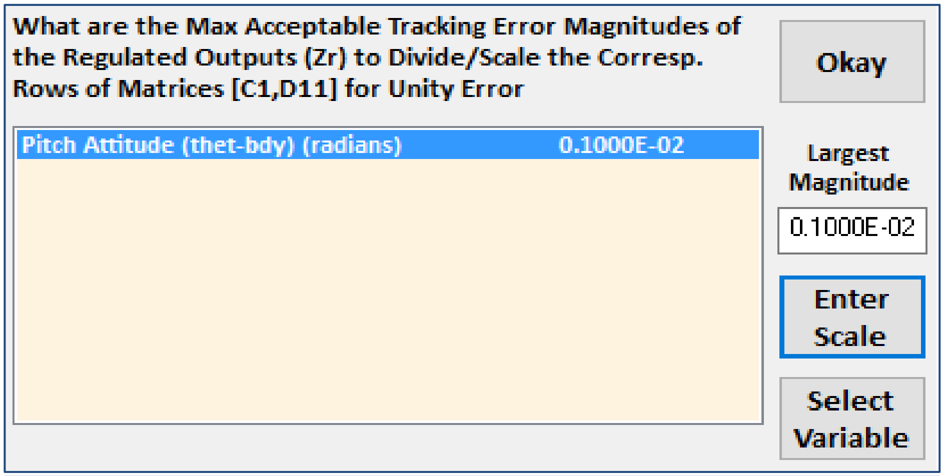
The next dialog is for defining the gains at the performance optimization criteria. That is, the maximum acceptable magnitude at the criteria outputs, which are: the pitch attitude θ, angle of attack α, θ-integral, and the 4 filter states. Reducing the gain value at a performance output produces better performance and smaller transient in that variable. Select one variable at a time, enter the gain and click on “Enter Scale”. When you finish click on “Okay” to go to the next dialog.



What are the Max Acceptable Magnitudes of Performance Criteria Outputs (Zo) to Divide and Scale the Corresp. Rows of Matrices (C1,D11) for Unity Outputs

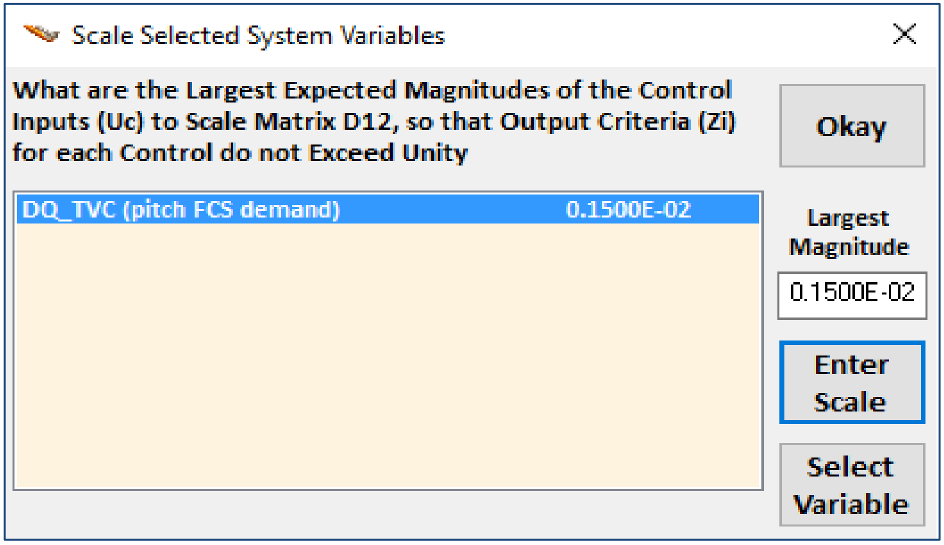
Pitch Attitude (thet-bdy) (radians)	0.1000E-02
Angle of attack, alfa, (radians)	0.7000E-01
Theta Integral	0.5000E-01
Filter State x1	0.3800E-02
Filter State x2	0.3800E-02
Filter State x3	0.3800E-02
Filter State x4	0.3800E-02

We must also enter a gain to define the max regulated output error (z_{re}). The last dialog is for defining the control magnitude because the control is also included in the optimization criteria. In this example we only have the pitch control demand.


What are the Max Acceptable Tracking Error Magnitudes of the Regulated Outputs (Zr) to Divide/Scale the Corresp. Rows of Matrices [C1,D11] for Unity Error

Pitch Attitude (thet-bdy) (radians) 0.1000E-02

Largest Magnitude: 0.1000E-02

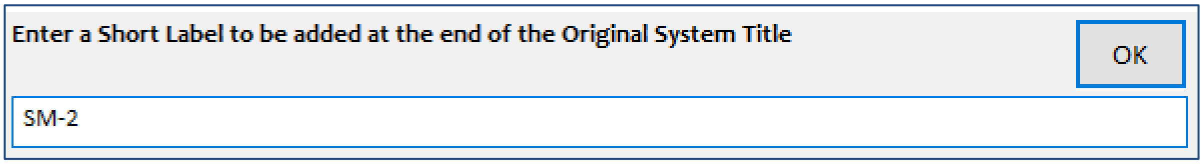

Scale Selected System Variables

What are the Largest Expected Magnitudes of the Control Inputs (Uc) to Scale Matrix D12, so that Output Criteria (Zi) for each Control do not Exceed Unity

DQ_TVC (pitch FCS demand) 0.1500E-02

Largest Magnitude: 0.1500E-02

The gains define the trade-off between performance/ sensitivity versus control bandwidth and stability. If we increase the control gain, we are telling the algorithm to allow more control which means bigger bandwidth and the system performance and speed will improve. Enter the two gains as before and click on "Okay". Finally enter a short label that will appear at the end of the SM title in the systems file.

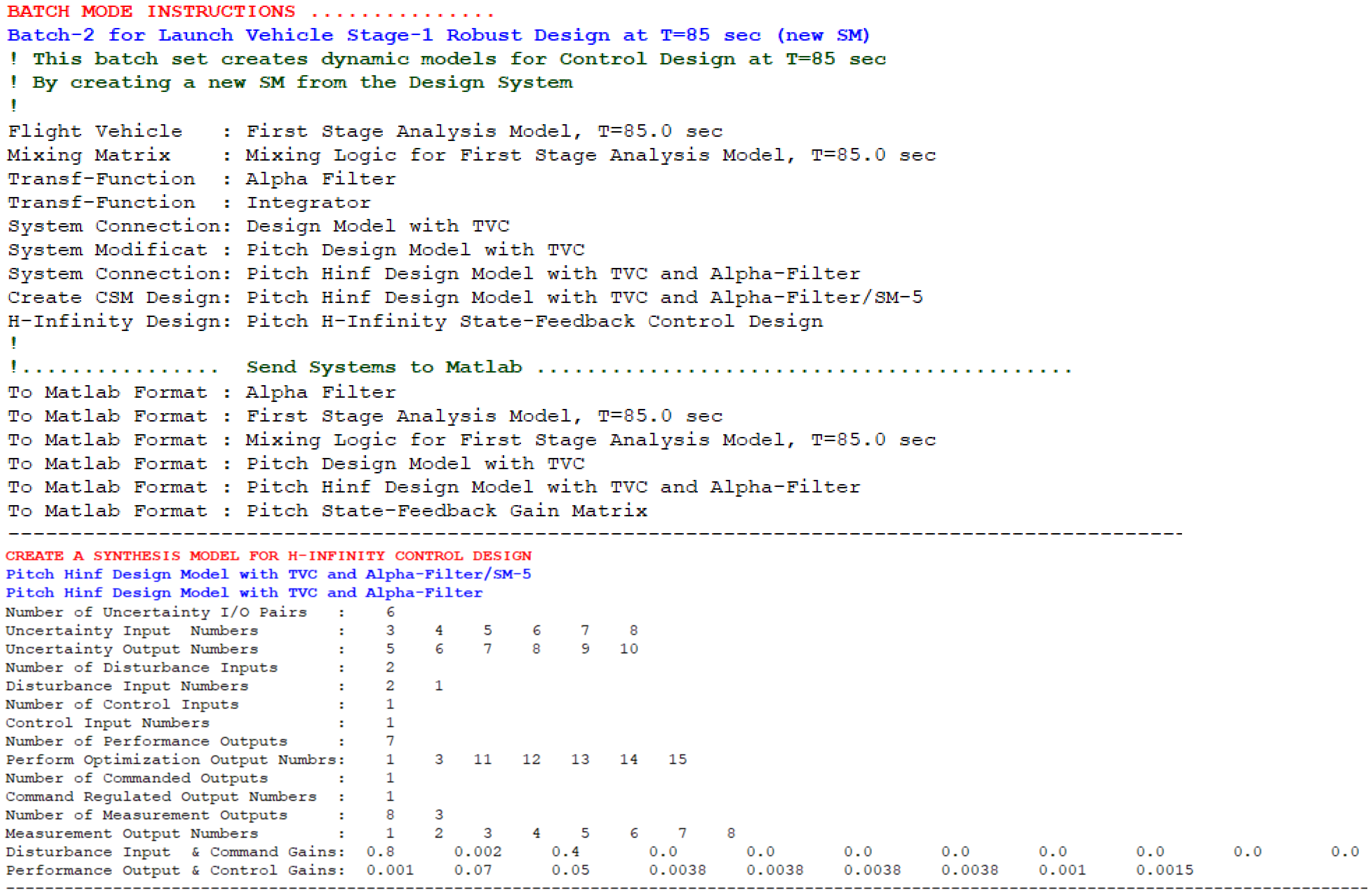

Enter a Short Label to be added at the end of the Original System Title

SM-2

Typically, several iterations are needed to converge to the desired trade-off between performance versus robustness. A simple, preliminary simulation model is often needed to evaluate the design. If we find that we are using too much control, we must reduce the corresponding control gain in the performance criteria output and repeat the design. If a regulated output such as vehicle attitude doesn't converge fast enough to its commanded value, the gain of the corresponding attitude criterion must be reduced. At the completion of the interactive SM creation process a SM creation dataset is automatically saved in the input file. This dataset can be processed later in batch to recreate the SM, as shown in Section 1.5.

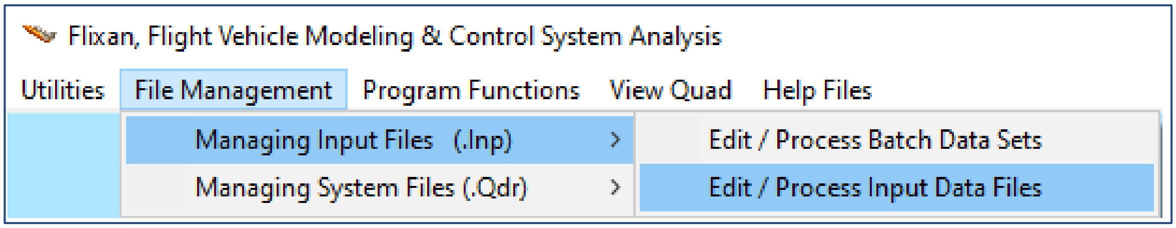
1.5 Creating the Synthesis Model in Batch Mode

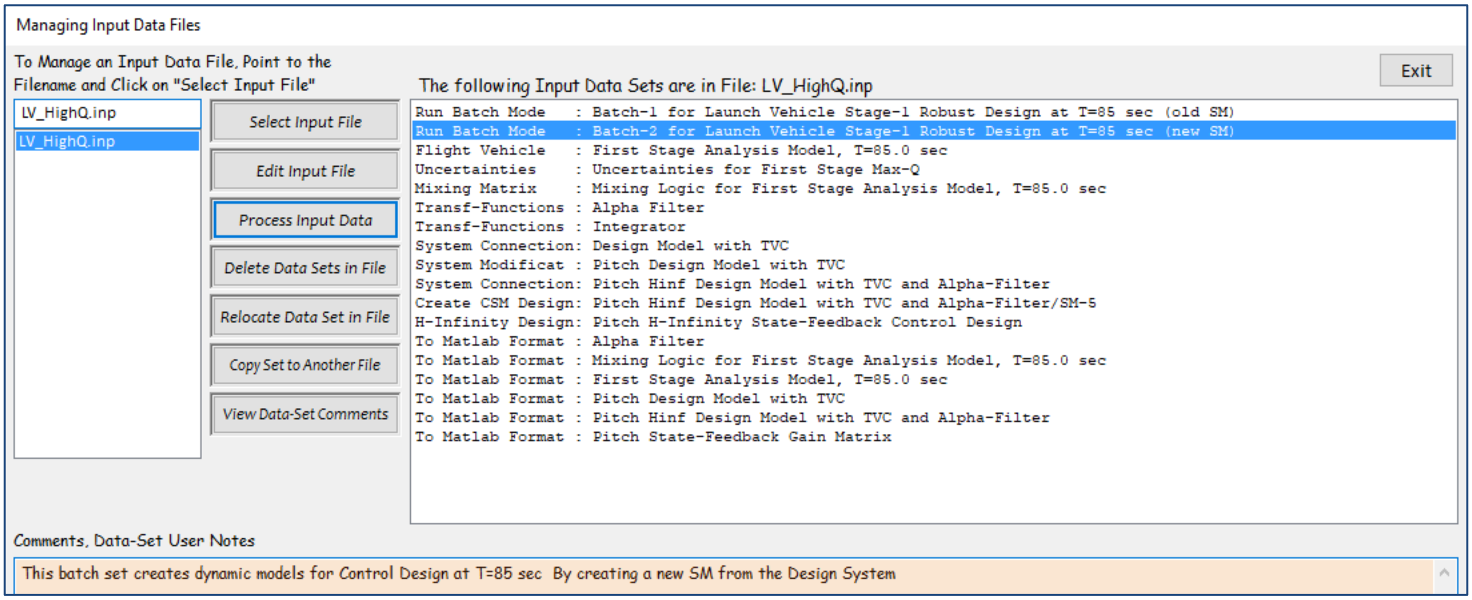
The SM creation time can be shortened by processing it in batch mode instead of creating the SM interactively, then modifying it and re-using it until satisfied with the results. The input file “LV_HighQ.inp” has a second batch dataset, title: “*Batch-2 for Launch Vehicle Stage-1 Robust Design at T=85 sec (new SM)”* which does not retain the old SM but includes a new input set “*Pitch Hinf Design Model with TVC and Alpha-Filter/SM-5*”, shown below. This CSM creation dataset automatically creates the SM from the design system “*Pitch Hinf Design Model with TVC and Alpha-Filter*”. It includes the input and output definitions of the uncertainties, the disturbances, control inputs, criteria outputs, measurements, and the corresponding performance optimization gains. It is processed by Flixan in batch to generate the 9-matrices Synthesis Model.



```
BATCH MODE INSTRUCTIONS ...............
Batch-2 for Launch Vehicle Stage-1 Robust Design at T=85 sec (new SM)
! This batch set creates dynamic models for Control Design at T=85 sec
! By creating a new SM from the Design System
!
Flight Vehicle    : First Stage Analysis Model, T=85.0 sec
Mixing Matrix     : Mixing Logic for First Stage Analysis Model, T=85.0 sec
Transf-Function   : Alpha Filter
Transf-Function   : Integrator
System Connection: Design Model with TVC
System Modificat : Pitch Design Model with TVC
System Connection: Pitch Hinf Design Model with TVC and Alpha-Filter
Create CSM Design: Pitch Hinf Design Model with TVC and Alpha-Filter/SM-5
H-Infinity Design: Pitch H-Infinity State-Feedback Control Design
!
!...............  Send Systems to Matlab ...........................................
To Matlab Format : Alpha Filter
To Matlab Format : First Stage Analysis Model, T=85.0 sec
To Matlab Format : Mixing Logic for First Stage Analysis Model, T=85.0 sec
To Matlab Format : Pitch Design Model with TVC
To Matlab Format : Pitch Hinf Design Model with TVC and Alpha-Filter
To Matlab Format : Pitch State-Feedback Gain Matrix
-----------------------------------------------------------------------------------------------
CREATE A SYNTHESIS MODEL FOR H-INFINITY CONTROL DESIGN
Pitch Hinf Design Model with TVC and Alpha-Filter/SM-5
Pitch Hinf Design Model with TVC and Alpha-Filter
Number of Uncertainty I/O Pairs    :    6
Uncertainty Input  Numbers         :    3    4    5    6    7    8
Uncertainty Output Numbers         :    5    6    7    8    9   10
Number of Disturbance Inputs       :    2
Disturbance Input Numbers          :    2    1
Number of Control Inputs           :    1
Control Input Numbers              :    1
Number of Performance Outputs      :    7
Perform Optimization Output Numbrs:    1    3   11   12   13   14   15
Number of Commanded Outputs        :    1
Command Regulated Output Numbers   :    1
Number of Measurement Outputs      :    8    3
Measurement Output Numbers         :    1    2    3    4    5    6    7    8
Disturbance Input  & Command Gains:  0.8      0.002     0.4       0.0       0.0       0.0       0.0       0.0       0.0       0.0       0.0
Performance Output & Control Gains:  0.001    0.07      0.05      0.0038    0.0038    0.0038    0.0038    0.001     0.0015
------------------------------------------------------------------------------------------------------------------------------------
```

To process the input file in batch mode, go to “*Edit/Process Input Data Files*”, select the project directory and the input datafile management dialog comes up, shown below. From the left menu select the input file “*LV_HighQ.inp*” and click on “*Select Input File*”. From the menu on the right select the second batch set that creates a new SM and it is the second title in that menu, and click on “*Process Input Data*” button to process the input file. This will also create the state-feedback gain Kqhi.





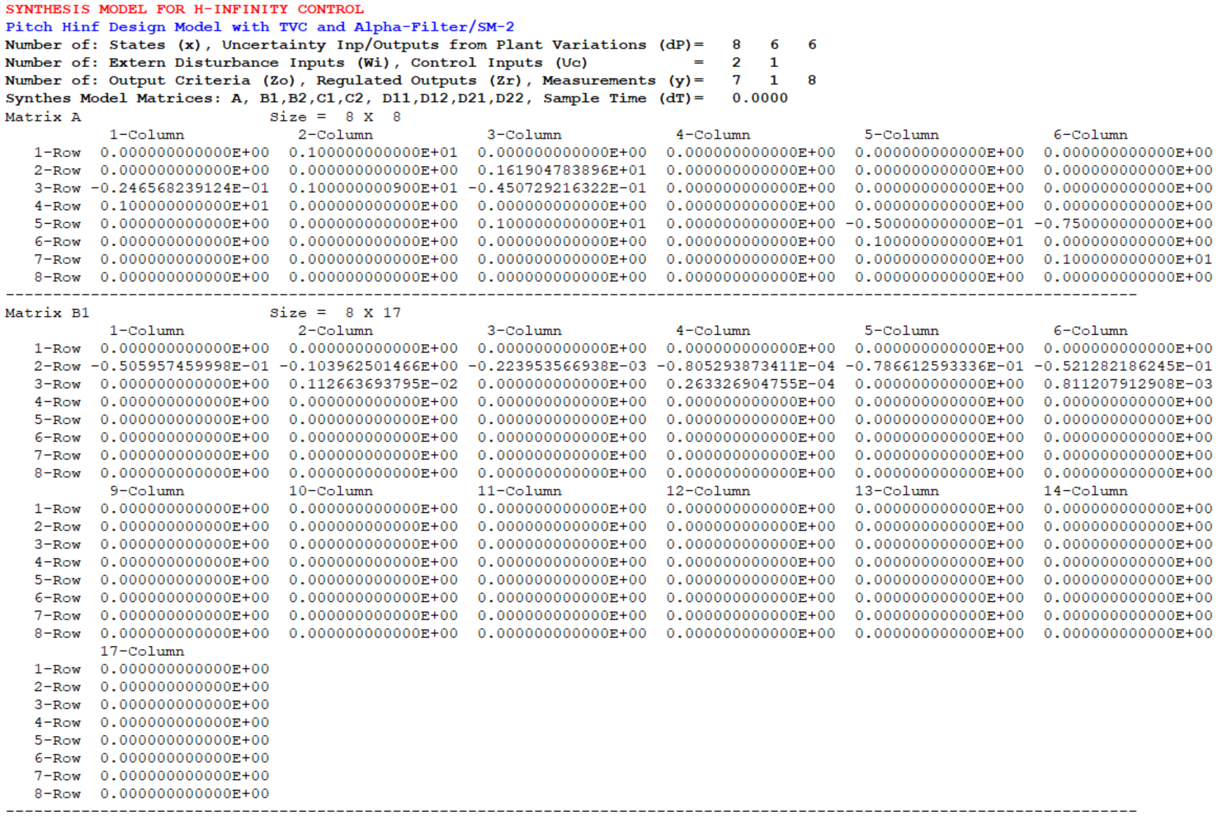
The following Input Data Sets are in File: LV_HighQ.inp

```
Run Batch Mode   : Batch-1 for Launch Vehicle Stage-1 Robust Design at T=85 sec (old SM)
Run Batch Mode   : Batch-2 for Launch Vehicle Stage-1 Robust Design at T=85 sec (new SM)
Flight Vehicle   : First Stage Analysis Model, T=85.0 sec
Uncertainties    : Uncertainties for First Stage Max-Q
Mixing Matrix    : Mixing Logic for First Stage Analysis Model, T=85.0 sec
Transf-Functions : Alpha Filter
Transf-Functions : Integrator
System Connection: Design Model with TVC
System Modificat : Pitch Design Model with TVC
System Connection: Pitch Hinf Design Model with TVC and Alpha-Filter
Create CSM Design: Pitch Hinf Design Model with TVC and Alpha-Filter/SM-5
H-Infinity Design: Pitch H-Infinity State-Feedback Control Design
To Matlab Format : Alpha Filter
To Matlab Format : Mixing Logic for First Stage Analysis Model, T=85.0 sec
To Matlab Format : First Stage Analysis Model, T=85.0 sec
To Matlab Format : Pitch Design Model with TVC
To Matlab Format : Pitch Hinf Design Model with TVC and Alpha-Filter
To Matlab Format : Pitch State-Feedback Gain Matrix
```

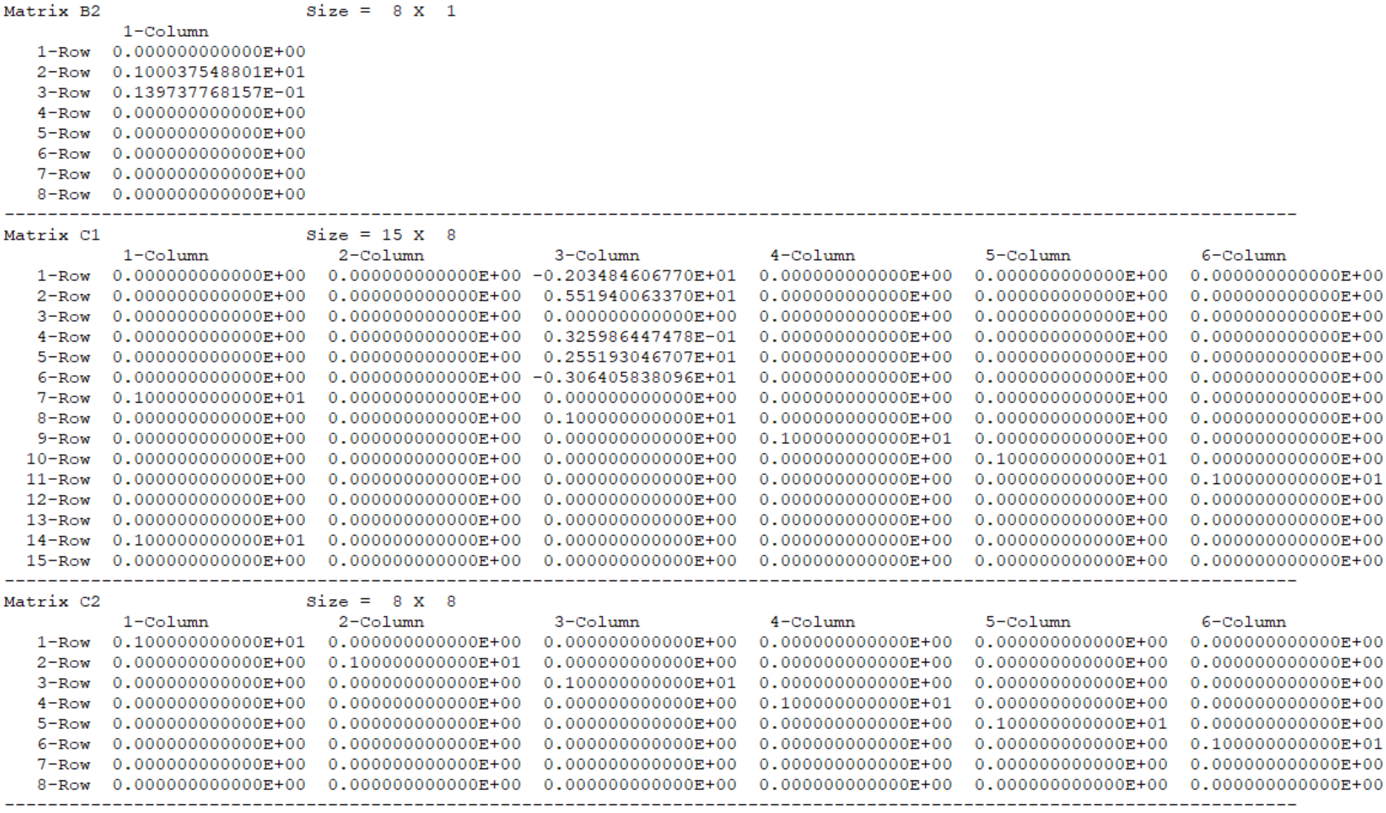
This batch set creates dynamic models for Control Design at T=85 sec By creating a new SM from the Design System

1.6 Systems File

Some sections of the Systems File: “*LV_HighQ.Qdr*” are shown below. The file includes the H-Infinity SM, the TVC Matrix, the augmented design system and the state-feedback gain.



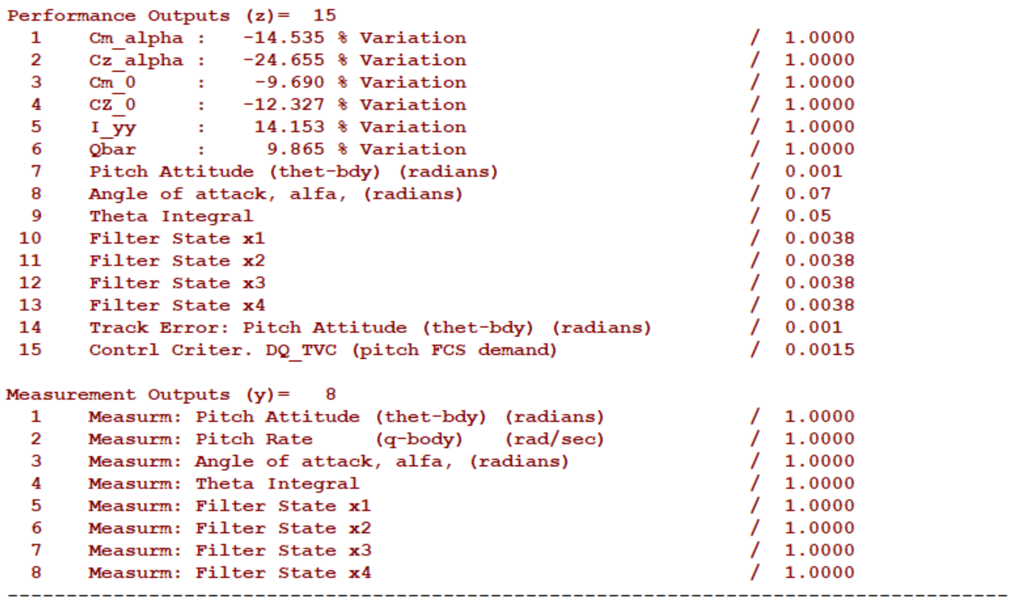
```
SYNTHESIS MODEL FOR H-INFINITY CONTROL
Pitch Hinf Design Model with TVC and Alpha-Filter/SM-2
Number of: States (x), Uncertainty Inp/Outputs from Plant Variations (dP)=   8   6   6
Number of: Extern Disturbance Inputs (Wi), Control Inputs (Uc)          =   2   1
Number of: Output Criteria (Zo), Regulated Outputs (Zr), Measurements (y)=   7   1   8
Synthes Model Matrices: A, B1,B2,C1,C2, D11,D12,D21,D22, Sample Time (dT)=   0.0000
Matrix A                    Size =  8 X  8
           1-Column             2-Column             3-Column             4-Column             5-Column             6-Column
   1-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.161904783896E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row -0.246568239124E-01  0.100000000900E+01 -0.450729216322E-01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00 -0.500000000000E-01 -0.750000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
   8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
-------------------------------------------------------------------------------------------------------------------------
Matrix B1                   Size =  8 X 17
           1-Column             2-Column             3-Column             4-Column             5-Column             6-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row -0.505957459998E-01 -0.103962501466E+00 -0.223953566938E-03 -0.805293873411E-04 -0.786612593336E-01 -0.521282186245E-01
   3-Row  0.000000000000E+00  0.112663693795E-02  0.000000000000E+00  0.263326904755E-04  0.000000000000E+00  0.811207912908E-03
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
           9-Column            10-Column            11-Column            12-Column            13-Column            14-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
          17-Column
   1-Row  0.000000000000E+00
   2-Row  0.000000000000E+00
   3-Row  0.000000000000E+00
   4-Row  0.000000000000E+00
   5-Row  0.000000000000E+00
   6-Row  0.000000000000E+00
   7-Row  0.000000000000E+00
   8-Row  0.000000000000E+00
-------------------------------------------------------------------------------------------------------------------------
```



```
Matrix B2                   Size =  8 X  1
           1-Column
   1-Row  0.000000000000E+00
   2-Row  0.100037548801E+01
   3-Row  0.139737768157E-01
   4-Row  0.000000000000E+00
   5-Row  0.000000000000E+00
   6-Row  0.000000000000E+00
   7-Row  0.000000000000E+00
   8-Row  0.000000000000E+00
-------------------------------------------------------------------------------------------------------------------------
Matrix C1                   Size = 15 X  8
           1-Column             2-Column             3-Column             4-Column             5-Column             6-Column
   1-Row  0.000000000000E+00  0.000000000000E+00 -0.203484606770E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.551940063370E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.325986447478E-01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.255193046707E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00 -0.306405838096E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   8-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   9-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
  10-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
  11-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
  12-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  13-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  14-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  15-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
-------------------------------------------------------------------------------------------------------------------------
Matrix C2                   Size =  8 X  8
           1-Column             2-Column             3-Column             4-Column             5-Column             6-Column
   1-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
-------------------------------------------------------------------------------------------------------------------------
```

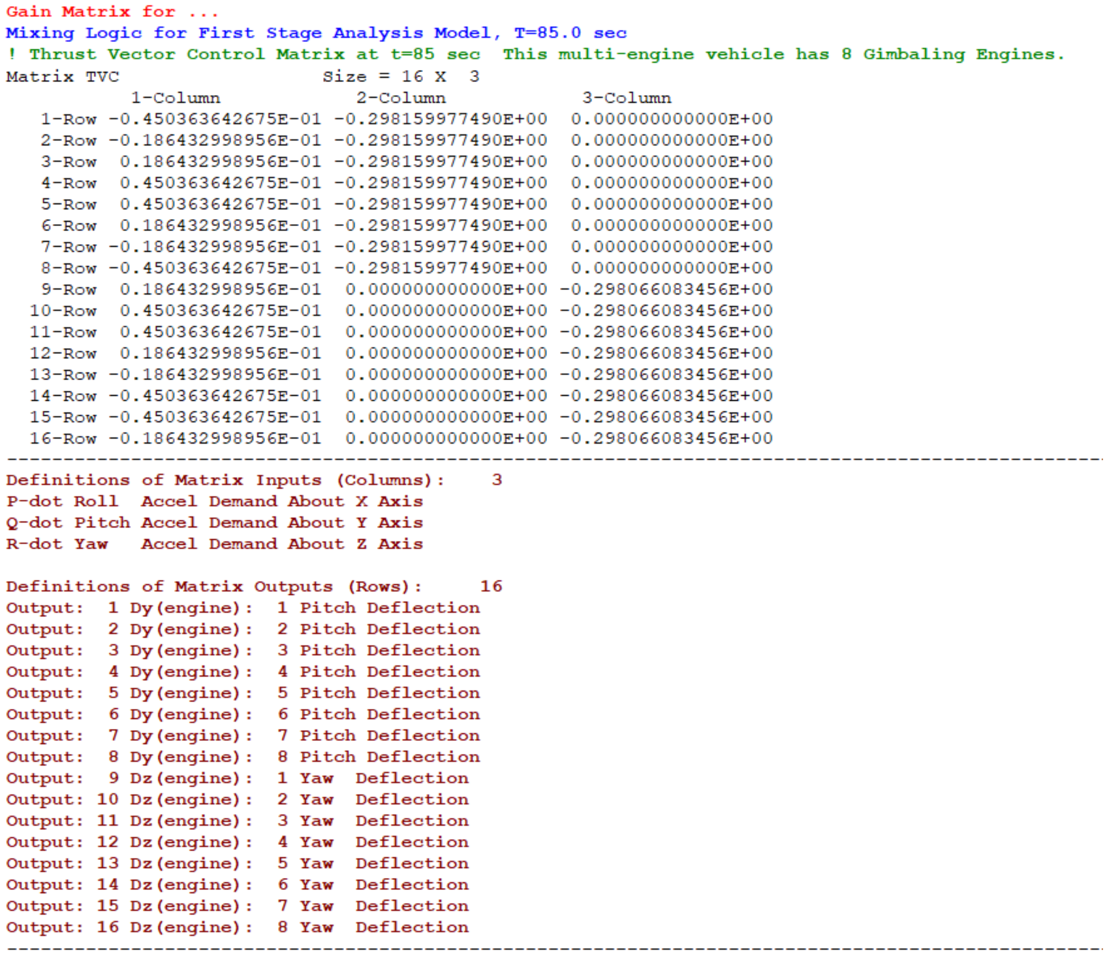
```
Matrix D11                 Size = 15 X 17
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   9-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  10-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  11-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  12-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  13-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  14-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  15-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
           9-Column           10-Column           11-Column           12-Column           13-Column           14-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   9-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  10-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  11-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  12-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  13-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  14-Row -0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  15-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
          17-Column
   1-Row  0.000000000000E+00
   2-Row  0.000000000000E+00
   3-Row  0.000000000000E+00
   4-Row  0.000000000000E+00
   5-Row  0.000000000000E+00
   6-Row  0.000000000000E+00
   7-Row  0.000000000000E+00
   8-Row  0.000000000000E+00
   9-Row  0.000000000000E+00
  10-Row  0.000000000000E+00
  11-Row  0.000000000000E+00
  12-Row  0.000000000000E+00
  13-Row  0.000000000000E+00
  14-Row  0.000000000000E+00
  15-Row  0.000000000000E+00
---------------------------------------------------------------------------------------
Matrix D12                 Size = 15 X  1
           1-Column
   1-Row  0.000000000000E+00
   2-Row  0.000000000000E+00
   3-Row  0.000000000000E+00
   4-Row  0.000000000000E+00
   5-Row  0.157678397445E+01
   6-Row  0.000000000000E+00
   7-Row  0.000000000000E+00
   8-Row  0.000000000000E+00
   9-Row  0.000000000000E+00
  10-Row  0.000000000000E+00
  11-Row  0.000000000000E+00
  12-Row  0.000000000000E+00
  13-Row  0.000000000000E+00
  14-Row  0.000000000000E+00
  15-Row  0.100000000000E+01
---------------------------------------------------------------------------------------
```

```
Matrix D21                  Size =  8 X 17
            1-Column              2-Column              3-Column              4-Column              5-Column              6-Column
   1-Row  0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00
   2-Row  0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00
   3-Row  0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00
   4-Row  0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00
   5-Row  0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00
   6-Row  0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00
   7-Row  0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00
   8-Row  0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00
            9-Column             10-Column             11-Column             12-Column             13-Column             14-Column
   1-Row  0.000000000000E+00 0.100000000000E+01 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00
   2-Row  0.000000000000E+00 0.000000000000E+00 0.100000000000E+01 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00
   3-Row  0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.100000000000E+01 0.000000000000E+00 0.000000000000E+00
   4-Row  0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.100000000000E+01 0.000000000000E+00
   5-Row  0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.100000000000E+01
   6-Row  0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00
   7-Row  0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00
   8-Row  0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00 0.000000000000E+00
           17-Column
   1-Row  0.000000000000E+00
   2-Row  0.000000000000E+00
   3-Row  0.000000000000E+00
   4-Row  0.000000000000E+00
   5-Row  0.000000000000E+00
   6-Row  0.000000000000E+00
   7-Row  0.000000000000E+00
   8-Row  0.100000000000E+01
------------------------------------------------------------------------------------------------------------------------
Matrix D22                  Size =  8 X  1
            1-Column
   1-Row  0.000000000000E+00
   2-Row  0.000000000000E+00
   3-Row  0.000000000000E+00
   4-Row  0.000000000000E+00
   5-Row  0.000000000000E+00
   6-Row  0.000000000000E+00
   7-Row  0.000000000000E+00
   8-Row  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Definition of Synthesis Model Variables                        Max Scaling Factors

States (x) ..........  =   8
  1    Pitch Attitude (thet-bdy) (radians)
  2    Pitch Rate     (q-body)   (rad/sec)
  3    Angle of attack, alfa, (radians)
  4    Theta Integral
  5    Filter State x1
  6    Filter State x2
  7    Filter State x3
  8    Filter State x4

Excitation Inputs (w)  =  17
  1    Cm_alpha :   -14.535 % Variation                        *  1.0000
  2    Cz_alpha :   -24.655 % Variation                        *  1.0000
  3    Cm_0     :    -9.690 % Variation                        *  1.0000
  4    CZ_0     :   -12.327 % Variation                        *  1.0000
  5    I_yy     :    14.153 % Variation                        *  1.0000
  6    Qbar     :     9.865 % Variation                        *  1.0000
  7    Wind Gust Azim, Elev Angles=(45, 90) (deg)              *  0.8
  8    DQ_TVC (pitch FCS demand)                               *  0.002
  9    Commd for Outpt: Pitch Attitude (thet-bdy) (radians)    *  0.400
 10    Noise at Output: Pitch Attitude (thet-bdy) (radians)    *  0.0000
 11    Noise at Output: Pitch Rate     (q-body)   (rad/sec)    *  0.0000
 12    Noise at Output: Angle of attack, alfa, (radians)       *  0.0000
 13    Noise at Output: Theta Integral                         *  0.0000
 14    Noise at Output: Filter State x1                        *  0.0000
 15    Noise at Output: Filter State x2                        *  0.0000
 16    Noise at Output: Filter State x3                        *  0.0000
 17    Noise at Output: Filter State x4                        *  0.0000

Control Inputs (u) ... =   1
  1    Control: DQ_TVC (pitch FCS demand)                      *  1.0000
```



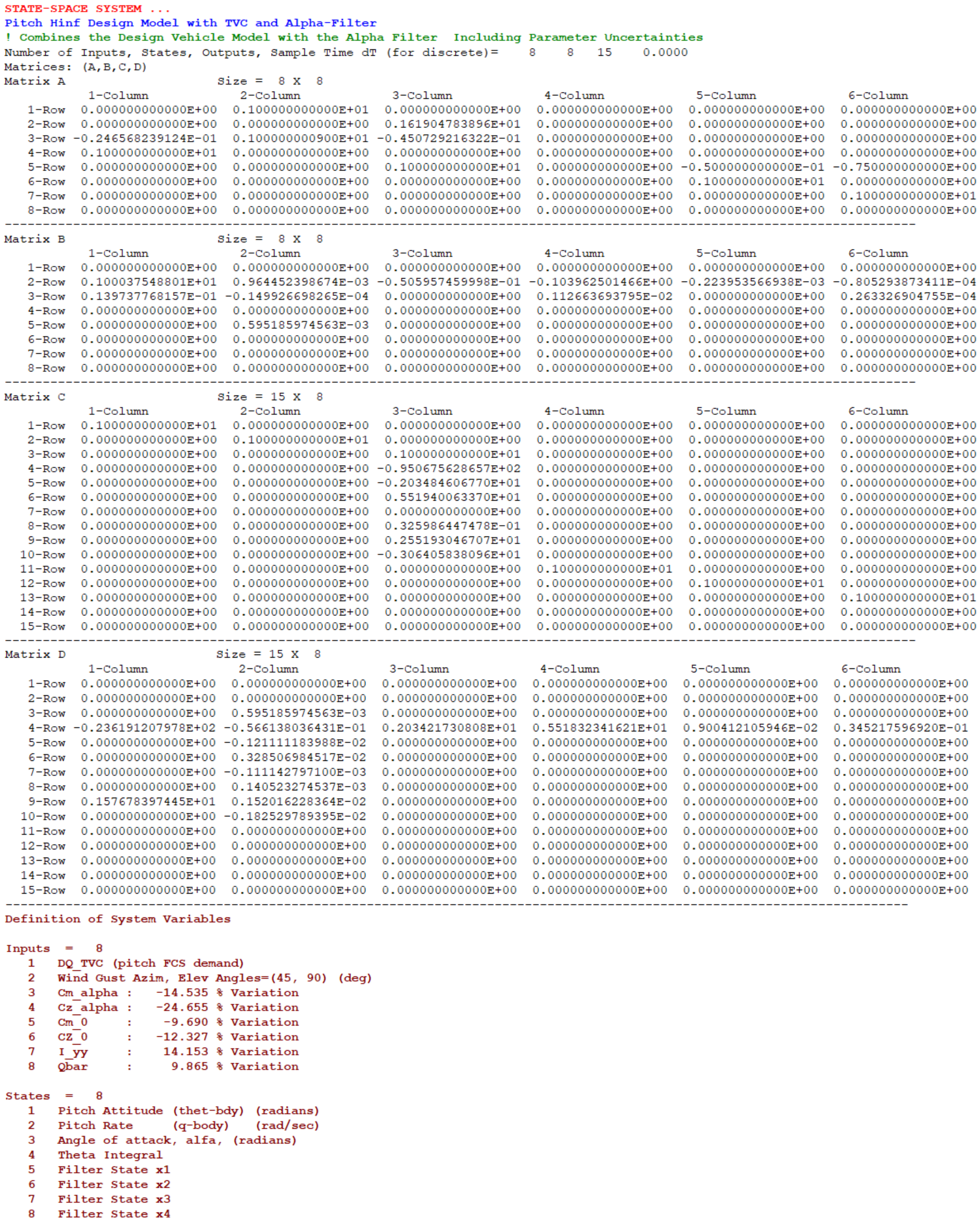
```
Performance Outputs (z)=  15
  1    Cm_alpha :    -14.535 % Variation                     /  1.0000
  2    Cz_alpha :    -24.655 % Variation                     /  1.0000
  3    Cm_0     :     -9.690 % Variation                     /  1.0000
  4    CZ_0     :    -12.327 % Variation                     /  1.0000
  5    I_yy     :     14.153 % Variation                     /  1.0000
  6    Qbar     :      9.865 % Variation                     /  1.0000
  7    Pitch Attitude (thet-bdy) (radians)                   /  0.001
  8    Angle of attack, alfa, (radians)                      /  0.07
  9    Theta Integral                                        /  0.05
 10    Filter State x1                                       /  0.0038
 11    Filter State x2                                       /  0.0038
 12    Filter State x3                                       /  0.0038
 13    Filter State x4                                       /  0.0038
 14    Track Error: Pitch Attitude (thet-bdy) (radians)      /  0.001
 15    Contrl Criter. DQ_TVC (pitch FCS demand)              /  0.0015

Measurement Outputs (y)=   8
  1    Measurm: Pitch Attitude (thet-bdy) (radians)          /  1.0000
  2    Measurm: Pitch Rate      (q-body)    (rad/sec)        /  1.0000
  3    Measurm: Angle of attack, alfa, (radians)             /  1.0000
  4    Measurm: Theta Integral                               /  1.0000
  5    Measurm: Filter State x1                              /  1.0000
  6    Measurm: Filter State x2                              /  1.0000
  7    Measurm: Filter State x3                              /  1.0000
  8    Measurm: Filter State x4                              /  1.0000
-------------------------------------------------------------------------------------
```

The scaling gains of the excitation inputs and performance criteria outputs are included on the right side, next to the corresponding variables. Below is the (16x3) TVC matrix.

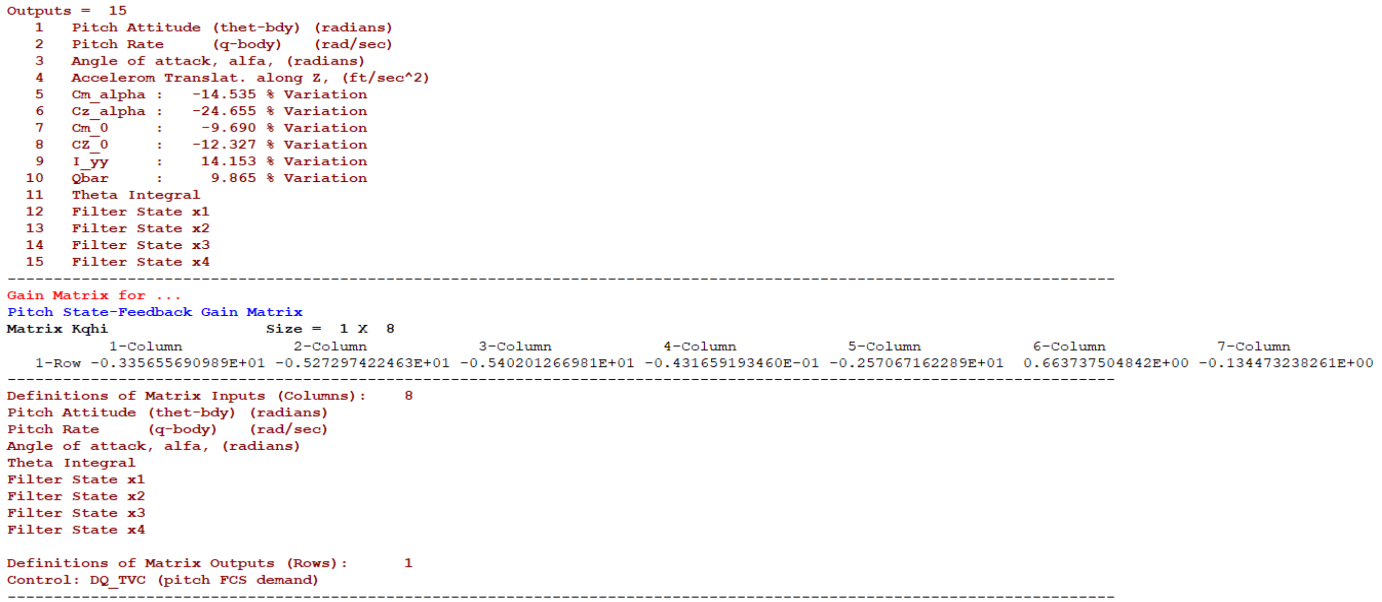


```
Gain Matrix for ...
Mixing Logic for First Stage Analysis Model, T=85.0 sec
! Thrust Vector Control Matrix at t=85 sec  This multi-engine vehicle has 8 Gimbaling Engines.
Matrix TVC                Size = 16 X  3
           1-Column             2-Column             3-Column
   1-Row -0.450363642675E-01 -0.298159977490E+00  0.000000000000E+00
   2-Row -0.186432998956E-01 -0.298159977490E+00  0.000000000000E+00
   3-Row  0.186432998956E-01 -0.298159977490E+00  0.000000000000E+00
   4-Row  0.450363642675E-01 -0.298159977490E+00  0.000000000000E+00
   5-Row  0.450363642675E-01 -0.298159977490E+00  0.000000000000E+00
   6-Row  0.186432998956E-01 -0.298159977490E+00  0.000000000000E+00
   7-Row -0.186432998956E-01 -0.298159977490E+00  0.000000000000E+00
   8-Row -0.450363642675E-01 -0.298159977490E+00  0.000000000000E+00
   9-Row  0.186432998956E-01  0.000000000000E+00 -0.298066083456E+00
  10-Row  0.450363642675E-01  0.000000000000E+00 -0.298066083456E+00
  11-Row  0.450363642675E-01  0.000000000000E+00 -0.298066083456E+00
  12-Row  0.186432998956E-01  0.000000000000E+00 -0.298066083456E+00
  13-Row -0.186432998956E-01  0.000000000000E+00 -0.298066083456E+00
  14-Row -0.450363642675E-01  0.000000000000E+00 -0.298066083456E+00
  15-Row -0.450363642675E-01  0.000000000000E+00 -0.298066083456E+00
  16-Row -0.186432998956E-01  0.000000000000E+00 -0.298066083456E+00
----------------------------------------------------------------------------------------------------
Definitions of Matrix Inputs (Columns):    3
P-dot Roll   Accel Demand About X Axis
Q-dot Pitch Accel Demand About Y Axis
R-dot Yaw    Accel Demand About Z Axis

Definitions of Matrix Outputs (Rows):     16
Output:  1 Dy(engine):  1 Pitch Deflection
Output:  2 Dy(engine):  2 Pitch Deflection
Output:  3 Dy(engine):  3 Pitch Deflection
Output:  4 Dy(engine):  4 Pitch Deflection
Output:  5 Dy(engine):  5 Pitch Deflection
Output:  6 Dy(engine):  6 Pitch Deflection
Output:  7 Dy(engine):  7 Pitch Deflection
Output:  8 Dy(engine):  8 Pitch Deflection
Output:  9 Dz(engine):  1 Yaw  Deflection
Output: 10 Dz(engine):  2 Yaw  Deflection
Output: 11 Dz(engine):  3 Yaw  Deflection
Output: 12 Dz(engine):  4 Yaw  Deflection
Output: 13 Dz(engine):  5 Yaw  Deflection
Output: 14 Dz(engine):  6 Yaw  Deflection
Output: 15 Dz(engine):  7 Yaw  Deflection
Output: 16 Dz(engine):  8 Yaw  Deflection
----------------------------------------------------------------------------------------------------
```

The following is the augmented pitch design system that includes the 4th order alpha-filter and the theta-integrator.



```
STATE-SPACE SYSTEM ...
Pitch Hinf Design Model with TVC and Alpha-Filter
! Combines the Design Vehicle Model with the Alpha Filter  Including Parameter Uncertainties
Number of Inputs, States, Outputs, Sample Time dT (for discrete)=    8    8   15    0.0000
Matrices: (A,B,C,D)
Matrix A                   Size =  8 X  8
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.161904783896E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row -0.246568239124E-01  0.100000000900E+01 -0.450729216322E-01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00 -0.500000000000E-01 -0.750000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
   8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
--------------------------------------------------------------------------------------------------------------------------
Matrix B                   Size =  8 X  8
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.100037548801E+01  0.964452398674E-03 -0.505957459998E-01 -0.103962501466E+00 -0.223953566938E-03 -0.805293873411E-04
   3-Row  0.139737768157E-01 -0.149926698265E-04  0.000000000000E+00  0.112663693795E-02  0.000000000000E+00  0.263326904755E-04
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.595185974563E-03  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
--------------------------------------------------------------------------------------------------------------------------
Matrix C                   Size = 15 X  8
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00 -0.950675628657E+02  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00 -0.203484606770E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.551940063370E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   8-Row  0.000000000000E+00  0.000000000000E+00  0.325986447478E-01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   9-Row  0.000000000000E+00  0.000000000000E+00  0.255193046707E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  10-Row  0.000000000000E+00  0.000000000000E+00 -0.306405838096E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  11-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
  12-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
  13-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
  14-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  15-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
--------------------------------------------------------------------------------------------------------------------------
Matrix D                   Size = 15 X  8
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.595185974563E-03  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row -0.236191207978E+02 -0.566138036431E-01  0.203421730808E+01  0.551832341621E+01  0.900412105946E-02  0.345217596920E-01
   5-Row  0.000000000000E+00 -0.121111183988E-02  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.328506984517E-02  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00 -0.111142797100E-03  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   8-Row  0.000000000000E+00  0.140523274537E-03  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   9-Row  0.157678397445E+01  0.152016228364E-02  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  10-Row  0.000000000000E+00 -0.182529789395E-02  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  11-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  12-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  13-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  14-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  15-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
--------------------------------------------------------------------------------------------------------------------------
Definition of System Variables

Inputs  =   8
   1   DQ_TVC (pitch FCS demand)
   2   Wind Gust Azim, Elev Angles=(45, 90) (deg)
   3   Cm_alpha :   -14.535 % Variation
   4   Cz_alpha :   -24.655 % Variation
   5   Cm_0     :    -9.690 % Variation
   6   CZ_0     :   -12.327 % Variation
   7   I_yy     :    14.153 % Variation
   8   Qbar     :     9.865 % Variation

States  =   8
   1   Pitch Attitude (thet-bdy) (radians)
   2   Pitch Rate      (q-body)   (rad/sec)
   3   Angle of attack, alfa, (radians)
   4   Theta Integral
   5   Filter State x1
   6   Filter State x2
   7   Filter State x3
   8   Filter State x4
```

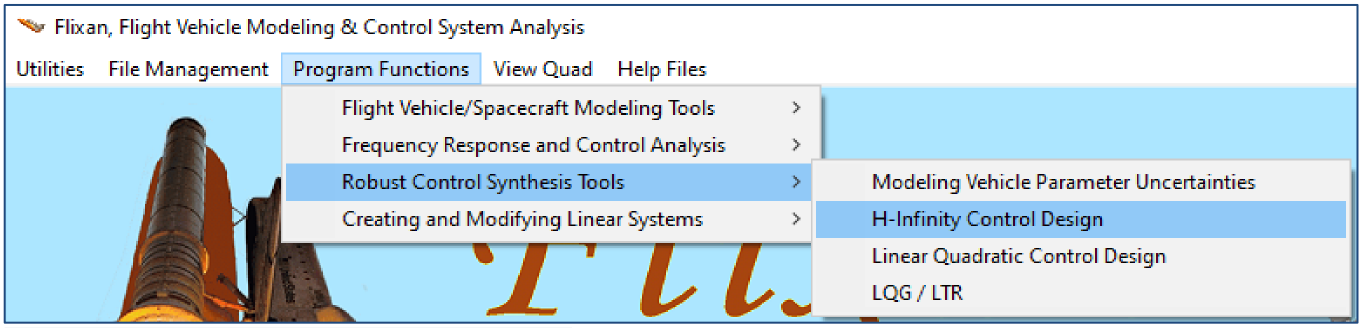


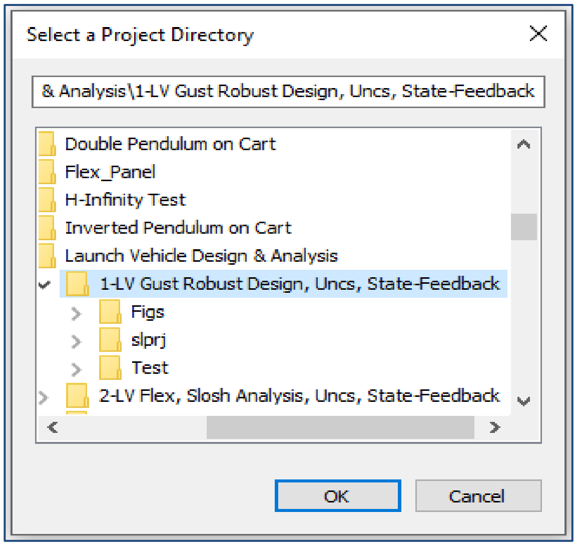
```
Outputs =  15
    1    Pitch Attitude  (thet-bdy)  (radians)
    2    Pitch Rate      (q-body)    (rad/sec)
    3    Angle of attack, alfa,  (radians)
    4    Accelerom Translat. along Z,  (ft/sec^2)
    5    Cm_alpha :    -14.535 % Variation
    6    Cz_alpha :    -24.655 % Variation
    7    Cm_0     :     -9.690 % Variation
    8    Cz_0     :    -12.327 % Variation
    9    I_yy     :     14.153 % Variation
   10    Qbar     :      9.865 % Variation
   11    Theta Integral
   12    Filter State x1
   13    Filter State x2
   14    Filter State x3
   15    Filter State x4
--------------------------------------------------------------------------------------------------
Gain Matrix for ...
Pitch State-Feedback Gain Matrix
Matrix Kqhi                   Size =  1 X  8
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column            7-Column
   1-Row -0.335655690989E+01 -0.527297422463E+01 -0.540201266981E+01 -0.431659193460E-01 -0.257067162289E+01  0.663737504842E+00 -0.134473238261E+00
--------------------------------------------------------------------------------------------------
Definitions of Matrix Inputs (Columns):     8
Pitch Attitude  (thet-bdy)  (radians)
Pitch Rate      (q-body)    (rad/sec)
Angle of attack, alfa,  (radians)
Theta Integral
Filter State x1
Filter State x2
Filter State x3
Filter State x4

Definitions of Matrix Outputs (Rows):       1
Control: DQ_TVC  (pitch FCS demand)
--------------------------------------------------------------------------------------------------
```

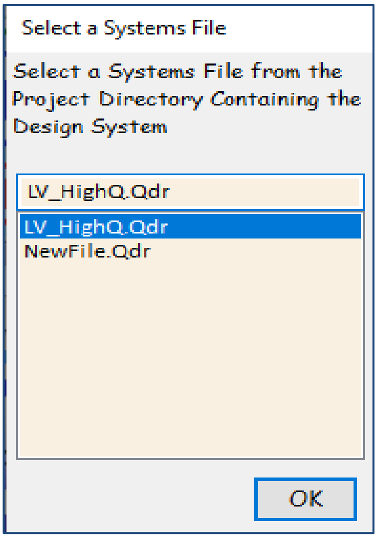
The (1x8) pitch state-feedback gain matrix Kqhi which is calculated by the H-Infinity algorithm is included at the bottom of the systems file.

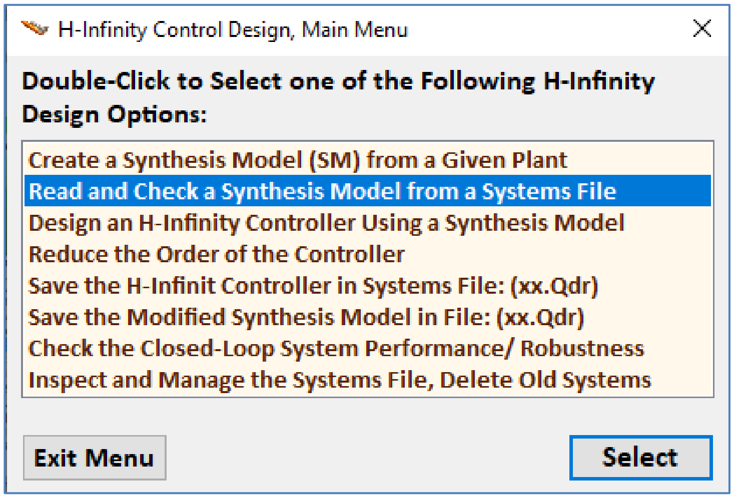
1.7 Designing the H-Infinity State-Feedback Controller

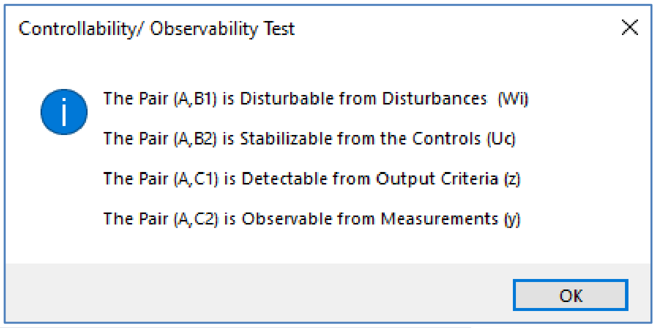
We will now use the SM to design the H-infinity controller interactively. We can also do it in batch mode by processing the batch datasets. Run again the H-infinity design program and from the main menu select the second option to read and process the SM which is already in file. From the next menu select its title. There is only one SM, and click on “Select”.









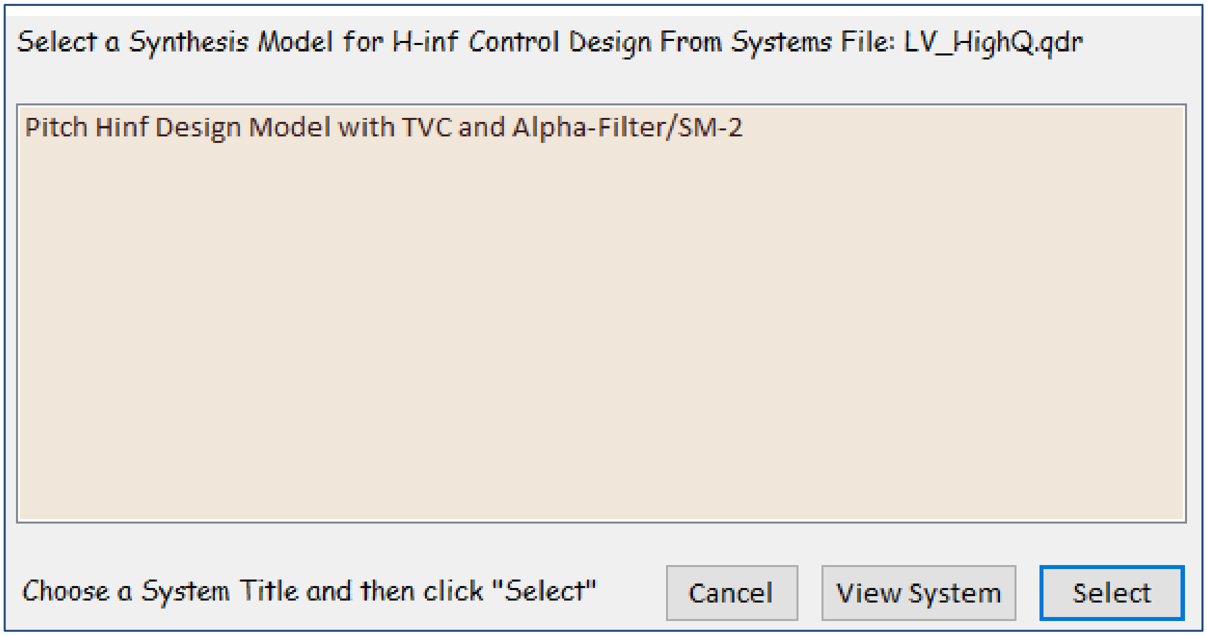


The Pair (A,B1) is Disturbable from Disturbances (Wi)

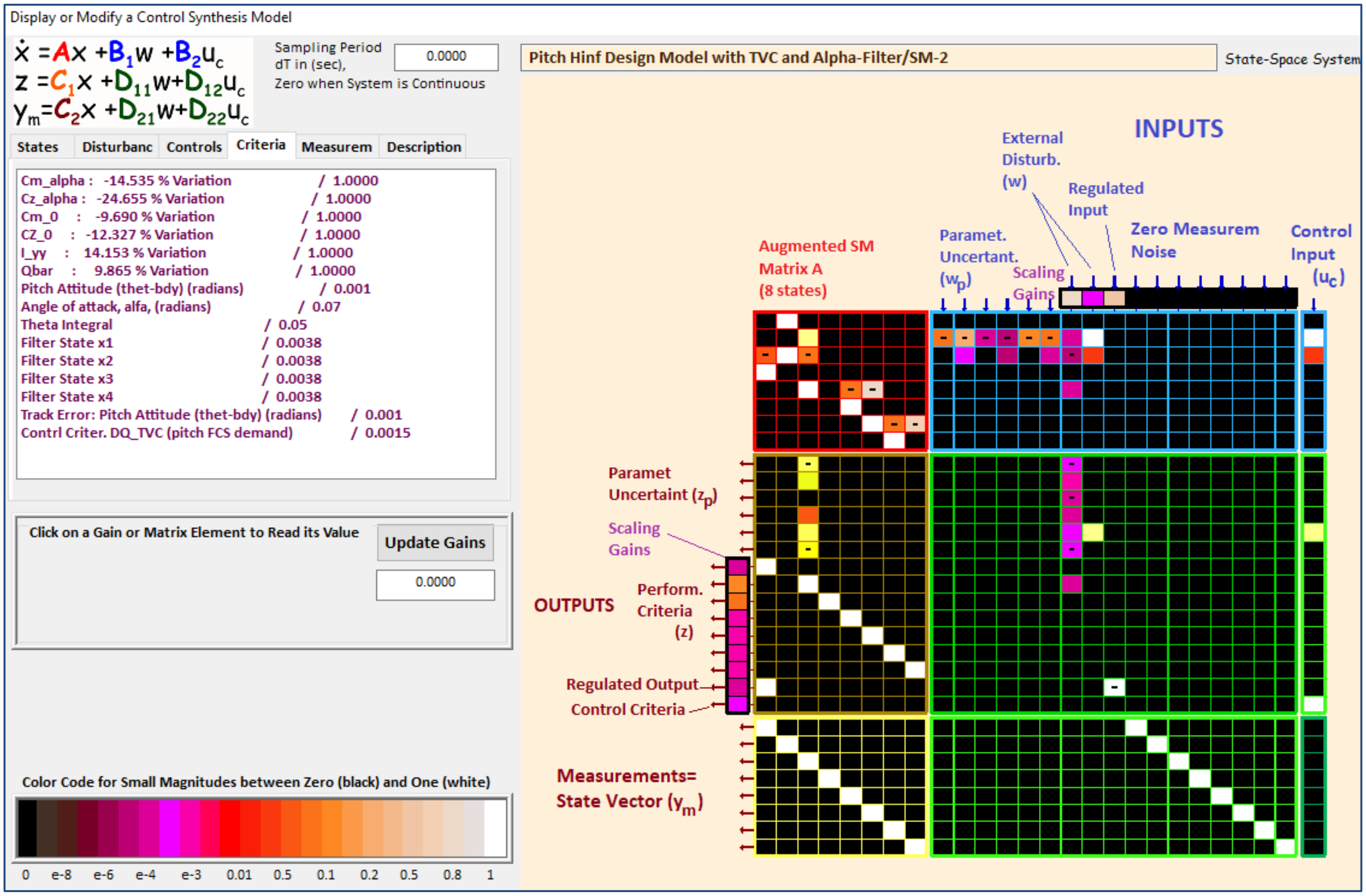
The Pair (A,B2) is Stabilizable from the Controls (Uc)

The Pair (A,C1) is Detectable from Output Criteria (z)

The Pair (A,C2) is Observable from Measurements (y)



The program confirms that the SM meets the expected observability and controllability requirements and displays the SM matrices graphically in system form in the dialog below. The 9 SM matrices appear color coded and the scaling gains that scale the disturbances and the criteria are also shown in the inputs and outputs. The A-matrix consists of 8 states. There are 6 uncertainty inputs (which have already been scaled to correspond to ±unity Δ and don't need scaling gains), 2 external disturbances, 1 command for a regulated output, 8 measurements noise inputs which are set to zero (black), and one pitch control input (dQ). We also have 6 uncertainty outputs (always the same as the uncertainty inputs), 7 performance criteria outputs, 1 criterion for the regulated output error, 1 control utilization criterion that penalizes the control magnitude, and 8 measurements which are also the state-vector because C_2 is the identity matrix.


Display or Modify a Control Synthesis Model

$$\dot{x} = Ax + B_1 w + B_2 u_c$$
$$z = C_1 x + D_{11} w + D_{12} u_c$$
$$y_m = C_2 x + D_{21} w + D_{22} u_c$$

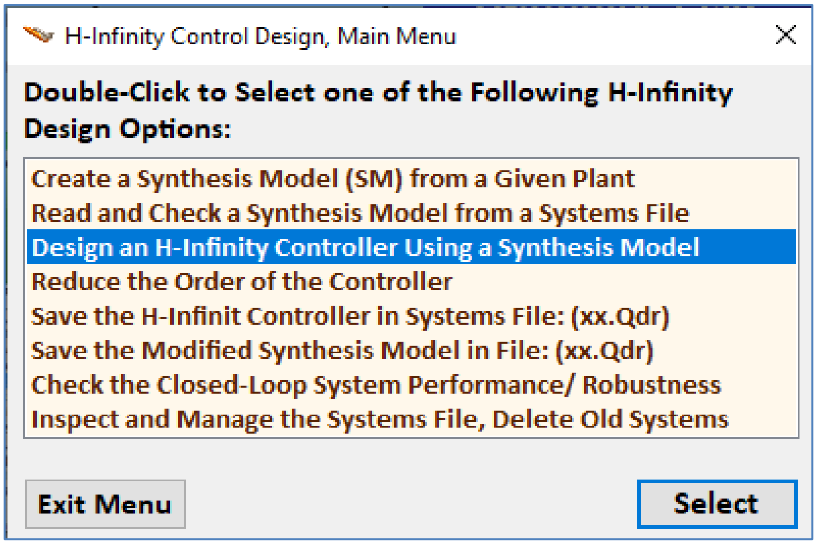
Sampling Period dT in (sec), Zero when System is Continuous: 0.0000

Pitch Hinf Design Model with TVC and Alpha-Filter/SM-2

```
Cm_alpha :   -14.535 % Variation              / 1.0000
Cz_alpha :   -24.655 % Variation             / 1.0000
Cm_0     :    -9.690 % Variation            / 1.0000
CZ_0     :   -12.327 % Variation            / 1.0000
I_yy     :    14.153 % Variation           / 1.0000
Qbar     :     9.865 % Variation           / 1.0000
Pitch Attitude (thet-bdy) (radians)           / 0.001
Angle of attack, alfa, (radians)           / 0.07
Theta Integral                         / 0.05
Filter State x1                        / 0.0038
Filter State x2                        / 0.0038
Filter State x3                        / 0.0038
Filter State x4                        / 0.0038
Track Error: Pitch Attitude (thet-bdy) (radians)       / 0.001
Contrl Criter. DQ_TVC (pitch FCS demand)          / 0.0015
```

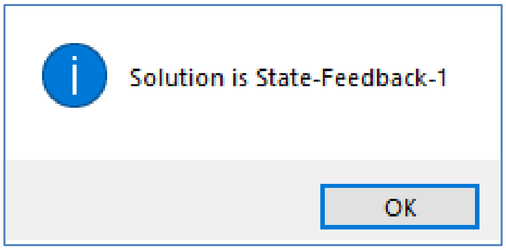
Click on a Gain or Matrix Element to Read its Value: 0.0000

Select the third option from the main menu to design the H-infinity controller, and click "Select". The program confirms that the solution is a state-feedback gain rather than dynamic.

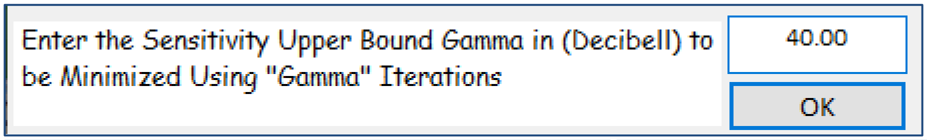

H-Infinity Control Design, Main Menu

Double-Click to Select one of the Following H-Infinity Design Options:

- Create a Synthesis Model (SM) from a Given Plant
- Read and Check a Synthesis Model from a Systems File
- Design an H-Infinity Controller Using a Synthesis Model
- Reduce the Order of the Controller
- Save the H-Infinit Controller in Systems File: (xx.Qdr)
- Save the Modified Synthesis Model in File: (xx.Qdr)
- Check the Closed-Loop System Performance/ Robustness
- Inspect and Manage the Systems File, Delete Old Systems

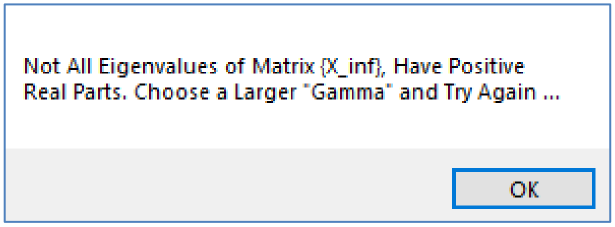

Solution is State-Feedback-1

Now we begin the iterative process of minimizing the upper bound γ of the infinity norm of the sensitivity transfer function between the disturbance inputs and the output criteria. We begin with an arbitrary large γ upper bound and try to find the smallest γ that will not violate the algorithm requirements. After 2-3 iterations we find that γ=44 (dB) works and we click on "No" meaning that we do not want to try another value but to accept the current controller.

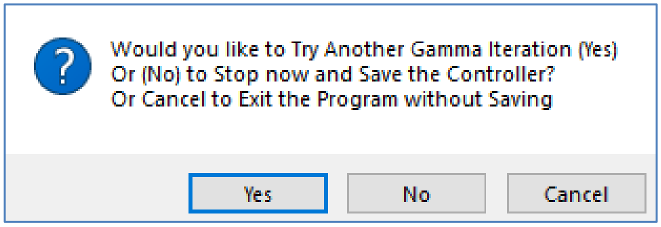


Enter the Sensitivity Upper Bound Gamma in (Decibell) to be Minimized Using "Gamma" Iterations

40.00



Not All Eigenvalues of Matrix {X_inf}, Have Positive Real Parts. Choose a Larger "Gamma" and Try Again ...



Would you like to Try Another Gamma Iteration (Yes)
Or (No) to Stop now and Save the Controller?
Or Cancel to Exit the Program without Saving

Figure-7 shows the closed-loop system poles with the control loop closed as in Figure-5, between the control inputs (u_c) and the measurements (y_m) via the state-feedback gain Kqhi. They are all stable as expected. We immediately notice that the control system has two complex pairs of poles near the disturbance frequency range. The green line corresponds to damping $\zeta=0.707$ and the red line to $\zeta=0.01$. They are used for damping reference. We return to the H-infinity main menu and at this point we can save the controller gain by clicking on "*Save the H-infinity Controller in Systems File (x.Qdr)*".

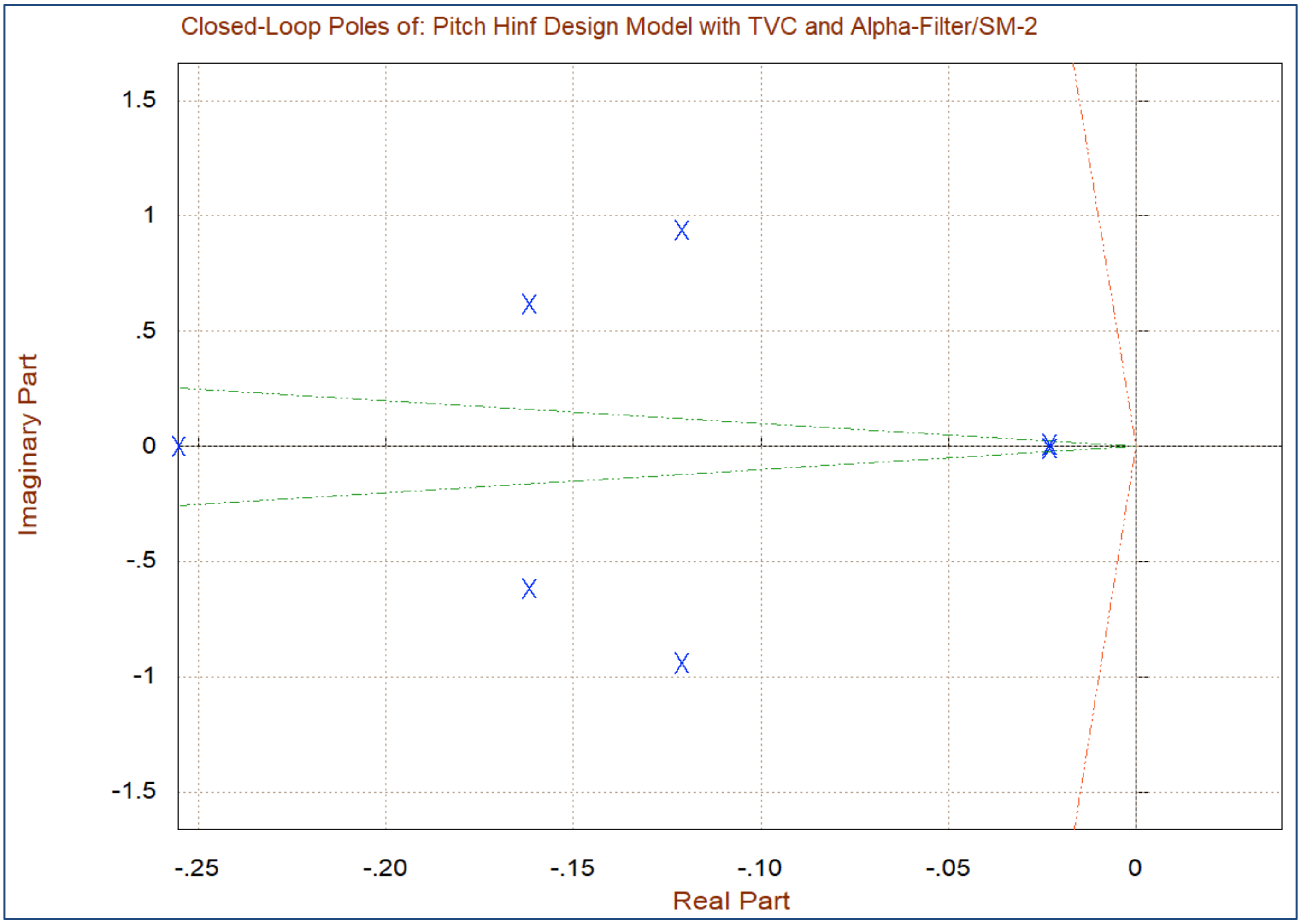
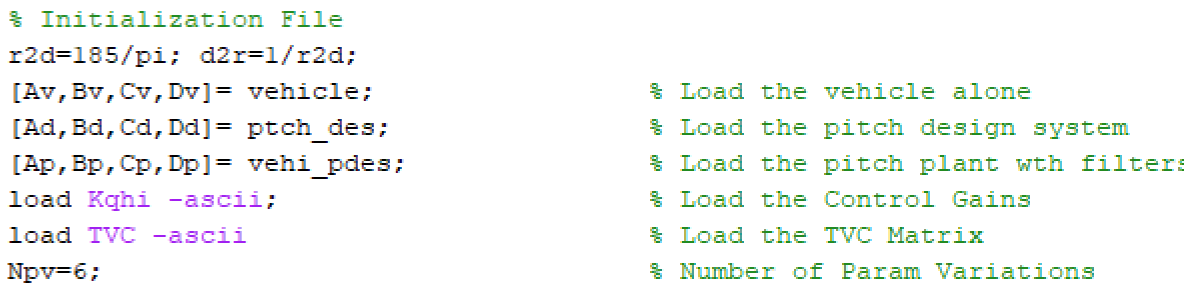


Figure 7 Closed-Loop System Poles

1.8 Control Analysis

We begin the control analysis by running the initialization file “init.m” which loads the vehicle systems, the TVC matrix and the H-infinity derived state-feedback gain from file “Kqhi.Mat”. We will use Matlab to analyze the control system stability and sensitivity to wind-gust disturbances and then analyze the system robustness to uncertainties. We have 6 parameter uncertainties in the pitch model. The analysis files are in directory “*Flixan\Control Analysis\Hinfinity\Examples\Launch Vehicle Design & Analysis\1-LV Gust Robust Design, Uncs, State-Feedback*”. In the analysis we will use Simulink models that include the augmented pitch system “*Pitch Hinf Design Model with TVC and Alpha-Filter*” loaded from file “*vehi_pdes*”.



```
% Initialization File
r2d=185/pi; d2r=1/r2d;
[Av,Bv,Cv,Dv]= vehicle;                % Load the vehicle alone
[Ad,Bd,Cd,Dd]= ptch_des;               % Load the pitch design system
[Ap,Bp,Cp,Dp]= vehi_pdes;              % Load the pitch plant wth filter
load Kqhi -ascii;                      % Load the Control Gains
load TVC -ascii                        % Load the TVC Matrix
Npv=6;                                 % Number of Param Variations
```

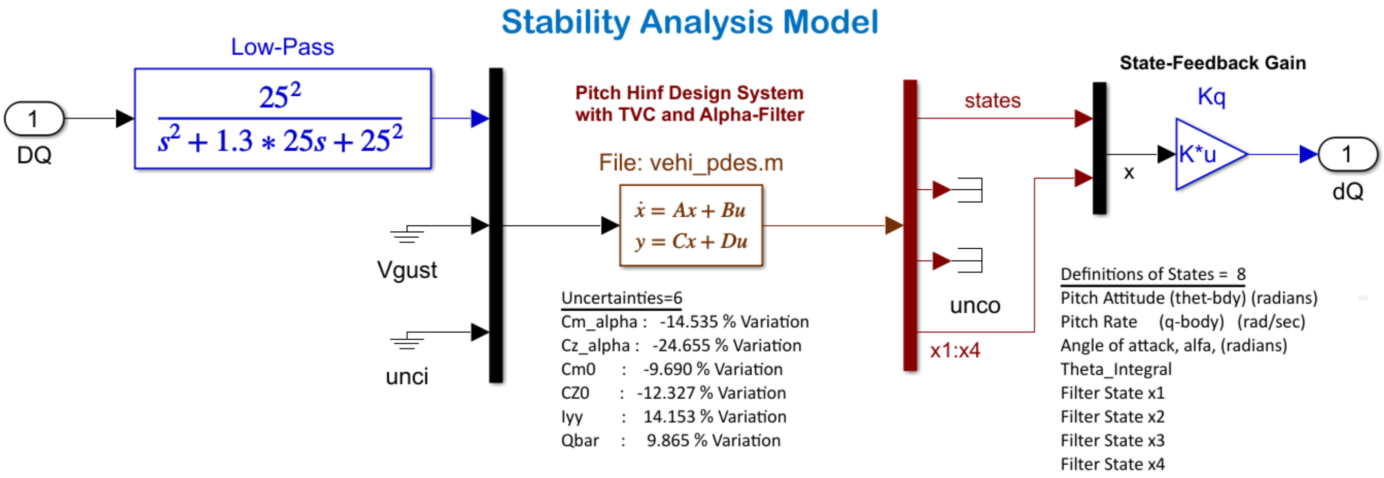


Figure 8 Stability Analysis Model in Simulink File “Open_Loop_1.slx”

Figure 8 shows the stability analysis model in file “*Open_Loop_1.slx*” that is used to calculate the open-loop Bode and Nichols plots. Sensitivity to gusts is analyzed using the closed-loop model “*Sensitivity.slx*”, shown in Figure-9, which includes the same system from file “*vehi_pdes.m*”. The gust disturbance input is scaled by multiplying it with the maximum expected wind-gust velocity which is 25 (feet/sec) and its output which is alpha dispersion is scaled by dividing it with the maximum allowed α angle, which is 4° or 0.07 (rad). The peak of the scaled sensitivity transfer function should, therefore, be less than 1 in order to satisfy the expected performance.

The script file “freq.m” shown below calculates the Nichols and Bode plots and also the Sigma plot of the sensitivity function. Figures 10 shows the open-loop Bode plot which has a big double resonance at the disturbance frequencies. The cross-over frequency is at 5 (rad/sec). Figure 11 shows the Nichols plot, the two resonances produced by the filter and the gain and phase margins.

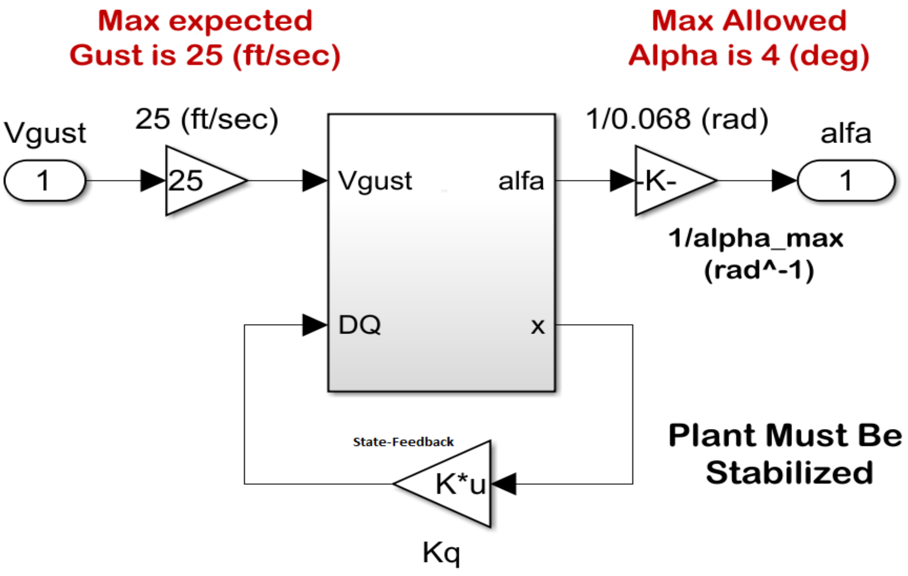


Figure 9 Sensitivity Analysis Model in File: "Sensitivity.slx"

```
% Stability and Sensitivity Analysis freq.m
init;
%[Af,Bf,Cf,Df]= linmod('filter');          % Linearize Gust Filter
[Ao,Bo,Co,Do]= linmod('Open_Loop_1');      % Linearize Open-Loop Simulink mo
[As,Bs,Cs,Ds]= linmod('Sensitivity');      % Linearize Sensitivity model
syso= ss(Ao,Bo,Co,Do);                     % Create Vehicle  SS System
syss= ss(As,Bs,Cs,Ds);                     % Create Vehicle  SS System
w=logspace(-2, 2, 6000);                   % Define Frequ Range
figure(10); nichols(syso,syso,w)           % Plot Nichol's Chart
figure(20); bode(syso,syso,w)              % Plot Bode
figure(30); sigma(syss,syss,w);
```

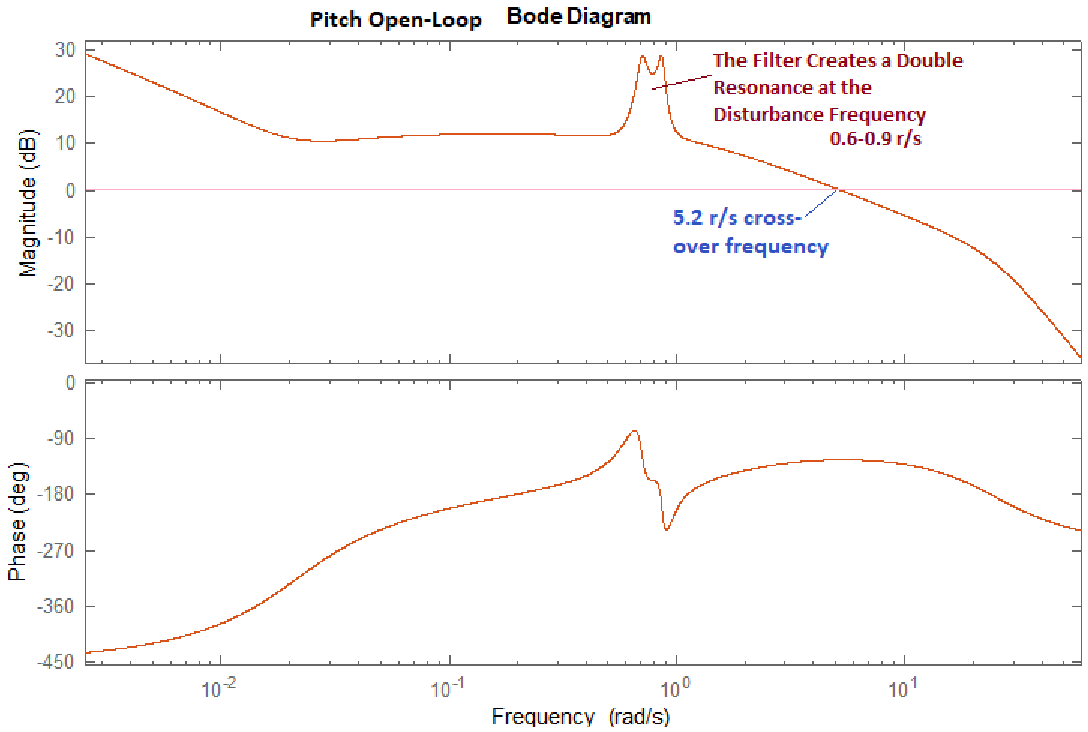


Figure 10 Open-Loop Bode Plot

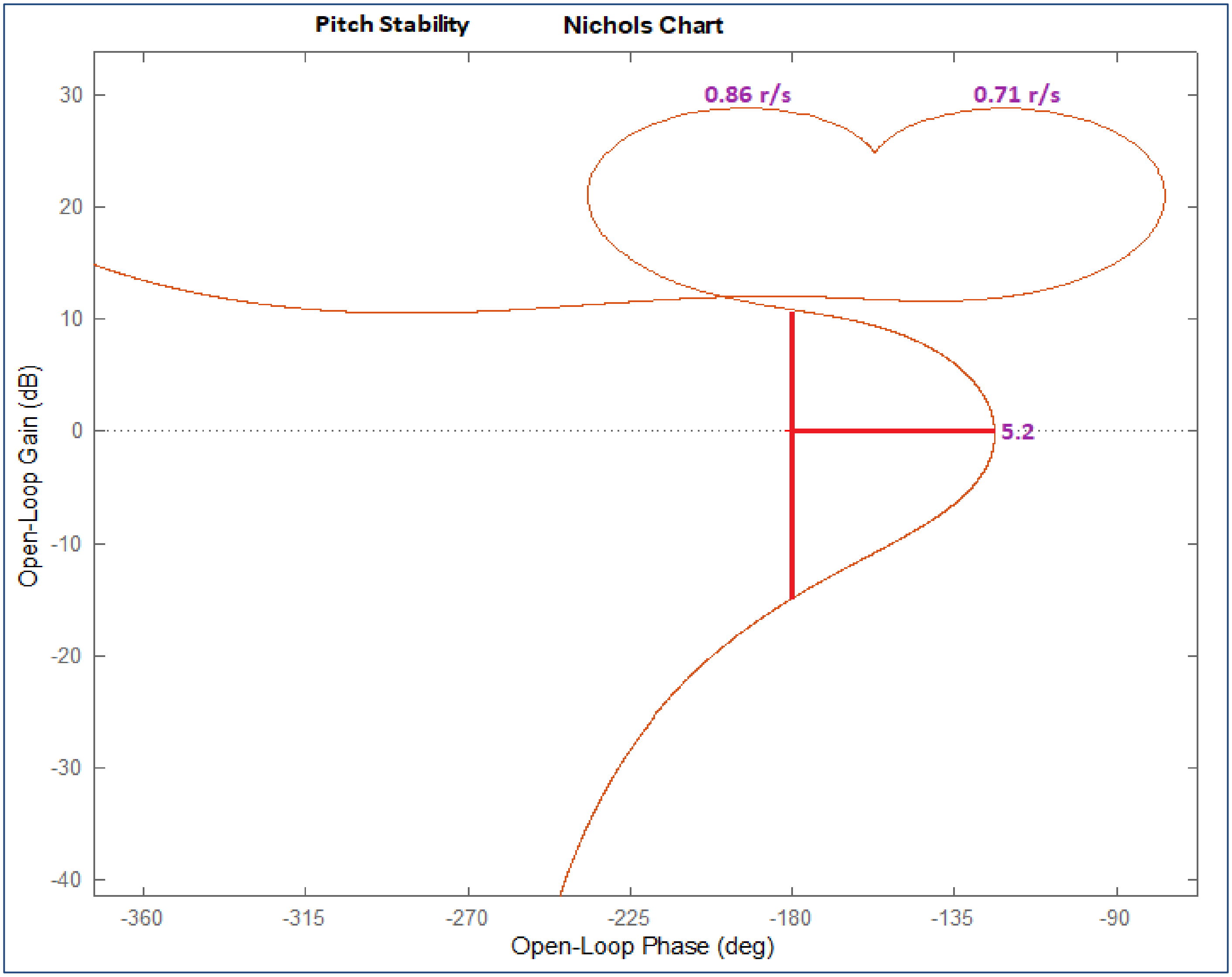
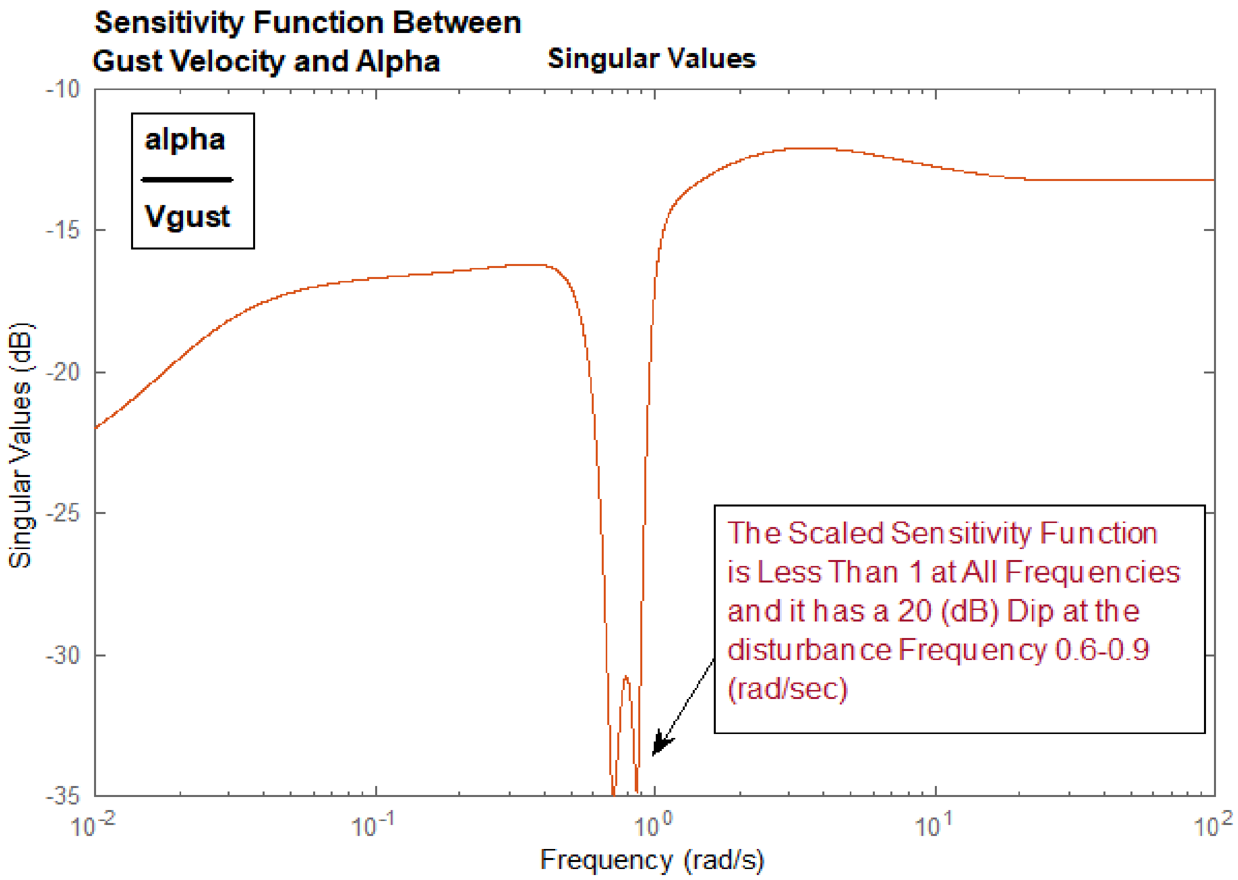


Figure 11 Nichols Plot Showing the Double Resonance and the Phase/ Gain Margins

Figure 12 shows the Sensitivity function calculated between the wind-gust velocity input and the angle of attack dispersion. The input and output of the transfer function are scaled as shown in Figure 9. The magnitude of the SV plot is less than one at all frequencies, as expected. In addition, the alpha-filter produces a further 20 (dB) dip in the sensitivity at the anticipated frequency range of the gust disturbances which is between 0.6 and 0.9 (rad/sec).

The two filter modes which are excited by the angle of attack behave like a counter-resonance against the disturbance. With the proper selection of feedback gains from states x1 to x4, which are obtained from the H-infinity solution, the filter modes are reducing the loading effects of the disturbance against the structure. Sensitivity is reduced by tuning the vehicle to oscillate at the disturbance frequency by turning against the oscillatory disturbance and thus minimizing α, as we shall see in the simulation.



The Scaled Sensitivity Function is Less Than 1 at All Frequencies and it has a 20 (dB) Dip at the disturbance Frequency 0.6-0.9 (rad/sec)

Figure 12 Sensitivity Function Between Gust Velocity and Angle of Attack

1.9 Robust Performance

We already proved from the Nichols charts that nominal stability is good. We also satisfy performance requirements regarding aero-loading with respect to wind-gusts. But can we satisfy both stability and performance when we have variations in some of the vehicle parameters? To answer that we need to analyze the system's robust performance. The control system satisfies robust performance criterion when the Structured Singular Value (SSV or μ) of the closed-loop system from (the 6 uncertainty inputs plus Vgust) to (the 6 uncertainty outputs plus alpha dispersion) is less than one at all frequencies. It should be less than one because the system is already normalized relative to the uncertainties block ±Δ, and also between Vgust and alpha, as already described. Robust Performance is calculated using the Simulink model "Robust-Performance.slx". It has the control loop closed and it is configured as shown in Figure-13 to perform this operation. The Matlab script "*Run_Robust_Performance.m*", shown below, calculates the system SSV frequency response between the combined inputs and the combined outputs and it has a magnitude less than 1 at all frequencies which means that robust performance is satisfied, as shown in Figure-14.

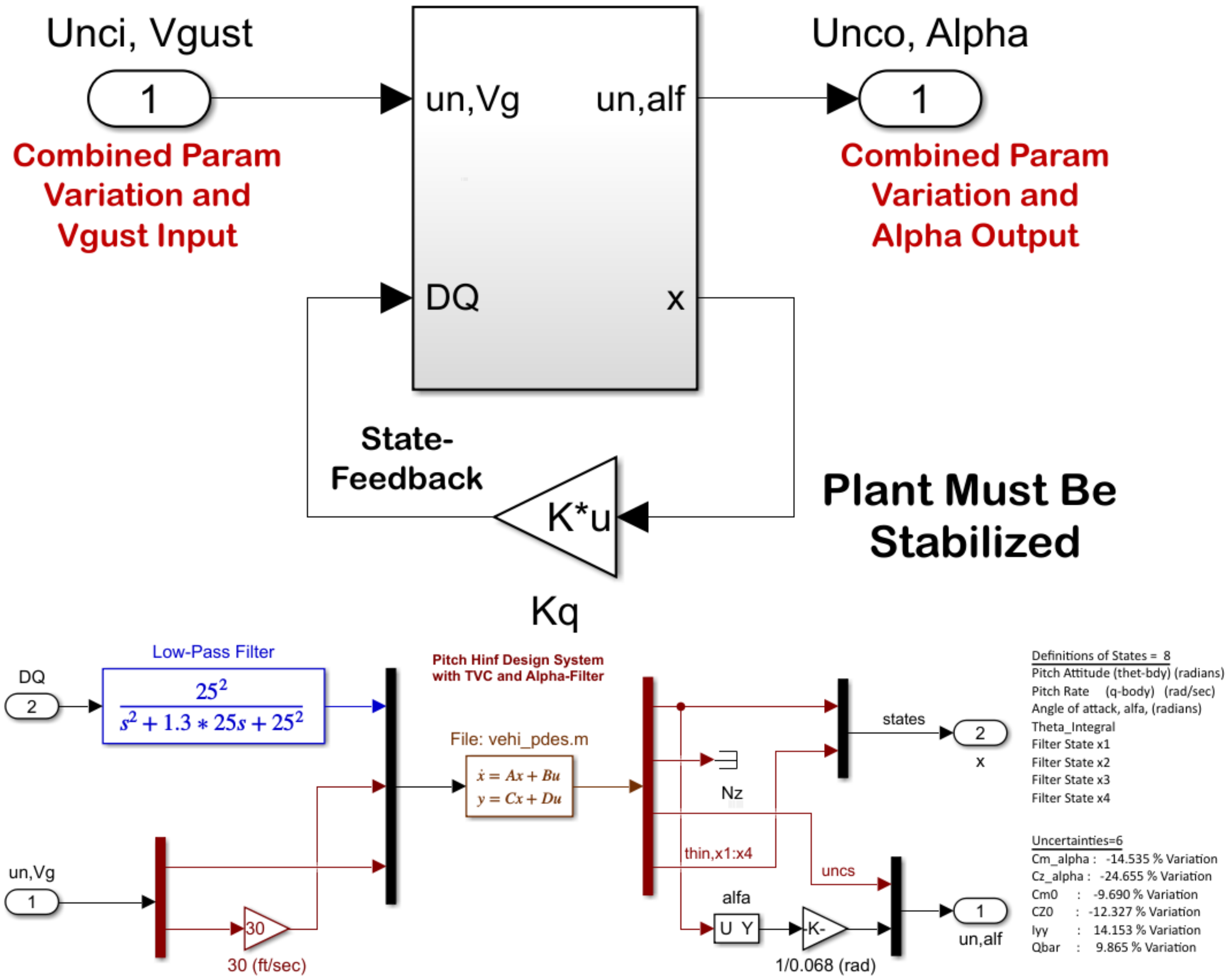


Figure 13 Robust Performance Analysis Model "Robust_Performance.slx"

```
% Robust Performance Analysis File
clear all; init
Npv=6;                                    % Number of Param Variation
w=logspace(-2,2,800);
[Acp,Bcp,Ccp,Dcp]=linmod('Robust_Performance');
eig(Acp)
sys=ss(Acp,Bcp,Ccp,Dcp);
sysf= frd(sys,w);
blk=[-ones(Npv+1,1), zeros(Npv+1,1)];
[bnd,muinfo]= mussv(sysf,blk);

ff= get(muinfo.bnds, 'frequency');
muu=get(muinfo.bnds, 'responsedata');
muu=squeeze(muu);
muu=muu(1,:);
loglog(ff,muu)
ylim([0.01,1.1])
ylabel('ssv')
xlabel('Frequency (rad/sec)')
```

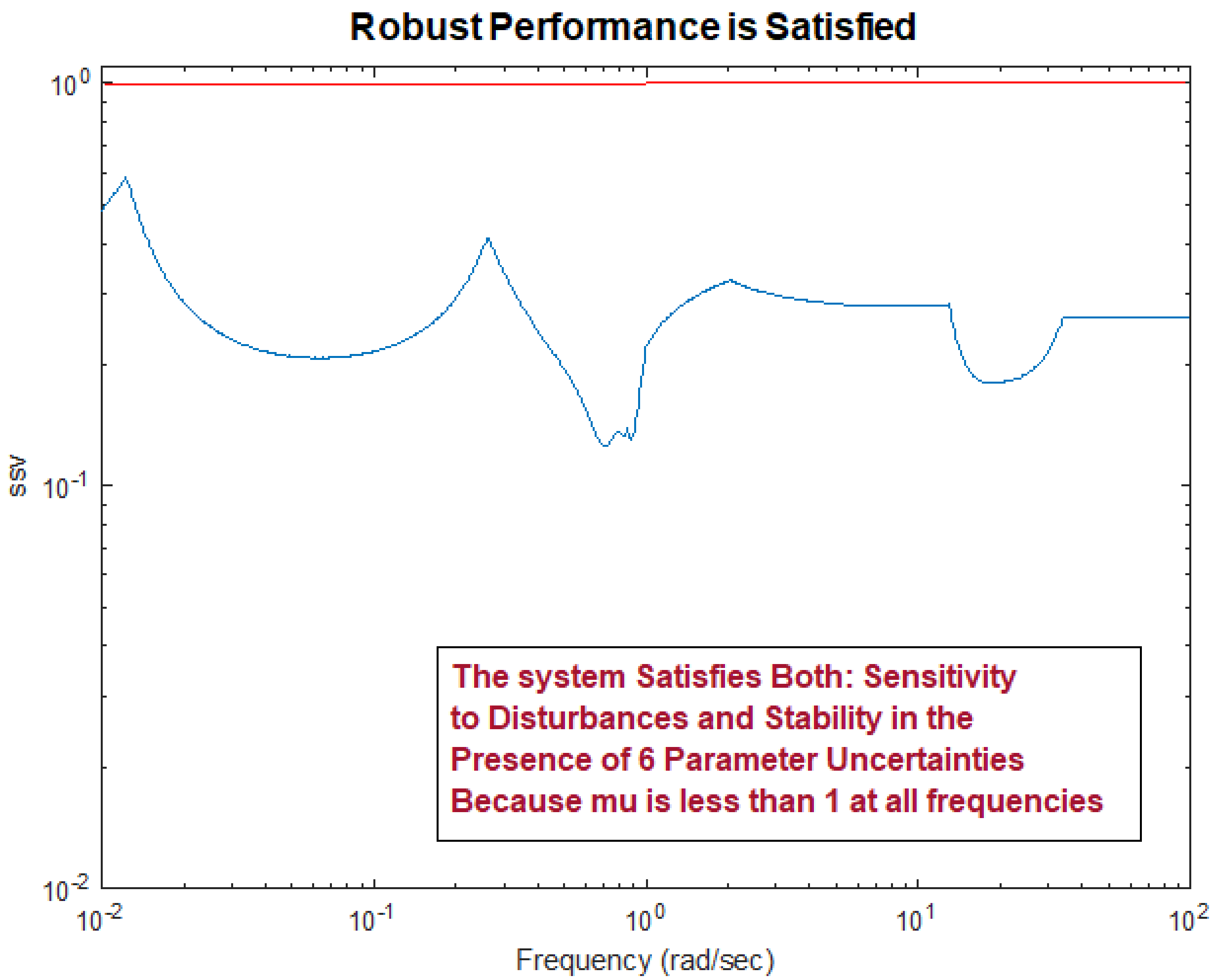


Figure 14 SSV Frequency Response Proves that the System Satisfies Both: Performance and Robustness to Uncertainties

1.10 Simulations

At high dynamic pressures the command following capability of the FCS is slow because the load-relief system is preventing aero-loading, plus the vehicle is not expected to be maneuvering during high dynamic pressures which lasts only 15-20 sec. The vehicle at high-Q maintains a steady or very slowly changing attitude relative to the wind. It should be able, however, to track small commands from guidance even at a very slow rate. Figure 15 shows the attitude response to a step attitude command. The α–filter causes an oscillatory transient but the attitude eventually converges to the command.

The effect of the disturbance filter is shown in Figure-16 where the vehicle is excited by an external oscillatory wind-gust disturbance of 0.75 (rad/sec) frequency. The top case includes the alpha-filter and the bottom case is without it. When the filter is included, the vehicle responds to the oscillatory wind-gust by turning towards it and thus reducing the alpha oscillation to an amplitude of 0.002 (rad). In the second case without the filter the oscillation is significantly higher at 0.011 (rad).

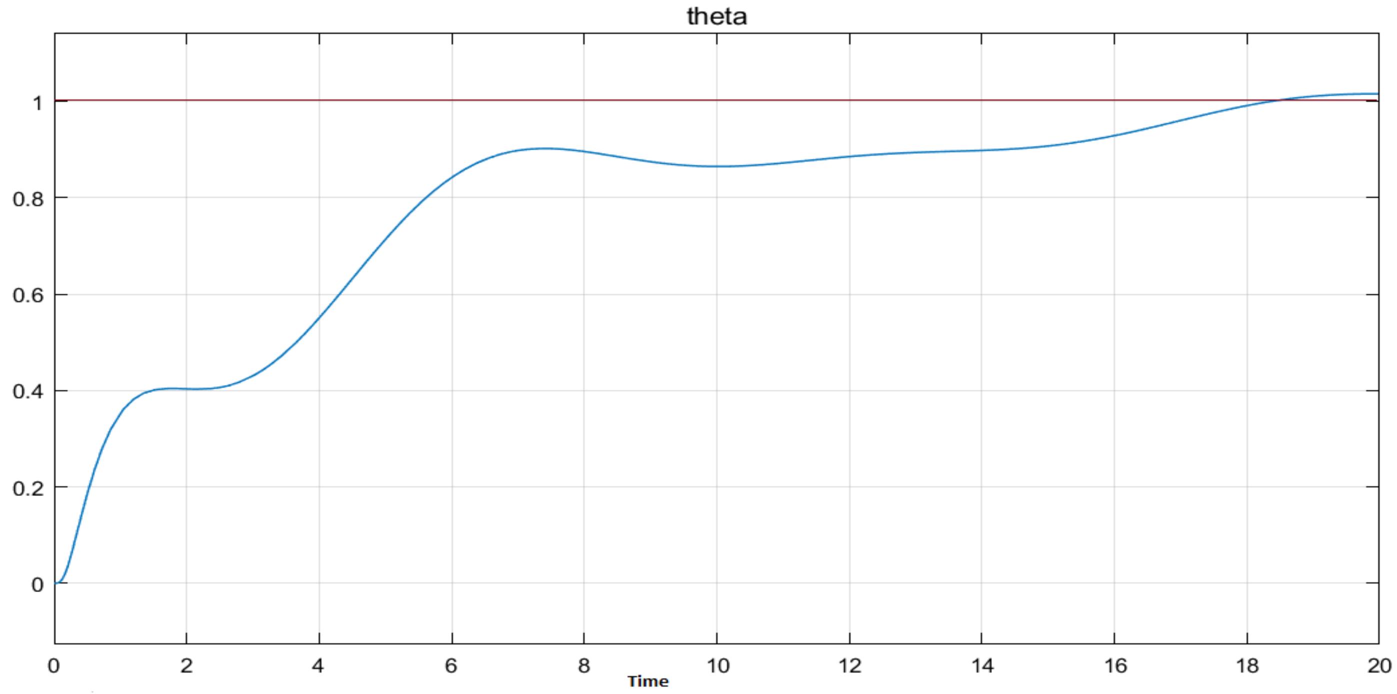


Figure 15 Attitude Response to a Step Attitude Command

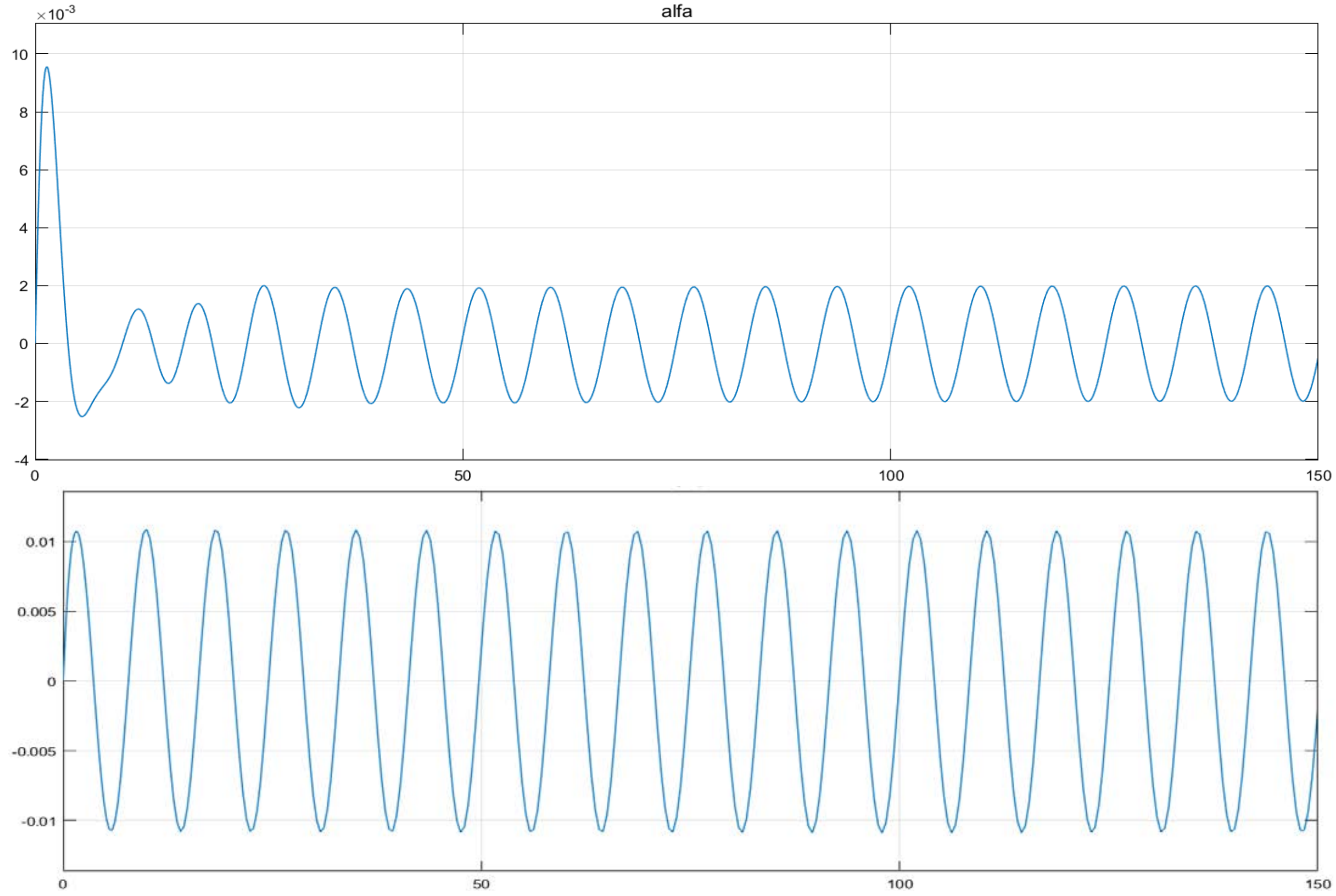


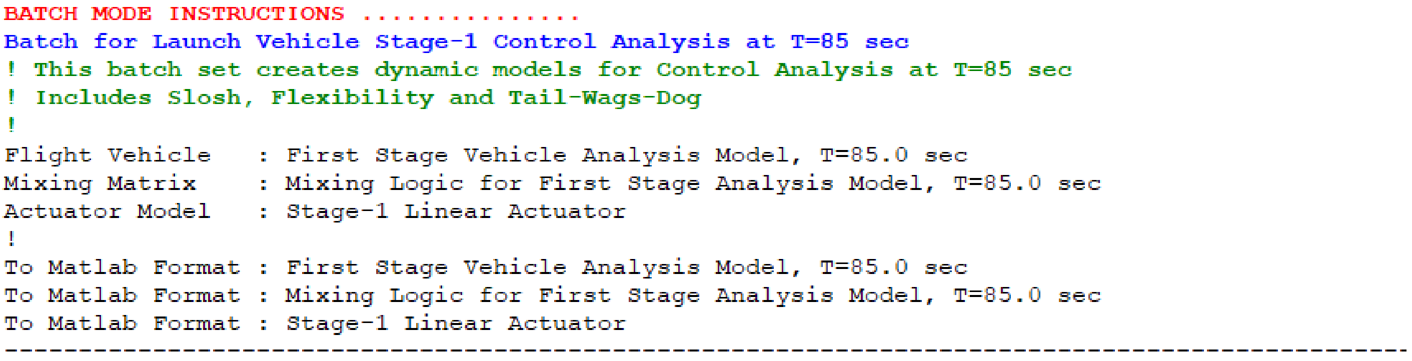
Figure 16 Alpha Response to a 0.75 r/s Oscillatory Wind-Gust Disturbance, With and Without the Alpha-Filter

2. Coupled Axes Analysis with Slosh and Flexibility

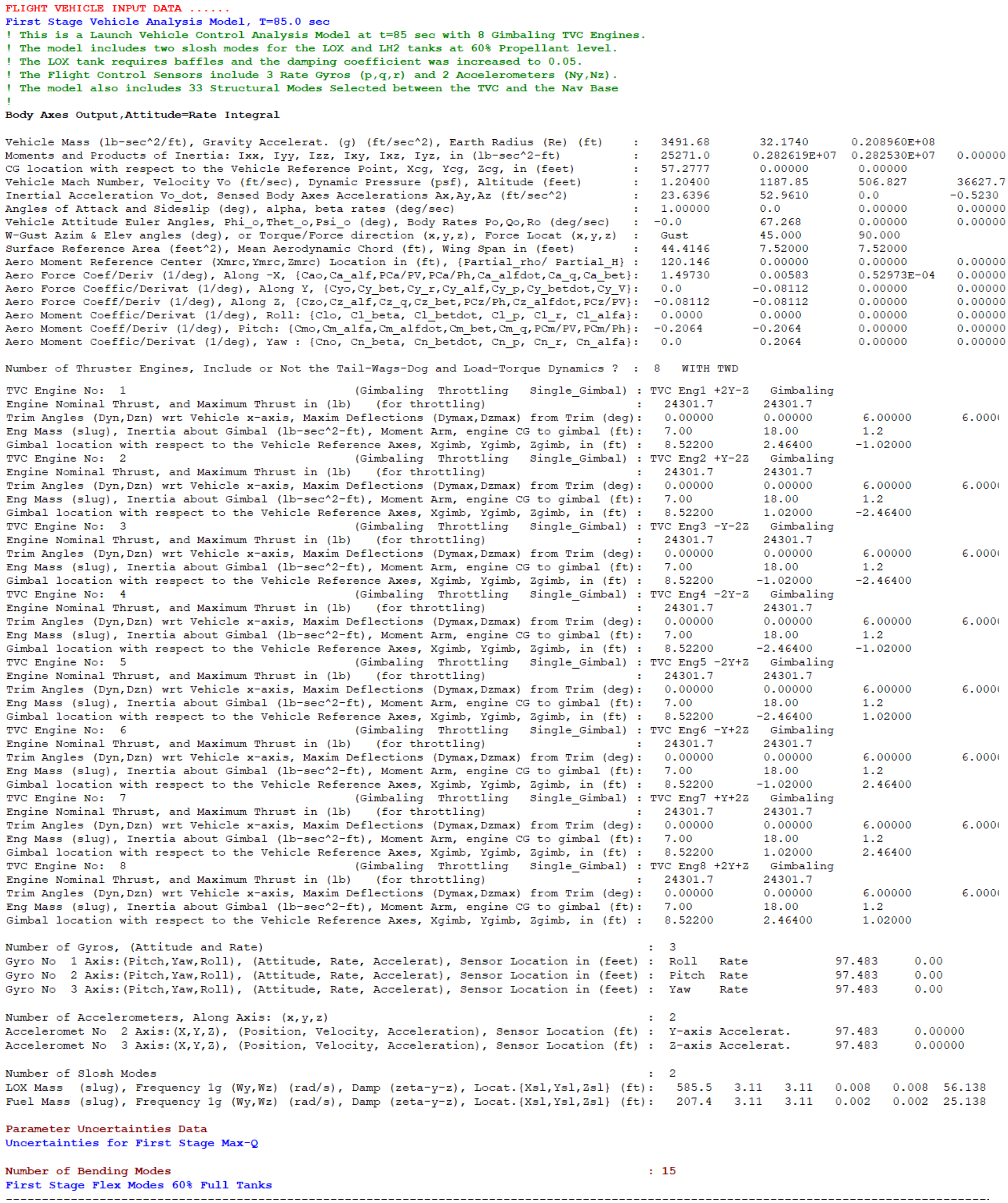
The preliminary state-feedback pitch design presented in Section 1 must now be analyzed with a coupled axes vehicle model that includes slosh and structural flexibility. Since our vehicle is cylindrical, the yaw dynamics and control system are identical to the pitch axis and we will include roll control. We will analyze stability in each axis, sensitivity to wind-gust disturbances and calculate the control system's response to attitude commands, as before. We will also calculate the control system's robustness to some parameter uncertainties. Detailed TVC actuator models will be included to drive the engine gimbals (2 per engine a total of 16 actuators). The analysis files are in directory "*Flixan\Control Analysis\Hinfinity\Examples\Launch Vehicle Design & Analysis\2-LV Flex, Slosh Analysis, Uncs, State-Feedback*". The Flixan data are in file: "*LV_Anal-T85.Inp*" and the systems and matrices are saved in file: "*LV_Anal_T85.Qdr*".

2.1 The Input File

The input data file "*LV_Anal_T85.Inp*" is shown below. It begins with a batch dataset for processing the entire file in batch mode. It consists of the vehicle model "*First Stage Vehicle Analysis Model, T=85.0 sec*" which includes two slosh modes and TWD dynamics. It is also combined with the modal data set "*First Stage Flex Modes 60% Full Tanks*" which is located at the bottom of the file. The modes have already been preselected, scaled and ready to be combined with the vehicle data. The vehicle also includes parameter uncertainties. An uncertainties dataset is included below the vehicle data that contains parameter variations from nominal values. Its title is "*Uncertainties for First Stage Max-Q*" and it contains 38 parameter variations. The uncertainties will later be split into 19 pitch and 19 lateral variations. There is a mixing logic dataset "*Mixing Logic for First Stage Analysis Model, T=85.0 sec*" that creates the TVC matrix from vehicle data. There is also an actuator dataset that contains actuator parameters for the 8 gimbaling engines, two actuators per engine. It is a "Simple Generic-B" type of actuator and its title is "*Stage-1 Linear Actuator*". The Flixan generated flex vehicle model with slosh and uncertainties is then exported into a Matlab file "vehicle_t85.m" and loaded into Matlab together with the TVC and actuator.

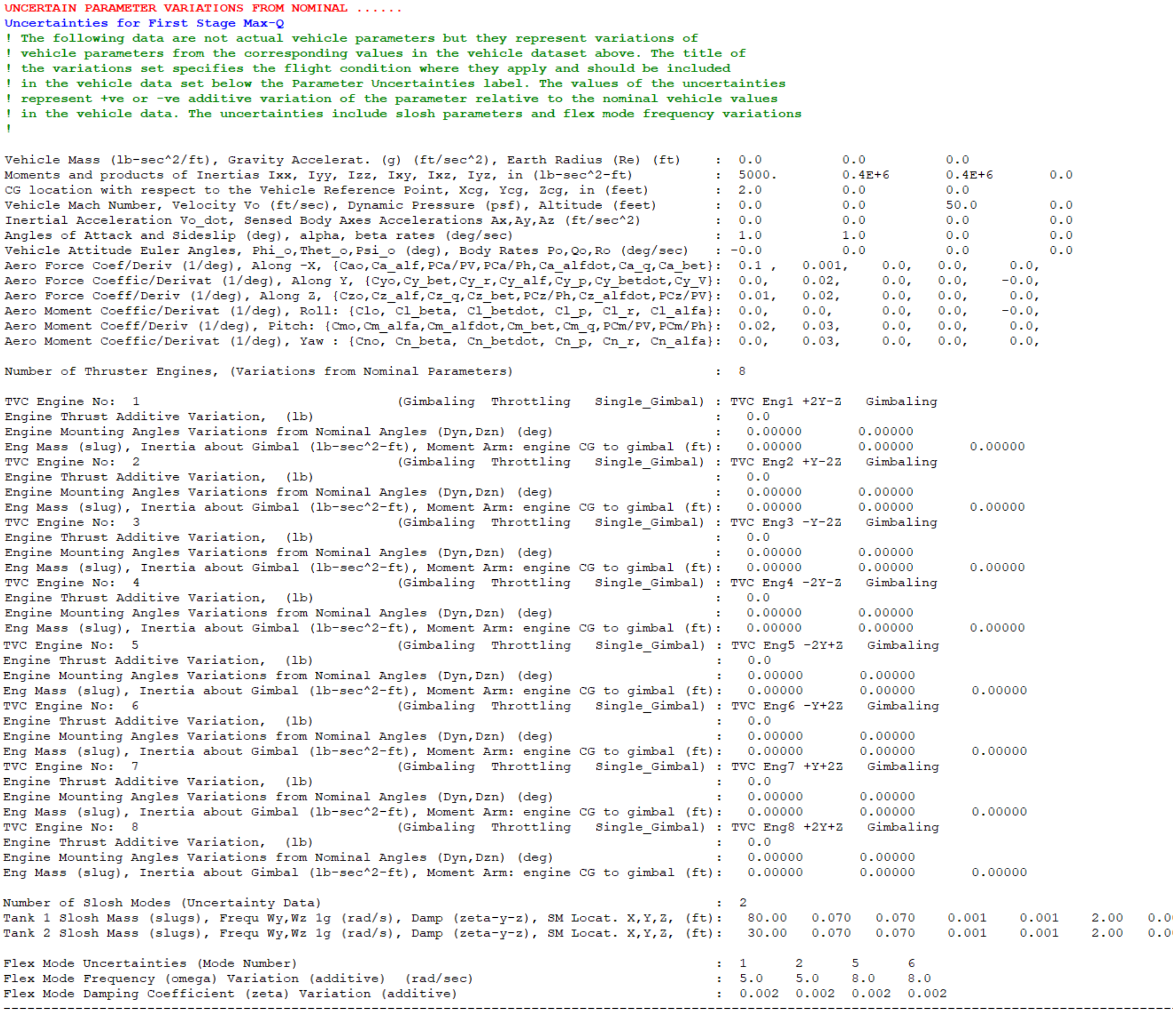


```
BATCH MODE INSTRUCTIONS ...............
Batch for Launch Vehicle Stage-1 Control Analysis at T=85 sec
! This batch set creates dynamic models for Control Analysis at T=85 sec
! Includes Slosh, Flexibility and Tail-Wags-Dog
!
Flight Vehicle    : First Stage Vehicle Analysis Model, T=85.0 sec
Mixing Matrix     : Mixing Logic for First Stage Analysis Model, T=85.0 sec
Actuator Model    : Stage-1 Linear Actuator
!
To Matlab Format : First Stage Vehicle Analysis Model, T=85.0 sec
To Matlab Format : Mixing Logic for First Stage Analysis Model, T=85.0 sec
To Matlab Format : Stage-1 Linear Actuator
--------------------------------------------------------------------------------------------
```



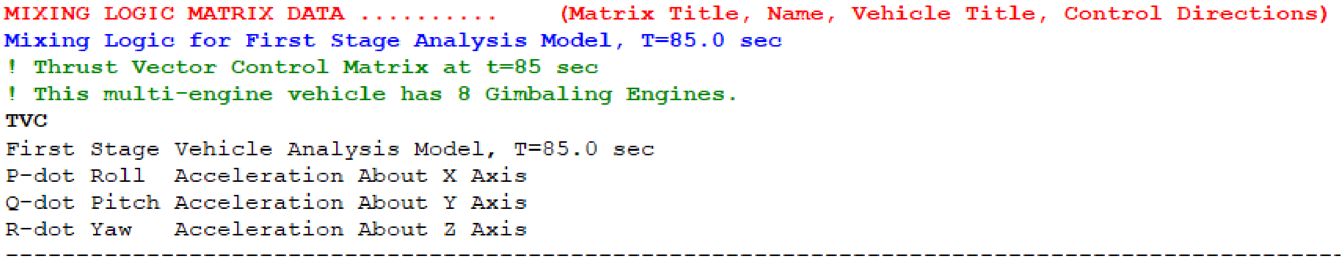
```
FLIGHT VEHICLE INPUT DATA ......
First Stage Vehicle Analysis Model, T=85.0 sec
! This is a Launch Vehicle Control Analysis Model at t=85 sec with 8 Gimbaling TVC Engines.
! The model includes two slosh modes for the LOX and LH2 tanks at 60% Propellant level.
! The LOX tank requires baffles and the damping coefficient was increased to 0.05.
! The Flight Control Sensors include 3 Rate Gyros (p,q,r) and 2 Accelerometers (Ny,Nz).
! The model also includes 33 Structural Modes Selected between the TVC and the Nav Base
!
Body Axes Output,Attitude=Rate Integral

Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft)    :  3491.68      32.1740      0.208960E+08
Moments and Products of Inertia: Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)          :  25271.0     0.282619E+07 0.282530E+07  0.00000
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)        :  57.2777      0.00000      0.00000
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)       :  1.20400      1187.85      506.827       36627.7
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)         :  23.6396      52.9610      0.0          -0.5230
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                         :  1.00000      0.0          0.00000       0.00000
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)   : -0.0          67.268       0.00000       0.00000
W-Gust Azim & Elev angles (deg), or Torque/Force direction (x,y,z), Force Locat (x,y,z)  :  Gust         45.000       90.000
Surface Reference Area (feet^2), Mean Aerodynamic Chord (ft), Wing Span in (feet)        :  44.4146      7.52000      7.52000
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial_rho/ Partial_H} :  120.146      0.00000      0.00000       0.00000
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:  1.49730      0.00583      0.52973E-04   0.00000
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}:  0.0         -0.08112      0.00000       0.00000
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}: -0.08112     -0.08112      0.00000       0.00000
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:  0.0000       0.0000       0.00000       0.00000
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}: -0.2064      -0.2064       0.00000       0.00000
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}:  0.0          0.2064       0.00000       0.00000

Number of Thruster Engines, Include or Not the Tail-Wags-Dog and Load-Torque Dynamics ?  :  8   WITH TWD

TVC Engine No:  1                          (Gimbaling  Throttling  Single_Gimbal) : TVC Eng1 +2Y-Z   Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)              :  24301.7      24301.7
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000      6.000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  7.00         18.00        1.2
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  8.52200      2.46400     -1.02000
TVC Engine No:  2                          (Gimbaling  Throttling  Single_Gimbal) : TVC Eng2 +Y-2Z   Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)              :  24301.7      24301.7
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000      6.000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  7.00         18.00        1.2
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  8.52200      1.02000     -2.46400
TVC Engine No:  3                          (Gimbaling  Throttling  Single_Gimbal) : TVC Eng3 -Y-2Z   Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)              :  24301.7      24301.7
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000      6.000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  7.00         18.00        1.2
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  8.52200     -1.02000     -2.46400
TVC Engine No:  4                          (Gimbaling  Throttling  Single_Gimbal) : TVC Eng4 -2Y-Z   Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)              :  24301.7      24301.7
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000      6.000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  7.00         18.00        1.2
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  8.52200     -2.46400     -1.02000
TVC Engine No:  5                          (Gimbaling  Throttling  Single_Gimbal) : TVC Eng5 -2Y+Z   Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)              :  24301.7      24301.7
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000      6.000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  7.00         18.00        1.2
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  8.52200     -2.46400      1.02000
TVC Engine No:  6                          (Gimbaling  Throttling  Single_Gimbal) : TVC Eng6 -Y+2Z   Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)              :  24301.7      24301.7
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000      6.000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  7.00         18.00        1.2
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  8.52200     -1.02000      2.46400
TVC Engine No:  7                          (Gimbaling  Throttling  Single_Gimbal) : TVC Eng7 +Y+2Z   Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)              :  24301.7      24301.7
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000      6.000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  7.00         18.00        1.2
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  8.52200      1.02000      2.46400
TVC Engine No:  8                          (Gimbaling  Throttling  Single_Gimbal) : TVC Eng8 +2Y+Z   Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)              :  24301.7      24301.7
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000      6.000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  7.00         18.00        1.2
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  8.52200      2.46400      1.02000

Number of Gyros, (Attitude and Rate)                                                       :  3
Gyro No  1 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Location in (feet) :  Roll   Rate          97.483     0.00
Gyro No  2 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Location in (feet) :  Pitch  Rate          97.483     0.00
Gyro No  3 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Location in (feet) :  Yaw    Rate          97.483     0.00

Number of Accelerometers, Along Axis: (x,y,z)                                              :  2
Acceleromet No  2 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Location (ft) :  Y-axis Accelerat.     97.483     0.00000
Acceleromet No  3 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Location (ft) :  Z-axis Accelerat.     97.483     0.00000

Number of Slosh Modes                                                                      :  2
LOX Mass  (slug), Frequency 1g (Wy,Wz) (rad/s), Damp (zeta-y-z), Locat.{Xsl,Ysl,Zsl} (ft):   585.5    3.11    3.11    0.008    0.008   56.138
Fuel Mass (slug), Frequency 1g (Wy,Wz) (rad/s), Damp (zeta-y-z), Locat.{Xsl,Ysl,Zsl} (ft):   207.4    3.11    3.11    0.002    0.002   25.138

Parameter Uncertainties Data
Uncertainties for First Stage Max-Q

Number of Bending Modes                                                                    : 15
First Stage Flex Modes 60% Full Tanks
-----------------------------------------------------------------------------------------------------------------------------
```

In addition to slosh, TWD, uncertainties and flexibility, the vehicle model includes 3 rate-gyros and 2 accelerometers that measure rigid and flex motion.



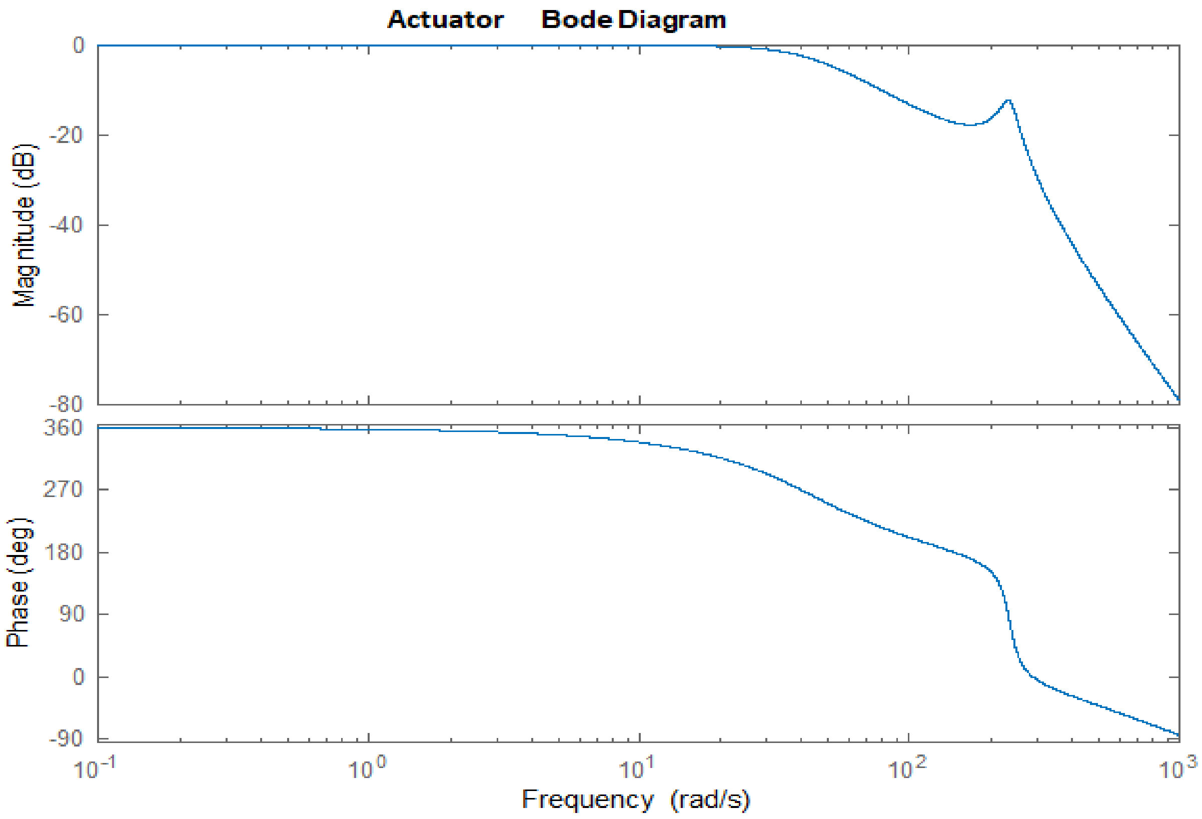
```
UNCERTAIN PARAMETER VARIATIONS FROM NOMINAL ......
Uncertainties for First Stage Max-Q
! The following data are not actual vehicle parameters but they represent variations of
! vehicle parameters from the corresponding values in the vehicle dataset above. The title of
! the variations set specifies the flight condition where they apply and should be included
! in the vehicle data set below the Parameter Uncertainties label. The values of the uncertainties
! represent +ve or -ve additive variation of the parameter relative to the nominal vehicle values
! in the vehicle data. The uncertainties include slosh parameters and flex mode frequency variations
!

Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft)    :  0.0          0.0          0.0
Moments and products of Inertias Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)          :  5000.        0.4E+6       0.4E+6       0.0
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)        :  2.0          0.0          0.0
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)       :  0.0          0.0          50.0         0.0
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)         :  0.0          0.0          0.0          0.0
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                         :  1.0          1.0          0.0          0.0
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)   : -0.0          0.0          0.0          0.0
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:  0.1 ,    0.001,     0.0,   0.0,     0.0,
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}:  0.0,     0.02,      0.0,   0.0,    -0.0,
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}:  0.01,    0.02,      0.0,   0.0,     0.0,
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:  0.0,     0.0,       0.0,   0.0,    -0.0,
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}:  0.02,    0.03,      0.0,   0.0,     0.0,
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}:  0.0,     0.03,      0.0,   0.0,     0.0,

Number of Thruster Engines, (Variations from Nominal Parameters)                         :  8

TVC Engine No:  1                          (Gimbaling  Throttling   Single_Gimbal) : TVC Eng1 +2Y-Z   Gimbaling
Engine Thrust Additive Variation,  (lb)                                             :   0.0
Engine Mounting Angles Variations from Nominal Angles (Dyn,Dzn) (deg)               :   0.00000       0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm: engine CG to gimbal (ft):   0.00000       0.00000       0.00000
TVC Engine No:  2                          (Gimbaling  Throttling   Single_Gimbal) : TVC Eng2 +Y-2Z   Gimbaling
Engine Thrust Additive Variation,  (lb)                                             :   0.0
Engine Mounting Angles Variations from Nominal Angles (Dyn,Dzn) (deg)               :   0.00000       0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm: engine CG to gimbal (ft):   0.00000       0.00000       0.00000
TVC Engine No:  3                          (Gimbaling  Throttling   Single_Gimbal) : TVC Eng3 -Y-2Z   Gimbaling
Engine Thrust Additive Variation,  (lb)                                             :   0.0
Engine Mounting Angles Variations from Nominal Angles (Dyn,Dzn) (deg)               :   0.00000       0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm: engine CG to gimbal (ft):   0.00000       0.00000       0.00000
TVC Engine No:  4                          (Gimbaling  Throttling   Single_Gimbal) : TVC Eng4 -2Y-Z   Gimbaling
Engine Thrust Additive Variation,  (lb)                                             :   0.0
Engine Mounting Angles Variations from Nominal Angles (Dyn,Dzn) (deg)               :   0.00000       0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm: engine CG to gimbal (ft):   0.00000       0.00000       0.00000
TVC Engine No:  5                          (Gimbaling  Throttling   Single_Gimbal) : TVC Eng5 -2Y+Z   Gimbaling
Engine Thrust Additive Variation,  (lb)                                             :   0.0
Engine Mounting Angles Variations from Nominal Angles (Dyn,Dzn) (deg)               :   0.00000       0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm: engine CG to gimbal (ft):   0.00000       0.00000       0.00000
TVC Engine No:  6                          (Gimbaling  Throttling   Single_Gimbal) : TVC Eng6 -Y+2Z   Gimbaling
Engine Thrust Additive Variation,  (lb)                                             :   0.0
Engine Mounting Angles Variations from Nominal Angles (Dyn,Dzn) (deg)               :   0.00000       0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm: engine CG to gimbal (ft):   0.00000       0.00000       0.00000
TVC Engine No:  7                          (Gimbaling  Throttling   Single_Gimbal) : TVC Eng7 +Y+2Z   Gimbaling
Engine Thrust Additive Variation,  (lb)                                             :   0.0
Engine Mounting Angles Variations from Nominal Angles (Dyn,Dzn) (deg)               :   0.00000       0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm: engine CG to gimbal (ft):   0.00000       0.00000       0.00000
TVC Engine No:  8                          (Gimbaling  Throttling   Single_Gimbal) : TVC Eng8 +2Y+Z   Gimbaling
Engine Thrust Additive Variation,  (lb)                                             :   0.0
Engine Mounting Angles Variations from Nominal Angles (Dyn,Dzn) (deg)               :   0.00000       0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm: engine CG to gimbal (ft):   0.00000       0.00000       0.00000

Number of Slosh Modes (Uncertainty Data)                                            :  2
Tank 1 Slosh Mass (slugs), Frequ Wy,Wz 1g (rad/s), Damp (zeta-y-z), SM Locat. X,Y,Z, (ft):   80.00   0.070   0.070    0.001    0.001    2.00   0.0
Tank 2 Slosh Mass (slugs), Frequ Wy,Wz 1g (rad/s), Damp (zeta-y-z), SM Locat. X,Y,Z, (ft):   30.00   0.070   0.070    0.001    0.001    2.00   0.0

Flex Mode Uncertainties (Mode Number)                                               :  1      2      5      6
Flex Mode Frequency (omega) Variation (additive)  (rad/sec)                         :  5.0    5.0    8.0    8.0
Flex Mode Damping Coefficient (zeta) Variation (additive)                           :  0.002  0.002  0.002  0.002
--------------------------------------------------------------------------------------------------------------------------------
```

There are uncertainties in the moments of inertia, Xcg location, alpha, beta, dynamic pressure, aero coefficients and derivatives, slosh masses, slosh frequencies, damping, x-location, bending mode frequencies and damping coefficients.



```
MIXING LOGIC MATRIX DATA ..........    (Matrix Title, Name, Vehicle Title, Control Directions)
Mixing Logic for First Stage Analysis Model, T=85.0 sec
! Thrust Vector Control Matrix at t=85 sec
! This multi-engine vehicle has 8 Gimbaling Engines.
TVC
First Stage Vehicle Analysis Model, T=85.0 sec
P-dot Roll  Acceleration About X Axis
Q-dot Pitch Acceleration About Y Axis
R-dot Yaw   Acceleration About Z Axis
-------------------------------------------------------------------------------------
```

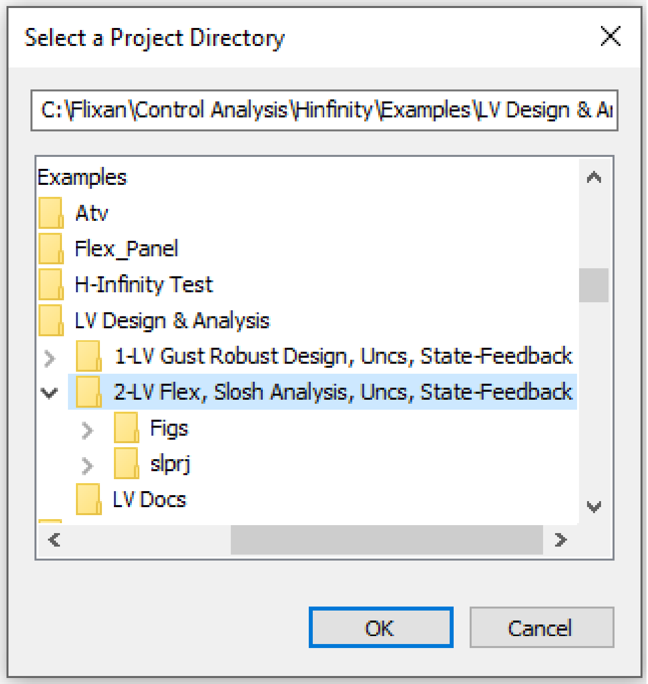
```
ACTUATOR INPUT DATA ............ SIMPLE GENERIC MODEL B
Stage-1 Linear Actuator
Symbol Parameter Description                         (Units)       Value
C(s)   Order of Pade Delay (0,1,2)                     ( - )       1, -0.001, 0.001
Ka     Gain of Amplifier ...................... (amps/volt)    28.0
Wsv    Bandwidth of the Linear Servo Actuator .  (rad/sec)     65.0
Kact   Actuator Stiffness (Piston+Oil+Electric)   (lb/ft)      2.4e+6
Klod   Stiffness at Surface or Nozzle Connection  (lb/ft)      1.2e+9
Kbck   Stiffness at Vehicle Backup Structure ..   (lb/ft)      8.0e+7
R      Moment Arm between Actuator Rod & Gimbal   (feet)       0.667
Jl     Load Inertia about the Gimbal .......... (ft-lb-s^2)    18.0
Kg     Load Gimbal Bearing Spring Constant      (ft-lb/rad)    0.0
Bg     Load Gimbal Bearing Viscous Damping .... (ft-lb-sec)    550.0
-------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Mixing Logic for First Stage Analysis Model, T=85.0 sec
Matrix TVC
-------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
First Stage Vehicle Analysis Model, T=85.0 sec
System
vehicle_t85.m
-------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Stage-1 Linear Actuator
System
actuator.m
-------------------------------------------------------------------------------------------
```



The figure above shows the actuator frequency response. It has a bandwidth of 30 (rad/sec). The mode at 210 r/s is caused by the combined backup structure, load, and shaft stiffnesses. The modal data for the first two pitch and yaw vehicle bending modes are shown below. Each frame includes frequency, damping and modal mass. Nodes are included at the 8 peripheral engine gimbals, gyro and accelerometer locations, and at the two slosh masses.

```
SELECTED MODAL DATA AND LOCATIONS FOR : 60% Full
First Stage Flex Modes 60% Full Tanks
! Flex Modes, First Stage 60% Full Tanks.
! Sensors are at the Top of LOX Tank at Node: 40015
! The Modes were selected between the TVC (Node:10001) and the IMU Locat. (Node:40015)

MODE#  1/  1, Frequency (rad/sec), Damping (zeta), Generalized Mass=    31.0         0.50000E-02    12.000
DEFINITION OF LOCATIONS (NODES)           phi along X   phi along Y   phi along Z   sigm about X  sigm about Y  sigm about Z

                              Node ID#    Modal Data at the  8 Engines, (x,y,z)...
S1 Engine No:1 +Y 0Z             18915   -0.12901D-01  -0.10186D+00  -0.56622D-01   0.34565D-03  -0.29891D-02   0.65573D-02
S1 Engine No:2 +Y +Z             18908   -0.14488D-01  -0.10161D+00  -0.57885D-01  -0.37952D-04  -0.41592D-02   0.64361D-02
S1 Engine No:3 0Y +Z             18909   -0.72726D-02  -0.10033D+00  -0.57407D-01  -0.40727D-03  -0.36961D-02   0.52490D-02
S1 Engine No:4 -Y +Z             18910    0.40387D-02  -0.10077D+00  -0.56226D-01  -0.54455D-03  -0.26467D-02   0.56298D-02
S1 Engine No:5 -Y 0Z             18911    0.12911D-01  -0.10186D+00  -0.56605D-01  -0.35178D-03  -0.29837D-02   0.65566D-02
S1 Engine No:6 -Y -Z             18912    0.14482D-01  -0.10163D+00  -0.57877D-01   0.23398D-04  -0.41620D-02   0.64502D-02
S1 Engine No:7 0Y -Z             18913    0.72833D-02  -0.10033D+00  -0.57400D-01   0.40945D-03  -0.36898D-02   0.52470D-02
S1 Engine No:8 +Y -Z             18914   -0.40344D-02  -0.10078D+00  -0.56242D-01   0.54324D-03  -0.26519D-02   0.56338D-02

                              Node ID#    Modal Data at the  3 Gyros ...
Stg-2 Tank Top, IMU Location     40015    0.36941D-05  -0.34870D-01  -0.19652D-01  -0.23159D-05   0.21456D-02  -0.38063D-02
Stg-2 Tank Top, IMU Location     40015    0.36941D-05  -0.34870D-01  -0.19652D-01  -0.23159D-05   0.21456D-02  -0.38063D-02
Stg-2 Tank Top, IMU Location     40015    0.36941D-05  -0.34870D-01  -0.19652D-01  -0.23159D-05   0.21456D-02  -0.38063D-02

                              Node ID#    Modal Data at the  2 Accelerometers, along (x,y,z)...
Stg-2 Tank Top, IMU Location     40015    0.36941D-05  -0.34870D-01  -0.19652D-01
Stg-2 Tank Top, IMU Location     40015    0.36941D-05  -0.34870D-01  -0.19652D-01

                              Node ID#    Modal Data at the  2 Slosh Masses...
LOX  Slosh Mass Locat.             601    0.36536D-05   0.61141D-01   0.34476D-01  -0.22194D-05   0.20894D-03  -0.37039D-03
Fuel Slosh Mass Locat.             600    0.35377D-05  -0.74025D-02  -0.41671D-02  -0.21259D-05  -0.24678D-02   0.43770D-02

                              Node ID#    Modal Data at the Disturbance Point
S1 Engine No:9 center            10001   -0.44302D-04  -0.10038D+00  -0.56615D-01   0.25363D-06  -0.27352D-02   0.48392D-02


MODE#  2/  2, Frequency (rad/sec), Damping (zeta), Generalized Mass=    31.1         0.50000E-02    12.000
DEFINITION OF LOCATIONS (NODES)           phi along X   phi along Y   phi along Z   sigm about X  sigm about Y  sigm about Z

                              Node ID#    Modal Data at the  8 Engines, (x,y,z)...
S1 Engine No:1 +Y 0Z             18915   -0.72993D-02  -0.57356D-01   0.10029D+00  -0.42673D-03   0.52327D-02   0.36689D-02
S1 Engine No:2 +Y +Z             18908    0.40224D-02  -0.56196D-01   0.10080D+00  -0.58175D-03   0.56656D-02   0.26598D-02
S1 Engine No:3 0Y +Z             18909    0.12928D-01  -0.56532D-01   0.10183D+00  -0.36583D-03   0.65404D-02   0.29636D-02
S1 Engine No:4 -Y +Z             18910    0.14517D-01  -0.57841D-01   0.10164D+00   0.21827D-04   0.64422D-02   0.41707D-02
S1 Engine No:5 -Y 0Z             18911    0.73118D-02  -0.57336D-01   0.10035D+00   0.39953D-03   0.52384D-02   0.36544D-02
S1 Engine No:6 -Y -Z             18912   -0.40337D-02  -0.56260D-01   0.10081D+00   0.53238D-03   0.56471D-02   0.26718D-02
S1 Engine No:7 0Y -Z             18913   -0.12929D-01  -0.56620D-01   0.10184D+00   0.33079D-03   0.65395D-02   0.29768D-02
S1 Engine No:8 +Y -Z             18914   -0.14509D-01  -0.57891D-01   0.10163D+00  -0.47824D-04   0.64644D-02   0.41674D-02

                              Node ID#    Modal Data at the  3 Gyros ...
Stg-2 Tank Top, IMU Location     40015   -0.53357D-07  -0.19659D-01   0.34858D-01  -0.14300D-04  -0.38062D-02  -0.21461D-02
Stg-2 Tank Top, IMU Location     40015   -0.53357D-07  -0.19659D-01   0.34858D-01  -0.14300D-04  -0.38062D-02  -0.21461D-02
Stg-2 Tank Top, IMU Location     40015   -0.53357D-07  -0.19659D-01   0.34858D-01  -0.14300D-04  -0.38062D-02  -0.21461D-02

                              Node ID#    Modal Data at the  2 Accelerometers, along (x,y,z)...
Stg-2 Tank Top, IMU Location     40015   -0.53357D-07  -0.19659D-01   0.34858D-01
Stg-2 Tank Top, IMU Location     40015   -0.53357D-07  -0.19659D-01   0.34858D-01

                              Node ID#    Modal Data at the  2 Slosh Masses...
LOX  Slosh Mass Locat.             601   -0.41928D-08   0.34473D-01  -0.61157D-01  -0.13703D-04  -0.37042D-03  -0.20870D-03
Fuel Slosh Mass Locat.             600    0.49613D-07  -0.41801D-02   0.74030D-02  -0.13123D-04   0.43780D-02   0.24680D-02

                              Node ID#    Modal Data at the Disturbance Point
S1 Engine No:9 center            10001    0.21092D-04  -0.56598D-01   0.10045D+00   0.18748D-04   0.48764D-02   0.27490D-02
```

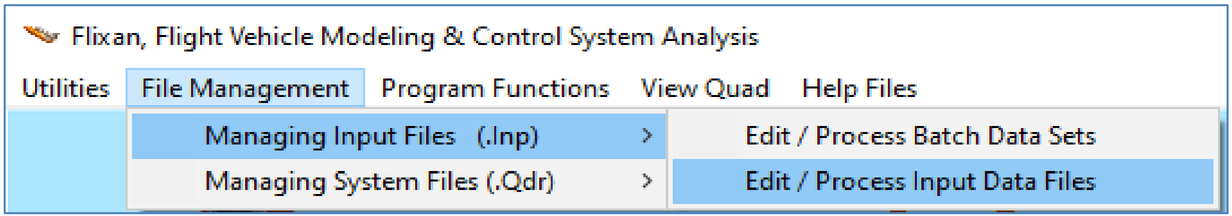
2.2 Processing the Input Data File in Batch

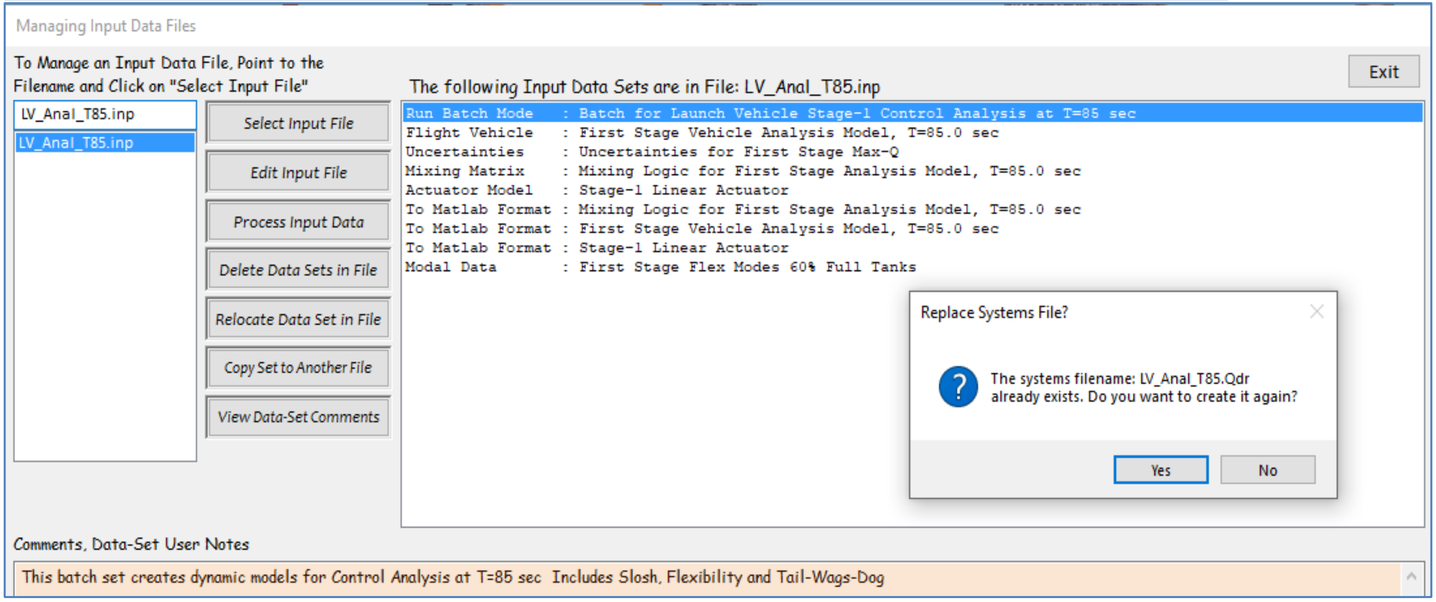


C:\Flixan\Control Analysis\Hinfinity\Examples\LV Design & Ai

We will now process the input file in batch mode by running the batch dataset located at the top of the file. Start the Flixan program and select the project directory: *"Flixan\Control Analysis\ Hinfinity\ Examples\Launch Vehicle Design & Analysis\ 2-LV Flex, Slosh Analysis, Uncs, State-Feedback"*. From the main menu select *"File Management"*, *"Managing Input Files"*, and then *"Edit/ Process Input Data Files"*, as shown below.

The following dialog comes up that includes two menus. The menu on the left side shows the input data files in the project directory. There is only one in this case. Highlight it and click on *"Select Input File"* button. The menu on the right shows the datasets which are in the input file. Select the batch set which is at the top of the list and click on *"Process Input Data"*.





To Manage an Input Data File, Point to the Filename and Click on "Select Input File"

LV_Anal_T85.inp

LV_Anal_T85.inp

The following Input Data Sets are in File: LV_Anal_T85.inp

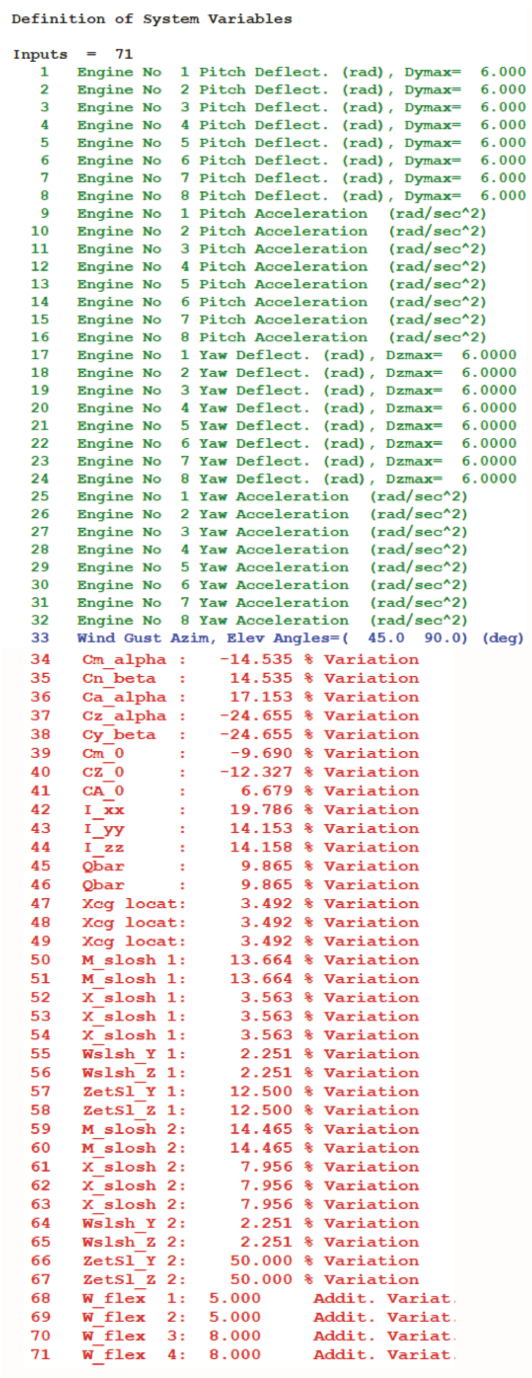
```
Run Batch Mode   : Batch for Launch Vehicle Stage-1 Control Analysis at T=85 sec
Flight Vehicle   : First Stage Vehicle Analysis Model, T=85.0 sec
Uncertainties    : Uncertainties for First Stage Max-Q
Mixing Matrix    : Mixing Logic for First Stage Analysis Model, T=85.0 sec
Actuator Model   : Stage-1 Linear Actuator
To Matlab Format : Mixing Logic for First Stage Analysis Model, T=85.0 sec
To Matlab Format : First Stage Vehicle Analysis Model, T=85.0 sec
To Matlab Format : Stage-1 Linear Actuator
Modal Data       : First Stage Flex Modes 60% Full Tanks
```

The systems filename: LV_Anal_T85.Qdr already exists. Do you want to create it again?

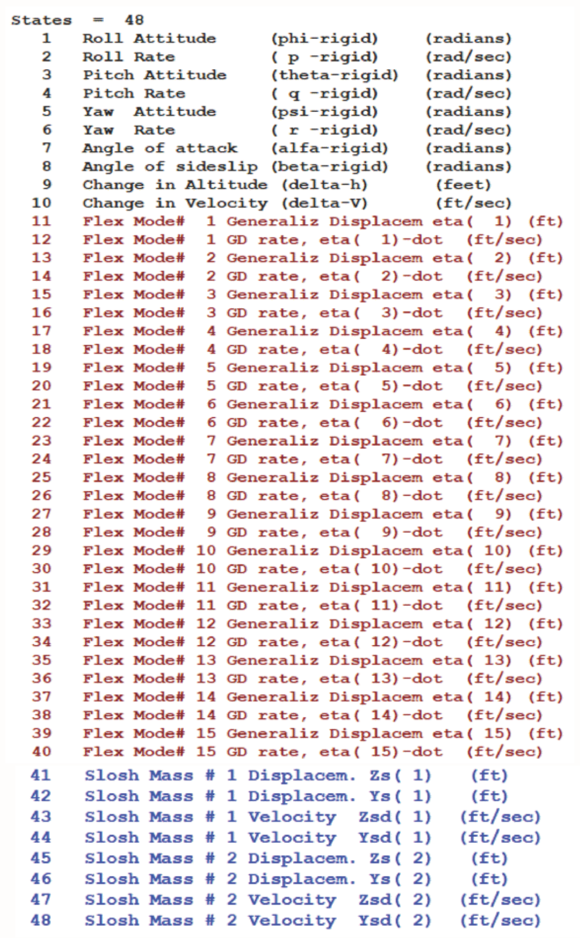
This batch set creates dynamic models for Control Analysis at T=85 sec Includes Slosh, Flexibility and Tail-Wags-Dog

In the following question, answer "Yes", which is okay to delete the old systems file and recreate it. The batch executes and creates the systems and matrices that can now be loaded into Matlab.

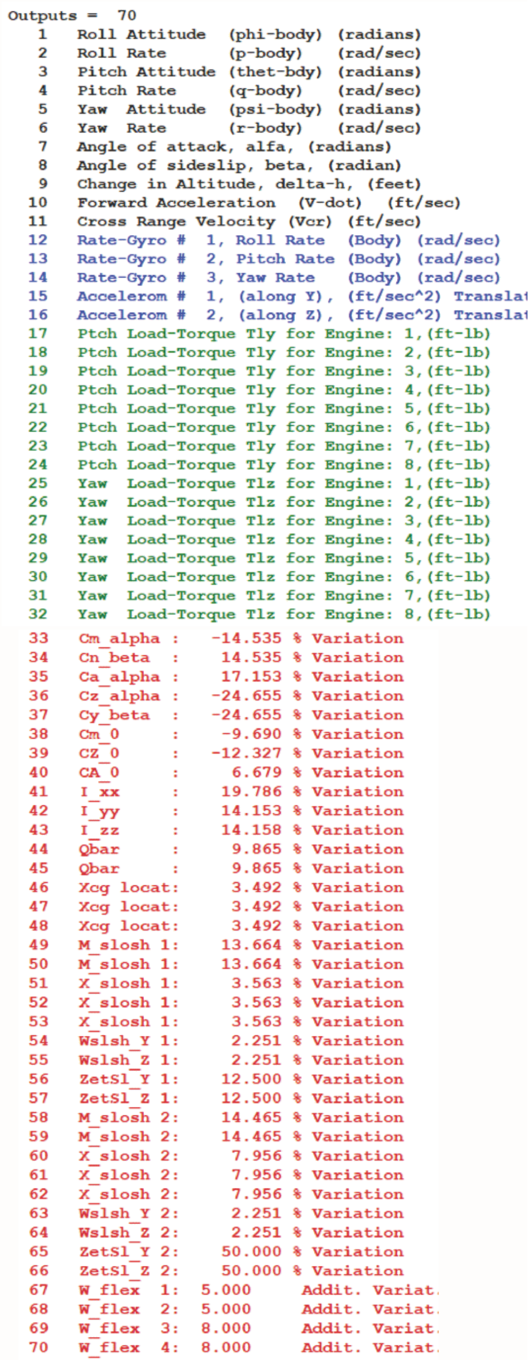
Definitions of Vehicle Inputs, States and Outputs



```
Definition of System Variables

Inputs   =  71
    1    Engine No  1 Pitch Deflect. (rad), Dymax=  6.000
    2    Engine No  2 Pitch Deflect. (rad), Dymax=  6.000
    3    Engine No  3 Pitch Deflect. (rad), Dymax=  6.000
    4    Engine No  4 Pitch Deflect. (rad), Dymax=  6.000
    5    Engine No  5 Pitch Deflect. (rad), Dymax=  6.000
    6    Engine No  6 Pitch Deflect. (rad), Dymax=  6.000
    7    Engine No  7 Pitch Deflect. (rad), Dymax=  6.000
    8    Engine No  8 Pitch Deflect. (rad), Dymax=  6.000
    9    Engine No  1 Pitch Acceleration  (rad/sec^2)
   10    Engine No  2 Pitch Acceleration  (rad/sec^2)
   11    Engine No  3 Pitch Acceleration  (rad/sec^2)
   12    Engine No  4 Pitch Acceleration  (rad/sec^2)
   13    Engine No  5 Pitch Acceleration  (rad/sec^2)
   14    Engine No  6 Pitch Acceleration  (rad/sec^2)
   15    Engine No  7 Pitch Acceleration  (rad/sec^2)
   16    Engine No  8 Pitch Acceleration  (rad/sec^2)
   17    Engine No  1 Yaw Deflect. (rad), Dzmax=  6.0000
   18    Engine No  2 Yaw Deflect. (rad), Dzmax=  6.0000
   19    Engine No  3 Yaw Deflect. (rad), Dzmax=  6.0000
   20    Engine No  4 Yaw Deflect. (rad), Dzmax=  6.0000
   21    Engine No  5 Yaw Deflect. (rad), Dzmax=  6.0000
   22    Engine No  6 Yaw Deflect. (rad), Dzmax=  6.0000
   23    Engine No  7 Yaw Deflect. (rad), Dzmax=  6.0000
   24    Engine No  8 Yaw Deflect. (rad), Dzmax=  6.0000
   25    Engine No  1 Yaw Acceleration  (rad/sec^2)
   26    Engine No  2 Yaw Acceleration  (rad/sec^2)
   27    Engine No  3 Yaw Acceleration  (rad/sec^2)
   28    Engine No  4 Yaw Acceleration  (rad/sec^2)
   29    Engine No  5 Yaw Acceleration  (rad/sec^2)
   30    Engine No  6 Yaw Acceleration  (rad/sec^2)
   31    Engine No  7 Yaw Acceleration  (rad/sec^2)
   32    Engine No  8 Yaw Acceleration  (rad/sec^2)
   33    Wind Gust Azim, Elev Angles=(  45.0  90.0)  (deg)
   34    Cm_alpha  :   -14.535 % Variation
   35    Cn_beta   :    14.535 % Variation
   36    Ca_alpha  :    17.153 % Variation
   37    Cz_alpha  :   -24.655 % Variation
   38    Cy_beta   :   -24.655 % Variation
   39    Cm_0      :    -9.690 % Variation
   40    CZ_0      :   -12.327 % Variation
   41    CA_0      :     6.679 % Variation
   42    I_xx      :    19.786 % Variation
   43    I_yy      :    14.153 % Variation
   44    I_zz      :    14.158 % Variation
   45    Qbar      :     9.865 % Variation
   46    Qbar      :     9.865 % Variation
   47    Xcg locat:      3.492 % Variation
   48    Xcg locat:      3.492 % Variation
   49    Xcg locat:      3.492 % Variation
   50    M_slosh 1:     13.664 % Variation
   51    M_slosh 1:     13.664 % Variation
   52    X_slosh 1:      3.563 % Variation
   53    X_slosh 1:      3.563 % Variation
   54    X_slosh 1:      3.563 % Variation
   55    Wslsh_Y 1:      2.251 % Variation
   56    Wslsh_Z 1:      2.251 % Variation
   57    ZetSl_Y 1:     12.500 % Variation
   58    ZetSl_Z 1:     12.500 % Variation
   59    M_slosh 2:     14.465 % Variation
   60    M_slosh 2:     14.465 % Variation
   61    X_slosh 2:      7.956 % Variation
   62    X_slosh 2:      7.956 % Variation
   63    X_slosh 2:      7.956 % Variation
   64    Wslsh_Y 2:      2.251 % Variation
   65    Wslsh_Z 2:      2.251 % Variation
   66    ZetSl_Y 2:     50.000 % Variation
   67    ZetSl_Z 2:     50.000 % Variation
   68    W_flex  1:  5.000       Addit. Variat.
   69    W_flex  2:  5.000       Addit. Variat.
   70    W_flex  3:  8.000       Addit. Variat.
   71    W_flex  4:  8.000       Addit. Variat.
```



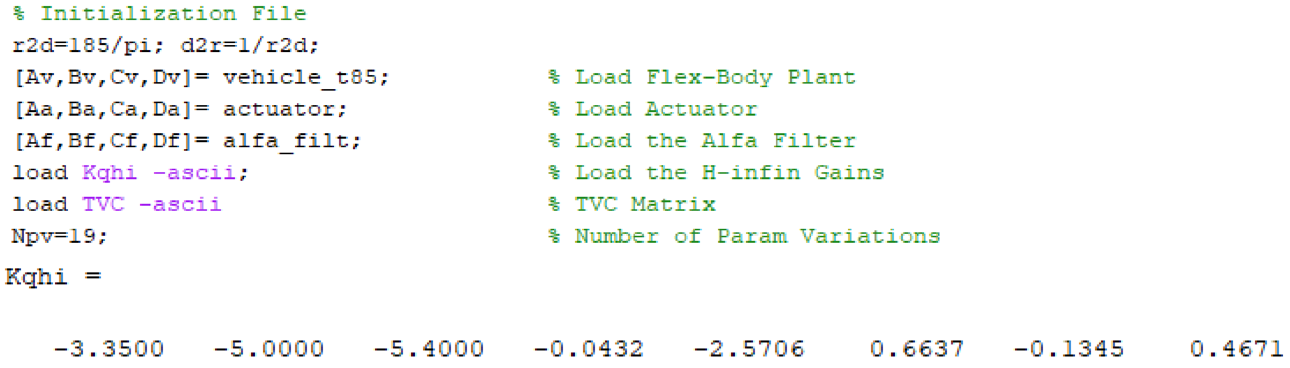
```
States   =  48
    1    Roll Attitude        (phi-rigid)      (radians)
    2    Roll Rate            ( p -rigid)      (rad/sec)
    3    Pitch Attitude       (theta-rigid)    (radians)
    4    Pitch Rate           ( q -rigid)      (rad/sec)
    5    Yaw  Attitude        (psi-rigid)      (radians)
    6    Yaw  Rate            ( r -rigid)      (rad/sec)
    7    Angle of attack      (alfa-rigid)     (radians)
    8    Angle of sideslip    (beta-rigid)     (radians)
    9    Change in Altitude   (delta-h)         (feet)
   10    Change in Velocity   (delta-V)         (ft/sec)
   11    Flex Mode#  1 Generaliz Displacem eta(  1)  (ft)
   12    Flex Mode#  1 GD rate, eta(  1)-dot  (ft/sec)
   13    Flex Mode#  2 Generaliz Displacem eta(  2)  (ft)
   14    Flex Mode#  2 GD rate, eta(  2)-dot  (ft/sec)
   15    Flex Mode#  3 Generaliz Displacem eta(  3)  (ft)
   16    Flex Mode#  3 GD rate, eta(  3)-dot  (ft/sec)
   17    Flex Mode#  4 Generaliz Displacem eta(  4)  (ft)
   18    Flex Mode#  4 GD rate, eta(  4)-dot  (ft/sec)
   19    Flex Mode#  5 Generaliz Displacem eta(  5)  (ft)
   20    Flex Mode#  5 GD rate, eta(  5)-dot  (ft/sec)
   21    Flex Mode#  6 Generaliz Displacem eta(  6)  (ft)
   22    Flex Mode#  6 GD rate, eta(  6)-dot  (ft/sec)
   23    Flex Mode#  7 Generaliz Displacem eta(  7)  (ft)
   24    Flex Mode#  7 GD rate, eta(  7)-dot  (ft/sec)
   25    Flex Mode#  8 Generaliz Displacem eta(  8)  (ft)
   26    Flex Mode#  8 GD rate, eta(  8)-dot  (ft/sec)
   27    Flex Mode#  9 Generaliz Displacem eta(  9)  (ft)
   28    Flex Mode#  9 GD rate, eta(  9)-dot  (ft/sec)
   29    Flex Mode# 10 Generaliz Displacem eta( 10)  (ft)
   30    Flex Mode# 10 GD rate, eta( 10)-dot  (ft/sec)
   31    Flex Mode# 11 Generaliz Displacem eta( 11)  (ft)
   32    Flex Mode# 11 GD rate, eta( 11)-dot  (ft/sec)
   33    Flex Mode# 12 Generaliz Displacem eta( 12)  (ft)
   34    Flex Mode# 12 GD rate, eta( 12)-dot  (ft/sec)
   35    Flex Mode# 13 Generaliz Displacem eta( 13)  (ft)
   36    Flex Mode# 13 GD rate, eta( 13)-dot  (ft/sec)
   37    Flex Mode# 14 Generaliz Displacem eta( 14)  (ft)
   38    Flex Mode# 14 GD rate, eta( 14)-dot  (ft/sec)
   39    Flex Mode# 15 Generaliz Displacem eta( 15)  (ft)
   40    Flex Mode# 15 GD rate, eta( 15)-dot  (ft/sec)
   41    Slosh Mass # 1 Displacem. Zs( 1)    (ft)
   42    Slosh Mass # 1 Displacem. Ys( 1)    (ft)
   43    Slosh Mass # 1 Velocity  Zsd( 1)   (ft/sec)
   44    Slosh Mass # 1 Velocity  Ysd( 1)   (ft/sec)
   45    Slosh Mass # 2 Displacem. Zs( 2)    (ft)
   46    Slosh Mass # 2 Displacem. Ys( 2)    (ft)
   47    Slosh Mass # 2 Velocity  Zsd( 2)   (ft/sec)
   48    Slosh Mass # 2 Velocity  Ysd( 2)   (ft/sec)
```



```
Outputs =  70
   1    Roll Attitude  (phi-body) (radians)
   2    Roll Rate      (p-body)   (rad/sec)
   3    Pitch Attitude (thet-bdy) (radians)
   4    Pitch Rate     (q-body)   (rad/sec)
   5    Yaw  Attitude  (psi-body) (radians)
   6    Yaw  Rate      (r-body)   (rad/sec)
   7    Angle of attack, alfa, (radians)
   8    Angle of sideslip, beta, (radian)
   9    Change in Altitude, delta-h, (feet)
  10    Forward Acceleration  (V-dot)  (ft/sec)
  11    Cross Range Velocity (Vcr) (ft/sec)
  12    Rate-Gyro #  1, Roll Rate  (Body) (rad/sec)
  13    Rate-Gyro #  2, Pitch Rate (Body) (rad/sec)
  14    Rate-Gyro #  3, Yaw Rate   (Body) (rad/sec)
  15    Accelerom #  1, (along Y), (ft/sec^2) Translat
  16    Accelerom #  2, (along Z), (ft/sec^2) Translat
  17    Ptch Load-Torque Tly for Engine: 1,(ft-lb)
  18    Ptch Load-Torque Tly for Engine: 2,(ft-lb)
  19    Ptch Load-Torque Tly for Engine: 3,(ft-lb)
  20    Ptch Load-Torque Tly for Engine: 4,(ft-lb)
  21    Ptch Load-Torque Tly for Engine: 5,(ft-lb)
  22    Ptch Load-Torque Tly for Engine: 6,(ft-lb)
  23    Ptch Load-Torque Tly for Engine: 7,(ft-lb)
  24    Ptch Load-Torque Tly for Engine: 8,(ft-lb)
  25    Yaw  Load-Torque Tlz for Engine: 1,(ft-lb)
  26    Yaw  Load-Torque Tlz for Engine: 2,(ft-lb)
  27    Yaw  Load-Torque Tlz for Engine: 3,(ft-lb)
  28    Yaw  Load-Torque Tlz for Engine: 4,(ft-lb)
  29    Yaw  Load-Torque Tlz for Engine: 5,(ft-lb)
  30    Yaw  Load-Torque Tlz for Engine: 6,(ft-lb)
  31    Yaw  Load-Torque Tlz for Engine: 7,(ft-lb)
  32    Yaw  Load-Torque Tlz for Engine: 8,(ft-lb)
  33    Cm_alpha  :   -14.535 % Variation
  34    Cn_beta   :    14.535 % Variation
  35    Ca_alpha  :    17.153 % Variation
  36    Cz_alpha  :   -24.655 % Variation
  37    Cy_beta   :   -24.655 % Variation
  38    Cm_0      :    -9.690 % Variation
  39    CZ_0      :   -12.327 % Variation
  40    CA_0      :     6.679 % Variation
  41    I_xx      :    19.786 % Variation
  42    I_yy      :    14.153 % Variation
  43    I_zz      :    14.158 % Variation
  44    Qbar      :     9.865 % Variation
  45    Qbar      :     9.865 % Variation
  46    Xcg locat:      3.492 % Variation
  47    Xcg locat:      3.492 % Variation
  48    Xcg locat:      3.492 % Variation
  49    M_slosh 1:     13.664 % Variation
  50    M_slosh 1:     13.664 % Variation
  51    X_slosh 1:      3.563 % Variation
  52    X_slosh 1:      3.563 % Variation
  53    X_slosh 1:      3.563 % Variation
  54    Wslsh_Y 1:      2.251 % Variation
  55    Wslsh_Z 1:      2.251 % Variation
  56    ZetSl_Y 1:     12.500 % Variation
  57    ZetSl_Z 1:     12.500 % Variation
  58    M_slosh 2:     14.465 % Variation
  59    M_slosh 2:     14.465 % Variation
  60    X_slosh 2:      7.956 % Variation
  61    X_slosh 2:      7.956 % Variation
  62    X_slosh 2:      7.956 % Variation
  63    Wslsh_Y 2:      2.251 % Variation
  64    Wslsh_Z 2:      2.251 % Variation
  65    ZetSl_Y 2:     50.000 % Variation
  66    ZetSl_Z 2:     50.000 % Variation
  67    W_flex  1:  5.000       Addit. Variat.
  68    W_flex  2:  5.000       Addit. Variat.
  69    W_flex  3:  8.000       Addit. Variat.
  70    W_flex  4:  8.000       Addit. Variat.
```

2.3 Loading the Files into Matlab

The Matlab script file "Init.m" loads the vehicle, TVC matrix, alpha-filter and the actuator models into Matlab. It also loads the control gains matrix that was calculated in section-1.

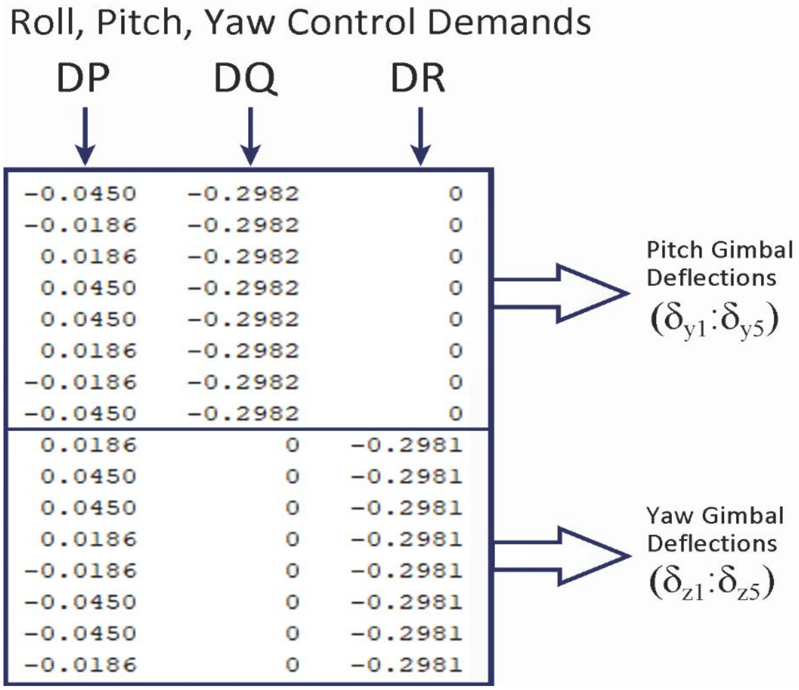


```
% Initialization File
r2d=185/pi; d2r=1/r2d;
[Av,Bv,Cv,Dv]= vehicle_t85;                 % Load Flex-Body Plant
[Aa,Ba,Ca,Da]= actuator;                    % Load Actuator
[Af,Bf,Cf,Df]= alfa_filt;                   % Load the Alfa Filter
load Kqhi -ascii;                           % Load the H-infin Gains
load TVC -ascii                             % TVC Matrix
Npv=19;                                     % Number of Param Variations
Kqhi =

   -3.3500   -5.0000   -5.4000   -0.0432   -2.5706    0.6637   -0.1345    0.4671
```

Figure 17 Initialization File and the State-Feedback Gain Matrix

2.4 TVC Matrix

The TVC matrix converts the roll, pitch and yaw FCS demands to gimbal deflections that command the 16 actuators (8-pitch and 8-yaw commands). It is calculated by the Flixan Mixing-Logic algorithm based on TVC thrust, max gimbal deflection, and gimbal geometry relative to vehicle CG.



```
 -0.0450   -0.2982         0
 -0.0186   -0.2982         0
  0.0186   -0.2982         0
  0.0450   -0.2982         0
  0.0450   -0.2982         0
  0.0186   -0.2982         0
 -0.0186   -0.2982         0
 -0.0450   -0.2982         0
  0.0186         0   -0.2981
  0.0450         0   -0.2981
  0.0450         0   -0.2981
  0.0186         0   -0.2981
 -0.0186         0   -0.2981
 -0.0450         0   -0.2981
 -0.0450         0   -0.2981
 -0.0186         0   -0.2981
```

Figure 18 The TVC Converts the Roll, Pitch and Yaw Demands to Gimbal Deflection Commands

2.5 Control System

The flight control system is shown in Figure-19. The pitch and yaw axes are identical and roll is a simple PD controller. The pitch and yaw controllers are (1x8) state-feedback. The first four states are vehicle states: attitude θ, pitch rate q, angle of attack α, and θ-integral. The other four states come from the 4th order alpha-filter which is implemented as a separate block. Low-pass filters and notch filters are included to attenuate the flex modes. It is the alpha-filter that provides disturbance attenuation at 0.6 to 0.9 (rad/sec).

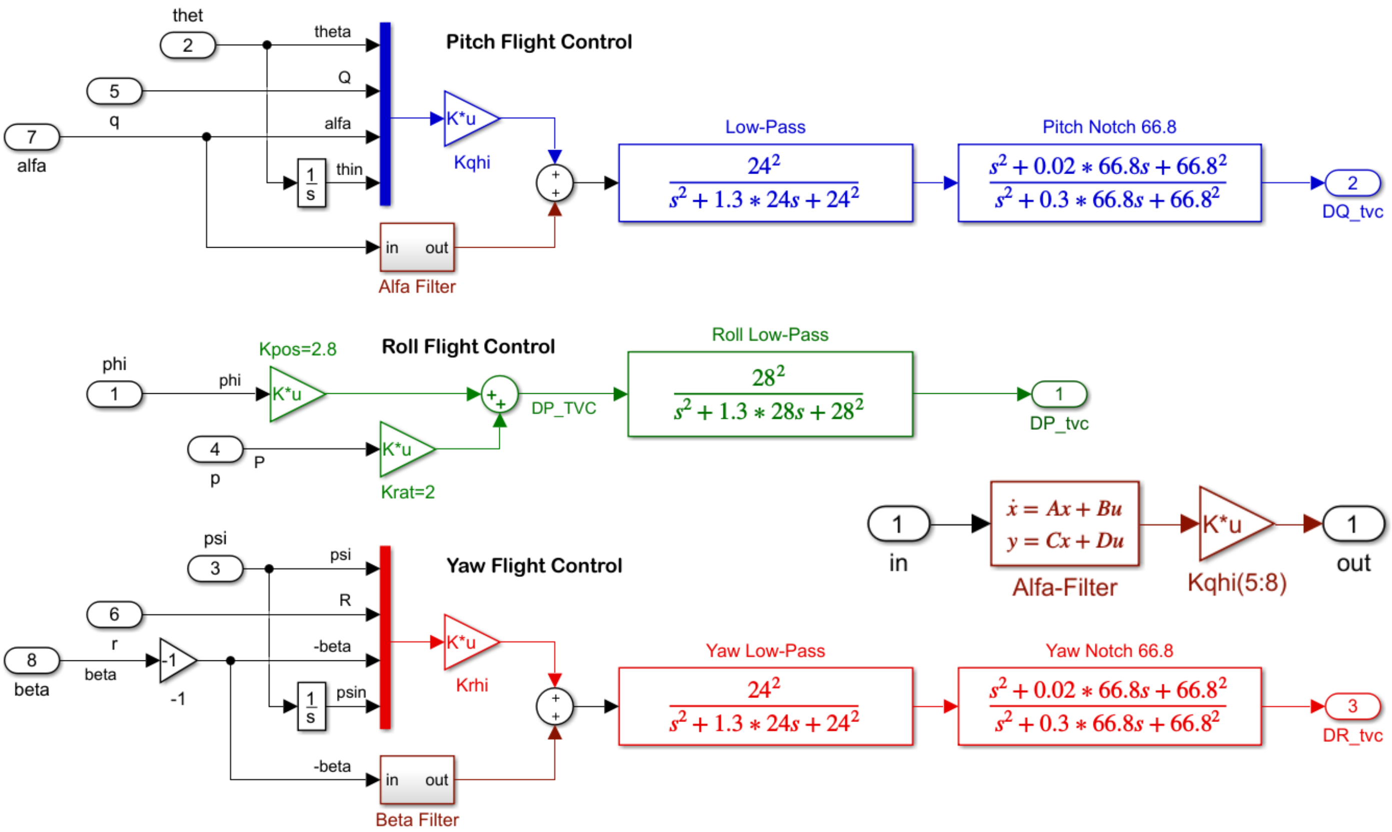


Figure 19 Flight Control System

2.6 Stability and Sensitivity Analysis

The open-loop Simulink model “Open_Loop_2.slx” in Figure 20 is used for analyzing stability, one loop at a time with the other two loops closed. It includes the vehicle, 16 actuators, the TVC matrix, and the control system of Figure 19. The vehicle system includes: slosh, structural flexibility, tail-wags-dog dynamics and load-torque feedback between the engine gimbals and the actuators. The file “freq.m” below uses the open-loop model to calculate the Bode and Nichols plots and analyze stability in pitch, yaw and roll, as shown in Figures 22 & 23. Figure 21 is the closed-loop sensitivity analysis model “Sensitiv_2.slx”. It consists of the same elements and it is used to analyze the control system’s sensitivity to wind-gust disturbances using Singular Value (Sigma) frequency response plots. The sensitivity model is normalized to unity. The gust velocity input is scaled by the largest wind-gust velocity 25 (ft/sec). The wind direction is defined in the input data to excite both pitch and yaw axes. The output consists of both (α & β) angles and is divided by the maximum allowed angles, which is 4°.

```
% Stability Analysis: freq.m
init;
[Ao,Bo,Co,Do]= linmod('Open_Loop_2');      % Open-Loop Stabil Anal Model
[As,Bs,Cs,Ds]= linmod('Sensitiv_2');       % Sensitivity to Gusts Model
syso= ss(Ao,Bo,Co,Do);                     % Create Vehicle  SS System
syss= ss(As,Bs,Cs,Ds);                     % Create Vehicle  SS System
w=logspace(-2, 3, 20000);                  % Define Frequ Range
figure(10); nichols(syso,syso,w)           % Plot Nichol's Chart
figure(20); bode(syso,syso,w)              % Plot Bode
figure(30); sigma(syss,syss,w);
```

The Sensitivity Function: (α,β)/Vgust is shown in Figure 24. Its magnitude is less than one at all frequencies and it has a dip at 0.6 – 0.9 (rad/sec), as expected, created by the α,β-filters.

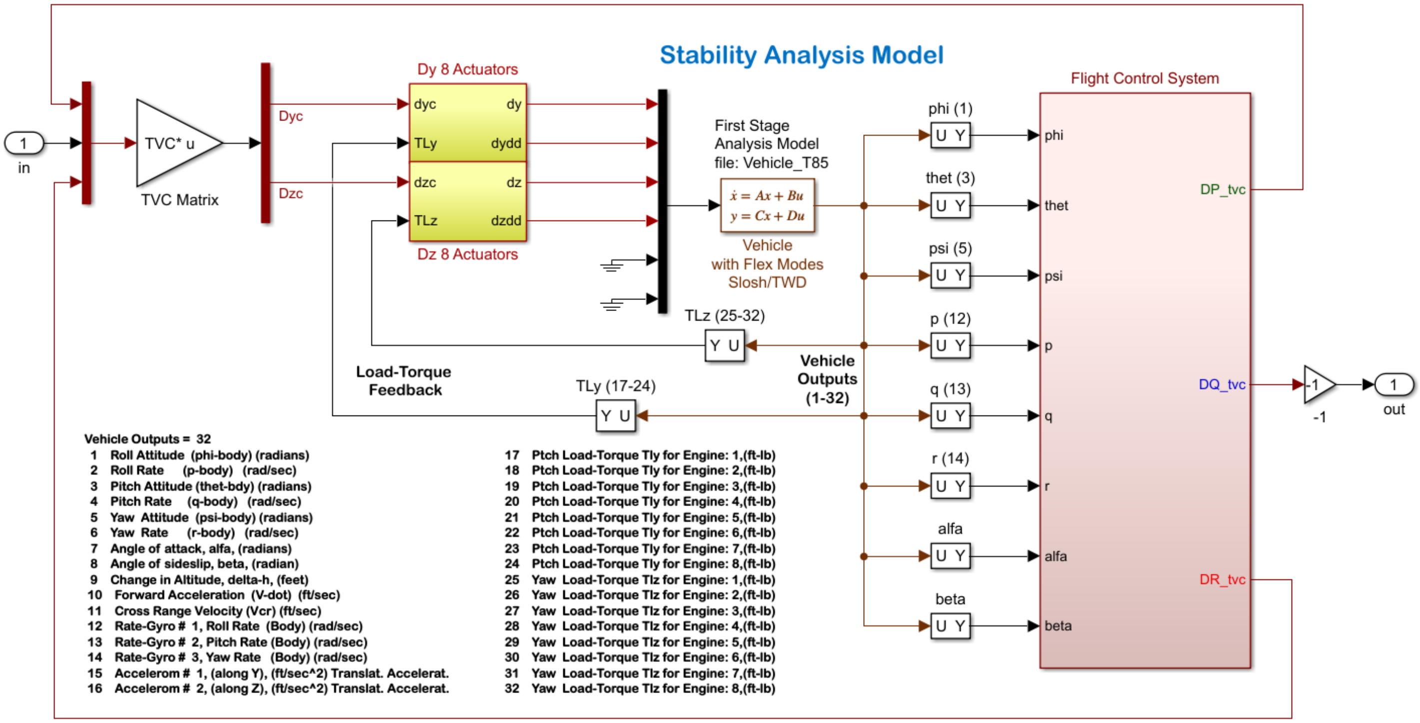


Figure 20 Coupled, Pitch and Lateral Model Used for Stability Analysis "Open_Loop_2.slx"

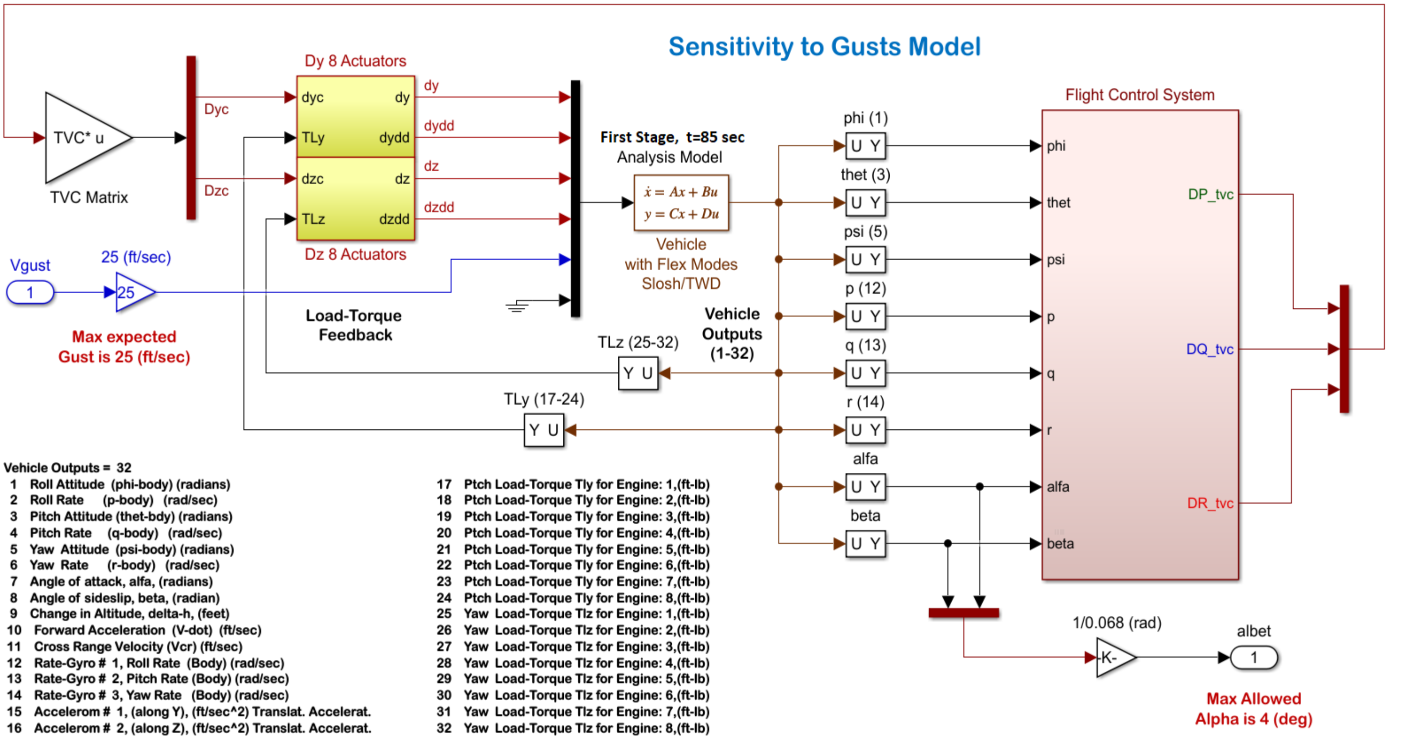


Figure 21 Scaled Sensitivity Analysis Model "Sensitiv.slx"

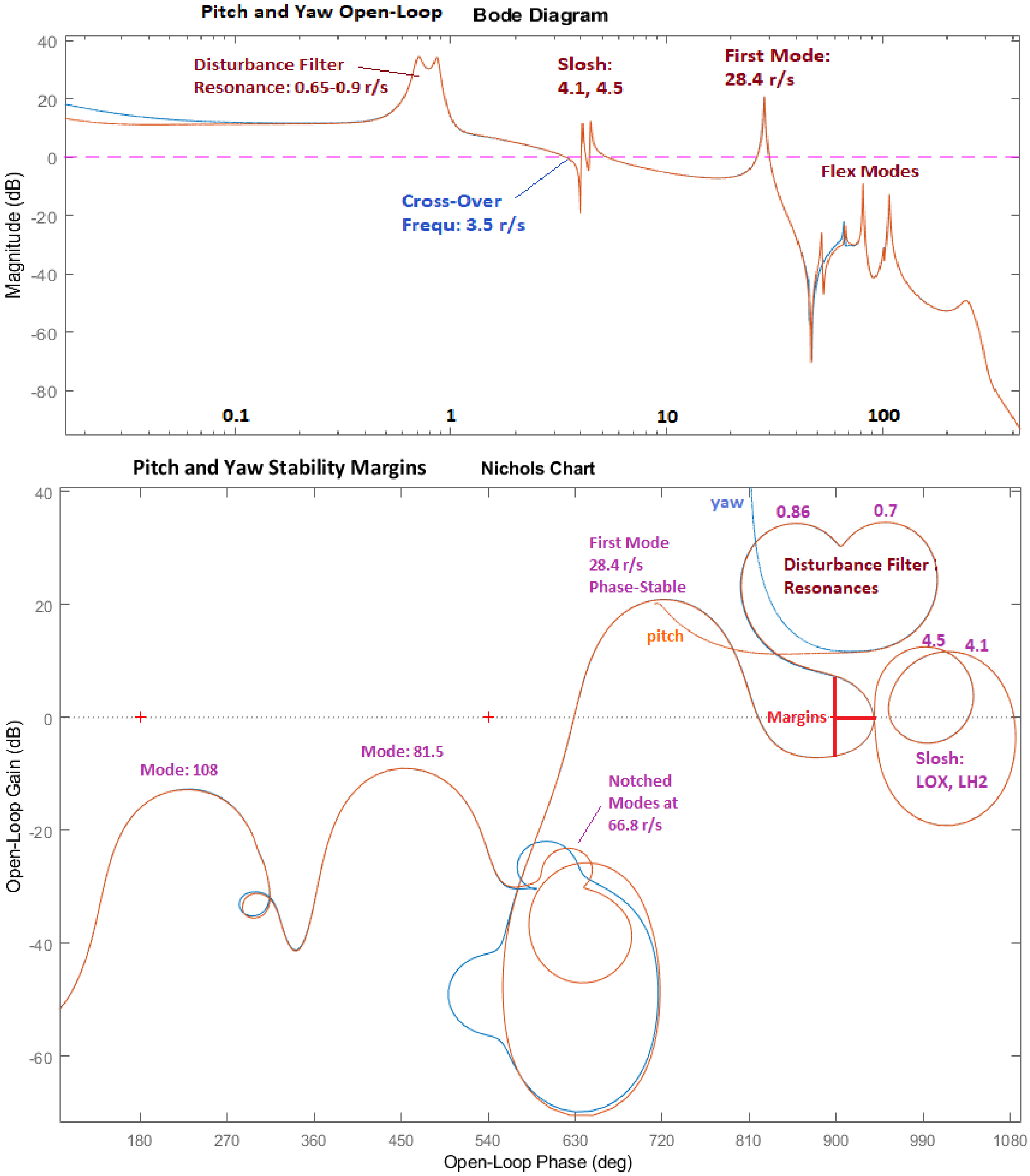


Figure 22 Pitch and Yaw Stability Analysis

The two slosh modes at 4.1 and 4.5 r/s are phase-stable. The first bending mode in pitch and yaw is strong but also phase-stabilized with the low-pass filter. An unstable mode at 66.8 r/s was notched with filters. The alpha-filter produces 2 strong resonances at 0.7 and 0.86 r/s.

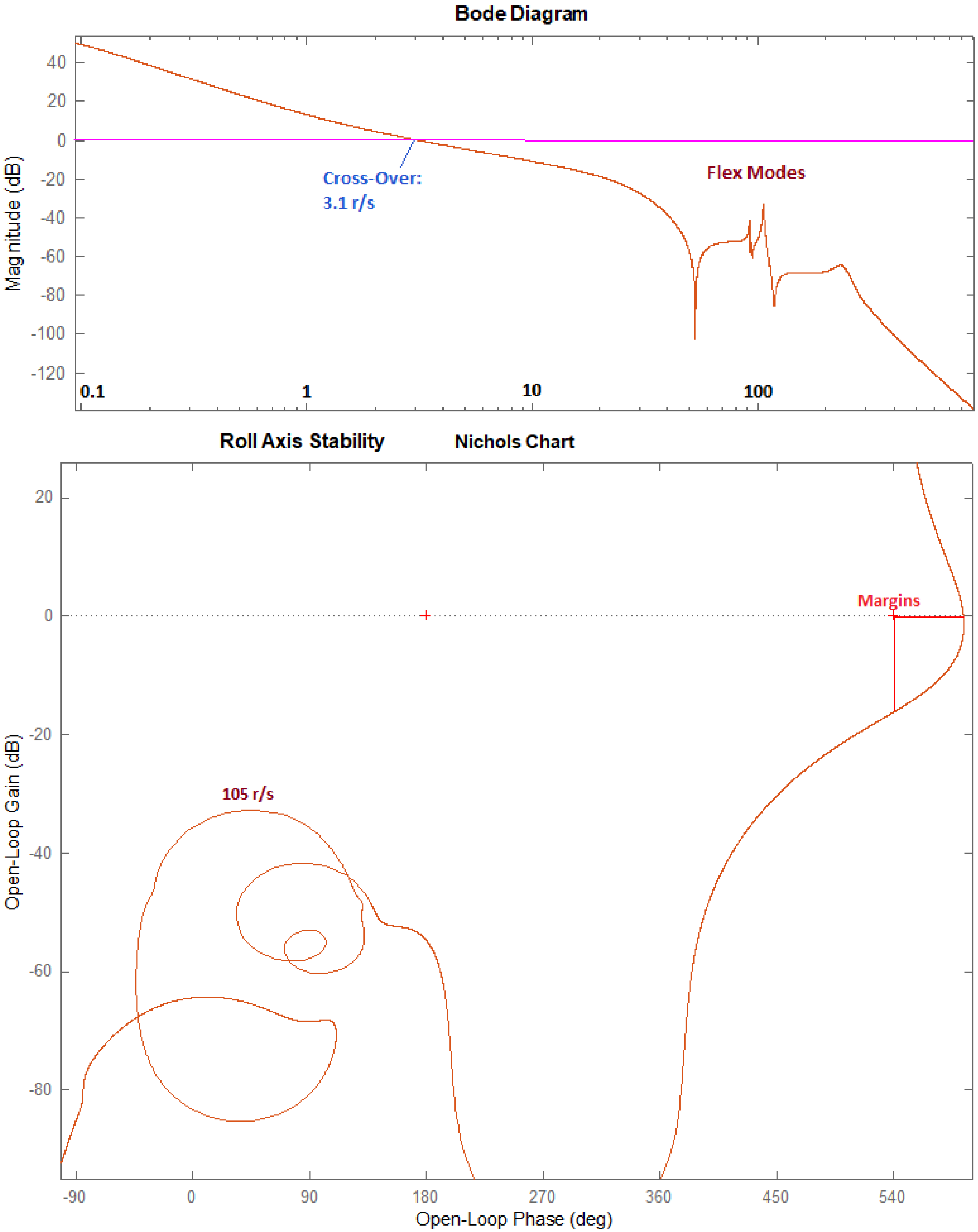


Figure 23 Roll Axis Stability Analysis

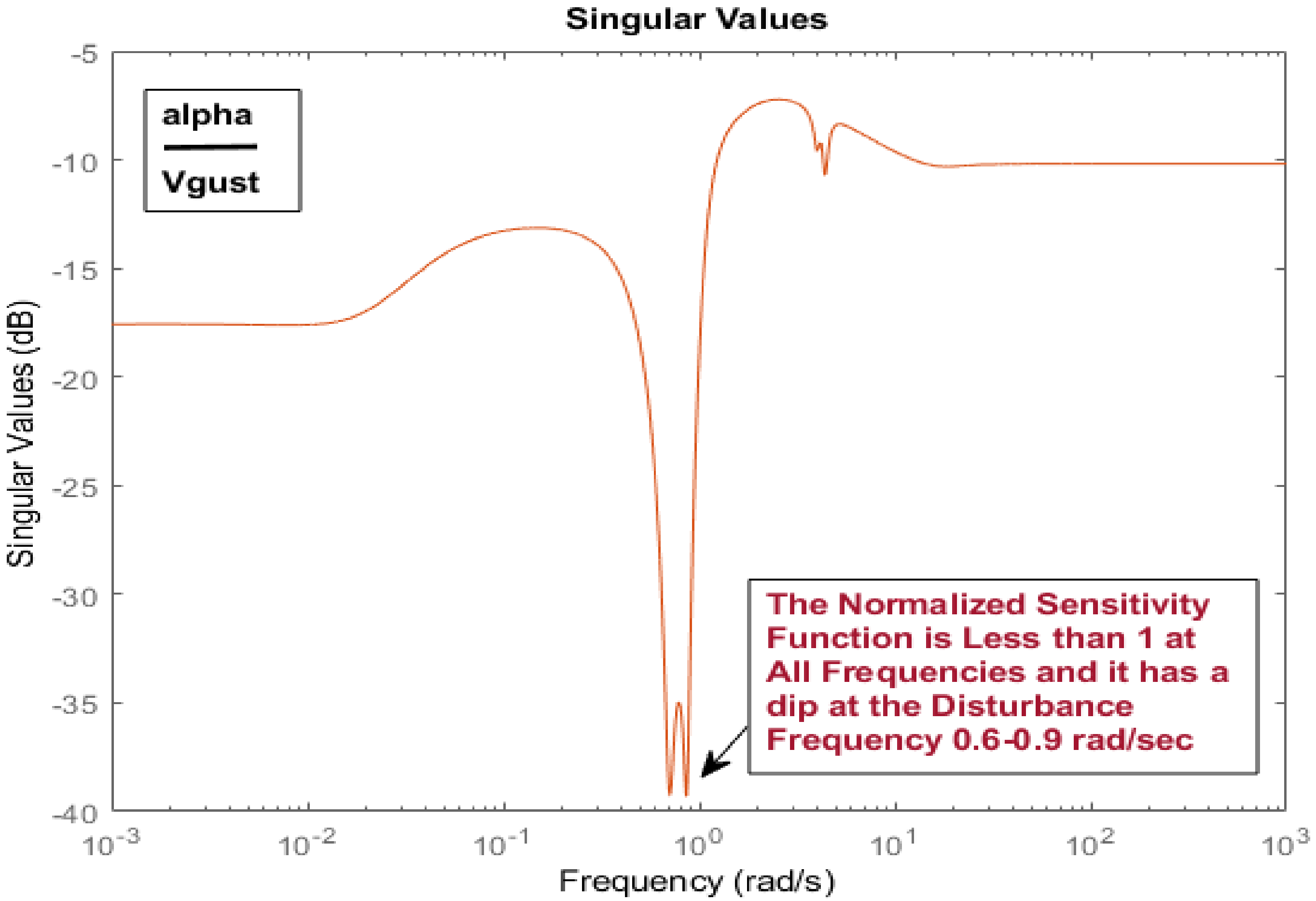
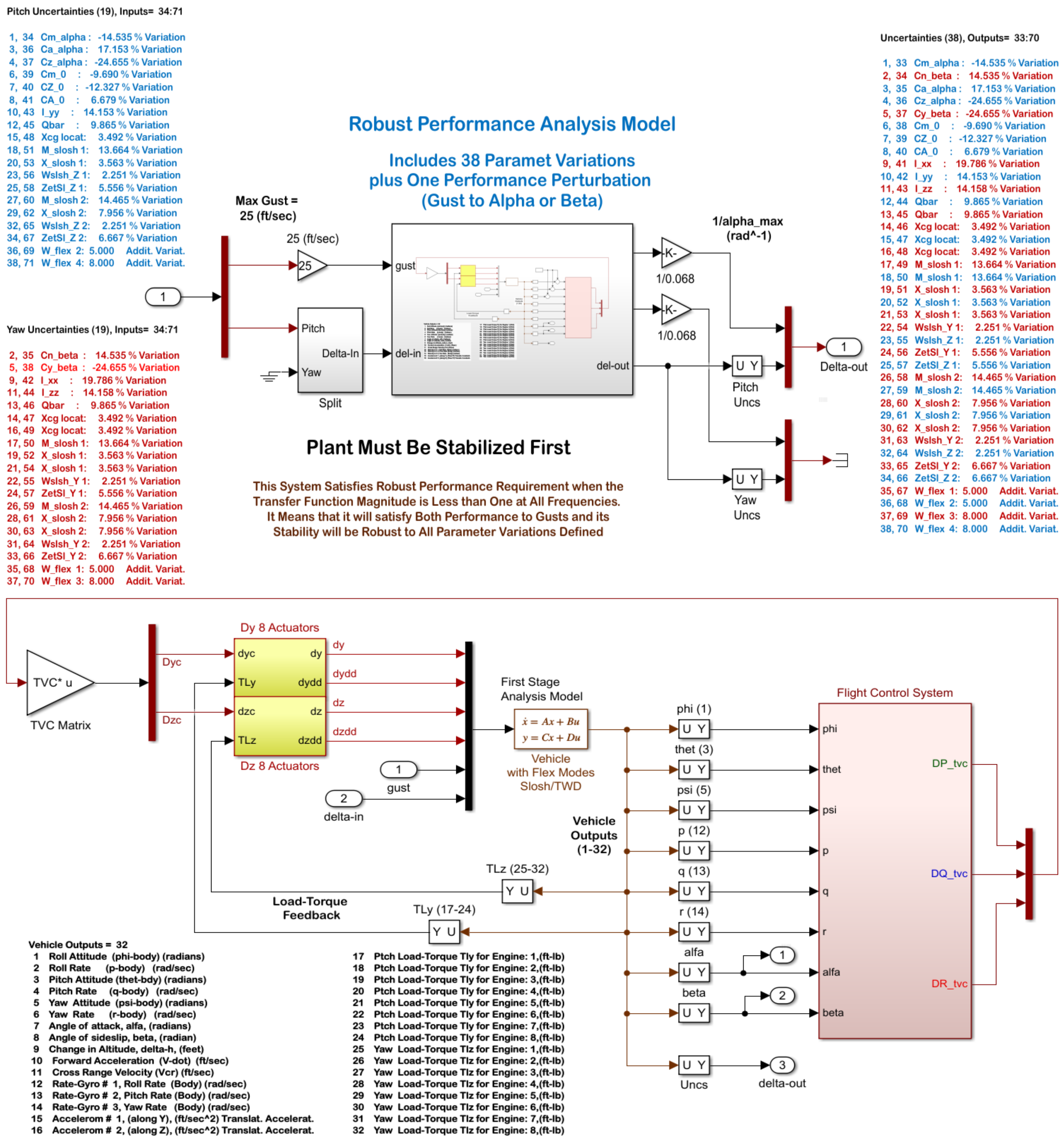


Figure 24 Normalized Sensitivity Function Between Gust Disturbance and (α, β) Output

2.6 Robustness to Uncertainties

The closed-loop Simulink model "*Robust_Perform.slx*" in Figure 25 is used to analyze Robust Performance which is the control system ability to satisfy both: robustness and good performance, which in this case is, to remain stable in the presence of structured uncertainties and to simultaneously satisfy sensitivity to wind-gusts. The inputs and outputs of this model are uncertainties. That is, inputs and outputs that connect to the uncertainties block Δ. There are 38 uncertainties which have been separated into 19 pitch and 19 lateral variations because the pitch variations affect only the pitch axis and the lateral affect only yaw and roll. They are already normalized for a unity block where the parameters vary ±1. So, we analyze pitch and lateral robustness separately by modifying the Simulink model, pitch robustness using the pitch i/o connections (as shown in Fig.25) and lateral robustness using the lateral i/o uncertainties. The model includes one additional i/o pair for analyzing sensitivity to gusts: the gust velocity input and alpha output, normalized to unity as already described. The system satisfies robust performance criterion when the Structured Singular Value (μ) of this system from the 19 uncertainty inputs plus Vgust to the 19 uncertainty outputs plus alpha, is less than one at all frequencies. The Matlab script "Run_Robust.m" performs this operation and calculates the SSV frequency response using the 19 variations which affect pitch in this case, and $\mu < 1$ at all frequencies, as shown in Figure-26, which satisfies the requirement.



Pitch Uncertainties (19), Inputs= 34:71

1, 34 Cm_alpha : -14.535 % Variation
3, 36 Ca_alpha : 17.153 % Variation
4, 37 Cz_alpha : -24.655 % Variation
6, 39 Cm_0 : -9.690 % Variation
7, 40 CZ_0 : -12.327 % Variation
8, 41 CA_0 : 6.679 % Variation
10, 43 I_yy : 14.153 % Variation
12, 45 Qbar : 9.865 % Variation
15, 48 Xcg locat: 3.492 % Variation
18, 51 M_slosh 1: 13.664 % Variation
20, 53 X_slosh 1: 3.563 % Variation
23, 56 Wslsh_Z 1: 2.251 % Variation
25, 58 ZetSl_Z 1: 5.556 % Variation
27, 60 M_slosh 2: 14.465 % Variation
29, 62 X_slosh 2: 7.956 % Variation
32, 65 Wslsh_Z 2: 2.251 % Variation
34, 67 ZetSl_Z 2: 6.667 % Variation
36, 69 W_flex 2: 5.000 Addit. Variat.
38, 71 W_flex 4: 8.000 Addit. Variat.

Yaw Uncertainties (19), Inputs= 34:71

2, 35 Cn_beta : 14.535 % Variation
5, 38 Cy_beta : -24.655 % Variation
9, 42 I_xx : 19.786 % Variation
11, 44 I_zz : 14.158 % Variation
13, 46 Qbar : 9.865 % Variation
14, 47 Xcg locat: 3.492 % Variation
16, 49 Xcg locat: 3.492 % Variation
17, 50 M_slosh 1: 13.664 % Variation
19, 52 X_slosh 1: 3.563 % Variation
21, 54 X_slosh 1: 3.563 % Variation
22, 55 Wslsh_Y 1: 2.251 % Variation
24, 57 ZetSl_Y 1: 5.556 % Variation
26, 59 M_slosh 2: 14.465 % Variation
28, 61 X_slosh 2: 7.956 % Variation
30, 63 X_slosh 2: 7.956 % Variation
31, 64 Wslsh_Y 2: 2.251 % Variation
33, 66 ZetSl_Y 2: 6.667 % Variation
35, 68 W_flex 1: 5.000 Addit. Variat.
37, 70 W_flex 3: 8.000 Addit. Variat.

Plant Must Be Stabilized First

This System Satisfies Robust Performance Requirement when the Transfer Function Magnitude is Less than One at All Frequencies. It Means that it will satisfy Both Performance to Gusts and its Stability will be Robust to All Parameter Variations Defined

Uncertainties (38), Outputs= 33:70

1, 33 Cm_alpha : -14.535 % Variation
2, 34 Cn_beta : 14.535 % Variation
3, 35 Ca_alpha : 17.153 % Variation
4, 36 Cz_alpha : -24.655 % Variation
5, 37 Cy_beta : -24.655 % Variation
6, 38 Cm_0 : -9.690 % Variation
7, 39 CZ_0 : -12.327 % Variation
8, 40 CA_0 : 6.679 % Variation
9, 41 I_xx : 19.786 % Variation
10, 42 I_yy : 14.153 % Variation
11, 43 I_zz : 14.158 % Variation
12, 44 Qbar : 9.865 % Variation
13, 45 Qbar : 9.865 % Variation
14, 46 Xcg locat: 3.492 % Variation
15, 47 Xcg locat: 3.492 % Variation
16, 48 Xcg locat: 3.492 % Variation
17, 49 M_slosh 1: 13.664 % Variation
18, 50 M_slosh 1: 13.664 % Variation
19, 51 X_slosh 1: 3.563 % Variation
20, 52 X_slosh 1: 3.563 % Variation
21, 53 X_slosh 1: 3.563 % Variation
22, 54 Wslsh_Y 1: 2.251 % Variation
23, 55 Wslsh_Z 1: 2.251 % Variation
24, 56 ZetSl_Y 1: 5.556 % Variation
25, 57 ZetSl_Z 1: 5.556 % Variation
26, 58 M_slosh 2: 14.465 % Variation
27, 59 M_slosh 2: 14.465 % Variation
28, 60 X_slosh 2: 7.956 % Variation
29, 61 X_slosh 2: 7.956 % Variation
30, 62 X_slosh 2: 7.956 % Variation
31, 63 Wslsh_Y 2: 2.251 % Variation
32, 64 Wslsh_Z 2: 2.251 % Variation
33, 65 ZetSl_Y 2: 6.667 % Variation
34, 66 ZetSl_Z 2: 6.667 % Variation
35, 67 W_flex 1: 5.000 Addit. Variat.
36, 68 W_flex 2: 5.000 Addit. Variat.
37, 69 W_flex 3: 8.000 Addit. Variat.
38, 70 W_flex 4: 8.000 Addit. Variat.

Vehicle Outputs = 32

1 Roll Attitude (phi-body) (radians)
2 Roll Rate (p-body) (rad/sec)
3 Pitch Attitude (thet-bdy) (radians)
4 Pitch Rate (q-body) (rad/sec)
5 Yaw Attitude (psi-body) (radians)
6 Yaw Rate (r-body) (rad/sec)
7 Angle of attack, alfa, (radians)
8 Angle of sideslip, beta, (radian)
9 Change in Altitude, delta-h, (feet)
10 Forward Acceleration (V-dot) (ft/sec)
11 Cross Range Velocity (Vcr) (ft/sec)
12 Rate-Gyro # 1, Roll Rate (Body) (rad/sec)
13 Rate-Gyro # 2, Pitch Rate (Body) (rad/sec)
14 Rate-Gyro # 3, Yaw Rate (Body) (rad/sec)
15 Accelerom # 1, (along Y), (ft/sec^2) Translat. Accelerat.
16 Accelerom # 2, (along Z), (ft/sec^2) Translat. Accelerat.
17 Ptch Load-Torque Tly for Engine: 1,(ft-lb)
18 Ptch Load-Torque Tly for Engine: 2,(ft-lb)
19 Ptch Load-Torque Tly for Engine: 3,(ft-lb)
20 Ptch Load-Torque Tly for Engine: 4,(ft-lb)
21 Ptch Load-Torque Tly for Engine: 5,(ft-lb)
22 Ptch Load-Torque Tly for Engine: 6,(ft-lb)
23 Ptch Load-Torque Tly for Engine: 7,(ft-lb)
24 Ptch Load-Torque Tly for Engine: 8,(ft-lb)
25 Yaw Load-Torque Tlz for Engine: 1,(ft-lb)
26 Yaw Load-Torque Tlz for Engine: 2,(ft-lb)
27 Yaw Load-Torque Tlz for Engine: 3,(ft-lb)
28 Yaw Load-Torque Tlz for Engine: 4,(ft-lb)
29 Yaw Load-Torque Tlz for Engine: 5,(ft-lb)
30 Yaw Load-Torque Tlz for Engine: 6,(ft-lb)
31 Yaw Load-Torque Tlz for Engine: 7,(ft-lb)
32 Yaw Load-Torque Tlz for Engine: 8,(ft-lb)

Figure 25 Robust Performance Analysis Model "Robust_Perform.slx" Configured for Pitch Analysis

For lateral robust performance analysis, we must modify the model input by disconnecting the "Demux" second output from pitch uncertainty inputs and connecting it to the second input of the "Split" block which is the yaw uncertainty inputs. The "Delta-out" output must also be disconnected from the top "Mux" and be connected to the bottom "Mux" which selects the beta angle and the yaw uncertainty outputs.

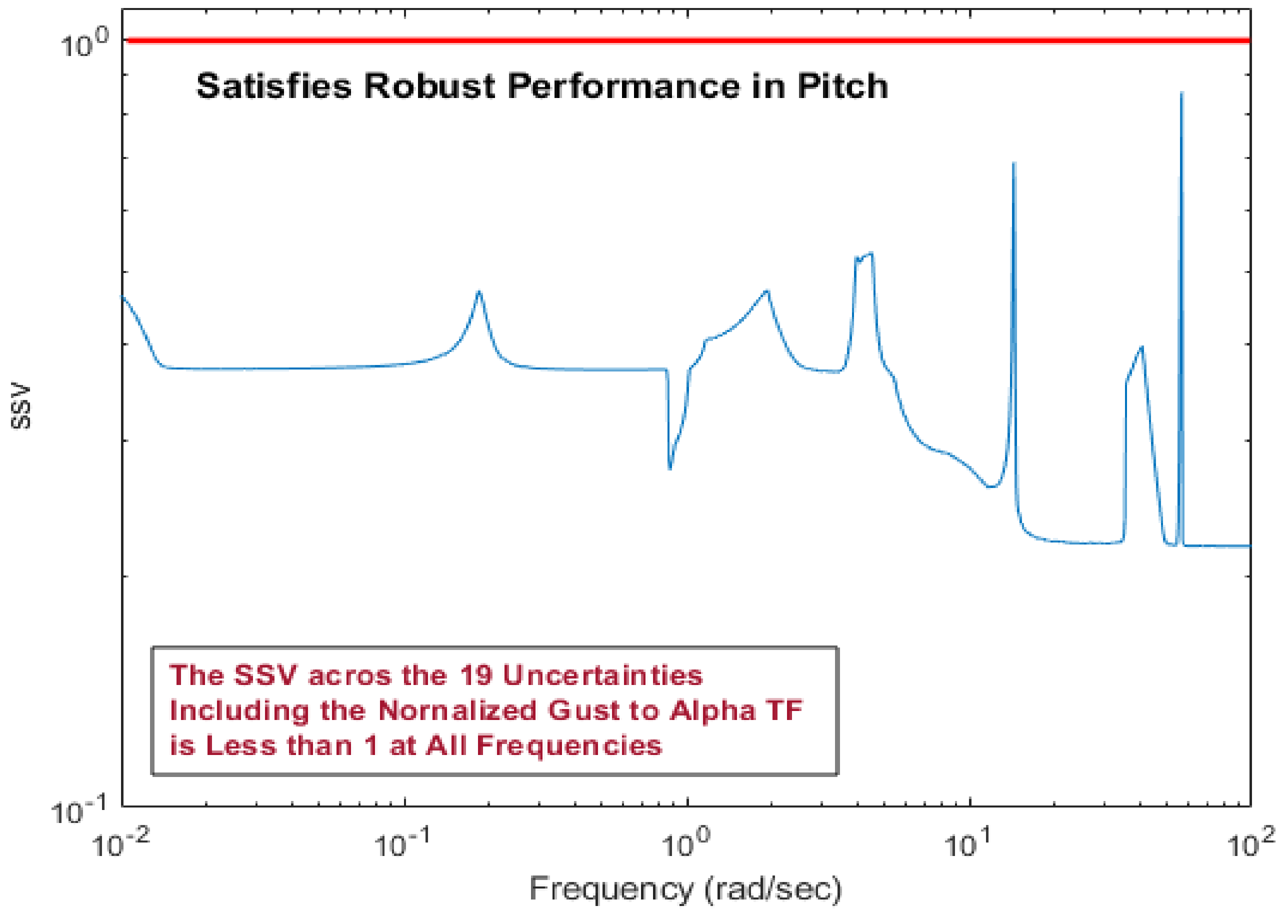
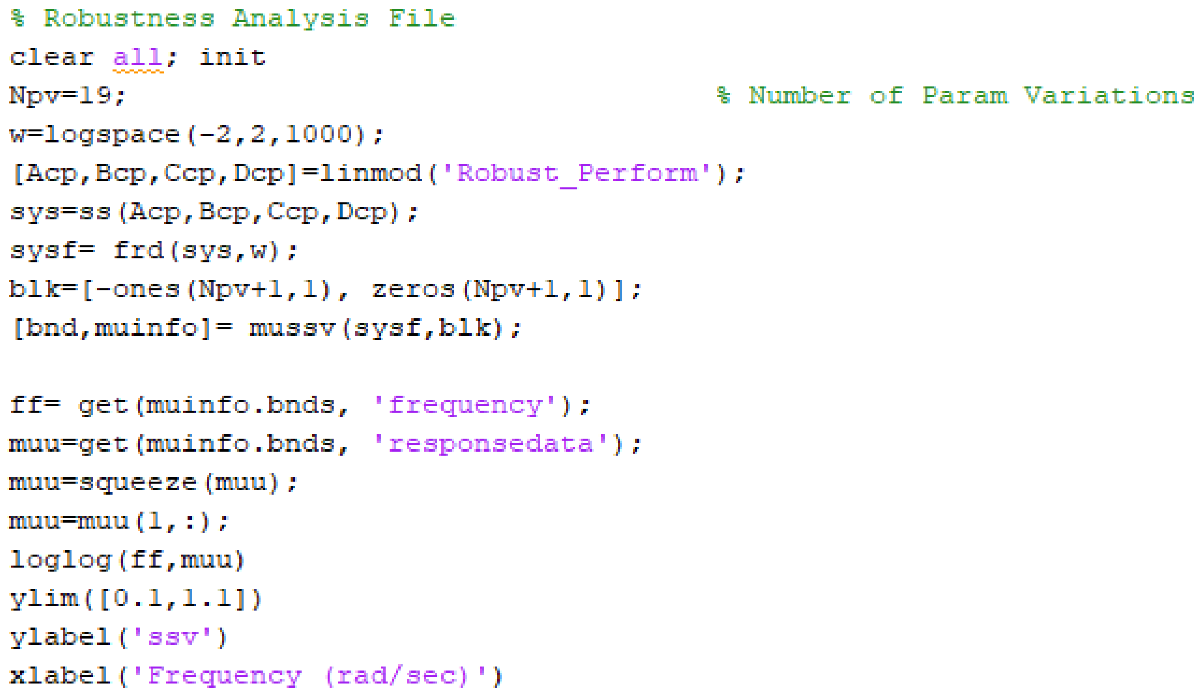


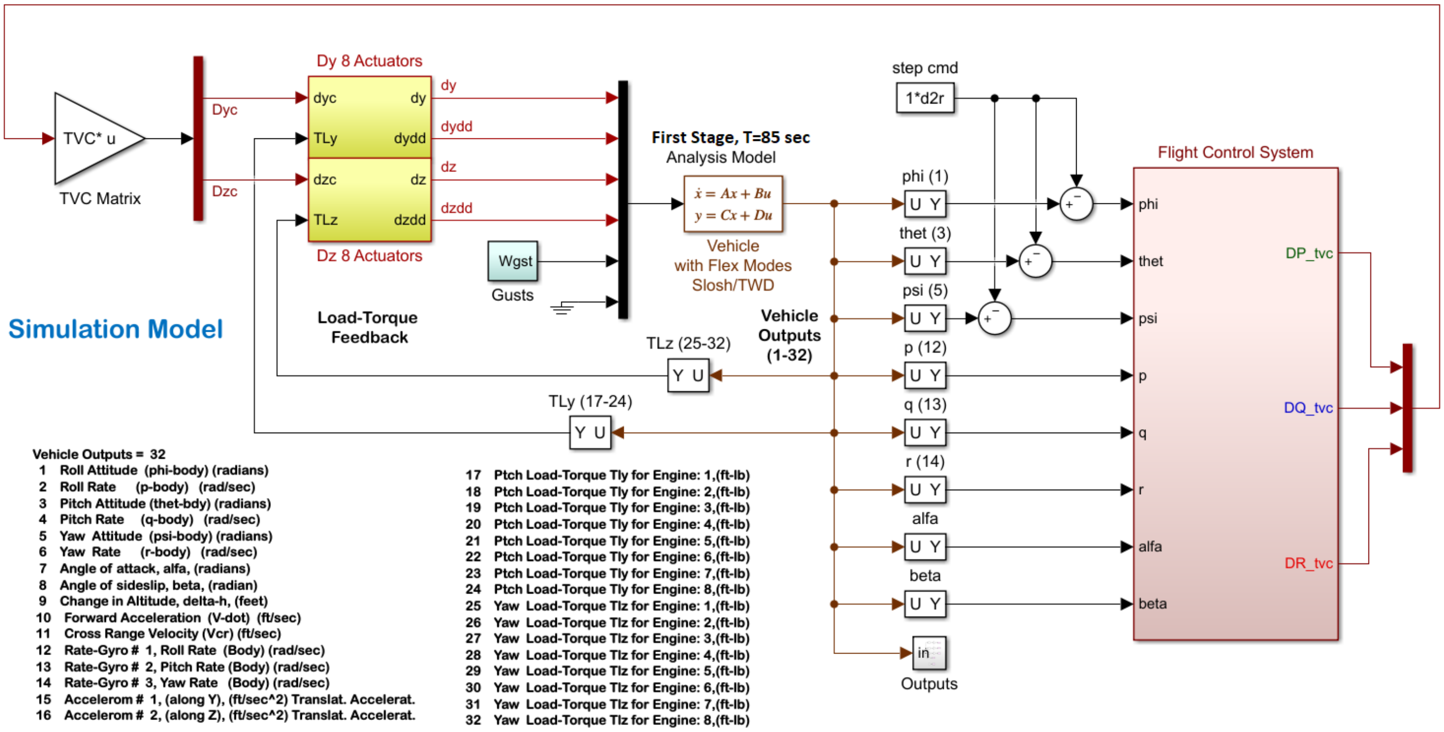
Figure 26 The Pitch System Satisfies Robust Performance Because the SSV between the 20 Inputs and the 20 Outputs Described is Less than 1, at All Frequencies



```
% Robustness Analysis File
clear all; init
Npv=19;                                    % Number of Param Variations
w=logspace(-2,2,1000);
[Acp,Bcp,Ccp,Dcp]=linmod('Robust_Perform');
sys=ss(Acp,Bcp,Ccp,Dcp);
sysf= frd(sys,w);
blk=[-ones(Npv+1,1), zeros(Npv+1,1)];
[bnd,muinfo]= mussv(sysf,blk);

ff= get(muinfo.bnds, 'frequency');
muu=get(muinfo.bnds, 'responsedata');
muu=squeeze(muu);
muu=muu(1,:);
loglog(ff,muu)
ylim([0.1,1.1])
ylabel('ssv')
xlabel('Frequency (rad/sec)')
```

2.7 Simulation Model and Results

The closed-loop Simulink model “*Simulat_2.slx*” in Figure 27 is used to perform time-slice simulations. The inputs are either (roll, pitch, yaw) attitude commands or wind-gust velocity disturbance in (feet/sec).



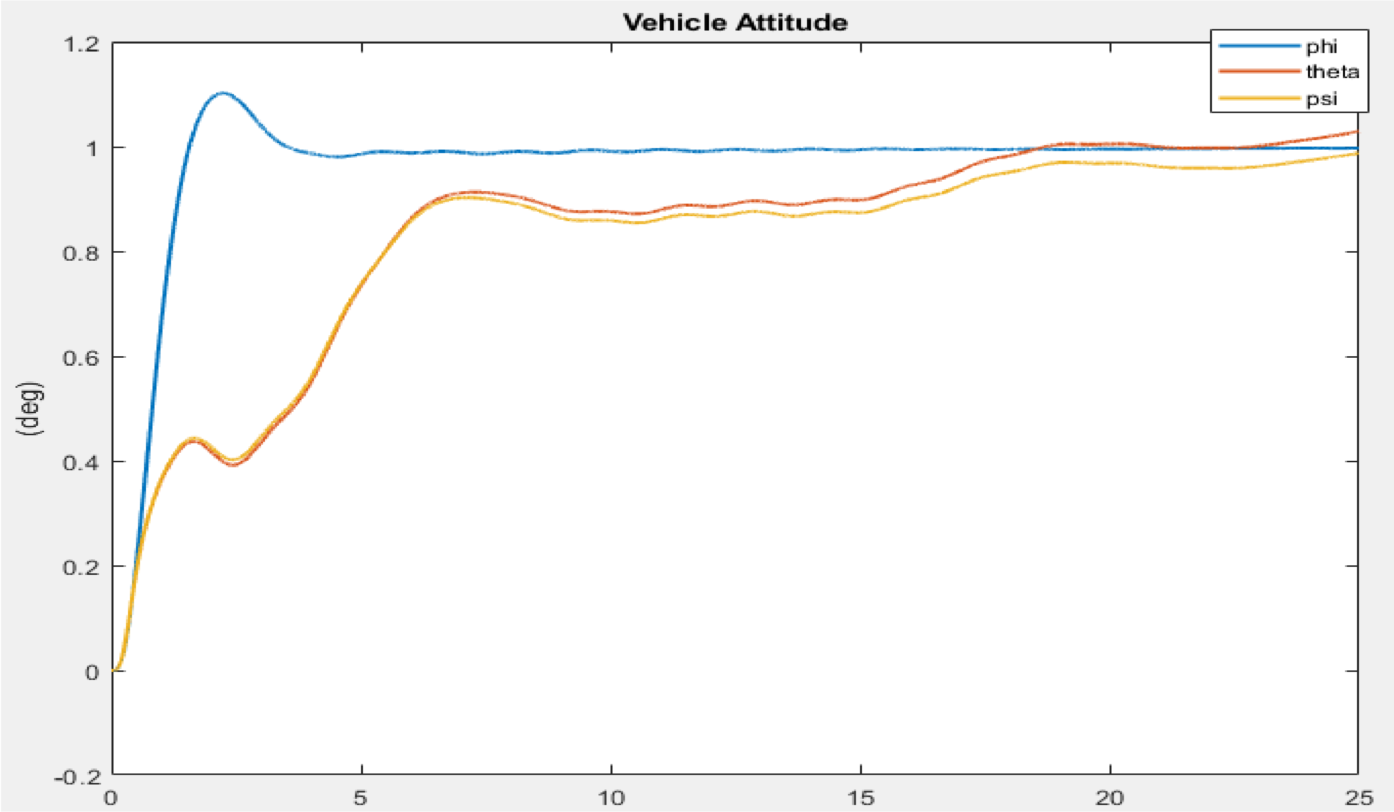
Vehicle Outputs = 32

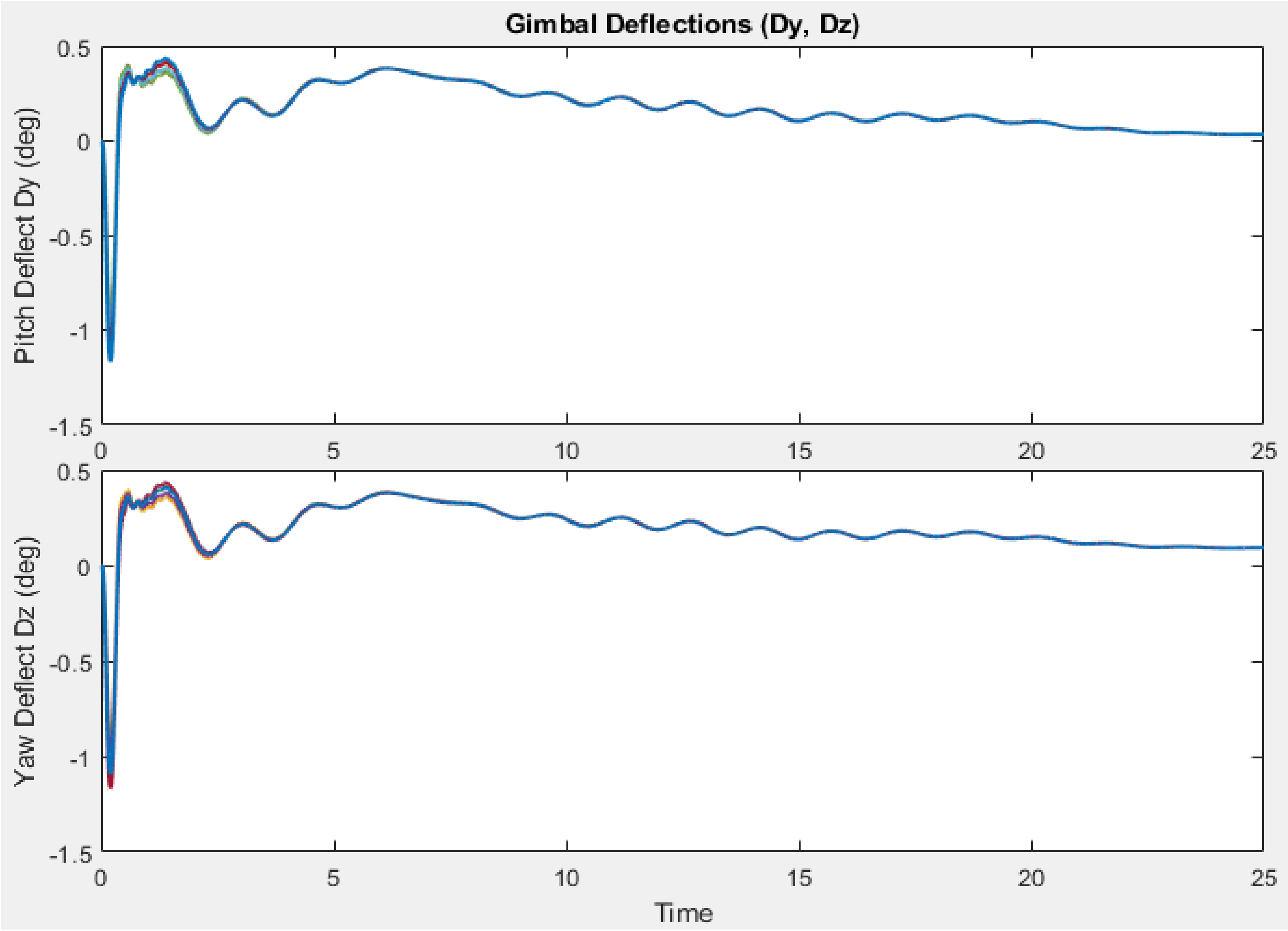
1	Roll Attitude (phi-body) (radians)
2	Roll Rate (p-body) (rad/sec)
3	Pitch Attitude (thet-bdy) (radians)
4	Pitch Rate (q-body) (rad/sec)
5	Yaw Attitude (psi-body) (radians)
6	Yaw Rate (r-body) (rad/sec)
7	Angle of attack, alfa, (radians)
8	Angle of sideslip, beta, (radian)
9	Change in Altitude, delta-h, (feet)
10	Forward Acceleration (V-dot) (ft/sec)
11	Cross Range Velocity (Vcr) (ft/sec)
12	Rate-Gyro # 1, Roll Rate (Body) (rad/sec)
13	Rate-Gyro # 2, Pitch Rate (Body) (rad/sec)
14	Rate-Gyro # 3, Yaw Rate (Body) (rad/sec)
15	Accelerom # 1, (along Y), (ft/sec^2) Translat. Accelerat.
16	Accelerom # 2, (along Z), (ft/sec^2) Translat. Accelerat.
17	Ptch Load-Torque Tly for Engine: 1,(ft-lb)
18	Ptch Load-Torque Tly for Engine: 2,(ft-lb)
19	Ptch Load-Torque Tly for Engine: 3,(ft-lb)
20	Ptch Load-Torque Tly for Engine: 4,(ft-lb)
21	Ptch Load-Torque Tly for Engine: 5,(ft-lb)
22	Ptch Load-Torque Tly for Engine: 6,(ft-lb)
23	Ptch Load-Torque Tly for Engine: 7,(ft-lb)
24	Ptch Load-Torque Tly for Engine: 8,(ft-lb)
25	Yaw Load-Torque Tlz for Engine: 1,(ft-lb)
26	Yaw Load-Torque Tlz for Engine: 2,(ft-lb)
27	Yaw Load-Torque Tlz for Engine: 3,(ft-lb)
28	Yaw Load-Torque Tlz for Engine: 4,(ft-lb)
29	Yaw Load-Torque Tlz for Engine: 5,(ft-lb)
30	Yaw Load-Torque Tlz for Engine: 6,(ft-lb)
31	Yaw Load-Torque Tlz for Engine: 7,(ft-lb)
32	Yaw Load-Torque Tlz for Engine: 8,(ft-lb)

Figure 27 Time-Slice Simulation Model “Simulat_2.slx”

Step Attitude Command:

Figures 28 show the response of the system to simultaneously applied unit step attitude commands in roll, pitch and yaw. Pitch and yaw attitudes converge slowly towards the commanded values because they are being prevented by a strong load-relief feedback from the angles of attack and sideslip. Slosh is excited by the maneuver but it is phase-stable and it gets damped out fast by gimbaling.


Vehicle Attitude
phi
theta
psi
(deg)
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
5
10
15
20
25


Gimbal Deflections (Dy, Dz)
Pitch Deflect Dy (deg)
0.5
0
-0.5
-1
-1.5
0
5
10
15
20
25
Yaw Deflect Dz (deg)
0.5
0
-0.5
-1
-1.5
0
5
10
15
20
25
Time

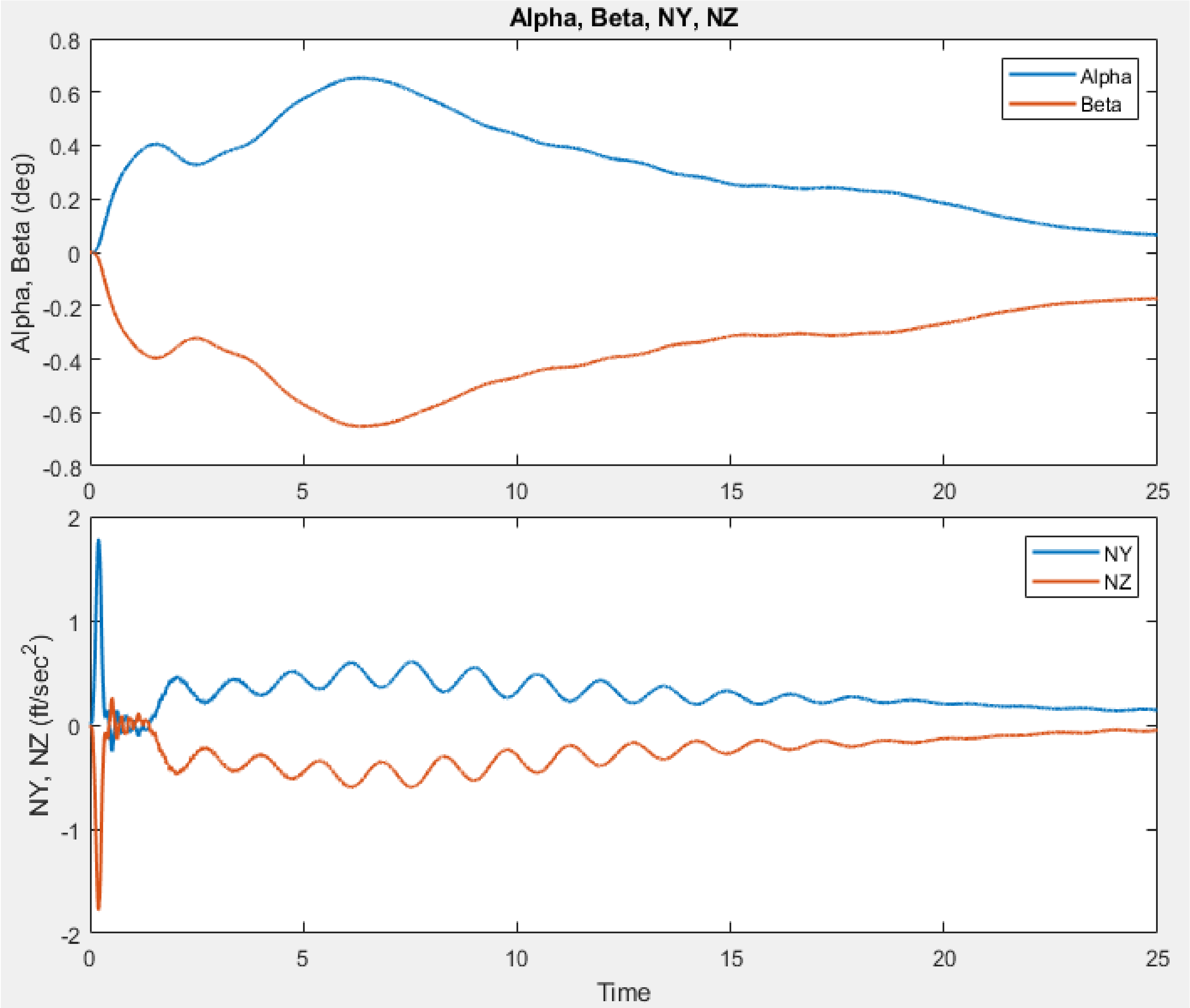


Figure 28 Response to Unit Step Attitude Command in Roll, Pitch and Yaw Together

Response to Sinusoidal Wind Disturbances: Figures 29 shows the effect of the (α,β)-filter to counteract external disturbances. The system is excited with an oscillatory wind disturbance consisting of frequencies between 0.65 to 0.85 r/s. The left side shows the response of the system without the filter and the right side shows the response of the system to the same disturbance with the filter included. The alpha loading without the filter is 5 times greater than what it is with the filter. Also, the gimbal deflections are almost twice as big without the filter. Notice also how the vehicle oscillates at twice higher rate and amplitude with the filter, even though α & β are smaller. It is because the system is maneuvering more by turning towards the oscillatory wind and thus reducing α & β. This however requires fast control and the limiting factors are the actuator bandwidth and the first flex mode.

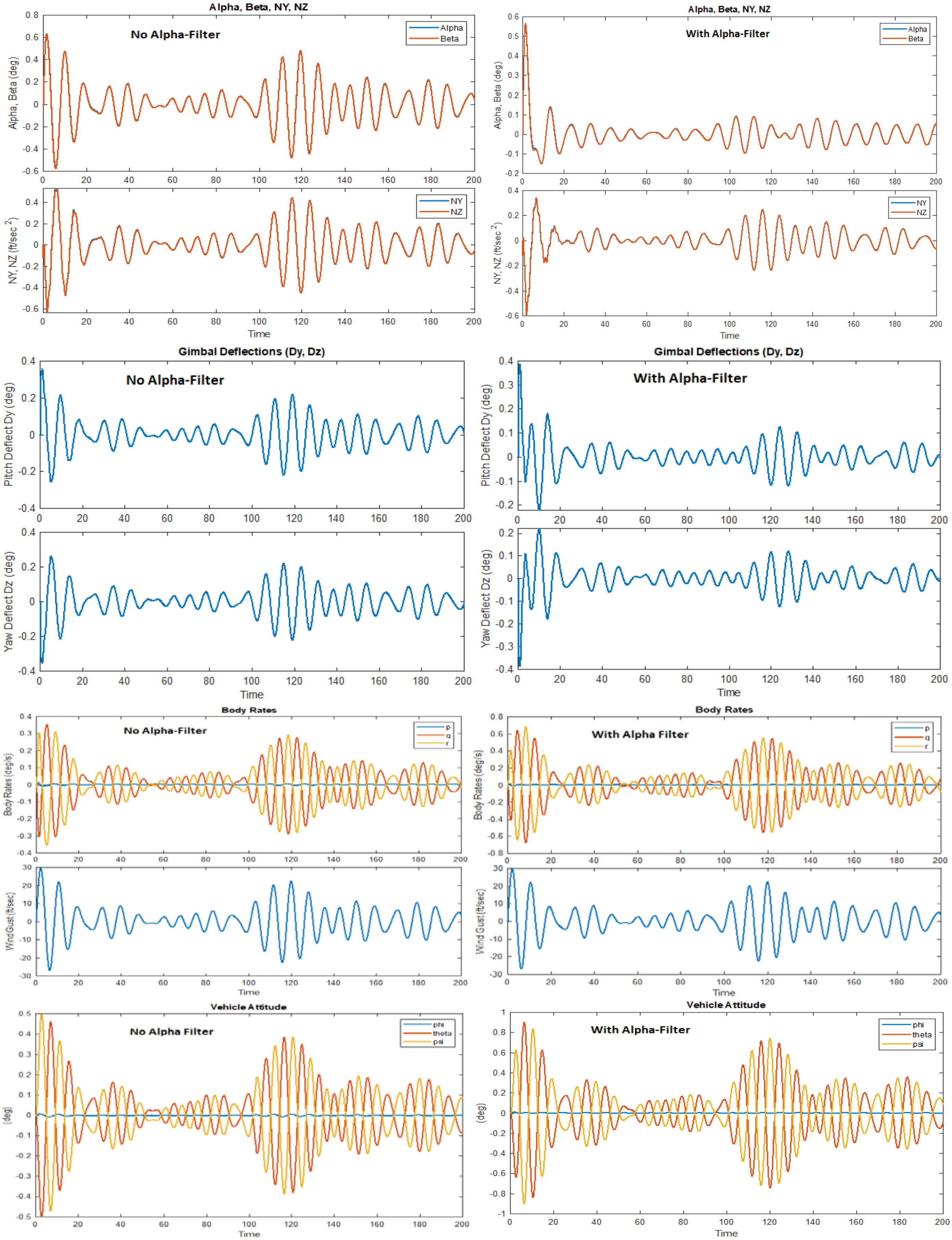
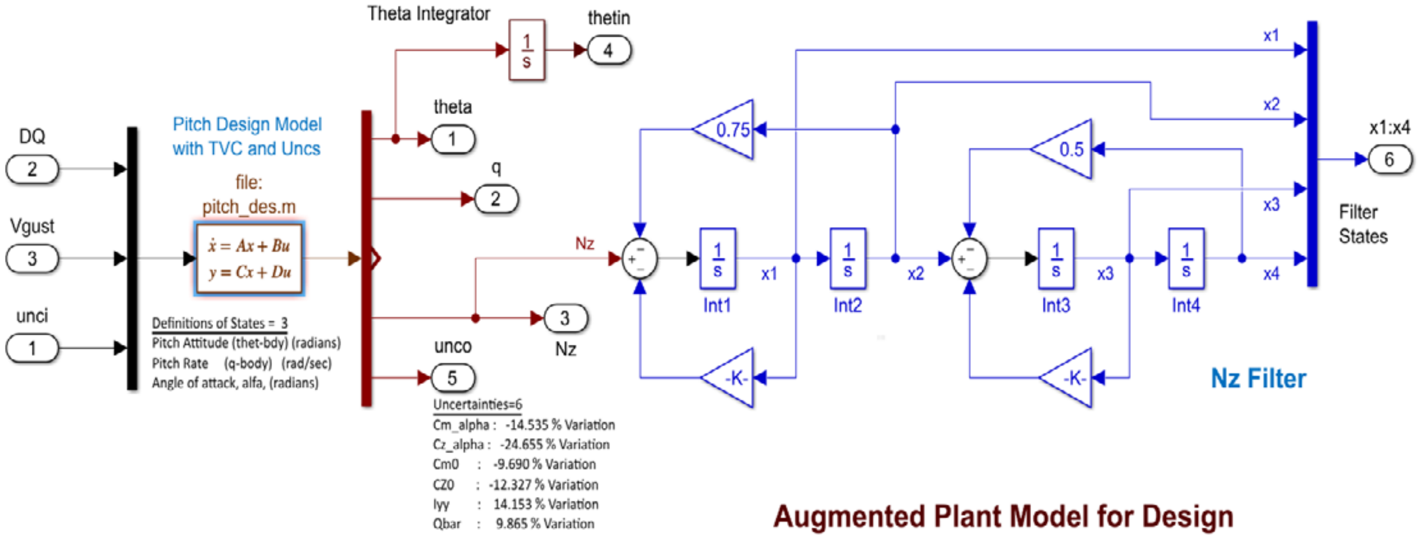


Figure 29 System Response to an Oscillatory Wind Disturbance, With and Without Filter

3.0 Rigid Body Design Using Accelerometer Feedback

In Section-1 we presented a state-feedback control design using attitude, rate and angle of attack. The alpha and beta states, however, are not always directly measurable and the nearest variables that can be used for load-relief feedback are the normal and lateral accelerometer measurements (Nz & Ny). The problem is that the Nz and Ny measurements are not members of the state-vector, and therefore, we cannot apply state-feedback in this case and we will need a state estimator. The input file in this example is "*LV_Estimator.Inp*", and it is located in directory "*Flixan\Control Analysis\Hinfinity\Examples\Launch Vehicle Design & Analysis\ 3-LV Gust Robust Design, Uncs, Output-Feedback*". The rigid vehicle model is "*First Stage Analysis Model, T=85.0 sec*", the same as in Section-1.



Augmented Plant Model for Design

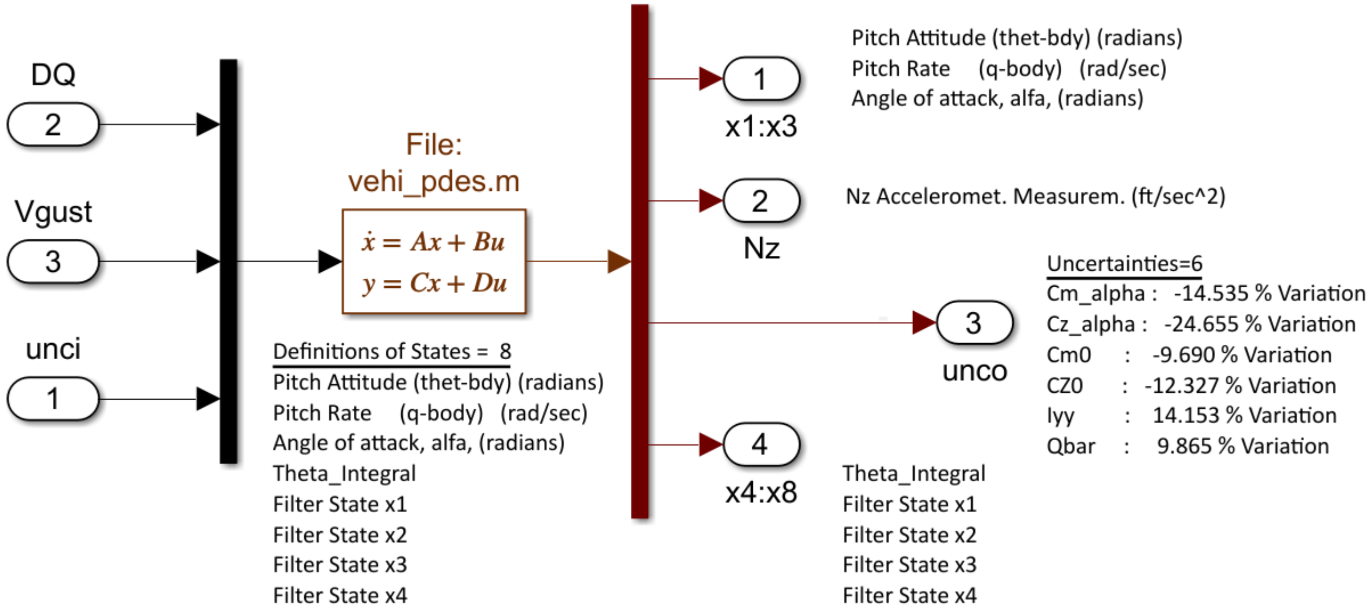


Figure 30 The Augmented Design System "Pitch H-inf Design Model with TVC and Nz-Filter" Includes the 4th Order Filter and Theta Integrator, Shown as a Combined State-Space System Below.

3.1 Creating the Design System and the Control Synthesis Model for H-Infinity

We begin by creating the pitch design system and augmenting it by adding the θ-integrator, and the 4th order filter, as we did in Section-1. However, the input to the filter now is the Nz acceleration, as shown in Figure 30a, and we call it "Nz-Filter". The filter amplifies the accelerometer measurement Nz in the frequency range of the disturbance: 0.6 to 0.9 (rad/sec) and the θ-integral improves the attitude tracking.

The augmented vehicle design system includes the TVC and it has 8 states: pitch attitude θ, body rate q, angle of attack α, θ-integral, and the 4 filer states. The normal acceleration Nz is included in the outputs. Similar to Section-1 the inputs and outputs include 6 i/o connections with the uncertainty block Δ. The augmented design system is shown combined in Figure-30b which includes the Flixan generated system title: "*Pitch Hinf Design Model with TVC and NZ-Filter*". This system will be used to create the Synthesis Model that will be used by the H-infinity algorithm. It is an interactive process of manually selecting the control and disturbance inputs and the criteria and measurement outputs and placing them in categories, as we shall see.

The SM is saved in the systems file "*LV_Estimator.Qdr*". Its title is "*Pitch Hinf Design Model with TVC and NZ-Filter/SM-3*". Figure-31 shows the Synthesis Model P(s) in block diagram form. It has 3 sets of input/ output vector pairs. The first set (w_p and z_p) are fictitious inputs and outputs that connect to the plant uncertainties that have been extracted to a block Δ. The B_1 and C_1 matrices of P(s) have already been scaled to match with the Δ block where each uncertainty element δ_i varies between ±1. The inputs w_p are treated as disturbances and the outputs z_p are included in the optimization criteria. The second input/ output set (w and z) are the actual external disturbances and performance criteria outputs. They are the wind-gust velocity (V_{gust}) and the angle of attack variation (α) respectively. The last set of I/O's are the measurement outputs (y_m) and the P(s) inputs (u_c) that connect with the dynamic control system K(s).

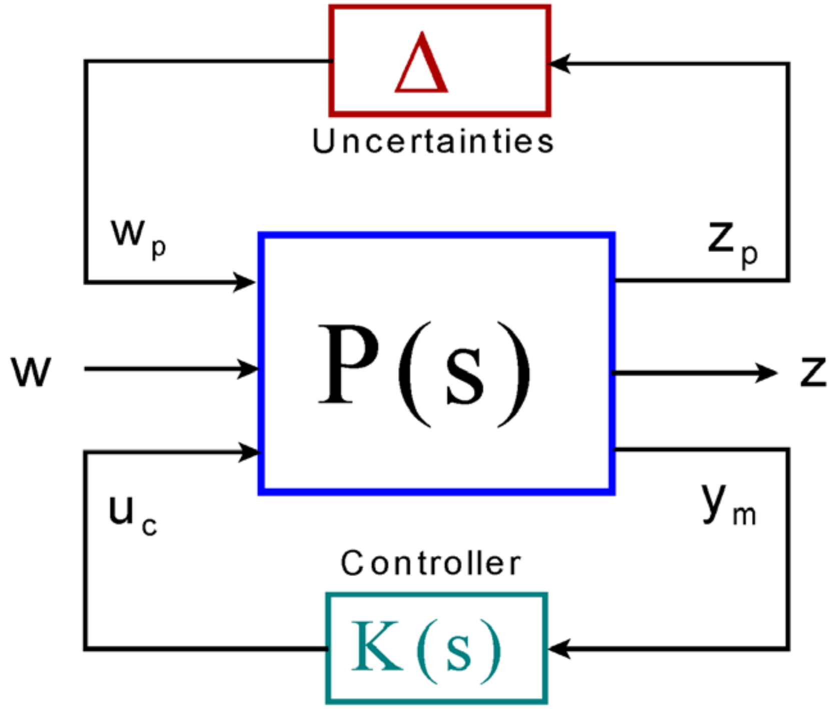
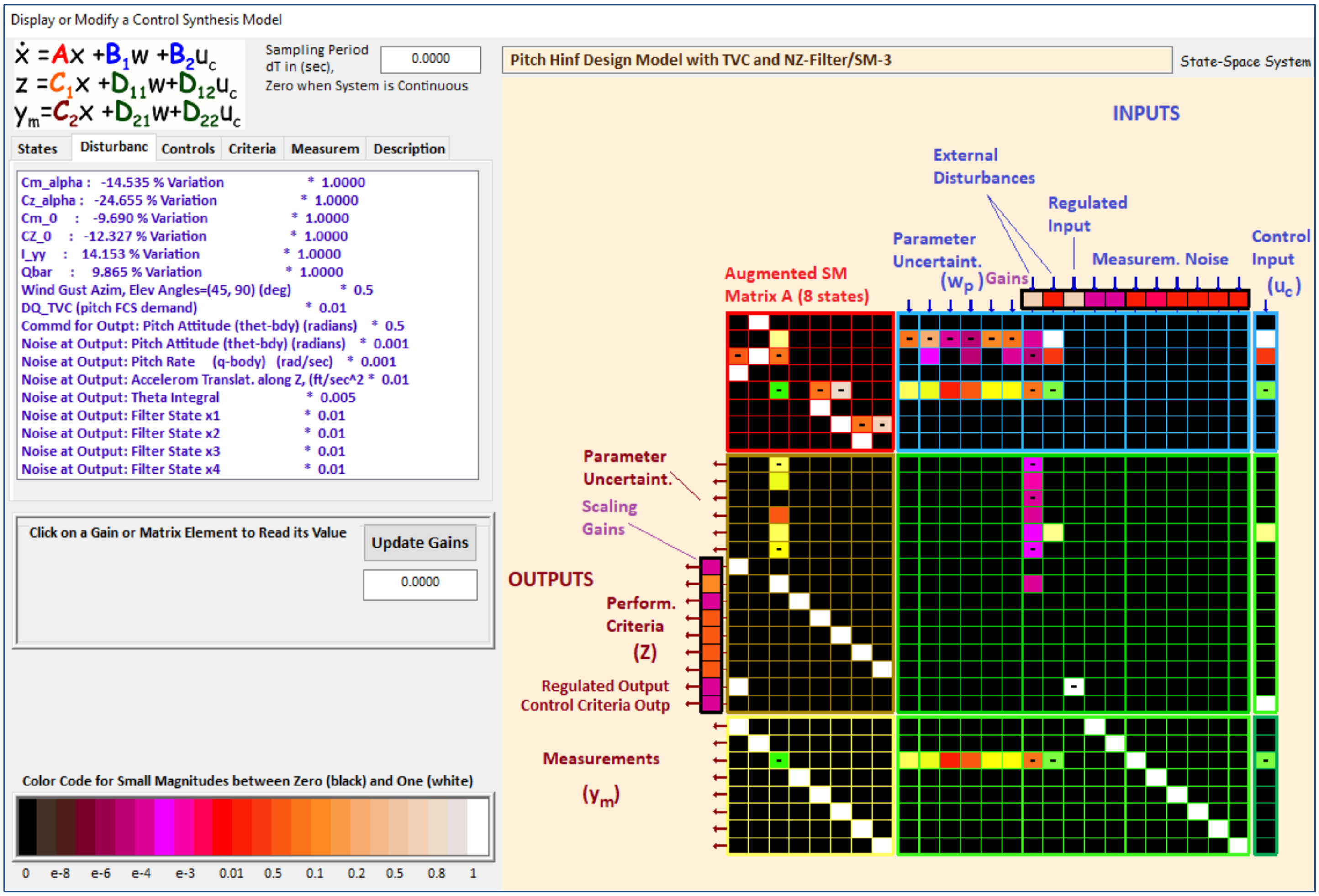


Figure 31 Synthesis Model

The SM used in the H-infinity design is shown in detail in Figure-32. It has 8 states, and it consists of 9 matrices and two sets of performance optimization gains. The gains are adjusted to satisfy control system bandwidth, stability, robustness and sensitivity to disturbances. In this design the measurements noise gains are not zero. The measurement noise in Section-1 was zero because we used state feedback. The matrix C_2 is no longer the identity matrix which requires a dynamic controller K(s) that uses output feedback instead of state feedback. The control system is saved in the systems file and its title is: "*H-Infin. Control for Launch Vehicle Output-Feedback System*".



Display or Modify a Control Synthesis Model

$$\dot{x} = Ax + B_1 w + B_2 u_c$$
$$z = C_1 x + D_{11} w + D_{12} u_c$$
$$y_m = C_2 x + D_{21} w + D_{22} u_c$$

Sampling Period dT in (sec), Zero when System is Continuous: 0.0000

Pitch Hinf Design Model with TVC and NZ-Filter/SM-3

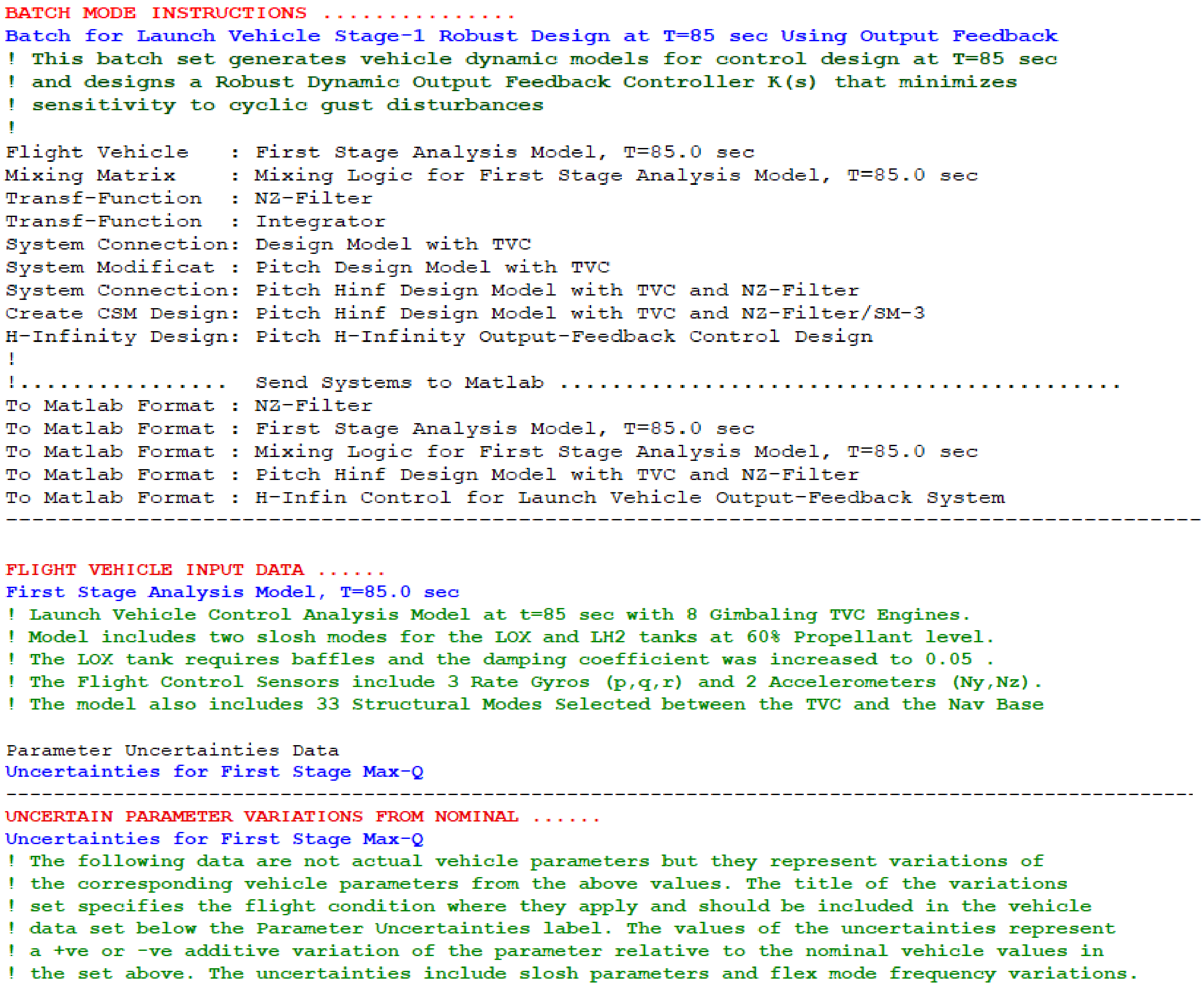
Cm_alpha :   -14.535 % Variation   * 1.0000
Cz_alpha :   -24.655 % Variation   * 1.0000
Cm_0   :   -9.690 % Variation   * 1.0000
CZ_0   :   -12.327 % Variation   * 1.0000
I_yy   :   14.153 % Variation   * 1.0000
Qbar   :   9.865 % Variation   * 1.0000
Wind Gust Azim, Elev Angles=(45, 90) (deg)   * 0.5
DQ_TVC (pitch FCS demand)   * 0.01
Commd for Outpt: Pitch Attitude (thet-bdy) (radians)   * 0.5
Noise at Output: Pitch Attitude (thet-bdy) (radians)   * 0.001
Noise at Output: Pitch Rate   (q-body)   (rad/sec)   * 0.001
Noise at Output: Accelerom Translat. along Z, (ft/sec^2 * 0.01
Noise at Output: Theta Integral   * 0.005
Noise at Output: Filter State x1   * 0.01
Noise at Output: Filter State x2   * 0.01
Noise at Output: Filter State x3   * 0.01
Noise at Output: Filter State x4   * 0.01

Click on a Gain or Matrix Element to Read its Value: 0.0000

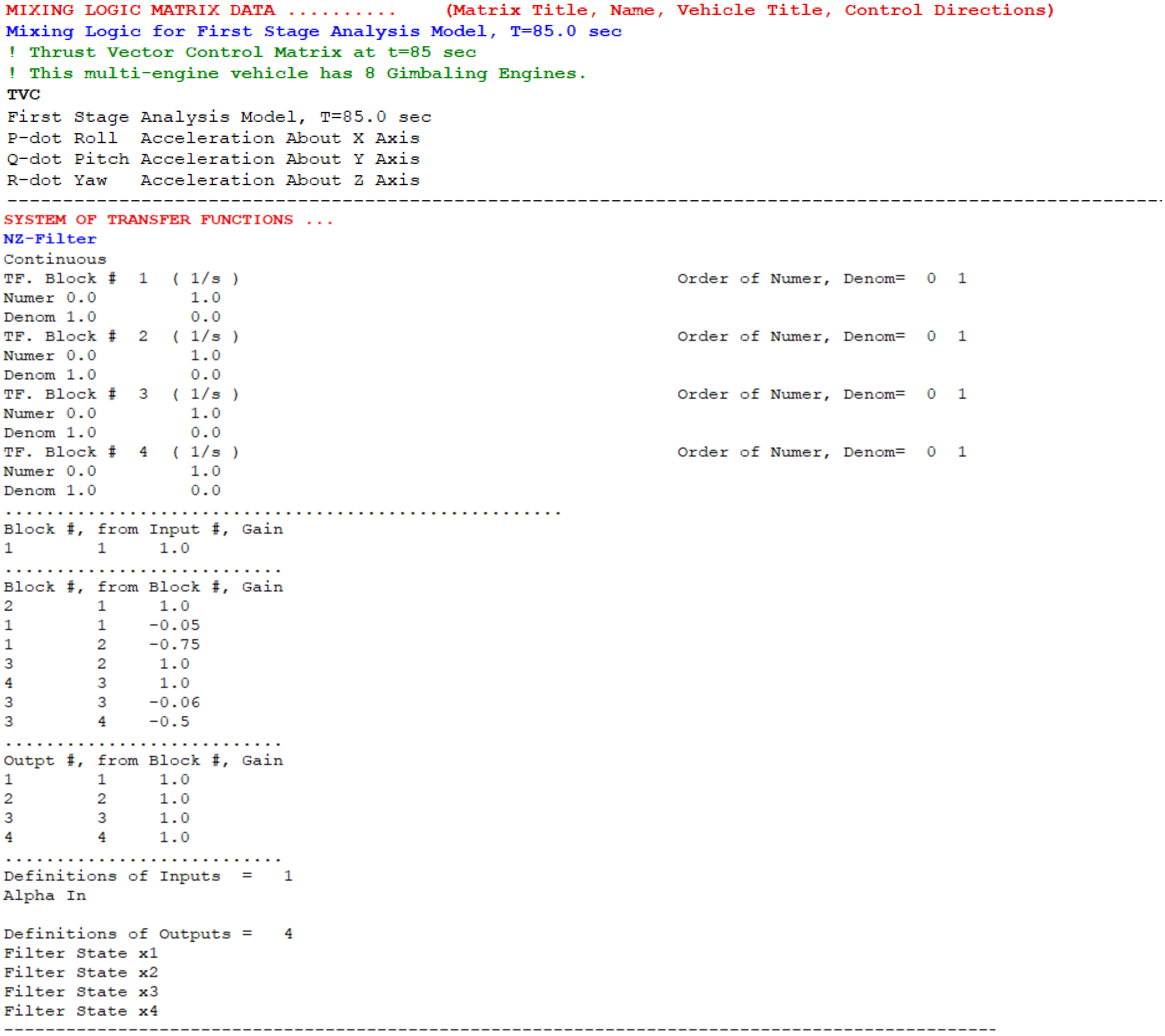
Figure 32 Synthesis Model in System Form with Performance Optimization Gains

3.2 Input File

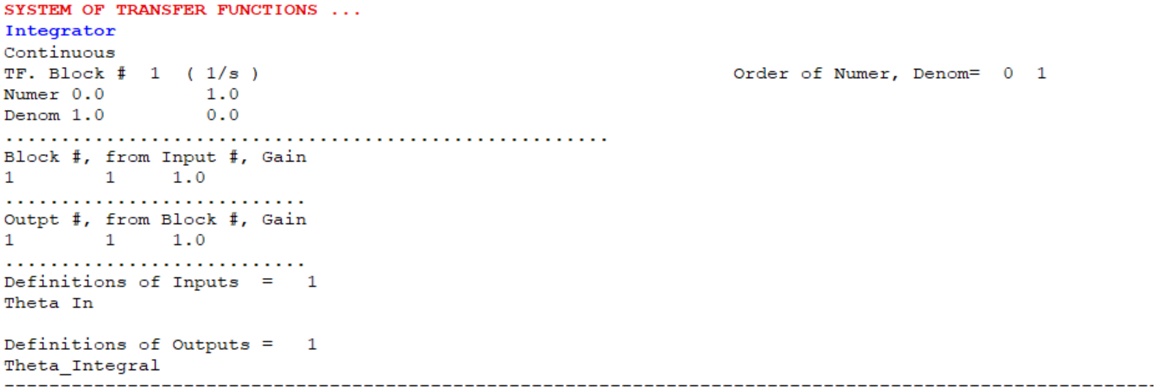
The input file in this design is "*LV_Estimator.Inp*". It contains the datasets used by the Flixan program to create the vehicle systems, matrices and the control system. It is similar to Section-1 file except that the accelerometer output is now used to drive the Nz-filter instead of alpha. The batch set can be used process the entire file in batch mode. There is a vehicle dataset "*First Stage Analysis Model, T=85.0 sec*" that generates the rigid vehicle system. A mixing logic dataset that creates the TVC matrix. The theta integrator and Nz-filter are implemented in transfer-function form and saved as state-space systems. The TVC matrix is then combined with the vehicle system to create the full axes system "*Design Model with TVC*". The pitch subsystem is extracted from the full system and saved in "*Pitch Design Model with TVC*". The pitch system is then augmented by including the Nz-filter and the θ-integrator, as shown in Figure-30a. Its title is "*Pitch H-inf Design Model with TVC and Nz-Filter*" also shown compact in Figure-30b. From this system we create the Synthesis Model: "*Pitch Hinf Design Model with TVC and NZ-Filter/SM-3*" by means of an interactive process by selecting inputs, outputs and gains as will be described in Section 3.3. The SM is also created in batch via the CSM creation dataset "*Pitch Hinf Design Model with TVC and NZ-Filter/SM-3*".



```
BATCH MODE INSTRUCTIONS ...............
Batch for Launch Vehicle Stage-1 Robust Design at T=85 sec Using Output Feedback
! This batch set generates vehicle dynamic models for control design at T=85 sec
! and designs a Robust Dynamic Output Feedback Controller K(s) that minimizes
! sensitivity to cyclic gust disturbances
!
Flight Vehicle    : First Stage Analysis Model, T=85.0 sec
Mixing Matrix     : Mixing Logic for First Stage Analysis Model, T=85.0 sec
Transf-Function   : NZ-Filter
Transf-Function   : Integrator
System Connection: Design Model with TVC
System Modificat : Pitch Design Model with TVC
System Connection: Pitch Hinf Design Model with TVC and NZ-Filter
Create CSM Design: Pitch Hinf Design Model with TVC and NZ-Filter/SM-3
H-Infinity Design: Pitch H-Infinity Output-Feedback Control Design
!
!................  Send Systems to Matlab ...........................................
To Matlab Format : NZ-Filter
To Matlab Format : First Stage Analysis Model, T=85.0 sec
To Matlab Format : Mixing Logic for First Stage Analysis Model, T=85.0 sec
To Matlab Format : Pitch Hinf Design Model with TVC and NZ-Filter
To Matlab Format : H-Infin Control for Launch Vehicle Output-Feedback System
------------------------------------------------------------------------------------------

FLIGHT VEHICLE INPUT DATA ......
First Stage Analysis Model, T=85.0 sec
! Launch Vehicle Control Analysis Model at t=85 sec with 8 Gimbaling TVC Engines.
! Model includes two slosh modes for the LOX and LH2 tanks at 60% Propellant level.
! The LOX tank requires baffles and the damping coefficient was increased to 0.05 .
! The Flight Control Sensors include 3 Rate Gyros (p,q,r) and 2 Accelerometers (Ny,Nz).
! The model also includes 33 Structural Modes Selected between the TVC and the Nav Base

Parameter Uncertainties Data
Uncertainties for First Stage Max-Q
------------------------------------------------------------------------------------------
UNCERTAIN PARAMETER VARIATIONS FROM NOMINAL ......
Uncertainties for First Stage Max-Q
! The following data are not actual vehicle parameters but they represent variations of
! the corresponding vehicle parameters from the above values. The title of the variations
! set specifies the flight condition where they apply and should be included in the vehicle
! data set below the Parameter Uncertainties label. The values of the uncertainties represent
! a +ve or -ve additive variation of the parameter relative to the nominal vehicle values in
! the set above. The uncertainties include slosh parameters and flex mode frequency variations.
```



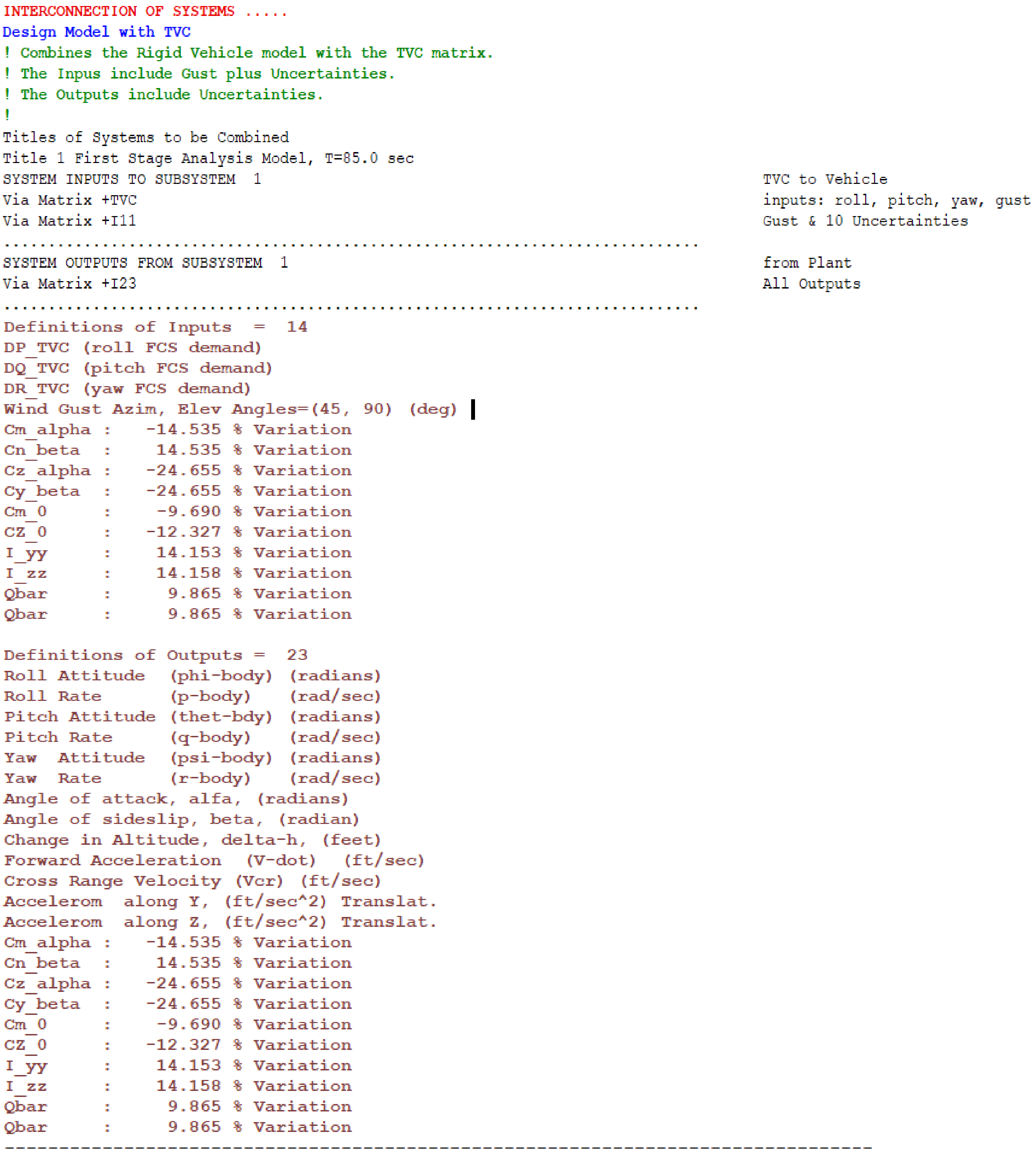
```
MIXING LOGIC MATRIX DATA ..........     (Matrix Title, Name, Vehicle Title, Control Directions)
Mixing Logic for First Stage Analysis Model, T=85.0 sec
! Thrust Vector Control Matrix at t=85 sec
! This multi-engine vehicle has 8 Gimbaling Engines.
TVC
First Stage Analysis Model, T=85.0 sec
P-dot Roll   Acceleration About X Axis
Q-dot Pitch Acceleration About Y Axis
R-dot Yaw    Acceleration About Z Axis
--------------------------------------------------------------------------------------------------------
SYSTEM OF TRANSFER FUNCTIONS ...
NZ-Filter
Continuous
TF. Block #   1   ( 1/s )                                        Order of Numer, Denom=   0   1
Numer 0.0          1.0
Denom 1.0          0.0
TF. Block #   2   ( 1/s )                                        Order of Numer, Denom=   0   1
Numer 0.0          1.0
Denom 1.0          0.0
TF. Block #   3   ( 1/s )                                        Order of Numer, Denom=   0   1
Numer 0.0          1.0
Denom 1.0          0.0
TF. Block #   4   ( 1/s )                                        Order of Numer, Denom=   0   1
Numer 0.0          1.0
Denom 1.0          0.0
.....................................................
Block #, from Input #, Gain
1        1     1.0
..........................
Block #, from Block #, Gain
2        1     1.0
1        1    -0.05
1        2    -0.75
3        2     1.0
4        3     1.0
3        3    -0.06
3        4    -0.5
..........................
Outpt #, from Block #, Gain
1        1     1.0
2        2     1.0
3        3     1.0
4        4     1.0
..........................
Definitions of Inputs  =    1
Alpha In

Definitions of Outputs =    4
Filter State x1
Filter State x2
Filter State x3
Filter State x4
-----------------------------------------------------------------------------------------------
```



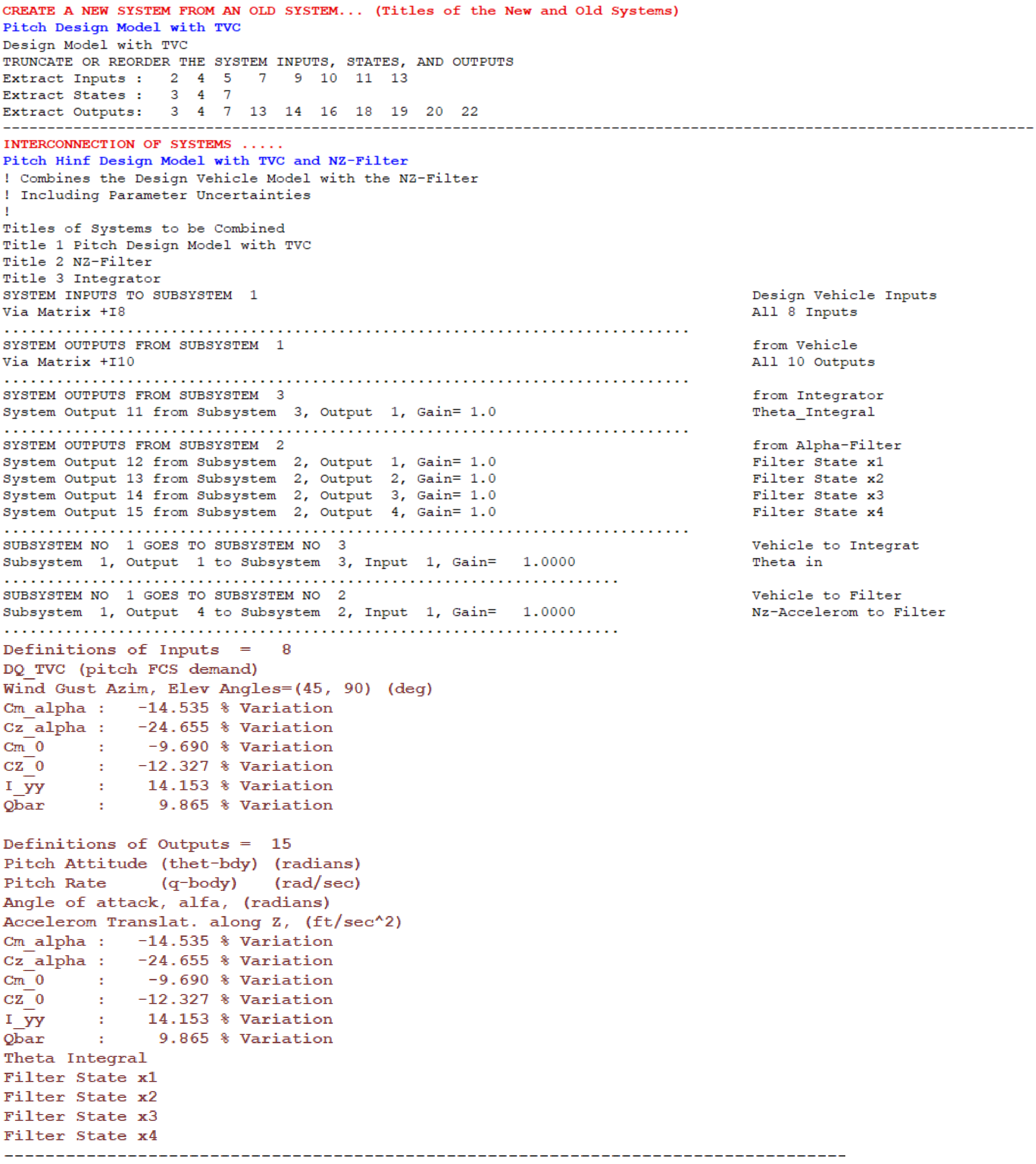
```
SYSTEM OF TRANSFER FUNCTIONS ...
Integrator
Continuous
TF. Block #   1   ( 1/s )                                        Order of Numer, Denom=   0   1
Numer 0.0          1.0
Denom 1.0          0.0
.....................................................
Block #, from Input #, Gain
1        1     1.0
..........................
Outpt #, from Block #, Gain
1        1     1.0
..........................
Definitions of Inputs  =    1
Theta In

Definitions of Outputs =    1
Theta_Integral
-----------------------------------------------------------------------------------------------
```

The Nz-filter above consists of two resonances which are excited from the Nz accelerometer output in order to amplify the frequencies between 0.6 to 0.9 (rad/sec) that is where the disturbances occur. The resonance states are optimized by the H-infinity algorithm which creates a control logic that attenuates excitations in that frequency range. The integrator is used to create the θ-integral state.

The interconnection dataset below combines the vehicle system with the TVC matrix to create a full axes system. The next dataset creates a reduced system “*Pitch Design Model with TVC*” by extracting only the pitch variables.



```
INTERCONNECTION OF SYSTEMS .....
Design Model with TVC
! Combines the Rigid Vehicle model with the TVC matrix.
! The Inpus include Gust plus Uncertainties.
! The Outputs include Uncertainties.
!
Titles of Systems to be Combined
Title 1 First Stage Analysis Model, T=85.0 sec
SYSTEM INPUTS TO SUBSYSTEM  1                                         TVC to Vehicle
Via Matrix +TVC                                                       inputs: roll, pitch, yaw, gust
Via Matrix +I11                                                       Gust & 10 Uncertainties
.............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                      from Plant
Via Matrix +I23                                                       All Outputs
.............................................................................
Definitions of Inputs  =  14
DP_TVC (roll FCS demand)
DQ_TVC (pitch FCS demand)
DR_TVC (yaw FCS demand)
Wind Gust Azim, Elev Angles=(45, 90) (deg)
Cm_alpha :   -14.535 % Variation
Cn_beta  :    14.535 % Variation
Cz_alpha :   -24.655 % Variation
Cy_beta  :   -24.655 % Variation
Cm_0     :    -9.690 % Variation
CZ_0     :   -12.327 % Variation
I_yy     :    14.153 % Variation
I_zz     :    14.158 % Variation
Qbar     :     9.865 % Variation
Qbar     :     9.865 % Variation

Definitions of Outputs =  23
Roll Attitude   (phi-body) (radians)
Roll Rate       (p-body)   (rad/sec)
Pitch Attitude  (thet-bdy) (radians)
Pitch Rate      (q-body)   (rad/sec)
Yaw  Attitude   (psi-body) (radians)
Yaw  Rate       (r-body)   (rad/sec)
Angle of attack, alfa, (radians)
Angle of sideslip, beta, (radian)
Change in Altitude, delta-h, (feet)
Forward Acceleration  (V-dot)  (ft/sec)
Cross Range Velocity (Vcr) (ft/sec)
Accelerom  along Y, (ft/sec^2) Translat.
Accelerom  along Z, (ft/sec^2) Translat.
Cm_alpha :   -14.535 % Variation
Cn_beta  :    14.535 % Variation
Cz_alpha :   -24.655 % Variation
Cy_beta  :   -24.655 % Variation
Cm_0     :    -9.690 % Variation
CZ_0     :   -12.327 % Variation
I_yy     :    14.153 % Variation
I_zz     :    14.158 % Variation
Qbar     :     9.865 % Variation
Qbar     :     9.865 % Variation
-------------------------------------------------------------------------------
```



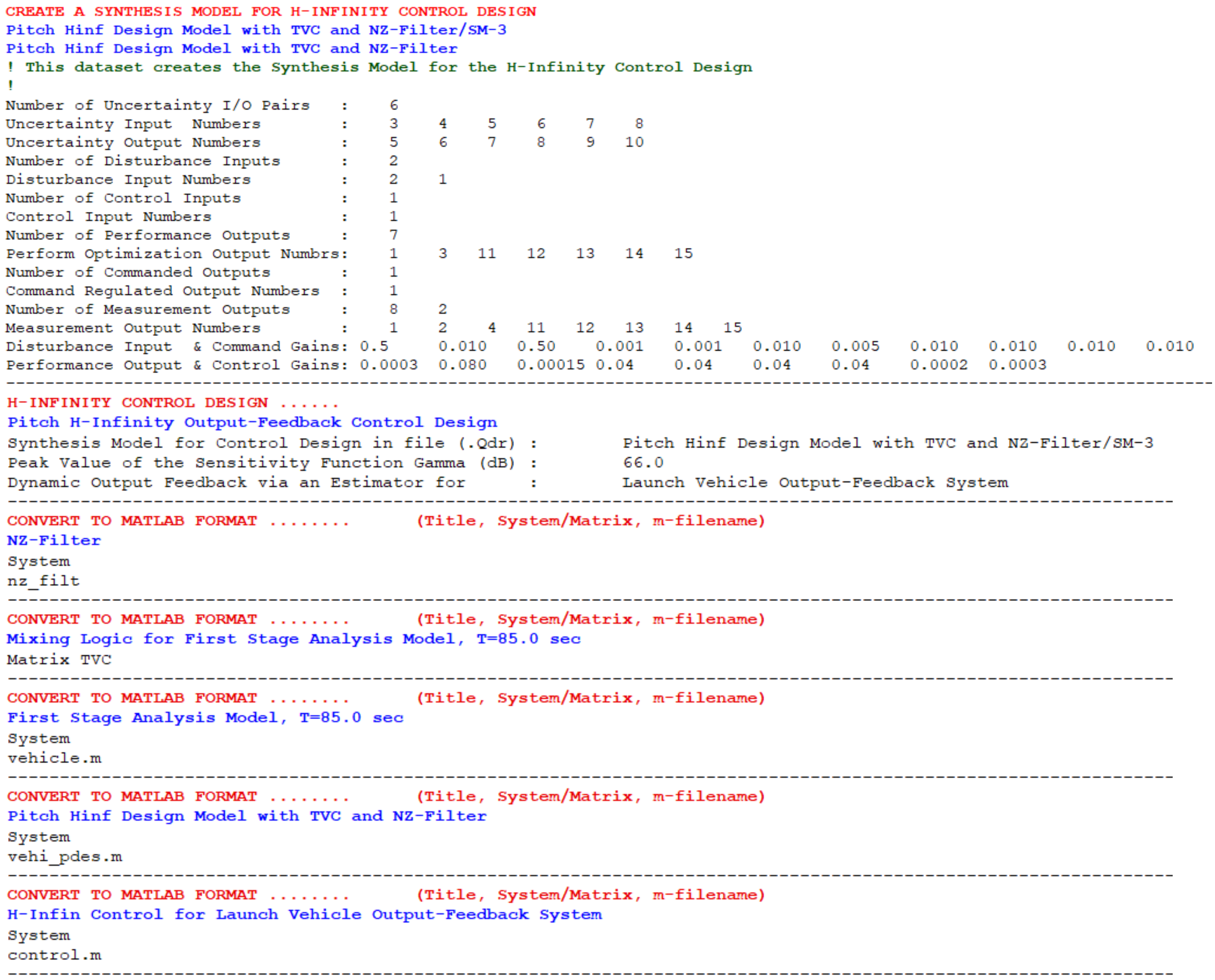
```
CREATE A NEW SYSTEM FROM AN OLD SYSTEM... (Titles of the New and Old Systems)
Pitch Design Model with TVC
Design Model with TVC
TRUNCATE OR REORDER THE SYSTEM INPUTS, STATES, AND OUTPUTS
Extract Inputs :   2  4  5   7   9  10  11  13
Extract States :   3  4  7
Extract Outputs:   3  4  7  13  14  16  18  19  20  22
-----------------------------------------------------------------------------------------------------------------------
INTERCONNECTION OF SYSTEMS .....
Pitch Hinf Design Model with TVC and NZ-Filter
! Combines the Design Vehicle Model with the NZ-Filter
! Including Parameter Uncertainties
!
Titles of Systems to be Combined
Title 1 Pitch Design Model with TVC
Title 2 NZ-Filter
Title 3 Integrator
SYSTEM INPUTS TO SUBSYSTEM  1                                                     Design Vehicle Inputs
Via Matrix +I8                                                                    All 8 Inputs
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                                  from Vehicle
Via Matrix +I10                                                                   All 10 Outputs
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  3                                                  from Integrator
System Output 11 from Subsystem  3, Output  1, Gain= 1.0                          Theta_Integral
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                                                  from Alpha-Filter
System Output 12 from Subsystem  2, Output  1, Gain= 1.0                          Filter State x1
System Output 13 from Subsystem  2, Output  2, Gain= 1.0                          Filter State x2
System Output 14 from Subsystem  2, Output  3, Gain= 1.0                          Filter State x3
System Output 15 from Subsystem  2, Output  4, Gain= 1.0                          Filter State x4
..............................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  3                                           Vehicle to Integrat
Subsystem  1, Output  1 to Subsystem  3, Input  1, Gain=   1.0000                 Theta in
......................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2                                           Vehicle to Filter
Subsystem  1, Output  4 to Subsystem  2, Input  1, Gain=   1.0000                 Nz-Accelerom to Filter
......................................................................
Definitions of Inputs  =   8
DQ_TVC (pitch FCS demand)
Wind Gust Azim, Elev Angles=(45, 90) (deg)
Cm_alpha :   -14.535 % Variation
Cz_alpha :   -24.655 % Variation
Cm_0     :    -9.690 % Variation
CZ_0     :   -12.327 % Variation
I_yy     :    14.153 % Variation
Qbar     :     9.865 % Variation

Definitions of Outputs =  15
Pitch Attitude (thet-bdy) (radians)
Pitch Rate     (q-body)   (rad/sec)
Angle of attack, alfa, (radians)
Accelerom Translat. along Z, (ft/sec^2)
Cm_alpha :   -14.535 % Variation
Cz_alpha :   -24.655 % Variation
Cm_0     :    -9.690 % Variation
CZ_0     :   -12.327 % Variation
I_yy     :    14.153 % Variation
Qbar     :     9.865 % Variation
Theta Integral
Filter State x1
Filter State x2
Filter State x3
Filter State x4
-------------------------------------------------------------------------------
```

This dataset above combines the pitch design model with the Nz-filter and the theta-integrator to create the augmented pitch design model that will be used to create the Synthesis Model.

The dataset below is used in batch mode in order to generate the SM from system "*Pitch Hinf Design Model with TVC and NZ-Filter*". It defines which inputs are controls and which ones are disturbances. Also, which outputs are measurements and which ones are criteria to be optimized by H-infinity. It includes also the performance adjustment gains. The SM is saved in the systems file "*LV_Estimator.Qdr*" under the title "*Pitch Hinf Design Model with TVC and NZ-Filter/SM-3*".

The next dataset performs the H-Infinity design. Its title is: "*Pitch H-Infinity Output-Feedback Control Design*" and it is using the SM "*Pitch Hinf Design Model with TVC and NZ-Filter/SM-3*" which is already in the systems file: "LV_Estimator.Qdr". It creates the H-infinity dynamic control system "*H-Infin. Control for Launch Vehicle Output-Feedback System*" which is also saved in the systems file. The upper bound gamma parameter is set to γ= 66 (dB). Values smaller than γ=64 (dB) violate the algorithm criteria.

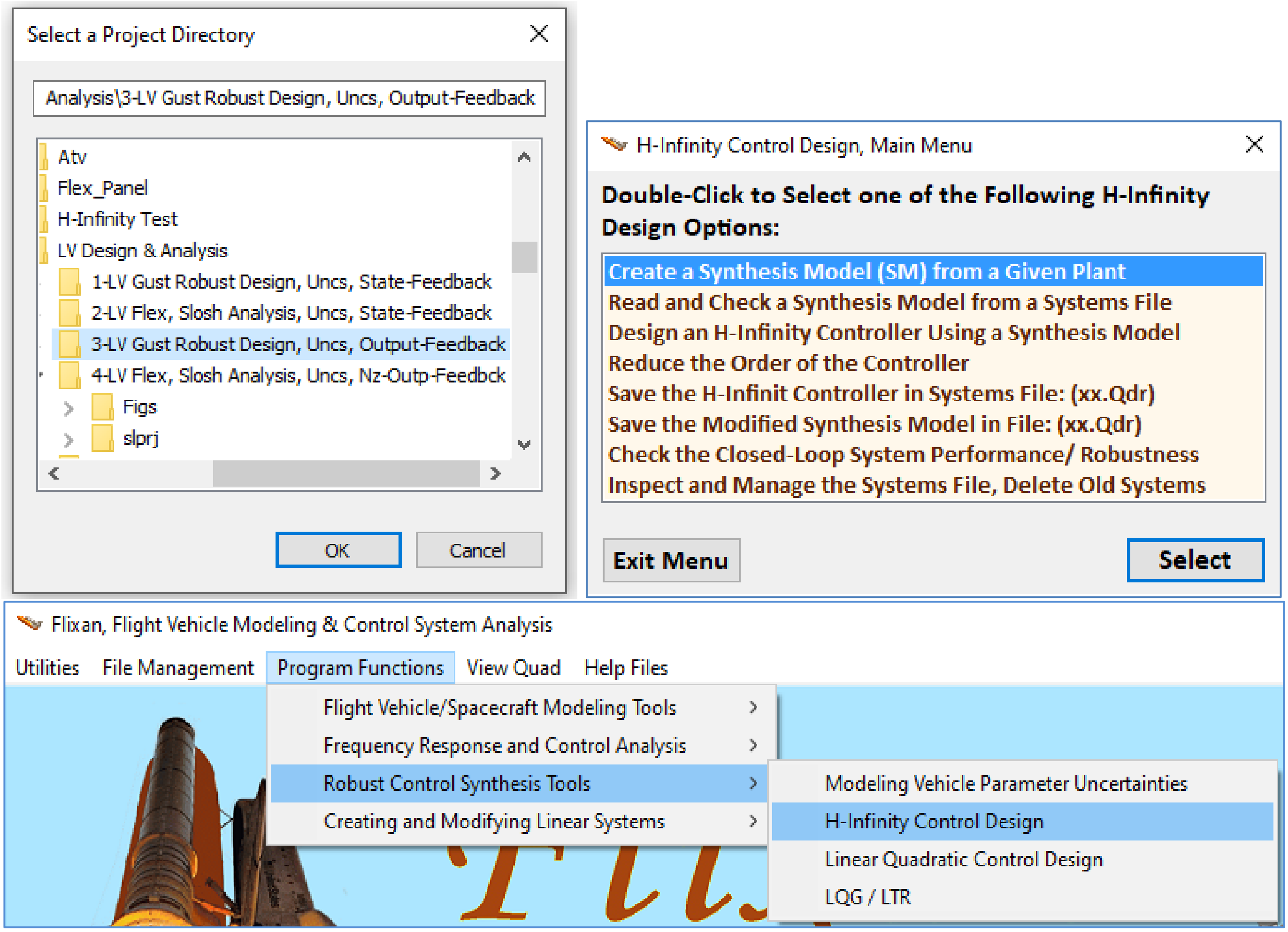
The following Matlab conversion datasets convert the Flixan generated systems and matrices from file "LV_Estimator.Qdr" to m-files and mat-files that can be loaded into Matlab for control analysis.

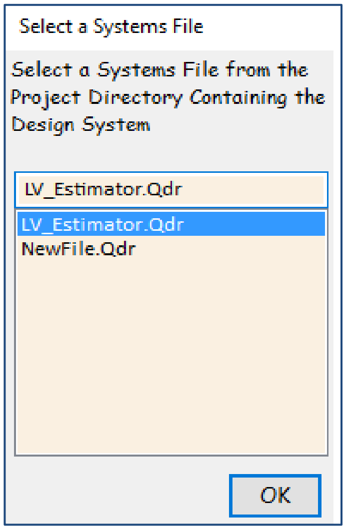


```
CREATE A SYNTHESIS MODEL FOR H-INFINITY CONTROL DESIGN
Pitch Hinf Design Model with TVC and NZ-Filter/SM-3
Pitch Hinf Design Model with TVC and NZ-Filter
! This dataset creates the Synthesis Model for the H-Infinity Control Design
!
Number of Uncertainty I/O Pairs   :    6
Uncertainty Input  Numbers        :    3    4    5    6    7    8
Uncertainty Output Numbers        :    5    6    7    8    9   10
Number of Disturbance Inputs      :    2
Disturbance Input Numbers         :    2    1
Number of Control Inputs          :    1
Control Input Numbers             :    1
Number of Performance Outputs     :    7
Perform Optimization Output Numbrs:    1    3   11   12   13   14   15
Number of Commanded Outputs       :    1
Command Regulated Output Numbers  :    1
Number of Measurement Outputs     :    8    2
Measurement Output Numbers        :    1    2    4   11   12   13   14   15
Disturbance Input  & Command Gains: 0.5     0.010   0.50    0.001   0.001   0.010   0.005   0.010   0.010   0.010   0.010
Performance Output & Control Gains: 0.0003  0.080   0.00015 0.04    0.04    0.04    0.04    0.0002  0.0003
------------------------------------------------------------------------------------------------------------------------
H-INFINITY CONTROL DESIGN ......
Pitch H-Infinity Output-Feedback Control Design
Synthesis Model for Control Design in file (.Qdr) :       Pitch Hinf Design Model with TVC and NZ-Filter/SM-3
Peak Value of the Sensitivity Function Gamma (dB) :       66.0
Dynamic Output Feedback via an Estimator for      :       Launch Vehicle Output-Feedback System
------------------------------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
NZ-Filter
System
nz_filt
------------------------------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Mixing Logic for First Stage Analysis Model, T=85.0 sec
Matrix TVC
------------------------------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
First Stage Analysis Model, T=85.0 sec
System
vehicle.m
------------------------------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Pitch Hinf Design Model with TVC and NZ-Filter
System
vehi_pdes.m
------------------------------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
H-Infin Control for Launch Vehicle Output-Feedback System
System
control.m
------------------------------------------------------------------------------------------------------------------------
```

3.3 Creating the Synthesis Model Interactively

We will now show how to generate the Synthesis Model in the systems file interactively. A CSM creation dataset is also saved in the input file that can be used to recreate the SM later in batch mode. The SM consists of 9 matrices and it includes vehicle dynamics and performance optimization parameters. It separates the inputs into controls and disturbances, and some of the inputs can be both. It also separates the outputs into measurements and optimization criteria, and some outputs can also be both. The performance adjustment gains define the trade-off between performance, sensitivity, robustness and stability margins that must be optimized by the H-infinity algorithm. The program uses interactive menus where the user defines which inputs are disturbances and which ones are controls. Also, which outputs are criteria to be optimized and which ones are measurements. This example is slightly different from the one in Section-1 because some of the measurements are not in the state-vector and we will require a state estimator. We begin by running the H-infinity design program and selecting the first option to create the SM. Use the menus to select the systems filename and the augmented design system.

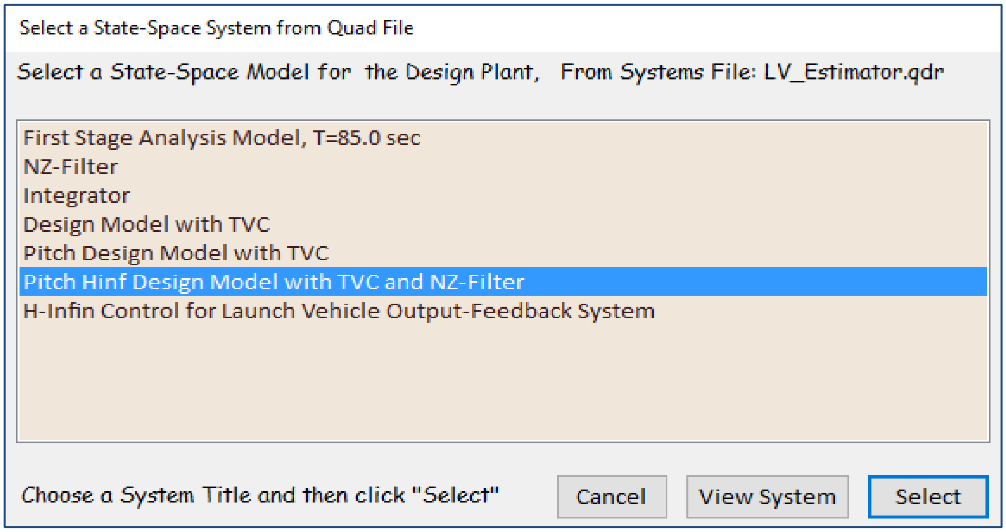



Select a Systems File

Select a Systems File from the Project Directory Containing the Design System

LV_Estimator.Qdr

LV_Estimator.Qdr
NewFile.Qdr

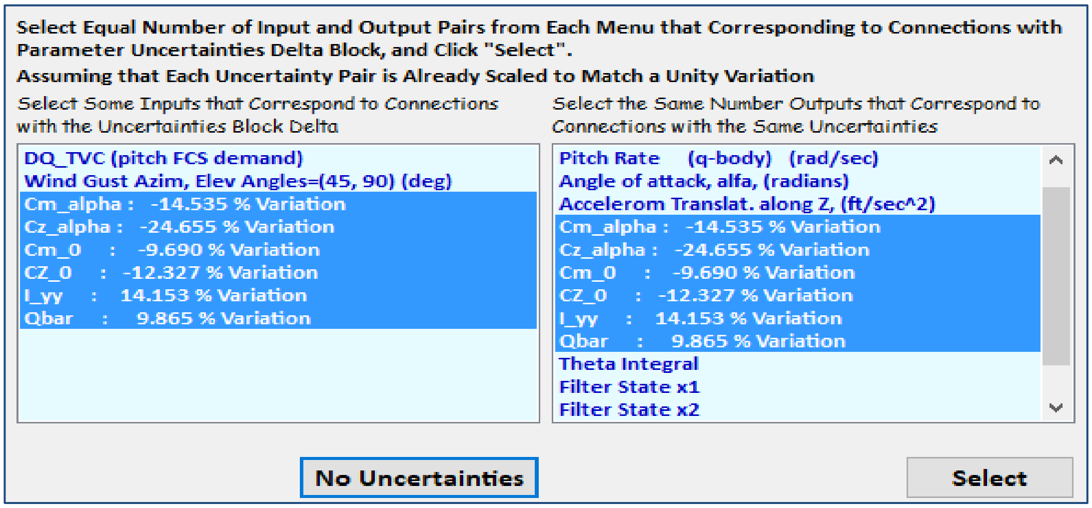

Select a State-Space System from Quad File

Select a State-Space Model for the Design Plant, From Systems File: LV_Estimator.qdr

First Stage Analysis Model, T=85.0 sec
NZ-Filter
Integrator
Design Model with TVC
Pitch Design Model with TVC
Pitch Hinf Design Model with TVC and NZ-Filter
H-Infin Control for Launch Vehicle Output-Feedback System

Choose a System Title and then click "Select"

The SM will be created from the augmented design system “*Pitch Hinf Design Model with TVC and NZ-Filter*”, by selecting and grouping some inputs and outputs using menus. In the next dialog use the left and right menus to select the 6 parameter variation pairs that connect with the uncertainty Δ block. They will be treated like disturbances and criteria outputs as described in the IFL method. Click on “Select”.


Select Equal Number of Input and Output Pairs from Each Menu that Corresponding to Connections with Parameter Uncertainties Delta Block, and Click "Select".
Assuming that Each Uncertainty Pair is Already Scaled to Match a Unity Variation

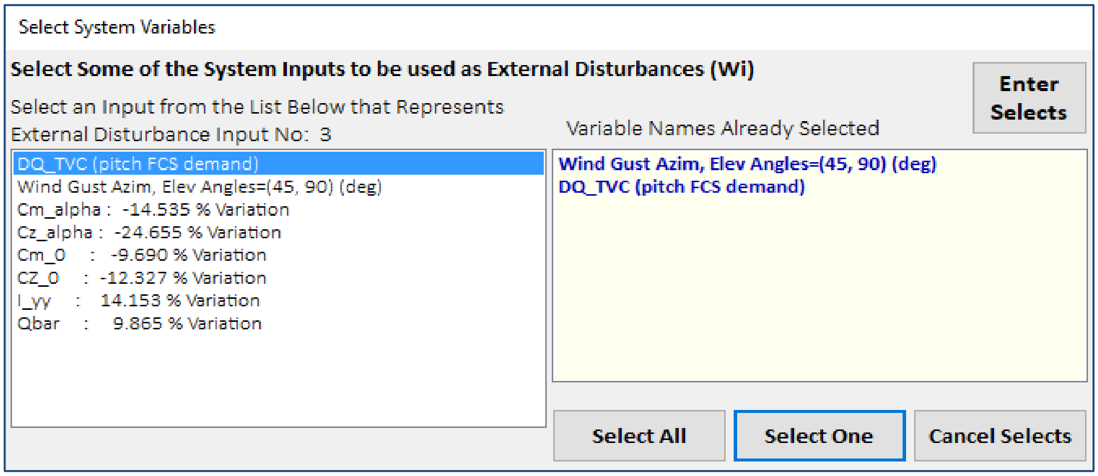
Select Some Inputs that Correspond to Connections with the Uncertainties Block Delta

DQ_TVC (pitch FCS demand)
Wind Gust Azim, Elev Angles=(45, 90) (deg)
Cm_alpha : -14.535 % Variation
Cz_alpha : -24.655 % Variation
Cm_0 : -9.690 % Variation
CZ_0 : -12.327 % Variation
I_yy : 14.153 % Variation
Qbar : 9.865 % Variation

Select the Same Number Outputs that Correspond to Connections with the Same Uncertainties

Pitch Rate (q-body) (rad/sec)
Angle of attack, alfa, (radians)
Accelerom Translat. along Z, (ft/sec^2)
Cm_alpha : -14.535 % Variation
Cz_alpha : -24.655 % Variation
Cm_0 : -9.690 % Variation
CZ_0 : -12.327 % Variation
I_yy : 14.153 % Variation
Qbar : 9.865 % Variation
Theta Integral
Filter State x1
Filter State x2

The next menu is for defining external disturbance inputs. The first two inputs are selected as disturbances. That is, the wind-gust velocity and noise at the control input DQ_tvc. Select one at a time, and click on “Enter Selects” to continue.


Select System Variables

Select Some of the System Inputs to be used as External Disturbances (Wi)

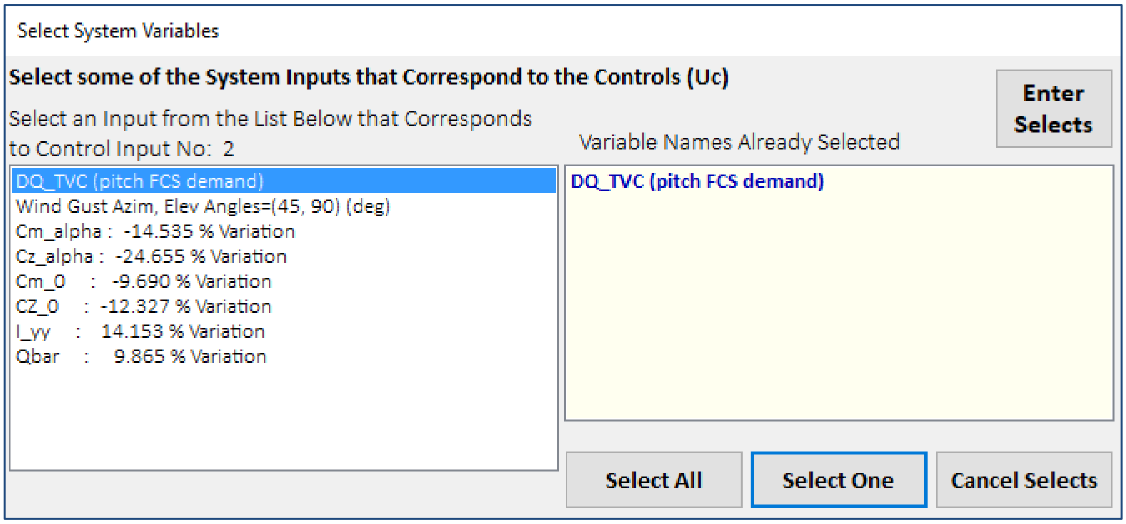
Select an Input from the List Below that Represents External Disturbance Input No: 3

DQ_TVC (pitch FCS demand)
Wind Gust Azim, Elev Angles=(45, 90) (deg)
Cm_alpha : -14.535 % Variation
Cz_alpha : -24.655 % Variation
Cm_0 : -9.690 % Variation
CZ_0 : -12.327 % Variation
I_yy : 14.153 % Variation
Qbar : 9.865 % Variation

Variable Names Already Selected

Wind Gust Azim, Elev Angles=(45, 90) (deg)
DQ_TVC (pitch FCS demand)

The next menu is for selecting the control inputs. There is only one control input, the pitch demand dQ to the TVC matrix. Select it and then click on “Enter Selects” to continue.



Select System Variables

Select some of the System Inputs that Correspond to the Controls (Uc)

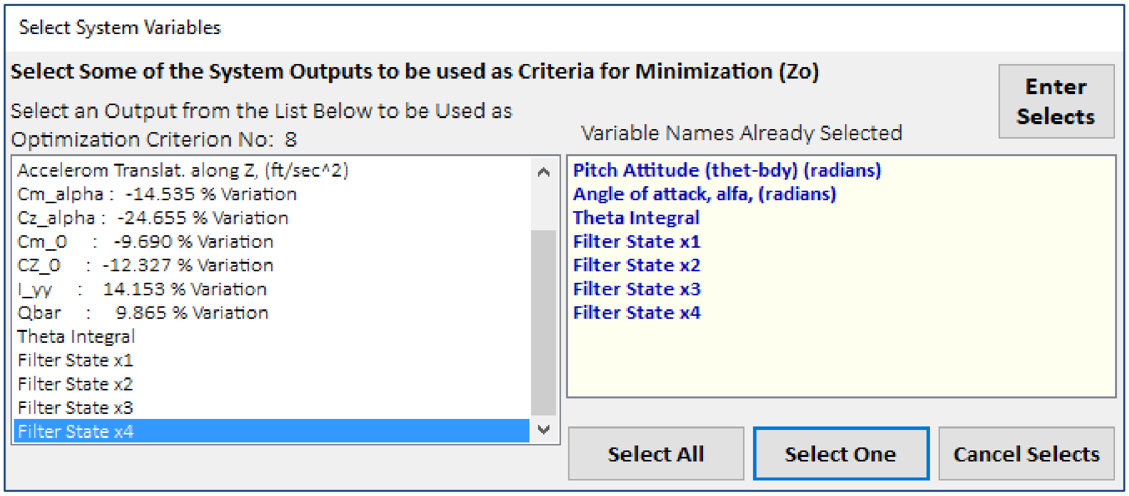
Select an Input from the List Below that Corresponds to Control Input No: 2

DQ_TVC (pitch FCS demand)
Wind Gust Azim, Elev Angles=(45, 90) (deg)
Cm_alpha : -14.535 % Variation
Cz_alpha : -24.655 % Variation
Cm_0 : -9.690 % Variation
CZ_0 : -12.327 % Variation
I_yy : 14.153 % Variation
Qbar : 9.865 % Variation

Variable Names Already Selected

DQ_TVC (pitch FCS demand)

The design system has several outputs. We will include only 7 outputs to be optimized which are also state variables. That is, the pitch attitude θ, the angle of attack α, θ-integral, and the four filter states, x1 to x4. Select one at a time and then click on “Enter Selects” to continue.



Select System Variables

Select Some of the System Outputs to be used as Criteria for Minimization (Zo)

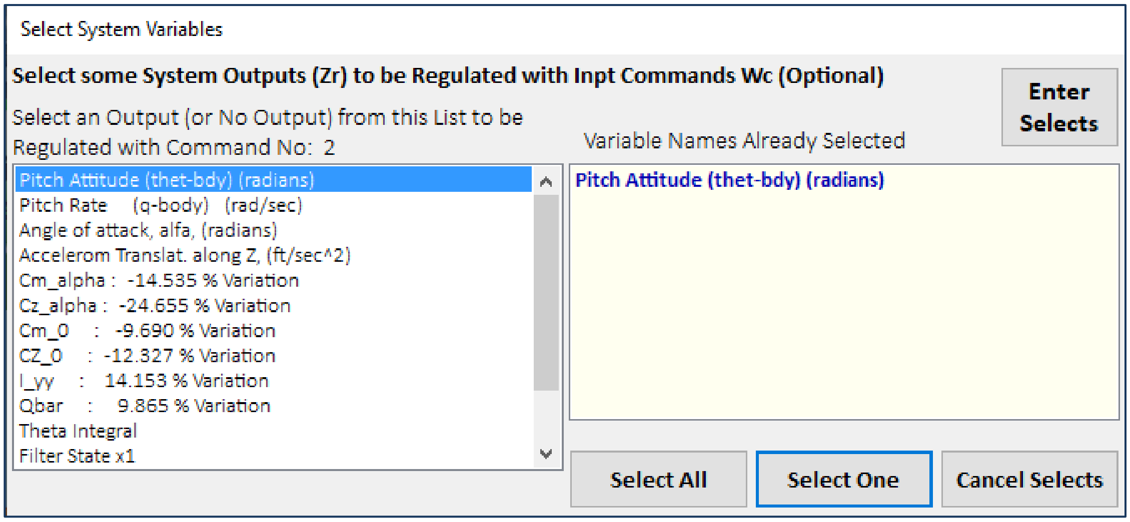
Select an Output from the List Below to be Used as Optimization Criterion No: 8

Accelerom Translat. along Z, (ft/sec^2)
Cm_alpha : -14.535 % Variation
Cz_alpha : -24.655 % Variation
Cm_0 : -9.690 % Variation
CZ_0 : -12.327 % Variation
I_yy : 14.153 % Variation
Qbar : 9.865 % Variation
Theta Integral
Filter State x1
Filter State x2
Filter State x3
Filter State x4

Variable Names Already Selected

Pitch Attitude (thet-bdy) (radians)
Angle of attack, alfa, (radians)
Theta Integral
Filter State x1
Filter State x2
Filter State x3
Filter State x4

In this design we have one output that will be regulated by command. In the next menu select the commanded output, which is the pitch attitude, and then click on “Enter Selects”.



Select System Variables

Select some System Outputs (Zr) to be Regulated with Inpt Commands Wc (Optional)

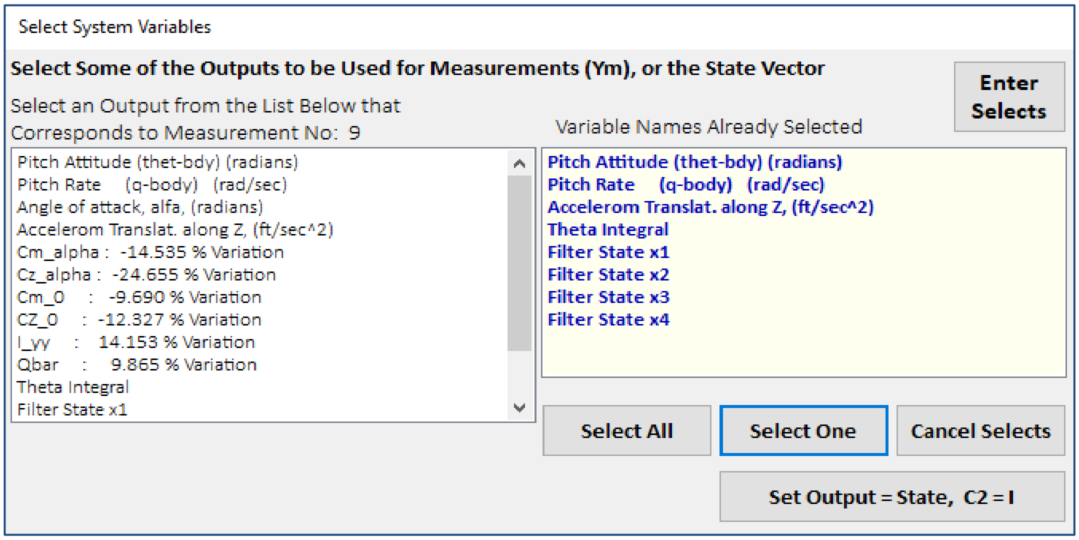
Select an Output (or No Output) from this List to be Regulated with Command No: 2

Pitch Attitude (thet-bdy) (radians)
Pitch Rate (q-body) (rad/sec)
Angle of attack, alfa, (radians)
Accelerom Translat. along Z, (ft/sec^2)
Cm_alpha : -14.535 % Variation
Cz_alpha : -24.655 % Variation
Cm_0 : -9.690 % Variation
CZ_0 : -12.327 % Variation
I_yy : 14.153 % Variation
Qbar : 9.865 % Variation
Theta Integral
Filter State x1

Variable Names Already Selected

Pitch Attitude (thet-bdy) (radians)

The next menu is used for selecting the output measurements. Select the pitch attitude and rate, the Nz acceleration, the θ-integral, and the four filter states x1-x4. Then click on “Enter Selects” to continue.



Select Some of the Outputs to be Used for Measurements (Ym), or the State Vector

Select an Output from the List Below that Corresponds to Measurement No: 9

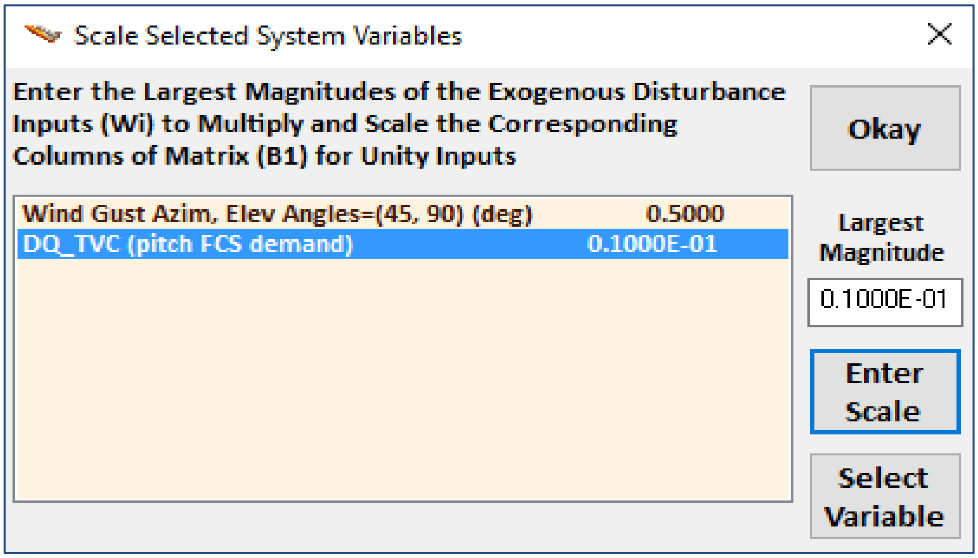
Pitch Attitude (thet-bdy) (radians)
Pitch Rate (q-body) (rad/sec)
Angle of attack, alfa, (radians)
Accelerom Translat. along Z, (ft/sec^2)
Cm_alpha : -14.535 % Variation
Cz_alpha : -24.655 % Variation
Cm_0 : -9.690 % Variation
CZ_0 : -12.327 % Variation
I_yy : 14.153 % Variation
Qbar : 9.865 % Variation
Theta Integral
Filter State x1

Variable Names Already Selected

Pitch Attitude (thet-bdy) (radians)
Pitch Rate (q-body) (rad/sec)
Accelerom Translat. along Z, (ft/sec^2)
Theta Integral
Filter State x1
Filter State x2
Filter State x3
Filter State x4

We have now finished defining the input and output variables and we must now enter the gains that will be used to scale them. The trade-off between control bandwidth and performance versus sensitivity to disturbances, stability and robustness to variations are defined in the optimization process by adjusting those gains which are “knobs” that scale the disturbance inputs and the criteria outputs and they can be changed in the next design iteration. Initially we don’t know what gains to choose that will produce the desired performance, so we begin by scaling the disturbance inputs by entering the maximum magnitudes of the expected disturbances in the input gains and for the output gains we enter the maximum allowable magnitude at each performance criterion. The control is also included in the criteria outputs and we must scale it by entering the maximum amount of control allowed. The measurements noise is also included in the disturbances vector and we must enter the maximum noise value at each measurement. The state estimator calculation requires the expected amount of noise.

In the dialog below enter the gains that will scale the disturbance inputs, which is the maximum expected disturbance at each input. Highlight the input, click on “Select Variable”, enter the noise magnitude, and click on “Enter Scale” to accept it, one at a time.

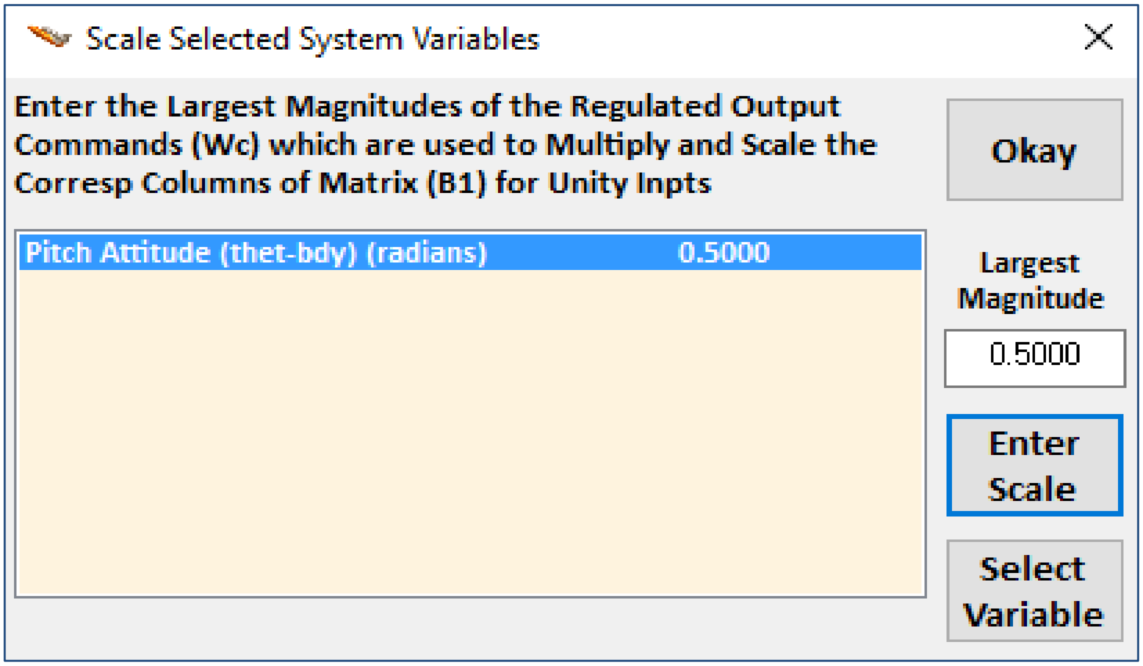


Enter the Largest Magnitudes of the Exogenous Disturbance Inputs (Wi) to Multiply and Scale the Corresponding Columns of Matrix (B1) for Unity Inputs

Wind Gust Azim, Elev Angles=(45, 90) (deg)	0.5000
DQ_TVC (pitch FCS demand)	0.1000E-01

Largest Magnitude: 0.1000E-01

In the dialog below enter the biggest expected magnitude of the control input. Click on “Enter Scale” and the value appears in the display next to the variable label. Then click on “Okay” to go to the next dialog.



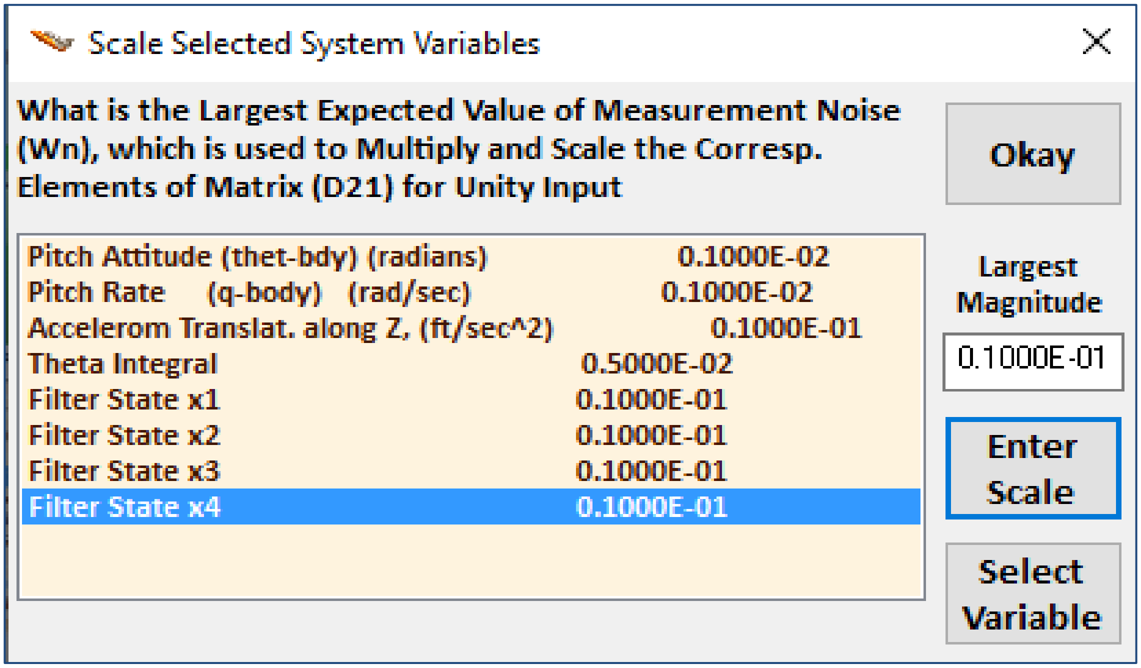
Scale Selected System Variables

Enter the Largest Magnitudes of the Regulated Output Commands (Wc) which are used to Multiply and Scale the Corresp Columns of Matrix (B1) for Unity Inpts

```
Pitch Attitude (thet-bdy) (radians)          0.5000
```

Largest Magnitude: 0.5000

The next dialog is used for entering the noise at the 8 measurements. The pitch attitude integral is also included in the measurements and the four filter states: x1 to x4. Select one variable at a time, enter the noise magnitude and click “Ener Scale”. When you finish, click “Okay” to go to the next dialog.



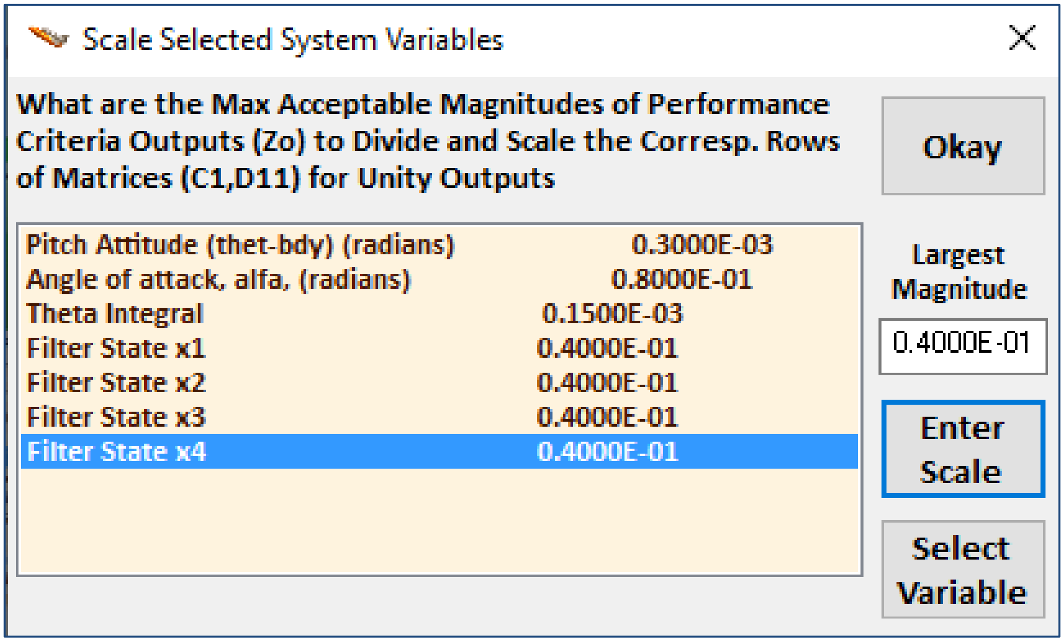
Scale Selected System Variables

What is the Largest Expected Value of Measurement Noise (Wn), which is used to Multiply and Scale the Corresp. Elements of Matrix (D21) for Unity Input

```
Pitch Attitude (thet-bdy) (radians)          0.1000E-02
Pitch Rate     (q-body)   (rad/sec)          0.1000E-02
Accelerom Translat. along Z, (ft/sec^2)      0.1000E-01
Theta Integral                    0.5000E-02
Filter State x1                   0.1000E-01
Filter State x2                   0.1000E-01
Filter State x3                   0.1000E-01
Filter State x4                   0.1000E-01
```

Largest Magnitude: 0.1000E-01

The next dialog is used to enter the gains at the performance optimization criteria. That is, the maximum acceptable magnitude of the performance outputs: pitch attitude θ, angle of attack α, θ-integral, and the 4 filter states. Reducing the gain value at a performance output improves performance and reduces transients of the corresponding variable, at the expense of bigger control. Select one variable at a time, enter the gain and click on "Enter Scale". When you finish click on "Okay" to go to the next dialog.

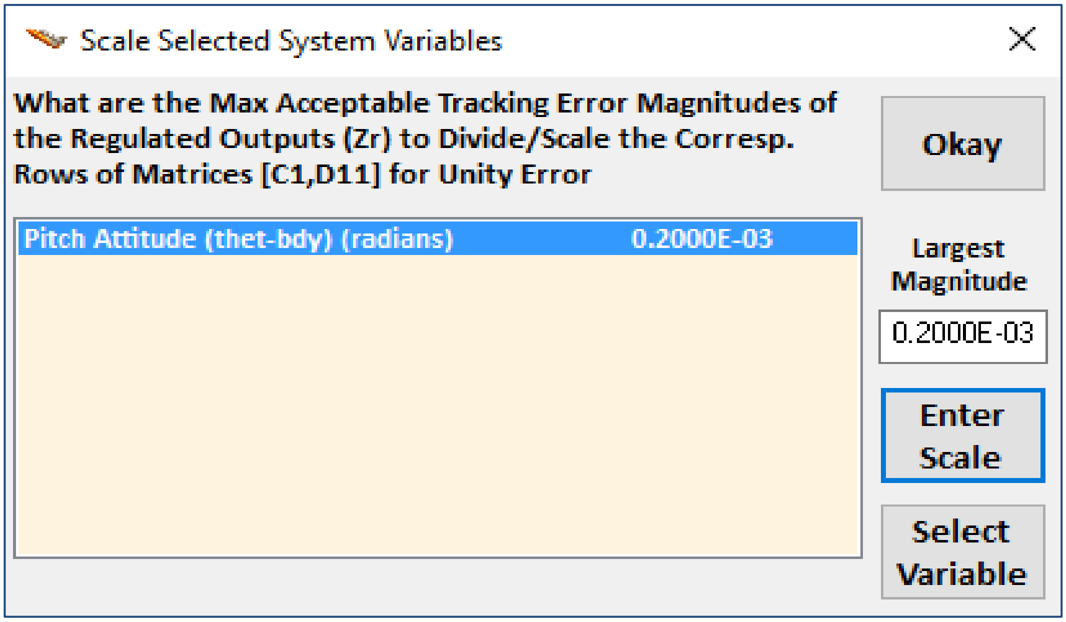


What are the Max Acceptable Magnitudes of Performance Criteria Outputs (Zo) to Divide and Scale the Corresp. Rows of Matrices (C1,D11) for Unity Outputs

```
Pitch Attitude (thet-bdy) (radians)      0.3000E-03
Angle of attack, alfa, (radians)        0.8000E-01
Theta Integral                     0.1500E-03
Filter State x1                   0.4000E-01
Filter State x2                   0.4000E-01
Filter State x3                   0.4000E-01
Filter State x4                   0.4000E-01
```

Largest Magnitude: 0.4000E-01

We must also enter a gain for the max error (z_{re}) of the regulated output, click on "Enter Scale" and then click on "Okay".

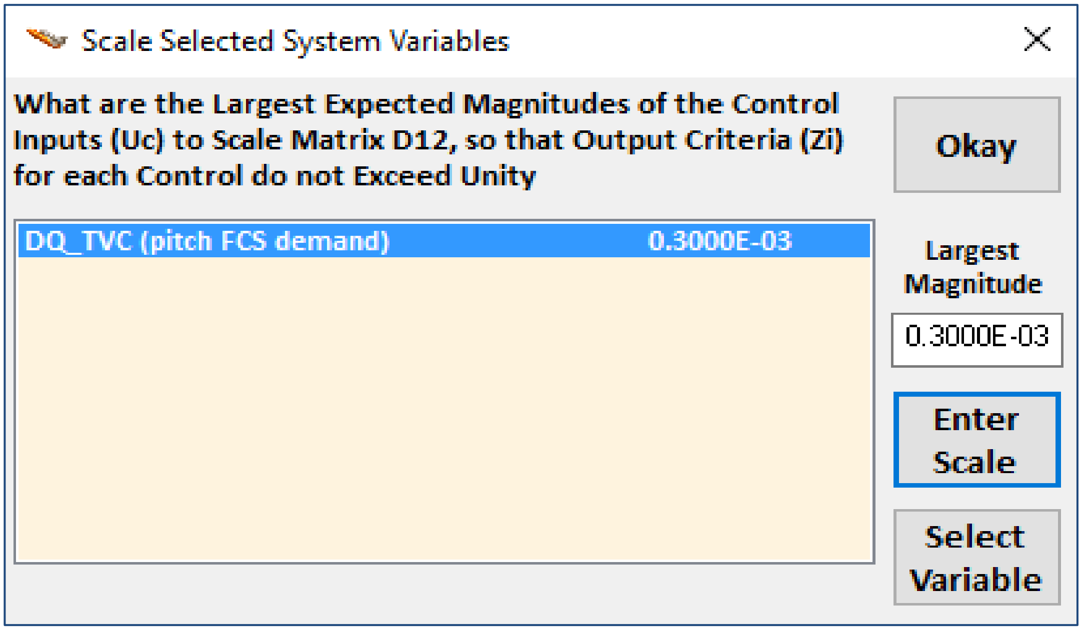


What are the Max Acceptable Tracking Error Magnitudes of the Regulated Outputs (Zr) to Divide/Scale the Corresp. Rows of Matrices [C1,D11] for Unity Error

```
Pitch Attitude (thet-bdy) (radians)      0.2000E-03
```

Largest Magnitude: 0.2000E-03

The last dialog is used for entering the maximum magnitude of the pitch control because the control is also included in the optimization criteria in order to penalize it. Enter the gains as before and click on "Okay".

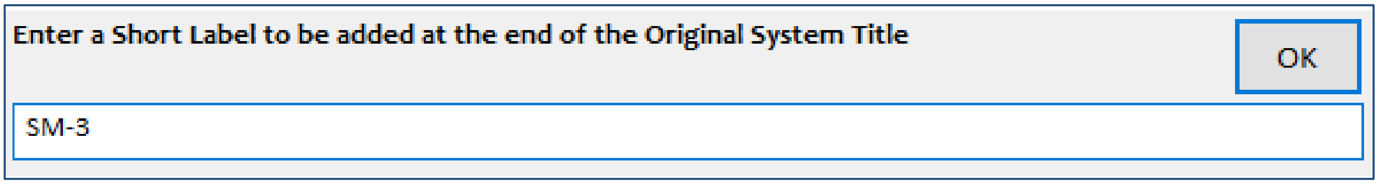


What are the Largest Expected Magnitudes of the Control Inputs (Uc) to Scale Matrix D12, so that Output Criteria (Zi) for each Control do not Exceed Unity

DQ_TVC (pitch FCS demand) 0.3000E-03

Largest Magnitude: 0.3000E-03

The performance optimization gains define the control system design requirements. If we increase the control scaling gain, we are telling the algorithm to allow more control which results in bigger bandwidth and faster system performance. If we increase the gains in the criteria, we are allowing bigger dispersions which produces a more sluggish response in the corresponding variable. Finally enter a short label that will appear at the end of the SM title in the systems file.



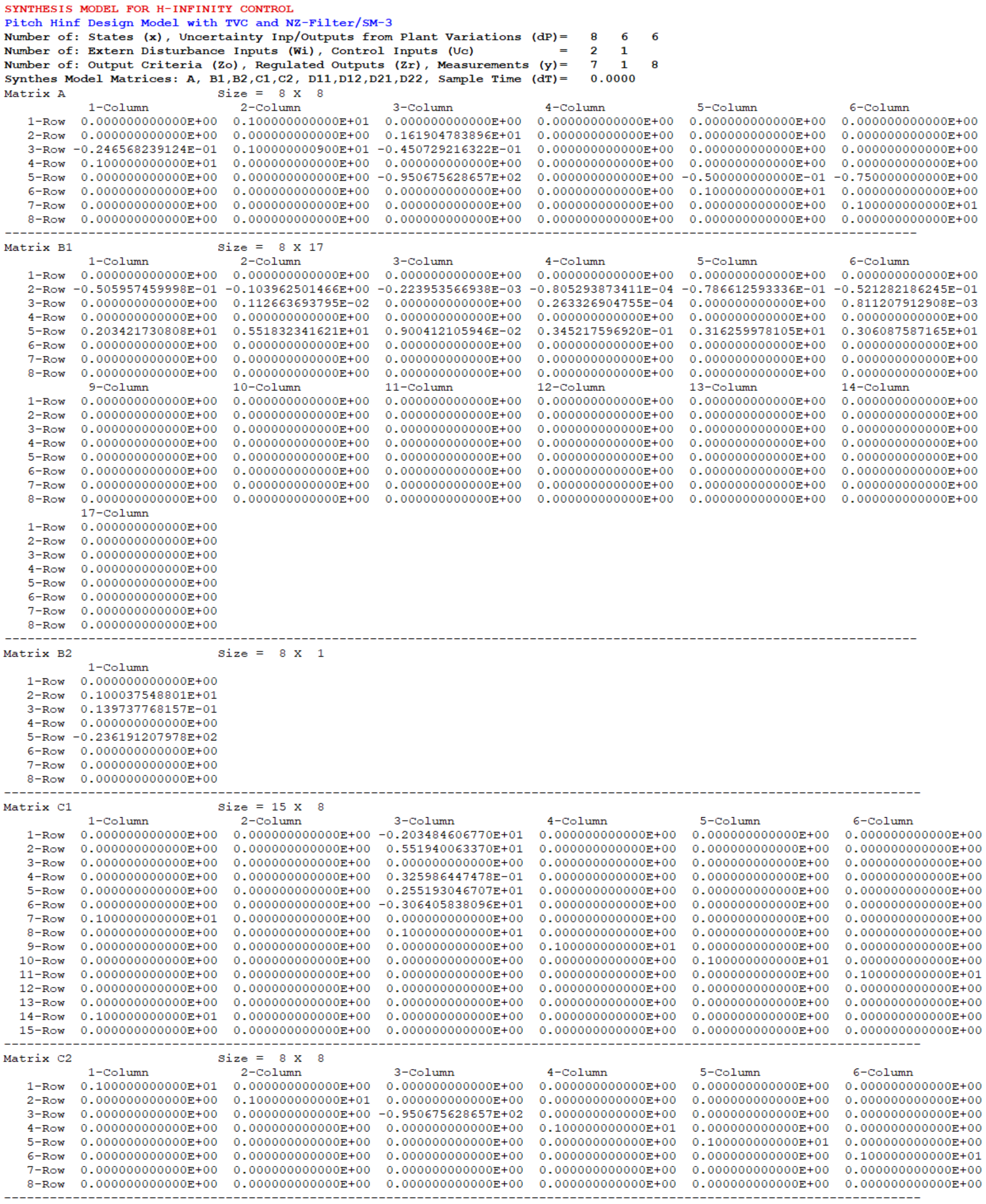
Enter a Short Label to be added at the end of the Original System Title

SM-3

Typically, several iterations are needed to converge to the desired trade-off between performance versus robustness. A simple, preliminary simulation model is often needed to evaluate the design. If we find that we are using too much control, we must reduce the corresponding control gain in the performance criteria output and repeat the design. If a regulated output such as vehicle attitude doesn't converge to its commanded value fast, the gain of the corresponding attitude criterion must be reduced.

3.4 Systems File

Some sections of the Systems File: “*LV_Estimator.Qdr*” are included below. The file includes the H-Infinity SM, the TVC Matrix, the augmented design system and the derived H-infinity control system.



```
SYNTHESIS MODEL FOR H-INFINITY CONTROL
Pitch Hinf Design Model with TVC and NZ-Filter/SM-3
Number of: States (x), Uncertainty Inp/Outputs from Plant Variations (dP)=   8   6   6
Number of: Extern Disturbance Inputs (Wi), Control Inputs (Uc)          =   2   1
Number of: Output Criteria (Zo), Regulated Outputs (Zr), Measurements (y)=   7   1   8
Synthes Model Matrices: A, B1,B2,C1,C2, D11,D12,D21,D22, Sample Time (dT)=   0.0000
Matrix A                   Size =  8 X  8
           1-Column              2-Column              3-Column              4-Column              5-Column              6-Column
   1-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.161904783896E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row -0.246568239124E-01  0.100000000900E+01 -0.450729216322E-01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00 -0.950675628657E+02  0.000000000000E+00 -0.500000000000E-01 -0.750000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
   8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Matrix B1                  Size =  8 X 17
           1-Column              2-Column              3-Column              4-Column              5-Column              6-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row -0.505957459998E-01 -0.103962501466E+00 -0.223953566938E-03 -0.805293873411E-04 -0.786612593336E-01 -0.521282186245E-01
   3-Row  0.000000000000E+00  0.112663693795E-02  0.000000000000E+00  0.263326904755E-04  0.000000000000E+00  0.811207912908E-03
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.203421730808E+01  0.551832341621E+01  0.900412105946E-02  0.345217596920E-01  0.316259978105E+01  0.306087587165E+01
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
           9-Column             10-Column             11-Column             12-Column             13-Column             14-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
          17-Column
   1-Row  0.000000000000E+00
   2-Row  0.000000000000E+00
   3-Row  0.000000000000E+00
   4-Row  0.000000000000E+00
   5-Row  0.000000000000E+00
   6-Row  0.000000000000E+00
   7-Row  0.000000000000E+00
   8-Row  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Matrix B2                  Size =  8 X  1
           1-Column
   1-Row  0.000000000000E+00
   2-Row  0.100037548801E+01
   3-Row  0.139737768157E-01
   4-Row  0.000000000000E+00
   5-Row -0.236191207978E+02
   6-Row  0.000000000000E+00
   7-Row  0.000000000000E+00
   8-Row  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Matrix C1                  Size = 15 X  8
           1-Column              2-Column              3-Column              4-Column              5-Column              6-Column
   1-Row  0.000000000000E+00  0.000000000000E+00 -0.203484606770E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.551940063370E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.325986447478E-01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.255193046707E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00 -0.306405838096E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   8-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   9-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
  10-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
  11-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
  12-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  13-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  14-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  15-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Matrix C2                  Size =  8 X  8
           1-Column              2-Column              3-Column              4-Column              5-Column              6-Column
   1-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00 -0.950675628657E+02  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
```

```
Matrix D11                  Size = 15 X 17
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   9-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  10-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  11-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  12-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  13-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  14-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  15-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
           9-Column           10-Column           11-Column           12-Column           13-Column           14-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   9-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  10-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  11-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  12-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  13-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  14-Row -0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  15-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
Matrix D12                  Size = 15 X  1
           1-Column
   1-Row  0.000000000000E+00
   2-Row  0.000000000000E+00
   3-Row  0.000000000000E+00
   4-Row  0.000000000000E+00
   5-Row  0.157678397445E+01
   6-Row  0.000000000000E+00
   7-Row  0.000000000000E+00
   8-Row  0.000000000000E+00
   9-Row  0.000000000000E+00
  10-Row  0.000000000000E+00
  11-Row  0.000000000000E+00
  12-Row  0.000000000000E+00
  13-Row  0.000000000000E+00
  14-Row  0.000000000000E+00
  15-Row  0.100000000000E+01
------------------------------------------------------------------------------------------------------------------------
Matrix D21                  Size =  8 X 17
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.203421730808E+01  0.551832341621E+01  0.900412105946E-02  0.345217596920E-01  0.316259978105E+01  0.306087587165E+01
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
           9-Column           10-Column           11-Column           12-Column           13-Column           14-Column
   1-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
          17-Column
   1-Row  0.000000000000E+00
   2-Row  0.000000000000E+00
   3-Row  0.000000000000E+00
   4-Row  0.000000000000E+00
   5-Row  0.000000000000E+00
   6-Row  0.000000000000E+00
   7-Row  0.000000000000E+00
   8-Row  0.100000000000E+01
------------------------------------------------------------------------------------------------------------------------
```

```
Matrix D22                  Size =   8 X   1
           1-Column
   1-Row   0.000000000000E+00
   2-Row   0.000000000000E+00
   3-Row  -0.236191207978E+02
   4-Row   0.000000000000E+00
   5-Row   0.000000000000E+00
   6-Row   0.000000000000E+00
   7-Row   0.000000000000E+00
   8-Row   0.000000000000E+00
-----------------------------------------------------------------------------------------
Definition of Synthesis Model Variables                             Max Scaling Factors

States (x) ..........  =   8
  1    Pitch Attitude (thet-bdy) (radians)
  2    Pitch Rate      (q-body)    (rad/sec)
  3    Angle of attack, alfa, (radians)
  4    Theta Integral
  5    Filter State x1
  6    Filter State x2
  7    Filter State x3
  8    Filter State x4

Excitation Inputs (w)  =  17
  1    Cm_alpha :   -14.535 % Variation                            *  1.0000
  2    Cz_alpha :   -24.655 % Variation                            *  1.0000
  3    Cm_0     :    -9.690 % Variation                            *  1.0000
  4    CZ_0     :   -12.327 % Variation                            *  1.0000
  5    I_yy     :    14.153 % Variation                            *  1.0000
  6    Qbar     :     9.865 % Variation                            *  1.0000
  7    Wind Gust Azim, Elev Angles=(45, 90)  (deg)                  *  0.5
  8    DQ_TVC (pitch FCS demand)                                    *  0.01
  9    Commd for Outpt: Pitch Attitude (thet-bdy) (radians)        *  0.5
 10    Noise at Output: Pitch Attitude (thet-bdy) (radians)        *  0.001
 11    Noise at Output: Pitch Rate      (q-body)   (rad/sec)       *  0.001
 12    Noise at Output: Accelerom Translat. along Z, (ft/sec^2     *  0.01
 13    Noise at Output: Theta Integral                             *  0.005
 14    Noise at Output: Filter State x1                            *  0.01
 15    Noise at Output: Filter State x2                            *  0.01
 16    Noise at Output: Filter State x3                            *  0.01
 17    Noise at Output: Filter State x4                            *  0.01

Control Inputs (u) ... =    1
  1    Control: DQ_TVC (pitch FCS demand)                           *  1.0000

Performance Outputs (z)=  15
  1    Cm_alpha :   -14.535 % Variation                            /  1.0000
  2    Cz_alpha :   -24.655 % Variation                            /  1.0000
  3    Cm_0     :    -9.690 % Variation                            /  1.0000
  4    CZ_0     :   -12.327 % Variation                            /  1.0000
  5    I_yy     :    14.153 % Variation                            /  1.0000
  6    Qbar     :     9.865 % Variation                            /  1.0000
  7    Pitch Attitude (thet-bdy) (radians)                          /  0.0003
  8    Angle of attack, alfa, (radians)                             /  0.08
  9    Theta Integral                                               /  0.00015
 10    Filter State x1                                              /  0.04
 11    Filter State x2                                              /  0.04
 12    Filter State x3                                              /  0.04
 13    Filter State x4                                              /  0.04
 14    Track Error: Pitch Attitude (thet-bdy) (radians)             /  0.0002
 15    Contrl Criter. DQ_TVC (pitch FCS demand)                     /  0.0003

Measurement Outputs (y)=   8
  1    Measurm: Pitch Attitude (thet-bdy) (radians)                 /  1.0000
  2    Measurm: Pitch Rate      (q-body)    (rad/sec)               /  1.0000
  3    Measurm: Accelerom Translat. along Z, (ft/sec^2)             /  1.0000
  4    Measurm: Theta Integral                                      /  1.0000
  5    Measurm: Filter State x1                                     /  1.0000
  6    Measurm: Filter State x2                                     /  1.0000
  7    Measurm: Filter State x3                                     /  1.0000
  8    Measurm: Filter State x4                                     /  1.0000
-----------------------------------------------------------------------------------------
```

```
Gain Matrix for ...
Mixing Logic for First Stage Analysis Model, T=85.0 sec
! Thrust Vector Control Matrix at t=85 sec  This multi-engine vehicle has 8 Gimbaling Engines.
Matrix TVC                  Size = 16 X  3
           1-Column              2-Column             3-Column
   1-Row -0.450363642675E-01 -0.298159977490E+00  0.000000000000E+00
   2-Row -0.186432998956E-01 -0.298159977490E+00  0.000000000000E+00
   3-Row  0.186432998956E-01 -0.298159977490E+00  0.000000000000E+00
   4-Row  0.450363642675E-01 -0.298159977490E+00  0.000000000000E+00
   5-Row  0.450363642675E-01 -0.298159977490E+00  0.000000000000E+00
   6-Row  0.186432998956E-01 -0.298159977490E+00  0.000000000000E+00
   7-Row -0.186432998956E-01 -0.298159977490E+00  0.000000000000E+00
   8-Row -0.450363642675E-01 -0.298159977490E+00  0.000000000000E+00
   9-Row  0.186432998956E-01  0.000000000000E+00 -0.298066083456E+00
  10-Row  0.450363642675E-01  0.000000000000E+00 -0.298066083456E+00
  11-Row  0.450363642675E-01  0.000000000000E+00 -0.298066083456E+00
  12-Row  0.186432998956E-01  0.000000000000E+00 -0.298066083456E+00
  13-Row -0.186432998956E-01  0.000000000000E+00 -0.298066083456E+00
  14-Row -0.450363642675E-01  0.000000000000E+00 -0.298066083456E+00
  15-Row -0.450363642675E-01  0.000000000000E+00 -0.298066083456E+00
  16-Row -0.186432998956E-01  0.000000000000E+00 -0.298066083456E+00
--------------------------------------------------------------------------------------------
Definitions of Matrix Inputs (Columns):     3
P-dot Roll  Accel Demand About X Axis
Q-dot Pitch Accel Demand About Y Axis
R-dot Yaw   Accel Demand About Z Axis

Definitions of Matrix Outputs (Rows):     16
Output:  1 Dy(engine):  1 Pitch Deflection
Output:  2 Dy(engine):  2 Pitch Deflection
Output:  3 Dy(engine):  3 Pitch Deflection
Output:  4 Dy(engine):  4 Pitch Deflection
Output:  5 Dy(engine):  5 Pitch Deflection
Output:  6 Dy(engine):  6 Pitch Deflection
Output:  7 Dy(engine):  7 Pitch Deflection
Output:  8 Dy(engine):  8 Pitch Deflection
Output:  9 Dz(engine):  1 Yaw  Deflection
Output: 10 Dz(engine):  2 Yaw  Deflection
Output: 11 Dz(engine):  3 Yaw  Deflection
Output: 12 Dz(engine):  4 Yaw  Deflection
Output: 13 Dz(engine):  5 Yaw  Deflection
Output: 14 Dz(engine):  6 Yaw  Deflection
Output: 15 Dz(engine):  7 Yaw  Deflection
Output: 16 Dz(engine):  8 Yaw  Deflection
--------------------------------------------------------------------------------------------
STATE-SPACE SYSTEM ...
NZ-Filter
Number of Inputs, States, Outputs, Sample Time dT (for discrete)=    1    4    4    0.0000
Matrices: (A,B,C,D)
Matrix A                    Size =  4 X  4
           1-Column             2-Column             3-Column             4-Column
   1-Row -0.500000000000E-01 -0.750000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.100000000000E+01 -0.600000000000E-01 -0.500000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
-----------------------------------------------------------------------------------------------
Matrix B                    Size =  4 X  1
           1-Column
   1-Row  0.100000000000E+01
   2-Row  0.000000000000E+00
   3-Row  0.000000000000E+00
   4-Row  0.000000000000E+00
-----------------------------------------------------------------------------------------------
Matrix C                    Size =  4 X  4
           1-Column             2-Column             3-Column             4-Column
   1-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
-----------------------------------------------------------------------------------------------
Matrix D                    Size =  4 X  1
           1-Column
   1-Row  0.000000000000E+00
   2-Row  0.000000000000E+00
   3-Row  0.000000000000E+00
   4-Row  0.000000000000E+00
-----------------------------------------------------------------------------------------------
Definition of System Variables

Inputs  =   1
   1   Alpha In

States  =   4
   1   State No:  1
   2   State No:  2
   3   State No:  3
   4   State No:  4

Outputs =   4
   1   Filter State x1
   2   Filter State x2
   3   Filter State x3
   4   Filter State x4
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STATE-SPACE SYSTEM ...
Integrator
Number of Inputs, States, Outputs, Sample Time dT (for discrete)=    1    1    1    0.0000
Matrices: (A,B,C,D)
Matrix A                    Size =  1 X  1
           1-Column
   1-Row  0.000000000000E+00
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Matrix B                    Size =  1 X  1
           1-Column
   1-Row  0.100000000000E+01
-------------------------------------------------------------------------------------------------------------
Matrix C                    Size =  1 X  1
           1-Column
   1-Row  0.100000000000E+01
-------------------------------------------------------------------------------------------------------------
Matrix D                    Size =  1 X  1
           1-Column
   1-Row  0.000000000000E+00
-------------------------------------------------------------------------------------------------------------
Definition of System Variables

Inputs  =   1
   1   Theta In

States  =   1
   1   State No:  1

Outputs =   1
   1   Theta_Integral
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STATE-SPACE SYSTEM ...
Pitch Design Model with TVC
Number of Inputs, States, Outputs, Sample Time dT (for discrete)=    8    3   10    0.0000
Matrices: (A,B,C,D)
Matrix A                    Size =  3 X  3
           1-Column            2-Column            3-Column
   1-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.161904783896E+01
   3-Row -0.246568239124E-01  0.100000000900E+01 -0.450729216322E-01
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Matrix B                    Size =  3 X  8
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.100037548801E+01  0.964452398674E-03 -0.505957459998E-01 -0.103962501466E+00 -0.223953566938E-03 -0.805293873411E-04
   3-Row  0.139737768157E-01 -0.149926698265E-04  0.000000000000E+00  0.112663693795E-02  0.000000000000E+00  0.263326904755E-04
------------------------------------------------------------------------------------------------------------------------
Matrix C                    Size = 10 X  3
           1-Column            2-Column            3-Column
   1-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
   4-Row  0.000000000000E+00  0.000000000000E+00 -0.950675628657E+02
   5-Row  0.000000000000E+00  0.000000000000E+00 -0.203484606770E+01
   6-Row  0.000000000000E+00  0.000000000000E+00  0.551940063370E+01
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   8-Row  0.000000000000E+00  0.000000000000E+00  0.325986447478E-01
   9-Row  0.000000000000E+00  0.000000000000E+00  0.255193046707E+01
  10-Row  0.000000000000E+00  0.000000000000E+00 -0.306405838096E+01
------------------------------------------------------------------------------------------------------------------------
Matrix D                    Size = 10 X  8
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.595185974563E-03  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row -0.236191207978E+02 -0.566138036431E-01  0.203421730808E+01  0.551832341621E+01  0.900412105946E-02  0.345217596920E-01
   5-Row  0.000000000000E+00 -0.121111183988E-02  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.328506984517E-02  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00 -0.111142797100E-03  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   8-Row  0.000000000000E+00  0.140523274537E-03  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   9-Row  0.157678397445E+01  0.152016228364E-02  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  10-Row  0.000000000000E+00 -0.182529789395E-02  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
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Definition of System Variables

Inputs  =   8
    1   DQ_TVC (pitch FCS demand)
    2   Wind Gust Azim, Elev Angles=(45, 90) (deg)
    3   Cm_alpha :   -14.535 % Variation
    4   Cz_alpha :   -24.655 % Variation
    5   Cm_0     :    -9.690 % Variation
    6   CZ_0     :   -12.327 % Variation
    7   I_yy     :    14.153 % Variation
    8   Qbar     :     9.865 % Variation

States  =   3
    1   Pitch Attitude (thet-bdy) (radians)
    2   Pitch Rate      (q-body)    (rad/sec)
    3   Angle of attack, alfa, (radians)

Outputs =  10
    1   Pitch Attitude (thet-bdy) (radians)
    2   Pitch Rate      (q-body)    (rad/sec)
    3   Angle of attack, alfa, (radians)
    4   Accelerom  along Z, (ft/sec^2) Translat.
    5   Cm_alpha :   -14.535 % Variation
    6   Cz_alpha :   -24.655 % Variation
    7   Cm_0     :    -9.690 % Variation
    8   CZ_0     :   -12.327 % Variation
    9   I_yy     :    14.153 % Variation
   10   Qbar     :     9.865 % Variation
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STATE-SPACE SYSTEM ...
Pitch Hinf Design Model with TVC and NZ-Filter
! Combines the Design Vehicle Model with the NZ-Filter Including Parameter Uncertainties
Number of Inputs, States, Outputs, Sample Time dT (for discrete)=    8     8    15     0.0000
Matrices: (A,B,C,D)
Matrix A                  Size =  8 X  8
          1-Column             2-Column             3-Column             4-Column             5-Column             6-Column
   1-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.161904783896E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row -0.246568239124E-01  0.100000000900E+01 -0.450729216322E-01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00 -0.950675628657E+02  0.000000000000E+00 -0.500000000000E-01 -0.750000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
   8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Matrix B                  Size =  8 X  8
          1-Column             2-Column             3-Column             4-Column             5-Column             6-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.100037548801E+01  0.964452398674E-03 -0.505957459998E-01 -0.103962501466E+00 -0.223953566938E-03 -0.805293873411E-04
   3-Row  0.139737768157E-01 -0.149926698265E-04  0.000000000000E+00  0.112663693795E-02  0.000000000000E+00  0.263326904755E-04
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row -0.236191207978E+02 -0.566138036431E-01  0.203421730808E+01  0.551832341621E+01  0.900412105946E-02  0.345217596920E-01
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
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Matrix C                  Size = 15 X  8
          1-Column             2-Column             3-Column             4-Column             5-Column             6-Column
   1-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00 -0.950675628657E+02  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00 -0.203484606770E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.551940063370E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   8-Row  0.000000000000E+00  0.000000000000E+00  0.325986447478E-01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   9-Row  0.000000000000E+00  0.000000000000E+00  0.255193046707E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  10-Row  0.000000000000E+00  0.000000000000E+00 -0.306405838096E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  11-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
  12-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
  13-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
  14-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  15-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
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Matrix D                  Size = 15 X  8
          1-Column             2-Column             3-Column             4-Column             5-Column             6-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.595185974563E-03  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row -0.236191207978E+02 -0.566138036431E-01  0.203421730808E+01  0.551832341621E+01  0.900412105946E-02  0.345217596920E-01
   5-Row  0.000000000000E+00 -0.121111183988E-02  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.328506984517E-02  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00 -0.111142797100E-03  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   8-Row  0.000000000000E+00  0.140523274537E-03  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   9-Row  0.157678397445E+01  0.152016228364E-02  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  10-Row  0.000000000000E+00 -0.182529789395E-02  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  11-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  12-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  13-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  14-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  15-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Definition of System Variables

Inputs  =   8
   1   DQ_TVC (pitch FCS demand)
   2   Wind Gust Azim, Elev Angles=(45, 90) (deg)
   3   Cm_alpha :   -14.535 % Variation
   4   Cz_alpha :   -24.655 % Variation
   5   Cm_0     :    -9.690 % Variation
   6   CZ_0     :   -12.327 % Variation
   7   I_yy     :    14.153 % Variation
   8   Qbar     :     9.865 % Variation

States  =   8
   1   Pitch Attitude (thet-bdy) (radians)
   2   Pitch Rate      (q-body)    (rad/sec)
   3   Angle of attack, alfa, (radians)
   4   Theta Integral
   5   Filter State x1
   6   Filter State x2
   7   Filter State x3
   8   Filter State x4

Outputs =  15
   1   Pitch Attitude (thet-bdy) (radians)
   2   Pitch Rate      (q-body)    (rad/sec)
   3   Angle of attack, alfa, (radians)
   4   Accelerom Translat. along Z, (ft/sec^2)
   5   Cm_alpha :   -14.535 % Variation
   6   Cz_alpha :   -24.655 % Variation
   7   Cm_0     :    -9.690 % Variation
   8   CZ_0     :   -12.327 % Variation
   9   I_yy     :    14.153 % Variation
  10   Qbar     :     9.865 % Variation
  11   Theta Integral
  12   Filter State x1
  13   Filter State x2
  14   Filter State x3
  15   Filter State x4
------------------------------------------------------
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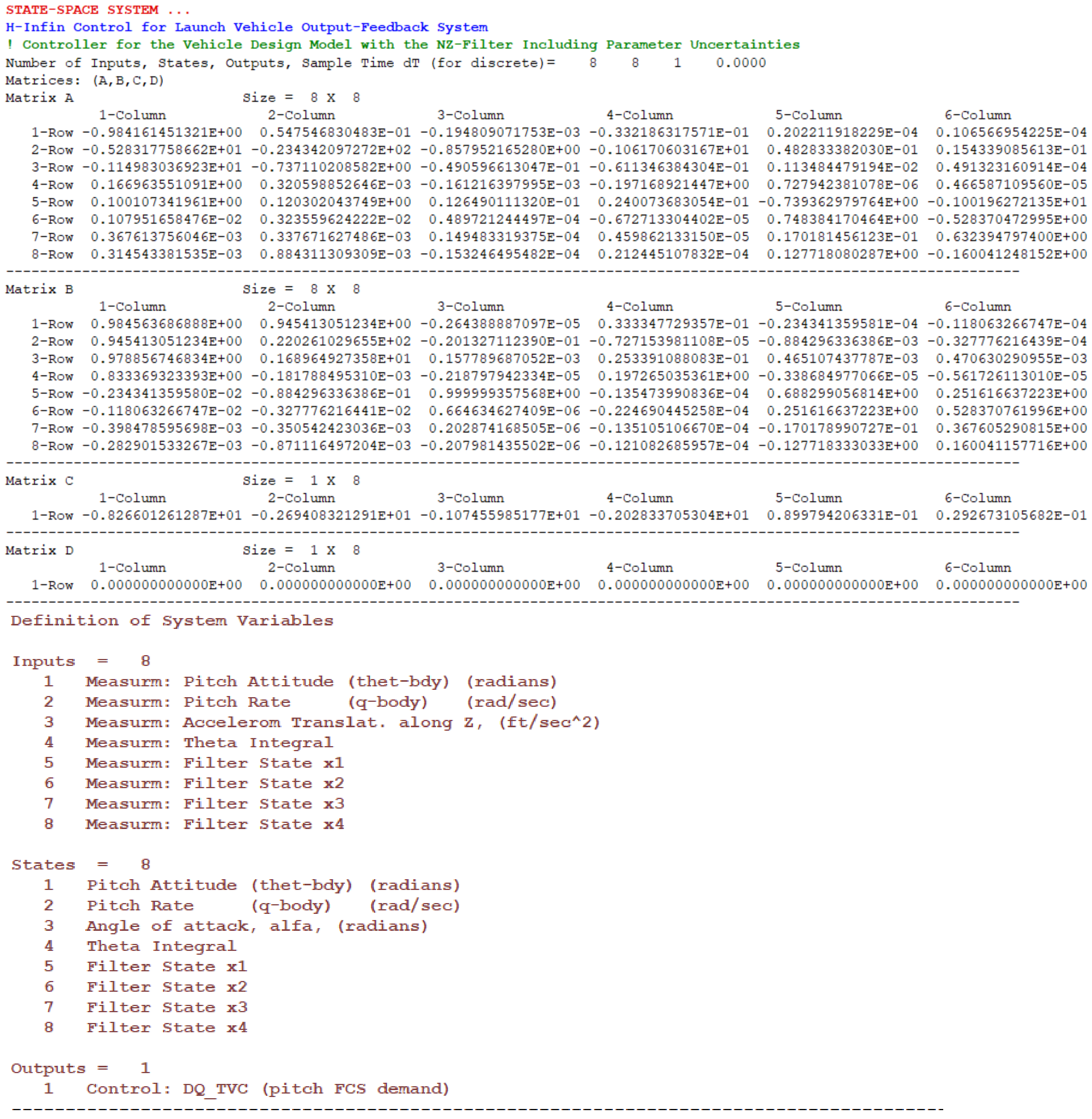

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STATE-SPACE SYSTEM ...
H-Infin Control for Launch Vehicle Output-Feedback System
! Controller for the Vehicle Design Model with the NZ-Filter Including Parameter Uncertainties
Number of Inputs, States, Outputs, Sample Time dT (for discrete)=    8    8    1    0.0000
Matrices: (A,B,C,D)
Matrix A                   Size =  8 X  8
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row -0.984161451321E+00  0.547546830483E-01 -0.194809071753E-03 -0.332186317571E-01  0.202211918229E-04  0.106566954225E-04
   2-Row -0.528317758662E+01 -0.234342097272E+02 -0.857952165280E+00 -0.106170603167E+01  0.482833382030E-01  0.154339085613E-01
   3-Row -0.114983036923E+01 -0.737110208582E+00 -0.490596613047E-01 -0.611346384304E-01  0.113484479194E-02  0.491323160914E-04
   4-Row  0.166963551091E+00  0.320598852646E-03 -0.161216397995E-03 -0.197168921447E+00  0.727942381078E-06  0.466587109560E-05
   5-Row  0.100107341961E+00  0.120302043749E+00  0.126490111320E-01  0.240073683054E-01 -0.739362979764E+00 -0.100196272135E+01
   6-Row  0.107951658476E-02  0.323559624222E-02  0.489721244497E-04 -0.672713304402E-05  0.748384170464E+00 -0.528370472995E+00
   7-Row  0.367613756046E-03  0.337671627486E-03  0.149483319375E-04  0.459862133150E-05  0.170181456123E-01  0.632394797400E+00
   8-Row  0.314543381535E-03  0.884311309309E-03 -0.153246495482E-04  0.212445107832E-04  0.127718080287E+00 -0.160041248152E+00
---------------------------------------------------------------------------------------------------------------------------
Matrix B                   Size =  8 X  8
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.984563686888E+00  0.945413051234E+00 -0.264388887097E-05  0.333347729357E-01 -0.234341359581E-04 -0.118063266747E-04
   2-Row  0.945413051234E+00  0.220261029655E+02 -0.201327112390E-01 -0.727153981108E-05 -0.884296336386E-03 -0.327776216439E-04
   3-Row  0.978856746834E+00  0.168964927358E+01  0.157789687052E-03  0.253391088083E-01  0.465107437787E-03  0.470630290955E-03
   4-Row  0.833369323393E+00 -0.181788495310E-03 -0.218797942334E-05  0.197265035361E+00 -0.338684977066E-05 -0.561726113010E-05
   5-Row -0.234341359580E-02 -0.884296336386E-01  0.999999357568E+00 -0.135473990836E-04  0.688299056814E+00  0.251616637223E+00
   6-Row -0.118063266747E-02 -0.327776216441E-02  0.664634627409E-06 -0.224690445258E-04  0.251616637223E+00  0.528370761996E+00
   7-Row -0.398478595698E-03 -0.350542423036E-03  0.202874168505E-06 -0.135105106670E-04 -0.170178990727E-01  0.367605290815E+00
   8-Row -0.282901533267E-03 -0.871116497204E-03 -0.207981435502E-06 -0.121082685957E-04 -0.127718333033E+00  0.160041157716E+00
---------------------------------------------------------------------------------------------------------------------------
Matrix C                   Size =  1 X  8
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row -0.826601261287E+01 -0.269408321291E+01 -0.107455985177E+01 -0.202833705304E+01  0.899794206331E-01  0.292673105682E-01
---------------------------------------------------------------------------------------------------------------------------
Matrix D                   Size =  1 X  8
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
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 Definition of System Variables

 Inputs  =   8
    1   Measurm: Pitch Attitude (thet-bdy) (radians)
    2   Measurm: Pitch Rate     (q-body)   (rad/sec)
    3   Measurm: Accelerom Translat. along Z, (ft/sec^2)
    4   Measurm: Theta Integral
    5   Measurm: Filter State x1
    6   Measurm: Filter State x2
    7   Measurm: Filter State x3
    8   Measurm: Filter State x4

 States  =   8
    1   Pitch Attitude (thet-bdy) (radians)
    2   Pitch Rate     (q-body)   (rad/sec)
    3   Angle of attack, alfa, (radians)
    4   Theta Integral
    5   Filter State x1
    6   Filter State x2
    7   Filter State x3
    8   Filter State x4

 Outputs =   1
    1   Control: DQ_TVC (pitch FCS demand)
 ------------------------------------------------------------------------------------
```
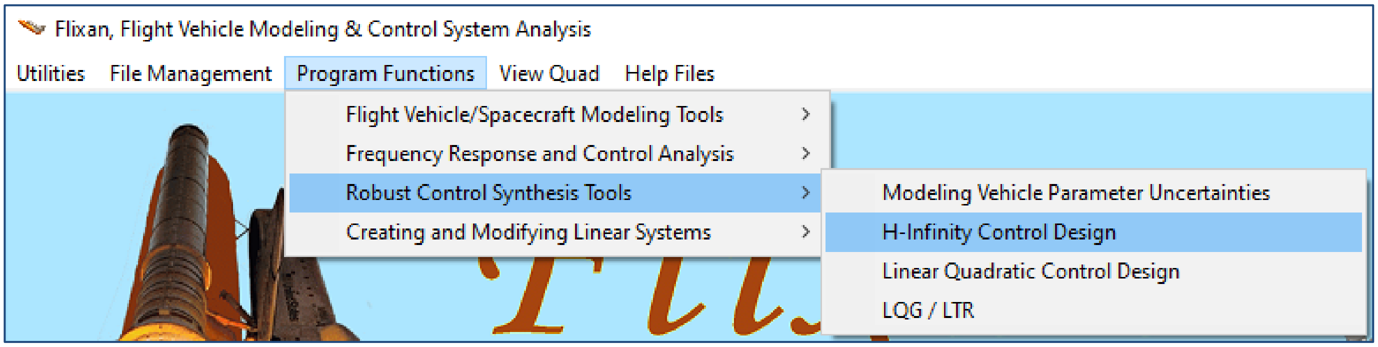
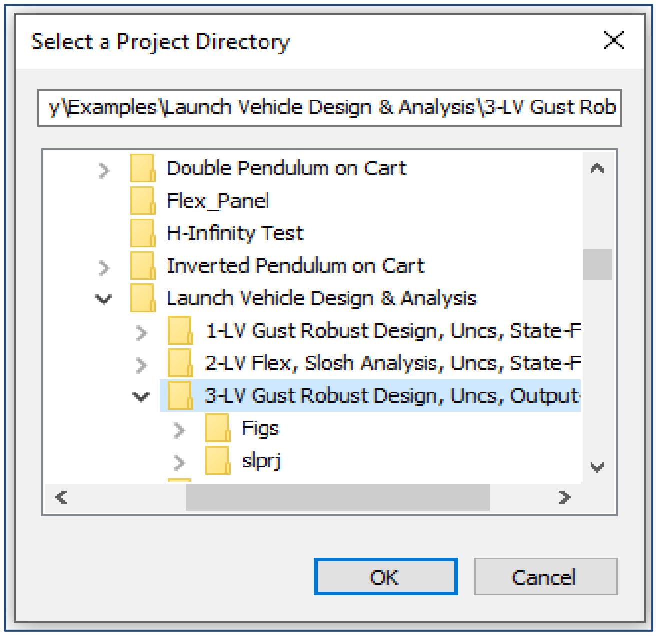

The 8th order, H-Infinity derived output-feedback control system is included at the bottom of the systems file, as shown above. Its title is "*H-Infin. Control for Launch Vehicle Output-Feedback System*".

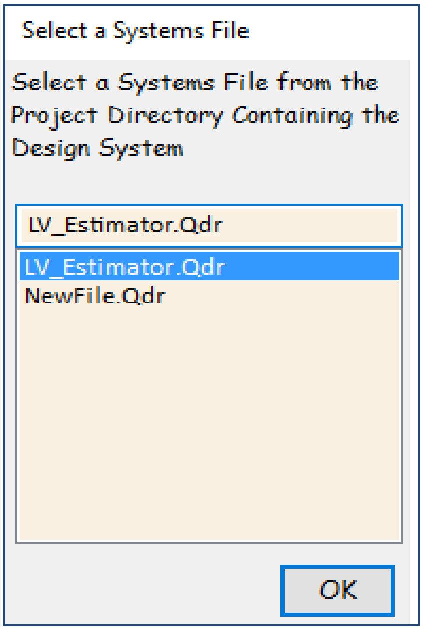
3.5 Designing the H-Infinity Dynamic Controller Interactively

We may now use the SM to design the H-infinity controller interactively. This can also be done in batch mode by processing the batch dataset. Start the H-infinity design program, select the project directory and systems file "*LV_Estimator.Qdr*", and from the H-infinity program main menu select the second option to read and process the SM which is already in file. From the next menu select its title, and click on "Select".



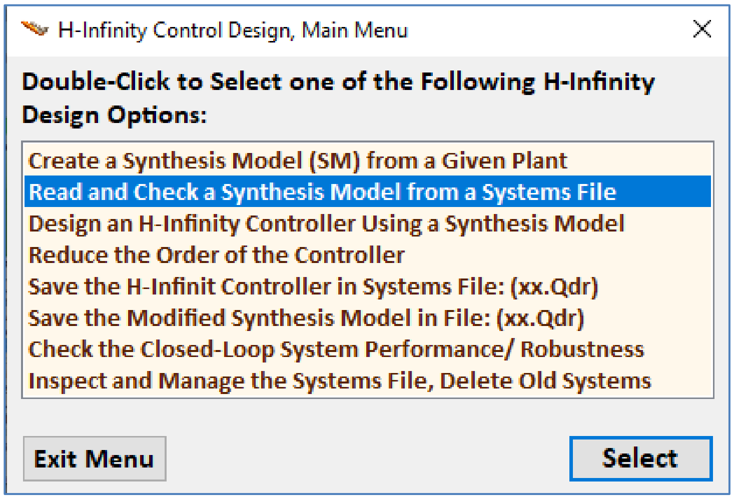


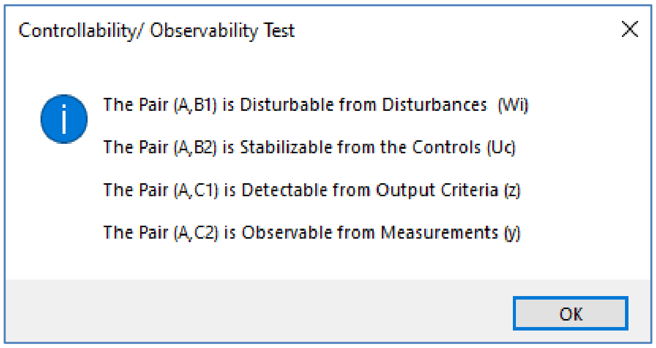
y\Examples\Launch Vehicle Design & Analysis\3-LV Gust Rob



Select a Systems File from the Project Directory Containing the Design System

LV_Estimator.Qdr





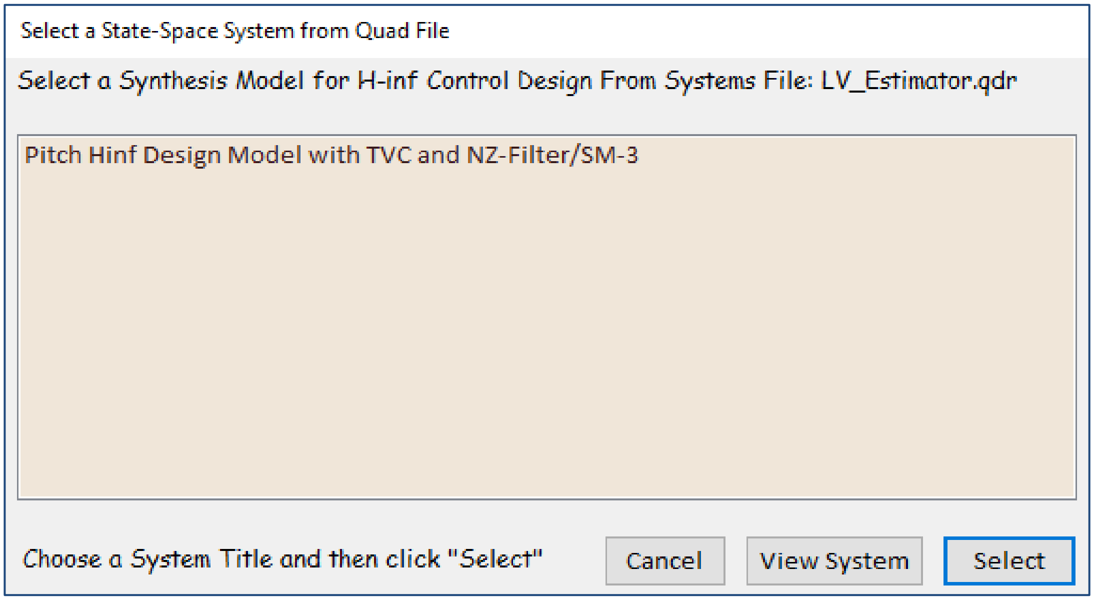
The Pair (A,B1) is Disturbable from Disturbances (Wi)

The Pair (A,B2) is Stabilizable from the Controls (Uc)

The Pair (A,C1) is Detectable from Output Criteria (z)

The Pair (A,C2) is Observable from Measurements (y)

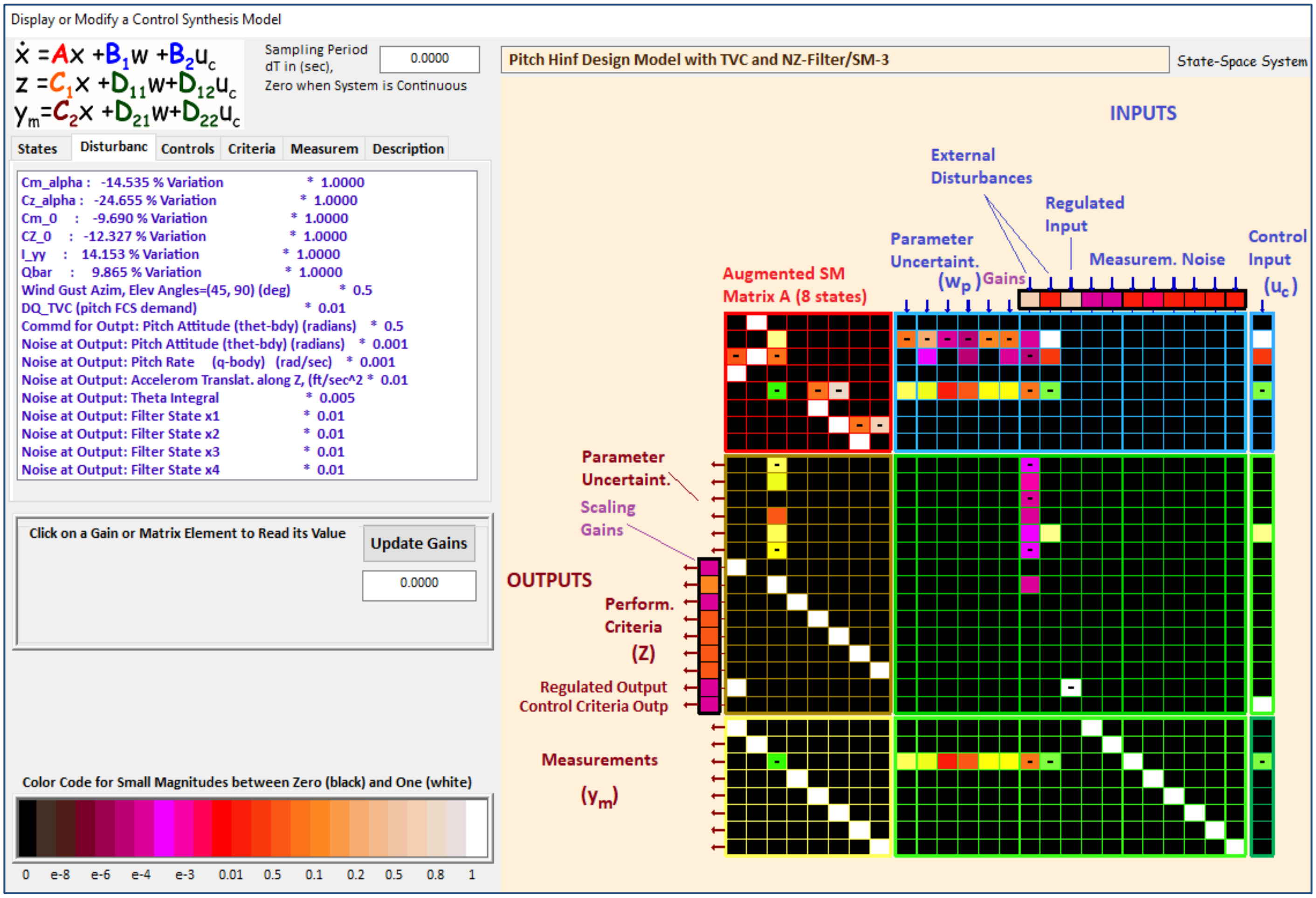
The program confirms that the SM satisfies the observability and controllability requirements and displays the SM matrices graphically in system form.



Select a Synthesis Model for H-inf Control Design From Systems File: LV_Estimator.qdr

Pitch Hinf Design Model with TVC and NZ-Filter/SM-3

Choose a System Title and then click "Select"



$$\dot{x} = Ax + B_1 w + B_2 u_c$$
$$z = C_1 x + D_{11} w + D_{12} u_c$$
$$y_m = C_2 x + D_{21} w + D_{22} u_c$$

Sampling Period dT in (sec), Zero when System is Continuous: 0.0000

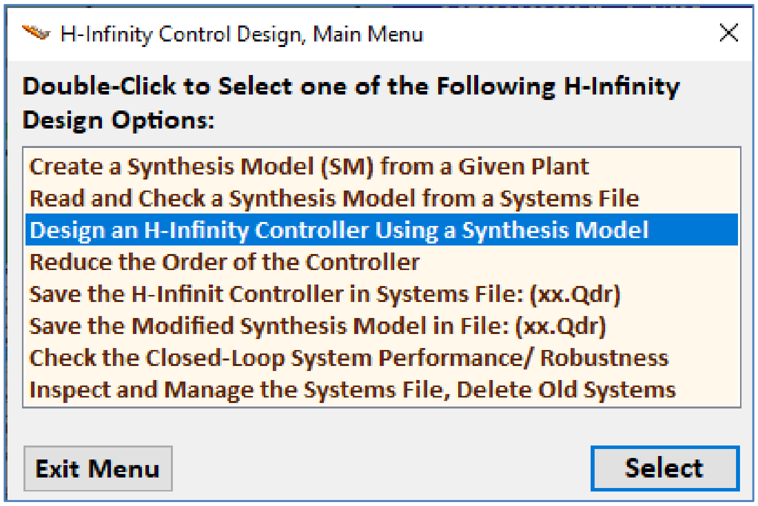
Pitch Hinf Design Model with TVC and NZ-Filter/SM-3

Cm_alpha : -14.535 % Variation * 1.0000
Cz_alpha : -24.655 % Variation * 1.0000
Cm_0 : -9.690 % Variation * 1.0000
CZ_0 : -12.327 % Variation * 1.0000
I_yy : 14.153 % Variation * 1.0000
Qbar : 9.865 % Variation * 1.0000
Wind Gust Azim, Elev Angles=(45, 90) (deg) * 0.5
DQ_TVC (pitch FCS demand) * 0.01
Commd for Outpt: Pitch Attitude (thet-bdy) (radians) * 0.5
Noise at Output: Pitch Attitude (thet-bdy) (radians) * 0.001
Noise at Output: Pitch Rate (q-body) (rad/sec) * 0.001
Noise at Output: Accelerom Translat. along Z, (ft/sec^2 * 0.01
Noise at Output: Theta Integral * 0.005
Noise at Output: Filter State x1 * 0.01
Noise at Output: Filter State x2 * 0.01
Noise at Output: Filter State x3 * 0.01
Noise at Output: Filter State x4 * 0.01

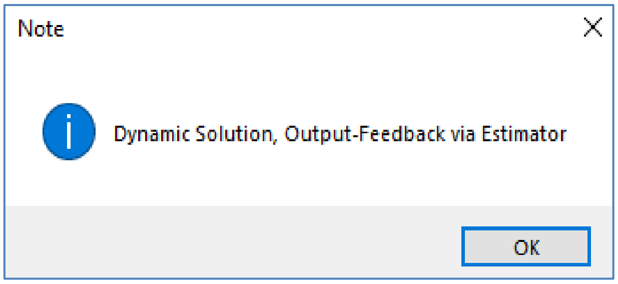
Click on a Gain or Matrix Element to Read its Value: 0.0000

The 9 SM matrices appear color coded and also the scaling gains that scale the disturbances and the criteria. The A-matrix consists of 8 states. There are 6 uncertainty inputs (w_p), 2 external disturbances (w), 1 command for a regulated output, 8 measurements noise inputs, and one control input (u_c). We also have 6 uncertainty outputs (z_p), 7 performance criteria (z), 1 criterion for a regulated output error (z_{re}), 1 criterion for monitoring the control utilization performance, and 8 measurements (y_m).

Select the third option from the main menu to design the H-infinity controller, and click “Select”. The program confirms that the solution is an output feedback which also produce a dynamic estimator.

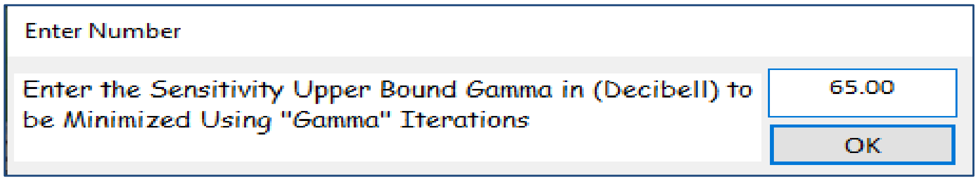


Double-Click to Select one of the Following H-Infinity Design Options:



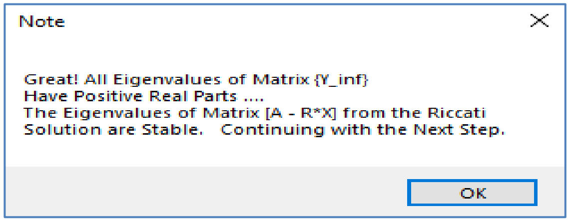
Dynamic Solution, Output-Feedback via Estimator

Now we begin the iterative process of minimizing the upper bound γ of the infinity norm of the sensitivity transfer function between the disturbance inputs and the output criteria. We begin with an arbitrary large γ upper bound and try to find the smallest γ that will not violate the algorithm requirements. After few iterations we find that γ=65 (dB) works and we click on “No” meaning that we do not want to try another value but to accept the current controller.

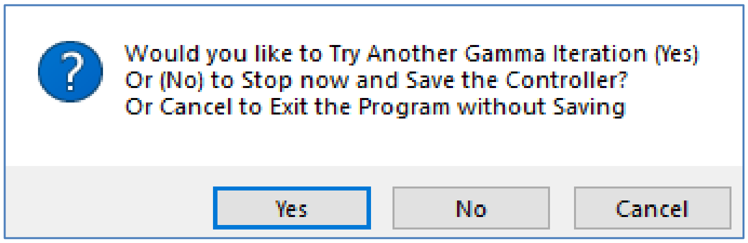


Enter the Sensitivity Upper Bound Gamma in (Decibell) to be Minimized Using "Gamma" Iterations

65.00



Great! All Eigenvalues of Matrix {Y_inf}
Have Positive Real Parts ....
The Eigenvalues of Matrix [A - R*X] from the Riccati
Solution are Stable.   Continuing with the Next Step.



Would you like to Try Another Gamma Iteration (Yes)
Or (No) to Stop now and Save the Controller?
Or Cancel to Exit the Program without Saving

Figure-33 shows the control system poles with the loop closed via the dynamic controller K(s) as in Figure-31. They are all stable as it should be expected. Note, that we have twice as many poles in comparison with Section-1 because of the estimator states, which were obtained using state-feedback. We now have four complex pairs of poles near the disturbance frequency range. The lines are used for damping coefficient ζ reference. The green line corresponds to ζ=0.707 and the red line to ζ=0.01. We return to the H-infinity main menu and we can save the controller gain by clicking on “*Save the H-infinity Controller in Systems File (x.Qdr)*”.

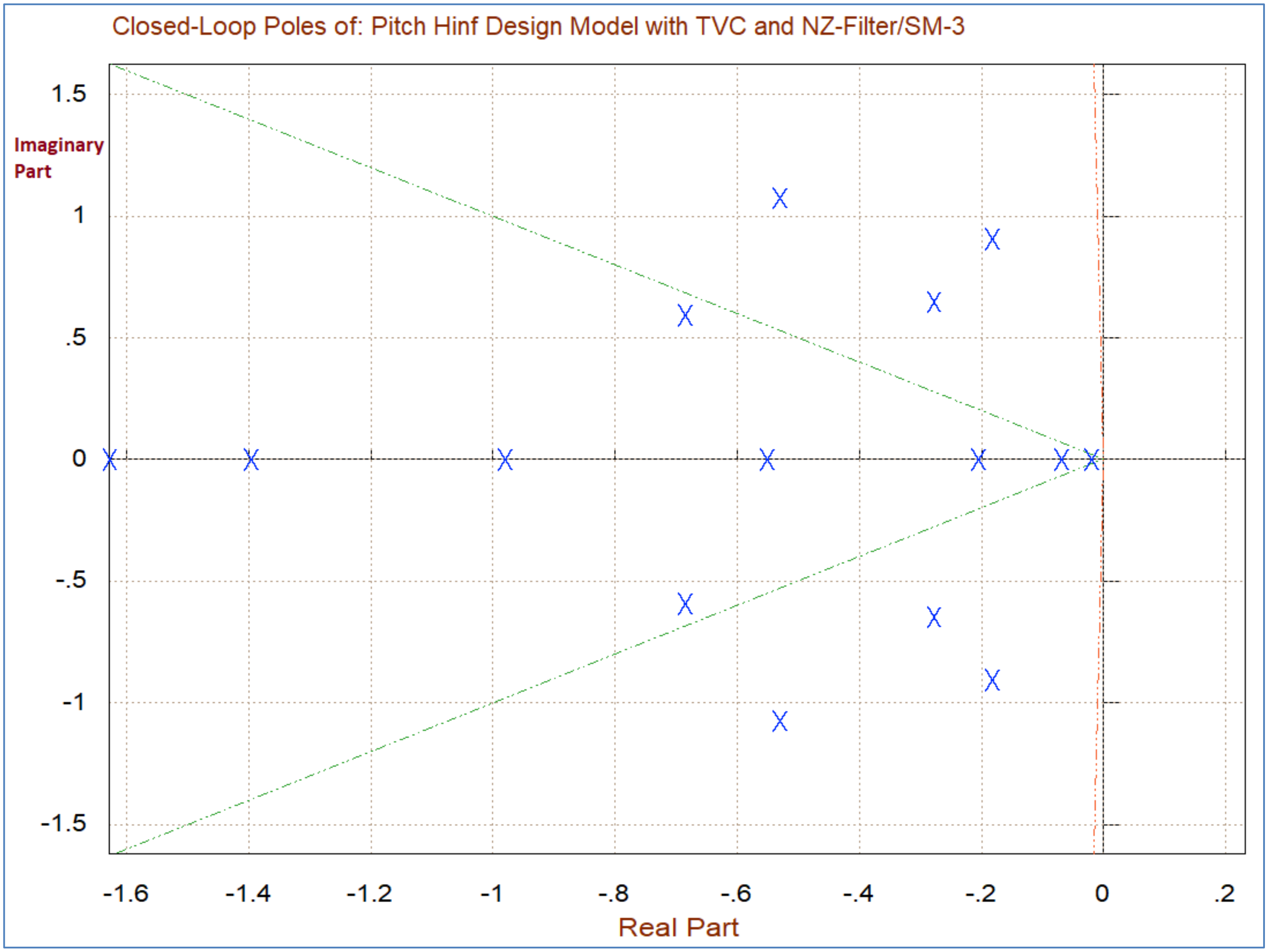
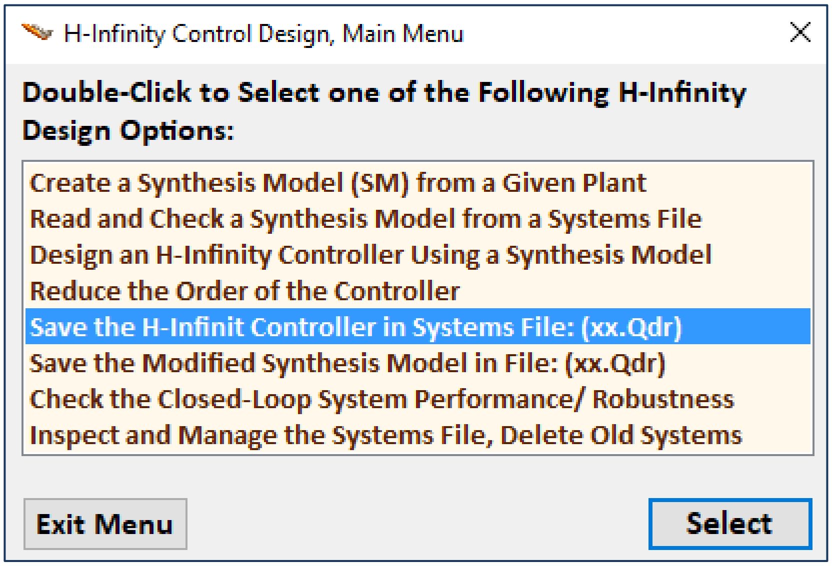


Figure 32 Closed-Loop System Poles

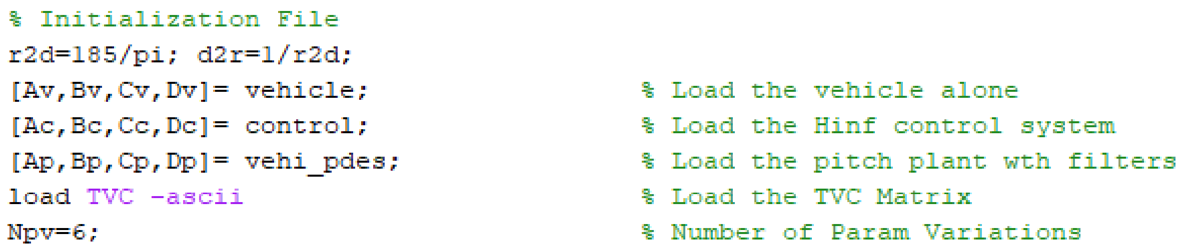


Double-Click to Select one of the Following H-Infinity Design Options:

- Create a Synthesis Model (SM) from a Given Plant
- Read and Check a Synthesis Model from a Systems File
- Design an H-Infinity Controller Using a Synthesis Model
- Reduce the Order of the Controller
- Save the H-Infinit Controller in Systems File: (xx.Qdr)
- Save the Modified Synthesis Model in File: (xx.Qdr)
- Check the Closed-Loop System Performance/ Robustness
- Inspect and Manage the Systems File, Delete Old Systems

3.6 Control Analysis

We begin the control analysis by running the initialization file “init.m” which loads the vehicle systems, the TVC matrix and the H-infinity control system that was saved in file “control.m”. We will use Matlab to analyze the control system stability and sensitivity to wind-gust disturbances and then analyze the system robustness to uncertainties. The analysis files are in directory “*Flixan\Control Analysis\Hinfinity\Examples\ Launch Vehicle Design & Analysis\3-LV Gust Robust Design, Uncs, Output-Feedback*”. In this analysis we will use simple Simulink models that include the augmented pitch system “*Pitch Hinf Design Model with TVC and NZ-Filter*” which is loaded from file “*vehi_pdes*”.



```
% Initialization File
r2d=185/pi; d2r=1/r2d;
[Av,Bv,Cv,Dv]= vehicle;                  % Load the vehicle alone
[Ac,Bc,Cc,Dc]= control;                  % Load the Hinf control system
[Ap,Bp,Cp,Dp]= vehi_pdes;                % Load the pitch plant wth filters
load TVC -ascii                          % Load the TVC Matrix
Npv=6;                                   % Number of Param Variations
```

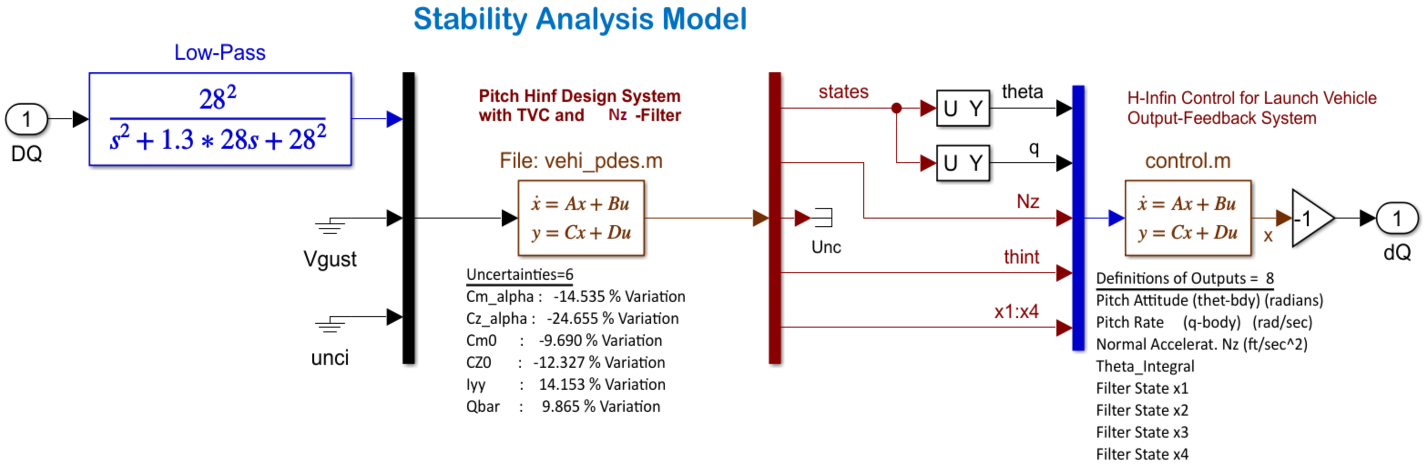


Figure 33 Pitch Stability Analysis Simulink Model in File “Open_Loop_3.slx”

Figure 34 shows the pitch stability analysis model in file “*Open_Loop_3.slx*” that is used to calculate the open-loop Bode and Nichols plots. It includes the system “*Pitch Hinf Design Model with TVC and NZ-Filter*” from file “*vehi_pdes.m*” which includes 6 parameter uncertainties for robustness analysis. Most of the control system inputs are also state variables. However, the Nz acceleration is used this time for feedback instead of alpha. A low-pass filter is also included to attenuate high frequencies. Sensitivity to gusts is analyzed using the closed-loop model “*Sensitiv_3.slx*”, shown in Figure-35, which includes the same vehicle system. The wind-gust input is scaled by multiplying it with the maximum expected gust velocity which is 25 (feet/sec) and the α-output is scaled by dividing it with the maximum allowed α dispersion angle, which is 4° or 0.07 (rad). The peak of the scaled sensitivity transfer function should, therefore, be less than 1 at all frequencies in order to satisfy good performance.

The script file “freq.m” shown below calculates the Nichols and Bode plots, and also the Sigma plot of the sensitivity function. Figure 35 shows the open-loop Bode plot that has a big double resonance at the disturbance frequencies: 0.6 to 0.9 r/s. The cross-over frequency is at 4.5 (rad/sec). The Nichols plot in Figure 36 shows the two resonances produced by the filter and the gain and phase margins.

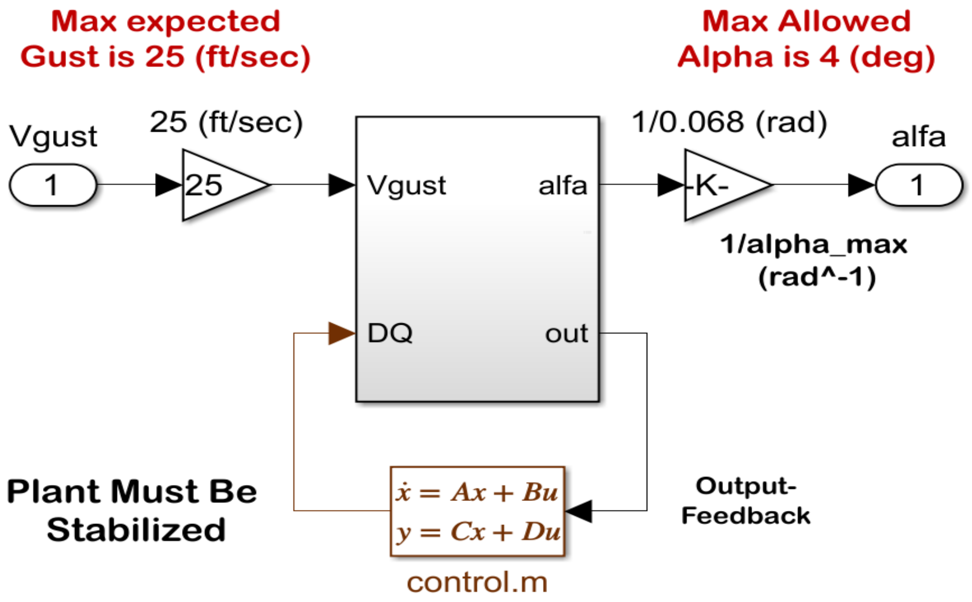
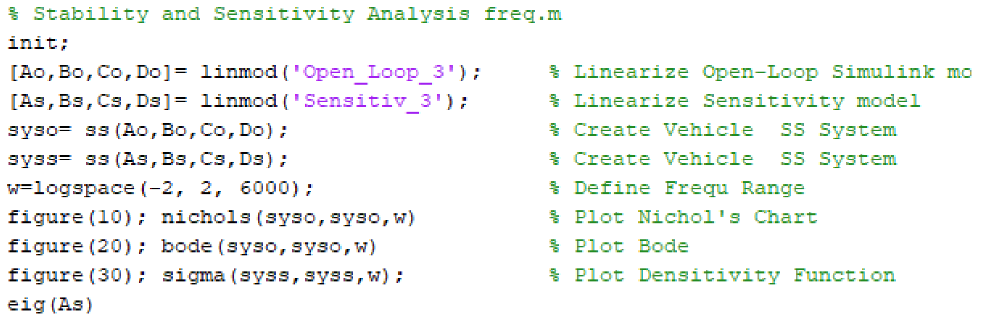


Figure 34 Sensitivity Analysis Model in File: "Sensitiv_3.slx"



```
% Stability and Sensitivity Analysis freq.m
init;
[Ao,Bo,Co,Do]= linmod('Open_Loop_3');      % Linearize Open-Loop Simulink mo
[As,Bs,Cs,Ds]= linmod('Sensitiv_3');       % Linearize Sensitivity model
syso= ss(Ao,Bo,Co,Do);                     % Create Vehicle  SS System
syss= ss(As,Bs,Cs,Ds);                     % Create Vehicle  SS System
w=logspace(-2, 2, 6000);                   % Define Frequ Range
figure(10); nichols(syso,syso,w)           % Plot Nichol's Chart
figure(20); bode(syso,syso,w)              % Plot Bode
figure(30); sigma(syss,syss,w);            % Plot Densitivity Function
eig(As)
```

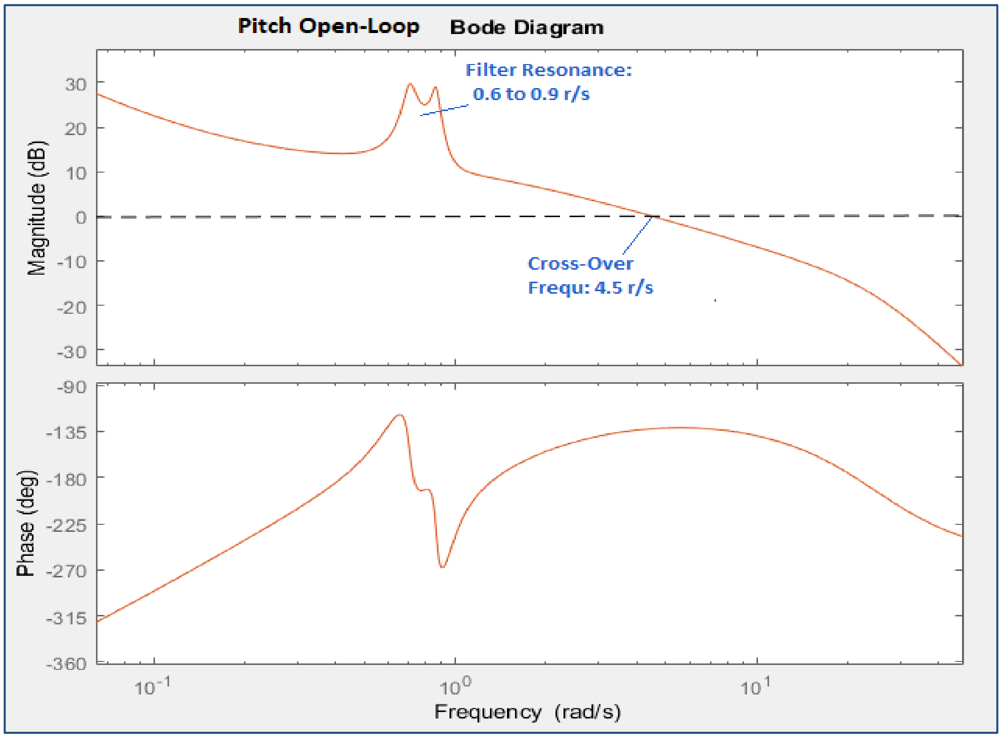


Figure 35 Open-Loop Bode Plot

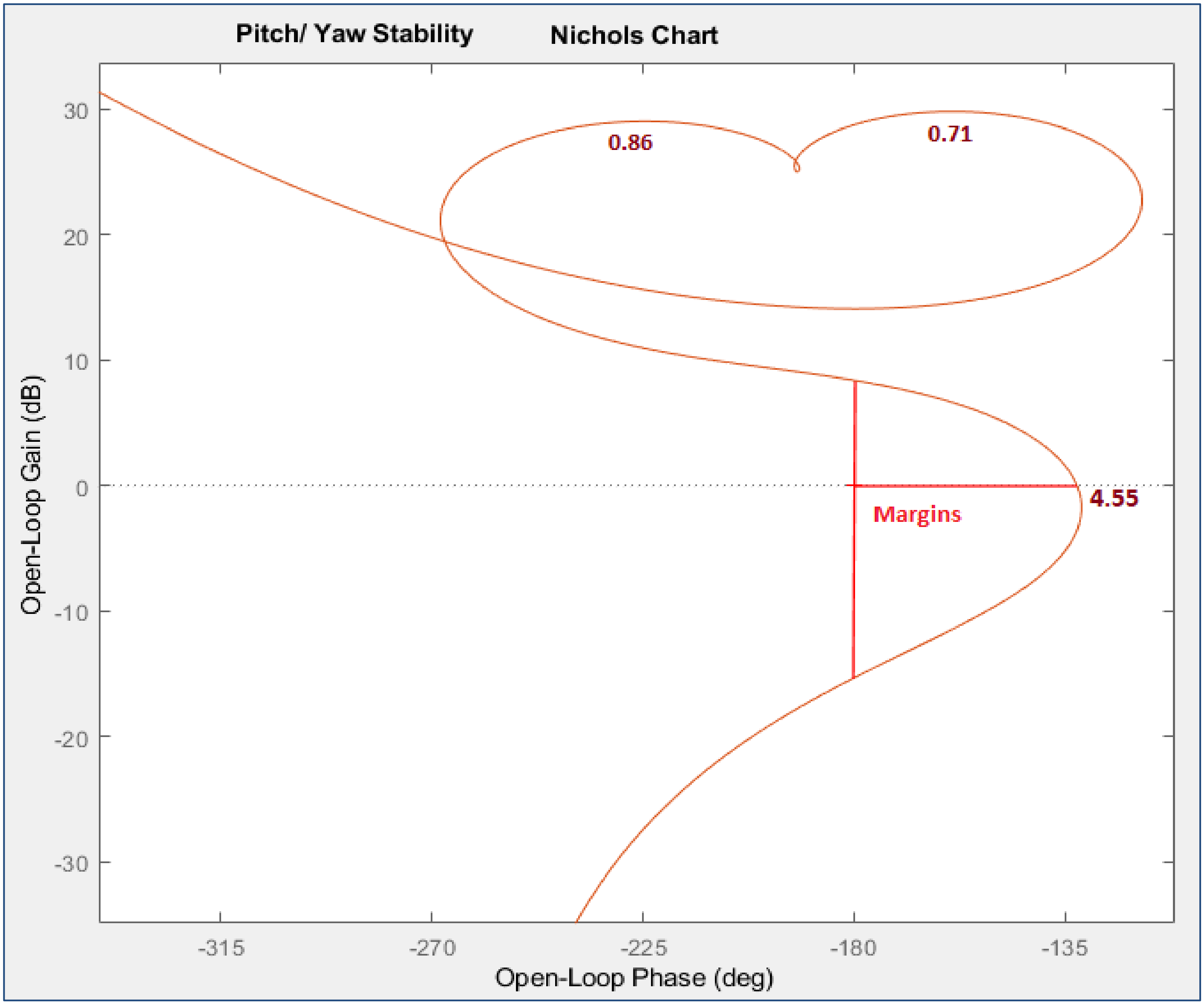
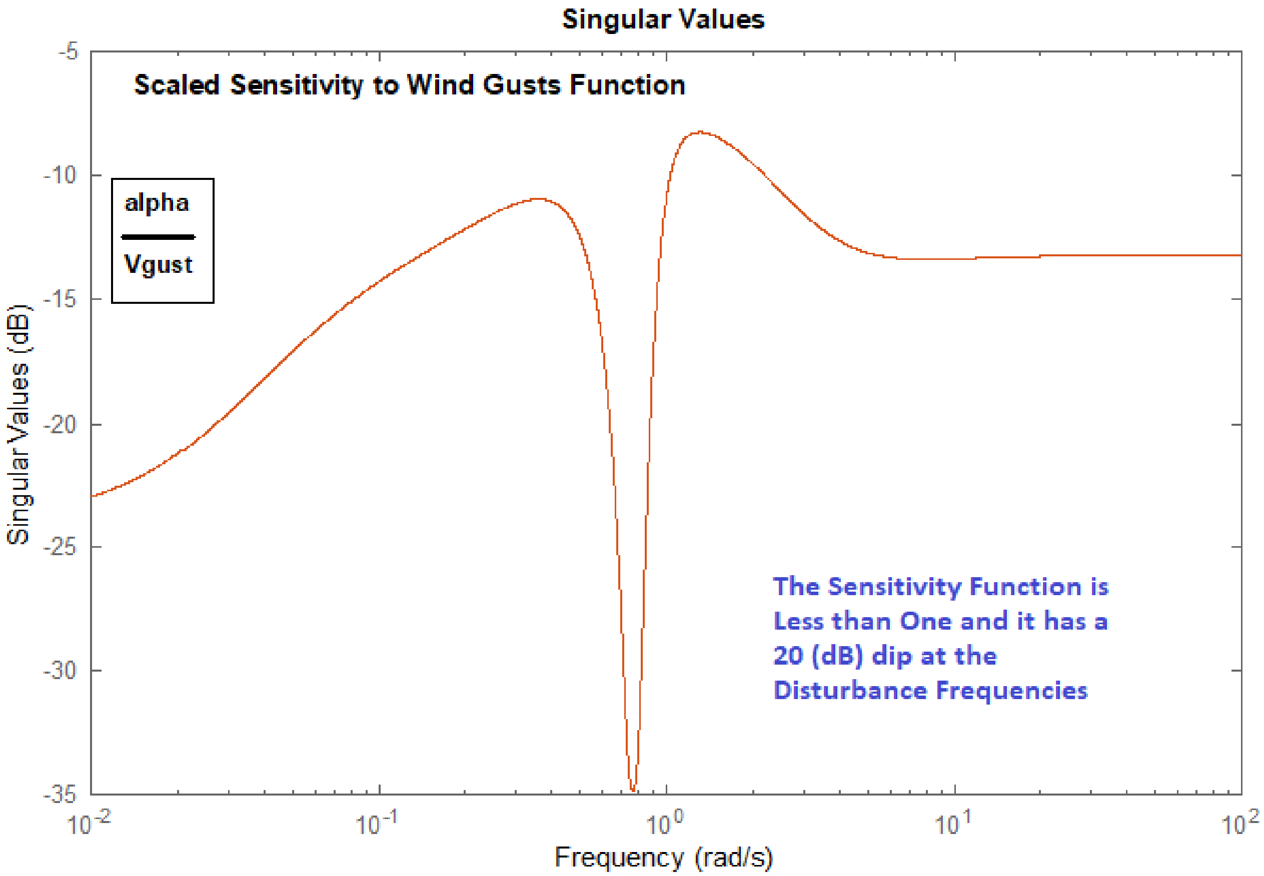


Figure 36 Nichols Plot Showing the Double Resonance and the Phase/ Gain Margins

Figure 37 shows the sensitivity function calculated between wind-gust velocity input and the angle of attack dispersion from nominal α. The system's input and output are scaled as shown in Figure 35. The magnitude of the SV plot is less than one at all frequencies, as expected. The Nz-filter produces an additional 25 (dB) sensitivity dip at the disturbance frequency range between 0.6 to 0.9 (rad/sec).

The two filter modes which are excited by the normal acceleration act like a counter-resonance against the disturbance. With the proper selection of feedback gains from states x1 to x4 which are obtained from the H-infinity solution, the filter modes are reducing the loading effects of the disturbance against the structure. Sensitivity is reduced by cycling the vehicle attitude towards the oscillatory wind disturbance and thus reducing the structural loading as we shall see in the simulation.

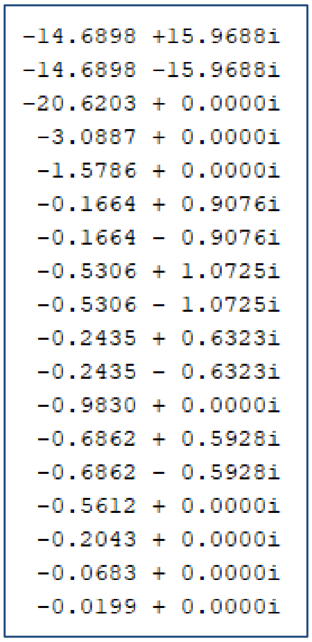


The Sensitivity Function is Less than One and it has a 20 (dB) dip at the Disturbance Frequencies

Figure 37 Sensitivity Function Between Gust Velocity and Angle of Attack

3.7 Robust Performance

We have already proven that closed-loop stability is nominally stable, see the eigenvalues on the right. We can also satisfy performance requirements regarding aero-loading with respect to wind-gusts. But can we satisfy both stability and performance when we have variations in some important parameters? To answer that we need to analyze robust performance. The system satisfies robust performance criterion when the (SSV or μ) of the closed-loop system from: (the 6 uncertainty inputs plus Vgust) to (the 6 uncertainty outputs plus alpha) is less than one at all frequencies, assuming that it is already normalized with respect to the uncertainties block Δ, and also between Vgust and alpha. Robust Performance is calculated using the Simulink model "Robust-Perform_3.slx" shown in Figure-38. The Matlab script "Run_Robust.m" below performs this operation and calculates the SSV frequency response which is less than 1 at all frequencies as shown in Figure-39.



```
-14.6898 +15.9688i
-14.6898 -15.9688i
-20.6203 + 0.0000i
 -3.0887 + 0.0000i
 -1.5786 + 0.0000i
 -0.1664 + 0.9076i
 -0.1664 - 0.9076i
 -0.5306 + 1.0725i
 -0.5306 - 1.0725i
 -0.2435 + 0.6323i
 -0.2435 - 0.6323i
 -0.9830 + 0.0000i
 -0.6862 + 0.5928i
 -0.6862 - 0.5928i
 -0.5612 + 0.0000i
 -0.2043 + 0.0000i
 -0.0683 + 0.0000i
 -0.0199 + 0.0000i
```

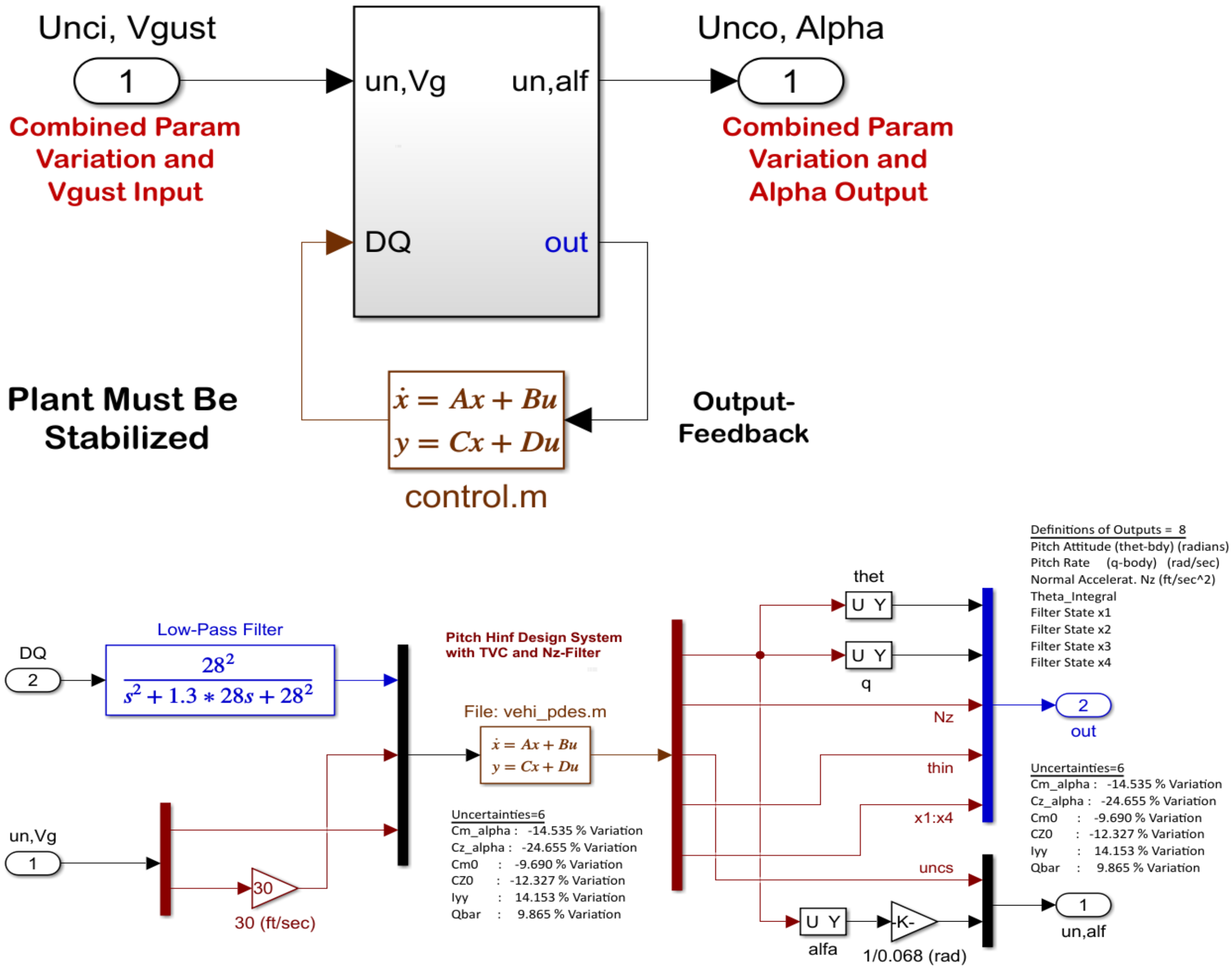


Figure 38 Robust Performance Analysis Model "Robust_Perform_3.slx"

```
% Robust Performance Analysis File
clear all; init
Npv=6;                              % Number of Param Variations
w=logspace(-2,2,800);
[Acp,Bcp,Ccp,Dcp]=linmod('Robust_Perform_3');
eig(Acp)
sys=ss(Acp,Bcp,Ccp,Dcp);
sysf= frd(sys,w);
blk=[-ones(Npv+1,1), zeros(Npv+1,1)];
[bnd,muinfo]= mussv(sysf,blk);

ff= get(muinfo.bnds, 'frequency');
muu=get(muinfo.bnds, 'responsedata');
muu=squeeze(muu);
muu=muu(1,:);
loglog(ff,muu)
ylim([0.01,1.1])
ylabel('ssv')
xlabel('Frequency (rad/sec)')
```

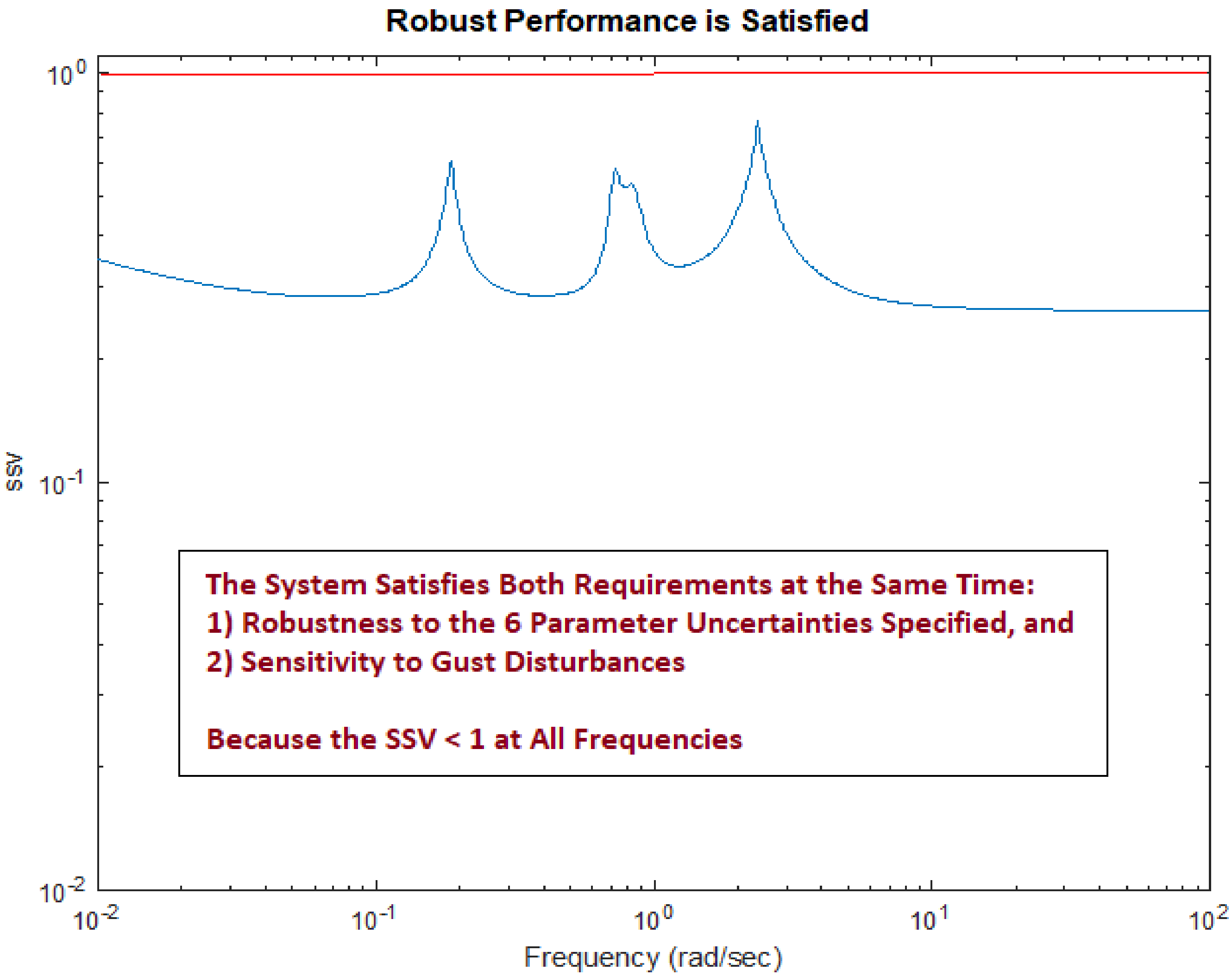


Figure 39 SSV Frequency Response Proves that the System Satisfies Both: Performance to Gusts and Robustness with respect to Uncertainties

3.8 Simulations

At high dynamic pressures the command following capability of the control system is slow because the load-relief system is preventing aero-loading and the vehicle is not expected to perform attitude maneuvers during that period which lasts only 15-20 sec, but the vehicle maintains a steady or very slowly changing attitude relative to the wind. It should be able, however, to track commands from guidance even at a slow rate. Figure 40 shows the attitude response to a step attitude command. The Nz-filter causes an oscillatory transient but the attitude eventually converges to the command.

The effect of the disturbance filter is shown in Figure-41, where the vehicle is excited with an external oscillatory wind-gust disturbance of 0.75 (rad/sec) frequency. The angle of attack and attitude responses are shown, without (left) and with the Nz-Filter (right). When the filter is included, the vehicle responds to the oscillatory wind-gust by turning towards it and thus reducing the alpha oscillation from an amplitude of 0.013 (rad) without filter to 0.002 (rad) with filter. The attitude oscillation amplitude increases from 0.0055 without filter to 0.02 (rad) with filter in order to maneuver against the wind and reduce the aero-loading.

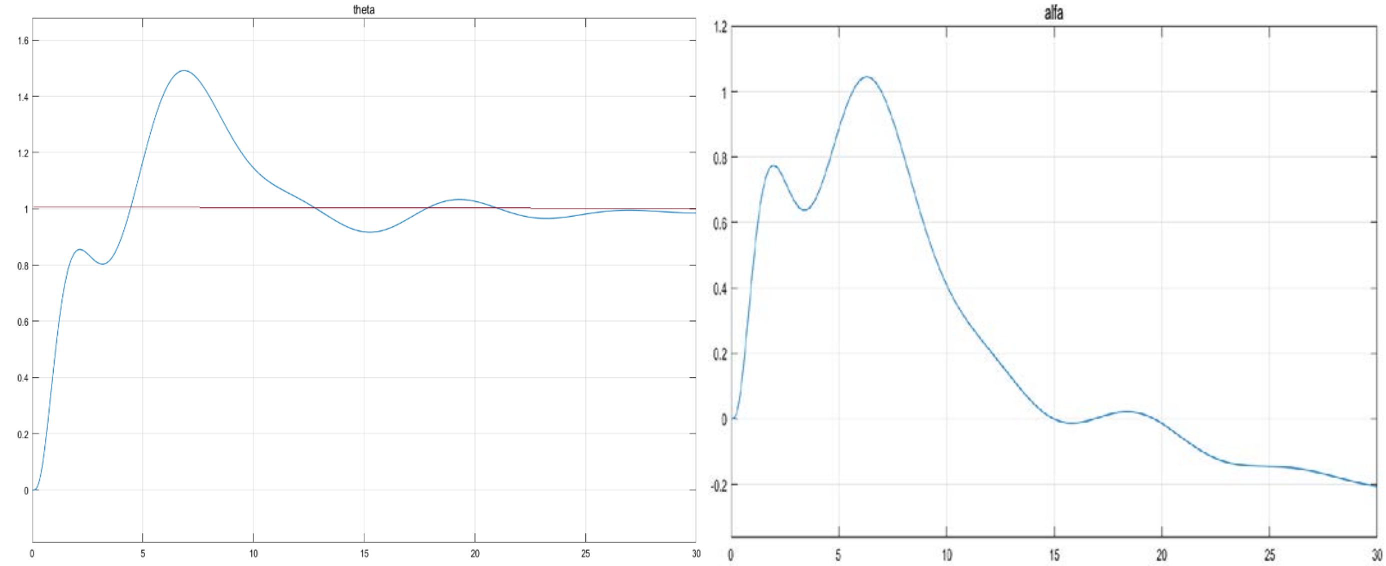


Figure 40 Attitude and Angle of Attack Response to a Step Attitude Command

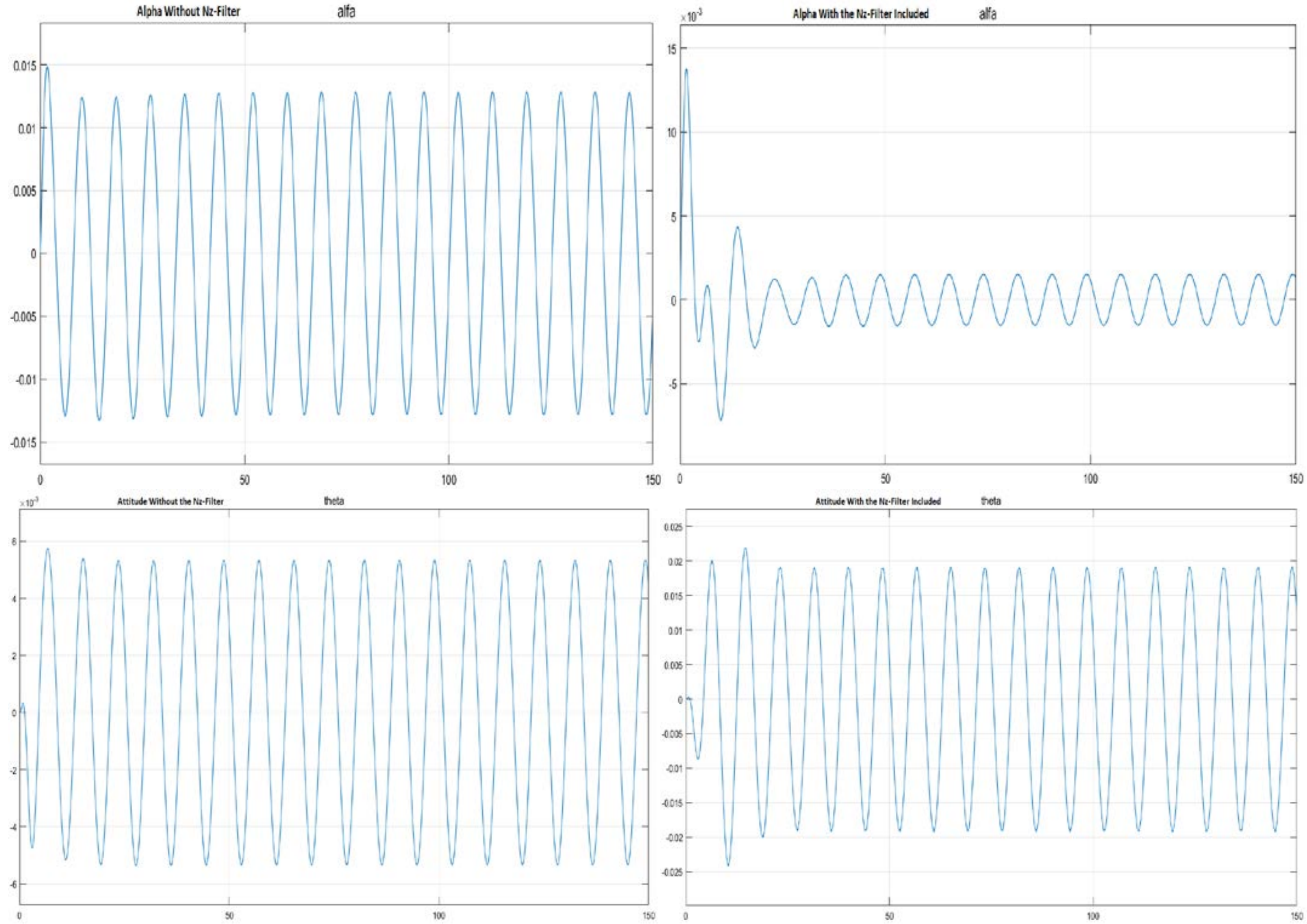


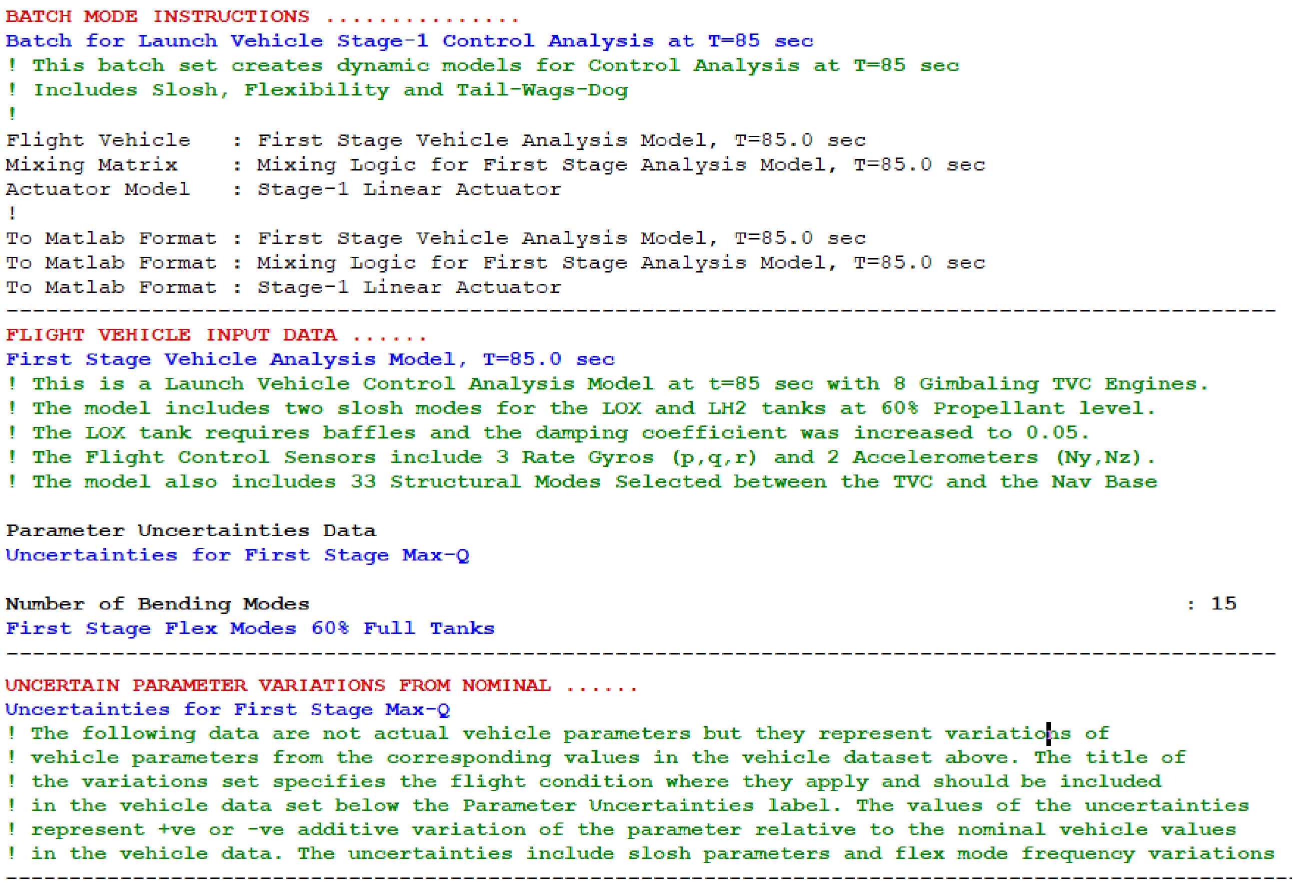
Figure 41 Alpha and Theta Response to a 0.75 r/s Oscillatory Wind-Gust Disturbance, Without and With the Nz-Filter. The Vehicle Maneuverability is Increased with Nz-Filter in Order to Reduce the Aero-Loading

4. Dynamic Controller Analysis with Slosh and Flexibility

The single axis output feedback control design presented in Section-3 will now be analyzed using the coupled axes vehicle model which includes slosh and structural flexibility. Pitch and yaw are identical and we will also include roll in the dynamic model and analysis. TVC and detailed actuator models are also included. We will analyze system stability, sensitivity to wind-gusts and calculate the control system's response to attitude commands and to cyclic disturbances, as before. We will also calculate the control system's robustness to parameter uncertainties. The analysis files are in directory "*Flixan\Control Analysis\Hinfinity\Examples\LV Design & Analysis\4-LV Flex, Slosh Analysis, Uncs, Nz-Outp-Feedbck*". The Flixan data are in file: "*LV_Flex_Anal.Inp*" and the systems and matrices are saved in file: "*LV_Flex-Anal.Qdr*".

4.1 The Input File

The input data file "*LV_Flex_Anal.Inp*" is partially shown below. It begins with a batch dataset for processing the entire file in batch mode. It includes the same vehicle model "*First Stage Vehicle Analysis Model, T=85.0 sec*" with two slosh modes, TWD dynamics and 15 flex modes from the modal data set "*First Stage Flex Modes 60% Full Tanks*" which is at the bottom of the file. The vehicle also includes uncertainties which are defined in dataset "*Uncertainties for First Stage Max-Q*" containing 38 parameter variations from nominal. The uncertainties will be split into 19 pitch and 19 lateral variations. There is a mixing logic dataset "*Mixing Logic for First Stage Analysis Model, T=85.0 sec*" that creates the TVC matrix, an actuator dataset "*Stage-1 Linear Actuator*" that contains actuator parameters for the 8 gimbaling engines, and Flixan to Matlab data conversion datasets, as before.



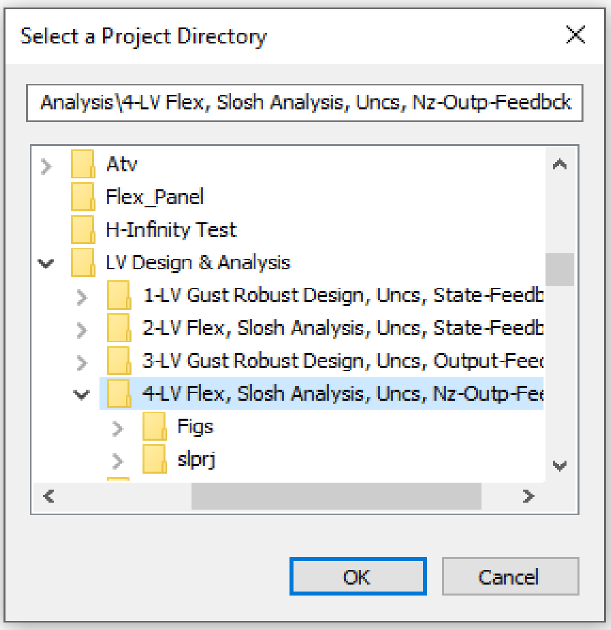
```
BATCH MODE INSTRUCTIONS ...............
Batch for Launch Vehicle Stage-1 Control Analysis at T=85 sec
! This batch set creates dynamic models for Control Analysis at T=85 sec
! Includes Slosh, Flexibility and Tail-Wags-Dog
!
Flight Vehicle    : First Stage Vehicle Analysis Model, T=85.0 sec
Mixing Matrix     : Mixing Logic for First Stage Analysis Model, T=85.0 sec
Actuator Model    : Stage-1 Linear Actuator
!
To Matlab Format : First Stage Vehicle Analysis Model, T=85.0 sec
To Matlab Format : Mixing Logic for First Stage Analysis Model, T=85.0 sec
To Matlab Format : Stage-1 Linear Actuator
-------------------------------------------------------------------------------------------------
FLIGHT VEHICLE INPUT DATA ......
First Stage Vehicle Analysis Model, T=85.0 sec
! This is a Launch Vehicle Control Analysis Model at t=85 sec with 8 Gimbaling TVC Engines.
! The model includes two slosh modes for the LOX and LH2 tanks at 60% Propellant level.
! The LOX tank requires baffles and the damping coefficient was increased to 0.05.
! The Flight Control Sensors include 3 Rate Gyros (p,q,r) and 2 Accelerometers (Ny,Nz).
! The model also includes 33 Structural Modes Selected between the TVC and the Nav Base

Parameter Uncertainties Data
Uncertainties for First Stage Max-Q

Number of Bending Modes                                                          : 15
First Stage Flex Modes 60% Full Tanks
-------------------------------------------------------------------------------------------------
UNCERTAIN PARAMETER VARIATIONS FROM NOMINAL ......
Uncertainties for First Stage Max-Q
! The following data are not actual vehicle parameters but they represent variations of
! vehicle parameters from the corresponding values in the vehicle dataset above. The title of
! the variations set specifies the flight condition where they apply and should be included
! in the vehicle data set below the Parameter Uncertainties label. The values of the uncertainties
! represent +ve or -ve additive variation of the parameter relative to the nominal vehicle values
! in the vehicle data. The uncertainties include slosh parameters and flex mode frequency variations
--------------------------------------------------------------------------------------------------
```

```
MIXING LOGIC MATRIX DATA ..........    (Matrix Title, Name, Vehicle Title, Control Directions)
Mixing Logic for First Stage Analysis Model, T=85.0 sec
! Thrust Vector Control Matrix at t=85 sec
! This multi-engine vehicle has 8 Gimbaling Engines.
TVC
First Stage Vehicle Analysis Model, T=85.0 sec
P-dot Roll  Acceleration About X Axis
Q-dot Pitch Acceleration About Y Axis
R-dot Yaw   Acceleration About Z Axis
-------------------------------------------------------------------------------------------------
ACTUATOR INPUT DATA ............ SIMPLE GENERIC MODEL B
Stage-1 Linear Actuator
Symbol Parameter Description                    (Units)       Value
C(s)   Order of Pade Delay (0,1,2)                ( - )       1, -0.001, 0.001
Ka     Gain of Amplifier ..................... (amps/volt)   28.0
Wsv    Bandwidth of the Linear Servo Actuator .  (rad/sec)    65.0
Kact   Actuator Stiffness (Piston+Oil+Electric)   (lb/ft)     2.4e+6
Klod   Stiffness at Surface or Nozzle Connection (lb/ft)     1.2e+9
Kbck   Stiffness at Vehicle Backup Structure ..   (lb/ft)     8.0e+7
R      Moment Arm between Actuator Rod & Gimbal   (feet)      0.667
Jl     Load Inertia about the Gimbal .......... (ft-lb-s^2)   18.0
Kg     Load Gimbal Bearing Spring Constant      (ft-lb/rad)   0.0
Bg     Load Gimbal Bearing Viscous Damping .... (ft-lb-sec)   550.0
-------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Mixing Logic for First Stage Analysis Model, T=85.0 sec
Matrix TVC
-------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
First Stage Vehicle Analysis Model, T=85.0 sec
System
vehicle_t85.m
-------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Stage-1 Linear Actuator
System
actuator.m
-------------------------------------------------------------------------------------------------
SELECTED MODAL DATA AND LOCATIONS FOR : 60% Full
First Stage Flex Modes 60% Full Tanks
! Flex Modes, First Stage 60% Full Tanks.
! Sensors are at the Top of LOX Tank at Node: 40015
! The Modes were selected between the TVC (Node:10001) and the IMU Locat. (Node:40015)

MODE#  1/  1, Frequency (rad/sec), Damping (zeta), Generalized Mass=    31.0         0.50000E-02      12.000
DEFINITION OF LOCATIONS (NODES)            phi along X   phi along Y   phi along Z   sigm about X  sigm about Y  sigm about Z

                              Node ID#     Modal Data at the  8 Engines, (x,y,z)...
S1 Engine No:1 +Y 0Z             18915    -0.12901D-01  -0.10186D+00  -0.56622D-01   0.34565D-03  -0.29891D-02   0.65573D-02
S1 Engine No:2 +Y +Z             18908    -0.14488D-01  -0.10161D+00  -0.57885D-01  -0.37952D-04  -0.41592D-02   0.64361D-02
S1 Engine No:3 0Y +Z             18909    -0.72726D-02  -0.10033D+00  -0.57407D-01  -0.40727D-03  -0.36961D-02   0.52490D-02
S1 Engine No:4 -Y +Z             18910     0.40387D-02  -0.10077D+00  -0.56226D-01  -0.54455D-03  -0.26467D-02   0.56298D-02
S1 Engine No:5 -Y 0Z             18911     0.12911D-01  -0.10186D+00  -0.56605D-01  -0.35178D-03  -0.29837D-02   0.65566D-02
S1 Engine No:6 -Y -Z             18912     0.14482D-01  -0.10163D+00  -0.57877D-01   0.23398D-04  -0.41620D-02   0.64502D-02
S1 Engine No:7 0Y -Z             18913     0.72833D-02  -0.10033D+00  -0.57400D-01   0.40945D-03  -0.36898D-02   0.52470D-02
S1 Engine No:8 +Y -Z             18914    -0.40344D-02  -0.10078D+00  -0.56242D-01   0.54324D-03  -0.26519D-02   0.56338D-02

                              Node ID#     Modal Data at the  3 Gyros ...
Stg-2 Tank Top, IMU Location     40015     0.36941D-05  -0.34870D-01  -0.19652D-01  -0.23159D-05   0.21456D-02  -0.38063D-02
Stg-2 Tank Top, IMU Location     40015     0.36941D-05  -0.34870D-01  -0.19652D-01  -0.23159D-05   0.21456D-02  -0.38063D-02
Stg-2 Tank Top, IMU Location     40015     0.36941D-05  -0.34870D-01  -0.19652D-01  -0.23159D-05   0.21456D-02  -0.38063D-02

                              Node ID#     Modal Data at the  2 Accelerometers, along (x,y,z)...
Stg-2 Tank Top, IMU Location     40015     0.36941D-05  -0.34870D-01  -0.19652D-01
Stg-2 Tank Top, IMU Location     40015     0.36941D-05  -0.34870D-01  -0.19652D-01

                              Node ID#     Modal Data at the  2 Slosh Masses...
LOX  Slosh Mass Locat.             601     0.36536D-05   0.61141D-01   0.34476D-01  -0.22194D-05   0.20894D-03  -0.37039D-03
Fuel Slosh Mass Locat.             600     0.35377D-05  -0.74025D-02  -0.41671D-02  -0.21259D-05  -0.24678D-02   0.43770D-02

                              Node ID#     Modal Data at the Disturbance Point
S1 Engine No:9 center            10001    -0.44302D-04  -0.10038D+00  -0.56615D-01   0.25363D-06  -0.27352D-02   0.48392D-02
-----------------------------------------------------------------------------------------------------------------------------
```

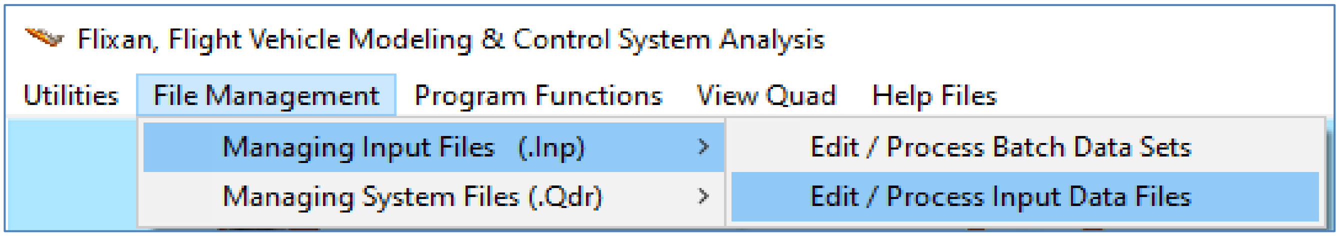
4.2 Processing the Input Data File in Batch

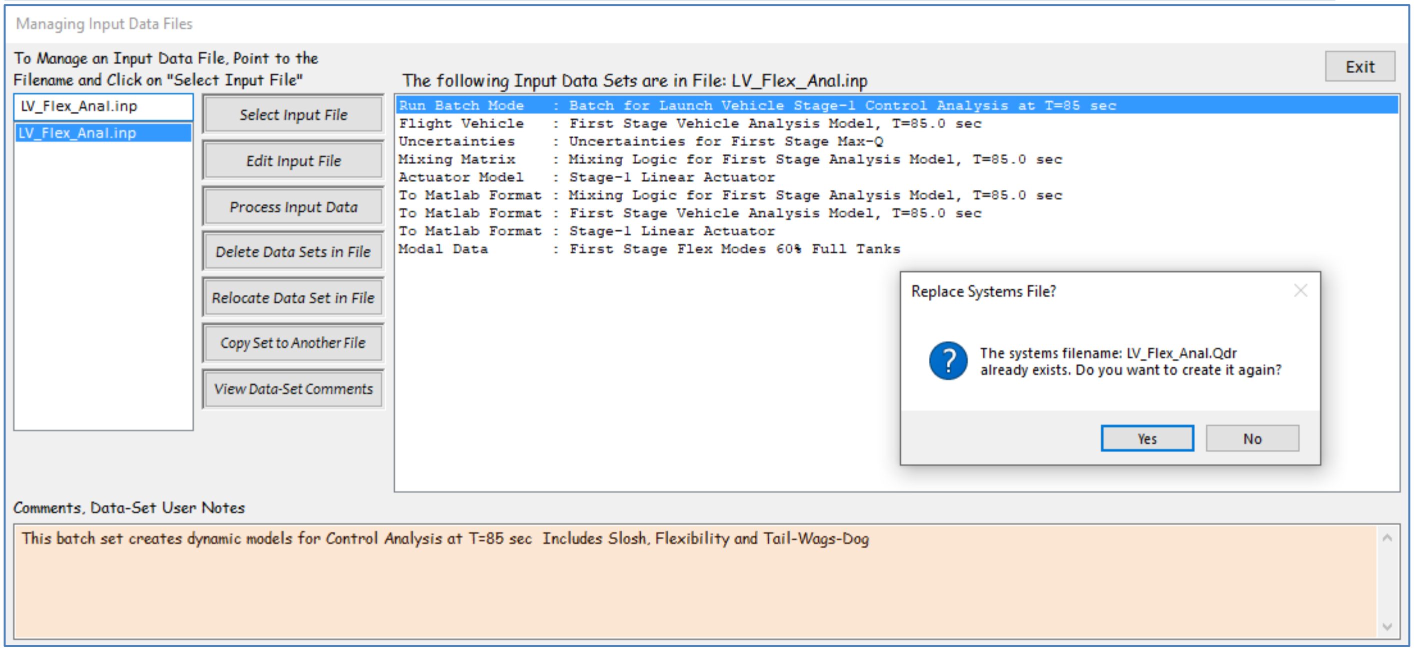
To process the input file in batch mode we start the Flixan program and select the project directory: “*Flixan\Control Analysis\ Hinfinity\ Examples\ LV Design & Analysis\4-LV Flex, Slosh Analysis, Uncs, Nz-Outp-Feedbck*”. From the main menu select “*File Management*”, “*Managing Input Files*”, and then “*Edit/ Process Input Data Files*”, as shown below.

The following dialog includes two menus. The menu on the left side shows the input data files in the project directory. There is only one file, highlight it and then click on “*Select Input File*” button. The menu on the right shows the datasets which are in the input file. Select the batch dataset title which is at the top of the list and click on “*Process Input Data*”.


Select a Project Directory

Analysis\4-LV Flex, Slosh Analysis, Uncs, Nz-Outp-Feedbck




Managing Input Data Files

To Manage an Input Data File, Point to the Filename and Click on "Select Input File"

The following Input Data Sets are in File: LV_Flex_Anal.inp

```
Run Batch Mode    : Batch for Launch Vehicle Stage-1 Control Analysis at T=85 sec
Flight Vehicle    : First Stage Vehicle Analysis Model, T=85.0 sec
Uncertainties     : Uncertainties for First Stage Max-Q
Mixing Matrix     : Mixing Logic for First Stage Analysis Model, T=85.0 sec
Actuator Model    : Stage-1 Linear Actuator
To Matlab Format  : Mixing Logic for First Stage Analysis Model, T=85.0 sec
To Matlab Format  : First Stage Vehicle Analysis Model, T=85.0 sec
To Matlab Format  : Stage-1 Linear Actuator
Modal Data        : First Stage Flex Modes 60% Full Tanks
```

Replace Systems File?

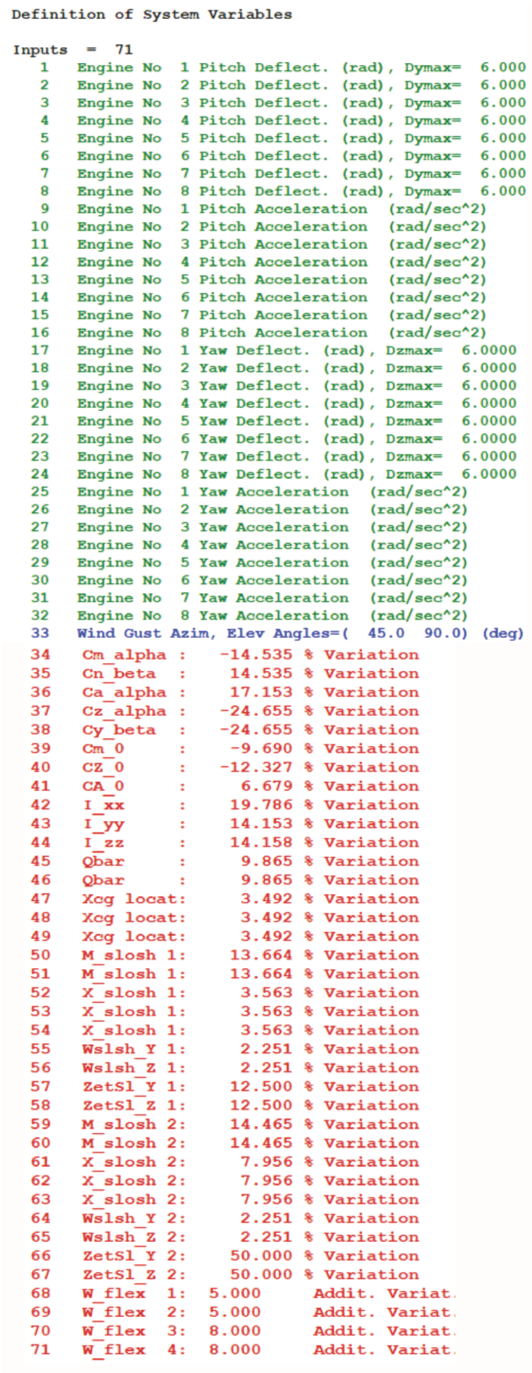
The systems filename: LV_Flex_Anal.Qdr already exists. Do you want to create it again?

Comments, Data-Set User Notes

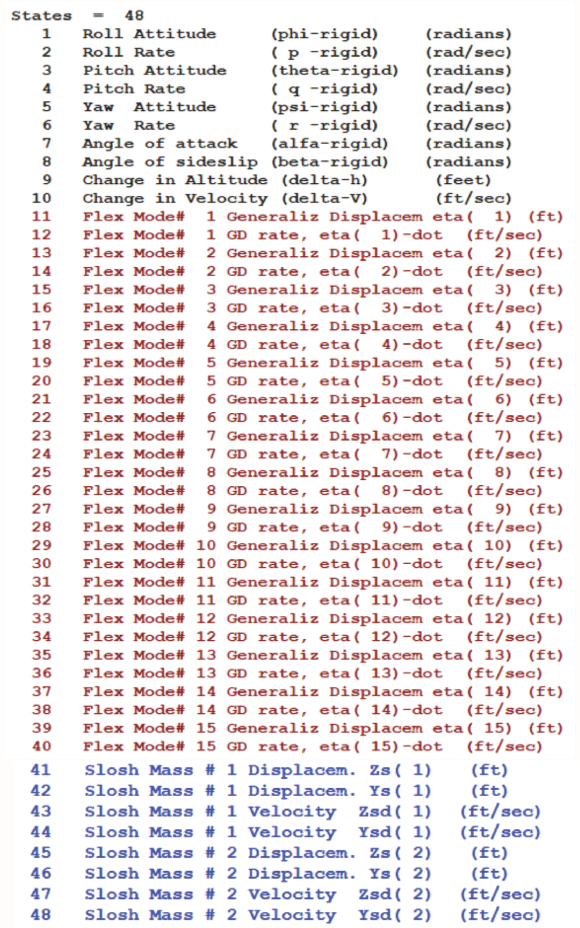
This batch set creates dynamic models for Control Analysis at T=85 sec Includes Slosh, Flexibility and Tail-Wags-Dog

In the following question, answer “Yes”, which is okay to delete and recreate the old systems file. The batch executes and creates the systems and matrices that can now be loaded into Matlab.

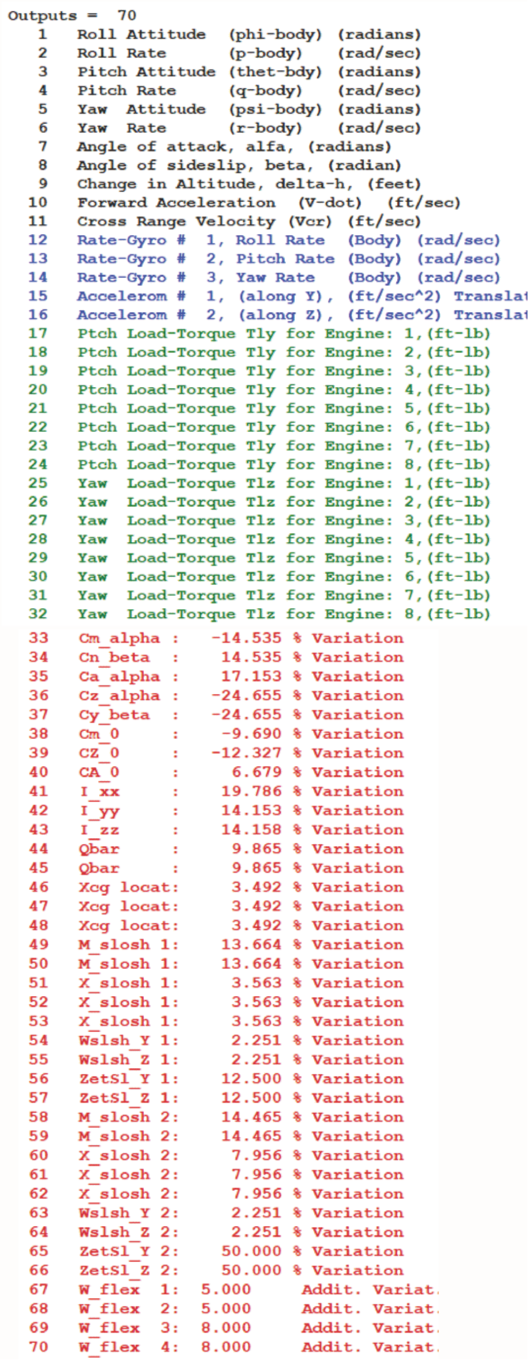
Definitions of Vehicle Inputs, States and Outputs



```
Definition of System Variables

Inputs  =  71
  1    Engine No  1 Pitch Deflect. (rad), Dymax=  6.000
  2    Engine No  2 Pitch Deflect. (rad), Dymax=  6.000
  3    Engine No  3 Pitch Deflect. (rad), Dymax=  6.000
  4    Engine No  4 Pitch Deflect. (rad), Dymax=  6.000
  5    Engine No  5 Pitch Deflect. (rad), Dymax=  6.000
  6    Engine No  6 Pitch Deflect. (rad), Dymax=  6.000
  7    Engine No  7 Pitch Deflect. (rad), Dymax=  6.000
  8    Engine No  8 Pitch Deflect. (rad), Dymax=  6.000
  9    Engine No  1 Pitch Acceleration  (rad/sec^2)
 10    Engine No  2 Pitch Acceleration  (rad/sec^2)
 11    Engine No  3 Pitch Acceleration  (rad/sec^2)
 12    Engine No  4 Pitch Acceleration  (rad/sec^2)
 13    Engine No  5 Pitch Acceleration  (rad/sec^2)
 14    Engine No  6 Pitch Acceleration  (rad/sec^2)
 15    Engine No  7 Pitch Acceleration  (rad/sec^2)
 16    Engine No  8 Pitch Acceleration  (rad/sec^2)
 17    Engine No  1 Yaw Deflect. (rad), Dzmax=  6.0000
 18    Engine No  2 Yaw Deflect. (rad), Dzmax=  6.0000
 19    Engine No  3 Yaw Deflect. (rad), Dzmax=  6.0000
 20    Engine No  4 Yaw Deflect. (rad), Dzmax=  6.0000
 21    Engine No  5 Yaw Deflect. (rad), Dzmax=  6.0000
 22    Engine No  6 Yaw Deflect. (rad), Dzmax=  6.0000
 23    Engine No  7 Yaw Deflect. (rad), Dzmax=  6.0000
 24    Engine No  8 Yaw Deflect. (rad), Dzmax=  6.0000
 25    Engine No  1 Yaw Acceleration  (rad/sec^2)
 26    Engine No  2 Yaw Acceleration  (rad/sec^2)
 27    Engine No  3 Yaw Acceleration  (rad/sec^2)
 28    Engine No  4 Yaw Acceleration  (rad/sec^2)
 29    Engine No  5 Yaw Acceleration  (rad/sec^2)
 30    Engine No  6 Yaw Acceleration  (rad/sec^2)
 31    Engine No  7 Yaw Acceleration  (rad/sec^2)
 32    Engine No  8 Yaw Acceleration  (rad/sec^2)
 33    Wind Gust Azim, Elev Angles=(  45.0   90.0)  (deg)
 34    Cm_alpha  :   -14.535 % Variation
 35    Cn_beta   :    14.535 % Variation
 36    Ca_alpha  :    17.153 % Variation
 37    Cz_alpha  :   -24.655 % Variation
 38    Cy_beta   :   -24.655 % Variation
 39    Cm_0      :    -9.690 % Variation
 40    CZ_0      :   -12.327 % Variation
 41    CA_0      :     6.679 % Variation
 42    I_xx      :    19.786 % Variation
 43    I_yy      :    14.153 % Variation
 44    I_zz      :    14.158 % Variation
 45    Qbar      :     9.865 % Variation
 46    Qbar      :     9.865 % Variation
 47    Xcg locat:      3.492 % Variation
 48    Xcg locat:      3.492 % Variation
 49    Xcg locat:      3.492 % Variation
 50    M_slosh 1:     13.664 % Variation
 51    M_slosh 1:     13.664 % Variation
 52    X_slosh 1:      3.563 % Variation
 53    X_slosh 1:      3.563 % Variation
 54    X_slosh 1:      3.563 % Variation
 55    Wslsh_Y 1:      2.251 % Variation
 56    Wslsh_Z 1:      2.251 % Variation
 57    ZetSl_Y 1:     12.500 % Variation
 58    ZetSl_Z 1:     12.500 % Variation
 59    M_slosh 2:     14.465 % Variation
 60    M_slosh 2:     14.465 % Variation
 61    X_slosh 2:      7.956 % Variation
 62    X_slosh 2:      7.956 % Variation
 63    X_slosh 2:      7.956 % Variation
 64    Wslsh_Y 2:      2.251 % Variation
 65    Wslsh_Z 2:      2.251 % Variation
 66    ZetSl_Y 2:     50.000 % Variation
 67    ZetSl_Z 2:     50.000 % Variation
 68    W_flex  1:   5.000       Addit. Variat.
 69    W_flex  2:   5.000       Addit. Variat.
 70    W_flex  3:   8.000       Addit. Variat.
 71    W_flex  4:   8.000       Addit. Variat.
```



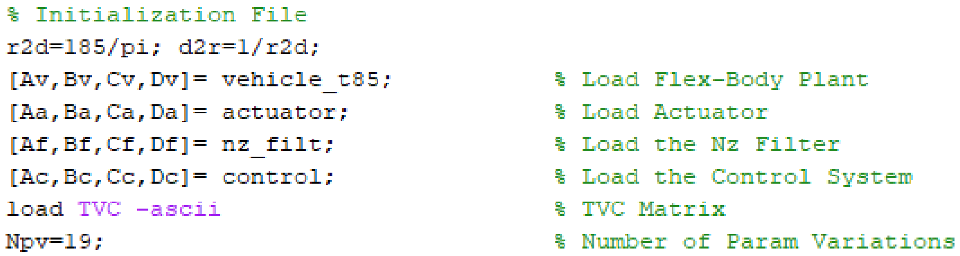
```
States   =  48
  1    Roll Attitude       (phi-rigid)       (radians)
  2    Roll Rate           ( p -rigid)       (rad/sec)
  3    Pitch Attitude      (theta-rigid)     (radians)
  4    Pitch Rate          ( q -rigid)       (rad/sec)
  5    Yaw  Attitude       (psi-rigid)       (radians)
  6    Yaw  Rate           ( r -rigid)       (rad/sec)
  7    Angle of attack     (alfa-rigid)      (radians)
  8    Angle of sideslip (beta-rigid)        (radians)
  9    Change in Altitude (delta-h)           (feet)
 10    Change in Velocity (delta-V)          (ft/sec)
 11    Flex Mode#  1 Generaliz Displacem eta(  1)  (ft)
 12    Flex Mode#  1 GD rate, eta(  1)-dot  (ft/sec)
 13    Flex Mode#  2 Generaliz Displacem eta(  2)  (ft)
 14    Flex Mode#  2 GD rate, eta(  2)-dot  (ft/sec)
 15    Flex Mode#  3 Generaliz Displacem eta(  3)  (ft)
 16    Flex Mode#  3 GD rate, eta(  3)-dot  (ft/sec)
 17    Flex Mode#  4 Generaliz Displacem eta(  4)  (ft)
 18    Flex Mode#  4 GD rate, eta(  4)-dot  (ft/sec)
 19    Flex Mode#  5 Generaliz Displacem eta(  5)  (ft)
 20    Flex Mode#  5 GD rate, eta(  5)-dot  (ft/sec)
 21    Flex Mode#  6 Generaliz Displacem eta(  6)  (ft)
 22    Flex Mode#  6 GD rate, eta(  6)-dot  (ft/sec)
 23    Flex Mode#  7 Generaliz Displacem eta(  7)  (ft)
 24    Flex Mode#  7 GD rate, eta(  7)-dot  (ft/sec)
 25    Flex Mode#  8 Generaliz Displacem eta(  8)  (ft)
 26    Flex Mode#  8 GD rate, eta(  8)-dot  (ft/sec)
 27    Flex Mode#  9 Generaliz Displacem eta(  9)  (ft)
 28    Flex Mode#  9 GD rate, eta(  9)-dot  (ft/sec)
 29    Flex Mode# 10 Generaliz Displacem eta( 10)  (ft)
 30    Flex Mode# 10 GD rate, eta( 10)-dot  (ft/sec)
 31    Flex Mode# 11 Generaliz Displacem eta( 11)  (ft)
 32    Flex Mode# 11 GD rate, eta( 11)-dot  (ft/sec)
 33    Flex Mode# 12 Generaliz Displacem eta( 12)  (ft)
 34    Flex Mode# 12 GD rate, eta( 12)-dot  (ft/sec)
 35    Flex Mode# 13 Generaliz Displacem eta( 13)  (ft)
 36    Flex Mode# 13 GD rate, eta( 13)-dot  (ft/sec)
 37    Flex Mode# 14 Generaliz Displacem eta( 14)  (ft)
 38    Flex Mode# 14 GD rate, eta( 14)-dot  (ft/sec)
 39    Flex Mode# 15 Generaliz Displacem eta( 15)  (ft)
 40    Flex Mode# 15 GD rate, eta( 15)-dot  (ft/sec)
 41    Slosh Mass # 1 Displacem. Zs( 1)    (ft)
 42    Slosh Mass # 1 Displacem. Ys( 1)    (ft)
 43    Slosh Mass # 1 Velocity   Zsd( 1)  (ft/sec)
 44    Slosh Mass # 1 Velocity   Ysd( 1)  (ft/sec)
 45    Slosh Mass # 2 Displacem. Zs( 2)    (ft)
 46    Slosh Mass # 2 Displacem. Ys( 2)    (ft)
 47    Slosh Mass # 2 Velocity   Zsd( 2)  (ft/sec)
 48    Slosh Mass # 2 Velocity   Ysd( 2)  (ft/sec)
```



```
Outputs =  70
  1    Roll Attitude  (phi-body)  (radians)
  2    Roll Rate      (p-body)    (rad/sec)
  3    Pitch Attitude (thet-bdy)  (radians)
  4    Pitch Rate     (q-body)    (rad/sec)
  5    Yaw  Attitude  (psi-body)  (radians)
  6    Yaw  Rate      (r-body)    (rad/sec)
  7    Angle of attack, alfa, (radians)
  8    Angle of sideslip, beta, (radian)
  9    Change in Altitude, delta-h, (feet)
 10    Forward Acceleration  (V-dot)  (ft/sec)
 11    Cross Range Velocity (Vcr)  (ft/sec)
 12    Rate-Gyro #  1, Roll Rate   (Body)  (rad/sec)
 13    Rate-Gyro #  2, Pitch Rate  (Body)  (rad/sec)
 14    Rate-Gyro #  3, Yaw Rate    (Body)  (rad/sec)
 15    Accelerom #  1, (along Y), (ft/sec^2) Translat
 16    Accelerom #  2, (along Z), (ft/sec^2) Translat
 17    Ptch Load-Torque Tly for Engine: 1,(ft-lb)
 18    Ptch Load-Torque Tly for Engine: 2,(ft-lb)
 19    Ptch Load-Torque Tly for Engine: 3,(ft-lb)
 20    Ptch Load-Torque Tly for Engine: 4,(ft-lb)
 21    Ptch Load-Torque Tly for Engine: 5,(ft-lb)
 22    Ptch Load-Torque Tly for Engine: 6,(ft-lb)
 23    Ptch Load-Torque Tly for Engine: 7,(ft-lb)
 24    Ptch Load-Torque Tly for Engine: 8,(ft-lb)
 25    Yaw  Load-Torque Tlz for Engine: 1,(ft-lb)
 26    Yaw  Load-Torque Tlz for Engine: 2,(ft-lb)
 27    Yaw  Load-Torque Tlz for Engine: 3,(ft-lb)
 28    Yaw  Load-Torque Tlz for Engine: 4,(ft-lb)
 29    Yaw  Load-Torque Tlz for Engine: 5,(ft-lb)
 30    Yaw  Load-Torque Tlz for Engine: 6,(ft-lb)
 31    Yaw  Load-Torque Tlz for Engine: 7,(ft-lb)
 32    Yaw  Load-Torque Tlz for Engine: 8,(ft-lb)
 33    Cm_alpha  :   -14.535 % Variation
 34    Cn_beta   :    14.535 % Variation
 35    Ca_alpha  :    17.153 % Variation
 36    Cz_alpha  :   -24.655 % Variation
 37    Cy_beta   :   -24.655 % Variation
 38    Cm_0      :    -9.690 % Variation
 39    CZ_0      :   -12.327 % Variation
 40    CA_0      :     6.679 % Variation
 41    I_xx      :    19.786 % Variation
 42    I_yy      :    14.153 % Variation
 43    I_zz      :    14.158 % Variation
 44    Qbar      :     9.865 % Variation
 45    Qbar      :     9.865 % Variation
 46    Xcg locat:      3.492 % Variation
 47    Xcg locat:      3.492 % Variation
 48    Xcg locat:      3.492 % Variation
 49    M_slosh 1:     13.664 % Variation
 50    M_slosh 1:     13.664 % Variation
 51    X_slosh 1:      3.563 % Variation
 52    X_slosh 1:      3.563 % Variation
 53    X_slosh 1:      3.563 % Variation
 54    Wslsh_Y 1:      2.251 % Variation
 55    Wslsh_Z 1:      2.251 % Variation
 56    ZetSl_Y 1:     12.500 % Variation
 57    ZetSl_Z 1:     12.500 % Variation
 58    M_slosh 2:     14.465 % Variation
 59    M_slosh 2:     14.465 % Variation
 60    X_slosh 2:      7.956 % Variation
 61    X_slosh 2:      7.956 % Variation
 62    X_slosh 2:      7.956 % Variation
 63    Wslsh_Y 2:      2.251 % Variation
 64    Wslsh_Z 2:      2.251 % Variation
 65    ZetSl_Y 2:     50.000 % Variation
 66    ZetSl_Z 2:     50.000 % Variation
 67    W_flex  1:   5.000       Addit. Variat.
 68    W_flex  2:   5.000       Addit. Variat.
 69    W_flex  3:   8.000       Addit. Variat.
 70    W_flex  4:   8.000       Addit. Variat.
```

4.3 Loading the Files into Matlab

We begin the analysis by running the Matlab script file “Init.m” which loads the vehicle, TVC matrix, actuator, Nz-filter and the control system into Matlab. The dynamic controller and the Nz-filter were derived in Section-3. The vehicle and actuator models are the same as those used in Section-2.



```
% Initialization File
r2d=185/pi; d2r=1/r2d;
[Av,Bv,Cv,Dv]= vehicle_t85;           % Load Flex-Body Plant
[Aa,Ba,Ca,Da]= actuator;              % Load Actuator
[Af,Bf,Cf,Df]= nz_filt;               % Load the Nz Filter
[Ac,Bc,Cc,Dc]= control;               % Load the Control System
load TVC -ascii                       % TVC Matrix
Npv=19;                               % Number of Param Variations
```

4.4 Control System

The flight control system is shown in Figures (42 & 43). The pitch and yaw controllers are dynamic and identical. Their inputs are: attitude, rate and normal acceleration, shown in detail in Fig.43. They include the Nz-filter and the integrator. Roll is a simple PD. In Figure 43 we have two slightly different versions of the pitch/yaw controller. The 4th order Nz-filter which is designed to provide disturbance attenuation in the 0.6 to 0.9 (rad/sec) frequency range, is implemented as a separate block. Low-pass and notch filters are included to attenuate flex modes.

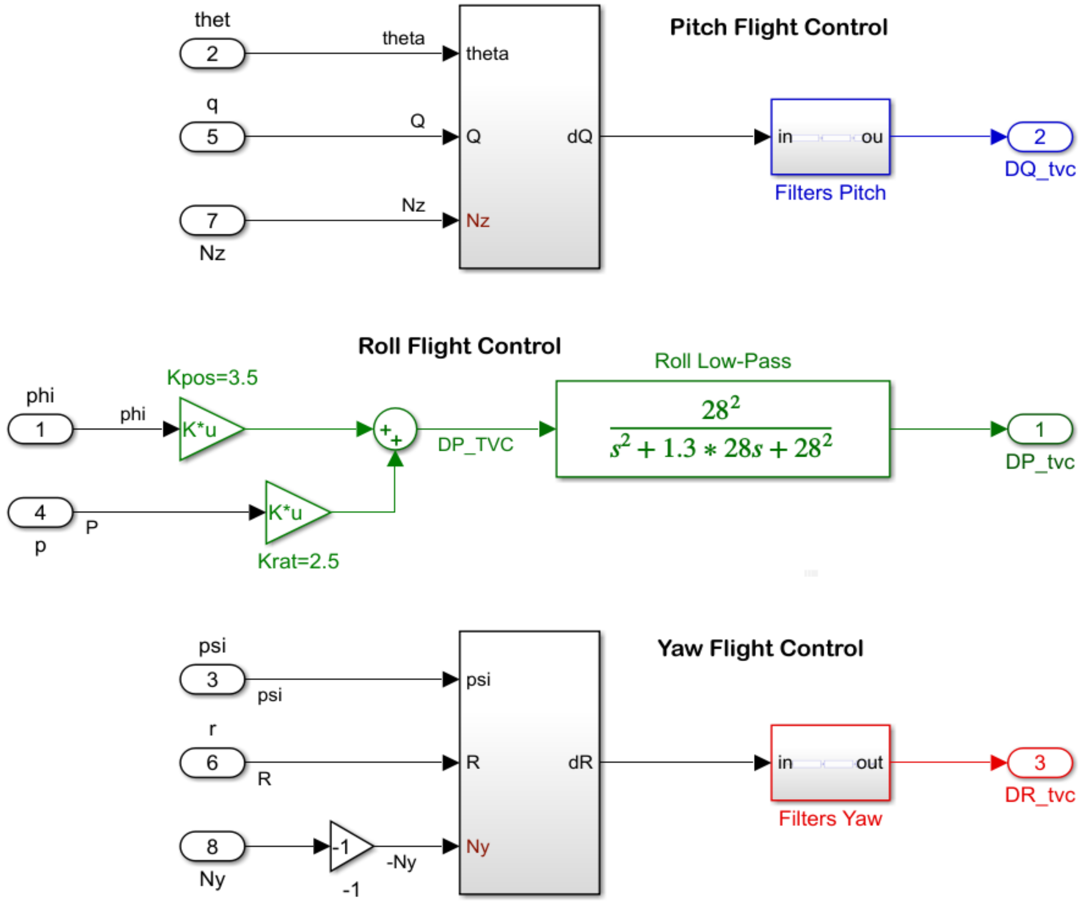


Figure 42 Roll, Pitch and Yaw Flight Control System

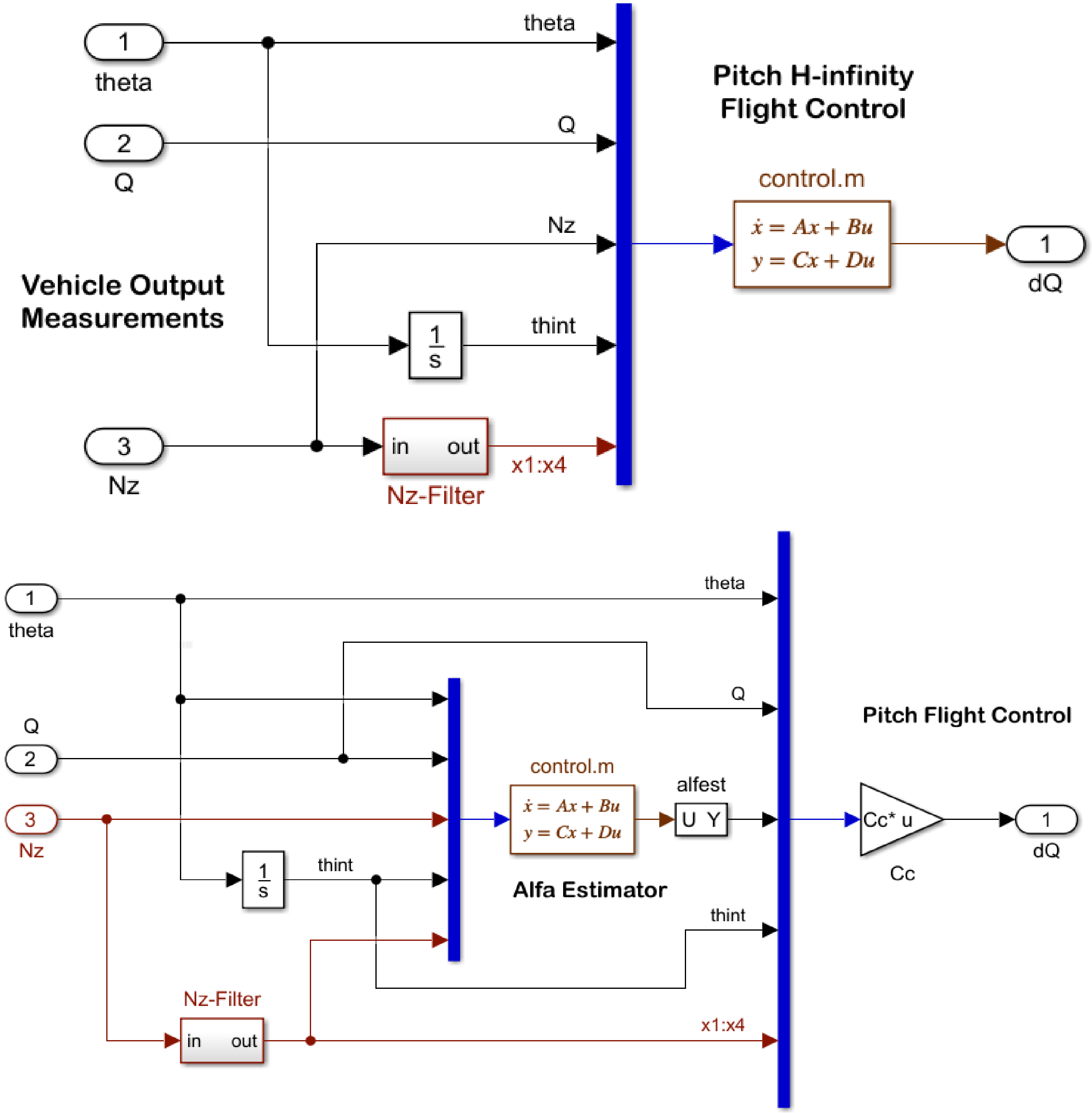


Figure 43 Two FCS Implementations

In the top version, the output-feedback dynamic controller is used exactly as it was derived in Section-3. The inputs are: attitude, rate, acceleration, theta-integral, and the four states x1:x4 coming from the Nz filter. In the second version at the bottom we take advantage that 7 out of the 8 plant model outputs and controller inputs, are also states. The dynamic controller is separated into an estimator which estimates alpha, and a state-feedback gain which is the controller matrix Cc. The 7 measurable states are fed directly into the state-feedback gain Cc. The estimator is only used to estimate alpha, the only remaining state that is not directly measurable. The results from both implementations are very similar.

4.5 Stability and Sensitivity Analysis

The open-loop Simulink model “Open_Loop_4.slx” in Figure 44 is used to analyze stability, as before. It includes the vehicle, 16 actuators, the TVC matrix, and the control system of Figure 42. The vehicle system includes: slosh, structural flexibility, tail-wags-dog dynamics and load-torque feedback between the engine gimbals and the actuators. The file “freq.m” uses the open-loop model to calculate the Bode and Nichols plots and analyze stability in pitch, yaw and roll, as shown in Figures 47 & 48.

```
% Stability Analysis: freq.m
init;
[Ao,Bo,Co,Do]= linmod('Open_Loop_4');      % Open-Loop Stabil Anal Model
[As,Bs,Cs,Ds]= linmod('Sensitiv_4');       % Sensitivity to Gusts Model
syso= ss(Ao,Bo,Co,Do);                     % Create Vehicle  SS System
syss= ss(As,Bs,Cs,Ds);                     % Create Vehicle  SS System
w=logspace(-2, 3, 20000);                  % Define Frequ Range
figure(10); nichols(syso,syso,w)           % Plot Nichol's Chart
figure(20); bode(syso,syso,w)              % Plot Bode
figure(30); sigma(syss,syss2,w);           % Plot SV Sensitivity Analysis
eig(As)
```

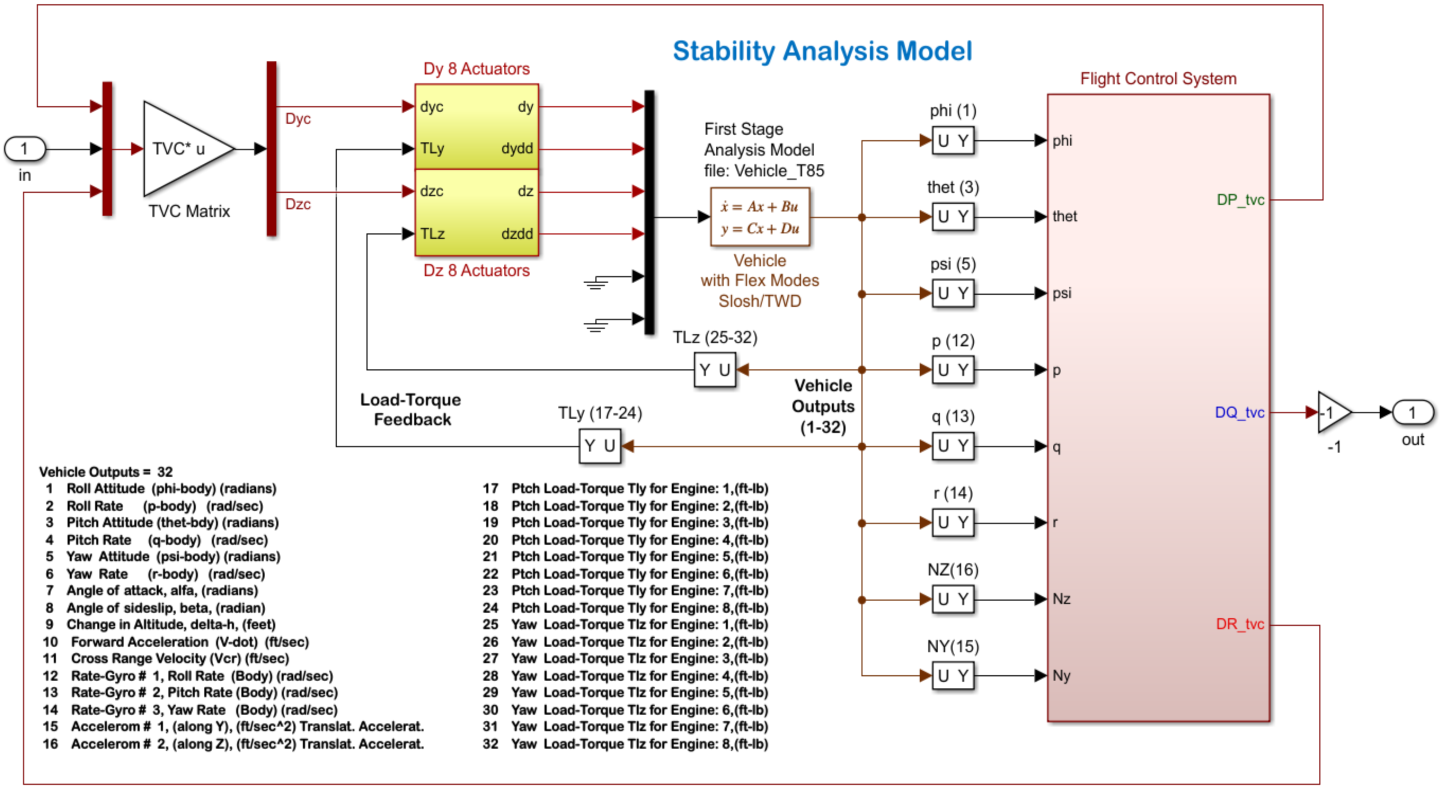
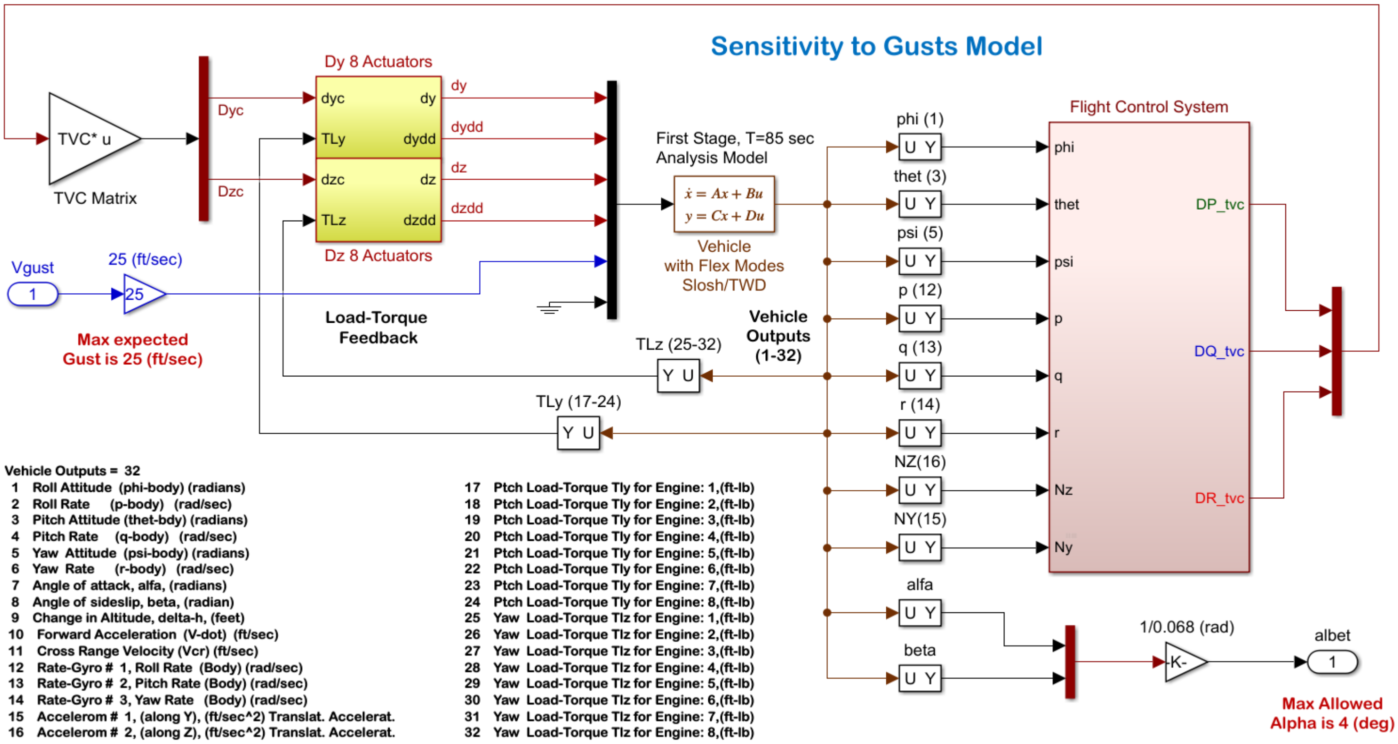


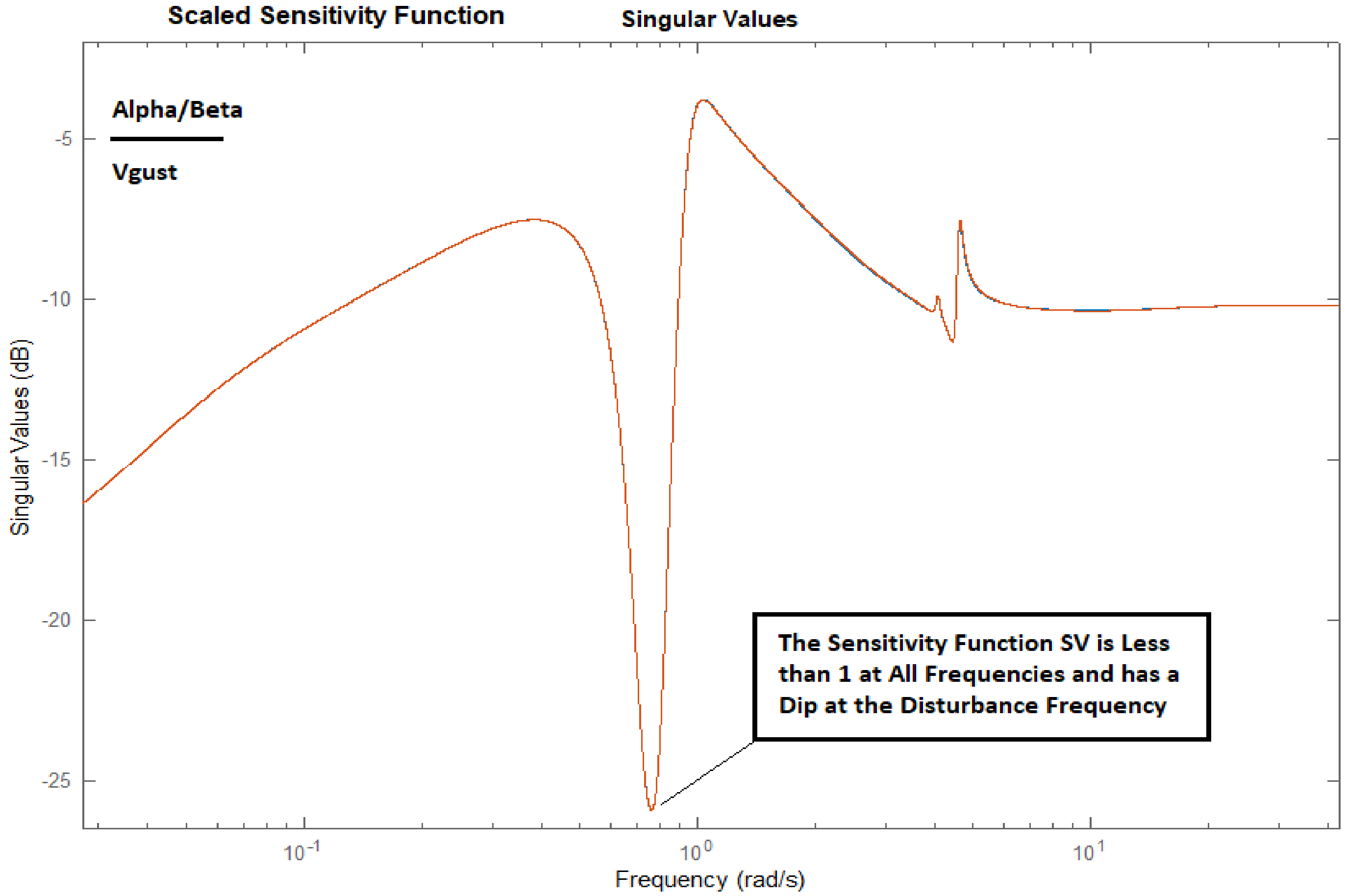
Figure 44 Coupled, Pitch and Lateral Model Used for Stability Analysis “Open_Loop_4.slx”

Figure 45 is the closed-loop sensitivity analysis model “Sensitiv_4.slx” that is used to analyze the control system’s sensitivity to wind-gust disturbances using SV frequency response plots. The sensitivity model is normalized to unity by scaling the input and output. The Sensitivity Function: $(\alpha,\beta)/V_{gust}$ is shown in Figure 46. Its magnitude is less than one at all frequencies and it has a dip at 0.6 – 0.9 (rad/sec), as expected, created by the accelerometer-filters.



Vehicle Outputs = 32
1 Roll Attitude (phi-body) (radians)
2 Roll Rate (p-body) (rad/sec)
3 Pitch Attitude (thet-bdy) (radians)
4 Pitch Rate (q-body) (rad/sec)
5 Yaw Attitude (psi-body) (radians)
6 Yaw Rate (r-body) (rad/sec)
7 Angle of attack, alfa, (radians)
8 Angle of sideslip, beta, (radian)
9 Change in Altitude, delta-h, (feet)
10 Forward Acceleration (V-dot) (ft/sec)
11 Cross Range Velocity (Vcr) (ft/sec)
12 Rate-Gyro # 1, Roll Rate (Body) (rad/sec)
13 Rate-Gyro # 2, Pitch Rate (Body) (rad/sec)
14 Rate-Gyro # 3, Yaw Rate (Body) (rad/sec)
15 Accelerom # 1, (along Y), (ft/sec^2) Translat. Accelerat.
16 Accelerom # 2, (along Z), (ft/sec^2) Translat. Accelerat.
17 Ptch Load-Torque Tly for Engine: 1,(ft-lb)
18 Ptch Load-Torque Tly for Engine: 2,(ft-lb)
19 Ptch Load-Torque Tly for Engine: 3,(ft-lb)
20 Ptch Load-Torque Tly for Engine: 4,(ft-lb)
21 Ptch Load-Torque Tly for Engine: 5,(ft-lb)
22 Ptch Load-Torque Tly for Engine: 6,(ft-lb)
23 Ptch Load-Torque Tly for Engine: 7,(ft-lb)
24 Ptch Load-Torque Tly for Engine: 8,(ft-lb)
25 Yaw Load-Torque Tlz for Engine: 1,(ft-lb)
26 Yaw Load-Torque Tlz for Engine: 2,(ft-lb)
27 Yaw Load-Torque Tlz for Engine: 3,(ft-lb)
28 Yaw Load-Torque Tlz for Engine: 4,(ft-lb)
29 Yaw Load-Torque Tlz for Engine: 5,(ft-lb)
30 Yaw Load-Torque Tlz for Engine: 6,(ft-lb)
31 Yaw Load-Torque Tlz for Engine: 7,(ft-lb)
32 Yaw Load-Torque Tlz for Engine: 8,(ft-lb)

Figure 45 Scaled Sensitivity Analysis Model “Sensitiv_4.slx”



The Sensitivity Function SV is Less than 1 at All Frequencies and has a Dip at the Disturbance Frequency

Figure 46 Normalized Sensitivity Function Between Gust Disturbance and (α, β) Output

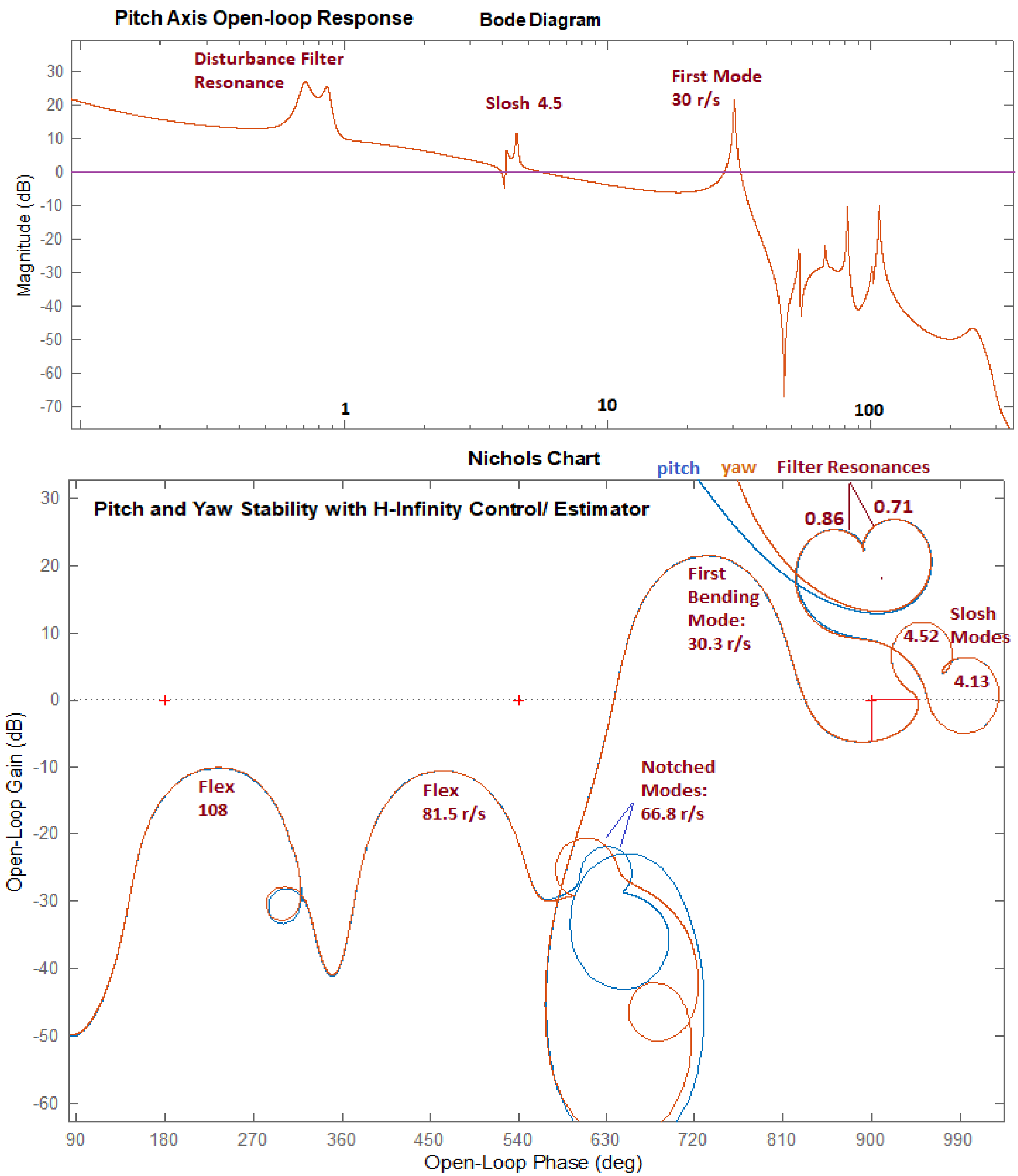


Figure 47 Pitch and Yaw Stability Analysis

The two slosh modes at 4.1 and 4.5 r/s are phase-stable. The first bending mode in pitch and yaw is strong but also phase-stabilized with the low-pass filter. An unstable flex mode at 66.8 r/s was attenuated with notched with filters. The Nz-filters produce 2 strong resonances at 0.71 and 0.86 r/s.

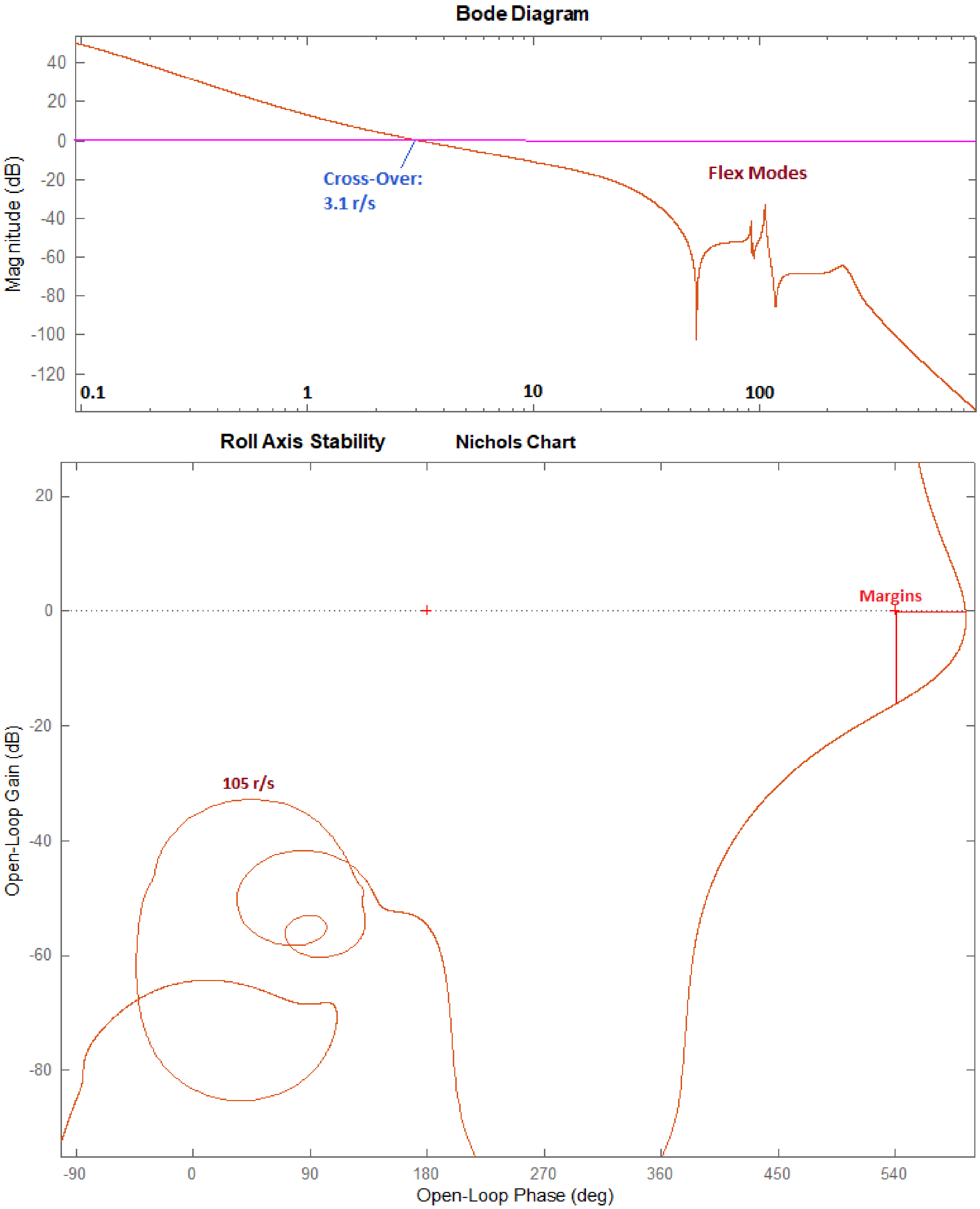
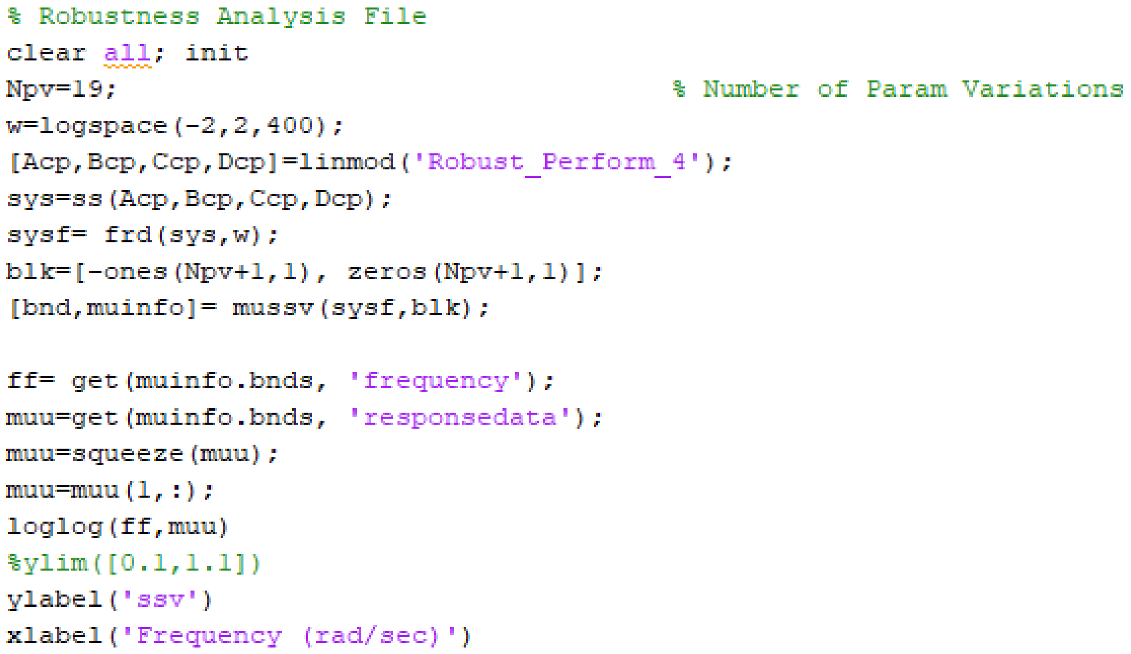


Figure 48 Roll Axis Stability Analysis

2.6 Robustness to Uncertainties

The closed-loop Simulink model "*Robust_Perform_4.slx*" in Figure 49 is used to analyze "Robust Performance" which is the control system ability to satisfy both: robustness and good performance, which in this case is, to remain stable in the presence of structured uncertainties and to simultaneously satisfy the sensitivity to wind-gusts requirement. The inputs and outputs of this model are uncertainties. That is, inputs and outputs that connect to the uncertainties block Δ. There are 38 uncertainties which have been separated into 19 pitch and 19 lateral variations because the pitch variations affect only the pitch axis and the lateral affect only yaw and roll. They are already normalized for a unity block where the parameters vary ±1. So, we analyze pitch and lateral robustness separately by modifying the Simulink model, pitch robustness using the pitch i/o connections (as shown in Fig.49) and lateral robustness using the lateral i/o uncertainties. The model includes one additional i/o pair for analyzing sensitivity to gusts: the gust velocity input and alpha output, normalized to unity as already described.

The system satisfies the robust performance criterion when the Structured Singular Value (μ) of this system from the 19 uncertainty inputs plus Vgust to the 19 uncertainty outputs plus alpha, is less than one at all frequencies. The Matlab script "Run_Robust.m" performs this operation and calculates the SSV frequency response using the 19 variations which affect pitch in this case, and finding that $\mu < 1$ at almost all frequencies, see Figure-50. This result almost satisfies the requirement, but not completely and we need some justification.



```
% Robustness Analysis File
clear all; init
Npv=19;                                         % Number of Param Variations
w=logspace(-2,2,400);
[Acp,Bcp,Ccp,Dcp]=linmod('Robust_Perform_4');
sys=ss(Acp,Bcp,Ccp,Dcp);
sysf= frd(sys,w);
blk=[-ones(Npv+1,1), zeros(Npv+1,1)];
[bnd,muinfo]= mussv(sysf,blk);

ff= get(muinfo.bnds, 'frequency');
muu=get(muinfo.bnds, 'responsedata');
muu=squeeze(muu);
muu=muu(1,:);
loglog(ff,muu)
%ylim([0.1,1.1])
ylabel('ssv')
xlabel('Frequency (rad/sec)')
```

Note that there is a small violation of the robust performance requirement at the slosh frequency. Slosh stability is sensitive to variations in slosh mass position relative to the vehicle CG and center of rotation. Note that the LOX mode is not perfectly phase-stable like the LH2, but it is half-way rotated towards instability which makes it vulnerable to variations in slosh mass position. This can be fixed by adding baffles and increasing the LOX damping or it can be ignored with some justifications. For example, the violation occurs only for a short period for the oscillations to grow or the disturbance caused by the slosh instability is negligible because the amplitude of slosh mass oscillation is limited by the tank radius.

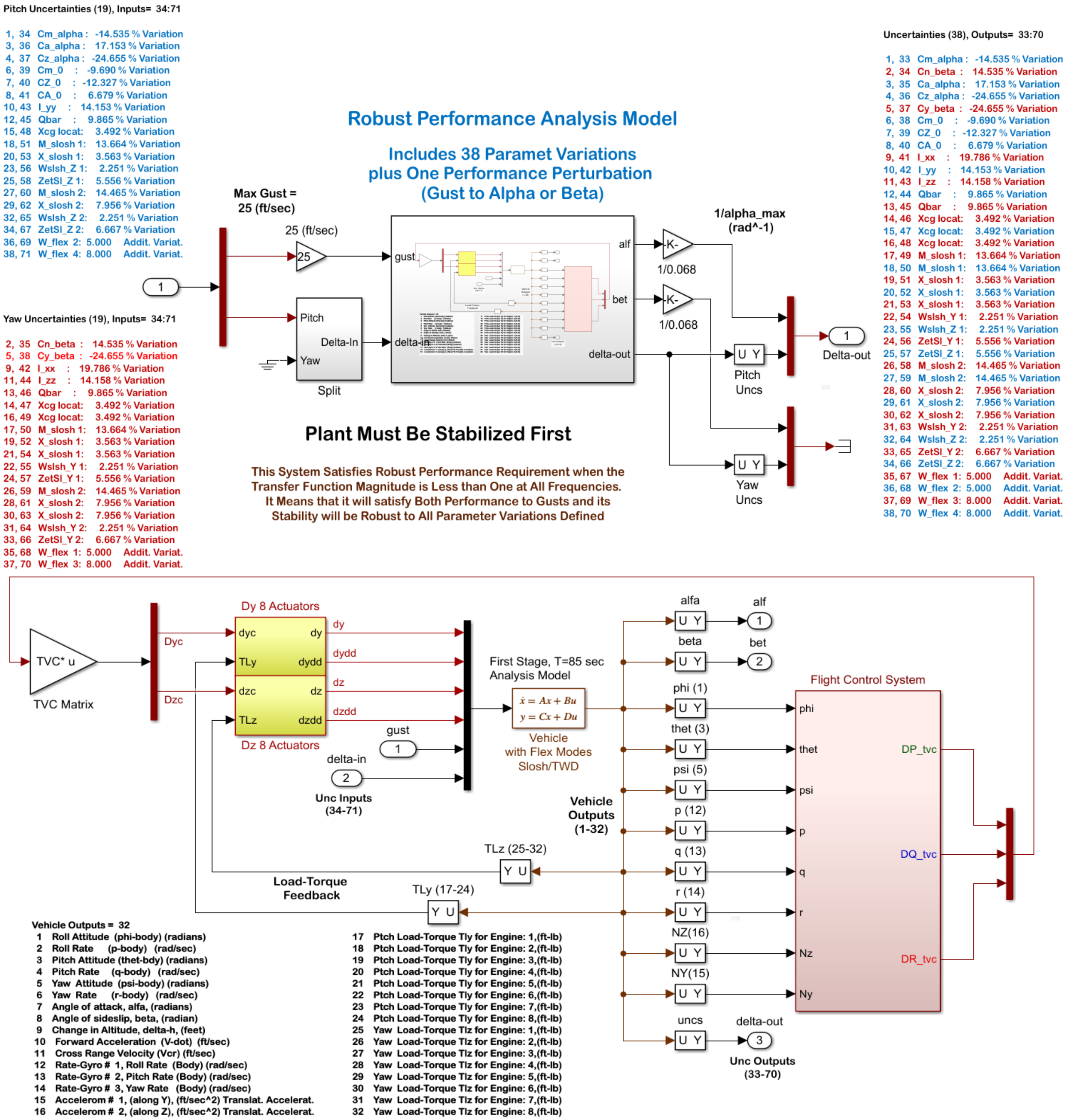


Figure 49 Robust Performance Analysis Model "Robust_Perform.slx" Configured for Pitch Analysis

For lateral robust performance analysis, we must modify the model input by disconnecting the "Demux" second output from pitch uncertainty inputs and connecting it to the second input of the "Split" block which is the yaw uncertainty inputs. The "Delta-out" output must also be disconnected from the top "Mux" and be connected to the bottom "Mux" which selects the beta angle and the yaw uncertainty outputs.

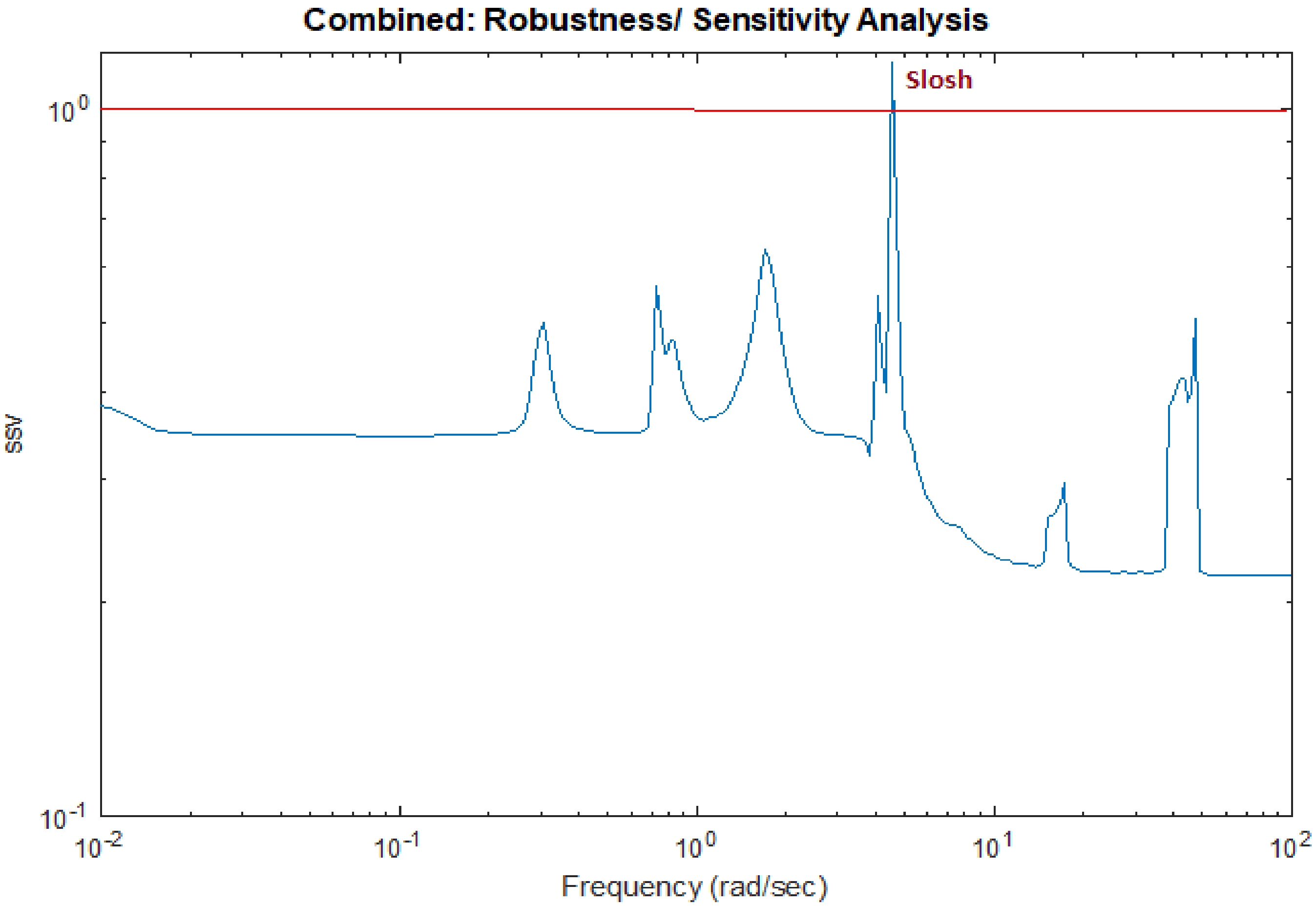


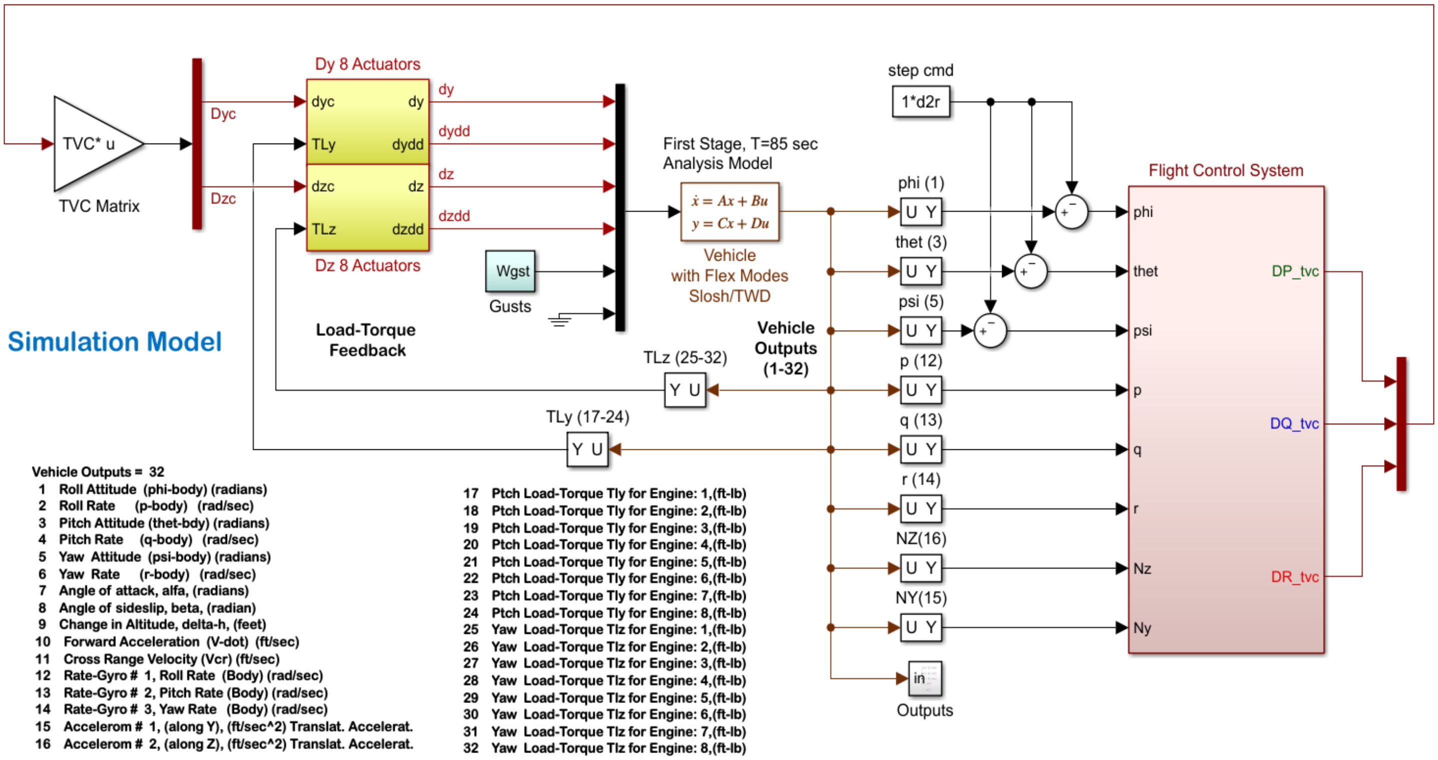
Figure 50 The System Nearly Satisfies Robust Performance in Pitch. The SSV between the 20 Inputs and 20 Outputs is Less than 1, at Almost All Frequencies, except Slosh at 4.5 (r/s). This Must be Fixed, Explained or Justified

2.7 Simulation Model and Results

The closed-loop Simulink model “*Simulat_4.slx*” in Figure 51 is used for time-slice simulations. The inputs are either (roll, pitch, yaw) attitude commands or wind-gust velocity disturbance in (feet/sec).

Step Attitude Command: Figures 52 show the response of the system to simultaneously applied unit step attitude commands in roll, pitch and yaw. Pitch and yaw attitudes converge slowly towards the commanded values because they are being prevented by a strong load-relief feedback from the angles of attack and sideslip. Slosh is excited by the maneuver but it is phase-stable and it gets damped out fast by gimbaling.

Response to Sinusoidal Wind Disturbances: Figure 53 shows the effect of the Nz-filter to counteract external disturbances. The system is excited with an oscillatory wind disturbance consisting of frequencies between 0.65 to 0.85 r/s. The left side shows the responses without the filter and the right side are the responses to the same disturbances with the filter included. The alpha loading without the filter is 5 times greater than what it is with the filter and the gimbal deflections are bigger without the filter. Notice also that the attitude oscillations with the filter are much bigger than without the filter. It is because the system is maneuvering more by turning towards the oscillatory wind and thus reducing α & β.



Vehicle Outputs = 32

1	Roll Attitude (phi-body) (radians)
2	Roll Rate (p-body) (rad/sec)
3	Pitch Attitude (thet-bdy) (radians)
4	Pitch Rate (q-body) (rad/sec)
5	Yaw Attitude (psi-body) (radians)
6	Yaw Rate (r-body) (rad/sec)
7	Angle of attack, alfa, (radians)
8	Angle of sideslip, beta, (radian)
9	Change in Altitude, delta-h, (feet)
10	Forward Acceleration (V-dot) (ft/sec)
11	Cross Range Velocity (Vcr) (ft/sec)
12	Rate-Gyro # 1, Roll Rate (Body) (rad/sec)
13	Rate-Gyro # 2, Pitch Rate (Body) (rad/sec)
14	Rate-Gyro # 3, Yaw Rate (Body) (rad/sec)
15	Accelerom # 1, (along Y), (ft/sec^2) Translat. Accelerat.
16	Accelerom # 2, (along Z), (ft/sec^2) Translat. Accelerat.
17	Ptch Load-Torque Tly for Engine: 1,(ft-lb)
18	Ptch Load-Torque Tly for Engine: 2,(ft-lb)
19	Ptch Load-Torque Tly for Engine: 3,(ft-lb)
20	Ptch Load-Torque Tly for Engine: 4,(ft-lb)
21	Ptch Load-Torque Tly for Engine: 5,(ft-lb)
22	Ptch Load-Torque Tly for Engine: 6,(ft-lb)
23	Ptch Load-Torque Tly for Engine: 7,(ft-lb)
24	Ptch Load-Torque Tly for Engine: 8,(ft-lb)
25	Yaw Load-Torque Tlz for Engine: 1,(ft-lb)
26	Yaw Load-Torque Tlz for Engine: 2,(ft-lb)
27	Yaw Load-Torque Tlz for Engine: 3,(ft-lb)
28	Yaw Load-Torque Tlz for Engine: 4,(ft-lb)
29	Yaw Load-Torque Tlz for Engine: 5,(ft-lb)
30	Yaw Load-Torque Tlz for Engine: 6,(ft-lb)
31	Yaw Load-Torque Tlz for Engine: 7,(ft-lb)
32	Yaw Load-Torque Tlz for Engine: 8,(ft-lb)

Figure 51 Time-Slice Simulation Model "Simulat_4.slx"

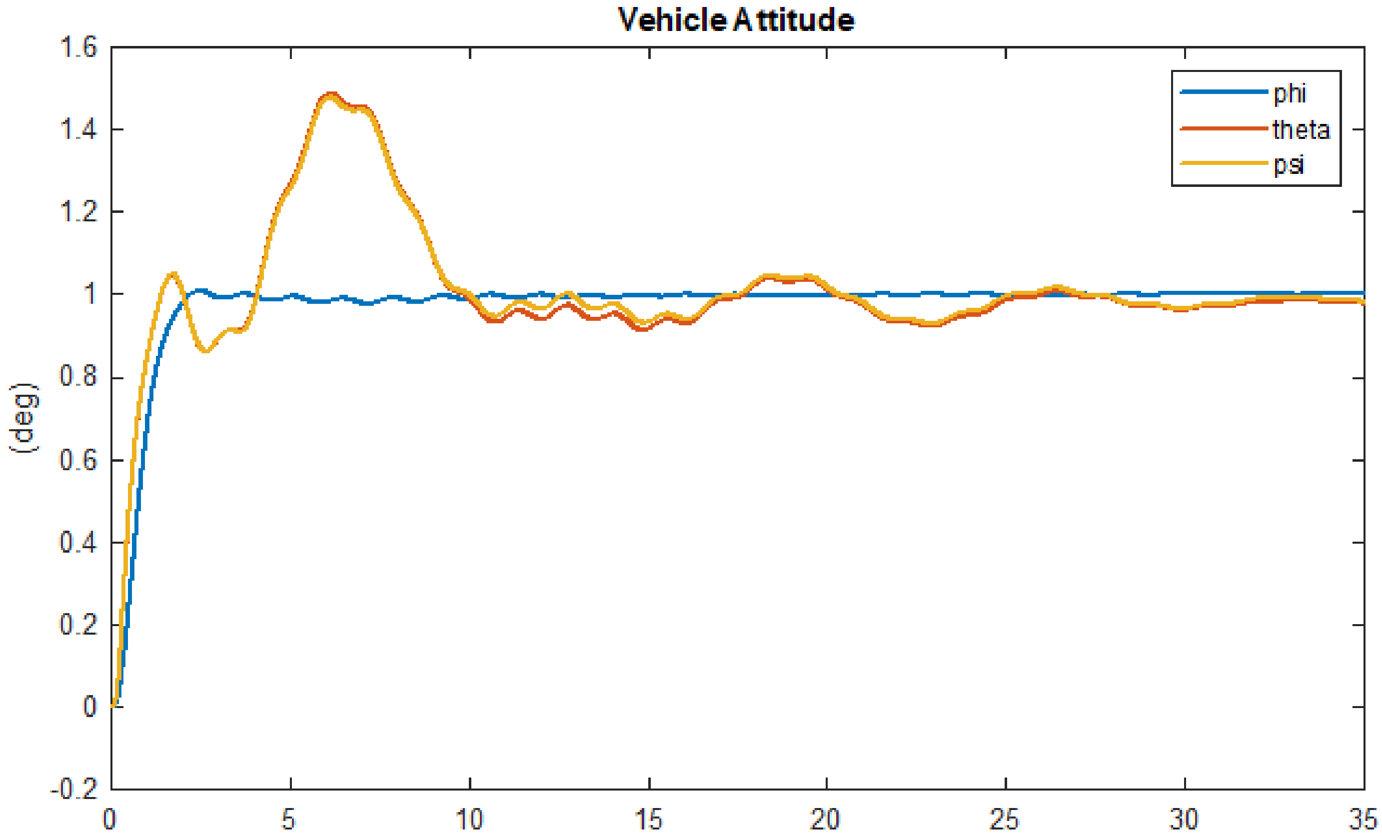
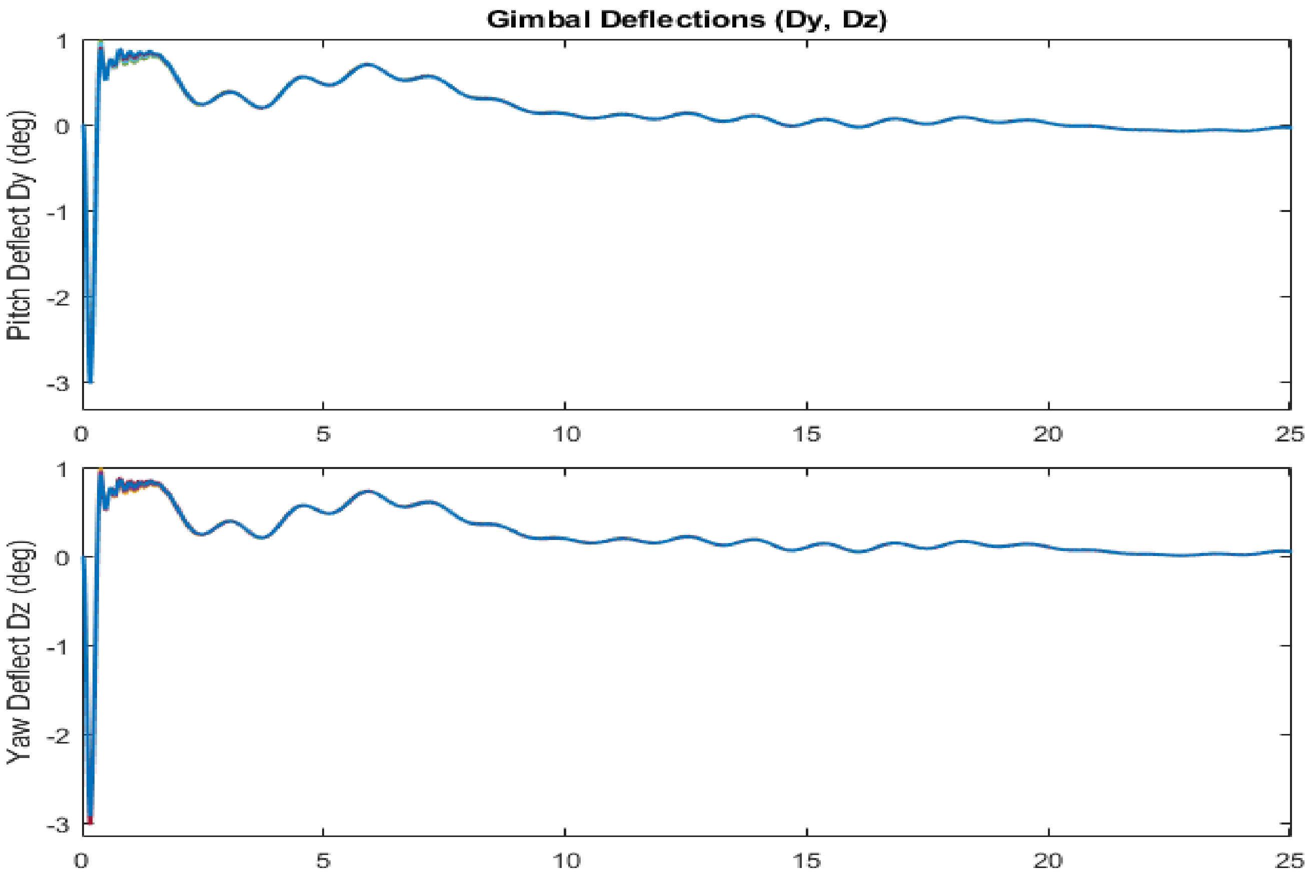
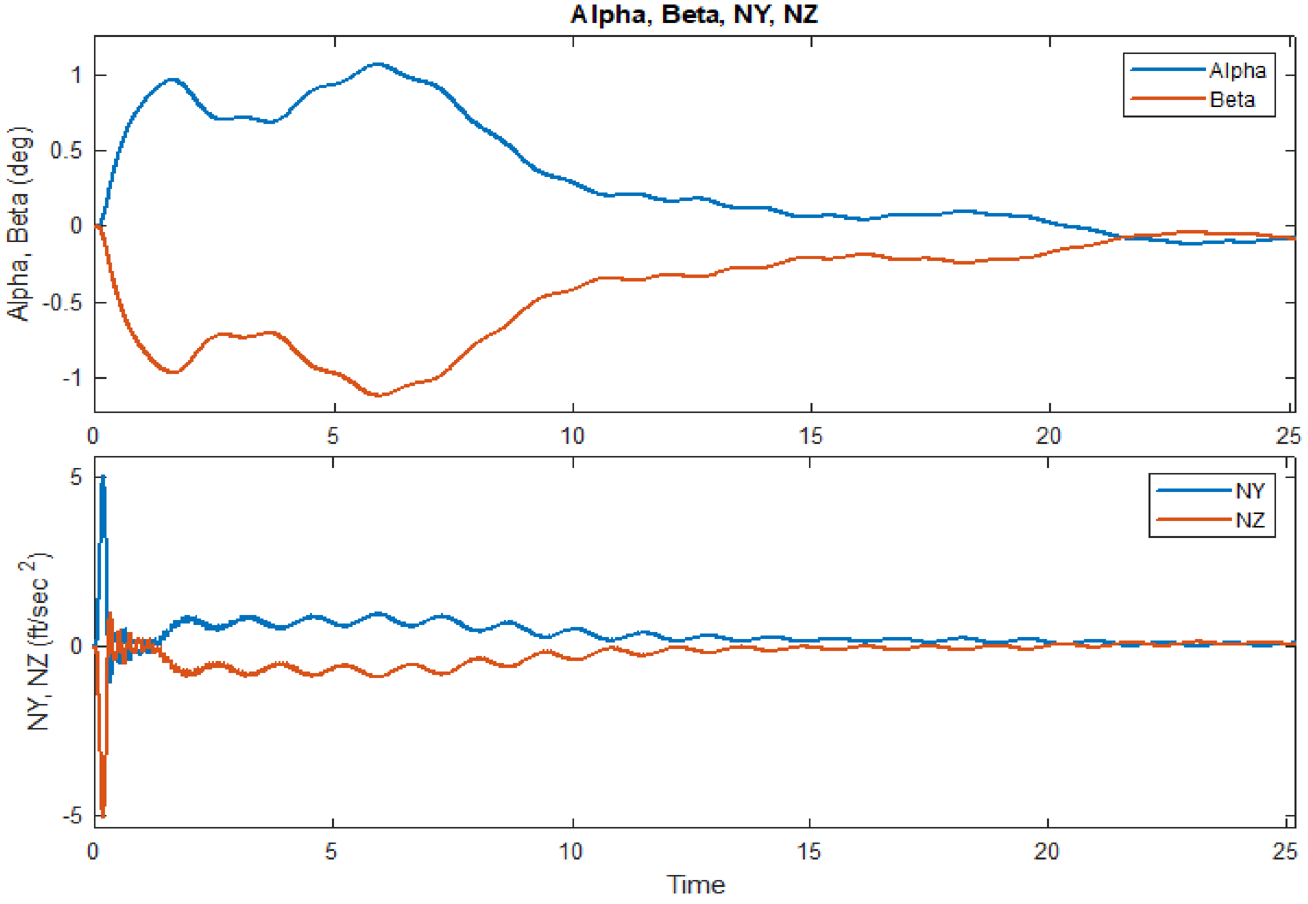


Figure 52 Response to Unit Step Attitude Command in Roll, Pitch and Yaw Together


Gimbal Deflections (Dy, Dz)
Pitch Deflect Dy (deg)
1
0
-1
-2
-3
0
5
10
15
20
25
Yaw Deflect Dz (deg)
1
0
-1
-2
-3
0
5
10
15
20
25


Alpha, Beta, NY, NZ
Alpha
Beta
Alpha, Beta (deg)
1
0.5
0
-0.5
-1
0
5
10
15
20
25
NY
NZ
NY, NZ (ft/sec 2)
5
0
-5
0
5
10
15
20
25
Time

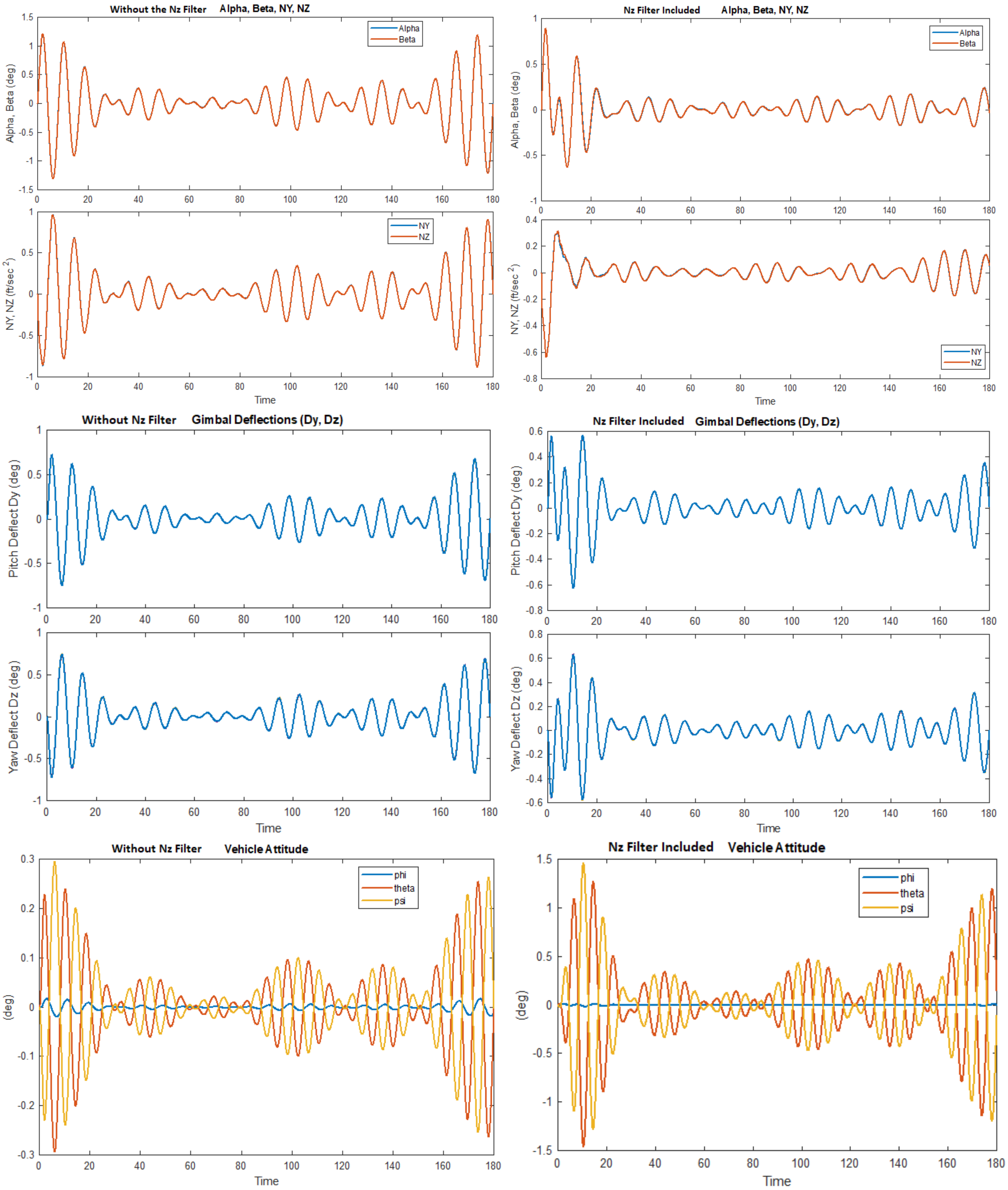


Figure 53 System Response to an Oscillatory Wind Disturbance, Without and With the Nz-Filter

Launch Vehicle Second Stage Design with Unstable Slosh

In this example we have a launch vehicle during second stage. The vehicle has one TVC engine that is used for pitch and yaw control. For roll control it has 8 reaction control jets. For fuel it has 2 tanks that contain liquid oxygen (LOX) and liquid hydrogen (LH2). We will analyze a time-slice during late 2nd stage when the propellant sloshing is significant because the vehicle is lighter. In addition, the LOX propellant sloshing is unstable because of its location relative to the vehicle CG and center of rotation.

When slosh is stable, which happens more frequently, PID control is usually sufficient to control the vehicle, but when it is unstable, the slosh mode is usually attenuated mechanically by adding baffles inside the tank. This is undesirable because it increases the weight of the vehicle and reduces fuel capacity. In this example we will try to solve the problem of slosh instability in the control system by designing an H-infinity dynamic controller that adjusts the phasing of the mode to stabilize it. We will also compare results against the PID design and discuss the advantages and disadvantages of the two approaches by using non-linear simulations that can evaluate the severity of unstable sloshing more accurately than linear models which rely on Nichols charts and stability margins.

In Section-1 we begin with a model that includes slosh but not flexibility. We will design an output feedback H-infinity control system and compare it against the PID system. We will analyze only the pitch axis because the yaw axis is identical and the roll control system will be examined in Sections 2 & 3. We will improve the control system robustness against slosh frequency variations by modeling the frequency uncertainties as external disturbances in the design model. In Section-2 we will include structural flexibility and tail-wag-dog dynamics in the model, analyze the system stability and design filters for flex modes. In Section-3 we will analyze the entire system further with non-linear simulations that include the spherical pendulum slosh model for both tanks. The mechanics of the spherical model will be described in detail. It will also help us estimate the minimum amount of slosh damping required to stabilize the PID controller in order to achieve an acceptable performance with reasonable oscillation amplitudes.

1.0 H-Infinity Control Design Including Slosh

In this section we will design an output feedback dynamic controller for the launch vehicle using H-infinity and compare it with a PID controller that was designed without consideration of the unstable slosh. We will only design the pitch axis because yaw is identical by symmetry. For roll control we will design an RCS system in Section 2 that uses reaction control jets. The input file for this analysis is "*Stg2_Des_T400.inp*" and it is located in directory "*Flixan\Control Analysis\Hinfinity\Examples\Stage-2 LV, Unstable Slosh\1-Robust Hinfin Control Design".* The title of the vehicle model is "*Second Stage Design Model with Slosh at T=400 sec*". It includes a 23,400 (lbf) TVC engine for pitch and yaw control and two slosh modes for the LOX and LH2 propellants. At this point it does not include TWD, actuators and flexibility. They will be added in Section-2 for more thorough analysis. There is no aerodynamics in this flight condition. The LOX mass is unstable when it is controlled with a simple PID system because it is located between the vehicle CG and the Center of Rotation. It means the oscillations are amplified by the TVC control system which leads to diverging slosh oscillations. But the LH2 mode is phase-stable. The H-infinity design system is augmented by including the pitch attitude state θ in order to improve attitude tracking for guidance. The H-infinity controller requires knowledge of the slosh modes in order to stabilize them. The four states of the two slosh modes are also included in the design model. Although only one slosh mode is unstable, we include them both because their frequencies are very close together and they interact significantly. The slosh states are also included in the outputs because they are needed in the performance criteria.

We will also try to introduce some robustness in the design by telling the optimization algorithm that we have 10% uncertainty in the slosh frequency. We do it in the vehicle model by extracting the uncertainty and treating it as an external disturbance. The vehicle dataset in the input file includes an uncertainties dataset that defines variations of the two slosh frequencies. It creates 2 additional uncertainty input/output pairs in the vehicle design system that correspond to 10% frequency variations. The additional input/output pairs are included in the optimization to produce a more robust design. The title of the pitch design system is "*Pitch Design Model with Slosh and Theta-Integral*" and we will use it to create the Synthesis Model "*Pitch Design Model with Slosh and Theta-Integral/SM-2*" by separating the inputs into disturbances and controls and the outputs into performance criteria and measurements. This is an interactive process that is accomplished by the H-infinity program but it will not be shown in this example. The systems generated by Flixan are saved in file "*Stg2_Des_T400.Qdr*". The SM is shown graphically in systems form in Figure-1. It consists of 9 matrices and the optimization gains are shown at the w-inputs and z-outputs. The w-inputs consist of: uncertainties, control disturbance, a command for a regulated output, and noise. The z-output criteria consist of: performance outputs, a regulated output error, and a control input penalization. The gains are adjusted for a satisfactory trade-off between control bandwidth versus stability and sensitivity to disturbances (both internal and external).

1.1 Input File

The input file "*Stg2_Des_T400.Inp*" contains the datasets used by the Flixan program to create the vehicle and the control system. There are two similar batch sets at the top of the file for fast processing the datasets. They both create the vehicle and H-infinity controller systems and export them for Matlab in files "vehicle.m" and "control.m". The first one "*Batch for calculating the Second Stage Design Models with Slosh (Keep Old SM)"* does not create a new SM but it retains the original SM in file "Stg2_Des_T400.Qdr". The second batch "*Batch for calculating the Second Stage Design Models with Slosh (Create New SM)"* creates a new SM and erases the previous one in the systems file. It replaces the "Retain CSM" statement with "Create CSM" dataset. Otherwise, they produce identical results.

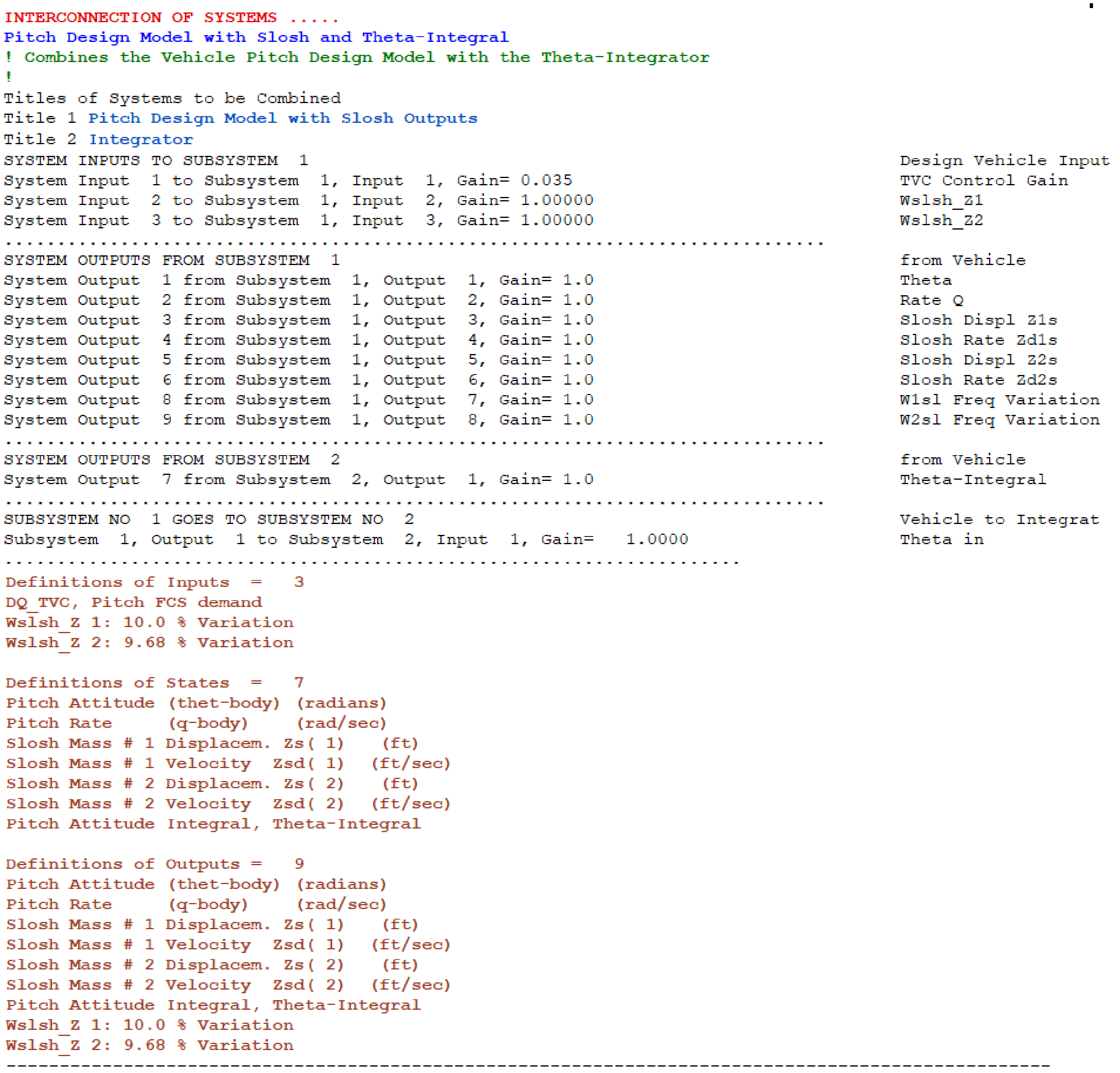
```
BATCH MODE INSTRUCTIONS ...............
Batch for calculating the Second Stage Design Models With Slosh (Keep Old SM)
! This batch set creates state-space systems for the rigid launch vehicle during second stage
! The Pitch Design Model is extracted from the full model. It includes Propellant Sloshing
! for the two tanks and Attitude Theta-Integral. It Retains the Previous Synthesis Model
!
Retain CSM        : Pitch Design Model with Slosh and Theta-Integral/SM-2
Flight Vehicle    : Second Stage Design Model with Slosh at T=400 sec
Transf-Functions : Integrator
System Modificat : Pitch Design Model
System Modificat : Pitch Design Model with Slosh Outputs
System Connection: Pitch Design Model with Slosh and Theta-Integral
H-Infinity Design: Output-Feedback H-Infinity Control Design
To Matlab Format : Pitch Design Model with Slosh and Theta-Integral
To Matlab Format : H-Infin Control for Stage-2 Launch Vehicle Output-Feedback
--------------------------------------------------------------------------------------------------------------------
BATCH MODE INSTRUCTIONS ...............
Batch for calculating the Second Stage Design Models With Slosh (Create New SM)
! This batch set creates state-space systems for the rigid launch vehicle during second stage
! The Pitch Design Model is extracted from the full model. It includes Propellant Sloshing
! for the two tanks and Attitude Theta-Integral. It Creates a New Synthesis Model Using Create CSM
!
Flight Vehicle    : Second Stage Design Model with Slosh at T=400 sec
Transf-Functions : Integrator
System Modificat : Pitch Design Model
System Modificat : Pitch Design Model with Slosh Outputs
System Connection: Pitch Design Model with Slosh and Theta-Integral
Create CSM Design: Pitch Design Model with Slosh and Theta-Integral/SM-2
H-Infinity Design: Output-Feedback H-Infinity Control Design
To Matlab Format : Pitch Design Model with Slosh and Theta-Integral
To Matlab Format : H-Infin Control for Stage-2 Launch Vehicle Output-Feedback
--------------------------------------------------------------------------------------------------------------------
```

```
FLIGHT VEHICLE INPUT DATA ......
Second Stage Design Model with Slosh at T=400 sec
! This is Second Stage vehicle model will be used for control design. It consists of one
! TVC engine and includes propellant sloshing. No flexibility.
!
Body Axes Output, Attitude=Rate Integral

Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft)     :   268.323       32.1740      0.208960E+08
Moments and Products of Inertia: Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)           :   1440.81       6012.39       6013.09       0.00000       0.00000
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)         :   90.6298       0.00000       0.00000
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)        :   8.41000       17853.4       0.10E-05      645542.0
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)          :   87.0943       87.3590       0.00000       0.00000
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                          :   5.28700       0.  0        -0.03          0.00000
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)    :   0.0           5.08700       0.00000       0.00000      -0.096
W-Gust Azim & Elev angles (deg), or Torque/Force direction (x,y,z), Force Locat (x,y,z)   :   Gust         45.00000      90.00000
Surface Reference Area (feet^2), Mean Aerodynamic Chord (ft), Wing Span in (feet)         :   44.1786       7.50000       7.50000
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial_rho/ Partial_H} :   105.431       0.00000       0.00000       0.00000
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:  0.537863      0.399947E-04  0.109956E-04  0.00000       0.00000
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}: -0.255853E-01 -0.394286E-01  0.00000      -0.500000E-03  0.00000
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}: -0.204966     -0.481700E-01  0.00000      -0.112857E-02  0.00000
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:  0.250000E-03  0.428571E-03  0.00000       0.00000       0.00000
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}: -0.232126     -0.656000E-01  0.00000      -0.271429E-02  0.00000
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}:  0.292534E-01  0.450000E-01  0.00000       0.00000       0.00000

Number of Thruster Engines, Include or Not the Tail-Wags-Dog and Load-Torque Dynamics ?  :  1   NO TWD

TVC Engine No:  1                                (Gimbaling  Throttling   Single_Gimbal) : Main Engine#1    Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)                     :   23440.5       23440.5
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):   0.00000       0.00000       6.00000       6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):   5.0000        12.00         1.3
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :   83.2150       0.00000       0.00000

Number of Gyros, (Attitude and Rate)                                                      :  3
Gyro No  1 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Location in (feet) :  Roll   Rate            97.438     0.00       0.00
Gyro No  2 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Location in (feet) :  Pitch  Rate            97.438     0.00       0.00
Gyro No  3 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Location in (feet) :  Yaw    Rate            97.438     0.00       0.00

Number of Slosh Modes                                                                     :  2
LOX Mass (slug), Frequenc 1g (Wy,Wz) (rad/s), Damp (zeta-y-z), Locat.{Xsl,Ysl,Zsl} (ft)  :  115.0  3.0  3.0  0.002  0.002  92.0  0.0  0.0
LH2 Mass (slug), Frequenc 1g (Wy,Wz) (rad/s), Damp (zeta-y-z), Locat.{Xsl,Ysl,Zsl} (ft)  :  42.0   3.1  3.1  0.002  0.002  85.2  0.0  0.0

Parameter Uncertainties Data
Slosh Uncertainties for Second Stage Launch Vehicle
--------------------------------------------------------------------------------------------------------------------------------------------
```

The uncertainties dataset below includes the amount of frequency variations for the two slosh modes which is about 10%. Each variation corresponds to a slosh parameter in the vehicle data. The variations of the non-changing parameters are zero. In this case we have 2 parameter variations that produce 2 additional inputs and 2 outputs in the vehicle system. They are scaled to correspond to a diagonal 2x2 uncertainty block Δ whose elements vary ±1.

```
UNCERTAIN PARAMETER VARIATIONS FROM NOMINAL ......
Slosh Uncertainties for Second Stage Launch Vehicle
! The following data are not actual vehicle parameters but they represent uncertainties
! of the corresponding parameters for the vehicle above. The title above these comments
! is the Uncertainties Title. The uncertainty values represent a +ve or -ve additive
! variation above or below the nominal values of the corresponding vehicle parameter.

Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft)    :  0.0    0.0     0.0
Moments and products of Inertias Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)         :  0.0    0.0     0.0    0.0    0.0     0.0
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)        :  0.0    0.0     0.0
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)       :  0.0    0.0     0.0    0.0
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)         :  0.0    0.0     0.0    0.0
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                          :  0.0    0.0     0.0    0.0
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)   :  0.0    0.0     0.0    0.0    0.0     0.0
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:  0.0    0.0     0.0    0.0    0.0     0.0     0.0
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}:  0.0    0.0     0.0    0.0    0.0     0.0     0.0
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}:  0.0    0.0     0.0    0.0    0.0     0.0     0.0
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:  0.0    0.0     0.0    0.0    0.0     0.0
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}:  0.0    0.0     0.0    0.0    0.0     0.0     0.0
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}:  0.0    0.0     0.0    0.0    0.0     0.0

Number of Thruster Engines, (Variations from Nominal Parameters)                          :  1

TVC Engine No:  1                                                                         :  TVC Engine
Engine Thrust Additive Variation,  (lb)                                                   :  0.0
Engine Mounting Angles Variations from Nominal Angles (Dyn,Dzn) (deg)                     :  0.0        0.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  0.0        0.0     0.0

Number of Slosh Modes (Uncertainties)                                                     :  2
LOX Slosh Mass (slugs), Frequ Wy,Wz 1g (rad/s), Damp (zeta-y-z), Locat {Xsl,Ysl,Zsl} (ft):  0.0  0.3  0.3  0.0  0.0  0.0  0.0  0.0
LH2 Slosh Mass (slugs), Frequ Wy,Wz 1g (rad/s), Damp (zeta-y-z), Locat {Xsl,Ysl,Zsl} (ft):  0.0  0.3  0.3  0.0  0.0  0.0  0.0  0.0
-------------------------------------------------------------------------------------------------------------
SYSTEM OF TRANSFER FUNCTIONS ...
Integrator
Continuous
TF. Block #  1  (1/s)                                 Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
---------------------------------------------------------------------------
Block #, from Input #, Gain
  1  1    1.00000
..........................
Outpt #, from Block #, Gain
  1  1    1.00000
..........................
-------------------------------------------------------------------------------
CREATE A NEW SYSTEM FROM AN OLD SYSTEM... (Titles of the New and Old Systems)
Pitch Design Model
Second Stage Design Model with Slosh at T=400 sec
! Pitch System with Slosh Modes is Extracted from the Vehicle system above
TRUNCATE OR REORDER THE SYSTEM INPUTS, STATES, AND OUTPUTS
Extract Inputs :   1   5   7
Extract States :   3   4   11  13  15  17
Extract Outputs:   3   4   14  16  18
-------------------------------------------------------------------------------
CREATE A NEW SYSTEM FROM AN OLD SYSTEM... (Titles of the New and Old Systems)
Pitch Design Model with Slosh Outputs
Pitch Design Model
! Extract More Outputs from the Previous System States
! Including rates such as: p_dot, r_dot, beta_dot
NEW SYSTEM OUTPUTS FROM PREVIOUS SYSTEM OUTPUTS/ STATES/ DERIVATIVES
From Old System Output:    1                    Pitch Attitude (thet-body) (radians)
From Old System Output:    2                    Pitch Rate     (q-body)    (rad/sec)
From Old System State :    3                    Slosh Mass # 1 Displacem. Zs( 1)    (ft)
From Old System State :    4                    Slosh Mass # 1 Velocity  Zsd( 1)  (ft/sec)
From Old System State :    5                    Slosh Mass # 2 Displacem. Zs( 2)    (ft)
From Old System State :    6                    Slosh Mass # 2 Velocity  Zsd( 2)  (ft/sec)
From Old System Output:    4                    Wslsh_Z 1: 10.0 % Frequ Variation
From Old System Output:    5                    Wslsh_Z 2: 9.68 % Frequ Variation
-------------------------------------------------------------------------------
```

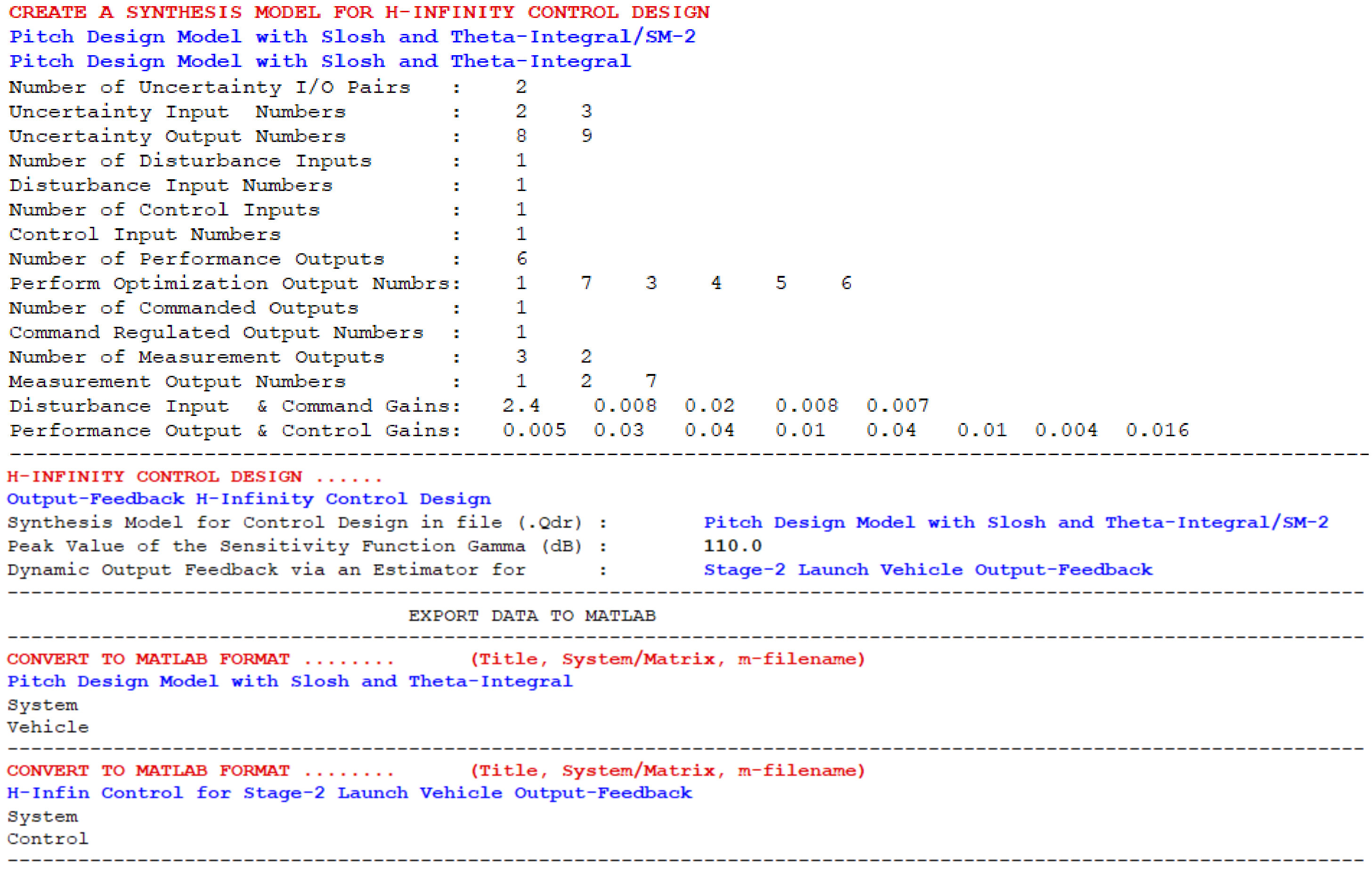
The control input and the 2 uncertainty inputs are included in the pitch design model. The outputs include the 3 measurements (θ, q, and θ-integral), the 4 slosh states (z_s displacements and z_s velocities relative to the tank center), and the two uncertainty outputs that correspond to the 2 slosh frequency variations. The above dataset extracts the 4 slosh variables from the system states and includes them in the design system outputs.



```
INTERCONNECTION OF SYSTEMS .....
Pitch Design Model with Slosh and Theta-Integral
! Combines the Vehicle Pitch Design Model with the Theta-Integrator
!
Titles of Systems to be Combined
Title 1 Pitch Design Model with Slosh Outputs
Title 2 Integrator
SYSTEM INPUTS TO SUBSYSTEM  1                                                Design Vehicle Input
System Input  1 to Subsystem  1, Input  1, Gain= 0.035                       TVC Control Gain
System Input  2 to Subsystem  1, Input  2, Gain= 1.00000                     Wslsh_Z1
System Input  3 to Subsystem  1, Input  3, Gain= 1.00000                     Wslsh_Z2
.............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                             from Vehicle
System Output  1 from Subsystem  1, Output  1, Gain= 1.0                     Theta
System Output  2 from Subsystem  1, Output  2, Gain= 1.0                     Rate Q
System Output  3 from Subsystem  1, Output  3, Gain= 1.0                     Slosh Displ Z1s
System Output  4 from Subsystem  1, Output  4, Gain= 1.0                     Slosh Rate Zd1s
System Output  5 from Subsystem  1, Output  5, Gain= 1.0                     Slosh Displ Z2s
System Output  6 from Subsystem  1, Output  6, Gain= 1.0                     Slosh Rate Zd2s
System Output  8 from Subsystem  1, Output  7, Gain= 1.0                     W1sl Freq Variation
System Output  9 from Subsystem  1, Output  8, Gain= 1.0                     W2sl Freq Variation
.............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                                             from Vehicle
System Output  7 from Subsystem  2, Output  1, Gain= 1.0                     Theta-Integral
.............................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2                                      Vehicle to Integrat
Subsystem  1, Output  1 to Subsystem  2, Input  1, Gain=   1.0000            Theta in
......................................................................
Definitions of Inputs  =   3
DQ_TVC, Pitch FCS demand
Wslsh_Z 1: 10.0 % Variation
Wslsh_Z 2: 9.68 % Variation

Definitions of States  =   7
Pitch Attitude (thet-body) (radians)
Pitch Rate     (q-body)    (rad/sec)
Slosh Mass # 1 Displacem. Zs( 1)   (ft)
Slosh Mass # 1 Velocity  Zsd( 1)  (ft/sec)
Slosh Mass # 2 Displacem. Zs( 2)   (ft)
Slosh Mass # 2 Velocity  Zsd( 2)  (ft/sec)
Pitch Attitude Integral, Theta-Integral

Definitions of Outputs =   9
Pitch Attitude (thet-body) (radians)
Pitch Rate     (q-body)    (rad/sec)
Slosh Mass # 1 Displacem. Zs( 1)   (ft)
Slosh Mass # 1 Velocity  Zsd( 1)  (ft/sec)
Slosh Mass # 2 Displacem. Zs( 2)   (ft)
Slosh Mass # 2 Velocity  Zsd( 2)  (ft/sec)
Pitch Attitude Integral, Theta-Integral
Wslsh_Z 1: 10.0 % Variation
Wslsh_Z 2: 9.68 % Variation
-----------------------------------------------------------------------------------------------------
```

The θ-integrator is included in the design model to create the final design system "*Pitch Design Model with Slosh and Theta-Integral*". The inputs and outputs include the 2 parameter variations that connect to the unity uncertainty block. The states and outputs include the 4 slosh states.

The Synthesis Model is also created in batch mode by processing the following dataset "*Pitch Design Model with Slosh and Theta-Integral/SM-2*". It separates the design model inputs into controls and disturbances, and the design model outputs into measurements and performance criteria, and it includes also the performance tuning gains. This dataset is automatically created by the program and saved in the input file after running the SM creation process interactively after selecting the first option from menu.

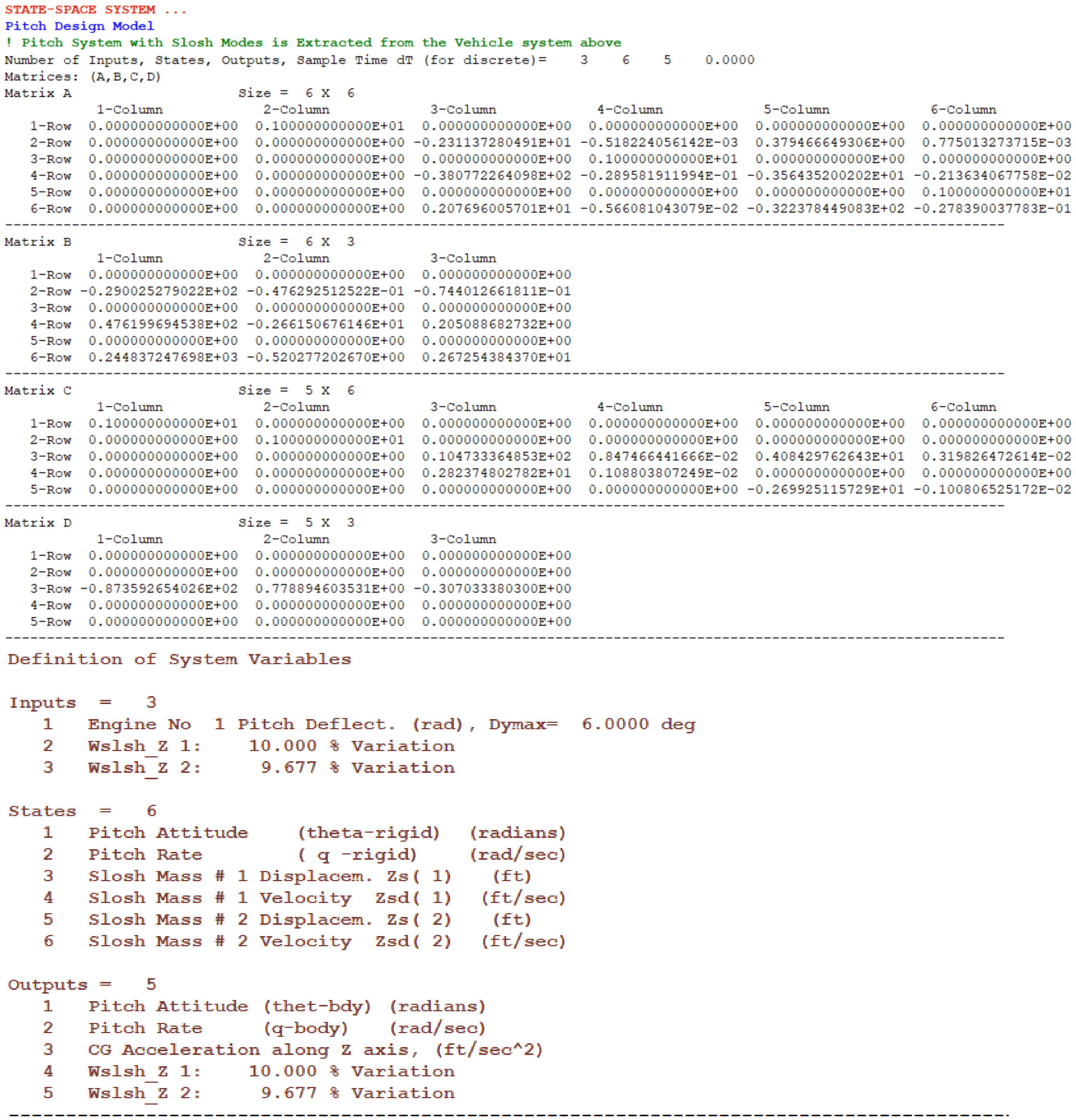


```
CREATE A SYNTHESIS MODEL FOR H-INFINITY CONTROL DESIGN
Pitch Design Model with Slosh and Theta-Integral/SM-2
Pitch Design Model with Slosh and Theta-Integral
Number of Uncertainty I/O Pairs     :    2
Uncertainty Input  Numbers          :    2    3
Uncertainty Output Numbers          :    8    9
Number of Disturbance Inputs        :    1
Disturbance Input Numbers           :    1
Number of Control Inputs            :    1
Control Input Numbers               :    1
Number of Performance Outputs       :    6
Perform Optimization Output Numbrs:      1    7    3    4    5    6
Number of Commanded Outputs         :    1
Command Regulated Output Numbers    :    1
Number of Measurement Outputs       :    3    2
Measurement Output Numbers          :    1    2    7
Disturbance Input  & Command Gains:   2.4     0.008   0.02    0.008   0.007
Performance Output & Control Gains:   0.005   0.03    0.04    0.01    0.04    0.01   0.004   0.016
----------------------------------------------------------------------------------------------------------
H-INFINITY CONTROL DESIGN ......
Output-Feedback H-Infinity Control Design
Synthesis Model for Control Design in file (.Qdr) :         Pitch Design Model with Slosh and Theta-Integral/SM-2
Peak Value of the Sensitivity Function Gamma (dB) :         110.0
Dynamic Output Feedback via an Estimator for      :         Stage-2 Launch Vehicle Output-Feedback
----------------------------------------------------------------------------------------------------------
                               EXPORT DATA TO MATLAB
----------------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........       (Title, System/Matrix, m-filename)
Pitch Design Model with Slosh and Theta-Integral
System
Vehicle
----------------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........       (Title, System/Matrix, m-filename)
H-Infin Control for Stage-2 Launch Vehicle Output-Feedback
System
Control
----------------------------------------------------------------------------------------------------------
```

The H-infinity design dataset "*Output-Feedback H-Infinity Control Design*" reads the SM "*Pitch Design Model with Slosh and Theta-Integral/SM-2*" from the systems file and creates the control system "*Stage-2 Launch Vehicle Output-Feedback*" which is also saved in the systems file. The upper magnitude bound of the sensitivity function is set to γ=110 (dB), smaller values violate the math conditions. The following datasets convert the Flixan generated systems from file *"Stg2_Des_T400.Qdr*" to m-files that can be loaded into Matlab for control analysis.

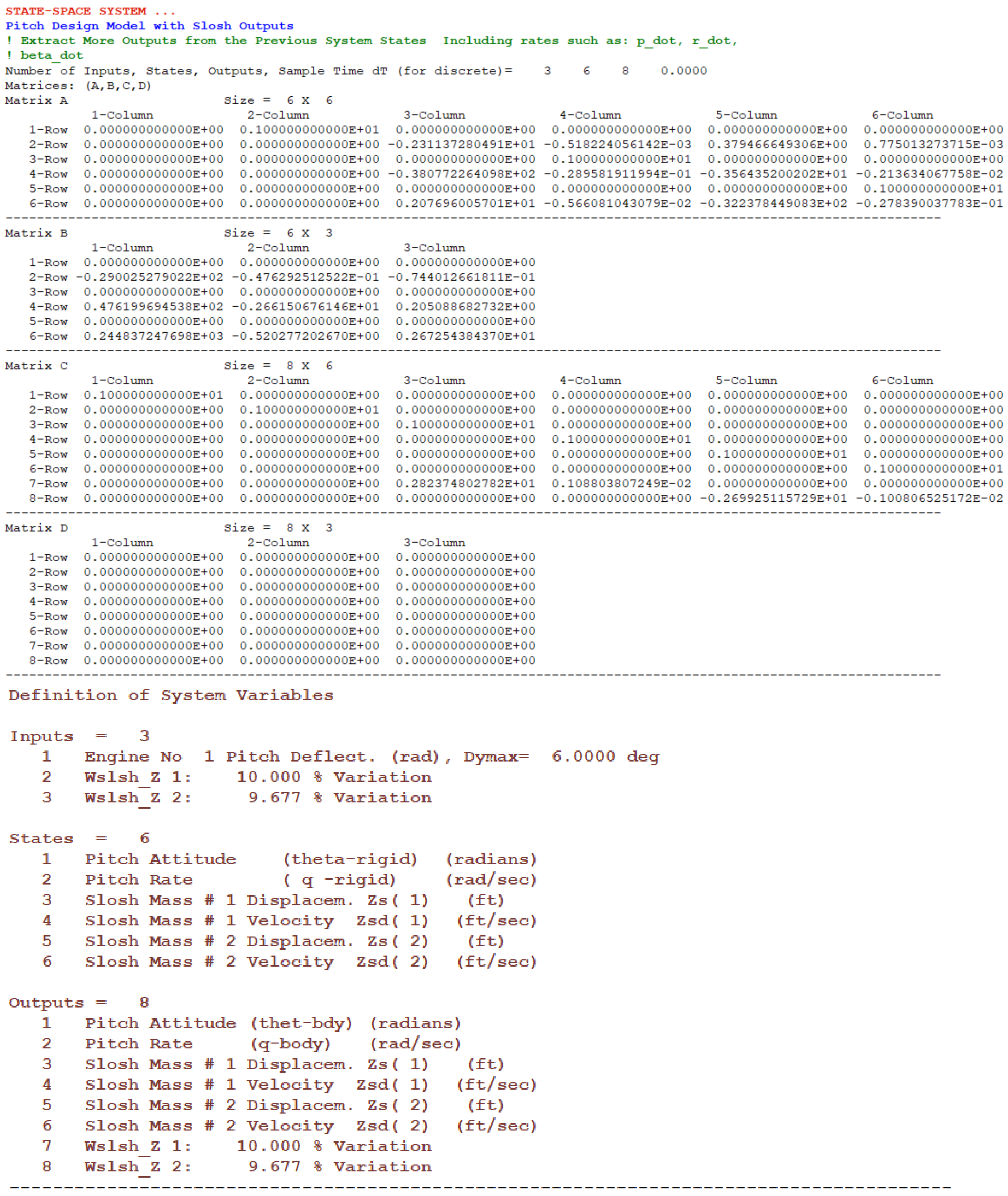
1.2 Systems File

Some of the systems in file: “*Stg2_Des_T400.Qdr*” are shown below. It includes the design vehicle system, the H-Infinity SM, and the H-infinity controller.



```
STATE-SPACE SYSTEM ...
Pitch Design Model
! Pitch System with Slosh Modes is Extracted from the Vehicle system above
Number of Inputs, States, Outputs, Sample Time dT (for discrete)=    3    6    5    0.0000
Matrices: (A,B,C,D)
Matrix A                    Size =  6 X  6
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00 -0.231137280491E+01 -0.518224056142E-03  0.379466649306E+00  0.775013273715E-03
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00 -0.380772264098E+02 -0.289581911994E-01 -0.356435200202E+01 -0.213634067758E-02
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
   6-Row  0.000000000000E+00  0.000000000000E+00  0.207696005701E+01 -0.566081043079E-02 -0.322378449083E+02 -0.278390037783E-01
------------------------------------------------------------------------------------------------------------------------
Matrix B                    Size =  6 X  3
           1-Column            2-Column            3-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row -0.290025279022E+02 -0.476292512522E-01 -0.744012661811E-01
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.476199694538E+02 -0.266150676146E+01  0.205088682732E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.244837247698E+03 -0.520277202670E+00  0.267254384370E+01
------------------------------------------------------------------------------------------------------------------------
Matrix C                    Size =  5 X  6
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.104733364853E+02  0.847466441666E-02  0.408429762643E+01  0.319826472614E-02
   4-Row  0.000000000000E+00  0.000000000000E+00  0.282374802782E+01  0.108803807249E-02  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00 -0.269925115729E+01 -0.100806525172E-02
------------------------------------------------------------------------------------------------------------------------
Matrix D                    Size =  5 X  3
           1-Column            2-Column            3-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row -0.873592654026E+02  0.778894603531E+00 -0.307033380300E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
 Definition of System Variables

 Inputs   =   3
    1   Engine No  1 Pitch Deflect. (rad), Dymax=  6.0000 deg
    2   Wslsh_Z 1:    10.000 % Variation
    3   Wslsh_Z 2:     9.677 % Variation

 States   =   6
    1   Pitch Attitude     (theta-rigid)  (radians)
    2   Pitch Rate         ( q -rigid)    (rad/sec)
    3   Slosh Mass # 1 Displacem. Zs( 1)    (ft)
    4   Slosh Mass # 1 Velocity  Zsd( 1)  (ft/sec)
    5   Slosh Mass # 2 Displacem. Zs( 2)    (ft)
    6   Slosh Mass # 2 Velocity  Zsd( 2)  (ft/sec)

 Outputs =   5
    1   Pitch Attitude (thet-bdy) (radians)
    2   Pitch Rate      (q-body)    (rad/sec)
    3   CG Acceleration along Z axis, (ft/sec^2)
    4   Wslsh_Z 1:    10.000 % Variation
    5   Wslsh_Z 2:     9.677 % Variation
------------------------------------------------------------------------------------------------------------------------
```

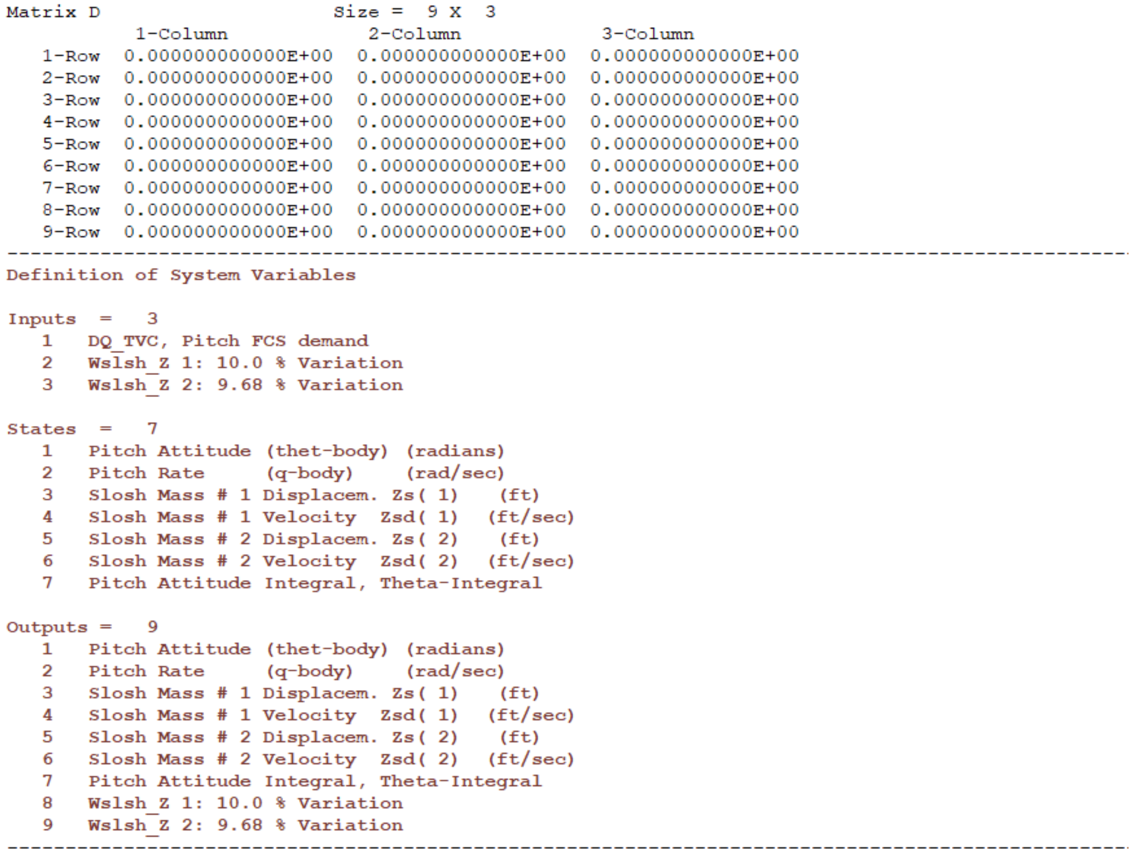
This is the original pitch system with slosh and uncertainty inputs and outputs which is extracted from the full vehicle system. The outputs do not include the slosh variables yet.



```
STATE-SPACE SYSTEM ...
Pitch Design Model with Slosh Outputs
! Extract More Outputs from the Previous System States  Including rates such as: p_dot, r_dot,
! beta_dot
Number of Inputs, States, Outputs, Sample Time dT (for discrete)=    3    6    8    0.0000
Matrices: (A,B,C,D)
Matrix A                  Size =  6 X  6
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00 -0.231137280491E+01 -0.518224056142E-03  0.379466649306E+00  0.775013273715E-03
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00 -0.380772264098E+02 -0.289581911994E-01 -0.356435200202E+01 -0.213634067758E-02
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
   6-Row  0.000000000000E+00  0.000000000000E+00  0.207696005701E+01 -0.566081043079E-02 -0.322378449083E+02 -0.278390037783E-01
-----------------------------------------------------------------------------------------------------------------------
Matrix B                  Size =  6 X  3
           1-Column            2-Column            3-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row -0.290025279022E+02 -0.476292512522E-01 -0.744012661811E-01
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.476199694538E+02 -0.266150676146E+01  0.205088682732E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.244837247698E+03 -0.520277202670E+00  0.267254384370E+01
-----------------------------------------------------------------------------------------------------------------------
Matrix C                  Size =  8 X  6
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column
   1-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
   7-Row  0.000000000000E+00  0.000000000000E+00  0.282374802782E+01  0.108803807249E-02  0.000000000000E+00  0.000000000000E+00
   8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00 -0.269925115729E+01 -0.100806525172E-02
-----------------------------------------------------------------------------------------------------------------------
Matrix D                  Size =  8 X  3
           1-Column            2-Column            3-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
-----------------------------------------------------------------------------------------------------------------------
Definition of System Variables

Inputs   =   3
   1   Engine No  1 Pitch Deflect. (rad), Dymax=  6.0000 deg
   2   Wslsh_Z 1:    10.000 % Variation
   3   Wslsh_Z 2:     9.677 % Variation

States   =   6
   1   Pitch Attitude    (theta-rigid)  (radians)
   2   Pitch Rate        ( q -rigid)    (rad/sec)
   3   Slosh Mass # 1 Displacem. Zs( 1)    (ft)
   4   Slosh Mass # 1 Velocity  Zsd( 1)  (ft/sec)
   5   Slosh Mass # 2 Displacem. Zs( 2)    (ft)
   6   Slosh Mass # 2 Velocity  Zsd( 2)  (ft/sec)

Outputs =   8
   1   Pitch Attitude (thet-bdy) (radians)
   2   Pitch Rate      (q-body)   (rad/sec)
   3   Slosh Mass # 1 Displacem. Zs( 1)    (ft)
   4   Slosh Mass # 1 Velocity  Zsd( 1)  (ft/sec)
   5   Slosh Mass # 2 Displacem. Zs( 2)    (ft)
   6   Slosh Mass # 2 Velocity  Zsd( 2)  (ft/sec)
   7   Wslsh_Z 1:    10.000 % Variation
   8   Wslsh_Z 2:     9.677 % Variation
-----------------------------------------------------------------------------------------
```

This is the pitch system with slosh and uncertainty inputs and outputs. The outputs now include the slosh variables.

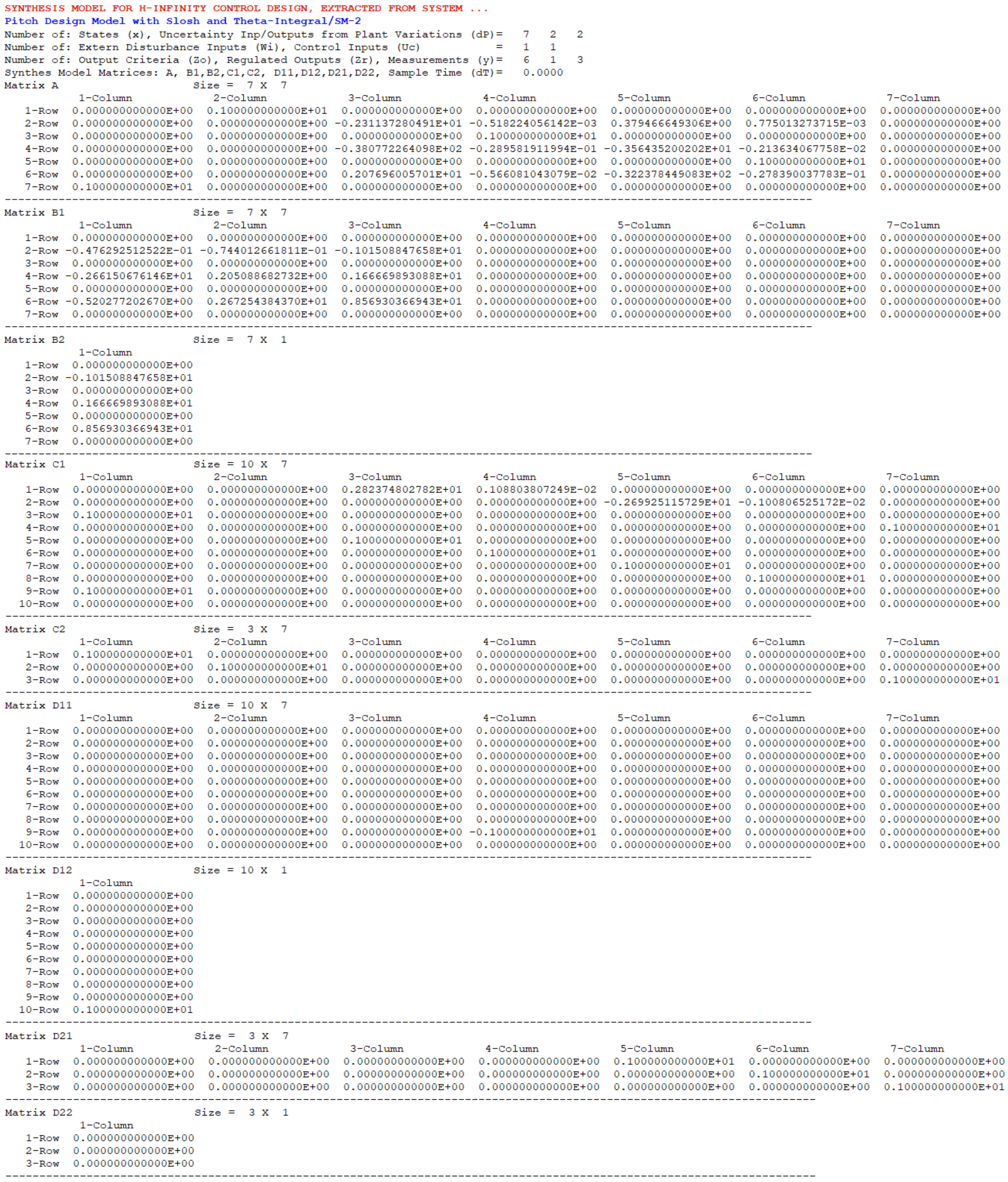
```
STATE-SPACE SYSTEM ...
Pitch Design Model with Slosh and Theta-Integral
! Combines the Vehicle Pitch Design Model with the Theta-Integrator
Number of Inputs, States, Outputs, Sample Time dT (for discrete)=    3    7    9    0.0000
Matrices: (A,B,C,D)
Matrix A                    Size =  7 X  7
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column            7-Column
   1-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.00000000
   2-Row  0.000000000000E+00  0.000000000000E+00 -0.231137280491E+01 -0.518224056142E-03  0.379466649306E+00  0.775013273715E-03  0.00000000
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.00000000
   4-Row  0.000000000000E+00  0.000000000000E+00 -0.380772264098E+02 -0.289581911994E-01 -0.356435200202E+01 -0.213634067758E-02  0.00000000
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.00000000
   6-Row  0.000000000000E+00  0.000000000000E+00  0.207696005701E+01 -0.566081043079E-02 -0.322378449083E+02 -0.278390037783E-01  0.00000000
   7-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.00000000
------------------------------------------------------------------------------------------------------------------------
Matrix B                    Size =  7 X  3
           1-Column            2-Column            3-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row -0.101508847658E+01 -0.476292512522E-01 -0.744012661811E-01
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.166669893088E+01 -0.266150676146E+01  0.205088682732E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.856930366943E+01 -0.520277202670E+00  0.267254384370E+01
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Matrix C                    Size =  9 X  7
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column            7-Column
   1-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.00000000
   2-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.00000000
   3-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.00000000
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.00000000
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.00000000
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.00000000
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.10000000
   8-Row  0.000000000000E+00  0.000000000000E+00  0.282374802782E+01  0.108803807249E-02  0.000000000000E+00  0.000000000000E+00  0.00000000
   9-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00 -0.269925115729E+01 -0.100806525172E-02  0.00000000
------------------------------------------------------------------------------------------------------------------------
```



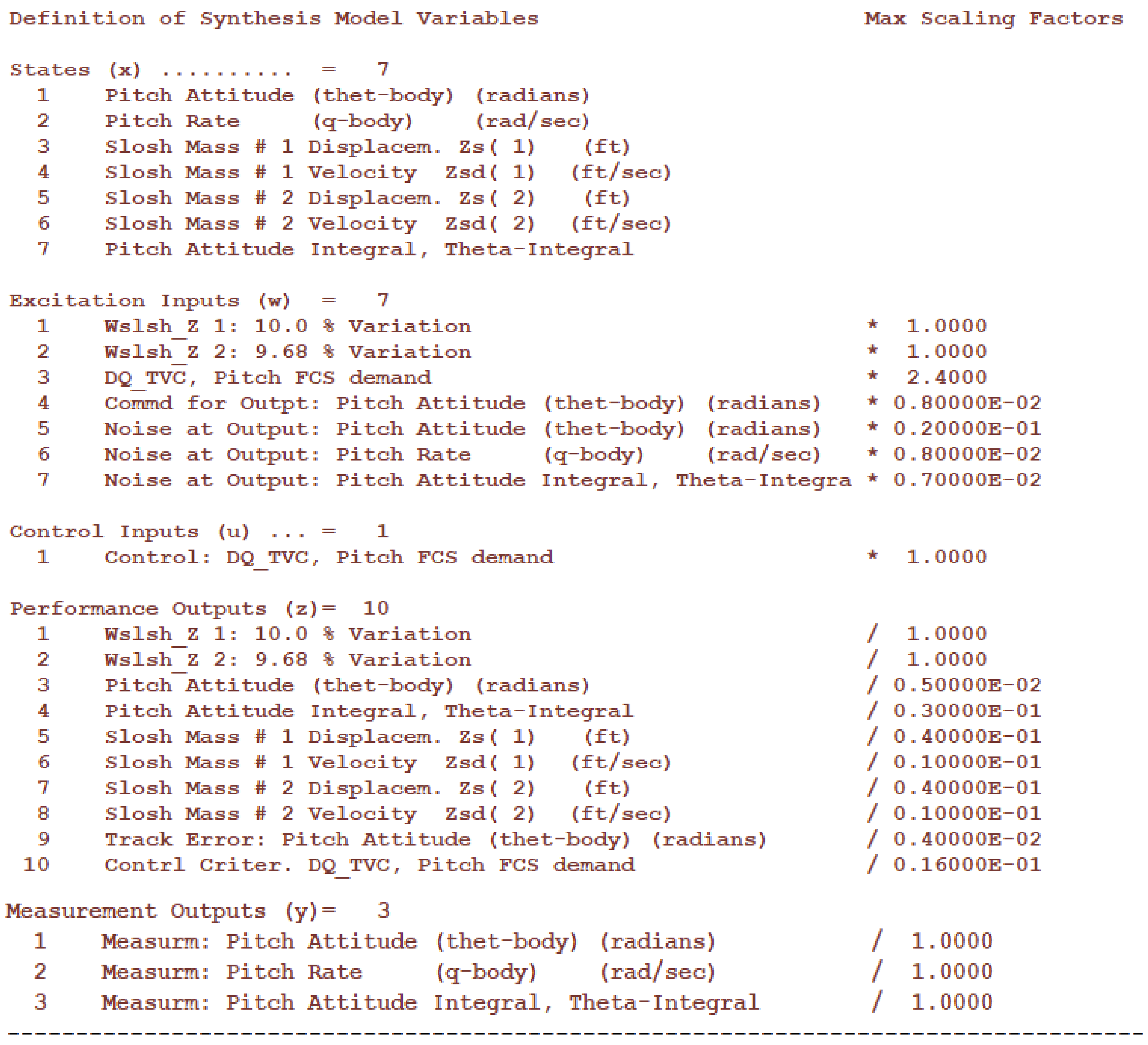
```
Matrix D                    Size =  9 X  3
           1-Column            2-Column            3-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   9-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
-------------------------------------------------------------------------------------------------
Definition of System Variables

Inputs  =    3
   1   DQ_TVC, Pitch FCS demand
   2   Wslsh_Z 1: 10.0 % Variation
   3   Wslsh_Z 2: 9.68 % Variation

States  =    7
   1   Pitch Attitude (thet-body) (radians)
   2   Pitch Rate     (q-body)    (rad/sec)
   3   Slosh Mass # 1 Displacem. Zs( 1)   (ft)
   4   Slosh Mass # 1 Velocity  Zsd( 1)  (ft/sec)
   5   Slosh Mass # 2 Displacem. Zs( 2)   (ft)
   6   Slosh Mass # 2 Velocity  Zsd( 2)  (ft/sec)
   7   Pitch Attitude Integral, Theta-Integral

Outputs =    9
   1   Pitch Attitude (thet-body) (radians)
   2   Pitch Rate     (q-body)    (rad/sec)
   3   Slosh Mass # 1 Displacem. Zs( 1)   (ft)
   4   Slosh Mass # 1 Velocity  Zsd( 1)  (ft/sec)
   5   Slosh Mass # 2 Displacem. Zs( 2)   (ft)
   6   Slosh Mass # 2 Velocity  Zsd( 2)  (ft/sec)
   7   Pitch Attitude Integral, Theta-Integral
   8   Wslsh_Z 1: 10.0 % Variation
   9   Wslsh_Z 2: 9.68 % Variation
-------------------------------------------------------------------------------------------------
```

The pitch design system is now augmented to include θ-integral in the states and in the outputs. Theta-integral is considered to be a measurement and it will also be included in the performance optimization criteria.

This is the Synthesis Model consisting of 9 matrices and it will be used by the algorithm to create the H-infinity control system.

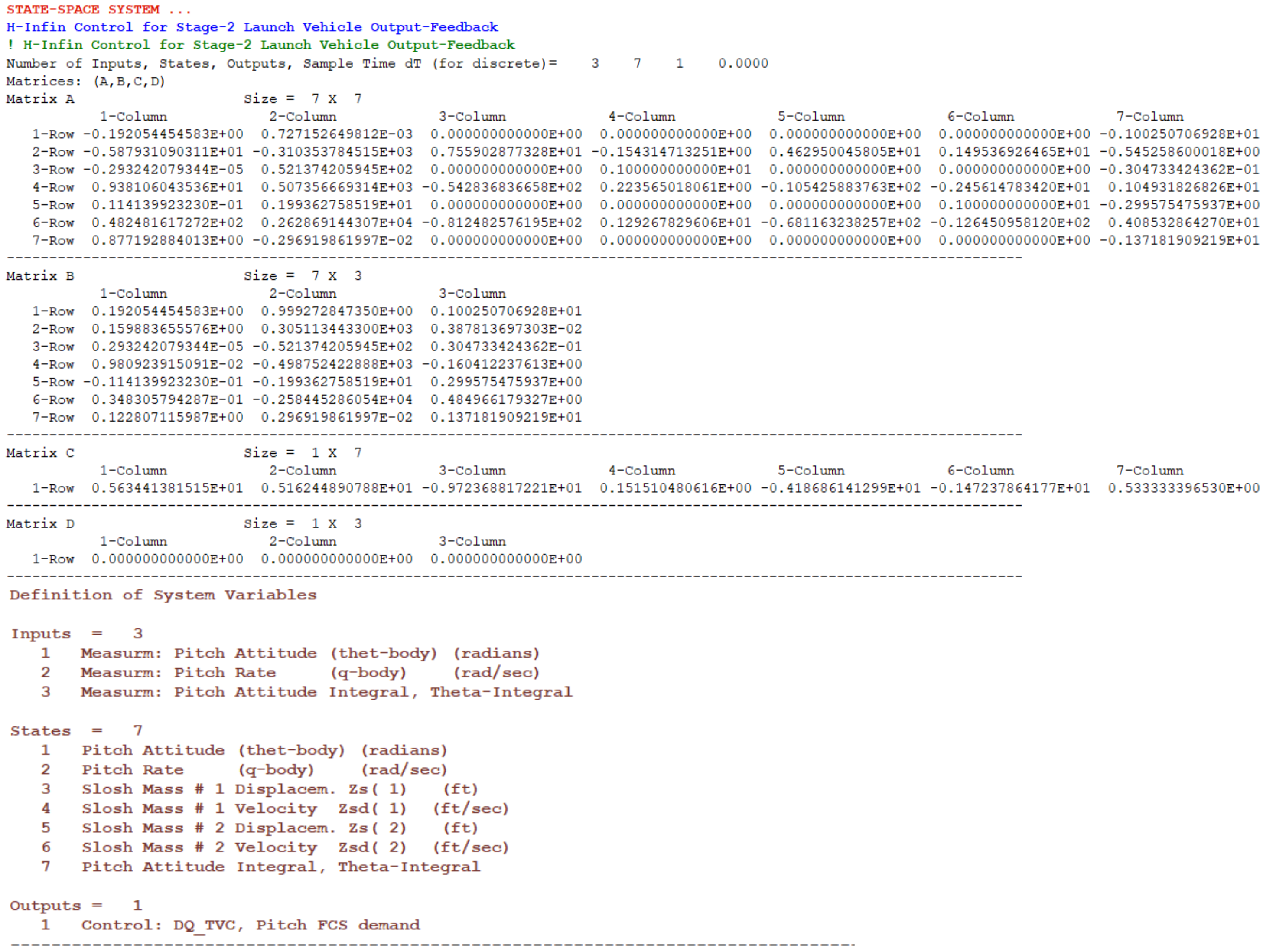


```
SYNTHESIS MODEL FOR H-INFINITY CONTROL DESIGN, EXTRACTED FROM SYSTEM ...
Pitch Design Model with Slosh and Theta-Integral/SM-2
Number of: States (x), Uncertainty Inp/Outputs from Plant Variations (dP)=   7   2   2
Number of: Extern Disturbance Inputs (Wi), Control Inputs (Uc)            =   1   1
Number of: Output Criteria (Zo), Regulated Outputs (Zr), Measurements (y)=   6   1   3
Synthes Model Matrices: A, B1,B2,C1,C2, D11,D12,D21,D22, Sample Time (dT)=   0.0000
Matrix A                    Size =  7 X  7
            1-Column            2-Column            3-Column            4-Column            5-Column            6-Column            7-Column
   1-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00 -0.231137280491E+01 -0.518224056142E-03  0.379466649306E+00  0.775013273715E-03  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00 -0.380772264098E+02 -0.289581911994E-01 -0.356435200202E+01 -0.213634067758E-02  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.207696005701E+01 -0.566081043079E-02 -0.322378449083E+02 -0.278390037783E-01  0.000000000000E+00
   7-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
-------------------------------------------------------------------------------------------------------------------------
Matrix B1                   Size =  7 X  7
            1-Column            2-Column            3-Column            4-Column            5-Column            6-Column            7-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row -0.476292512522E-01 -0.744012661811E-01 -0.101508847658E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row -0.266150676146E+01  0.205088682732E+00  0.166669893088E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row -0.520277202670E+00  0.267254384370E+01  0.856930366943E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
-------------------------------------------------------------------------------------------------------------------------
Matrix B2                   Size =  7 X  1
            1-Column
   1-Row  0.000000000000E+00
   2-Row -0.101508847658E+01
   3-Row  0.000000000000E+00
   4-Row  0.166669893088E+01
   5-Row  0.000000000000E+00
   6-Row  0.856930366943E+01
   7-Row  0.000000000000E+00
-------------------------------------------------------------------------------------------------------------------------
Matrix C1                   Size = 10 X  7
            1-Column            2-Column            3-Column            4-Column            5-Column            6-Column            7-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.282374802782E+01  0.108803807249E-02  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00 -0.269925115729E+01 -0.100806525172E-02  0.000000000000E+00
   3-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
   5-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   9-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  10-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
-------------------------------------------------------------------------------------------------------------------------
Matrix C2                   Size =  3 X  7
            1-Column            2-Column            3-Column            4-Column            5-Column            6-Column            7-Column
   1-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
-------------------------------------------------------------------------------------------------------------------------
Matrix D11                  Size = 10 X  7
            1-Column            2-Column            3-Column            4-Column            5-Column            6-Column            7-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   9-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00 -0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  10-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
-------------------------------------------------------------------------------------------------------------------------
Matrix D12                  Size = 10 X  1
            1-Column
   1-Row  0.000000000000E+00
   2-Row  0.000000000000E+00
   3-Row  0.000000000000E+00
   4-Row  0.000000000000E+00
   5-Row  0.000000000000E+00
   6-Row  0.000000000000E+00
   7-Row  0.000000000000E+00
   8-Row  0.000000000000E+00
   9-Row  0.000000000000E+00
  10-Row  0.100000000000E+01
-------------------------------------------------------------------------------------------------------------------------
Matrix D21                  Size =  3 X  7
            1-Column            2-Column            3-Column            4-Column            5-Column            6-Column            7-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
-------------------------------------------------------------------------------------------------------------------------
Matrix D22                  Size =  3 X  1
            1-Column
   1-Row  0.000000000000E+00
   2-Row  0.000000000000E+00
   3-Row  0.000000000000E+00
-------------------------------------------------------------------------------------------------------------------------
```



```
Definition of Synthesis Model Variables                          Max Scaling Factors

States (x) .........  =   7
  1    Pitch Attitude (thet-body) (radians)
  2    Pitch Rate     (q-body)    (rad/sec)
  3    Slosh Mass # 1 Displacem. Zs( 1)   (ft)
  4    Slosh Mass # 1 Velocity  Zsd( 1)  (ft/sec)
  5    Slosh Mass # 2 Displacem. Zs( 2)   (ft)
  6    Slosh Mass # 2 Velocity  Zsd( 2)  (ft/sec)
  7    Pitch Attitude Integral, Theta-Integral

Excitation Inputs (w)  =   7
  1    Wslsh_Z 1: 10.0 % Variation                             *  1.0000
  2    Wslsh_Z 2: 9.68 % Variation                             *  1.0000
  3    DQ_TVC, Pitch FCS demand                                *  2.4000
  4    Commd for Outpt: Pitch Attitude (thet-body) (radians)   * 0.80000E-02
  5    Noise at Output: Pitch Attitude (thet-body) (radians)   * 0.20000E-01
  6    Noise at Output: Pitch Rate     (q-body)    (rad/sec)   * 0.80000E-02
  7    Noise at Output: Pitch Attitude Integral, Theta-Integra * 0.70000E-02

Control Inputs (u) ... =   1
  1    Control: DQ_TVC, Pitch FCS demand                       *  1.0000

Performance Outputs (z)=  10
  1    Wslsh_Z 1: 10.0 % Variation                             /  1.0000
  2    Wslsh_Z 2: 9.68 % Variation                             /  1.0000
  3    Pitch Attitude (thet-body) (radians)                    / 0.50000E-02
  4    Pitch Attitude Integral, Theta-Integral                 / 0.30000E-01
  5    Slosh Mass # 1 Displacem. Zs( 1)   (ft)                 / 0.40000E-01
  6    Slosh Mass # 1 Velocity  Zsd( 1)  (ft/sec)              / 0.10000E-01
  7    Slosh Mass # 2 Displacem. Zs( 2)   (ft)                 / 0.40000E-01
  8    Slosh Mass # 2 Velocity  Zsd( 2)  (ft/sec)              / 0.10000E-01
  9    Track Error: Pitch Attitude (thet-body) (radians)       / 0.40000E-02
 10    Contrl Criter. DQ_TVC, Pitch FCS demand                 / 0.16000E-01

Measurement Outputs (y)=   3
  1    Measurm: Pitch Attitude (thet-body) (radians)           /  1.0000
  2    Measurm: Pitch Rate     (q-body)    (rad/sec)           /  1.0000
  3    Measurm: Pitch Attitude Integral, Theta-Integral        /  1.0000
----------------------------------------------------------------------------------------
```

A list of input and output variables is included at the bottom of the Synthesis Model. On the right side of the variables the performance optimization gains are included that scale the corresponding disturbance inputs and criteria outputs. Each disturbance gain multiplies the corresponding excitation input column in the B1 and D11 matrices. Each performance gains divides the corresponding criteria output row in the C1 and D11 matrices. The control and measurement gains are set to 1. Also, the parameter variation gains are set to 1 because they are already normalized for ±unity variations.



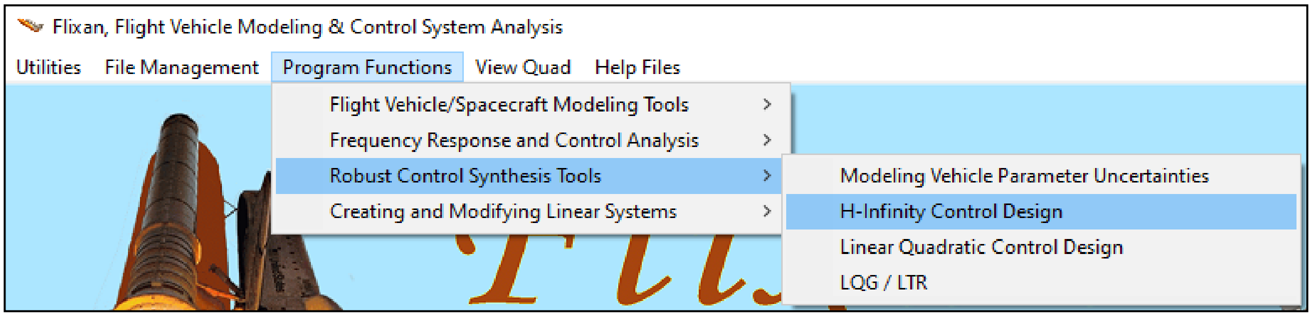
```
STATE-SPACE SYSTEM ...
H-Infin Control for Stage-2 Launch Vehicle Output-Feedback
! H-Infin Control for Stage-2 Launch Vehicle Output-Feedback
Number of Inputs, States, Outputs, Sample Time dT (for discrete)=    3    7    1    0.0000
Matrices: (A,B,C,D)
Matrix A                    Size =  7 X  7
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column            7-Column
   1-Row -0.192054454583E+00  0.727152649812E-03  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00 -0.100250706928E+01
   2-Row -0.587931090311E+01 -0.310353784515E+03  0.755902877328E+01 -0.154314713251E+00  0.462950045805E+01  0.149536926465E+01 -0.545258600018E+00
   3-Row -0.293242079344E-05  0.521374205945E+02  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00 -0.304733424362E-01
   4-Row  0.938106043536E+01  0.507356669314E+03 -0.542836836658E+02  0.223565018061E+00 -0.105425883763E+02 -0.245614783420E+01  0.104931826826E+01
   5-Row  0.114139923230E-01  0.199362758519E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01 -0.299575475937E+00
   6-Row  0.482481617272E+02  0.262869144307E+04 -0.812482576195E+02  0.129267829606E+01 -0.681163238257E+02 -0.126450958120E+02  0.408532864270E+01
   7-Row  0.877192884013E+00 -0.296919861997E-02  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00 -0.137181909219E+01
-------------------------------------------------------------------------------------------------------------------------
Matrix B                    Size =  7 X  3
           1-Column            2-Column            3-Column
   1-Row  0.192054454583E+00  0.999272847350E+00  0.100250706928E+01
   2-Row  0.159883655576E+00  0.305113443300E+03  0.387813697303E-02
   3-Row  0.293242079344E-05 -0.521374205945E+02  0.304733424362E-01
   4-Row  0.980923915091E-02 -0.498752422888E+03 -0.160412237613E+00
   5-Row -0.114139923230E-01 -0.199362758519E+01  0.299575475937E+00
   6-Row  0.348305794287E-01 -0.258445286054E+04  0.484966179327E+00
   7-Row  0.122807115987E+00  0.296919861997E-02  0.137181909219E+01
-------------------------------------------------------------------------------------------------------------------------
Matrix C                    Size =  1 X  7
           1-Column            2-Column            3-Column            4-Column            5-Column            6-Column            7-Column
   1-Row  0.563441381515E+01  0.516244890788E+01 -0.972368817221E+01  0.151510480616E+00 -0.418686141299E+01 -0.147237864177E+01  0.533333396530E+00
-------------------------------------------------------------------------------------------------------------------------
Matrix D                    Size =  1 X  3
           1-Column            2-Column            3-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
-------------------------------------------------------------------------------------------------------------------------
Definition of System Variables

Inputs  =   3
   1   Measurm: Pitch Attitude (thet-body) (radians)
   2   Measurm: Pitch Rate      (q-body)    (rad/sec)
   3   Measurm: Pitch Attitude Integral, Theta-Integral

States  =   7
   1   Pitch Attitude (thet-body) (radians)
   2   Pitch Rate     (q-body)    (rad/sec)
   3   Slosh Mass # 1 Displacem. Zs( 1)   (ft)
   4   Slosh Mass # 1 Velocity  Zsd( 1)  (ft/sec)
   5   Slosh Mass # 2 Displacem. Zs( 2)   (ft)
   6   Slosh Mass # 2 Velocity  Zsd( 2)  (ft/sec)
   7   Pitch Attitude Integral, Theta-Integral

Outputs =   1
   1   Control: DQ_TVC, Pitch FCS demand
-----------------------------------------------------------------------------------
```

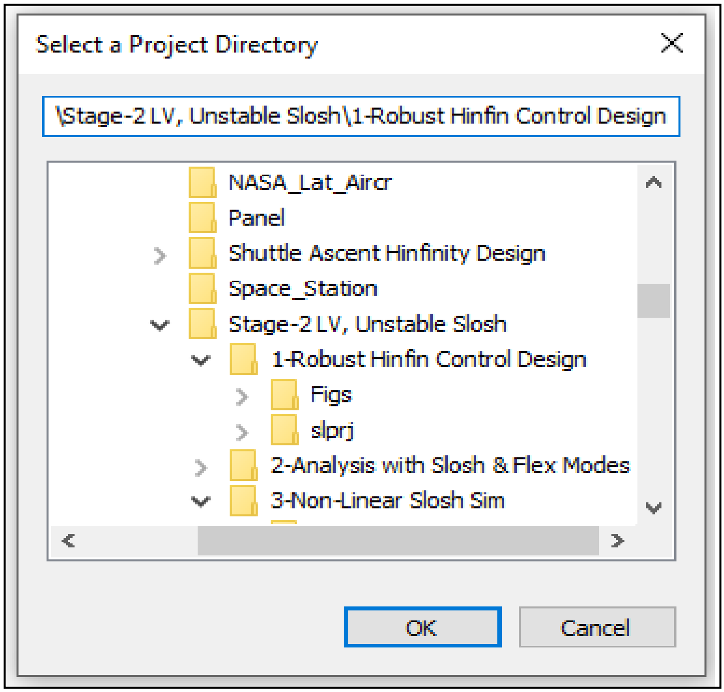
This is the H-Infinity control system that reads the 3 measurements, it estimates the 7-states vector and calculates the pitch TVC command (dQ). The matrix C is actually a state-feedback from the estimated state. It is expected to stabilize the slosh modes and to reduce sensitivity to slosh frequency variations. We will use it to analyze stability and compare it against the PID controller.

The PID is just a 3-states gain vector Kq that is feeding back: attitude, body rate, and attitude integral, where, Kq=[1.1, 1.3, 0.17].

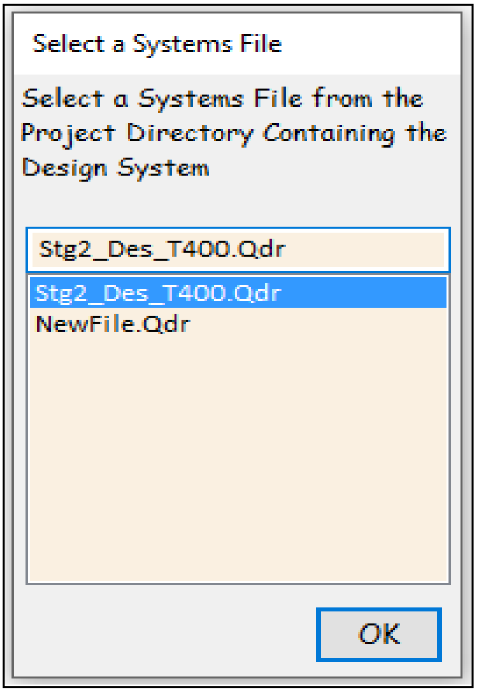
1.3 Designing the H-Infinity Controller Interactively

We will now use the SM which is already in file "*Stg2_Des_T400.Qdr*" to design the H-infinity controller interactively. We can also do it in batch mode by processing one of the batch datasets. Run the H-infinity design program, select the project folder, the systems file, and from the main menu select the second option to read and process the SM. From the next menu select the only SM title and click on "Select".



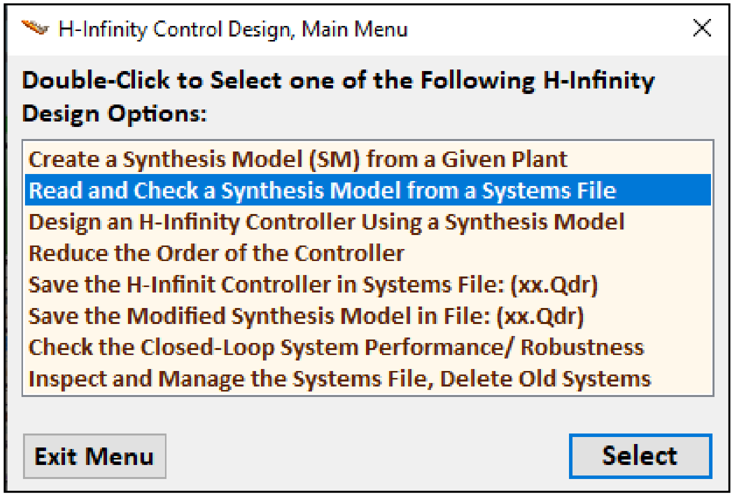


\Stage-2 LV, Unstable Slosh\1-Robust Hinfin Control Design

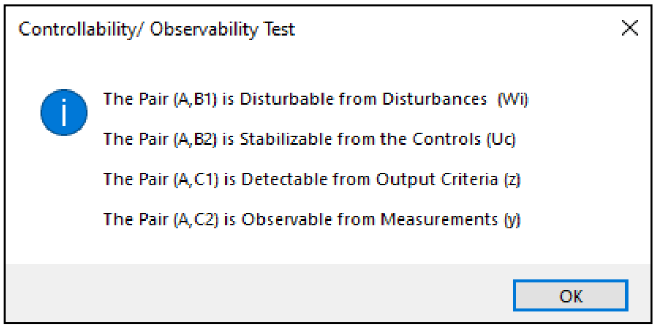


Select a Systems File from the Project Directory Containing the Design System

Stg2_Des_T400.Qdr



Double-Click to Select one of the Following H-Infinity Design Options:

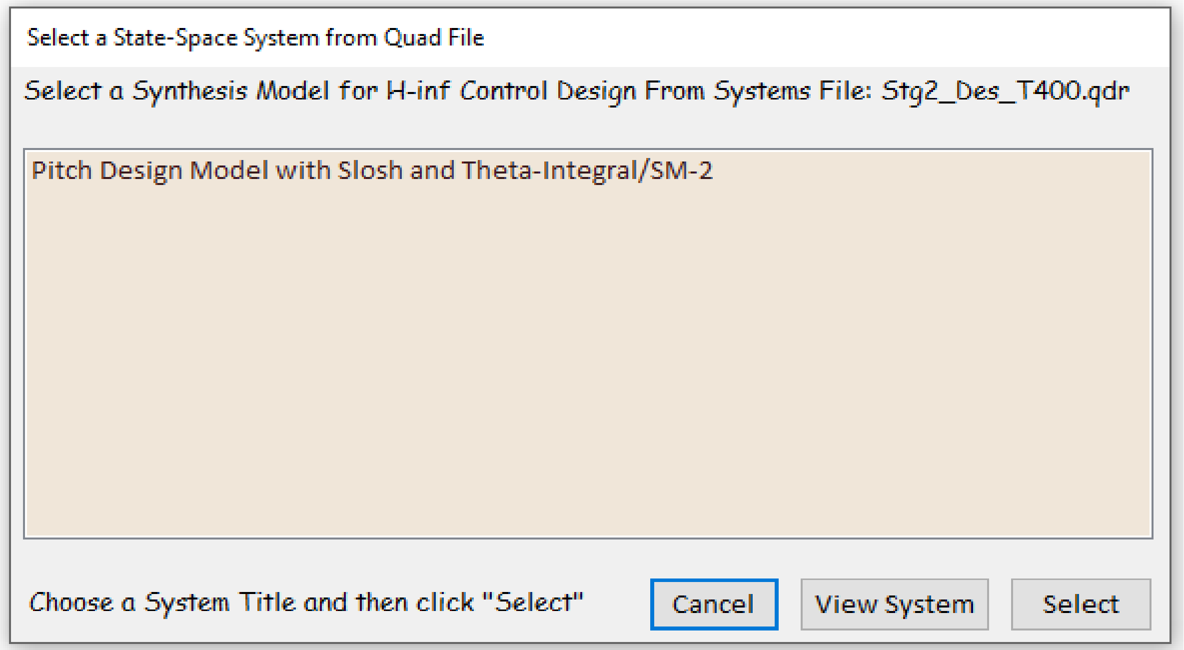


The Pair (A,B1) is Disturbable from Disturbances (Wi)

The Pair (A,B2) is Stabilizable from the Controls (Uc)

The Pair (A,C1) is Detectable from Output Criteria (z)

The Pair (A,C2) is Observable from Measurements (y)

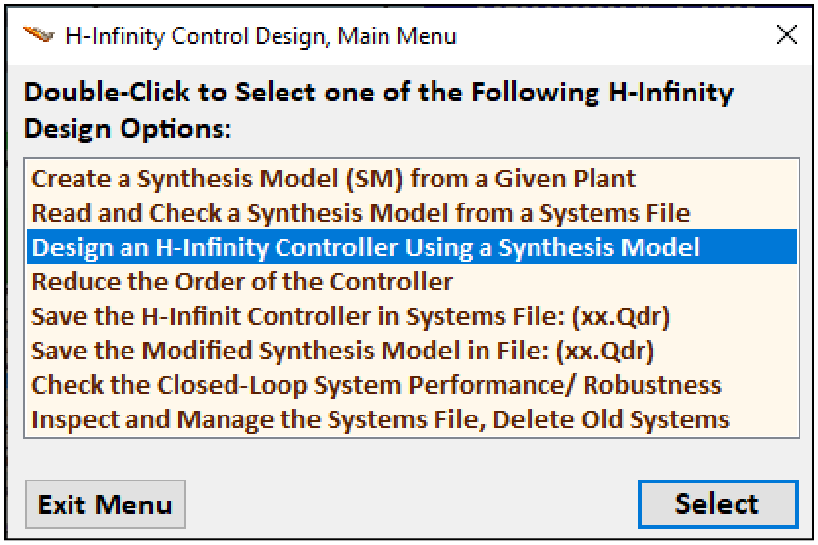

Select a State-Space System from Quad File

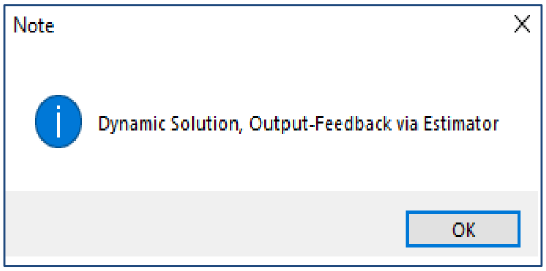
Select a Synthesis Model for H-inf Control Design From Systems File: Stg2_Des_T400.qdr

Pitch Design Model with Slosh and Theta-Integral/SM-2

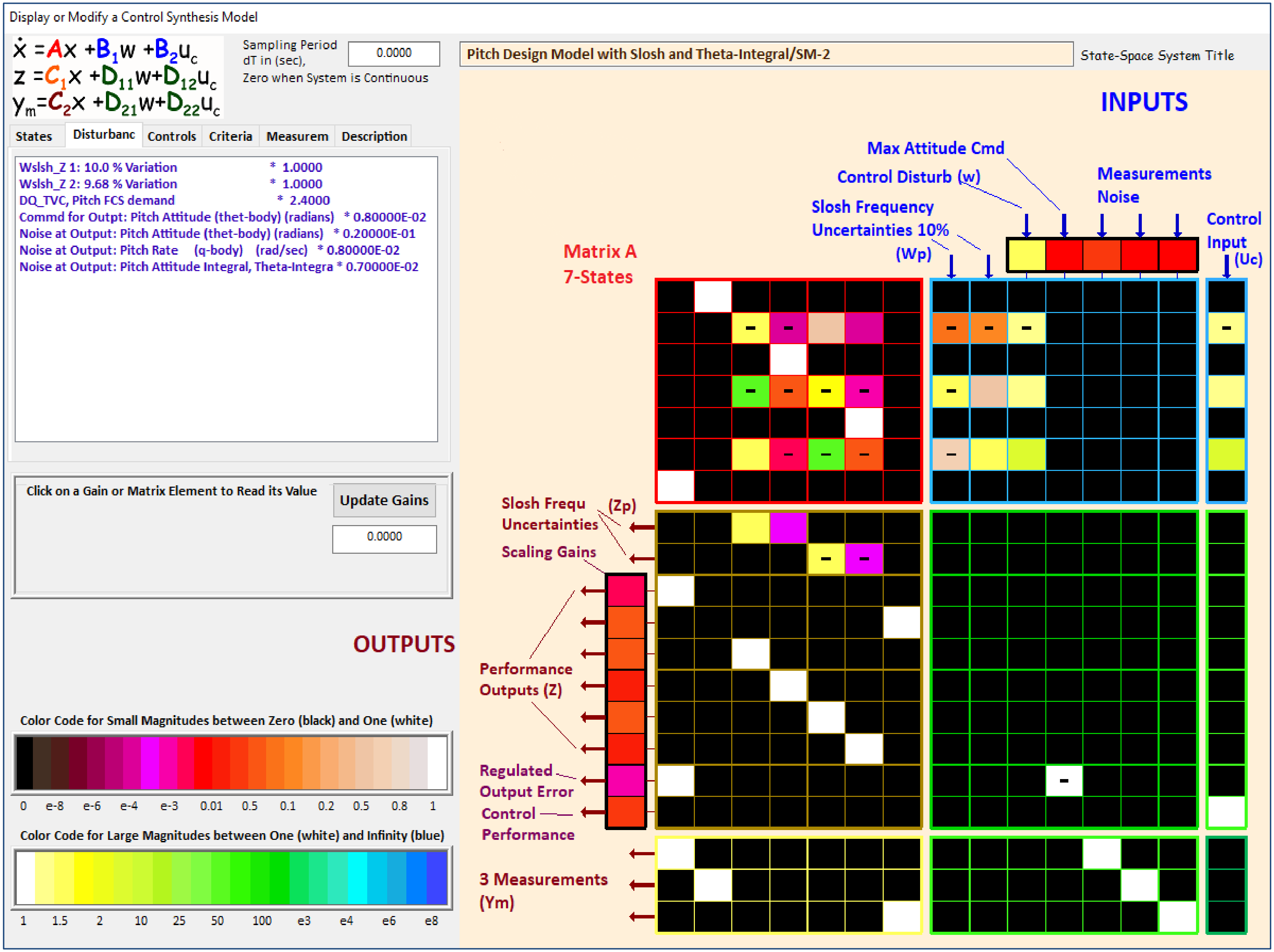
Choose a System Title and then click "Select"

The program confirms that the SM meets the expected observability and controllability requirements and displays the SM matrices graphically in system form, in Figure-1. The 9 SM matrices appear color coded and the performance optimization gains that scale the disturbances and the criteria are also shown in the inputs and outputs. The A-matrix consists of 7 states. There are 2 uncertainty inputs (w_p) for the two slosh frequency variations which are already scaled to correspond to ±unity Δ and they don't need scaling gains. There is 1 control disturbance (w), 1 θ-command for a regulated output θ-error, noise for the 3 measurements, and the TVC control input (u_c). In the outputs we have 2 uncertainty outputs (z_p), always the same as the uncertainty inputs, 6 performance criteria (z), 1 criterion for the regulated output error (z_{re}), 1 control utilization criterion that penalizes the control magnitude, and 3 measurements (y_m). Select the third option from the main menu to design the H-infinity controller, and click "Select". The program confirms that the solution is an output feedback gain dynamic controller that includes a state estimator.




Note

Dynamic Solution, Output-Feedback via Estimator



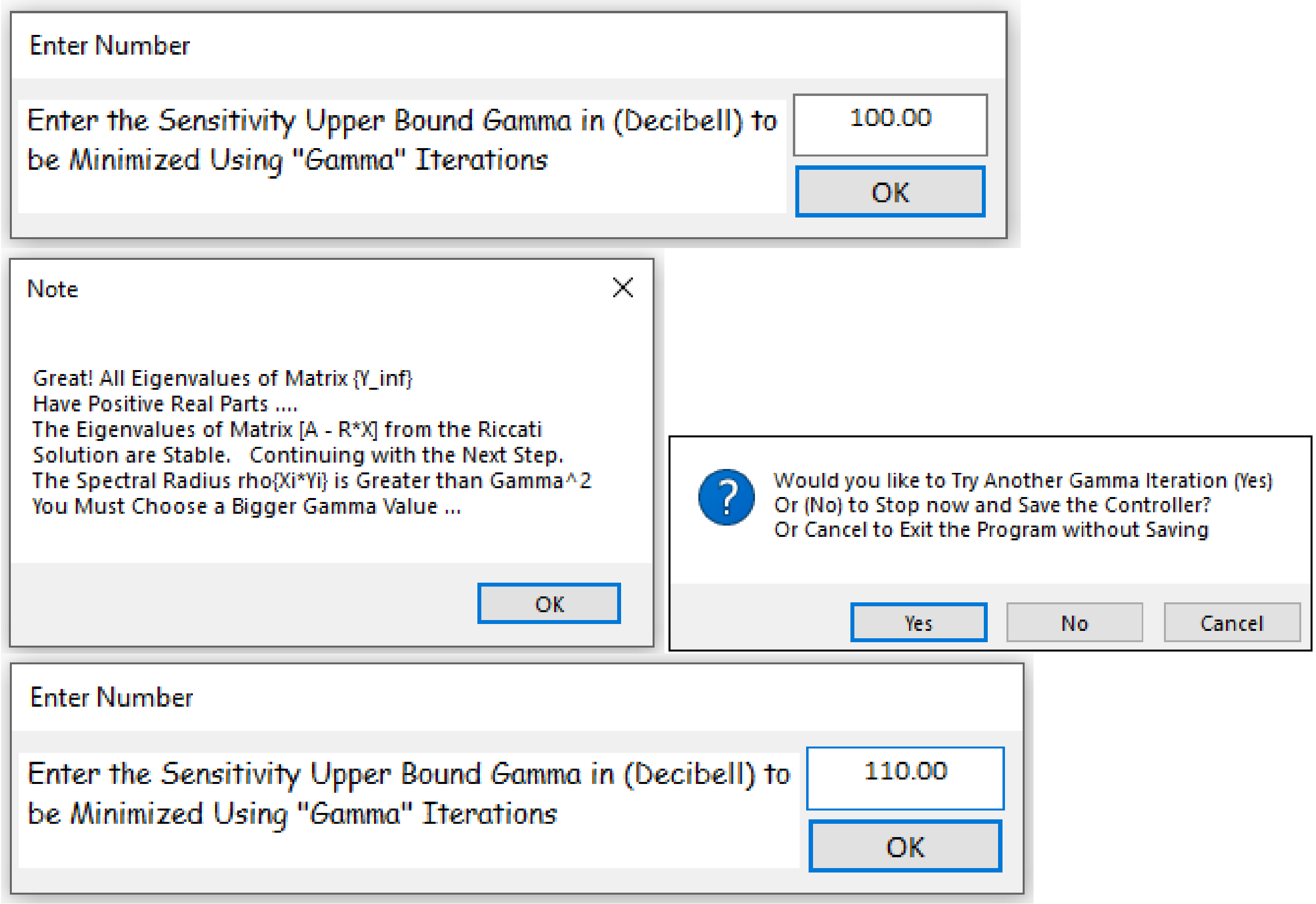
$$\dot{x} = Ax + B_1 w + B_2 u_c$$
$$z = C_1 x + D_{11} w + D_{12} u_c$$
$$y_m = C_2 x + D_{21} w + D_{22} u_c$$

Pitch Design Model with Slosh and Theta-Integral/SM-2

```
Wslsh_Z 1: 10.0 % Variation                    * 1.0000
Wslsh_Z 2: 9.68 % Variation                    * 1.0000
DQ_TVC, Pitch FCS demand                        * 2.4000
Commd for Outpt: Pitch Attitude (thet-body) (radians)  * 0.80000E-02
Noise at Output: Pitch Attitude (thet-body) (radians)  * 0.20000E-01
Noise at Output: Pitch Rate    (q-body)   (rad/sec)  * 0.80000E-02
Noise at Output: Pitch Attitude Integral, Theta-Integra * 0.70000E-02
```

Figure 1 Synthesis Model in Systems Form

Now we begin the iterative process of trying to minimize the upper bound γ of the infinity norm of the sensitivity transfer function between the input disturbances vector and the output criteria vector. We begin with an arbitrary large upper bound γ and try to find the smallest γ that will not violate the algorithm requirements. After 2-3 iterations we find that γ=110 (dB) works and we click on "No" meaning that we do not want to try another value but to accept the current controller.


Enter Number

Enter the Sensitivity Upper Bound Gamma in (Decibell) to be Minimized Using "Gamma" Iterations

100.00

Note

Great! All Eigenvalues of Matrix {Y_inf}
Have Positive Real Parts ....
The Eigenvalues of Matrix [A - R*X] from the Riccati
Solution are Stable.   Continuing with the Next Step.
The Spectral Radius rho{Xi*Yi} is Greater than Gamma^2
You Must Choose a Bigger Gamma Value ...

Would you like to Try Another Gamma Iteration (Yes)
Or (No) to Stop now and Save the Controller?
Or Cancel to Exit the Program without Saving

Enter Number

Enter the Sensitivity Upper Bound Gamma in (Decibell) to be Minimized Using "Gamma" Iterations

110.00

Figure-2 shows the closed-loop system poles with the control loop closed between the control inputs (u_c) and the measurements (y_m) via the H-infinity control system. All the poles are stable as expected. There are 3 low damped complex pairs of poles near the slosh frequencies, which is 6 (rad/sec). The red line corresponds to damping coefficient ζ=0.01. There are also 2 well damped complex pairs of poles near the ζ=0.707 green line. We return to the H-infinity main menu and save the controller gain by clicking on "*Save the H-infinity Controller in Systems File (x.Qdr)*". The title of the controller in the systems file is: "*H-Infin. Control for Stage-2 Launch Vehicle Output-Feedback*".

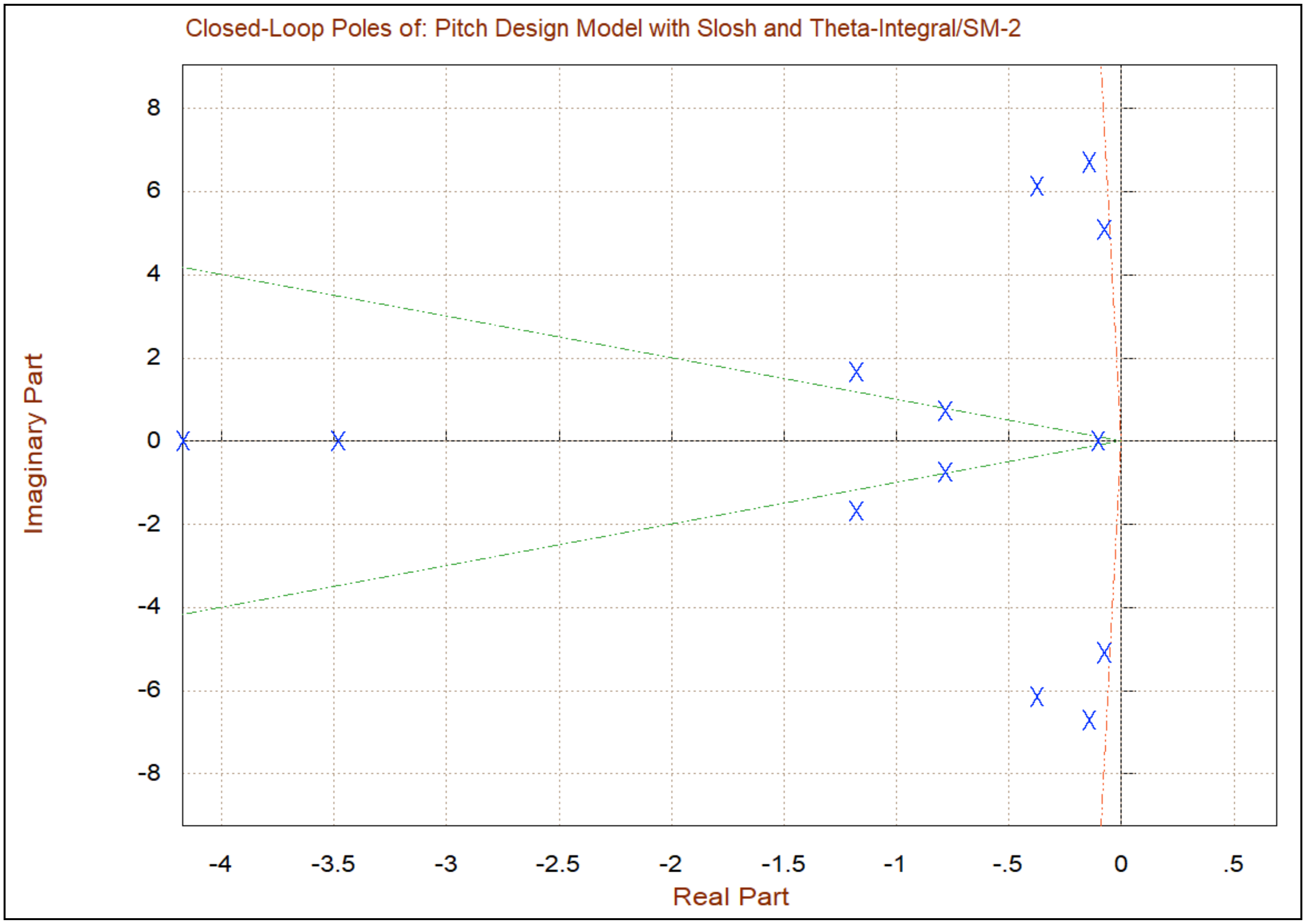
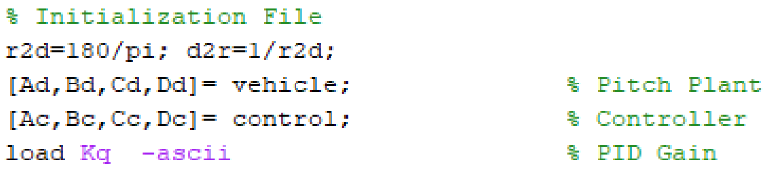


Figure 2 Closed-Loop System Poles

1.4 Control Analysis

We begin the control analysis by running the initialization file "init.m" which loads the vehicle system, the H-infinity controller (control.m), and the PID state-feedback gain controller (Kq). We will use Matlab to analyze stability by calculating the frequency response of the two open-loop systems, use simulations to analyze their responses to attitude change commands and compare the two designs. The H-infinity design is implemented in Simulink files "*Open_Loop_1.slx*" and "*Sim_1.slx*". The PID design is implemented in files "*Open_Loop_2.slx*" and "*Sim_2.slx*". The vehicle system used in this analysis is also the design system.



```
% Initialization File
r2d=180/pi; d2r=1/r2d;
[Ad,Bd,Cd,Dd]= vehicle;                    % Pitch Plant
[Ac,Bc,Cc,Dc]= control;                    % Controller
load Kq  -ascii                            % PID Gain
```

Figure 3 shows the two Simulink models used for analyzing stability in the frequency domain. The Matlab script file “freq.m” calculates the open-loop Bode and Nichols plots. The 3 inputs to both controllers are: (θ-attitude, q-rate and θ-integral). The H∞ controller however includes a state-estimator. That’s what makes it dynamic. A 110 (rad/sec) low-pass filter is also included in both.

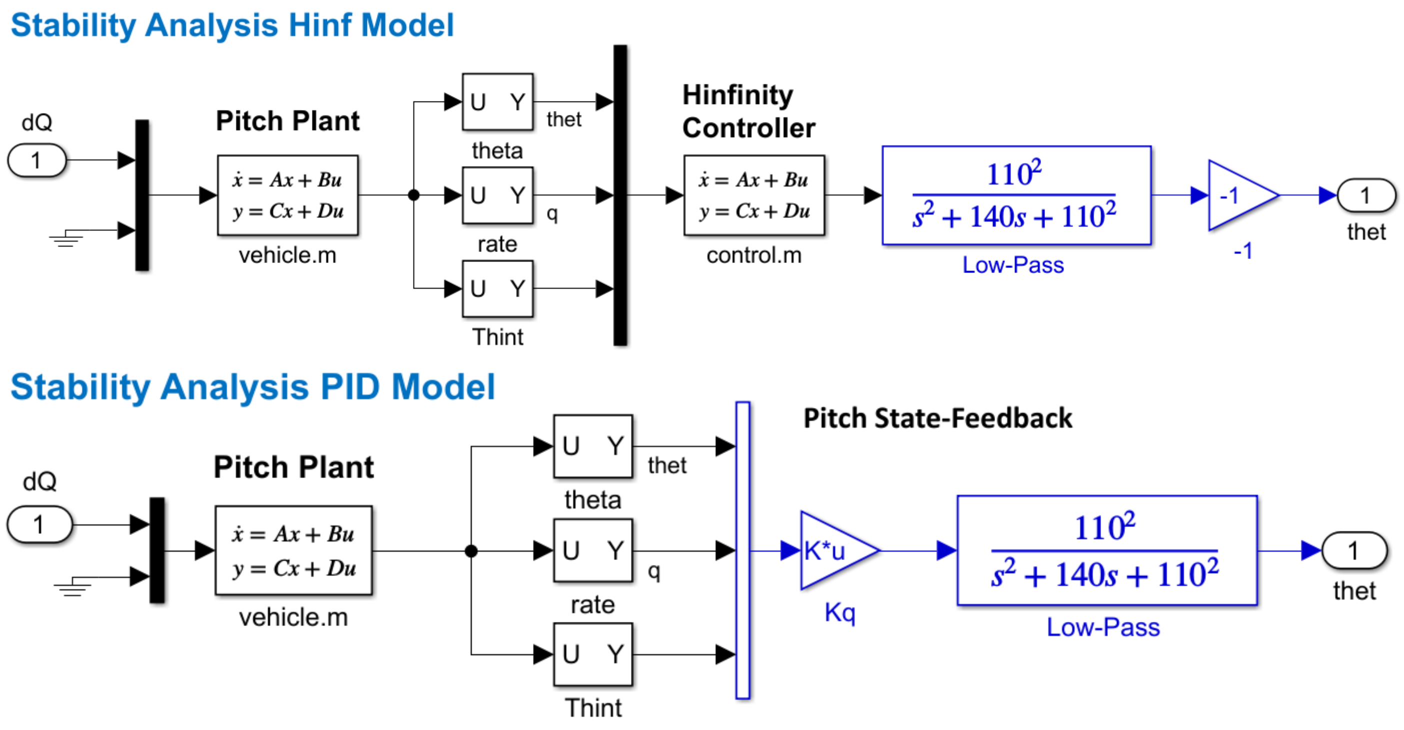
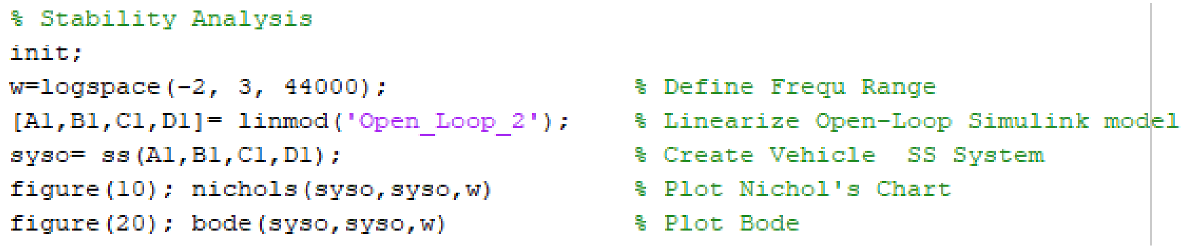


Figure 3 Stability Analysis Model in Simulink File “Open_Loop_1.slx”

Figure 4 shows the open-loop Bode and Nichols plots of the PID state feedback controller. The two modes are very close in frequency (5.84 and 6.02 r/s). The damping on both slosh modes was set to ζ=0.002 assuming no baffles. The LOX mode is very unstable because it was designed without slosh consideration. Figure 5 shows the same plots calculated with the H-infinity model. The 6.02 (r/s) LOX mode is no longer encircling the critical + point but it is phase-stable. It has shifted to the right by the introduction of a 4.6 r/s controller mode, and the phase and gain margins are great.



```
% Stability Analysis
init;
w=logspace(-2, 3, 44000);                    % Define Frequ Range
[A1,B1,C1,D1]= linmod('Open_Loop_2');        % Linearize Open-Loop Simulink model
syso= ss(A1,B1,C1,D1);                       % Create Vehicle  SS System
figure(10); nichols(syso,syso,w)             % Plot Nichol's Chart
figure(20); bode(syso,syso,w)                % Plot Bode
```

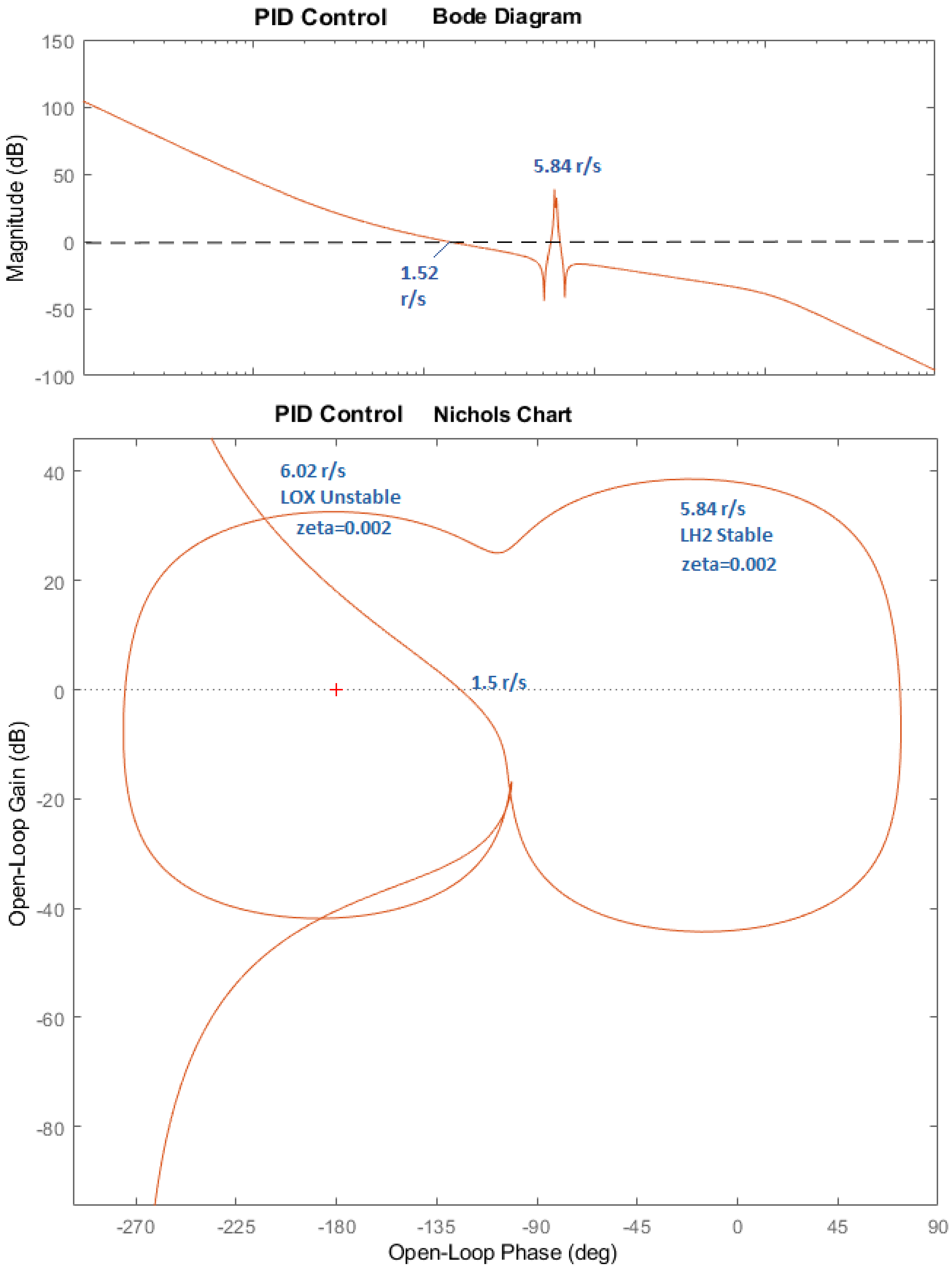


Figure 4 Bode and Nichols Plots Showing that the LOX Slosh Mode is Very Unstable when Using the PID Controller

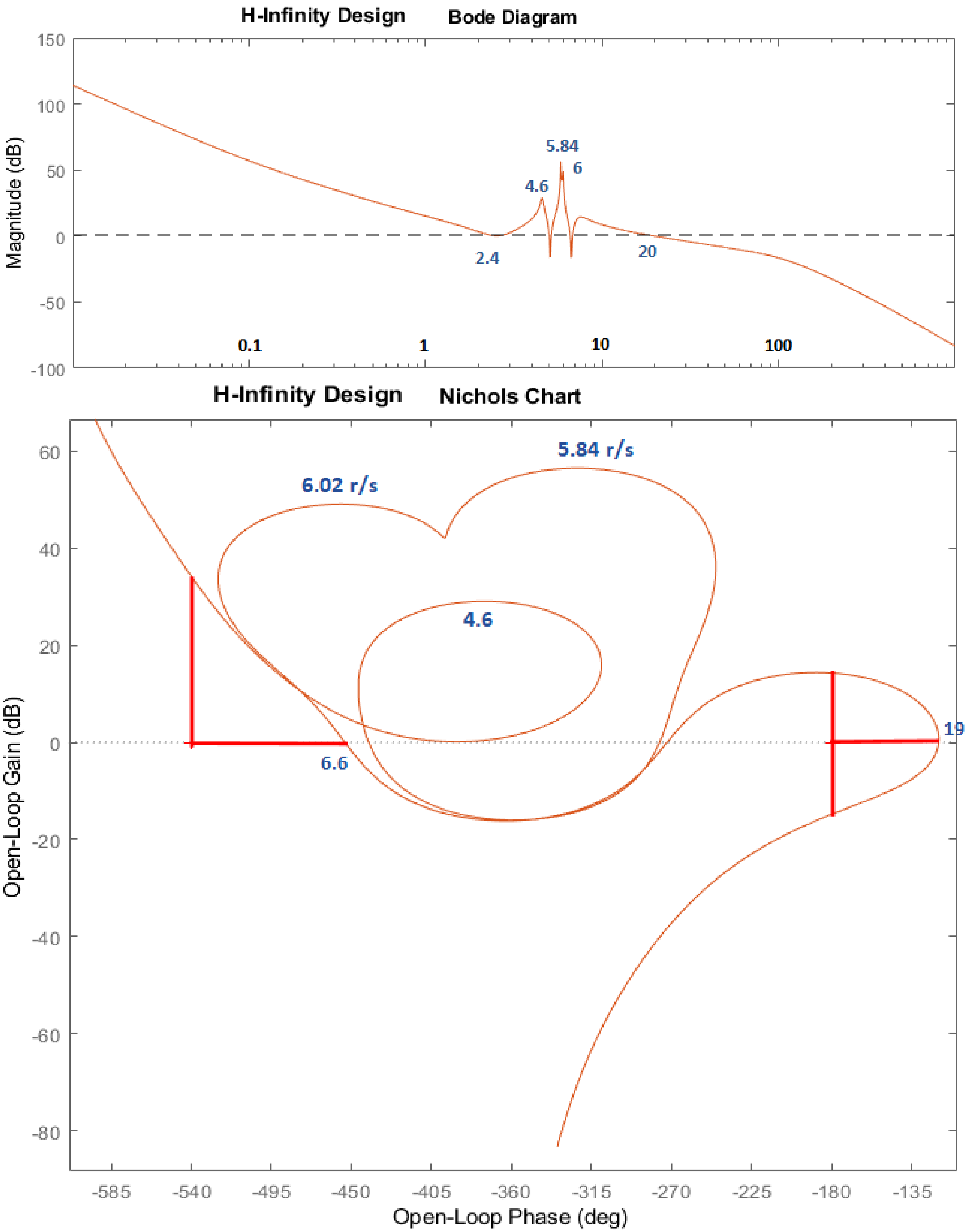


Figure 5 Bode and Nichols Plots Show that the H-Infinity Controller Stabilizes Both Slosh Modes with plenty of Margins

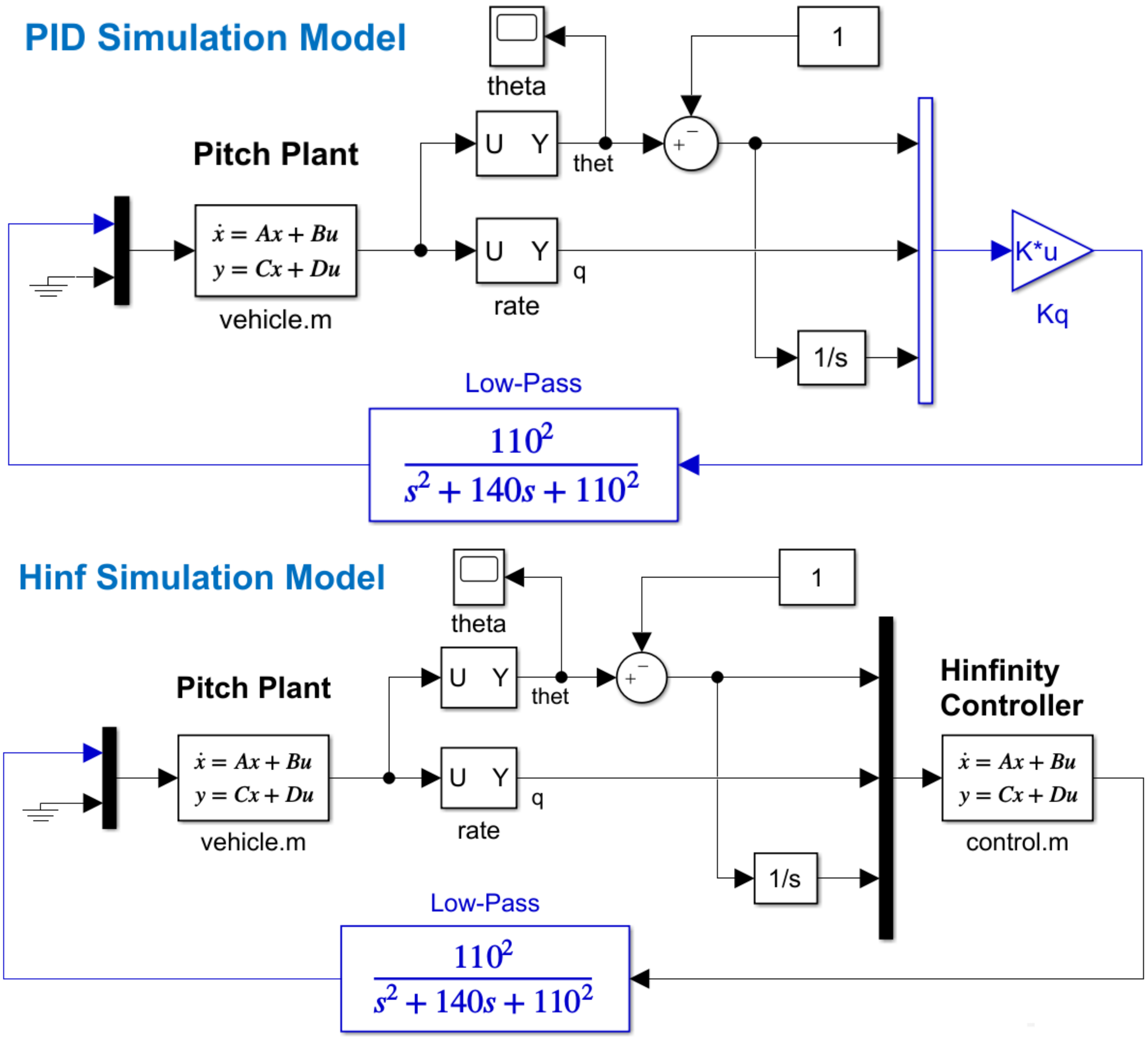


Figure 6 Two Simulation Models "Sim_2" and "Sim_1" for PID Control and H-Infinity

The two simulation models in Figure-6 use the same vehicle system "*Pitch Design Model with Slosh and Theta-Integral*" but different controls. Sim_1.slx includes the H-infinity control system "*H-Infin Control for Stage-2 Launch Vehicle Output-Feedback*" which is stable and Sim_2.slx includes the PID state-feedback gain Kq which is unstable due to sloshing. Figure-7 shows the step responses of both systems. The PID system "*Sim_2.slx*" has an oscillatory divergence at 6 (rad/sec) which is the LOX frequency.

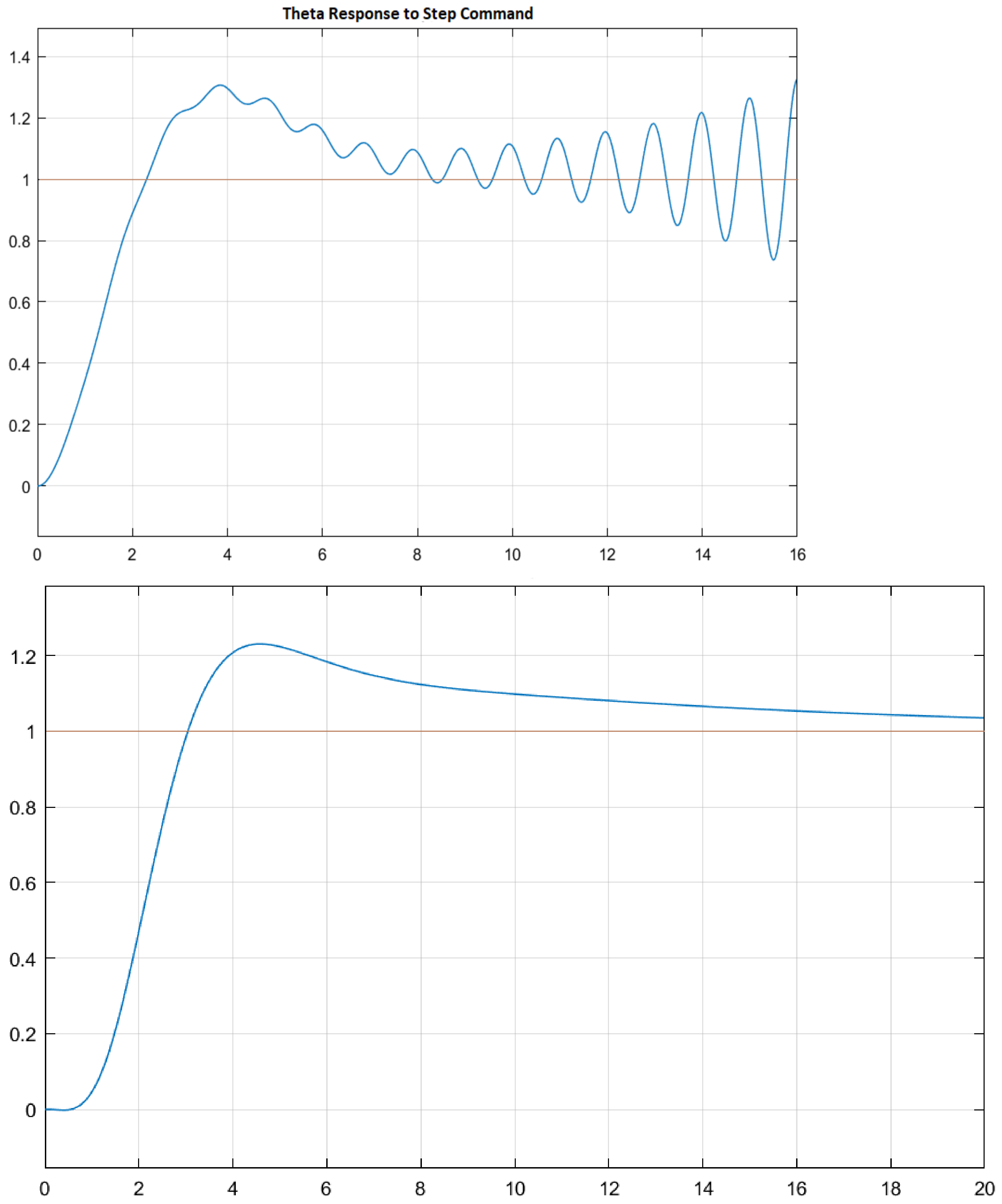


Figure 7 Closed-Loop System Response to Attitude Step Command, Stable versus Unstable Systems

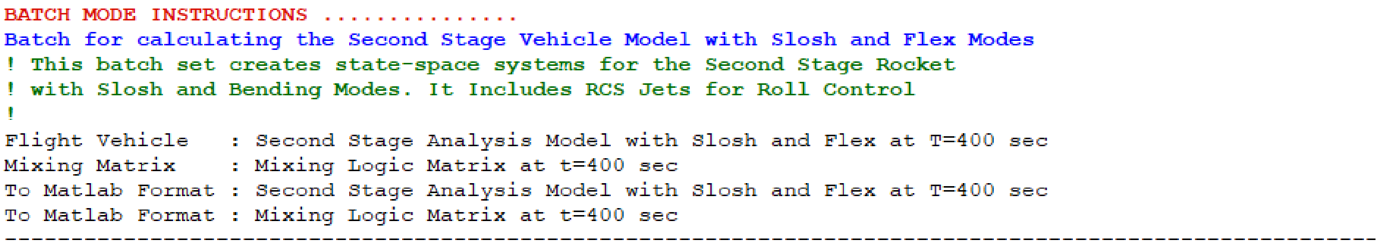
2.0 Detailed Control Analysis

We will continue analyzing the 2nd stage vehicle in more detail by including structural flexibility, slosh, actuators, TWD dynamics, and load-torque feedback which is the dynamic coupling between the vehicle and actuators. The work files for this analysis are in this directory: "*Flixan\Control Analysis\Hinfinity\ Examples\ Stage-2 LV, Unstable Slosh\2-Analysis with Slosh & Flex Modes*". The input file is "*Stg2_Anal_T400.inp*" that contains the vehicle data including the flex modes. The title of the vehicle dataset is "*Second Stage Analysis Model with Slosh and Flex at T=400 sec*". The purpose of the vehicle system in Section-1 was to create a robust control design. The vehicle in this section is used to analyze stability and performance by including RCS thrusters and a lot of other stuff missing from the design model. The design model is kept simple in order to produce a low order control system. Modifications may be needed later by adding filters or adjusting the gains.

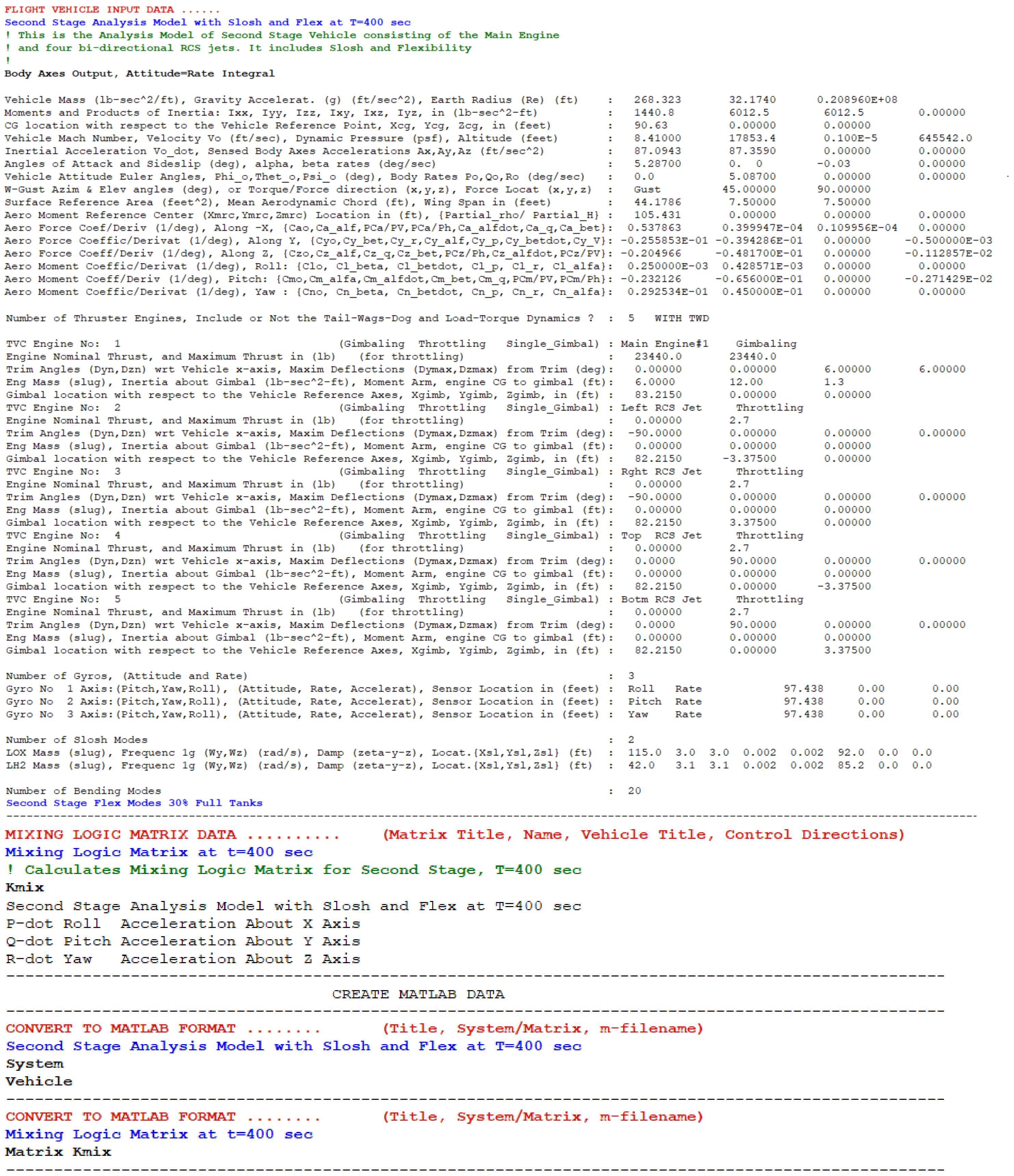
This time we will analyze the full vehicle, roll, pitch and yaw using both controllers: the stabilizing H-Infinity and the simple PID which allows LOX to become unstable. The yaw axis is the same as the pitch axis. For roll we will include the RCS control system in the simulations and describe the phase-plane control system with the jet selection logic. The slosh modes are included in the Flixan vehicle in both pitch z_s and yaw y_s directions. In Section-3 slosh will be replaced with an external spherical pendulum model. Structural bending is included with 20 flex modes. The vehicle data is set "WITH TWD" flag which creates gimbal acceleration inputs to implement the tail-wag-dog dynamics. It generates also load-torque feedback outputs that couples dynamically the vehicle with the TVC actuator. We will use two actuator models, a linear model for the stability analysis and a non-linear actuator for the simulation model. We will analyze linear stability and show simulation results.

2.1 Input File

The Flixan input file in this section is "*Stg2_Anal_T400.inp*" and the systems are saved in file "*Stg2_Anal-T400.inp*". The vehicle has one gimbaling TVC engine and 4 bi-directional thrusters that represent the 8 actual jets. They are activated by the jet-selection logic and their thrusts are either zero or ±2.7 (lbf). A set of flex modes is included at the bottom of the input file. Its title is "*Second Stage Flex Modes at 30% Full Tanks*". The modes are already selected and scaled to match the units of vehicle data. They are processed together with the vehicle data to produce the flex vehicle. The batch set is on the top of the file and it can be used to process the entire file. The Mixing Logic dataset creates the mixing logic matrix but we will only use the 2x2 TVC section that affects pitch and yaw. We ignore the part of the matrix that affects the jets because we are using a separate logic for roll.



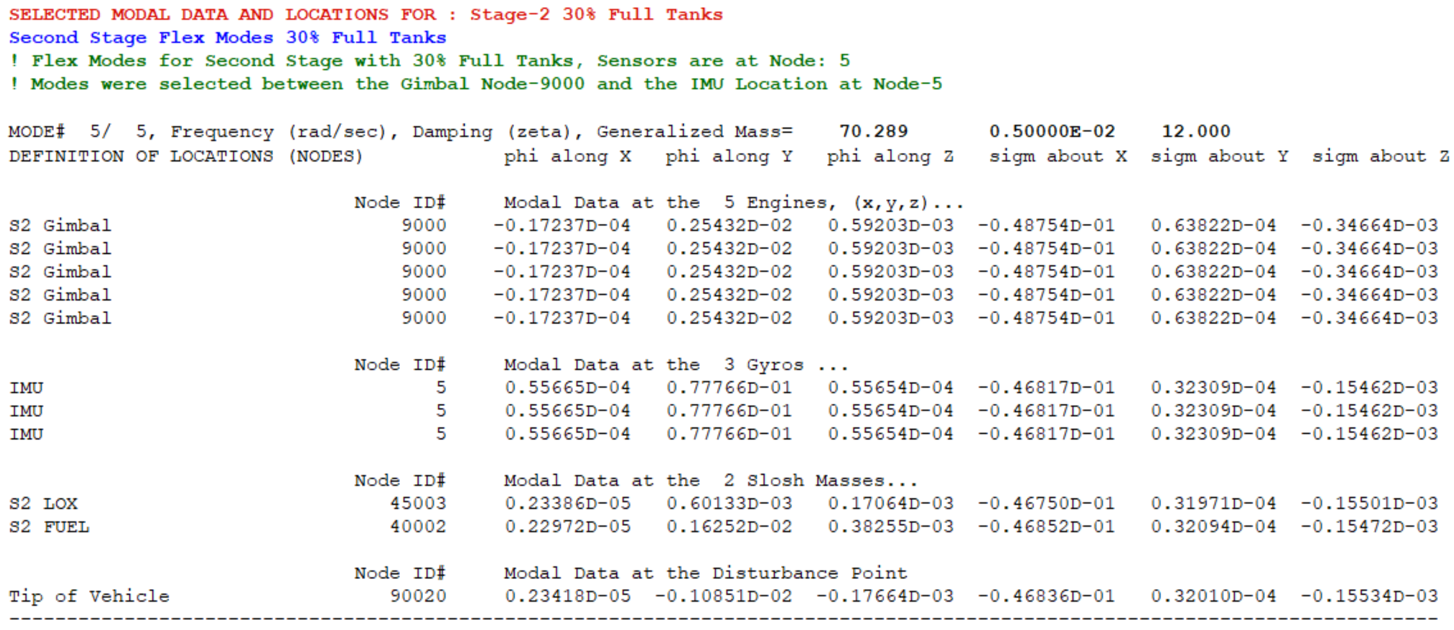
```
BATCH MODE INSTRUCTIONS ...............
Batch for calculating the Second Stage Vehicle Model with Slosh and Flex Modes
! This batch set creates state-space systems for the Second Stage Rocket
! with Slosh and Bending Modes. It Includes RCS Jets for Roll Control
!
Flight Vehicle    : Second Stage Analysis Model with Slosh and Flex at T=400 sec
Mixing Matrix     : Mixing Logic Matrix at t=400 sec
To Matlab Format : Second Stage Analysis Model with Slosh and Flex at T=400 sec
To Matlab Format : Mixing Logic Matrix at t=400 sec
------------------------------------------------------------------------------------------------------
```



```
FLIGHT VEHICLE INPUT DATA ......
Second Stage Analysis Model with Slosh and Flex at T=400 sec
! This is the Analysis Model of Second Stage Vehicle consisting of the Main Engine
! and four bi-directional RCS jets. It includes Slosh and Flexibility
!
Body Axes Output, Attitude=Rate Integral

Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft)    :  268.323      32.1740     0.208960E+08
Moments and Products of Inertia: Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)          :  1440.8       6012.5       6012.5       0.00000
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)        :  90.63        0.00000      0.00000
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)       :  8.41000      17853.4      0.100E-5     645542.0
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)         :  87.0943      87.3590      0.00000      0.00000
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                         :  5.28700      0.  0       -0.03         0.00000
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)   :  0.0          5.08700      0.00000      0.00000
W-Gust Azim & Elev angles (deg), or Torque/Force direction (x,y,z), Force Locat (x,y,z)  :  Gust        45.00000     90.00000
Surface Reference Area (feet^2), Mean Aerodynamic Chord (ft), Wing Span in (feet)        :  44.1786      7.50000      7.50000
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial_rho/ Partial_H} :  105.431      0.00000      0.00000      0.00000
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:  0.537863     0.399947E-04 0.109956E-04  0.00000
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}: -0.255853E-01 -0.394286E-01 0.00000     -0.500000E-03
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}: -0.204966     -0.481700E-01 0.00000     -0.112857E-02
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:  0.250000E-03 0.428571E-03 0.00000      0.00000
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}: -0.232126     -0.656000E-01 0.00000     -0.271429E-02
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}:  0.292534E-01 0.450000E-01 0.00000      0.00000

Number of Thruster Engines, Include or Not the Tail-Wags-Dog and Load-Torque Dynamics ?  :  5   WITH TWD

TVC Engine No:  1                         (Gimbaling  Throttling   Single_Gimbal) : Main Engine#1    Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)             :  23440.0      23440.0
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000      6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  6.0000       12.00        1.3
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  83.2150      0.00000      0.00000
TVC Engine No:  2                         (Gimbaling  Throttling   Single_Gimbal) : Left RCS Jet     Throttling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)             :  0.00000      2.7
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg): -90.0000      0.00000      0.00000      0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  0.00000      0.00000      0.00000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  82.2150     -3.37500      0.00000
TVC Engine No:  3                         (Gimbaling  Throttling   Single_Gimbal) : Rght RCS Jet     Throttling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)             :  0.00000      2.7
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg): -90.0000      0.00000      0.00000      0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  0.00000      0.00000      0.00000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  82.2150      3.37500      0.00000
TVC Engine No:  4                         (Gimbaling  Throttling   Single_Gimbal) : Top  RCS Jet     Throttling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)             :  0.00000      2.7
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.0000       90.0000      0.00000      0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  0.00000      0.00000      0.00000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  82.2150      0.00000     -3.37500
TVC Engine No:  5                         (Gimbaling  Throttling   Single_Gimbal) : Botm RCS Jet     Throttling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)             :  0.00000      2.7
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.0000       90.0000      0.00000      0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  0.00000      0.00000      0.00000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  82.2150      0.00000      3.37500

Number of Gyros, (Attitude and Rate)                                                     :  3
Gyro No  1 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Location in (feet) :  Roll   Rate          97.438    0.00      0.00
Gyro No  2 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Location in (feet) :  Pitch  Rate          97.438    0.00      0.00
Gyro No  3 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Location in (feet) :  Yaw    Rate          97.438    0.00      0.00

Number of Slosh Modes                                                                    :  2
LOX Mass (slug), Frequenc 1g (Wy,Wz) (rad/s), Damp (zeta-y-z), Locat.{Xsl,Ysl,Zsl} (ft)  :  115.0  3.0  3.0  0.002  0.002  92.0  0.0  0.0
LH2 Mass (slug), Frequenc 1g (Wy,Wz) (rad/s), Damp (zeta-y-z), Locat.{Xsl,Ysl,Zsl} (ft)  :  42.0   3.1  3.1  0.002  0.002  85.2  0.0  0.0

Number of Bending Modes                                                                  :  20
Second Stage Flex Modes 30% Full Tanks
--------------------------------------------------------------------------------------------------------------------------
MIXING LOGIC MATRIX DATA ..........     (Matrix Title, Name, Vehicle Title, Control Directions)
Mixing Logic Matrix at t=400 sec
! Calculates Mixing Logic Matrix for Second Stage, T=400 sec
Kmix
Second Stage Analysis Model with Slosh and Flex at T=400 sec
P-dot Roll  Acceleration About X Axis
Q-dot Pitch Acceleration About Y Axis
R-dot Yaw   Acceleration About Z Axis
-------------------------------------------------------------------------------------------
                     CREATE MATLAB DATA
-------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Second Stage Analysis Model with Slosh and Flex at T=400 sec
System
Vehicle
-------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Mixing Logic Matrix at t=400 sec
Matrix Kmix
-------------------------------------------------------------------------------------------
```

The vehicle system and the TVC matrix are saved in files “vehicle.m” and “Kmix.mat” in order to be loaded into Matlab.

The first mode of the modal data set "*Second Stage Flex Modes at 30% Full Tanks*" is shown below. It includes nodes at the TVC gimbal, the 4 jets, the IMU, and the 2 slosh masses.



```
SELECTED MODAL DATA AND LOCATIONS FOR : Stage-2 30% Full Tanks
Second Stage Flex Modes 30% Full Tanks
! Flex Modes for Second Stage with 30% Full Tanks, Sensors are at Node: 5
! Modes were selected between the Gimbal Node-9000 and the IMU Location at Node-5

MODE#  5/   5, Frequency (rad/sec), Damping (zeta), Generalized Mass=    70.289      0.50000E-02    12.000
DEFINITION OF LOCATIONS (NODES)             phi along X   phi along Y   phi along Z   sigm about X  sigm about Y  sigm about Z

                              Node ID#    Modal Data at the  5 Engines, (x,y,z)...
S2 Gimbal                         9000   -0.17237D-04   0.25432D-02   0.59203D-03  -0.48754D-01   0.63822D-04  -0.34664D-03
S2 Gimbal                         9000   -0.17237D-04   0.25432D-02   0.59203D-03  -0.48754D-01   0.63822D-04  -0.34664D-03
S2 Gimbal                         9000   -0.17237D-04   0.25432D-02   0.59203D-03  -0.48754D-01   0.63822D-04  -0.34664D-03
S2 Gimbal                         9000   -0.17237D-04   0.25432D-02   0.59203D-03  -0.48754D-01   0.63822D-04  -0.34664D-03
S2 Gimbal                         9000   -0.17237D-04   0.25432D-02   0.59203D-03  -0.48754D-01   0.63822D-04  -0.34664D-03

                              Node ID#    Modal Data at the  3 Gyros ...
IMU                                  5    0.55665D-04   0.77766D-01   0.55654D-04  -0.46817D-01   0.32309D-04  -0.15462D-03
IMU                                  5    0.55665D-04   0.77766D-01   0.55654D-04  -0.46817D-01   0.32309D-04  -0.15462D-03
IMU                                  5    0.55665D-04   0.77766D-01   0.55654D-04  -0.46817D-01   0.32309D-04  -0.15462D-03

                              Node ID#    Modal Data at the  2 Slosh Masses...
S2 LOX                           45003    0.23386D-05   0.60133D-03   0.17064D-03  -0.46750D-01   0.31971D-04  -0.15501D-03
S2 FUEL                          40002    0.22972D-05   0.16252D-02   0.38255D-03  -0.46852D-01   0.32094D-04  -0.15472D-03

                              Node ID#    Modal Data at the Disturbance Point
Tip of Vehicle                   90020    0.23418D-05  -0.10851D-02  -0.17664D-03  -0.46836D-01   0.32010D-04  -0.15534D-03
-----------------------------------------------------------------------------------------------------------------------------
```

2.2 Control Analysis

For stability analysis we use two open-loop models, see Figure-8. The model "Open_Loop_1.slx" is using the H∞ controller and the model "Open_Loop_2.slx" uses the PID. They are almost identical except for the control systems, shown in Figure-9. Only the pitch and yaw loops are included. The Phase-Plane logic for roll RCS control is non-linear and it is not included in linear analysis.

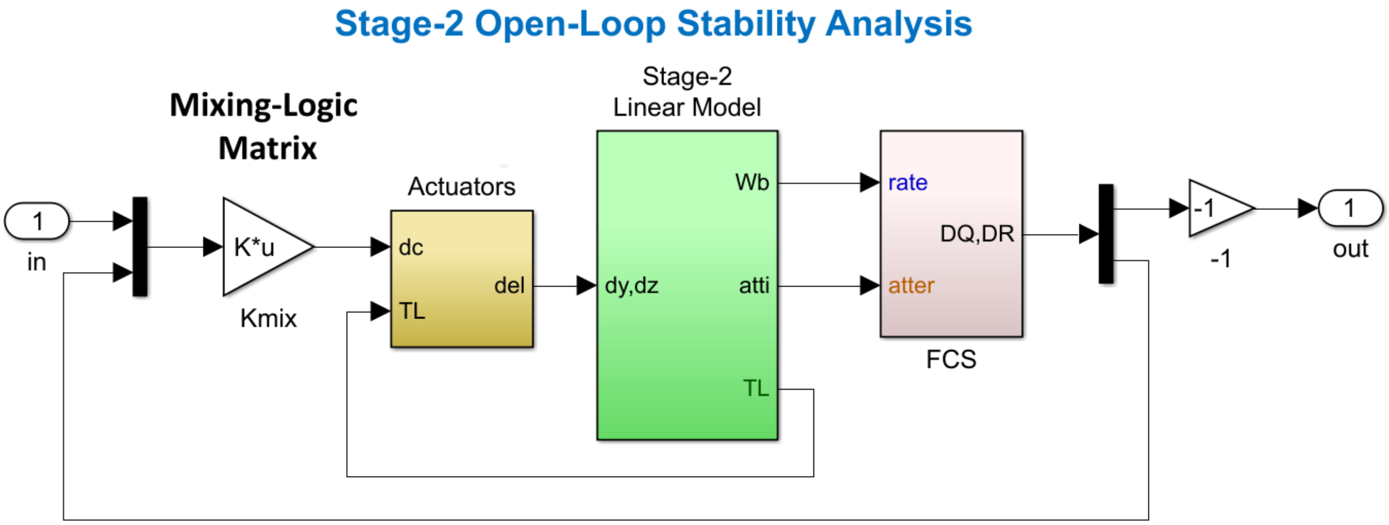


Figure 8 Stability Analysis Model Configured for Pitch Analysis

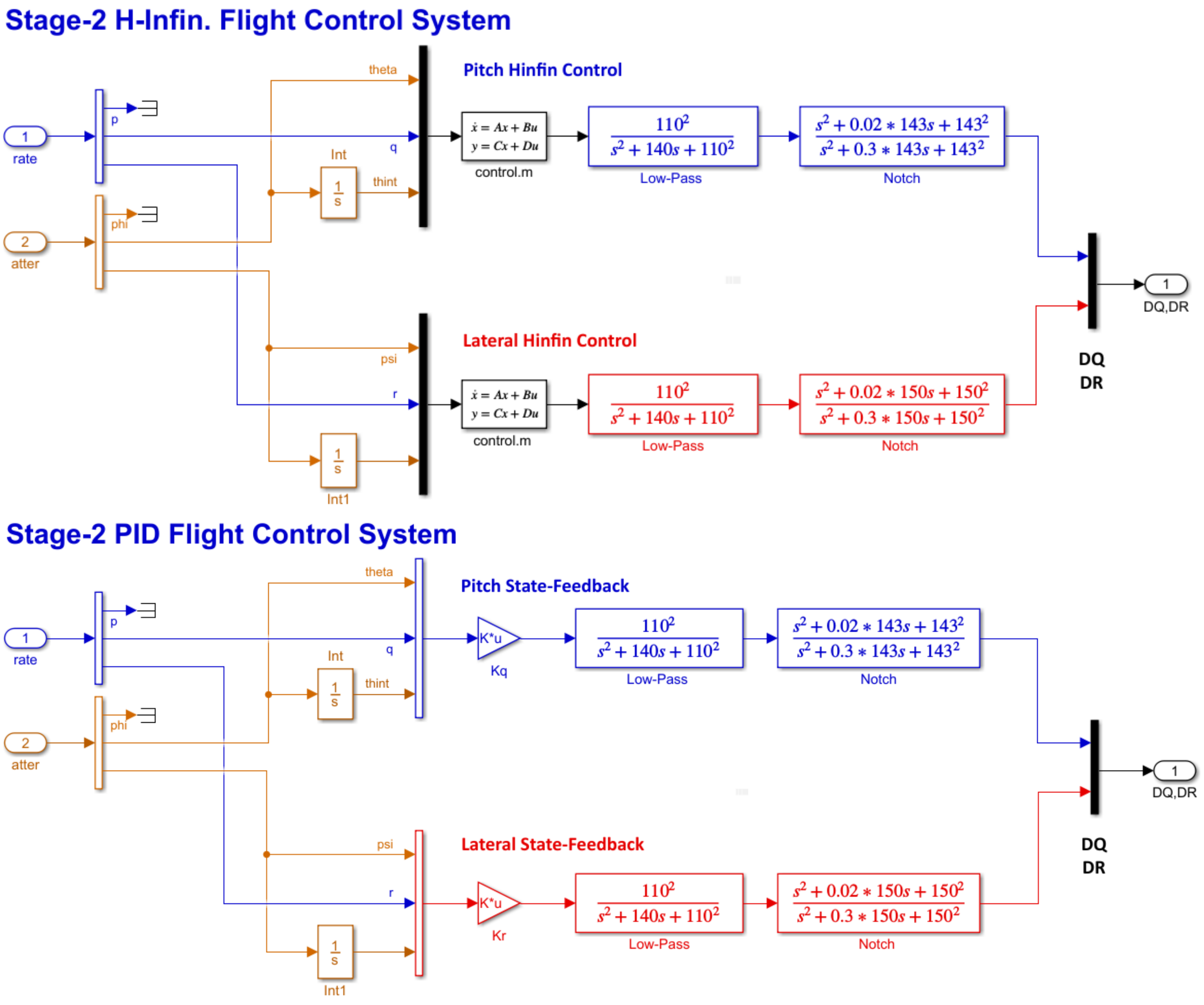


Figure 9 The Two Flight Control Systems, H∞ and PID

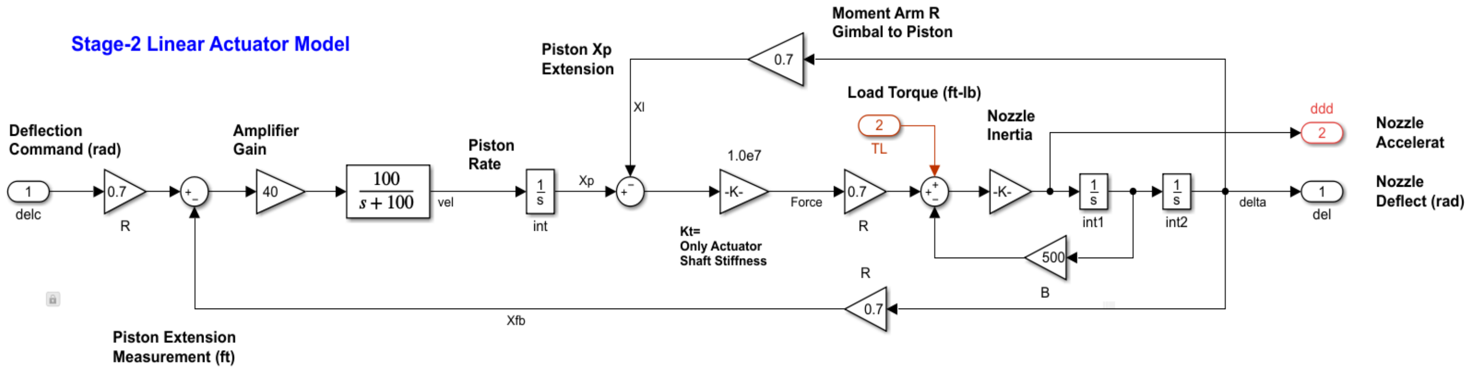
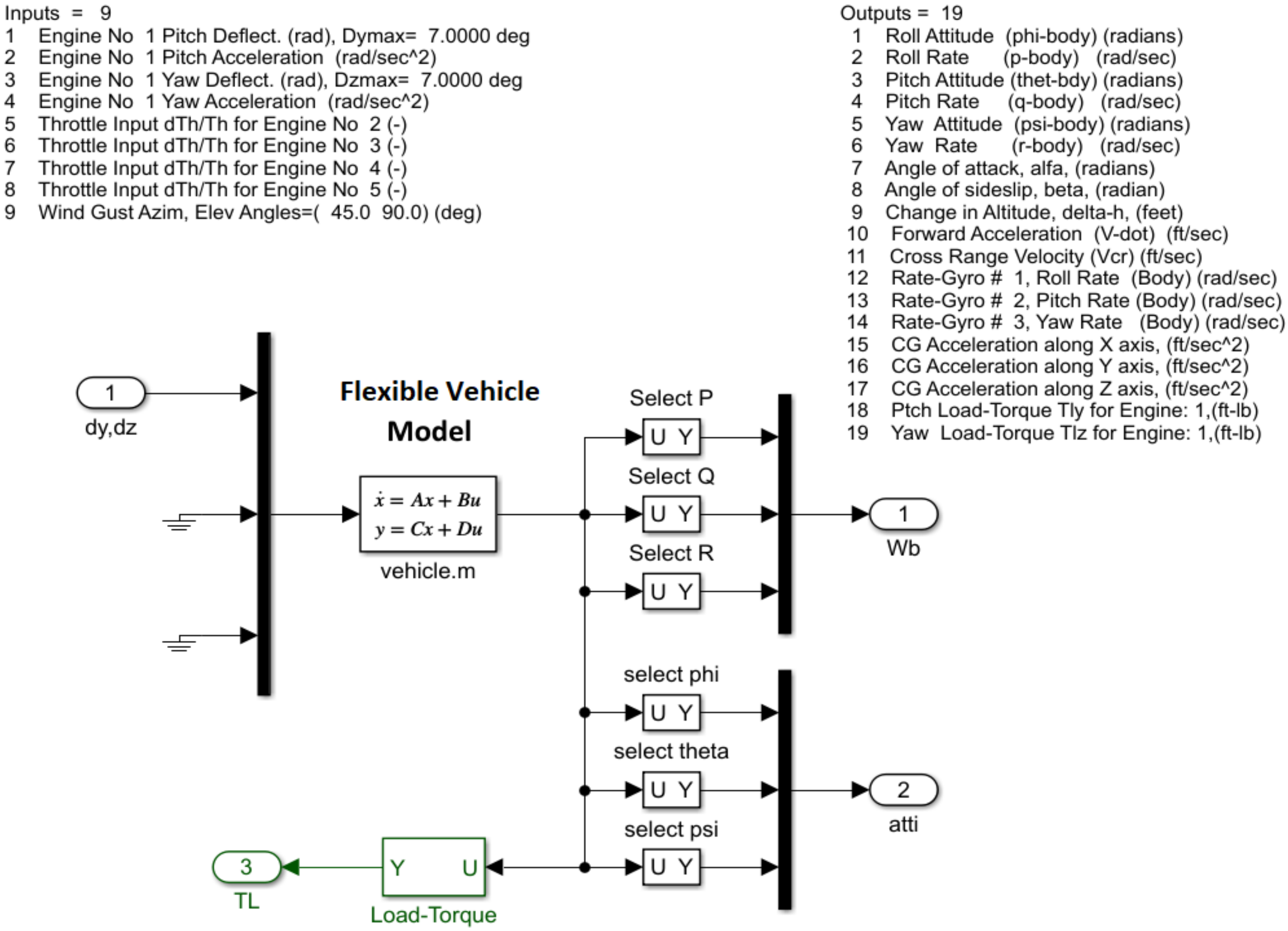


Figure 10 Pitch and Yaw TVC Actuator



```
Inputs  =   9
1    Engine No  1 Pitch Deflect. (rad), Dymax=  7.0000 deg
2    Engine No  1 Pitch Acceleration  (rad/sec^2)
3    Engine No  1 Yaw Deflect. (rad), Dzmax=  7.0000 deg
4    Engine No  1 Yaw Acceleration  (rad/sec^2)
5    Throttle Input dTh/Th for Engine No  2 (-)
6    Throttle Input dTh/Th for Engine No  3 (-)
7    Throttle Input dTh/Th for Engine No  4 (-)
8    Throttle Input dTh/Th for Engine No  5 (-)
9    Wind Gust Azim, Elev Angles=(  45.0  90.0) (deg)
```

```
Outputs =  19
 1    Roll Attitude  (phi-body) (radians)
 2    Roll Rate      (p-body)   (rad/sec)
 3    Pitch Attitude (thet-bdy) (radians)
 4    Pitch Rate     (q-body)   (rad/sec)
 5    Yaw  Attitude  (psi-body) (radians)
 6    Yaw  Rate      (r-body)   (rad/sec)
 7    Angle of attack, alfa, (radians)
 8    Angle of sideslip, beta, (radian)
 9    Change in Altitude, delta-h, (feet)
10    Forward Acceleration  (V-dot)  (ft/sec)
11    Cross Range Velocity (Vcr) (ft/sec)
12    Rate-Gyro #  1, Roll Rate  (Body) (rad/sec)
13    Rate-Gyro #  2, Pitch Rate (Body) (rad/sec)
14    Rate-Gyro #  3, Yaw Rate   (Body) (rad/sec)
15    CG Acceleration along X axis, (ft/sec^2)
16    CG Acceleration along Y axis, (ft/sec^2)
17    CG Acceleration along Z axis, (ft/sec^2)
18    Ptch Load-Torque Tly for Engine: 1,(ft-lb)
19    Yaw  Load-Torque Tlz for Engine: 1,(ft-lb)
```

Figure 11 Flexible Vehicle System "Second Stage Analysis Model with Slosh and Flex at T=400 sec" Generated from Flixan

The flex vehicle system is loaded into Matlab from file "vehicle.m" using the script file "init.m". It includes gimbal acceleration inputs for the TWD implementation. The outputs include pitch and yaw load-torques at the TVC gimbal which are fed back into the actuator load-torque inputs. The rate-gyro outputs (12, 13, 14) are used for measurement because they include flexibility, instead of the rigid rate outputs (2, 4, 6). For attitude feedback we use the rigid outputs (1, 3, and 5). The TWD produces a dip in the frequency response at 49 (r/s). Figures 12 and 13 show the frequency response stability analysis results for the PID against the H_∞ controllers. They are calculated by running the file "freq.m".

```
% Initialize Orbital Parameters
d2r= pi/180; r2d=180/pi;
[Av, Bv, Cv, Dv]= vehicle;          % Load Vehicle Analysis Model
[Ac, Bc, Cc, Dc]= control;          % Load Hinfin Controller
load Kmix  -ascii
load Kq    -ascii;                  % Load the PID Gains

nt=8; Thr=2.7; Tsw=10;              % Number of thrusters, Thrust
Mdel=0.005;                         % Measurement Delay
x1_ini= [-0,0, -0,0, 0,0]*d2r;      % State Initialization
x2_ini=zeros(1,52);                 % Other State Initialization
x2_ini=[x1_ini,x2_ini];
Ixx=1440.8; Iyy=6012.5; Xcg=90.63;  % Phase-Plane Parameters
```

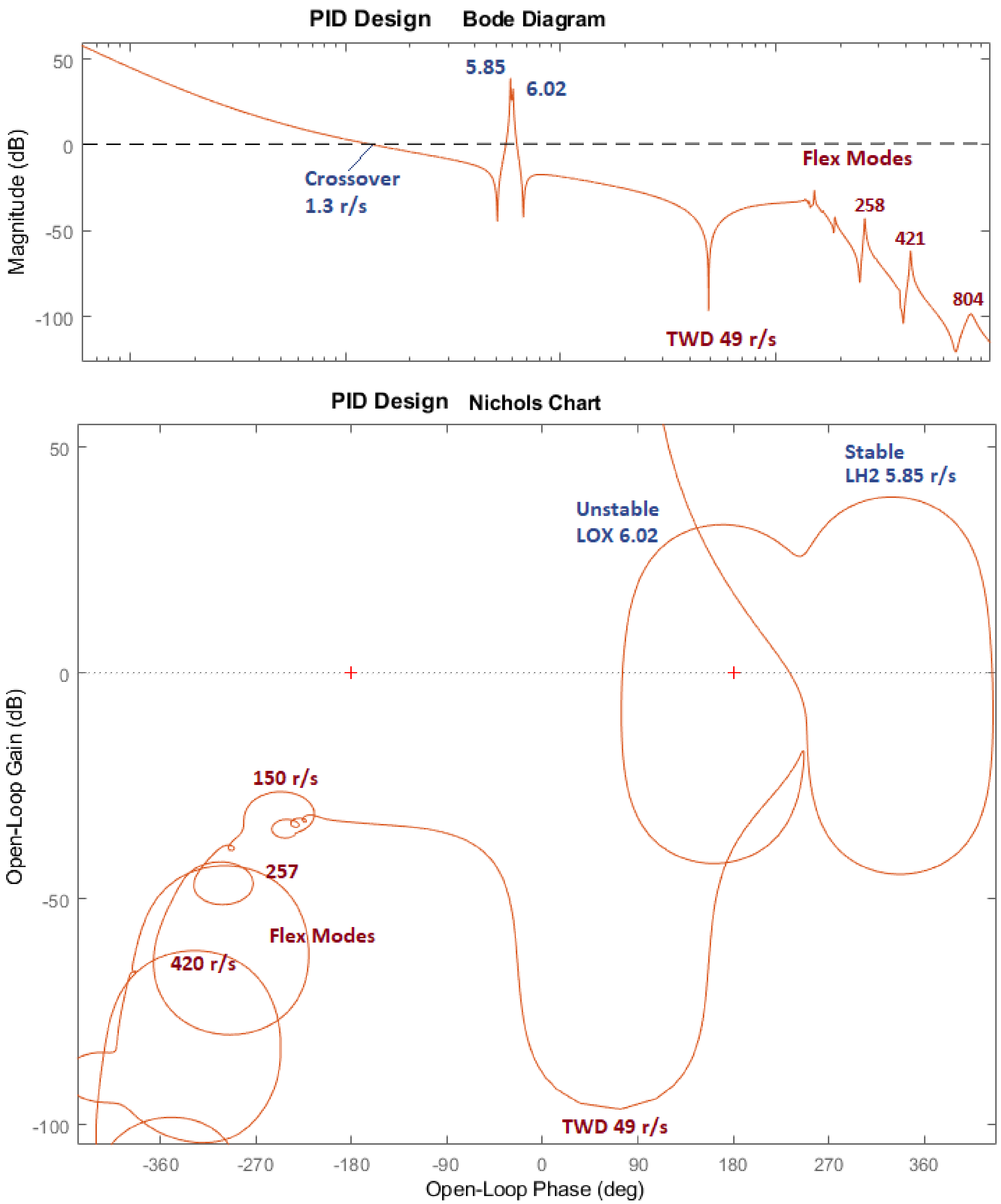


Figure 12 Stability Analysis with Flexibility and Slosh Using the PID Controller. Shows that LOX is Unstable. The 150 r/s Mode is attenuated with a Notch Filter

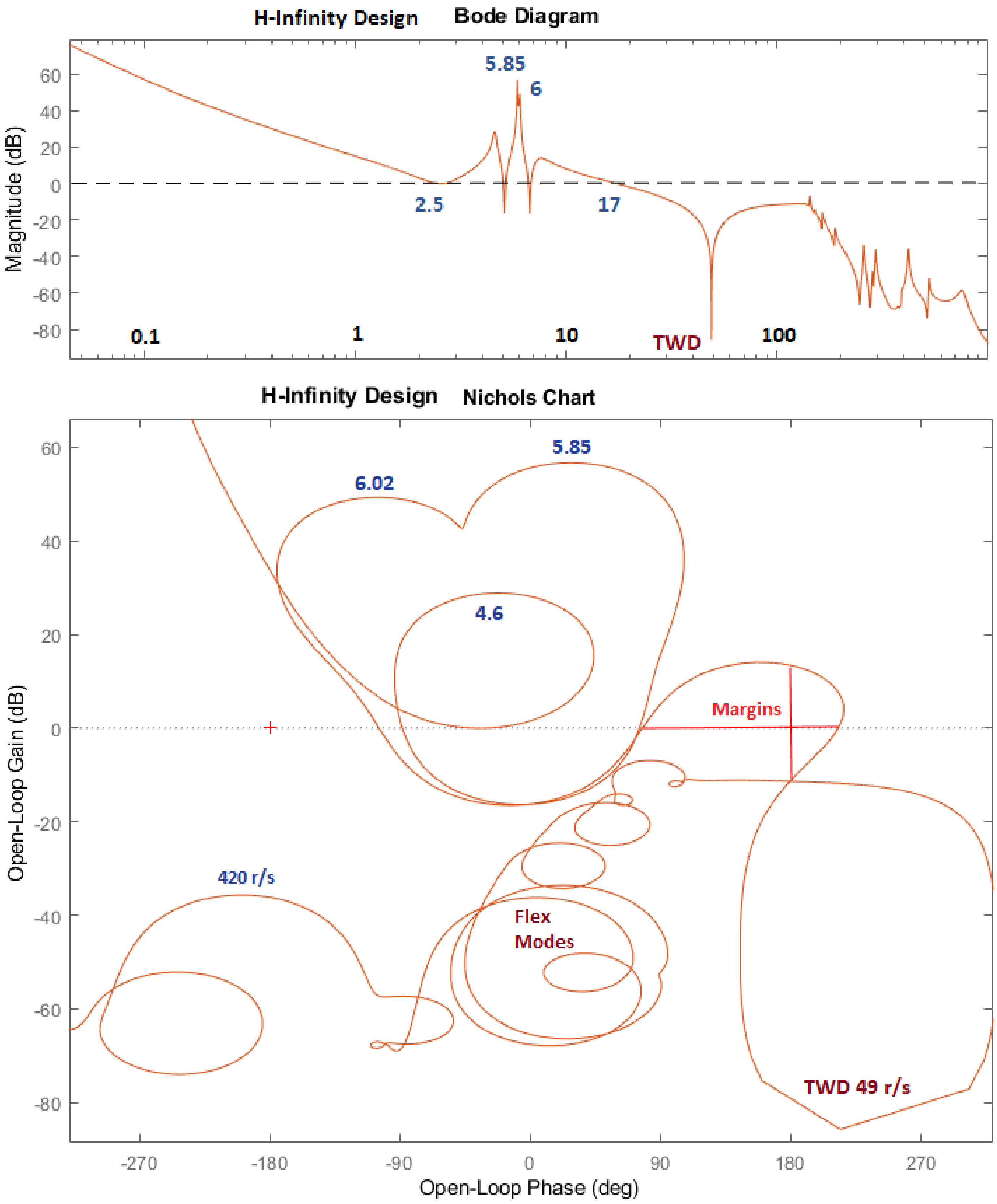


Figure 13 Stability Analysis with Flexibility and Slosh Using the H-Infinity Controller. Both Slosh Modes are Phase Stabilized. The 143 r/s Mode is attenuated with a Notch Filter

2.3 Simulation Models

We have two simulation models which are very similar, see Figure-14, but they use the two different controllers. The model "*Sim_2.slx*" has the PID system and it is unstable, as expected, and the model "*Sim_1.slx*" uses the H_∞ system and it is stable. They both use the Flixan generated vehicle with slosh and flexibility. They also use the non-linear actuator, see Figure-15, and the RCS control system that controls the roll axis.

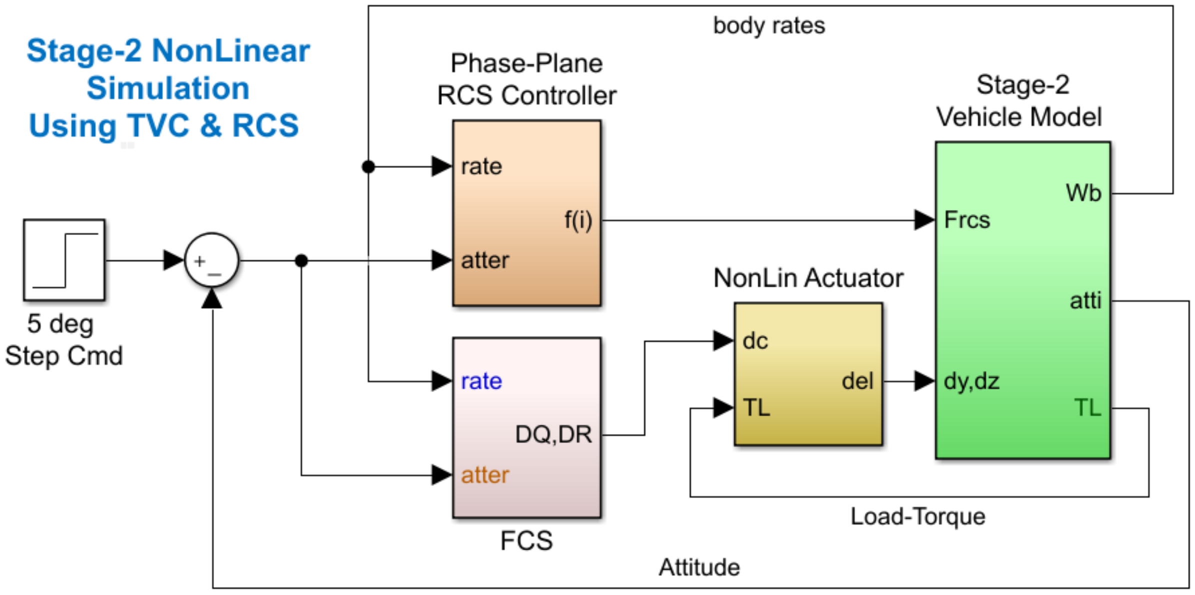


Figure 14 Simulation Model "Sim_1.slx" with Slosh and Flexibility

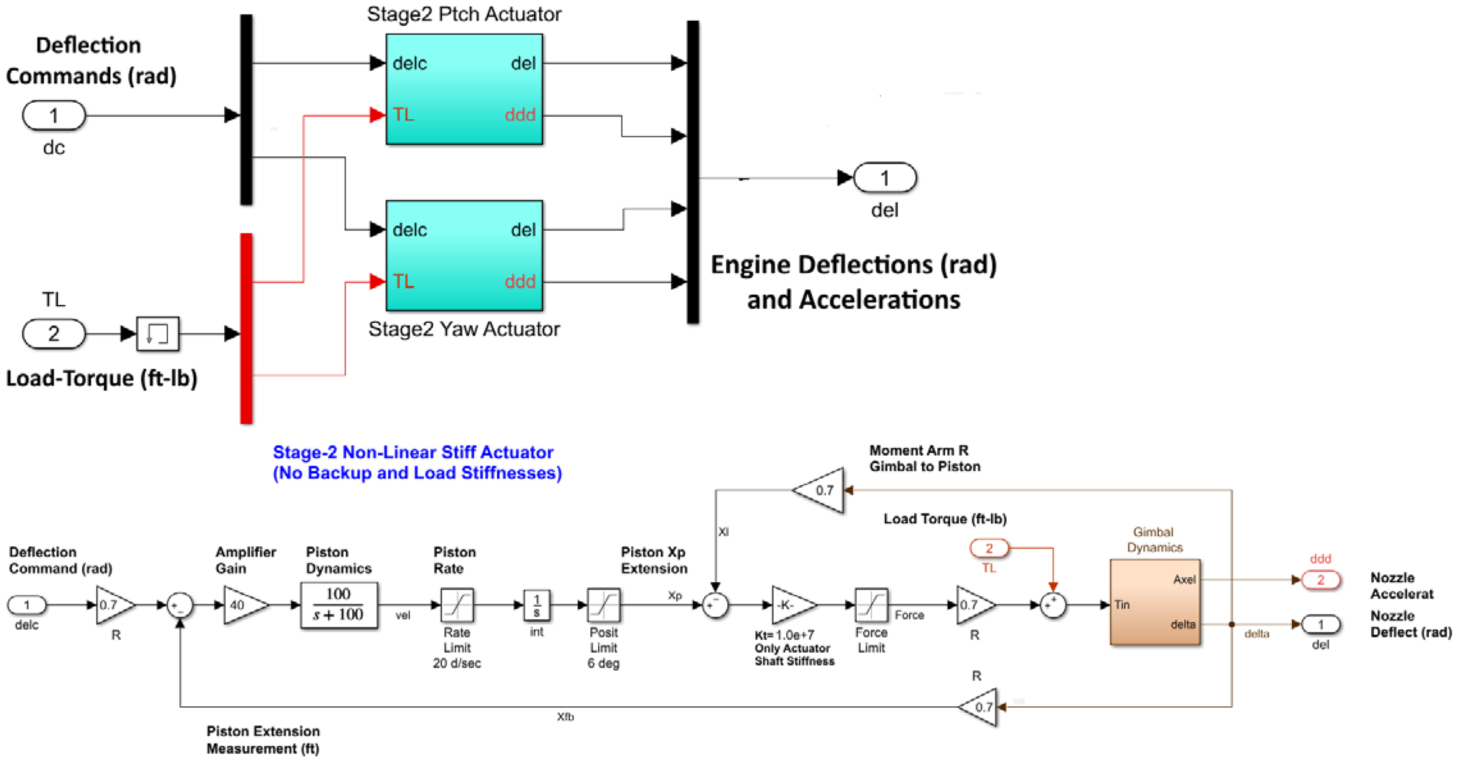
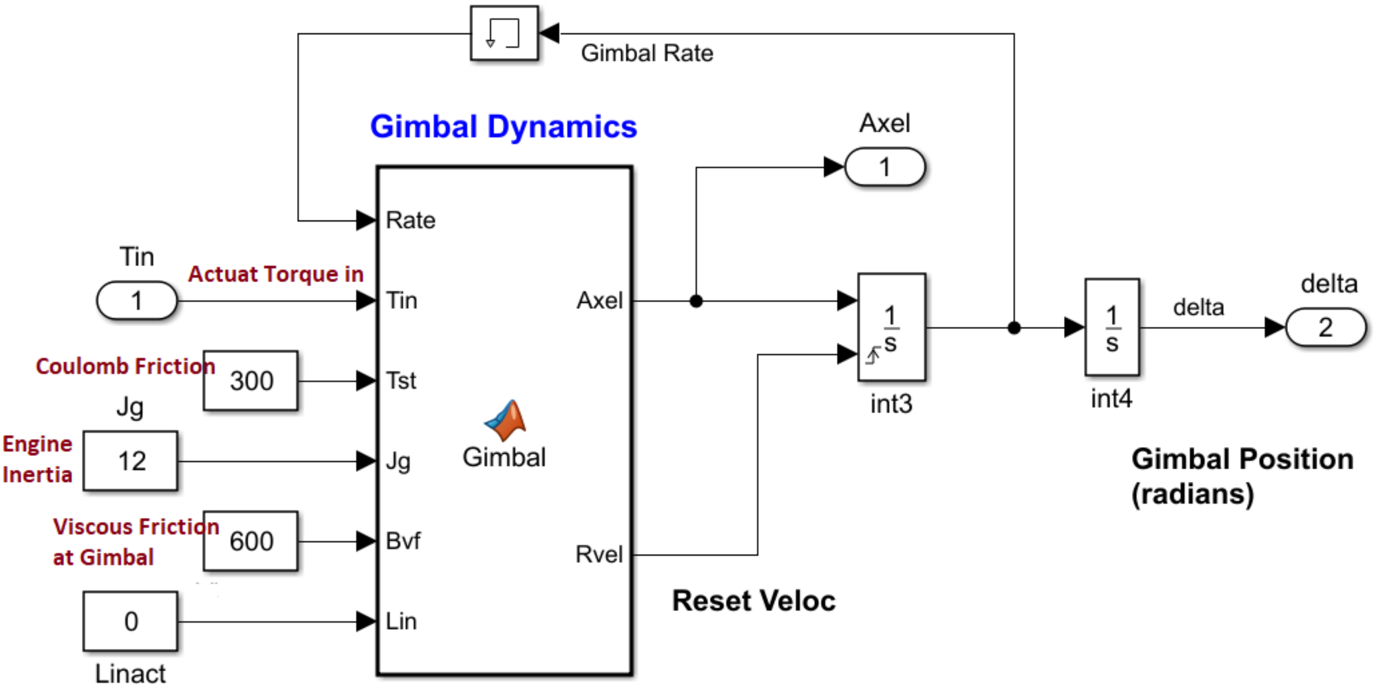
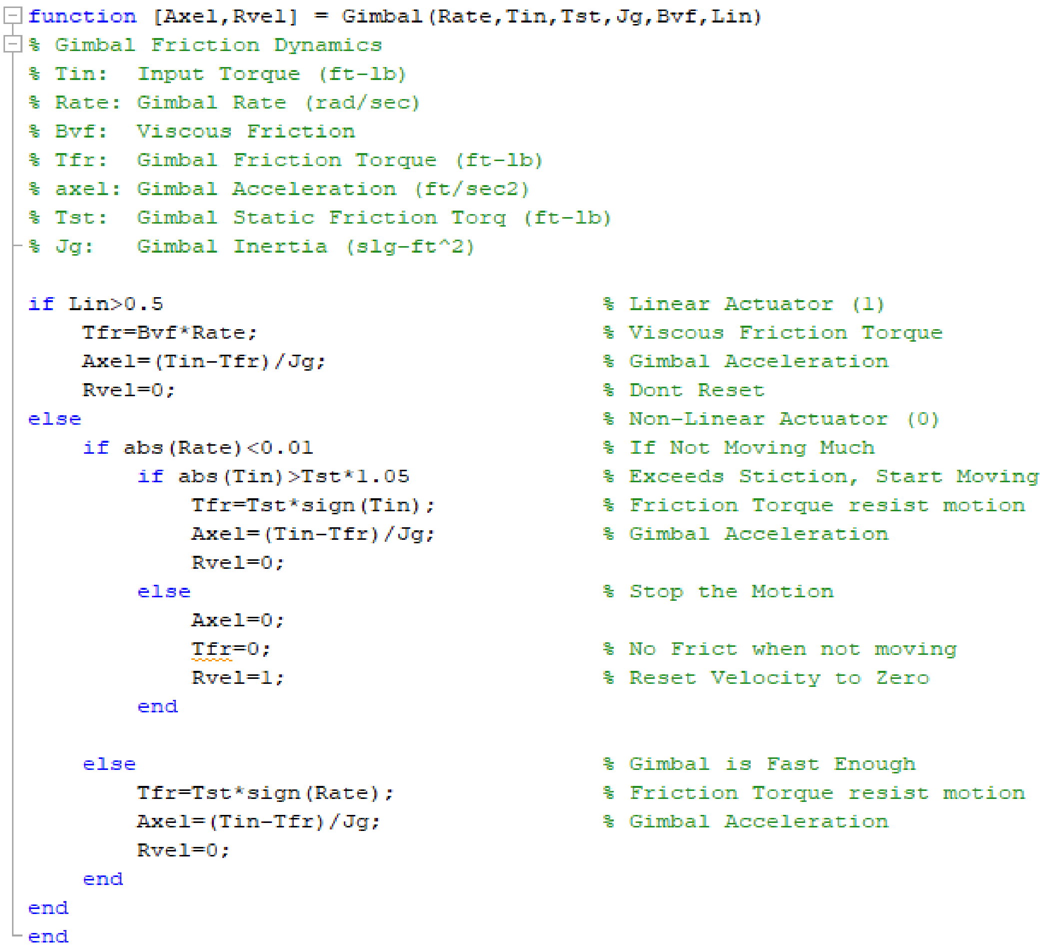


Figure 15 Non-Linear Pitch and Yaw TVC Actuator with Gimbal Dynamics Block





```
function [Axel,Rvel] = Gimbal(Rate,Tin,Tst,Jg,Bvf,Lin)
% Gimbal Friction Dynamics
% Tin:  Input Torque (ft-lb)
% Rate: Gimbal Rate (rad/sec)
% Bvf:  Viscous Friction
% Tfr:  Gimbal Friction Torque (ft-lb)
% axel: Gimbal Acceleration (ft/sec2)
% Tst:  Gimbal Static Friction Torq (ft-lb)
% Jg:   Gimbal Inertia (slg-ft^2)

if Lin>0.5                                  % Linear Actuator (1)
    Tfr=Bvf*Rate;                           % Viscous Friction Torque
    Axel=(Tin-Tfr)/Jg;                      % Gimbal Acceleration
    Rvel=0;                                 % Dont Reset
else                                        % Non-Linear Actuator (0)
    if abs(Rate)<0.01                       % If Not Moving Much
        if abs(Tin)>Tst*1.05                % Exceeds Stiction, Start Moving
            Tfr=Tst*sign(Tin);              % Friction Torque resist motion
            Axel=(Tin-Tfr)/Jg;              % Gimbal Acceleration
            Rvel=0;
        else                                % Stop the Motion
            Axel=0;
            Tfr=0;                          % No Frict when not moving
            Rvel=1;                         % Reset Velocity to Zero
        end

    else                                    % Gimbal is Fast Enough
        Tfr=Tst*sign(Rate);                 % Friction Torque resist motion
        Axel=(Tin-Tfr)/Jg;                  % Gimbal Acceleration
        Rvel=0;
    end
end
end
```

Figure 16 Non-Linear Actuator Gimbal Dynamics which Includes Static and Viscous Friction

The non-linear actuator model includes the non-linearity at the gimbal due to static (Coulomb) friction. It includes also viscous friction. The gimbal is implemented as a separate block that includes the Matlab function “*Gimbal*”, shown in Figure-16. The actuator model also includes shaft position and rate limits.

Reaction Control System

The RCS control system block activates the 8 jets for roll control and it is shown in detail in Figure-17. The logic uses phase-plane and a jet selection logic to determine which jets to fire in order to correct attitude. It activates only 0 to 4 jets at a time (out of 8 available) as a function of rate and attitude errors. The RCS logic comprises of two Matlab functions: the phase-plane and the jet-select logic, which are shown in Figure-18. The 8 output thrust forces (0 or 2.7 lbf) firing along ±Y axis and ±Z axis, are converted to 4 bidirectional throttles ±2.7 (lbf) which are inputs to the 4 throttles of the Flixan vehicle because it accepts also negative forces. Note, one bidirectional jet represents two real back-to-back firing jets.

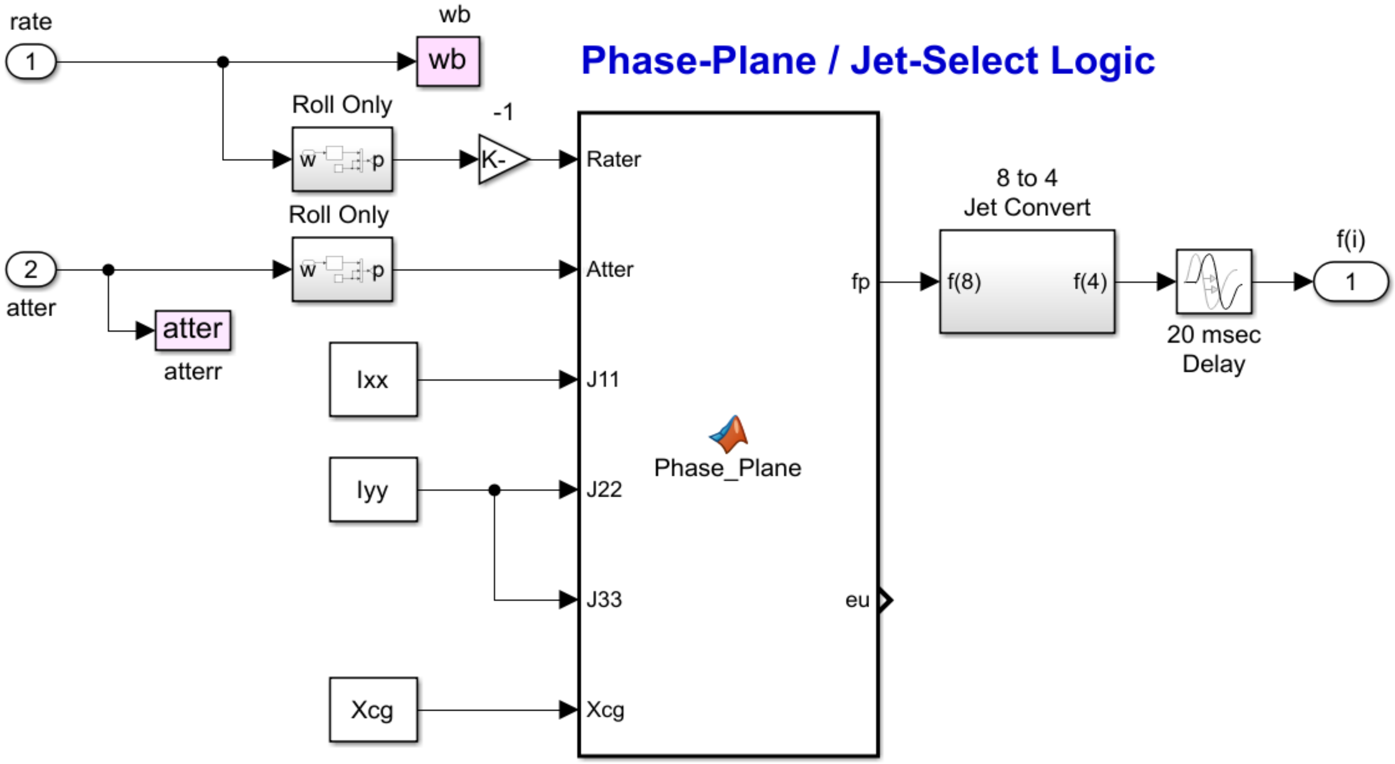
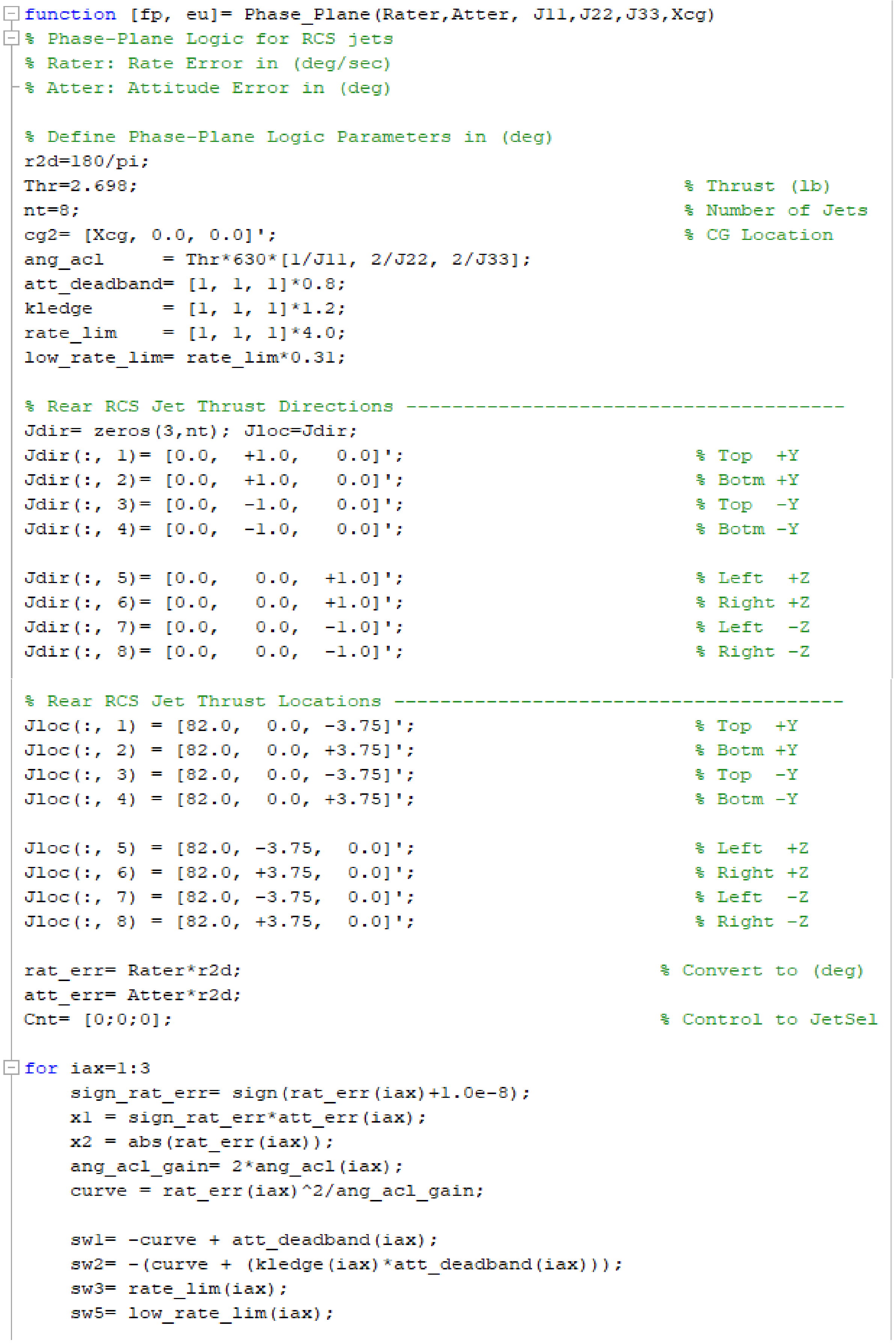
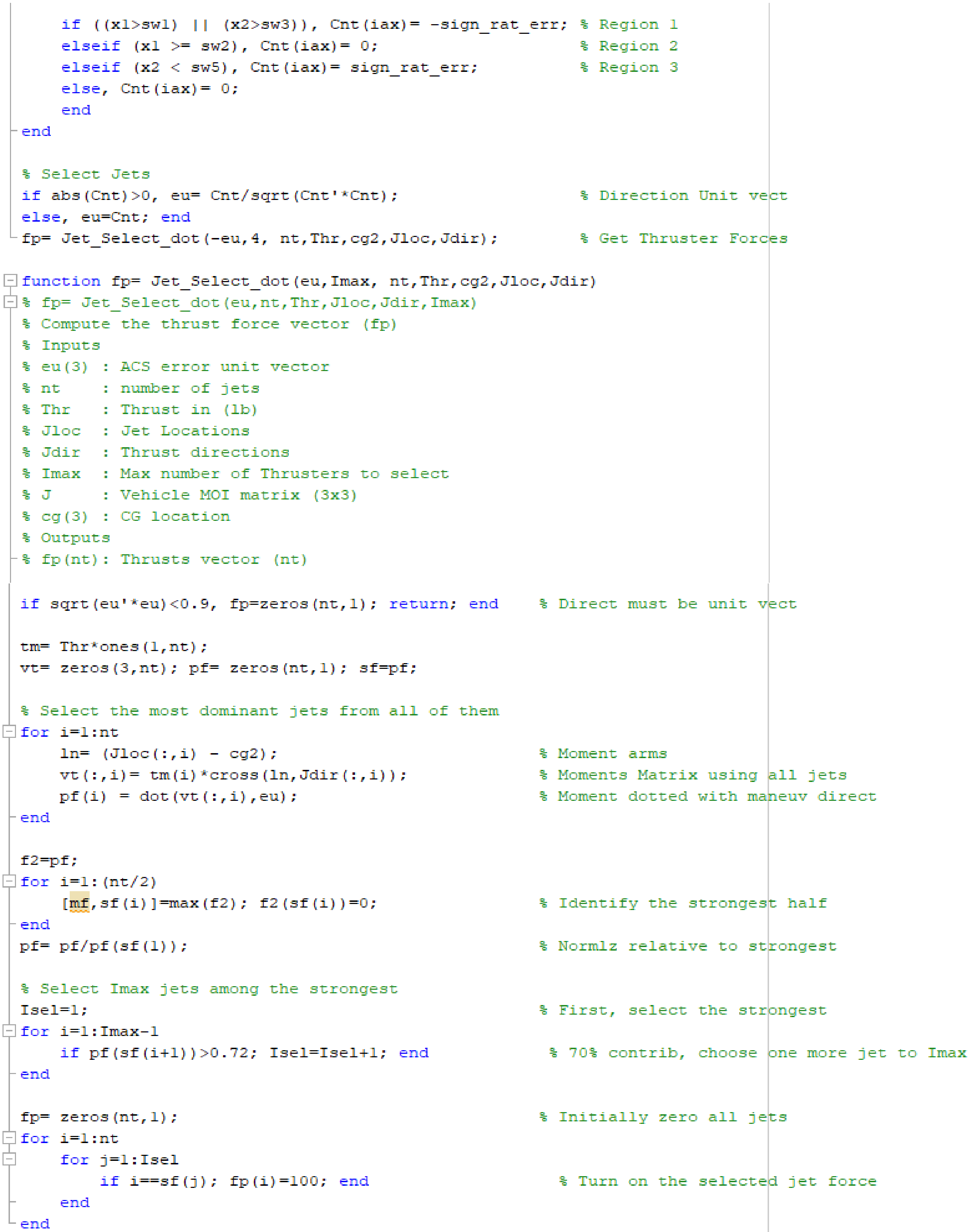


Figure 17 Reaction Control System for Roll



```
function [fp, eu]= Phase_Plane(Rater,Atter, J11,J22,J33,Xcg)
% Phase-Plane Logic for RCS jets
% Rater: Rate Error in (deg/sec)
% Atter: Attitude Error in (deg)

% Define Phase-Plane Logic Parameters in (deg)
r2d=180/pi;
Thr=2.698;                                                % Thrust (lb)
nt=8;                                                     % Number of Jets
cg2= [Xcg, 0.0, 0.0]';                                    % CG Location
ang_acl     = Thr*630*[1/J11, 2/J22, 2/J33];
att_deadband= [1, 1, 1]*0.8;
kledge      = [1, 1, 1]*1.2;
rate_lim    = [1, 1, 1]*4.0;
low_rate_lim= rate_lim*0.31;

% Rear RCS Jet Thrust Directions ---------------------------------------
Jdir= zeros(3,nt); Jloc=Jdir;
Jdir(:, 1)= [0.0,  +1.0,   0.0]';                          % Top  +Y
Jdir(:, 2)= [0.0,  +1.0,   0.0]';                          % Botm +Y
Jdir(:, 3)= [0.0,  -1.0,   0.0]';                          % Top  -Y
Jdir(:, 4)= [0.0,  -1.0,   0.0]';                          % Botm -Y

Jdir(:, 5)= [0.0,   0.0,  +1.0]';                          % Left  +Z
Jdir(:, 6)= [0.0,   0.0,  +1.0]';                          % Right +Z
Jdir(:, 7)= [0.0,   0.0,  -1.0]';                          % Left  -Z
Jdir(:, 8)= [0.0,   0.0,  -1.0]';                          % Right -Z

% Rear RCS Jet Thrust Locations ----------------------------------------
Jloc(:, 1) = [82.0,  0.0, -3.75]';                         % Top  +Y
Jloc(:, 2) = [82.0,  0.0, +3.75]';                         % Botm +Y
Jloc(:, 3) = [82.0,  0.0, -3.75]';                         % Top  -Y
Jloc(:, 4) = [82.0,  0.0, +3.75]';                         % Botm -Y

Jloc(:, 5) = [82.0, -3.75,  0.0]';                         % Left  +Z
Jloc(:, 6) = [82.0, +3.75,  0.0]';                         % Right +Z
Jloc(:, 7) = [82.0, -3.75,  0.0]';                         % Left  -Z
Jloc(:, 8) = [82.0, +3.75,  0.0]';                         % Right -Z

rat_err= Rater*r2d;                                     % Convert to (deg)
att_err= Atter*r2d;
Cnt= [0;0;0];                                           % Control to JetSel

for iax=1:3
    sign_rat_err= sign(rat_err(iax)+1.0e-8);
    x1 = sign_rat_err*att_err(iax);
    x2 = abs(rat_err(iax));
    ang_acl_gain= 2*ang_acl(iax);
    curve = rat_err(iax)^2/ang_acl_gain;

    sw1= -curve + att_deadband(iax);
    sw2= -(curve + (kledge(iax)*att_deadband(iax)));
    sw3= rate_lim(iax);
    sw5= low_rate_lim(iax);
```



```
    if ((x1>sw1) || (x2>sw3)), Cnt(iax)= -sign_rat_err; % Region 1
    elseif (x1 >= sw2), Cnt(iax)= 0;                     % Region 2
    elseif (x2 < sw5), Cnt(iax)= sign_rat_err;           % Region 3
    else, Cnt(iax)= 0;
    end
end

% Select Jets
if abs(Cnt)>0, eu= Cnt/sqrt(Cnt'*Cnt);                   % Direction Unit vect
else, eu=Cnt; end
fp= Jet_Select_dot(-eu,4, nt,Thr,cg2,Jloc,Jdir);         % Get Thruster Forces

function fp= Jet_Select_dot(eu,Imax, nt,Thr,cg2,Jloc,Jdir)
% fp= Jet_Select_dot(eu,nt,Thr,Jloc,Jdir,Imax)
% Compute the thrust force vector (fp)
% Inputs
% eu(3) : ACS error unit vector
% nt    : number of jets
% Thr   : Thrust in (lb)
% Jloc  : Jet Locations
% Jdir  : Thrust directions
% Imax  : Max number of Thrusters to select
% J     : Vehicle MOI matrix (3x3)
% cg(3) : CG location
% Outputs
% fp(nt): Thrusts vector (nt)

if sqrt(eu'*eu)<0.9, fp=zeros(nt,1); return; end    % Direct must be unit vect

tm= Thr*ones(1,nt);
vt= zeros(3,nt); pf= zeros(nt,1); sf=pf;

% Select the most dominant jets from all of them
for i=1:nt
    ln= (Jloc(:,i) - cg2);                          % Moment arms
    vt(:,i)= tm(i)*cross(ln,Jdir(:,i));             % Moments Matrix using all jets
    pf(i) = dot(vt(:,i),eu);                        % Moment dotted with maneuv direct
end

f2=pf;
for i=1:(nt/2)
    [mf,sf(i)]=max(f2); f2(sf(i))=0;                % Identify the strongest half
end
pf= pf/pf(sf(1));                                   % Normlz relative to strongest

% Select Imax jets among the strongest
Isel=1;                                             % First, select the strongest
for i=1:Imax-1
    if pf(sf(i+1))>0.72; Isel=Isel+1; end            % 70% contrib, choose one more jet to Imax
end

fp= zeros(nt,1);                                    % Initially zero all jets
for i=1:nt
    for j=1:Isel
        if i==sf(j); fp(i)=100; end                  % Turn on the selected jet force
    end
end
```

Figure 18 Phase-Plane and Jet Selection Logic for the RCS Jets

Simulation Results

Figures (19-21) show the results obtained using the H∞ Simulink model "*Sim_1.slx*" when it is commanded to perform an attitude change of [5°, 5°, -5°] in roll, pitch, and yaw respectively. The PID system in "*Sim_2.slx*" is unstable (not shown).

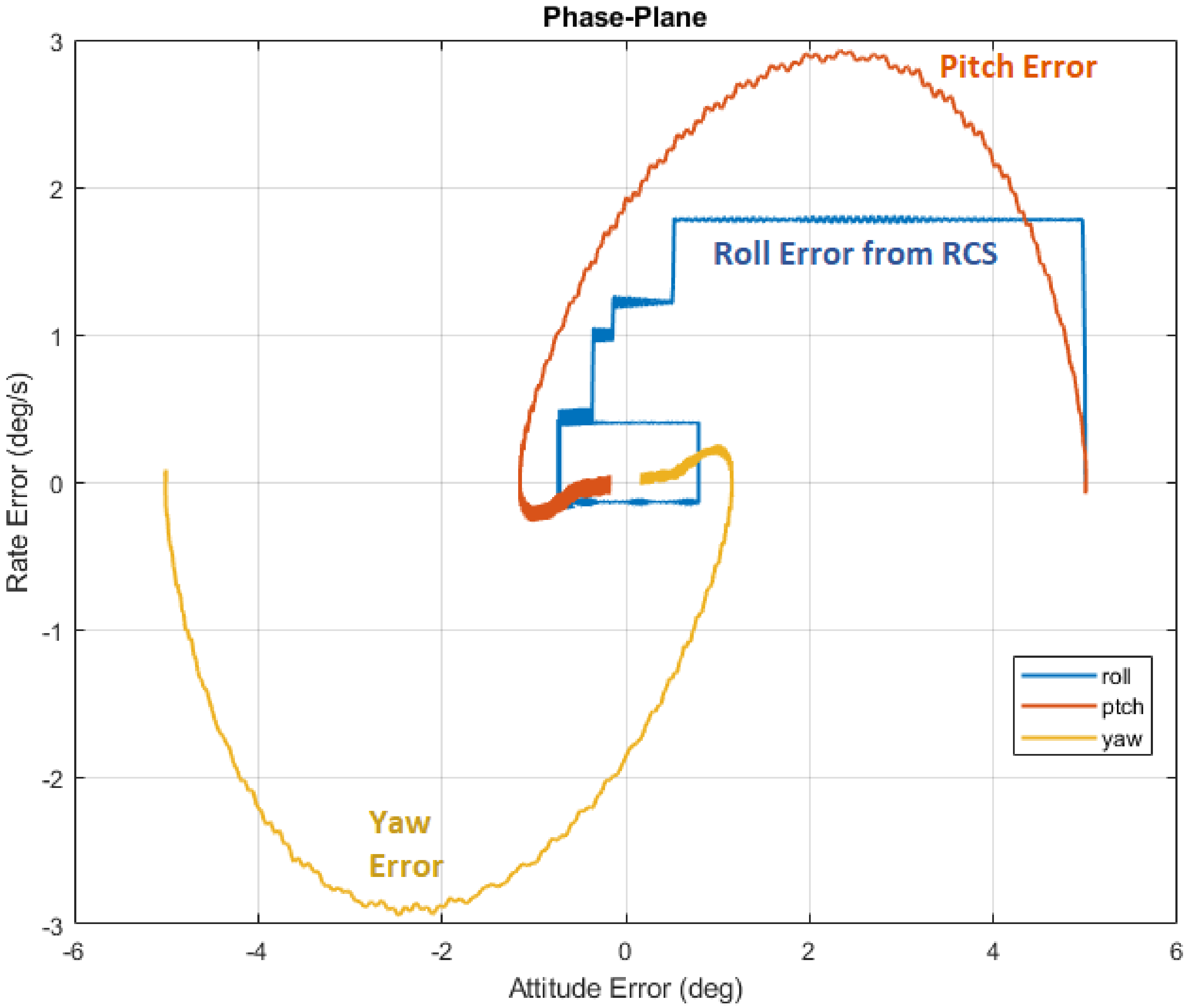


Figure 19 Phase-Plane Showing the Attitude and Rate Errors Decaying Towards Zero

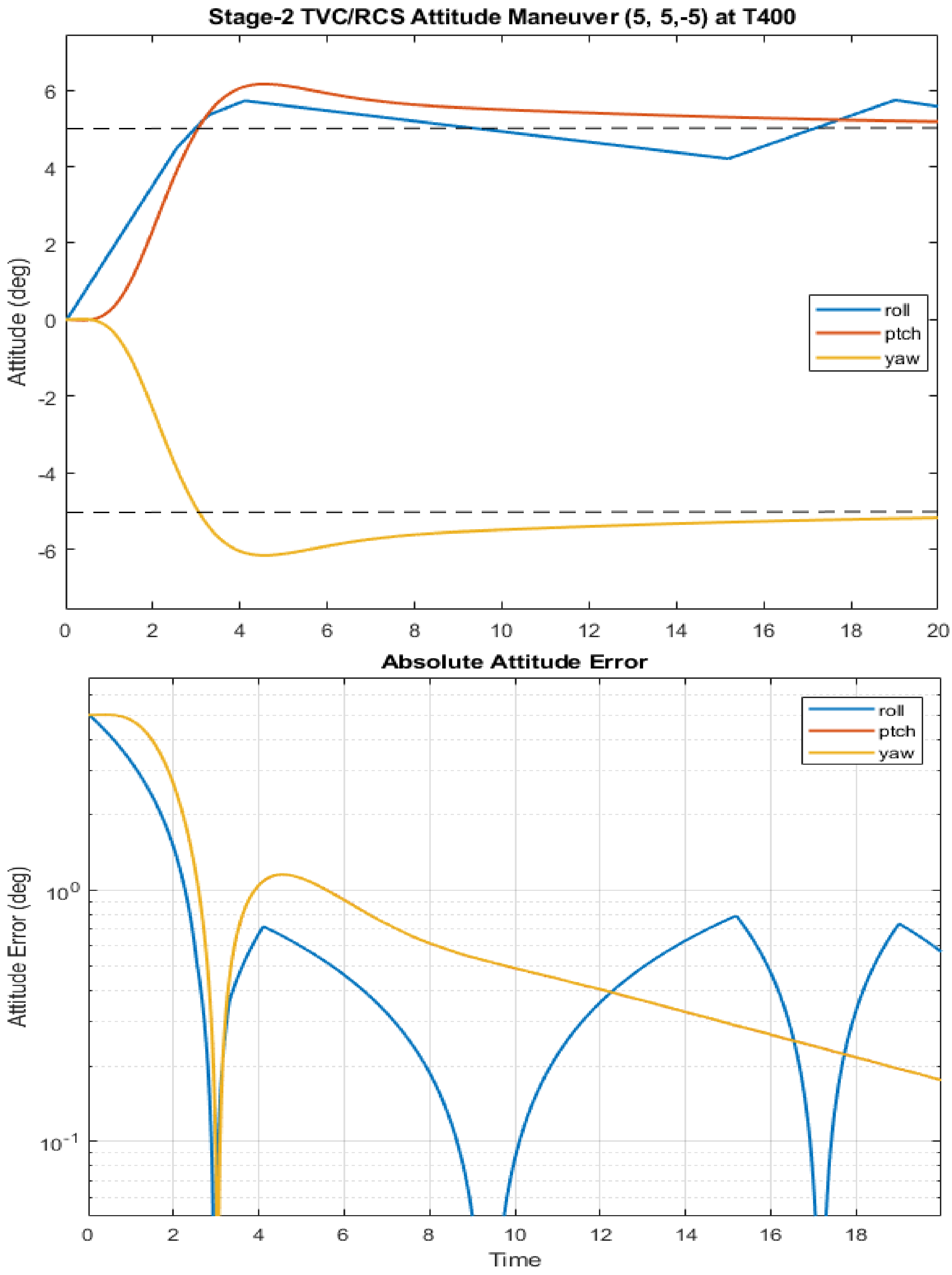


Figure 20 The Attitude Error Decreases in Pitch and Yaw. Max Roll Error is 0.8°

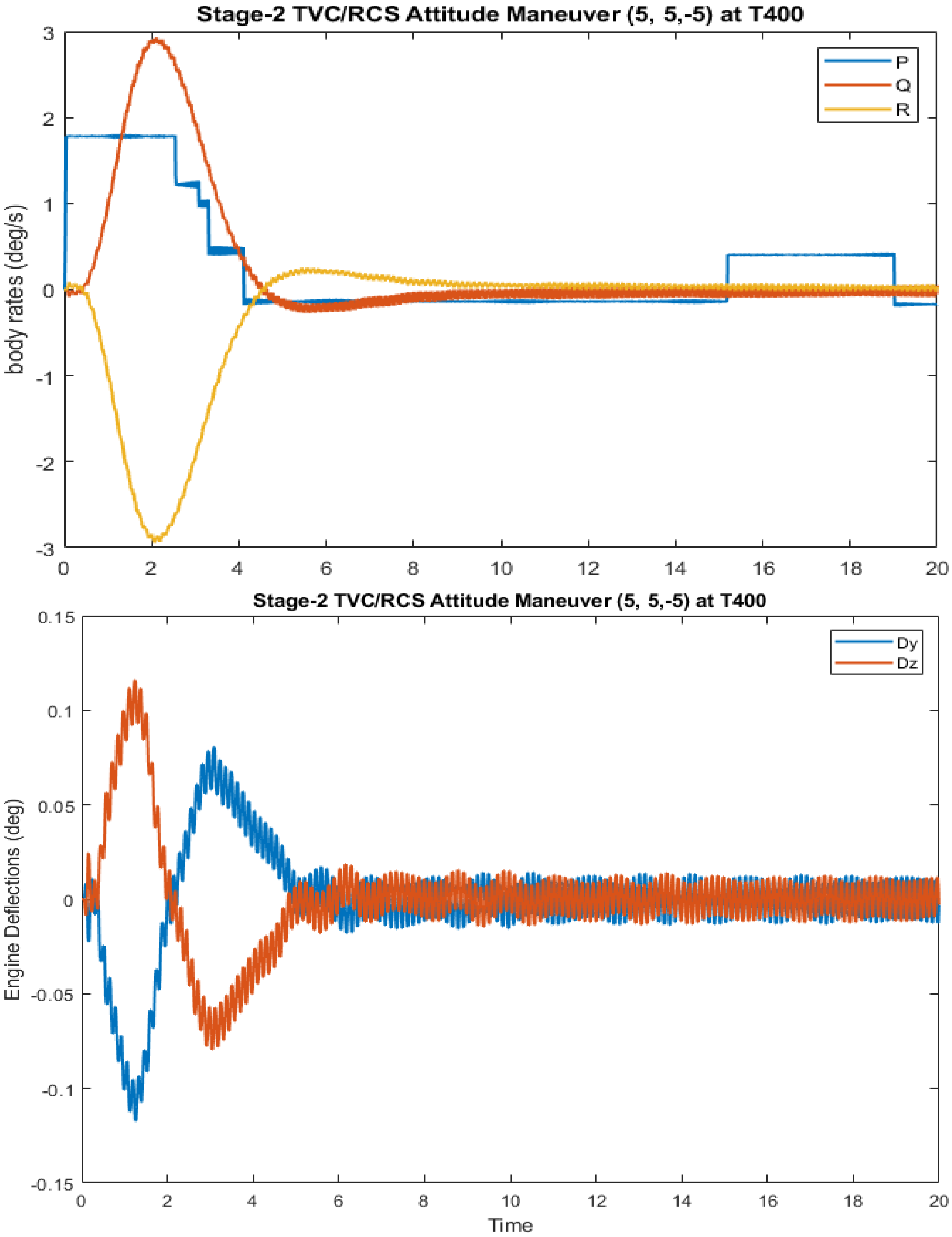


Figure 21 Body Rates and Gimbal Deflections. There is a Small Limit-Cycle due to Static Friction in the Actuators Coupling with Flexibility

3- Simulations Using Non-Linear Slosh Models

In this section we will develop simulation models for the flexible vehicle combined with two sloshing propellant models for the LOX and LH2 tanks. We will use non-linear spherical pendulum slosh models to describe the motion of the slosh mass inside the tanks and compare the PID controller, which is unstable because it was designed without consideration of slosh, against the dynamic H-infinity controller which was designed by considering the slosh modes in the design as already described. Our emphasis in this section is to demonstrate the benefits of using the spherical pendulum model which is effectively more accurate than the spring-mass model for unstable slosh situations because it limits the deflection of the slosh mass. The work files for this section are in directory "*Flixan\Control Analysis\Hinfinity\ Examples\ Stage-2 LV, Unstable Slosh\3-Non-Linear Slosh Sim*" and there are 3 Simulink models there. The first model is in file "*Stg2_LinSlsh_Sim_Hinf.slx*" which includes linear slosh subsystems implemented in Matlab and the H-infinity controller. It is used to compare the linear against the non-linear slosh models. The second model in file "*Stg2_NonLin_Sim_Pid.slx*" includes the non-linear spherical pendulum models and the PID controller and it is obviously unstable. The third model in file "*Stg2_NonLin_Sim_Hinf.slx*" includes the non-linear spherical pendulum model and the H-infinity controller which is stable. In the following sections we will describe the simulation models, present simulation results and analyze the efficiency of the slosh models. We begin with the linear spring-mass model which in this case it is not included in the Flixan vehicle but it is wrapped externally around the vehicle model in "*Stg2_LinSlsh_Sim_Hinf.slx*". We will develop equations of motion for the spherical pendulum model which are included in "*Stg2_NonLin-Sim_Hinf.slx*". We will describe the simulation models, analyze 3 different cases and present simulation results, and finally discuss the results and use them to make conclusions about the tank design.

Linear Spring-Mass Slosh Model

Let us begin with the simple spring-mass analogy model. When a vehicle is accelerating the motion of the propellant inside the tank can be approximated with a mass attached to the vehicle via a spring. The slosh mass displacement z_s relative to the tank is calculated by a 2nd order differential equation, where a_z is the vehicle normal acceleration at the location of the slosh mass.

$$\ddot{z}_s[s^2 + 2\zeta\omega_s s + \omega_s^2] = -a_z \qquad (1)$$

The slosh frequency is $\omega_s = \sqrt{\frac{A_x}{l_p}}$ that is, axial vehicle acceleration A_x over the pendulum length l_p

The force applied to the vehicle by the slosh mass is $F_z = -m_s(\omega_s^2 z_s + 2\zeta\omega_s \dot{z}_s)$, where m_s is the slosh mass. Two 2nd order slosh equations are needed for each propellant tank in order to describe the slosh mass relative motion in two directions y_s and z_s perpendicular to the acceleration vector Ax. The Matlab function "Slosh_Lin" is used to implement the linear slosh model for the LOX and LH2 tanks. The Flixan vehicle system in this case includes flexibility but the propellant slosh modes are not included.

Figure 22 shows the block diagram of the linear LOX model which is included in the Simulink model “Stg2_LinSlsh_Sim_Hinf.slx”. The two slosh subsystems are combined with the vehicle in the Simulink model. The inputs to the slosh block are vehicle accelerations at the slosh mass and the outputs are reaction forces which are applied to the vehicle and also moments due to slosh mass deflection z_s coupling with the Ax acceleration.

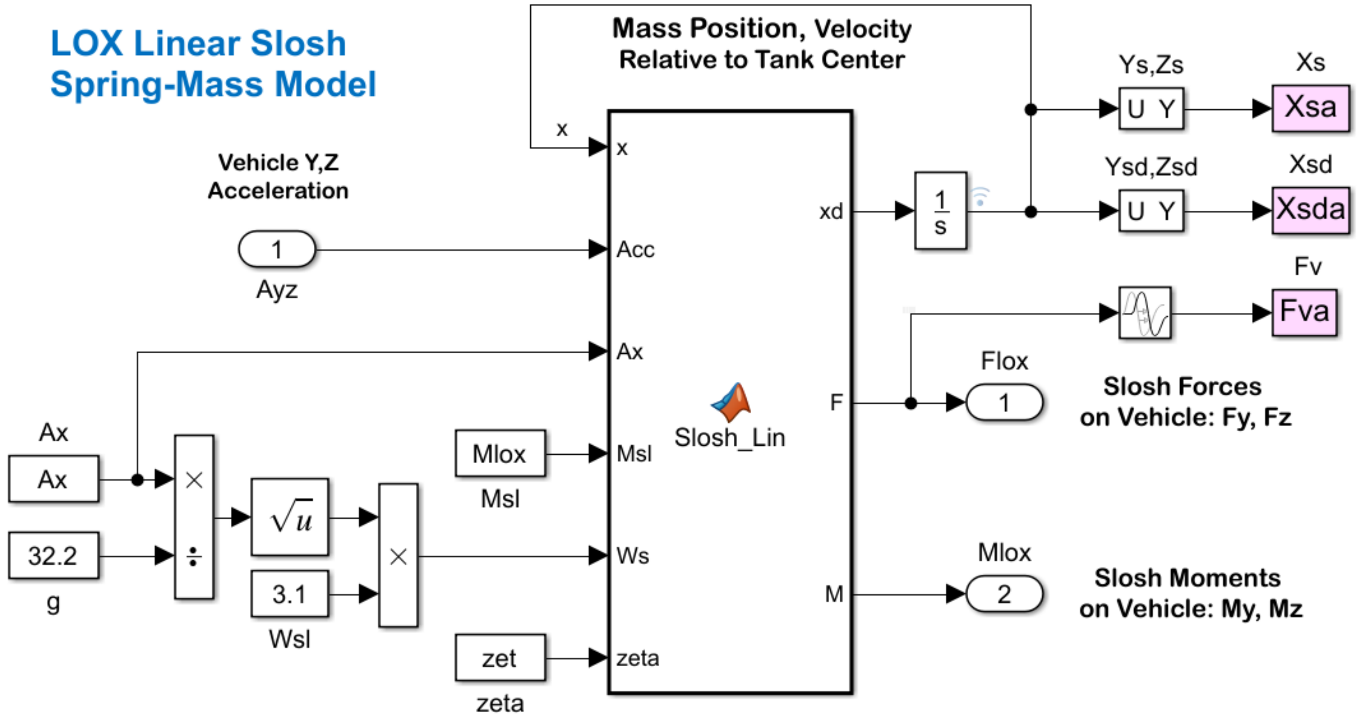
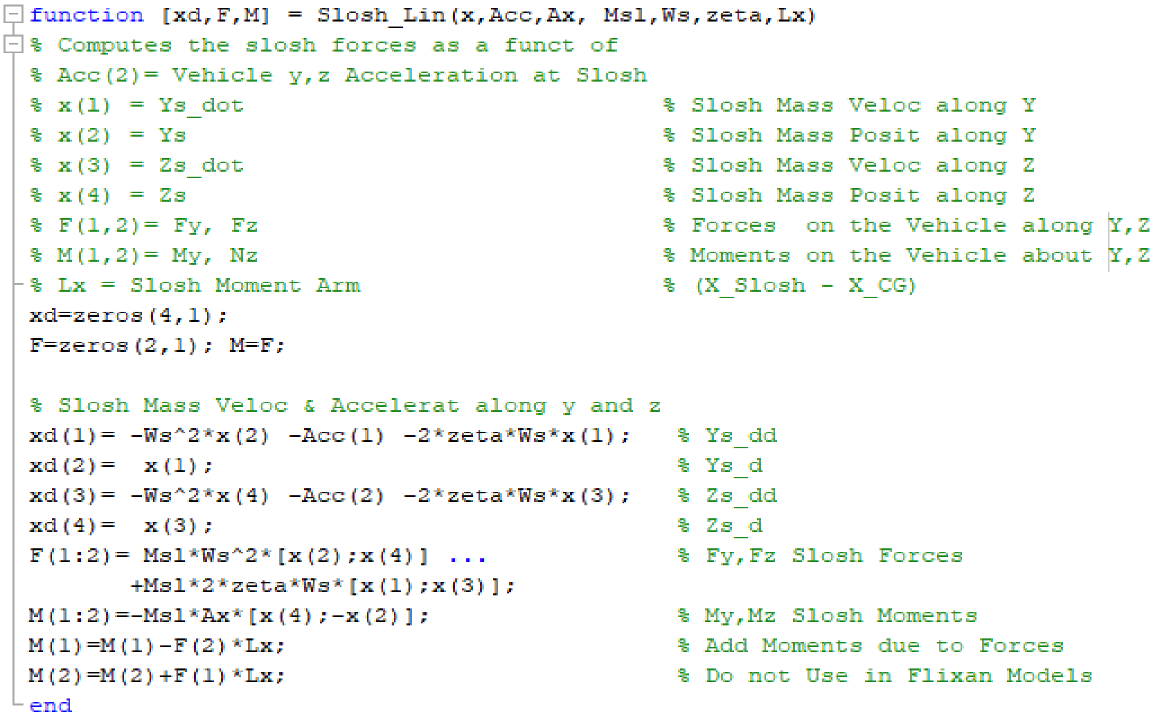


Figure 22 Implementation of Linear Slosh in Simulink Using a Matlab Function



```
function [xd,F,M] = Slosh_Lin(x,Acc,Ax, Msl,Ws,zeta,Lx)
% Computes the slosh forces as a funct of
% Acc(2)= Vehicle y,z Acceleration at Slosh
% x(1)  = Ys_dot                          % Slosh Mass Veloc along Y
% x(2)  = Ys                              % Slosh Mass Posit along Y
% x(3)  = Zs_dot                          % Slosh Mass Veloc along Z
% x(4)  = Zs                              % Slosh Mass Posit along Z
% F(1,2)= Fy, Fz                          % Forces  on the Vehicle along Y,Z
% M(1,2)= My, Nz                          % Moments on the Vehicle about Y,Z
% Lx = Slosh Moment Arm                   % (X_Slosh - X_CG)
xd=zeros(4,1);
F=zeros(2,1); M=F;

% Slosh Mass Veloc & Accelerat along y and z
xd(1)= -Ws^2*x(2) -Acc(1) -2*zeta*Ws*x(1);  % Ys_dd
xd(2)=  x(1);                               % Ys_d
xd(3)= -Ws^2*x(4) -Acc(2) -2*zeta*Ws*x(3);  % Zs_dd
xd(4)=  x(3);                               % Zs_d
F(1:2)= Msl*Ws^2*[x(2);x(4)] ...            % Fy,Fz Slosh Forces
       +Msl*2*zeta*Ws*[x(1);x(3)];
M(1:2)=-Msl*Ax*[x(4);-x(2)];                % My,Mz Slosh Moments
M(1)=M(1)-F(2)*Lx;                          % Add Moments due to Forces
M(2)=M(2)+F(1)*Lx;                          % Do not Use in Flixan Models
end
```

Non-Linear Pendulum Slosh Model

The spring mass model is useful for linear analysis but when slosh is unstable the linear model is not sufficient to evaluate the situation. When unstable, the slosh mass will not diverge to infinity because its deflection is limited by the tank radius and, therefore, the force on the vehicle will be bounded. If we assume a spherical pendulum analogy where the slosh mass is suspended from a pivot point located on the tank centerline, the slosh mass won't even be able to reach near the tank walls. It will only swing up to 45° before the wave breaks and the oscillations will begin growing again from a lower amplitude. The force on the vehicle is applied at the pivot. Another advantage of the non-linear pendulum analogy is that it includes the centripetal forces produced by the angular velocity of the slosh mass as it spins around the tank. It allows us to analyze swirling type of dynamic instabilities when the mass develops a vortex motion and produces a centripetal disturbance force on the vehicle that couples with the TVC control system and further aggravates the spinning. The linear model includes only the reaction forces due to the mass acceleration. The non-linear model will show if the instability damps out or diverges further. Figure-23 shows the propellant inside a tank. We assume that the propellant consists of two parts: (a) a solid mass near the bottom of the tank that does not move relative to the tank, and (b) a sloshing part that oscillates like a pendulum. Its center of mass is a little below the surface. The pendulum oscillations are excited by the vehicle normal and lateral acceleration components A_Z and A_Y at the slosh mass, and the motion of the mass is along two lateral directions perpendicular to the acceleration vector A_X.

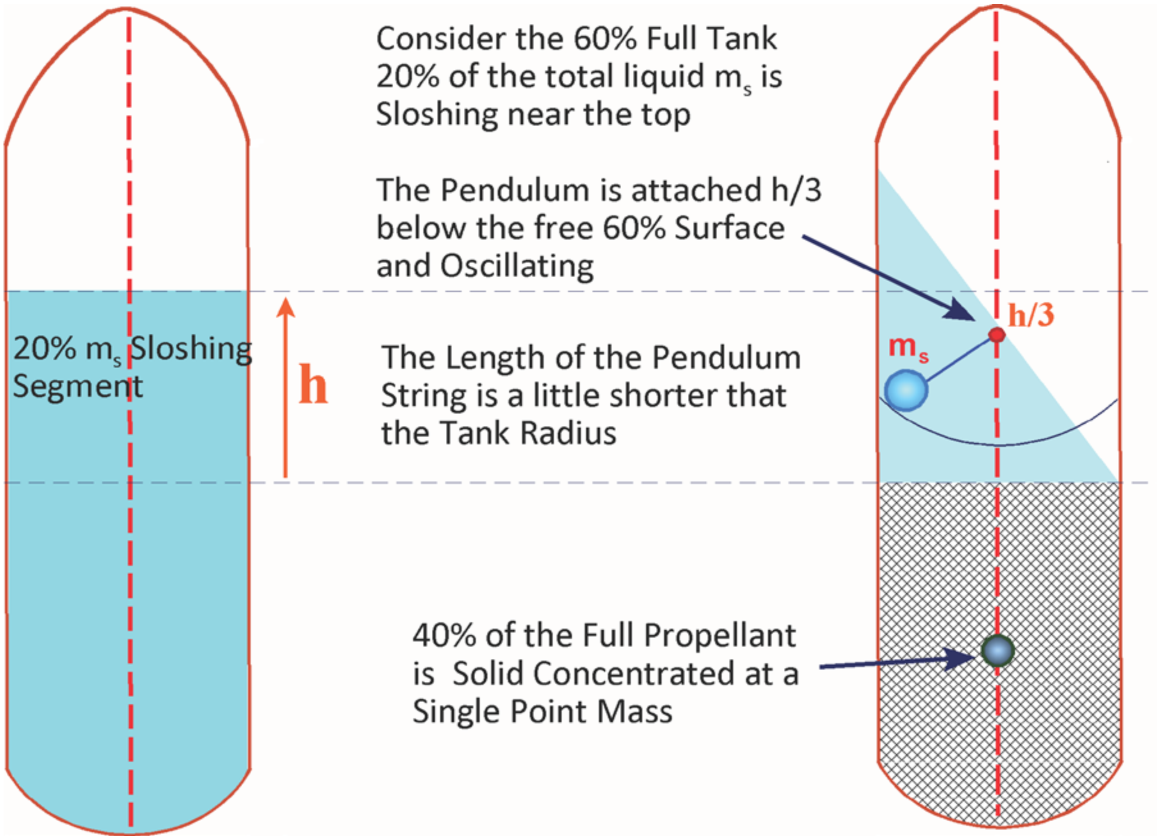


Figure 23 Pendulum Slosh Model

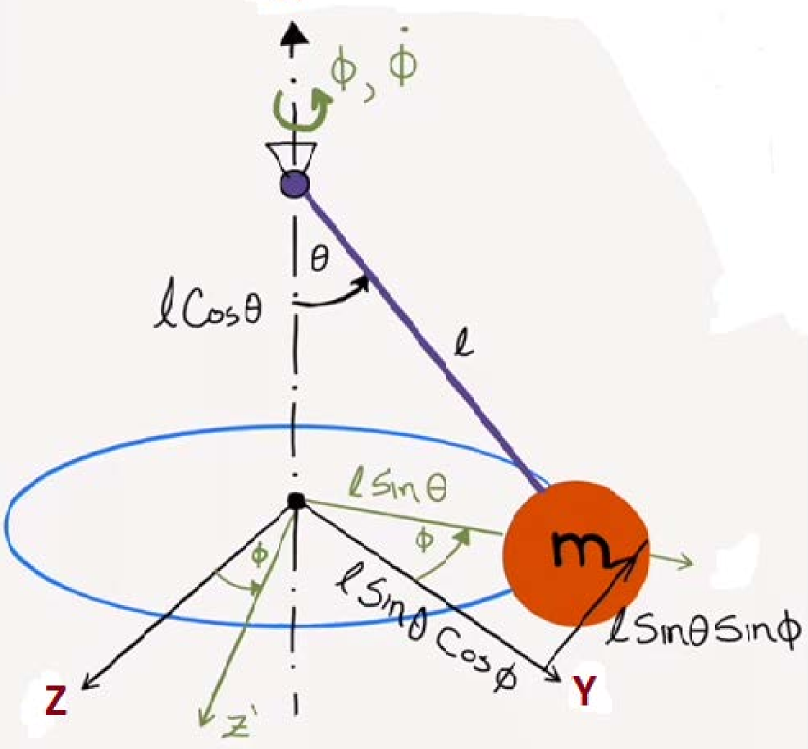
The weight of the solid propellant is rigidly attached to the vehicle mass properties at the fixed point on the tank centerline and it is included in the calculations of the vehicle CG and moments of inertia. The sloshing mass is not included because its effect is captured by the forces applied in the dynamic model. The motion of the slosh mass is circular about the center of rotation which is located at a point on the tank centerline. It is approximated with a pendulum of length l_p a little shorter (approx. 3/4) of the tank radius. The string is non-elastic, attached at the pivot point a little below the propellant surface. In Figure-23 the tank is at 60% fill level. We assume that only the top 20% of the propellant is sloshing and 40% is rigidly attached at a point mass on the tank centerline near the bottom.

Spherical Pendulum Slosh Model

Propellant sloshing can be approximated by the circular motion of a slosh mass that swings about a center of rotation which is the pendulum pivot point and this is where the reaction force is applied to the vehicle, in the opposite direction of the mass acceleration. There is an axial component force due to steady vehicle x-acceleration and two lateral forces Fy and Fz applied along the vehicle y and z axes. The pendulum motion is excited by the vehicle normal and lateral Az and Ay accelerations at the pivot and the pendulum produces the Fy and Fz forces which are applied back to the vehicle at the pivot. This is a mechanical feedback that can produce instability. The non-linear pendulum model can be used to assess the severity of the instability. The tank radius bounds the oscillation amplitude and the instability converges to a limit-cycle. The amplitude of the oscillation strongly depends on the length of the pendulum l_p and the coefficient of friction of the liquid mass sliding along the tank surface. This model can be used to determine the minimum damping coefficient for acceptable amplitudes of oscillation.

Figure-24 shows the spherical pendulum. The mass can swing in two directions perpendicular to the acceleration Ax, along the y and z axes. The displacement of the mass can be resolved in two rotation angles: a vertical pendulum rotation angle θ along a longitude, and a horizontal rotation ϕ about the x-axis along a latitude. The pendulum angle θ is measured from the vertical and it is always greater than zero. The angle ϕ is measured counter-clockwise from the projection of l_p on the y-z plane.

Figure 24 Spherical Pendulum



The pendulum is initialized at (θ_0 , ϕ_0) and the motion is excited by the accelerations A_y and A_z of the pivot point along the y and z axes. The acceleration of the pivot relative to the mass can be resolved into two acceleration components that produce vertical and lateral moments on the pendulum: an axial acceleration A_ξ that produces a vertical moment, and a tangential acceleration A_τ that produces a lateral rotation moment about the x-axis. So, we have two pendulum moment equations.

Vertical Moment Equation

Equation (2) is the moment for the vertical motion and calculates the pendulum angle θ. It is excited on the RHS by the axial component of vehicle acceleration A_ξ towards the slosh mass which produces the vertical moment. There is also a friction force $D.V_\theta$ of the mass as it slides with velocity V_θ against along the surface which produces an opposing torque, where D is the viscous friction coefficient.

$$ml^2\ddot{\theta} - ml^2\dot{\phi}^2\cos\theta\sin\theta + mlA_{\mathrm{X}}\sin\theta = -mlA_\xi\cos\theta - Dl^2\dot{\theta} \qquad (2)$$

$$\ddot{\theta} = +\dot{\phi}^2\cos\theta\sin\theta - \frac{A_{\mathrm{X}}}{l}\sin\theta - \frac{A_\xi}{l}\cos\theta - \frac{D}{m}\dot{\theta} \qquad (3)$$

For small angles and without any lateral motion this equation reduces to

$$\ddot{\theta} + 2\zeta\omega\dot{\theta} + \omega^2\theta = -\frac{A_\xi}{l} \qquad (4)$$

Where the oscillation frequency: $\omega^2 = \frac{A_{\mathrm{X}}}{l}$ and $D = 2\zeta\omega\, m$, where ζ is the damping coefficient. The D coefficient is selected to produce a ζ =0.002. The pendulum length l is a little smaller than the tank radius.

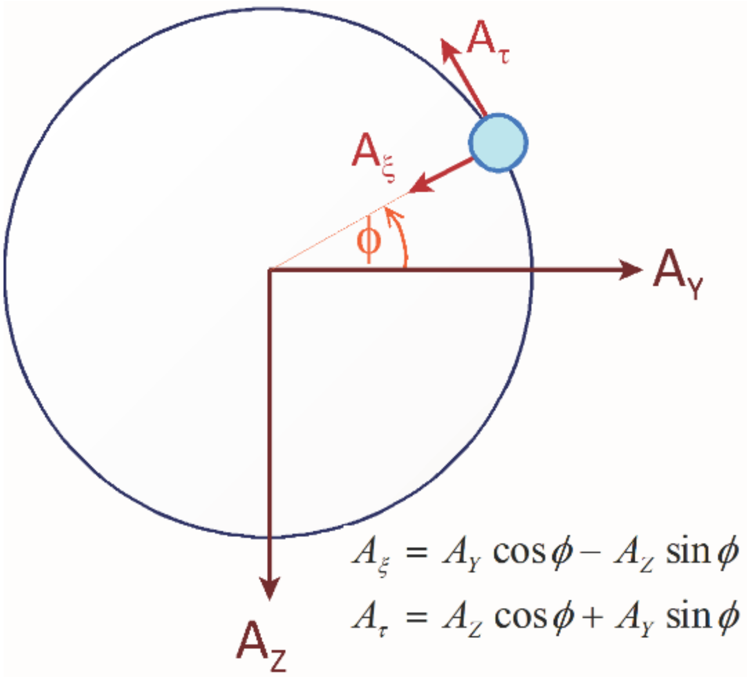


Figure 25 Top View. The Vehicle Normal and Lateral Accelerations are Resolved into Axial and Tangential Relative Accelerations

Lateral Moment Equation

In the lateral direction the spin moment about x is given in equation (5) which calculates the rotational angle ϕ about the tank centerline x. It is excited by the torque produced by the relative tangential acceleration A_τ between the pivot and the mass, which is perpendicular to the A_ξ acceleration. There is also a viscous friction force $D.V_\phi$ due to the horizontal velocity component V_ϕ producing a negative torque.

$$ml^2\ddot{\phi}\,\sin^2\theta + 2\,ml^2\dot{\theta}\dot{\phi}\,\cos\theta\sin\theta = +mlA_\mathrm{t}\sin\theta - D\dot{\phi}l^2\sin^2\theta \qquad (5)$$

$$\ddot{\phi} = -2\,\dot{\theta}\dot{\phi}\,\frac{\cos\theta}{\sin\theta} + \frac{A_\mathrm{t}}{l\sin\theta} - \frac{D}{m}\dot{\phi} \qquad (6)$$

Slosh Mass Kinematics Relative to Tank Centerline Attachment:

$$Y_s = l\,\sin\theta\cos\phi$$
$$X_s = -l\,\cos\theta$$
$$Z_s = -l\,\sin\theta\sin\phi$$

Slosh Mass Velocities:

$$\dot{Y}_s = +l\dot{\theta}\cos\theta\cos\phi - l\,\dot{\phi}\sin\theta\sin\phi$$
$$\dot{Z}_s = -l\dot{\theta}\cos\theta\sin\phi - l\,\dot{\phi}\sin\theta\cos\phi$$

Slosh Mass Accelerations Relative to Tank:

$$\ddot{Y}_s/_l = +\ddot{\theta}\cos\theta\cos\phi - \ddot{\phi}\sin\theta\sin\phi - \left(\dot{\theta}^2+\dot{\phi}^2\right)\sin\theta\cos\phi - 2\dot{\theta}\dot{\phi}\cos\theta\sin\phi$$
$$\ddot{Z}_s/_l = -\ddot{\theta}\cos\theta\sin\phi - \ddot{\phi}\sin\theta\cos\phi + \left(\dot{\theta}^2+\dot{\phi}^2\right)\sin\theta\sin\phi - 2\dot{\theta}\dot{\phi}\cos\theta\cos\phi$$

Slosh Forces on the Vehicle:

$$F_Y = m_s\left(\ddot{Y}_s + A_{Yt}\right)$$

Mass x Inertial Acceleration

$$F_Z = m_s\left(\ddot{Z}_s + A_{Zt}\right)$$

Slosh Moments on the Vehicle:

$$\boxed{\begin{aligned} M_{YSL} &= -\sum_{i=1}^{Nsl}\left\{F_{Zsi}\,l_{sxi} + m_s(i)A_X z_{si}\right\} \\ N_{ZSL} &= +\sum_{i=1}^{Nsl}\left\{F_{Ysi}\,l_{sxi} + m_s(i)A_X y_{si}\right\} \end{aligned}}$$

Spherical Pendulum Model in Simulink

The equations of motion of the spherical pendulum slosh model are implemented in the Simulink model "*Stg2_NonLin_Sim_Hinf.slx*" which is shown in Figure-26. It includes the H-Infinity flight control for the TVC engine which is identical for both pitch and yaw. It also includes pitch and yaw non-linear TVC actuators. The phase-plane reaction control system is also included for roll control. There is an identical simulation in file "*Stg2_NonLin_Sim_Pid.slx*" that uses the PID controller.

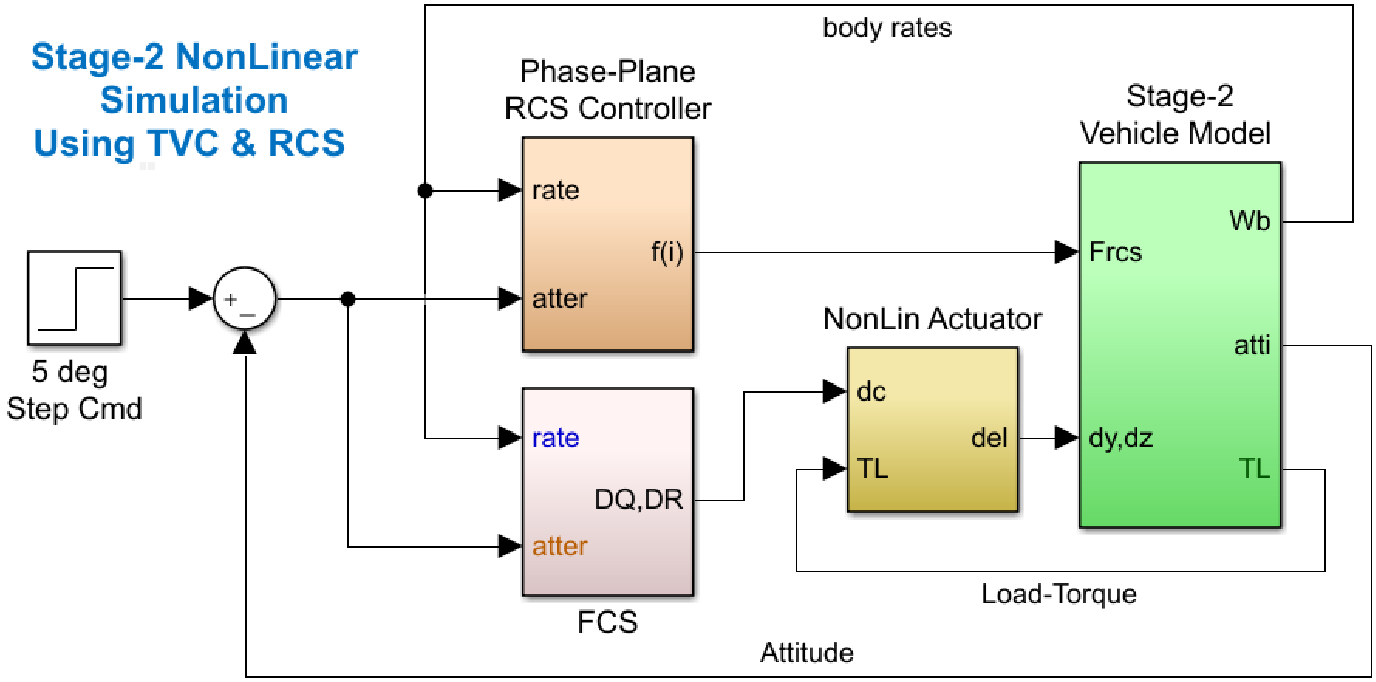


Figure 26 Non-Linear Simulation "Stg2_NonLin_Sim_Hinf.slx" which Includes the Spherical Pendulum Slosh Models

The green vehicle block is shown in detail in Figure-27. The two slosh subsystems are combined with the Flixan vehicle, that includes flex but no slosh, to simulate the dynamic coupling with the sloshing propellants. The pendulum slosh equations are coded in Matlab functions. Figure 28 shows the LOX Spherical Pendulum block and Figure-29 shows the equations of motion in Matlab. They consist of two 2nd order differential equations for the two pendulum rotations which are non-linearly coupled together. That is, the vertical rotation θ and the lateral rotation about the tank x-axis ϕ. They are excited by the vehicle lateral and normal accelerations A_y and A_z at the tank. The 2 relative accelerations are transformed into an axial component $A\xi$ that excites the vertical motion and a tangential component $A\tau$ that excites the lateral motion. The angular rates and accelerations produce the forces and moments which are applied back to the vehicle. The simulations are initialized by the function "init.m" which loads the systems and initializes the vehicle parameters. The pendulum angles of the spherical models are initialized at $\theta = 4°$, the rotational angle $\phi = 45°$, and the rotation rate x-axis is initialized at $\dot{\phi}$=10 (deg/sec). This will excite the pendulum swing and vortex motions.

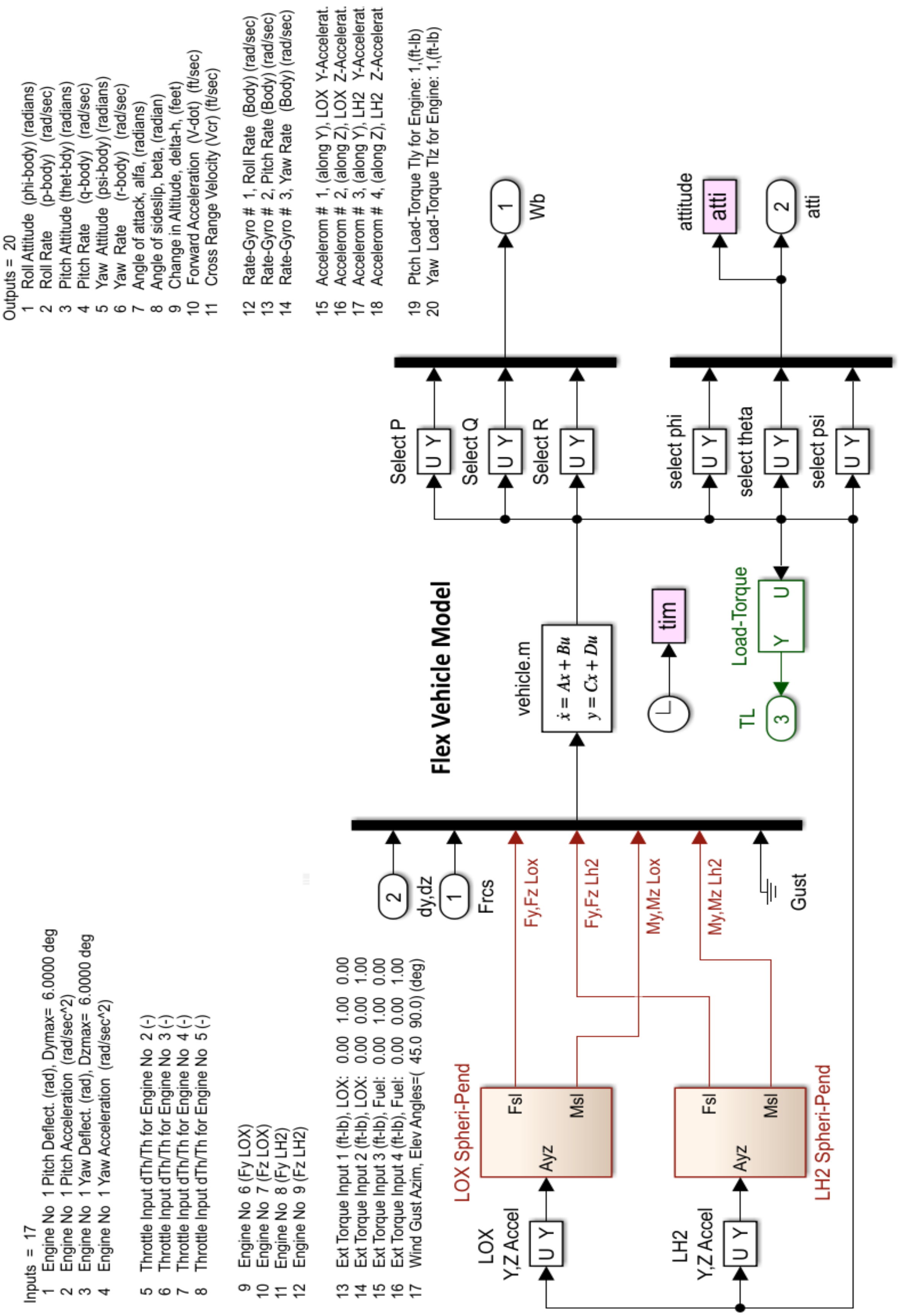
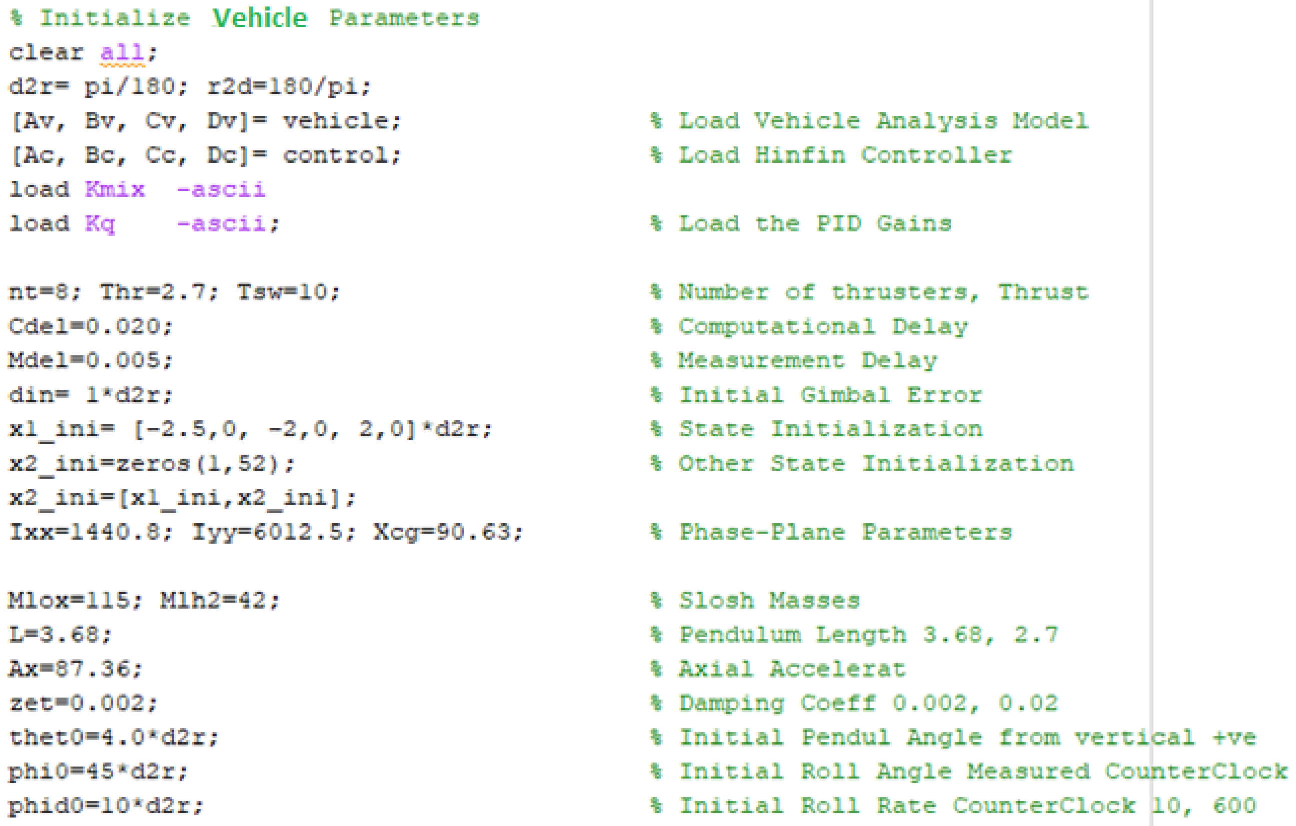


Figure 27 Vehicle Dynamics in the Simulink Model. It Includes the Flixan Vehicle State-Space System with Structural Flexibility and the Two Spherical Pendulum Models for the LOX and LH2 Tanks



```
% Initialize  Vehicle  Parameters
clear all;
d2r= pi/180; r2d=180/pi;
[Av, Bv, Cv, Dv]= vehicle;                % Load Vehicle Analysis Model
[Ac, Bc, Cc, Dc]= control;                % Load Hinfin Controller
load Kmix  -ascii
load Kq    -ascii;                        % Load the PID Gains

nt=8; Thr=2.7; Tsw=10;                    % Number of thrusters, Thrust
Cdel=0.020;                               % Computational Delay
Mdel=0.005;                               % Measurement Delay
din= 1*d2r;                               % Initial Gimbal Error
x1_ini= [-2.5,0, -2,0, 2,0]*d2r;          % State Initialization
x2_ini=zeros(1,52);                       % Other State Initialization
x2_ini=[x1_ini,x2_ini];
Ixx=1440.8; Iyy=6012.5; Xcg=90.63;        % Phase-Plane Parameters

Mlox=115; Mlh2=42;                        % Slosh Masses
L=3.68;                                   % Pendulum Length 3.68, 2.7
Ax=87.36;                                 % Axial Accelerat
zet=0.002;                                % Damping Coeff 0.002, 0.02
thet0=4.0*d2r;                            % Initial Pendul Angle from vertical +ve
phi0=45*d2r;                              % Initial Roll Angle Measured CounterClock
phid0=10*d2r;                             % Initial Roll Rate CounterClock 10, 600
```

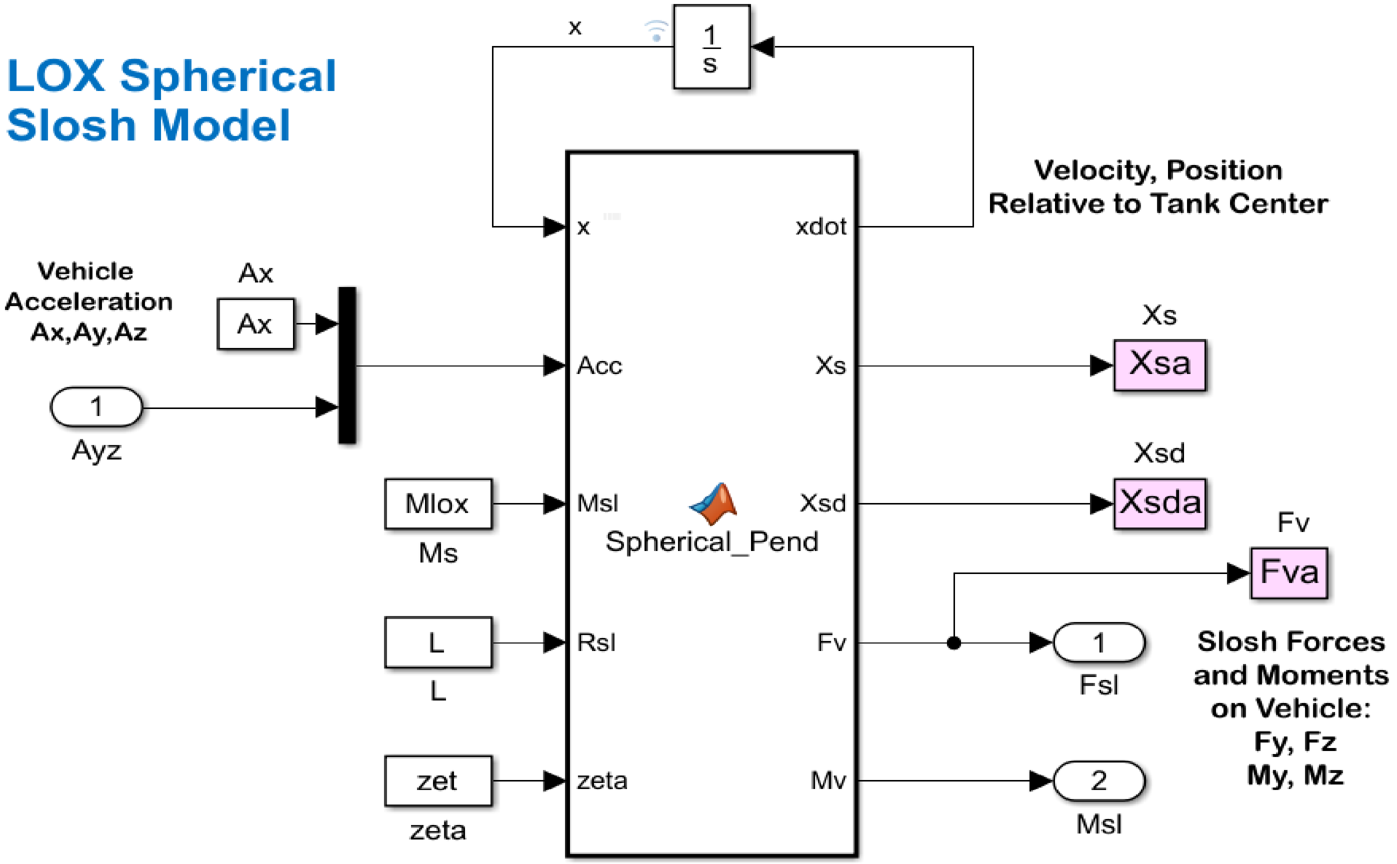


Figure 28 Spherical Pendulum Subsystem in Simulink which Includes the Matlab Function

```matlab
function [xdot,Xs,Xsd,Fv,Mv]= Spherical_Pend(x,Acc,Msl,Rsl,zeta)
% Spherical Pendulum Slosh Model
% [xdot,Xs,Xsd,Fv,Mv]= Spherical_Pend(x,Acc,Msl,Rsl,zeta)
% Calculates the Slosh Forces on the Tank as a function of
% Vehicle acceleration at the Tank
%
% State Variables (x)
% x(1:2)   = Xsd Slosh Mass Ysd,Zsd Velocity wrt Tank
% x(3:4)   = Xs Slosh Mass Ys,Zs Position wrt Tank
% Acc(3}   = Vehicle Linear Ax,Ay,Az Accelerat at Tank
% Msl      = Slosh Mass (slug)
% Rsl      = Pendulum length (ft)

xdot=zeros(4,1);                                      % Initialize
thd= x(1);                                            % Theta-dot
phd= x(2);                                            % Phi-dot
the= x(3);                                            % Theta
phi= x(4);                                            % Phi
Ax= Acc(1);                                           % Y-accelerat at Tank
Ay= Acc(2);                                           % Y-accelerat at Tank
Az= Acc(3);                                           % Z-accelerat at Tank
Wsl=sqrt(Ax/Rsl);                                     % Slosh Frequency (r/
Sth=sin(the); Cth=cos(the);
Sph=sin(phi); Cph=cos(phi);

% Axial and Tangential Accelerations
Aksi= Ay*Cph - Az*Sph;                                % Axial   Accel
Atau= Az*Cph + Ay*Sph;                                % Tangent Accel

% Vertical and Horizontal Accelerations
thdd=(phd^2)*Cth*Sth -(Ax/Rsl)*Sth -(Aksi/Rsl)*Cth -2*zeta*Wsl*thd;
phdd=-2*thd*phd*(Cth/Sth) + Atau/(Rsl*Sth) -2*zeta*Wsl*phd;

% Slosh Mass Displacements Relative to Tank Ys, Zs
%Xs=-Rsl*Cth;
Ys= Rsl*Sth*Cph;
Zs=-Rsl*Sth*Sph; Xs=[Ys;Zs];

% Slosh Mass Velocities Relative to Tank Ys, Zs
Ysd= Rsl*(thd*Cth*Cph - phd*Sth*Sph);
Zsd=-Rsl*(thd*Cth*Sph + phd*Sth*Cph); Xsd=[Ysd;Zsd];

% Slosh Mass Accelerations Relative to Tank Ysd, Zsd
Ysdd= Rsl*(+thdd*Cth*Cph - phdd*Sth*Sph -2*thd*phd*Cth*Sph -(thd^2+phd^2)*Sth*Cph);
Zsdd= Rsl*(-thdd*Cth*Sph - phdd*Sth*Cph -2*thd*phd*Cth*Cph +(thd^2+phd^2)*Sth*Sph);

% Lateral Forces on the Vehicle = Mass x Inert Acceleration
Fv=-Msl*([Ysdd+Ay; Zsdd+Az]);                        % Forces on the Vehicle
Mv= Msl*Ax*[-Zs; Ys];                                % Additional Moments due to Ax

% State Derivatives
xdot(1:2)= [thdd;phdd];                              % X-dot-dot
xdot(3:4)= [thd; phd];                               % X-dot[4]
end
```

Figure 29 Function “Spherical_Pend” that Codes the Equations of Motion of the Spherical Pendulum in Matlab

Reaction Control System

The RCS control system block activates the 8 jets for roll control and it is shown in detail in Figure-17. The logic uses phase-plane and a jet selection logic to determine which jets to fire in order to correct attitude. It activates only 0 to 4 jets at a time (out of 8 available) as a function of rate and attitude errors. The RCS logic consists of two Matlab functions: the phase-plane and the jet-select logic, which are shown in Figure-18. The 8 output thrust forces (0 or 2.7 lbf) firing along ±Y axis and ±Z axis, are converted to 4 bidirectional throttles ±2.7 (lbf) which are inputs to the 4 throttles of the Flixan vehicle because it accepts also negative forces. Note, one bidirectional jet represents two real back-to-back firing jets.

Simulation Results

We will now use the two Simulink models that include the spherical pendulum equations to analyze 3 special cases, compare results, obtain conclusions that could not be investigated by using classical linear control analysis, and determine the required damping for baffles that will produce acceptable performance even with unstable slosh.

Case-1: PID versus H-Infinity

We will first compare the model "*Stg2_NonLin_Sim_Pid.slx*" that uses the classical PID controller against the model "*Stg2-NonLin_Sim_Hinf.slx*" that uses the H-infinity controller to prove that the H-infinity controller is still superior to the PID, even when using the non-linear pendulum model and non-linear actuators. Both models are initialized with the same pendulum angles and commanded to perform ±5° attitude maneuvers. The control systems are controlling all 3 axes, roll, pitch and yaw. The pitch and yaw controllers are identical. The roll axis is using the RCS jets and the roll attitude and rate responses (shown in blue) are jumpy because of the jet firing. The PID controller is unstable, as expected, in pitch and yaw, and the divergence reaches unacceptably high amplitudes. The H-infinity controller stabilizes the slosh modes and the oscillations decay. Figures (30.1 to 30.7) compare results between the unstable model (top figures) against the stable H-infinity model (bottom figures).

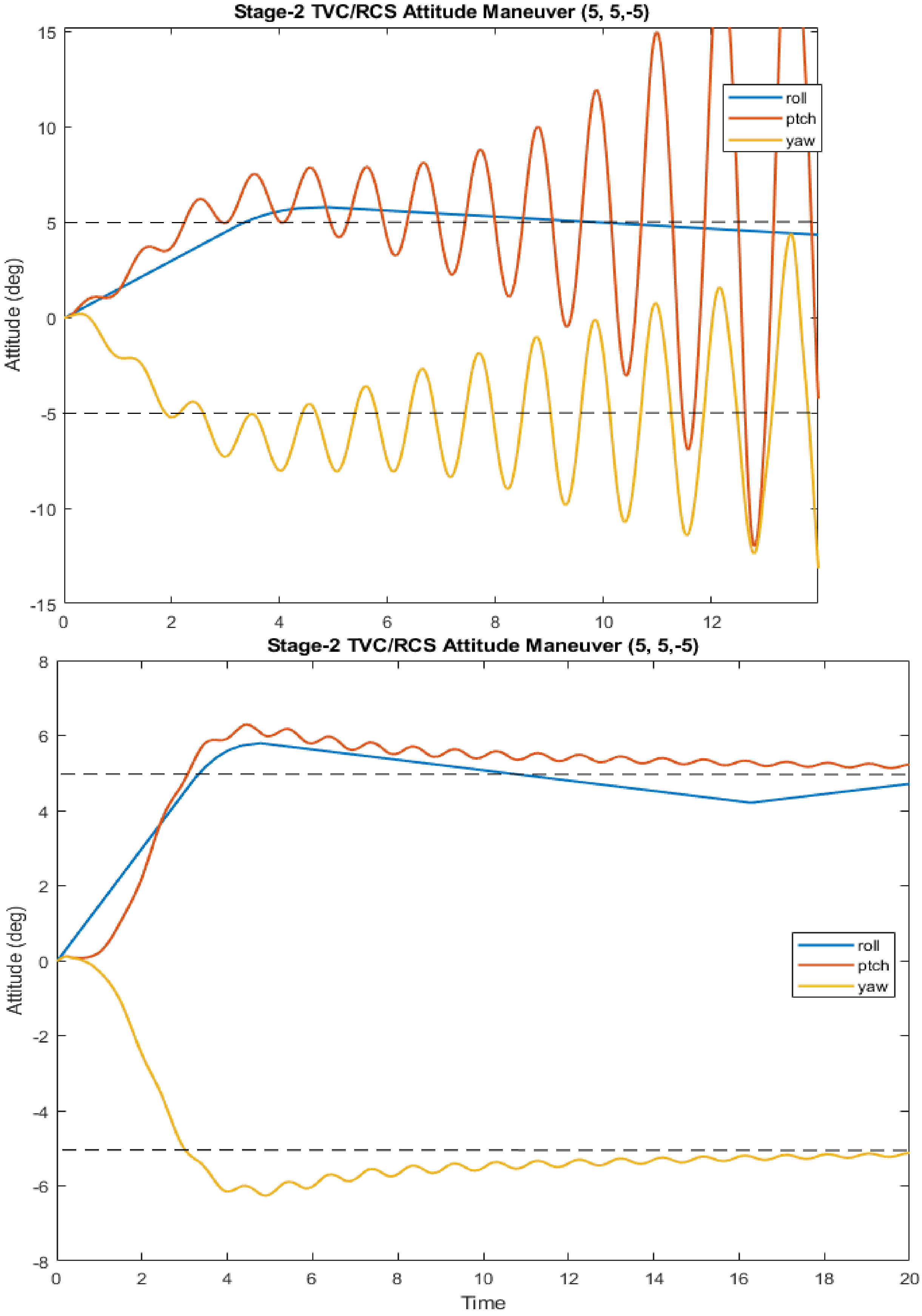


Figure 30-1 Attitude Response to Commanded Values. The PID System Diverges, the H-Infinity is Stable, the Oscillations Decay and the Attitude Converges to Commands. The Roll Axis (Blue) is Controlled by RCS Jets

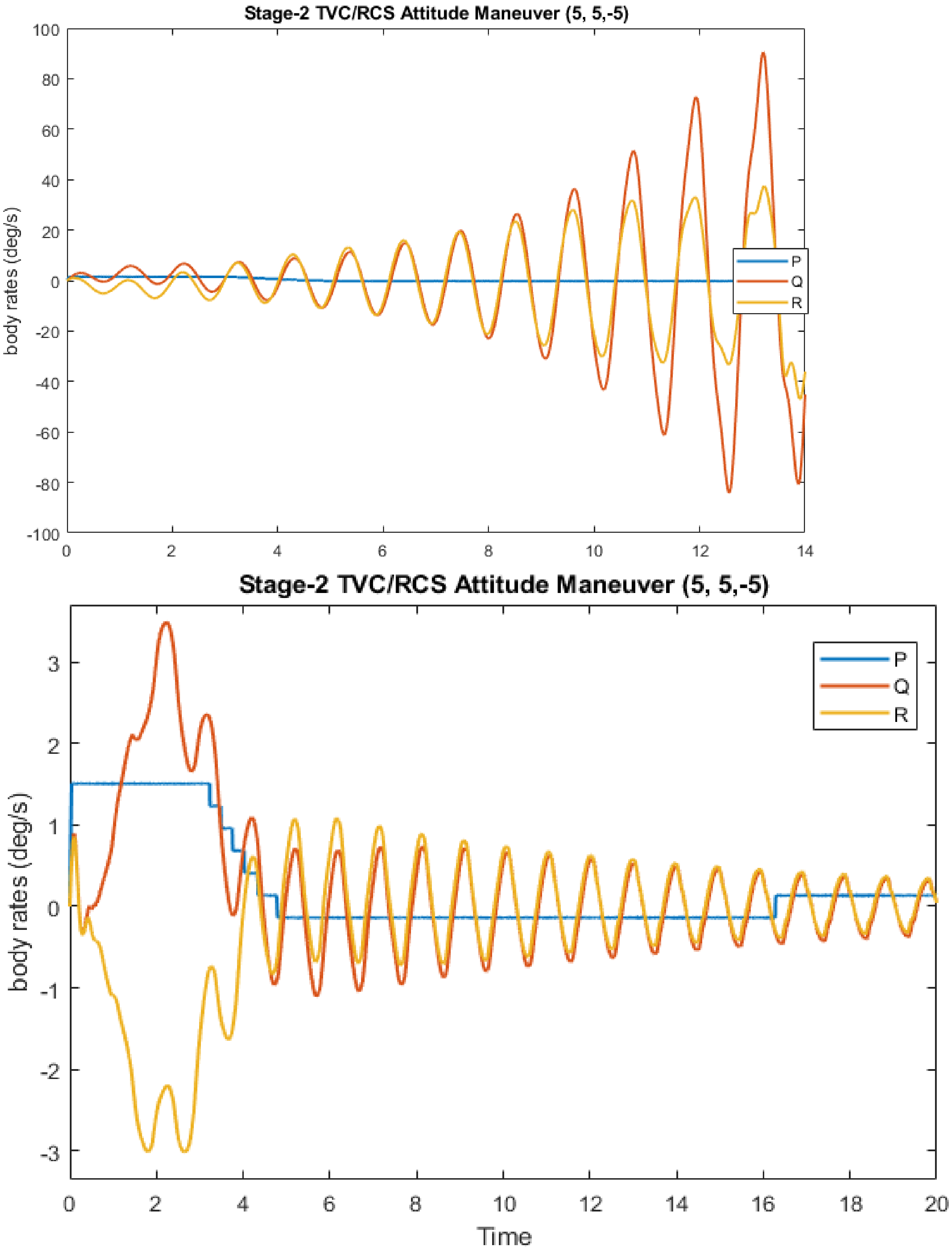


Figure 30.2 The Body Rates of the H-Infinity System Converge Towards Zero. The PID System is Unstable

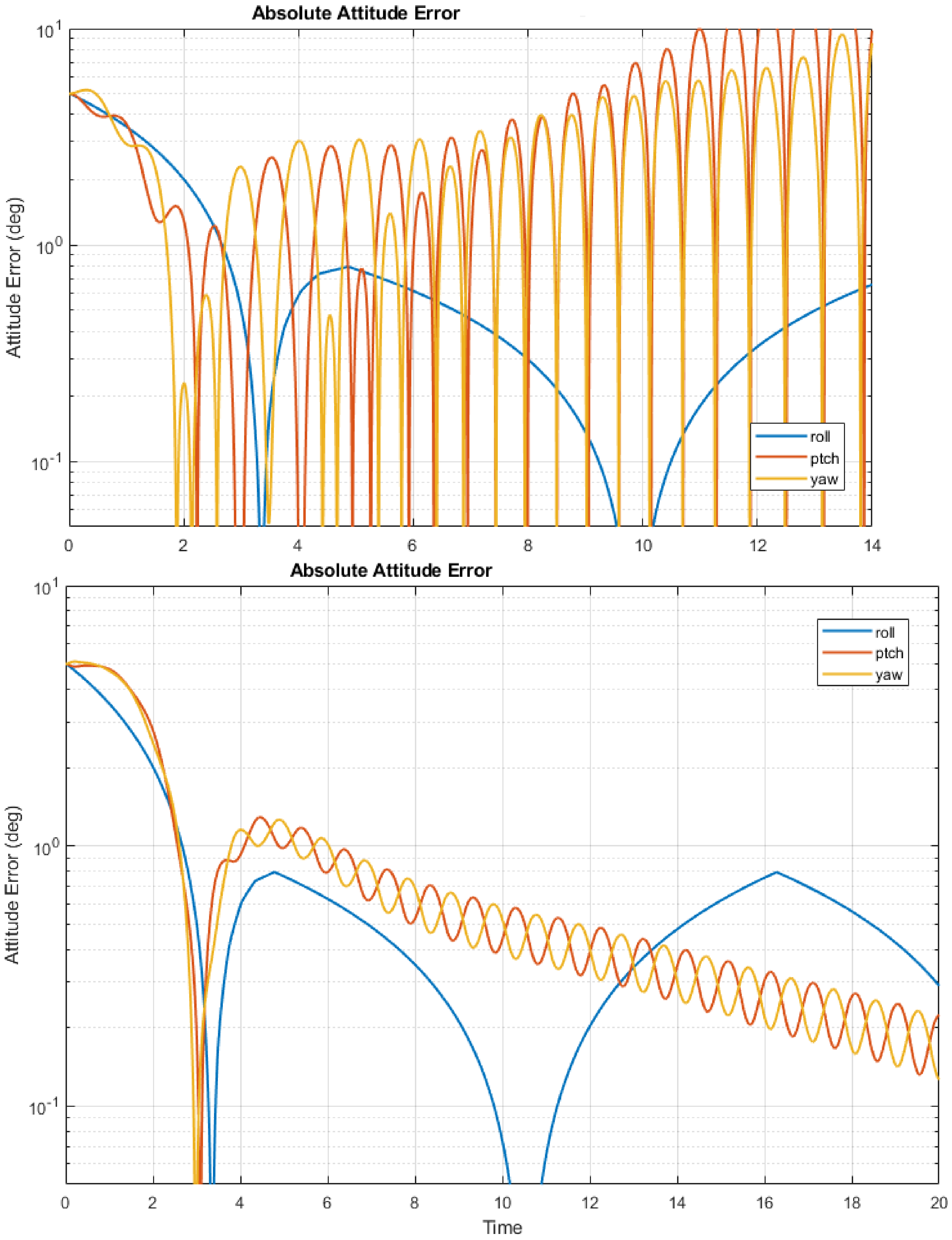


Figure 30.3 Attitude Error Magnitudes in Logarithmic Scale

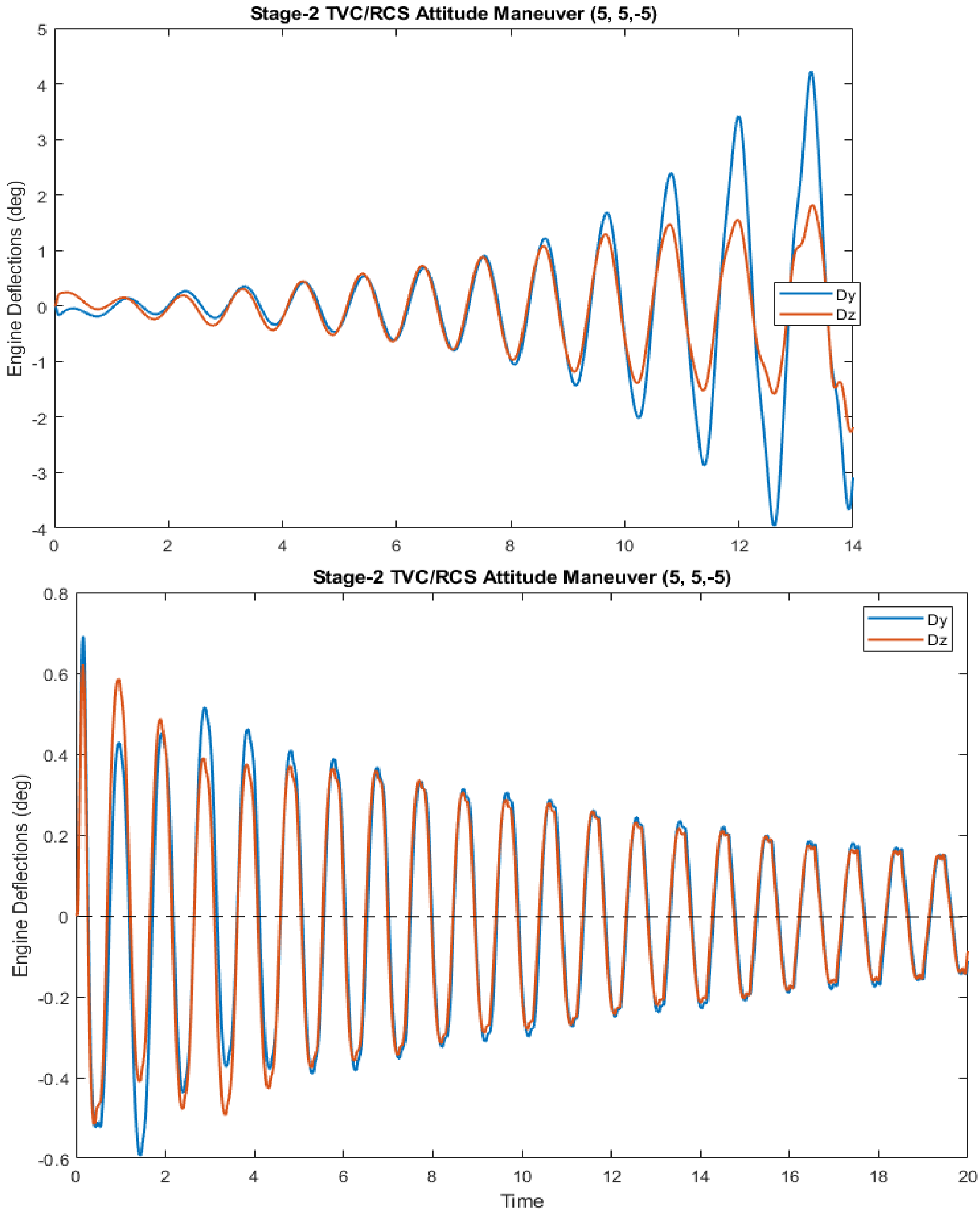


Figure 30.4 Engine Gimbal Deflections in (deg)

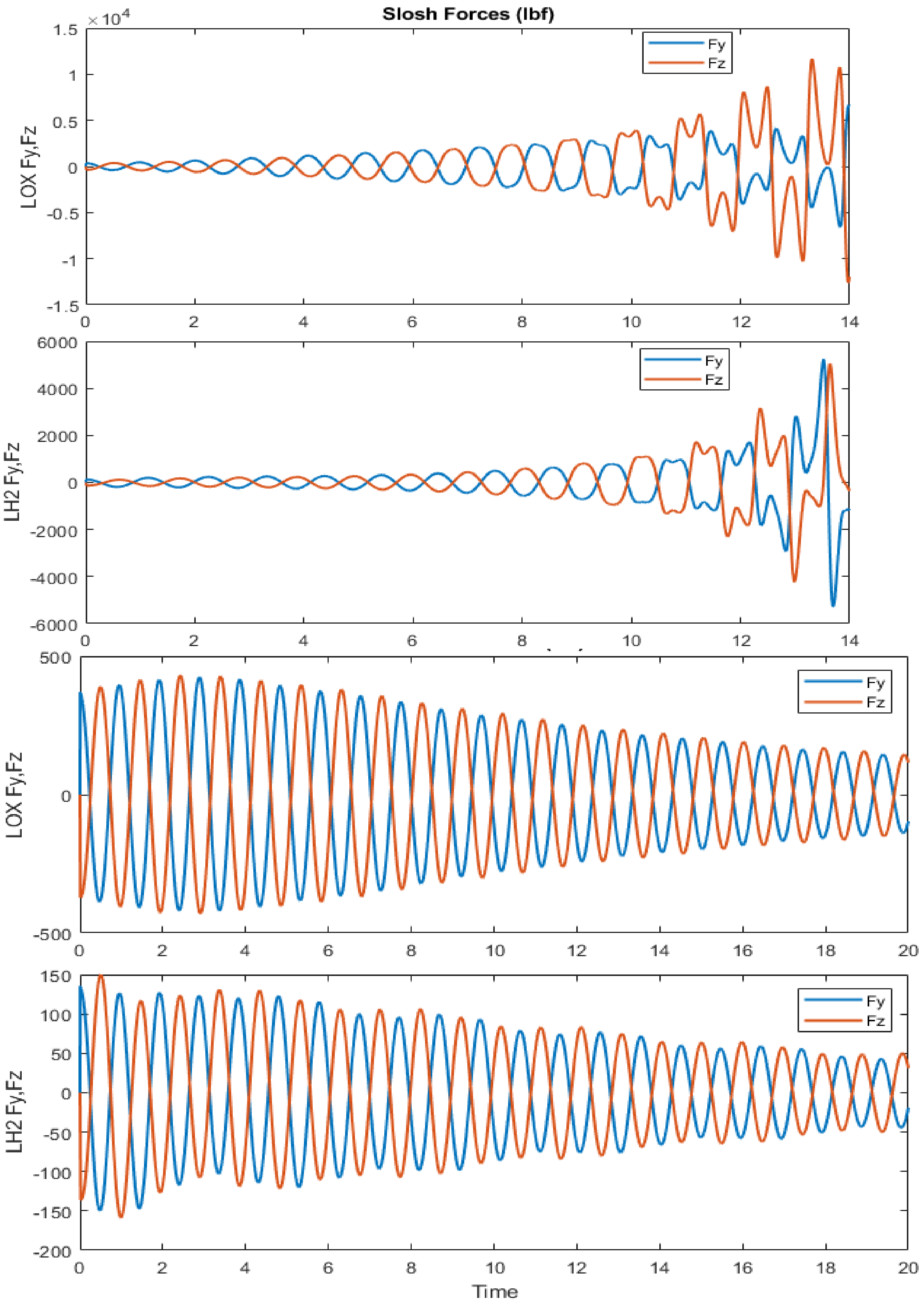


Figure 30.5 LOX and LH2 Slosh Forces Against the Tank

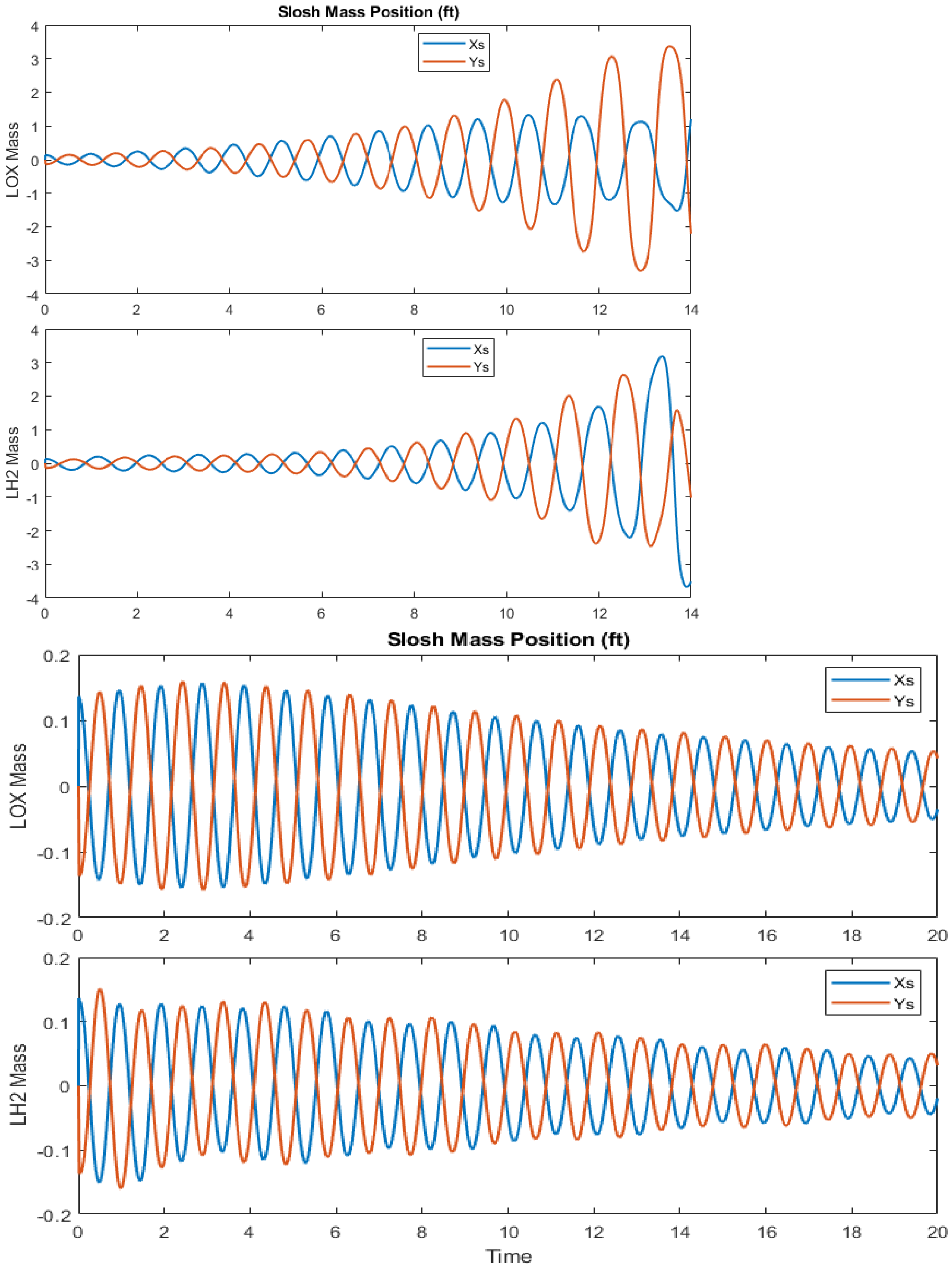


Figure 30.6 Slosh Mass Normal Deflections Along the Y and Z Axes

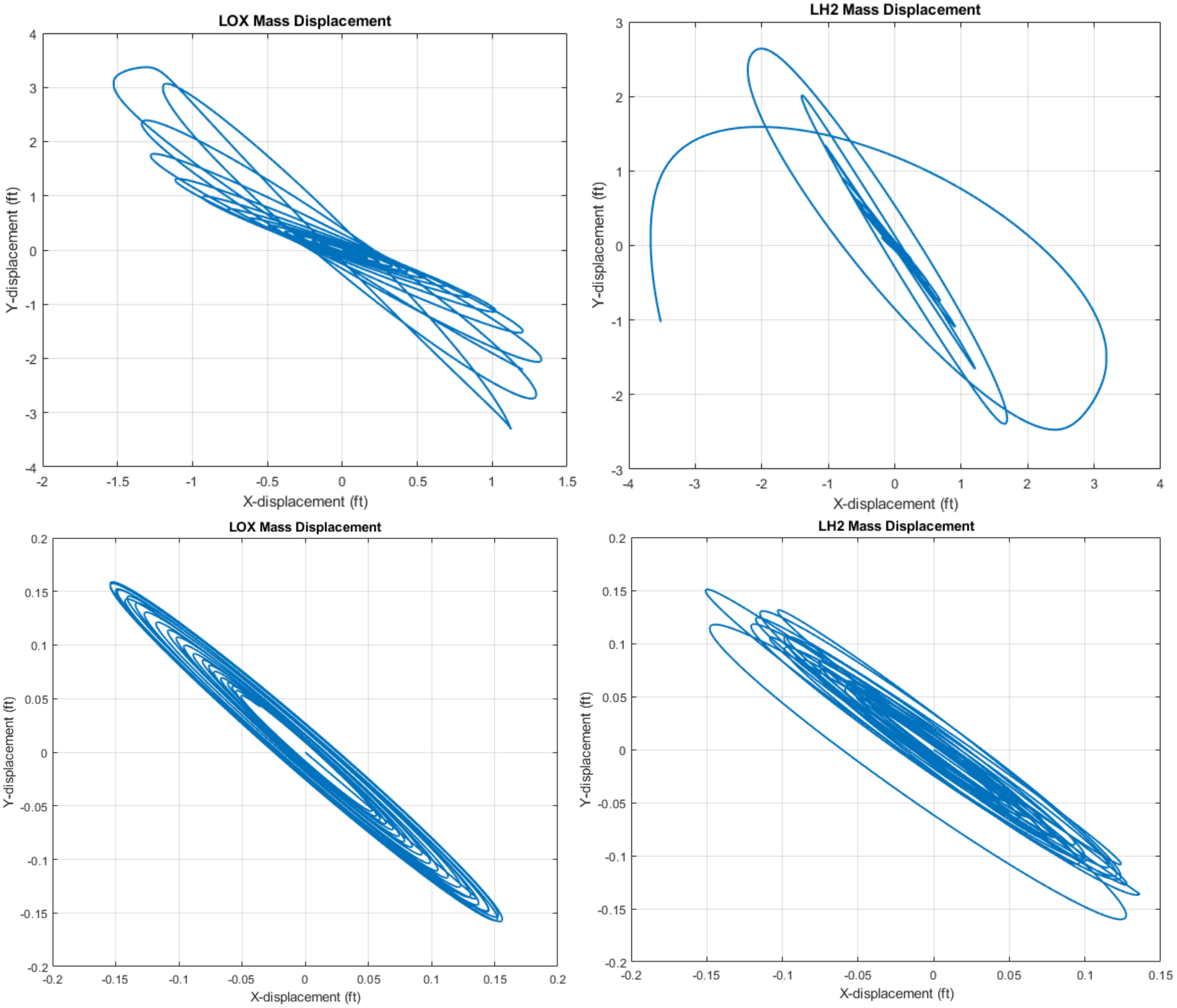


Figure 30.7 The Slosh Mass Deflections in the Unstable Model Above Reach Almost to Maximum (3 .6 feet) which is almost θ=90°. They Decay in the Stable Model Below

Case-2: Increasing the PID Damping

The next step is to use the PID/ spherical pendulum simulation to demonstrate that increasing damping in the tank improves performance even with phase-unstable slosh and the spherical pendulum model can be used to determine the amount of damping and number of baffles necessary to achieve satisfactory performance. The H-infinity requires knowledge of the slosh frequency which is not a very robust solution when you are not sure about it. You can even make it worse if you design around the wrong frequency. The simulation "Stg2_NonLin_Sim_Pid.slx" is used to determine how much baffles we need to add in order to survive with a PID controller and unstable slosh. We don't want to add too many baffles in the tanks because of weight, so we will have to tolerate a certain amount of oscillation in attitude, gimbals, and rates. Adding baffles around the tank walls (like shelves) effectively increases the damping coefficient and slightly reduces the pendulum length to the point that the instability converges to a limit-cycle that can be tolerated performance wise. This information cannot be obtained from the linear model because it is still unstable even with increased damping, although less. In this case we increased zeta from ζ=0.002 to ζ=0.02, shortened the pendulum length from l_p=3.68 to l_p=2.7, with an initial pendulum angle θ=4°, lateral angle ϕ=45°, and a small spin rate $\dot{\phi} = 10$ (deg/sec). This mechanical modification makes the oscillation levels acceptable even when using the PID controller which was unstable before.

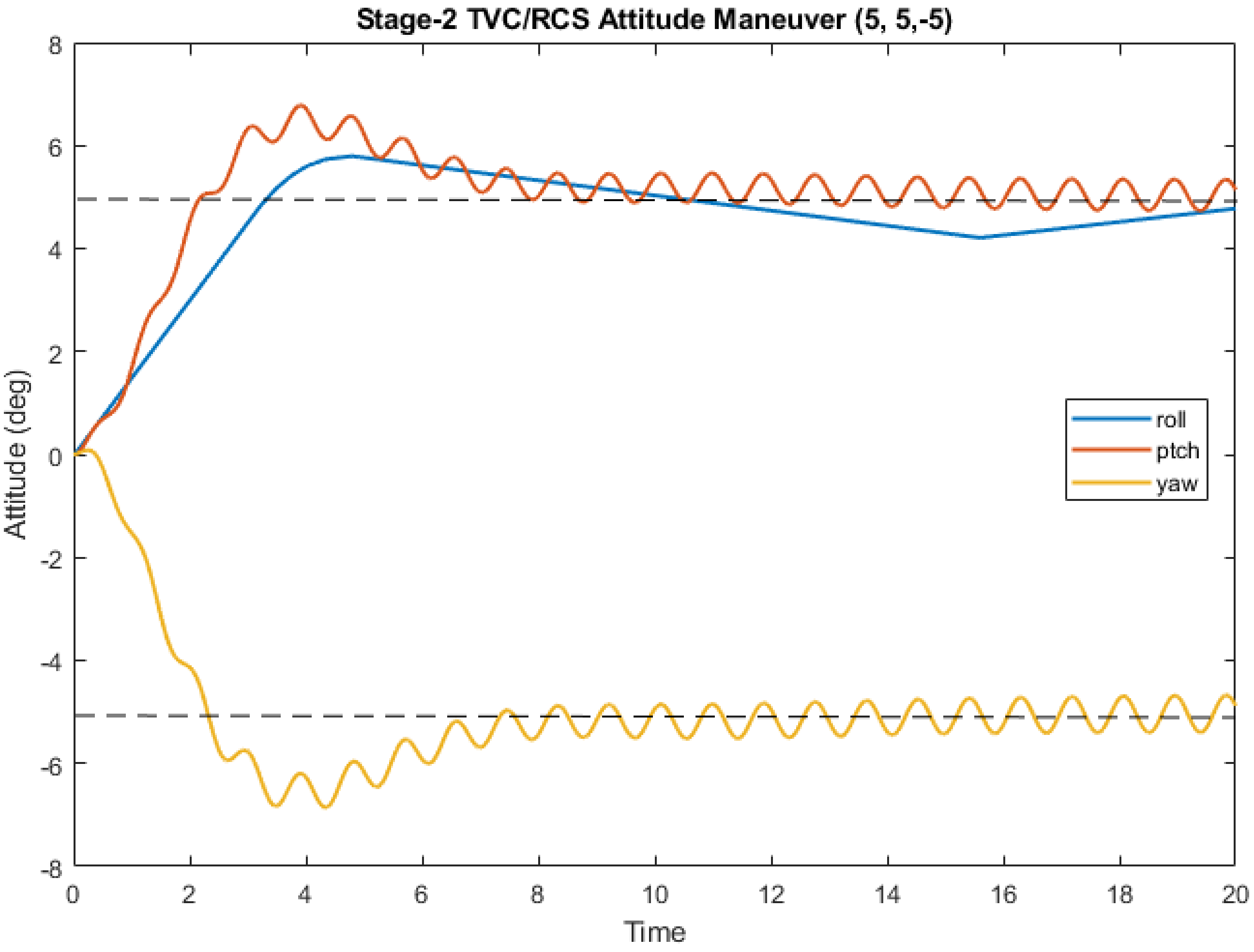


Figure 31.1 The PID Attitude Oscillations Now Converge to Small Limit Cycles Because the Damping was Increased

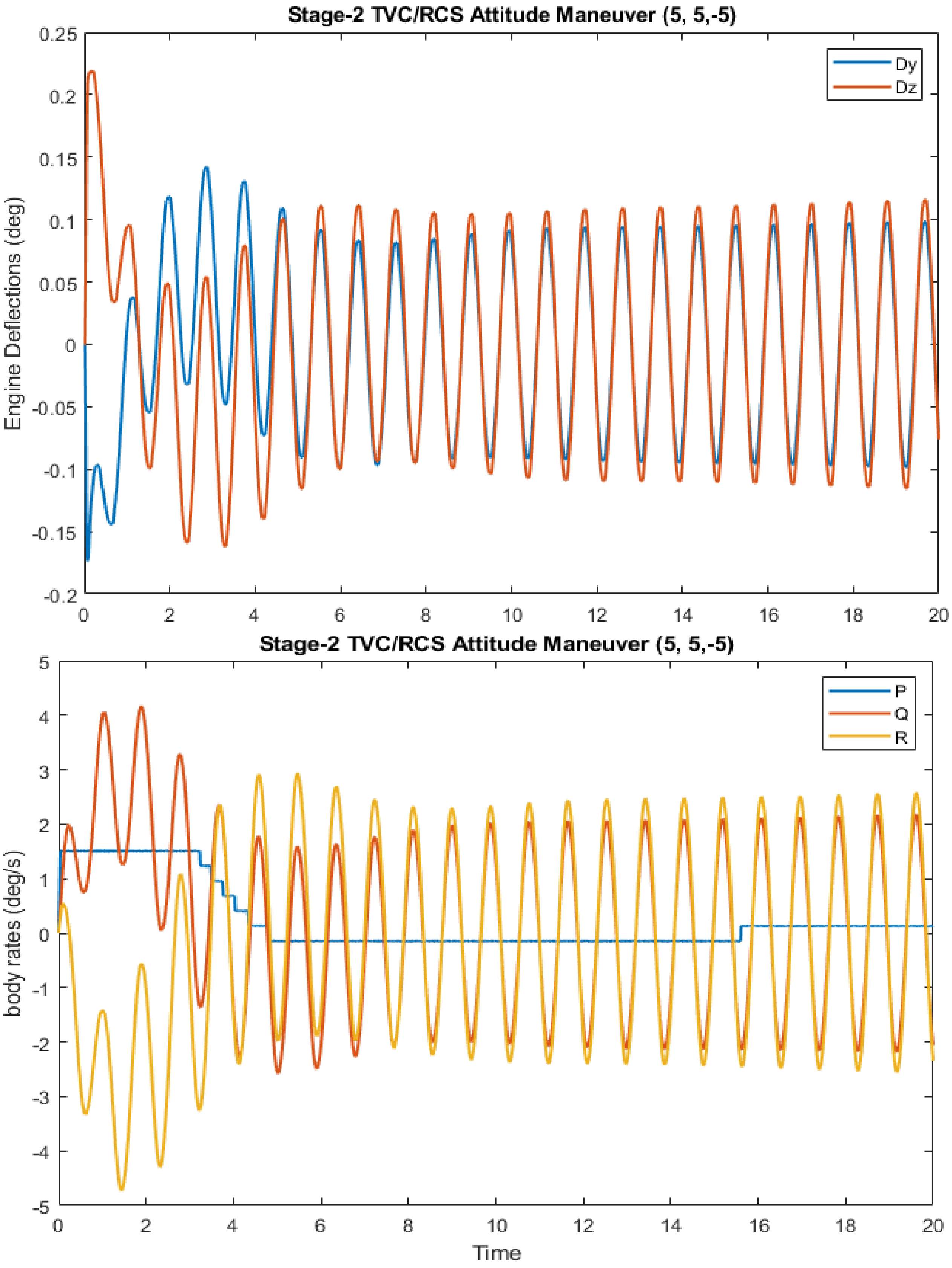


Figure 31.2 Attitude Errors and Body Rates Oscillate Around Zero with Small and Acceptable Amplitudes

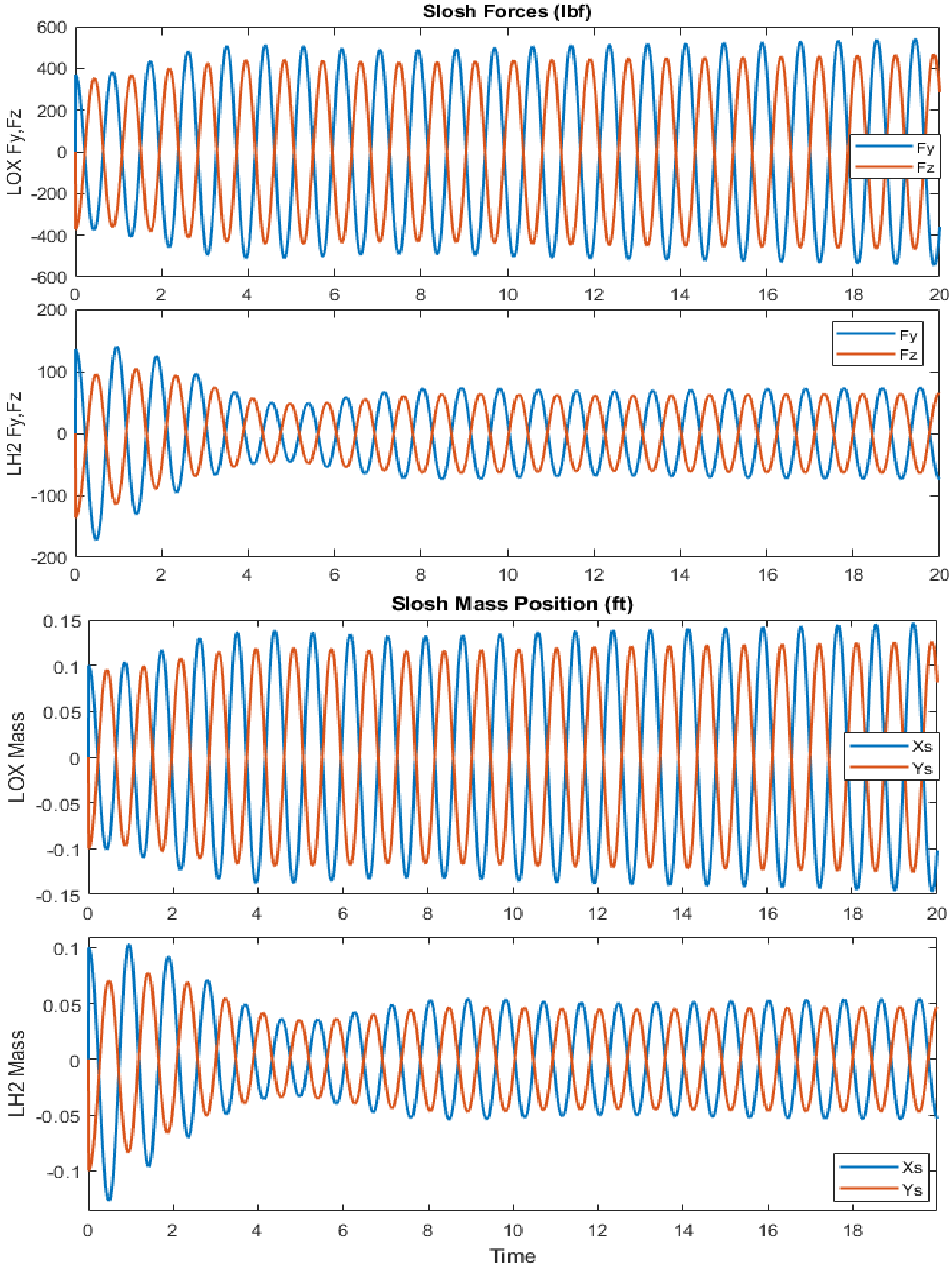


Figure 31.3 Slosh Mass Oscillatory Displacements Ys and Zs inside the tanks, along the Y and Z axes are Small and Steady

Case-3: Vortex Instabilities Using the PID

Another advantage of using the spherical pendulum simulation is to analyze the control system's robustness to vortex type of slosh instabilities where the slosh masses are swirling around inside the tank leading to large oscillations. Unacceptable instabilities can occur when the slosh mass develops a high spin rate $\dot{\phi}$ around the tank x-axis, pushing against the tank wall and producing high centripetal disturbance forces that couple with the TVC control system and become further excited to unacceptable amplitudes.

We are curious to examine the amount of slosh mass vortex rates that can be tolerated by the control system before they diverge to very large oscillation amplitudes. What happens when you increase the mass angular rate around the tank? Will the control system be able to stand against this vortex type of disturbance, decay and converge to an acceptable limit-cycle or is it going to diverge further? We want to pursue the latter case so we initialize at a very high spin rate $\dot{\phi}$=600 (rad/sec) and a bigger pendulum angle θ=50°. Everything else is the same as in Case-2 that uses the PID controller. The simulation results in Figures (32.1 to 32.4) show that the oscillations do not decay even though the damping is pretty high but they diverge to higher pendulum angles and unacceptable amplitudes. This is caused by the dynamic coupling of slosh with the control system.

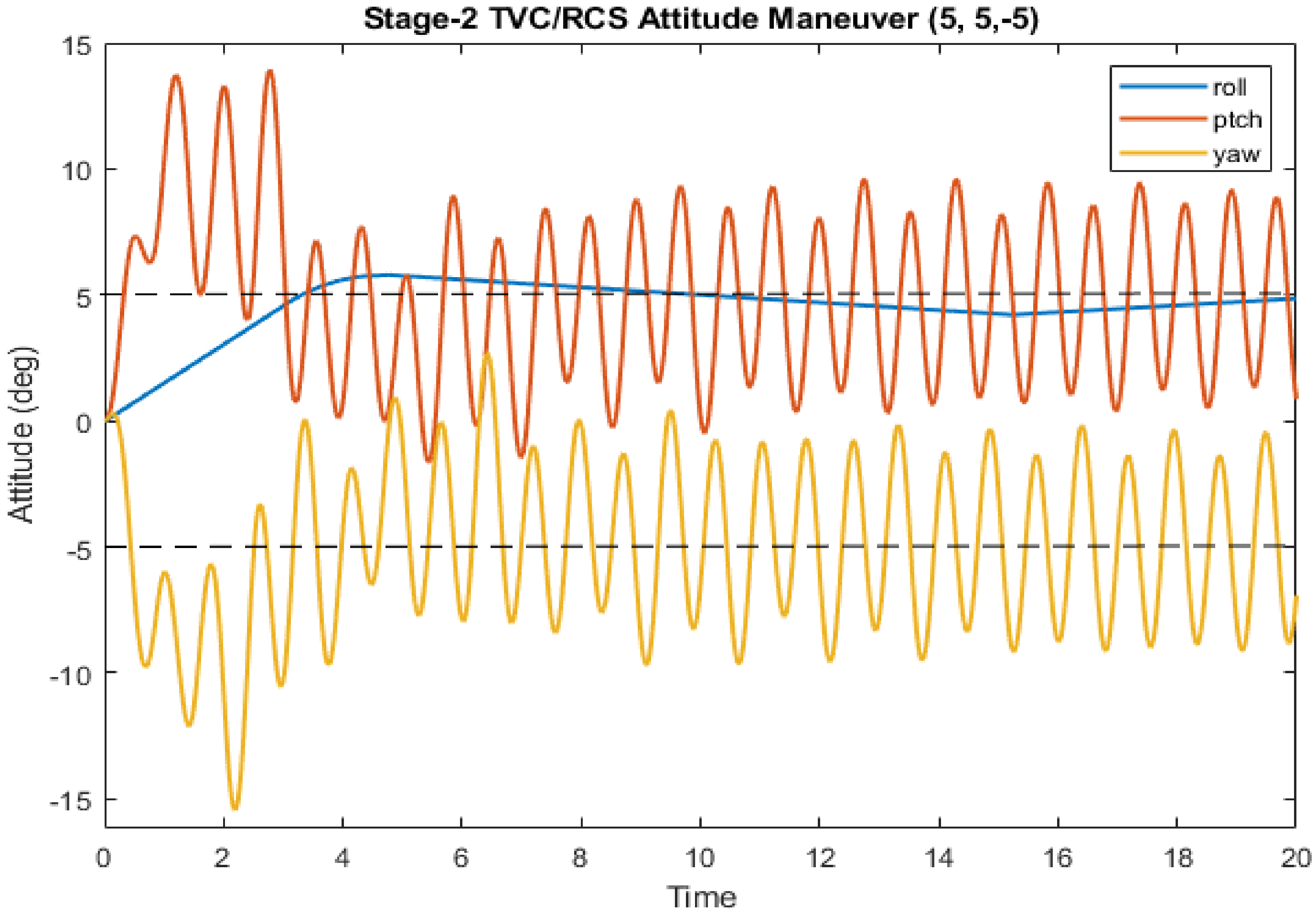


Figure 32.1 The Attitude Oscillations Converge to Unacceptably High Lint-Cycles

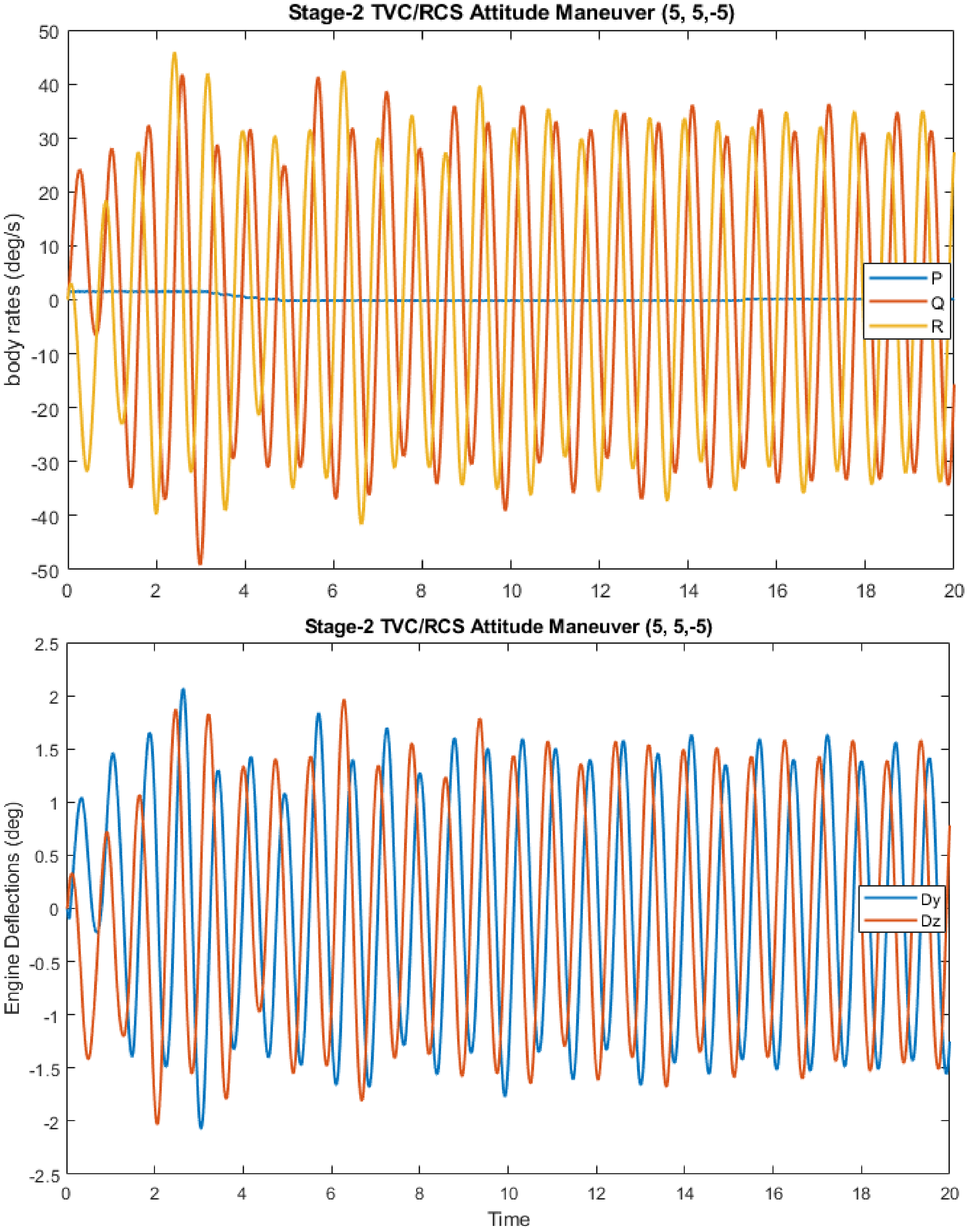


Figure 32.2 The Body Rates and Gimbal Deflections Oscillate at Steady but Very Large Amplitudes

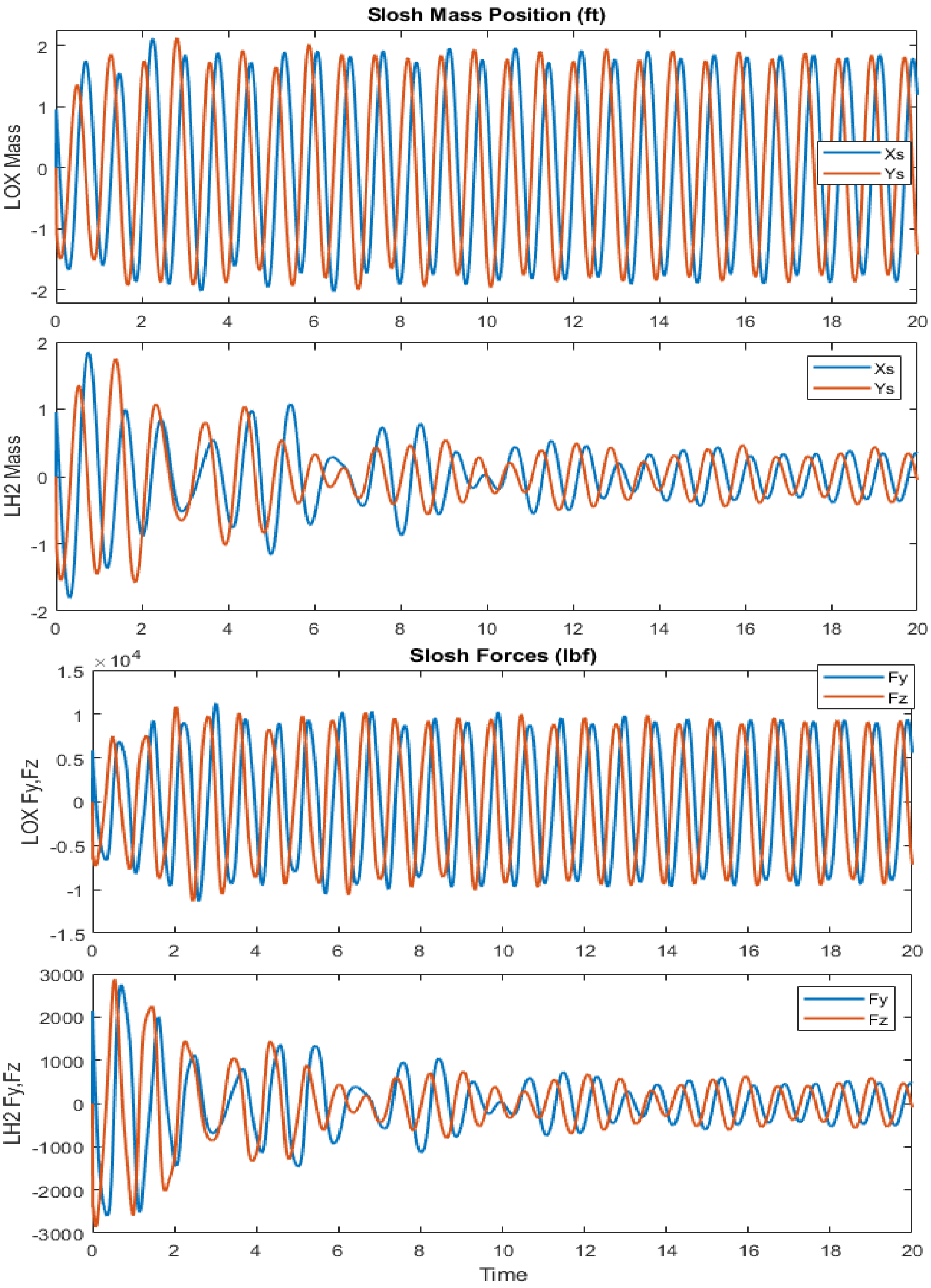


Figure 32.3 The LOX Slosh Mass Deflections Oscillations are approaching to almost Max Amplitude 2.7 (feet) because LOX is the Unstable Mode. This is like a pendulum angle θ = ±70°. The LH2 Mode is Stable but it is excited by the LOX Oscillations.

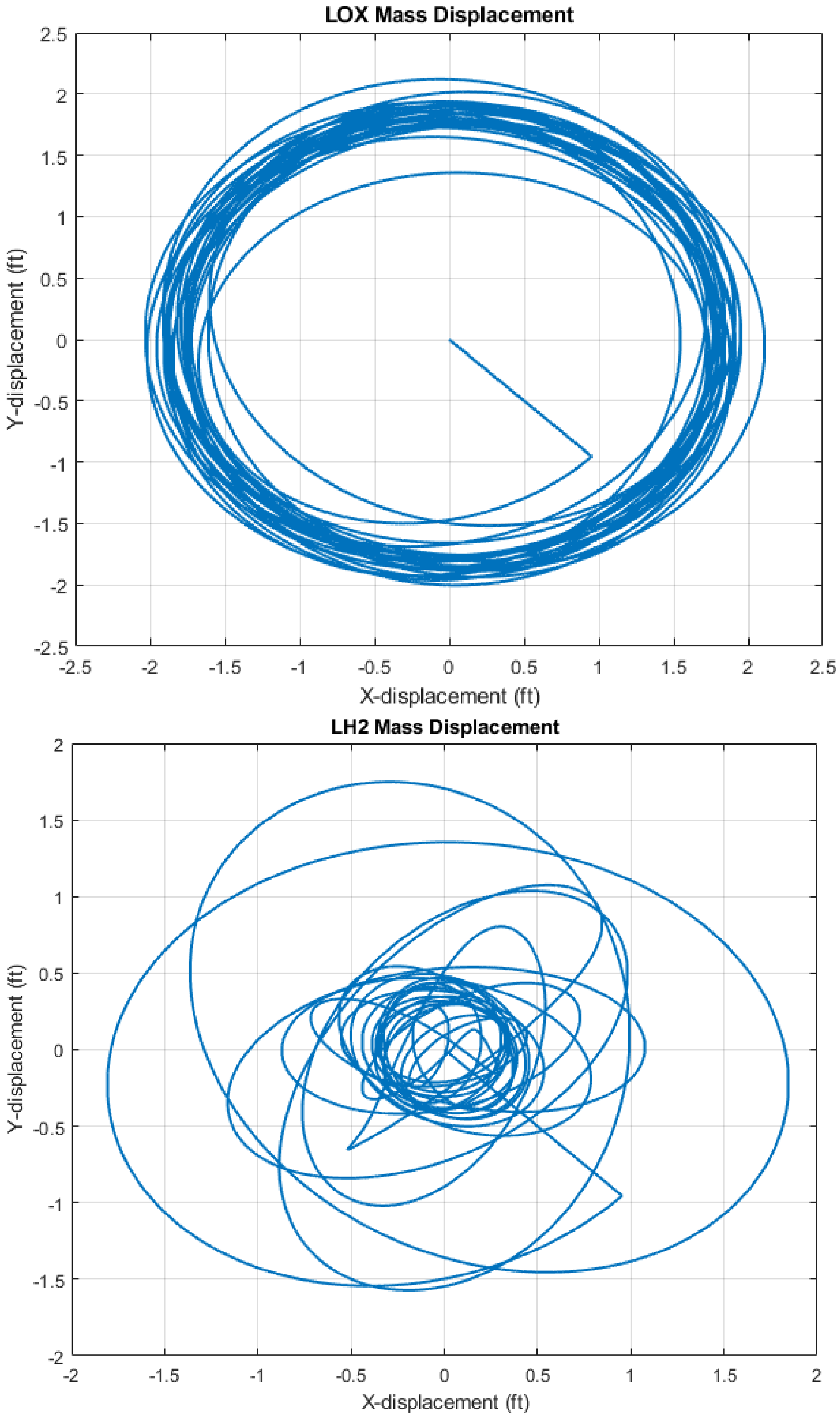
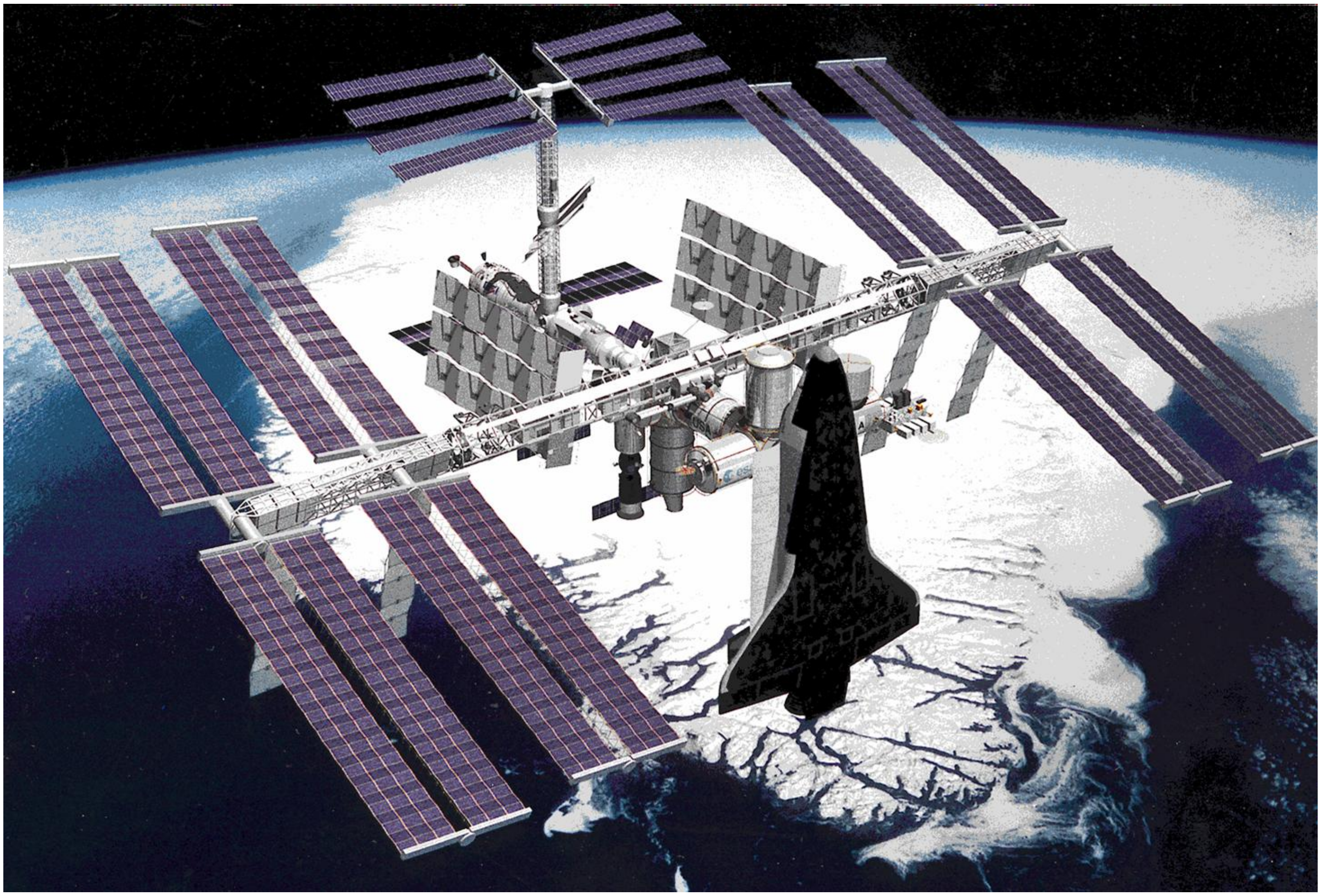


Figure 32.4 Pendulum Displacements Ys and Zs perpendicular to the Tank Centerline. It shows large deflections of more than 2 (feet). They cannot exceed 2.7 (ft) because this corresponds to θ = ±90°

Space Station Attitude and CMG Momentum Control Design



In this example we will use the H-Infinity method and tools to design the control system of a large and flexible Space Station that is in orbit around the earth. It has a circular orbit and its attitude is maintained constant relative to the Local Vertical Local Horizontal (LVLH) frame, where the x-axis is in the direction of the velocity vector, the z-axis is pointing towards the earth's center and the y-axis towards the right solar array. The Space Station consists of a truss structure, some attached modules for the crew, equipment, experiments, etc. which are located near the center of the structure, and two rotating solar arrays which are pointing to the sun and complete one rotation per orbit relative to the station. The attitude is controlled by reaction jets (RCS) and control moment gyros (CMGs) depending on the operations. In the mode of operation that we will analyze, the attitude control system uses only CMGs to provide torques that stabilize the spacecraft and maintain a steady attitude relative to the LVLH. The CMGs are momentum exchange devices located near the center of the structure but they have limited torque and momentum capability. The CMG torque in this situation is not used to maneuver the spacecraft attitude but to keep it constant under the influence of cyclic aerodynamic torque disturbances which occur at orbital rate. The momentum is the integral of torque, which means, that the CMGs can only supply torque for a limited amount of time before they saturate and when they do, their supplied torque drops to zero and they must be desaturated by applying a torque on the vehicle in the opposite to the saturation direction. Momentum dump is achieved either by firing RCS jets or by using gravity gradient torque.

The aero disturbances consist of steady torques combined with cyclic components which excite the spacecraft attitude to oscillations. To react against the steady torques the spacecraft attitude must be adjusted in order to produce gravity gradient torques that counteract the bias torque. This adjustment must be performed by the control system. The cyclic disturbance torques must also be counteracted by the control system. There are two frequency components associated with the disturbances: one is at orbital rate ω_o due to the difference in atmospheric density between the sunny and the dark sides of the earth and the second component is at twice the orbital rate $2\omega_o$ produced by drag variation due to the rotation of the solar arrays.

In Section-1 we will design a control system that stabilizes the spacecraft attitude and uses gravity gradient to manage the CMG momentum. By managing the momentum, we mean preventing the CMGs from reaching saturation and allowing the torque and momentum of the CMG cluster to cycle about a zero average as they react against the cyclic aero disturbances. In this mode of operation, the attitude converges to the Torque Equilibrium Attitude (TEA), which is the orientation at which the secular (non-cyclic) aero torques are balancing against the gravity-gradient torques. In Section-2 we will include structural flexibility in the model, analyze stability, design bending filters, and demonstrate the momentum management concept by simulating the spacecraft response to aerodynamic disturbance torques using the same linear model with flex modes. In Section-3 we will demonstrate the ACS design on a 6DOF simulation in Simulink using non-linear equations of motion.

1.1 Spacecraft Equations Orbiting Earth in the LVLH Frame

The linearized Equations 1.1 are derived from the non-linear Equations 3.1 and they describe the space-station motion in the LVLH frame. The body rates are replaced with LVLH rates and this model will be used for the control design and linear simulations. The equations include gravity gradient torque which is calculated from the spacecraft LVLH attitude (ϕ, θ, ψ). The inputs are control torque T_c and disturbance torque T_d. There is also a bias torque caused by the cross-products of inertia. The CMG momentum (h_x, h_y, h_z) and momentum integral are also included in the design model in order to control the CMG momentum. The 12 spacecraft states are: LVLH rates, LVLH attitudes, CMG momentum, and CMG momentum integral.

$$\begin{bmatrix} I_{XX} & I_{XY} & I_{XZ} \\ I_{XY} & I_{YY} & I_{YZ} \\ I_{XZ} & I_{YZ} & I_{ZZ} \end{bmatrix} \begin{pmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{pmatrix} = \omega_o \begin{bmatrix} 0 & 2I_{YZ} & I_{XX} - I_{YY} + I_{ZZ} \\ -2I_{YZ} & 0 & 2I_{XY} \\ -I_{XX} + I_{YY} - I_{ZZ} & -2I_{XY} & 0 \end{bmatrix} \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix}$$

$$+\omega_o^2 \begin{bmatrix} 4(I_{ZZ} - I_{YY}) & 3I_{XY} & -I_{XZ} \\ 4I_{XY} & 3(I_{ZZ} - I_{XX}) & I_{YZ} \\ -4I_{XZ} & -3I_{YZ} & (I_{XX} - I_{YY}) \end{bmatrix} \begin{pmatrix} \phi \\ \theta \\ \psi \end{pmatrix} + \omega_o^2 \begin{pmatrix} -4I_{YZ} \\ 3I_{XZ} \\ I_{XY} \end{pmatrix} + T_d + T_c$$

$$\begin{pmatrix} \dot{h}_X \\ \dot{h}_Y \\ \dot{h}_Z \end{pmatrix}_{CMG} = \omega_o \begin{pmatrix} h_Z \\ 0 \\ -h_X \end{pmatrix}_{CMG} - T_c$$

Equation 1.1 Dynamic Equations Linearized in the LVLH frame

1.2 Momentum Stabilization Using Gravity Gradient Torques

When the vehicle average attitude is at the TEA, the external aero and gravity torques balance out, the CMG momentum oscillates but it does not diverge to saturation. The momentum is cyclic because the control torque is cyclic because it counteracts cyclic disturbances. The purpose of the attitude control system is not to command the spacecraft attitude but to stabilize it at the TEA and to attenuate the attitude oscillations which are caused by the aerodynamic disturbances. It also stabilizes the CMG momentum. This is accomplished by including feedback from the CMG momentum in order to prevent it from diverging. The attitude in this case is allowed to drift and it naturally converges to the TEA because this is the attitude where the external torques balance and the momentum does not diverge. Momentum feedback plus additional momentum integral feedback produce zero average momentum as it oscillates around zero. This is a continuous momentum desaturation process and it is very attractive because it does not require RCS propellant but it uses gravity gradient to adjust the spacecraft attitude to prevent the CMG momentum from building up. It requires, however, sufficient knowledge of the vehicle dynamics and its mass properties for the derivation of the control gains.

1.3 Control Design Model

The H-infinity method will be used to design the state-feedback gain matrix that stabilizes this highly coupled dynamic system. The vehicle design model won't be decoupled into pitch and lateral, as we do in other examples, but a single controller gain matrix will merge all 3 axes by receiving feedback from all coupled vehicle states, stabilizing not only attitude but also the CMG momentum. In order to synthesize the H-infinity controller, we must first create the design model from Equations 1.1 and then the 9-matrices Synthesis Model. The design model is shown in Figure 1.1. It consists of the vehicle equations augmented with momentum integrators and two additional filters.

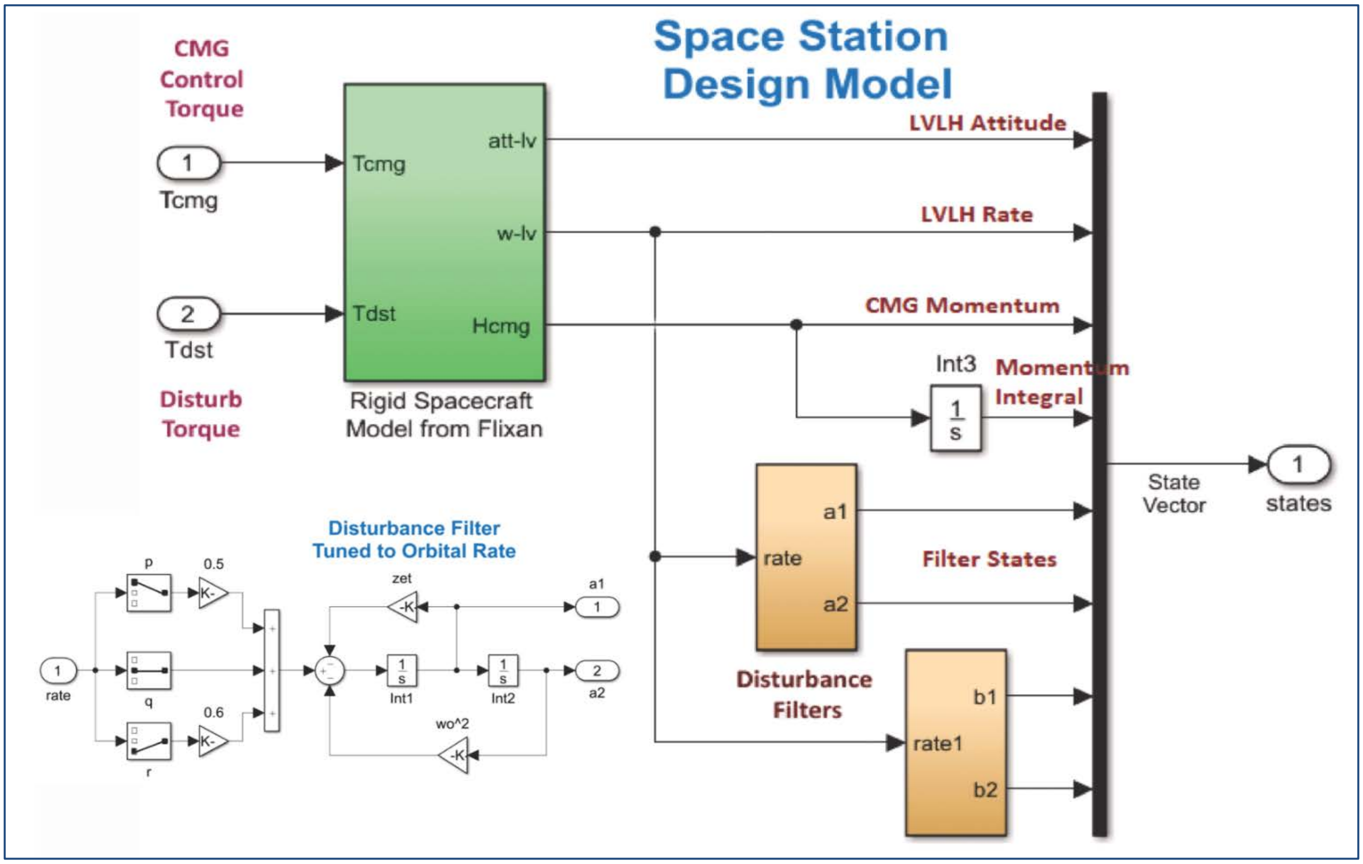
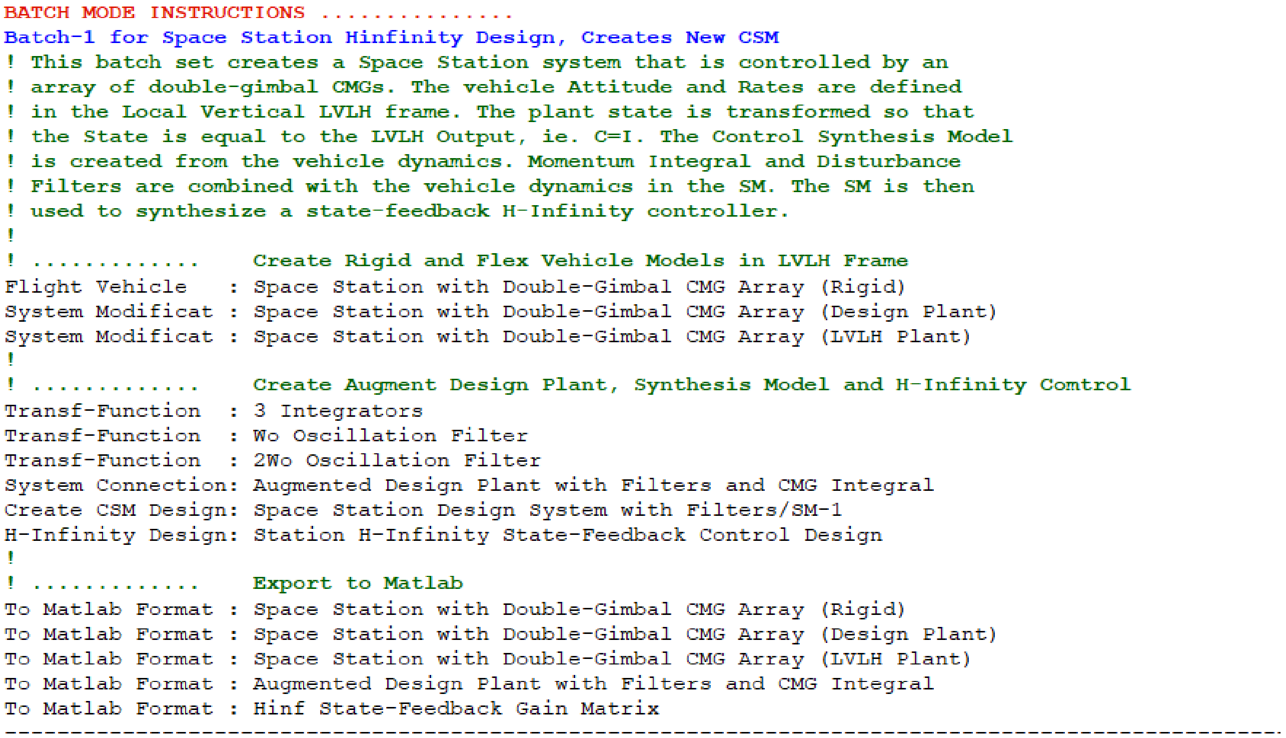


Figure 1.1 Control Design Model

The purpose of the disturbance filters is to attenuate the two disturbances which occur at constant frequencies ω_o and $2\omega_o$. They are 2nd order resonances which are tuned to those frequencies and they introduce 4 additional states in the design model, a total of 16 states. The two filter resonances are excited by the vehicle rates because the rates are sensitive to the disturbances. They amplify the system's response at the disturbance frequencies and the filter states are penalized by the optimization algorithm, and hence, the derived control system reduces sensitivity at the disturbance frequencies. Each filter is excited by rates from all 3 directions. The 9-matrices Synthesis Model is created from the design model via an interactive process that has been described in other examples. It includes control design parameters in addition to the vehicle dynamics. The analysis files for this section are in folder "*Flixan\Control Analysis\Hinfinity\ Examples\ 6-Space Station Attitude & Momentum Control Design\1-Hinfinity Control Design*". The Flixan input file "*Space_Station-1.Inp*" includes the vehicle dynamics and creates the space-station system, the design and synthesis models and also computes the H-infinity controller which is saved in m-file Kpqr. The systems are saved in file "*Space_Station_1.Qdr*" and they are also converted and loaded into Matlab.

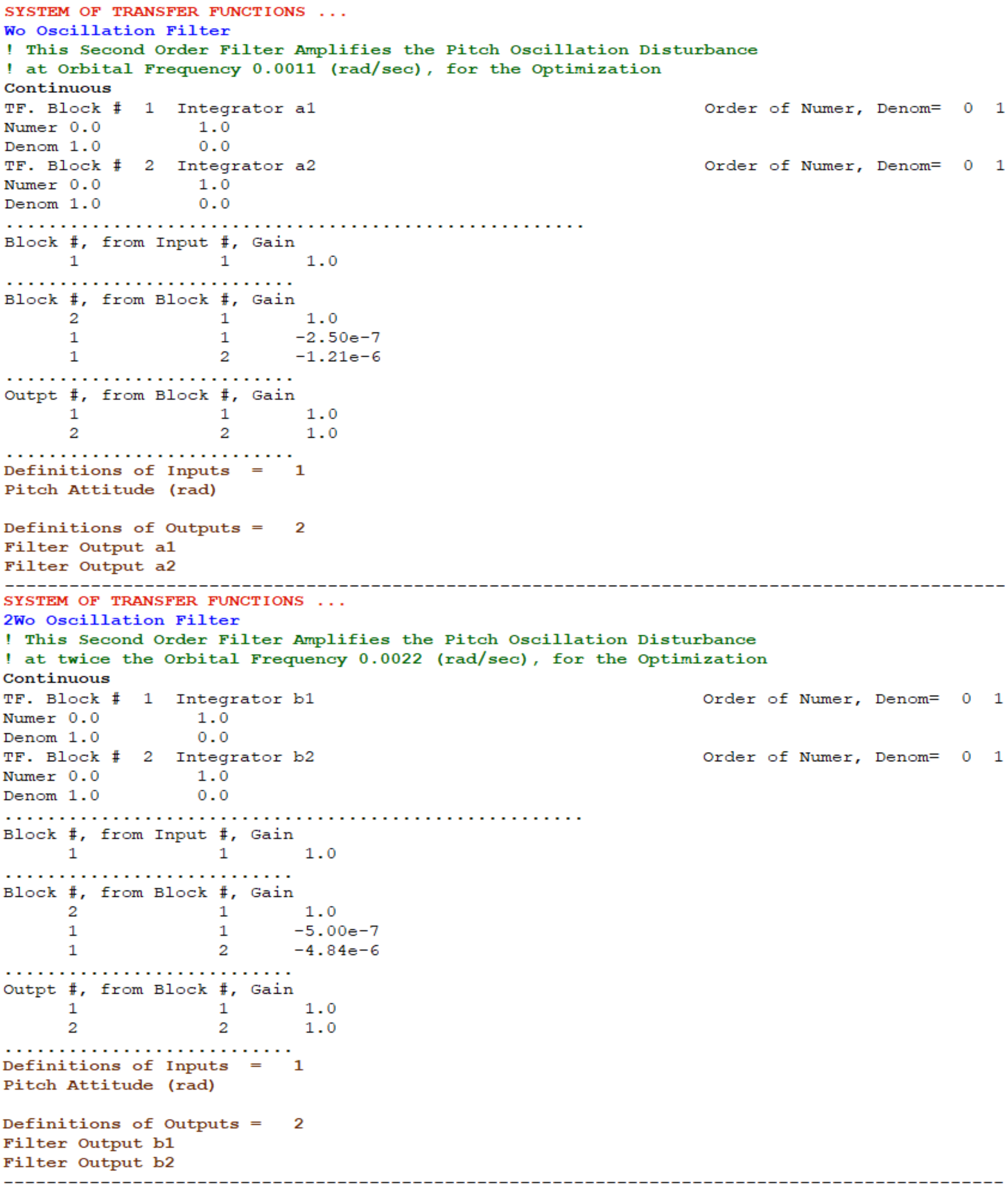
1.4 Input File

There are two batch sets in the input file which are very similar. The first one "*Batch-1 for Space Station Hinfinity Design, Creates New CSM*" creates a new synthesis model using an already existing "*Create CSM Design*" set and overwrites the previous SM and controller. The second batch "*Batch-2 for Space Station Hinfinity Design, Uses Old CSM*" preserves and uses the previous SM to creates a new controller. The Flixan generated vehicle system is "*Space Station with Double-Gimbal CMG Array (Rigid)*". The inputs are 3 disturbance torques (roll, pitch, yaw) and 3 control torques in body axes from a 3-axes momentum exchange device with zero momentum bias. The vehicle attitude and rate outputs are set by the flag line to be in the LVLH frame. The next two datasets are used to remove the extraneous vehicle states and to transform the state-vector from body to LVLH in order to match the output vector for convenience, that is, C=I. The transformed vehicle title is "*Space Station with Double-Gimbal CMG Array (LVLH Plant)*".



```
BATCH MODE INSTRUCTIONS ...............
Batch-1 for Space Station Hinfinity Design, Creates New CSM
! This batch set creates a Space Station system that is controlled by an
! array of double-gimbal CMGs. The vehicle Attitude and Rates are defined
! in the Local Vertical LVLH frame. The plant state is transformed so that
! the State is equal to the LVLH Output, ie. C=I. The Control Synthesis Model
! is created from the vehicle dynamics. Momentum Integral and Disturbance
! Filters are combined with the vehicle dynamics in the SM. The SM is then
! used to synthesize a state-feedback H-Infinity controller.
!
! ............    Create Rigid and Flex Vehicle Models in LVLH Frame
Flight Vehicle   : Space Station with Double-Gimbal CMG Array (Rigid)
System Modificat : Space Station with Double-Gimbal CMG Array (Design Plant)
System Modificat : Space Station with Double-Gimbal CMG Array (LVLH Plant)
!
! ............    Create Augment Design Plant, Synthesis Model and H-Infinity Comtrol
Transf-Function  : 3 Integrators
Transf-Function  : Wo Oscillation Filter
Transf-Function  : 2Wo Oscillation Filter
System Connection: Augmented Design Plant with Filters and CMG Integral
Create CSM Design: Space Station Design System with Filters/SM-1
H-Infinity Design: Station H-Infinity State-Feedback Control Design
!
! ............    Export to Matlab
To Matlab Format : Space Station with Double-Gimbal CMG Array (Rigid)
To Matlab Format : Space Station with Double-Gimbal CMG Array (Design Plant)
To Matlab Format : Space Station with Double-Gimbal CMG Array (LVLH Plant)
To Matlab Format : Augmented Design Plant with Filters and CMG Integral
To Matlab Format : Hinf State-Feedback Gain Matrix
-------------------------------------------------------------------------------
```

```
----------------------------------------------------------------------------------------------------
BATCH MODE INSTRUCTIONS ...............
Batch-2 for Space Station Hinfinity Design, Uses Old CSM
! This batch set creates a Space Station system that is controlled by
! an array of double-gimbal CMGs. The vehicle Attitude and Rates are
! defined in the Local Vertical LVLH frame. The plant state is transformed
! so that the State is equal to the LVLH Output, ie. C=I. The original
! Synthesis Model is retained from the previous *.Qdr file and reused to
! synthesize a state-feedback H-Infinity controller.
!
! ............     Create Rigid and Flex Vehicle Models in LVLH Frame
Retain CSM        : Space Station Design System with Filters/SM-1
Flight Vehicle    : Space Station with Double-Gimbal CMG Array (Rigid)
System Modificat  : Space Station with Double-Gimbal CMG Array (Design Plant)
System Modificat  : Space Station with Double-Gimbal CMG Array (LVLH Plant)
!
! ............     Create Augment Plant, Design H-Infin Control using previous SM
Transf-Function   : 3 Integrators
Transf-Function   : Wo Oscillation Filter
Transf-Function   : 2Wo Oscillation Filter
System Connection: Augmented Design Plant with Filters and CMG Integral
H-Infinity Design: Station H-Infinity State-Feedback Control Design
!
! ............     Export to Matlab
To Matlab Format  : Space Station with Double-Gimbal CMG Array (Rigid)
To Matlab Format  : Space Station with Double-Gimbal CMG Array (Design Plant)
To Matlab Format  : Space Station with Double-Gimbal CMG Array (LVLH Plant)
To Matlab Format  : Augmented Design Plant with Filters and CMG Integral
To Matlab Format  : Hinf State-Feedback Gain Matrix
----------------------------------------------------------------------------------------------------
FLIGHT VEHICLE INPUT DATA ......
Space Station with Double-Gimbal CMG Array (Rigid)
!
! The Space Station state-space system is created using the vehicle modeling program.
! It includes an array of double-gimbal control moment gyros, 3 rate gyros, 3 attitude
! sensors, and 4 accelerometers. The Station is initialized at the Local Vertical Local
! Horizontal (LVLH) attitude and it has a negative pitch rate of -0.063 (rad/sec) which is
! equal to the orbital rate. The vehicle rates are computed relative to the LVLH frame.
! A constant bias torque 7.40153 (ft-lb) is applied in the direction (-0.1969, 0.5963, 0.7781)
! representing the steady-state gyroscopic and gravity-gradient torques due to the constant
! pitch rate
Body Axes Output, LVLH Attitude & Rate

Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft)     :  6200.0       32.174       0.20896E+08
Moments and products of Inertias Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)           :  0.119416e+9  0.40408e+8   0.110166e+9, 0.476e+7,    0.1216e+7,    0.30121e+6
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)         :  0.0          0.0          0.0
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)        :  0.0          25500.0      0.0001       700000.0
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)          :  0.0          0.0          0.0          0.0
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                          :  0.0          0.0          0.0          0.0
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)    :  0.0000       0.000        0.0000       0.0000       -0.063        0.0000
W-Gust Azim & Elev angles (deg), or Torque/Force direction (x,y,z), Force Locat (x,y,z)   :  Torque       0.196972     -0.596376    -0.778163
Surface Reference Area (feet^2), Mean Aerodynamic Chord (ft), Wing Span in (feet)         :  0.0          1.0          1.0
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial_rho/ Partial_H} :  0.0          0.0          0.0          -0.0
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:  0.0          0.0          0.0          0.0          0.0           0.0000
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}:  0.0          -0.0         0.0000       0.0000       0.0000        0.0000
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}:  0.0          -0.0         0.0000       0.0000       0.0000        0.0000
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:  0.0          -0.0         0.0000       0.0000       0.0000        0.0000
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}:  0.0          -0.0         0.0000       0.0000       0.0000        0.0000
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}:  0.0          0.0          0.0000       0.0000       0.0000        0.0000

Number of External Torques on the Vehicle                                                 :  3
Torque No  1  Direction (x, y, z)                                                         :  1.0          0.0          0.0
Torque No  2  Direction (x, y, z)                                                         :  0.0          1.0          0.0
Torque No  3  Direction (x, y, z)                                                         :  0.0          0.0          1.0

Double Gimbal Control Moment Gyro System  (3-axes), Initial Momentum (x,y,z) (ft-lb-sec) :  Yes          0.0     0.0     0.0

Number of Gyros, (Attitude and Rate)                                                       :  6
Gyro No  1 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 2     :  Roll   Rate          0.0        82.0       0.0
Gyro No  2 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 2     :  Pitch  Rate          0.0        82.0       0.0
Gyro No  3 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 2     :  Yaw    Rate          0.0        82.0       0.0
Gyro No  1 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 2     :  Roll   Attitude      0.0        82.0       0.0
Gyro No  2 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 2     :  Pitch  Attitude      0.0        82.0       0.0
Gyro No  3 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 2     :  Yaw    Attitude      0.0        82.0       0.0

Number of Accelerometers, Along Axes: (x,y,z)                                              :  4
Acceleromet No  1 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 2     :  X-axis Accelerat.    0.0      232.35    18.0
Acceleromet No  2 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 2     :  Z-axis Accelerat.    0.0      232.35    18.0
Acceleromet No  3 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 4     :  X-axis Accelerat.    0.0     -224.63    16.0
Acceleromet No  4 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 4     :  Z-axis Accelerat.    0.0     -224.63    16.0
----------------------------------------------------------------------------------------------------------------------
CREATE A NEW SYSTEM FROM AN OLD SYSTEM... (Titles of the New and Old Systems)
Space Station with Double-Gimbal CMG Array (Design Plant)
Space Station with Double-Gimbal CMG Array (Rigid)
! Create the Station Design Model by Reducing the Rigid-Body Model
! Inputs are the 3 CMG Control Torques and 3 Disturbance Torques about (x,y,z)
! Outputs are: LVLH attitude, rates, and CMG Momentum
!
TRUNCATE OR REORDER THE SYSTEM INPUTS, STATES, AND OUTPUTS
Extract Inputs :   1   2   3   4   5   6
Extract States :   1   3   5   2   4   6  11  12  13
Extract Outputs:   1   3   5   2   4   6  16  17  18
----------------------------------------------------------------------------------------------------------
CREATE A NEW SYSTEM FROM AN OLD SYSTEM... (Titles of the New and Old Systems)
Space Station with Double-Gimbal CMG Array (LVLH Plant)
Space Station with Double-Gimbal CMG Array (Design Plant)
! Transform the design plant. Keep the same inputs and states and make the
! States Vector equal to the Outputs vector
!
SYSTEM TRANSFORMATION, STATES EQUAL TO OUTPUTS
-----------------------------------------------------------------------------------------------------------
```



```
SYSTEM OF TRANSFER FUNCTIONS ...
Wo Oscillation Filter
! This Second Order Filter Amplifies the Pitch Oscillation Disturbance
! at Orbital Frequency 0.0011 (rad/sec), for the Optimization
Continuous
TF. Block #  1  Integrator a1                                    Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
TF. Block #  2  Integrator a2                                    Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
......................................................
Block #, from Input #, Gain
      1              1        1.0
...........................
Block #, from Block #, Gain
      2              1        1.0
      1              1       -2.50e-7
      1              2       -1.21e-6
...........................
Outpt #, from Block #, Gain
      1              1        1.0
      2              2        1.0
...........................
Definitions of Inputs  =   1
Pitch Attitude (rad)

Definitions of Outputs =   2
Filter Output a1
Filter Output a2
---------------------------------------------------------------------------------------------
SYSTEM OF TRANSFER FUNCTIONS ...
2Wo Oscillation Filter
! This Second Order Filter Amplifies the Pitch Oscillation Disturbance
! at twice the Orbital Frequency 0.0022 (rad/sec), for the Optimization
Continuous
TF. Block #  1  Integrator b1                                    Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
TF. Block #  2  Integrator b2                                    Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
......................................................
Block #, from Input #, Gain
      1              1        1.0
...........................
Block #, from Block #, Gain
      2              1        1.0
      1              1       -5.00e-7
      1              2       -4.84e-6
...........................
Outpt #, from Block #, Gain
      1              1        1.0
      2              2        1.0
...........................
Definitions of Inputs  =   1
Pitch Attitude (rad)

Definitions of Outputs =   2
Filter Output b1
Filter Output b2
---------------------------------------------------------------------------------------------
```

The two disturbance filters and the CMG momentum integrators are implemented in transfer-function form and converted into state-space. They are combined with the vehicle system to create the augmented design plant “*Augmented Design Plant with Filters and CMG Integral*”. This system will be used to create the 9 matrices SM via the CSM creation dataset “*Space Station Design System with Filters/SM-1*” and it is saved in the systems file. The SM is finally used in batch mode to create the H-infinity controller by processing the H-infinity dataset “*Station H-Infinity State-Feedback Control Design*” which is set up to produce a state-feedback controller (not dynamic) using the asymptotic algorithm (2). The (3x16) controller matrix K_{pqr} is saved in the systems file under the title “*Hinf State-Feedback Gain Matrix*”. The vehicle systems and controller gain are also converted into Matlab format and used in the analysis.

```
SYSTEM OF TRANSFER FUNCTIONS ...
3 Integrators
! Used to calculate the CMG Momentum Integral for the Optimization
Continuous
TF. Block #  1  Integrator x                              Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
TF. Block #  2  Integrator y                              Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
TF. Block #  3  Integrator z                              Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
.....................................................
Block #, from Input #, Gain
      1             1        1.0
      2             2        1.0
      3             3        1.0
..........................
Outpt #, from Block #, Gain
      1             1        1.0
      2             2        1.0
      3             3        1.0
..........................
Definitions of Inputs  =   3
CMG Momentum-X
CMG Momentum-Y
CMG Momentum-Z

Definitions of Outputs =   3
CMG Momentum-X Integral
CMG Momentum-Y Integral
CMG Momentum-Z Integral
-------------------------------------------------------------------------------------------
CREATE A SYNTHESIS MODEL FOR H-INFINITY CONTROL DESIGN
Space Station Design System with Filters/SM-1
Augmented Design Plant with Filters and CMG Integral
! Create the H-Infinity Synthesis Model from the Vehicle Design Plant
!
Number of Uncertainty I/O Pairs    :    0
Number of Disturbance Inputs       :    6
Disturbance Input Numbers          :    1    2    3    4    5    6
Number of Control Inputs           :    3
Control Input Numbers              :    1    2    3
Number of Performance Outputs      :   16
Perform Optimization Output Numbrs:    1    2    3    4    5    6    7    8    9   10   11   12   13   14   15   16
Number of Commanded Outputs        :    0
Number of Measurement Outputs      :   16    3
Measurement Output Numbers         :    1    2    3    4    5    6    7    8    9   10   11   12   13   14   15   16
Disturbance Input  & Command Gains: 0.0001 0.0001 0.0001 0.02   0.02   0.02    0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
Performance Output & Control Gains: 0.2e-4  0.001 0.1e-4 0.5e-4 0.001  0.1e-3  1000.0 1000.0 1000.0  4000.0 4000.0 4000.0   2.0  2.0  2.0  2.0  0.2  0.2  0.2
---------------------------------------------------------------------------------------------------------------------------------
H-INFINITY CONTROL DESIGN ......
Station H-Infinity State-Feedback Control Design
Synthesis Model for Control Design in file (.Qdr) :        Space Station Design System with Filters/SM-1
Peak Value of the Sensitivity Function Gamma (dB) :        30.0
Riccati Solver Algorithm (1 or 2)                 :        2
State-Feedback Control Solution via Gain Kqhinf   :Kpqr    Hinf State-Feedback Gain Matrix
-------------------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Space Station with Double-Gimbal CMG Array (Rigid)
System
vehicle_rigid
-------------------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Space Station with Double-Gimbal CMG Array (LVLH Plant)
System
lvlh_plant
-------------------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Space Station with Double-Gimbal CMG Array (Design Plant)
System
design_plant
-------------------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Augmented Design Plant with Filters and CMG Integral
System
augm_plant
-------------------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Hinf State-Feedback Gain Matrix
Matrix Kpqr
-------------------------------------------------------------------------------------------------------------
```

```
INTERCONNECTION OF SYSTEMS .....
Augmented Design Plant with Filters and CMG Integral
! Augment the Design Plant by including the Momentum Integral states
! and the two Oscillation Filters
!
Titles of Systems to be Combined          (Found in File Comb_tst.Qdr)
Title 1 Space Station with Double-Gimbal CMG Array (LVLH Plant)
Title 2 3 Integrators
Title 3 Wo Oscillation Filter                                                                            Wo  Filter
Title 4 2Wo Oscillation Filter                                                                           2Wo Filter
SYSTEM INPUTS TO SUBSYSTEM  1                                                                            Station
Via Matrix +I06                                                                                          Control, Disturb Inputs
.............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                                             9 Original Plant States
System Output  1 from Subsystem  1, Output  1, Gain= 1.0                                     3 LVLH Attitudes
System Output  2 from Subsystem  1, Output  2, Gain= 1.0
System Output  3 from Subsystem  1, Output  3, Gain= 1.0
System Output  4 from Subsystem  1, Output  4, Gain= 1.0                                     3 LVLH Rates
System Output  5 from Subsystem  1, Output  5, Gain= 1.0
System Output  6 from Subsystem  1, Output  6, Gain= 1.0
System Output  7 from Subsystem  1, Output  7, Gain= 1.0                                     3 CMG Momentum
System Output  8 from Subsystem  1, Output  8, Gain= 1.0
System Output  9 from Subsystem  1, Output  9, Gain= 1.0
.............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                                                             from Integrators
System Output 10 from Subsystem  2, Output  1, Gain= 1.0                                     CMG-X Momentum Integral
System Output 11 from Subsystem  2, Output  2, Gain= 1.0                                     CMG-Y Momentum Integral
System Output 12 from Subsystem  2, Output  3, Gain= 1.0                                     CMG-Z Momentum Integral
.............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  3                                                             from Wo Filter
System Output 13 from Subsystem  3, Output  1, Gain= 1.0                                     a1
System Output 14 from Subsystem  3, Output  2, Gain= 1.0                                     a2
.............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  4                                                             from 2Wo Filter
System Output 15 from Subsystem  4, Output  1, Gain= 1.0                                     b1
System Output 16 from Subsystem  4, Output  2, Gain= 1.0                                     b2
.............................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2                                                      Momentum to Integrators
Subsystem  1, Output  7 to Subsystem  2, Input  1, Gain=  1.0                                Roll  CMG Momentum
Subsystem  1, Output  8 to Subsystem  2, Input  2, Gain=  1.0                                Pitch CMG Momentum
Subsystem  1, Output  9 to Subsystem  2, Input  3, Gain=  1.0                                Yaw   CMG Momentum
.....................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  3                                                       1 to Wo Filter
Subsystem  1, Output  4 to Subsystem  3, Input  1, Gain=  0.5                                 Roll Rate
Subsystem  1, Output  5 to Subsystem  3, Input  1, Gain=  1.0                                 Pitch Rate
Subsystem  1, Output  6 to Subsystem  3, Input  1, Gain=  0.6                                 Yaw Rate
......................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  4                                                       1 to 2Wo Filter
Subsystem  1, Output  4 to Subsystem  4, Input  1, Gain=  0.2                                 Roll Rate
Subsystem  1, Output  5 to Subsystem  4, Input  1, Gain=  1.0                                 Pitch Rate
Subsystem  1, Output  6 to Subsystem  4, Input  1, Gain=  0.6                                 Yaw Rate
......................................................................
Definitions of Inputs  =    6
Roll CMG Control Torque (ft-lb)
Ptch CMG Control Torque (ft-lb)
Yaw  CMG Control Torque (ft-lb)
External Roll  Torque (ft-lb)
External Pitch Torque (ft-lb)
External Yaw   Torque (ft-lb)

Definitions of States =   16
Roll Attitude  (phi-LVLH)  (radians)
Pitch Attitude (thet-LVLH) (radians)
Yaw  Attitude  (psi-LVLH)  (radians)
Roll Rate      (p-lvlh)    (rad/sec)
Pitch Rate     (q-lvlh)    (rad/sec)
Yaw  Rate      (r-lvlh)    (rad/sec)
CMG Momentum in X-axis (ft-lb-sec)
CMG Momentum in Y-axis (ft-lb-sec)
CMG Momentum in Z-axis (ft-lb-sec)
CMG Momentum Integral X (ft-lb-sec^2)
CMG Momentum Integral Y (ft-lb-sec^2)
CMG Momentum Integral Z (ft-lb-sec^2)
Wo  Oscillation Filter Output (a1)
Wo  Oscillation Filter Output (a2)
2Wo Oscillation Filter Output (b1)
2Wo Oscillation Filter Output (b2)

Definitions of Outputs =  16
Roll Attitude  (phi-LVLH)  (radians)
Pitch Attitude (thet-LVLH) (radians)
Yaw  Attitude  (psi-LVLH)  (radians)
Roll Rate      (p-lvlh)    (rad/sec)
Pitch Rate     (q-lvlh)    (rad/sec)
Yaw  Rate      (r-lvlh)    (rad/sec)
CMG Momentum in X-axis (ft-lb-sec)
CMG Momentum in Y-axis (ft-lb-sec)
CMG Momentum in Z-axis (ft-lb-sec)
CMG Momentum Integral X (ft-lb-sec^2)
CMG Momentum Integral Y (ft-lb-sec^2)
CMG Momentum Integral Z (ft-lb-sec^2)
Wo  Oscillation Filter Output (a1)
Wo  Oscillation Filter Output (a2)
2Wo Oscillation Filter Output (b1)
2Wo Oscillation Filter Output (b2)
-------------------------------------------------------------------------------------------------------------------------------------------
```

1.5 Systems File

The systems file "*Space_Station_1.Qdr*" is big and we are only showing the Synthesis Model consisting of 9 matrices: {A, B_1, B_2, C_1, C_2, D_{11}, D_{12}, D_{21}, D_{22}} and the controller gain matrix K_{pqr} created by the H-infinity program.

```
SYNTHESIS MODEL FOR H-INFINITY CONTROL
Space Station Design System with Filters/SM-1
Number of: States (x), Uncertainty Inp/Outputs from Plant Variations (dP)=  16   0   0
Number of: Extern Disturbance Inputs (Wi), Control Inputs (Uc)            =   6   3
Number of: Output Criteria (Zo), Regulated Outputs (Zr), Measurements (y)=  16   0  16
Synthes Model Matrices: A, B1,B2,C1,C2, D11,D12,D21,D22, Sample Time (dT)=   0.0000
Matrix A                        Size = 16 X 16
            1-Column            2-Column            3-Column            4-Column            5-Column            6-Column            7-Column            8-Column
  1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
  4-Row  0.281645313103E-05 -0.178452852871E-06  0.211408793673E-07 -0.192267156368E-04 -0.457316082801E-05  0.173991809591E-02  0.000000000000E+00  0.000000000000E+00
  5-Row -0.237287262456E-06 -0.851272836576E-06 -0.568405290346E-10  0.516940033292E-07  0.169203008517E-06 -0.539506272118E-04  0.000000000000E+00  0.000000000000E+00
  6-Row  0.838193221192E-07  0.561972118348E-08  0.867313610974E-06 -0.188834172528E-02  0.949682969008E-04  0.190575126283E-04  0.000000000000E+00  0.000000000000E+00
  7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  9-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00 -0.109955749940E-02  0.000000000000E+00
 10-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
 11-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
 12-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 13-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.500000000000E+00  0.100000000000E+01  0.600000000000E+00  0.000000000000E+00  0.000000000000E+00
 14-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 15-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.200000000000E+00  0.100000000000E+01  0.600000000000E+00  0.000000000000E+00  0.000000000000E+00
 16-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
            9-Column           10-Column           11-Column           12-Column           13-Column           14-Column           15-Column           16-Column
  1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  7-Row  0.109955749940E-02  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  9-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 10-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 11-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 12-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 13-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00 -0.250000000000E-06 -0.121000000000E-05  0.000000000000E+00  0.000000000000E+00
 14-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 15-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00 -0.500000000000E-06 -0.484000000000E-05
 16-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------------
Matrix B1                       Size = 16 X 22
            1-Column            2-Column            3-Column            4-Column            5-Column            6-Column            7-Column            8-Column
  1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  4-Row  0.841459997857E-08  0.991939443191E-09  0.955915223719E-10  0.841459997857E-08  0.991939443191E-09  0.955915223719E-10  0.000000000000E+00  0.000000000000E+00
  5-Row  0.991939443191E-09  0.248650120598E-07  0.789335062128E-10  0.991939443191E-09  0.248650120598E-07  0.789335062128E-10  0.000000000000E+00  0.000000000000E+00
  6-Row  0.955915223719E-10  0.789335062128E-10  0.907848169901E-08  0.955915223719E-10  0.789335062128E-10  0.907848169901E-08  0.000000000000E+00  0.000000000000E+00
  7-Row -0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  8-Row  0.000000000000E+00 -0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  9-Row  0.000000000000E+00  0.000000000000E+00 -0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 10-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 11-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 12-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 13-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 14-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 15-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 16-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
            9-Column           10-Column           11-Column           12-Column           13-Column           14-Column           15-Column           16-Column
  1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  9-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 10-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 11-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 12-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 13-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 14-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 15-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 16-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
           17-Column           18-Column           19-Column           20-Column           21-Column           22-Column
  1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  9-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 10-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 11-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 12-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 13-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 14-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 15-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 16-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------------
Matrix B2                       Size = 16 X  3
            1-Column            2-Column            3-Column
  1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  4-Row  0.841459997857E-08  0.991939443191E-09  0.955915223719E-10
  5-Row  0.991939443191E-09  0.248650120598E-07  0.789335062128E-10
  6-Row  0.955915223719E-10  0.789335062128E-10  0.907848169901E-08
  7-Row -0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
  8-Row  0.000000000000E+00 -0.100000000000E+01  0.000000000000E+00
  9-Row  0.000000000000E+00  0.000000000000E+00 -0.100000000000E+01
 10-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 11-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 12-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 13-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 14-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 15-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
 16-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------------
```

```
Matrix C1                Size = 19 X 16
          1-Column            2-Column            3-Column            4-Column            5-Column            6-Column            7-Column            8-Column
   1-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
   9-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  10-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  11-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  12-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  13-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  14-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  15-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  16-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  17-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  18-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  19-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
          9-Column            10-Column           11-Column           12-Column           13-Column           14-Column           15-Column           16-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   9-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  10-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  11-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  12-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  13-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  14-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
  15-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
  16-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
  17-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  18-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  19-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
----------------------------------------------------------------------------------------------------------------------------
Matrix C2                Size = 16 X 16
          1-Column            2-Column            3-Column            4-Column            5-Column            6-Column            7-Column            8-Column
   1-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
   9-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  10-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  11-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  12-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  13-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  14-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  15-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  16-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
          9-Column            10-Column           11-Column           12-Column           13-Column           14-Column           15-Column           16-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   9-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  10-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  11-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  12-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  13-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  14-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
  15-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
  16-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
----------------------------------------------------------------------------------------------------------------------------
Matrix D11               Size = 19 X 22
          1-Column            2-Column            3-Column            4-Column            5-Column            6-Column            7-Column            8-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   9-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  10-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  11-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  12-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  13-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  14-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  15-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  16-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  17-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  18-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  19-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
          9-Column            10-Column           11-Column           12-Column           13-Column           14-Column           15-Column           16-Column
   1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   9-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  10-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  11-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  12-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  13-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  14-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  15-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  16-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  17-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  18-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
  19-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
```

```
            17-Column         18-Column         19-Column         20-Column         21-Column         22-Column
    1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    9-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   10-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   11-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   12-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   13-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   14-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   15-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   16-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   17-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   18-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   19-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
Matrix D12               Size = 19 X  3
             1-Column          2-Column          3-Column
    1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    9-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   10-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   11-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   12-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   13-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   14-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   15-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   16-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   17-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   18-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   19-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
------------------------------------------------------------------------------------------------------------------------
Matrix D21               Size = 16 X 22
             1-Column          2-Column          3-Column          4-Column          5-Column          6-Column          7-Column          8-Column
    1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
    2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
    3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    9-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   10-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   11-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   12-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   13-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   14-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   15-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   16-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
             9-Column         10-Column         11-Column         12-Column         13-Column         14-Column         15-Column         16-Column
    1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    3-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    4-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    5-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
    9-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   10-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
   11-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   12-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   13-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   14-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   15-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   16-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
            17-Column         18-Column         19-Column         20-Column         21-Column         22-Column
    1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    9-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   10-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   11-Row  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   12-Row  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   13-Row  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   14-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00  0.000000000000E+00
   15-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01  0.000000000000E+00
   16-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00  0.100000000000E+01
------------------------------------------------------------------------------------------------------------------------
Matrix D22               Size = 16 X  3
             1-Column          2-Column          3-Column
    1-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    2-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    3-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    4-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    5-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    6-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    7-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    8-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
    9-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   10-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   11-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   12-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   13-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   14-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   15-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
   16-Row  0.000000000000E+00  0.000000000000E+00  0.000000000000E+00
------------------------------------------------------------------------------------------------------------------------
```

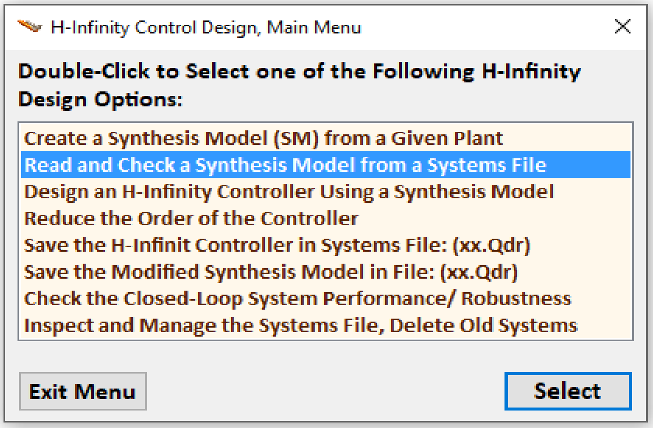
```
Definition of Synthesis Model Variables                        Max Scaling Factors

States (x) ..........  =  16
  1    Roll Attitude  (phi-LVLH)  (radians)
  2    Pitch Attitude (thet-LVLH) (radians)
  3    Yaw  Attitude  (psi-LVLH)  (radians)
  4    Roll Rate      (p-lvlh)    (rad/sec)
  5    Pitch Rate     (q-lvlh)    (rad/sec)
  6    Yaw  Rate      (r-lvlh)    (rad/sec)
  7    CMG Momentum in X-axis (ft-lb-sec)
  8    CMG Momentum in Y-axis (ft-lb-sec)
  9    CMG Momentum in Z-axis (ft-lb-sec)
 10    CMG Momentum Integral X (ft-lb-sec^2)
 11    CMG Momentum Integral Y (ft-lb-sec^2)
 12    CMG Momentum Integral Z (ft-lb-sec^2)
 13    Wo  Oscillation Filter Output (a1)
 14    Wo  Oscillation Filter Output (a2)
 15    2Wo Oscillation Filter Output (b1)
 16    2Wo Oscillation Filter Output (b2)

Excitation Inputs (w)  =  22
  1    Roll CMG Control Torque (ft-lb)                         *  0.0001
  2    Ptch CMG Control Torque (ft-lb)                         *  0.0001
  3    Yaw  CMG Control Torque (ft-lb)                         *  0.0001
  4    External Roll  Torque (ft-lb)                           *  0.02
  5    External Pitch Torque (ft-lb)                           *  0.02
  6    External Yaw   Torque (ft-lb)                           *  0.02
  7    Noise at Output: Roll Attitude  (phi-LVLH)  (radians)   *  0.0000
  8    Noise at Output: Pitch Attitude (thet-LVLH) (radians)   *  0.0000
  9    Noise at Output: Yaw  Attitude  (psi-LVLH)  (radians)   *  0.0000
 10    Noise at Output: Roll Rate      (p-lvlh)    (rad/sec)   *  0.0000
 11    Noise at Output: Pitch Rate     (q-lvlh)    (rad/sec)   *  0.0000
 12    Noise at Output: Yaw  Rate      (r-lvlh)    (rad/sec)   *  0.0000
 13    Noise at Output: CMG Momentum in X-axis (ft-lb-sec)     *  0.0000
 14    Noise at Output: CMG Momentum in Y-axis (ft-lb-sec)     *  0.0000
 15    Noise at Output: CMG Momentum in Z-axis (ft-lb-sec)     *  0.0000
 16    Noise at Output: CMG Momentum Integral X (ft-lb-sec^2)  *  0.0000
 17    Noise at Output: CMG Momentum Integral Y (ft-lb-sec^2)  *  0.0000
 18    Noise at Output: CMG Momentum Integral Z (ft-lb-sec^2)  *  0.0000
 19    Noise at Output: Wo  Oscillation Filter Output (a1)     *  0.0000
 20    Noise at Output: Wo  Oscillation Filter Output (a2)     *  0.0000
 21    Noise at Output: 2Wo Oscillation Filter Output (b1)     *  0.0000
 22    Noise at Output: 2Wo Oscillation Filter Output (b2)     *  0.0000

Control Inputs (u) ... =   3
  1    Control: Roll CMG Control Torque (ft-lb)                *  1.0000
  2    Control: Ptch CMG Control Torque (ft-lb)                *  1.0000
  3    Control: Yaw  CMG Control Torque (ft-lb)                *  1.0000

Performance Outputs (z)=  19
  1    Roll Attitude  (phi-LVLH)  (radians)                    /  0.2E-04
  2    Pitch Attitude (thet-LVLH) (radians)                    /  0.001
  3    Yaw  Attitude  (psi-LVLH)  (radians)                    /  0.1E-04
  4    Roll Rate      (p-lvlh)    (rad/sec)                    /  0.5E-04
  5    Pitch Rate     (q-lvlh)    (rad/sec)                    /  0.001
  6    Yaw  Rate      (r-lvlh)    (rad/sec)                    /  0.0001
  7    CMG Momentum in X-axis (ft-lb-sec)                      /  1000.0
  8    CMG Momentum in Y-axis (ft-lb-sec)                      /  1000.0
  9    CMG Momentum in Z-axis (ft-lb-sec)                      /  1000.0
 10    CMG Momentum Integral X (ft-lb-sec^2)                   /  4000.0
 11    CMG Momentum Integral Y (ft-lb-sec^2)                   /  4000.0
 12    CMG Momentum Integral Z (ft-lb-sec^2)                   /  4000.0
 13    Wo  Oscillation Filter Output (a1)                      /  2.0
 14    Wo  Oscillation Filter Output (a2)                      /  2.0
 15    2Wo Oscillation Filter Output (b1)                      /  2.0
 16    2Wo Oscillation Filter Output (b2)                      /  2.0
 17    Contrl Criter. Roll CMG Control Torque (ft-lb)          /  0.20
 18    Contrl Criter. Ptch CMG Control Torque (ft-lb)          /  0.20
 19    Contrl Criter. Yaw  CMG Control Torque (ft-lb)          /  0.20

Measurement Outputs (y)=  16
  1    Measurm: Roll Attitude  (phi-LVLH)  (radians)           /  1.0000
  2    Measurm: Pitch Attitude (thet-LVLH) (radians)           /  1.0000
  3    Measurm: Yaw  Attitude  (psi-LVLH)  (radians)           /  1.0000
  4    Measurm: Roll Rate      (p-lvlh)    (rad/sec)           /  1.0000
  5    Measurm: Pitch Rate     (q-lvlh)    (rad/sec)           /  1.0000
  6    Measurm: Yaw  Rate      (r-lvlh)    (rad/sec)           /  1.0000
  7    Measurm: CMG Momentum in X-axis (ft-lb-sec)             /  1.0000
  8    Measurm: CMG Momentum in Y-axis (ft-lb-sec)             /  1.0000
  9    Measurm: CMG Momentum in Z-axis (ft-lb-sec)             /  1.0000
 10    Measurm: CMG Momentum Integral X (ft-lb-sec^2)          /  1.0000
 11    Measurm: CMG Momentum Integral Y (ft-lb-sec^2)          /  1.0000
 12    Measurm: CMG Momentum Integral Z (ft-lb-sec^2)          /  1.0000
 13    Measurm: Wo  Oscillation Filter Output (a1)             /  1.0000
 14    Measurm: Wo  Oscillation Filter Output (a2)             /  1.0000
 15    Measurm: 2Wo Oscillation Filter Output (b1)             /  1.0000
 16    Measurm: 2Wo Oscillation Filter Output (b2)             /  1.0000
--------------------------------------------------------------------------------------------------
```

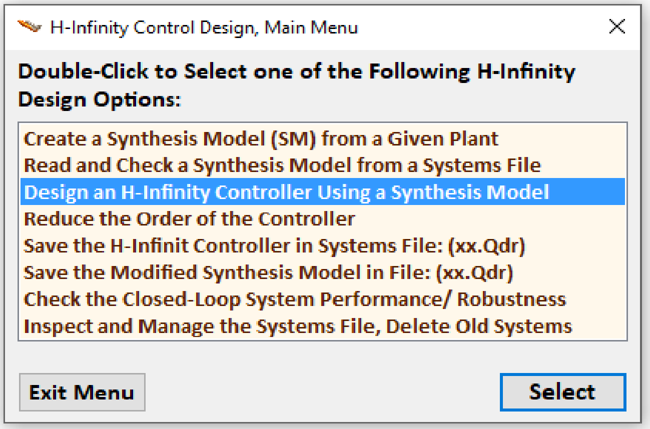
The scaling gains for the excitation inputs and the performance criteria are shown on the right side of the corresponding variables. They are design parameters which are defined by the designer and they trade control bandwidth versus performance and stability margins. The measurements noise in this example is set to zero because we are designing state-feedback controller without an estimator.

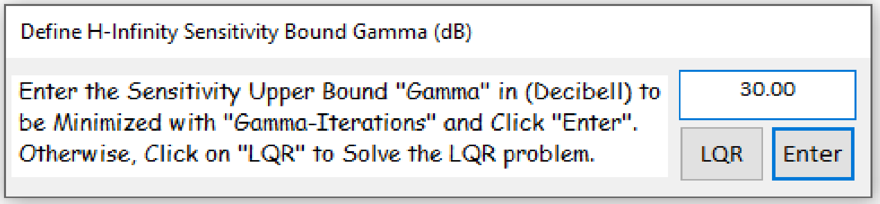
```
Gain Matrix for ...
Hinf State-Feedback Gain Matrix
Matrix Kpqr                 Size =  3 X 16
           1-Column           2-Column           3-Column           4-Column           5-Column           6-Column           7-Column           8-Column
   1-Row -0.930068467390E+04  0.340370624773E+03  0.148581418299E+05 -0.119512109618E+08  0.847459944063E+06  0.674659038235E+07 -0.830024957058E-01  0.100112711187E-01
   2-Row -0.159410689707E+04 -0.174162346994E+03 -0.170024865765E+04  0.104237381798E+07  0.683912447813E+06 -0.154059181119E+07  0.956380582491E-02  0.284352105101E-01
   3-Row -0.104719378973E+05  0.510580617056E+03 -0.186731829961E+05 -0.148445094925E+07  0.403689216103E+05 -0.648586322584E+07 -0.132514801730E-01 -0.818547755754E-03
           9-Column          10-Column          11-Column          12-Column          13-Column          14-Column          15-Column          16-Column
   1-Row  0.599927866185E-01  0.121571638029E-04  0.781257991037E-05  0.478661058647E-04  0.625943469320E+00 -0.192812453907E-01 -0.361856355489E+01  0.760354326309E-03
   2-Row -0.136704929611E-01 -0.155398899169E-05  0.493844325117E-04 -0.766565519823E-05  0.433831195019E+02 -0.854301977880E-01 -0.102234472349E+02 -0.961461709285E-01
   3-Row -0.394443698347E-01 -0.484745534279E-04  0.376097922514E-06  0.122503099602E-04  0.123334116765E+01 -0.219114533656E-02  0.653674227094E-01 -0.405444396055E-02
------------------------------------------------------------------------------------------------------------------------
Definitions of Matrix Inputs (Columns):   16
Roll Attitude  (phi-LVLH)  (radians)
Pitch Attitude (thet-LVLH) (radians)
Yaw  Attitude  (psi-LVLH)  (radians)
Roll Rate      (p-lvlh)    (rad/sec)
Pitch Rate     (q-lvlh)    (rad/sec)
Yaw  Rate      (r-lvlh)    (rad/sec)
CMG Momentum in X-axis (ft-lb-sec)
CMG Momentum in Y-axis (ft-lb-sec)
CMG Momentum in Z-axis (ft-lb-sec)
CMG Momentum Integral X (ft-lb-sec^2)
CMG Momentum Integral Y (ft-lb-sec^2)
CMG Momentum Integral Z (ft-lb-sec^2)
Wo  Oscillation Filter Output (a1)
Wo  Oscillation Filter Output (a2)
2Wo Oscillation Filter Output (b1)
2Wo Oscillation Filter Output (b2)

Definitions of Matrix Outputs (Rows):      3
Control: Roll CMG Control Torque (ft-lb)
Control: Ptch CMG Control Torque (ft-lb)
Control: Yaw  CMG Control Torque (ft-lb)
------------------------------------------------------------------------------------------------------------------------
```

1.6 Interactive Control design

The H-infinity design can also be processed interactively where we can see the Synthesis Model in graphic form and the closed-loop system poles, see Figure 1.2.

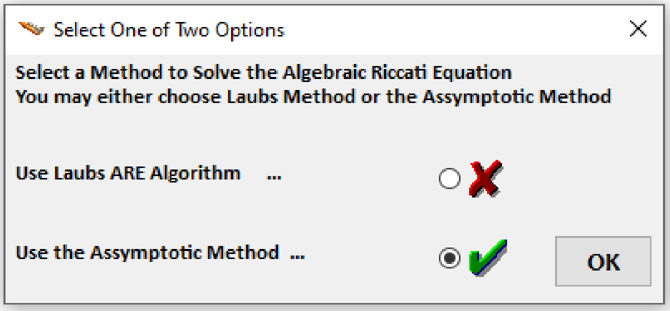





Define H-Infinity Sensitivity Bound Gamma (dB)

Enter the Sensitivity Upper Bound "Gamma" in (Decibell) to be Minimized with "Gamma-Iterations" and Click "Enter". Otherwise, Click on "LQR" to Solve the LQR problem.

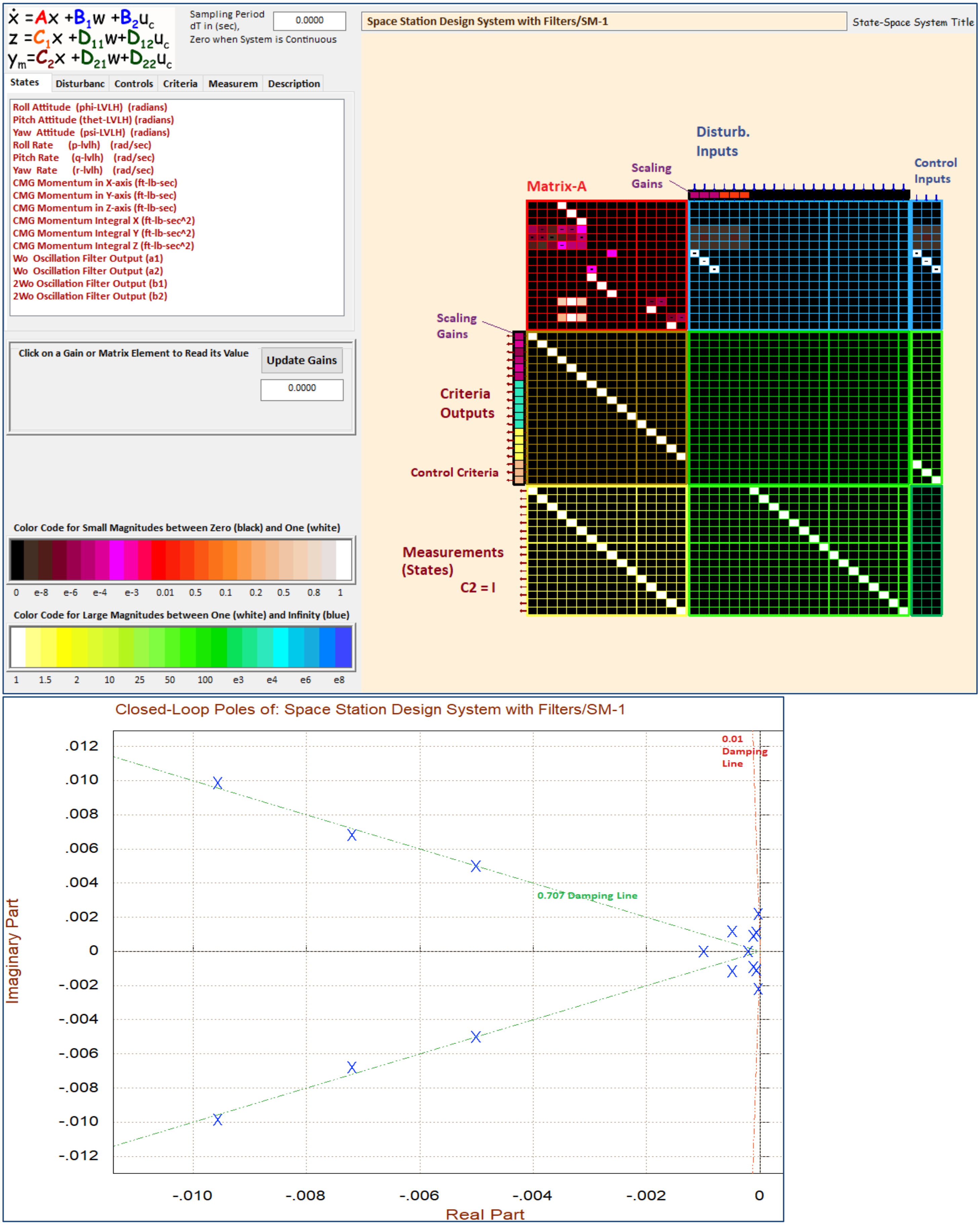
30.00


Select One of Two Options

Select a Method to Solve the Algebraic Riccati Equation
You may either choose Laubs Method or the Assymptotic Method

Use Laubs ARE Algorithm ... ○ ✗

Use the Assymptotic Method ... ● ✓



$$\dot{x} = Ax + B_1 w + B_2 u_c$$
$$z = C_1 x + D_{11} w + D_{12} u_c$$
$$y_m = C_2 x + D_{21} w + D_{22} u_c$$

Sampling Period dT in (sec), Zero when System is Continuous: 0.0000

State-Space System Title: Space Station Design System with Filters/SM-1

States:
- Roll Attitude (phi-LVLH) (radians)
- Pitch Attitude (thet-LVLH) (radians)
- Yaw Attitude (psi-LVLH) (radians)
- Roll Rate (p-lvlh) (rad/sec)
- Pitch Rate (q-lvlh) (rad/sec)
- Yaw Rate (r-lvlh) (rad/sec)
- CMG Momentum in X-axis (ft-lb-sec)
- CMG Momentum in Y-axis (ft-lb-sec)
- CMG Momentum in Z-axis (ft-lb-sec)
- CMG Momentum Integral X (ft-lb-sec^2)
- CMG Momentum Integral Y (ft-lb-sec^2)
- CMG Momentum Integral Z (ft-lb-sec^2)
- Wo Oscillation Filter Output (a1)
- Wo Oscillation Filter Output (a2)
- 2Wo Oscillation Filter Output (b1)
- 2Wo Oscillation Filter Output (b2)

Click on a Gain or Matrix Element to Read its Value: 0.0000

Figure 1.2 Synthesis Model with Scaling Gains in Graphic Systems Form and Closed-Loop Poles

1.7 Simulation Models

Figure 1.3 shows a simple simulation model "*Simple_Sim.slx*" that includes the augmented system "*Augmented Design Plant with Filters and CMG Integral*" from file "*augm_plant.m*". It is used during the design process to test the control gains and optimize the system performance while adjusting the H-infinity scaling parameters. The file "init.m" is used to load the vehicle and control systems into Matlab and to initialize the simulation models. The augmented 16-state vector is fed back via the (3x16) state-feedback gain K_{pqr} to produce the CMG control torque T_C that controls the spacecraft in roll, pitch and yaw.

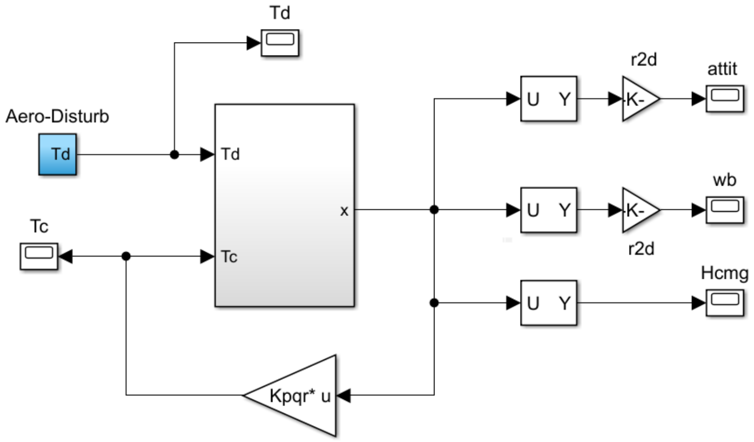
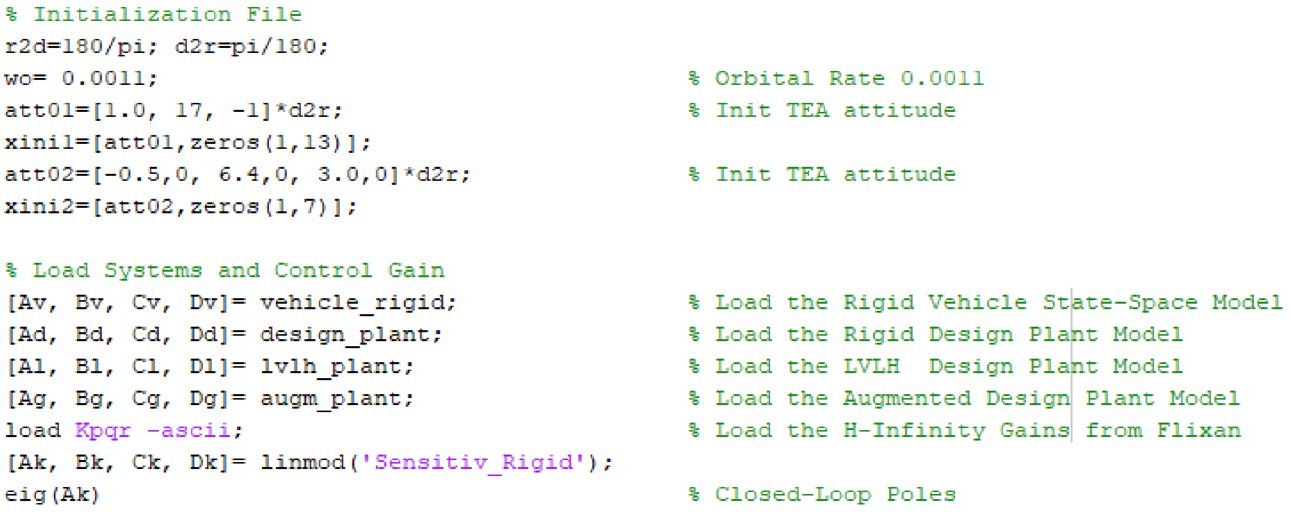


Figure 1.3 Simple Simulation Model "Simple_Sim.slx" Used for Preliminary Control Evaluation



```
% Initialization File
r2d=180/pi; d2r=pi/180;
wo= 0.0011;                                     % Orbital Rate 0.0011
att01=[1.0, 17, -1]*d2r;                        % Init TEA attitude
xini1=[att01,zeros(1,13)];

att02=[-0.5,0, 6.4,0, 3.0,0]*d2r;               % Init TEA attitude
xini2=[att02,zeros(1,7)];

% Load Systems and Control Gain
[Av, Bv, Cv, Dv]= vehicle_rigid;                % Load the Rigid Vehicle State-Space Model
[Ad, Bd, Cd, Dd]= design_plant;                 % Load the Rigid Design Plant Model
[Al, Bl, Cl, Dl]= lvlh_plant;                   % Load the LVLH  Design Plant Model
[Ag, Bg, Cg, Dg]= augm_plant;                   % Load the Augmented Design Plant Model
load Kpqr -ascii;                               % Load the H-Infinity Gains from Flixan
[Ak, Bk, Ck, Dk]= linmod('Sensitiv_Rigid');
eig(Ak)                                         % Closed-Loop Poles
```

A better closed-loop simulation model is in file "*Sim_TEA_Rigid.slx*" shown in Figure 1.4, which includes the vehicle system "*Space Station with Double-Gimbal CMG Array (Rigid)*" in file "*vehicle_rigid.m*" and it is used to analyze the system's response to cyclic disturbances T_d. The momentum integrators and the two disturbance augmentation resonances, that were included in the design model, are now becoming filters and they are part of the attitude control system. The green spacecraft subsystem is shown in detail in Figure 1.5 which includes also the CMG dynamics implemented as 2nd order transfer-functions. The output attitude and rates are in the LVLH frame. The bias torque due to the cross-products of inertia is also included as external torque of 7.4015 (ft-lb) along the direction: (0.19697, -0.59637, -0.77816) which is specified in the vehicle input dataset.

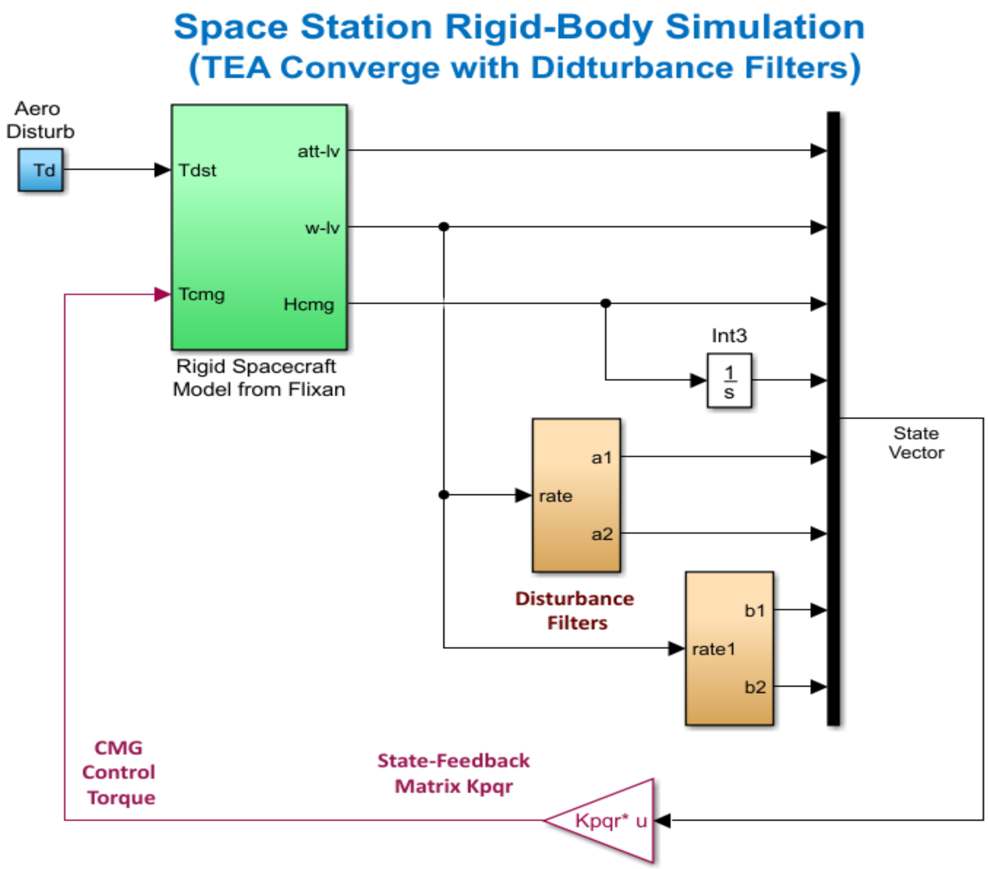
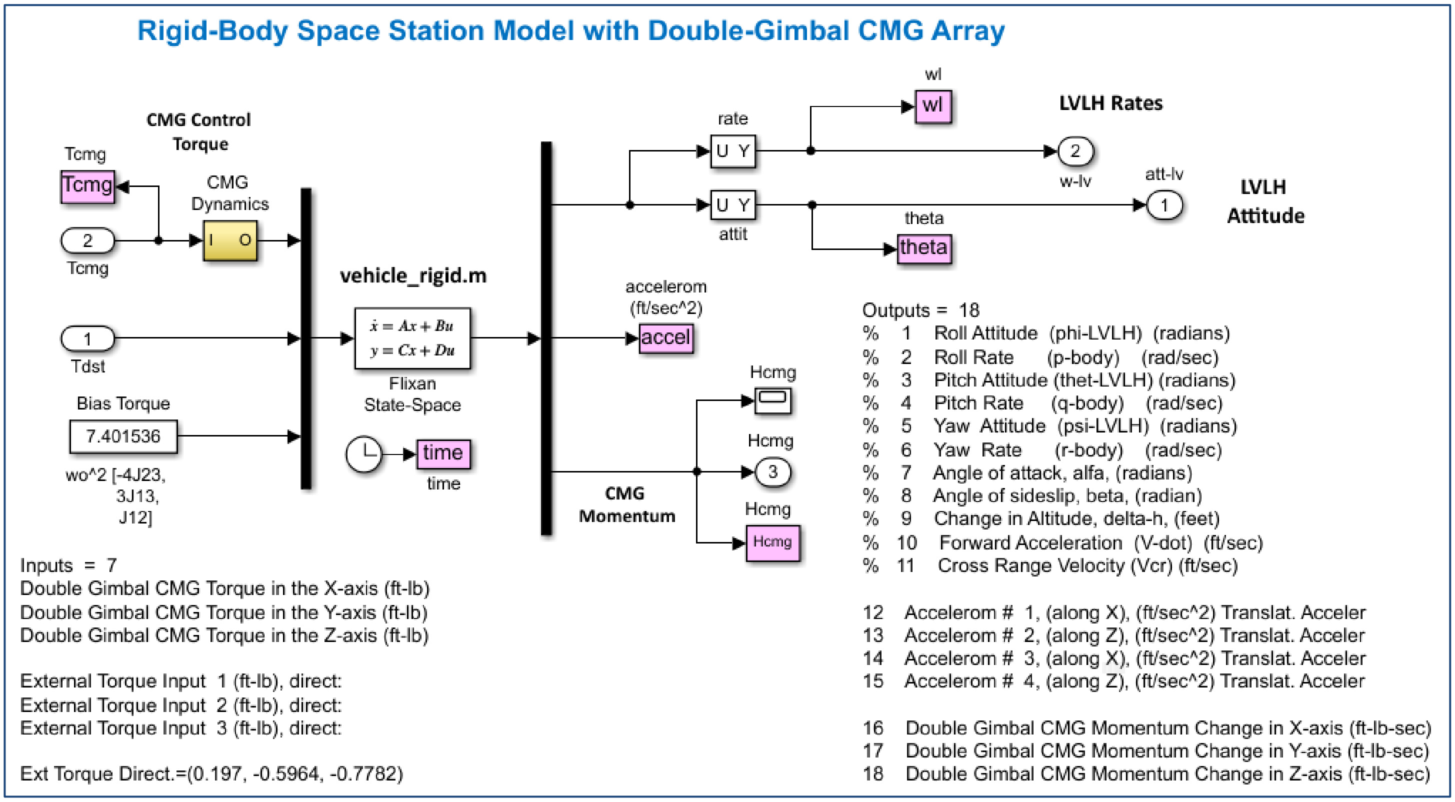


Figure 1.4 Closed-Loop Simulation Model "Sim_TEA_Rigid.slx"



Inputs = 7
Double Gimbal CMG Torque in the X-axis (ft-lb)
Double Gimbal CMG Torque in the Y-axis (ft-lb)
Double Gimbal CMG Torque in the Z-axis (ft-lb)

External Torque Input 1 (ft-lb), direct:
External Torque Input 2 (ft-lb), direct:
External Torque Input 3 (ft-lb), direct:

Ext Torque Direct.=(0.197, -0.5964, -0.7782)

Outputs = 18
% 1 Roll Attitude (phi-LVLH) (radians)
% 2 Roll Rate (p-body) (rad/sec)
% 3 Pitch Attitude (thet-LVLH) (radians)
% 4 Pitch Rate (q-body) (rad/sec)
% 5 Yaw Attitude (psi-LVLH) (radians)
% 6 Yaw Rate (r-body) (rad/sec)
% 7 Angle of attack, alfa, (radians)
% 8 Angle of sideslip, beta, (radian)
% 9 Change in Altitude, delta-h, (feet)
% 10 Forward Acceleration (V-dot) (ft/sec)
% 11 Cross Range Velocity (Vcr) (ft/sec)

12 Accelerom # 1, (along X), (ft/sec^2) Translat. Acceler
13 Accelerom # 2, (along Z), (ft/sec^2) Translat. Acceler
14 Accelerom # 3, (along X), (ft/sec^2) Translat. Acceler
15 Accelerom # 4, (along Z), (ft/sec^2) Translat. Acceler

16 Double Gimbal CMG Momentum Change in X-axis (ft-lb-sec)
17 Double Gimbal CMG Momentum Change in Y-axis (ft-lb-sec)
18 Double Gimbal CMG Momentum Change in Z-axis (ft-lb-sec)

Figure 1.5 Vehicle Block uses the Flixan Generated Rigid Vehicle System "Space Station with Double-Gimbal CMG Array (Rigid)"

1.8 Attitude Control System Stability

The Stability of the system is calculated using the open-loop model "*OpenLoop_TEA_Rigid.slx*", shown in Figure 1.6. It has the control loop broken at the CMG control torque input. The frequency response for each axis is calculated separately by opening one loop at a time and closing the other two. In the configuration shown below, the Simulink model is set up for analyzing roll stability. The Matlab file "freq.m" calculates the frequency responses separately for each axis. The Nichols plots for the roll, pitch and yaw axes are shown in figure 1.7. All axes are stable with plenty of margins.

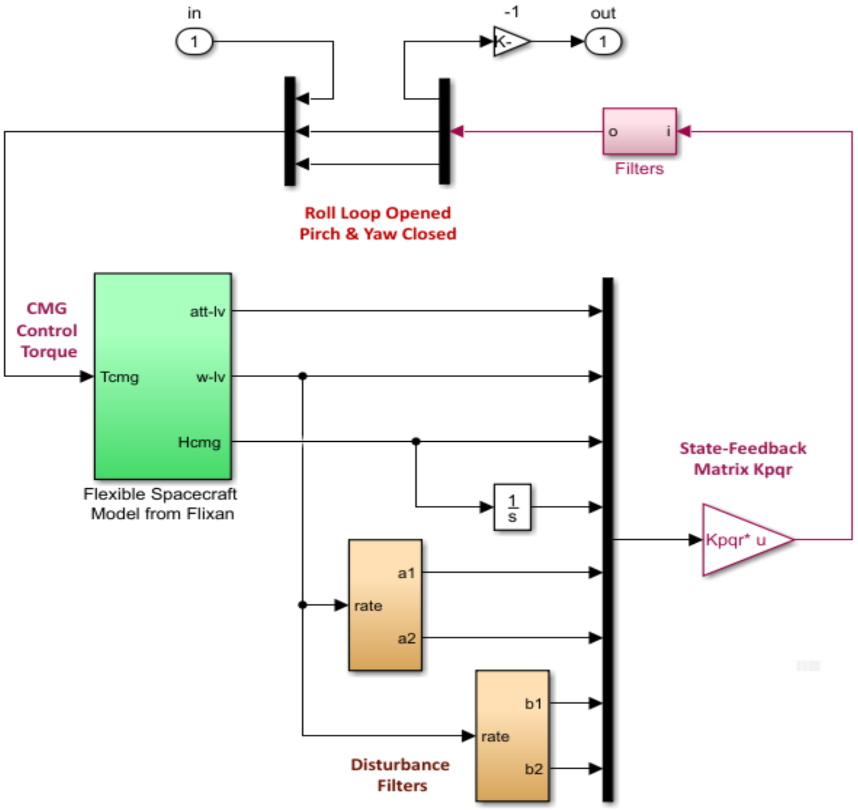
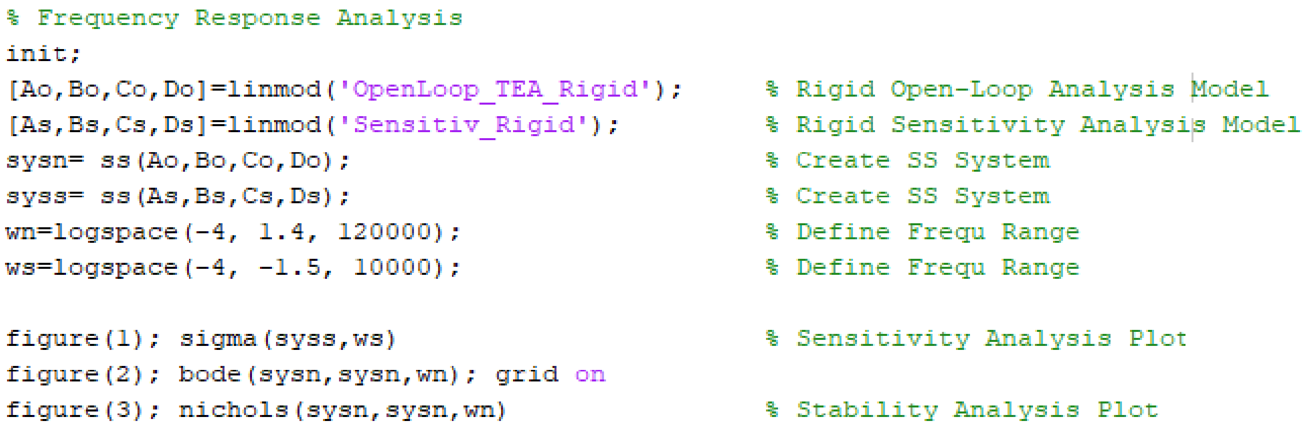


Figure 1.6 Open-Loop Analysis Model "OpenLoop_TEA_Rigid.slx"



```
% Frequency Response Analysis
init;
[Ao,Bo,Co,Do]=linmod('OpenLoop_TEA_Rigid');      % Rigid Open-Loop Analysis Model
[As,Bs,Cs,Ds]=linmod('Sensitiv_Rigid');          % Rigid Sensitivity Analysis Model
sysn= ss(Ao,Bo,Co,Do);                           % Create SS System
syss= ss(As,Bs,Cs,Ds);                           % Create SS System
wn=logspace(-4, 1.4, 120000);                    % Define Frequ Range
ws=logspace(-4, -1.5, 10000);                    % Define Frequ Range

figure(1); sigma(syss,ws)                        % Sensitivity Analysis Plot
figure(2); bode(sysn,sysn,wn); grid on
figure(3); nichols(sysn,sysn,wn)                 % Stability Analysis Plot
```

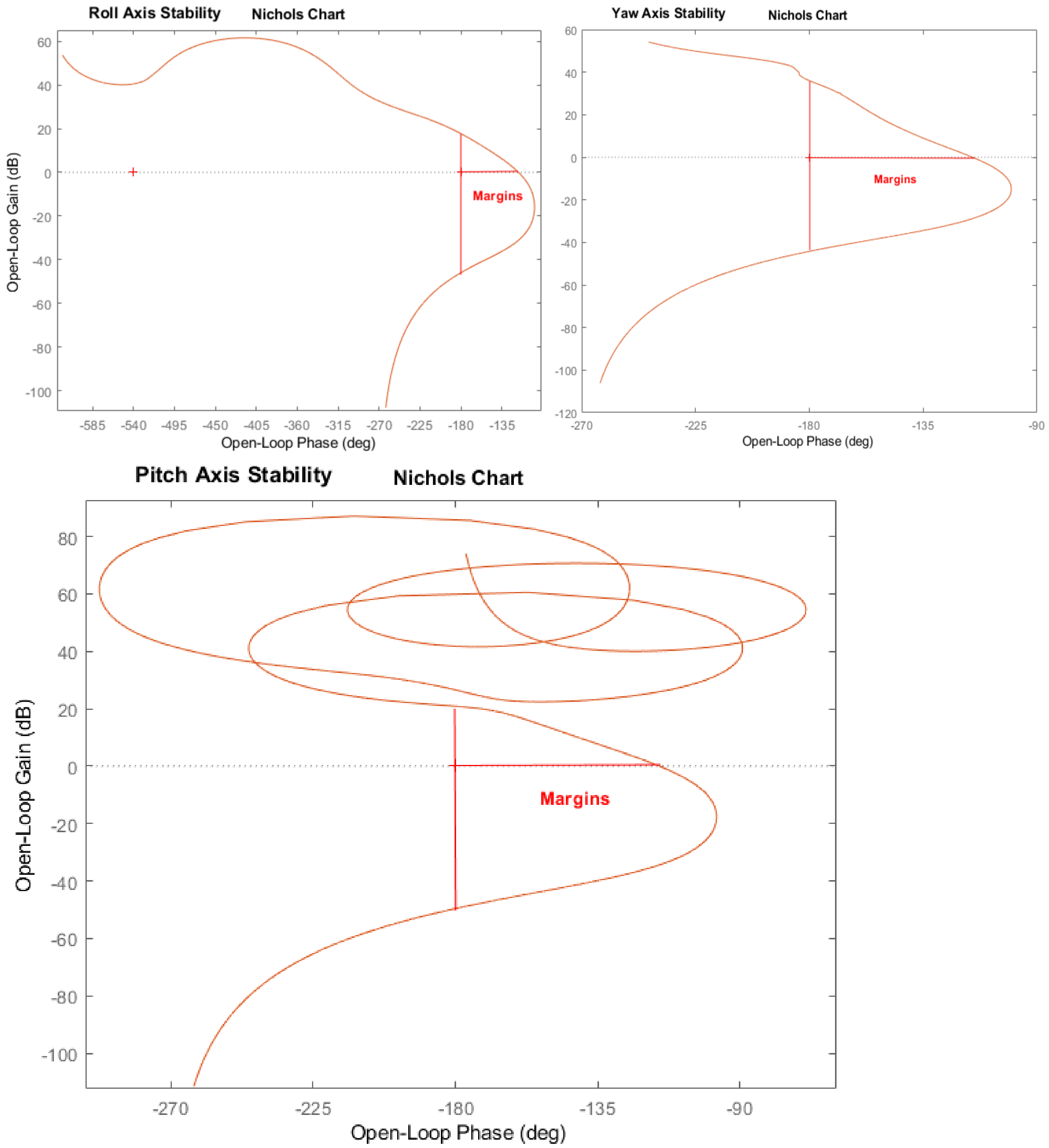


Figure 1.7 Nichols Charts Show Plenty of Stability Margins in All 3 Axes

The filters introduce low-damped modes at the two disturbance frequencies ω_o and $2\omega_o$ which eventually produce the expected disturbance attenuation at those frequencies when the control loops are closed, as we shall see in the sensitivity analysis.

1.9 Sensitivity Analysis to Disturbances

Sensitivity analysis of the CMG control system to disturbances is performed to analyze the spacecraft response to external excitations. The Space Station must provide a "disturbance free" environment for micro-gravity experiments. Sensitivity analysis in the frequency domain using singular value plots is a good indication of the amount of disturbance isolation achieved between external excitation and sensors. In this case, between the disturbance torque Td and the rate measurements.

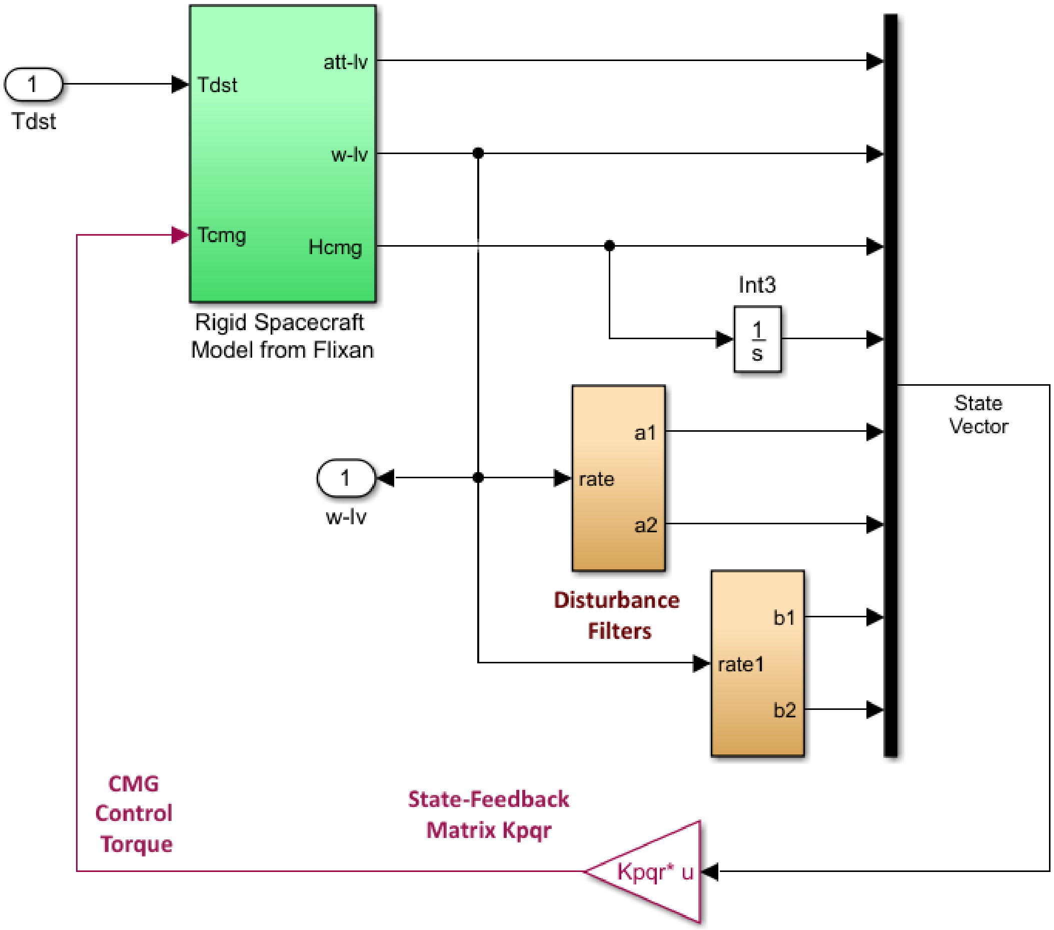


Figure 1.8 Sensitivity Analysis Model "Sensitiv_Rigid.slx"

The closed-loop model "*Sensitiv_Rigid.slx*" in Figure 1.8 is used by the same script "freq.m" to calculate the Singular Values frequency response (sigma) between the disturbances Td and the LVLH rate outputs w_{lv} which is shown in Figure 1.9. Notice the very sharp reduction in the sensitivity response, two deep and narrow notches, occurring at the two disturbance frequencies. It means that the disturbance attenuation achieved by the filters is strong at those frequencies. The 3 curves shown in the sigma-plots are because the response is calculated from a 3-vector input to a 3-vector output and, therefore, there are 3 SV (sigmas) generated at each frequency point. In this case we care more about the biggest value because it represents the maximum amplification between input and output.

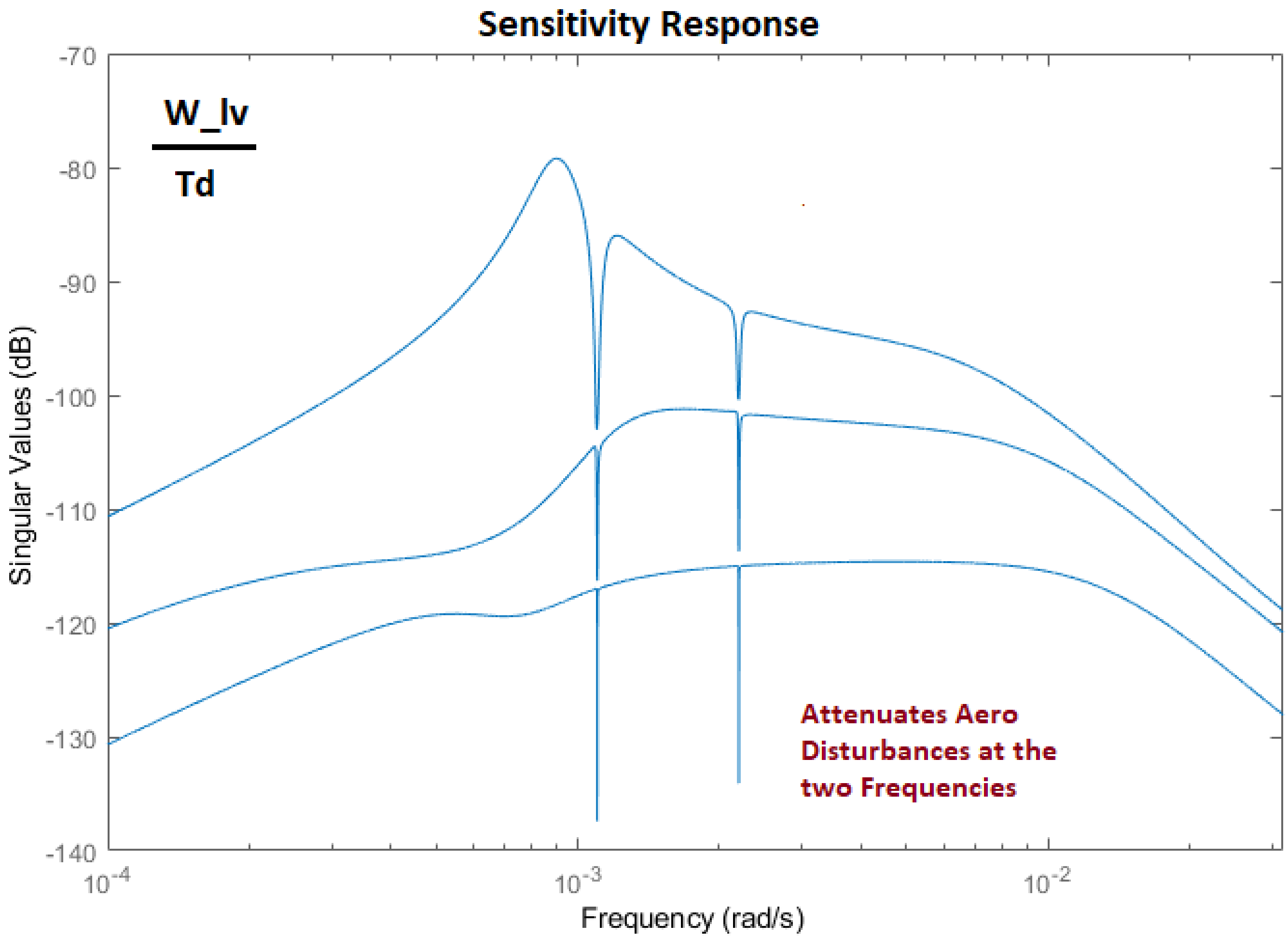
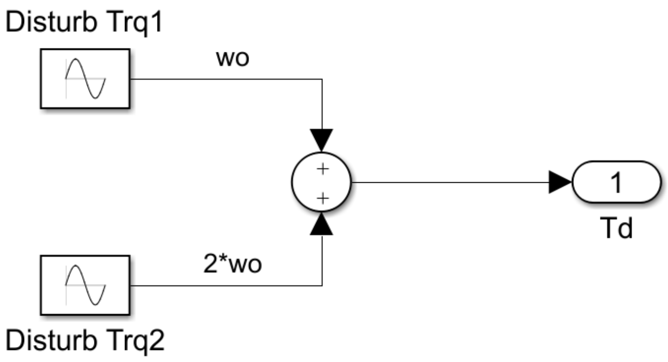


Figure 1.9 Sensitivity of Vehicle Rate to Disturbance Torques

1.10 Simulation

We will now use the simulation model "*Sim_TEA_Rigid.slx*" to calculate the system response to the aerodynamic disturbances Td which is cyclic with two frequency components and a bias torque, as shown in Figure 1.10.



$$T_d = \begin{bmatrix} 1+\sin\omega_o t+0.5\sin 2\omega_o t \\ 8+3\sin\omega_o t+0.5\sin 2\omega_o t \\ 1+\sin\omega_o t+0.5\sin 2\omega_o t \end{bmatrix}$$

Figure 1.10 Aerodynamic Disturbance Torque in Roll, Pitch and Yaw

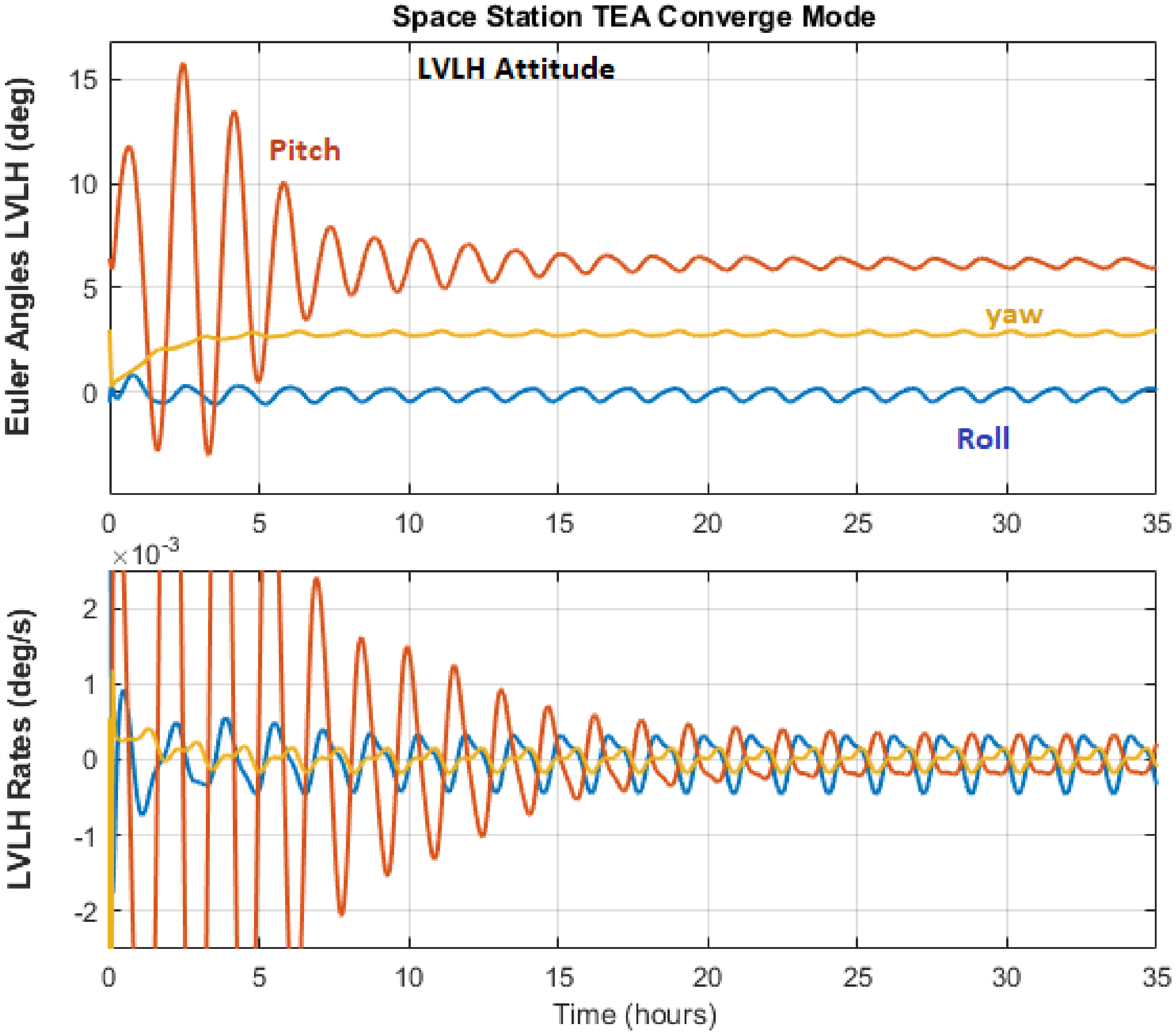


Figure 1.11 LVLH Attitudes and Rates Responding to the Cyclic Disturbance Torque

In Figure 1.11 the attitude and rate responses to the oscillating disturbance are initially very large. Eventually the filters are tuned-in to the disturbance and they produce the oscillatory torques necessary to counteract it. At steady-state the attitude oscillations are very small, less than 0.6°. It takes, however, about a day to tune. In Figure 1.12 the CMG torque at steady-state is oscillating between ±3 (ft-lb). The phasing of the oscillations is automatically adjusted to attenuate the disturbance. The momentum is also oscillating symmetrically about zero and it does not diverge, but it is below 3,000 (ft-lb-sec). The y and z accelerations at 2 vehicle locations are also shown.

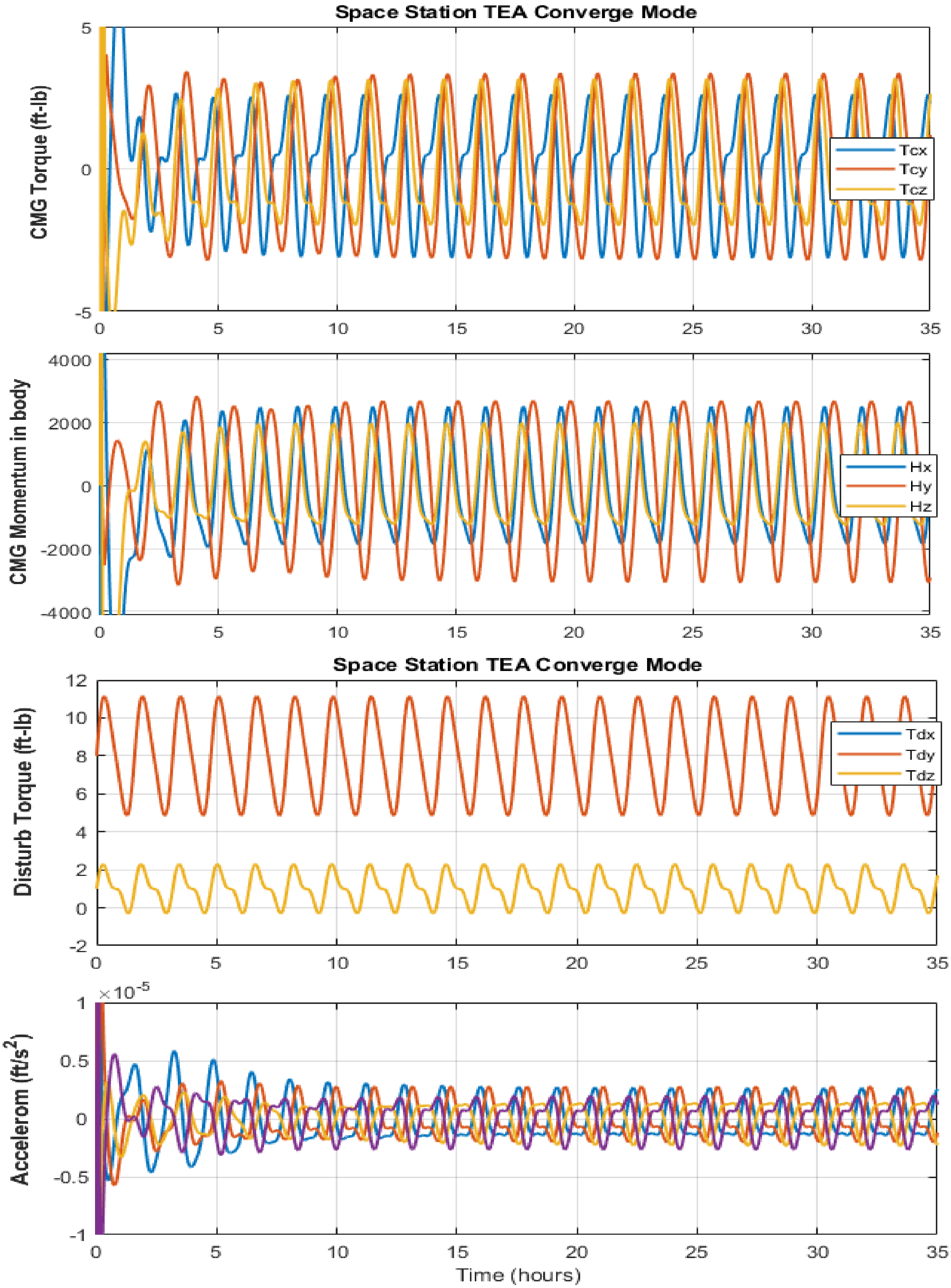


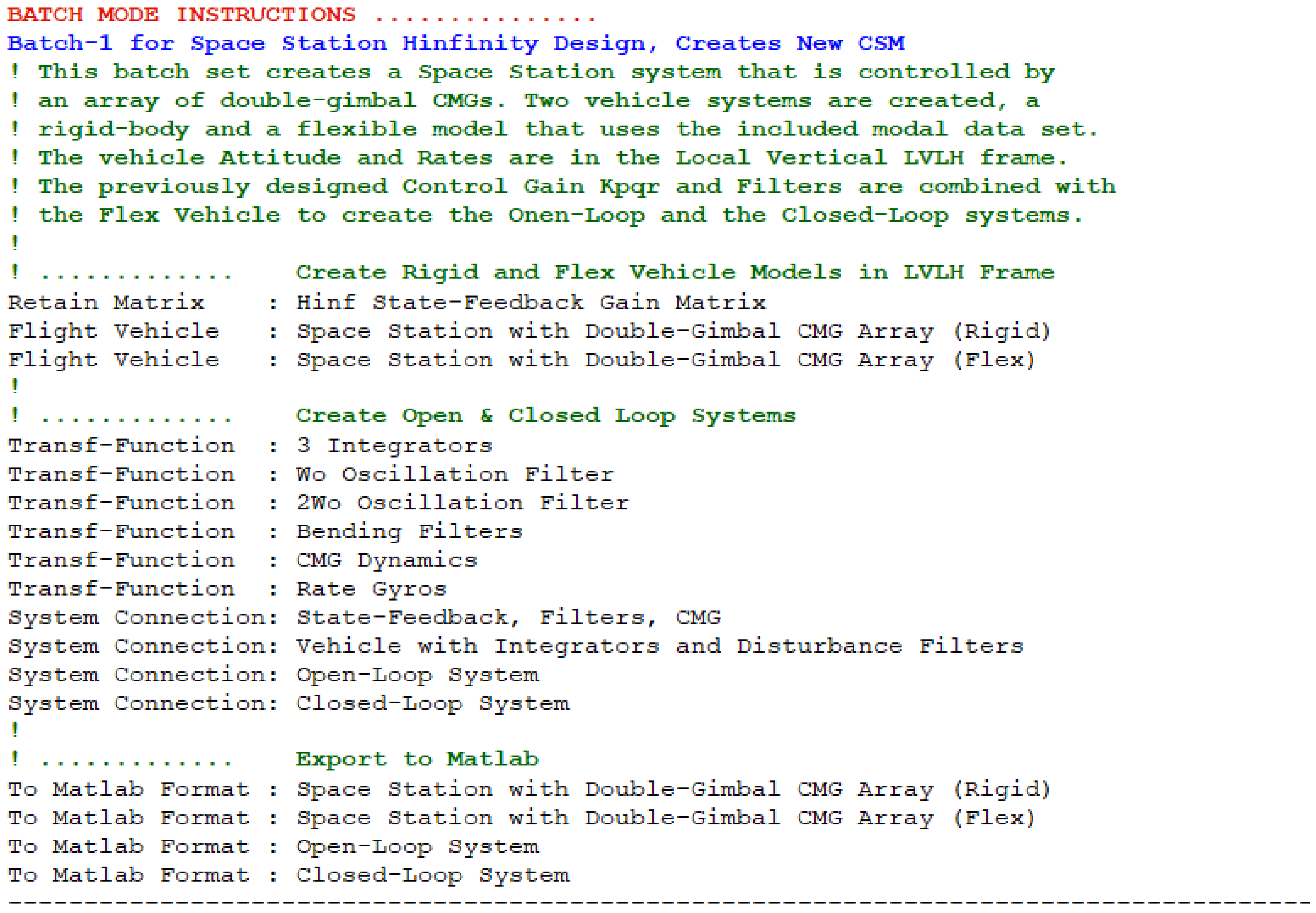
Figure 1.12 CMG Torque and Momentum Responses

2.0 Analysis with Bending Modes

To validate the ACS design, we must also analyze the system with structural modes obtained from a finite elements program. In this section we will include 34 flex modes into the Flixan vehicle model and use the same state-feedback controller designed in Section-1 to analyze open-loop stability and the closed-loop system response to disturbances. Bending filters will also be included in the control system to improve the stability margins. Two additional systems will be created in the Flixan files: an open-loop system for stability analysis and a closed-loop system for simulations.

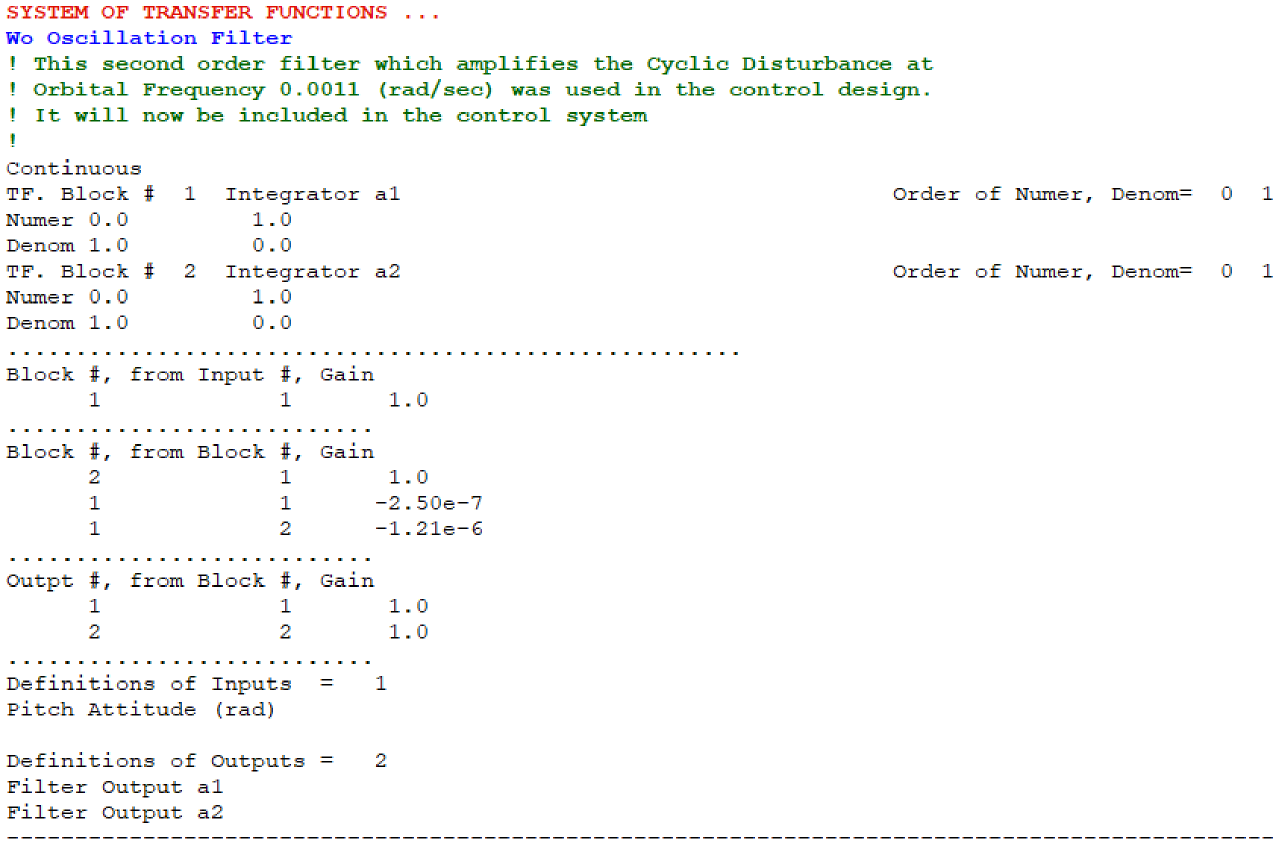
2.1 Input Data File

The analysis files for this section are in folder: "*Flixan\ Control Analysis\Hinfinity\Examples\6-Space Station Attitude & Momentum Control Design\2-Linear Analysis with Flexibility*". The Flixan input data file is "*Space_Station_2.Inp*". It includes datasets containing the vehicle dynamics, CMG and sensor dynamics, flex modes, filters, and also creates the open-loop and closed-loop systems. It begins with a batch dataset that processes the datasets in batch mode and saves them in in file "*Space_Station_2.Qdr*". They are also converted into Matlab format for analysis. There are two vehicle datasets: a rigid and a flex with 34 bending modes. The controller matrix Kpqr from Section-1 is retained in the systems file.

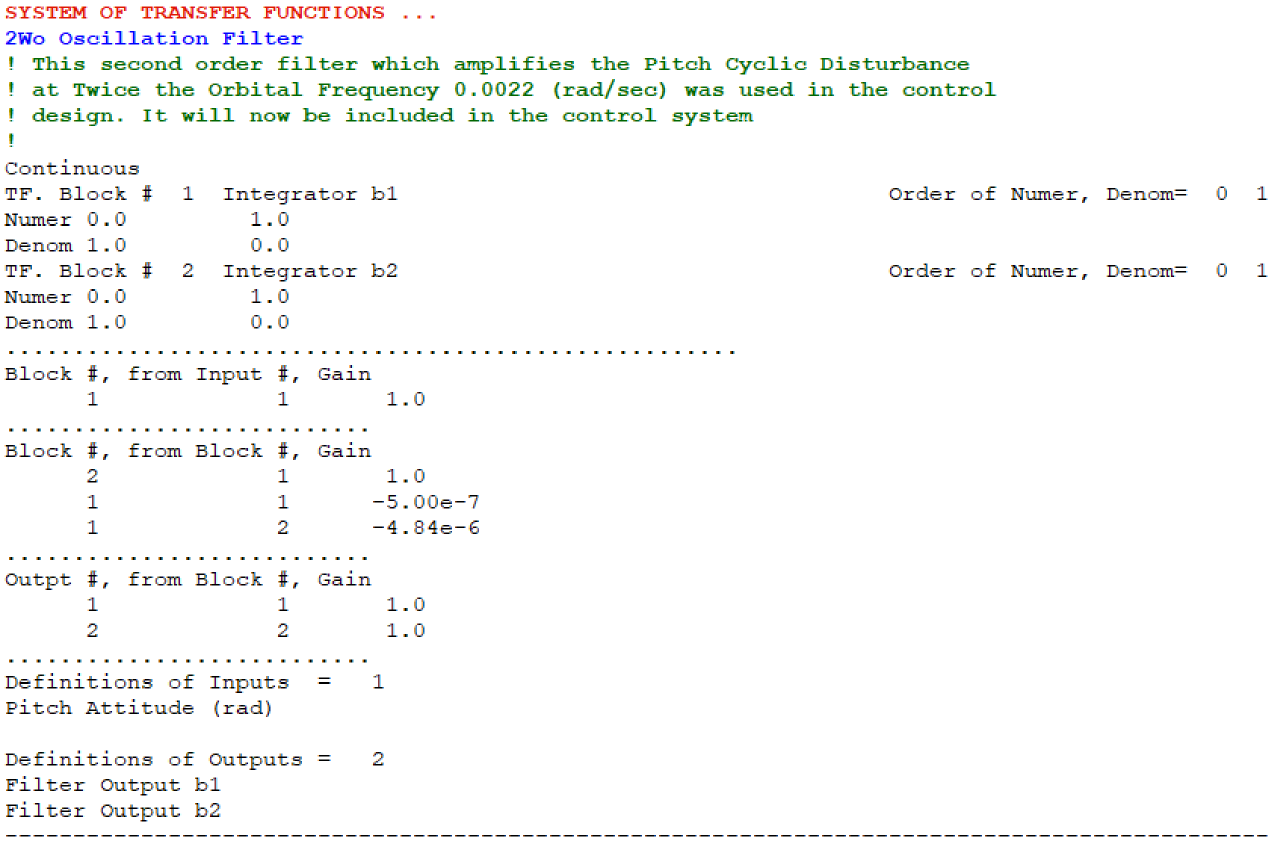


```
BATCH MODE INSTRUCTIONS ...............
Batch-1 for Space Station Hinfinity Design, Creates New CSM
! This batch set creates a Space Station system that is controlled by
! an array of double-gimbal CMGs. Two vehicle systems are created, a
! rigid-body and a flexible model that uses the included modal data set.
! The vehicle Attitude and Rates are in the Local Vertical LVLH frame.
! The previously designed Control Gain Kpqr and Filters are combined with
! the Flex Vehicle to create the Onen-Loop and the Closed-Loop systems.
!
! ............    Create Rigid and Flex Vehicle Models in LVLH Frame
Retain Matrix     : Hinf State-Feedback Gain Matrix
Flight Vehicle    : Space Station with Double-Gimbal CMG Array (Rigid)
Flight Vehicle    : Space Station with Double-Gimbal CMG Array (Flex)
!
! ............    Create Open & Closed Loop Systems
Transf-Function   : 3 Integrators
Transf-Function   : Wo Oscillation Filter
Transf-Function   : 2Wo Oscillation Filter
Transf-Function   : Bending Filters
Transf-Function   : CMG Dynamics
Transf-Function   : Rate Gyros
System Connection: State-Feedback, Filters, CMG
System Connection: Vehicle with Integrators and Disturbance Filters
System Connection: Open-Loop System
System Connection: Closed-Loop System
!
! ............    Export to Matlab
To Matlab Format : Space Station with Double-Gimbal CMG Array (Rigid)
To Matlab Format : Space Station with Double-Gimbal CMG Array (Flex)
To Matlab Format : Open-Loop System
To Matlab Format : Closed-Loop System
---------------------------------------------------------------------------------------
```

```
FLIGHT VEHICLE INPUT DATA ......
Space Station with Double-Gimbal CMG Array (Rigid)
!
! The Space Station state-space system is created using the vehicle modeling program.
! It includes an array of double-gimbal control moment gyros, 3 rate gyros, 3 attitude
! sensors, and 4 accelerometers. The Station is initialized at the Local Vertical Local
! Horizontal (LVLH) attitude and it has a negative pitch rate of -0.063 (rad/sec) which is
! equal to the orbital rate. The vehicle rates are computed relative to the LVLH frame.
! A constant bias torque 7.40153 (ft-lb) is applied in the direction (-0.1969, 0.5963, 0.7781)
! representing the steady-state gyroscopic and gravity-gradient torques due to the constant
! pitch rate
!
Body Axes Output, LVLH Attitude & Rate

Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft)     :   6200.0       32.174       0.20896E+08
Moments and products of Inertias Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)           :   0.119416e+9  0.40408e+8   0.110166e+9, 0.476e+7,    0.1216e+7,   0.30121e+6
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)         :   0.0          0.0          0.0
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)        :   0.0          25500.0      0.0001       700000.0
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)          :   0.0          0.0          0.0          0.0
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                          :   0.0          0.0          0.0          0.0
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)    :   0.0000       0.000        0.0000       0.0000       -0.063       0.0000
W-Gust Azim & Elev angles (deg), or Torque/Force direction (x,y,z), Force Locat (x,y,z)   :   Torque       0.196972    -0.596376    -0.778163
Surface Reference Area (feet^2), Mean Aerodynamic Chord (ft), Wing Span in (feet)         :   0.0          1.0          1.0
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial_rho/ Partial_H} :   0.0          0.0          0.0         -0.0
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:   0.0          0.0          0.0          0.0          0.0          0.0000
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}:   0.0         -0.0          0.0000       0.0000       0.0000       0.0000
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}:   0.0         -0.0          0.0000       0.0000       0.0000       0.0000
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:   0.0         -0.0          0.0000       0.0000       0.0000       0.0000
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}:   0.0         -0.0          0.0000       0.0000       0.0000       0.0000
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}:   0.0          0.0          0.0000       0.0000       0.0000       0.0000

Number of External Torques on the Vehicle                                                 :  3
Torque No  1  Direction (x, y, z)                                                         :  1.0          0.0         0.0
Torque No  2  Direction (x, y, z)                                                         :  0.0          1.0         0.0
Torque No  3  Direction (x, y, z)                                                         :  0.0          0.0         1.0

Double Gimbal Control Moment Gyro System  (3-axes), Initial Momentum (x,y,z) (ft-lb-sec) :  Yes          0.0    0.0    0.0

Number of Gyros, (Attitude and Rate)                                                       :  6
Gyro No  1 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,    Node 2    :  Roll   Rate            0.0         82.0      0.0
Gyro No  2 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,    Node 2    :  Pitch  Rate            0.0         82.0      0.0
Gyro No  3 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,    Node 2    :  Yaw    Rate            0.0         82.0      0.0
Gyro No  1 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,    Node 2    :  Roll   Attitude        0.0         82.0      0.0
Gyro No  2 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,    Node 2    :  Pitch  Attitude        0.0         82.0      0.0
Gyro No  3 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,    Node 2    :  Yaw    Attitude        0.0         82.0      0.0

Number of Accelerometers, Along Axes: (x,y,z)                                              :  4
Acceleromet No  1 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 2    :  X-axis Accelerat.      0.0       232.35    18.0
Acceleromet No  2 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 2    :  Z-axis Accelerat.      0.0       232.35    18.0
Acceleromet No  3 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 4    :  X-axis Accelerat.      0.0      -224.63    16.0
Acceleromet No  4 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 4    :  Z-axis Accelerat.      0.0      -224.63    16.0
-------------------------------------------------------------------------------------------------------------------------------------------------------------------------
FLIGHT VEHICLE INPUT DATA ......
Space Station with Double-Gimbal CMG Array (Flex)
!
! The Flexible Space Station state-space system is created using the vehicle modeling
! program. It includes an array of double-gimbal control moment gyros, 3 rate gyros,
! 3 attitude sensors, and 4 accelerometers. The Station is initialized at the Local Vertical
! Local Horizontal (LVLH) attitude and it has a negative pitch rate of -0.063 (rad/sec) which
! is equal to the orbital rate. The vehicle rates are computed relative to the LVLH frame.
! A constant bias torque 7.40153 (ft-lb) is applied in the direction (-0.1969, 0.5963, 0.7781)
! representing the steady-state gyroscopic and gravity-gradient torques due to the constant
! pitch rate
!
Body Axes Output, LVLH Attitude

Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft)     :   6200.0       32.174       0.20896E+08
Moments and products of Inertias Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)           :   0.119416e+9  0.40408e+8   0.110166e+9, 0.476e+7,    0.1216e+7,   0.30121e+6
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)         :   0.0          0.0          0.0
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)        :   0.0          25500.0      0.0001       700000.0
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)          :   0.0          0.0          0.0          0.0
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                          :   0.0          0.0          0.0          0.0
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)    :   0.0000       0.000        0.0000       0.0000       -0.063       0.0000
W-Gust Azim & Elev angles (deg), or Torque/Force direction (x,y,z), Force Locat (x,y,z)   :   Torque       0.196972    -0.596376    -0.778163
Surface Reference Area (feet^2), Mean Aerodynamic Chord (ft), Wing Span in (feet)         :   0.0          1.0          1.0
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial_rho/ Partial_H} :   0.0          0.0          0.0         -0.0
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:   0.0          0.0          0.0          0.0          0.0          0.0000
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}:   0.0         -0.0          0.0000       0.0000       0.0000       0.0000
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}:   0.0         -0.0          0.0000       0.0000       0.0000       0.0000
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:   0.0         -0.0          0.0000       0.0000       0.0000       0.0000
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}:   0.0         -0.0          0.0000       0.0000       0.0000       0.0000
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}:   0.0          0.0          0.0000       0.0000       0.0000       0.0000

Number of External Torques on the Vehicle                                                 :  3
Torque No  1  Direction (x, y, z)                                                         :  1.0          0.0         0.0
Torque No  2  Direction (x, y, z)                                                         :  0.0          1.0         0.0
Torque No  3  Direction (x, y, z)                                                         :  0.0          0.0         1.0

Double Gimbal Control Moment Gyro System  (3-axes), Initial Momentum (x,y,z) (ft-lb-sec) :  Yes          0.0    0.0    0.0

Number of Gyros, (Attitude and Rate)                                                       :  6
Gyro No  1 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,    Node 2    :  Roll   Rate            0.0         82.0      0.0
Gyro No  2 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,    Node 2    :  Pitch  Rate            0.0         82.0      0.0
Gyro No  3 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,    Node 2    :  Yaw    Rate            0.0         82.0      0.0
Gyro No  1 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,    Node 2    :  Roll   Attitude        0.0         82.0      0.0
Gyro No  2 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,    Node 2    :  Pitch  Attitude        0.0         82.0      0.0
Gyro No  3 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,    Node 2    :  Yaw    Attitude        0.0         82.0      0.0

Number of Accelerometers, Along Axes: (x,y,z)                                              :  4
Acceleromet No  1 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 2    :  X-axis Accelerat.      0.0       232.35    18.0
Acceleromet No  2 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 2    :  Z-axis Accelerat.      0.0       232.35    18.0
Acceleromet No  3 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 4    :  X-axis Accelerat.      0.0      -224.63    16.0
Acceleromet No  4 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 4    :  Z-axis Accelerat.      0.0      -224.63    16.0

Number of Bending Modes                                                                    : 34
Space Station 34 Flex Modes with Double-Gimbal CMG Array
-------------------------------------------------------------------------------------------------------------------------------------------------------------------------
```



```
SYSTEM OF TRANSFER FUNCTIONS ...
Wo Oscillation Filter
! This second order filter which amplifies the Cyclic Disturbance at
! Orbital Frequency 0.0011 (rad/sec) was used in the control design.
! It will now be included in the control system
!
Continuous
TF. Block #  1  Integrator a1                                    Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
TF. Block #  2  Integrator a2                                    Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
......................................................
Block #, from Input #, Gain
      1             1       1.0
...........................
Block #, from Block #, Gain
      2             1       1.0
      1             1      -2.50e-7
      1             2      -1.21e-6
...........................
Outpt #, from Block #, Gain
      1             1       1.0
      2             2       1.0
...........................
Definitions of Inputs  =   1
Pitch Attitude (rad)

Definitions of Outputs =   2
Filter Output a1
Filter Output a2
---------------------------------------------------------------------------------------------
```



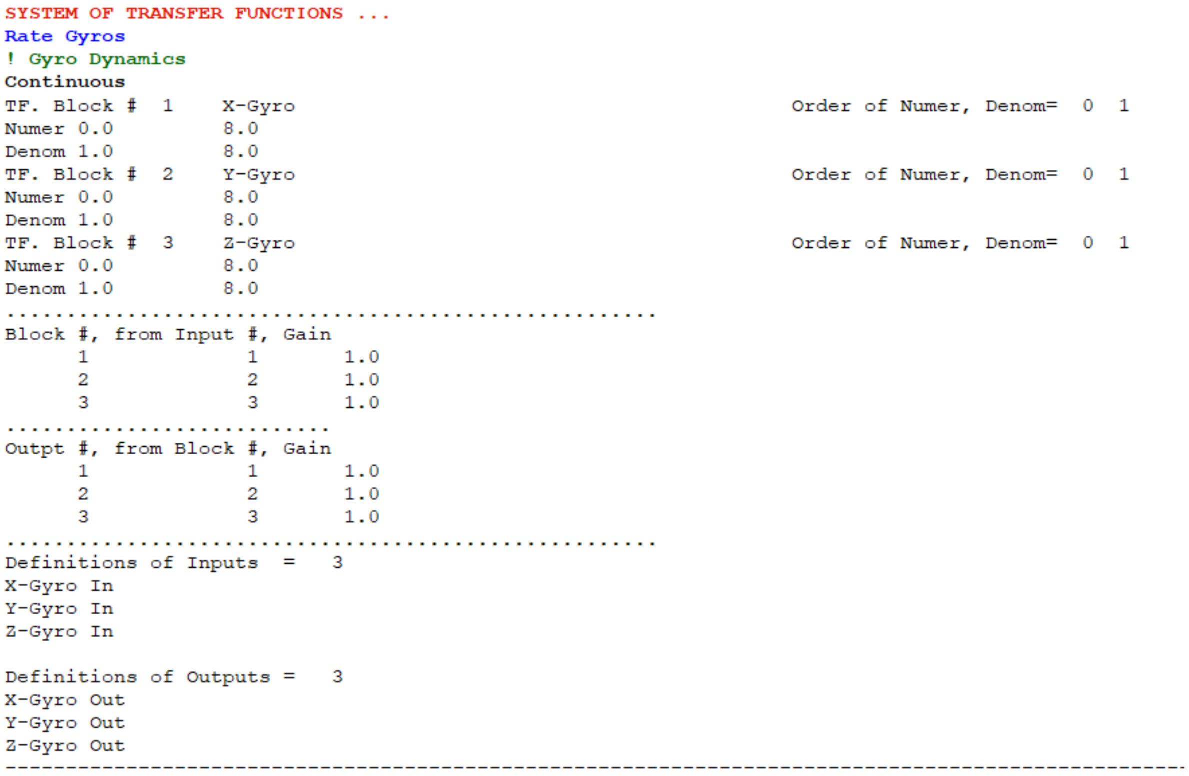
```
SYSTEM OF TRANSFER FUNCTIONS ...
2Wo Oscillation Filter
! This second order filter which amplifies the Pitch Cyclic Disturbance
! at Twice the Orbital Frequency 0.0022 (rad/sec) was used in the control
! design. It will now be included in the control system
!
Continuous
TF. Block #  1  Integrator b1                                    Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
TF. Block #  2  Integrator b2                                    Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
......................................................
Block #, from Input #, Gain
      1             1       1.0
...........................
Block #, from Block #, Gain
      2             1       1.0
      1             1      -5.00e-7
      1             2      -4.84e-6
...........................
Outpt #, from Block #, Gain
      1             1       1.0
      2             2       1.0
...........................
Definitions of Inputs  =   1
Pitch Attitude (rad)

Definitions of Outputs =   2
Filter Output b1
Filter Output b2
---------------------------------------------------------------------------------------------
```

These are the two disturbance filters and the CMG momentum integrators that were described in Section-1. The bending filters are also implemented in transfer-function form.

```
SYSTEM OF TRANSFER FUNCTIONS ...
3 Integrators
! It is used to calculate the CMG Momentum Integral which is needed for
! the Kpqr state-feedback. It prevents the Momentum Bias and the Momentum
! oscillations will cycle about zero average
!
Continuous
TF. Block #  1  Integrator x                                      Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
TF. Block #  2  Integrator y                                      Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
TF. Block #  3  Integrator z                                      Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
.......................................................
Block #, from Input #, Gain
      1             1       1.0
      2             2       1.0
      3             3       1.0
...........................
Outpt #, from Block #, Gain
      1             1       1.0
      2             2       1.0
      3             3       1.0
...........................
Definitions of Inputs  =   3
CMG Momentum-X
CMG Momentum-Y
CMG Momentum-Z

Definitions of Outputs =   3
CMG Momentum-X Integral
CMG Momentum-Y Integral
CMG Momentum-Z Integral
----------------------------------------------------------------------------------------------------
SYSTEM OF TRANSFER FUNCTIONS ...
Bending Filters
! Low-Pass and Notch Filters for attebuating the Bending Modes
!
Continuous
TF. Block #  1    Roll-1                                           Order of Numer, Denom=  2  2
Numer 1.0         0.039       1.69
Denom 1.0         0.39        1.69
TF. Block #  2    Roll-2                                           Order of Numer, Denom=  0  2
Numer 0.0         0.0         0.25
Denom 1.0         0.675       0.25
TF. Block #  3    Pitch                                            Order of Numer, Denom=  0  2
Numer 0.0         0.0         0.25
Denom 1.0         0.675       0.25
TF. Block #  4    Yaw-1                                            Order of Numer, Denom=  2  2
Numer 1.0         0.0348      1.345
Denom 1.0         0.348       1.345
TF. Block #  5    Yaw-2                                            Order of Numer, Denom=  0  2
Numer 0.0         0.0         0.36
Denom 1.0         0.81        0.36
.......................................................
Block #, from Input #, Gain
      1             1         1.0
      3             2         1.0
      4             3         1.0
...........................
Block #, from Block #, Gain
      2             1         1.0
      5             4         1.0
...........................
Outpt #, from Block #, Gain
      1             2         1.0
      2             3         1.0
      3             5         1.0
...........................
Definitions of Inputs  =   3
Roll  Error in
Pitch Error in
Yaw   Error in

Definitions of Outputs =   3
Roll  Error Out
Pitch Error Out
Yaw   Error Out
-----------------------------------------------------------------------------------------------
```

```
SYSTEM OF TRANSFER FUNCTIONS ...
CMG Dynamics
! Low-Pass Filters Representing the CMG Dynamics
!
Continuous
TF. Block #  1    X-CMG                                            Order of Numer, Denom=  0  2
Numer 0.0         0.0         4.0
Denom 1.0         3.2         4.0
TF. Block #  2    Y-CMG                                            Order of Numer, Denom=  0  2
Numer 0.0         0.0         4.0
Denom 1.0         3.2         4.0
TF. Block #  3    Z-CMG                                            Order of Numer, Denom=  0  2
Numer 0.0         0.0         4.0
Denom 1.0         3.2         4.0
......................................................
Block #, from Input #, Gain
      1             1       1.0
      2             2       1.0
      3             3       1.0
...........................
Outpt #, from Block #, Gain
      1             1       1.0
      2             2       1.0
      3             3       1.0
...........................
Definitions of Inputs  =   3
CMG X-Torque In
CMG Y-Torque In
CMG Z-Torque In

Definitions of Outputs =   3
CMG X-Torque Out
CMG Y-Torque Out
CMG Z-Torque Out
----------------------------------------------------------------------------------------------------
```

```
INTERCONNECTION OF SYSTEMS .....
State-Feedback, Filters, CMG
! Combines the CMG dynamics with the Bending Filters
!
Titles of Systems to be Combined
Title 1 Bending Filters
Title 2 CMG Dynamics
SYSTEM INPUTS TO SUBSYSTEM  1                                                                               Filters
Via Matrix +I03                                                                                             All 3 Inputs
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                                                                            CMGs
Via Matrix +I03                                                                                             3-Torques
..............................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2                                                                     Filters to CMG Torques
Subsystem  1, Output  1 to Subsystem  2, Input  1, Gain=  1.0
Subsystem  2, Output  2 to Subsystem  2, Input  2, Gain=  1.0
Subsystem  3, Output  3 to Subsystem  2, Input  3, Gain=  1.0
..............................................................................
Definitions of Inputs  =   3
CMG X-Torque in
CMG Y-Torque in
CMG Z-Torque in

Definitions of Outputs =   3
CMG X-Torque Out
CMG Y-Torque Out
CMG Z-Torque Out
-----------------------------------------------------------------------------------------------------------------------------------
```



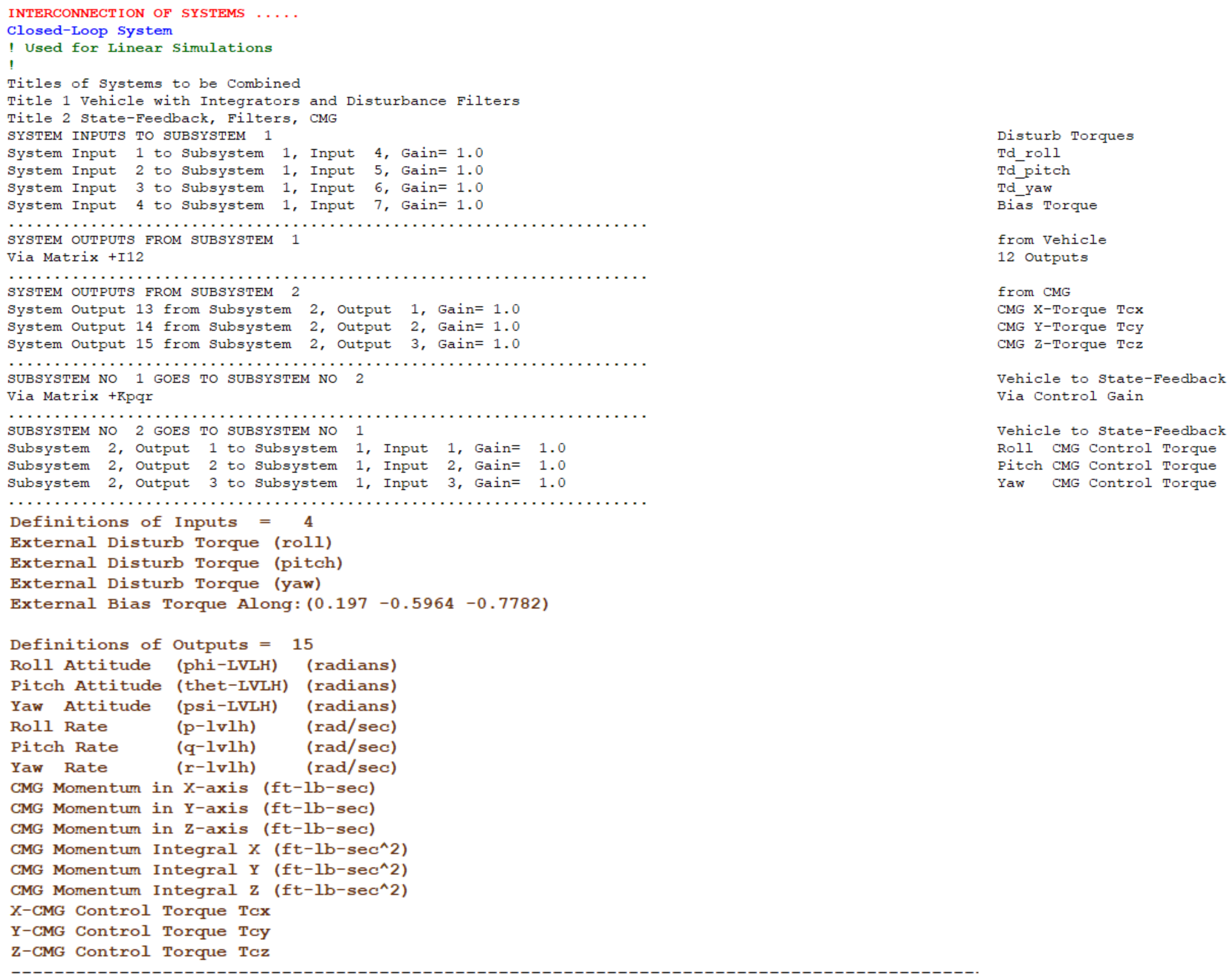
```
SYSTEM OF TRANSFER FUNCTIONS ...
Rate Gyros
! Gyro Dynamics
Continuous
TF. Block #  1    X-Gyro                                           Order of Numer, Denom=   0  1
Numer 0.0         8.0
Denom 1.0         8.0
TF. Block #  2    Y-Gyro                                           Order of Numer, Denom=   0  1
Numer 0.0         8.0
Denom 1.0         8.0
TF. Block #  3    Z-Gyro                                           Order of Numer, Denom=   0  1
Numer 0.0         8.0
Denom 1.0         8.0
......................................................
Block #, from Input #, Gain
      1             1       1.0
      2             2       1.0
      3             3       1.0
...........................
Outpt #, from Block #, Gain
      1             1       1.0
      2             2       1.0
      3             3       1.0
......................................................
Definitions of Inputs  =   3
X-Gyro In
Y-Gyro In
Z-Gyro In

Definitions of Outputs =   3
X-Gyro Out
Y-Gyro Out
Z-Gyro Out
--------------------------------------------------------------------------------------------------
```

The CMG dynamics are implemented as 2nd order transfer-functions of 2 (rad/sec) bandwidth “*CMG Dynamics*”. One transfer-function per axis. The rate-gyro dynamics are also implemented as 1st order filters of 8 (rad/sec) bandwidth, “Rate Gyros”. The bending filters and the CMG dynamics are combined together into this intermediate system “*State-Feedback, Filters, CMG*”.

The flex vehicle “*Space Station with Double-Gimbal CMG Array (Flex)”* is then combined with the disturbance filters, integrators and the rate gyros to create this system: “*Vehicle with Integrators and Disturbance Filters*”.

The two systems: “*Vehicle with Integrators and Disturbance Filters*” and “*State-Feedback, Filters, CMG*” are finally combined together to create two new systems: “*Open-Loop System*” and “*Closed-Loop System*” which are also exported into Matlab for stability analysis and simulations.

```
INTERCONNECTION OF SYSTEMS .....
Vehicle with Integrators and Disturbance Filters
!
Titles of Systems to be Combined
Title 1 Space Station with Double-Gimbal CMG Array (Flex)                                                   Spacecraft
Title 2 3 Integrators                                                                                       Momentum Integral
Title 3 Wo Oscillation Filter                                                                               Wo  Filter
Title 4 2Wo Oscillation Filter                                                                              2Wo Filter
Title 5 Rate Gyros                                                                                          3-Gyros
SYSTEM INPUTS TO SUBSYSTEM  1                                                                               Filters
Via Matrix +I07                                                                                             All Inputs
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                                                            Spacecraft Outputs
System Output  1 from Subsystem  1, Output  1, Gain= 1.0                                                    phi_lvlh
System Output  2 from Subsystem  1, Output  3, Gain= 1.0                                                    thet_lvlh
System Output  3 from Subsystem  1, Output  5, Gain= 1.0                                                    psi_lvlh
System Output  7 from Subsystem  1, Output 22, Gain= 1.0                                                    X-CMG Momentum
System Output  8 from Subsystem  1, Output 23, Gain= 1.0                                                    Y-CMG Momentum
System Output  9 from Subsystem  1, Output 24, Gain= 1.0                                                    Z-CMG Momentum
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  5                                                                            from Rate Gyros
System Output  4 from Subsystem  5, Output  1, Gain= 1.0                                                    p_lvlh rate
System Output  5 from Subsystem  5, Output  2, Gain= 1.0                                                    q_lvlh rate
System Output  6 from Subsystem  5, Output  3, Gain= 1.0                                                    r_lvlh rate
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                                                                            from Integrators
System Output 10 from Subsystem  2, Output  1, Gain= 1.0                                                    CMG-X Momentum Integral
System Output 11 from Subsystem  2, Output  2, Gain= 1.0                                                    CMG-Y Momentum Integral
System Output 12 from Subsystem  2, Output  3, Gain= 1.0                                                    CMG-Z Momentum Integral
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  3                                                                            from Wo Filter
System Output 13 from Subsystem  3, Output  1, Gain= 1.0                                                    a1
System Output 14 from Subsystem  3, Output  2, Gain= 1.0                                                    a2
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  4                                                                            from 2Wo Filter
System Output 15 from Subsystem  4, Output  1, Gain= 1.0                                                    b1
System Output 16 from Subsystem  4, Output  2, Gain= 1.0                                                    b2
..............................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  5                                                                     Vehi to Gyros
Subsystem  1, Output 12 to Subsystem  5, Input  1, Gain=  1.0                                               p_lvlh rate
Subsystem  1, Output 13 to Subsystem  5, Input  2, Gain=  1.0                                               q_lvlh rate
Subsystem  1, Output 14 to Subsystem  5, Input  3, Gain=  1.0                                               r_lvlh rate
Subsystem  1, Output  5 to Subsystem  5, Input  1, Gain=  0.0011                                            p_lvlh + wo*psi
Subsystem  1, Output  1 to Subsystem  5, Input  3, Gain= -0.0011                                            r_lvlh - wo*phi
......................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2                                                                     Momentum to Integrators
Subsystem  1, Output 22 to Subsystem  2, Input  1, Gain=  1.0                                               Roll  CMG Momentum
Subsystem  1, Output 23 to Subsystem  2, Input  2, Gain=  1.0                                               Pitch CMG Momentum
Subsystem  1, Output 24 to Subsystem  2, Input  3, Gain=  1.0                                               Yaw   CMG Momentum
......................................................................
SUBSYSTEM NO  5 GOES TO SUBSYSTEM NO  3                                                                     5 to Wo Filter
Subsystem  5, Output  1 to Subsystem  3, Input  1, Gain=  0.5                                               Roll Rate
Subsystem  5, Output  2 to Subsystem  3, Input  1, Gain=  1.0                                               Pitch Rate
Subsystem  5, Output  3 to Subsystem  3, Input  1, Gain=  0.6                                               Yaw Rate
......................................................................
SUBSYSTEM NO  5 GOES TO SUBSYSTEM NO  4                                                                     5 to 2Wo Filter
Subsystem  5, Output  1 to Subsystem  4, Input  1, Gain=  0.2                                               Roll Rate
Subsystem  5, Output  2 to Subsystem  4, Input  1, Gain=  1.0                                               Pitch Rate
Subsystem  5, Output  3 to Subsystem  4, Input  1, Gain=  0.6                                               Yaw Rate
......................................................................
Definitions of Inputs  =   7
Double Gimbal CMG Torque in the X-axis (ft-lb)
Double Gimbal CMG Torque in the Y-axis (ft-lb)
Double Gimbal CMG Torque in the Z-axis (ft-lb)
External Disturb Torque (roll)
External Disturb Torque (pitch)
External Disturb Torque (yaw)
External Torque Direct.=(0.1970 -0.5964 -0.7782)

Definitions of Outputs =  16
Roll Attitude  (phi-LVLH)  (radians)
Pitch Attitude (thet-LVLH) (radians)
Yaw  Attitude  (psi-LVLH)  (radians)
Roll Rate      (p-lvlh)    (rad/sec)
Pitch Rate     (q-lvlh)    (rad/sec)
Yaw  Rate      (r-lvlh)    (rad/sec)
CMG Momentum in X-axis (ft-lb-sec)
CMG Momentum in Y-axis (ft-lb-sec)
CMG Momentum in Z-axis (ft-lb-sec)
CMG Momentum Integral X (ft-lb-sec^2)
CMG Momentum Integral Y (ft-lb-sec^2)
CMG Momentum Integral Z (ft-lb-sec^2)
Wo  Oscillation Filter Output (a1)
Wo  Oscillation Filter Output (a2)
2Wo Oscillation Filter Output (b1)
2Wo Oscillation Filter Output (b2)
-------------------------------------------------------------------------------------------------------------------------------------
```



```
INTERCONNECTION OF SYSTEMS .....
Closed-Loop System
! Used for Linear Simulations
!
Titles of Systems to be Combined
Title 1 Vehicle with Integrators and Disturbance Filters
Title 2 State-Feedback, Filters, CMG
SYSTEM INPUTS TO SUBSYSTEM  1                                                      Disturb Torques
System Input  1 to Subsystem  1, Input  4, Gain= 1.0                               Td_roll
System Input  2 to Subsystem  1, Input  5, Gain= 1.0                               Td_pitch
System Input  3 to Subsystem  1, Input  6, Gain= 1.0                               Td_yaw
System Input  4 to Subsystem  1, Input  7, Gain= 1.0                               Bias Torque
.......................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                                   from Vehicle
Via Matrix +I12                                                                    12 Outputs
.......................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                                                   from CMG
System Output 13 from Subsystem  2, Output  1, Gain= 1.0                           CMG X-Torque Tcx
System Output 14 from Subsystem  2, Output  2, Gain= 1.0                           CMG Y-Torque Tcy
System Output 15 from Subsystem  2, Output  3, Gain= 1.0                           CMG Z-Torque Tcz
.......................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2                                            Vehicle to State-Feedback
Via Matrix +Kpqr                                                                   Via Control Gain
.......................................................................
SUBSYSTEM NO  2 GOES TO SUBSYSTEM NO  1                                            Vehicle to State-Feedback
Subsystem  2, Output  1 to Subsystem  1, Input  1, Gain=  1.0                      Roll  CMG Control Torque
Subsystem  2, Output  2 to Subsystem  1, Input  2, Gain=  1.0                      Pitch CMG Control Torque
Subsystem  2, Output  3 to Subsystem  1, Input  3, Gain=  1.0                      Yaw   CMG Control Torque
.......................................................................
Definitions of Inputs  =   4
External Disturb Torque (roll)
External Disturb Torque (pitch)
External Disturb Torque (yaw)
External Bias Torque Along:(0.197 -0.5964 -0.7782)

Definitions of Outputs =  15
Roll Attitude  (phi-LVLH)  (radians)
Pitch Attitude (thet-LVLH) (radians)
Yaw  Attitude  (psi-LVLH)  (radians)
Roll Rate      (p-lvlh)    (rad/sec)
Pitch Rate     (q-lvlh)    (rad/sec)
Yaw  Rate      (r-lvlh)    (rad/sec)
CMG Momentum in X-axis (ft-lb-sec)
CMG Momentum in Y-axis (ft-lb-sec)
CMG Momentum in Z-axis (ft-lb-sec)
CMG Momentum Integral X (ft-lb-sec^2)
CMG Momentum Integral Y (ft-lb-sec^2)
CMG Momentum Integral Z (ft-lb-sec^2)
X-CMG Control Torque Tcx
Y-CMG Control Torque Tcy
Z-CMG Control Torque Tcz
-------------------------------------------------------------------------------------------
```

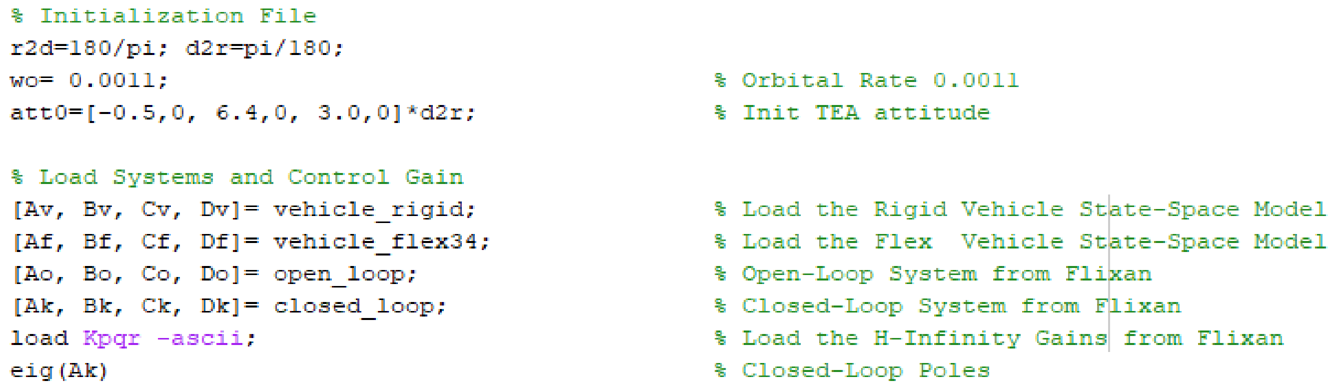
The closed-loop system will be used for simulations. Its inputs are: 3 disturbance torques in roll, pitch and yaw, and one bias torque. The direction of the bias torque is (0.19697, -0.59637, -0.77816) which is defined in the vehicle data file. The bias torque is caused by the cross-products of inertia. It does not affect stability or performance but it affects the torque equilibrium attitude (TEA). If you don’t include it, the steady-state response won’t match the response obtained from the non-linear simulation. The open-loop system has all 3 control loops opened at the CMG torque inputs. It will be used for stability analysis by opening one loop at a time and closing the other two.

```
INTERCONNECTION OF SYSTEMS .....
Open-Loop System
! Used for Stability Analysis
!
Titles of Systems to be Combined
Title 1 Vehicle with Integrators and Disturbance Filters
Title 2 State-Feedback, Filters, CMG
SYSTEM INPUTS TO SUBSYSTEM  1                                                                                Control Torques
System Input  1 to Subsystem  1, Input  1, Gain= 1.0                                                         Tc_roll
System Input  2 to Subsystem  1, Input  2, Gain= 1.0                                                         Tc_pitch
System Input  3 to Subsystem  1, Input  3, Gain= 1.0                                                         Tc_yaw
......................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                                                                             from Filters
Via Matrix +I03                                                                                              All 3
......................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2                                                                      Vehicle to State-Feedback
Via Matrix +Kpqr                                                                                              All 3
......................................................................
Definitions of Inputs  =   3
Control Torque (roll)
Control  Torque (pitch)
Control  Torque (yaw)

Definitions of Outputs =   3
Control Torque (roll)
Control  Torque (pitch)
Control  Torque (yaw)
-----------------------------------------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Space Station with Double-Gimbal CMG Array (Rigid)
System
vehicle_rigid
-----------------------------------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Space Station with Double-Gimbal CMG Array (Flex)
System
vehicle_flex34
-----------------------------------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Open-Loop System
System
open_loop
-----------------------------------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Closed-Loop System
System
closed_loop
----------------------------------------------------------------------------------------------------------------------------------
SELECTED MODAL DATA AND LOCATIONS FOR : 34 Modes
Space Station 34 Flex Modes with Double-Gimbal CMG Array
! 34 modes were selected from all directions. The first 6 Rigid-Body Modes were not included

MODE#  1/  7, Frequency (rad/sec), Damping (zeta), Generalized Mass=    1.0097       0.50000E-02    12.000
DEFINITION OF LOCATIONS (NODES)              phi along X   phi along Y   phi along Z   sigm about X  sigm about Y  sigm about Z

                               Node ID#     Modal Data at the Double Gimbal GMGs Cluster...
Right Keel/Boom Intrsect, CMG      1000     0.11384D-02  -0.58736D-04   0.96470D-04  -0.23552D-06  -0.30827D-04   0.65746D-04

                               Node ID#     Modal Data at the  3 External Torque Points...
Center Habitat Module              9007     0.29012D-02  -0.26348D-04   0.31307D-04   0.30160D-06  -0.43846D-05  -0.97127D-07
Center Habitat Module              9007     0.29012D-02  -0.26348D-04   0.31307D-04   0.30160D-06  -0.43846D-05  -0.97127D-07
Center Habitat Module              9007     0.29012D-02  -0.26348D-04   0.31307D-04   0.30160D-06  -0.43846D-05  -0.97127D-07

                               Node ID#     Modal Data at the  6 Gyros ...
Right Boom (Sensor Assembly)       6001    -0.14682D-03  -0.65626D-04   0.90529D-04  -0.37958D-06  -0.33769D-04   0.86246D-04
Right Boom (Sensor Assembly)       6001    -0.14682D-03  -0.65626D-04   0.90529D-04  -0.37958D-06  -0.33769D-04   0.86246D-04
Right Boom (Sensor Assembly)       6001    -0.14682D-03  -0.65626D-04   0.90529D-04  -0.37958D-06  -0.33769D-04   0.86246D-04
Right Boom (Sensor Assembly)       6001    -0.14682D-03  -0.65626D-04   0.90529D-04  -0.37958D-06  -0.33769D-04   0.86246D-04
Right Boom (Sensor Assembly)       6001    -0.14682D-03  -0.65626D-04   0.90529D-04  -0.37958D-06  -0.33769D-04   0.86246D-04
Right Boom (Sensor Assembly)       6001    -0.14682D-03  -0.65626D-04   0.90529D-04  -0.37958D-06  -0.33769D-04   0.86246D-04

                               Node ID#     Modal Data at the  4 Accelerometers, along (x,y,z)...
Right Solar Array Boom             6070    -0.25921D-01  -0.73995D-04  -0.57584D-04
Right Solar Array Boom             6070    -0.25921D-01  -0.73995D-04  -0.57584D-04
Left SA Boom, Extreme end          5070    -0.26691D-01   0.68332D-05  -0.15103D-03
Left SA Boom, Extreme end          5070    -0.26691D-01   0.68332D-05  -0.15103D-03

                               Node ID#     Modal Data at the Disturbance Point
Module, (Front Dock)               9018     0.28820D-02  -0.73445D-04   0.35876D-03   0.49346D-06  -0.54146D-05  -0.28216D-06
```

The modal data set is already selected from the finite elements model output and properly scaled to match the vehicle parameters. They are included at the bottom of the input file and the dataset title "*Space Station 34 Flex Modes with Double-Gimbal CMG Array*" is also included at the bottom of the vehicle dataset.

2.2 Stability Analysis

The system stability is calculated using the open-loop model "*OpenLoop_TEA_Flex.slx*", shown in Figure 2.1. It includes the bending filters and it has the control loop broken at the CMG control torque input. The initialization file loads the systems and the state-feedback matrix Kpqr into Matlab. The frequency response for each axis is calculated by opening one loop at a time and closing the other two, as shown for roll. The Matlab file "freq.m" calculates the frequency responses for each axis.



```
% Initialization File
r2d=180/pi; d2r=pi/180;
wo= 0.0011;                                     % Orbital Rate 0.0011
att0=[-0.5,0, 6.4,0, 3.0,0]*d2r;                % Init TEA attitude

% Load Systems and Control Gain
[Av, Bv, Cv, Dv]= vehicle_rigid;                % Load the Rigid Vehicle State-Space Model
[Af, Bf, Cf, Df]= vehicle_flex34;               % Load the Flex  Vehicle State-Space Model
[Ao, Bo, Co, Do]= open_loop;                    % Open-Loop System from Flixan
[Ak, Bk, Ck, Dk]= closed_loop;                  % Closed-Loop System from Flixan
load Kpqr -ascii;                               % Load the H-Infinity Gains from Flixan
eig(Ak)                                         % Closed-Loop Poles
```

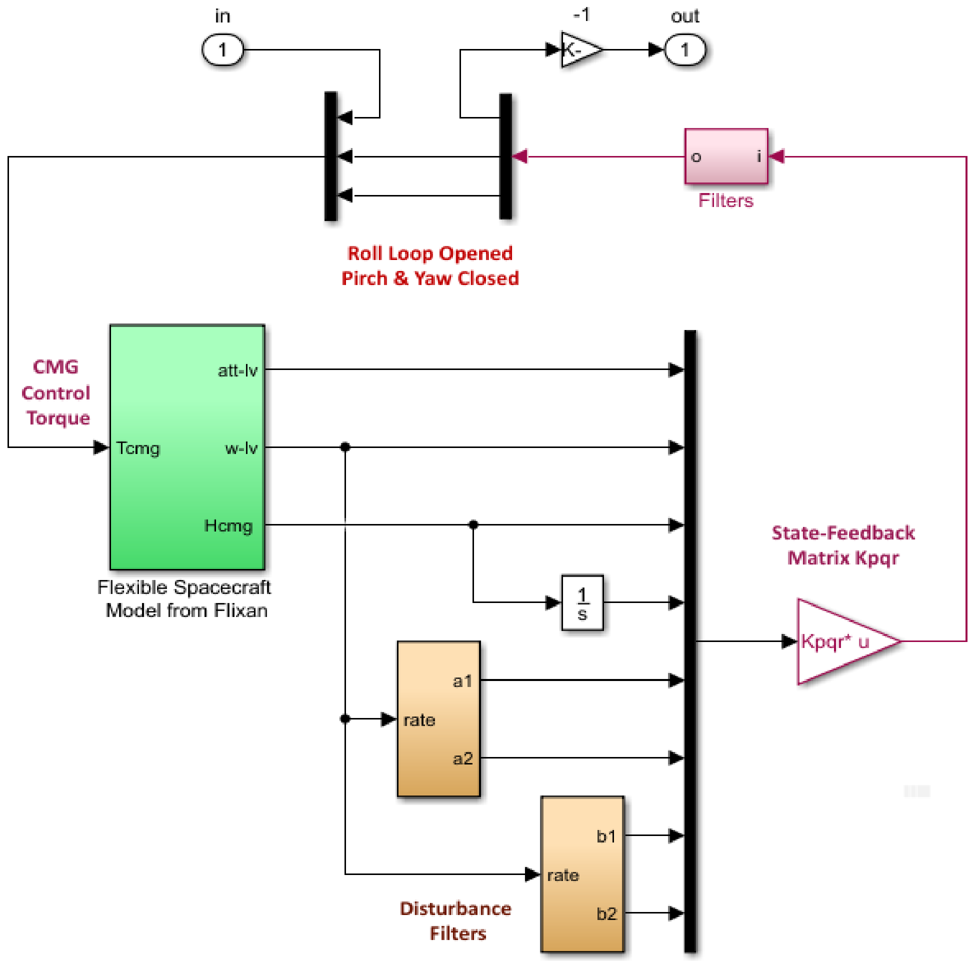


Figure 2.1 Stability Analysis Model "OpenLoop_TEA_Flex.slx"

The flex spacecraft dynamics block is shown in detail in Figure 2.2. It includes the CMG dynamics and the Flixan generated vehicle system *"Space Station with Double-Gimbal CMG Array (Flex)"* that was saved in file "*vehicle_flex34.m*". It is similar to Figure 1.5 but it includes a transformation which is needed to convert the body rates with flexibility (outputs: 12,13,14) to LVLH rates with flex, because the controller gain Kpqr was designed to feed-back LVLH rates. In Figure 1.5 the rigid rates are already in the LVLH.

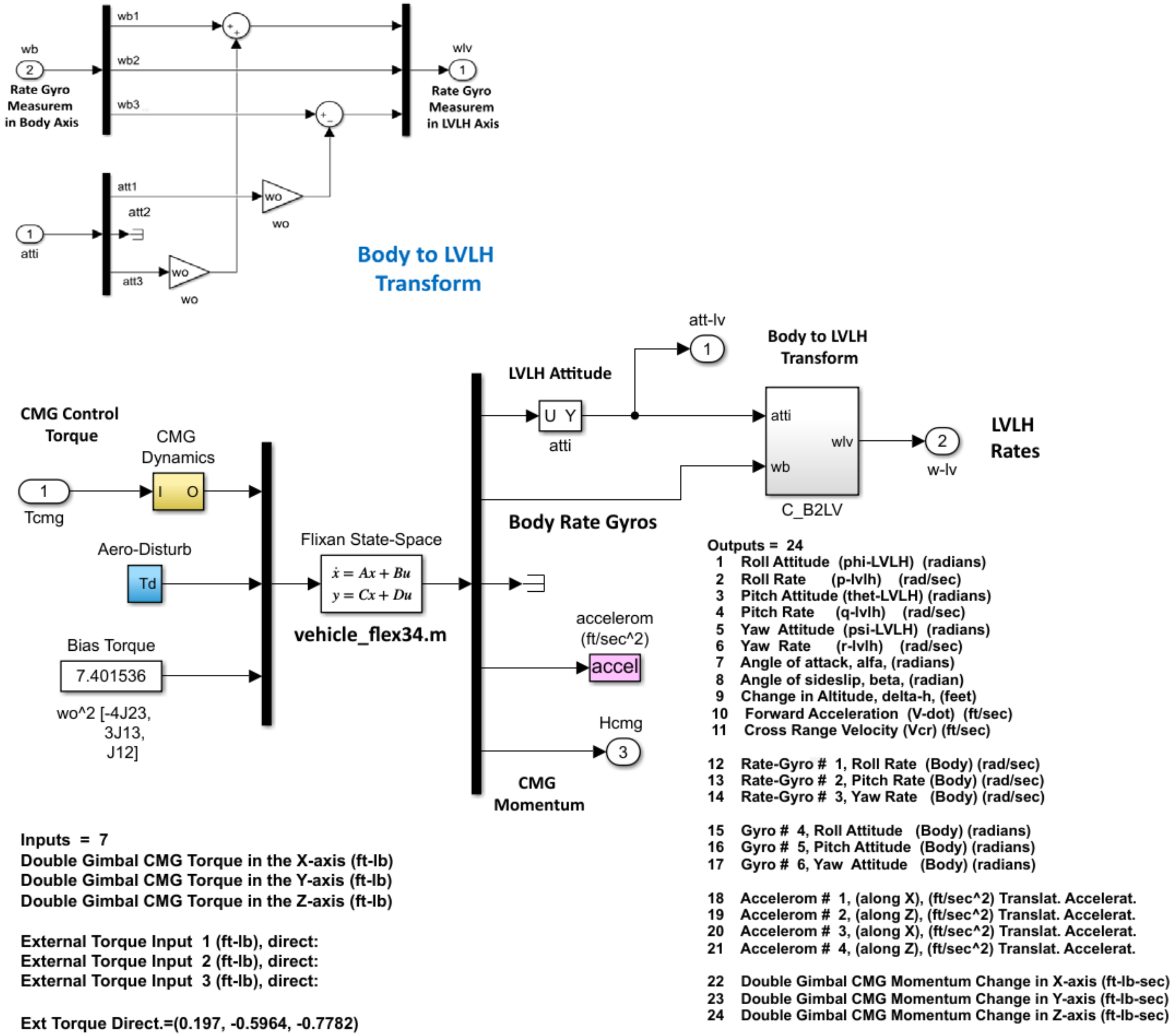


Figure 2.2 Flex Vehicle Block Includes the Flixan System from file "vehicle_flex34.m"

The Bode and Nichols plots are shown in Figures 2.3 to 2.5. There is sufficient gain and phase margins in all 3 axes and all modes are gain stabilized with more than 20 (dB) gain margin. Filter resonances at the disturbance frequencies, ω_o and $2\omega_o$ are introduced by the two disturbance filters.

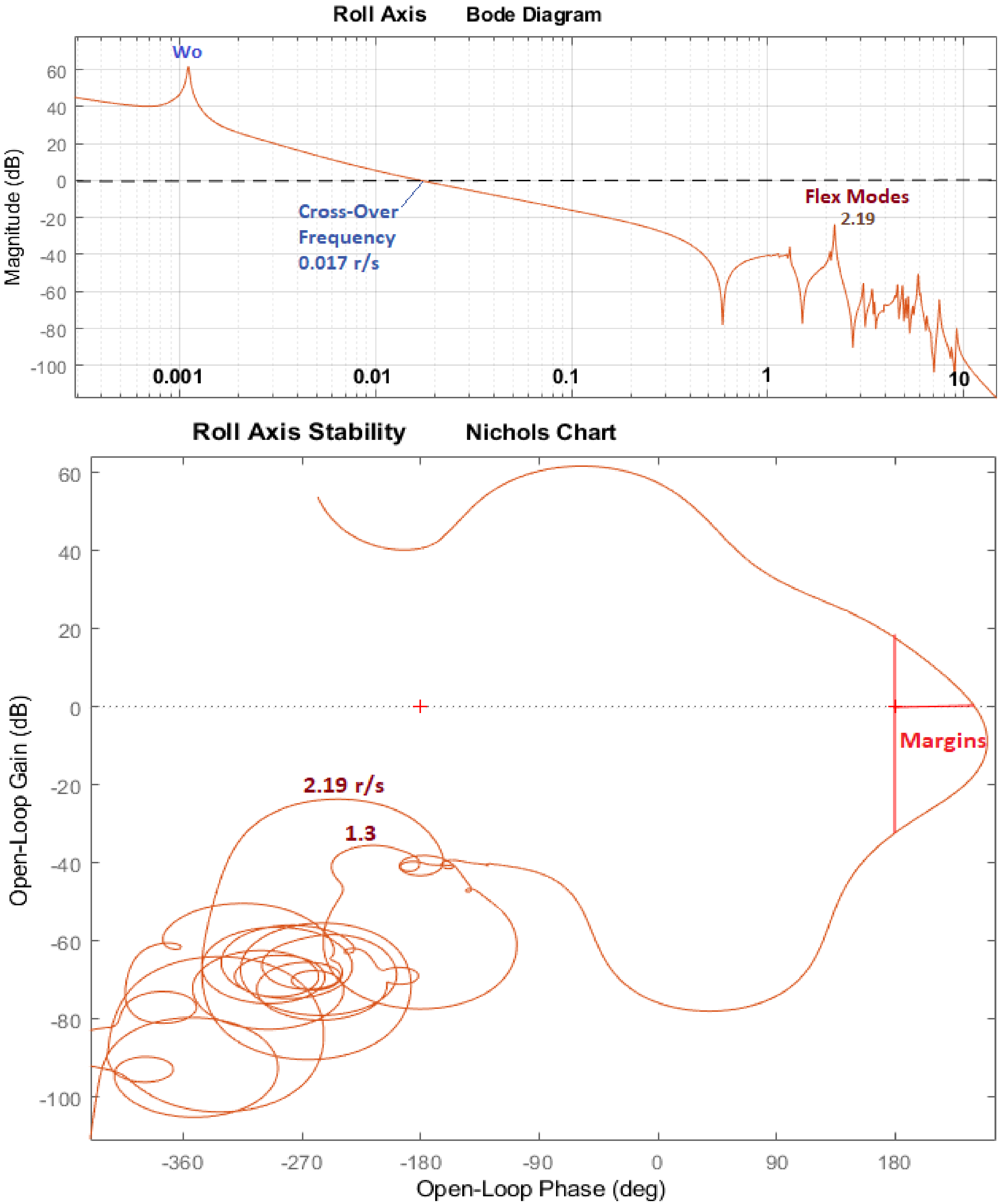


Figure 2.3 Roll Axis Stability

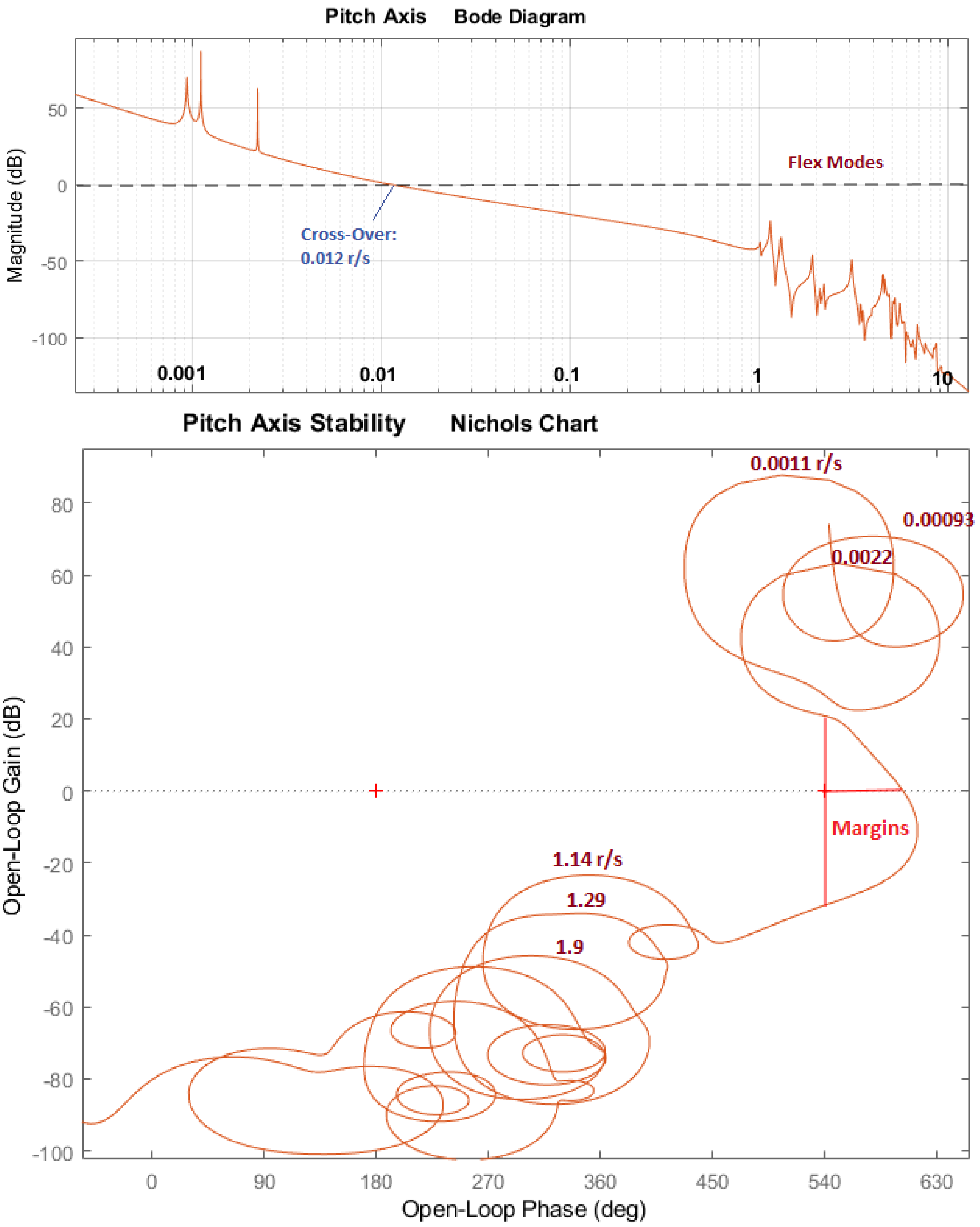


Figure 2.4 Pitch Axis Stability, Shows Resonances at ω_o and $2\omega_o$ Introduced by the Disturbance Filters.

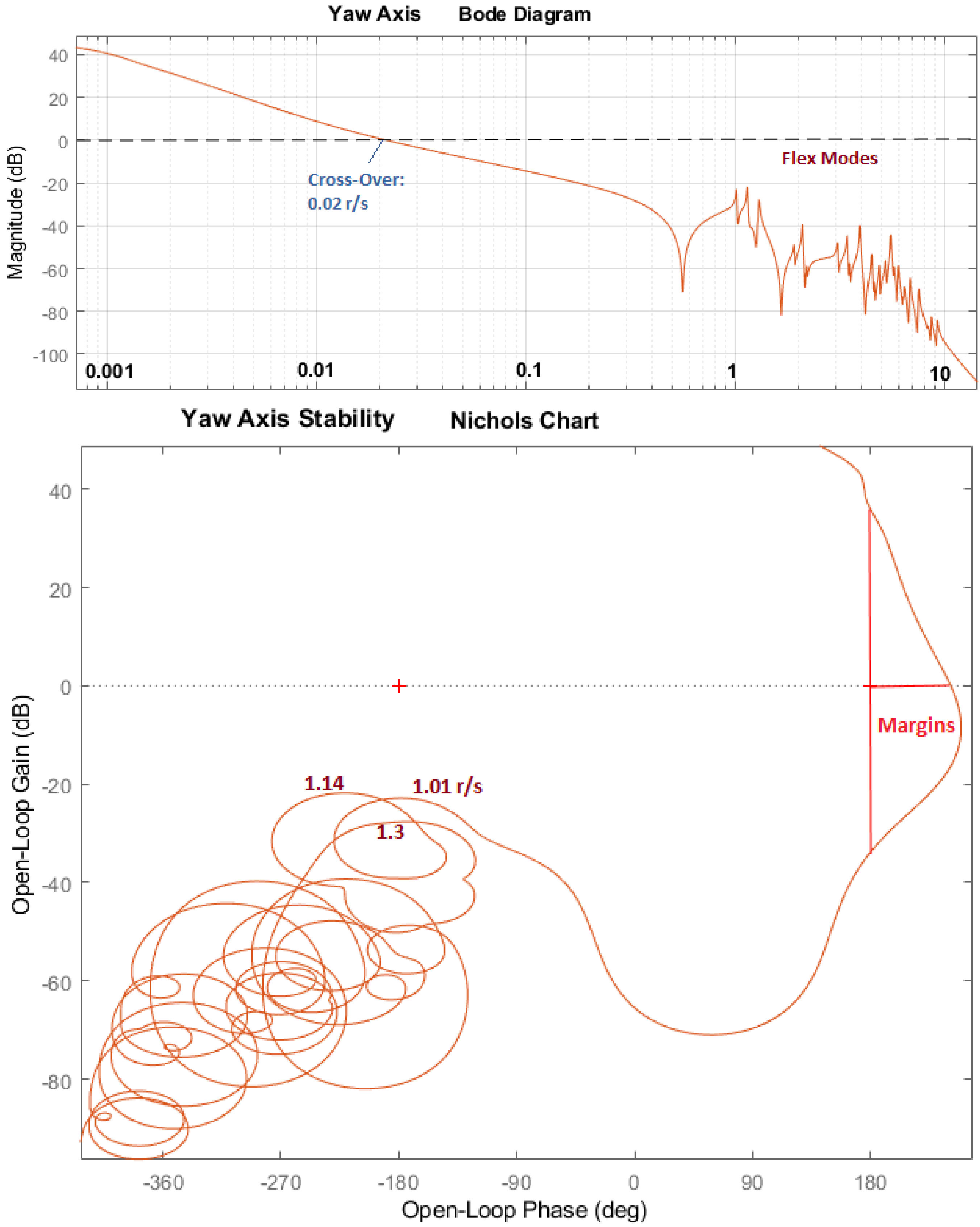
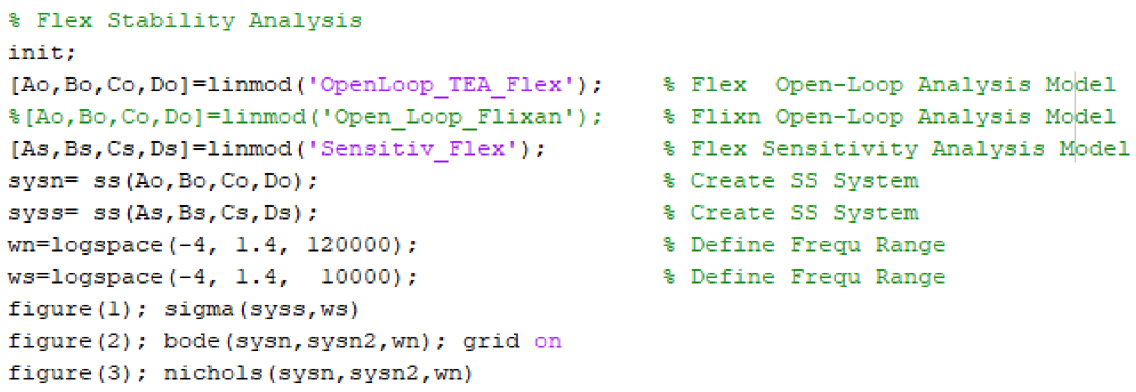


Figure 2.5 Yaw Axis Stability



```
% Flex Stability Analysis
init;
[Ao,Bo,Co,Do]=linmod('OpenLoop_TEA_Flex');      % Flex  Open-Loop Analysis Model
%[Ao,Bo,Co,Do]=linmod('Open_Loop_Flixan');      % Flixn Open-Loop Analysis Model
[As,Bs,Cs,Ds]=linmod('Sensitiv_Flex');          % Flex Sensitivity Analysis Model
sysn= ss(Ao,Bo,Co,Do);                          % Create SS System
syss= ss(As,Bs,Cs,Ds);                          % Create SS System
wn=logspace(-4, 1.4, 120000);                   % Define Frequ Range
ws=logspace(-4, 1.4,  10000);                   % Define Frequ Range
figure(1); sigma(syss,ws)
figure(2); bode(sysn,sysn2,wn); grid on
figure(3); nichols(sysn,sysn2,wn)
```

Figure 2.6 Frequency Response m-file “freq.m”

2.3 Sensitivity to External Disturbances

The sensitivity analysis model “*Sensitiv_Flex.slx*” is similar to the rigid model “*Sensitiv_Rigid.slx*” in Figure 1.8 but it includes the vehicle system with flexibility. It is used to analyze the rate sensitivity to disturbance torques as it was described in Section 1.9. Figure 2.7 shows the sensitivity response with flex modes. It is obtained by running the script “freq.m” which calculates the Singular Values frequency response (sigma) between the disturbances Td and the LVLH rate outputs ω_{lv}. The disturbance due to the flex modes excitation is significant in the frequency range between 1 and 10 (rad/sec). In the low frequency region, the sensitivity is similar to Figure 1.9 with two sharp notches at the disturbance frequencies.

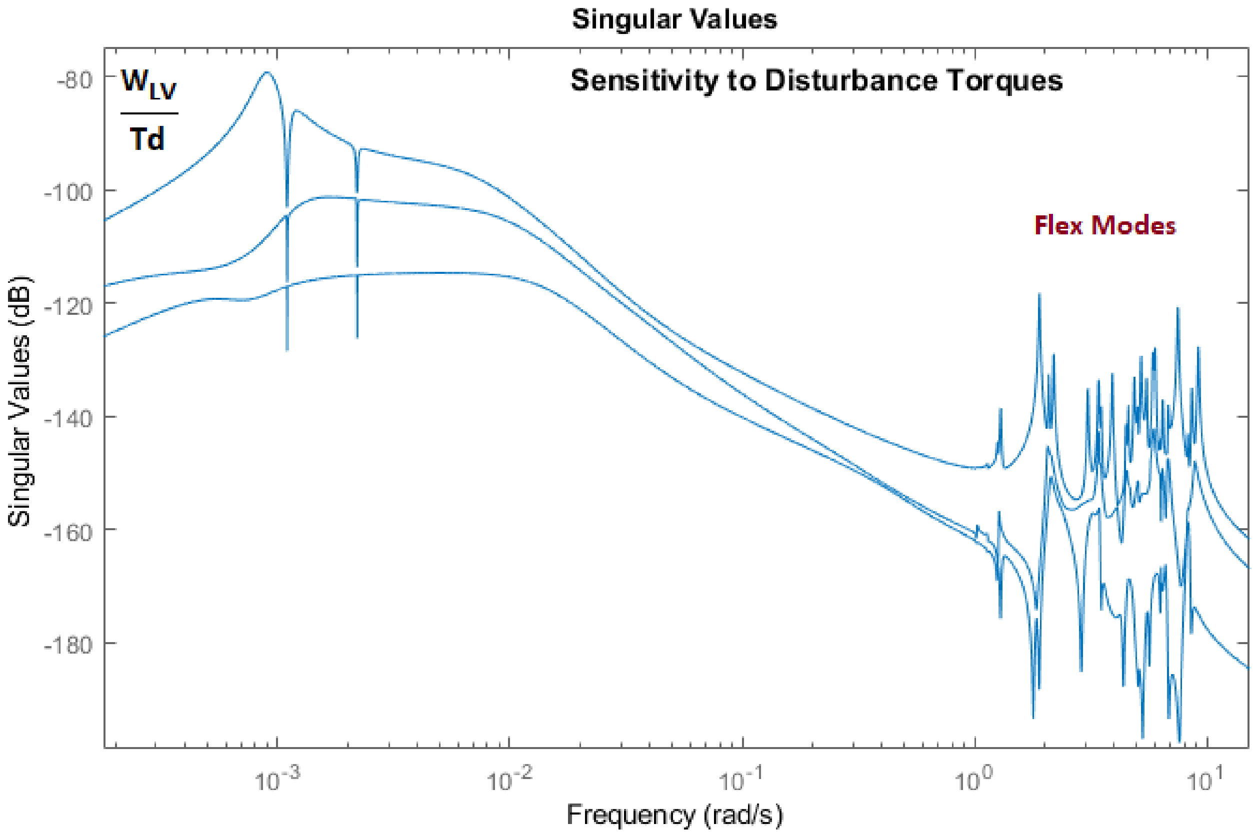


Figure 2.7 Rate Sensitivity to Disturbance Torques

2.4 Simulation Model

The simulation model “*Sim_TEA_Flex.slx*” is shown in Figure 2.8. It is similar to “*Sim_TEA_Rigid.slx*” but it includes the flex vehicle system “vehicle_flex34.m” and the bending filters, Fig. 2.9. Also, the momentum integrators and the disturbance filters. It is excited by the aero torques and the bias torque.

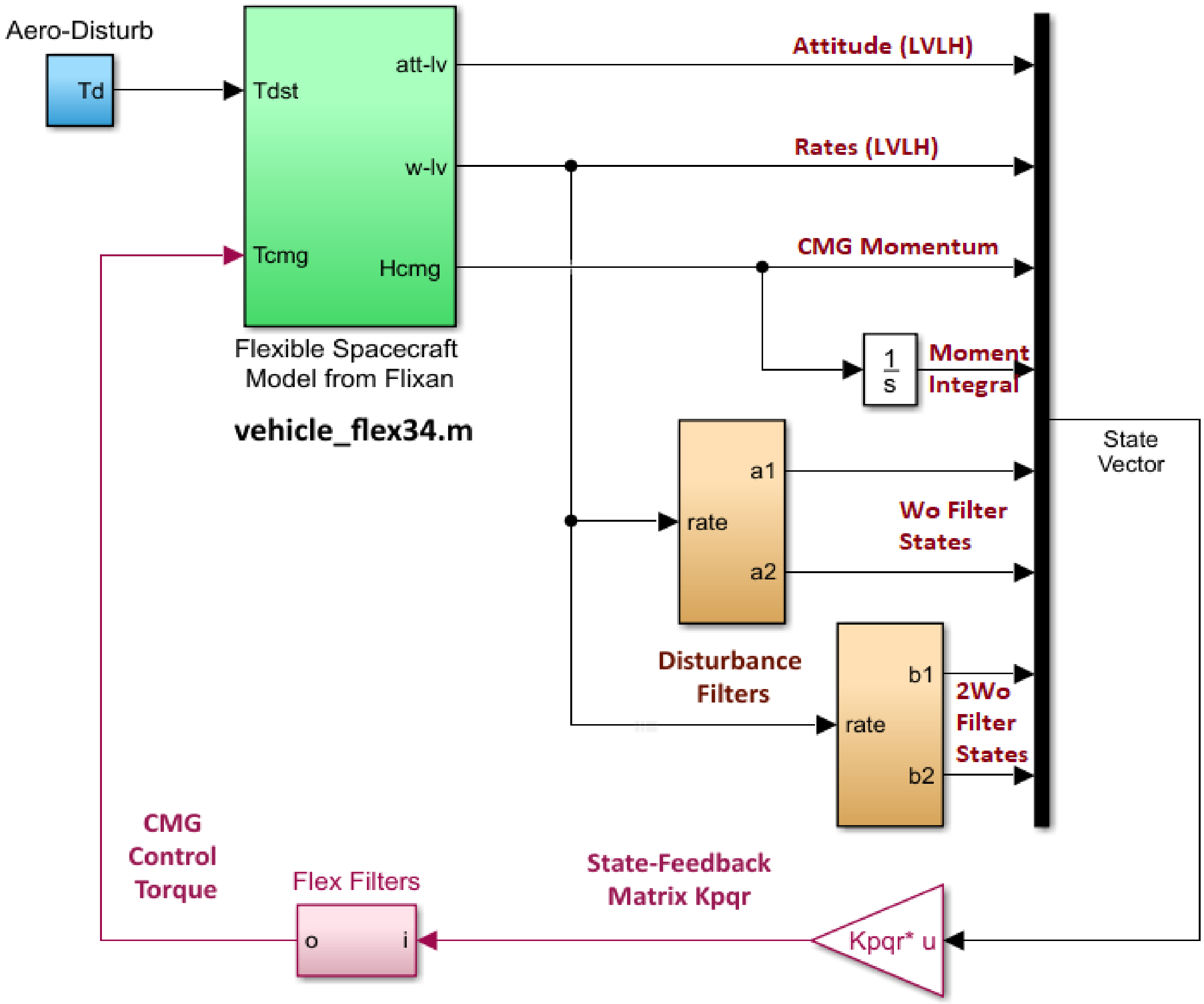


Figure 2.8 Simulation Model “Sim_TEA_Flex.slx”

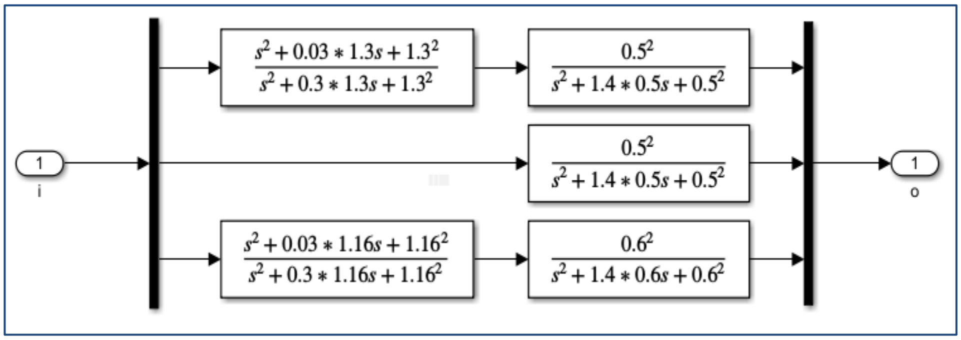


Figure 2.9 Bending Filters

2.5 Simulation Results

The simulation results in Figure 2.10 obtained from the simulation model “*Sim_TEA_Flex.slx*” are identical to the rigid-body results in Figure 1.11.

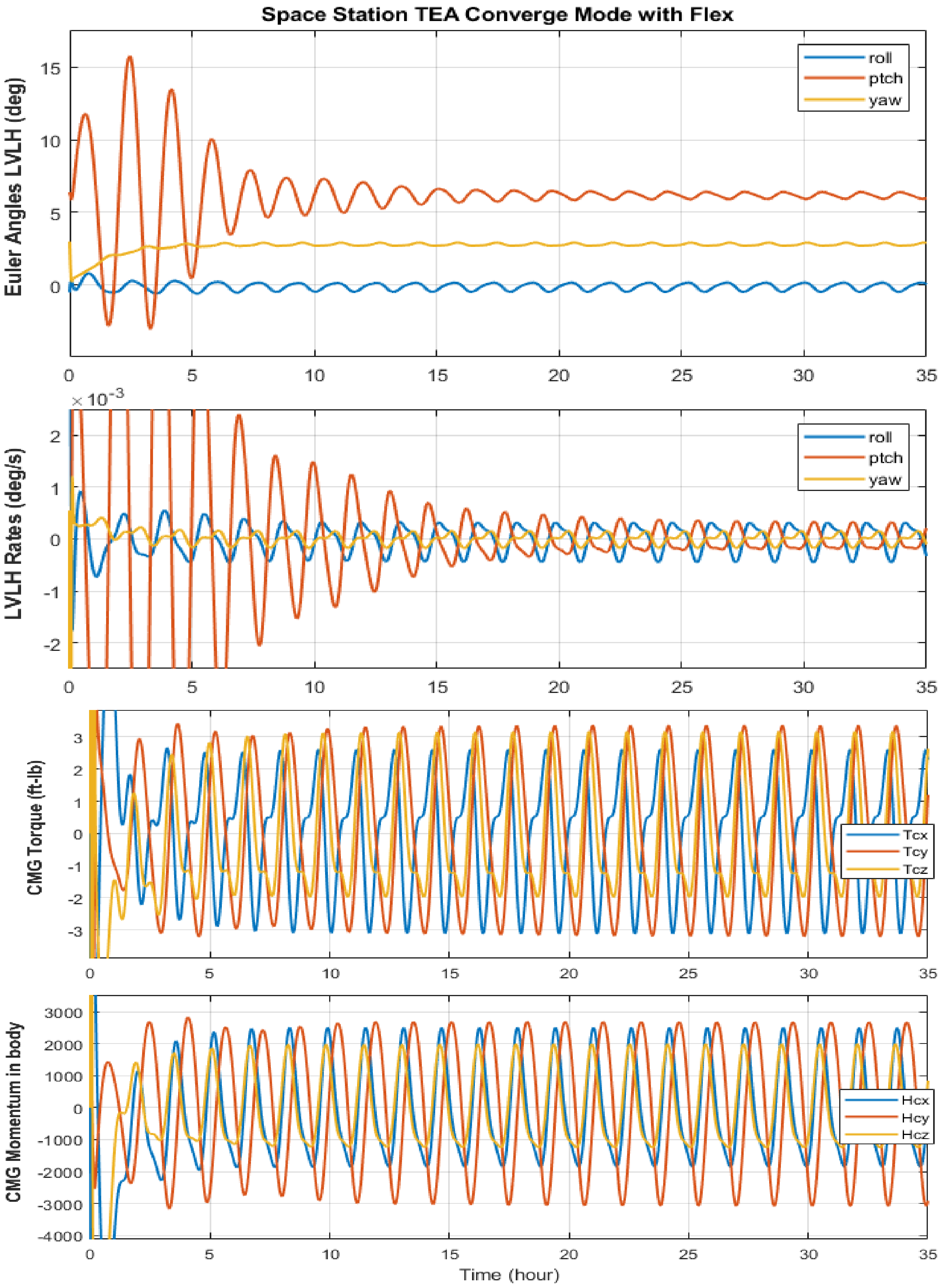


Figure 2 LVLH Attitude Converges to the TEA and CMG Torque and Momentum Oscillate Around Zero

2.6 Analyzing the Flixan Generated Open and Closed Loop Systems

Similar stability analysis and simulation results can be obtained by using the "*Open-Loop System*" and the "*Closed-Loop System*" that were combined together by the Flixan program. They are implemented in Simulink models "*Open_Loop_Flixan.slx*" and "*Closed_Loop_Flixan.slx*". They were used to validate the previous Simulink models.

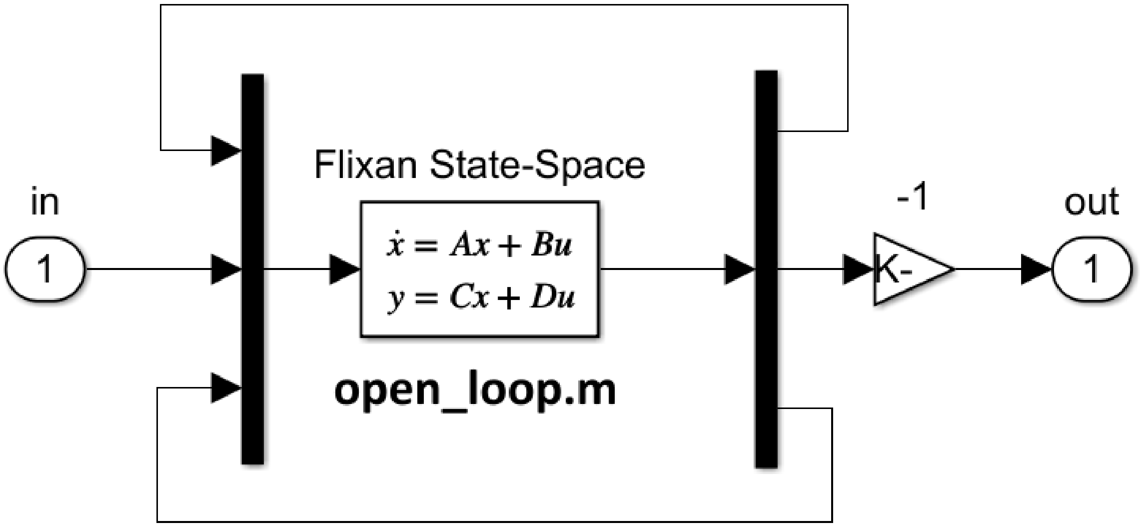


Figure 3 Open-Loop Stability Analysis Model "Open_Loop_Flixan.slx" Using the Flixan Generated "Open-Loop System". Configured for Pitch Analysis

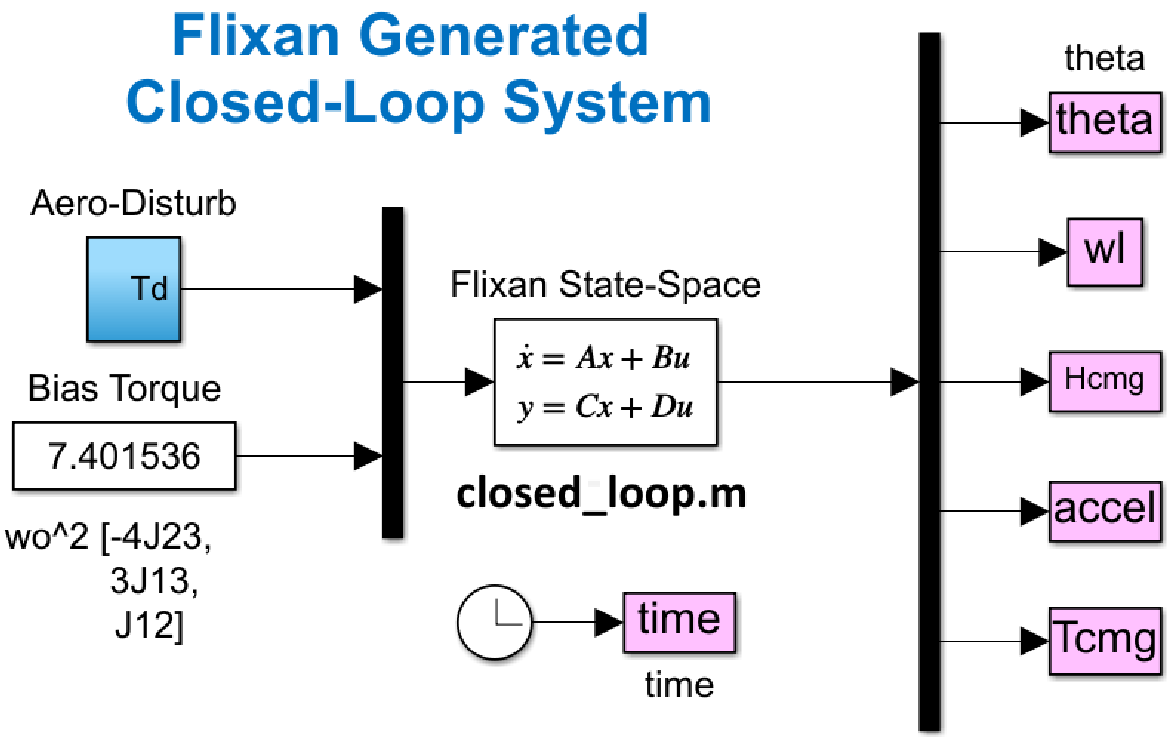


Figure 4 Simulation Model "Closed_Loop_Flixan.slx" Using the Flixan Generated "Closed-Loop System".

3.1 Non-Linear Simulation

The design is finally validated with a 6DOF simulation using Simulink. The non-linear equations 3.1 describe the Space Station rigid body dynamics in the LVLH frame which rotates at orbital rate. The spacecraft ACS attempts to keep a constant attitude relative to the LVLH.

$$I\,\underline{\dot{\omega}} = -\left(\underline{\omega}\times I\,\underline{\omega}\right) + 3\omega_o^2\left(\underline{c}\times I\,\underline{c}\right) + \underline{T}_d + \underline{T}_c$$

$$I = \begin{bmatrix} I_{XX} & I_{XY} & I_{XZ} \\ I_{XY} & I_{YY} & I_{YZ} \\ I_{XZ} & I_{YZ} & I_{ZZ} \end{bmatrix}; \quad c = \begin{bmatrix} -\sin\theta\cos\psi \\ \cos\phi\sin\theta\sin\psi + \sin\phi\cos\theta \\ -\sin\phi\sin\theta\sin\psi + \cos\phi\cos\theta \end{bmatrix}$$

$$\begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix}_{LVLH} = \frac{1}{\cos\psi}\begin{bmatrix} \cos\psi & -\cos\phi\sin\psi & \sin\phi\sin\psi \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\cos\psi & \cos\phi\cos\psi \end{bmatrix}\omega + \begin{pmatrix} 0 \\ \omega_o \\ 0 \end{pmatrix}$$

$$\dot{h}_{CMG} = -\left(\omega \times h_{CMG}\right) - T_c$$

Equation 3.1 Non-Linear Equations of Motion of a Spacecraft in the Rotating LVLH frame

Where: $\underline{\omega}$ is the spacecraft body rate (not LVLH), and ω_o = 0.0011 (rad/sec) is the orbital rate. The first two non-liner terms on the RHS of the top moment equation are the gyroscopic and gravity gradient torques. T_c and T_d are the CMG control and external disturbance torques applied to the spacecraft. The attitude kinematics equation calculates the Euler angles (ϕ, θ, ψ) relative to the LVLH frame by integrating the body rates $\underline{\omega}$. The spacecraft attitude is initialized in the LVLH frame and the pitch rate is initialized at $-\omega_o$ (rad/sec). The CMG array provides the control torque T_c to the spacecraft in all 3 directions. An equal and opposite torque is applied to the CMG cluster. The bottom part of Equation 3.1 calculates the rate of change of the CMG momentum as a function of the CMG control torque T_c.

The aerodynamic disturbance torque T_d consists of steady torques and also cyclic components that excite the spacecraft attitude to oscillations. The cyclic components are due to variations in aero drag. There are two frequency components associated with the disturbance torques: one is at orbital rate ω_o due to the difference in atmospheric density between the sunny and the dark sides of the earth and the second component is at twice the orbital rate $2\omega_o$ caused by drag variation due to the solar arrays rotation. There is less drag when the arrays are horizontal and more drag when they are vertical. The aero disturbance is stronger in pitch direction because the moment-arm distance between the vehicle CG and the center of pressure. The roll, pitch, and yaw disturbance torque T_d used in the simulations is:

$$T_d = \begin{bmatrix} 1 + \sin\omega_o t + 0.5\sin 2\omega_o t \\ 8 + 3\sin\omega_o t + 0.5\sin 2\omega_o t \\ 1 + \sin\omega_o t + 0.5\sin 2\omega_o t \end{bmatrix} \tag{3.2}$$

The Matlab analysis files for Section 3 are located in folder: *"Flixan\Control Analysis\Hinfinity\ Examples\ 6-Space Station Attitude & Momentum Control Design\3-Non-Linear 6DOF Simulation".* The non-linear Equations 3.1 are coded in Matlab function "*Rigbod-Dynam-LVLH.m*" which is included in the simulation model "*Sim_NonLin_TEA.slx*" shown in Figure 3.1. The inputs to the spacecraft system are: 3 CMG control torques T_c and 3 disturbance torques T_d in roll, pitch and yaw. There are 15 outputs which are: 3 body rates, 3 vehicle attitudes relative to the LVLH frame, the CMG momentum in body axes (3), the combined system momentum (3), and the 3 vehicle rates in the LVLH frame. The gravity gradient dynamics are calculated internally as a function of the Euler angles. The momentum integrators and the two filters are also included in the control law to complete the state vector for feedback via the gain K_{pqr}.

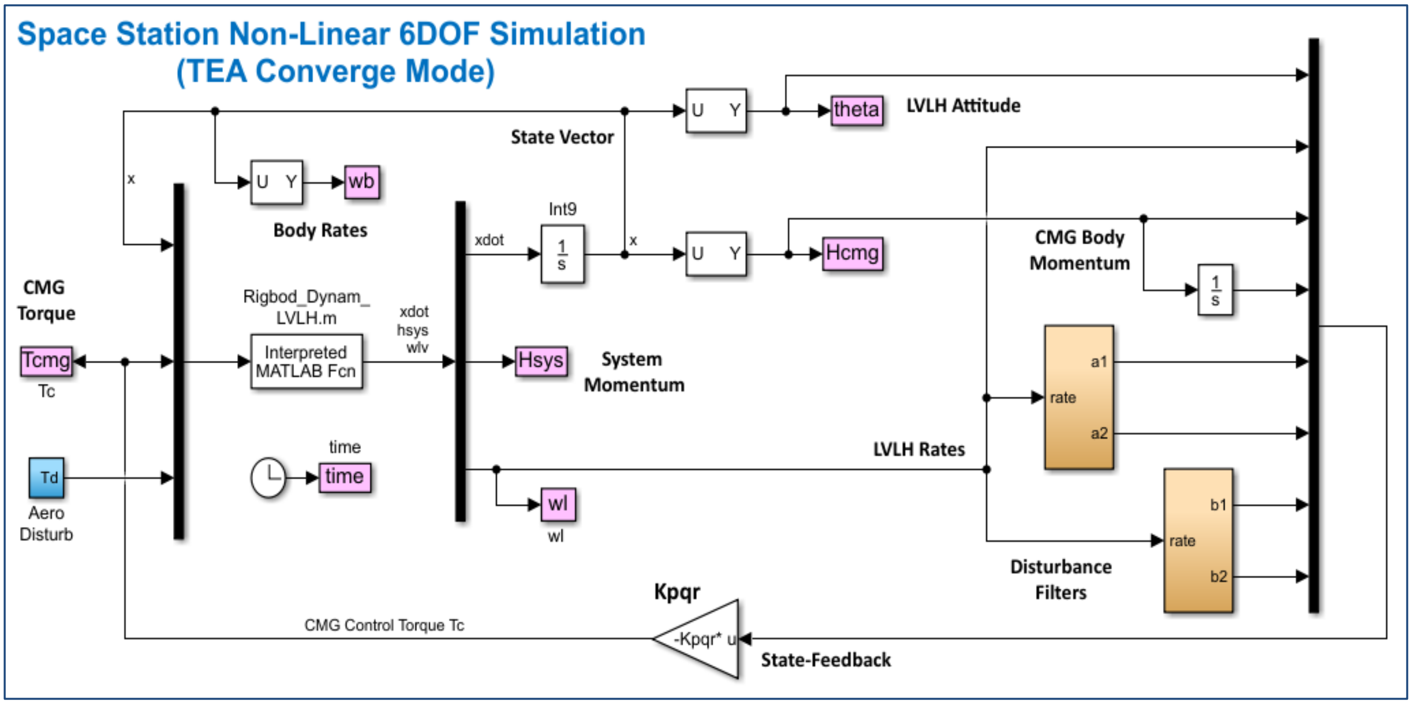
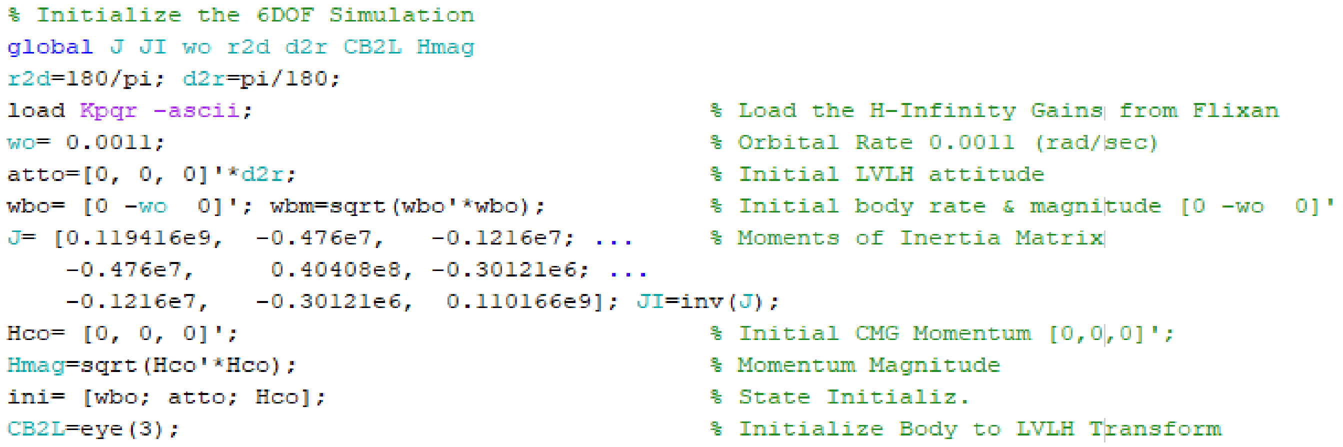
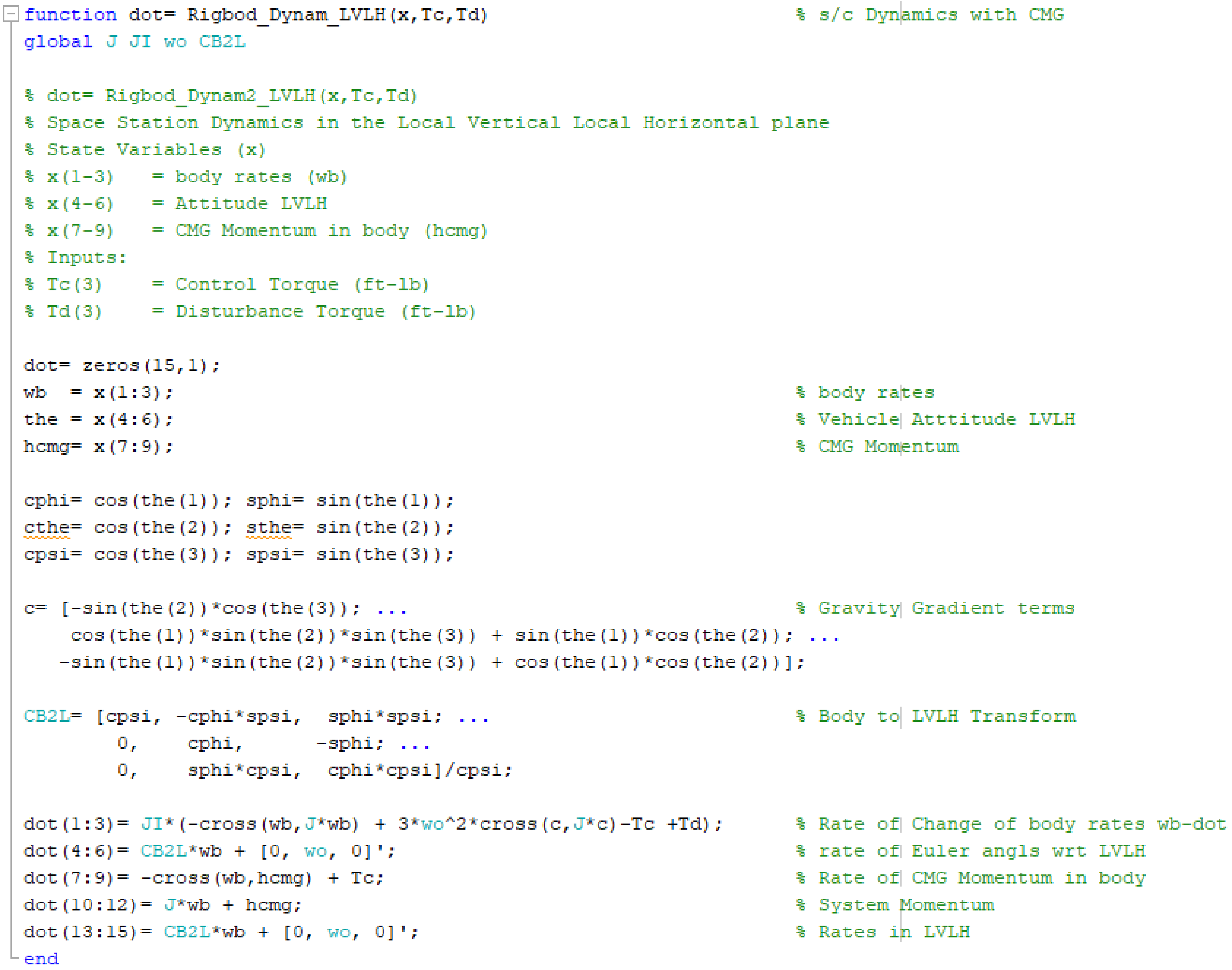


Figure 3.1 6DOF Simulation Model "Sim_NonLin_TEA.slx"

The simulation is initialized by the script "init.m" which loads the state-feedback matrix Kpqr and the spacecraft parameters. The vehicle dynamics function "*Rigbod_Dynam_LVLH.m*" which implements equations 3.1 is shown in Figure 3.2. It also calculates the transformation matrix CB2L which is used to transform the vehicle attitude and rate from body $\underline{\omega}$ to LVLH frame $\underline{\omega}_{lv}$. The function calculates the derivatives of the 9-state-vector which are updated by an integrator loop around the function.



```
% Initialize the 6DOF Simulation
global J JI wo r2d d2r CB2L Hmag
r2d=180/pi; d2r=pi/180;
load Kpqr -ascii;                                 % Load the H-Infinity Gains from Flixan
wo= 0.0011;                                       % Orbital Rate 0.0011 (rad/sec)
atto=[0, 0, 0]'*d2r;                              % Initial LVLH attitude
wbo= [0 -wo  0]'; wbm=sqrt(wbo'*wbo);             % Initial body rate & magnitude [0 -wo  0]'
J= [0.119416e9,  -0.476e7,   -0.1216e7; ...       % Moments of Inertia Matrix
    -0.476e7,     0.40408e8, -0.30121e6; ...
    -0.1216e7,   -0.30121e6,  0.110166e9]; JI=inv(J);
Hco= [0, 0, 0]';                                  % Initial CMG Momentum [0,0,0]';
Hmag=sqrt(Hco'*Hco);                              % Momentum Magnitude
ini= [wbo; atto; Hco];                            % State Initializ.
CB2L=eye(3);                                      % Initialize Body to LVLH Transform
```

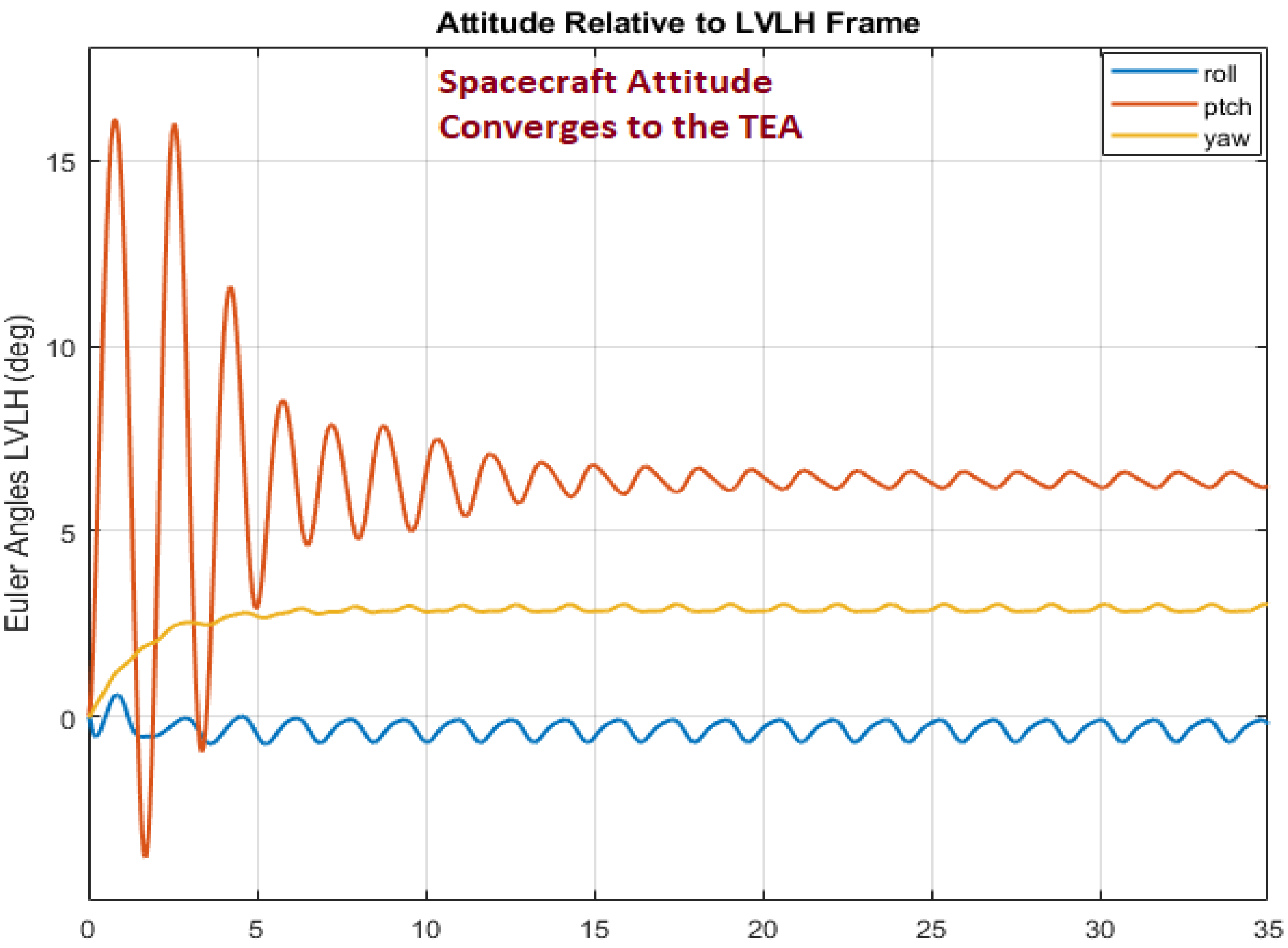


```
function dot= Rigbod_Dynam_LVLH(x,Tc,Td)                     % s/c Dynamics with CMG
global J JI wo CB2L

% dot= Rigbod_Dynam2_LVLH(x,Tc,Td)
% Space Station Dynamics in the Local Vertical Local Horizontal plane
% State Variables (x)
% x(1-3)    = body rates (wb)
% x(4-6)    = Attitude LVLH
% x(7-9)    = CMG Momentum in body (hcmg)
% Inputs:
% Tc(3)     = Control Torque (ft-lb)
% Td(3)     = Disturbance Torque (ft-lb)

dot= zeros(15,1);
wb  = x(1:3);                                               % body rates
the = x(4:6);                                               % Vehicle Atttitude LVLH
hcmg= x(7:9);                                               % CMG Momentum

cphi= cos(the(1)); sphi= sin(the(1));
cthe= cos(the(2)); sthe= sin(the(2));
cpsi= cos(the(3)); spsi= sin(the(3));

c= [-sin(the(2))*cos(the(3)); ...                           % Gravity Gradient terms
    cos(the(1))*sin(the(2))*sin(the(3)) + sin(the(1))*cos(the(2)); ...
   -sin(the(1))*sin(the(2))*sin(the(3)) + cos(the(1))*cos(the(2))];

CB2L= [cpsi, -cphi*spsi,  sphi*spsi; ...                    % Body to LVLH Transform
        0,    cphi,       -sphi; ...
        0,    sphi*cpsi,  cphi*cpsi]/cpsi;

dot(1:3)= JI*(-cross(wb,J*wb) + 3*wo^2*cross(c,J*c)-Tc +Td);      % Rate of Change of body rates wb-dot
dot(4:6)= CB2L*wb + [0, wo, 0]';                            % rate of Euler angls wrt LVLH
dot(7:9)= -cross(wb,hcmg) + Tc;                             % Rate of CMG Momentum in body
dot(10:12)= J*wb + hcmg;                                    % System Momentum
dot(13:15)= CB2L*wb + [0, wo, 0]';                          % Rates in LVLH
end
```

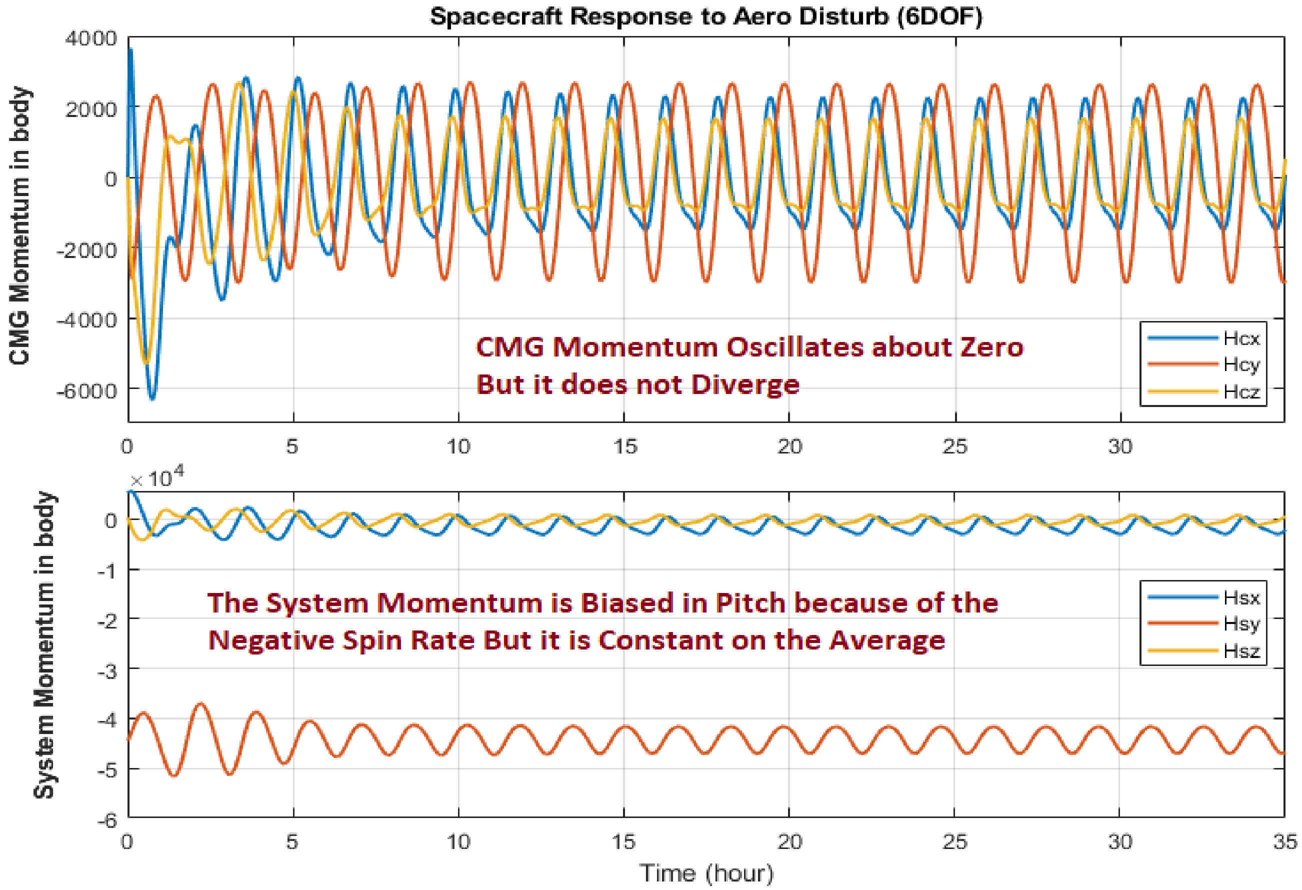
Figure 3.2 Equations of Motion Implemented in Function "Rigbod_Dynam_LVLH"

3.2 Simulation Results

The simulation results in Figure 3.3 show the Space Station response to the aero disturbances and demonstrate the capability of the control system to stabilize attitude and regulate the CMG momentum. The attitude is initialized at zero in the LVLH frame and the initial pitch body rate is set to negative orbital rate, that is (0, -ω_o, 0)'. The spacecraft attitude responds to the cyclic aerodynamic disturbance and oscillates mainly in pitch. The disturbance filters are eventually tuned to the two disturbance frequencies and the CMGs react by applying a counteracting torque that attenuates the attitude oscillations to less than 0.6° peak-to-peak. The attitude converges to the TEA which is (-0.5°, 6.3°, 3.0°) in roll, pitch and yaw, where the external torques balance in all directions. The CMG momentum after an initial transient it does not grow to saturation but it converges to a steady unbiased oscillation about zero. This is achieved by the feedback of CMG momentum and momentum-integral. The body rate and system momentum are negatively biased mainly in pitch because the spacecraft has a negative rate, which is mostly in pitch as it rotates at negative orbital rate. The rate also couples slightly in the other two axes because of the small roll and yaw Euler angles.

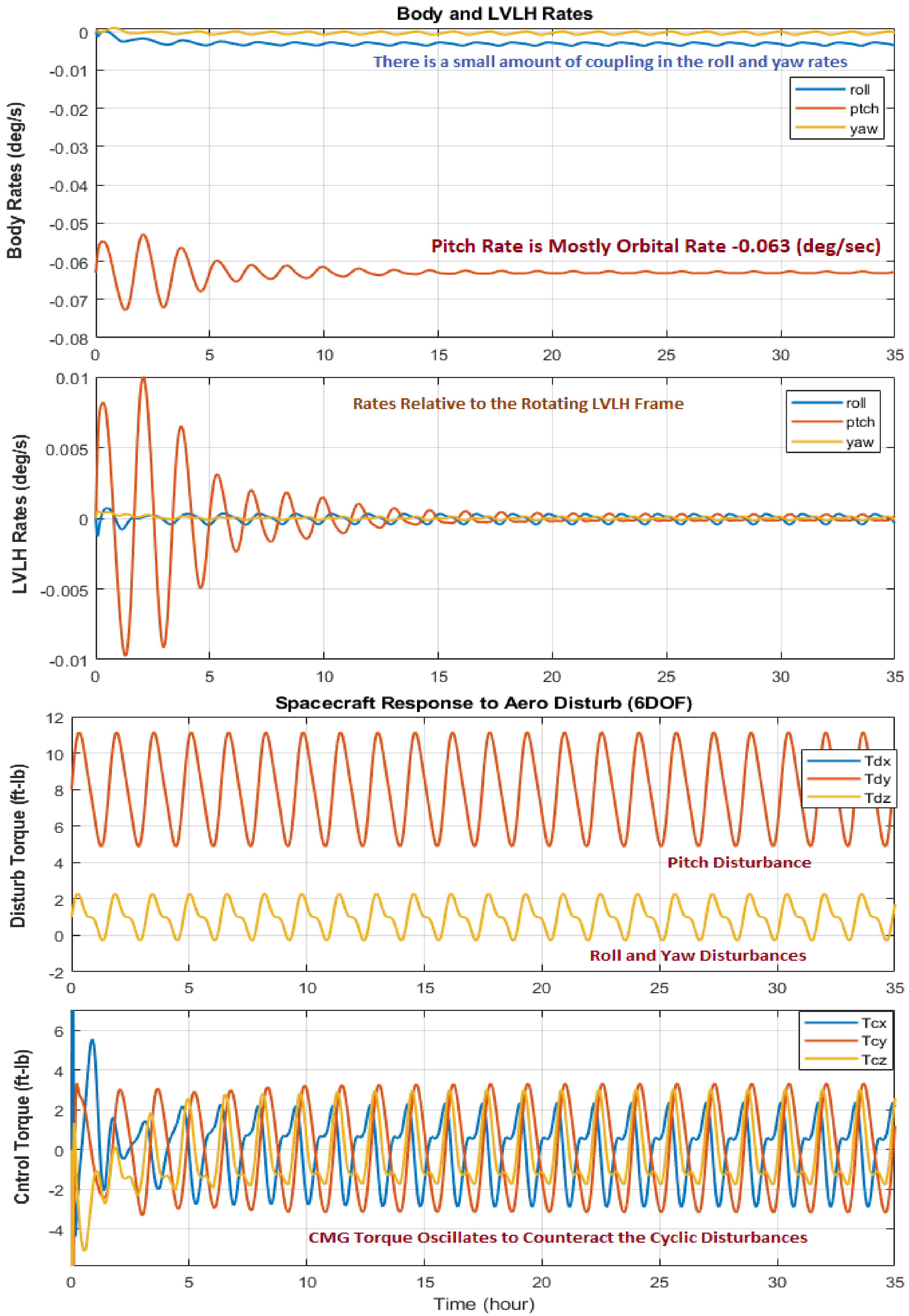


Spacecraft Attitude Converges to the TEA



CMG Momentum Oscillates about Zero But it does not Diverge

The System Momentum is Biased in Pitch because of the Negative Spin Rate But it is Constant on the Average



Body and LVLH Rates

There is a small amount of coupling in the roll and yaw rates

Pitch Rate is Mostly Orbital Rate -0.063 (deg/sec)

Rates Relative to the Rotating LVLH Frame

Spacecraft Response to Aero Disturb (6DOF)

Pitch Disturbance

Roll and Yaw Disturbances

CMG Torque Oscillates to Counteract the Cyclic Disturbances

Figure 3.3 6DOF Simulation Response to Aero Disturbance Torques