

# Robustness Analysis of a Lifting Body Aircraft



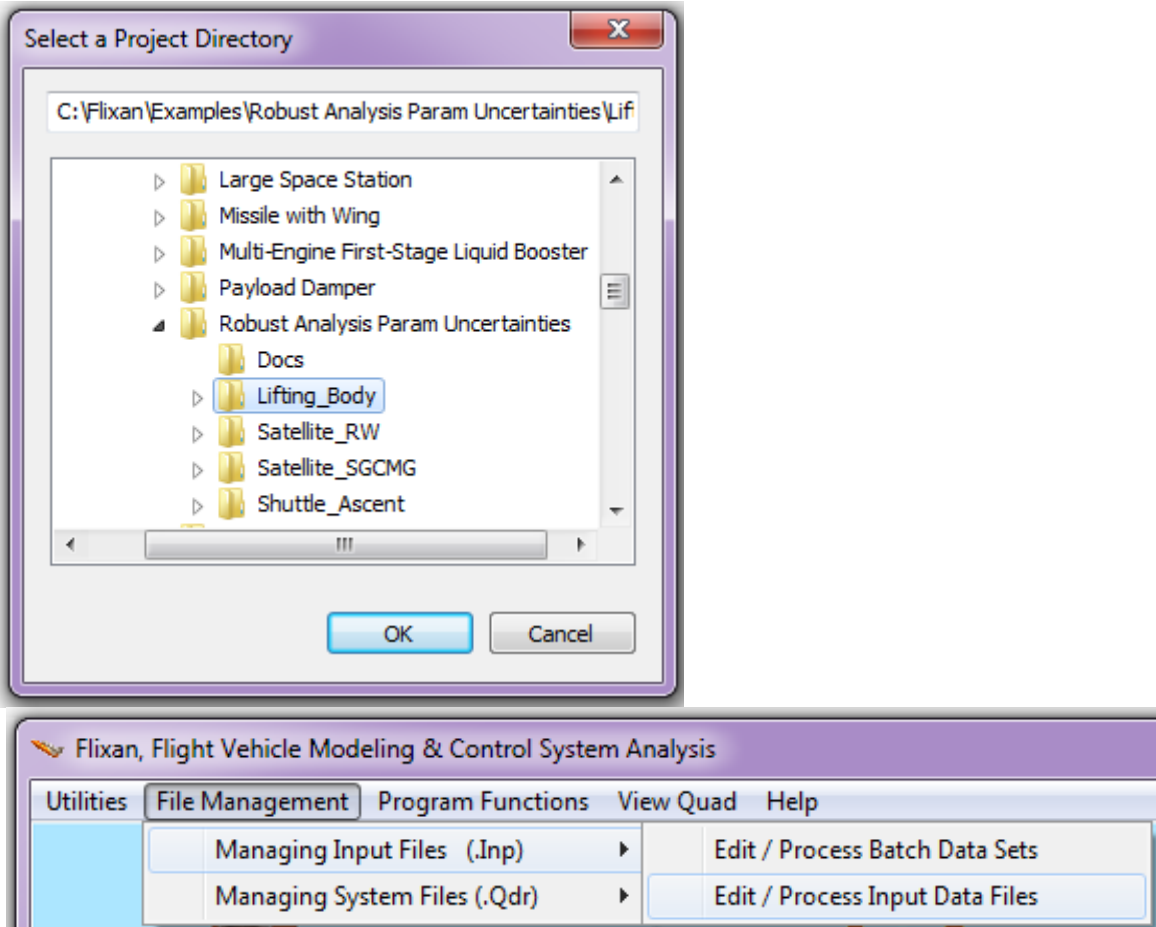
In this example we have a lifting body glider vehicle at 0.8 Mach and 20° angle of attack. The vehicle has four aero-surfaces, an Elevon for pitch control, and a combination of Aileron and two Rudders for lateral control. It also has RCS jets which are not used in this robustness analysis. The dynamic model is excited by a wind gust input that affects both pitch and lateral dynamics. The plant model has 33 parameters which are uncertain. They are defined by an estimated value and a maximum variation expected from the estimated value. We are going to generate a dynamic model for this flight vehicle and simulate its response to a wind gust. We will also create an uncertainty model that includes connections with the normalized uncertainties block  $\Delta$ , and analyze robustness with respect to uncertainties using  $\mu$ -analysis.

## Modeling

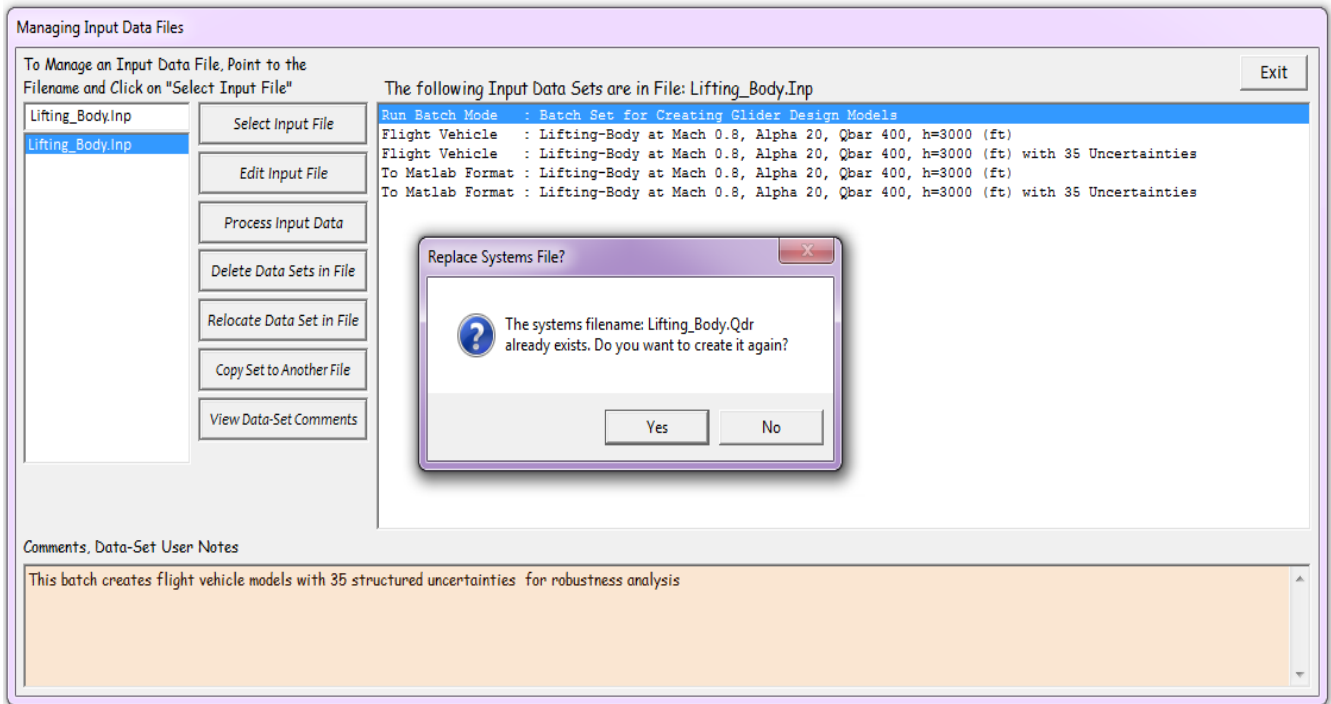
The analysis files for this example are in folder “*C:Flixan\Examples\Robust Analysis Param Uncertainties\ Lifting-Body*”. The vehicle data is in file “*Lifting\_Body.Inp*”. This file contains two sets of flight vehicle input data for implementing the vehicle dynamic model using two different approaches: a standard flight vehicle model with a title: “*Lifting-Body at Mach 0.8, Alpha 20, Qbar 400, h=3000 (ft)*”, and a set of vehicle data that includes the parameter uncertainties which are bounded in magnitude and saved in a separate data-set. The title of the second vehicle data set is “*Lifting-Body at Mach 0.8, Alpha 20, Qbar 400, h=3000 (ft) with 35 Uncertainties*”. The vehicle data-set points to the title of the uncertainties data-set which includes magnitudes of maximum variations of each parameter from its nominal value. It is assumed that the vehicle parameters may vary plus or minus from their nominal values, but the variation of each parameter cannot exceed the specified maximum magnitude. The uncertainties are processed by the vehicle modeling program together with the vehicle data, and the resulting dynamic model includes additional input/ output pairs that connect to the “pulled out” uncertainties block  $\Delta$ .

The Flixan program recognizes the uncertainties data-sets by the label “*UNCERTAIN PARAMETER VARIATIONS FROM NOMINAL ...*” which is located at the top of the data set. There is also a title for the uncertainties set below this label: “*Uncertainties at Mach=0.8, Alpha=20*”, which is used to reference and relate the uncertainties data with the vehicle input data. The uncertainties title must be included in the vehicle data-set that below the line: “*Parameter Uncertainties Data*”, otherwise, the dynamic model will not include the IFL input/ output connections. The variations of the parameters which are known and do not vary are set to zero in the uncertainties data-set and they are not extracted in the  $\Delta$  block. In this example the uncertainties data contains 33 parameter variations. However, two of the uncertainties, the Xcg and the Zcg are of rank-2 dependency and they couple to both: pitch and lateral states. They are extracted, therefore, as two uncertainties each, a total of four uncertainties and input/ output interconnections.

In this example the input file “*Lifting\_Body.Inp*” will be processed in batch mode. At the top of the file there is a batch set of data that we can run to generate the vehicle state-space models in file “*Lifting\_Body.Qdr*”. It also converts them to Matlab functions that can be loaded into Matlab to perform the  $\mu$ -analysis. Start the Flixan program, and select the project folder “*... \Examples \Robust Analysis Param Uncertainties \Lifting\_Body*” as shown below. Then, from the Flixan main menu, go to “*File Management*”, “*Manage Input Files*”, and “*Edit/ Process Input Data*”, as shown below.



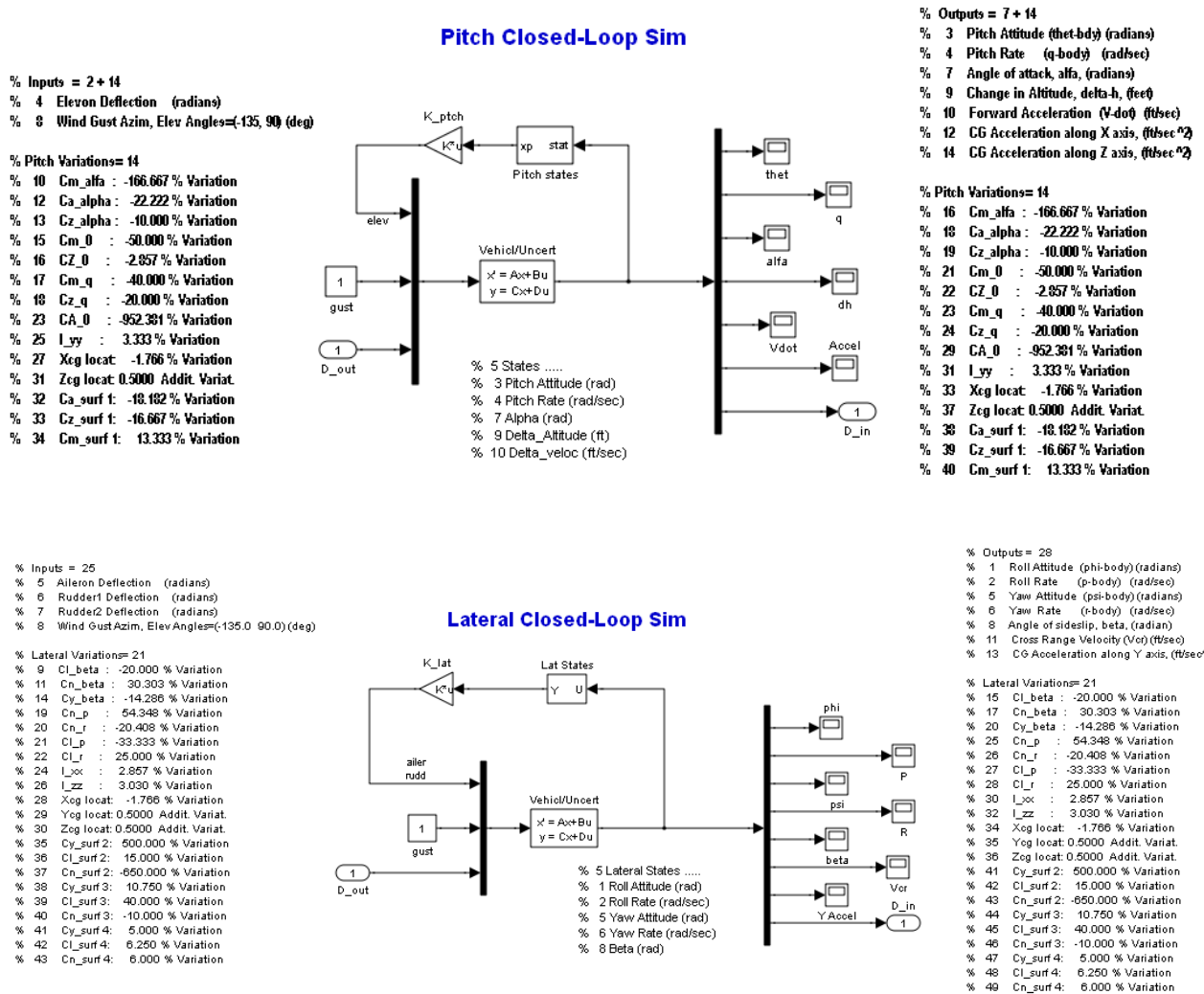
The following dialog is used for processing input data files and it has two menus. From the file selection menu on the left select the input file “*Lifting\_Body.Inp*” and click on “Select Input File”. The menu on the right shows the titles of the data-sets which are included in this input file. Select the first title which is a batch “*Batch Set for Creating Glider Design Models*” and click on “Process Input Data” button to process the batch set. The Flixan program will generate the systems and it will save them in systems file “*Lifting\_Body.Qdr*”.



It will also convert the systems to Matlab function format for the  $\mu$ -analysis. The vehicle system without uncertainties is saved in file “*Vehicle\_Nom.m*”, and the system with uncertainties is saved in file “*Vehicle\_35Unc.m*”. Both systems contain pitch and lateral coupled vehicle dynamics. The second system in addition to the standard inputs and outputs of the first system, it includes also the 35 pairs of inputs and outputs that connect to the uncertainties  $\Delta$  block. This is the scaled diagonal block that contains the normalized uncertainties which vary between -1 and +1. Some of the uncertainties couple mainly with the pitch states, and some uncertainties couple with the lateral states. The CG variations, however, couple with both: pitch and lateral states, as expected.

## μ-Analysis

The Matlab script file “run.m” performs the analysis in Matlab. It loads the coupled system “Vehicle\_35Unc.m”, and decouples it into pitch and lateral subsystems using the Matlab file “Split.m”. It separates also the uncertainties input/output pairs into pitch and lateral uncertainties. It synthesizes also an LQR state-feedback control law for the pitch and the lateral systems independently using the Matlab m-file “K\_des.m”. The folder also contains two Simulink models for simulation and μ-analysis, “Closed\_Loop\_Pitch.Mdl” for pitch, and “Closed\_Loop-Lateral.Mdl” for lateral analysis. They are shown in figures (1 & 2) below.



Figures 1 and 2, Closed-Loop Models Used for μ-Analysis

The two systems have their control loops closed via the state-feedback gains  $K_p$  and  $K_l$ . The 35 uncertainties were separated to 14 pitch and 21 lateral uncertainties, as shown above. The uncertainty inputs and outputs are combined into vectors which are supposedly connected to the diagonal uncertainty block  $\Delta$  (not shown).

Robustness analysis is performed by calculating the  $\mu$  frequency response between the input and the output uncertainty vectors for the pitch and the lateral systems separately. The Matlab file “run.m” calculates the  $\mu$  frequency response, assuming real and not complex parameter variations (complex are easier to calculate but they are too conservative), and plots  $\mu$  versus frequency, as shown in Figure 3. Robustness is guaranteed when  $\mu$  is less than 1 at all frequencies for both the pitch and the lateral systems. It means that the pitch and lateral systems will be stable despite all the parameter uncertainties which have bounded and known magnitudes.

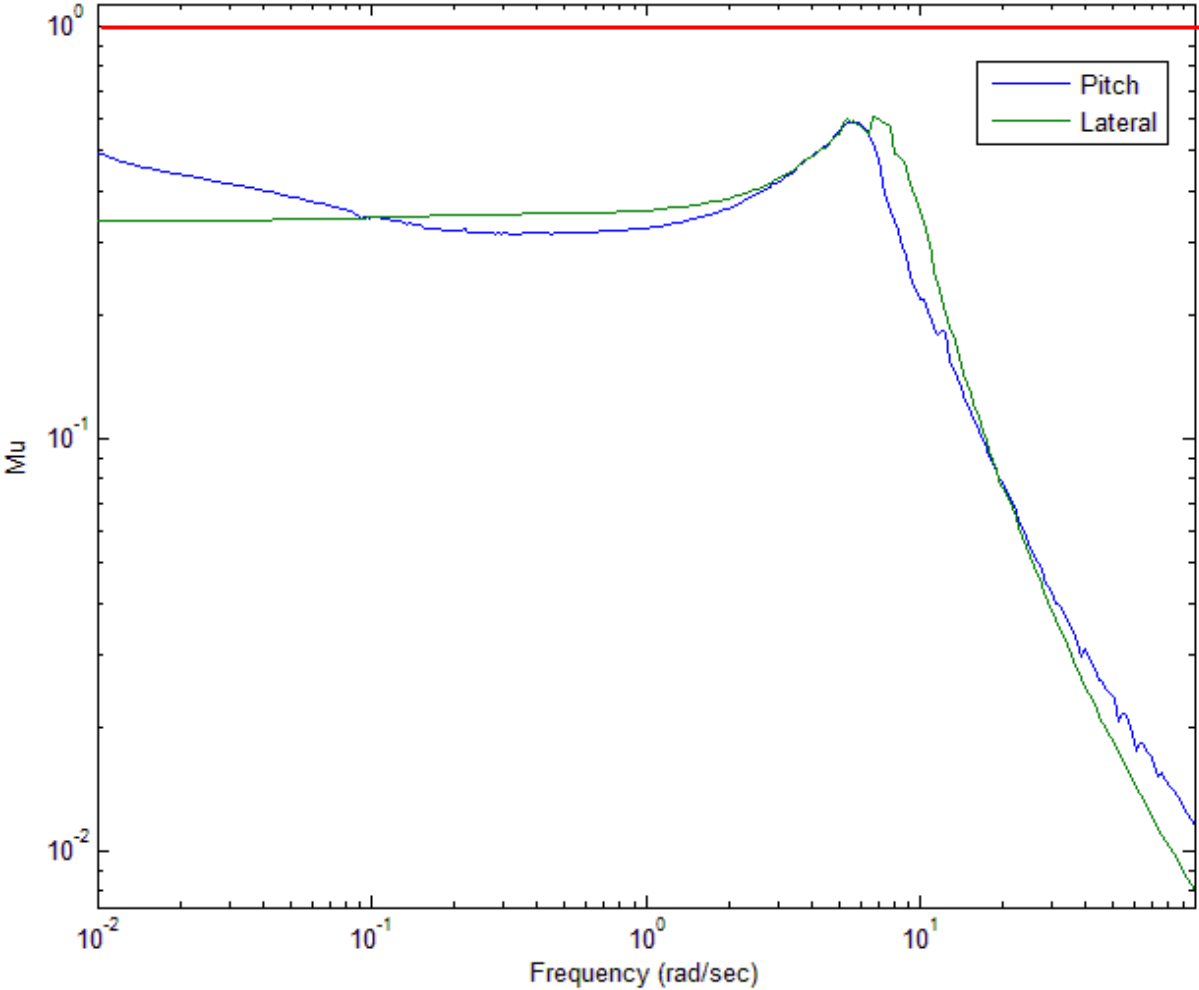


Figure 3, Mu frequency response for 14 pitch and 21 lateral uncertainties, showing that the control system is robust to the parameter variations.

## Simulation Model

The Simulink model “*Closed\_Loop\_Sim.mdl*”, shown in Figure 4, is used for the pitch and lateral coupled simulation. The system “*Lifting-Body at Mach 0.8, Alpha 20, Qbar 400, h=3000 (ft) with 35 Uncertainties*” is loaded into Matlab from file “*Vehicle-35Unc.m*” and included as a state-space model. The inputs and outputs are shown in the Figure, and the control loops are closed via the derived state-feedback gains  $K_p$  and  $K_l$ . The system also has the 35 uncertainty input and output pairs at the bottom of the list. Actually, there are 33 uncertainties but as we already mentioned the  $X_{cg}$  and the  $Z_{cg}$  couple into both: pitch and lateral dynamics, and are extracted, therefore, as two separate uncertainties each.

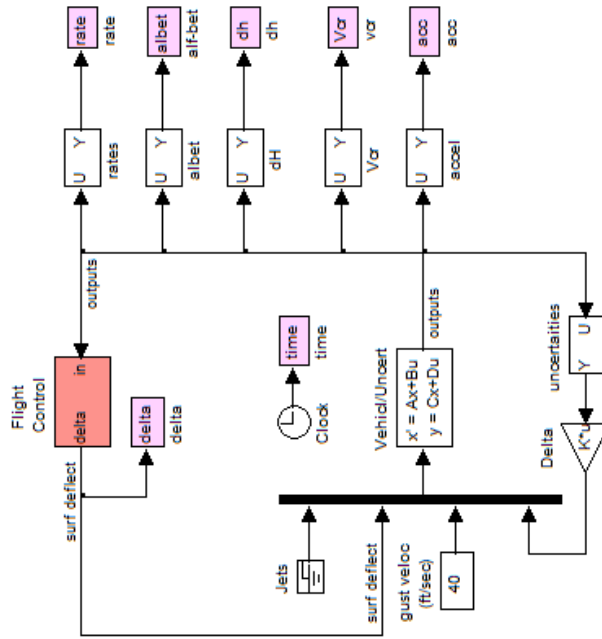
The parameter uncertainties loop via the diagonal matrix “Delta” that represents the normalized uncertainties, is shown closed in the model. When the uncertainties loop is opened the system behaves like the nominal system which is closed-loop stable since the gains were “tailored” around this original system. According to our  $\mu$ -analysis, the closed-loop system is robust to the uncertainties, and therefore, it should remain stable after the uncertainties loop is closed, as long as the absolute values of the diagonal elements of matrix “Delta” are less than “1”. Remember, that Delta is normalized because the actual parameter variations were absorbed inside the plant model. This simulation model can be used to verify system robustness to uncertainties by varying the diagonal elements of the Delta matrix and checking stability (as long as they vary between -1 and +1). The following figures show the nominal system responses to a 40 ft/sec wind gust step. The gust is applied in a direction that excites both, pitch and lateral states. After running the simulation model use the Matlab script “*pl.m*” to plot the data.

```

% Outputs = 14 + 35
% 1 Roll Attitude (phi-body) (radians)
% 2 Roll Rate (p-body) (rad/sec)
% 3 Pitch Attitude (thet-body) (radians)
% 4 Pitch Rate (q-body) (rad/sec)
% 5 Yaw Attitude (psi-body) (radians)
% 6 Yaw Rate (r-body) (rad/sec)
% 7 Angle of attack, alfa, (radians)
% 8 Angle of sideslip, beta, (radian)
% 9 Change in Altitude, delta-h, (feet)
% 10 Forward Acceleration (V-dot) (ft/sec)
% 11 Cross Range Velocity (Vcr) (ft/sec)
% 12 CG Acceleration along X axis, (ft/sec^2)
% 13 CG Acceleration along Y axis, (ft/sec^2)
% 14 CG Acceleration along Z axis, (ft/sec^2)
% Param Variations= 35
% 15 CL_beta : 20.000 % Variation
% 16 Cm_alfa : 166.667 % Variation
% 17 Cn_beta : 30.303 % Variation
% 18 Ca_alpha : 22.222 % Variation
% 19 Cz_alpha : 10.000 % Variation
% 20 Cy_beta : 14.286 % Variation
% 21 Cm_0 : -50.000 % Variation
% 22 CZ_0 : -2.857 % Variation
% 23 Cm_q : -40.000 % Variation
% 24 Cz_q : -20.000 % Variation
% 25 Cn_p : 54.348 % Variation
% 26 Cn_r : -20.408 % Variation
% 27 CL_p : -33.333 % Variation
% 28 CL_r : 25.000 % Variation
% 29 CA_0 : -238.095 % Variation
% 30 L_xx : 2.857 % Variation
% 31 L_yy : 3.333 % Variation
% 32 L_zz : 3.030 % Variation
% 33 Xcg locat: -1.766 % Variation
% 34 Ycg locat: -1.766 % Variation
% 35 Zcg locat: 0.5000 Addit. Variat.
% 36 Zcg locat: 0.5000 Addit. Variat.
% 37 Zcg locat: 0.5000 Addit. Variat.
% 38 Ca_surf1: 18.182 % Variation
% 39 Cz_surf1: 16.667 % Variation
% 40 Cm_surf1: 13.333 % Variation
% 41 Cy_surf2: 500.000 % Variation
% 42 CL_surf2: 15.000 % Variation
% 43 Cn_surf2: -650.000 % Variation
% 44 Cy_surf3: 10.750 % Variation
% 45 CL_surf3: 40.000 % Variation
% 46 Cn_surf3: -10.000 % Variation
% 47 Cy_surf4: 5.000 % Variation
% 48 CL_surf4: 6.250 % Variation
% 49 Cn_surf4: 6.000 % Variation

```

### Simulation Model



```

% 10 States .....
% 1 Roll Attitude (rad)
% 2 Roll Rate (rad/sec)
% 3 Pitch Attitude (rad)
% 4 Pitch Rate (rad/sec)
% 5 Yaw Attitude (rad)
% 6 Yaw Rate (rad/sec)
% 7 Alpha (rad)
% 8 Beta (rad)
% 9 Delta_Altitude (ft)
% 10 Delta_veloc (ft/sec)

```

```

% Inputs = 8 + 35
% 1 Throttle Input dThr/Th for Engine No 1 (-)
% 2 Throttle Input dThr/Th for Engine No 2 (-)
% 3 Throttle Input dThr/Th for Engine No 3 (-)
% 4 Elevon Surface No 1 Deflect. (radians)
% 5 Aileron Surface No 2 Deflect. (radians)
% 6 Rudder-1 Surface No 3 Deflect. (radians)
% 7 Rudder-2 Surface No 4 Deflect. (radians)
% 8 Wind Gust Azim, Elev Angles=(-135, 90) (deg)

```

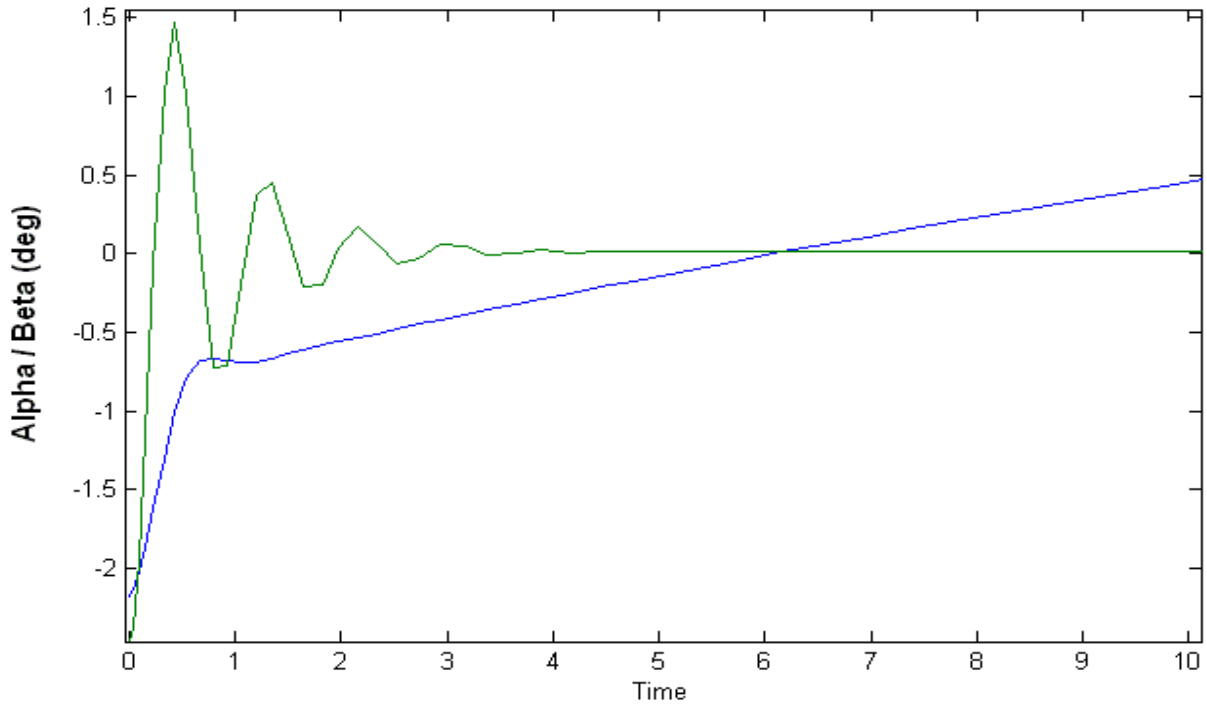
```

% Param Variations = 35
% 9 CL_beta : 20.000 % Variation
% 10 Cm_alfa : 166.667 % Variation
% 11 Cn_beta : 30.303 % Variation
% 12 Ca_alpha : 22.222 % Variation
% 13 Cz_alpha : 10.000 % Variation
% 14 Cy_beta : 14.286 % Variation
% 15 Cm_0 : -50.000 % Variation
% 16 CZ_0 : -2.857 % Variation
% 17 Cm_q : -40.000 % Variation
% 18 Cz_q : -20.000 % Variation
% 19 Cn_p : 54.348 % Variation
% 20 Cn_r : -20.408 % Variation
% 21 CL_p : -33.333 % Variation
% 22 CL_r : 25.000 % Variation
% 23 CA_0 : -238.095 % Variation
% 24 L_xx : 2.857 % Variation
% 25 L_yy : 3.333 % Variation
% 26 L_zz : 3.030 % Variation
% 27 Xcg locat: -1.766 % Variation
% 28 Ycg locat: -1.766 % Variation
% 29 Zcg locat: 0.5000 Addit. Variat.
% 30 Zcg locat: 0.5000 Addit. Variat.
% 31 Zcg locat: 0.5000 Addit. Variat.
% 32 Ca_surf1: 18.182 % Variation
% 33 Cz_surf1: 16.667 % Variation
% 34 Cm_surf1: 13.333 % Variation
% 35 Cy_surf2: 500.000 % Variation
% 36 CL_surf2: 15.000 % Variation
% 37 Cn_surf2: -650.000 % Variation
% 38 Cy_surf3: 10.750 % Variation
% 39 CL_surf3: 40.000 % Variation
% 40 Cn_surf3: -10.000 % Variation
% 41 Cy_surf4: 5.000 % Variation
% 42 CL_surf4: 6.250 % Variation
% 43 Cn_surf4: 6.000 % Variation

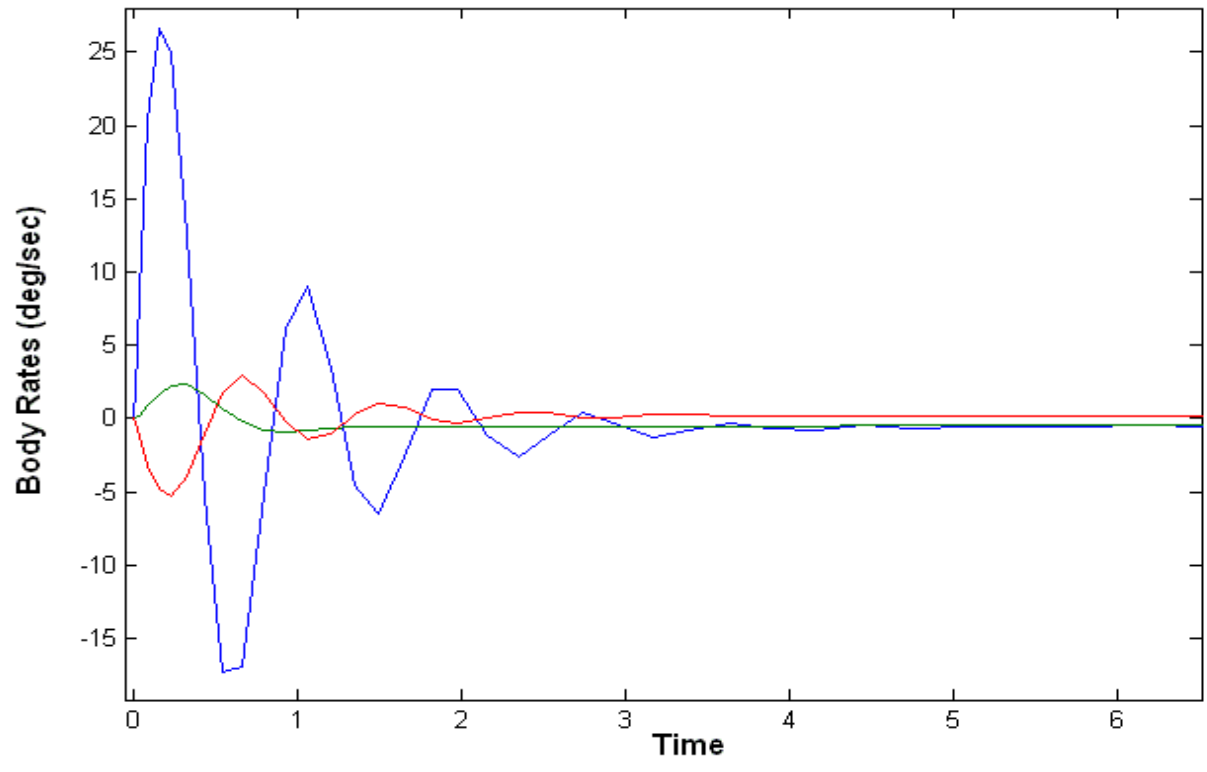
```

Figure 4 Time Domain Simulation Model with Control and Uncertainties Loops Closed

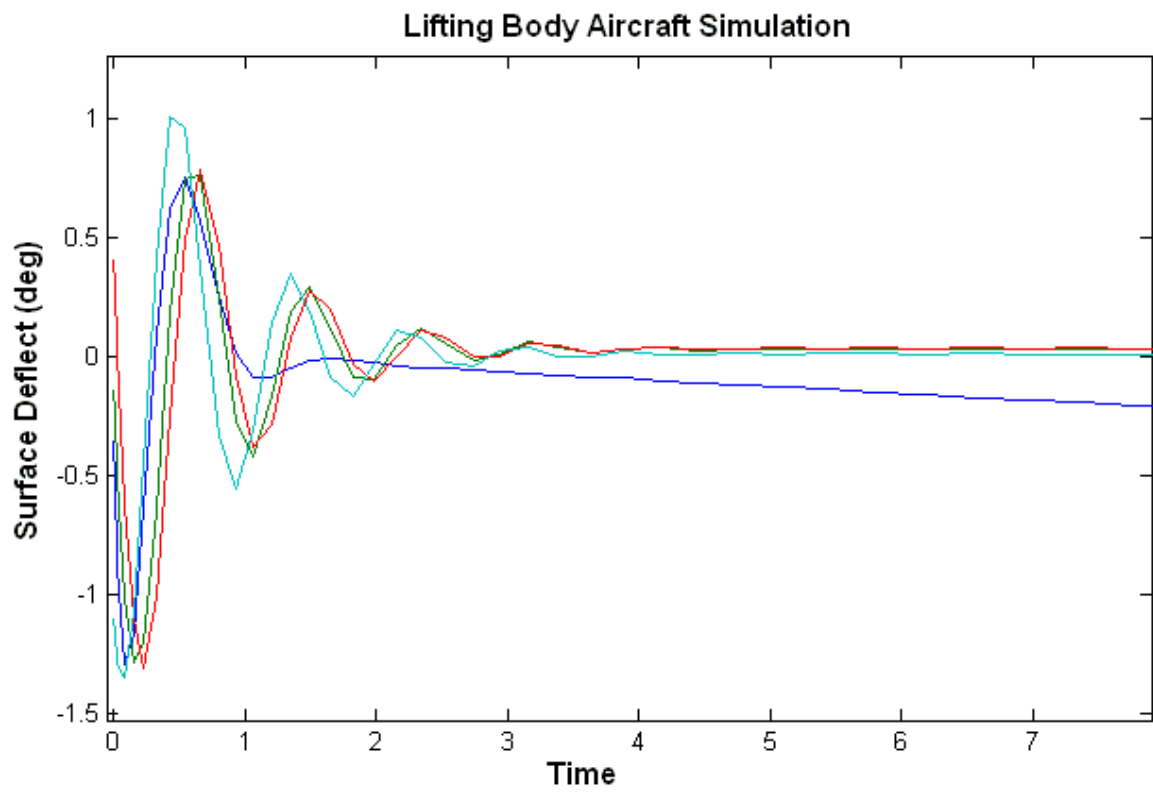
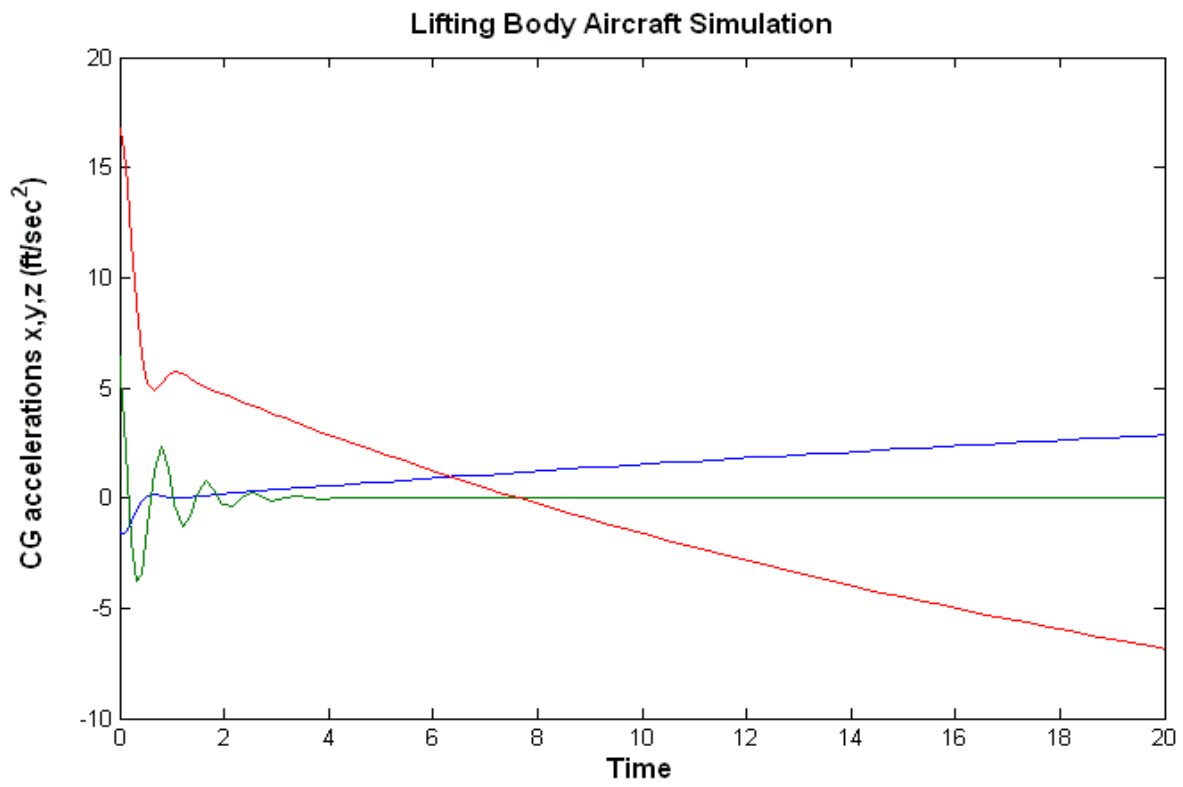
Angle of Attack/ Sideslip Response to 40 ft/sec Gust Shear



Lifting Body Aircraft Simulation







# Space Shuttle Robustness Analysis during Second Stage

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This example analyzes the Space Shuttle vehicle during second stage, immediately after the solid rocket booster separation. Although in this time period the Shuttle is almost outside the atmosphere, there is still a considerable amount of dynamic pressure for not ignoring the aerodynamics. We will develop a dynamic model of the vehicle including structural flexibility and propellant sloshing dynamics. The model is uncertain with 45 parameter variations. In addition to the standard state variables it includes IFL inputs and outputs that connect to the normalized uncertainty block  $\Delta$ . It will be used to perform robustness analysis.



The analysis data files are in folder “\Flixan\Examples\Robust Analysis Param Uncertainties\Shuttle\_Ascent”, and the vehicle input data is in file: “Shuttle\_Stg2.Inp”. The vehicle is controlled by the 3 Shuttle main engines which are gimbaling in pitch and yaw. The tail-wag-dog effect is included in the dynamic model by setting the flag in the engine data “WITH TWD”. A linear main engine actuator model is also included in the input data file. A gust disturbance is applied in the model perpendicular to the vehicle x axis, skewed to excite both: pitch and lateral directions. The input data file “Shuttle\_Stg2.Inp” contains two sets of vehicle data, a set that generates the standard flight vehicle model with title: “Space Shuttle Second Stage at T=123 sec”, and a set of vehicle data that generates a similar vehicle model with additional inputs and outputs for the uncertainty model. The title of the second data-set is “Space Shuttle Second Stage at T=123 sec (With Uncertain Param)”, and requires also a set of parameter uncertainties data which is also included in the input file “Shuttle\_Stg2.Inp”. Its title is “Uncertainties for Shuttle Ascent Second Stage”. It contains maximum variations in vehicle parameters, such as: moments of inertia, angles of attack, sideslip, aero coefficients, engine thrusts, slosh uncertainties, and flex mode frequency and damping variations. The title of the uncertainties data is included in the vehicle data, below the line “Parameter Uncertainties Data”.

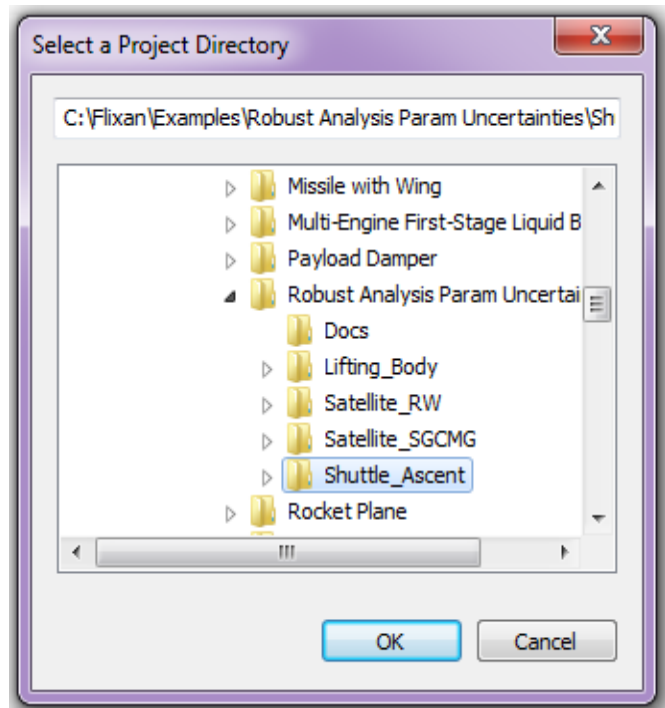
The dynamic models include 20 flexible structural modes. The modes are already preselected from a big finite elements model and a smaller number of selected modes are included in file “Shuttle\_Stg2.Inp”. The title of the selected modes set is: “Shuttle Second Stage at T=123 sec, Mixed Modes”. The mode selection process is not shown in this example. The Flight Vehicle Modeling program will process the uncertainties data set together with the vehicle data and the slosh and flex modes to create the augmented state-space model. In this example, the vehicle, flex modes, and the uncertainties data-sets already exist and they are saved in the input file “Shuttle\_Stg2.Inp”. The Flixan program will process the input data and it will create a state-space model of the vehicle augmented with the additional inputs and outputs necessary to connect with the uncertainty block  $\Delta$ . The augmented system will be saved in systems file “Shuttle\_Stg2.Qdr”.

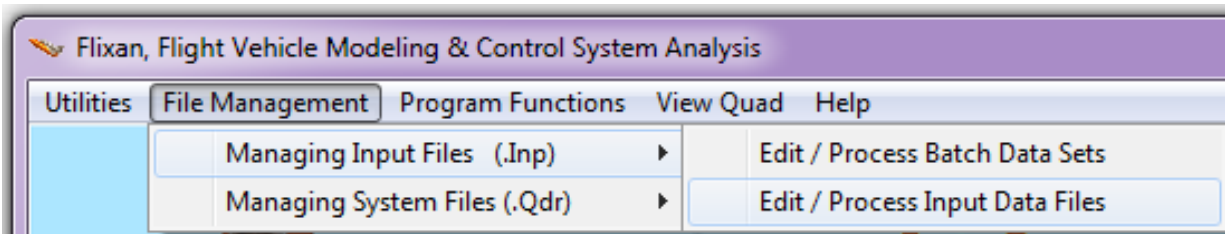
Notice, that the input/ output pairs that connect to the uncertainty block  $\Delta$  is greater than the number of parameter variations. This is because some of the uncertainties, such as: engine thrusts, slosh masses, and slosh positions are higher than rank-1 dependency and they couple to both pitch and lateral dynamics. Those parameters are extracted more than once because they couple in both pitch and lateral equations. This model will be used to perform robustness analysis using  $\mu$  methods.

## Processing the Data Files in Batch Mode

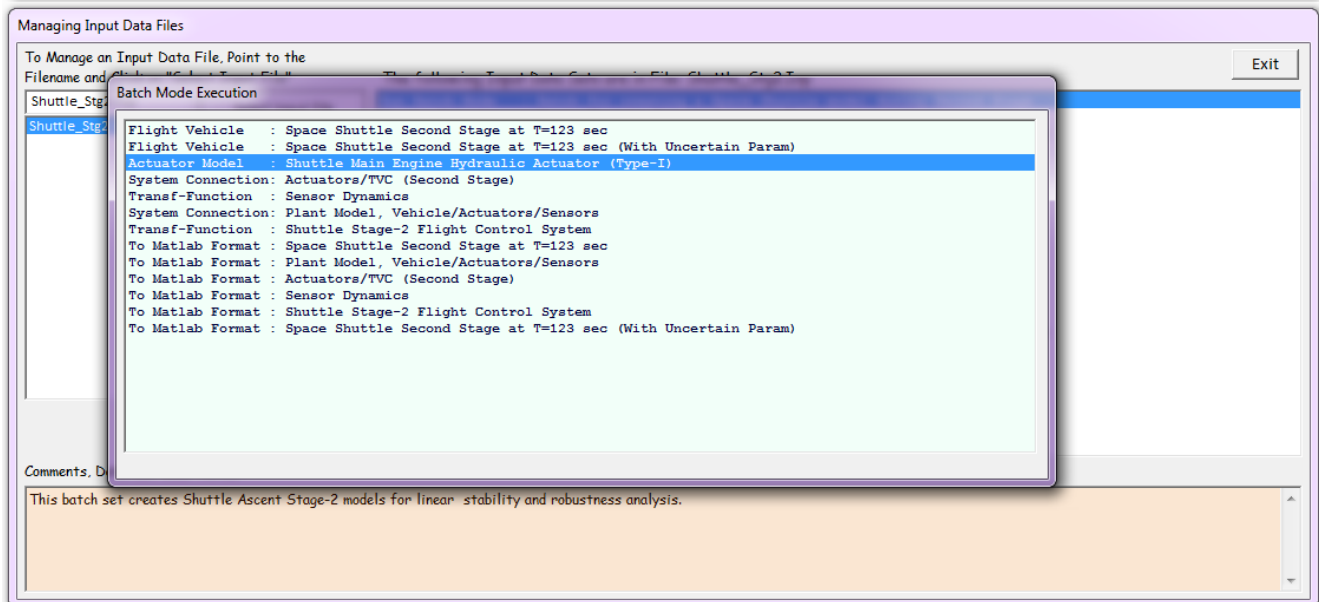
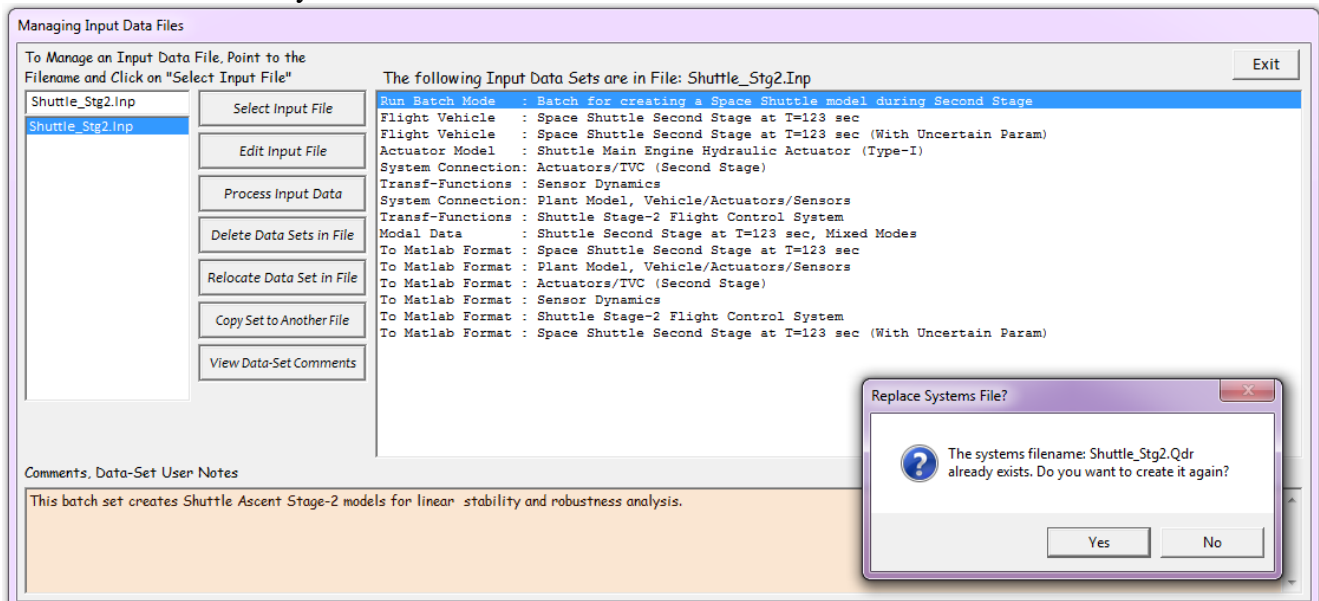
The input file for this example “*Shuttle\_Stg2.Inp*” is already prepared. It includes vehicle data, structural data, uncertainties, actuators, sensor models, the flight control system and systems interconnection data. It will be processed interactively in batch mode in order to generate the systems for the analysis. At the top of the file there is a batch set that automates the processing of the data-sets. It creates the state-space models and converts them to Matlab format for further analysis using Matlab. It begins by creating the two vehicle models: a standard model and an uncertainty model that includes the parameter variations data. The uncertain model is dynamically identical to the regular model but it includes one additional input/ output pair for each uncertainty. The batch generates also actuator models for the three SSME engines that include acceleration outputs for coupling the actuators with the acceleration inputs in the vehicle model. The acceleration inputs generate the tail-wag-dog dynamics. The actuator modeling program is used for generating the actuator state-space model that has a title: “*Shuttle Main Engine Hydraulic Actuator (Type-I)*”. This system is duplicated six times for the 3 main engines, 3 pitch and 3 yaw actuators. It is also combined with the TVC gains to form a single system “*Actuators/TVC (Second Stage)*”.

The batch also creates dynamic models for the gyro and IMU sensors using transfer-function block interconnections, its title is “*Sensor Dynamics*”. It combines the vehicle, actuators and sensor subsystems together into a single system “*Plant Model, Vehicle/ Actuators/ Sensors*”, and generates also the flight control system from transfer-function blocks “*Shuttle Stage-2 Flight Control System*”. The systems generated are saved in the systems file “*Shuttle\_Stg2.Qdr*”. The systems from the systems file are also re-formatted to Matlab m-functions to be analyzed for robustness in Matlab. The vehicle models are saved in files “*Vehicle.m*”, and “*Vehicle\_Unc.m*”. The sensors system is saved in “*Sensors.m*”. The combined actuator and TVC system is saved in “*Actor\_tvc.m*”. The flight control system is saved in “*Controller.m*”. The combined plant model is saved in “*Plant.m*”. These systems are loaded into Matlab by executing the script file “*run.m*”.





From the Flixan main menu go to “*Edit/ Process Input Data Files*”. The following dialog comes up and it is used for processing input data files. It has two menus. From the file selection menu on the left select the input file “*Shuttle\_Stg2.Inp*” and click on “*Select Input File*”. The menu on the right shows the titles of the data-sets which are included in this input file. Select the first title which is a batch “*Batch Set for Creating a Space Shuttle Model During Second Stage*” and click on “*Process Input Data*” button to process the batch set. The Flixan program will generate the systems and it will save them in systems file “*Shuttle\_Stg2.Qdr*”. A display window momentarily pops up highlighting the batch commands as they execute.



## Control Analysis

The folder “\Examples\Robust Analysis Param Uncertainties\Shuttle\_Ascent” includes several Simulink models used in this robustness analysis. The file “run.m” below loads the systems, and performs linear stability and robustness  $\mu$ -analysis. One of the simulation models is “Closed\_Loop\_Plant.mdl”, shown in figure (1a) below. It uses the combined plant model “Plant.m”, and the flight control system “Controller.m” in closed-loop form. The plant subsystem and the controller are shown in figures (1b and 1c). Figure 2 shows its transient responses to a wind gust step.

```
% Computes the Open-Loop Frequency Response for Flight Control Analysis
% Loop is opened at the Flight Control System Output
d2r=pi/180; r2d=180/pi;
Npv=45; % Number of Param Variations
% Load the control loop subsystems
[Av, Bv, Cv, Dv]= vehicle; % Vehicle Nominal Model
[Au, Bu, Cu, Du]= vehicle_unc; % Vehicle Model with Uncertainties
[At, Bt, Ct, Dt]= actor_tvc; % Actuator + TVC Model
[As, Bs, Cs, Ds]= sensors; % Sensors Model
[Ac, Bc, Cc, Dc]= controller; % Controller
[Ap, Bp, Cp, Dp]= plant; % Combined Plant Model

label='Stage-2 Yaw Axis Stability, (Pitch & Roll Closed)';
[Ao,Bo,Co,Do]= linmod('Open_Loop'); % Frequ Response Model (z-domain)
sys=ss(Ao,Bo,Co,Do); % Perform Linearization
w=logspace(-2,2,12000); % and Frequ domain analysis
figure(1); Nichols(sys,w); title(label)
figure(2); Bode(sys,w); title(label)

w=logspace(-2,2,500);
[Acp,Bcp,Ccp,Dcp]=linmod('Closed_Loop_Unc');
sys=ss(Acp,Bcp,Ccp,Dcp);
sysf= FRD(sys,w);
blk=[-ones(Npv,1), zeros(Npv,1)];
[bnd,muinfo]= mussv(sysf,blk);

ff= get(muinfo.bnds, 'frequency');
muu=get(muinfo.bnds, 'responsedata');
muu=squeeze(muu);
muu=muu(1,:);
loglog(ff,muu)
xlabel('Frequency (rad/sec)')
ylabel('ssv')
```

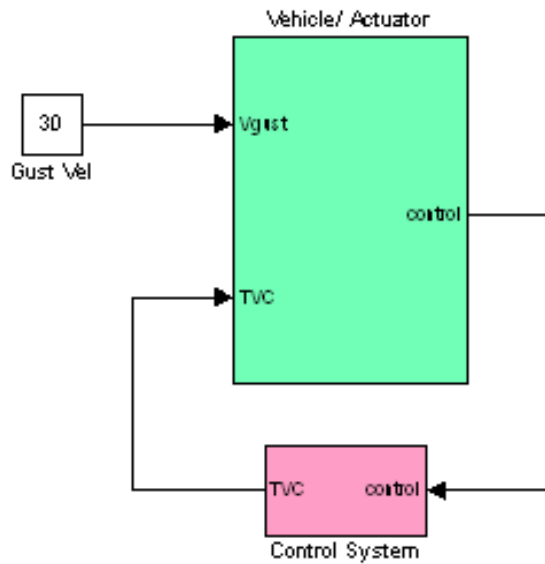


Figure (1a) Plant Model and Flight Control System in Closed-Loop Form

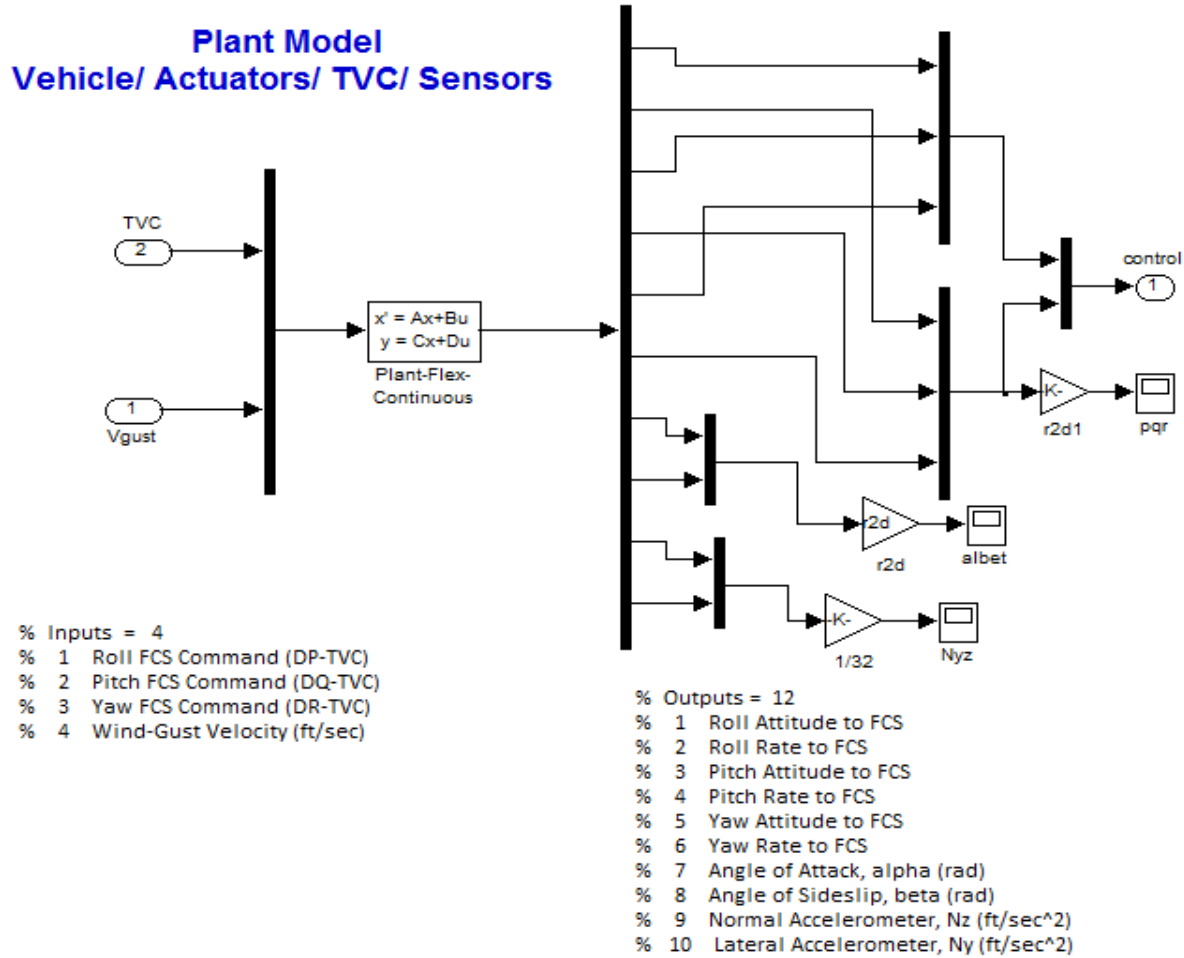


Figure (1b) Plant model from file "plant.m" consisting of Vehicle, Actuators, TVC, and Sensor dynamics

## Attitude and Rate Errors

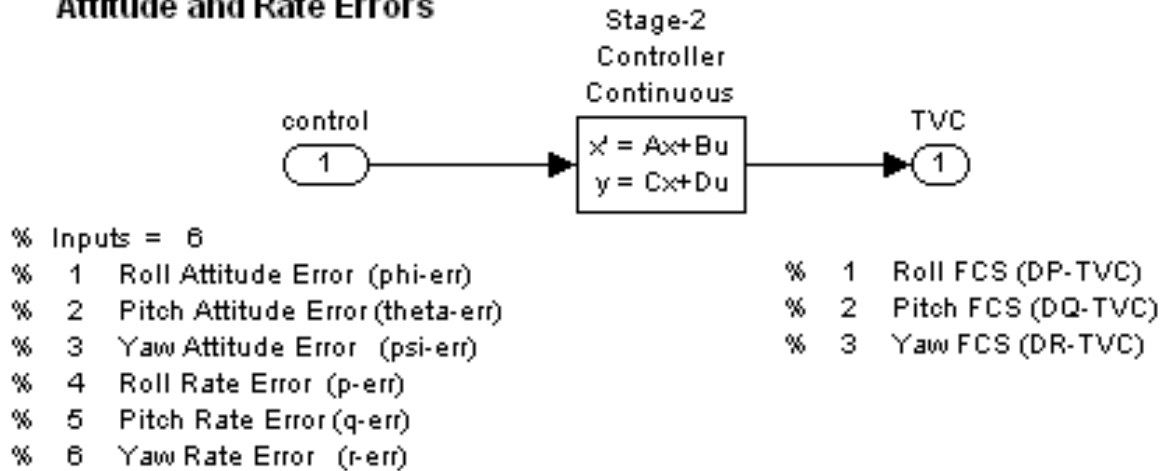


Figure 1(c) Shuttle Second Stage flight control system converted to State-Space form

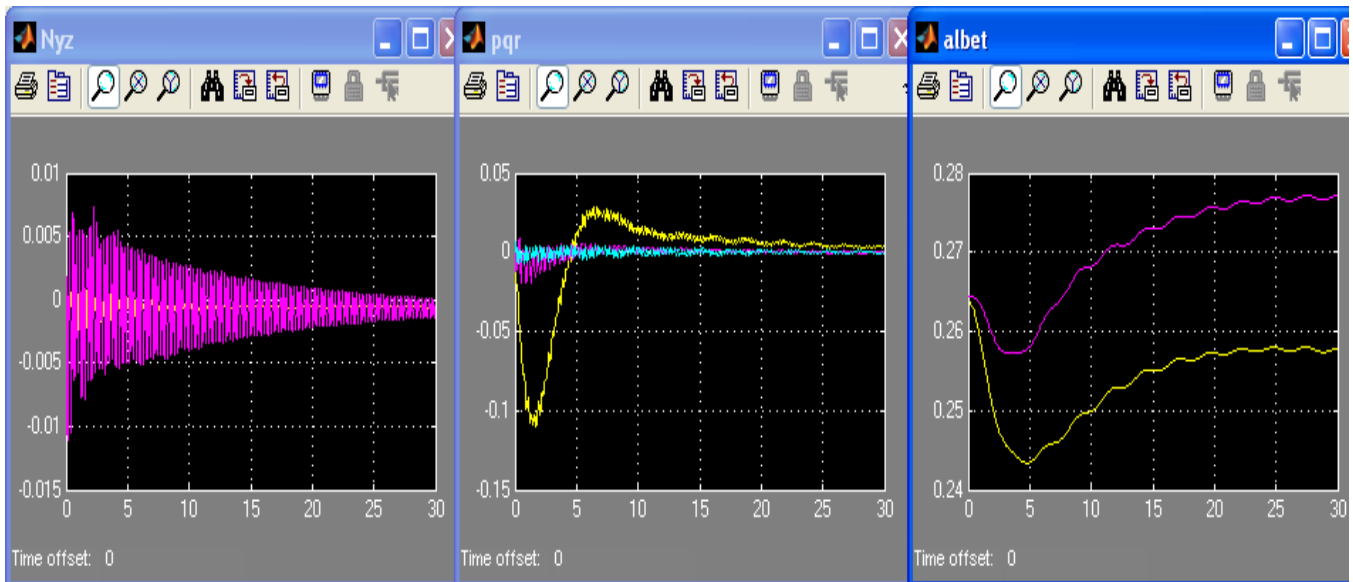


Figure 2 Vehicle Responses to a Wind-Gust Step

## Robustness Analysis Model

Figure 3 shows the closed-loop Simulink model “*Closed\_Loop\_Unc.mdl*” that it is used for analyzing robustness, and it includes the uncertainty input/output attachments. It consists of the following subsystems:

1. Vehicle model “*Space Shuttle Second Stage at T=123 sec (With Uncertain Param)*” which is loaded into Matlab from file “*Vehicle\_Unc.m*”, and it is shown in detail in Figure 3b.
2. Actuators/TVC model is loaded from file “*Actor\_tvc.m*” and it shown in detail in Figure 3c
3. Sensors, and
4. Control System

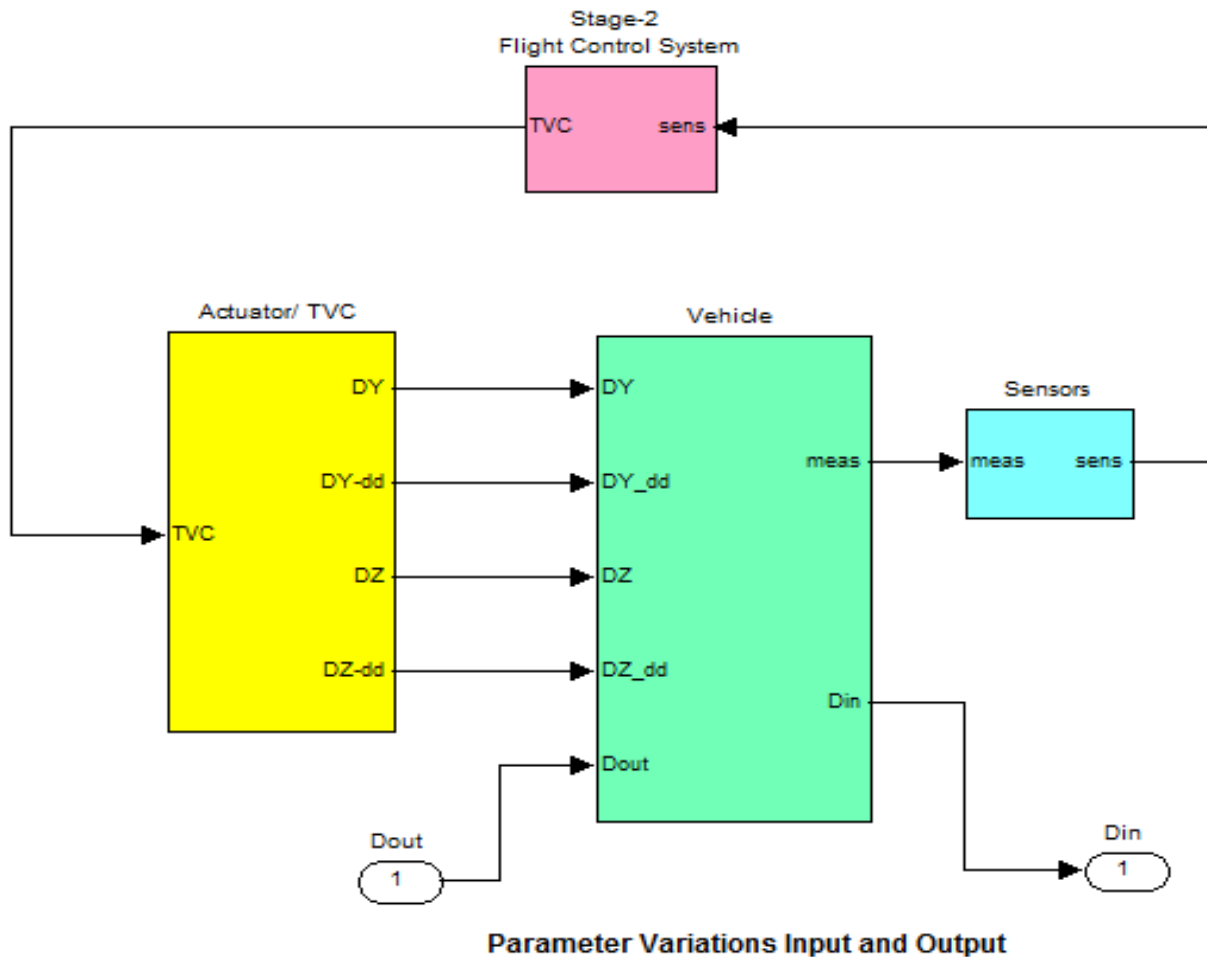
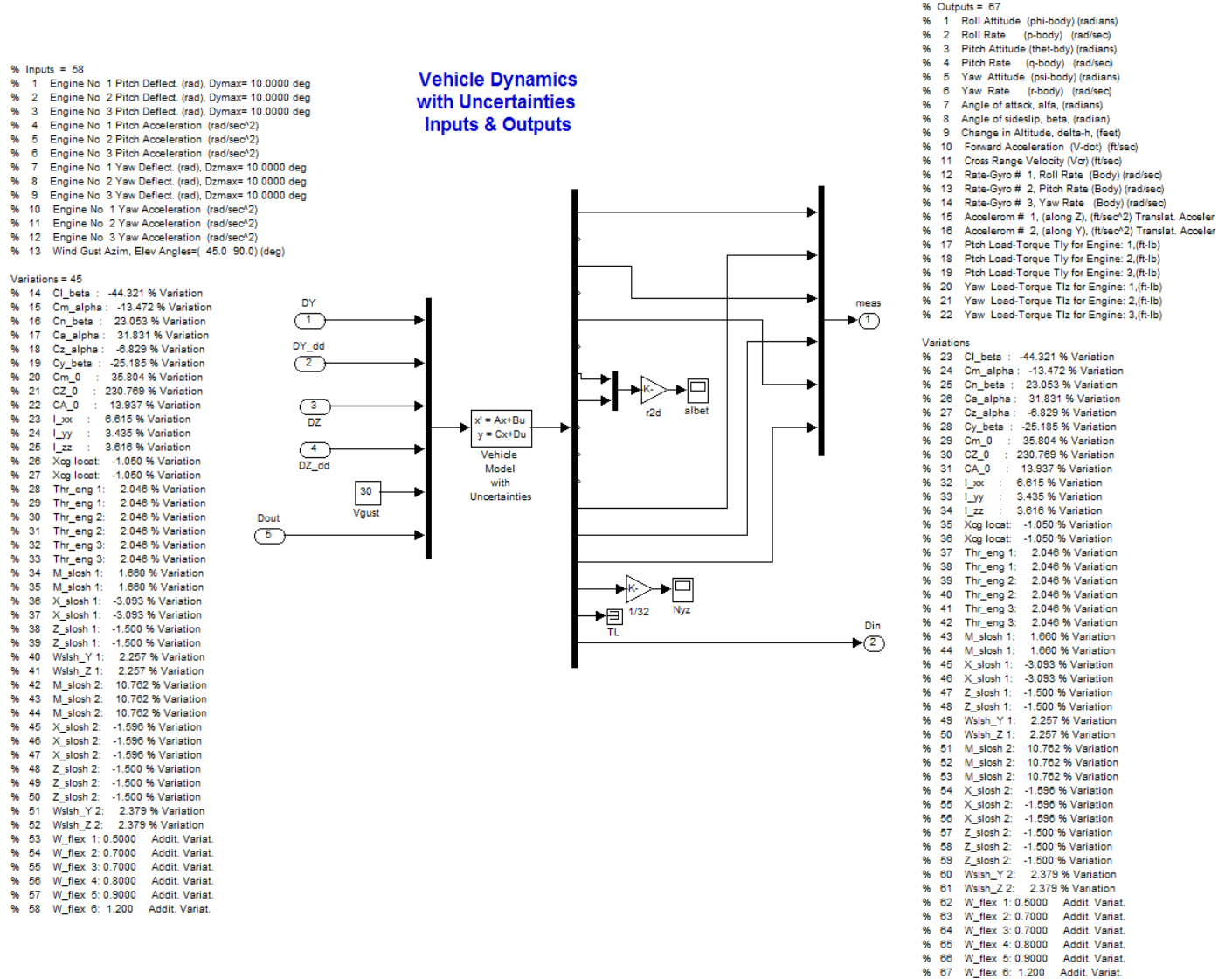


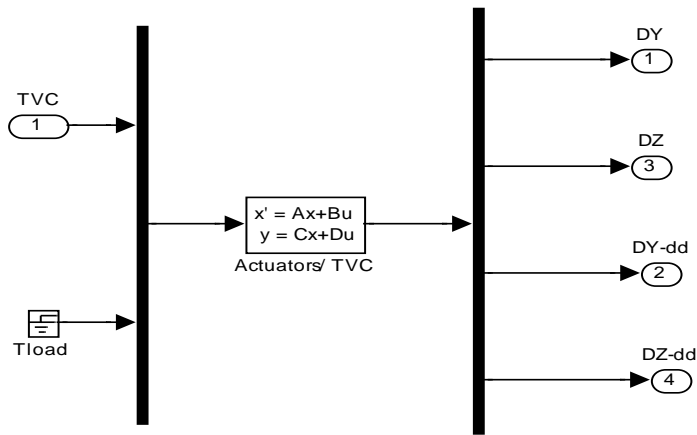
Figure 3, Shuttle vehicle model with the flight control system loop closed. It includes the input and output vectors that connect to the uncertainty block  $\Delta$ .



The dynamic model includes the input and output attachments that connect to the uncertainty block  $\Delta$ . The system's uncertainty inputs and outputs are scaled so that the elements ( $\delta_i$ ) of the  $\Delta$  block vary between -1 and +1. This model is used in robustness analysis by calculating the ( $\mu$ ) structural singular value (SSV) frequency response between the uncertainty inputs and outputs. According to small gain theory, the system is robust to the uncertainties (defined by the parameter variations) when the SSV of the closed-loop system, as seen across  $\Delta$ , is less than one at all frequencies.



**Figure 3b, State-space model consisting of vehicle dynamics plus 45 additional input/ output pairs that connect across the uncertainty block  $\Delta$ .**



```

% Inputs = 9
% 1 Roll FCS Command (DPTVC)
% 2 Pitch FCS Command (DQTVc)
% 3 Yaw FCS Command (DRTVC)
% 4 Engine 1 Pitch Load-Torque TLY1
% 5 Engine 2 Pitch Load-Torque TLY2
% 6 Engine 3 Pitch Load-Torque TLY3
% 7 Engine 1 Yaw Load-Torque TLZ1
% 8 Engine 2 Yaw Load-Torque TLZ2
% 9 Engine 3 Yaw Load-Torque TLZ3

% Outputs = 12
% 1 Engine 1 Pitch Deflection (rad)
% 2 Engine 2 Pitch Deflection (rad)
% 3 Engine 3 Pitch Deflection (rad)
% 4 Engine 1 Yaw Deflection (rad)
% 5 Engine 2 Yaw Deflection (rad)
% 6 Engine 3 Yaw Deflection (rad)
% 7 Engine 1 Pitch Accelerat (rad/sec^2)
% 8 Engine 2 Pitch Accelerat (rad/sec^2)
% 9 Engine 3 Pitch Accelerat (rad/sec^2)
% 10 Engine 1 Yaw Accelerat (rad/sec^2)
% 11 Engine 2 Yaw Accelerat (rad/sec^2)
% 12 Engine 3 Yaw Accelerat (rad/sec^2)

```

Figure 3c, State-space system consisting of a combination of 6 engine actuators, 3 pitch and 3 yaw, including the TVC. The gimbal acceleration outputs drive the tail-wag-dog dynamics. The Load-Torque feedback is ignored.

## Open-Loop Frequency Response Analysis

The following Simulink model “*Open\_Loop.mdl*”, shown in Figure 4, is used for calculating open-loop frequency responses and analyzing classical stability by measuring the gain and phase margins. The configuration shown below (yaw loop opened, roll and pitch loops closed) is used for calculating the frequency response and stability margins in the yaw direction, as shown in the Nichols plot of Figure 5. The block diagram must be modified accordingly for calculating the pitch or roll frequency responses and stability margins.

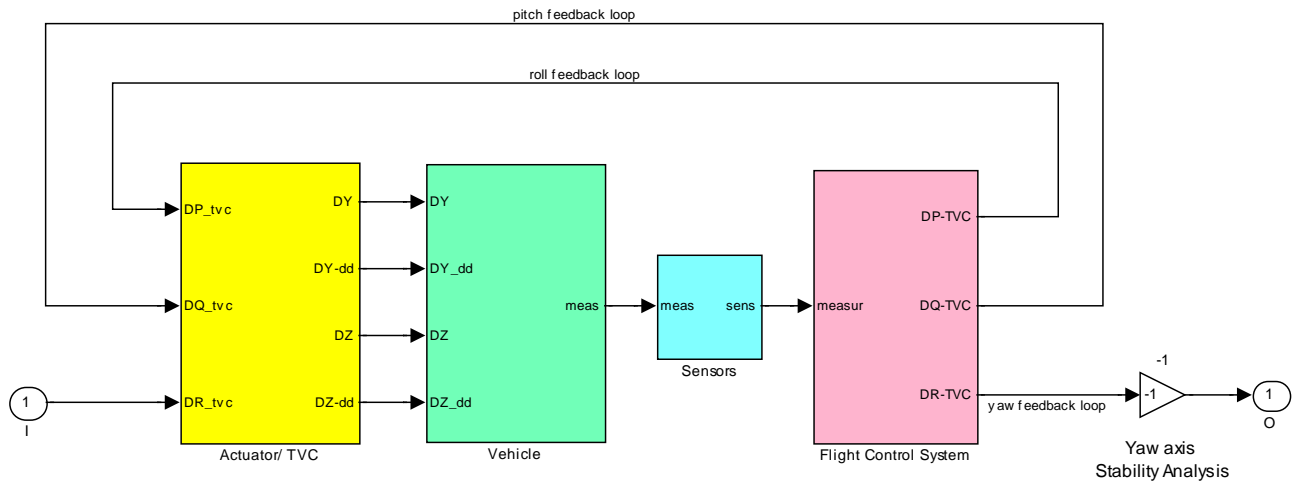
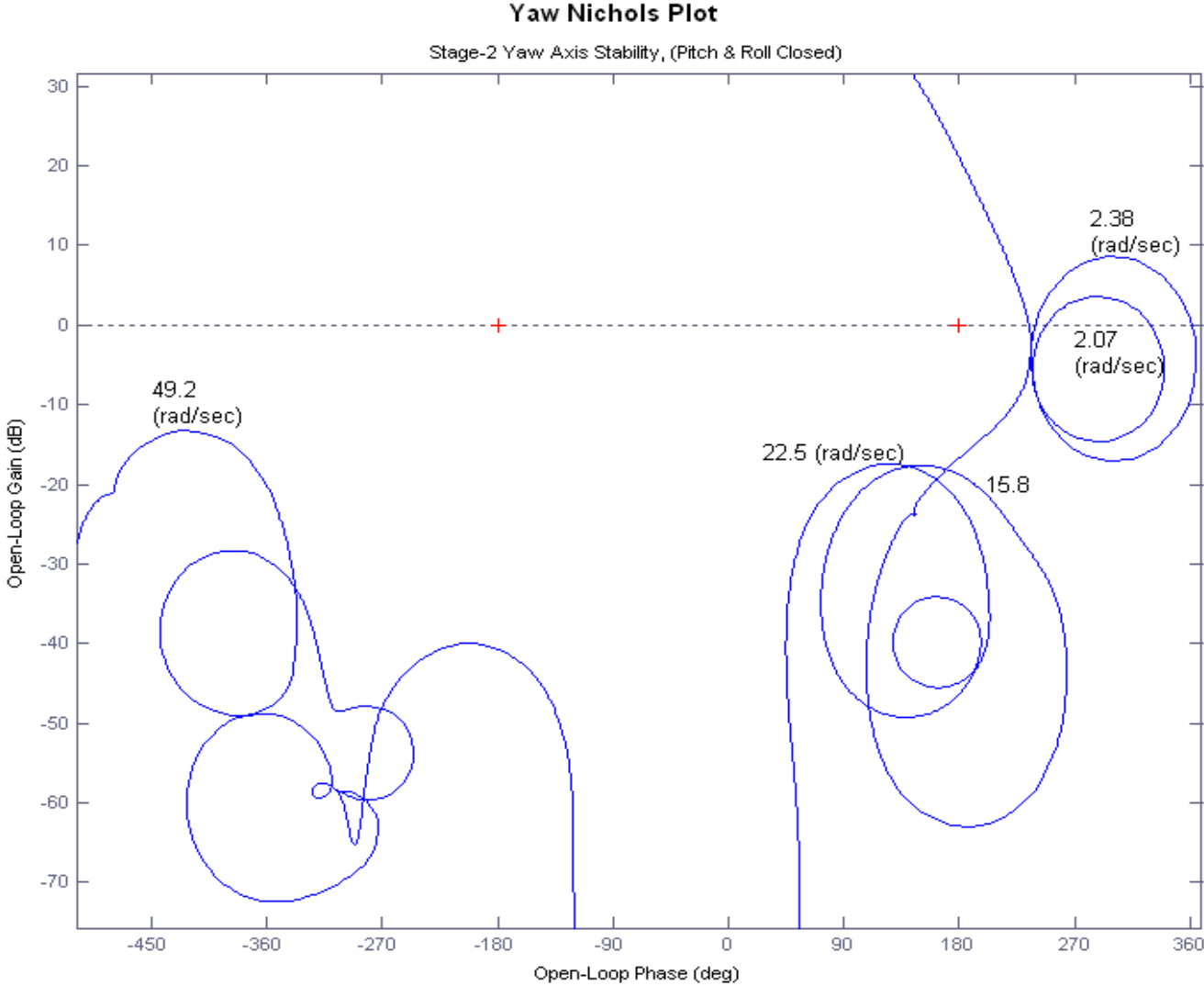


Figure 4, Open-Loop Model for Linear Stability Analysis

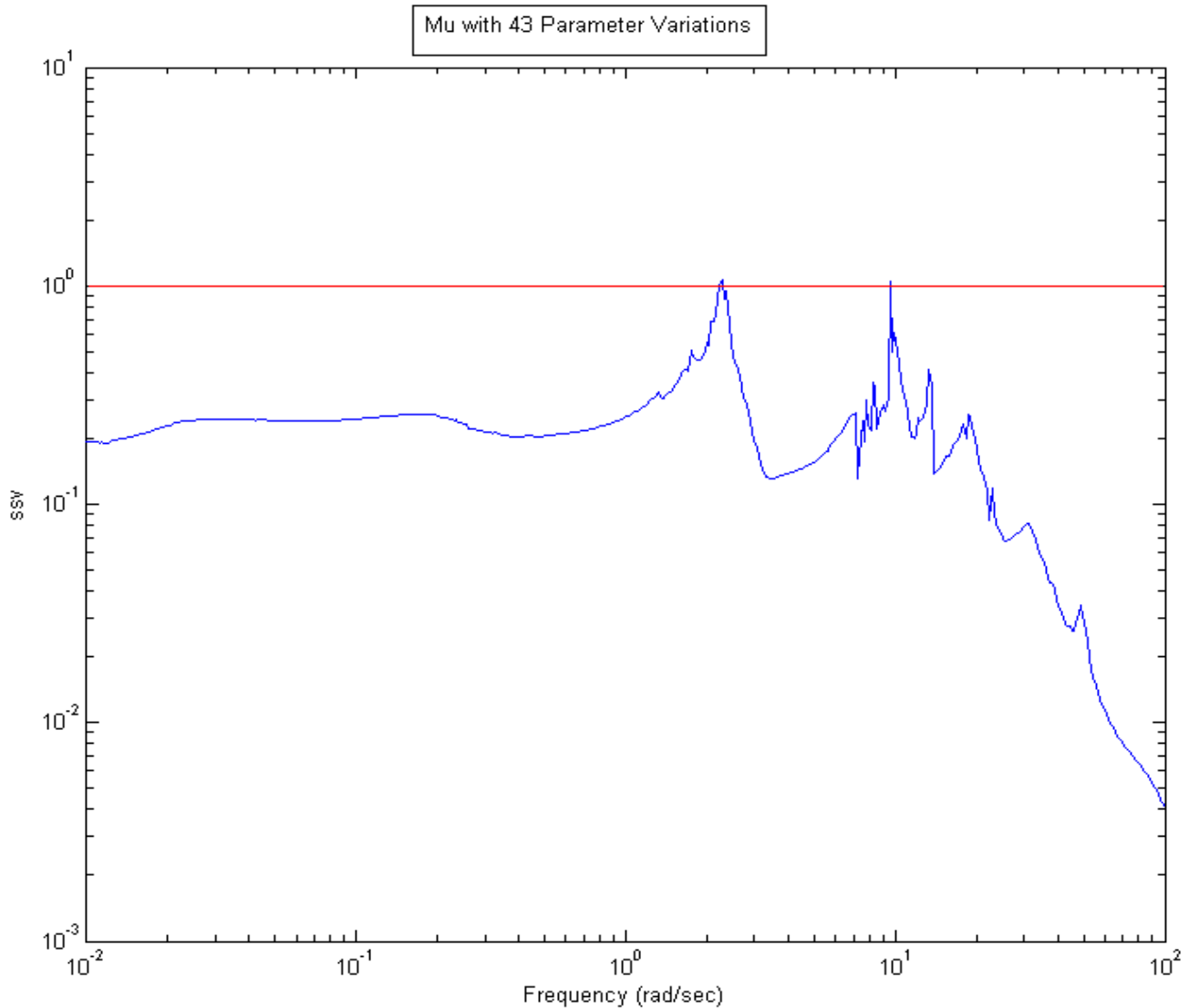
The Matlab file “run.m” uses the open-loop model “*Open\_Loop.mdl*” described in Figure-4 to calculate the frequency response and use the Nichols chart in Figure-5 to perform classical stability analysis.



**Figure 5, Yaw axis Nichols plot shows slosch and flex resonances. It also shows that the flight control system has sufficient phase and gain margins**

## Mu Analysis

The Matlab script file “run.m” also uses the closed-loop model with uncertainties “*Closed\_Loop-Unc.mdl*” to calculate the  $\mu$  frequency response across the scaled  $\Delta$  block, as shown in Figure 6. The structural singular value plot shows that the closed-loop system barely satisfies the  $\mu[M(\omega)] < 1$  requirement for robustness. It means that the control system is capable of tolerating the specified uncertainties without becoming unstable. The Matlab  $\mu$ -analysis algorithm assumes real parameter variations. Note that complex variations are too conservative in this case due to the low damped resonances and it would violate the robustness criterion.



**Figure 6,  $\mu$ -analysis Shows that the Control System Satisfies the Robustness Requirements**

# Robustness Analysis of a Flexible Satellite with Reaction Wheels



In this example we will analyze robustness of a flexible satellite that is controlled by 3 reaction wheels. The project files are in folder “\Flixan\Examples\Robust Analysis Param Uncertainties\ Satellite\_RW”. The input data file that contains the satellite parameters is “Satellite\_RW.Inp”. Its title is “Flex Satellite with Reaction Wheels”. This data set in addition to vehicle mass properties it contains also: the spin axis direction of the 3 reaction wheels, their initial rotational speeds in (rpm) which are zero in this case, and the moment of inertia for each wheel about its spin axis, which is 0.1 (slug-ft<sup>2</sup>). It contains also 3 rate gyros, 3 attitude sensors, and 2 accelerometer sensors. The satellite model also contains 60 structural flexibility modes. The modes are already pre-processed and selected from a finite elements model and the selected modes are included in the input data file “Satellite\_RW.Inp”. The flex modes title is: “Flex Satellite with Reaction Wheels (60 Flex Modes)”. The block of data for each mode contains the mode frequency and the mode shapes at the locations of the 3 reaction wheels, the 6 gyros, and the 2 accelerometers. The modal data title is also included in the satellite input data-set, under the line “Number of Flex Modes: 60”.

The input file “Satellite\_RW.Inp” also includes the uncertainties data-set located below the satellite input data. The Flixan program recognizes the parameter variations data-set by the label “UNCERTAIN PARAMETER VARIATIONS FROM NOMINAL ...”. The uncertainties title is “Uncertainties for the Flex Satellite with Reaction Wheels” and it is located below the label. It is also included in the satellite data below the line “Parameter Uncertainties Data”. The uncertainties data consists of variations in the moments of inertia, in the RW spin axis directions, the RW momentum bias, and the rotor moment of inertia. Variations in some of the flex mode frequencies are also included. The flex mode numbers that have uncertain frequencies appear in line “Flex Mode Uncertainties (Mode Number): 1 3 4 5 6 42 43 50”. The corresponding max frequency variations are included in the next line “Flex Mode Frequency Variation (additive): 0.3 0.4 0.6 2.0 2.0 10.0 10.0 10.0”.

**FLIGHT VEHICLE INPUT DATA .....**

**Flex Satellite with Reaction Wheels**

! This is a simple Satellite model that uses 3 Reaction Wheels, one wheel per axis.  
! It contains also 60 flex modes. The modal data are included in bottom of file.  
! Title of Parameter Uncertainties data-set is also included for IFL modeling.  
! The augmented spacecraft model is used to analyze robustness to structural uncertainties  
!

**Body Axes Output, Euler Angles**

Vehicle Mass (lb-sec<sup>2</sup>/ft), Gravity Accelerat. (g) (ft/sec<sup>2</sup>), Earth Radius (Re) (ft) : 50.0 0.0001 0.20896E+08  
Moments and products of Inertias Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec<sup>2</sup>-ft) : 1045.3, 641.02, 505.05,  
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet) : -0.05, 0.1, 22.625  
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet) : 0.0 29000.0 0.0001  
Inertial Acceleration Vo\_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec<sup>2</sup>) : 0.0 0.0 0.0 0.0  
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec) : 0.0 0.0  
Vehicle Attitude Euler Angles, Phi\_o,Thet\_o,Psi\_o (deg), Body Rates Po,Qo,Ro (deg/sec) : 0.0 0.0 0.0 0.0, -0.0647,  
External Torque and direction about (x,y,z) : Torque 0.0 1.0 0.0  
Surface Reference Area (feet<sup>2</sup>), Mean Aerodynamic Chord (ft), Wing Span in (feet) : 0.0 1.0 1.0  
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial\_rho/ Partial\_H} : 0.0 0.0 0.0 0.0  
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca\_alf,PCa/PV,PCa/Ph,Ca\_alfdot,Ca\_q,Ca\_bet} : 0.0 0.0 0.0 0.0  
Aero Force Coeff/Derivat (1/deg), Along Y, {Cyo,Cy\_bet,Cy\_r,Cy\_alf,Cy\_p,Cy\_betdot,Cy\_V} : 0.0 -0.0 0.0 0.0 0.0  
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz\_alf,Cz\_q,Cz\_bet,PCz/Ph,Cz\_alfdot,PCz/PV} : 0.0 0.0 0.0 0.0  
Aero Moment Coeff/Derivat (1/deg), Roll: {Clo, Cl\_beta, Cl\_betdot, Cl\_p, Cl\_r, Cl\_alfa} : 0.0 0.0 0.0 0.0 0.0  
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm\_alfa,Cm\_alfdot,Cm\_bet,Cm\_q,PCm/PV,PCm/Ph} : 0.0 0.0 0.0 0.0 0.0  
Aero Moment Coeff/Derivat (1/deg), Yaw : {Cno, Cn\_beta, Cn\_betdot, Cn\_p, Cn\_r, Cn\_alfa} : 0.0 0.0 0.0 0.0 0.0  
  
Number of Thruster Engines, RCS Jets, No Engine Gimbaling : 0  
  
Number of Reaction Wheels : 3  
RW No: 1, Spin Direct. Unit Vect, Init Rot Speed (rpm), Rotor Mom of Inertia (slg-ft<sup>2</sup>) : 1.0 0.0 0.0 0.0 0.1  
RW No: 2, Spin Direct. Unit Vect, Init Rot Speed (rpm), Rotor Mom of Inertia (slg-ft<sup>2</sup>) : 0.0 1.0 0.0 0.0 0.1  
RW No: 3, Spin Direct. Unit Vect, Init Rot Speed (rpm), Rotor Mom of Inertia (slg-ft<sup>2</sup>) : 0.0 0.0 1.0 0.0 0.1  
  
Number of Gyros, (Attitude and Rate) : 6  
Gyro No 1 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 6 : Roll Rate -0.747 0.114  
Gyro No 2 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 6 : Pitch Rate -0.747 0.114  
Gyro No 3 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 6 : Yaw Rate -0.747 0.114  
Gyro No 4 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 6 : Roll Attitu -0.747 0.114  
Gyro No 5 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 6 : Pitch Attitu -0.747 0.114  
Gyro No 6 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 6 : Yaw Attitu -0.747 0.114  
  
Number of Accelerometers, Along Axes: (x,y,z) : 2  
Acceleromet No 1 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 7 : X-axis Accel 0.338 0.64 25  
Acceleromet No 2 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 7 : Y-axis Accel 0.338 0.64 25

**Parameter Uncertainties Data**

**Uncertainties for the Flex Satellite with Reaction Wheels**

Number of Bending Modes : 60  
**Flex Satellite with Reaction Wheels (60 Flex Modes)**

**UNCERTAIN PARAMETER VARIATIONS FROM NOMINAL .....**

**New Uncertainties for Satellite**

! This is a new set of parameter variations for a Satellite with three Reaction Wheels. The wheel  
! moment of inertia about the rotor, and the spin axis direction are not known very accurately.  
! Also the wheel speed is unknown. There are also uncertainties in the spacecraft moments of  
! inertia, and also in the flex mode frequencies and damping coefficient (zeta).

Vehicle Mass (lb-sec<sup>2</sup>/ft), Gravity Accelerat. (g) (ft/sec<sup>2</sup>), Earth Radius (Re) (ft) : 0.00 0.0 0.0  
Moments and products of Inertias Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec<sup>2</sup>-ft) : 50.0 30.0 25.0 0.0  
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet) : 0.0 0.0 0.0  
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet) : 0.00 0.0 0.0  
Inertial Acceleration Vo\_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec<sup>2</sup>) : 0.0 0.0 0.0 0.0  
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec) : 0.0 0.0 0.0 0.0  
Vehicle Attitude Euler Angles, Phi\_o,Thet\_o,Psi\_o (deg), Body Rates Po,Qo,Ro (deg/sec) : 0.0 0.0 0.0 0.0  
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca\_alf,PCa/PV,PCa/Ph,Ca\_alfdot,Ca\_q,Ca\_bet} : 0.0 0.0 0.0 0.0  
Aero Force Coeff/Derivat (1/deg), Along Y, {Cyo,Cy\_bet,Cy\_r,Cy\_alf,Cy\_p,Cy\_betdot,Cy\_V} : 0.0 0.0 0.0 0.0  
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz\_alf,Cz\_q,Cz\_bet,PCz/Ph,Cz\_alfdot,PCz/PV} : 0.0 0.0 0.0 0.0  
Aero Moment Coeff/Derivat (1/deg), Roll: {Clo, Cl\_beta, Cl\_betdot, Cl\_p, Cl\_r, Cl\_alfa} : 0.0 0.0 0.0 0.0  
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm\_alfa,Cm\_alfdot,Cm\_bet,Cm\_q,PCm/PV,PCm/Ph} : 0.0 0.0 0.0 0.0  
Aero Moment Coeff/Derivat (1/deg), Yaw : {Cno, Cn\_beta, Cn\_betdot, Cn\_p, Cn\_r, Cn\_alfa} : 0.0 0.0 0.0 0.0  
  
Number of Reaction Wheels, (Variations from Nominal Wheel Parameters) : 3  
RW No: 1, Spin Direct. Unit Vect, Init Rot Speed (rpm), Rotor Mom of Inertia (slg-ft<sup>2</sup>) : 0.0 0.1 0.1 10.0 0.02  
RW No: 2, Spin Direct. Unit Vect, Init Rot Speed (rpm), Rotor Mom of Inertia (slg-ft<sup>2</sup>) : 0.1 0.0 0.1 10.0 0.02  
RW No: 3, Spin Direct. Unit Vect, Init Rot Speed (rpm), Rotor Mom of Inertia (slg-ft<sup>2</sup>) : 0.1 0.1 0.0 10.0 0.02  
  
Flex Mode Uncertainties (Mode Number) : 1 3 4 5 6 42  
43 50  
Flex Mode Frequency (omega) Variation (additive) (rad/sec) : 0.3 0.4 0.6 2.0 2.5 25  
Flex Mode Damping Coefficient (zeta) Variation (additive) : 0.001 0. 0.001 0.0 0.001 0.

**SELECTED MODAL DATA AND LOCATIONS FOR : Pre-Selected  
Flex Satellite with Reaction Wheels (60 Flex Modes)**

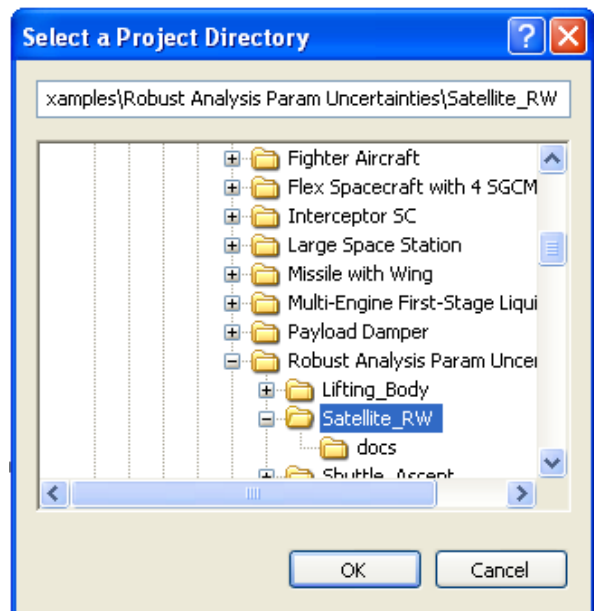
! The flex modes used in this analysis are the same modes that were selected and used in a previous flex spacecraft analysis. We start from Mode #7 because the first 6 rigid-body modes are not included in the modal data because the rigid-body dynamics are included in the vehicle. ! The Nodes correspond to vehicle locations defined in the vehicle data above.

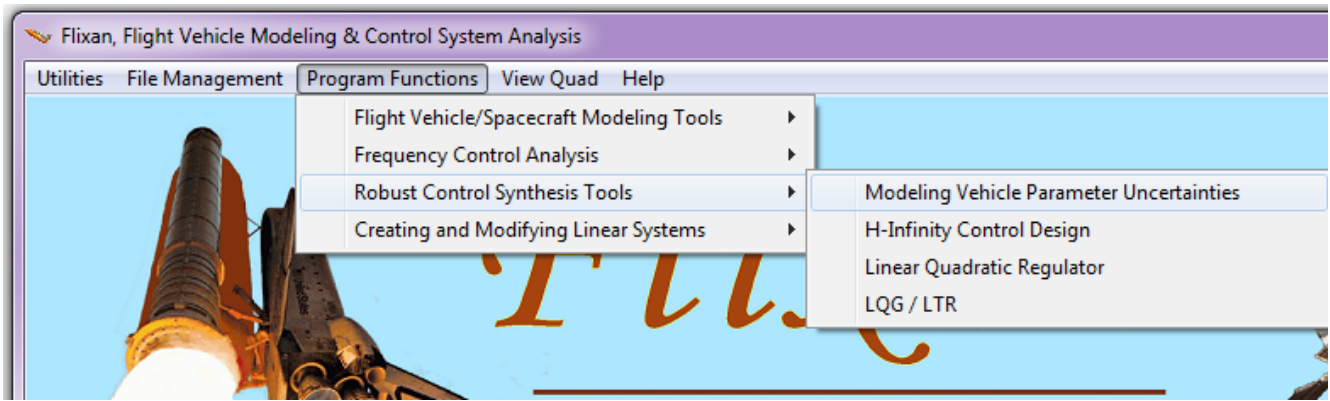
MODE#	1/ 7,	Frequency (rad/sec),	Damping (zeta),	Generalized Mass=	3.0998	0.50000E-02	12.000	
DEFINITION OF LOCATIONS (NODES)		phi along X	phi along Y	phi along Z	sigm about X	sigm about Y	sigm	
	Node ID#	Modal Data at the 3 Reaction Wheels...						
Reaction Wheel #1 CG	58041	-0.20700D+00	-0.43105D-03	0.41763D-01	-0.11013D-02	0.64943D-01	-	
Reaction Wheel #2 CG	58042	-0.20783D+00	-0.16859D-02	-0.31758D-01	-0.11002D-02	0.64942D-01	-	
Reaction Wheel #3 CG	58043	-0.20632D+00	-0.17810D-02	-0.38780D-01	-0.11033D-02	0.64942D-01	-	
	Node ID#	Modal Data at the 6 Gyros ...						
Inertial Attitude Sensors	31001	0.51466D-01	0.39036D-02	0.38773D-01	-0.11017D-02	0.64943D-01	-	
Inertial Attitude Sensors	31001	0.51466D-01	0.39036D-02	0.38773D-01	-0.11017D-02	0.64943D-01	-	
Inertial Attitude Sensors	31001	0.51466D-01	0.39036D-02	0.38773D-01	-0.11017D-02	0.64943D-01	-	
Inertial Attitude Sensors	31001	0.51466D-01	0.39036D-02	0.38773D-01	-0.11017D-02	0.64943D-01	-	
Inertial Attitude Sensors	31001	0.51466D-01	0.39036D-02	0.38773D-01	-0.11017D-02	0.64943D-01	-	
Inertial Attitude Sensors	31001	0.51466D-01	0.39036D-02	0.38773D-01	-0.11017D-02	0.64943D-01	-	
	Node ID#	Modal Data at the 2 Accelerometers, along (x,y,z)...						
Accelerometers	31002	0.51889D-01	0.27108D-02	-0.32279D-01				
Accelerometers	31002	0.51889D-01	0.27108D-02	-0.32279D-01				
	Node ID#	Modal Data at the Disturbance Point						
RCS Jet #8 (+X)	98008	-0.19474D+00	0.14764D-02	0.14237D+00	-0.11013D-02	0.64937D-01		

## Generating the Uncertain Satellite State-Space Model

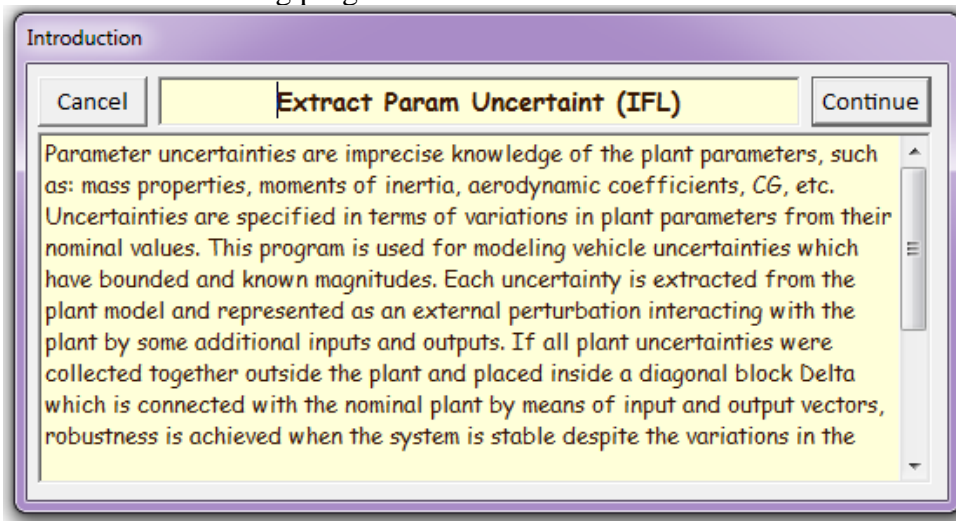
There are several ways of processing this input data file in Flixan and to create the satellite state-space model with the additional IFL uncertainty I/Os. It can either be processed by running the batch set which is located on the top of file “*Satellite\_RW.Inp*” similar to the previous two examples, or it can be processed by running the flight vehicle modeling program just like we would normally run a typical flight vehicle input data. The uncertainties data will also be processed because they are referenced by their title in the vehicle data. Notice, that if the uncertainties title is not included in the vehicle data they will be ignored and the vehicle model will not include the additional input/ output pairs that connect to the  $\Delta$  block. In this example we will use the third option which is to run the parameter uncertainties modeling program. The uncertainties are already in the input file.

Start the Flixan program and select the current project folder: “\Flixan\Examples\Robust Analysis Param Uncertainties\Satellite\_RW”. Then from the main menu select “*Program Functions*”, “*Robust Control Synthesis Tools*”, and “*Modeling Vehicle Parameter Uncertainties*”, as shown below.

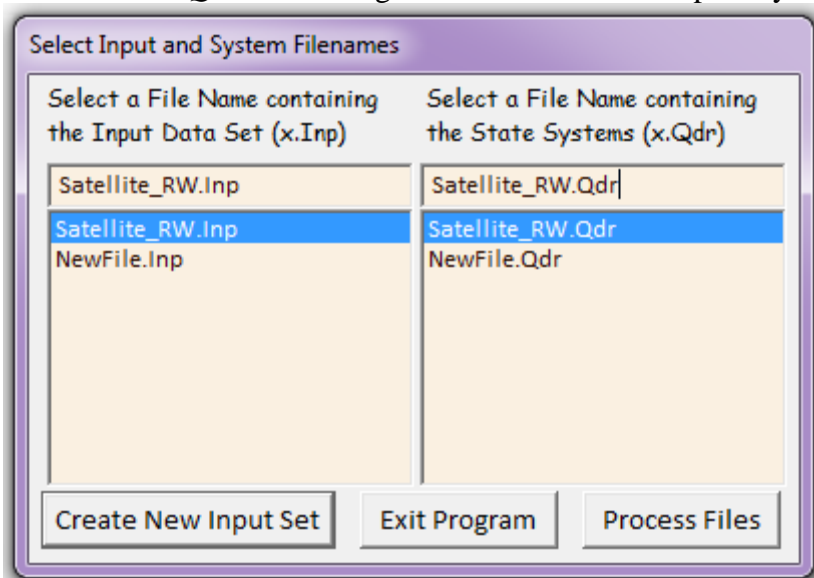




The following window presents an introductory paragraph that describes the function of the parameter uncertainties modeling program. Click on “*Continue*” to continue.

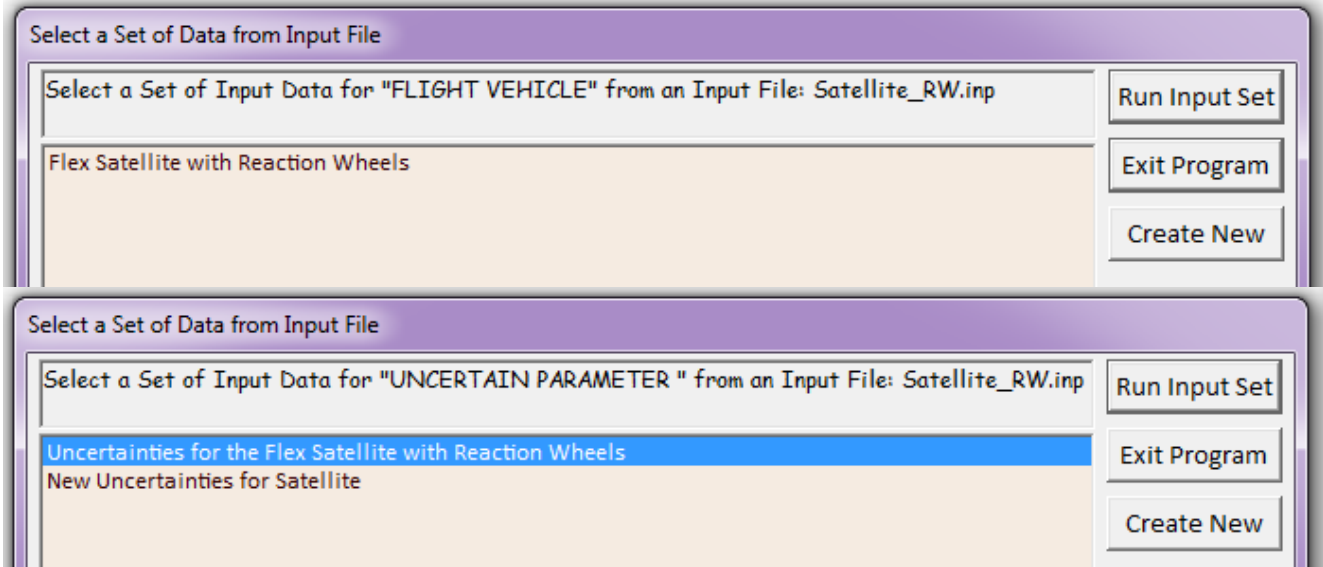


Select the already prepared satellite input data file “*Satellite\_RW.Inp*” and the systems file “*Satellite\_RW.Qdr*” for saving the flex satellite state-space system and click on “*Process Files*”.





The following menu shows the titles of flight vehicle input data-sets which are in file “Satellite\_RW.Inp”. In this case there is only one set, we select “Flex Satellite with Reaction Wheels” and click on “Run Input Set”. Similarly, the following menu shows the titles of uncertainty data-sets in the input file. Select the title “Uncertainties for the Flex Satellite with Reaction Wheels” and click on “Run Input Set”.



The following dialog consists of various tabs and displays for validating or modifying the uncertainties data. The uncertainties title appears on the top. Each tab consists of a group of data. In the mass properties tab below the satellite mass properties and the max variations are shown. The nominal vehicle data are in the light blue fields on the left and the max parameter variations are on the right. The user may also check the reaction wheel data, and the flex mode variations by clicking in the appropriate tabs.

Flight Vehicle Parameter Uncertainties

**Uncertainties Title**  
**Uncertainties for the Flex Satellite with Reaction Wheels**

Gimbal Engines or Jets. Include Tail-Wags-Dog? 
 Reaction Wheels? 
 Structure Bending Modes

Rotating Control Surfaces. Include Tail-Wags-Dog? 
 Single Gimbal CMGs? 
 Fuel Sloshing Modes

Reaction Wheels	Single Gimbal CMGs	Double Gimbal CMG System	Fuel Slosh	Flex Modes	User Comments
Mass Properties	Trajectory Data	Aero Force Coeffs	Aero Moment Coeffs	Control Surfaces	Gimbal Engines/ RCS

**Vehicle Mass Properties**

	Nominal Data	Param. Variations
Vehicle Mass in (slugs)	50.00000	0.00000
Moments of Inertia in (slugs-ft <sup>2</sup> )	Ixx	1045.300
	Iyy	641.0200
	Izz	505.0500
Products of Inertia in (slugs-ft <sup>2</sup> )	Ixy	0.00000
	Ixz	-19.00000
	Iyz	-40.00000
CG Location in (feet)	Xcg	-0.5000000E-01
	Ycg	0.1000000
	Zcg	22.62500

Flight Vehicle Parameter Uncertainties

**Uncertainties Title**  
**Uncertainties for the Flex Satellite with Reaction Wheels**

Gimbal Engines or Jets. Include Tail-Wags-Dog? 
 Reaction Wheels? 
 Structure Bending Modes

Rotating Control Surfaces. Include Tail-Wags-Dog? 
 Single Gimbal CMGs? 
 Fuel Sloshing Modes

Mass Properties	Trajectory Data	Aero Force Coeffs	Aero Moment Coeffs	Control Surfaces	Gimbal Engines/ RCS
Reaction Wheels	Single Gimbal CMGs	Double Gimbal CMG System	Fuel Slosh	Flex Modes	User Comments

**This Vehicle has 3 Reaction/Momentum Wheels**

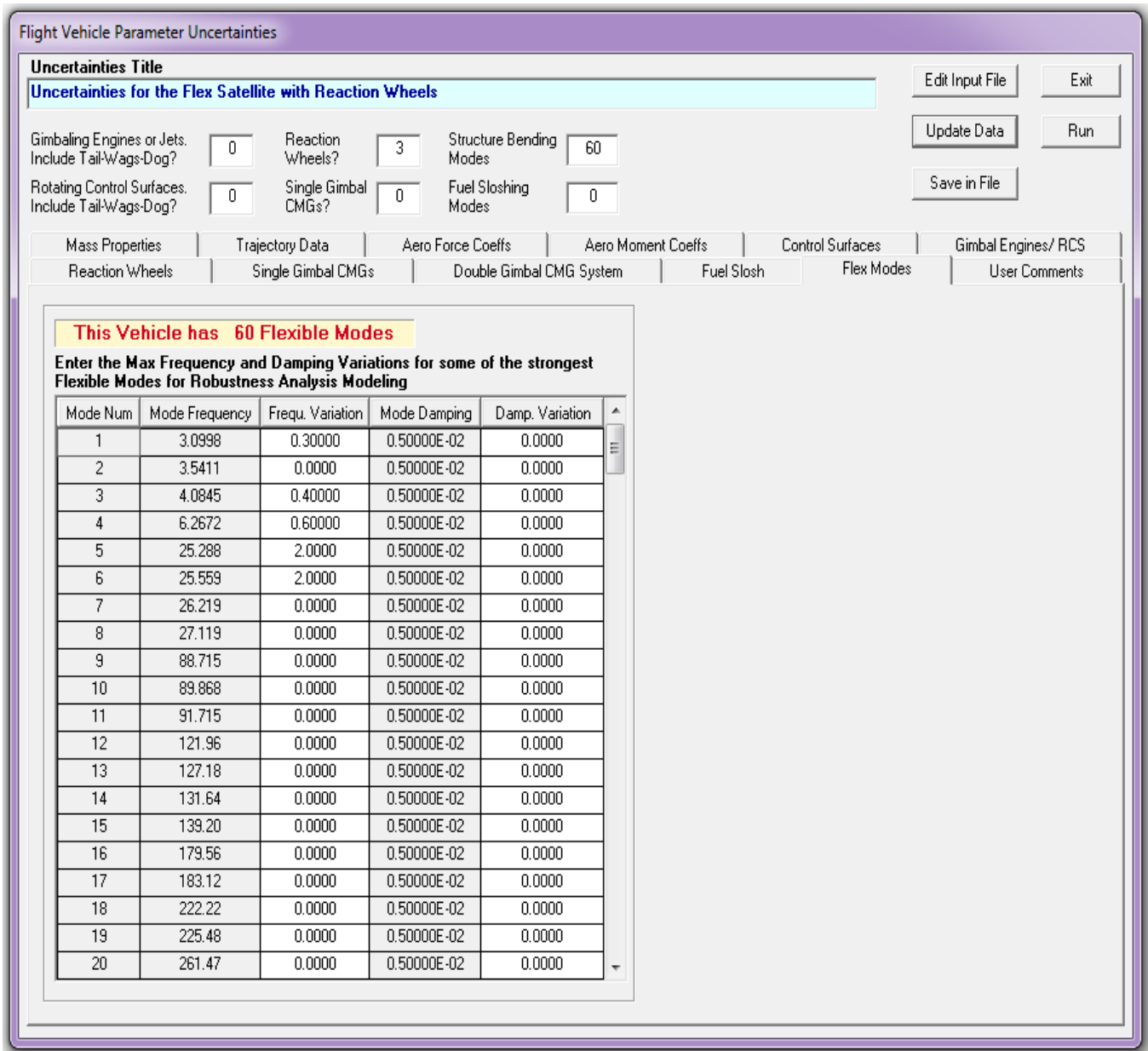
**Spin Direction, Rotor Inertia, and Initial rpm Speed for RW No: 2**

RW Spin Direction Unit Vector

	Nominal Values	Enter Param. Variations
X	0.0000	0.10000
Y	1.0000	0.0000
Z	0.0000	0.10000

RW Rotor Mom. Inertia

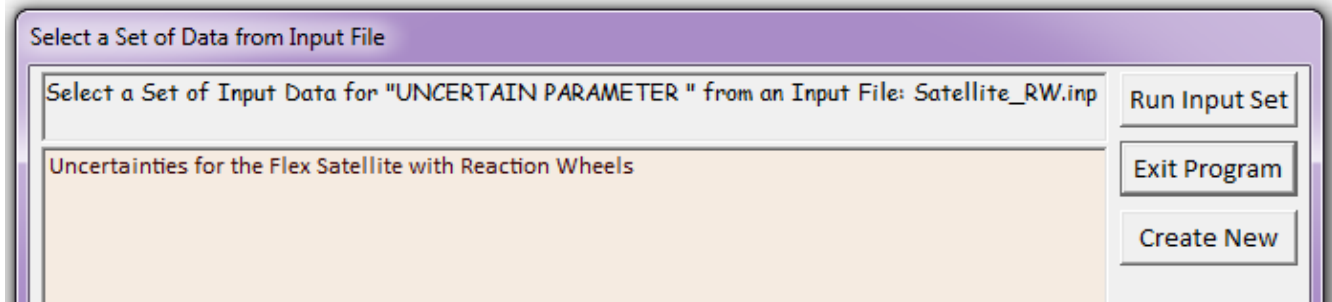
Initial RW Speed (rpm)



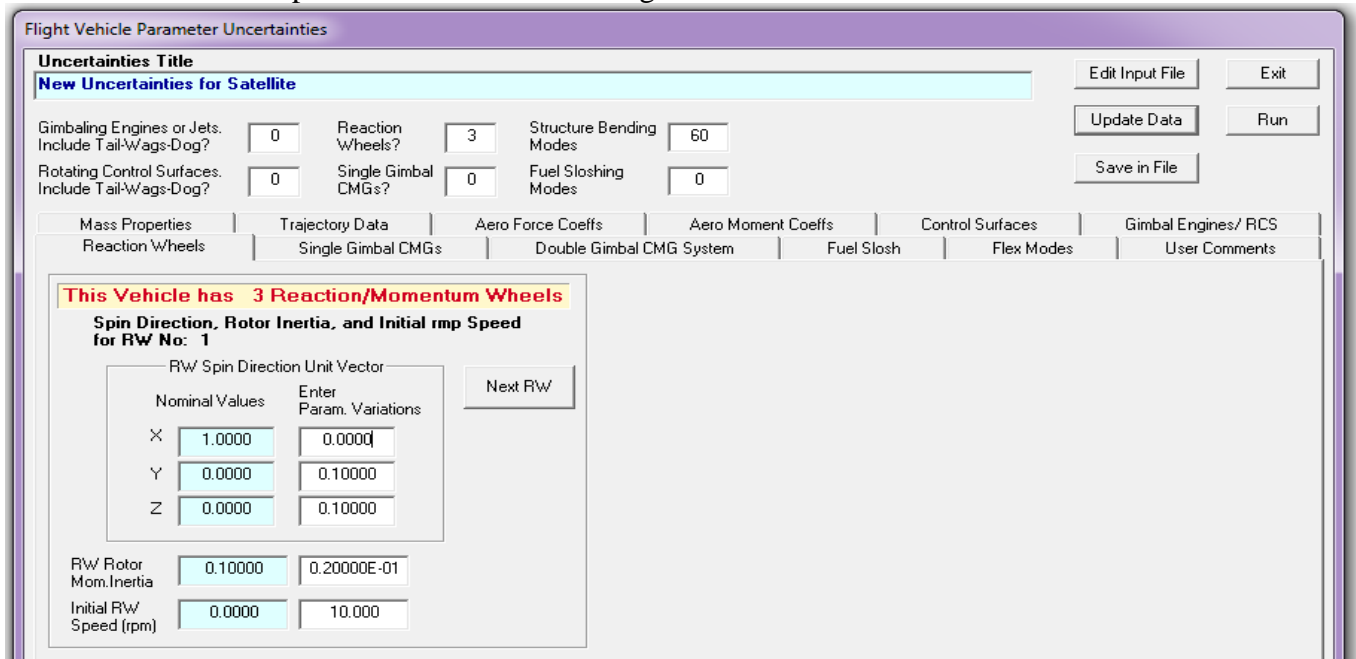
Click on “Run” and the program generates the flex satellite state-space model and it will save it in file “*Satellite\_RW.Qdr*”. This system is augmented because it includes the 28 additional inputs and the 28 additional outputs that connect with the "pulled out" uncertainty block  $\Delta$ . It was also converted to Matlab state-space function m-file format “*satellite\_60flx.m*” for  $\mu$ -analysis in Matlab. In our  $\mu$ -analysis we don’t physically connect it with a  $\Delta$  block but we simply calculate the  $\mu(\omega)$  between the 28 inputs and outputs. Notice, that the input/ output pairs connecting to the uncertainty block  $\Delta$  is greater than the number of parameter variations. This is because some of the uncertainties, such as: moments of inertia, RW momentum bias, etc, are higher than rank-1 dependency coupling into more than one direction, and they appear in 2 or 3 equations.

## Creating a new set of Uncertainties Data

To create a new set of parameter variations start the Flixan program as before, select the current project folder: “\Flixan\Examples\Robust Analysis Param Uncertainties\ Satellite\_RW”. From the main menu select “Program Functions”, “Robust Control Synthesis Tools”, and “Modeling Vehicle Parameter Uncertainties”. Select the same input data file “Satellite\_RW.Inp” and systems file “Satellite\_RW.Qdr”, like before. From the menu that selects the title of the vehicle data select again “Flex Satellite with Reaction Wheels” and click on “Run Input Set”, like before. However, now from the menu that selects an uncertainties title, do not select and run an already existing uncertainties data-set, like before, but click on “Create New” button instead.



This will activate the uncertainties data preparation utility and it will display the following dialog for entering the parameter variations data in the appropriate fields. The dialog consists of several sub-dialogs selectable by tabs. On the top of the main dialog we must enter the new uncertainties title “New Uncertainties for Satellite” and click on “Update Data”. The update button does not save the data in file but it updates the internal memory because, otherwise, it may get erased when you change tabs. The dialog also shows that the satellite has 3 reaction wheels, and 60 flex modes. Click on the “Reaction Wheels” tab and it will show the reaction wheel parameters. In the fields on the right side you may enter the parameter variations for each wheel separately. Starting from wheel #1, we enter a small amount of uncertainty in the direction of the spin axis (x, y, z), an uncertainty in the rotor inertia, and in the initial reaction wheel speed  $\pm 10$  (rpm). Click on “Next RW” and repeat for the second and third RW. Click on “Update Data” before selecting the next tab which is “Flex Modes”.



The Flex Modes tab consists of 5 columns. You may only enter data in the 3<sup>rd</sup> and 5<sup>th</sup> columns which are the frequency and damping uncertainties. The first column is the mode numbers as listed in the input data file. The 2<sup>nd</sup> column is the mode frequencies in (rad/sec). In the 3<sup>rd</sup> column you may enter the frequency variations, obviously, not in every single mode but only in the problematic modes that dominate in the flex dynamics. The 4<sup>th</sup> column shows the modal damping and you can't change it. In the 5<sup>th</sup> column you may enter the modal damping variations which are relative to the nominal damping values. Make sure that the variations are reasonable, for example, not bigger than the actual damping coefficients. Click on the "Update Data" and select the mass properties tab to enter variations in the vehicle moments of inertia.

Flight Vehicle Parameter Uncertainties

**Uncertainties Title**  
 New Uncertainties for Satellite

Gimbal Engines or Jets. Include Tail-Wags-Dog? 0    
  Reaction Wheels? 3    
  Structure Bending Modes 60  
 Rotating Control Surfaces. Include Tail-Wags-Dog? 0    
  Single Gimbal CMGs? 0    
  Fuel Sloshing Modes 0

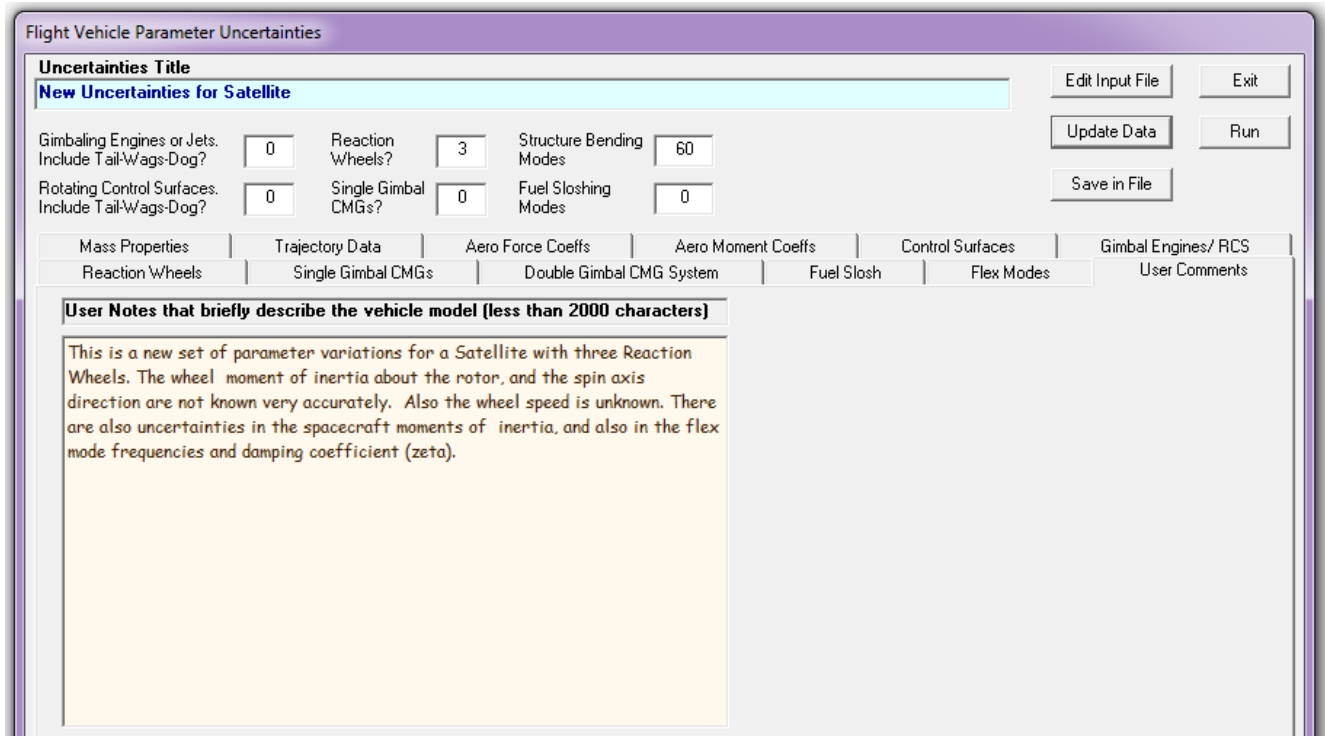
                     
                   

**This Vehicle has 60 Flexible Modes**

**Enter the Max Frequency and Damping Variations for some of the strongest Flexible Modes for Robustness Analysis Modeling**

Mode Num	Mode Frequency	Frequ. Variation	Mode Damping	Damp. Variation
1	3.0998	0.30000	0.50000E-02	0.10000E-02
2	3.5411	0.0000	0.50000E-02	0.0000
3	4.0845	0.40000	0.50000E-02	0.0000
4	6.2672	0.60000	0.50000E-02	0.10000E-02
5	25.288	2.0000	0.50000E-02	0.0000
6	25.559	2.5000	0.50000E-02	0.10000E-02
7	26.219	0.0000	0.50000E-02	0.0000
8	27.119	0.0000	0.50000E-02	0.0000
9	88.715	0.0000	0.50000E-02	0.0000
10	89.868	0.0000	0.50000E-02	0.0000
11	91.715	0.0000	0.50000E-02	0.0000
12	121.96	0.0000	0.50000E-02	0.0000
13	127.18	0.0000	0.50000E-02	0.0000
14	131.64	0.0000	0.50000E-02	0.0000
15	139.20	0.0000	0.50000E-02	0.0000
16	179.56	0.0000	0.50000E-02	0.0000
17	183.12	0.0000	0.50000E-02	0.0000
18	222.22	0.0000	0.50000E-02	0.0000
19	225.48	0.0000	0.50000E-02	0.0000
20	261.47	0.0000	0.50000E-02	0.0000

Finally you may enter some user notes for book-keeping purposes. Click on the “User Comments” tab, and in the following field comes up where you may enter a short description of the newly created parameter variations data set. This description note will be included as comments in the new uncertainty data-set in file “*Satellite\_RW.Inp*”. Remember again to click on the “Update Data” button after completing the note. Finally, when the parameter variations data entering is complete in all tabs, click on “Save in File” button to save the uncertainty data in file “*Satellite\_RW.Inp*”, under the new title and user comments. You may finally click on “Run” to process the vehicle and uncertainties data, and to create the augmented vehicle system in file “*Satellite\_RW.Qdr*”.



## Analysis

The Matlab script file “run.m” loads the flex satellite state-space model from Matlab function “*satellite\_60flx.m*”. It loads also the spacecraft moments of inertia matrix which is used in the attitude control laws to provide the same ACS bandwidth in all directions. The Simulink model “*Open-Loop.mdl*”, in Figure (1), is used to perform open-loop frequency response analysis in order to determine the ACS phase and gain margins. In this case it is configured for yaw axis analysis, with pitch and roll axes loops closed. The yaw Nichols is shown in figure (2), showing also the phase and gain margins.

```

close all
r2d=180/pi; d2r=pi/180;
[Au, Bu, Cu, Du] = satellite_60flx;
npv=28; % Number of Param Variation
J= [1045.3, 0, 19; ... % Moments of Inertia Matrix
    0, 641.02, 40; ...
    19, 40, 505.05]; JI=inv(J);

label='Yaw Axis Stability, (Pitch & Roll Closed)';
[Ao,Bo,Co,Do]=linmod('Open_Loop'); sys=ss(Ao,Bo,Co,Do);
w=logspace(-2,3,12000); % and Frequ domain analysis
figure(11); Nichols(sys,w); title(label)
figure(12); Bode(sys,w); title(label)

w=logspace(-3,3,300); % and Frequ domain analysis
[Acl,Bcl,Ccl,Dcl]=linmod('Closed_Loop'); sys=ss(Acl,Bcl,Ccl,Dcl);
mu=ssv(Acl,Bcl,Ccl,Dcl,w);
figure(13); loglog(w,mu);

[Acl,Bcl,Ccl,Dcl]=linmod('Closed_Loop'); sys=ss(Acl,Bcl,Ccl,Dcl);
sysf= FRD(sys,w);
blk=[-ones(npv,1), zeros(npv,1)];
[bnd,muinfo]= mussv(sysf,blk);

ff= get(muinfo.bnds, 'frequency');
muu=get(muinfo.bnds, 'responsedata');
muu=squeeze(muu);
muu=muu(1,:);
figure(14); loglog(ff,muu)
xlabel('Frequency (rad/sec)')
ylabel('ssv')

```

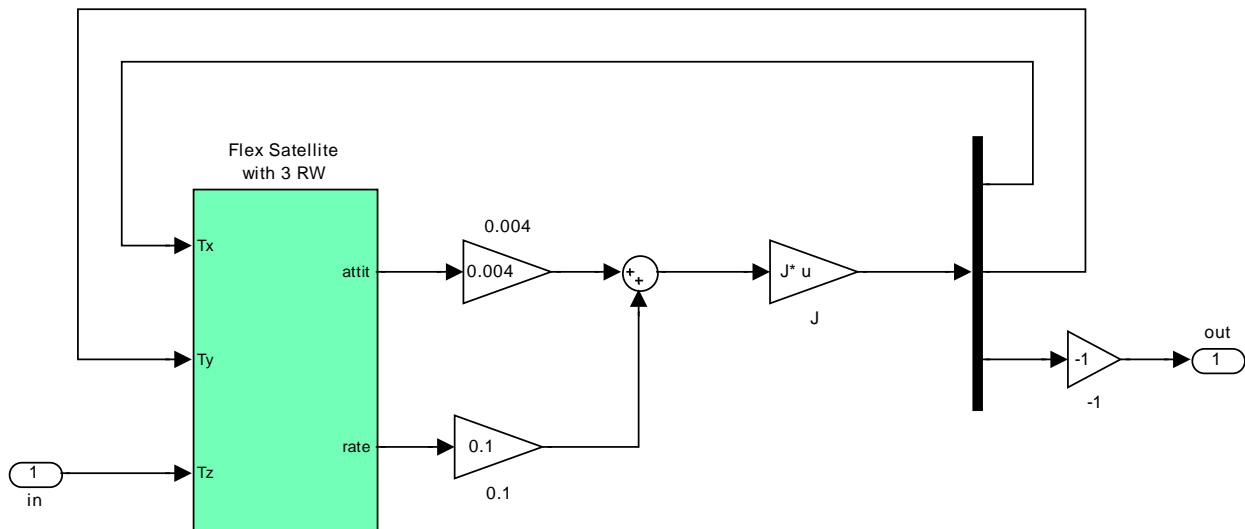
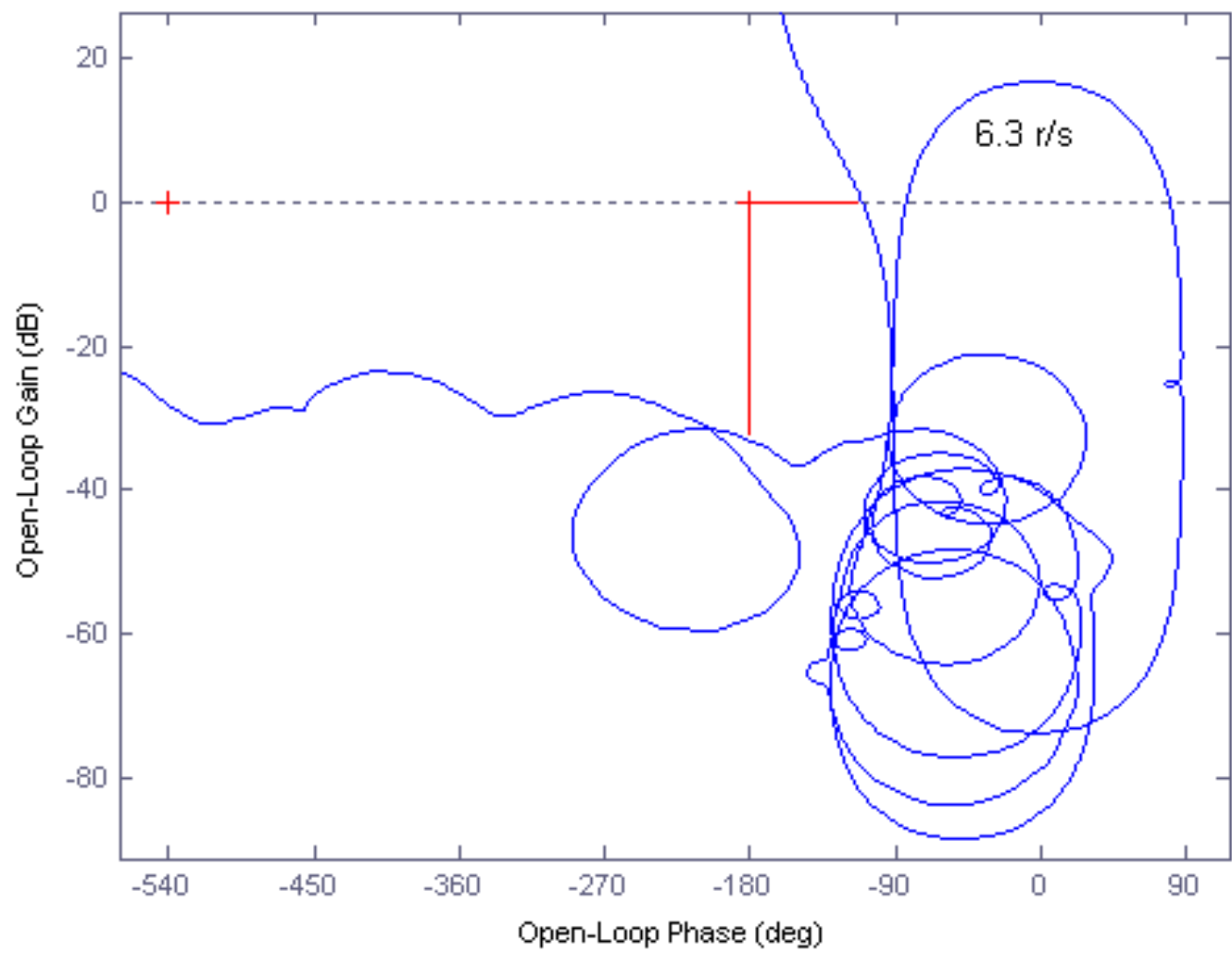


Figure (1) Simulink Model "Open\_Loop.Mdl" used for Open-Loop Stability Analysis (configured here for yaw analysis)

Yaw Axis Stability, (Pitch & Roll Closed)





The Simulink model in Figure (3) is “*Closed\_Loop.mdl*” and the green satellite block includes the Flixan generated flex vehicle with uncertainties system "satellite\_60flx.m". The satellite dynamics model (green block) is shown in detail in figure (4), and the controller is a simple PD control law. The file “run.m” calculates also the structural singular value ( $\mu$ ) frequency response of the closed-loop system with the loop opened across the parameter variations block  $\Delta$ , see Figure (3). It calculates the ( $\mu$ ) using two different methods, (a) assuming that the variations in block  $\Delta$  are complex with magnitudes less than 1 (this method is usually too conservative for low damped resonances), and (b) assuming that the variations in block  $\Delta$  are real numbers with magnitudes varying between  $\pm 1$  (slower to compute).

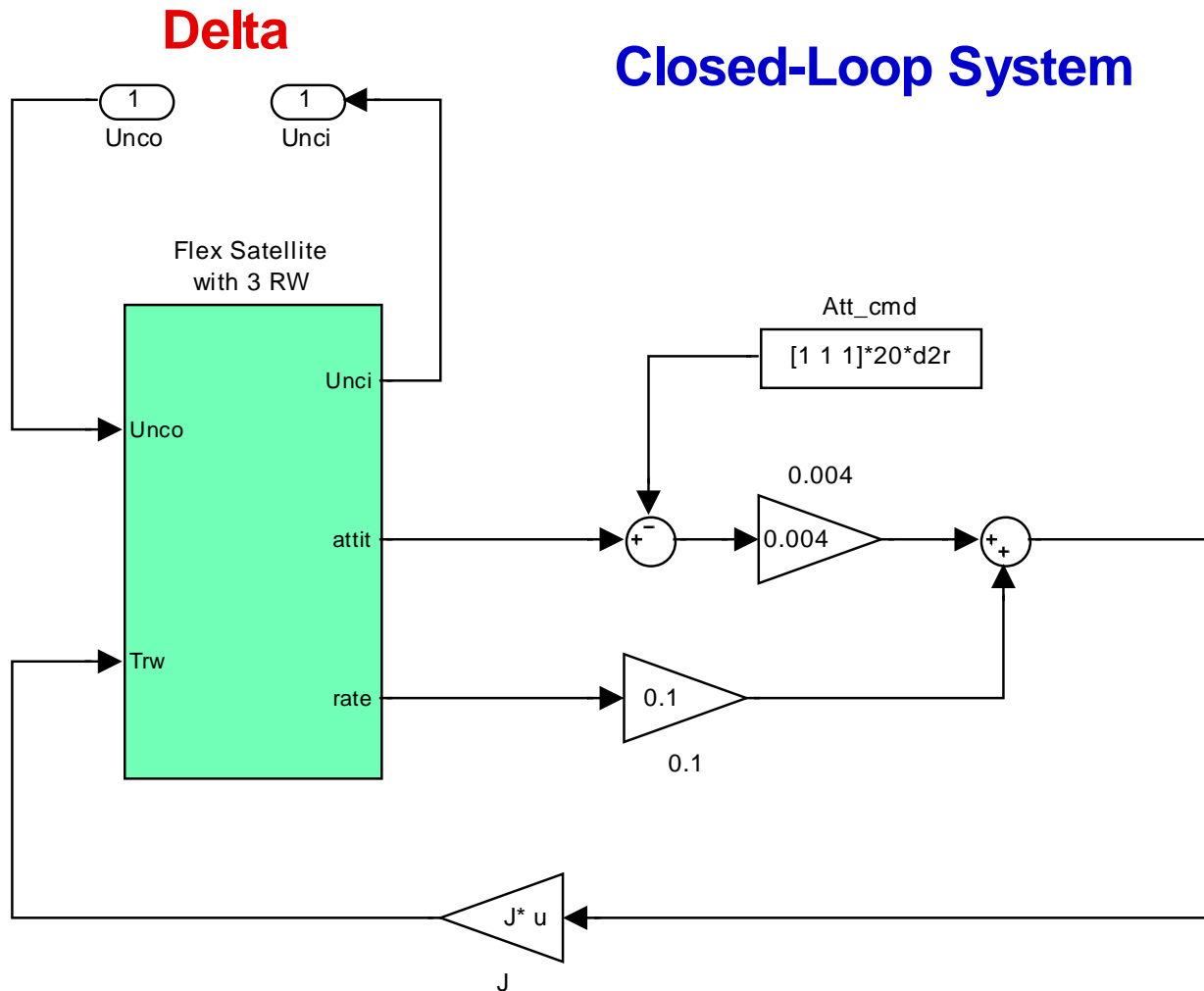


Figure (3) Closed-Loop Simulation Model “*Closed\_Loop.mdl*” uses a simple PD control law.

### Flex Satellite with 3 Reaction Wheels

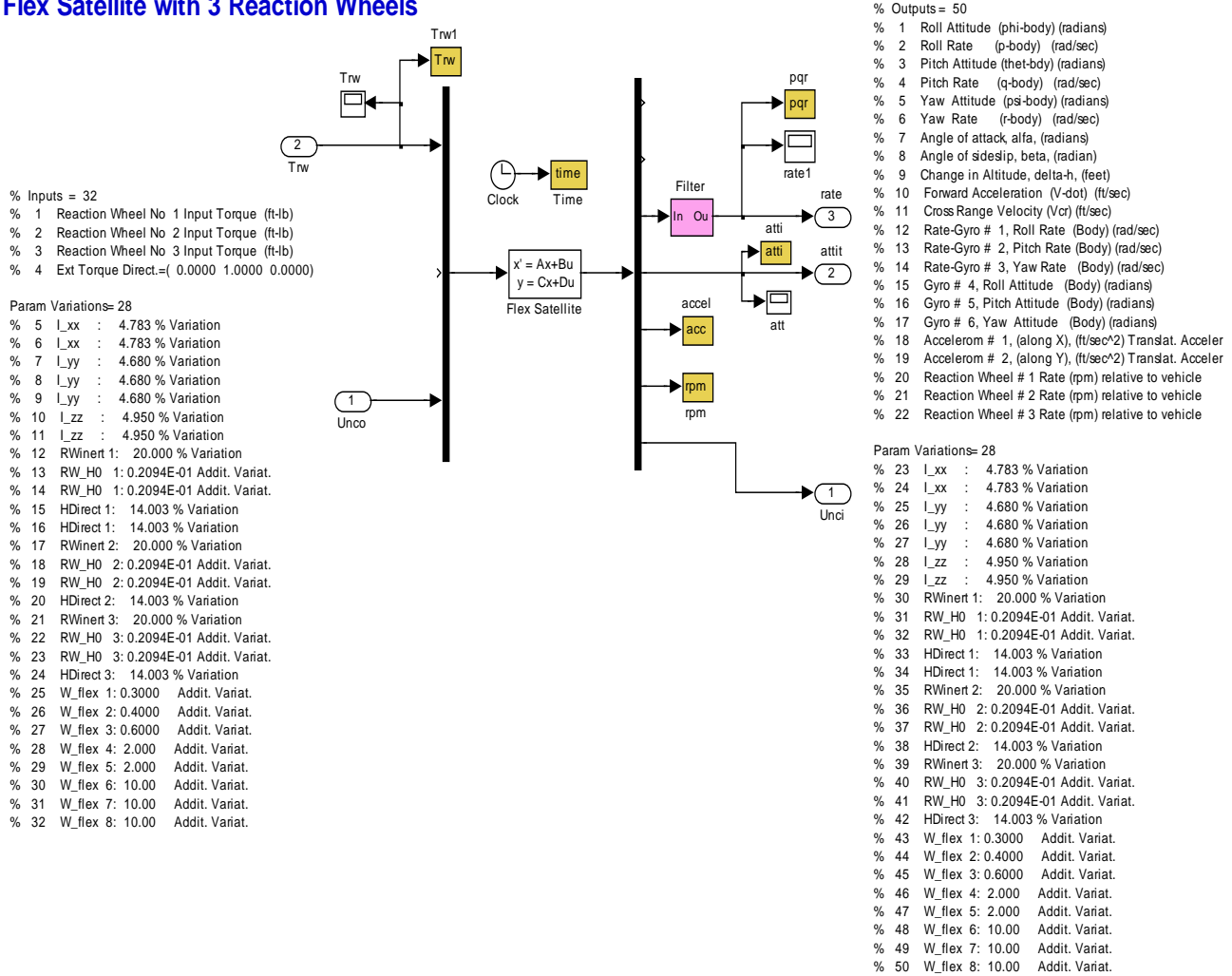
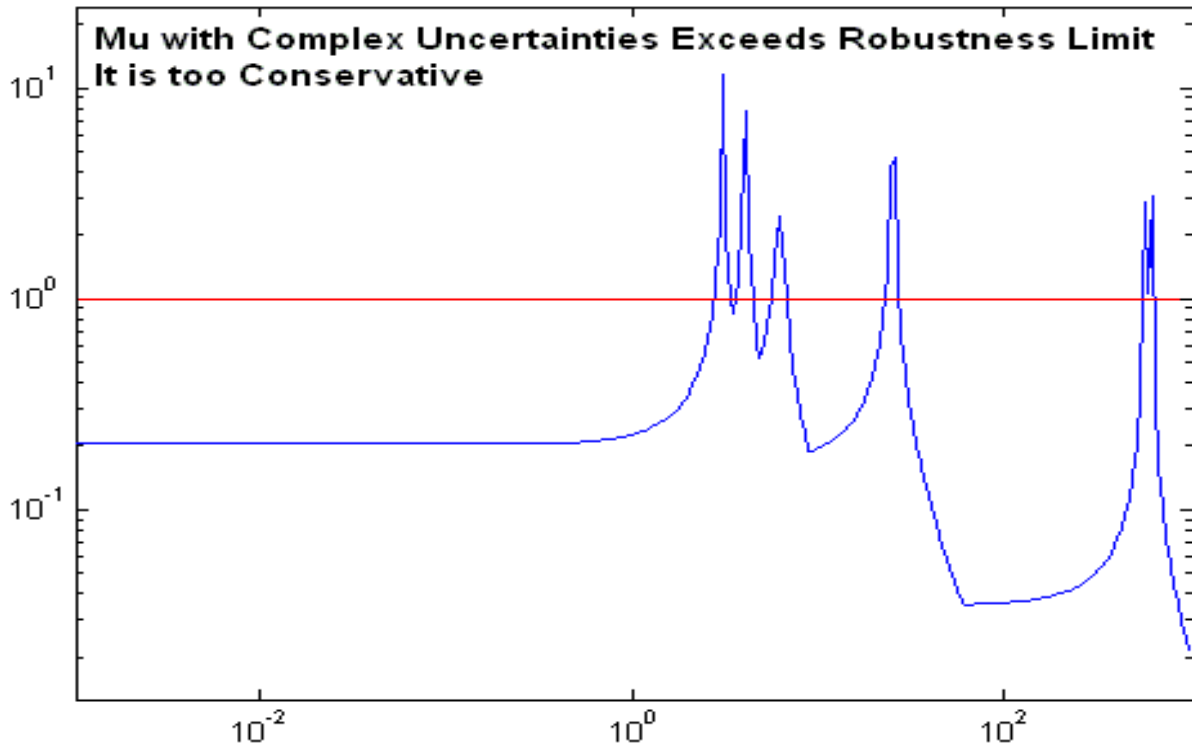


Figure (4) Spacecraft dynamics subsystem uses the uncertain state-space model “Satellite\_60flx.m” which provides the input/ output pairs that couple with the variations block Δ

Figure (5) shows the μ-analysis results of the closed-loop system between the uncertainty inputs and the uncertainty outputs. The results show comparison between the two methods used to calculate the SSV, (a) using complex parameter variations, which is easier to calculate but too conservative, and (b) using real parameter variations which is slower to compute, but not as conservative as the complex variations. The complex variations being conservative violate our robustness requirement, but the (μ) with real variations satisfy the requirement of being less than one at all frequencies, with plenty of margin. It means that the system maintains stability in presence of all possible parameter variations as long as the variations lie between -1 and +1. Remember, that the individual max parameter variations were used during system creation to scale the uncertainty inputs and outputs, and the uncertainty block is, therefore, normalized assuming that its individual elements vary between -1 and +1.

### Mu Across the Uncertainty Block Delta



### Mu Across the Uncertainty Block Delta

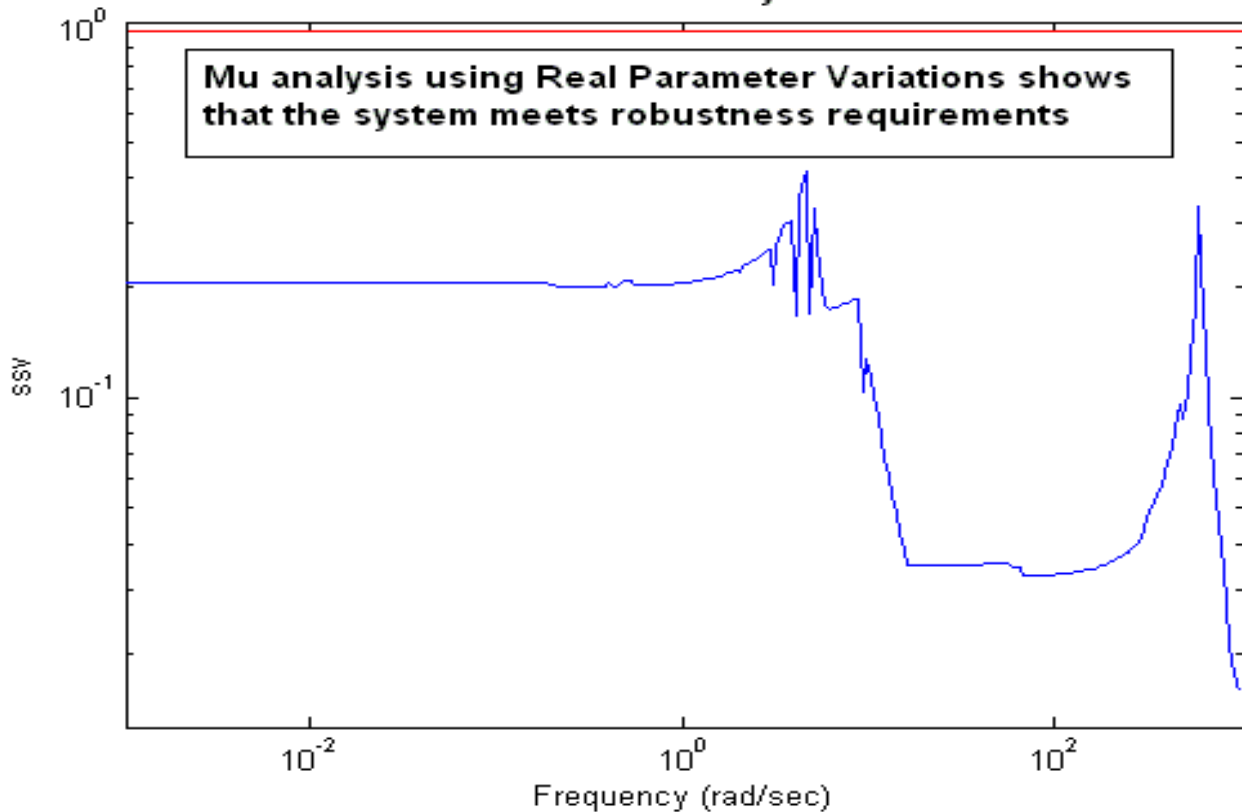


Figure (5) SSV of the Closed-Loop system as seen across the perturbation block  $\Delta$ . Analysis using complex variations is too conservative and violates robustness requirement, but real variations meets robustness requirement

## Simulation Results

The following simulation results are obtained by using the Simulink model “*Closed-Loop.mdl*”. It is commanded to perform a simultaneous 20 (deg) attitude maneuver in all 3 directions. The Matlab file “*pl.m*” is used to plot the data. Figure (6) shows the attitude and rate responses in all 3 axes which are very similar. Flexibility is also visible in yaw (red).

Figure (7) shows the reaction wheel torques in (ft-lb) and rates in (rpm), versus time during the attitude maneuver. It also shows the response of the pitch and yaw accelerometers which are sensitive to structure flexibility.

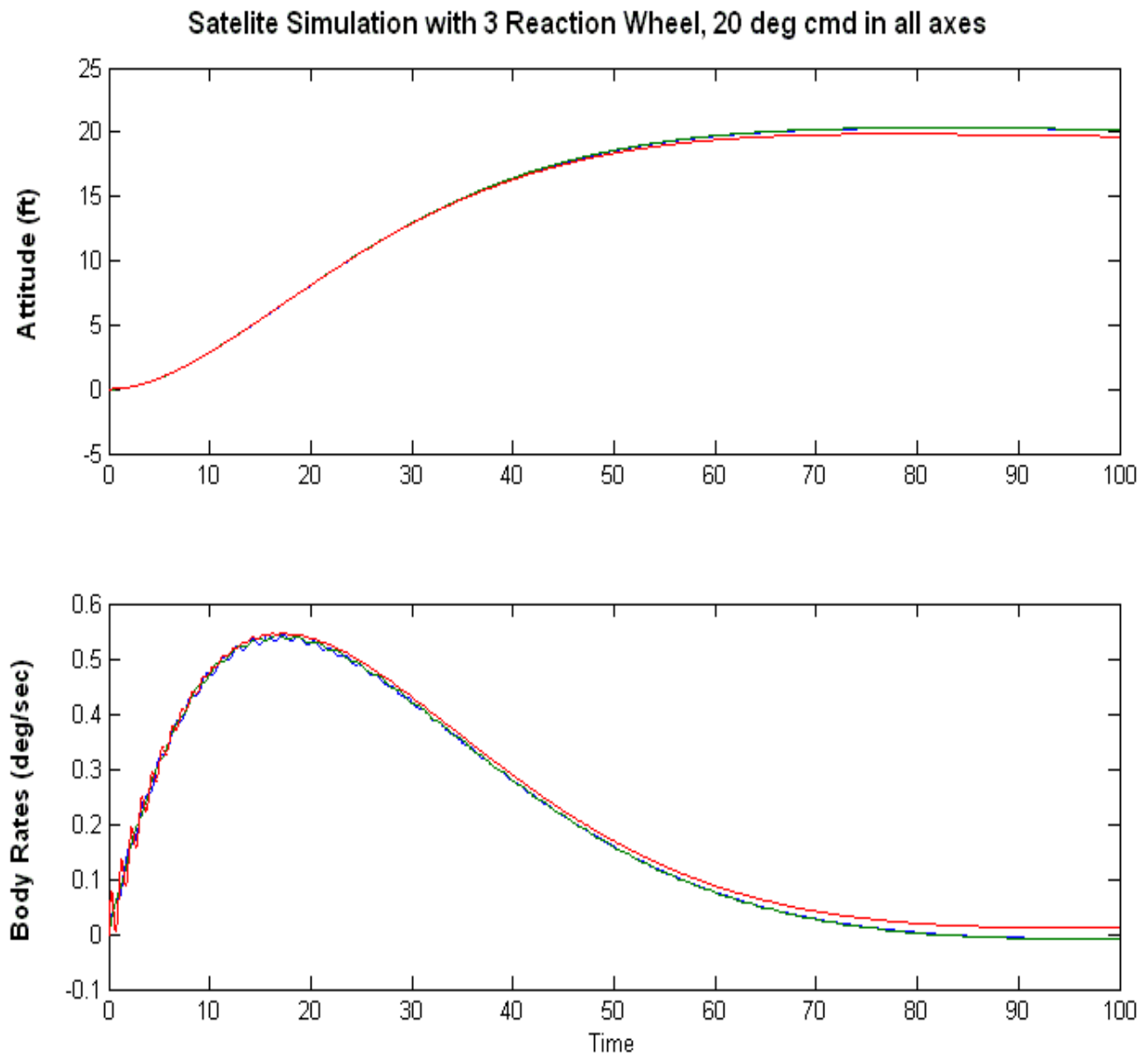
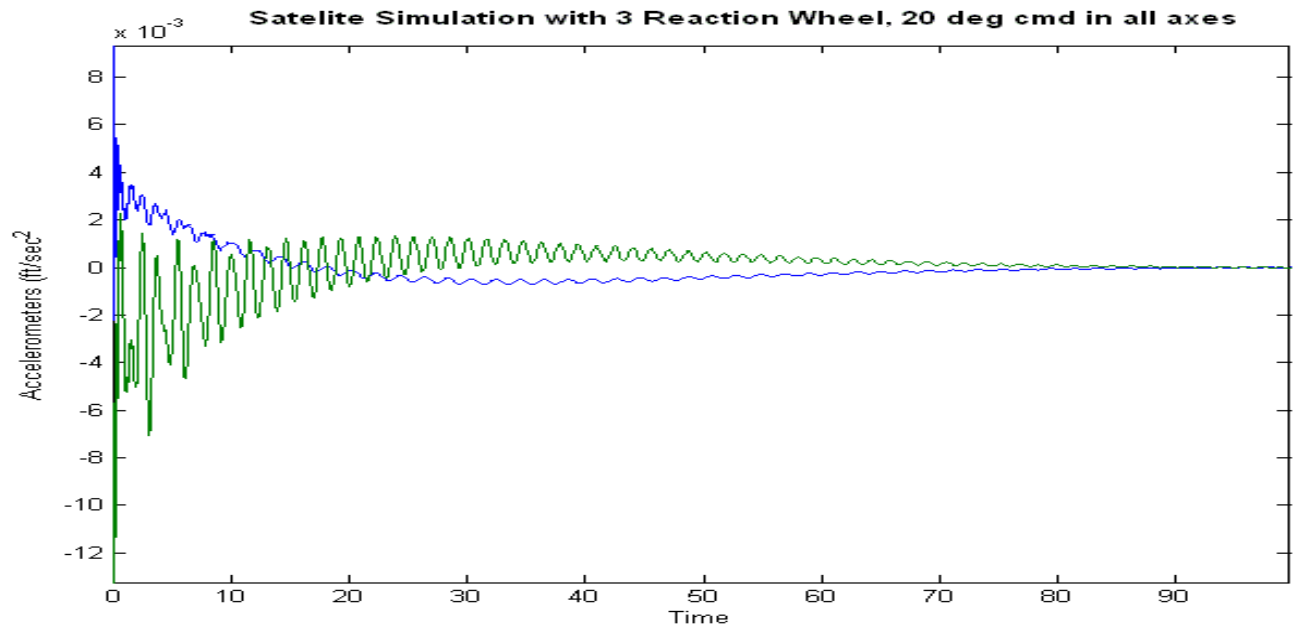
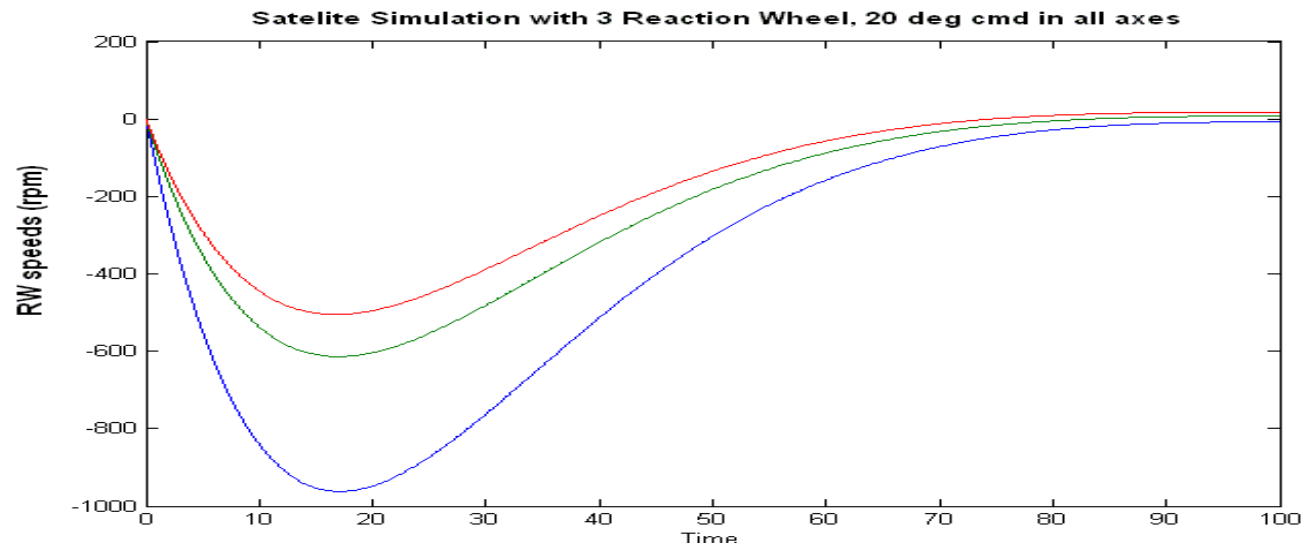
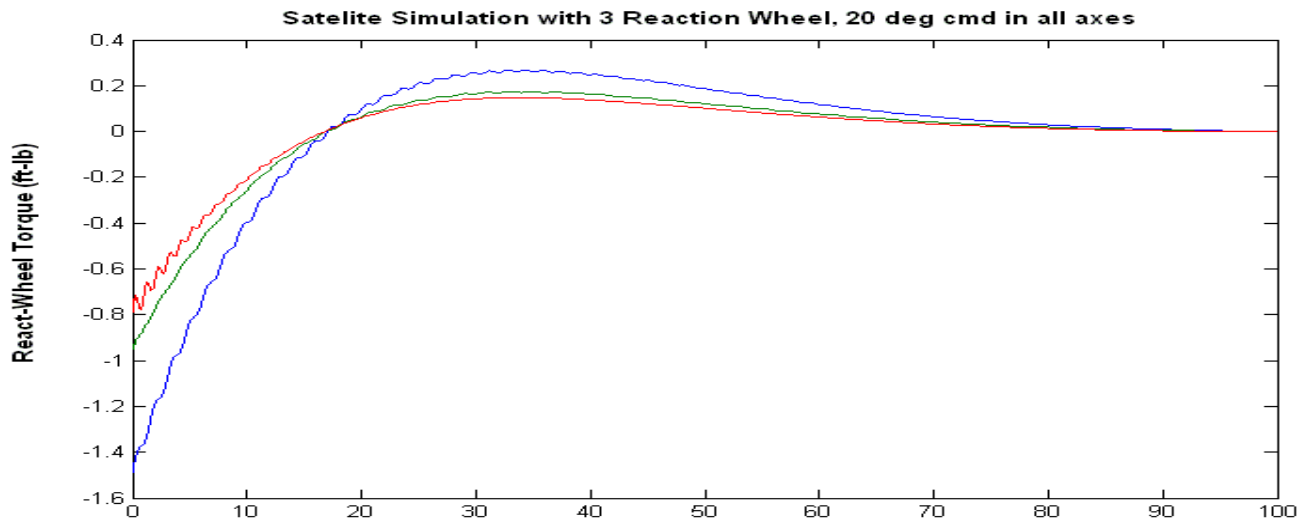


Figure (6) Attitude and rate response to the 20 (deg) command in all directions. It shows that the bandwidth is the same in all directions.

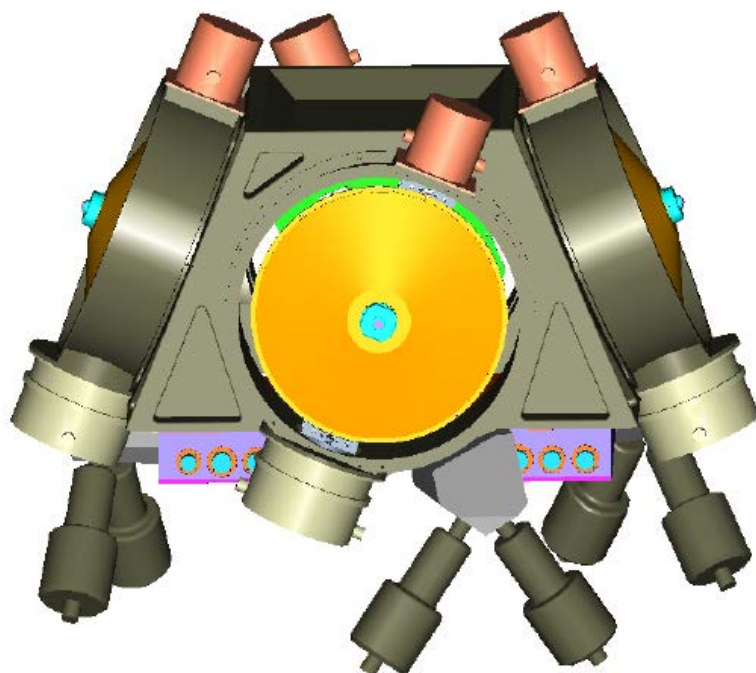


# Robustness Analysis of a Flexible Satellite with Single Gimbal Control Moment Gyros

---

In this example we have a flexible spacecraft that is controlled by a cluster of four Single-Gimbal CMGs. The dynamics of a spacecraft with CMGs is highly non-linear and it is a function of the CMG gimbal angles. In this analysis, however, we will use a linearized model generated by the Flixan flight vehicle modeling program and also use linear control laws to analyze stability and robustness at a fixed gimbal angle positions. The analysis is performed at the initialization gimbal positions where the total system momentum is zero. The four SGCMGs have the same momentum 1200 (ft-lb-sec) and they are mounted on a four-sided pyramid configuration, as shown in Figure 2. The pyramid angle ( $\beta$ ) is  $68^\circ$ . The four gimbaling directions are perpendicular to the surfaces of the pyramid and the initial gimbal angles are zero, that is, where the four momentum vectors ( $h_i$ ) are parallel to the base of the pyramid, producing a total CMG momentum equal to zero. The positions shown in Figure 2 represent the CMG initialization state, and we will analyze the system stability and robustness to parameter uncertainties relative to this position. The Flight Vehicle Modeling program (FVMP) is used to create dynamic models in this initial orientation. The gimbal angles, gimbal directions, and momentum reference directions are inputs to the program. It is also possible to create linear models in other CMG orientations for analysis. The control analysis will be performed in Matlab.

Figure 1, shows a cluster of four SG-CMGs mounted on a rigid support structure which is isolated from the spacecraft structure by means of vibration isolation struts. Notice that, the pyramid mounting structure of Figure 2 is a standard arrangement used for visualization purposes. The actual mounting of the CMGs are not necessarily as shown in the pyramid, but they can be translated anywhere on the spacecraft, as long as, the gimbal direction vectors and the momentum reference directions are parallel to those shown in the pyramid model. The momentum reference directions for each CMG are defined to be the momentum directions when the gimbal angles are zero.



In our example, the FVMP creates two spacecraft models, a nominal model, and a similar model that has 61 additional inputs and outputs for the 61 parameter uncertainties. Dynamically, both models are the same, but the second model includes the additional 61 inputs and outputs that would connect with the parameter uncertainties that have been "pulled-out" in the normalized  $\Delta$  block. We will use the nominal model to analyze stability and to prove that the nominal system is stable and then we will use the model with the uncertainties to analyze robustness to variations. We must, therefore, design the control system, close the control loop in order to stabilize the system without commands, and perform  $\mu$ -analysis across the connections with the uncertainty inputs and outputs to make sure that there is no combination of the uncertainties (which may vary between  $\pm 1$ ) that will drive it unstable.

The analysis is performed in folder "C:\Flixan\Examples\Robust Analysis Param Uncertainties\Satellite\_SGCMG". The spacecraft parameters are in the input file "FlexSc\_4CMG.Inp". The file includes also a set of modal data consisting of 40 selected modes. The mode selection process is not shown here because it is fully described in other examples. The input file also includes the parameter uncertainties in a separate dataset. Its title is: "Uncertainties for Flexible Agile Spacecraft with 4 SG-CMG". The uncertainties are additive variations to the nominal spacecraft parameters. Notice that, not all parameters should be varied and the user must use caution in selecting which parameters to vary, because when we perturb some parameters it does not create a plant variation and causes errors. In this example we shall vary the spacecraft moments and products of inertia, the frequencies in 8 flex modes, which are the strongest. We also include uncertainty in the CMG momentum  $\pm 50$  (ft-lb-sec), the momentum direction, the initial gimbal angle  $\delta_0$ , the CMG moment of inertia about the gimbal axis  $J_g$ , and the pyramid surface orientation angles  $(\beta, \gamma)$ . We did not include variations in the gimbal directions and the CMG inertias ( $J_s, J_o$ ) because they do not create plant variations in this example.

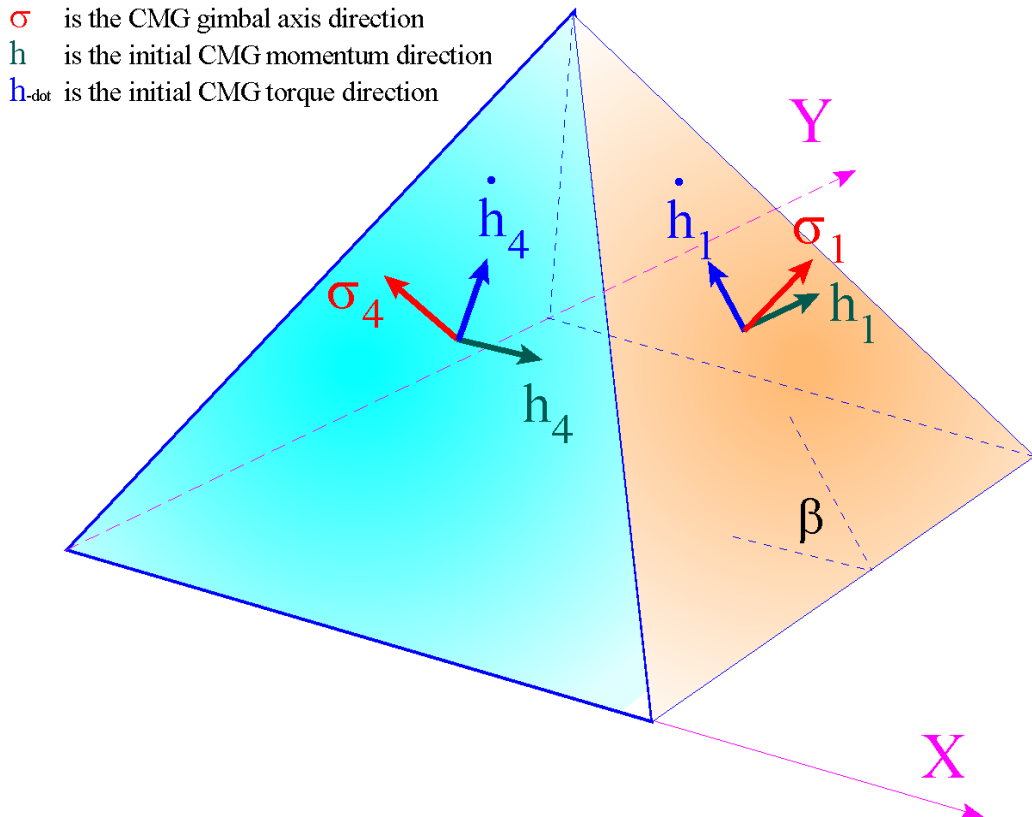


Figure 2 Array of four CMGs in a Pyramid Configuration

# Spacecraft Model Creation

We will now process the input file "FlexSc-4CMG.Inp" that already contains the spacecraft data, the uncertainties, and the modal data.

## FLIGHT VEHICLE INPUT DATA .....

Flexible Agile Spacecraft with 4 SG-CMG (Uncertainties)

! This is a Flex Spacecraft model that includes four 1200 (ft-lb-sec) SGCMGs

! Uncertainties in the mass properties and in the CMGs are also included

Body Axes Output, Attitude=Euler Angles

```
Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft) : 202.05 32.17 0.20896E+08
Moments and products of Inertias Ixx, Iyy, Izz, -Ixy, -Ixz, -Iyz, in (lb-sec^2-ft) : 1.41E+4 1.3E+4 0.1759E+4
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet) : 0.0 -0.20975 0.62
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet) : 0.0 25500.0 0.0 700000.
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2) : 0.0 0.0 0.0 0.0
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec) : 0.0 0.0 0.0 0.0
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec) : 0.0 0.0 0.0 0.0
External Force direction unit vect: (x,y,z), Force application point: (x,y,z) (feet) : Force 1.0 0.0 0.0 -12.1
Surface Reference Area (feet^2), Mean Aerodynamic Chord (ft), Wing Span in (feet) : 1.0 1.0 1.0
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial_rho/ Partial_H} : 0.0 -0.209 0.6201 0.0
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_g,Ca_bet} : 0.0 0.0 0.0 0.0 0.0
Aero Force Coeff/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V} : 0.0 0.0 0.0 0.0 0.0
Aero Moment Coeff/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa} : 0.0 0.0 0.0 0.0 0.0
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_g,PCm/PV,PCm/Ph} : 0.0 0.0 0.0 0.0 0.0
Aero Moment Coeff/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa} : 0.0 0.0 0.0 0.0 0.0
```

```
Number of Single Gimbal Control Moment Gyros (SG CMG) ? : 4
CMG: 1 Angular Momentum magnitude H0 (ft-lb-sec), Initial Gimbal Angle (delta) (deg) : 1200.0 0.0
Gimbal Direction Vector (m) : 0.927183 0.0 0.3746
Momentum Reference Direction Vector (r) : 0.0 1.0 0.0
SGCMG Surface Orientation angles in Pyramid (beta & gamma), (Figure 2.2) : 68.0 90.0
SGCMG Moments of Inertia (Js, Jg, Jo) about: spin, gimbal and output axes : 1.2 0.6 0.8
CMG: 2 Angular Momentum magnitude H0 (ft-lb-sec), Initial Gimbal Angle (delta) (deg) : 1200.0 0.0
Gimbal Direction Vector (m) : 0.0 0.927 0.3746
Momentum Reference Direction Vector (r) : -1.0 0.0 0.0
SGCMG Surface Orientation angles in Pyramid (beta & gamma), (Figure 2.2) : 68.0 180.0
SGCMG Moments of Inertia (Js, Jg, Jo) about: spin, gimbal and output axes : 1.2 0.6 0.8
CMG: 3 Angular Momentum magnitude H0 (ft-lb-sec), Initial Gimbal Angle (delta) (deg) : 1200.0 0.0
Gimbal Direction Vector (m) : -0.927183 0.0 0.3746
Momentum Reference Direction Vector (r) : 0.0 -1.0 0.0
SGCMG Surface Orientation angles in Pyramid (beta & gamma), (Figure 2.2) : 68.0 270.0
SGCMG Moments of Inertia (Js, Jg, Jo) about: spin, gimbal and output axes : 1.2 0.6 0.8
CMG: 4 Angular Momentum magnitude H0 (ft-lb-sec), Initial Gimbal Angle (delta) (deg) : 1200.0 0.0
Gimbal Direction Vector (m) : 0.0 -0.9271 0.3746
Momentum Reference Direction Vector (r) : 1.0 0.0 0.0
SGCMG Surface Orientation angles in Pyramid (beta & gamma), (Figure 2.2) : 68.0 0.0
SGCMG Moments of Inertia (Js, Jg, Jo) about: spin, gimbal and output axes : 1.2 0.6 0.8
```

```
Number of Gyros, (Attitude and Rate) : 9
Gyro No 1 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 2 : Roll Attit 6.53 0.0 -1.1
Gyro No 2 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 2 : Pitch Attit 6.53 0.0 -1.1
Gyro No 3 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 2 : Yaw Attit 6.53 0.0 -1.1
Gyro No 4 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 2 : Roll Rate 6.53 0.0 -1.1
Gyro No 5 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 2 : Pitch Rate 6.53 0.0 -1.1
Gyro No 6 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 2 : Yaw Rate 6.53 0.0 -1.1
Gyro No 7 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 1 : Roll Rate -2.93 0.2 -0.2
Gyro No 8 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 1 : Pitch Rate -2.93 0.2 -0.2
Gyro No 9 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat, Node 1 : Yaw Rate -2.93 0.2 -0.2
```

```
Number of Accelerometers, Along Axes: (x,y,z) : 6
Acceleromet No 1 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 2 : X-axis Accel 6.5 0.0 -1.1
Acceleromet No 2 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 2 : Y-axis Accel 6.5 0.0 -1.1
Acceleromet No 2 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 2 : Z-axis Accel 6.5 0.0 -1.1
Acceleromet No 4 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 4 : X-axis Accel 5.2 0.0 -2.0
Acceleromet No 5 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 4 : Y-axis Accel 5.2 0.0 -2.0
Acceleromet No 6 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 4 : Z-axis Accel 5.2 0.0 -2.0
```

## Parameter Uncertainties Data

Uncertainties for Flexible Agile Spacecraft with 4 SG-CMG

Number of Bending Modes : 40

Flexible Agile Spacecraft with 4 SG-CMG, All Flex Modes



**UNCERTAIN PARAMETER VARIATIONS FROM NOMINAL .....**

**Uncertainties for Flexible Agile Spacecraft with 4 SG-CMG**

! This is a set of max parameter variations for the Agile Spacecraft with 4 SG-CMGs.  
! We perturb the vehicle moments of inertia matrix, the momentum reference direction  
! by introducing components in the zero skewed directions, we do not vary the gimbal  
! direction vectors because in this case they do not create deltas. We also perturb  
! the CMG surface orientation angles beta and gamma, and the CMG moment of inertia Jg  
! about the gimbal axis. The other two moments of inertia, Js and Jo, do not create deltas.  
! We should only perturb parameters that create plant variations, otherwise errors occur.

Vehicle Mass (lb-sec<sup>2</sup>/ft), Gravity Accelerat. (g) (ft/sec<sup>2</sup>), Earth Radius (Re) (ft) : 0.0 0.0 0.0  
Moments and products of Inertias Ixx, Iyy, Izz, -Ixy,-Ixz,-Iyz, in (lb-sec<sup>2</sup>-ft) : 160.0 150.0 25.0, 0.4,  
0.2, 1.8  
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet) : 0.0 0.0 0.0  
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet) : 0.0 0.0 0.0 0.0  
Inertial Acceleration Vo\_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec<sup>2</sup>) : 0.0 0.0 0.0 0.0  
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec) : 0.0 0.0 0.0 0.0  
Vehicle Attitude Euler Angles, Phi\_o,Theta\_o,Psi\_o (deg), Body Rates Po,Qo,Ro (deg/sec) : 0.0 0.0 0.0 0.0  
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca\_alf,PCa/PV,PCa/Ph,Ca\_alfdot,Ca\_q,Ca\_bet}: 0.0, 0.0, 0.0, 0.0, 0.0,  
Aero Force Coef/Deriv (1/deg), Along Z, {Czo,Cz\_alf,Cz\_q,Cz\_bet,PCz/Ph,Cz\_alfdot,PCz/PV}: 0.0, 0.0, 0.0, 0.0, 0.0,  
Aero Moment Coeff/Derivat (1/deg), Roll: {Clo, Cl\_beta, Cl\_betdot, Cl\_p, Cl\_r, Cl\_alfa}: 0.0, 0.0, 0.0, 0.0, 0.0,  
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm\_alfa,Cm\_alfdot,Cm\_bet,Cm\_q,PCm/PV,PCm/Ph}: 0.0, 0.0, 0.0, 0.0, 0.0,  
Aero Moment Coeff/Derivat (1/deg), Yaw : {Cno, Cn\_beta, Cn\_betdot, Cn\_p, Cn\_r, Cn\_alfa}: 0.0, 0.0, 0.0, 0.0, 0.0,

Number of Single Gimbal Control Moment Gyros (SG CMG) Variations ? : 4  
CMG: 1 Angular Momentum magnitude H0 (ft-lb-sec), Initial Gimbal Angle (delta) (deg) : 50.0 1.0  
Gimbal Direction Vector (m) variation : 0.0 0.0 0.0  
Momentum Reference Direction Vector (r) variation : 0.1 0.0 0.1  
SGCMG Surface Orientation angle in Pyramid (beta & gamma) variations,(Figure 2.2) : 1.5 1.5  
SGCMG Moments of Inertia (Js, Jg, Jo) variat. about: spin, gimbal and output axes : 0.0 0.06 0.0  
CMG: 2 Angular Momentum magnitude H0 (ft-lb-sec), Initial Gimbal Angle (delta) (deg) : 50.0 1.0  
Gimbal Direction Vector (m) variation : 0.0 0.0 0.0  
Momentum Reference Direction Vector (r) variation : 0.0 0.1 0.1  
SGCMG Surface Orientation angle in Pyramid (beta & gamma) variations,(Figure 2.2) : 1.5 1.5  
SGCMG Moments of Inertia (Js, Jg, Jo) variat. about: spin, gimbal and output axes : 0.0 0.06 0.0  
CMG: 3 Angular Momentum magnitude H0 (ft-lb-sec), Initial Gimbal Angle (delta) (deg) : 50.0 1.0  
Gimbal Direction Vector (m) variation : 0.0 0.0 0.0  
Momentum Reference Direction Vector (r) variation : 0.1 0.0 0.1  
SGCMG Surface Orientation angle in Pyramid (beta & gamma) variations,(Figure 2.2) : 1.5 1.5  
SGCMG Moments of Inertia (Js, Jg, Jo) variat. about: spin, gimbal and output axes : 0.0 0.06 0.0  
CMG: 4 Angular Momentum magnitude H0 (ft-lb-sec), Initial Gimbal Angle (delta) (deg) : 50.0 1.0  
Gimbal Direction Vector (m) variation : 0.0 0.0 0.0  
Momentum Reference Direction Vector (r) variation : 0.0 0.1 0.1  
SGCMG Surface Orientation angle in Pyramid (beta & gamma) variations,(Figure 2.2) : 1.5 1.5  
SGCMG Moments of Inertia (Js, Jg, Jo) variat. about: spin, gimbal and output axes : 0.0 0.06 0.0

Flex Mode Uncertainties (Mode Number) : 1 4 6 24 26 33 34  
Flex Mode Frequency Variation (additive) : 0.4 0.7 1.2 3.0 3.2 3.4 4.0

**-----  
SELECTED MODAL DATA AND LOCATIONS FOR :**

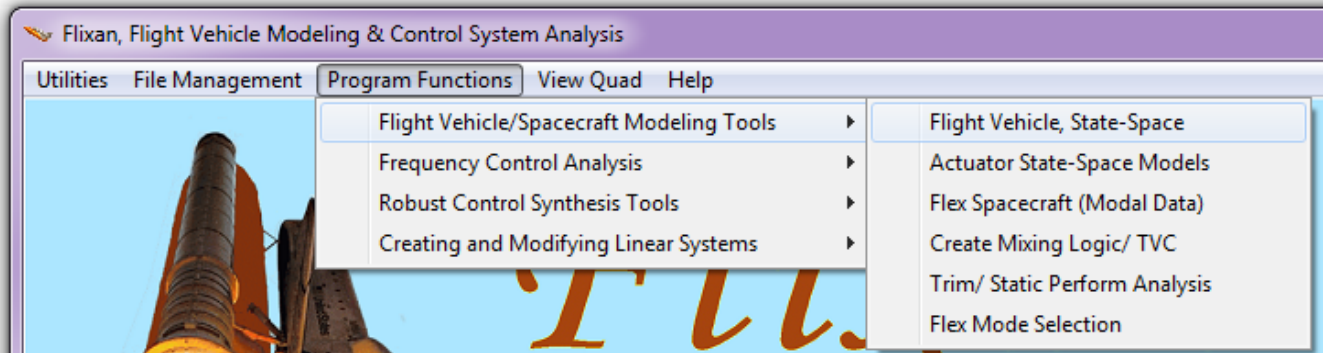
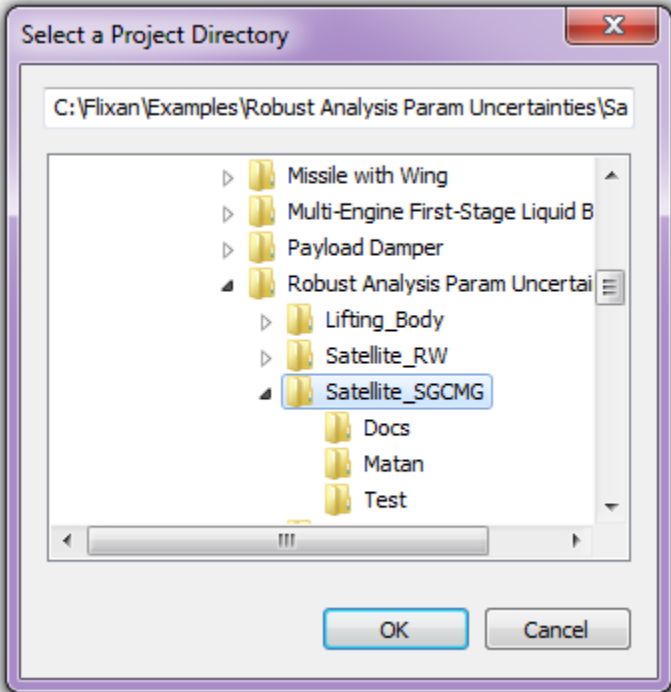
**Flexible Agile Spacecraft with 4 SG-CMG, All Flex Modes**

! All the flex modes are selected except for the first 6 rigid-body modes. A total of 40  
! structural modes.

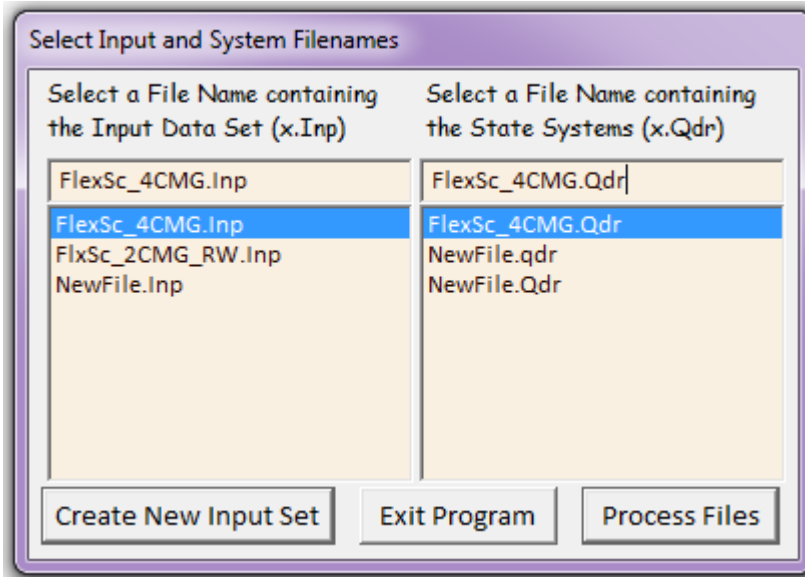
MODE#	1/ 7,	Frequency (rad/sec),	Damping (zeta),	Generalized Mass=	3.1905	0.30000E-02	12.000	
DEFINITION OF LOCATIONS (NODES)		phi along X	phi along Y	phi along Z	sigm about X	sigm about Y	sigm	
	Node ID#	Modal Data at the 4 Single Gimbal GMGs...						
Control Moment Gyros (CMG) Loc	6	0.10717D-02	-0.16639D-01	-0.31137D-01	-0.42835D-01	-0.29132D-02	-	
Control Moment Gyros (CMG) Loc	6	0.10717D-02	-0.16639D-01	-0.31137D-01	-0.42835D-01	-0.29132D-02	-	
Control Moment Gyros (CMG) Loc	6	0.10717D-02	-0.16639D-01	-0.31137D-01	-0.42835D-01	-0.29132D-02	-	
Control Moment Gyros (CMG) Loc	6	0.10717D-02	-0.16639D-01	-0.31137D-01	-0.42835D-01	-0.29132D-02	-	
	Node ID#	Modal Data at the 9 Gyros ...						
Attitude, Rate, Accelerom Sens	2	0.45900D-02	-0.76587D-01	0.46744D-01	-0.42835D-01	-0.29132D-02	-	
Attitude, Rate, Accelerom Sens	2	0.45900D-02	-0.76587D-01	0.46744D-01	-0.42835D-01	-0.29132D-02	-	
Attitude, Rate, Accelerom Sens	2	0.45900D-02	-0.76587D-01	0.46744D-01	-0.42835D-01	-0.29132D-02	-	
Attitude, Rate, Accelerom Sens	2	0.45900D-02	-0.76587D-01	0.46744D-01	-0.42835D-01	-0.29132D-02	-	
Attitude, Rate, Accelerom Sens	2	0.45900D-02	-0.76587D-01	0.46744D-01	-0.42835D-01	-0.29132D-02	-	
Attitude, Rate, Accelerom Sens	2	0.45900D-02	-0.76587D-01	0.46744D-01	-0.42835D-01	-0.29132D-02	-	
Pointing Antena	1	0.19735D-02	-0.34853D-01	0.29482D-01	-0.42912D-01	-0.26934D-02	-	
Pointing Antena	1	0.19735D-02	-0.34853D-01	0.29482D-01	-0.42912D-01	-0.26934D-02	-	
Pointing Antena	1	0.19735D-02	-0.34853D-01	0.29482D-01	-0.42912D-01	-0.26934D-02	-	

	Node ID#	Modal Data at the 6 Accelerometers, along (x,y,z)...				
Attitude, Rate, Accelerom Sens	2	0.45900D-02	-0.76587D-01	0.46744D-01		
Attitude, Rate, Accelerom Sens	2	0.45900D-02	-0.76587D-01	0.46744D-01		
Attitude, Rate, Accelerom Sens	2	0.45900D-02	-0.76587D-01	0.46744D-01		
Solar Array Hinge	4	-0.28312D-01	0.44163D-02	0.23976D+00		
Solar Array Hinge	4	-0.28312D-01	0.44163D-02	0.23976D+00		
Solar Array Hinge	4	-0.28312D-01	0.44163D-02	0.23976D+00		
	Node ID#	Modal Data at the Disturbance Point				
Reboost Engine Thruster 110 (1	5	0.16319D-02	-0.22063D-01	-0.79956D-01	-0.43084D-01	-0.27097D-02

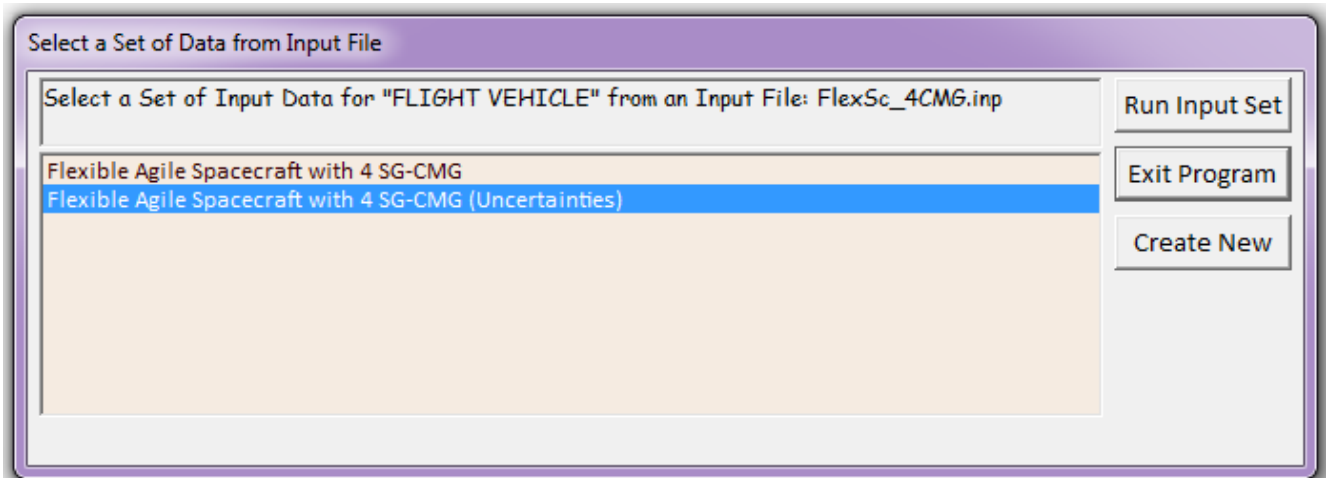
We begin the Flixan program and select the project folder: “\Flixan\ Examples\Robust Analysis Param Uncertainties\Satellite\_SGCMG”. Unlike the previous examples, this time we will create the uncertainty model using the FVMP. From the Flixan main menu select “Program Functions”, “Flight Vehicle/ Spacecraft Modeling Tools”, and “Flight Vehicle State-Space Model”, as shown below.



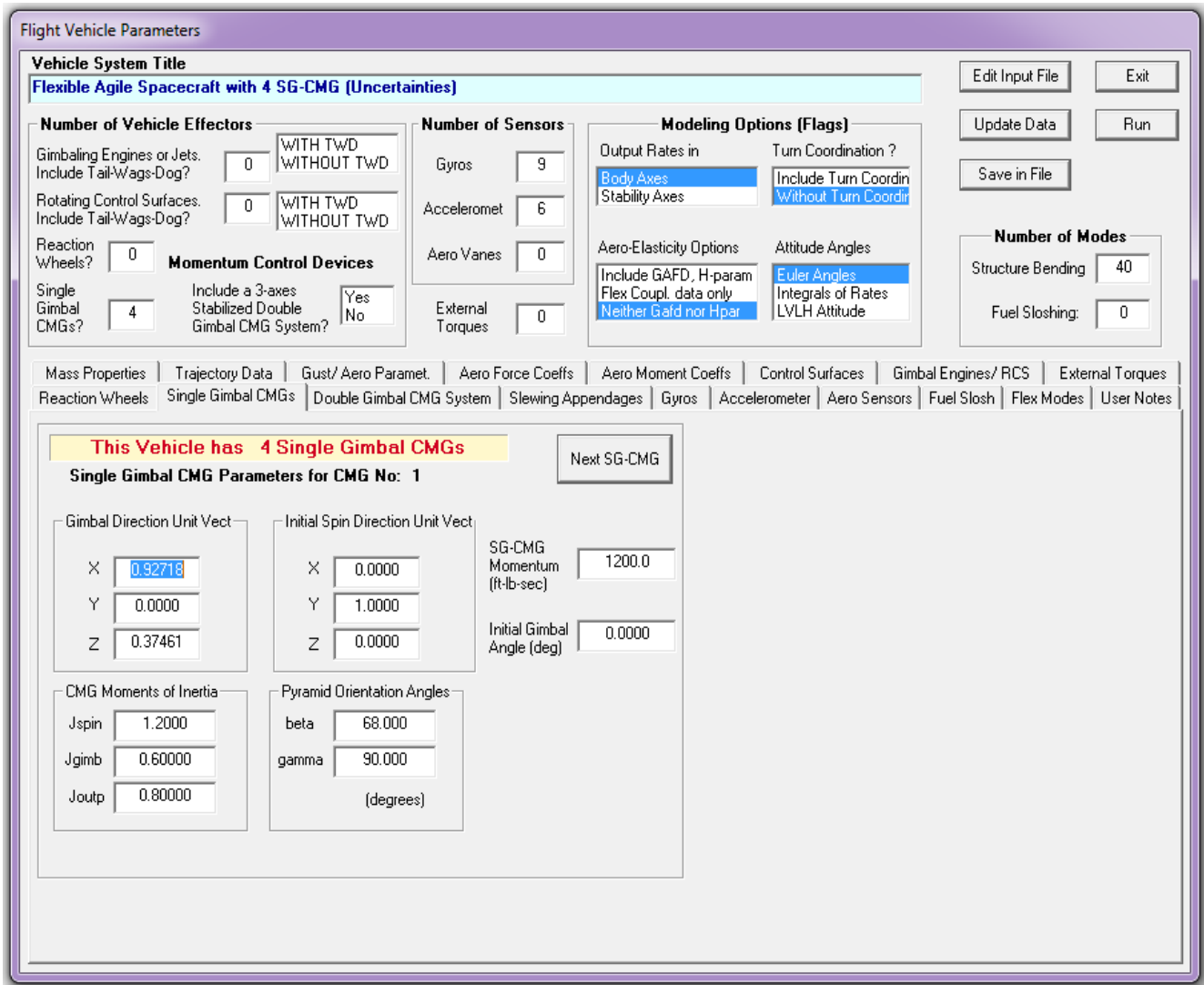
From the filename selection menu select the input data file “*FlexSc\_4CMG.Inp*”, and the output systems file “*FlexSc\_4CMG.Qdr*”, and click on "Select Files" button.



From the following menu select the spacecraft data title "*Flexible Agile Spacecraft with 4 SG-CMG (Uncertainties)*" that includes references to the uncertainties and to the modal data sets. So when we process the data via the FVMP we do not need to select the uncertainties data-set because it is included in the spacecraft data. The spacecraft data set contains the spacecraft and CMG input data for the initial CMG gimbal configuration. Click on "*Run Input Set*" to process the data.



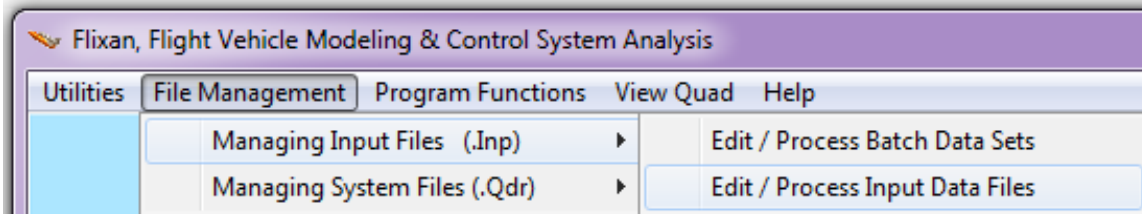
The following dialog is generated by the vehicle modeling program and it consists of several tabs for browsing and modifying the spacecraft data. Each tab consists of a group of data. The following tab shows the SG-CMG data.



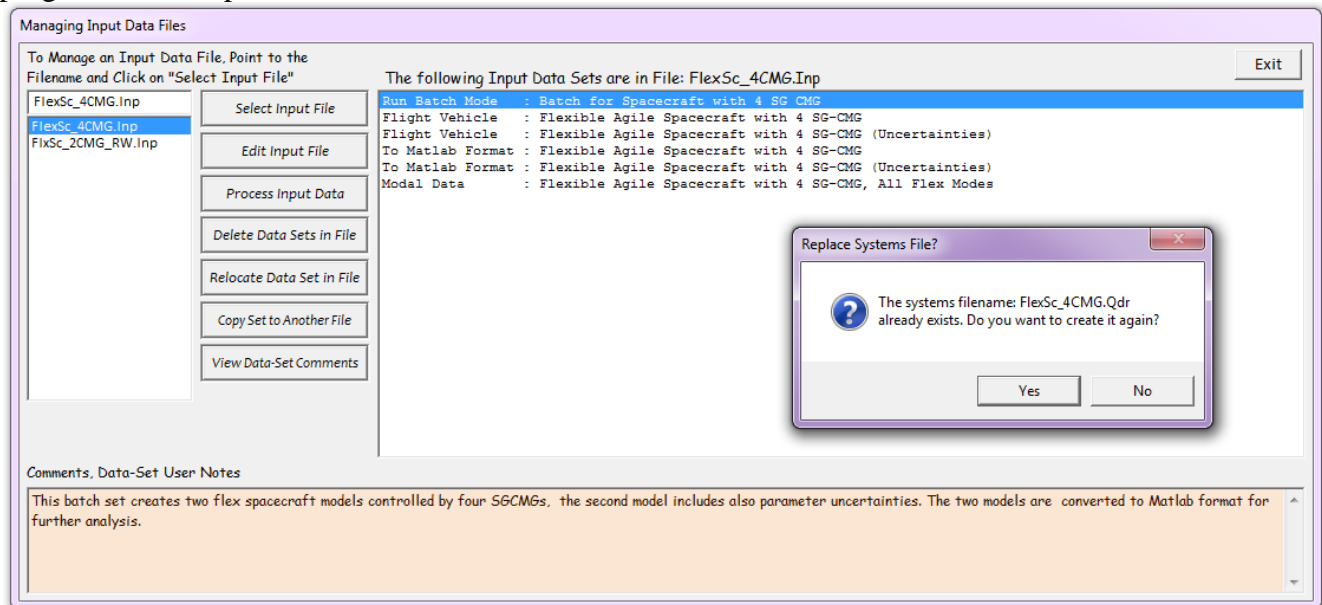
Click on “Run” and the program will create a linearized state-space model for the agile satellite and it will save it in file “*FlexSc\_4CMG.Qdr*”. This system is also converted as a Matlab function in m-file: “*sc\_4cmg\_flex\_unc.m*” that will be used to perform  $\mu$ -analysis in Matlab. This system includes the 61 additional inputs and the 61 additional outputs that connect to the uncertainty block  $\Delta$ . In our  $\mu$ -analysis we don’t connect it with a  $\Delta$  block but we simply compute the  $\mu$  frequency response between the 61 inputs and outputs. Notice, that the input/ output pairs connecting to the uncertainty block  $\Delta$  is greater than the number of parameter variations. This is because some of the uncertainties are higher than rank-1 dependency, and they couple into more than one direction since they appear in 2 equations. The products of inertia, for example, couple in two directions. Since we do not decouple the system but analyze roll, pitch, and yaw axes together, it is acceptable to treat them as two separate parameter variations although they originate from a single uncertainty.

## Batch Processing

Batch processing is the easiest and fastest method for generating the spacecraft models. The input file “*FlexSc\_4CMG.Inp*” includes a batch set “*Batch for Spacecraft with 4 SG CMG*” located on the top of the input data file that can be processed to generate the systems as follows. Start the Flixan program and select the folder “*Examples\Robust Analysis Param Uncertainties\Satellite\_SGCMG*” as before. Then, from the Flixan main menu go to “*File Management*”, “*Manage Input Files*”, and “*Process/ Edit Input Data*”, as shown below.



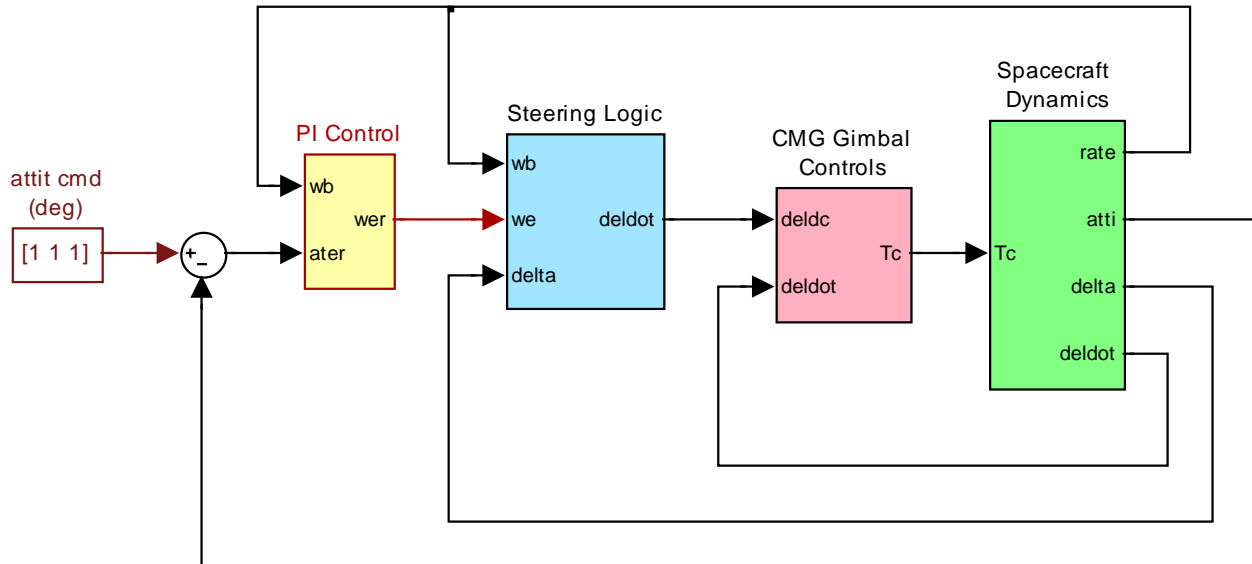
The following Menu/ Dialog comes up, and from the file selection menu on the left select the input file “*FlexSc\_4CMG.Inp*”. The menu on the right shows the titles of the data-sets which are included in the input file. Select the batch on the top “*Batch for Spacecraft with 4 SG CMG*”, and click on “*Process Input Data*” to process the input file. The Flixan program generates the systems and saves them in systems file “*FlexSc\_4CMG.Qdr*”. If a previous version of the systems file already exists in folder the program will ask permission to recreate it, answer "Yes".



The batch performs the following operations. It creates a nominal spacecraft model with 40 flex modes, title: “*Flexible Agile Spacecraft with 4 SG-CMG*” in file “*FlexSc\_4CMG.Qdr*”. It also creates the uncertainty model with the 61 parameter variations, title: “*Flexible Agile Spacecraft with 4 SG-CMG (Uncertainties)*”. The two state-space models are then converted to Matlab system functions, “*sc\_4cmg\_flex.m*” and “*sc\_4cmg\_flex\_unc.m*”, respectively that can be loaded into Matlab. Both systems contain roll, pitch, and yaw coupled vehicle dynamics. The second system, in addition to the standard inputs and outputs present in the first system, it includes also the 61 pairs of inputs and outputs that connect to the uncertainties  $\Delta$  block. This is the diagonal block that contains the normalized uncertainties that vary between -1 and +1. Some of the uncertainties couple only in one axis, but some uncertainties couple in more than one axis.

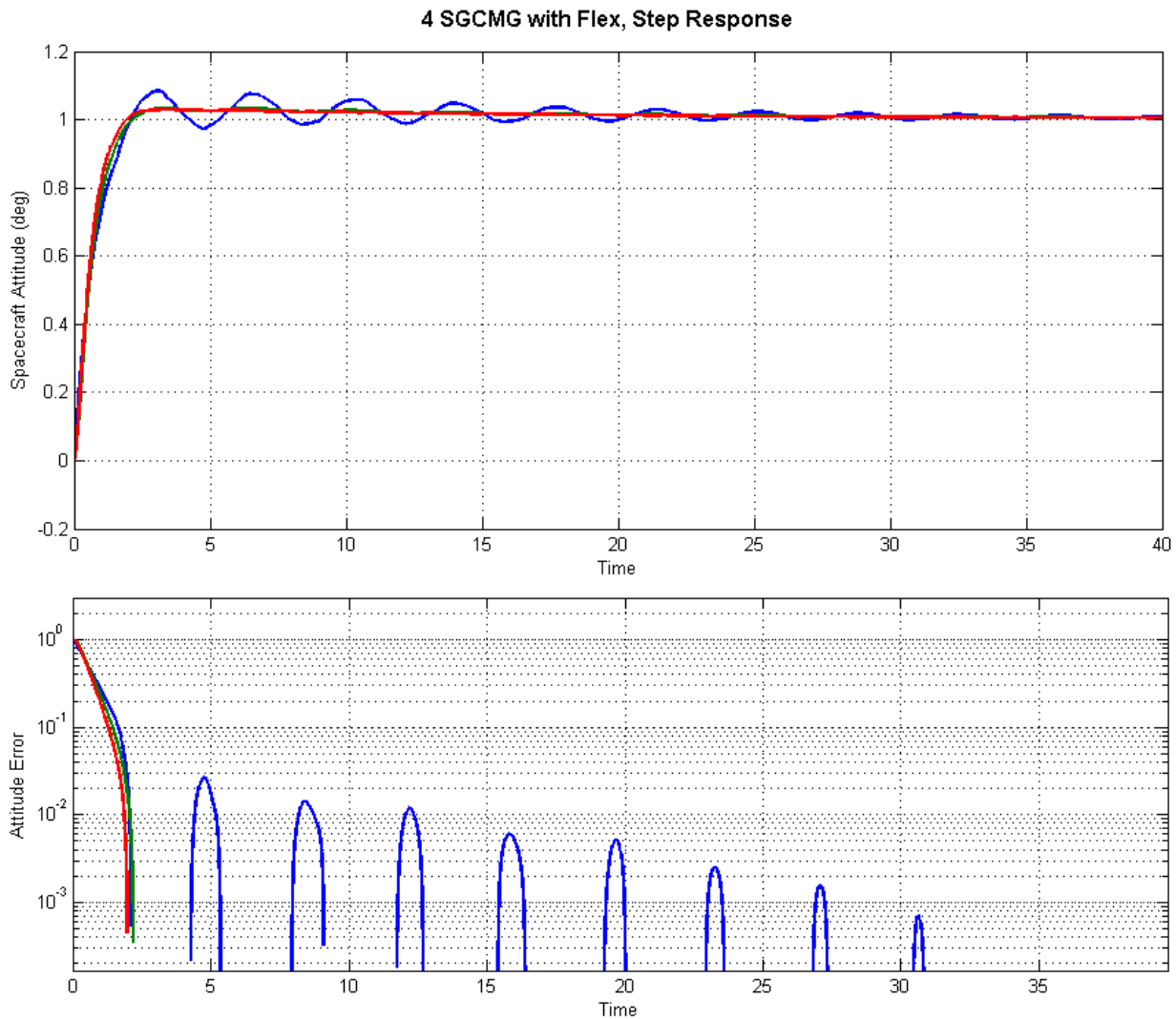
## Simulation Model

Now, let us take a look at the linear simulation model in file "*Lin\_Flex\_Sim.Mdl*", shown in Figure (2), that uses the nominal spacecraft system from file "*sc\_4cmg\_flex.m*", without the uncertainties. This simulation model is initialized using file "*start.m*", which also loads the two spacecraft systems into the Matlab workspace.



**Figure (2) Simulink model for the Flex Agile Spacecraft with 4 SG-CMGs, in file "*Lin\_Flex\_Sim.mdl*"**

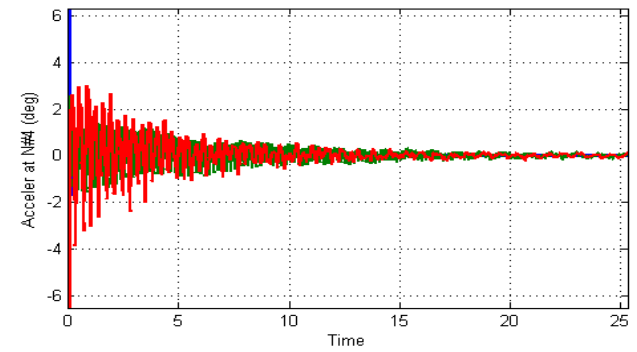
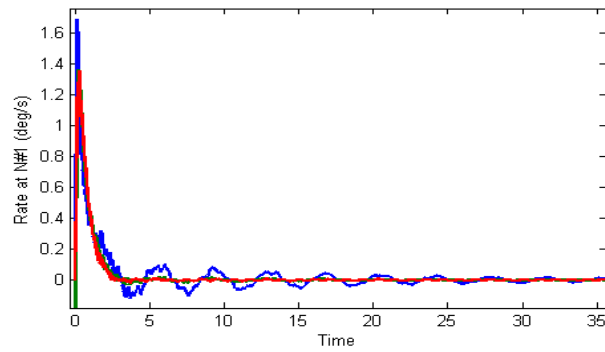
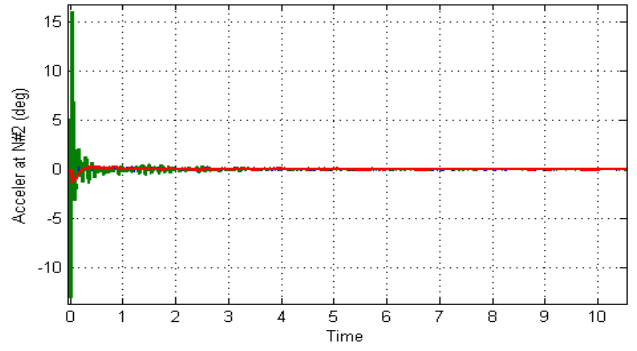
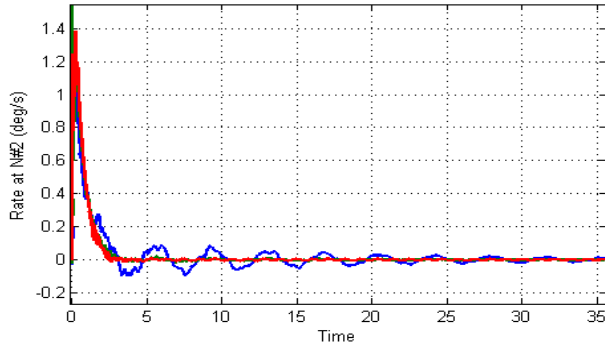
The spacecraft dynamics (green) block consists of the state-space system "*sc\_4cmg\_flex.m*" (without uncertainties). The input ( $T_c$ ) is a vector of four CMG gimbal torques which control the gimbal rates. The outputs are: spacecraft attitude, rates, CMG gimbal angles, and gimbal rates. The gimbal rate commands come from the CMG steering logic and the gimbal rate control system provides the gimbal torques required to control the rates. The purpose of the steering logic is to control the spacecraft rate by creating gimbal rate commands at the 4 CMG gimbals. The inputs to the steering logic are: spacecraft rates, gimbal angles, and spacecraft rate error. The attitude control system is a simple PI. The (D) part of the PID is included in the steering. In the simulation the spacecraft is commanded to perform a one degree rotation in all 3 directions. The purpose of the simulation is to demonstrate nominal system stability.



**Figure (3a) Spacecraft attitude response to one degree command in all 3 directions**

Figure (3a) shows a stable attitude response to  $1^\circ$  command in all 3 directions. The roll axis (blue) takes longer to settle because flexibility is stronger in roll. Figure (3b) shows the spacecraft rate and acceleration at two separate locations with different flex mode sensitivity. Figure (3c) shows the gimbal angles and gimbal rates. It also shows the CMG momentum. The roll momentum oscillates as the CMGs respond to the roll structural oscillations.

4 SGCMG with Flex



4 SGCMG with Flex

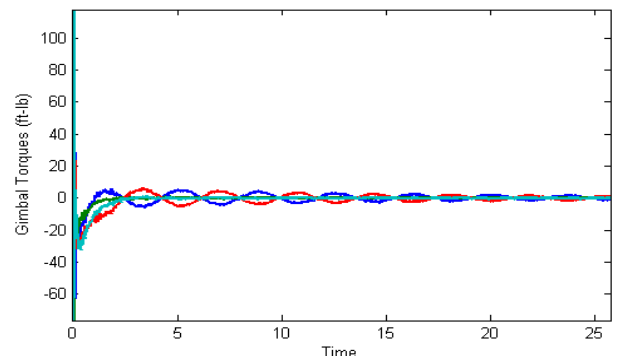
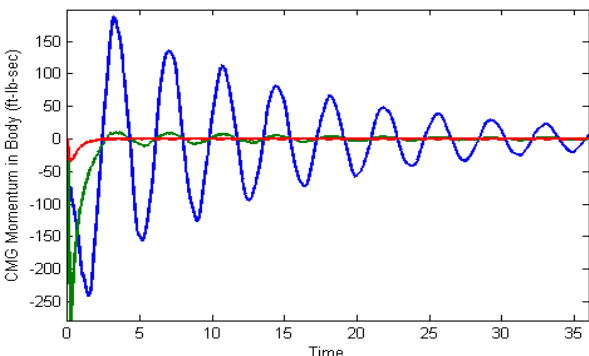
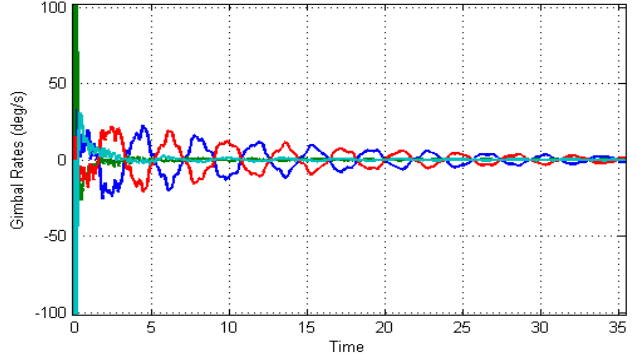
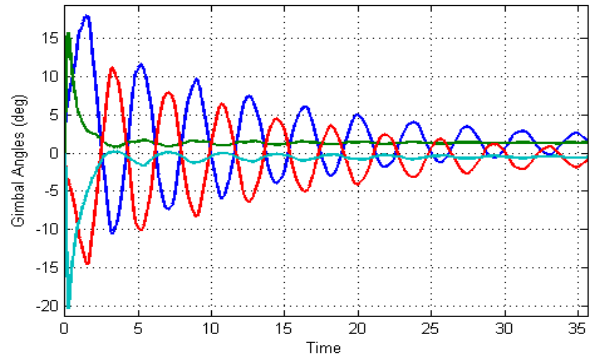


Figure (3b & 3c) Spacecraft response to the one degree command in 3 directions



## Linear Stability Analysis

A similar Simulink model "*Open\_Loop.mdl*" shown in Figure (4) is used to perform linear stability analysis and to determine the phase and gain margins in the nominal CMG configuration. It consists of the same subsystems as the simulation model having three control loops (roll, pitch, yaw) two of which are closed and one opened. The file "*freq.m*" performs the open-loop frequency response analysis. The Matlab script linearizes this system across the opened input and output using the `linmod` function, it calculates its frequency response and plots the Nichols charts, as shown in Figures (5a-5c).

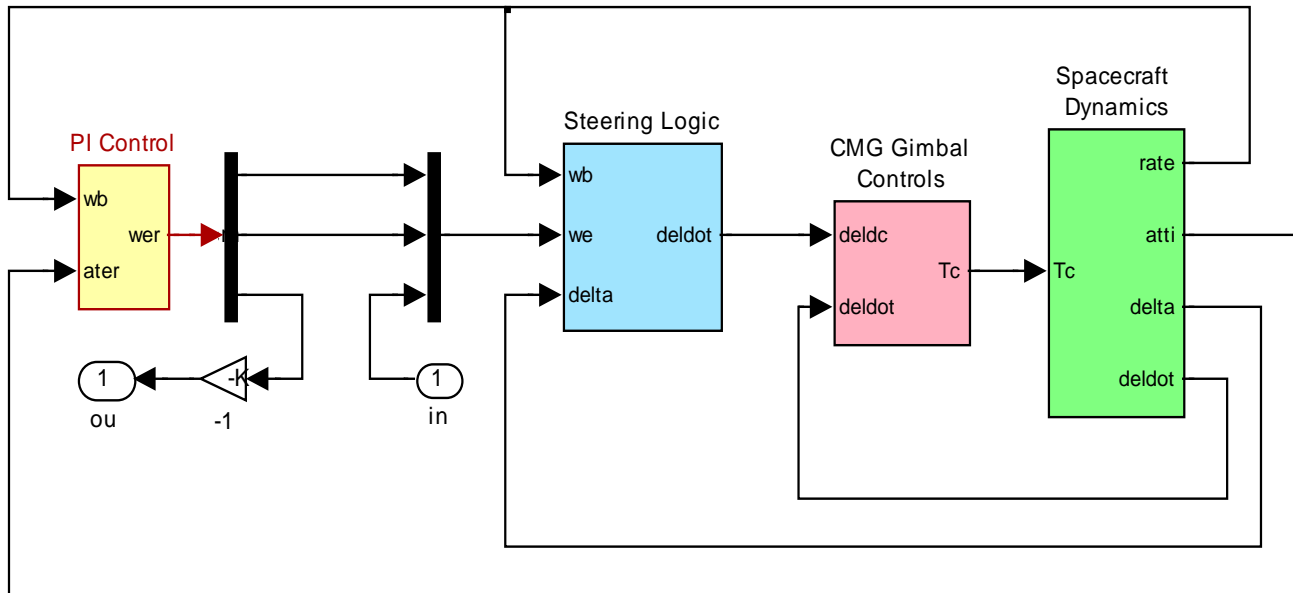
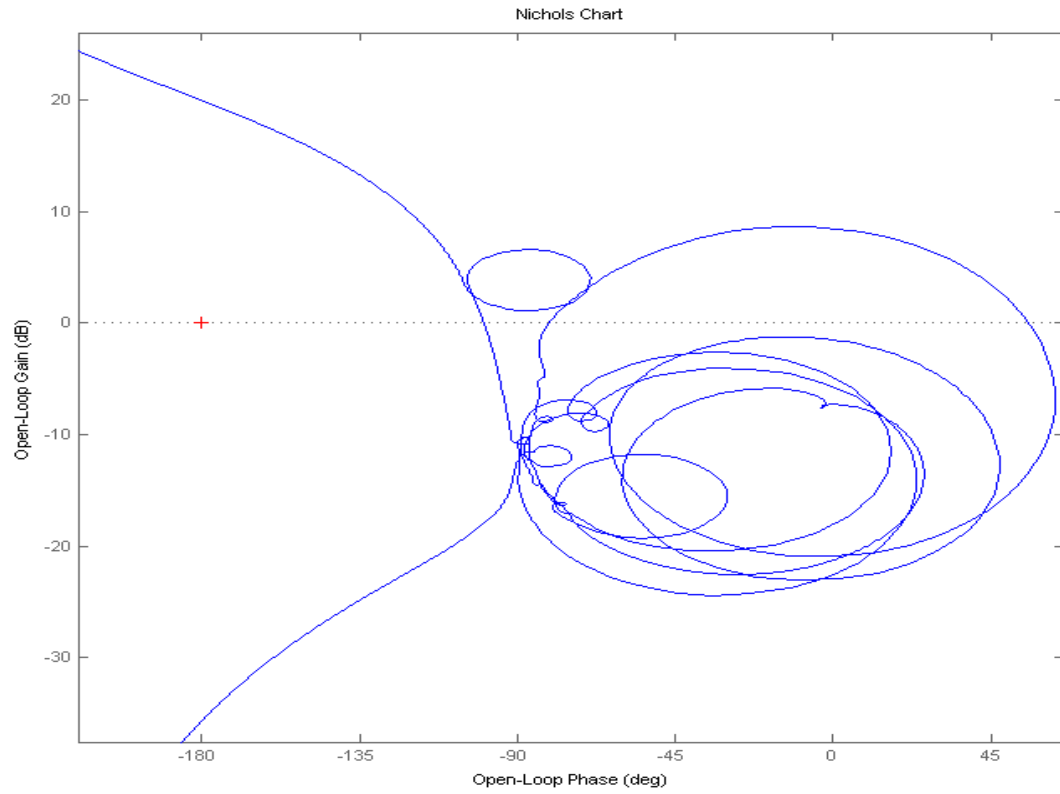
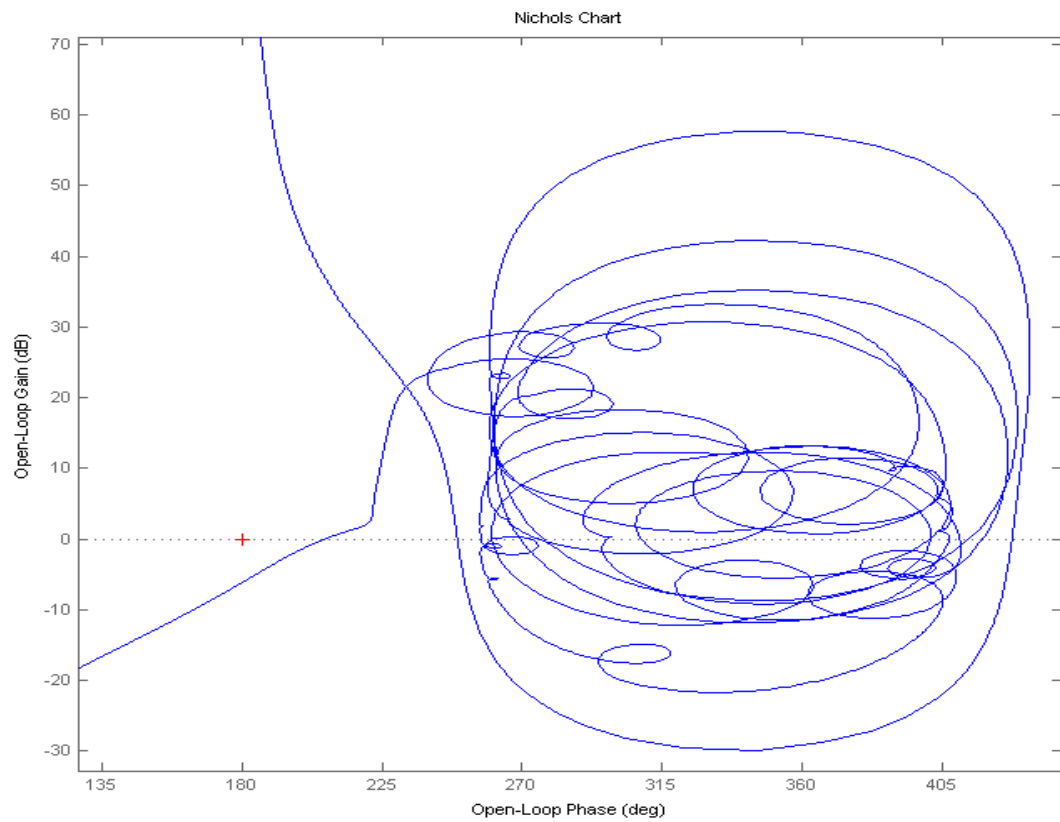


Figure (4) Simulink model "*Open\_Loop.mdl*" used for linear stability analysis

### Nominal System Stability Margins in the Yaw Axis



### Nominal System Stability Margins in the Roll Axis



### Nominal System Stability Margins in the Pitch Axis

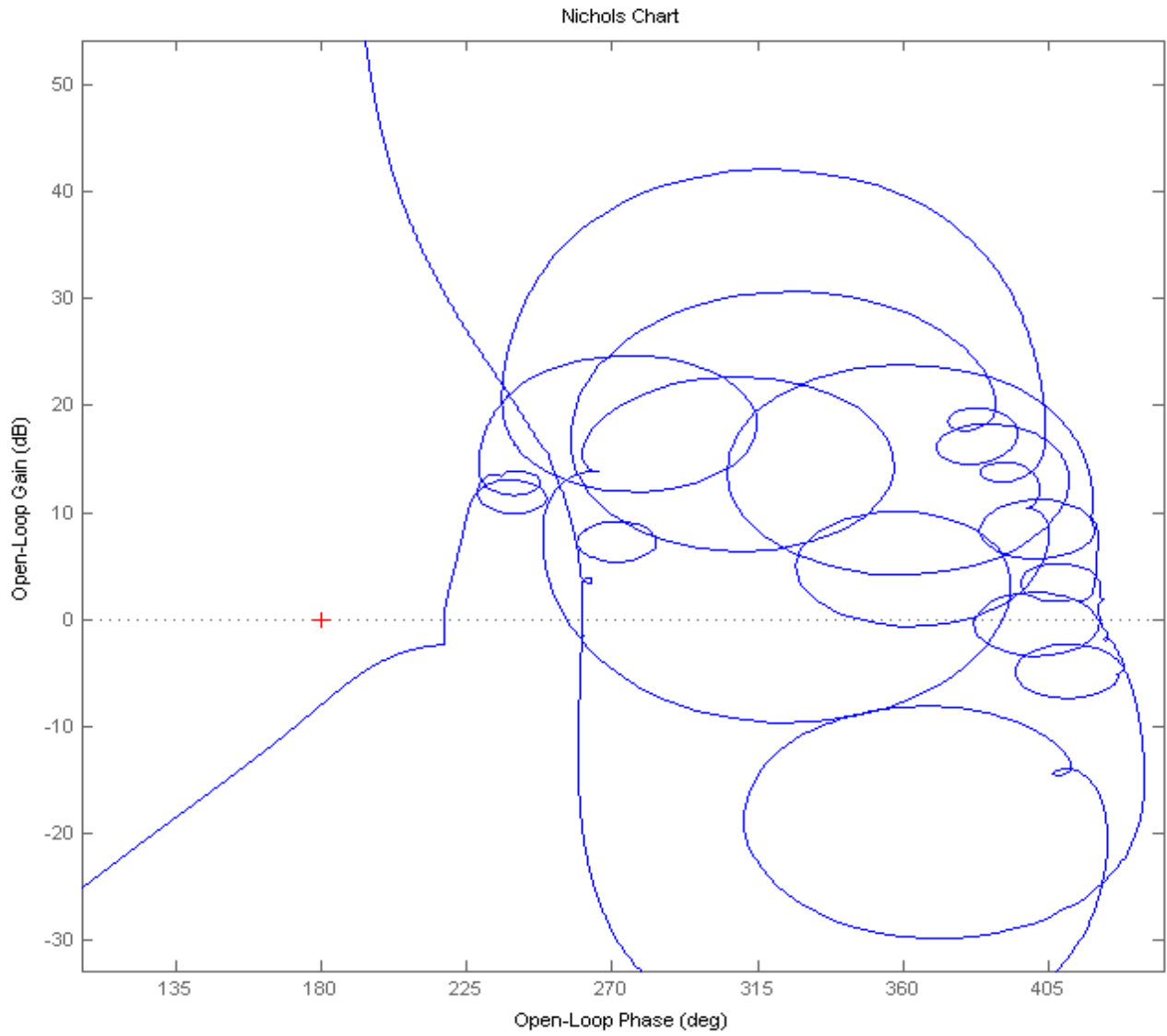
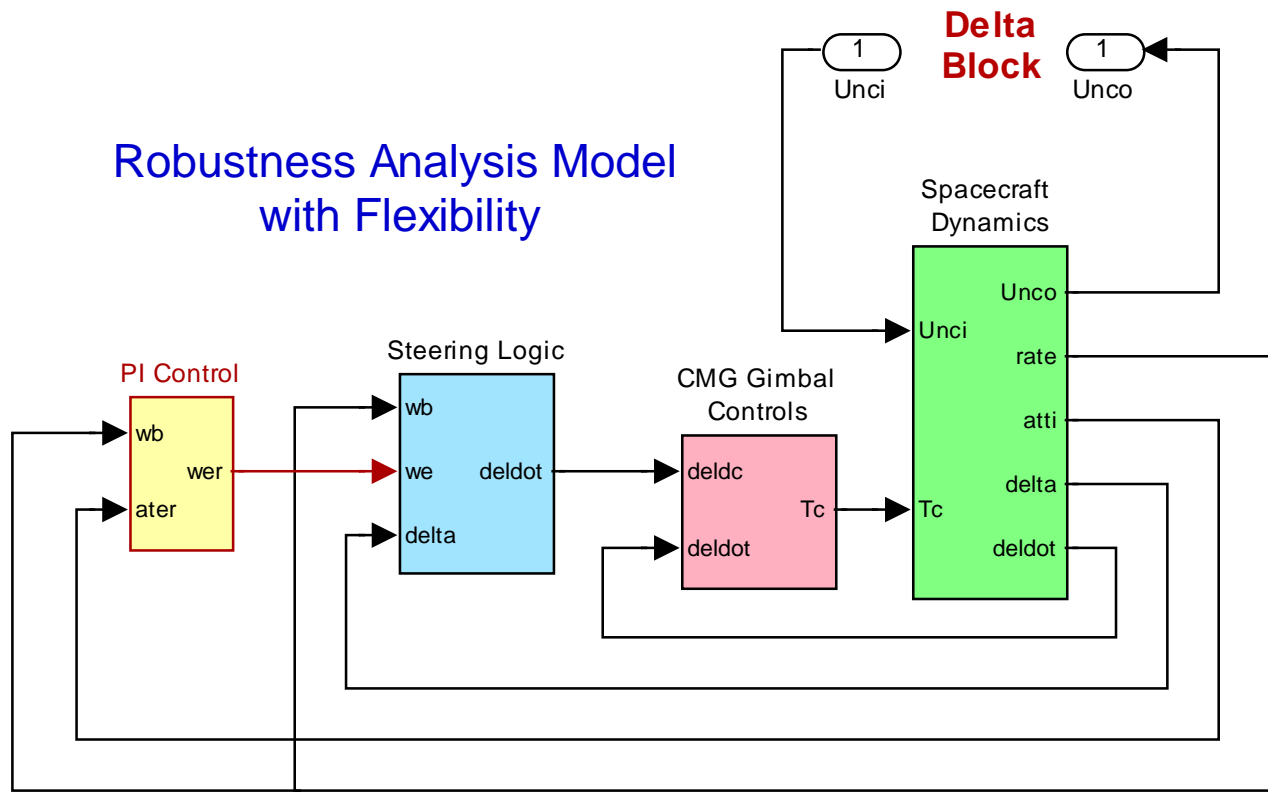


Figure 5c System is nominally stable in all 3 axes

## Robustness Analysis

Robustness analysis is performed by closing the attitude control loops (assuming that the closed-loop system is stable) and by calculating the  $\mu$ -frequency response of the system across the normalized perturbations block  $\Delta$ . Since the scaled diagonal elements of  $\Delta$  do not vary more than  $\pm 1$ , the system is assumed to be robust when the SSV of the closed-loop system across the diagonal perturbations is less than one at all frequencies. The following Simulink model "Robust-Anal.mdl" is used to calculate the SSV.



**Figure (6) Simulink model "Robust\_Anal.mdl" used to calculate system robustness to parameter variations**

The green spacecraft dynamics block now includes the additional inputs and outputs that connect to the uncertainty block  $\Delta$ . The control system, steering logic, and CMG gimbal control subsystems are the same as in previous models. The spacecraft subsystem is shown in detail in Figure (7). It uses the state-space system "*sc\_4cmg\_flex\_unc.m*" which includes the 61 additional inputs and outputs that connect with the IFL block. The Matlab script "freq.m" also calculates the  $\mu$ -frequency response of the linearized system in Figure (6) assuming that the parameter variations are "real" (not "complex" because complex variations are very conservative especially with flex mode variations). Figure (8) shows the  $\mu$ -frequency response across the  $\Delta$  block. It is less than one at all frequencies. We therefore conclude that the system is robust to the parameter variations.

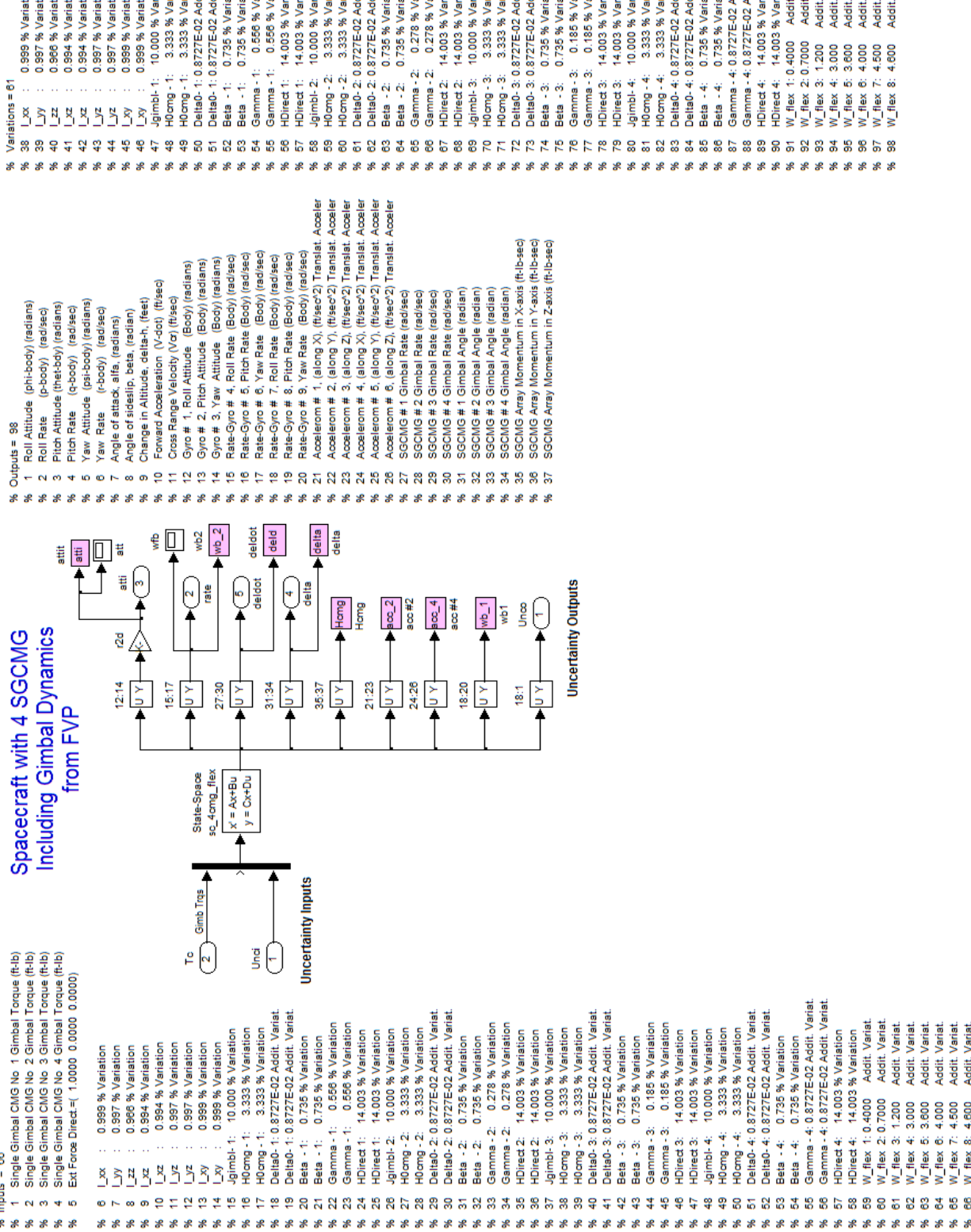
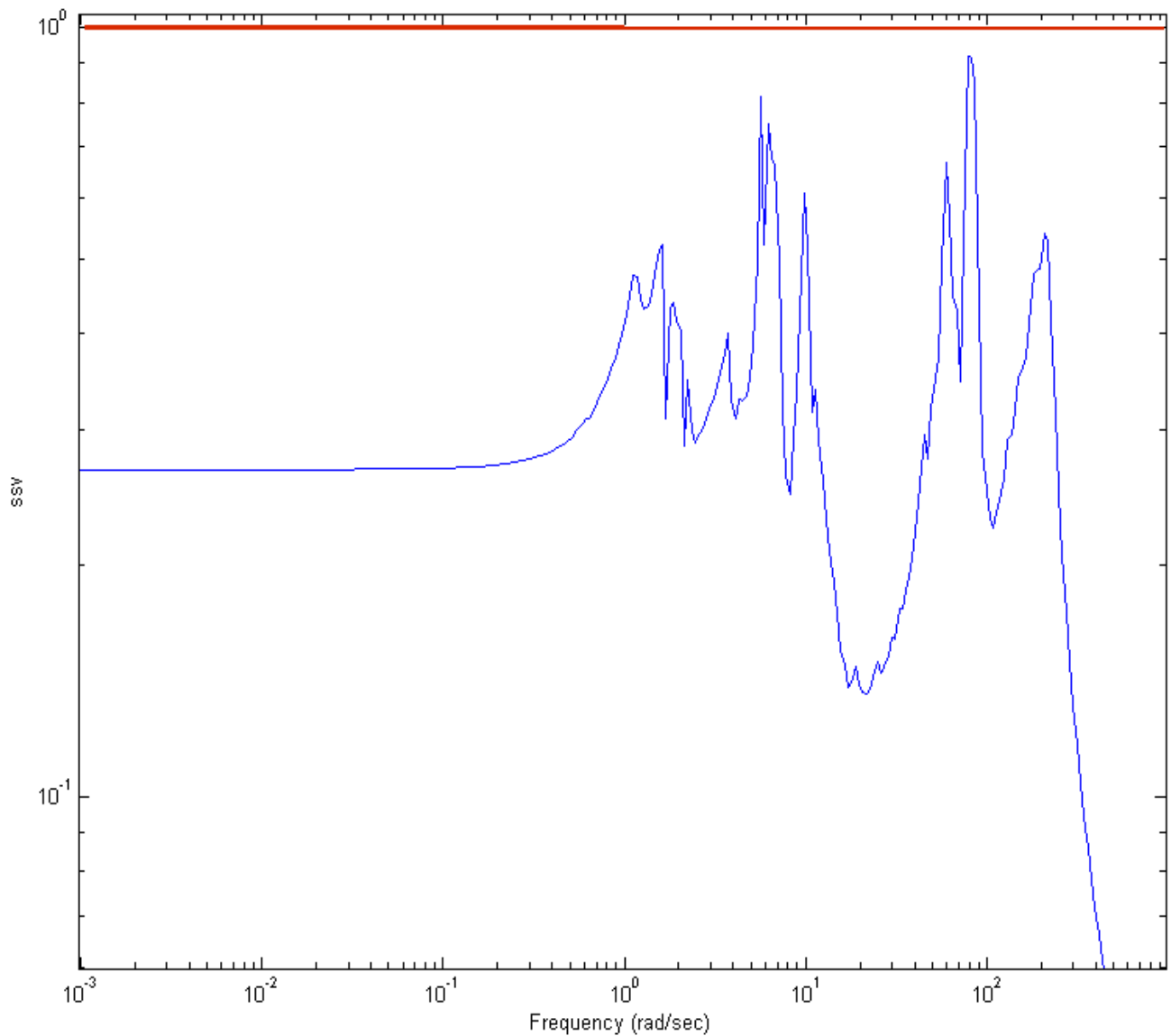


Figure (7) Spacecraft System from "sc\_4cmg\_flex\_unc.m" that includes the uncertainty inputs and outputs.

### Structured Singular Value Analysis of the Agile Spacecraft with 4 SGCMG using 61 Uncertainties



**Figure (8) Structured Singular Value frequency response across the perturbations block  $\Delta$  is less than one at all frequencies. The system, therefore, is robust to the parameter variations.**