# Single Stage to Space Rocket Plane



In this example we will design a single stage rocket with wings that takes off vertically like a launch vehicle, goes into space, and re-enters the atmosphere to land on a runway like a Space Shuttle. It looks like a space-plane that operates in three phases: ascent, on-orbit, and re-entry. In this analysis, however, we will only study the ascent phase, create models, design a flight control system for different flight conditions along the ascent trajectory and analyze its performance in those critical flight conditions to make sure that flight vehicle controllability, performance and stability are satisfactory throughout ascent.

The vehicle shape is similar to the one shown in the above Figure. During ascent it is controlled by 6 gimbaling engines, each producing a thrust of 430,000 (lb) at Max-Q. All engines are identical and they gimbal in pitch and yaw. Since the vehicle mass properties change with fuel level, a thrust vector control (TVC) logic is calculated at each flight condition to convert the roll, pitch, and yaw flight control commands to gimbal deflections for each engine, and the TVC gains vary with time. The engines consume liquid oxygen (LOX) and liquid hydrogen (LH2) and, therefore, the vehicle carries two big tanks for the two propellants which are sloshing and they produce a significant amount of disturbance forces on the vehicle that must be analyzed for stability. Structural flexibility and tail-wag-dog dynamics are also included in the models. The space-plane also has Elevon and Rudder aerosurfaces which are only used during descent.

# 1.0 Vehicle Configuration Overview

The vehicle configuration is shown in Figure 1, which shows the locations of the 6 TVC engines and the propellant tanks. The LH2 tank is bigger and it is located in the front section. The LOX tank is smaller but heavier and it is located towards the rear. Slosh dynamics of the two propellants are represented by spring-mass models which are included in the analysis models. We will analyze the slosh interaction with the vehicle dynamics including structural flexibility to make sure that oscillatory instabilities do not occur. The slosh parameters were obtained from separate mechanical models that calculate the slosh masses, locations, damping coefficients and frequencies as a function of the tank geometry, fuel level and propellant density. The vehicle CG is towards the rear which makes the vehicle statically unstable. It varies but it is mostly near the center of the LOX tank. The analysis will show that the LOX mode is not always phase-stable (unlike our Space Shuttle example) but it has tendency towards instability at some flight conditions, and baffles will be required inside the LOX tank to increase the damping coefficient. Structural flexibility is also included in the analysis models but the emphasis in this example is not to demonstrate flexibility and modal data selection since this process has already been demonstrated in other examples. For this reason the same set of flex modes is used in all flight cases. The flex modes were obtained from a finite elements model and have already been selected, re-formatted and ready to be processed in Flixan. The analysis files for this example are in folder "*Flixan\Examples\Space Plane Launch Vehicle*" and it consists of two parts: (a) the control design in subdirectory "*System Design*" which is based on simple models, and (b) the subdirectory "*Analysis Models*" which analyzes stability and performance with flexibility and propellant sloshing.
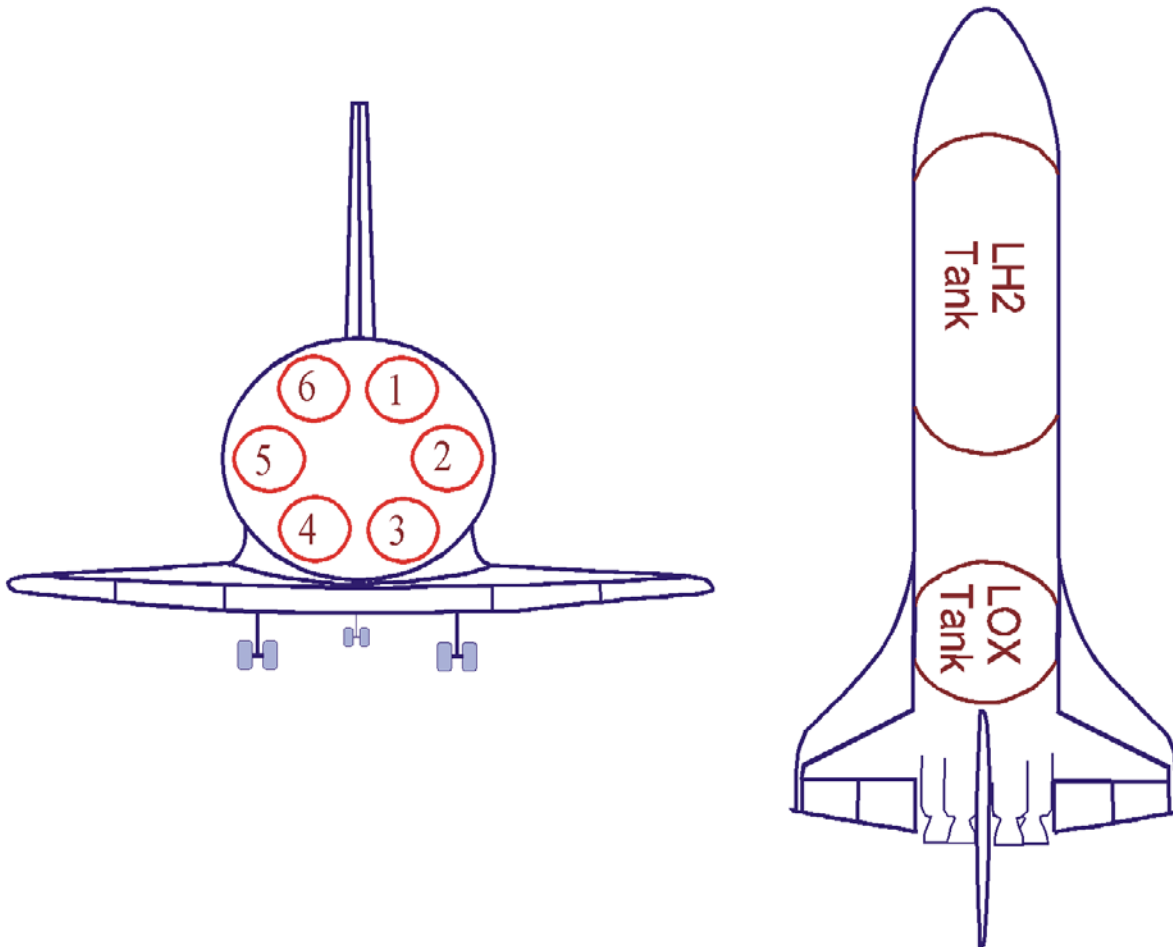


**Figure 1 Rocket-Plane Concept**

## 2.0 Preliminary Vehicle Modeling and Data Processing

The purpose of this example is to demonstrate the use of Flixan program to perform a systematic control design of an ascent vehicle and analyze its stability and performance at multiple flight conditions along a trajectory. For design, input and system files are created at each flight condition and placed in separate subdirectories under "*Space Plane Launch Vehicle\System Design*". The input files (.Inp) include the vehicle modeling, mixing-logic, systems combination, LQR control design, and Matlab conversion datasets, plus a batch processing dataset for fast data processing. The systems files (.Qdr) include the plant models, the TVC matrices, and the LQR weight matrices. The analysis consists of several steps:

- Seven critical flight conditions are selected along the trajectory to create dynamic models of the space plane (lift-off at T0, T20, T56, Max-Q at T82, T108, T160, and engine cut-off at T300. The state-space models at the critical flight conditions are created using the flight vehicle modeling program.
- Mixing Logic (12 x 3) TVC matrices are also created by the Flixan program at each flight condition, which convert the roll, pitch, and yaw flight control commands to 6-pitch and 6-yaw TVC deflections. The TVC matrix is then combined with the vehicle plant at each flight condition to create a simplified design plant "*Design Model with TVC*".
- The design model is separated into pitch and lateral control designs and the LQR method is used to design the flight control gains. Some additional attitude-integral states are included in the design models for better attitude tracking. The pitch design model consists of pitch attitude, pitch rate, and angle of attack ($\theta$, q, $\alpha$) states and it is augmented to include also the pitch attitude integral. The input is pitch TVC demand (DQ_tvc), and its title is "*Augmented Pitch Design Model*". The lateral design model includes: roll attitude and rate ($\phi$, p), yaw attitude and rate ($\psi$, r) and angle of sideslip ($\beta$) states and it is augmented to include also the yaw attitude integral. The 2 inputs are roll and yaw TVC demands (DP_tvc, DR_tvc). Its title is "*Augmented Lateral Design Model*".
- The augmented design models are then used to calculate the pitch and lateral state-feedback gains at each flight condition using the LQR method. There are two LQR Control Design datasets for pitch and lateral control design. The state and control weight matrices Qc and Rc are read from the systems file and the state-feedback gain matrices Kq and Kpr are also saved in the same systems file. Each flight condition is designed and analyzed in a separate subdirectory (T0, T20, etc.). The plant models and control gains are also converted to Matlab format for further analysis.

A sample input file: "*Design_T20.Inp*" for the t=20 sec case is shown below and it consists of different types of datasets which are processed by the Flixan program. The batch "*Batch for Space-Plane Control Design at T=20 sec*" is used to process the entire file and to generate the plant models and gains in file "*Design_T20.Qdr*". It preserves the 4 LQR weighting matrices in the systems file, creates the plant model "*Space-Plane Launch Vehicle at T=20 sec Design Model*", the TVC matrix "*Mixing Logic for the Space-Plane at T=20 sec*", and combines the plant model with the TVC to create the system: "*Design Model with TVC*". This combination simplifies the control design because it reduces the design plant inputs. The files for the other 6 flight conditions are very similar. The LQR weight matrices were adjusted for each case based on analysis to achieve good performance. The flags line on the top of the vehicle dataset defines the vehicle attitude equal to the integral of the body rates. This is because the attitude commands from guidance are not Euler commands relative to a fixed frame, but they are small attitude increments relative to the current vehicle body position.

! This batch set creates Pitch and Lateral Space-Plane models
! and performs LQR Control Design
!
Retain Matrix    : Pitch State Weight Matrix Qc (4x4)
Retain Matrix    : Pitch Control Weight Matrix Rc
Retain Matrix    : Lateral State Weight Matrix Qc (6x6)
Retain Matrix    : Lateral Control Weight Matrix Rc (2x2)
!
Flight Vehicle   : Space-Plane Launch Vehicle at T=20 sec Design Model
Mixing Matrix    : Mixing Logic for the Space-Plane at T=20 sec
System Connection: Design Model with TVC
Transf-Functions : Integrator
!
System Modificat : Pitch Design Model
System Connection: Augmented Pitch Design Model
LQR Control Des  : Pitch LQR Control Design
!
System Modificat : Lateral Design Model
System Connection: Augmented Lateral Design Model
LQR Control Des  : Lateral LQR Control Design
!
To Matlab Format : Design Model with TVC
To Matlab Format : Pitch Design Model
To Matlab Format : Lateral Design Model
To Matlab Format : Pitch LQR State-Feedback Controller
To Matlab Format : Lateral LQR State-Feedback Controller
--------------------------------------------------------------------------------
FLIGHT VEHICLE INPUT DATA ......
Space-Plane Launch Vehicle at T=20 sec Design Model
Body Axes Output, Attitude=Rate Integral

Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft)   :   58979.6, 32.17, 0.2089E+08
Moments and products of Inertias Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)         :   0.392E+7 0.7976E+8 0.79E+8
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)        :  -137.0,  0.0,   0.22
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)      :   0.5, 157.0, 28.0,  1576.0
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)        :   41.4,  41.4, 0.0,  -0.2
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                        :  -0.64, 0.0,  0.0,  0.0
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)  :   0.0, 89.8, 0.0,  0.0, 0.0,
Wind Gust Vel wrt Vehi (Azim & Elev) angles (deg), or Force(lb), Torque(ft-lb), locat:xyz:  Gust    45.0      90.0
Surface Reference Area (feet^2), Mean Aerodynamic Chord (ft), Wing Span in (feet)       :   3500, 45.9, 86.0
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial_rho/ Partial_H} :  -123.17, 0.0,  2.9, 0.0
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:  0.0333, -3.491e-5, 0.0,
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}:  0.0, -0.044, 0.0, 0.0,0.0,
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}:  -0.0959, -0.0547, 0.0, 0.0,
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:  0.0, -0.0013, 0.0, 0.0,
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}:  0.0034, -0.0063, 0.0, 0.0,
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}:  0.0, 8.552e-4, 0.0, 0.0,

Number of Thruster Engines, Include or Not the Tail-Wags-Dog and Load-Torque Dynamics ? :  6   NO TWD

TVC Engine No:  1                                (Gimbaling  Throttling   Single_Gimbal) : TVC Upper Right  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                     : 408755.0      408755.0
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): -5.00   0.0  10.0  10.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft): 63.8  690.0  3.70
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -173.50 5.4160  -9.3810

TVC Engine No:  2                                (Gimbaling  Throttling   Single_Gimbal) : TVC Right        Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                     : 408755.0      408755.0
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): -5.0  0.00 10.0  10.00
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft): 63.8 690.00 3.70
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -173.50 10.833  0.0000

TVC Engine No:  3                                (Gimbaling  Throttling   Single_Gimbal) : TVC Low Right    Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                     : 408755.0      408755.0
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): -5.00   0.00  10.0  10.00
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft): 63.800  690.00    3.7000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -173.50  5.4160   9.3810

TVC Engine No:  4                                (Gimbaling  Throttling   Single_Gimbal) : TVC Low Left     Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                     : 408755.0      408755.0
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): -5.0  0.0  10.0  10.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft): 63.8   690.0  3.7000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -173.50  -5.4160 9.3810

TVC Engine No:  5                                (Gimbaling  Throttling   Single_Gimbal) : TVC Left         Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                     : 408755.0      408755.0
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): -5.0  0.0  10.0  10.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft): 63.8  690.0   3.70
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -173.50  -10.833   0.0

4

```
TVC Engine No:  6                              (Gimbaling Throttling  Single_Gimbal) : TVC Upper Left   Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                 : 408755.0      408755.0
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): -5.0   0.0  10.0   10.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  63.8  690.00    3.70
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) : -173.50 -5.4160 -9.381

Number of Gyros, (Attitude and Rate)                                               :  3
Gyro No 1 Axis:(Pitch,Yaw,Roll), (Attitude or Rate), Location in Vehicle Axes  (feet)  : Pitch  Rate -10.0, 0.0, 0.0
Gyro No 2 Axis:(Pitch,Yaw,Roll), (Attitude or Rate), Location in Vehicle Axes  (feet)  : Yaw    Rate -10.0, 0.0, 0.0
Gyro No 3 Axis:(Pitch,Yaw,Roll), (Attitude or Rate), Location in Vehicle Axes  (feet)  : Roll   Rate -10.0, 0.0, 0.0

Number of Bending Modes                                                            :  0
-----------------------------------------------------------------------------------------------------------
MIXING LOGIC MATRIX DATA ..........    (Matrix Title, Name, Vehicle Title, Control Directions)
Mixing Logic for the Space-Plane at T=20 sec
! Generates the Thrust Vector Control Matrix for a Launch Vehicle at T=20 sec
! This vehicle has 6 Gimbaling Engines.
TVC
Space-Plane Launch Vehicle at T=20 sec Design Model
P-dot Roll  Acceleration About X Axis
Q-dot Pitch Acceleration About Y Axis
R-dot Yaw   Acceleration About Z Axis
-----------------------------------------------------------------------------------------------------------
INTERCONNECTION OF SYSTEMS .....
Design Model with TVC
! Combines the Vehicle Model with the TVC Matrix
Titles of Systems to be Combined
Title 1 Space-Plane Launch Vehicle at T=20 sec Design Model
SYSTEM INPUTS TO SUBSYSTEM  1                                              TVC to Vehicle
Via Matrix +TVC                                                           roll, pitch, yaw
............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                          Vehicle Outputs
Via Matrix +I8                                                           First 8 Outputs
............................................................................
Definitions of Inputs  =  3
Roll  TVC Demand from Flight Control
Pitch TVC Demand from Flight Control
Yaw   TVC Demand from Flight Control

Definitions of Outputs =  8
Roll Attitude  (phi)    (radians)
Roll Rate      (p-body) (rad/sec)
Pitch Attitude (theta)  (radians)
Pitch Rate     (q-body) (rad/sec)
Yaw  Attitude  (psi)    (radians)
Yaw  Rate      (r-body) (rad/sec)
Angle of attack, alfa,  (radians)
Angle of sideslip, beta, (radian)
-----------------------------------------------------------------------------------------------------------
SYSTEM OF TRANSFER FUNCTIONS ...
Integrator
Continuous
TF. Block #  1  (1/s)                                         Order of Numer, Denom=  0  1
Numer 0.0        1.0
Denom 1.0        0.0
-----------------------------------------------------------------------------------------------
Block #, from Input #, Gain
  1  1   1.00000
...........................
Outpt #, from Block #, Gain
  1  1   1.00000
...........................
```

The inputs to the "Design Model with TVC" are: roll, pitch and yaw demands from the flight control system outputs instead of using 12 gimbal deflections. The system modification utility is used to split the design model to two state-space systems "*Pitch Design Model*" and "*Lateral Design Model*", which are converted to Matlab m-functions in files "*pitch_rb_t20.m*" and "*later_rb_t20.m*", for LQR synthesis.

```
--------------------------------------------------------------------------------
                                  PITCH DESIGN
--------------------------------------------------------------------------------
CREATE A NEW SYSTEM FROM AN OLD SYSTEM... (Titles of the New and Old Systems)
Pitch Design Model
Design Model with TVC
! The 3-state pitch rigid body system is extracted from the coupled RB system above
TRUNCATE OR REORDER THE SYSTEM INPUTS, STATES, AND OUTPUTS
Extract Inputs :    2
Extract States :    3    4    7
Extract Outputs:    3    4    7
--------------------------------------------------------------------------------
INTERCONNECTION OF SYSTEMS .....
Augmented Pitch Design Model
! Create a 4-State Augmented Pitch Model that Includes Theta-Integral
!
Titles of Systems to be Combined
Title 1 Pitch Design Model
Title 2 Integrator
SYSTEM INPUTS TO SUBSYSTEM  1                                    Pitch Design
System Input  1 to Subsystem  1, Input  1, Gain= 1.0            Delta Command
...............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                 Vehicle Plant
System Output  1 from Subsystem  1, Output  1, Gain= 1.0        theta
System Output  2 from Subsystem  1, Output  2, Gain= 1.0        q -pitch rate
System Output  3 from Subsystem  1, Output  3, Gain= 1.0        alpha
...............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                                 Integrator
System Output  4 from Subsystem  2, Output  1, Gain= 1.0        theta-integrator
...............................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2                          Plant to Integr
Subsystem  1, Output  1 to Subsystem  2, Input  1, Gain=  1.0000  theta
...............................................................................
Definitions of Inputs  =    1
Pitch TVC Command (DQ_tvc)

Definitions of Outputs =    4
Pitch Attitude, theta  (rad)
Pitch Rate, q (rad/sec)
Angle of Attack, alpha (rad)
Theta-Integral (rad-sec)
--------------------------------------------------------------------------------
LINEAR QUADRATIC REGULATOR STATE-FEEDBACK CONTROL DESIGN
Pitch LQR Control Design
Plant Model Used to Design the Control System from:     Augmented Pitch Design Model
Criteria Optimization Output is Matrix C
State Penalty Weight (Qc) is Matrix:   Qc4            Pitch State Weight Matrix Qc (4x4)
Control Penalty Weight (Rc) is Matrix: Rc             Pitch Control Weight Matrix Rc
Continuous LQR Solution Using Laub Method
LQR State-Feedback Control Gain Matrix Kq_t20         Pitch LQR State-Feedback Controller
--------------------------------------------------------------------------------
```

The "*Pitch Design Model*" is extracted from the original plant: "*Design Model with TVC*" and an additional state (θ-integral) is included to create the "*Augmented Pitch Design Model*". This augmentation produces a control system that better tracks the pitch attitude. The dataset "*Pitch LQR Control Design*" performs the LQR design and it saves the Pitch LQR State-Feedback Controller matrix in Kq_t20 in the systems file.

```
----------------------------------------------------------------------------------------
                                  LATERAL DESIGN
----------------------------------------------------------------------------------------
CREATE A NEW SYSTEM FROM AN OLD SYSTEM... (Titles of the New and Old Systems)
Lateral Design Model
Design Model with TVC
! The lateral rigid body system is extracted from the coupled RB system
TRUNCATE OR REORDER THE SYSTEM INPUTS, STATES, AND OUTPUTS
Extract Inputs :   1   3
Extract States :   1   2   5   6   8
Extract Outputs:   1   2   5   6   8
----------------------------------------------------------------------------------------
INTERCONNECTION OF SYSTEMS .....
Augmented Lateral Design Model
! Create a 6-State Augmented Lateral Model that Includes Psi-Integral
!
Titles of Systems to be Combined
Title 1 Lateral Design Model
Title 2 Integrator
SYSTEM INPUTS TO SUBSYSTEM  1                                         Lateral Design Model
System Input  1 to Subsystem  1, Input  1, Gain= 1.0                  Roll TVC Demand
System Input  2 to Subsystem  1, Input  2, Gain= 1.0                  Yaw  TVC Demand
..............................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                      Vehicle Plant
System Output  1 from Subsystem  1, Output  1, Gain= 1.0             Roll Attit (phi)
System Output  2 from Subsystem  1, Output  2, Gain= 1.0             Roll Rate  (p-body)
System Output  3 from Subsystem  1, Output  3, Gain= 1.0             Yaw  Attitude(psi)
System Output  4 from Subsystem  1, Output  4, Gain= 1.0             Yaw  Rate  (r-body)
System Output  5 from Subsystem  1, Output  5, Gain= 1.0             Angle of sideslip,
..............................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                                      Integrator
System Output  6 from Subsystem  2, Output  1, Gain= 1.0             Psi-integral
..............................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2                               Plant to Integr
Subsystem  1, Output  3 to Subsystem  2, Input  1, Gain=   1.0000    Psi
..............................................................
Definitions of Inputs  =   2
Roll TVC Command (DP_tvc)
Yaw  TVC Command (DR_tvc)

Definitions of Outputs =   6
Roll Attitude  (phi)    (radians)
Roll Rate      (p-body) (rad/sec)
Yaw  Attitude  (psi)    (radians)
Yaw  Rate      (r-body) (rad/sec)
Angle of sideslip, beta, (radian)
Yaw, Psi-Integral       (rad-sec)
--------------------------------------------------------------------------------
LINEAR QUADRATIC REGULATOR STATE-FEEDBACK CONTROL DESIGN
Lateral LQR Control Design
Plant Model Used to Design the Control System from:        Augmented Lateral Design Model
Criteria Optimization Output is Matrix C
State Penalty Weight (Qc) is Matrix:   Qc6                 Lateral State Weight Matrix Qc (6x6)
Control Penalty Weight (Rc) is Matrix: Rc2                 Lateral Control Weight Matrix Rc (2x2)
Continuous LQR Solution Using Laub Method
LQR State-Feedback Control Gain Matrix Kpr_t20            Lateral LQR State-Feedback Controller
--------------------------------------------------------------------------------
```

Similar in the lateral case: The "*Lateral Design Model*" is extracted from the original model: "*Design Model with TVC*" and an additional state (ψ-integral) is included in the "*Augmented Lateral Design Model*". This augmentation produces a control system that better tracks the yaw attitude. The dataset "*Lateral LQR Control Design*" performs the LQR design and it saves the Lateral LQR State-Feedback Controller matrix in Kpr_t20 in the systems file.

```
CONVERT TO MATLAB FORMAT ........        (Title, System/Matrix, m-filename)
Pitch Design Model
System
pitch_rb_t20.m
-------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........        (Title, System/Matrix, m-filename)
Lateral Design Model
System
later_rb_t20.m
-------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........        (Title, System/Matrix, m-filename)
Design Model with TVC
System
plant_t20.m
-------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........        (Title, System/Matrix, m-filename)
Pitch LQR State-Feedback Controller
Matrix Kq_t20
-------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........        (Title, System/Matrix, m-filename)
Lateral LQR State-Feedback Controller
Matrix Kpr_t20
-------------------------------------------------------------------------------------
```

The plant models, the pitch and lateral state-feedback matrices are saved in Matlab format in folder "*Space Plane Launch Vehicle\System Design\T20*" for further stability analysis and simulations using Matlab to evaluate the performance of the flight control system and adjust the weight matrices if necessary. This input file can be processed in batch mode as follows:

Start the Flixan program and select the directory of the corresponding design flight case. From the main menu, select "File Management", "Managing Input Files (.Inp)", and "Edit/ Process Input Data Files", as shown.
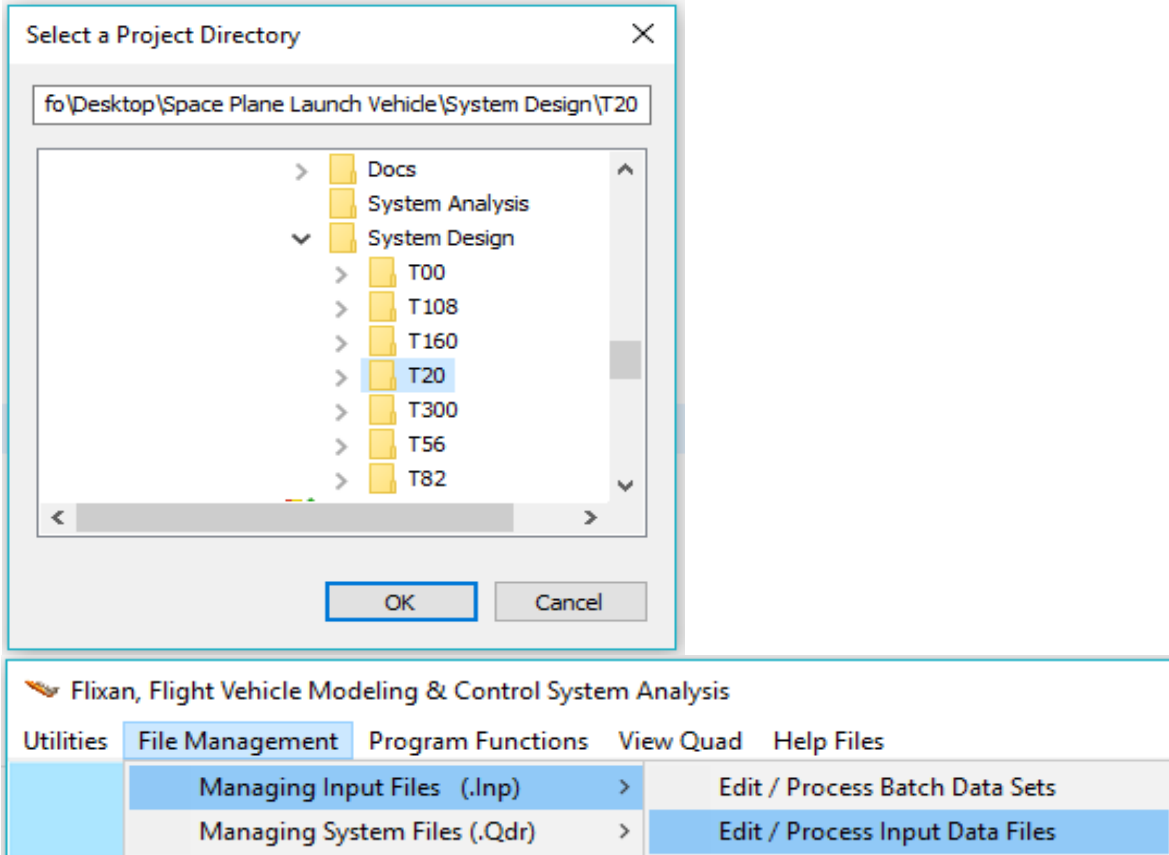
The input file manager utility dialog comes up and from the left menu select the only input file: "*Design_T20.Inp*" in the T20 folder, and click on "*Select Input File*" button. The menu on the right shows the titles of all datasets which are included in this file and to be processed by the Flixan program. Each title is processed by a Flixan utility shown on the left side of the title. Select the top title which is the batch set that will process all the other datasets, and click on "*Process Input Data*" button. Answer "Yes" in the next question to replace the previously created data in the systems file "*Design_T20.Qdr*". It will preserve, however, the LQR weight matrices.
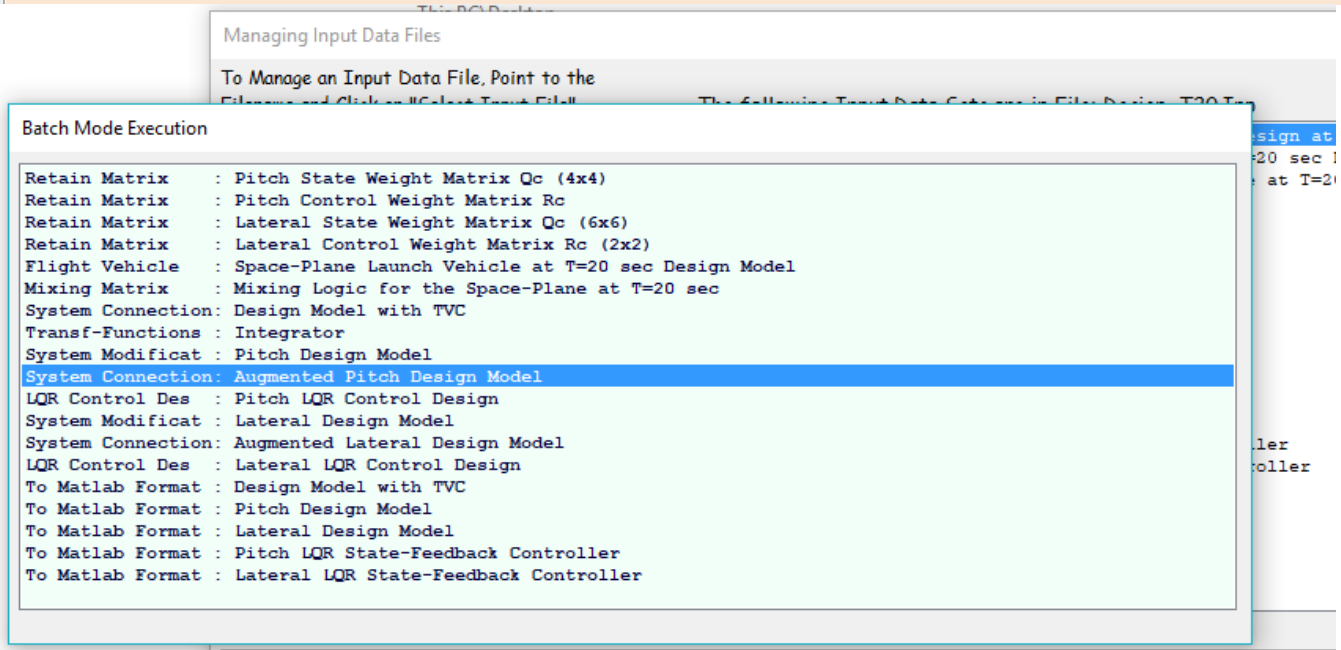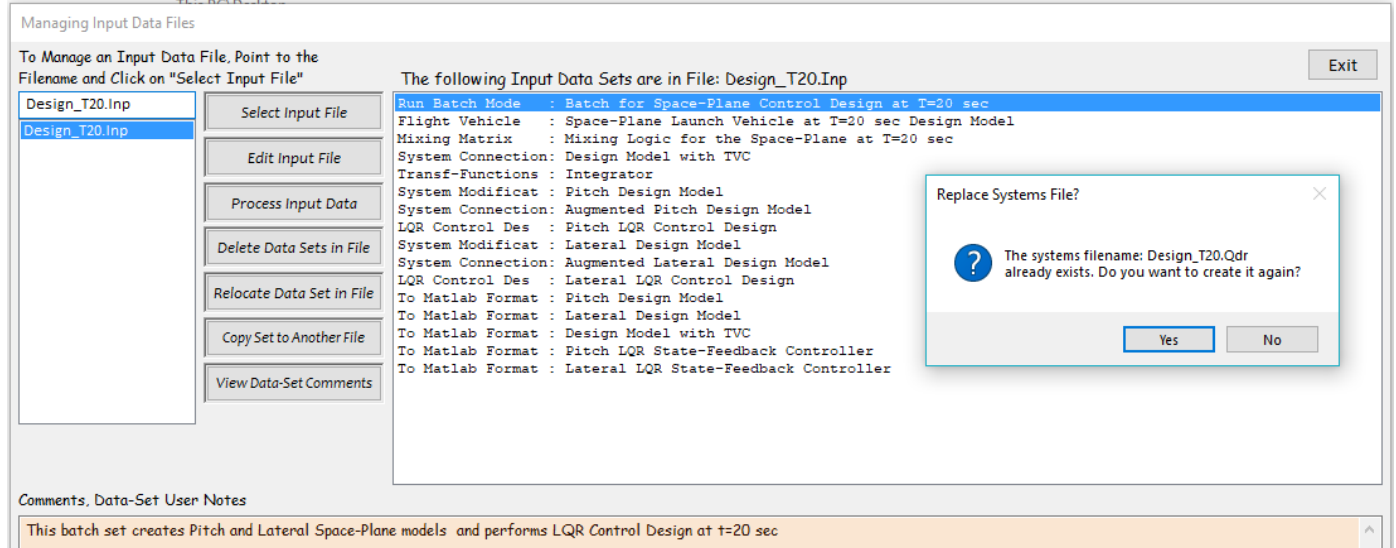




The program will process all the datasets referenced by the batch and it will save the systems and matrices in file: "*Design_T20.Qdr*". It also saves the data in Matlab format for control analysis. A display list pops-up highlighting each dataset while it is executing, shown above, and it disappears when the batch execution is complete. Click "Exit" to close Flixan program and go to Matlab for the control analysis.

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# 3.0 Control Design

The Flixan generated vehicle systems used for LQR state-feedback control design are simple rigid-body models and they do not include slosh, bending, tail-wags-dog, and accelerometer sensors. This is an initial design and the gains will be adjusted later in the analysis to accommodate the flex vehicle model that includes also sensor dynamics. The pitch and lateral LQD design models are shown in Figures (3.1, and 3.2). The input to the pitch design model is pitch flight control demand and the 4 outputs are: pitch attitude θ, rate q, angle of attack α, and θ-integral, which is intended to improve pitch attitude tracking. This is also implemented in the Simulink model "*Pdes_int.mdl*", shown below.

## Augmented Pitch Design Model with theta integral
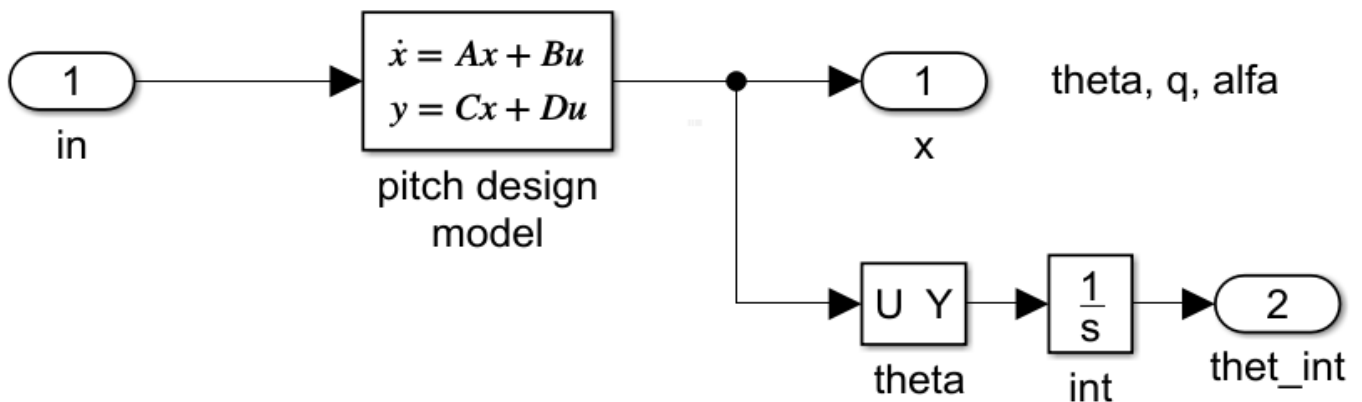


**Figure 3.1 Four-State Pitch Synthesis Model "Augmented Pitch Design Model" in File "Pdes_int.mdl"**

The lateral design model inputs are roll and yaw flight control demands. The six outputs which are also equal to the states are: roll attitude φ, roll rate p, yaw attitude ψ, yaw rate r, angle of sideslip β, and ψ-integral, which is intended to improve yaw tracking. This is also implemented in the Simulink model "*Ldes_int.mdl*", shown below.

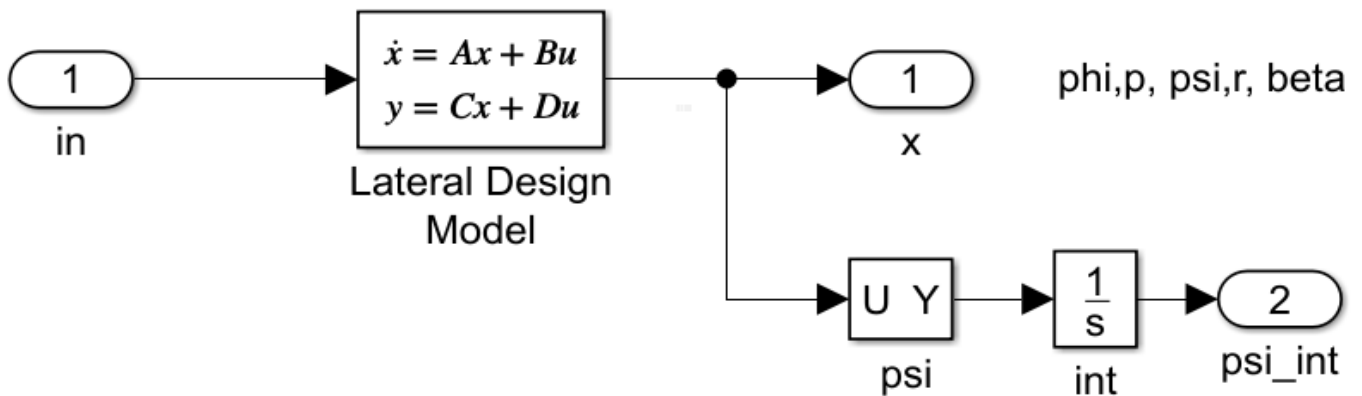## Augmented Lateral Design Model with yaw attitude integral



**Figure 3.2 Six-State Lateral Synthesis Model "Augmented Lateral Design Model" in File "Ldes_int.mdl"**

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The LQR design can also be performed in Matlab. An m-file script is included in folder T20 that performs the LQR design using Matlab. It first loads the pitch and lateral design plants from files "*pitch_rb_t20.m*" and "*later_rb_t20.m*". It augments the state vectors by adding the pitch and yaw attitude integrals, as in Figures (3.1, and 3.2). Then it performs the LQR control design using the diagonal Q and R matrices. The elements in the Q and R matrices are adjusted according to the closed-loop system performance in response to step commands or external disturbances. The pitch and lateral state-feedback gains are saved in files "Kq_t20.mat" and "Kpr_t20.mat" respectively.

```
% LQR Design for Space Plane at T=20 sec, file: des_t20.m
% Pitch Axis
[Ap,Bp,Cp,Dp]= pitch_rb_t20;                    % Load the Pitch Design Model
[Api,Bpi,Cpi,Dpi]= linmod('Pdes_int');          % Linearize Open-Loop model that uses combined plant
sys=ss(Api,Bpi,Cpi,Dpi);
Qp=diag([0.001, 0.001, 0.0001, 0.0001]);        % State & Control Weights
Rp=40;
[Kq,S,E] = lqr(sys,Qp,Rp)
save Kq_t20.mat Kq -ascii                        % Save the LQR gains in Kpqr.mat
% Kq = [0.0262    0.3012    0.0196    0.0016]    % KM_theta, q, alpha, thet_int

% Lateral Axis
[Al,Bl,Cl,Dl]= later_rb_t20;                     % Load the Lateral Design Model
[Ali,Bli,Cli,Dli]= linmod('Ldes_int');           % Linearize Open-Loop model that uses combined plant
sys=ss(Ali,Bli,Cli,Dli);
Ql=diag([0.01  0.1  0.1  0.1  0.001  0.0001]);   % X Weights(phi,p, psi,r, beta, psi_int)
Rl=diag([1  1]*40);                              % U Weights
[Kpr,S,E] = lqr(sys,Ql,Rl)
save Kpr_t20.mat Kpr -ascii                       % Save the LQR gains in Kpqr.mat
% Kpr = [
%   0.0158    0.1846    0.0009   -0.0011    0.0018    0.0000
%  -0.0002   -0.0014    0.0612    0.4237   -0.0284    0.0016]; ! KN_phi, p, psi, r, beta, ps_int
[Ad,Bd,Cd,Dd]= plant_t20;
```

The last statement in m-file "*des_t20.m*" loads the fully coupled rigid-body plant "*Design Model with TVC*" which is in file "*plant_t20.m*". This system is used in the simulation model of Figure 3.3, which is in file "*Sim_RB.mdl*". It is a preliminary simulation model used to evaluate the closed-loop system's step responses and to adjust the Q and R weights in the design. In Figure 3.3, the vehicle state-vector which originally consisted of 3 pitch and 5 lateral states is augmented with the introduction of the (θ-integral) and the (ψ-integral) states. We have no access to the engine inputs because the TVC matrix is already included inside the plant. The pitch states are fed-back to the plant via the pitch gain matrix Kq, and the lateral states are fed-back via the lateral gain matrix Kpr. The only inputs to the closed-loop plant are: pitch and yaw attitude commands.

In the high dynamic pressure region of the trajectory only the pitch and yaw axes are commanded from guidance because flight-path and directional guidance are more important than roll attitude, and the roll axis is commanded at zero. This is because in the high Qbar region there is a strong roll and yaw aerodynamic coupling due to Clβ and it is not easy to command yaw and roll independently from each other. Therefore, yaw performance is accomplished at the expense of roll performance. A yaw attitude command produces a transient in roll that eventually decays to zero. This doesn't mean that roll stability is compromised because lateral LQR stabilizes both axes.
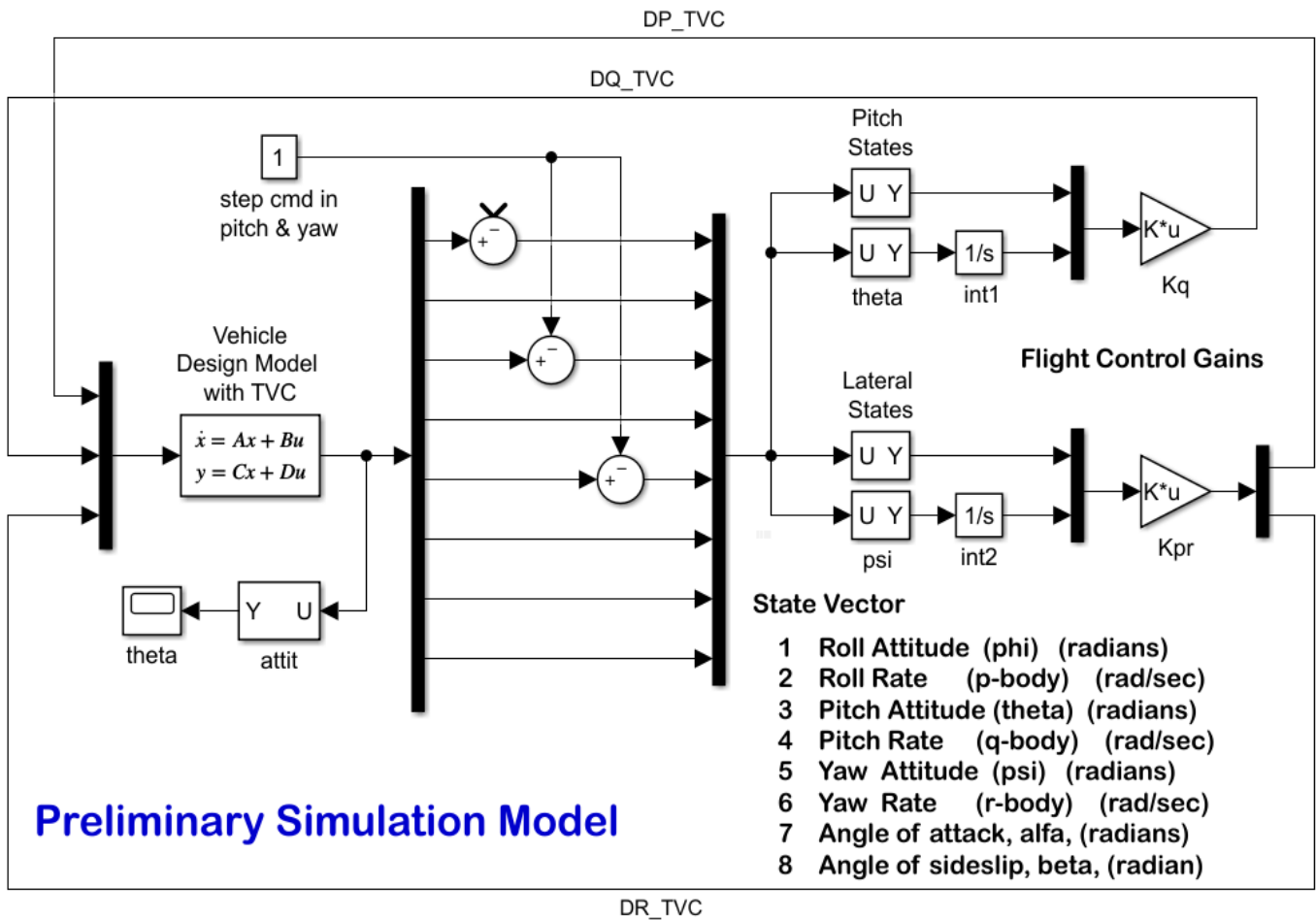
**Figure 3.3 Preliminary Simulation Model "*Sim_RB.mdl*" used for Performance Evaluation**

The control design process is as follows: For the T20 example case, we typically begin with a set of Q and R weight matrices in file "*des_t20.m*". We generate the state-feedback matrices and try them in Fig. 3.3 simulation model by applying attitude step commands. If the step responses are sluggish or too slow in a certain direction we go back in the design file and increase the attitude weight or attitude integral weight, or reduce the alpha weight or control weight. If on the other hand a very high bandwidth system is produced and the response is faster than expected, we go back and reduce the attitude, rate, or attitude integral weights, or increase the control penalty, until we obtain a satisfactory response with a reasonably small amount of control magnitude. The same process is repeated 7 times for the 7 control design cases in each folder, starting with "*des_t0.m*" in folder T00, to "*des_t300.m*" in folder T300. The Simulink models are similar for all flight conditions.

Notice that in Figure 3.4, the control system response to step commands at t=82 sec, which is at Max-Q, shown below, is much slower in comparison with t=20 sec, shown above. This is expected because during the high-Q region the load-relief feedback from α and β angles is increased and the emphasis is not in maneuvering but in alleviating the normal and lateral loads on the structure. The yaw attitude change also produces a big roll transient that eventually decays. During the high-Q period the vehicle receives zero or very small attitude increment commands. When the dynamic pressure recedes, the command following performance is considerably improved.
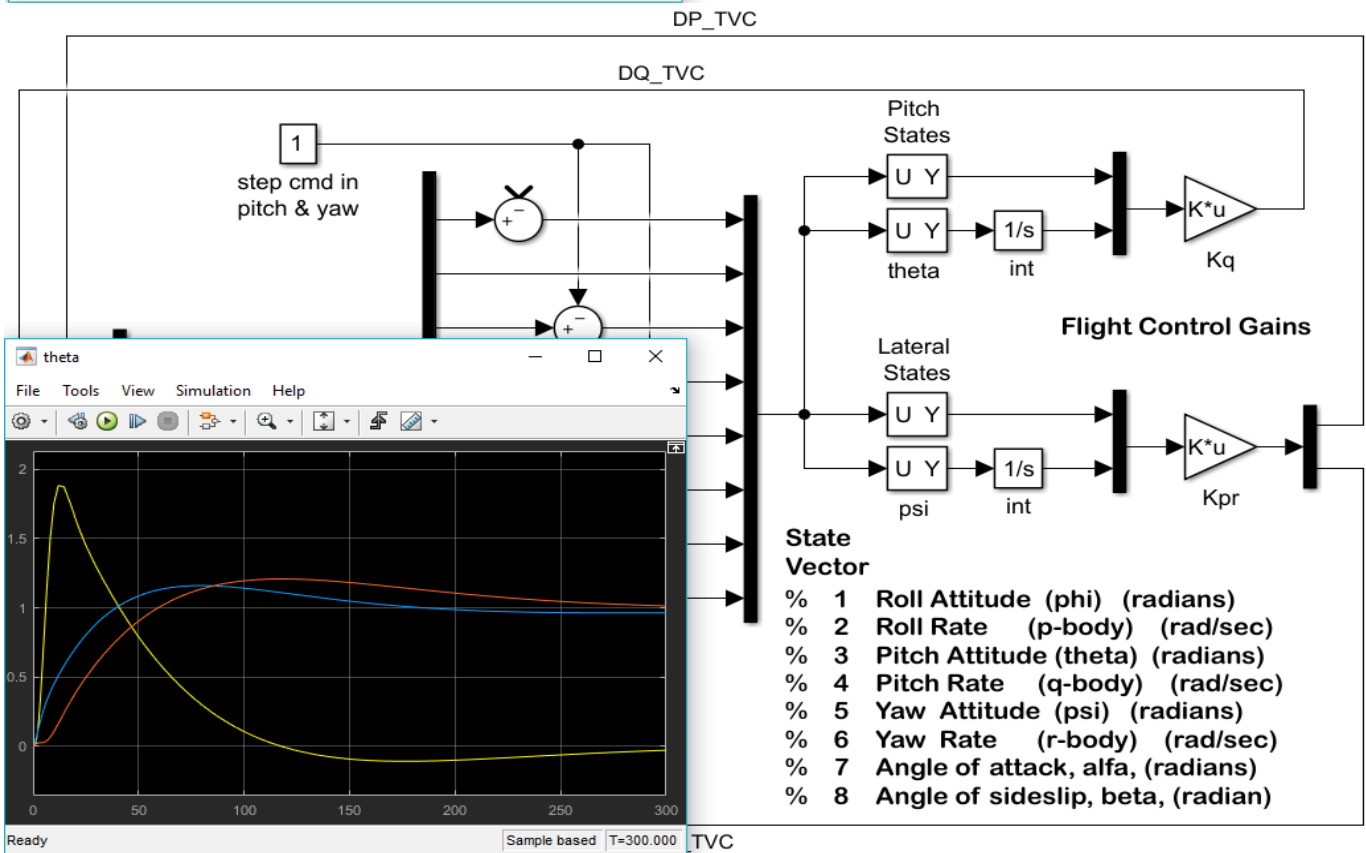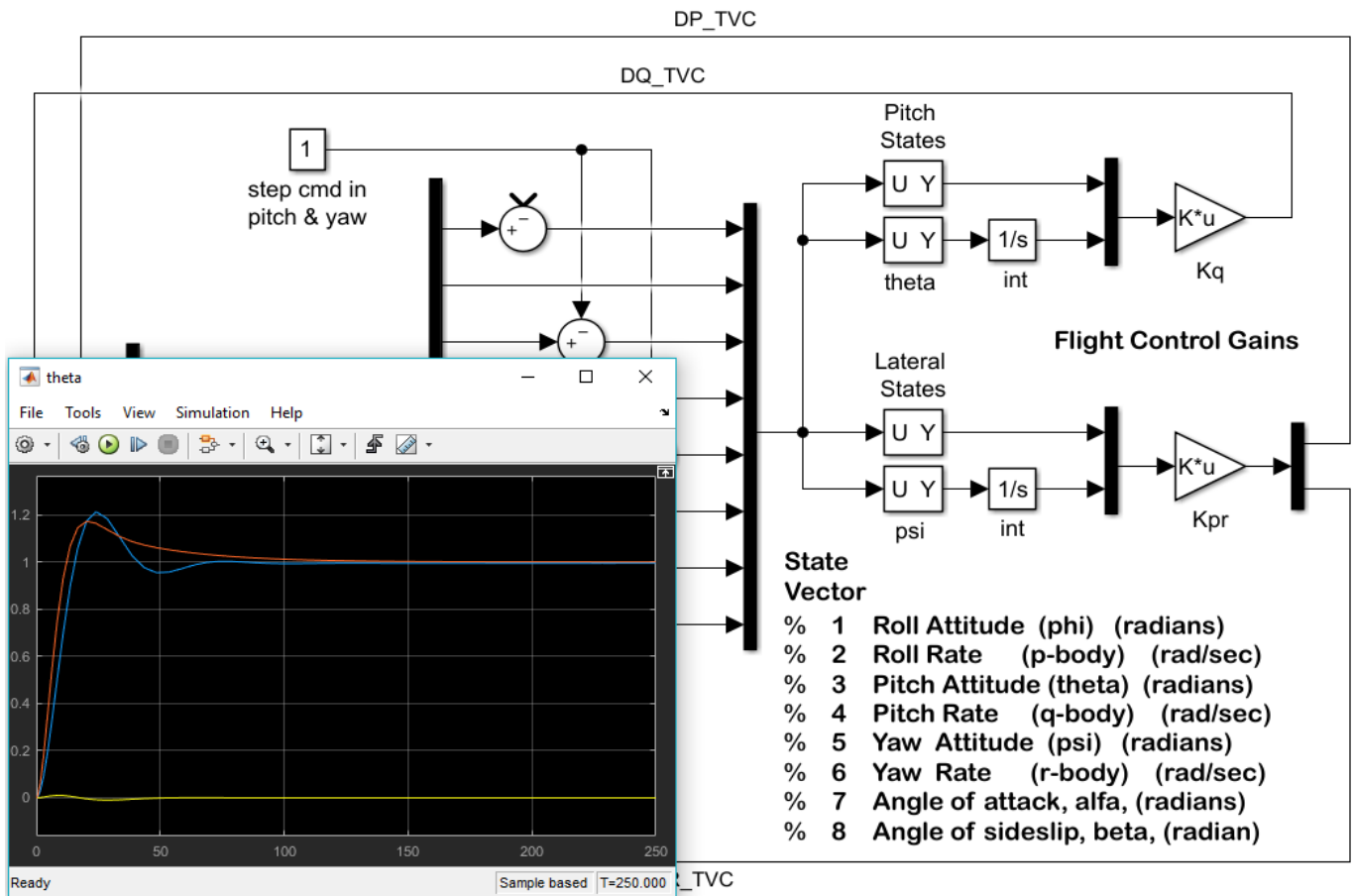
**Figure 3.4 System Response to Pitch and Yaw Attitude Increment Commands at T20 and at T82 (Max-Q)**

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# 4.0 Preliminary Stability Analysis

The design, however, is not complete without analyzing the system stability in the frequency domain. That is, checking the phase and gain margins, cross-over frequencies, etc. This open-loop analysis is performed in Matlab using the Simulink model "*Open_RB.mdl*" shown in Figure 4.1 that uses the same plant model "*plant_t20.m*" as in the simulation model. Figure 4.1 is shown in this case configured for yaw axis analysis where the yaw loop is opened and the pitch and roll loops are closed. The Matlab m-file "freq.m" (shown below) loads the plant model, the state-feedback gains, and calculates the frequency response across the opened yaw loop. It then plots the Bode and Nichols charts which are used to check the system stability. The Figures in 4.2 show the 3 Nichols plots for the roll, pitch and yaw directions.
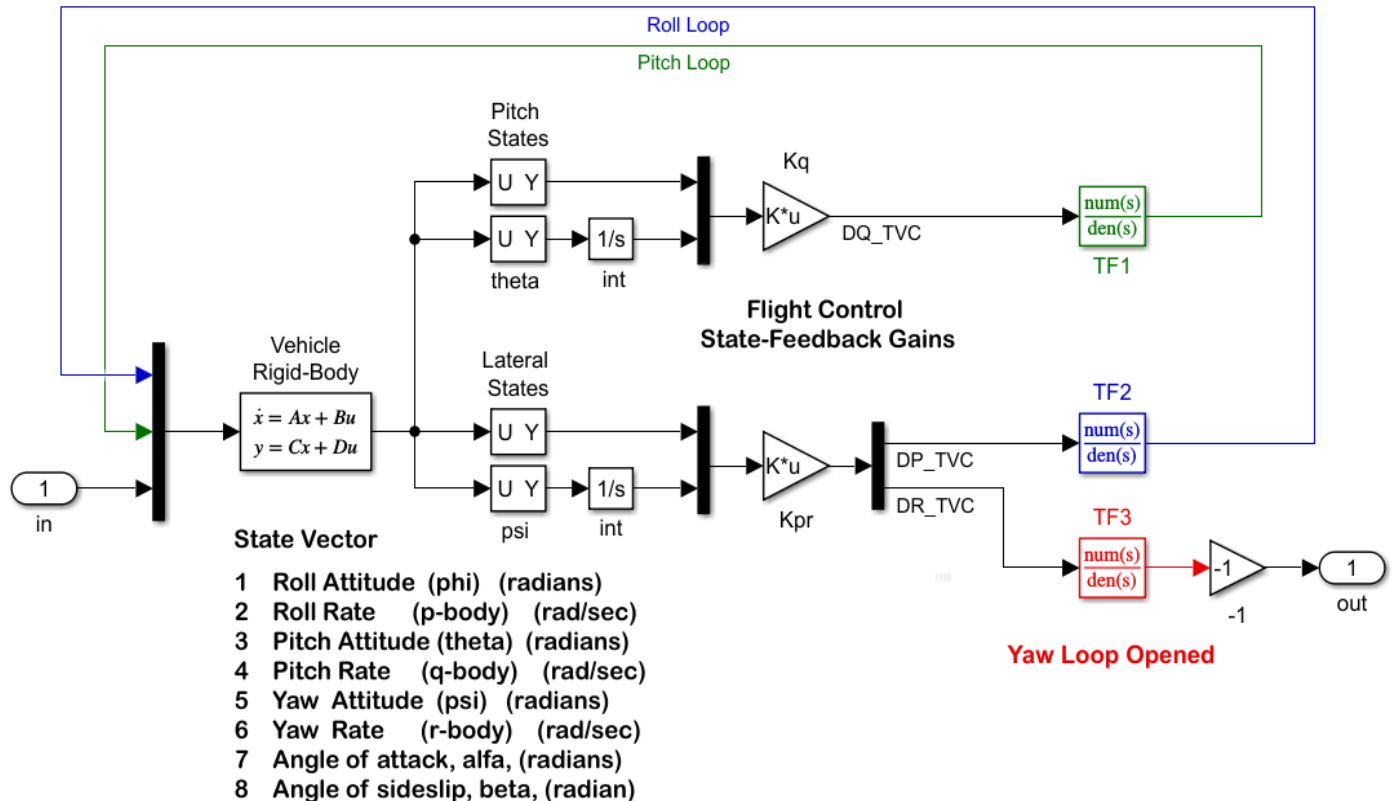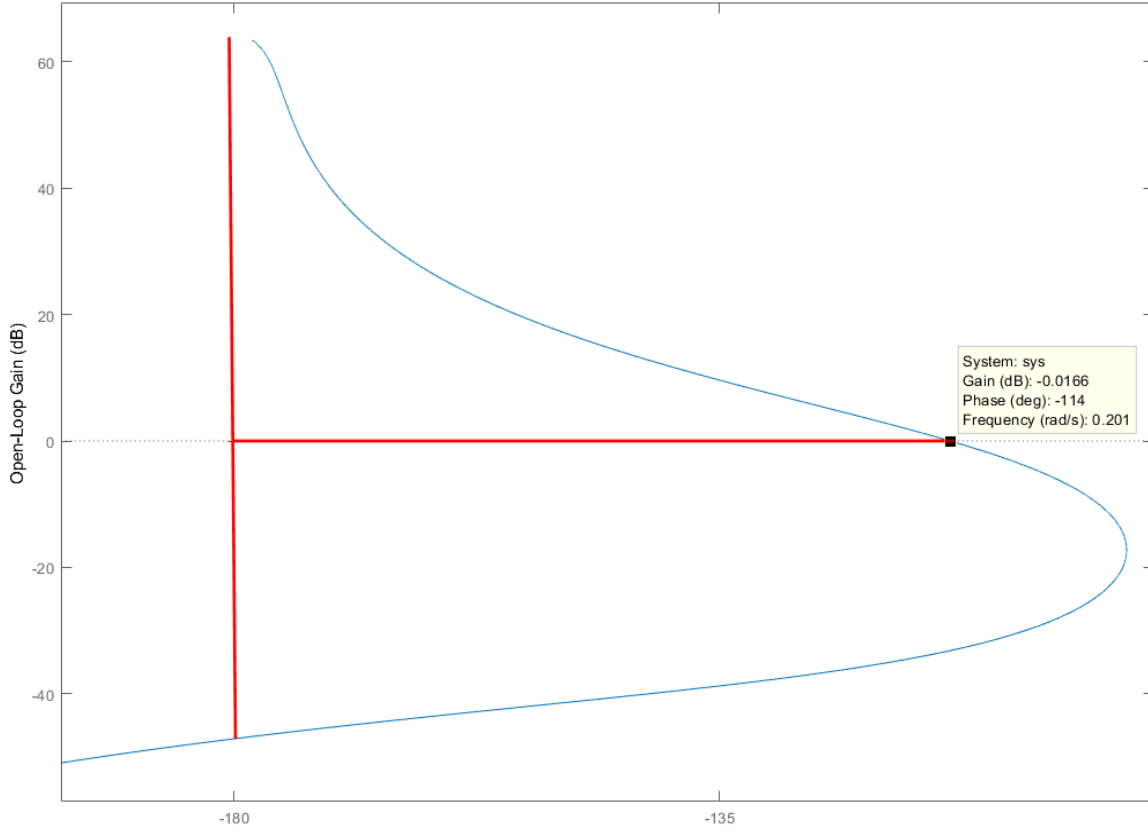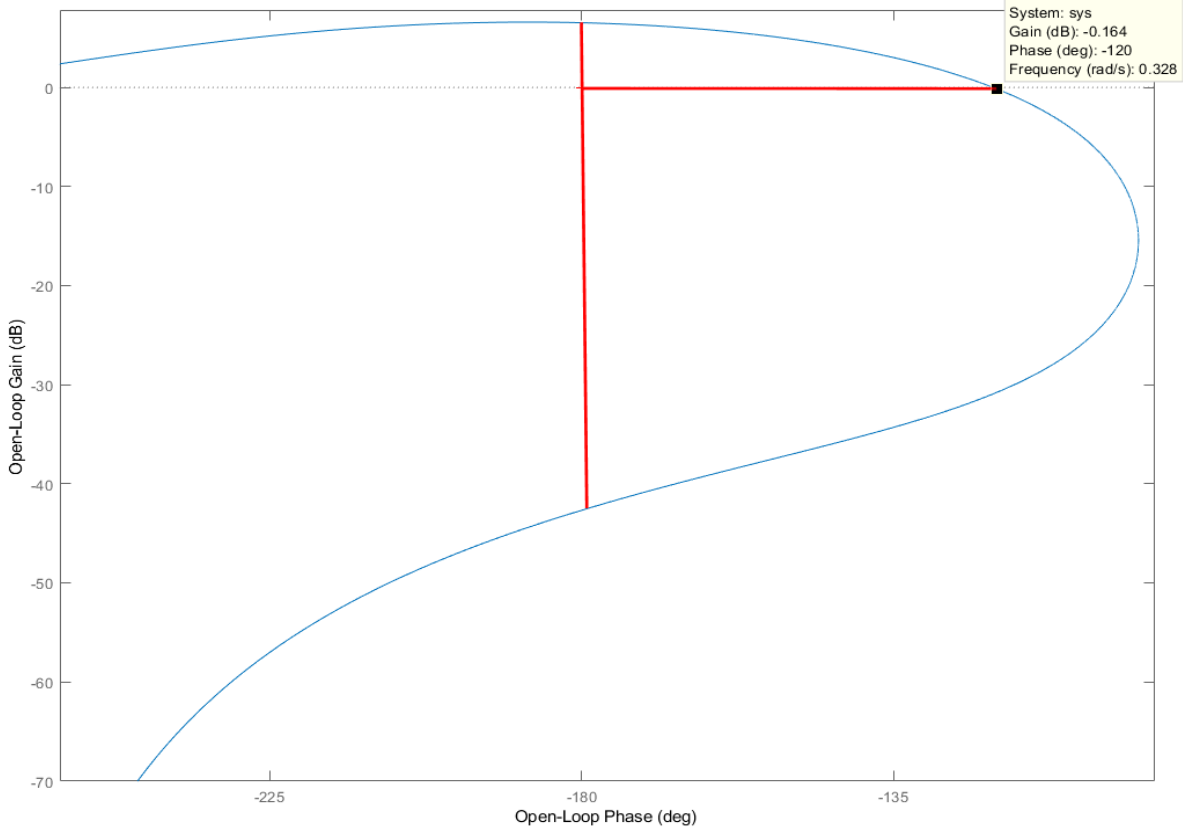


**Figure 4.1 Preliminary Stability Analysis Model "*Open_RB.mdl*", Shown with the Yaw Loop Opened**

```
% Preliminary Stability Analysis file: frequ.m
[Ad,Bd,Cd,Dd]= plant_t20;              % Load Rigid-Body Plant
load Kq_t20  -ascii                    % Load State-Feedback Matrices
load Kpr_t20 -ascii
[A1,B1,C1,D1]= linmod('Open_RB');      % Linearize Open-Loop Simulink model
sys= ss(A1,B1,C1,D1);                  % Create SS System
w=logspace(-3, 3, 24000);              % Define Frequ Range
figure(1); nichols(sys,w)             % Plot Nichol's Chart
figure(2); bode(sys,w)                % Plot Nichol's Chart
```

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## Roll Loop Stability Margins

### Nichols Chart



System: sys
Gain (dB): -0.0166
Phase (deg): -114
Frequency (rad/s): 0.201

Open-Loop Gain (dB)

## Pitch Loop Stability Margins

### Nichols Chart



System: sys
Gain (dB): -0.164
Phase (deg): -120
Frequency (rad/s): 0.328

Open-Loop Gain (dB)

Open-Loop Phase (deg)

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## Yaw Loop Stability Margins
### Nichols Chart
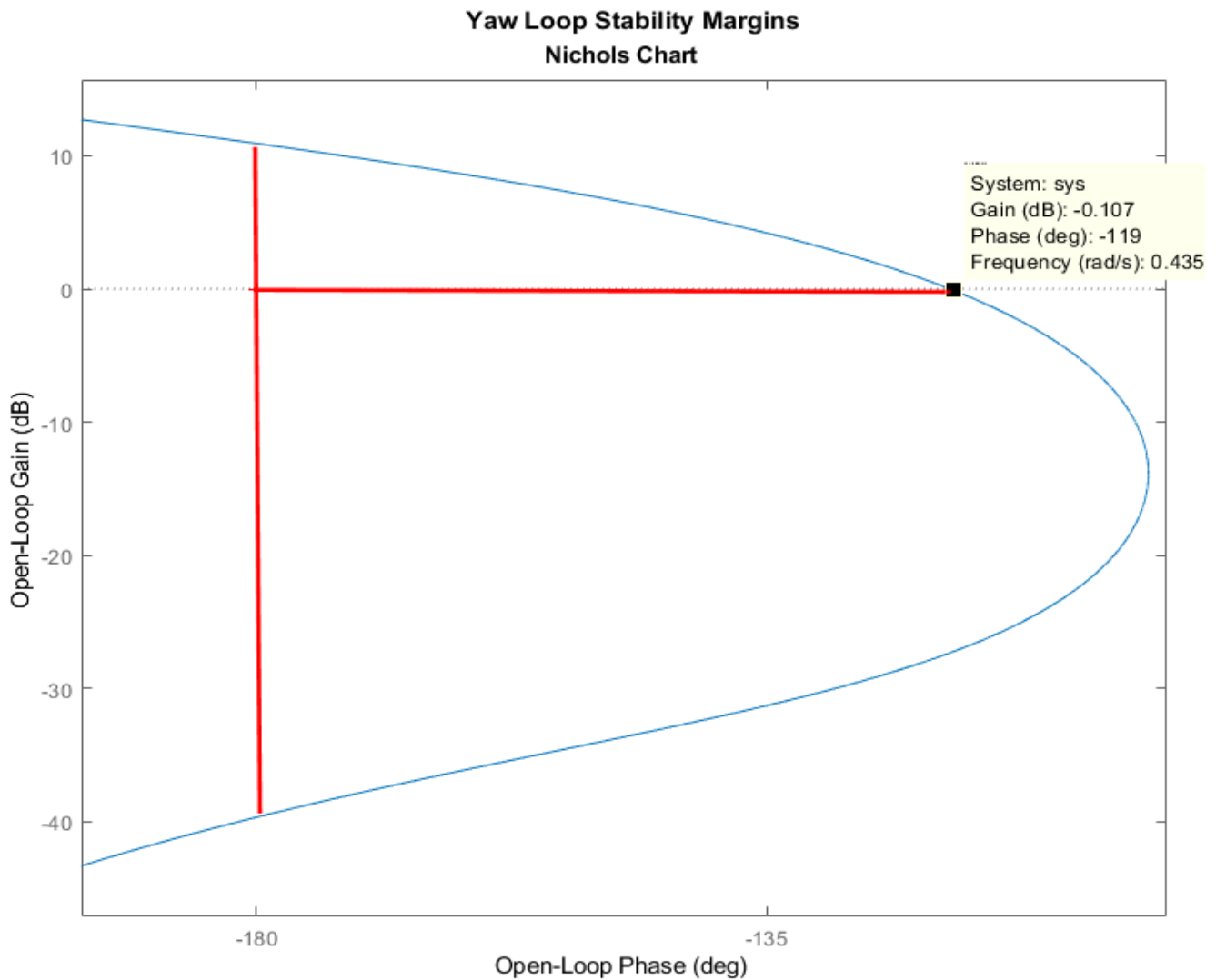


**Figure 4.2 Roll, Pitch and Yaw Nichols Charts Showing the System Stability Gain and Phase Margins at T=20 sec**

Note that, the gains that will be used in the control analysis are slightly different from the design gains obtained using the preliminary models. This is because in the actual vehicle the angles of attack and sideslip are not measured directly because the vehicle does not have a vane sensor and we shall use instead normal and lateral accelerometer (Nz and Ny) feedback. Nz and Ny measurements are approximately scaled mirror images of ($\alpha$ and $\beta$) respectively, assuming of course that the accelerometers are located at the vehicle center of mass, which we do in this case, because we assume that this problem can be compensated if we know the position of the CG relative to the accelerometer.

## 5.0 Detailed Analysis Models

The input data files for the detailed vehicle analysis models at the critical flight conditions have already been prepared. They are located in folder "*C:\Flixan\Examples\Space Plane Launch Vehicle\Analysis Models*" and they can be processed in batch mode. The folder includes 7 subdirectories for each flight condition (T0, T20, etc.). Each input file contains datasets that call different Flixan applications to create systems for analyzing the corresponding flight condition. They create flight vehicle and TVC actuator models, calculate the TVC matrix, implement the flight control system, combine the subsystems together for open-loop analysis and for closed-loop simulations, and convert the systems to Matlab

format for further analysis. Propellant sloshing, structural flexibility and tail-wag-dog dynamics are now included in the vehicle models. The control system is defined by transfer function interconnections and it is converted into state-space form. The feedback inputs to the control system are vehicle attitude and rate errors, plus normal and lateral accelerations measured at the CG. The gains that were obtained in the previous section using $\alpha$ and $\beta$ feedback are scaled for accelerometer measurements. The introduction of slosh, flexibility and TWD changes the dynamic behavior of the vehicle in comparison with the rigid-body model and causes some stability degradation. Some of the gains are therefore adjusted in order to accommodate the additional dynamics that were ignored in the original design and compensate against stability deterioration. A lead-lag filter is also included in the yaw channel to compensate propellant sloshing. It is not practical to include TWD, slosh, and flexibility in the control design model because it complicates the design, the controller size and it would not be very robust to uncertainties. We will now describe the various systems which are generated by Flixan and the corresponding input datasets. For example, let us take a look at the Max-Q input file "*Space_Plane_T82.Inp*" in folder T82. The other 6 flight conditions are very similar.

**Batch Set**: At the top of each input file there is a batch set that can be used to process the vehicle data in batch mode instead of processing them interactively one set at a time. Interactive processing of the datasets is slower and it is often used when starting a new project or for debugging.

```
BATCH MODE INSTRUCTIONS ...............
Batch for preparing the Space-Plane Launch Vehicle Models at T=82 sec
! This batch set creates Space Plane Simulation and Analysis models
! at T=82 sec (Max-Q)
!
Actuator Model    : TVC Actuator
System Connection : System of Six TVC Actuators
Transf-Function   : Wind Gust Shaping
Flight Vehicle    : Space-Plane Launch Vehicle at T=82 sec (Max-Q), with Slosh and Flex
Mixing Matrix     : Mixing Logic for the Space-Plane Launch Vehicle at T=82 sec
System Connection : Plant Model at T=82 sec, Vehicle/ Actuators
System Connection : Plant Model at T=82 sec, Vehicle/ Actuators/ TVC
Transf-Function   : Flight Control System at T=82 sec
System Connection : Open-Loop System at T=82 sec
System Connection : Closed-Loop Simulation System at T=82 sec
!
To Matlab Format : TVC Actuator
To Matlab Format : System of Six TVC Actuators
To Matlab Format : Space-Plane Launch Vehicle at T=82 sec (Max-Q), with Slosh and Flex
To Matlab Format : Mixing Logic for the Space-Plane Launch Vehicle at T=82 sec
To Matlab Format : Plant Model at T=82 sec, Vehicle/ Actuators/ TVC
To Matlab Format : Flight Control System at T=82 sec
To Matlab Format : Open-Loop System at T=82 sec
To Matlab Format : Closed-Loop Simulation System at T=82 sec
-----------------------------------------------------------------------------------------
ACTUATOR INPUT DATA ............ HYDRAULIC TYPE B
TVC Actuator
! Engine Nozzle Hydraulic Actuator Model
Symbol        Parameter Description            (Units)     Value
C(s)   Order of Compensat:(0,1,2), Coefficients   (---)       0
Kav    Total Gain of Amplifier + Torque Motor (ft-lb/rad) 1.5
Kact   Power Valve and Actuator Gain .........(ft/s/ft-lb)   90.0
Kt     Piston Ram Stiffness ................... (lb/ft)    0.35e+7
R      Moment Arm of Actuator Ram from Gimbal  (feet)       2.48
Ie     Engine Inertia about Gimbal ............(ft-lb-s^2)  690
Be     Engine Viscous Damping .................(ft-lb-sec)  16500.
Kb     Engine Gimbal Bearing Spring Constant  (ft-lb/rad)   27160.0
Kl     Engine Mount Structural Stiffness ...... (lb/ft)    0.35e+7
Tc     Different. Pressure Feedbk Time Constant (seconds) 0.12660
Kdpf   Differ Pressure Feedbk Linearizat. Gain (ft-lb/lb) 0.2e-6
Kfb    Position Feedback Gain ................. (rad/feet)  0.403
-----------------------------------------------------------------------
```

**TVC Actuators**: The first system created by Flixan is the "TVC Actuator" using the actuator modeling program. The actuator parameters dataset is included in the input data file, and it is a hydraulic type (B) simple fourth order model (see hydraulic actuator type B in the actuator documentation). The model includes the piston and nozzle rotational dynamics. The two inputs are: nozzle deflection command in (radian), and Load-Torque in (ft-lb). The outputs are: nozzle deflection in (radians), nozzle rate, and nozzle acceleration. Only the deflection and acceleration outputs are used in this model. The $\delta/\delta_{com}$ frequency response of the actuator is shown in Figure 5.1.
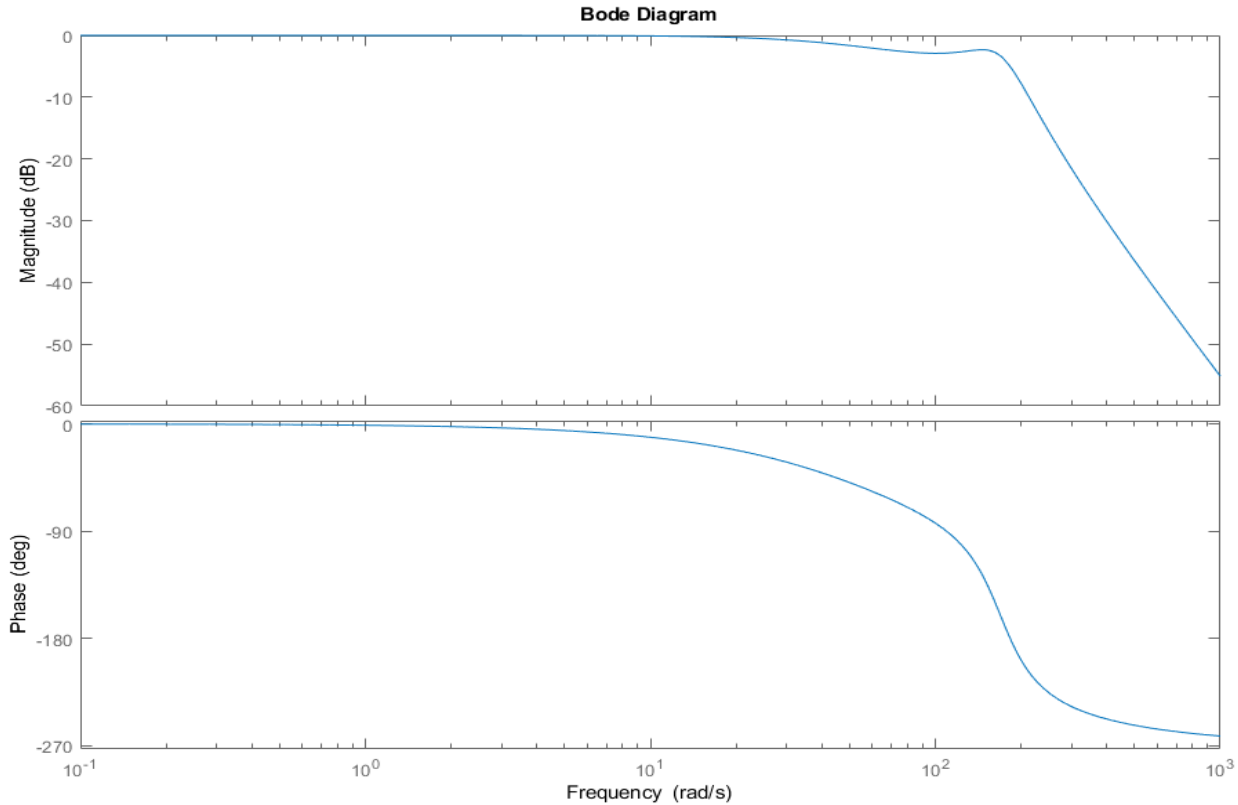


**Figure 5.1 Frequency Response of the TVC Actuator**

The next set of data is a systems interconnection that creates 6 identical actuator models in parallel for the 6 TVC engines. It duplicates the same actuator 6 times for the 6 engines and saves the combined system under the title "*System of Six TVC Actuators*". The gimbal acceleration outputs are required for the tail-wag-dog implementation. The TVC actuator model is also converted to Matlab format and used in the analysis.
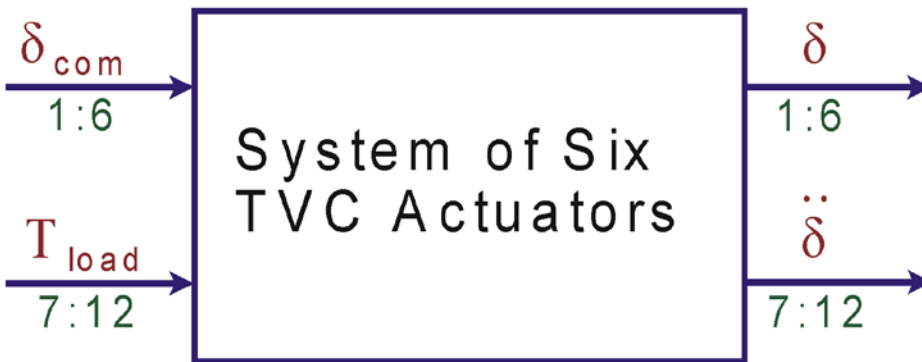


**Figure 5.2 System of six Actuators**

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INTERCONNECTION OF SYSTEMS .....
**System of Six TVC Actuators**
**! Combination of 6 TVC Actuators in Parallel**
Titles of Systems to be Combined
Title 1 TVC Actuator
Title 2 TVC Actuator
Title 3 TVC Actuator
Title 4 TVC Actuator
Title 5 TVC Actuator
Title 6 TVC Actuator
```
SYSTEM INPUTS TO SUBSYSTEM  1                                         Engine # 1
System Input  1 to Subsystem  1, Input  1, Gain= 1.00000              Command Input
System Input  7 to Subsystem  1, Input  2, Gain= 1.00000              Load-Torque
.....................................................................
SYSTEM INPUTS TO SUBSYSTEM  2                                         Engine # 2
System Input  2 to Subsystem  2, Input  1, Gain= 1.00000              Command Input
System Input  8 to Subsystem  2, Input  2, Gain= 1.00000              Load-Torque
.....................................................................
SYSTEM INPUTS TO SUBSYSTEM  3                                         Engine # 3
System Input  3 to Subsystem  3, Input  1, Gain= 1.00000              Command Input
System Input  9 to Subsystem  3, Input  2, Gain= 1.00000              Load-Torque
.....................................................................
SYSTEM INPUTS TO SUBSYSTEM  4                                         Engine # 4
System Input  4 to Subsystem  4, Input  1, Gain= 1.00000              Command Input
System Input 10 to Subsystem  4, Input  2, Gain= 1.00000              Load-Torque
.....................................................................
SYSTEM INPUTS TO SUBSYSTEM  5                                         Engine # 5
System Input  5 to Subsystem  5, Input  1, Gain= 1.00000              Command Input
System Input 11 to Subsystem  5, Input  2, Gain= 1.00000              Load-Torque
.....................................................................
SYSTEM INPUTS TO SUBSYSTEM  6                                         Engine # 6
System Input  6 to Subsystem  6, Input  1, Gain= 1.00000              Command Input
System Input 12 to Subsystem  6, Input  2, Gain= 1.00000              Load-Torque
.....................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                      Engine # 1
System Output  1 from Subsystem  1, Output  1, Gain= 1.00000          Deflection
System Output  7 from Subsystem  1, Output  3, Gain= 1.00000          Acceleration
.....................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                                      Engine # 2
System Output  2 from Subsystem  2, Output  1, Gain= 1.00000          Deflection
System Output  8 from Subsystem  2, Output  3, Gain= 1.00000          Acceleration
.....................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  3                                      Engine # 3
System Output  3 from Subsystem  3, Output  1, Gain= 1.00000          Deflection
System Output  9 from Subsystem  3, Output  3, Gain= 1.00000          Acceleration
.....................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  4                                      Engine # 4
System Output  4 from Subsystem  4, Output  1, Gain= 1.00000          Deflection
System Output 10 from Subsystem  4, Output  3, Gain= 1.00000          Acceleration
.....................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  5                                      Engine # 5
System Output  5 from Subsystem  5, Output  1, Gain= 1.00000          Deflection
System Output 11 from Subsystem  5, Output  3, Gain= 1.00000          Acceleration
.....................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  6                                      Engine # 6
System Output  6 from Subsystem  6, Output  1, Gain= 1.00000          Deflection
System Output 12 from Subsystem  6, Output  3, Gain= 1.00000          Acceleration
.....................................................................
```
**Definitions of Inputs  =  12**
**Engine # 1 delta command (rad)**
**Engine # 2 delta command (rad)**
**Engine # 3 delta command (rad)**
**Engine # 4 delta command (rad)**
**Engine # 5 delta command (rad)**
**Engine # 6 delta command (rad)**
**Engine # 1 Load-Torque (ft-lb)**
**Engine # 2 Load-Torque (ft-lb)**
**Engine # 3 Load-Torque (ft-lb)**
**Engine # 4 Load-Torque (ft-lb)**
**Engine # 5 Load-Torque (ft-lb)**
**Engine # 6 Load-Torque (ft-lb)**

```
Definitions of Outputs =  12
Engine # 1 deflection (rad)
Engine # 2 deflection (rad)
Engine # 3 deflection (rad)
Engine # 4 deflection (rad)
Engine # 5 deflection (rad)
Engine # 6 deflection (rad)
Engine # 1 accelerat. (rad/sec^2)
Engine # 2 accelerat. (rad/sec^2)
Engine # 3 accelerat. (rad/sec^2)
Engine # 4 accelerat. (rad/sec^2)
Engine # 5 accelerat. (rad/sec^2)
Engine # 6 accelerat. (rad/sec^2)
-------------------------------------------------------------------------------------------
```

**Wind-Gust**: The transfer function dataset below is a wind-gust low-pass filter used for smoothing a 33 (feet/sec) square pulse and shaping it to look like a more natural wind-gust impulse.

```
SYSTEM OF TRANSFER FUNCTIONS ...
Wind Gust Shaping
Continuous
TF. Block #  1  0.6^2/(s^2 +0.5s +0.6^2)                         Order of Numer, Denom=  0  2
Numer 0.0        0.0        0.36
Denom 1.0        0.6        0.36
......................................................
Block #, from Input #, Gain
  1  1    1.0
...........................
Outpt #, from Block #, Gain
  1  1    1.0
...........................
-------------------------------------------------------------------------------------------
```

**Vehicle Data**: The dataset below contains the flight vehicle input data at Max-Q. It is similar to the design dataset used in Section 3, but it includes propellant sloshing parameters for the two propellant tanks and structural flexibility. It uses two spring-mass slosh models for the LOX and the LH2 tanks. The input data for each slosh model are: slosh mass, slosh frequencies in y and z directions (in this case they are equal), damping (in y and z), and slosh mass positions (un-deflected). The slosh frequencies in the input data are assumed to be at 1 g. They are scaled by the program proportionally to the square root of the acceleration. The damping coefficients are higher than typical slosh damping because the tanks contain baffles. Otherwise, in some cases the slosh modes would be unstable without baffles. The model also uses a set of 5 flex modes that have already been selected and included in the input file as a separate dataset "*Space-Plane Launch Vehicle Pitch Flex Modes*". The flags line below the title and the comment lines indicate that the state vector is in the body axis, and that the vehicle attitude ($\phi$, $\theta$, $\psi$) are equal to the integrals of the body rates. This is acceptable because the attitude commands or small increments from guidance are received relative to the current vehicle attitude. The moment reference center (MRC) for the aero data is located at X= -123.17 (feet) and the CG is further back at X= -139 (ft) which makes the vehicle statically unstable. It is flying at zero alpha, at Mach 1.16, and the dynamic pressure is 470 (psf). It uses 6 gimbaling engines of 438500 (lb) of thrust each, which are tilted or trimmed -5° in pitch.

**Space-Plane Launch Vehicle at T=82 sec (Max-Q), with Slosh and Flex**
**! This model is for a Single Stage to Orbit type Launch Vehicle which takes off like a rocket and**
**! comes back and lands like an airplane. The data correspond to a flight condition during the**
**! ascent phase at high dynamic pressure. The vehicle has six TVC liquid fuel engines and two**
**! propellant tanks carrying liquid oxygen and liquid hydrogen. The fuel tanks cause a significant**
**! amount of sloshing. For flight control it uses gyros and accelerometers measurements.**
**!**
**Body Axes Output, Attitude=Rate Integral**

```
Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft)  :  46882.6, 32.17, 0.208E+8
Moments and products of Inertias Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)        :  0.392E+7 0.64E+8 0.643E+8
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)       :  -139.0,   0.0,    0.3
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)      :  1.16, 1130.0, 470.0,
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)        :  49.7, 49.6, 0.0,  -5.3
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                        :  0.0, 0.0, 0.0,  0.0
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)  :  0.0, 58.6, 0.0,  0.0,
Wind Gust Vel wrt Vehi (Azim & Elev) angles (deg), or Force(lb), Torque(ft-lb), locat:xyz: Gust   45.0    90.0
Surface Reference Area (feet^2), Mean Aerodynamic Chord (ft), Wing Span in (feet)       :  3500.0, 45.92, 86.080
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial_rho/ Partial_H} : -123.17,  0.0, 2.916,
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:  0.162,   0.0037, 0.0, 0.0,
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}:  0.0,   -0.0421, 0.0, 0.0,
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}: -0.1386, -0.0691, 0.0, 0.0,
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:  0.0,   -0.0032, 0.0, 0.0,
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}: -0.0377, -0.0177, 0.0, 0.0,
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}:  0.0,    6.98e-4, 0.0, 0.0,


Number of Thruster Engines, Include or Not the Tail-Wags-Dog and Load-Torque Dynamics ? :  6   WITH TWD


TVC Engine No:  1                              (Gimbaling  Throttling   Single_Gimbal) : TVC Upper Right  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                    :  438502.0     438502.0
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): -5.0  0.00  10.0  10.00
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  63.8  690.00    3.7000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  -173.50  5.4160  -9.3810

TVC Engine No:  2                              (Gimbaling  Throttling   Single_Gimbal) : TVC Right        Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                    :  438502.0     438502.0
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): -5.0  0.0  10.00  10.00
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  63.8   690.0  3.7
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  -173.50  10.833    0.0

TVC Engine No:  3                              (Gimbaling  Throttling   Single_Gimbal) : TVC Low Right    Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                    :  438502.0     438502.0
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): -5.0  0.0   10.0  10.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  63.80  690.00    3.70
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  -173.50  5.4160  9.3810

TVC Engine No:  4                              (Gimbaling  Throttling   Single_Gimbal) : TVC Low Left     Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                    :  438502.0     438502.0
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): -5.0   0.0   10.0  10.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  63.8   690.00  3.7
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  -173.50 -5.4160  9.3810

TVC Engine No:  5                              (Gimbaling  Throttling   Single_Gimbal) : TVC Left         Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                    :  438502.0    438502.0
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): -5.0   0.00 10.0   10.00
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  63.80  690.00    3.7000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  -173.50  -10.833   0.00

TVC Engine No:  6                              (Gimbaling  Throttling   Single_Gimbal) : TVC Upper Left   Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                    :  438502.0     438502.0
Mounting Angles wrt Vehicle (Dyn,Dzn), Maximum Deflections from Mount (Dymax,Dzmax) (deg): -5.0   0.0  10.0  10.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  63.8  690.00  3.7000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  -173.50  -5.4160  -9.3810


Number of Gyros, (Attitude and Rate)                                                   :  3
Gyro No 1 Axis:(Pitch,Yaw,Roll), (Attitude or Rate), Location in Vehicle Axes  (feet)  :  Roll Rate   -10.0, 0.0, 0.0
Gyro No 2 Axis:(Pitch,Yaw,Roll), (Attitude or Rate), Location in Vehicle Axes  (feet)  :  Pitch Rate  -10.0, 0.0, 0.0
Gyro No 3 Axis:(Pitch,Yaw,Roll), (Attitude or Rate), Location in Vehicle Axes  (feet)  :  Yaw Rate    -10.0, 0.0, 0.0


Number of Accelerometers, Along Axis: (x,y,z)                                          :  2
Acceleromet No 1 Axis:(X,Y,Z), (Position,Velocity,Accelerat), Location in (feet)       :  Y-axis Acceler   -139.0,
Acceleromet No 2 Axis:(X,Y,Z), (Position,Velocity,Accelerat), Location in (feet)       :  Z-axis Acceler   -139.0,


Number of Vane Sensors, (Measuring Alpha or Beta)                                      :  1
Vane No 1 Location in Vehicle Axes  (feet)                                             :  -10, 0.0, 0.0


Number of Slosh Modes                                                                  :  2
LOX Slosh Mass (slug), Frequ Wy,Wz @1g (rad/s), Damp (zeta-y-z), Locat {Xsl,Ysl,Zsl} (ft): 12200.0, 2.1, 2.1, 0.01,
LH2 Slosh Mass (slug), Frequ Wy,Wz @1g (rad/s), Damp (zeta-y-z), Locat {Xsl,Ysl,Zsl} (ft): 825.0,   2.4, 2.4, 0.03,


Number of Bending Modes                                                                :  5
```
**Space-Plane Launch Vehicle Pitch Flex Modes**
----------------------------------------------------------------------------------------------------------

The wind-gust disturbance is defined to be perpendicular to the vehicle x-axis (90º elevation), and it is coming at 45º azimuth, between the +y and the +z axes, thus, exciting both pitch and lateral directions. The tail-wag-dog flag is now turned "on" for the 6 gimbaling engines (it was "off" in the design model). This will create additional gimbal acceleration inputs and load-torque outputs in the vehicle system to couple with the actuators. The nozzle masses and nozzle inertia are also used for the TWD implementation. The vehicle model uses 3 rate gyros (roll, pitch, yaw) and two lateral and normal accelerometers located at the CG for flight control. It also has a vane sensor for measuring the angles of attack and sideslip at the front. The Ny and Nz accelerometer measurements are scaled and used to replace the α and β feedback signals. The vane measurements are only used for reference since the vehicle doesn't have an air-data probe. The vehicle modeling program will process the dataset, create the vehicle state-space system: "*Space-Plane Launch Vehicle at T=82 sec (Max-Q), with Slosh and Flex*" that will be used in the analysis, and it will save it in file "*Space_Plane_T82.Qdr*".

**TVC Mixing Logic Matrix**: The mixing logic program creates the TVC matrix from the vehicle data. The TVC matrix converts the roll, pitch and yaw demands from the flight control system to pitch and yaw gimbal deflection commands for the 6 engines. The mixing logic dataset below calls the mixing logic program to calculate the (12x3) TVC matrix. Its title is "*Mixing Logic for the Space-Plane Launch Vehicle*". The program uses some of the vehicle data such as: engine thrusts, max gimbal deflections and the vehicle geometry to generate the TVC matrix and the flight vehicle title must be included in the mixing logic dataset. The matrix name is TVC. The last three lines define the control inputs to the TVC. In this case we have roll, pitch, and yaw acceleration demands from flight control which are converted to pitch and yaw gimbal deflection commands for the 6 engines. The TVC matrix is also saved in file "*Space_Plane_T82.Qdr*" and it is used by the systems interconnection utility to create the plant model.

```
MIXING LOGIC MATRIX DATA ..........     (Matrix Title, Name, Vehicle Title, Control Directions)
Mixing Logic for the Space-Plane Launch Vehicle at T=82 sec
! Thrust Vector Control Matrix for a Launch Vehicle at Max_Q.
! This multi-engine vehicle has 6 Engines.
TVC
Space-Plane Launch Vehicle at T=82 sec (Max-Q), with Slosh and Flex
P-dot Roll  Acceleration About X Axis
Q-dot Pitch Acceleration About Y Axis
R-dot Yaw   Acceleration About Z Axis
------------------------------------------------------------------------------------------------
```

**Plant Model**: The next two datasets are system interconnections that create the plant model "*Plant Model at T=82 sec, Vehicle/ Actuators/ TVC*" which consists of the vehicle, actuators, wind-gust and the TVC subsystems combined. The plant model is generated in two steps for convenience. In the first step we combine the vehicle with the actuator dynamics together. Two of the six actuator packs described earlier are included: a six-pack for the pitch gimbal deflections and another six-pack for the yaw deflections, see Figure 5.3. It includes the "tail-wag-dog" and the "load-torque feedback" dynamics by closing the load-torque feedback loops between the vehicle load-torque outputs (at the gimbal) and the actuators. The title of this first combination subsystem is "*Plant Model at T=82 sec, Vehicle/ Actuators*". Notice that the interconnection algorithm does not have any problem calculating the load-torque which is an algebraic feedback loop (a common problem in Matlab). In the second step we combine the plant created from step-1 (Figure 5.3) with the TVC matrix and the gust shaping filter, as shown in Figure 5.4. The title is: "*Plant Model at T=82 sec, Vehicle/ Actuators/ TVC*". The gust filter is a low-pass second order which is implemented in transfer function form: "*Wind Gust Shaping*" and is used for smoothing the wind-gust disturbance pulse. The inputs to this system are: roll, pitch, and yaw δ(rate) demands from the flight control system, and the wind gust velocity pulse in (feet/sec) which is typically a short duration square pulse. The direction of the wind-gust relative to the vehicle body is defined in the input data as already described.
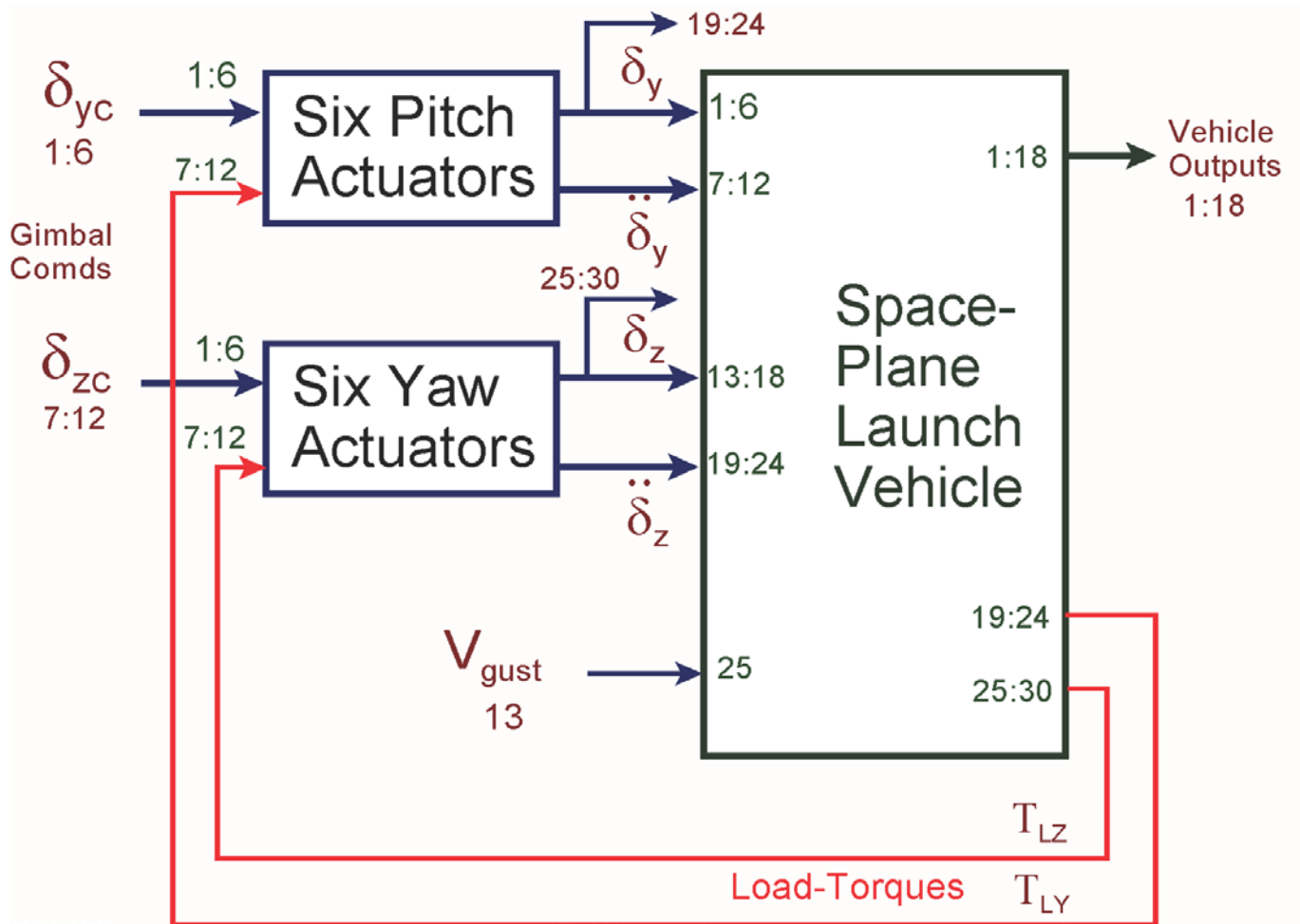
**Figure 5.3 Step-1 System Interconnection "*Plant Model at T=82 sec, Vehicle/ Actuators* "**
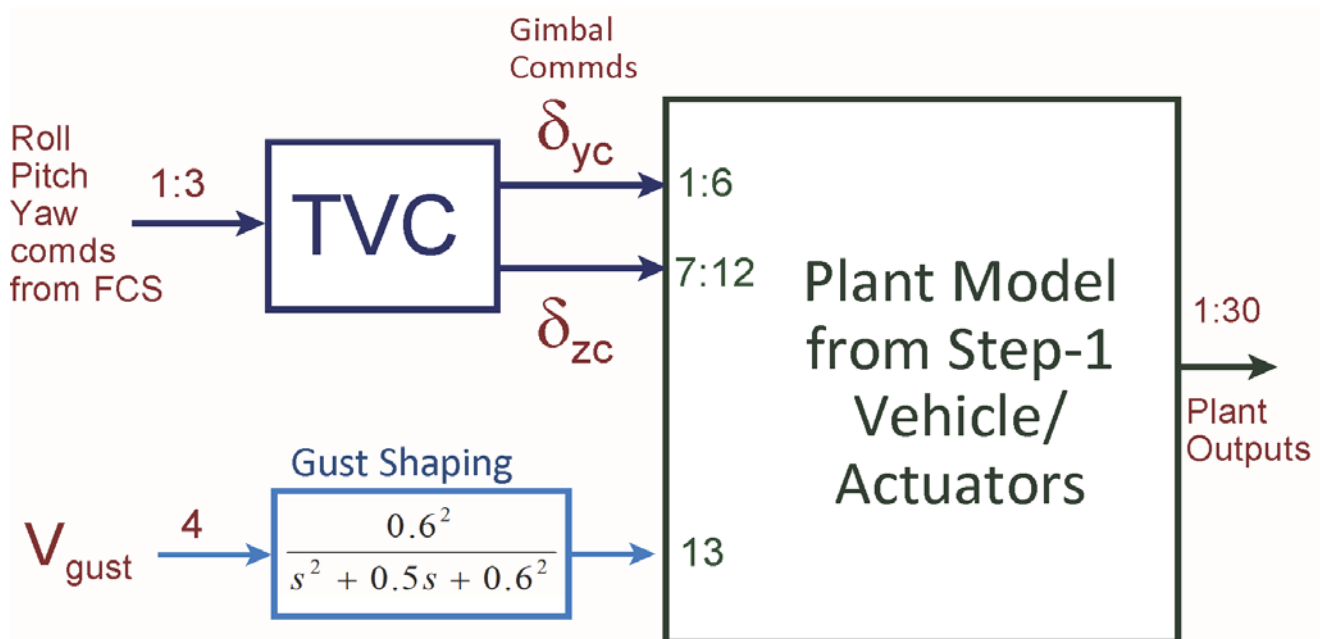


**Figure 5.4 Step-2 Systems Interconnection "*Plant Model at T=82 sec, Vehicle/ Actuators/ TVC*"**

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```
INTERCONNECTION OF SYSTEMS .....
Plant Model at T=82 sec, Vehicle/ Actuators
! Combines the Vehicle with the Actuators
Titles of Systems to be Combined
Title 1 Space-Plane Launch Vehicle at T=82 sec (Max-Q), with Slosh and Flex
Title 2 System of Six TVC Actuators
Title 3 System of Six TVC Actuators
SYSTEM INPUTS TO SUBSYSTEM  2                                             To Pitch Actuators
System Input  1 to Subsystem  2, Input  1, Gain= 1.00000
System Input  2 to Subsystem  2, Input  2, Gain= 1.00000
System Input  3 to Subsystem  2, Input  3, Gain= 1.00000
System Input  4 to Subsystem  2, Input  4, Gain= 1.00000
System Input  5 to Subsystem  2, Input  5, Gain= 1.00000
System Input  6 to Subsystem  2, Input  6, Gain= 1.00000
.....................................................................
SYSTEM INPUTS TO SUBSYSTEM  3                                             To Yaw Actuators
System Input  7 to Subsystem  3, Input  1, Gain= 1.00000
System Input  8 to Subsystem  3, Input  2, Gain= 1.00000
System Input  9 to Subsystem  3, Input  3, Gain= 1.00000
System Input 10 to Subsystem  3, Input  4, Gain= 1.00000
System Input 11 to Subsystem  3, Input  5, Gain= 1.00000
System Input 12 to Subsystem  3, Input  6, Gain= 1.00000
.....................................................................
SYSTEM INPUTS TO SUBSYSTEM  1                                             Wind-Gust
System Input 13 to Subsystem  1, Input 25, Gain= 1.0
.....................................................................
SUBSYSTEM NO  2 GOES TO SUBSYSTEM NO  1                                   Pitch Actuat to Vehicle
Subsystem  2, Output  1 to Subsystem  1, Input  1, Gain= 1.0             delta_y1    deflects
Subsystem  2, Output  2 to Subsystem  1, Input  2, Gain= 1.0             delta_y2
Subsystem  2, Output  3 to Subsystem  1, Input  3, Gain= 1.0             delta_y3
Subsystem  2, Output  4 to Subsystem  1, Input  4, Gain= 1.0             delta_y4
Subsystem  2, Output  5 to Subsystem  1, Input  5, Gain= 1.0             delta_y5
Subsystem  2, Output  6 to Subsystem  1, Input  6, Gain= 1.0             delta_y6
Subsystem  2, Output  7 to Subsystem  1, Input  7, Gain= 1.0             dy1_dd      accels
Subsystem  2, Output  8 to Subsystem  1, Input  8, Gain= 1.0             dy2_dd
Subsystem  2, Output  9 to Subsystem  1, Input  9, Gain= 1.0             dy3_dd
Subsystem  2, Output 10 to Subsystem  1, Input 10, Gain= 1.0             dy4_dd
Subsystem  2, Output 11 to Subsystem  1, Input 11, Gain= 1.0             dy5_dd
Subsystem  2, Output 12 to Subsystem  1, Input 12, Gain= 1.0             dy6_dd
.....................................................................
SUBSYSTEM NO  3 GOES TO SUBSYSTEM NO  1                                   Yaw Actuat to Vehicle
Subsystem  3, Output  1 to Subsystem  1, Input 13, Gain= 1.0             delta_z1    deflects
Subsystem  3, Output  2 to Subsystem  1, Input 14, Gain= 1.0             delta_z2
Subsystem  3, Output  3 to Subsystem  1, Input 15, Gain= 1.0             delta_z3
Subsystem  3, Output  4 to Subsystem  1, Input 16, Gain= 1.0             delta_z4
Subsystem  3, Output  5 to Subsystem  1, Input 17, Gain= 1.0             delta_z5
Subsystem  3, Output  6 to Subsystem  1, Input 18, Gain= 1.0             delta_z6
Subsystem  3, Output  7 to Subsystem  1, Input 19, Gain= 1.0             dz1_dd      accels
Subsystem  3, Output  8 to Subsystem  1, Input 20, Gain= 1.0             dz2_dd
Subsystem  3, Output  9 to Subsystem  1, Input 21, Gain= 1.0             dz3_dd
Subsystem  3, Output 10 to Subsystem  1, Input 22, Gain= 1.0             dz4_dd
Subsystem  3, Output 11 to Subsystem  1, Input 23, Gain= 1.0             dz5_dd
Subsystem  3, Output 12 to Subsystem  1, Input 24, Gain= 1.0             dz6_dd
.....................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2                                   Load-Torq TLY to Act
Subsystem  1, Output 19 to Subsystem  2, Input  7, Gain= 1.0             TL_y1
Subsystem  1, Output 20 to Subsystem  2, Input  8, Gain= 1.0             TL_y2
Subsystem  1, Output 21 to Subsystem  2, Input  9, Gain= 1.0             TL_y3
Subsystem  1, Output 22 to Subsystem  2, Input 10, Gain= 1.0             TL_y4
Subsystem  1, Output 23 to Subsystem  2, Input 11, Gain= 1.0             TL_y5
Subsystem  1, Output 24 to Subsystem  2, Input 12, Gain= 1.0             TL_y6
.....................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  3                                   Load-Torq TLZ to Act
Subsystem  1, Output 25 to Subsystem  3, Input  7, Gain= 1.0             TL_z1
Subsystem  1, Output 26 to Subsystem  3, Input  8, Gain= 1.0             TL_z2
Subsystem  1, Output 27 to Subsystem  3, Input  9, Gain= 1.0             TL_z3
Subsystem  1, Output 28 to Subsystem  3, Input 10, Gain= 1.0             TL_z4
Subsystem  1, Output 29 to Subsystem  3, Input 11, Gain= 1.0             TL_z5
Subsystem  1, Output 30 to Subsystem  3, Input 12, Gain= 1.0             TL_z6
.....................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                          Vehicle Outputs
Via Matrix +I18
.....................................................................
```

```
SYSTEM OUTPUTS FROM SUBSYSTEM  2                                                            Pitch Gimbal Deflects
System Output 19 from Subsystem  2, Output  1, Gain= 1.0                                    dy-1
System Output 20 from Subsystem  2, Output  2, Gain= 1.0                                    dy-2
System Output 21 from Subsystem  2, Output  3, Gain= 1.0                                    dy-3
System Output 22 from Subsystem  2, Output  4, Gain= 1.0                                    dy-4
System Output 23 from Subsystem  2, Output  5, Gain= 1.0                                    dy-5
System Output 24 from Subsystem  2, Output  6, Gain= 1.0                                    dy-6
.....................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  3                                                            Yaw Gimbal Deflects
System Output 25 from Subsystem  3, Output  1, Gain= 1.0                                    dz-1
System Output 26 from Subsystem  3, Output  2, Gain= 1.0                                    dz-2
System Output 27 from Subsystem  3, Output  3, Gain= 1.0                                    dz-3
System Output 28 from Subsystem  3, Output  4, Gain= 1.0                                    dz-4
System Output 29 from Subsystem  3, Output  5, Gain= 1.0                                    dz-5
System Output 30 from Subsystem  3, Output  6, Gain= 1.0                                    dz-6
.....................................................................
```

**Definitions of Inputs  =  13**
**Engine Pitch Deflect Comd delta y_1 (rad)**
**Engine Pitch Deflect Comd delta y_2 (rad)**
**Engine Pitch Deflect Comd delta y_3 (rad)**
**Engine Pitch Deflect Comd delta y_4 (rad)**
**Engine Pitch Deflect Comd delta y_5 (rad)**
**Engine Pitch Deflect Comd delta y_6 (rad)**
**Engine Yaw Deflect Commnd delta z_1 (rad)**
**Engine Yaw Deflect Commnd delta z_2 (rad)**
**Engine Yaw Deflect Commnd delta z_3 (rad)**
**Engine Yaw Deflect Commnd delta z_4 (rad)**
**Engine Yaw Deflect Commnd delta z_5 (rad)**
**Engine Yaw Deflect Commnd delta z_6 (rad)**
**Wind-Gust Velocity**

**Definitions of Outputs =  30**
**Roll Attitude  (phi)    (radians)**
**Roll Rate       (p-body) (rad/sec)**
**Pitch Attitude (thet)   (radians)**
**Pitch Rate      (q-body) (rad/sec)**
**Yaw  Attitude  (psi)    (radians)**
**Yaw  Rate       (r-body) (rad/sec)**
**Angle of attack, alfa,   (radian)**
**Angle of sideslip, beta, (radian)**
**Change in Altitude, delta-h, (feet)**
**Forward Acceleration  (V-dot)  (ft/sec)**
**Cross Range Velocity (Vcr) (ft/sec)**
**Rate-Gyro #  1, Roll Rate  (Body) (rad/sec)**
**Rate-Gyro #  2, Pitch Rate (Body) (rad/sec)**
**Rate-Gyro #  3, Yaw Rate   (Body) (rad/sec)**
**Accelerom #  1, (along Y), (ft/sec^2) Translat. Accelerat.**
**Accelerom #  2, (along Z), (ft/sec^2) Translat. Accelerat.**
**Alpha Sensed at Vane # 1 (radian)**
**Beta  Sensed at Vane # 1 (radian)**
**Engine #1 Pitch Deflection dy-1 (rad)**
**Engine #2 Pitch Deflection dy-2 (rad)**
**Engine #3 Pitch Deflection dy-3 (rad)**
**Engine #4 Pitch Deflection dy-4 (rad)**
**Engine #5 Pitch Deflection dy-5 (rad)**
**Engine #6 Pitch Deflection dy-6 (rad)**
**Engine #1 Yaw Deflection dz-1 (rad)**
**Engine #2 Yaw Deflection dz-2 (rad)**
**Engine #3 Yaw Deflection dz-3 (rad)**
**Engine #4 Yaw Deflection dz-4 (rad)**
**Engine #5 Yaw Deflection dz-5 (rad)**
**Engine #6 Yaw Deflection dz-6 (rad)**
------------------------------------------------------------------------------------------------

```
INTERCONNECTION OF SYSTEMS .....
Plant Model at T=82 sec, Vehicle/ Actuators/ TVC
! Combines the Vehicle with Actuators, TVC and Wind-Gust
Titles of Systems to be Combined
Title 1 Plant Model at T=82 sec, Vehicle/ Actuators
Title 2 Wind Gust Shaping
SYSTEM INPUTS TO SUBSYSTEM  1                                           TVC to Vehicle
Via Matrix +TVC                                                        roll, pitch, yaw
.....................................................................
SYSTEM INPUTS TO SUBSYSTEM  2                                          To Pitch Actuators
System Input  4 to Subsystem  2, Input  1, Gain= 1.0                  Wind-Gust Shaper
.....................................................................
SUBSYSTEM NO  2 GOES TO SUBSYSTEM NO  1                                Gust Input
Subsystem  2, Output  1 to Subsystem  1, Input 13, Gain= 1.0
.....................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                       Vehicle Outputs
Via Matrix +I30                                                        All Outputs
.....................................................................
Definitions of Inputs  =  4
Roll  TVC Demand from Flight Control
Pitch TVC Demand from Flight Control
Yaw   TVC Demand from Flight Control
Wind-Gust Velocity

Definitions of Outputs =  30
Roll Attitude  (phi)    (radians)
Roll Rate      (p-body) (rad/sec)
Pitch Attitude (thet)   (radians)
Pitch Rate     (q-body) (rad/sec)
Yaw  Attitude  (psi)    (radians)
Yaw  Rate      (r-body) (rad/sec)
Angle of attack, alfa,   (radian)
Angle of sideslip, beta, (radian)
Change in Altitude, delta-h, (feet)
Forward Acceleration  (V-dot)  (ft/sec)
Cross Range Velocity (Vcr) (ft/sec)
Rate-Gyro #  1, Roll Rate  (Body) (rad/sec)
Rate-Gyro #  2, Pitch Rate (Body) (rad/sec)
Rate-Gyro #  3, Yaw Rate   (Body) (rad/sec)
Accelerom #  1, (along Y), (ft/sec^2) Translat. Accelerat.
Accelerom #  2, (along Z), (ft/sec^2) Translat. Accelerat.
Alpha Sensed at Vane # 1 (radian)
Beta  Sensed at Vane # 1 (radian)
Engine #1 Pitch Deflection dy-1 (rad)
Engine #2 Pitch Deflection dy-2 (rad)
Engine #3 Pitch Deflection dy-3 (rad)
Engine #4 Pitch Deflection dy-4 (rad)
Engine #5 Pitch Deflection dy-5 (rad)
Engine #6 Pitch Deflection dy-6 (rad)
Engine #1 Yaw Deflection dz-1 (rad)
Engine #2 Yaw Deflection dz-2 (rad)
Engine #3 Yaw Deflection dz-3 (rad)
Engine #4 Yaw Deflection dz-4 (rad)
Engine #5 Yaw Deflection dz-5 (rad)
Engine #6 Yaw Deflection dz-6 (rad)
-------------------------------------------------------------------------------------------
```

**Flight Control System:** The next dataset implements the flight control system which is defined in transfer-function form and it is converted by Flixan to state-space form to be combined with other systems. Its title is "*Flight Control System at T=82 sec*" and consists of three control loops (roll, pitch, yaw). It includes gains that were designed in Section 3, low-pass and notch filters, and integrators, as shown in Figure 5.5. The inputs to the flight control system are vehicle attitudes, rates, normal and lateral accelerations. The original FCS was designed for α and β load-relief feedback but the gains are scaled for Nz and Ny feedback instead because α and β are not easy to measure and the signals are similar. The 3 outputs are roll, pitch and yaw errors that command the TVC matrix. The TVC is part of the flight control system but it is included in the plant model for convenience in order to avoid too many interconnections. The FCS gains were obtained by the LQR method as already described in Section 2 but they were slightly modified to accommodate bending, slosh, and TWD dynamics that were left out from the design model. Integrators are included in the pitch and yaw attitude feedback loops for better attitude tracking. Low-pass filters are also included and a notch-filter in pitch for flex-mode attenuation. The gains are scheduled as a function of time.
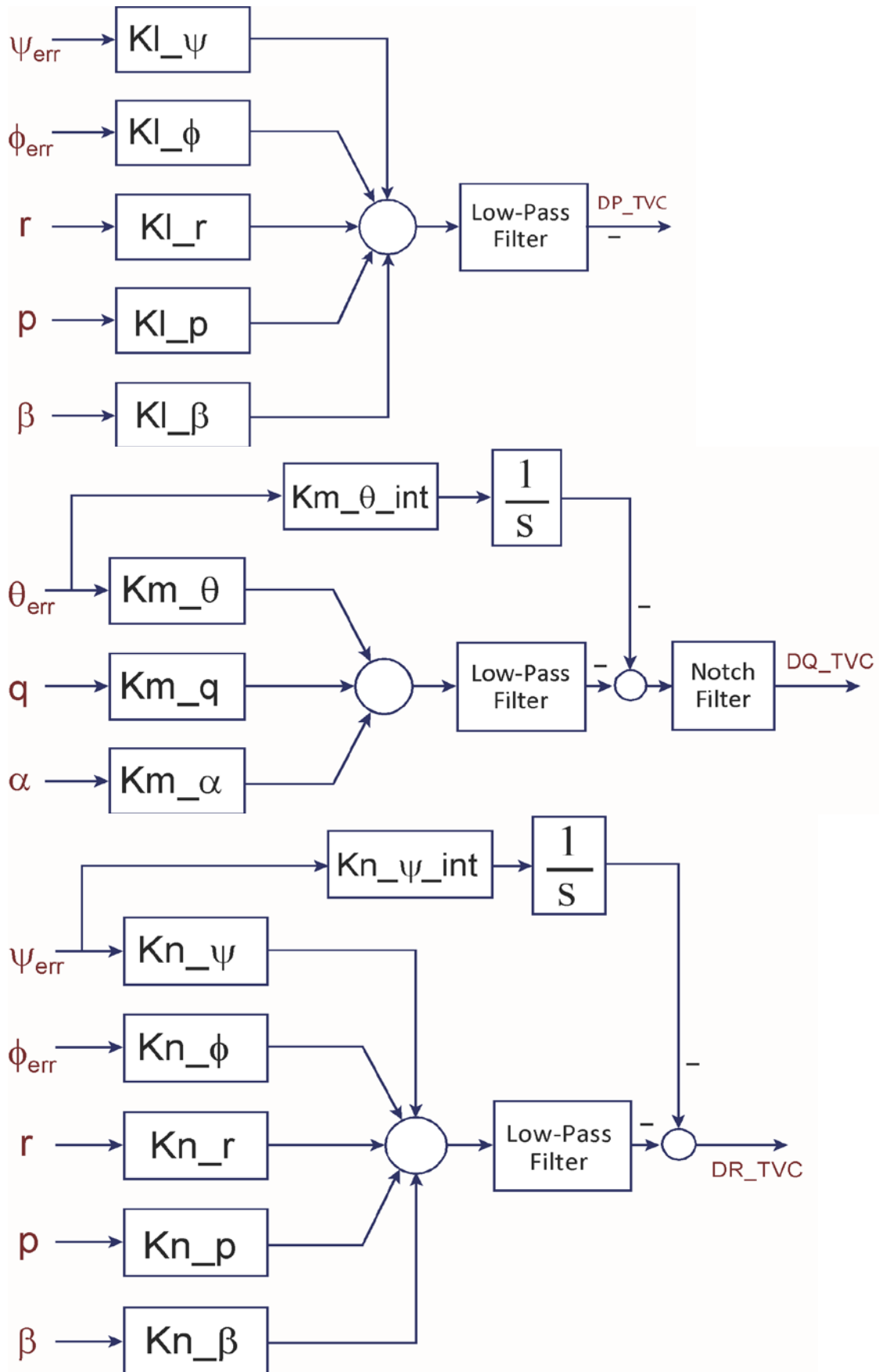
**Figure 5.5 Roll, Pitch and Yaw Flight Control System**

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The flight control system at high-Q attempts to trade between tracking the guidance attitude commands versus aerodynamic loading due to α and β angles relative to the air-flow. During periods of low dynamic pressure attitude tracking is more important and the attitude gains are emphasized in comparison with the load-relief gains which are reduced because you don't care as much about aero loading. But when the dynamic pressure is high, structural loading becomes an important issue and the load-relief feedback gains from the accelerometers are increased more in comparison with the attitude gains. At Max-Q the vehicle attempts to fly at zero α and β to minimize aero loading at the expense of flight-path tracking performance which is poor until the dynamic pressure is somewhat reduced.

```
SYSTEM OF TRANSFER FUNCTIONS ...
Flight Control System at T=82 sec
Continuous
TF. Block #  1 Roll Low-Pass                          Order of Numer, Denom=  0  2
Numer 0.0         0.0        2500.0
Denom 1.0        70.0        2500.0
TF. Block #  2 Ptch Low-Pass                          Order of Numer, Denom=  0  2
Numer 0.0         0.0        2500.0
Denom 1.0        70.0        2500.0
TF. Block #  3 Yaw Low-Pass                           Order of Numer, Denom=  0  2
Numer 0.0         0.0        2200.0
Denom 1.0        65.0        2200.0
TF. Block #  4 Lead-Lag @ 7 rad/s                     Order of Numer, Denom=  0  1
Numer 0.06        1.0
Denom 0.03        1.0
TF. Block #  5 Theta Integrator                       Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
TF. Block #  6 Psi Integrator                         Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
TF. Block #  7 Notch at 35.5 (rad)                    Order of Numer, Denom=  2  2
Numer 1.0         0.2485     1260.0
Denom 1.0         6.6        1089.0
.................................................
Block #, from Input #, Gain
 1        1        0.113                               KL_phi    0.113
 1        3        0.084                               KL_psi    0.084
 1        4        0.358                               KL_p      0.358
 1        6        0.578                               KL_r      0.578
 1        8       -1.45                                KL_beta  -1.45
 2        2        0.073                               KM_theta  0.073
 2        5        1.1                                 KM_q      1.1
 2        7        0.544                               KM_alfa   0.544
 3        1        0.01                                KN_phi    0.01
 3        3        0.057                               KN_psi    0.057
 3        4        0.16                                KN_p      0.16
 3        6        2.0                                 KN_r      2.0
 3        8       -2.2                                 KN_beta  -2.2
 5        2        0.0                                 KM_theta_integr 0.0
 6        3        0.0                                 KN_psi_integr   0.0
..........................
Block #, from Block #, Gain
 4        3        1.0
 7        2        1.0                                 Pitch Low-Pass to Notch
..........................
Outpt #, from Block #, Gain
 1        1       -1.1                                 DP_TVC
 2        7       -1.0                                 Pitch Notch  to DQ_TVC
 2        5       -1.0                                 Pitch Integr to DQ_TVC
 3        4       -0.7                                 Yaw Lead-Lag to DR_TVC
 3        6       -0.7                                 Psi Integrat to DR_TVC
..........................
```

```
Definitions of Inputs  =   8
Roll Attitude   (radians)
Pitch Attitude  (radians)
Yaw  Attitude   (radians)
Roll Rate       (rad/sec)
Pitch Rate      (rad/sec)
Yaw  Rate       (rad/sec)
Angle of attack, alfa, (radians)
Angle of sideslip, beta, (radian)

Definitions of Outputs =   3
Roll Flight Control Output DP_TVC
Ptch Flight Control Output DQ_TVC
Yaw  Flight Control Output DR_TVC
-------------------------------------------------------------------------------------------
```

Our final steps are to combine the plant model of Figure 5.4 with the flight control system in Figure 5.5 in two configurations and to create two different systems: (a) an open-loop system "*Open-Loop System at T=82 sec*" that will be used to calculate frequency responses for analyzing stability, and (b) a closed-loop system "*Closed-Loop Simulation System at T=82 sec*" that will be used for simulations and analyzing the system's response to commands and to wind-gust excitations. The Nz and Ny accelerations signals from the plant model are scaled to approximate α and β in order to be applied to the flight control inputs.

**Open Loop System:** The open-loop system is described in Section 7, and it is used for calculating frequency responses and analyzing system stability. It has 3 inputs and 3 outputs consisting of all three (roll, pitch, yaw) control loops broken at the TVC input. The title of the interconnection dataset is "*Open-Loop System at T=82 sec*". It uses scaled Nz and Ny signals (outputs 16 and 15) instead of the expected α and β (outputs 7 and 8) to connect between the plant and the flight control system inputs.

```
INTERCONNECTION OF SYSTEMS .....
Open-Loop System at T=82 sec
Titles of Systems to be Combined
Title 1 Plant Model at T=82 sec, Vehicle/ Actuators/ TVC
Title 2 Flight Control System at T=82 sec
SYSTEM INPUTS TO SUBSYSTEM  1                                                    TVC
Via Matrix +I3
.................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2                                          Plant to FCS
Subsystem  1, Output  1 to Subsystem  2, Input  1, Gain= 1.0                     phi
Subsystem  1, Output  3 to Subsystem  2, Input  2, Gain= 1.0                     theta
Subsystem  1, Output  5 to Subsystem  2, Input  3, Gain= 1.0                     psi
Subsystem  1, Output 12 to Subsystem  2, Input  4, Gain= 1.0                     p   12
Subsystem  1, Output 13 to Subsystem  2, Input  5, Gain= 1.0                     q   13
Subsystem  1, Output 14 to Subsystem  2, Input  6, Gain= 1.0                     r   14
Subsystem  1, Output 16 to Subsystem  2, Input  7, Gain= -0.008                  alfa frm Nz 16 -0.0083
Subsystem  1, Output 15 to Subsystem  2, Input  8, Gain= -0.009                  beta frm Ny 15 -0.0092
.................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                                                 Vehicle Outputs
Via Matrix +I3                                                                   All Outputs
.................................................................
Definitions of Inputs  =  3
Roll  TVC Demand from Flight Control
Pitch TVC Demand from Flight Control
Yaw   TVC Demand from Flight Control

Definitions of Outputs =   3
Roll Flight Control Output DP_TVC
Ptch Flight Control Output DQ_TVC
Yaw  Flight Control Output DR_TVC
-------------------------------------------------------------------------------------------
```

**Closed-Loop System:** This system, shown in Figure 5.6, is also a combination of the same two state-space systems: the plant model which includes the TVC and the gust shaping filter and the flight control system. It has the 3 control loops closed and it will be used for simulations in Section 8. It includes the δ-attitude commands and the wind-gust disturbance inputs. Its outputs consist of the 30 outputs defined earlier in the plant model. The title of the interconnection dataset is "*Closed-Loop Simulation System at T=82 sec*" which is also its title in the systems file. The two systems are also converted to Matlab format in order to be loaded into Matlab for analysis. The Matlab files created are "*open_lp_T82.m*" and "*sim_cl-T82.m*".

```
INTERCONNECTION OF SYSTEMS .....
Closed-Loop Simulation System at T=82 sec
Titles of Systems to be Combined
Title 1 Plant Model at T=82 sec, Vehicle/ Actuators/ TVC
Title 2 Flight Control System at T=82 sec
SYSTEM INPUTS TO SUBSYSTEM  2                                         Flight Control System
Via Matrix -I3                                                       Phi, Theta, Psi
Commands
...........................................................................
SYSTEM INPUTS TO SUBSYSTEM  1                                         To Plant
System Input  4 to Subsystem  1, Input  4, Gain= 1.0                  Wind-Gust Input
...........................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                      Plant Outputs
Via Matrix +I30                                                      All Outputs
...........................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2                               Plant to FCS
Subsystem  1, Output  1 to Subsystem  2, Input  1, Gain= 1.0          phi
Subsystem  1, Output  3 to Subsystem  2, Input  2, Gain= 1.0          theta
Subsystem  1, Output  5 to Subsystem  2, Input  3, Gain= 1.0          psi
Subsystem  1, Output 12 to Subsystem  2, Input  4, Gain= 1.0          p   12
Subsystem  1, Output 13 to Subsystem  2, Input  5, Gain= 1.0          q   13
Subsystem  1, Output 14 to Subsystem  2, Input  6, Gain= 1.0          r   14
Subsystem  1, Output 16 to Subsystem  2, Input  7, Gain= -0.008       alfa frm Nz 16 -0.0083
Subsystem  1, Output 15 to Subsystem  2, Input  8, Gain= -0.009       beta frm Ny 15 -0.0092
...........................................................................
SUBSYSTEM NO  2 GOES TO SUBSYSTEM NO  1                               FCS to Plant
Via Matrix +I3                                                       DPQR_TVC
...........................................................................
Definitions of Inputs  =  4
Roll  Attitude Command (rad)
Pitch Attitude Command (rad)
Yaw   Attitude Command (rad)
Wind-Gust Velocity Disturbance (ft/sec)

Definitions of Outputs =  30
Roll Attitude  (phi)    (radians)
Roll Rate      (p-body) (rad/sec)
Pitch Attitude (thet)   (radians)
Pitch Rate     (q-body) (rad/sec)
Yaw  Attitude  (psi)    (radians)
Yaw  Rate      (r-body) (rad/sec)
Angle of attack,   alfa, (radian)
Angle of sideslip, beta, (radian)
Change in Altitude, delta-h, (feet)
Forward Acceleration  (V-dot)  (ft/sec)
Cross Range Velocity (Vcr) (ft/sec)
Rate-Gyro #  1, Roll Rate  (Body) (rad/sec)
Rate-Gyro #  2, Pitch Rate (Body) (rad/sec)
Rate-Gyro #  3, Yaw Rate   (Body) (rad/sec)
Accelerom #  1, (along Y), (ft/sec^2) Translat. Accelerat.
Accelerom #  2, (along Z), (ft/sec^2) Translat. Accelerat.
Alpha Sensed at Vane # 1 (radian)
Beta  Sensed at Vane # 1 (radian)
Engine #1 Pitch Deflection dy-1 (rad)
Engine #2 Pitch Deflection dy-2 (rad)
Engine #3 Pitch Deflection dy-3 (rad)
Engine #4 Pitch Deflection dy-4 (rad)
Engine #5 Pitch Deflection dy-5 (rad)
Engine #6 Pitch Deflection dy-6 (rad)
Engine #1 Yaw Deflection dz-1 (rad)
Engine #2 Yaw Deflection dz-2 (rad)
Engine #3 Yaw Deflection dz-3 (rad)
Engine #4 Yaw Deflection dz-4 (rad)
Engine #5 Yaw Deflection dz-5 (rad)
Engine #6 Yaw Deflection dz-6 (rad)
```
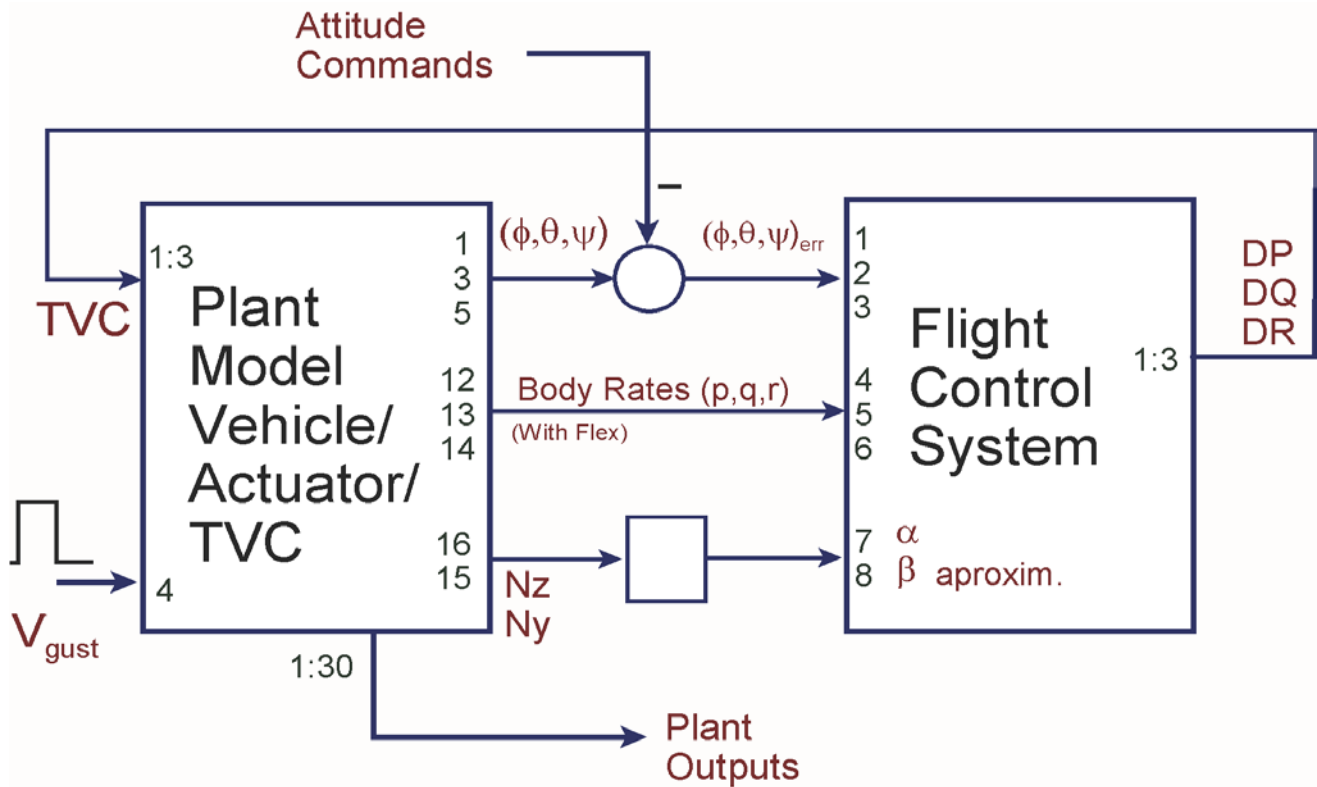
**Figure 5.6 Closed-Loop Interconnection System "*Closed-Loop Simulation System at T=82 sec*"**

**Modal Data**: The dataset below contains the pre-selected modes that will be combined with the vehicle data to generate the flex model. Its title: "*Space-Plane Launch Vehicle Pitch Flex Modes*" is also included at the bottom of the vehicle dataset in order to associate the modes with the vehicle data. The dataset consists of 5 modes but only the first mode frame is shown. Each mode consists of: frequency, damping coefficient, and generalized mass. It also includes mode shapes (3-rotations and 3-translations) at the TVC gimbals, the sensors, and the slosh mass locations.

```
SELECTED MODAL DATA AND LOCATIONS FOR THE Pitch Axis
Space-Plane Launch Vehicle Pitch Flex Modes

MODE # 1/  4, Frequency (rad/sec), Damping (zeta), Generalized Mass=    18.448      0.50000E-02    80.000
DEFINITION OF LOCATIONS (NODES)             phi along X   phi along Y   phi along Z   sigm about X  sigm about Y  sigm
about Z

                        Node ID#    Modal Data at the  6 Engines
Engine Number 1             7       0.21341E-02  -0.73032E-09   0.81331E-02  -0.12972E-08   0.71835E-04
Engine Number 2             7       0.21341E-02  -0.73032E-09   0.81331E-02  -0.12972E-08   0.71835E-04
Engine Number 3             7       0.21341E-02  -0.73032E-09   0.81331E-02  -0.12972E-08   0.71835E-04
Engine Number 4             7       0.21341E-02  -0.73032E-09   0.81331E-02  -0.12972E-08   0.71835E-04
Engine Number 5             7       0.21341E-02  -0.73032E-09   0.81331E-02  -0.12972E-08   0.71835E-04
Engine Number 6             7       0.21341E-02  -0.73032E-09   0.81331E-02  -0.12972E-08   0.71835E-04

                        Node ID#    Modal Data at the  3 Gyros ...
NAV BASE (roll)             2      -0.66786E-02   0.49597E-02   0.12942E-02   0.58008E-03  -0.37565E-03
NAV BASE (pitch)            2      -0.66786E-02   0.49597E-02   0.12942E-02   0.58008E-03  -0.37565E-03
NAV BASE (yaw)              2      -0.66786E-02   0.49597E-02   0.12942E-02   0.58008E-03  -0.37565E-03

                        Node ID#    Modal Data at the  2 Accelerometers, along (x,y,z)...
NAV BASE (Y)                2      -0.66786E-02   0.49597E-02   0.12942E-02   0.58008E-03  -0.37565E-03
NAV BASE (Z)                2      -0.66786E-02   0.49597E-02   0.12942E-02   0.58008E-03  -0.37565E-03

                        Node ID#    Modal Data at the  1 Vane Aero Sensor, along (y,z)...
Vane                        9       0.00000E+00   0.87617E-09  -0.86682E-02   0.00000E+00   0.34565E-03  -

                        Node ID#    Modal Data at the  2 Slosh Masses, along (y,z)...
LOX Mass                    5       0.00000E+00   0.67617E-09  -0.76682E-02   0.00000E+00   0.00000E+00
LH2 Mass                    6       0.00000E+00   0.86842E-10   0.13886E-01   0.00000E+00   0.00000E+00

                        Node ID#    Modal Data at the Disturbance Point
Dist                        6       0.00000E+00   0.86842E-10   0.13886E-01   0.00000E+00   0.00000E+00
-----------------------------------------------------------------------------------------------------------
```

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**Matlab Conversions:** At the bottom of the input data file: "*Space_Plane_T82.Inp*" there are additional "Convert to Matlab" datasets that convert some systems and the TVC matrix from file "*Space_Plane-T82.Qdr*" to Matlab m-files that can easily be loaded into Matlab for analysis. The second line of each set contains the title of the system to be converted. The third line specifies either a system or a matrix. The fourth line is the filename, such as, "*Sim_cl_T82.m*" where the closed-loop system will be saved as m-file and will be loaded into Matlab.

```
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
TVC Actuator
System
actuator.m
--------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
System of Six TVC Actuators
System
Actuat_6.m
--------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Space-Plane Launch Vehicle at T=82 sec (Max-Q), with Slosh and Flex
System
Vehicle.m
--------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Plant Model at T=82 sec, Vehicle/ Actuators/ TVC
System
Plant.m
--------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Mixing Logic for the Space-Plane Launch Vehicle at T=82 sec
Matrix TVC
--------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Open-Loop System at T=82 sec
System
open_lp_T82.m
--------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Closed-Loop Simulation System at T=82 sec
System
Sim_cl_T82.m
--------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Flight Control System at T=82 sec
System
fcs.m
--------------------------------------------------------------------------------------------
```

# 6.0 Processing the Vehicle Data Files in Batch Mode

At this point we assume that the input data files for all flight condition have already been prepared, including the interconnections, the flight control laws datasets, etc. and they are all saved in 7 separate folders corresponding to the trajectory flight times: (T0, T20, T56, T82, T108, T160, and T300) in folder: "*Examples\Space Plane Launch Vehicle\Analysis Models*". The input filenames are: "*Space_Plane_T0.Inp*", "*Space_Plane_T20.Inp*", etc. and they are ready to be processed in batch mode. For example, to process the Max-Q case at T=82 (sec), run the Flixan program and select the subdirectory where the input file is located, then from the main menu, select "*File Management*", "*Managing Input Files (.Inp)*", and "*Edit/ Process Input Data Files*", as shown below.
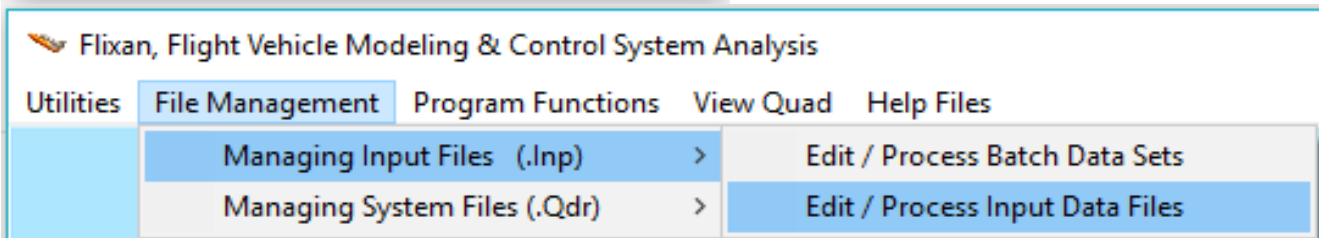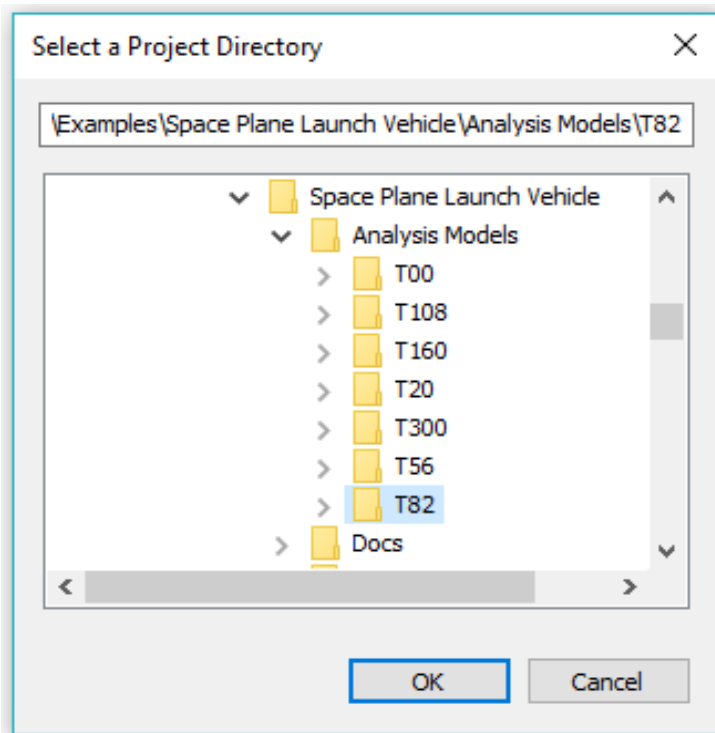
The input file manager utility dialog comes up and from the left menu select the only input file: "*Space_Plane_T82.Inp*" in the T82 folder, and click on "*Select Input File*" button. The menu on the right shows the titles of all datasets which are included in this file and ready to be processed by the Flixan program. Each dataset can be processed by a Flixan utility shown on the left side of the title. Select the top title which is the batch and it will process all the other datasets, and click on "*Process Input Data*" button. Answer "Yes" in the next question to replace the previously created data in the systems file "*Space_Plane_T82.Qdr*".

The program will process all the datasets referenced by the batch and it will save the systems and matrices in file: "*Space_Plane_T82.Qdr*". It also saves the data in Matlab format for control analysis. A display list pops-up highlighting each dataset while it is executing. Click "Exit" to close Flixan program and go to Matlab for the control analysis. This process is repeated for the remaining flight conditions and the corresponding plants, TVC matrices, control systems, open-loop and closed-loop systems are saved in the corresponding folders to be analyzed using Matlab/ Simulink, as it will be shown in the following sections.
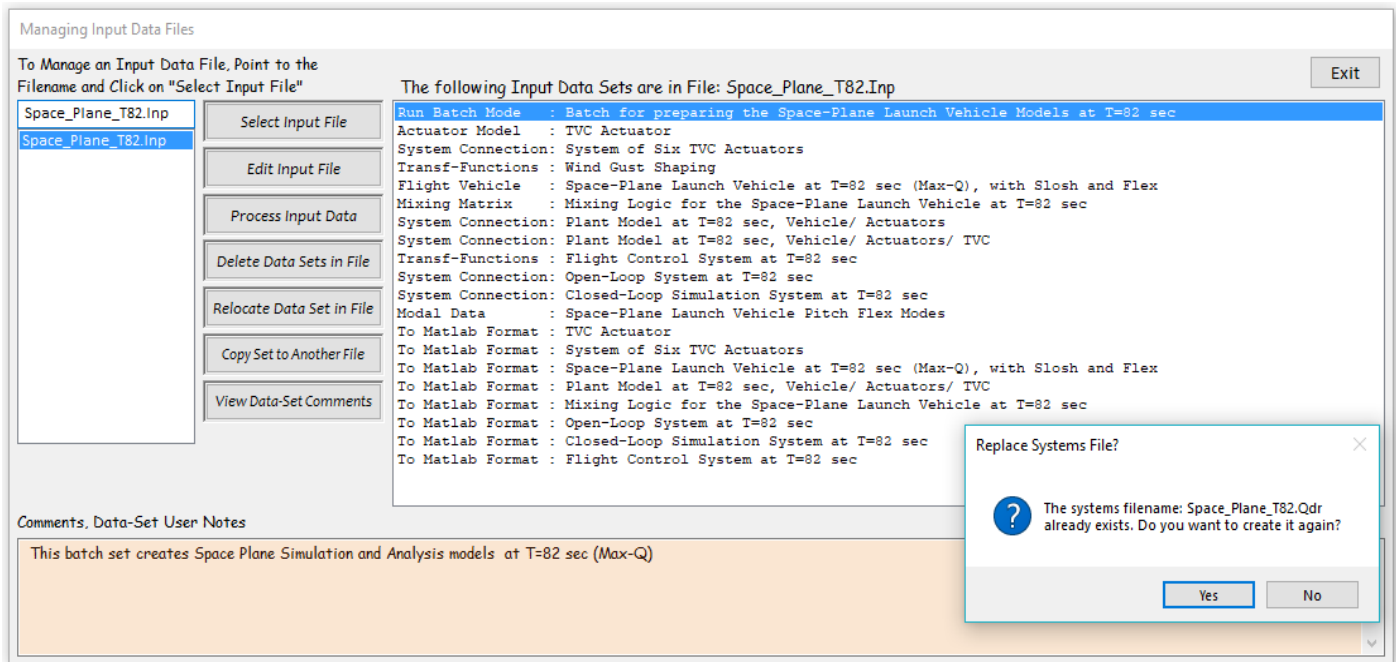
## 7.0 Frequency Response Analysis

The frequency domain stability analysis is performed using Matlab. We assume that all the systems have already been created and saved in the corresponding folder, as described in Section 6. The Matlab initialization file "init.m" loads the systems, gains, and the TVC matrix into Matlab, and the file "freq.m" calculates and plots the frequency response.

```
% Sim Initialization file init.m, at t=82 sec
close all
r2d=180/pi;
[Av,Bv,Cv,Dv]= vehicle;                    % Vehicle Model (Slosh,Flex)
[Aa,Ba,Ca,Da]= actuat_6;                   % 6-Actuators
[Ac,Bc,Cc,Dc]= fcs;                        % Flight Control
[Ao,Bo,Co,Do]= open_lp_t82;                % Load Combind Open-Loop Systm
[As,Bs,Cs,Ds]= sim_cl_t82;                 % Closed-Loop System
load TVC -ascii                            % TVC Matrix


Km_the=0.073; Km_the_int=0.0;              % Pitch Gains .0162 .0017 1.19 .562
Km_q=1.1; Km_alf=0.544;
Kl_phi=0.113; Kl_psi=0.084; Kl_p=0.358;    % Roll Gains
Kl_r= 0.578; Kl_bet=-1.45;                 % Kl_r= 0.578; Kl_bet=-1.45;
Kn_phi=0.01; Kn_psi= 0.057; Kn_p= 0.16;    % Yaw Gains .01, .057, .158
Kn_r=2.0; Kn_bet=-2.2; Kn_psi_int=0.0;     % 2.163, -2.44,


% File freq.m calculates the Frequency Response
close all
r2d=180/pi; init
%[A1,B1,C1,D1]= linmod('OpenLoop_1');      % Linearize Open-Loop model-1
[A1,B1,C1,D1]= linmod('OpenLoop_2');       % Linearize Open-Loop model-2
w=logspace(-3, 3, 44000);                  % Define Frequ Range
sys0= ss(A1,B1,C1,D1);                     % Create SS System
figure(1); nichols(sys0,w)                 % Plot Nichol's Chart
figure(2); bode(sys0,w)                    % Plot Bode Plot
sys82=sys0; save sys82;
```

Two separate Simulink models are used to calculate the frequency response for comparison. The model "*OpenLoop_2.mdl*" shown in Figure 7.1 is using the system "*Open-Loop System at T=82 sec*" from file "*open_lp_t82.m*" where the subsystems were combined together using Flixan, as already described, and the model "*OpenLoop_1.mdl*", shown in Figure 7.2 where the vehicle, actuators, TVC, and flight control subsystems generated from Flixan are combined together using Simulink, as shown below. Both systems are identical and they are used to calculate the open-loop frequency responses and plot the Bode and Nichols Charts. Since the systems consist of 3 loops, only one loop is opened at a time. In this case, the pitch loop is shown opened and the two lateral loops are closed.



## Open-Loop System
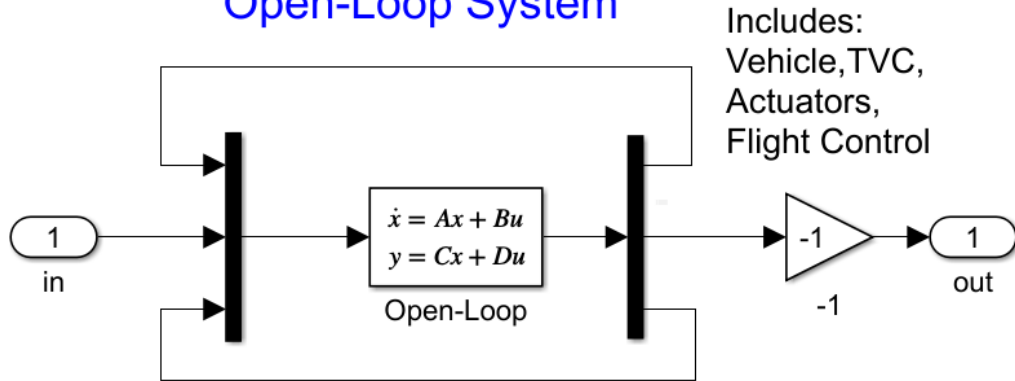
Includes:
Vehicle, TVC,
Actuators,
Flight Control

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

Open-Loop

Inputs = 3
Roll  TVC Demand from Flight Control
Pitch TVC Demand from Flight Control
Yaw   TVC Demand from Flight Control

Outputs = 3
Roll Flight Control Output DP_TVC
Ptch Flight Control Output DQ_TVC
Yaw  Flight Control Output DR_TVC

**Figure 7.1 Simulink Model "OpenLoop_1.Mdl" Configured for Pitch Axis Stability Analysis**
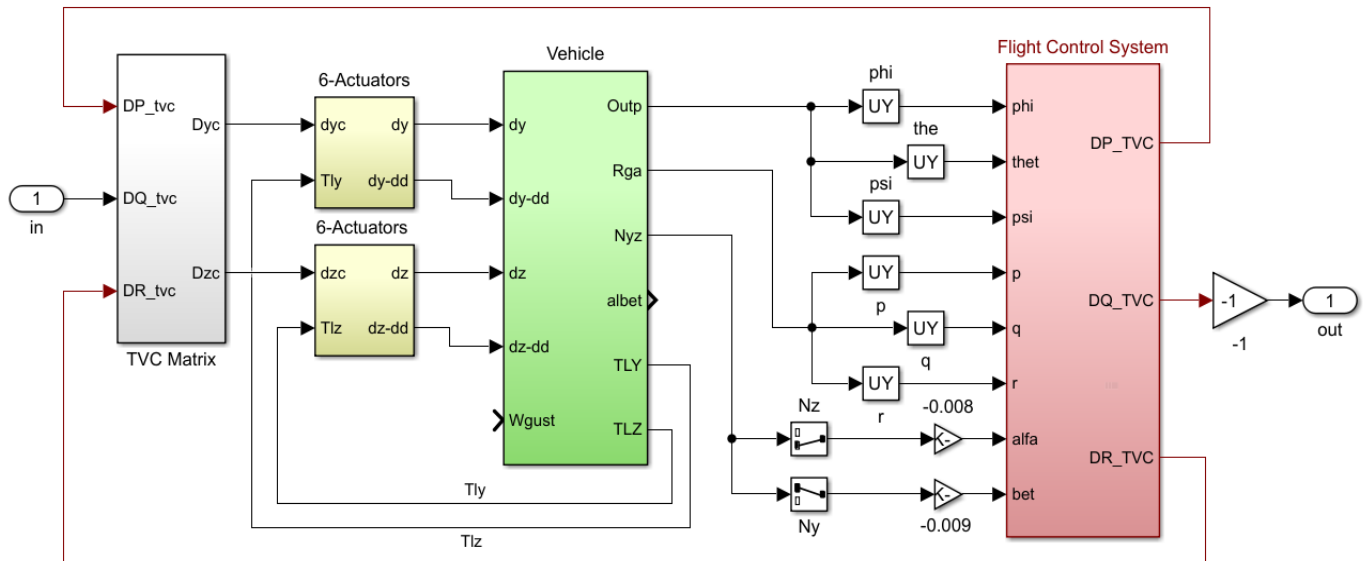


**Figure 7.2 Simulink Model "OpenLoop_2.Mdl" Configured for Pitch Axis Stability Analysis**

We will analyze stability in pitch and yaw using the above models which produce identical results, beginning with pitch axis and then yaw. For yaw analysis we must close the pitch and roll loops, open the yaw loop and calculate the frequency response across the opened DR_tvc terminals.

Figure 7.1 shows the pitch axis Bode plots at three High-Q flight conditions. The slosh modes appear at around 3 (rad/sec) frequency. Flexibility is more visible in the pitch axis because the modes are predominantly symmetric. The tail-wags-dog zero is also visible at 40 (rad/sec). The Nichols plot in Figure 7.2 shows the system stability for the three High-Q cases.
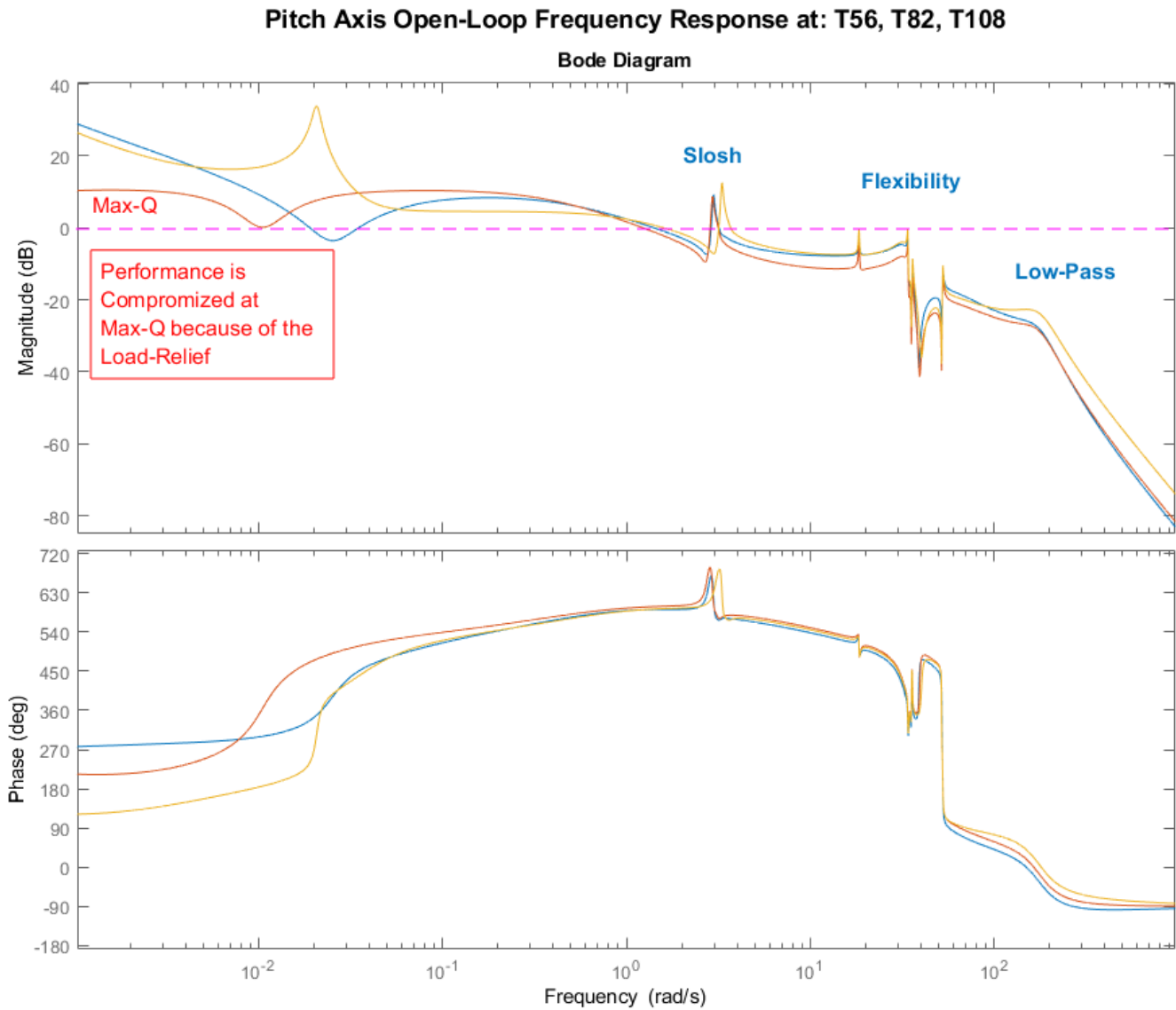


**Figure 7.1 Pitch Axis Frequency Response at three High-Q Flight Conditions**
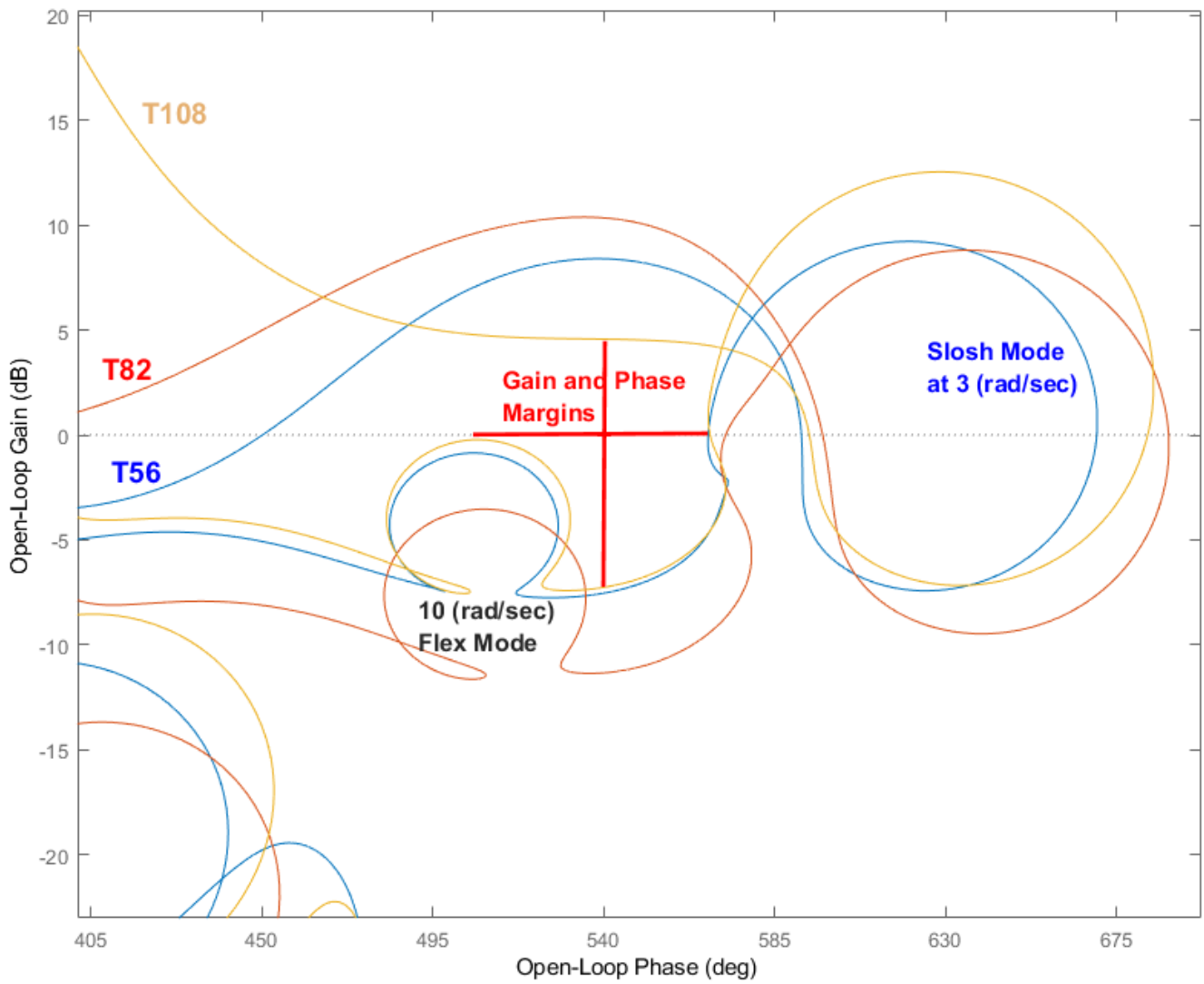
**Figure 7.2 Pitch Axis Stability at three High-Q Flight Conditions**

**Figure 7.3 Pitch Stability for two Low Dynamic Pressure Cases, at T=0 and at T=300 (sec)**

The Nichols plots in Figure 7.3 shows the pitch axis stability for the two low dynamic pressure flight conditions

**Figure 7.4 Yaw Axis Frequency Response at three High-Q Flight Conditions**

The yaw axis shows similar frequency responses. At Max-Q the amplitude is reduced at low frequencies indicating a compromised command following performance because of the load-relief action. Propellant sloshing is at 3 (rad/sec) and the tail-wags-dog zero is at 40 (rad/sec).
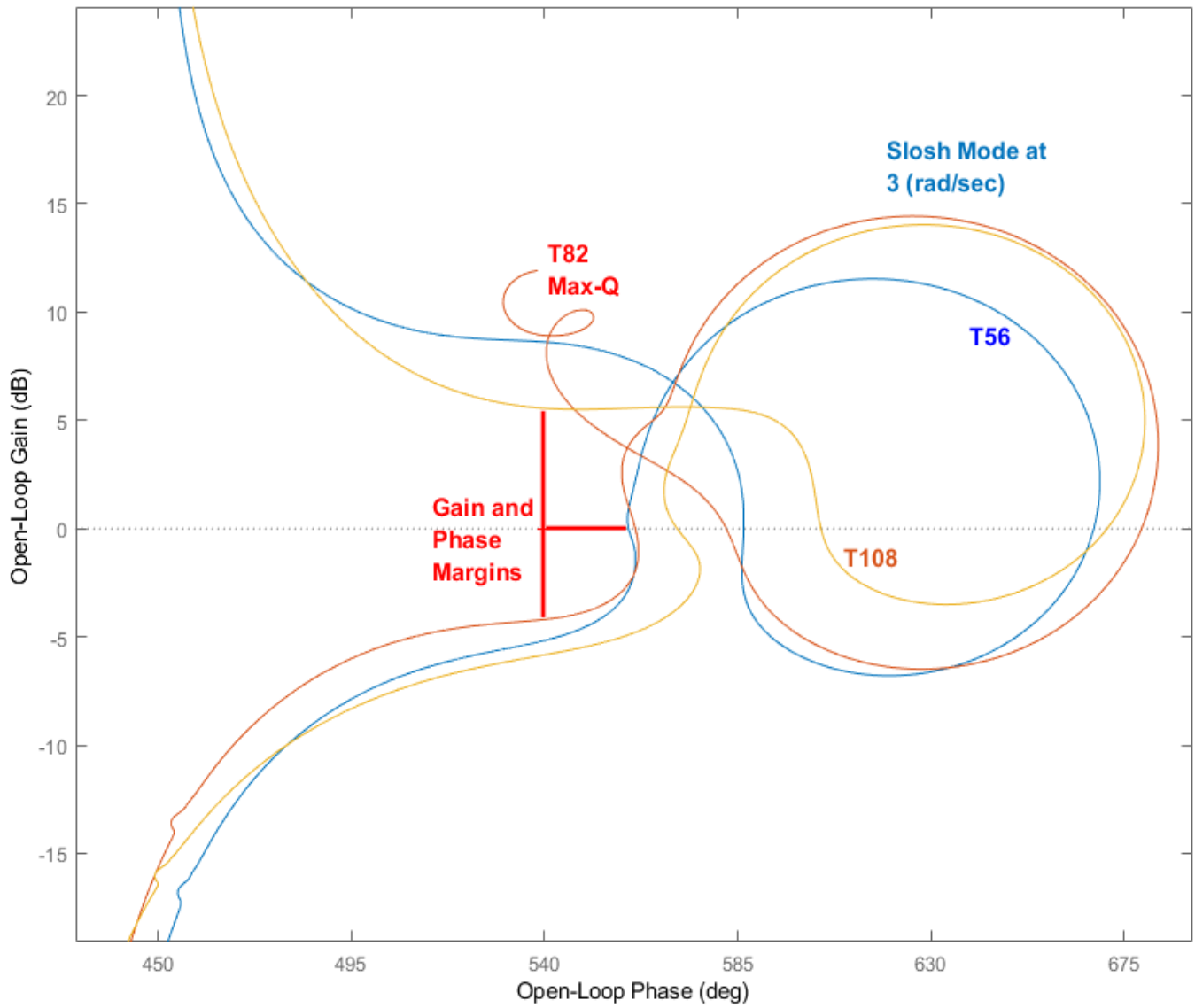
**Figure 7.5 Yaw Axis Stability at three High-Q Flight Conditions**

**Yaw Axis Stability at Lift-Off and at T300 (low Q cases)**

Nichols Chart
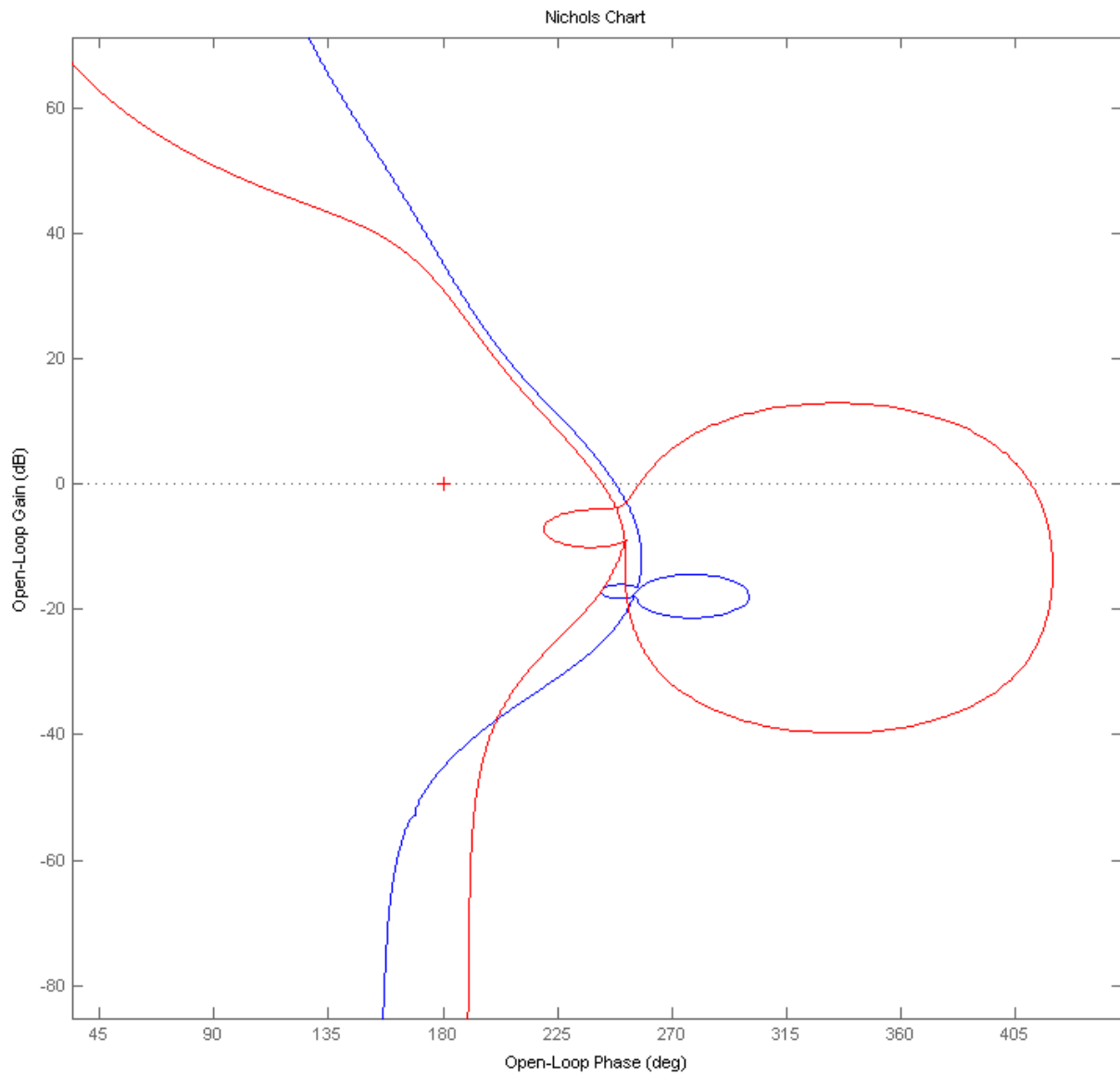
**Figure 7.6 Yaw Stability at two Low Dynamic Pressure Cases, at T=0 and at T=300 (sec)**

# 8.0 Detailed Simulations

There are two simulation models "*Sim1.mdl*" and "*Sim2.mdl*" for each flight condition in the corresponding subdirectories (T0, T20, T56, etc.). They include TWD, flex, and propellant sloshing dynamics. For example, in the lift-off case which is in folder T0, the model "Sim1.mdl", shown in Figure 8.1 consists of vehicle, actuators, TVC, and flight control subsystems generated from Flixan which are combined together using Simulink. The model "Sim2.mdl" shown in Figure 8.2 includes the Flixan generated system: "*Closed-Loop Simulation System at Lift-Off*" that is loaded from m-file "*sim_cl_t0.m*". The script file "init.m" loads the state-space systems, gains, and TVC matrix in Matlab and initializes the simulations. In the simulations we can either apply a roll, pitch, and yaw attitude commands or a 30 (ft/sec) wind-gust pulse, which is square and it is smoothed by a low-pass filter to make it look like a realistic gust. The roll attitude is commanded only in periods of low dynamic pressure. The file "pl.m" plots the simulation results.
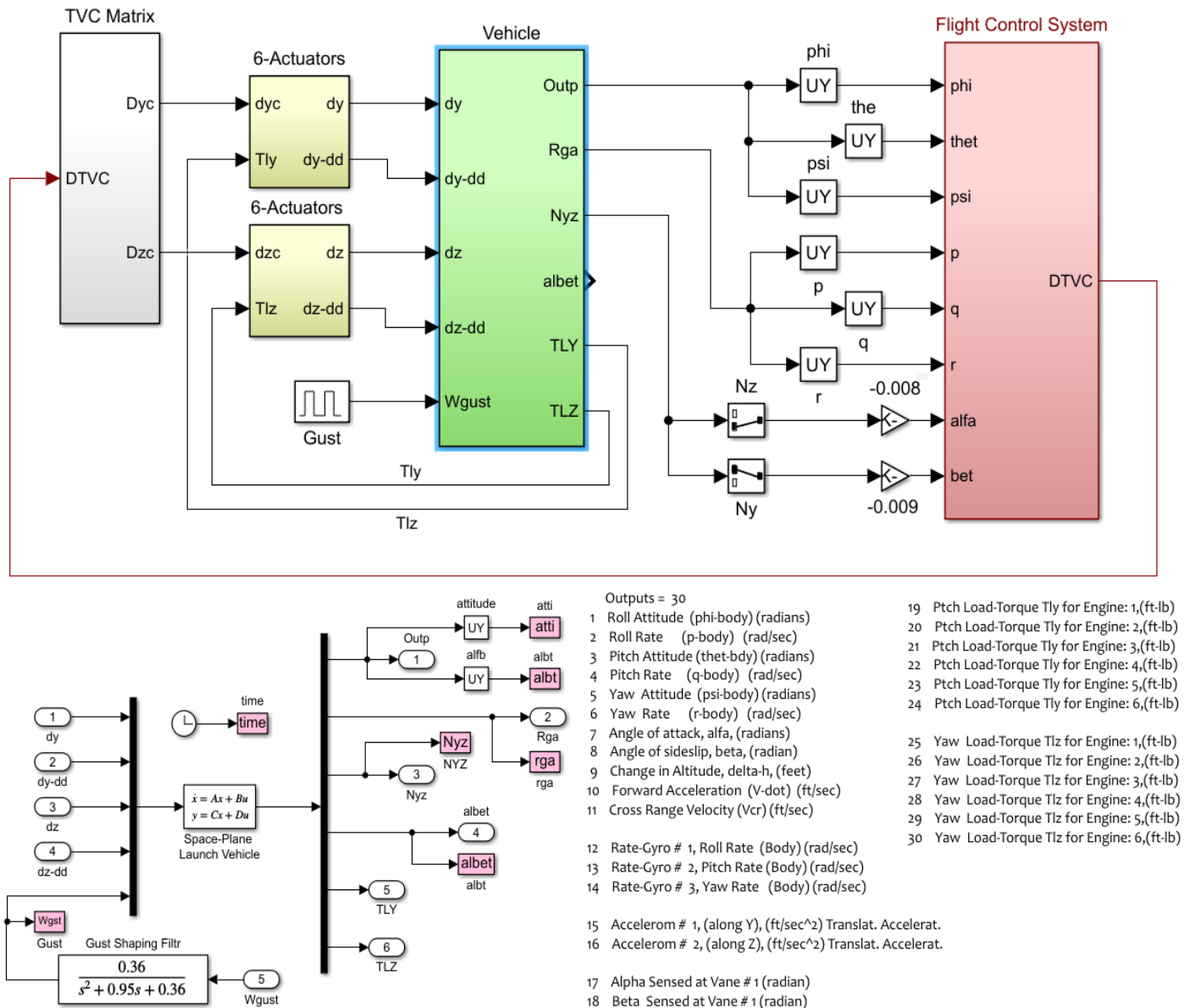


**Figure 8.1 Closed-Loop Simulation Model "Sim1.mdl" with Vehicle Subsystem in Detail**

In the second model the attitude commands are applied at the closed-loop system inputs (1:3). A square wind-gust pulse is applied at input #4 for a duration of 5 (sec). The wind-gust filter is already included in the closed-loop system. The definitions of the system inputs and outputs are shown in the Simulink model. The results from the two simulation models are identical.
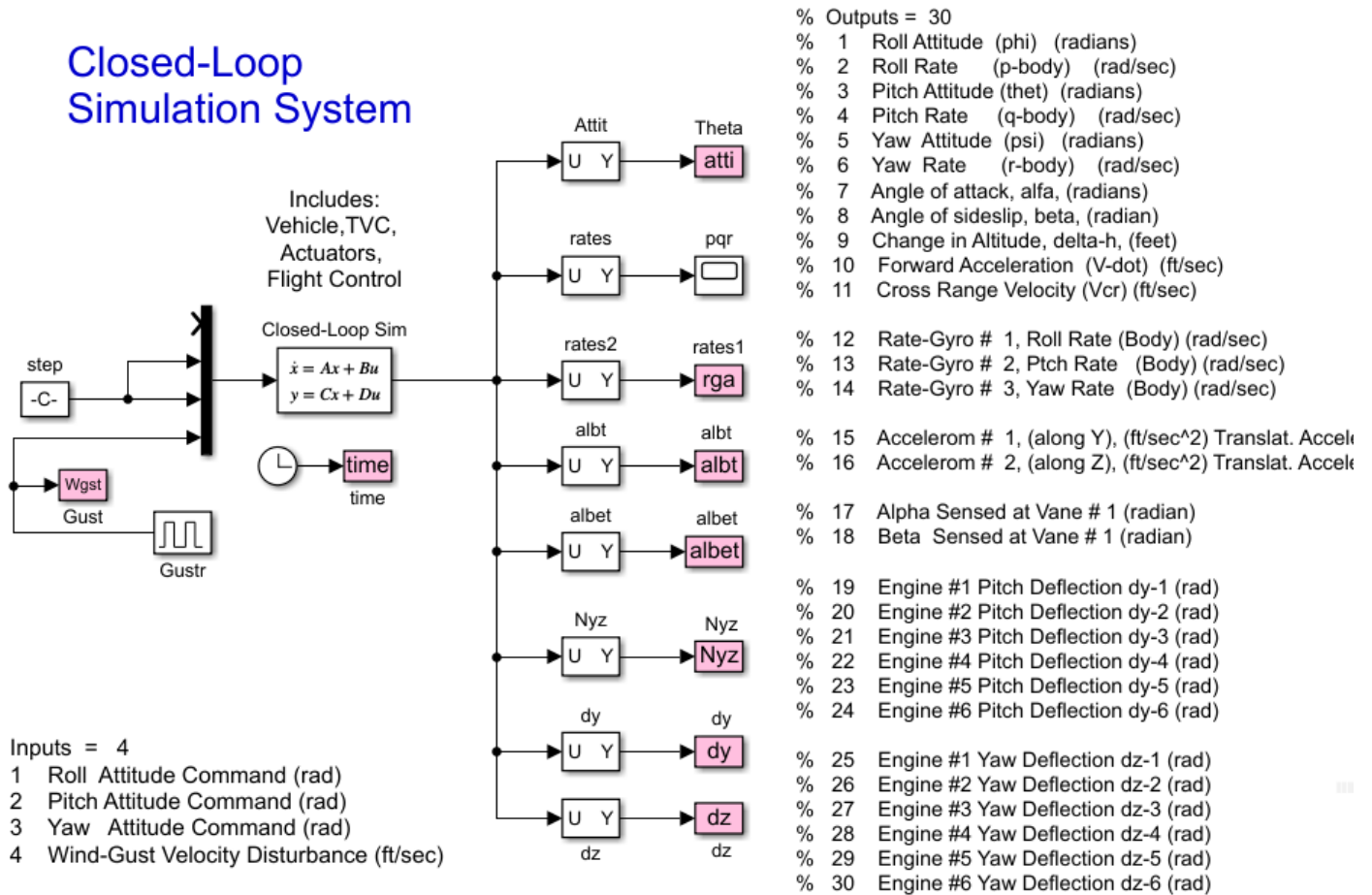
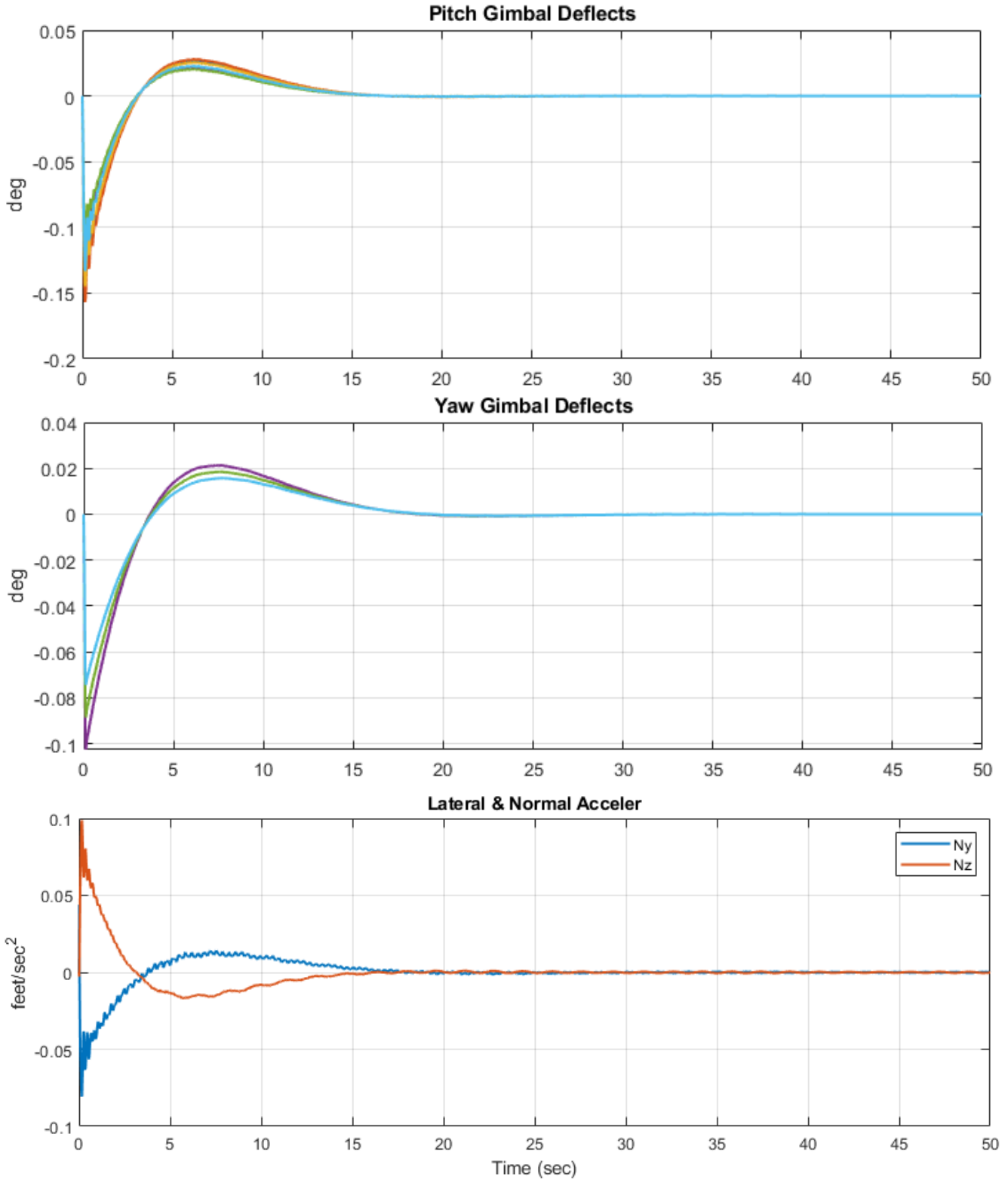

**Figure 8.2 Closed-Loop Simulation Model "Sim2.mdl"**

We will now demonstrate simulation results from three flight conditions:

1. A lift-off case from folder T0 where the vehicle is commanded to perform an increment of 1° in roll, pitch and yaw attitudes.
2. A response to a wind-gust at Max-Q case, without commands, which is at T=82 sec.
3. A simultaneous response to command and gust case at T=108 sec, where the vehicle is commanded 1° in pitch and yaw attitude with gust

The results verify our design specs. That is, acceptable attitude response to guidance commands during periods of medium to low dynamic pressures and an emphasis on load-relief action during periods of high Q-bar which causes sluggishness in vehicle responses to attitude commands.

**8.1 Lift-Off Case:** The vehicle is commanded to perform 1 (deg) attitude inclement in roll, pitch and yaw simultaneously. The response time is approximately 10 sec. It was obtained using the simulation models in file T0.



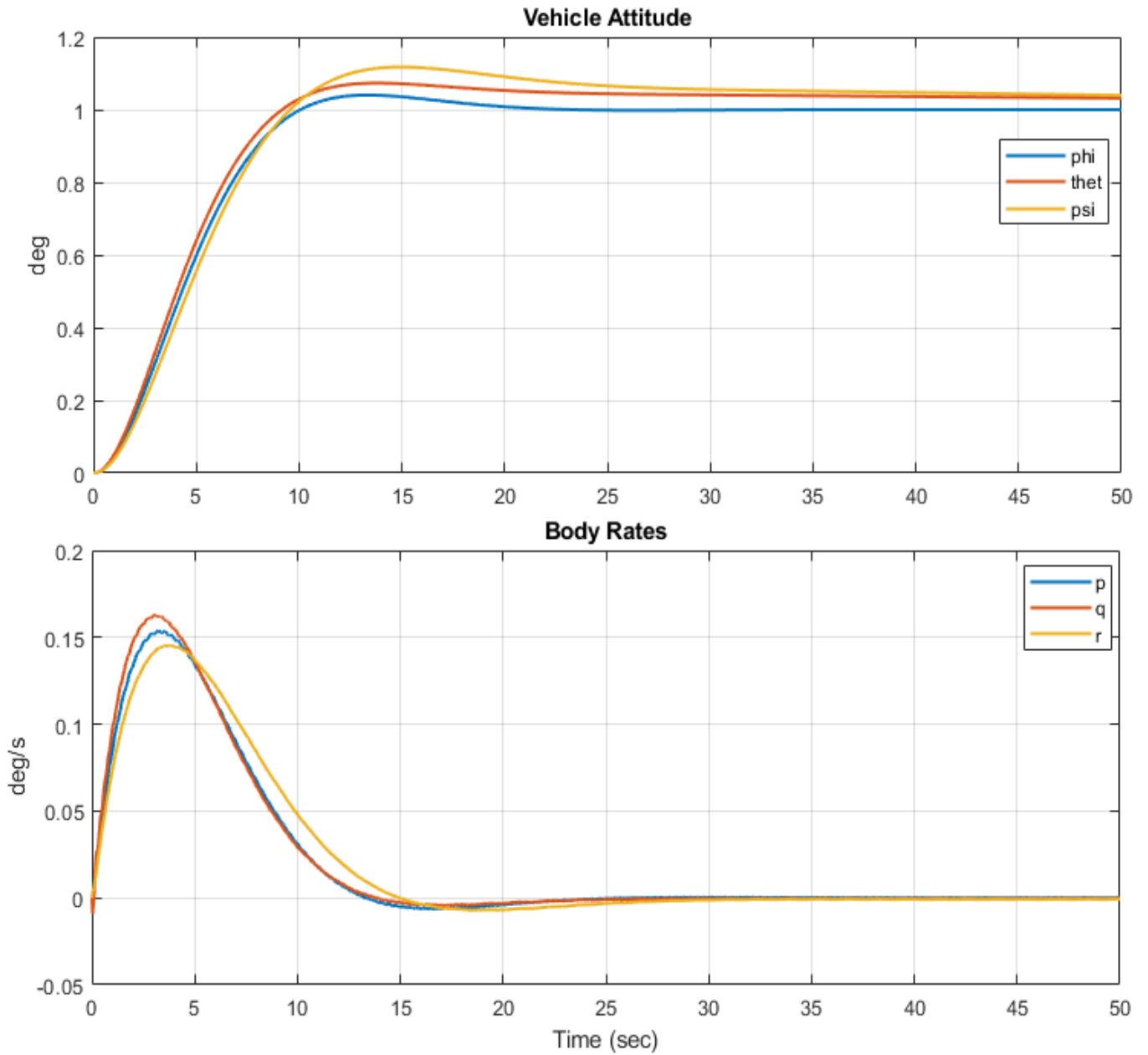Unit Step Attitude Response at Lift-Off (Roll, Pitch, Yaw)

**Figure 8.3 Vehicle Responds to Attitude Commands at Lift-Off**

**8.2 Max-Q Case:** During Max-Q, which occurs at T=82 sec, the vehicle does not receive any attitude increment commands from guidance. In this example it is excited by a 30 (ft/sec) wind-gust and the flight control system responds to the disturbance. The gust direction is perpendicular to the x axis, along the –z and –y directions exciting all directions. Notice how the accelerations (Nz and Ny) are almost mirror images of ($\alpha$ and $\beta$). This analogy applies only at high dynamic pressures but this is also where we need it for load-relief. The accelerometers are more sensitive to sloshing. The pitch and yaw dispersions due to the gust are small but the roll transient is 5° because the sideslip excites roll through the big vertical tail in the back and low roll inertia. The gimbal deflections are small but you can see the slosh effect on the pitch gimbal deflections.
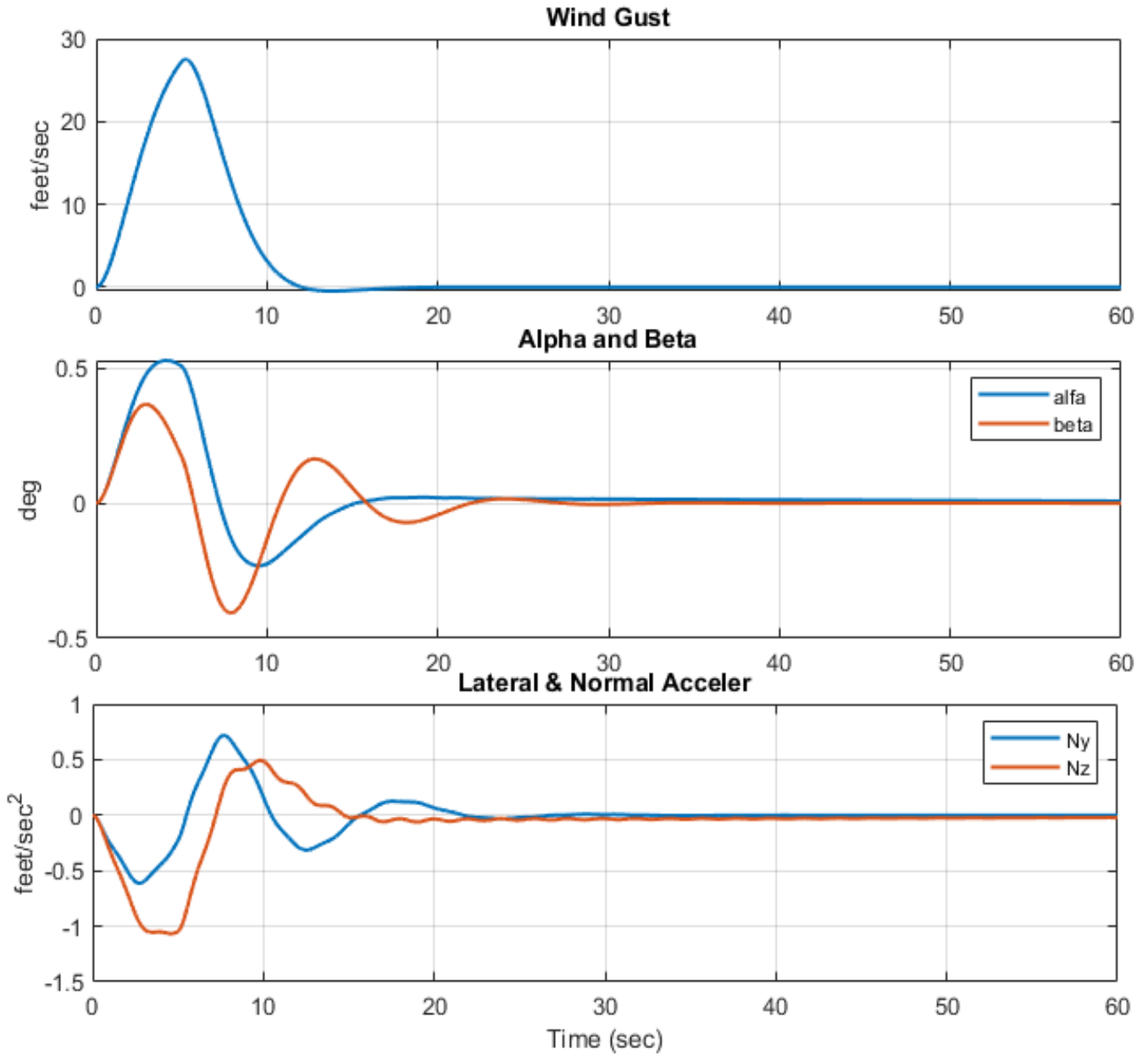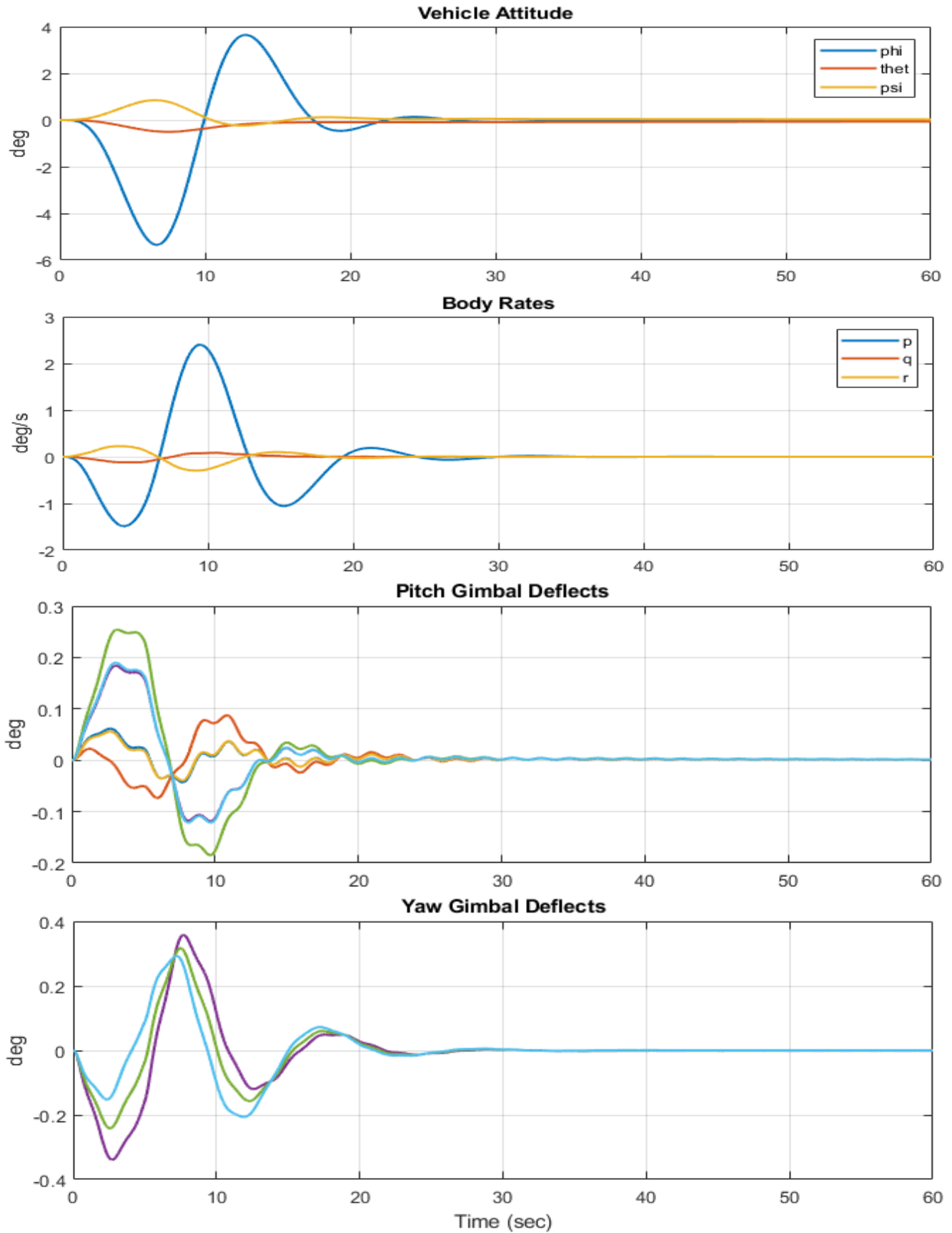


**Figure 8.4 System's Response to Wind-Gust at Max-Q**

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# Wind-Gust Response at Max-Q

**8.3 T108 Case:** At T=108 sec the dynamic pressure is somewhat reduced, although still high, and the vehicle response to attitude commands is improved. The command performance is significantly better at low dynamic pressures such as near lift-off or at T300. In this flight condition which was obtained from folder T108, we are receiving attitude command increments in pitch and yaw assuming to be tracking a trajectory. The vehicle is also excited by a gust a few seconds after the attitude commands. The pitch and yaw attitudes respond to the 1° command, although the sideslip causes a 2° transient in roll that decays. The gust is applied at 60 sec and it causes 1° transient in roll which also decays. The TVC gimbals respond to both, command and gust excitations. Propellant sloshing affects the accelerometer measurements and gimbal responses.
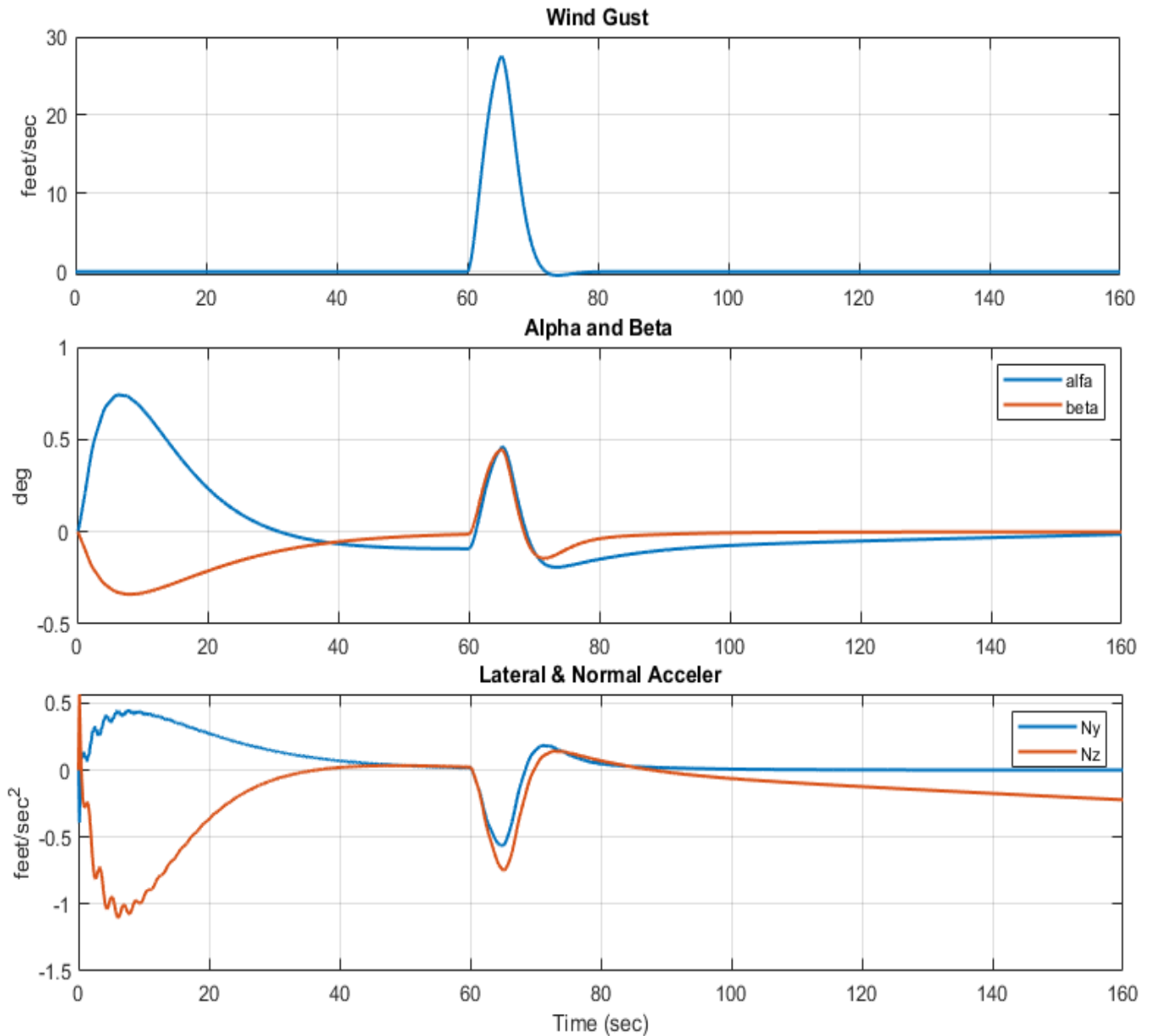


**Figure 8.5 System's Response to Commands and Wind-Gust at T=108 sec**

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# Wind-Gust and Step Attitude Response at T=108 sec