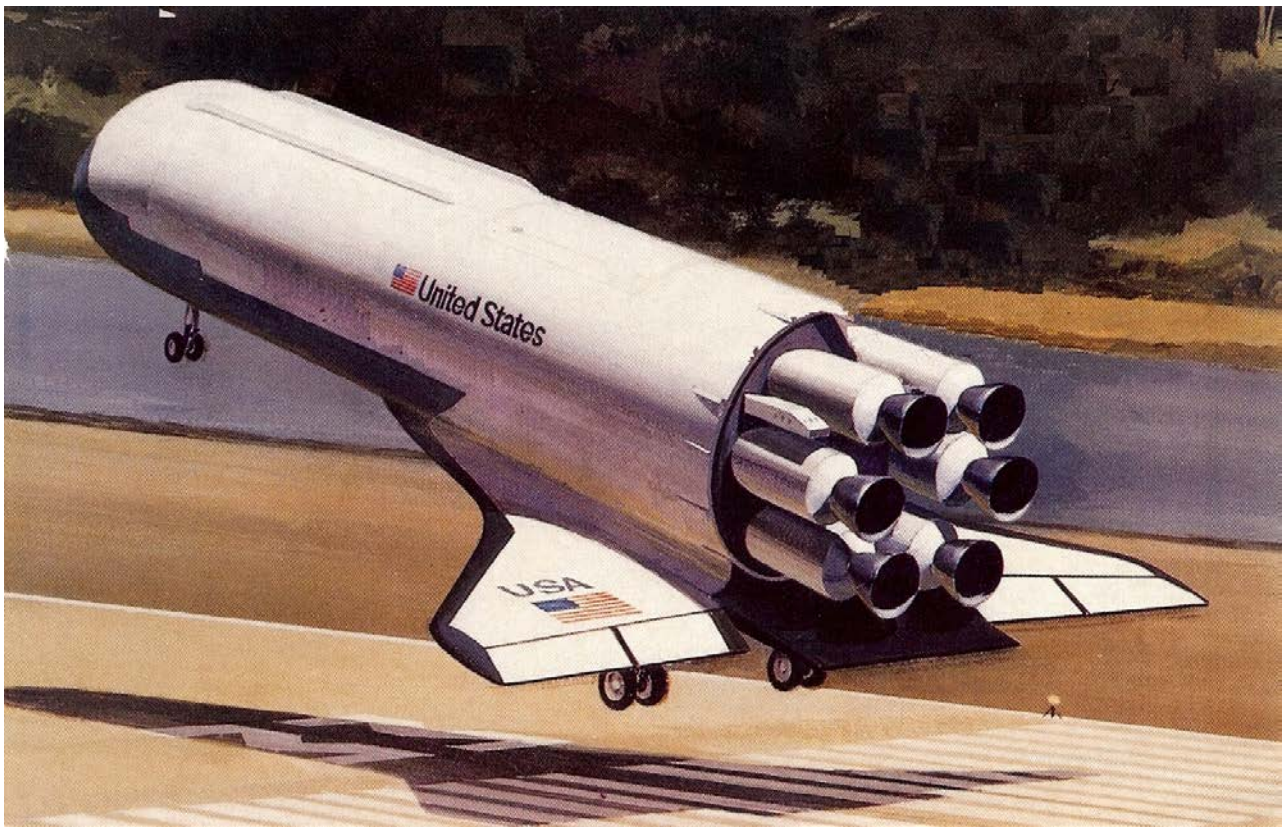


# Reusable Launch Vehicle



In this example we will analyze a launch vehicle with wings similar to the one shown in this picture. It takes off vertically like a launch vehicle, reaches up to low earth orbits and returns to land like a glider without engine power. During ascent it uses 6 engines that provide a total 350,000 (lb) of thrust. The engines are not the same but of different types and thrusts, deliberately selected for the purpose of demonstrating the Trim program capability to combine different engines. The engines configuration is shown in Figure 1. The first two engines are gimbaling  $\pm 14^\circ$  in pitch and  $\pm 12^\circ$  yaw and they have constant thrusts of 70,000 (lb) each. Engines #3 and #4 are also gimbaling but only  $\pm 5^\circ$  in pitch and yaw and they can also vary their thrusts between: 9,200 to 100,000 (lb) each. Engines #5 and #6 can also vary their thrusts from 9,200 to 100,000 (lb) each, but they are fixed and do not gimbal. The launch vehicle also uses two elevons which are located at the wings. During ascent the elevons are used to control roll because the gimbaling engines are too close together and the moment arm is too short for roll control. Notice, that this vehicle does not have a vertical tail and rudder. During descent the engines are not active but the vehicle uses the two elevons for pitch and roll control and two lateral thruster jets for yaw control which are located near the front. Two different trajectories were created for analyzing the vehicle performance during ascent and descent. This launch vehicle is a great example for demonstrating the effectors combination methodology that was described in Section 5. The mixing logic matrix optimally combines multiple types of effectors together and optimizes control authority because it takes into consideration the control capability of each effector in the control directions.

In the following sections we will use the Trim program to analyze the vehicle performance during ascent and descent phases in separate sections and different folders. We will evaluate its performance and stability parameters, its capability to trim against aero disturbances, analyze effects due to lack of lateral symmetry caused by the  $Y_{CG}$  offset, engine and actuator failures, We will create linear models, design flight controls, and analyze its dynamic performance.

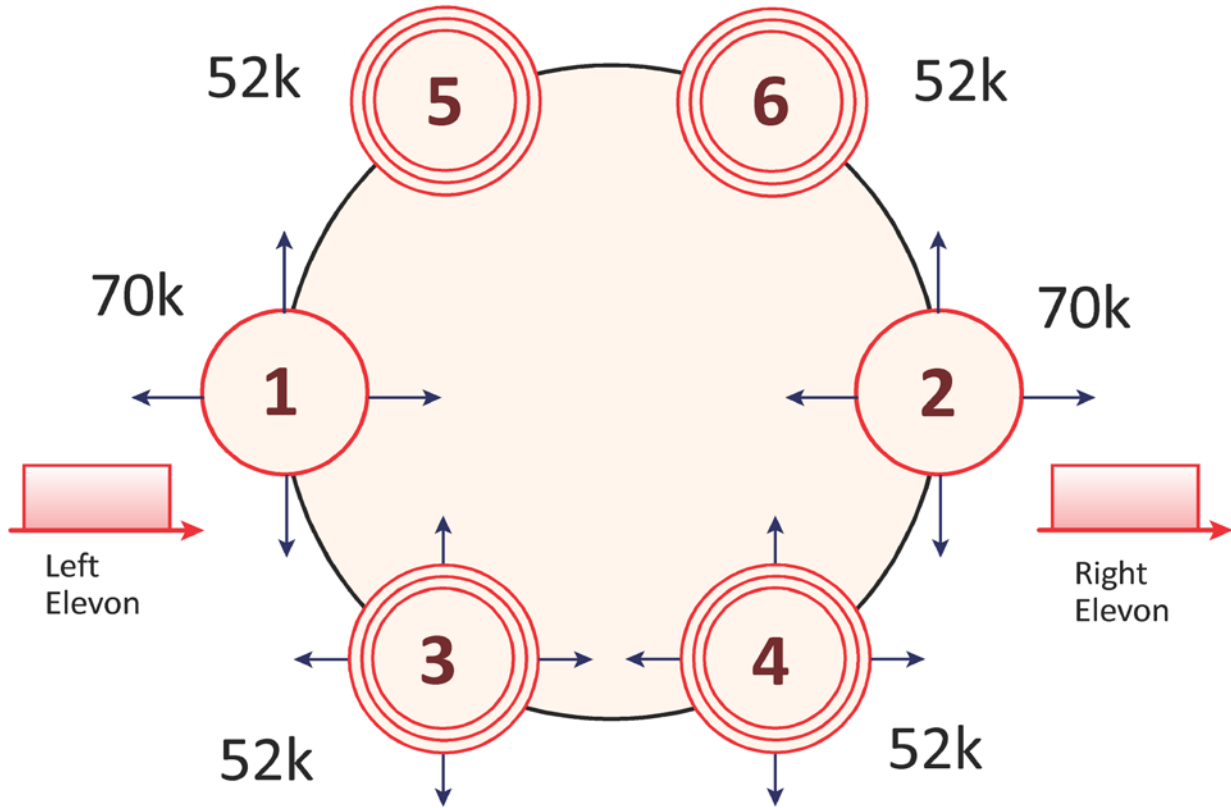


Figure 1 The Vehicle Uses Six Main Engines and Two Elevons during Ascent

### 1.0 Ascent Analysis Files

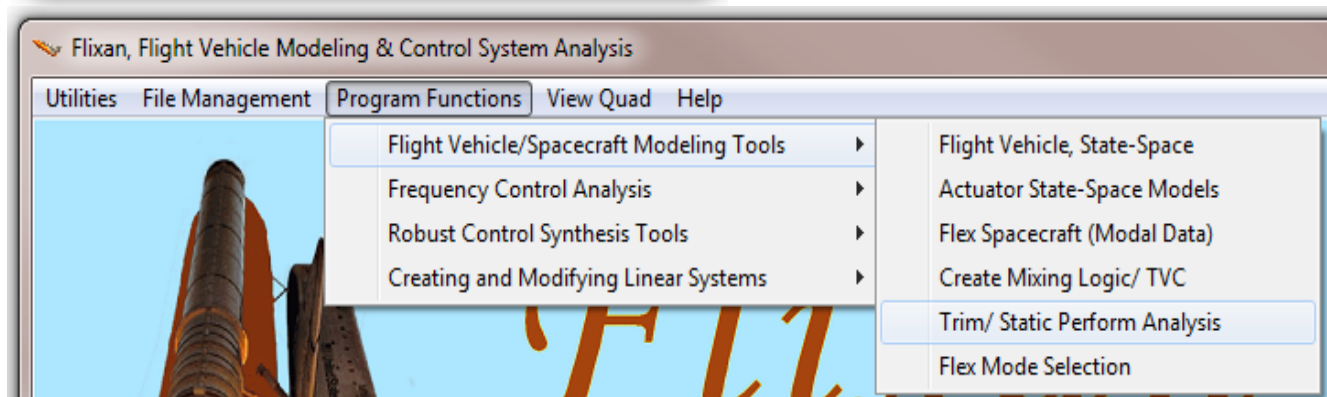
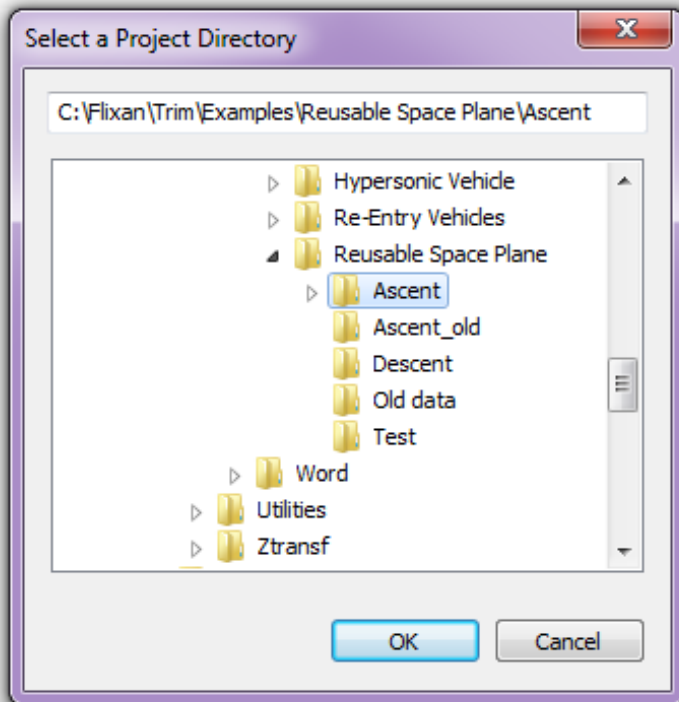
The input data files for the ascent analysis are already prepared for the Trim program. They are located in folder "*C:\Flixan\Trim\Examples\Reusable Space Plane\Ascent*", and it contains the following files:

- **Trajectory Files:** There are two trajectory files containing only the boost phase of the trajectories during the period when the engines are firing. In both cases the vehicle takes off vertically and reaches an altitude of approximately 260,000 (feet), but the maneuvers are very different. The first trajectory is in file "*ReSP\_Ascent\_A.Traj*" and performs a 180° pitch maneuver at high altitude, and the second trajectory is in file "*ReSP\_Ascent-B.Traj*".
- **Mass Properties:** The vehicle mass properties are located in file "*ReSP.Mass*". It contains the inertias and CG location as a function of vehicle mass (slugs) in descending magnitude.

- **TVC/Engine Data:** The data for the 6 main engines, such as: thrusts, locations, trim angles, max deflections, throttle parameters, etc. are located in file "*ReSP6.Engn*". All 6 engines are located in the back of the vehicle with a  $-2^\circ$  tilt in pitch ( $D_Y=-2^\circ$ ,  $D_Z=0^\circ$ ). The first two engines have a constant thrust of 70,000 (lb) and they do not throttle but are gimbaling  $14^\circ$  in pitch and  $12^\circ$  yaw. The next two engines have a nominal thrust of 52,000 (lb) and they can vary their thrust  $\pm 95\%$  from nominal. They can also gimbal  $5^\circ$  in pitch and yaw. Engines #5 and #6 are not gimbaling but only throttling. They have a nominal thrust of 52,000 (lb) each (at zero throttle command) and they can vary their thrust  $\pm 95\%$  from nominal.
- **Basic Aero Data:** The basic aerodynamic coefficients file in this example is poor. We only have force and moment coefficients at two Mach numbers, 3 betas and 12 alpha positions. They are located in file "*ReSP.Aero*".
- **Aero-Surface Data:** This vehicle has two Elevons aerosurfaces, and the aerodynamic increment coefficients for the left and right Elevons are in file "*ReSP.Delt*". Each aerosurface includes increment coefficients at 4 positions. The aerosurfaces Mach #, alpha, and beta values correspond to the same values as in the basic aero data.
- **Damping Derivatives:** The damping derivatives in this example are in file "*ReSP.Damp*". They correspond to the same Mach numbers and alphas as the basic aero data.
- **Hinge Moment Coefficients:** In this example we also have a hinge moment coefficients file "*ReSP.HMco*". It corresponds to the same Mach numbers, alphas, betas, and aerosurface increments as in the aerosurface data.
- **Propellant Sloshing Data:** This launch vehicle contains two propellant tanks: liquid oxygen (LOX) and liquid hydrogen (LH2) tanks. Although the Trim program itself does not analyze sloshing, the slosh data, however, are transferred to the flight vehicle modeling program and used for creating slosh dynamic models. The slosh data file "*ReSP.Slsh*" contains data for the two propellant tanks as a function of the vehicle mass in (slugs). The data consist of: slosh masses, slosh frequencies (at 1g acceleration), damping coefficients, and the location of the slosh masses in vehicle coordinates. The slosh frequencies are proportional to the square root of the acceleration. They are converted by the vehicle modeling program from 1g to the frequencies that correspond to the vehicle acceleration.

## 1.1 Trajectory (a) Analysis

It is a good practice to familiarize ourselves with the trajectory by taking a good look at it before analyzing the vehicle performance. We start the Flixan program and select the project folder that contains the analysis files. Then from the Flixan main menu select "Program Functions", "Flight Vehicle/ Spacecraft Modeling Tools", and then "Trim/ Static Performance Analysis", as shown below.



From the filename selection dialog below, select the input files, already described, that will be used by the Trim program in the ascent analysis. Select the first trajectory file "ReSP\_Ascent\_A.Traj", which is saved in this folder, and click OK. From the Trim main menu select option-2 for plotting the trajectory parameters.



Select One Data File from Each Menu Category

The following analysis requires some data files to be selected from the current project directory. Select one data file for each category, (some of the categories are optional).

<b>Mass Properties</b> ReSP.Mass	<b>Surface Hinge Moments</b> ReSP.HMco
<b>Trajectory Data</b> ReSP_Ascent_A.Traj	<b>Aero Damping Derivat</b> ReSP.Damp
<b>Basic Aero Data</b> ReSP.Aero	<b>Propulsion Data</b> ReSP6.Engn
<b>Contr Surface Aero Coeff</b> ReSP1.Delt	<b>Aero Uncertainties</b> NO DATA FILE
<b>Slosh Parameters</b> ReSP.Slsh	

OK

Enter Filenames

Enter a File Name containing the Input Data (xxx.Inp)	Enter a File Name containing the Quadruple Data (xxx.Qdr)
NewFile.Inp	NewFile.Qdr
NewFile.inp	NewFile.qdr
NewFile2.inp	NewFile.Qdr
NewFile.Inp	

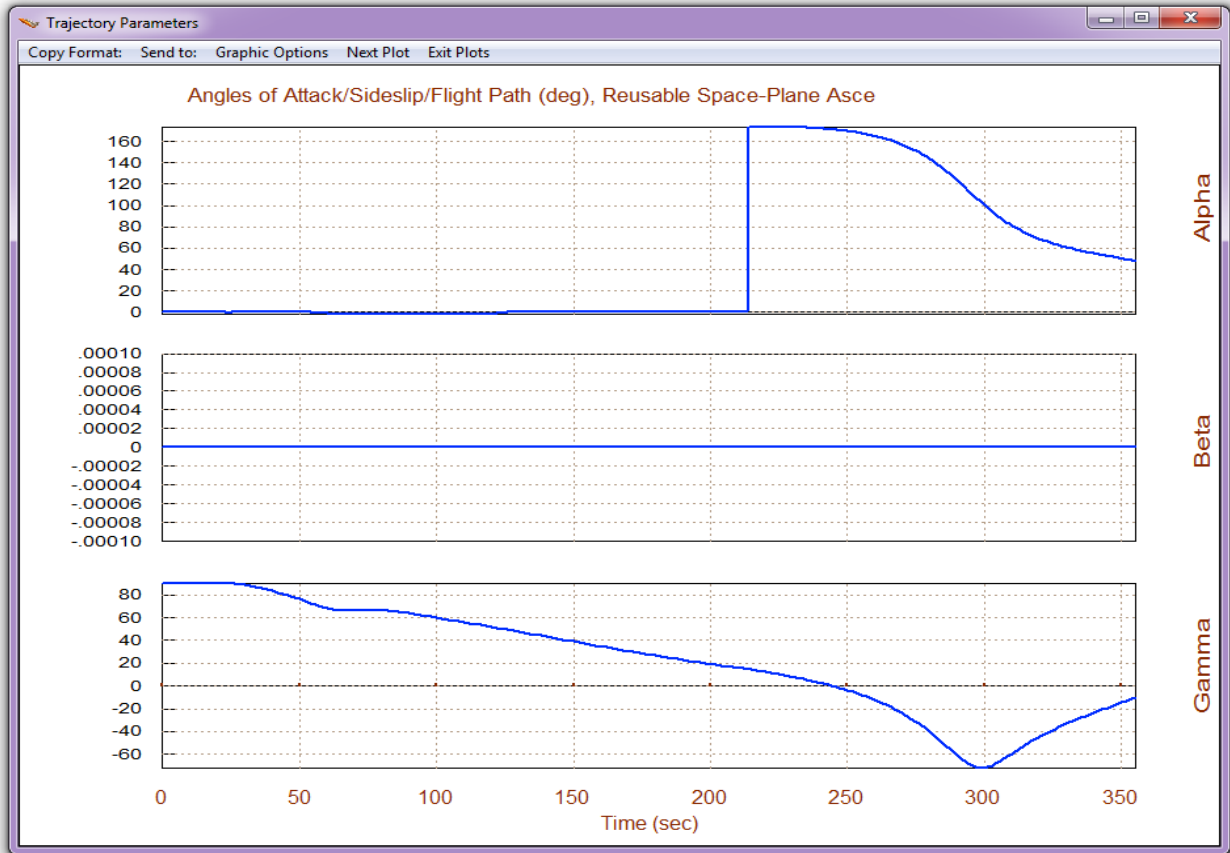
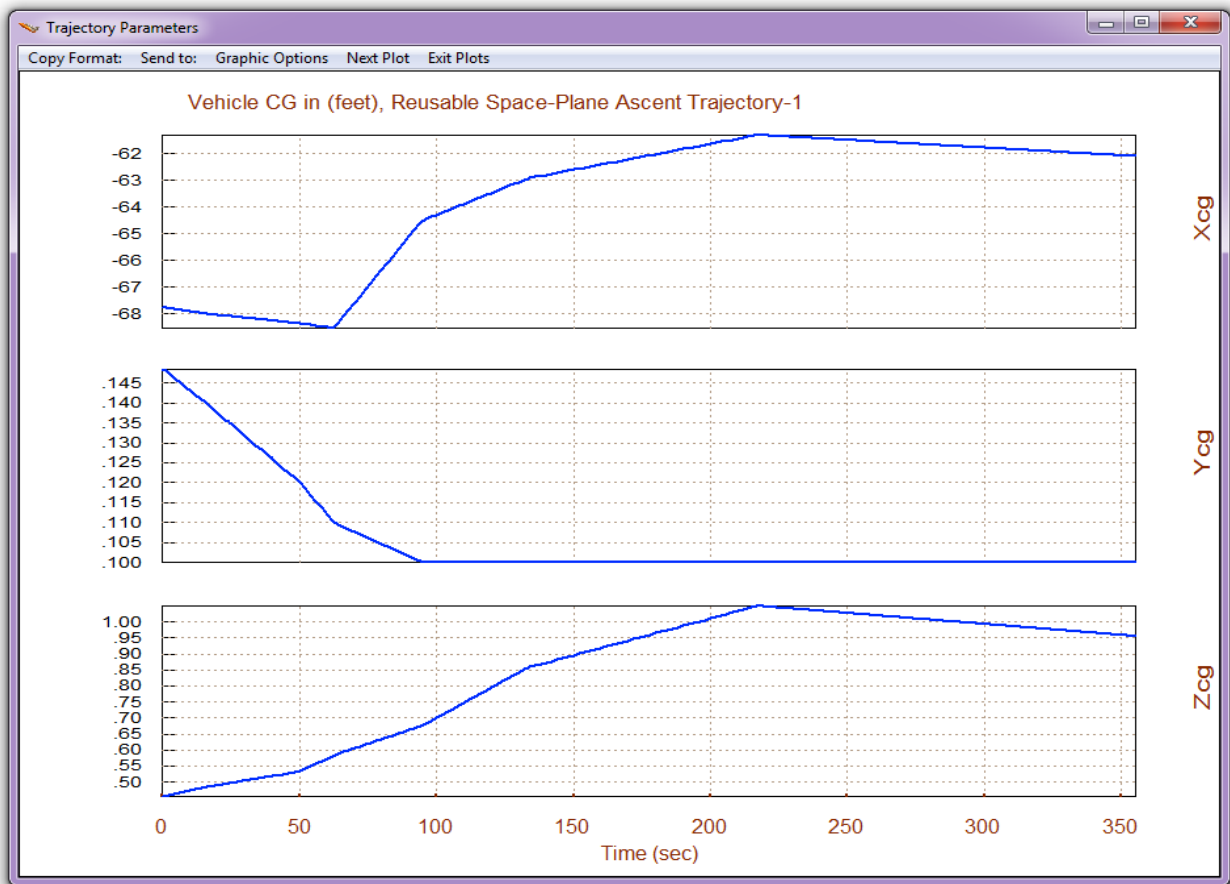
Create New Input Set    Exit Program    Select Files

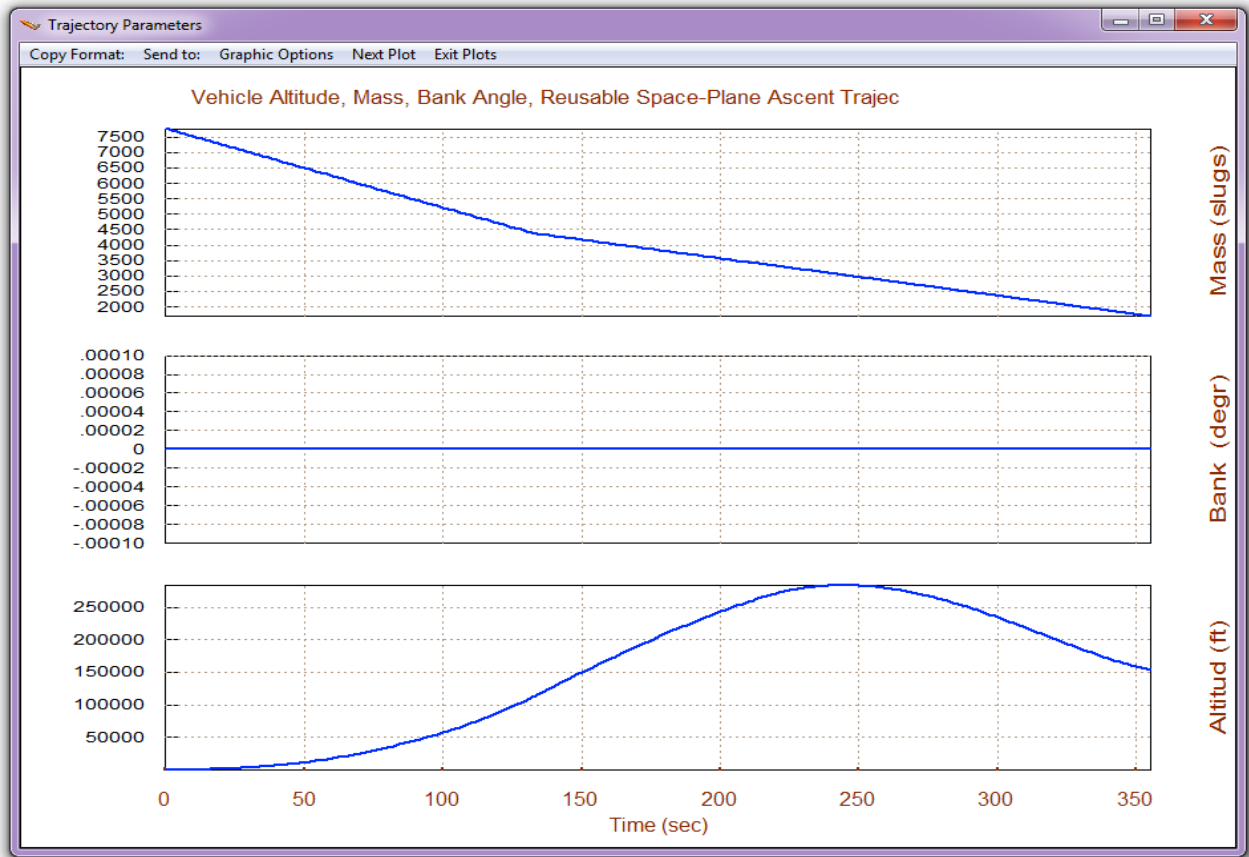
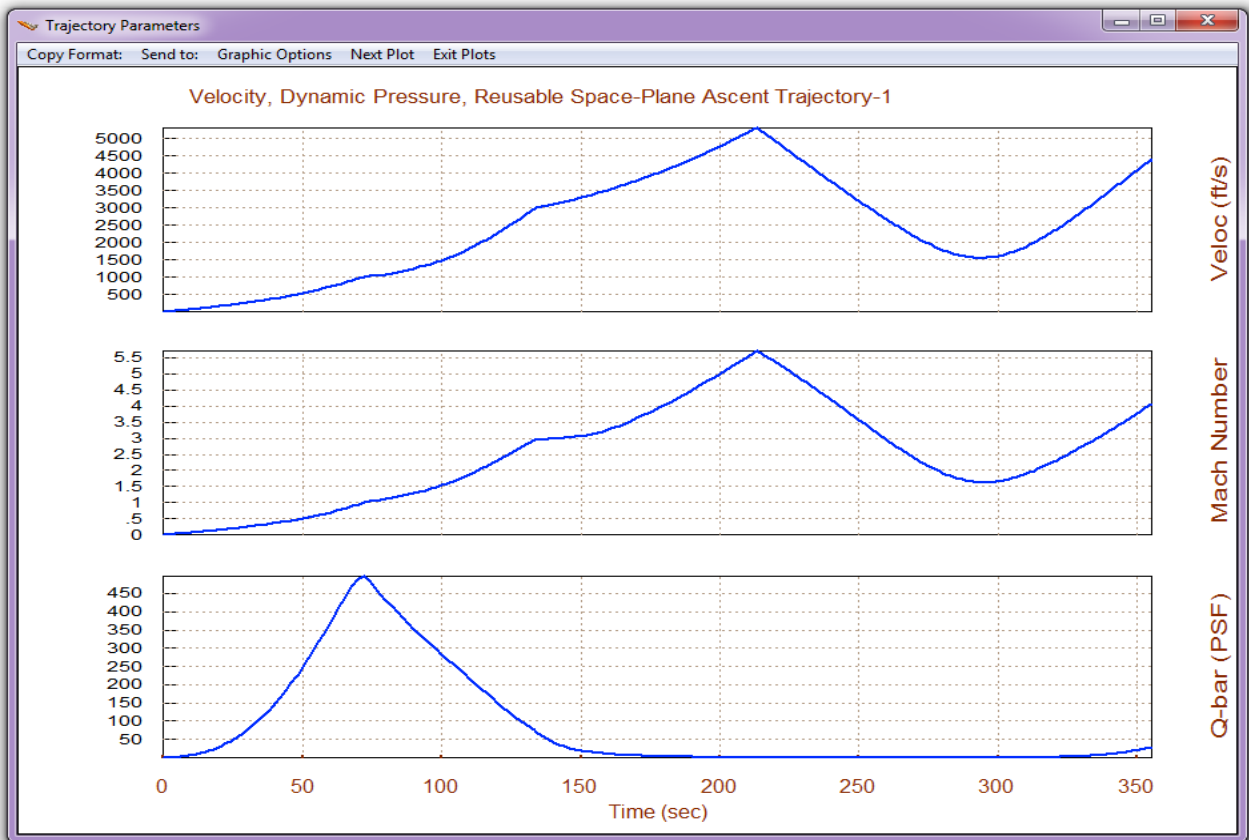
Main Trim Menu

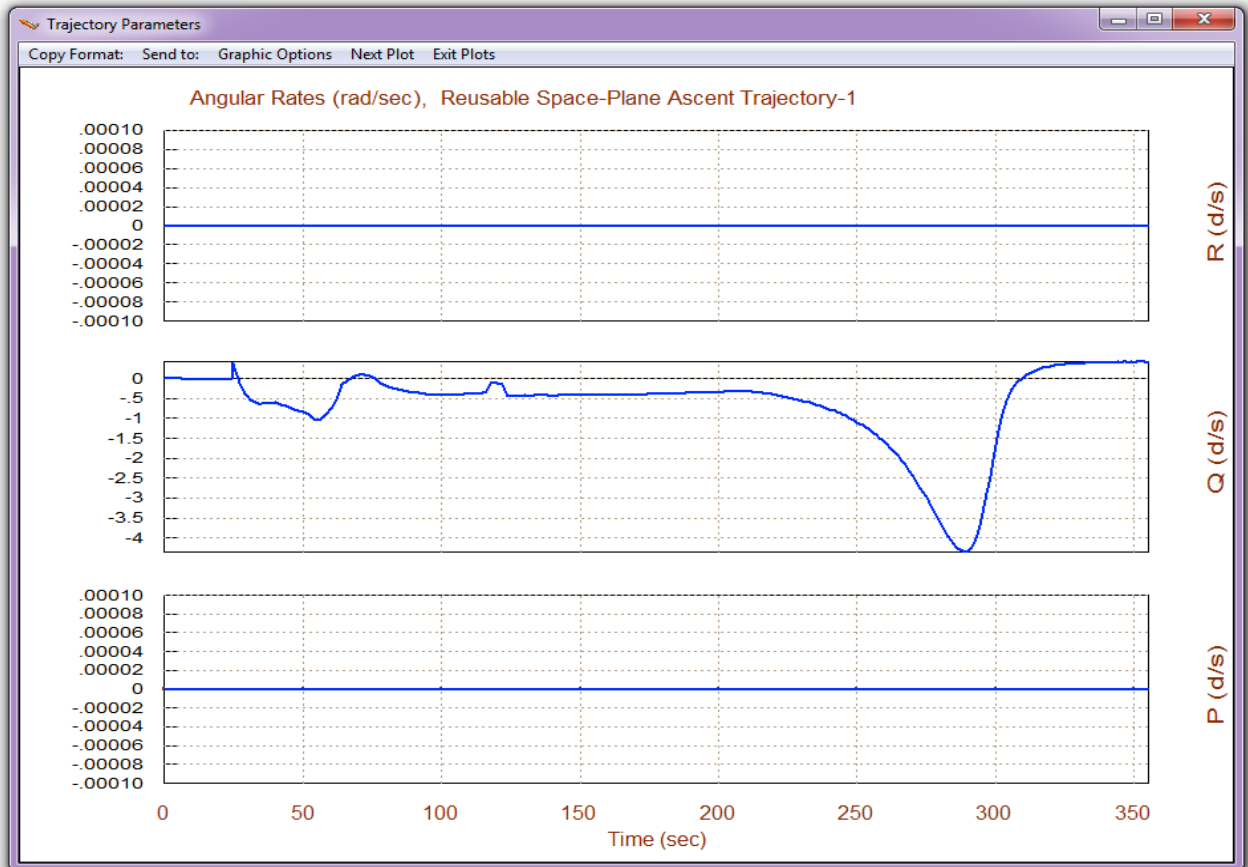
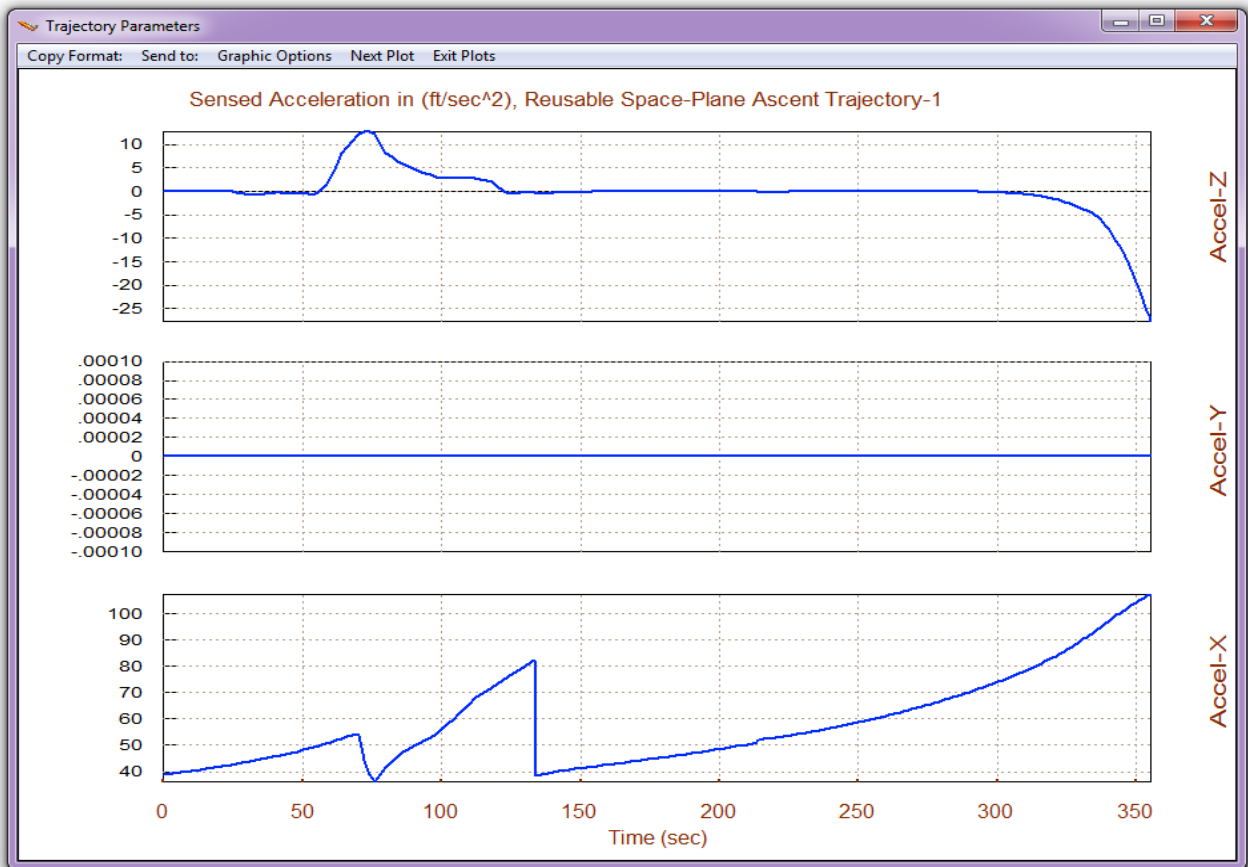
Select one of the following options

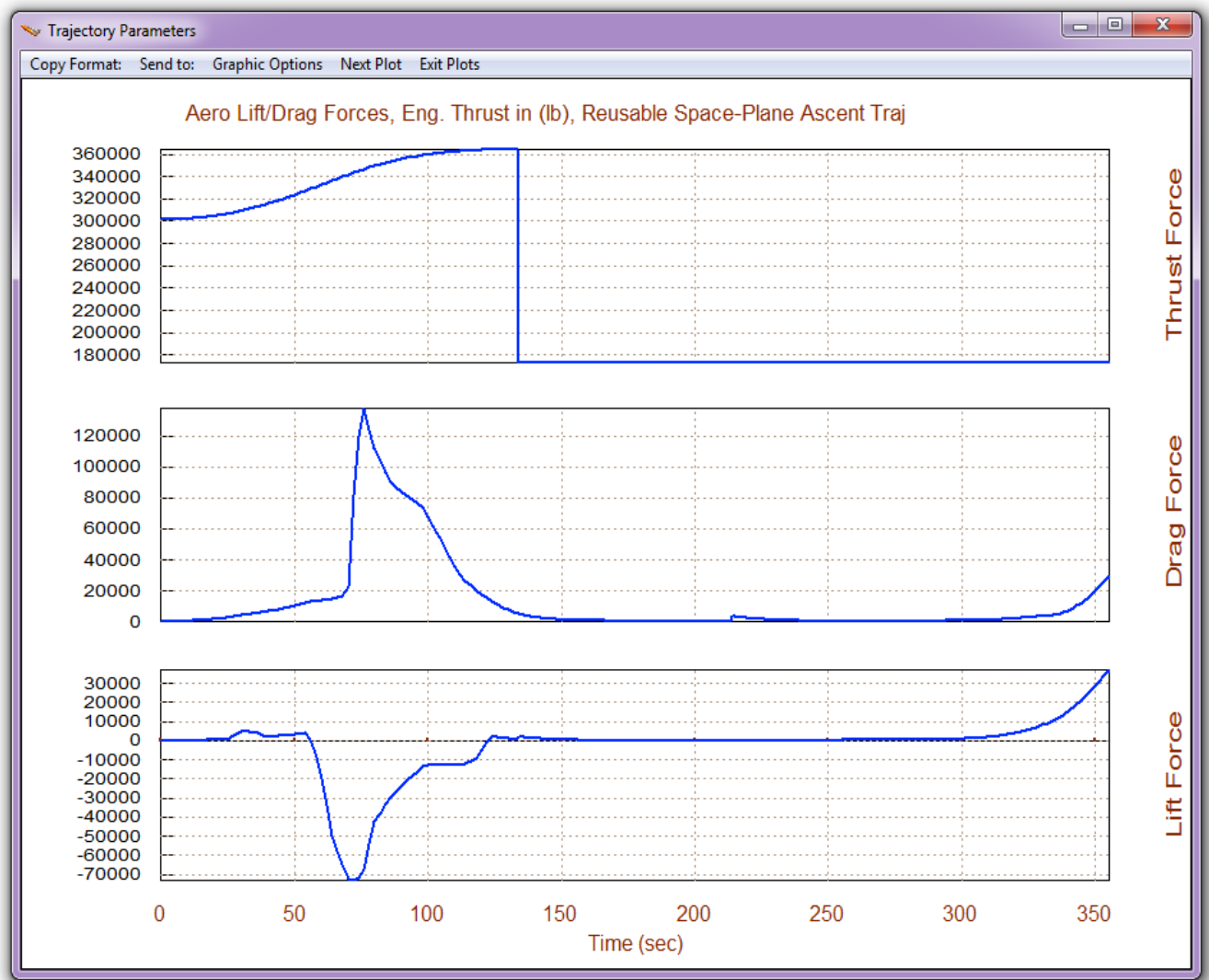
Exit    OK

1. Plot Aero Coefficients, Derivatives, and Control Surface Increments
2. Plot Trajectory Parameters Versus Time from the Trajectory File ".Traj"
3. Trim the Effector Deflections to Balance the Vehicle Moments and Forces
4. Create an Effector Mixing Logic or a TVC Matrix (Kmix)
5. State-Space Modeling of the Flight Vehicle at Selected Times
6. Performance and Stability Parameter Plots Along Trajectory Time
7. Landing and Pull-Up Maneuverability, plus, Inertial Coupling Effects
8. Moments at the Hinges of Control Surfaces Along the Trajectory Time
9. View and Modify Vehicle Data (CG, MRC, TVC, Surfaces) for Dispersion Analysis
10. Contour Plots (Mach versus Alpha) for Performance, Control Authority Analysis
11. Vector Diagrams for Maneuverability & Stability at Selected Flight Conditions
12. Plot and Compare Previous Data Files (Traject, Trim, Perform, Hinge Moment)









The above trajectory parameters show that the CG location is varying with time because the vehicle weight and distribution changes as the fuel is depleted. There is also 0.14 (feet) offset in the  $Y_{CG}$  which upsets the lateral symmetry. The flight-path  $\gamma$  starts at  $90^\circ$  because the vehicle takes-off vertically from the launch pad, and it gradually reduces to negative values. The max dynamic pressure reaches about 500 (psf), and the altitude reaches up to 260,000 (feet). The thrust increases to 360,000 (lb) and it is reduced to 180,000 (lb) at  $t=130$  (sec). When the dynamic pressure is low the vehicle rotates  $180^\circ$ , pointing its x-axis opposite to the velocity vector with  $\alpha=180^\circ$ . Eventually the angle of attack is gradually reduced as the vehicle continues to rotate, and to begin its descent trajectory.

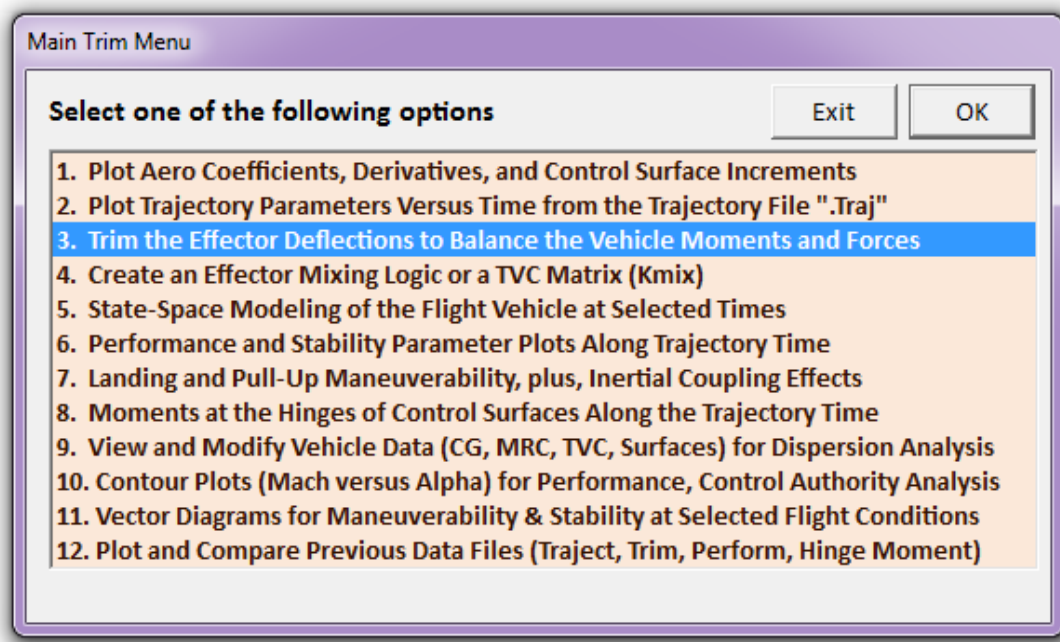
## 1.2 Trim Analysis along the Trajectory (A)

During ascent the space-plane uses the two flaps and 6 engines. The flaps are used mainly for roll control because the TVC does not have sufficient control authority in roll. The 6 engines are in file "*ReSP6.Engn*" and they are different, as already described. The Trim program calculates the throttle commands, the TVC deflections, and the aero-surface deflections required to balance the

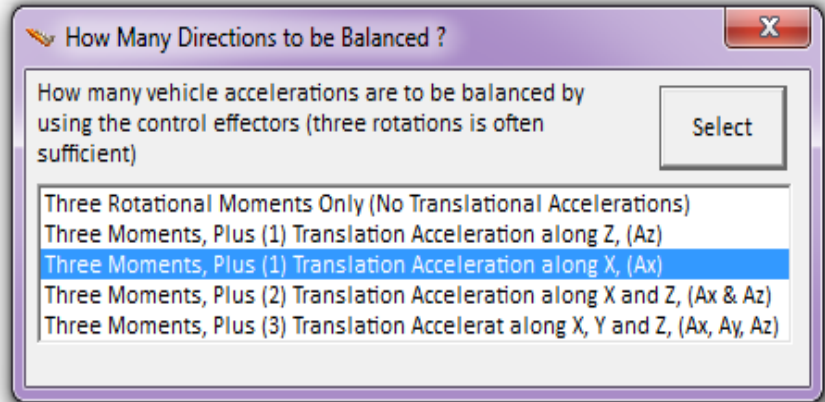
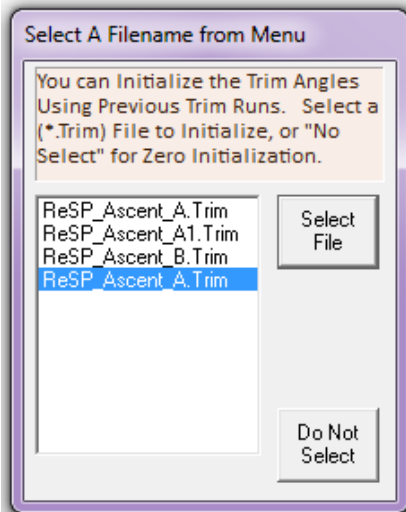


moments and the axial force along the trajectory. The throttle parameter of the 4 throttling engines is 0.95 which defines the maximum thrust variation from nominal. During trimming the throttle commands define the actual thrust variation from nominal as a function of time that is required to balance the moments and the acceleration. They vary between zero and  $\pm 1$  to adjust the thrust.

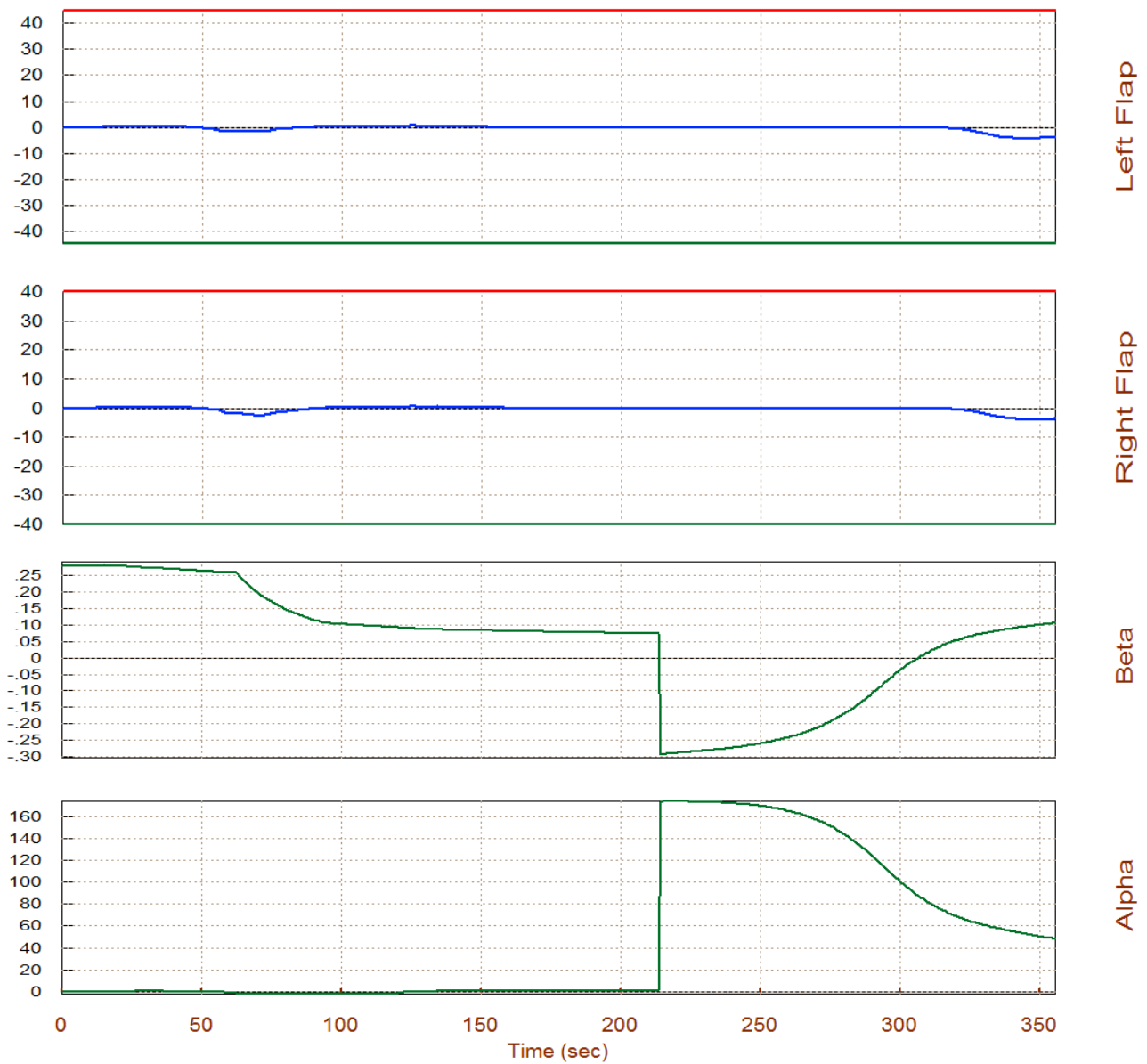
Return to the Trim main menu and choose option-3 to trim the effectors. There are a total of 14 effectors in this configuration. From the next two menu/dialogs, do not select an initialization file, and in the menu that selects the directions to be balanced choose the third option to trim along 3 moments, plus axial acceleration since the vehicle has a variable thrust capability. The program calculates the engine and surface deflections and the thrust variations required to balance the 3 moments and the axial acceleration along the trajectory. The deflections are based on the individual effector control capability.



After trimming, beta is no longer zero as in the original trajectory. The  $+Y_{CG}$  offset causes the vehicle to yaw slightly creating  $0.25^\circ$  beta sideslip. The four TVC engines are, therefore, balancing the yaw moment due to beta by rotating approx  $0.3^\circ$  in yaw ( $\delta_z$ ). The pitch deflections of the four engines ( $\delta_y$ ) are not the same because they are balancing the rolling moment. The thrusts of the throttling engines vary significantly when trimming in order to match the acceleration defined in the trajectory. There is also a small difference in the deflections of the two flaps which help in balancing the rolling moments generated due to beta and the  $Y_{CG}$  offset.



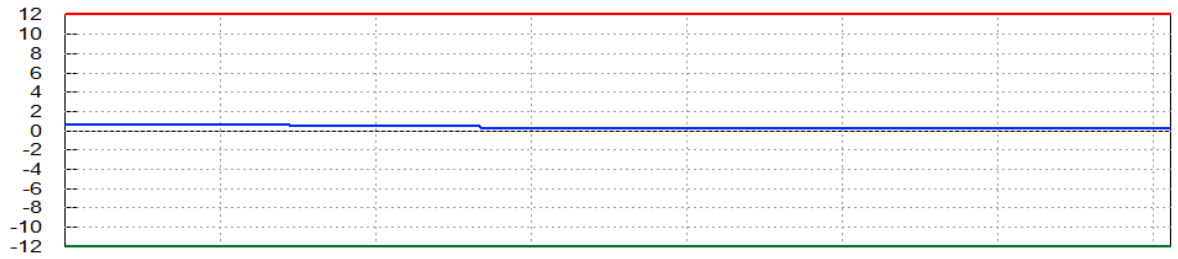
Surface & Engine Deflections/ Thrusts, Reusable Space-Plane Ascent Trajectory-1



Surface & Engine Deflections/ Thrusts, Reusable Space-Plane Ascent Trajectory-1



Dy\_Engine 1



Dz\_Engine 1



Dy\_Engine 2



Dz\_Engine 2



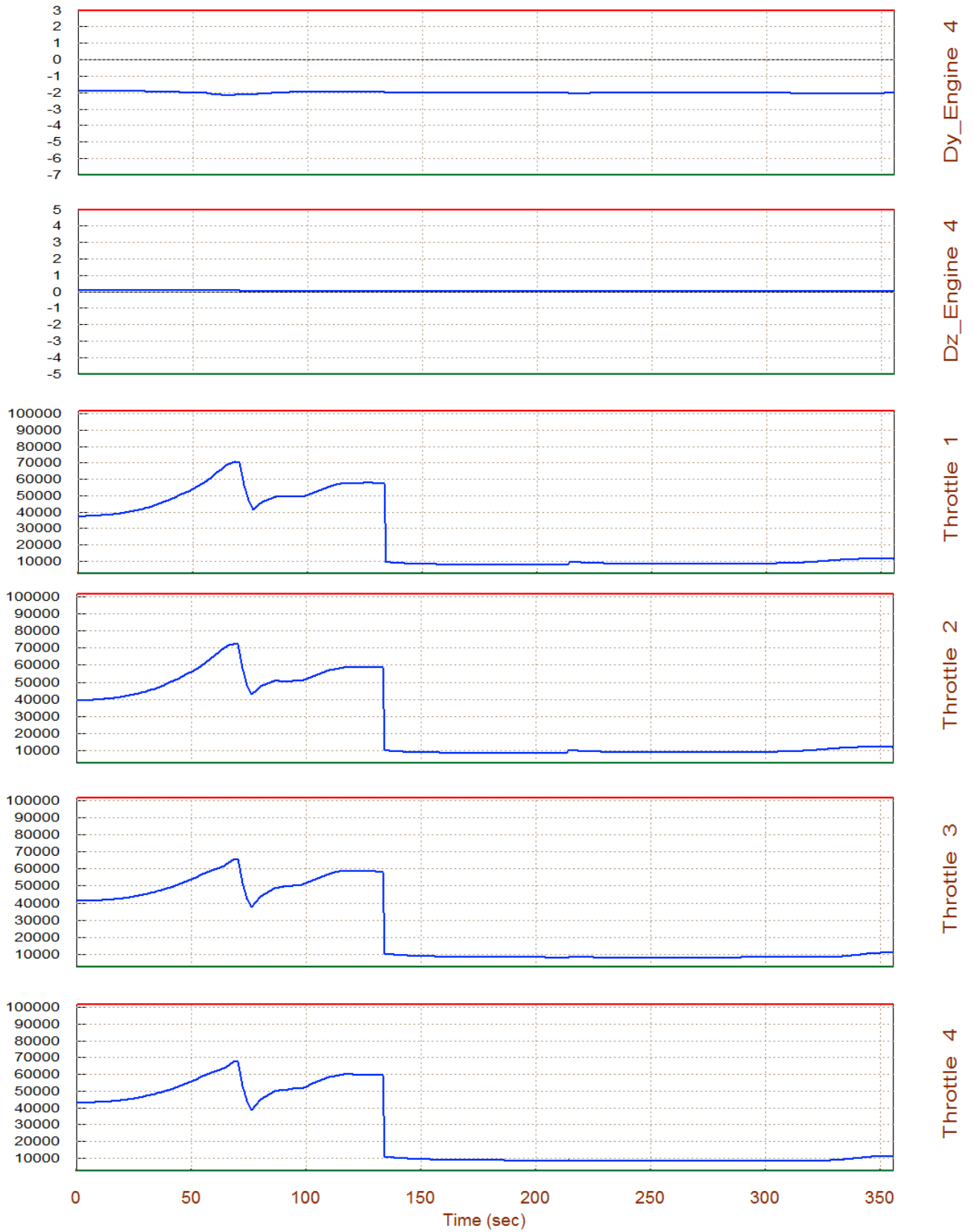
Dy\_Engine 3



Dz\_Engine 3

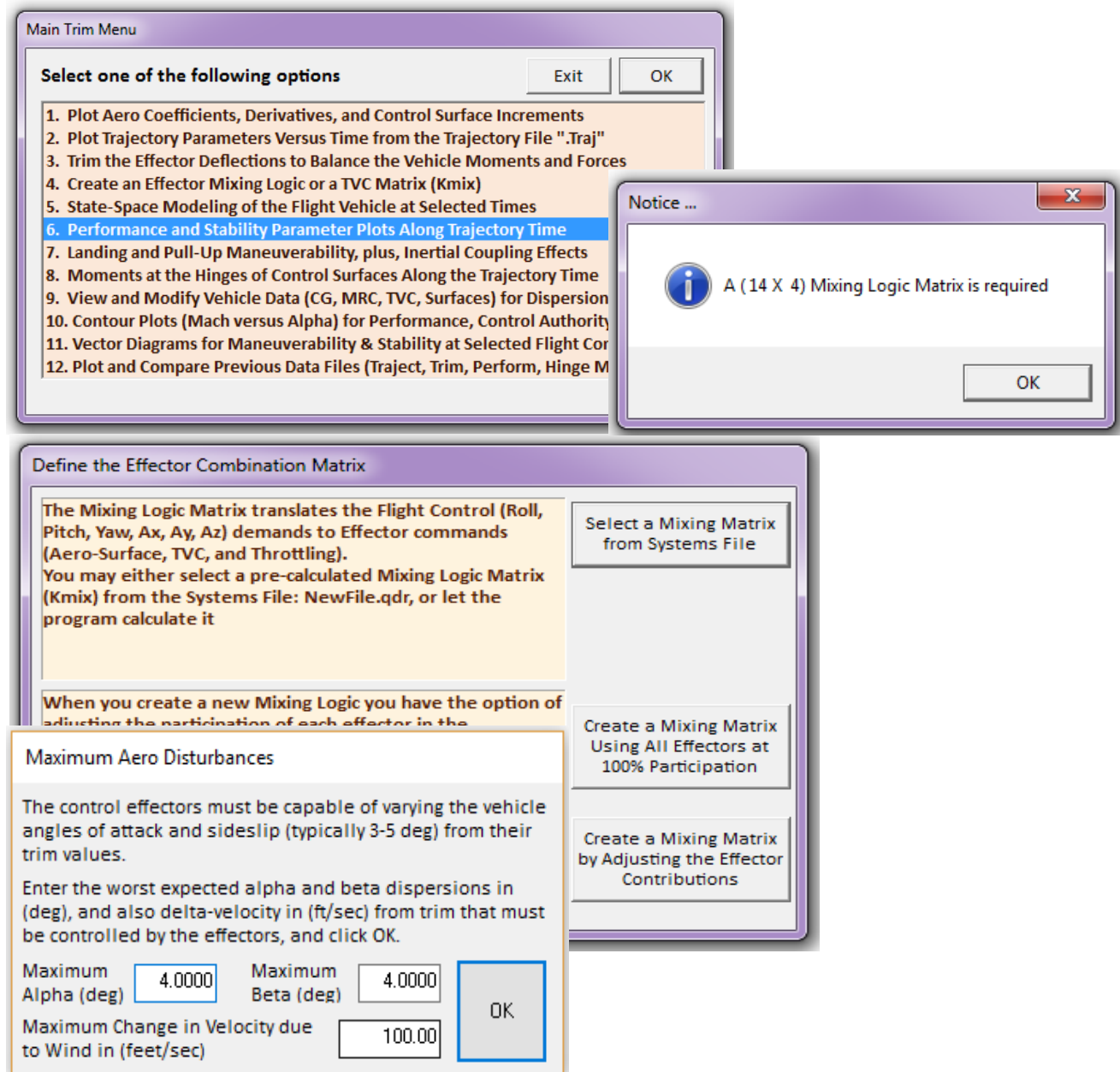
0 50 100 150 200 250 300 350  
Time (sec)

### Surface & Engine Deflections/ Thrusts, Reusable Space-Plane Ascent Trajectory-1

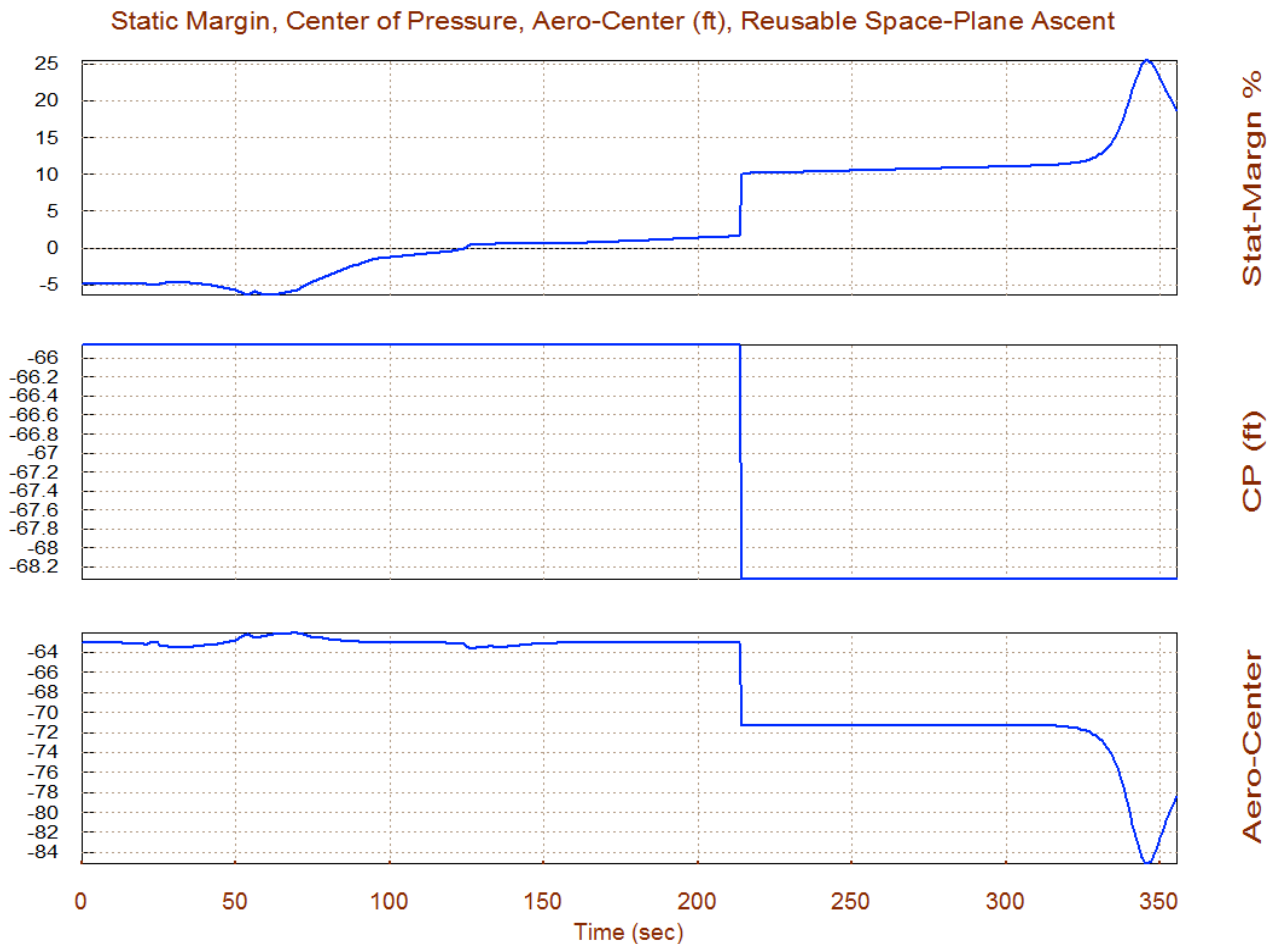


### 1.3 Performance Analysis of Trajectory (A)

Let us now analyze vehicle static performance parameters along the first trajectory, as described in Section 3. Return to the main menu and select option-6 to check the performance and stability parameters along the trajectory time. However, before evaluating the vehicle performance the program needs to know how the effectors will combine together to trim along the four directions. The flight control system will also independently control these four directions. A mixing logic matrix defines the effector allocation along the four control directions. The program can either read a matrix from the systems file or calculate it from the vehicle parameters. In this case we allow the program calculate a Kmix along the trajectory by selecting the 2<sup>nd</sup> option "Calculate a Mixing Matrix Using All Effectors at 100% Participation".



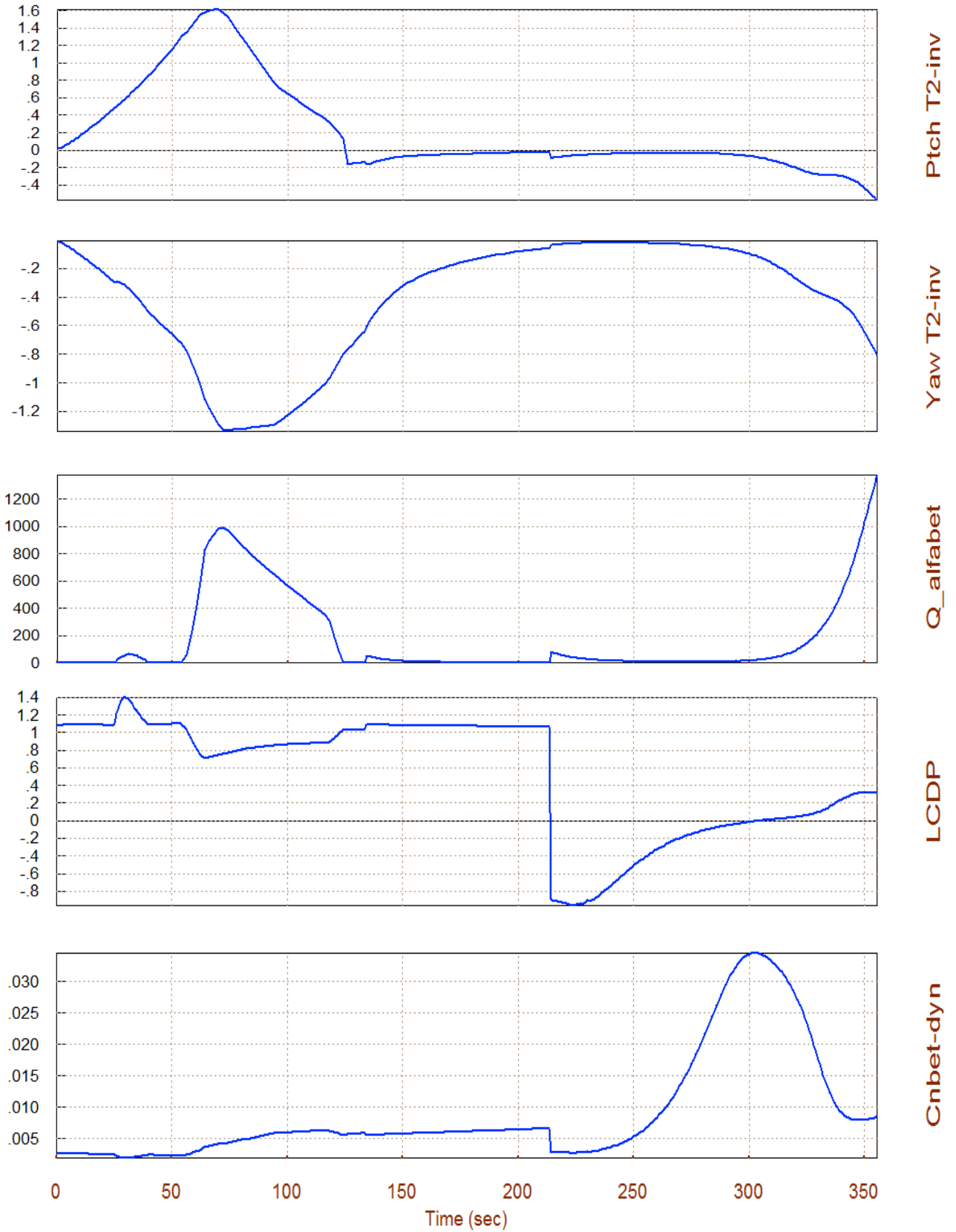


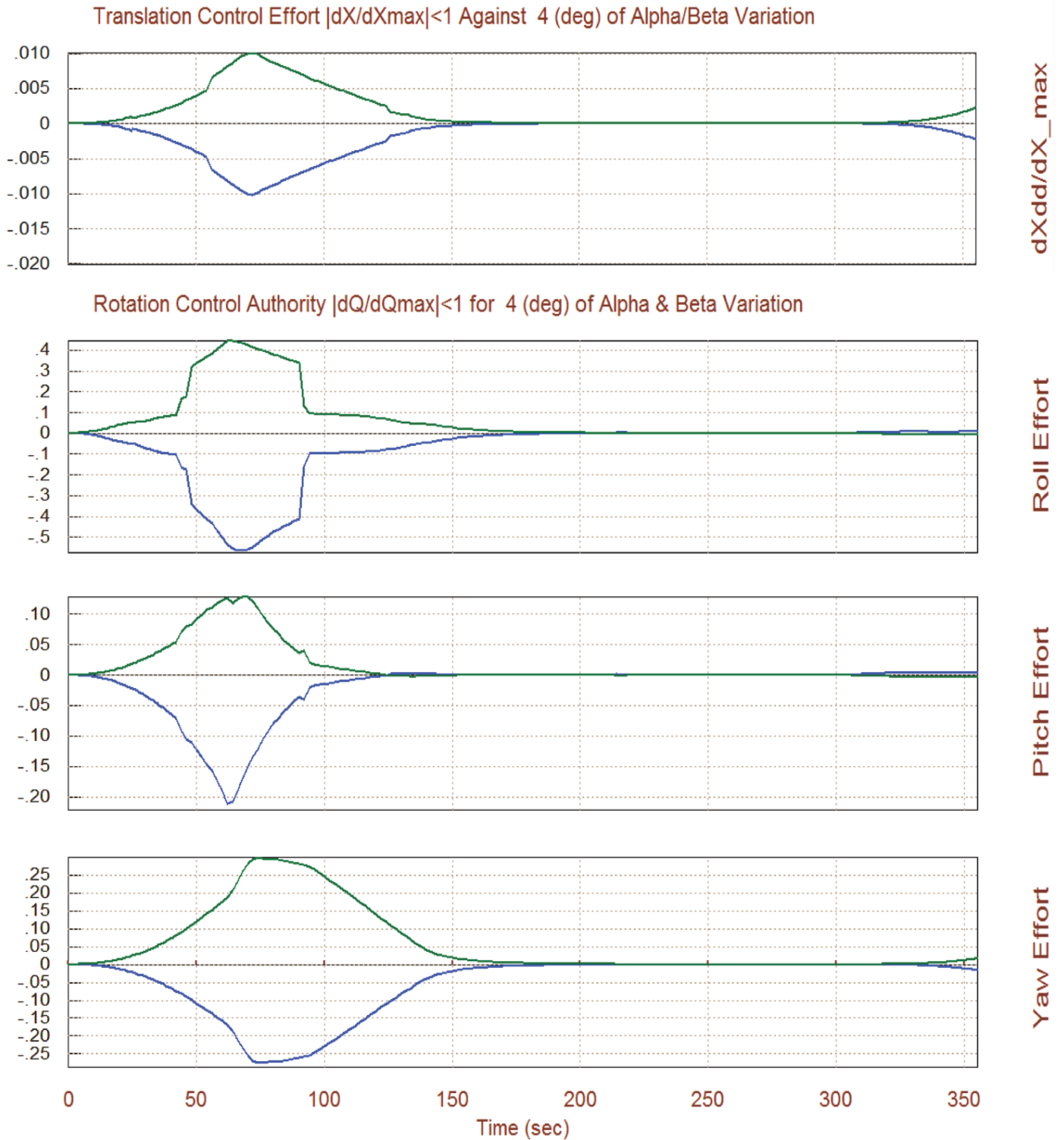


The effector combination matrix must have 4 inputs corresponding to the 4 control directions (3 rotations plus x-acceleration), and 14 outputs (8 TVC, 4 throttling, and 2 aero-surfaces). In the next dialog you must define the expected disturbances. Enter the maximum dispersions in the angles of attack and sideslip  $\alpha_{\max}=\pm 4^\circ$ ,  $\beta_{\max}=\pm 4^\circ$ , and air-speed variation  $V_{\max}=\pm 100$  (ft/sec). These are aerodynamic disturbances generated due to wind-shear or maneuvering. They produce moments and forces that the vehicle should have the control power to overcome. The effector system control authority is measured by its capability to counteract them.

The static margin is telling us that this launch vehicle is unstable in pitch during the first 130 seconds of the trajectory and later on it becomes stable. From the pitch T2-inverse plot we see that the max instability occurs at 70 seconds and the time to double amplitude  $T_2=0.62$  (sec). In the lateral direction, however, it is always stable with a positive  $C_n\beta$ -dynamic. The yaw T2-inverse curve is negative (stable) and the Dutch-Roll resonance peaks to 1.3 (rad/sec) at 75 sec. The combined (Q-alpha and Q-beta) loading at ( $\alpha_{\max}=4^\circ$ ,  $\beta_{\max}=4^\circ$ ) is acceptable. The LCDP parameter looks great up to 210 sec and then it suddenly changes sign a couple of times because of the large pitch maneuver. This sign reversal in the LCDP implies that the control gains must also change sign after the  $180^\circ$  maneuver. This is not a surprise since the entire control law must be changed during this low dynamic pressure large angle maneuver using a combination of RCS and TVC.

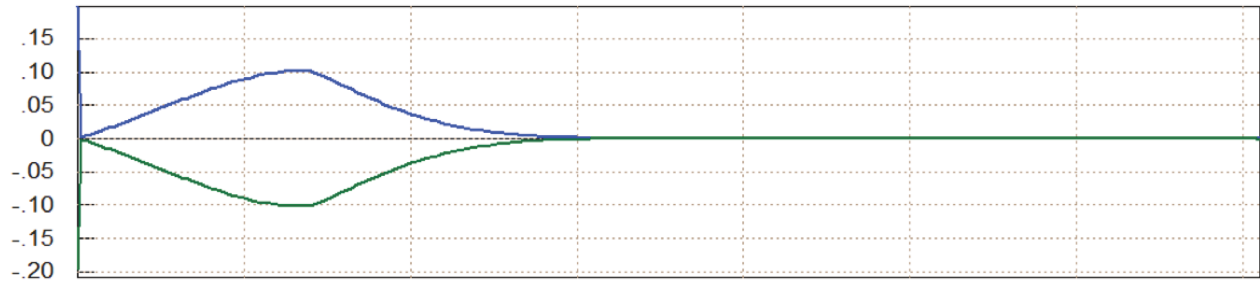
Short-Period (w)/ Time-to-Double-Ampl-Inverse (/sec), Q\_alpha\_beta (deg-lb/ft^2)





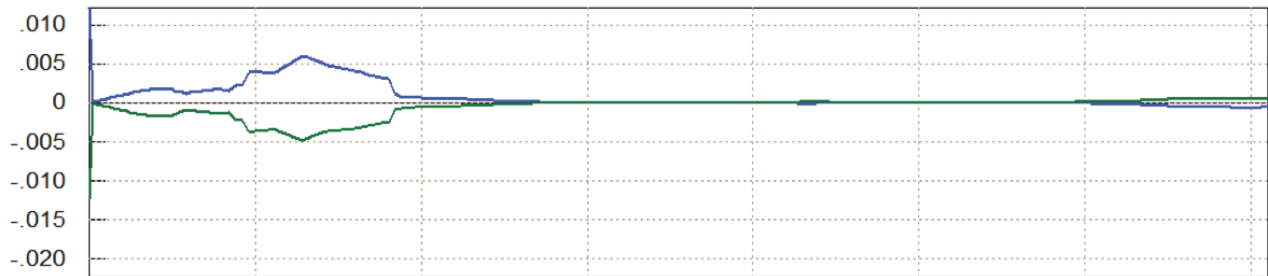
The control effort magnitude against the  $\pm\alpha_{max}$  and  $\pm\beta_{max}$  dispersions from trim is less than 50% in all four directions (roll, pitch, yaw, and x-acceleration). It means that the effector system has plenty of control authority for maneuvering against the anticipated variations. The blue curves represent the control effort when the  $\alpha_{max}$  and  $\beta_{max}$  dispersion angles from trim are positive and the green curves is when they are negative.

Translation Control Effort  $|dX/dX_{max}| < 1$  Against +Vmax & -Vmax AirSpeed Variation

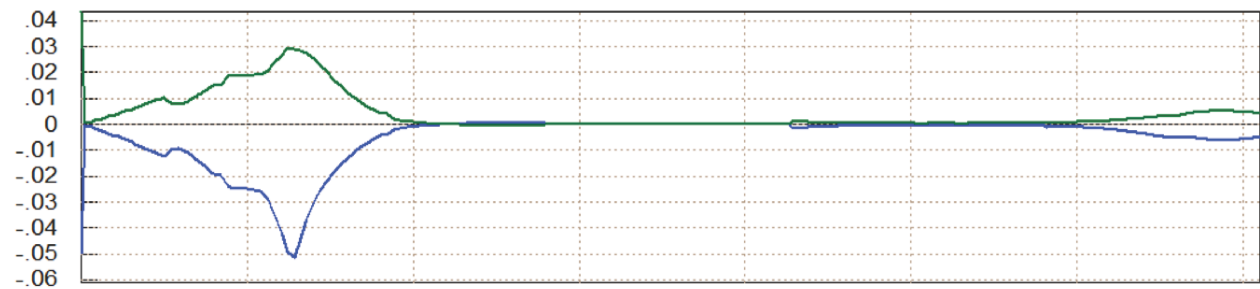


$dX_{dd}/dX_{max}$

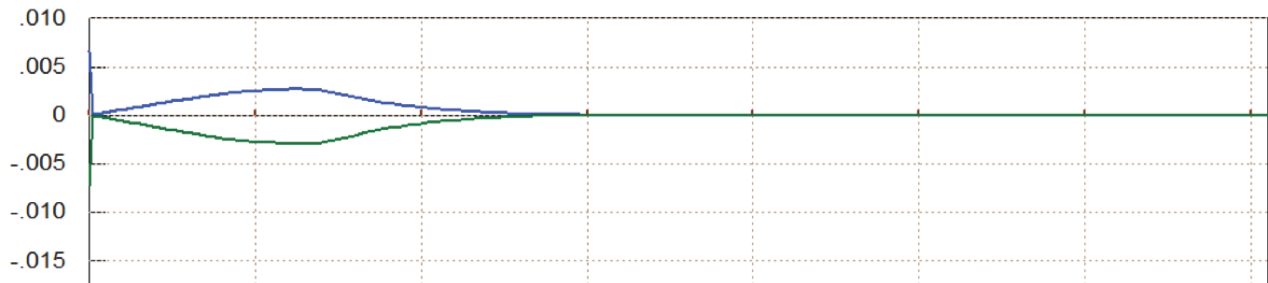
Rotational Control Authority  $|dQ/dQ_{max}| < 1$  Against +Vmax & -Vmax Veloc Variations



Roll Effort



Pitch Effort

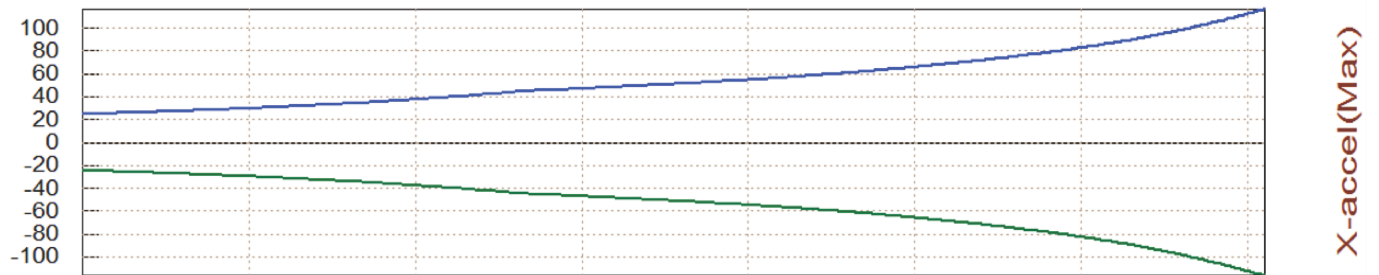


Yaw Effort

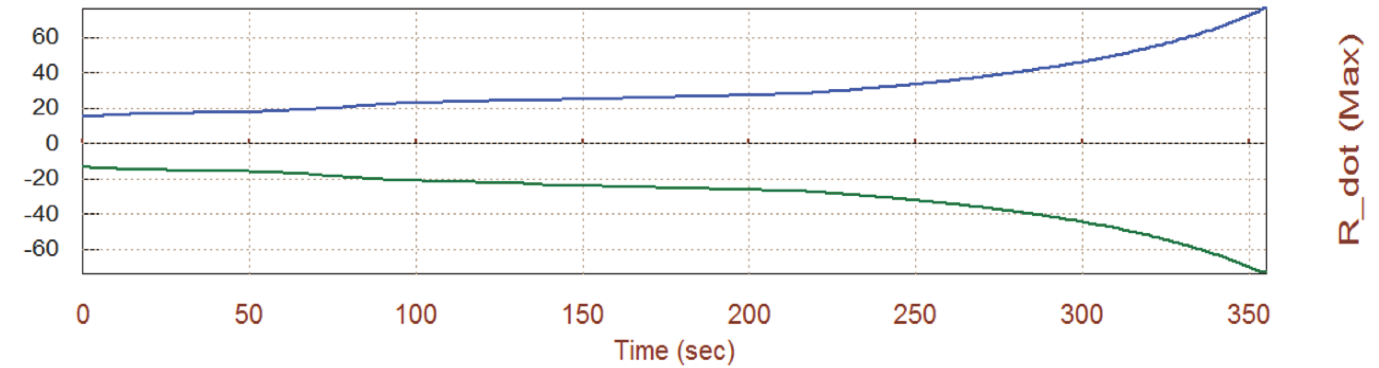
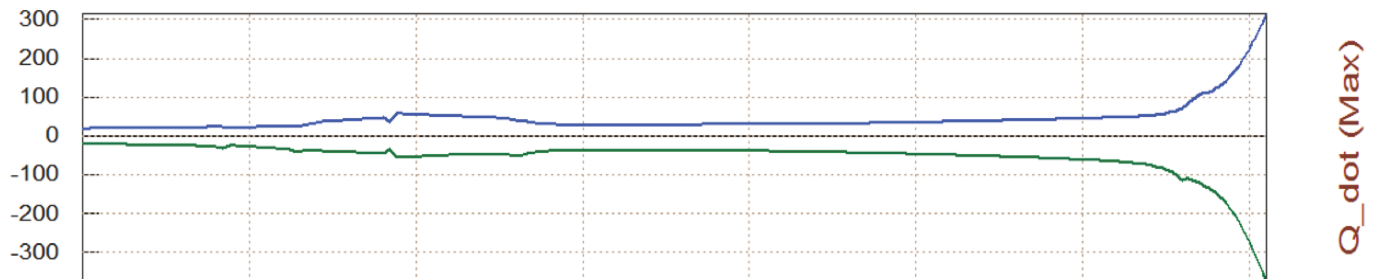
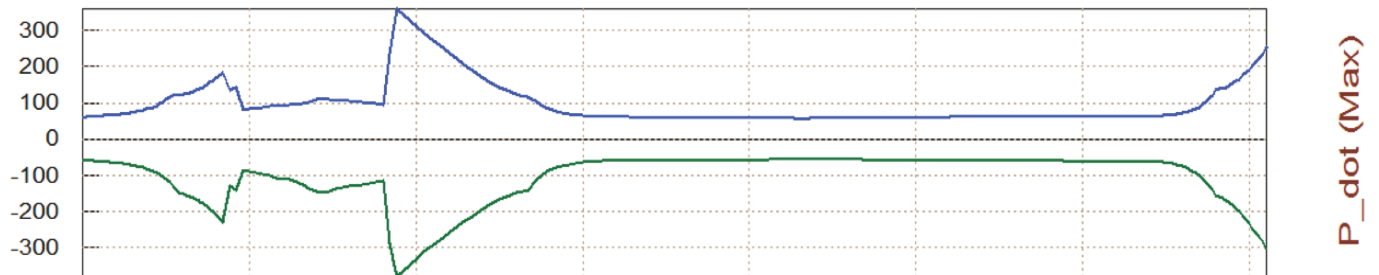
0 50 100 150 200 250 300 350  
Time (sec)

The control effort magnitude is also small against air-speed variations. It means that the effector system has plenty of control authority to counteract  $\pm 100$  (feet/sec) variations from nominal speed  $V_0$  in all four directions (roll, pitch, yaw, and x-acceleration). The blue curves represent the control effort when the variation is a head-wind and the green curves when it is a tail-wind.

Max Linear Accelerations in (ft/sec<sup>2</sup>), at Maximum +ve and -ve Control Demands



Max Angular Accelerations (rad/sec<sup>2</sup>), at Maximum +ve and -ve Control Demands

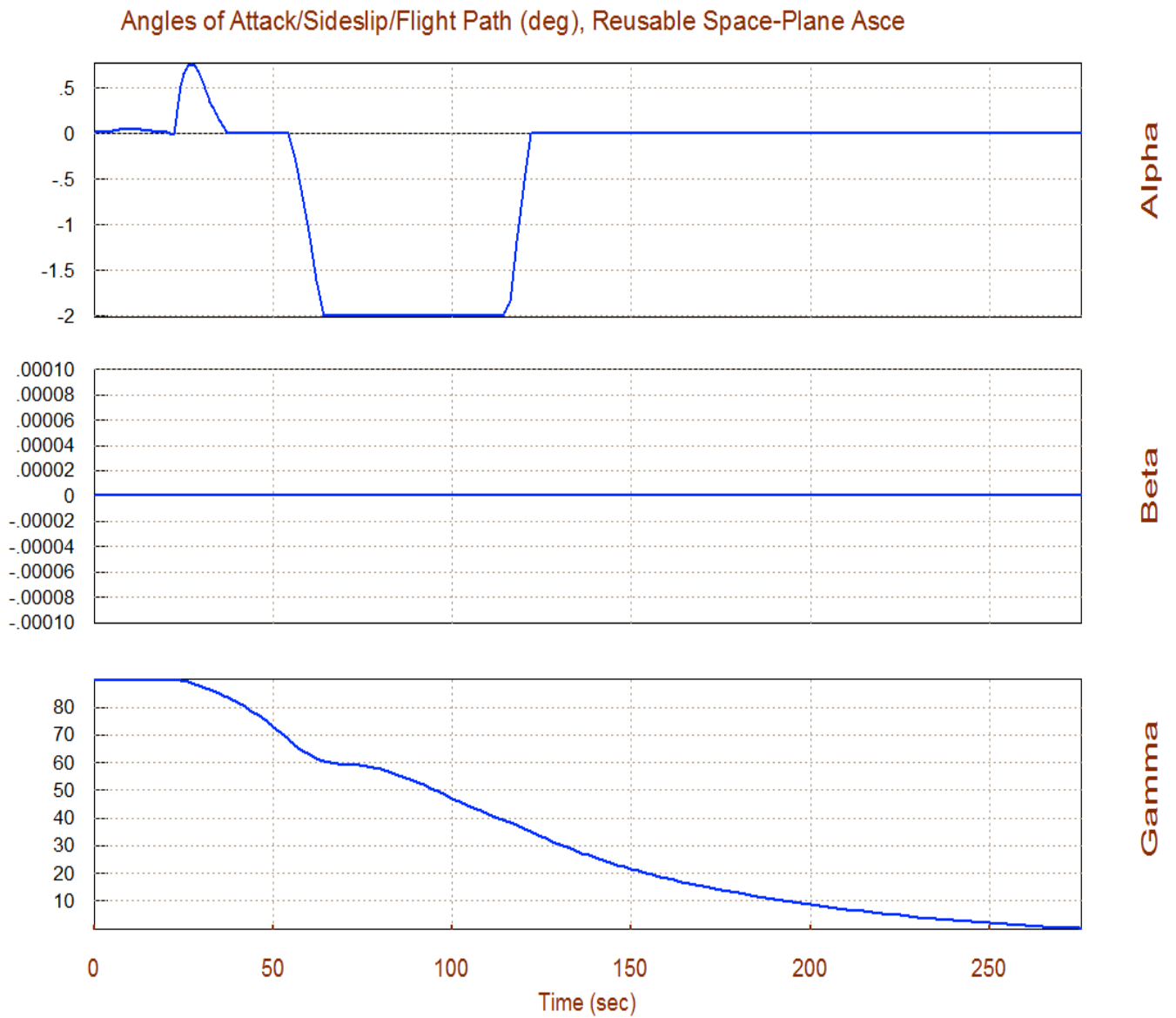


This plot shows the maximum angular accelerations in (rad/sec<sup>2</sup>) attained in roll, pitch, and yaw and also the maximum translational x-acceleration in (feet/sec<sup>2</sup>) attained when the control demands are maximized. The blue curves are positive accelerations due to max positive demands and the green curves are negative accelerations due to max negative demands.

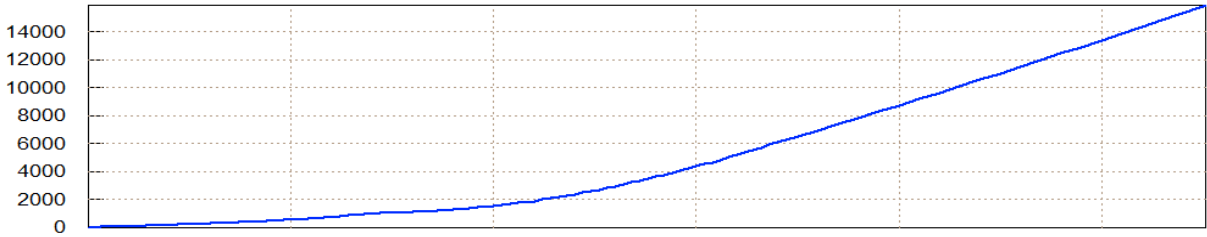


## 1.4 Trajectory (B) Analysis

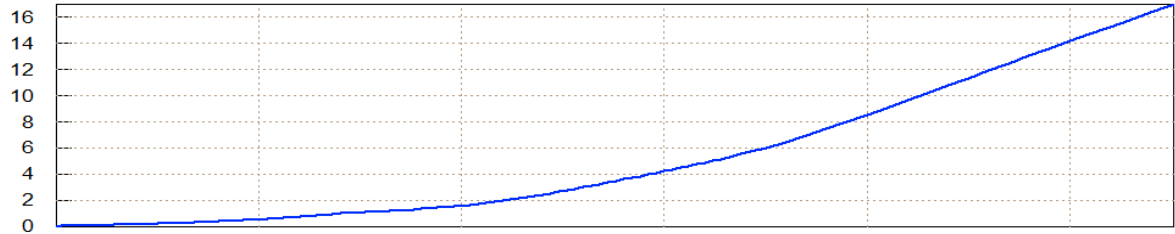
Let us now take a look at another trajectory and repeat the Trim analysis to analyze the vehicle performance along the second trajectory. We must rerun Flixan, select the same project folder but this time in the filename selection menus we must select the second trajectory file "*ReSP\_Ascent\_B.Traj*". Let us first take a look at the trajectory. From the Trim main menu select option-2 for plotting the trajectory parameters. This trajectory is more benign than the previous because it avoids the large 180° pitch maneuver and the thrust does not drop rapidly from 360,000 to 180,000 (lb), as it was in the first trajectory, but it is gradually reduced to zero. Also, the angle of attack is reasonably small, unlike the large alpha maneuver that we saw in the first trajectory.



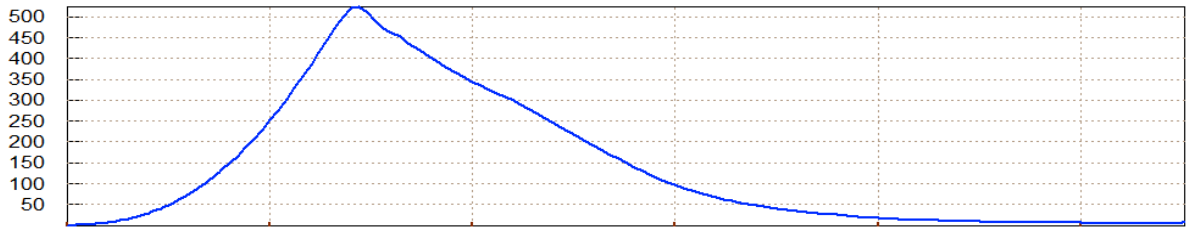
Velocity, Dynamic Pressure, Reusable Space-Plane Ascent Trajectory-2



Veloc (ft/s)

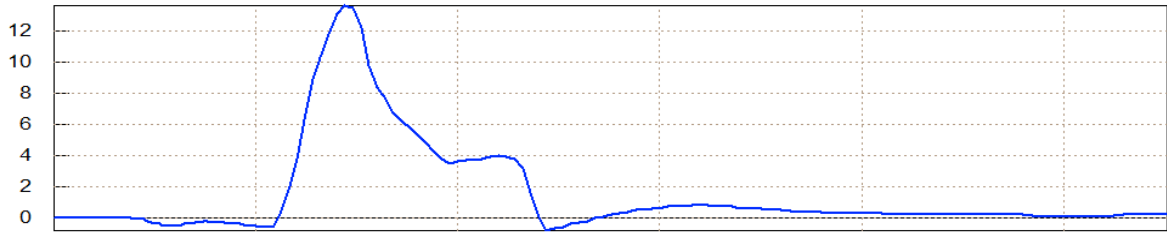


Mach Number

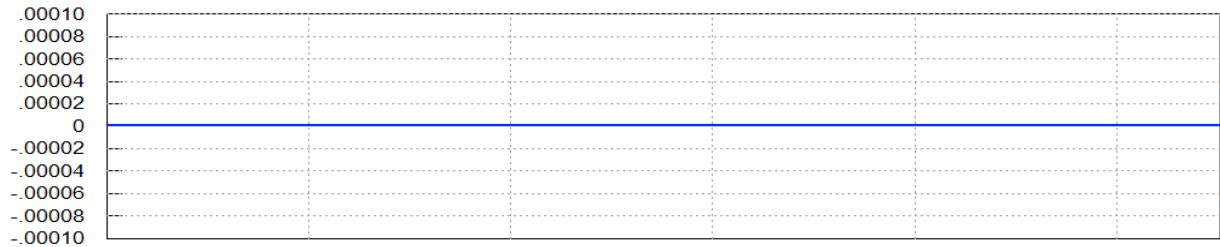


Q-bar (PSF)

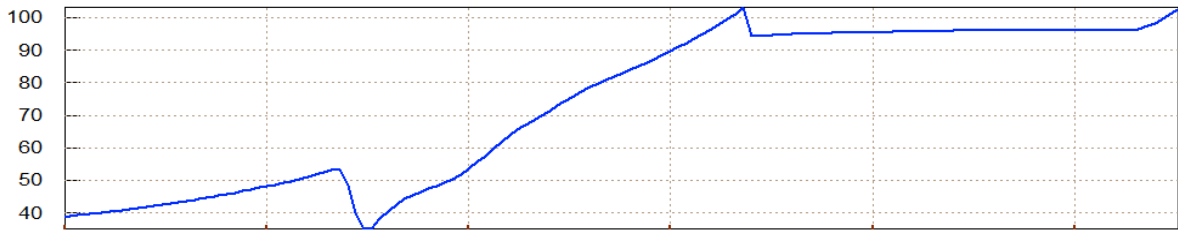
Sensed Acceleration in (ft/sec^2), Reusable Space-Plane Ascent Trajectory-2



Accel-Z



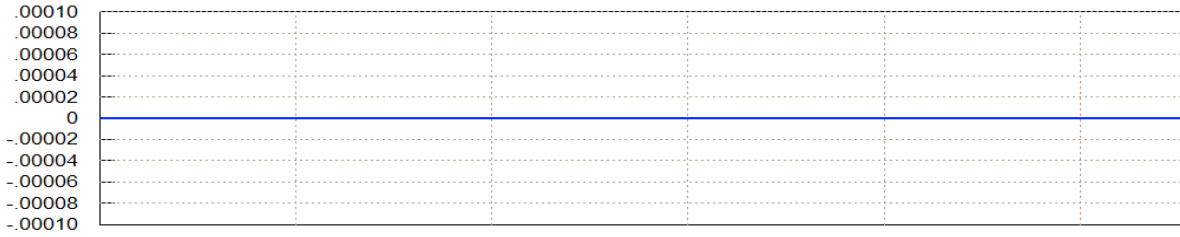
Accel-Y



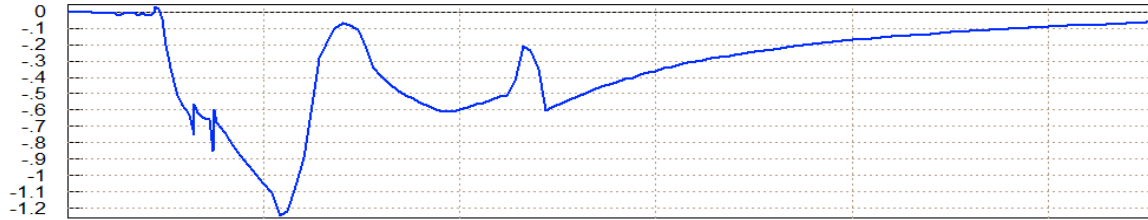
Accel-X

Time (sec)

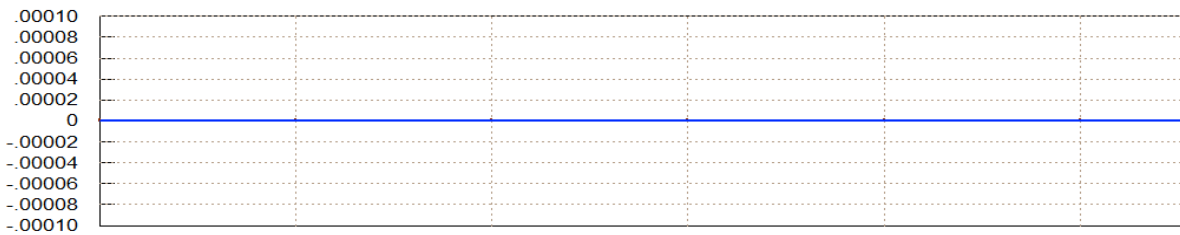
Angular Rates (rad/sec), Reusable Space-Plane Ascent Trajectory-2



R (d/s)

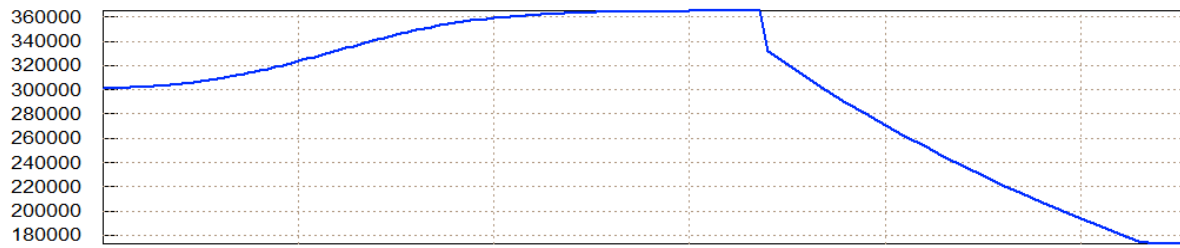


Q (d/s)

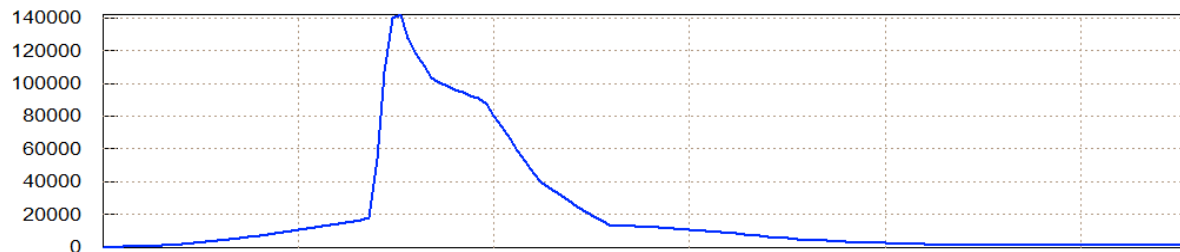


P (d/s)

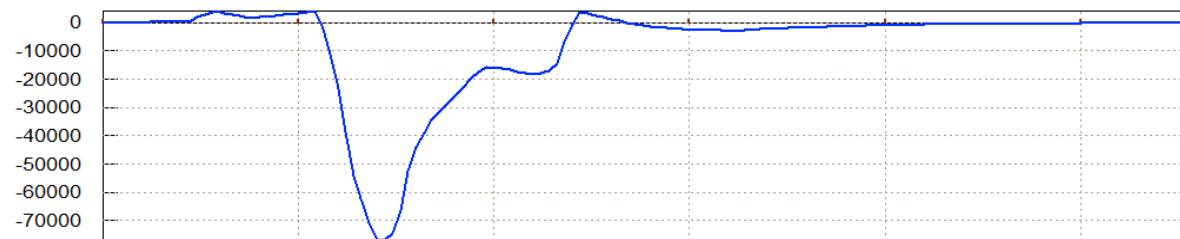
Aero Lift/Drag Forces, Eng. Thrust in (lb), Reusable Space-Plane Ascent Traj



Thrust Force



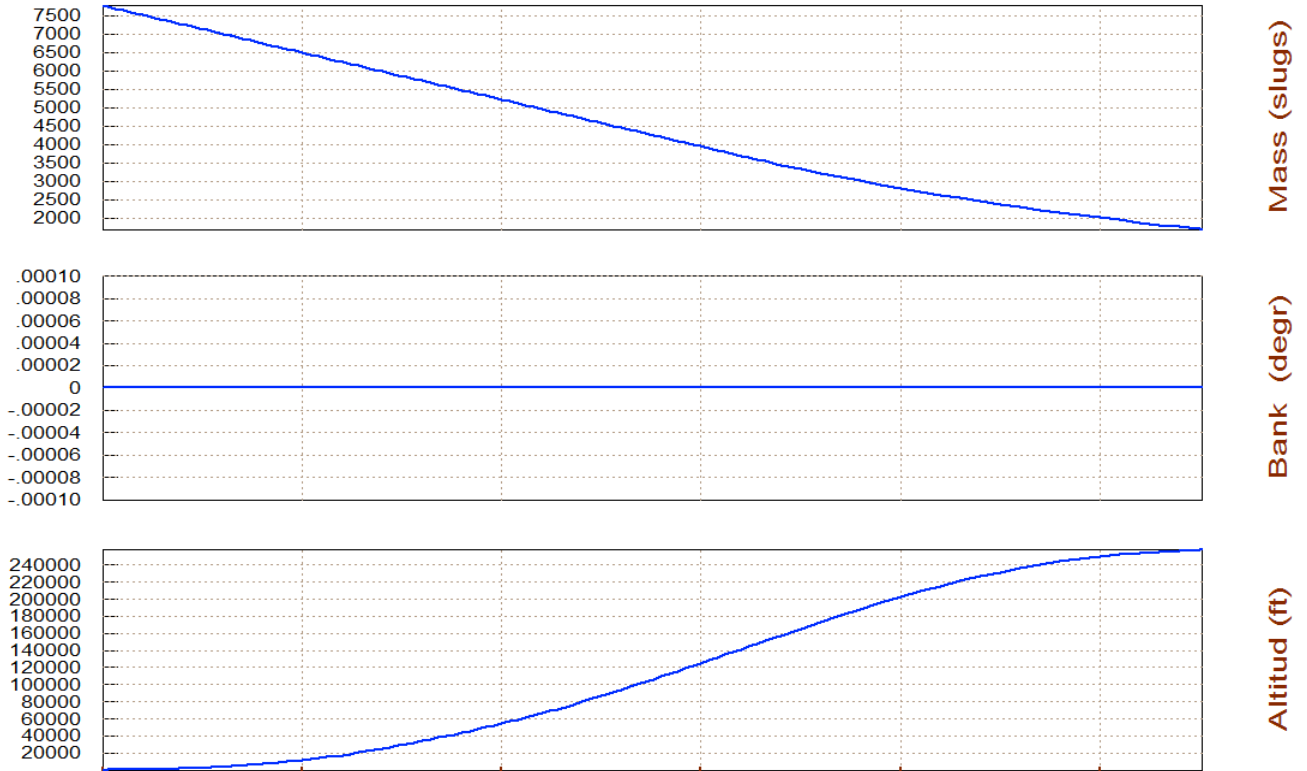
Drag Force



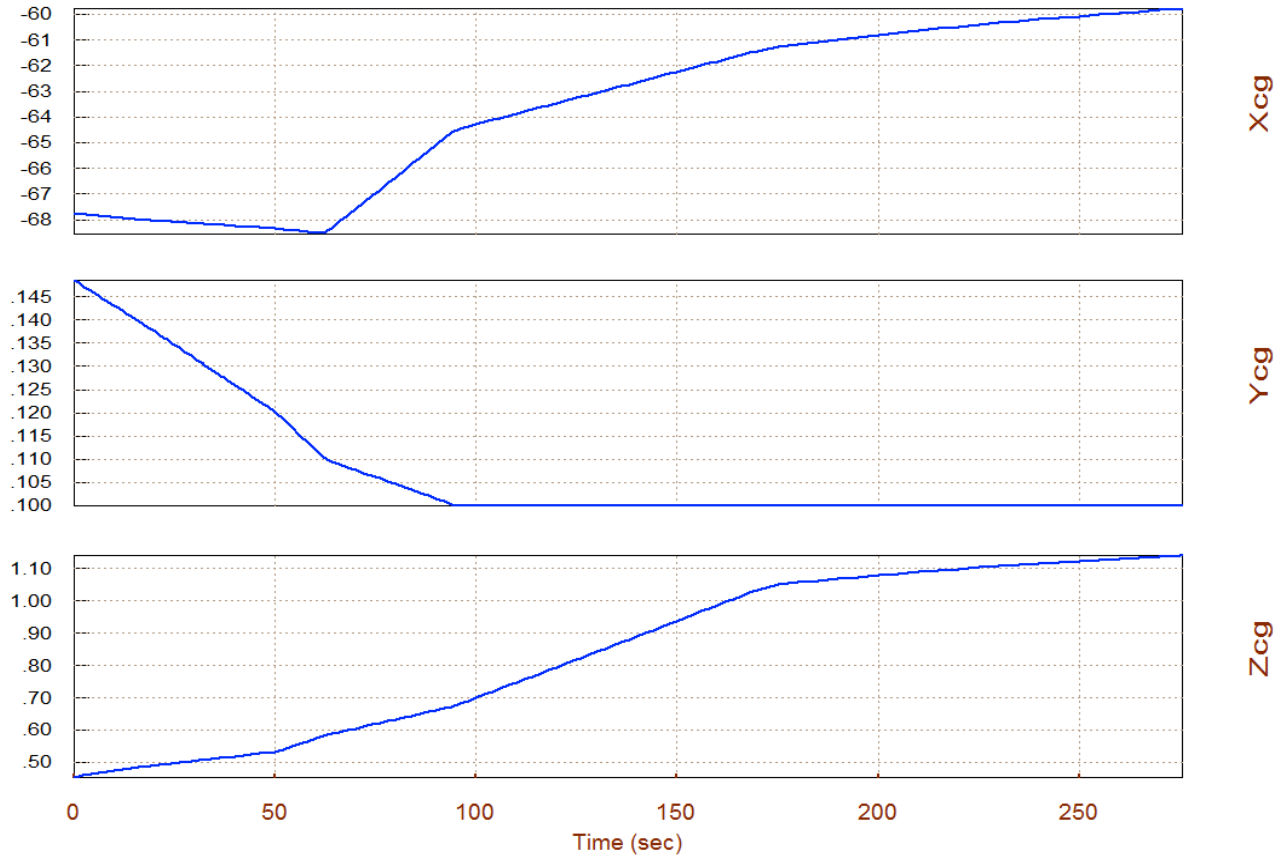
Lift Force

0 50 100 150 200 250  
Time (sec)

Vehicle Altitude, Mass, Bank Angle, Reusable Space-Plane Ascent Trajec

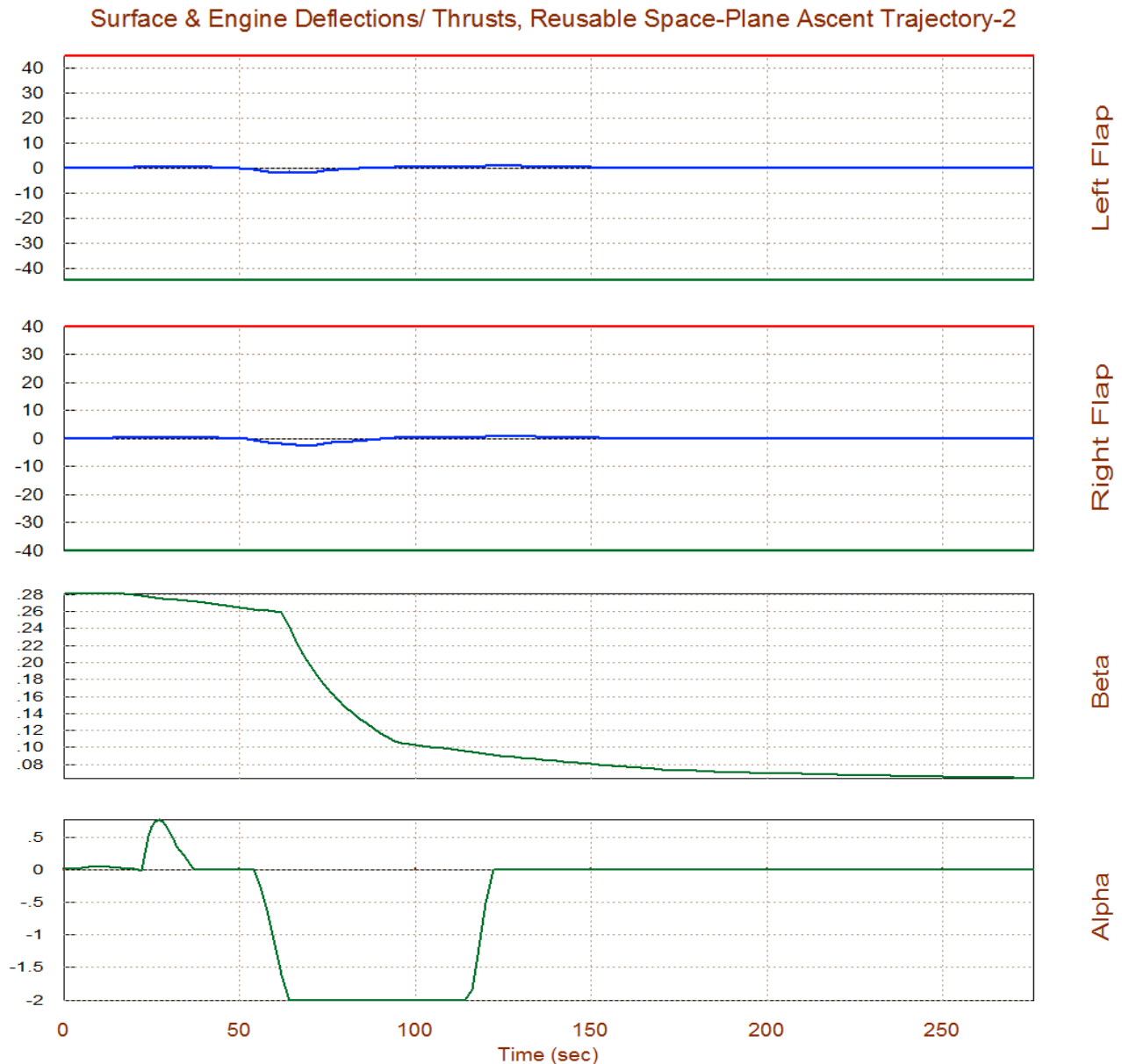


Vehicle CG in (feet), Reusable Space-Plane Ascent Trajectory-2



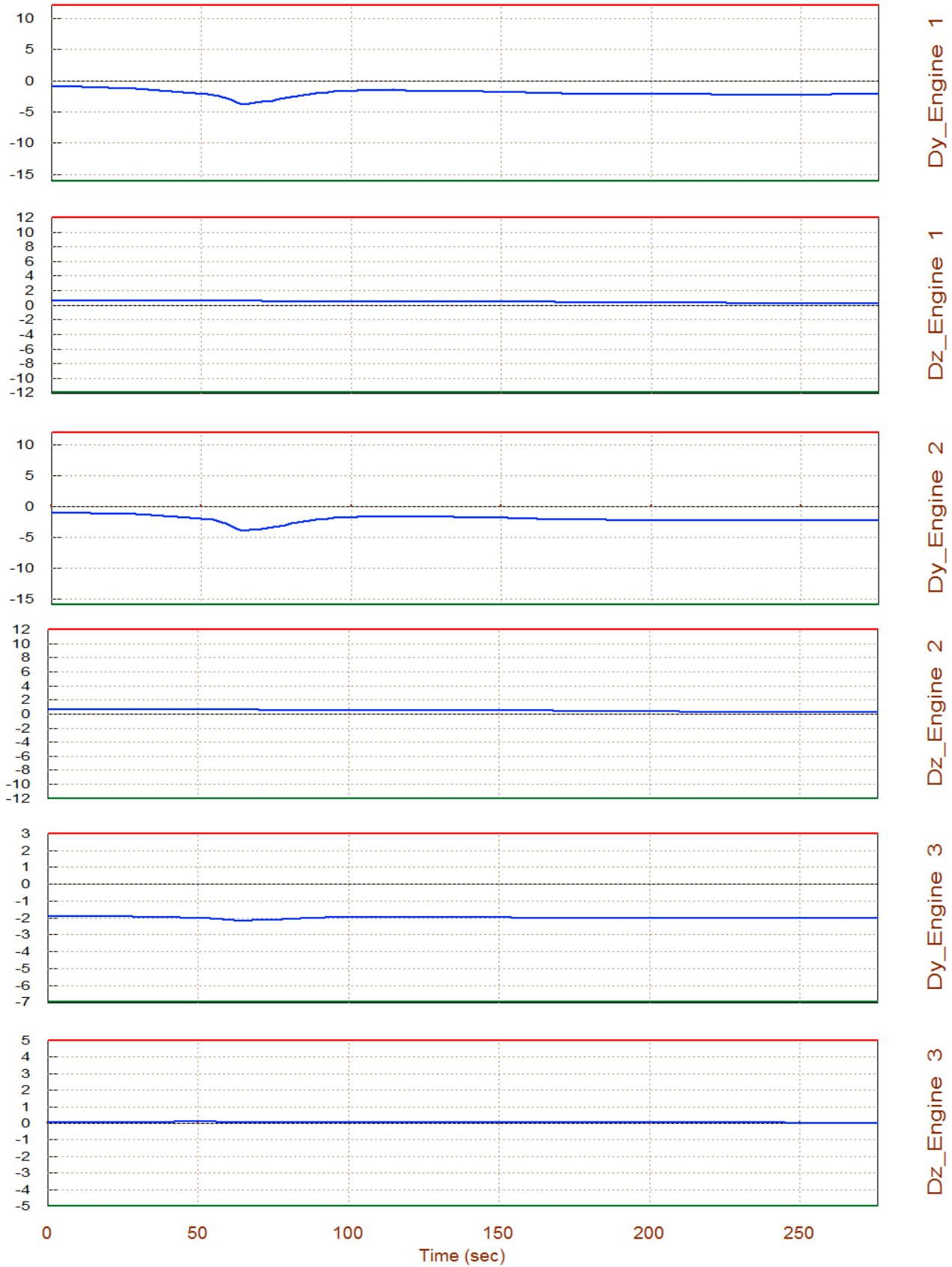
## 1.5 Trim Analysis along Trajectory (B)

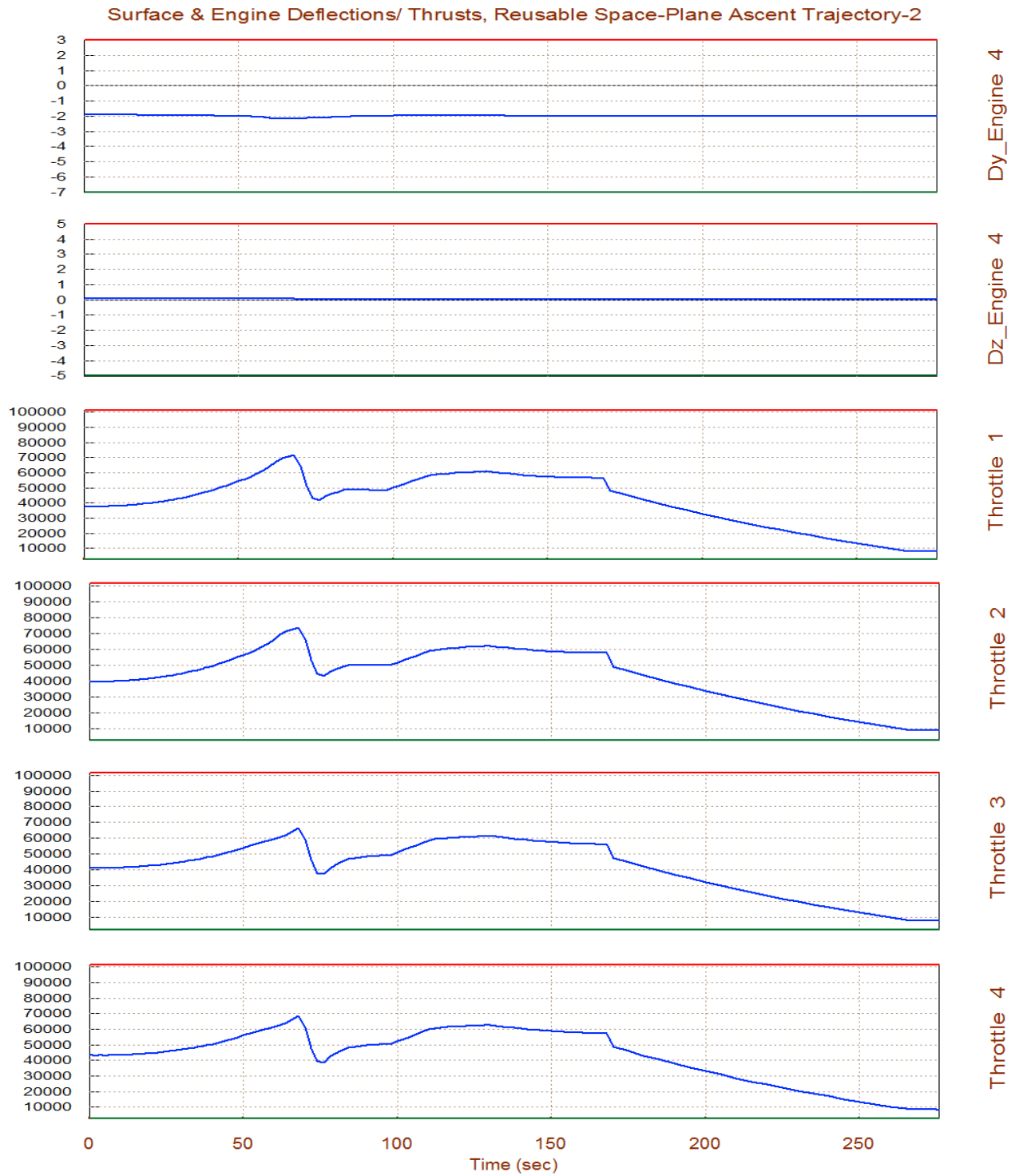
Our first step is to determine the effectors trim angles along the second ascent trajectory. The vehicle uses all six engines and the two elevons to trim and the program still uses the same effector files, only the trajectory has changed and, obviously, the trim history of the 2 flaps and the 6 engines thrusts and deflections will be different from the first trajectory. Return to the Trim main menu and choose option-3 to trim the effectors as in Section 1.2. From the menu/dialogs, do not select an initialization file, but trim from zero because it is the first trim using this trajectory. Then choose to trim along the 3 moments including the axial acceleration, like before. The program will calculate the new effector deflections and thrust variations required to trim the moments and axial force along trajectory (b), shown below. Also, similar to the previous trajectory the  $+Y_{CG}$  offset causes the vehicle to yaw, creating a sideslip  $\beta$ .





### Surface & Engine Deflections/ Thrusts, Reusable Space-Plane Ascent Trajectory-2





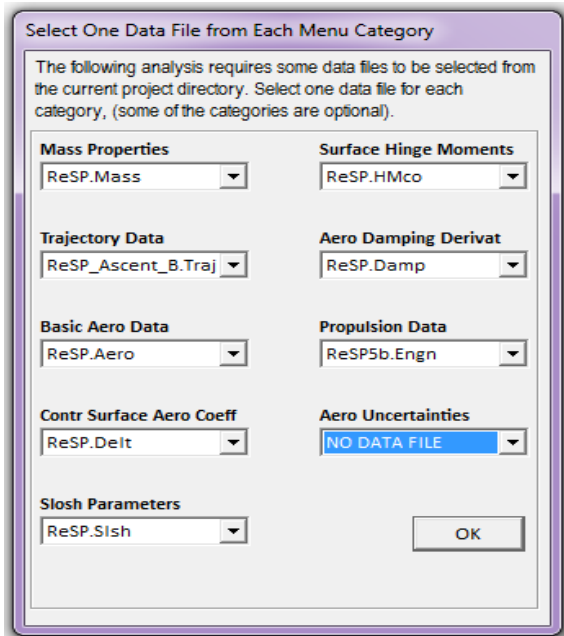
The TVC engines are tilted  $0.3^\circ$  in yaw ( $\delta_z$ ) to counteract the yaw moment due to  $\beta$ . They are also pitching in the negative direction to balance the pitching moment along the trajectory. The pitch deflections of the TVC engines  $\delta_y$  are different between the left and right sides, and also the two Elevon deflections are slightly different, because they are trimming the rolling moment generated due to the lack of lateral symmetry. The thrusts of the 4 throttling engines are also varying during trim as they try to match the acceleration defined in the trajectory.

## 1.6 Analyzing Engine Failures

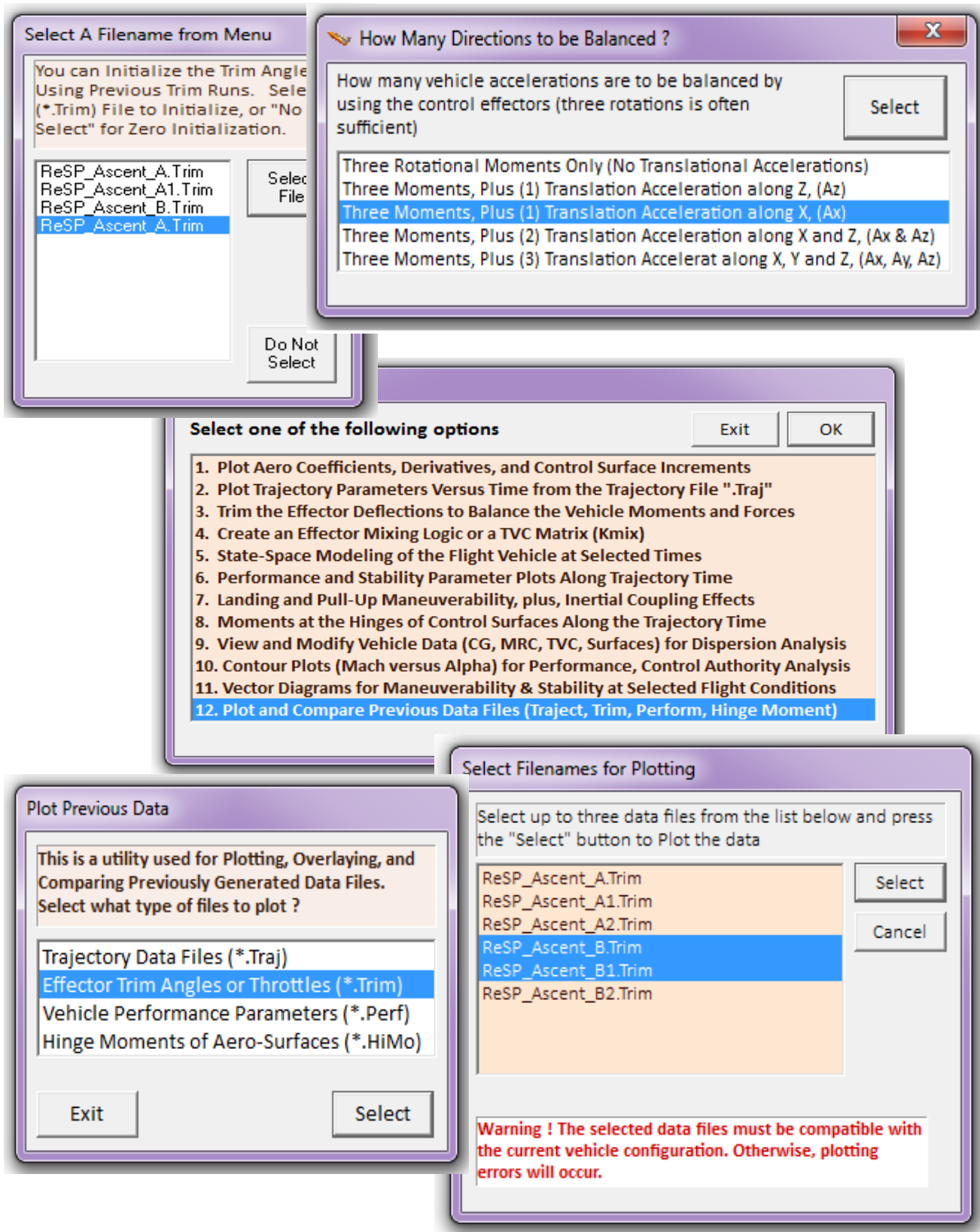
Engine failures are not uncommon in launch vehicles with multiple engines and in most cases they should be able to complete their missions in the event of at least one engine failure. Failures may be due to a loss of thrust or a TVC actuator failure. In the analysis that follows we will demonstrate that we are able to trim and to control the vehicle in the event of loss of thrust from one of the 6 engines or in the event of an actuator failing to gimbal. We will fail one of the throttling engines, the last one #6, and see if the remaining 5 engines in combination with the aerosurfaces are able to compensate and to continue trimming the vehicle along the same trajectory. It is easy to fail an engine in Trim, either by removing one of the engines from the engines data file, or by setting its effectiveness to zero. The second method is more convenient because it makes it easier when co-plotting and comparing the data (before and after the failure). We are still keeping engine #6 in file, but its thrust and throttling capability are set to a very small number that is essentially eliminating its control authority. Instead of modifying the original engines data file "*ReSP6.Engn*", it makes more sense to copy it under a different name "*ReSP5b.Engn*" and modify that file instead. So let us restart the Trim program and select the failed engine thrust file.

Reusable Launch Vehicle data for the 6 Engines  
 Engine #6 Has Lost its Thrust. Normally it was Throttling but Not Gimbaling

Engine Description,	Thrust (lb)	Mass (slug)	Ieng (slug-ft <sup>2</sup> )	Mom Arm (ft)	Location (x,y,z) (feet)	Mounting Angles Elevat, Azimuth	(Dy, Dz) (degr)	Max Deflection Dym,Dzm (deg)	Max Thrott1 (0-1)
Main Engine#1	70000.0	100.0	1500.0	2.8	-89.0 -4.8 0.0	-2.0 0.0	14.0 12.0	0.0	
Main Engine#2	70000.0	100.0	1500.0	2.8	-89.0 +4.8 0.0	-2.0 0.0	14.0 12.0	0.0	
Main Engine#3	52000.0	100.0	1500.0	2.8	-89.0 -2.25 +3.9	-2.0 0.0	5.0 5.0	0.95	
Main Engine#4	52000.0	100.0	1500.0	2.8	-89.0 +2.25 +3.9	-2.0 0.0	5.0 5.0	0.95	
Main Engine#5	52000.0	100.0	1500.0	2.8	-89.0 -2.25 -3.9	-2.0 0.0	0.0 0.0	0.95	
Main Engine#6	0.01	100.0	1500.0	2.8	-89.0 +2.25 -3.9	-2.0 0.0	0.0 0.0	0.01	

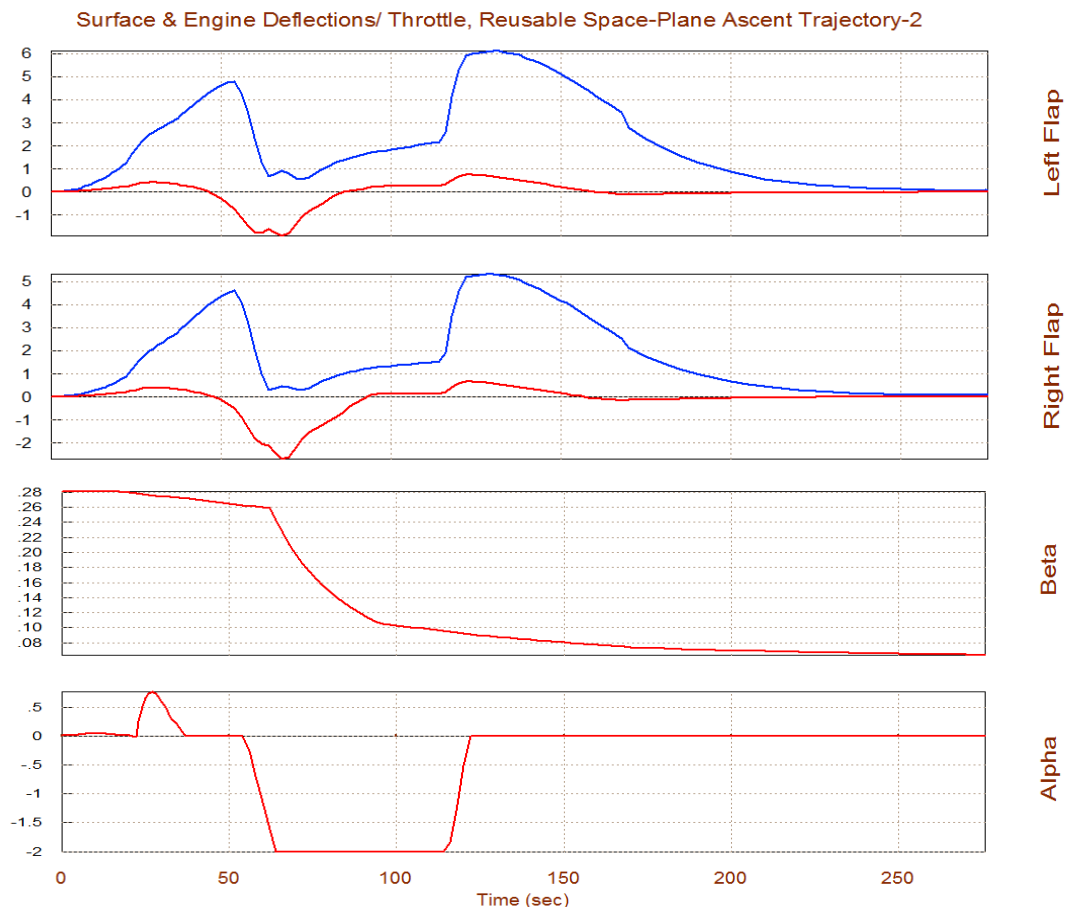


We must trim again along 3 rotations plus axial acceleration using the second trajectory and the 5 engines file this time, without selecting an initialization file, as before. When trimming is complete return to the main menu and chose option-12 to co-plot and compare the two trim data files: the original trim file "*ReSP\_Ascent\_B1.Trim*" where all 6 engines are operating, against the latest file "*ReSP\_Ascent\_B.Trim*" that has engine #6 disabled.

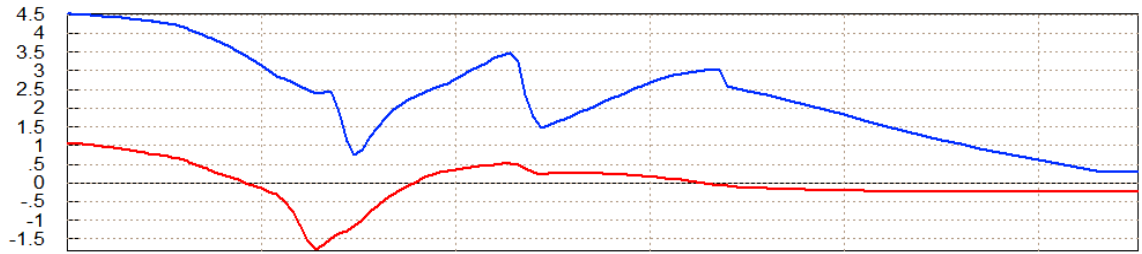


There is a sideslip beta due the  $+Y_{CG}$  offset like before. Only the throttle values instead of actual thrusts are shown in the comparison plots. The red curves are the effector positions from the original 6 engines file (saved by the program in *ReSP\_Ascent\_B1.Trim*), and the blue curves are from the recently created trim file with the failed engine #6 (*ReSP\_Ascent\_B.Trim*). Both trim files were obtained from trajectory (b). The throttle value and also the thrust of Engine #6 (Throttle 4) are zero because of the modification in the failed engine file. Here are some interesting points to notice between the nominal and the failed engine cases.

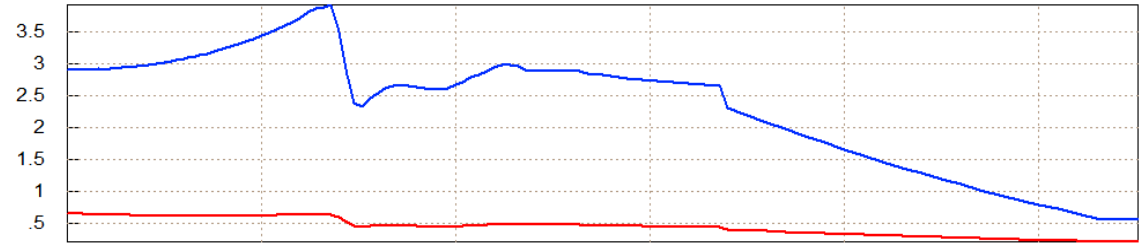
- In the failed engine thrust case, the throttle value and also the thrust of Engine #6 (which is the 4<sup>th</sup> throttling engine) are zero.
- The new trimming with the failed engine increased the throttle levels on the remaining 3 throttling engines (Throttle # 1 to 3), as expected, in order to make up for the failed engine thrust as needed in order to match the expected trajectory acceleration.
- The yaw deflections ( $\delta_{z1}$ ,  $\delta_{z2}$ ,  $\delta_{z3}$ ,  $\delta_{z4}$ ) of the four gimbaling engines are bigger in order to provide negative yawing moment and to make up against the positive yawing moment produced by the absence of engine #6 thrust.
- The pitch deflections ( $\delta_{y1}$ ,  $\delta_{y2}$ ,  $\delta_{y3}$ ,  $\delta_{y4}$ ) in the four TVC engines are also increased in order to make up for the lack of symmetry.
- The flap deflections are also increased and the difference between left and right flaps is greater in order to counteract the rolling moment caused due to the lack of lateral symmetry.



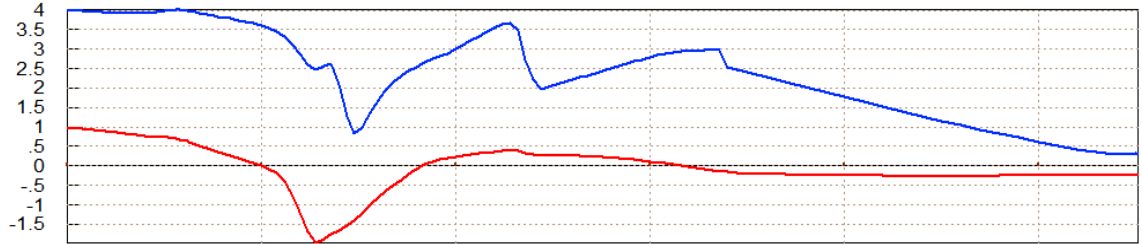
Surface & Engine Deflections/ Throttle, Reusable Space-Plane Ascent Trajectory-2



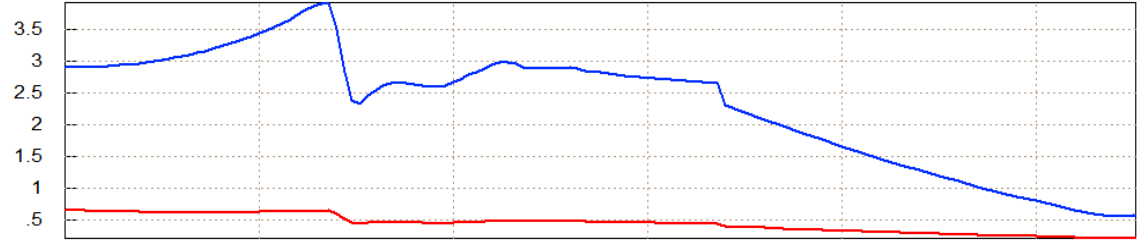
Dy\_Engine 1



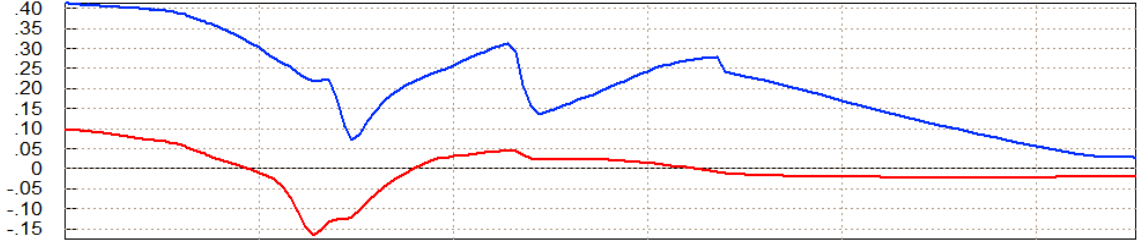
Dz\_Engine 1



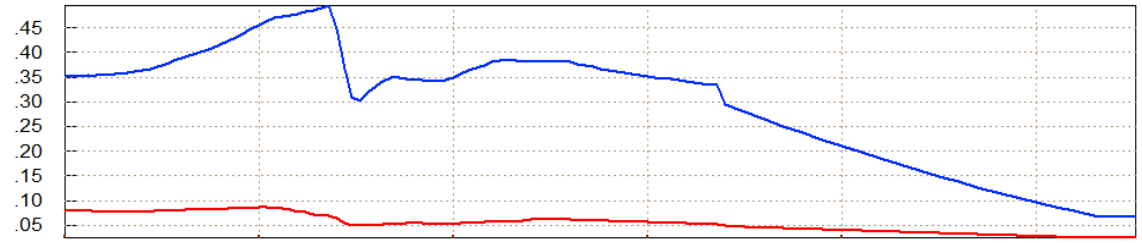
Dy\_Engine 2



Dz\_Engine 2



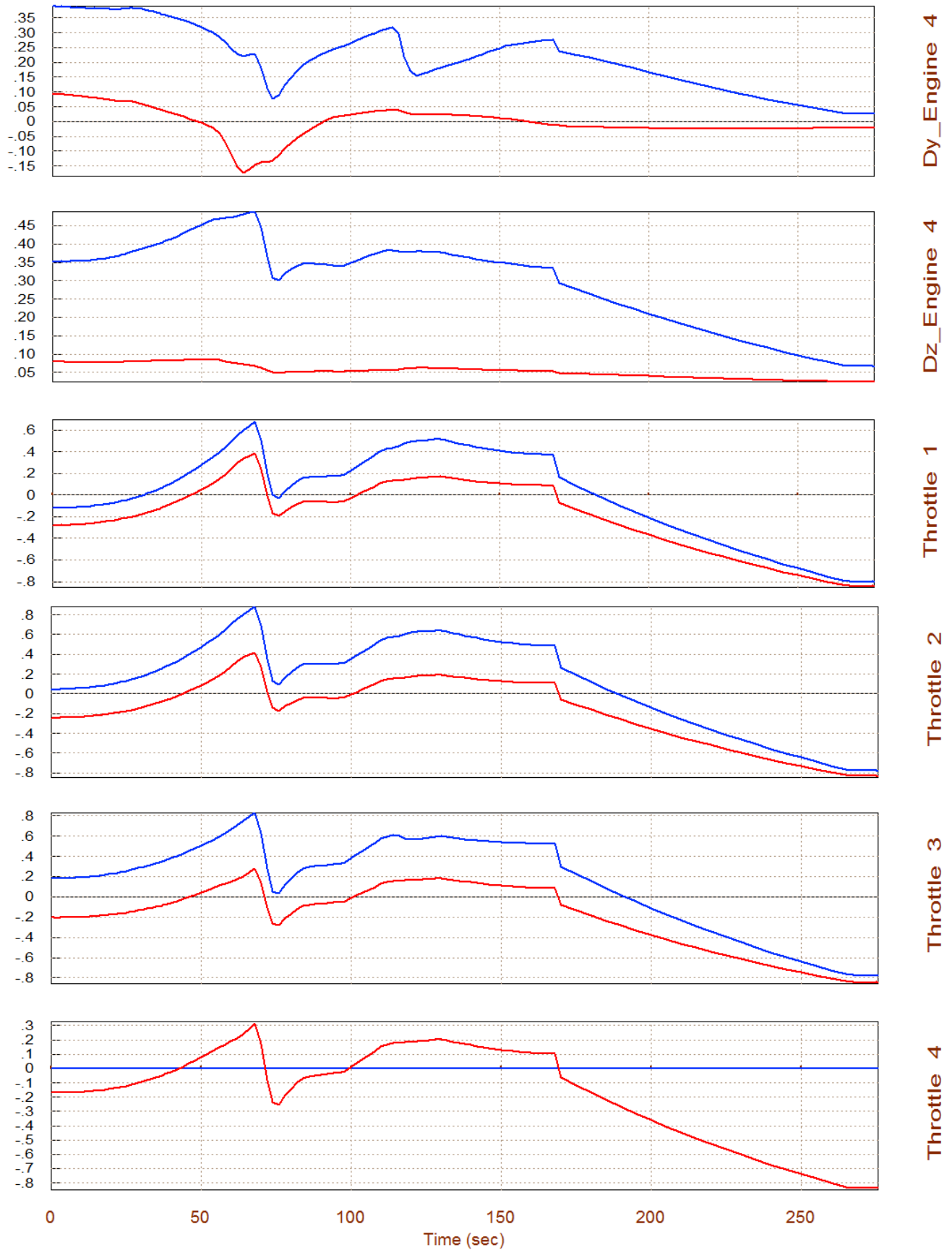
Dy\_Engine 3



Dz\_Engine 3

0 50 100 150 200 250  
Time (sec)

### Surface & Engine Deflections/ Throttle, Reusable Space-Plane Ascent Trajectory-2





## Engine Actuator Failure

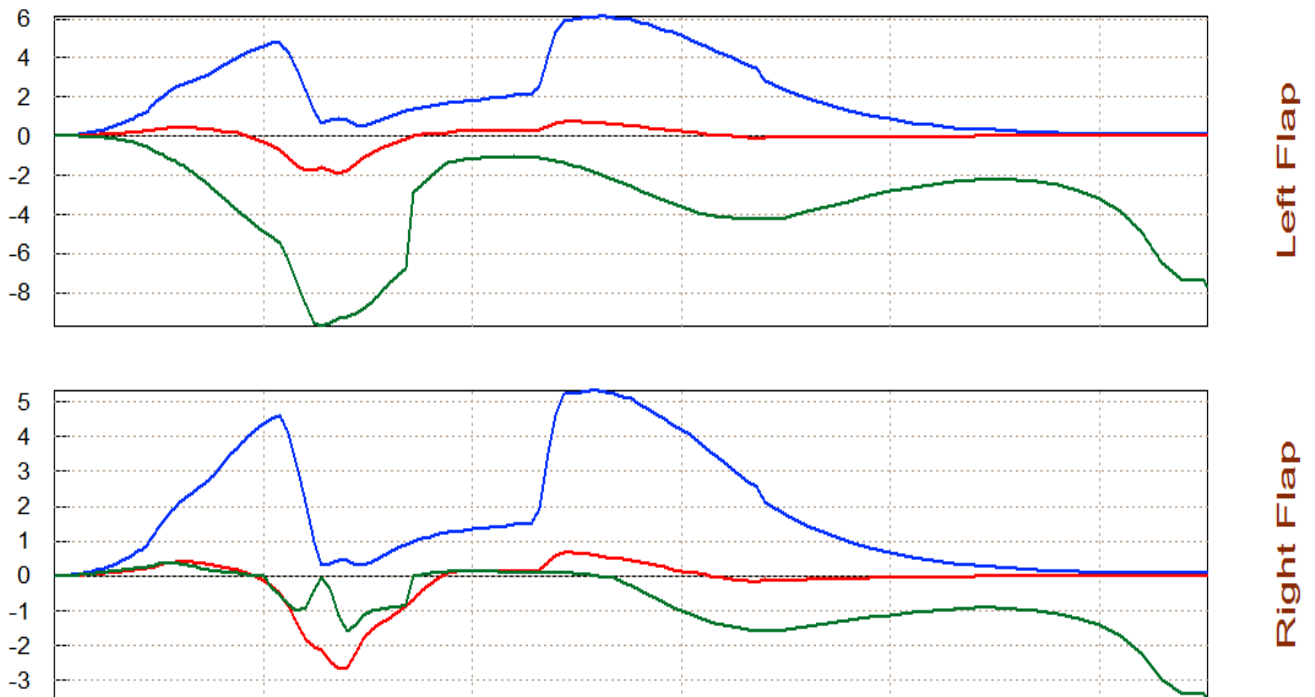
Now let's take a look at a case where the actuators of a gimbaling engine fail to gimbal and the engine is stuck at a constant full thrust and at a fixed non-zero position  $\delta\gamma=-0.5^\circ$  and  $\delta z=-0.5^\circ$ . This situation is defined in a separate engines file "*ReSP5c.Engn*", shown below.

Reusable Space-Plane Data for the 6 Engines  
 Engine #1 Actuator failure (No Gimbaling, No Throttling, Fixed Thrust, Fixed Gimbal Position)

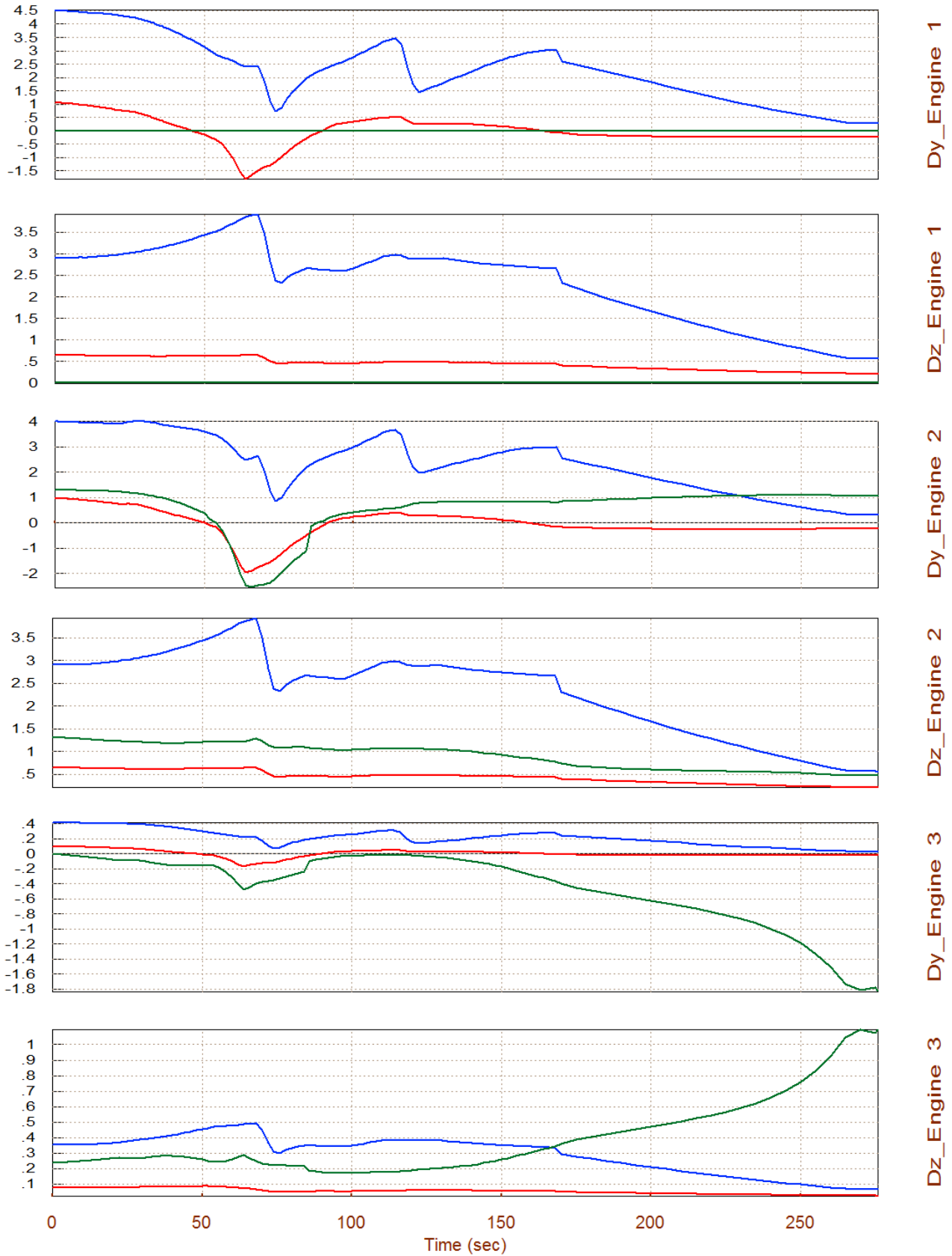
Engine Description, Thrust (lb)	Mass (slug)	Ieng (slug-ft <sup>2</sup> )	Mom Arm (Ft)	Location (x,y,z) (feet)	Mounting Angles (Dy, Dz) Elevat, Azimuth (degr)	Max Deflection Dym,Dzm (deg)	Max Thrott1 (0-1)
Main Engine#1	70000.0	100.0	1500.0	2.8 -89.0 -4.8 0.0	-0.5 -0.5	0.001 0.001	0.0
Main Engine#2	70000.0	100.0	1500.0	2.8 -89.0 +4.8 0.0	-2.0 0.0	14.0 12.0	0.0
Main Engine#3	52000.0	100.0	1500.0	2.8 -89.0 -2.25 +3.9	-2.0 0.0	5.0 5.0	0.95
Main Engine#4	52000.0	100.0	1500.0	2.8 -89.0 +2.25 +3.9	-2.0 0.0	5.0 5.0	0.95
Main Engine#5	52000.0	100.0	1500.0	2.8 -89.0 -2.25 -3.9	-2.0 0.0	0.0 0.0	0.95
Main Engine#6	52000.0	100.0	1500.0	2.8 -89.0 +2.25 -3.9	-2.0 0.0	0.0 0.0	0.95

The trimming process is repeated using the failed actuator file and the results are shown by the green curves in the plots below, using option-12. The red and blue curves are the same as before. The red is from the original 6 engines file "*ReSP6.Engn*", the blue is from the failed thrust file "*ReSP5b.Engn*", and green is from the failed actuator file "*ReSP5c.Engn*". Note that the deflections in option-12 are measured from the engine bias positions, so the zero deflections of TVC engine #1 (green curves) corresponds to the failed positions ( $\delta\gamma=-0.5^\circ$ ,  $\delta z=-0.5^\circ$ ). The green curves show that the remaining TVC engines and aerosurfaces are deflecting further to make up for the disability of the Engine #1 to gimbal.

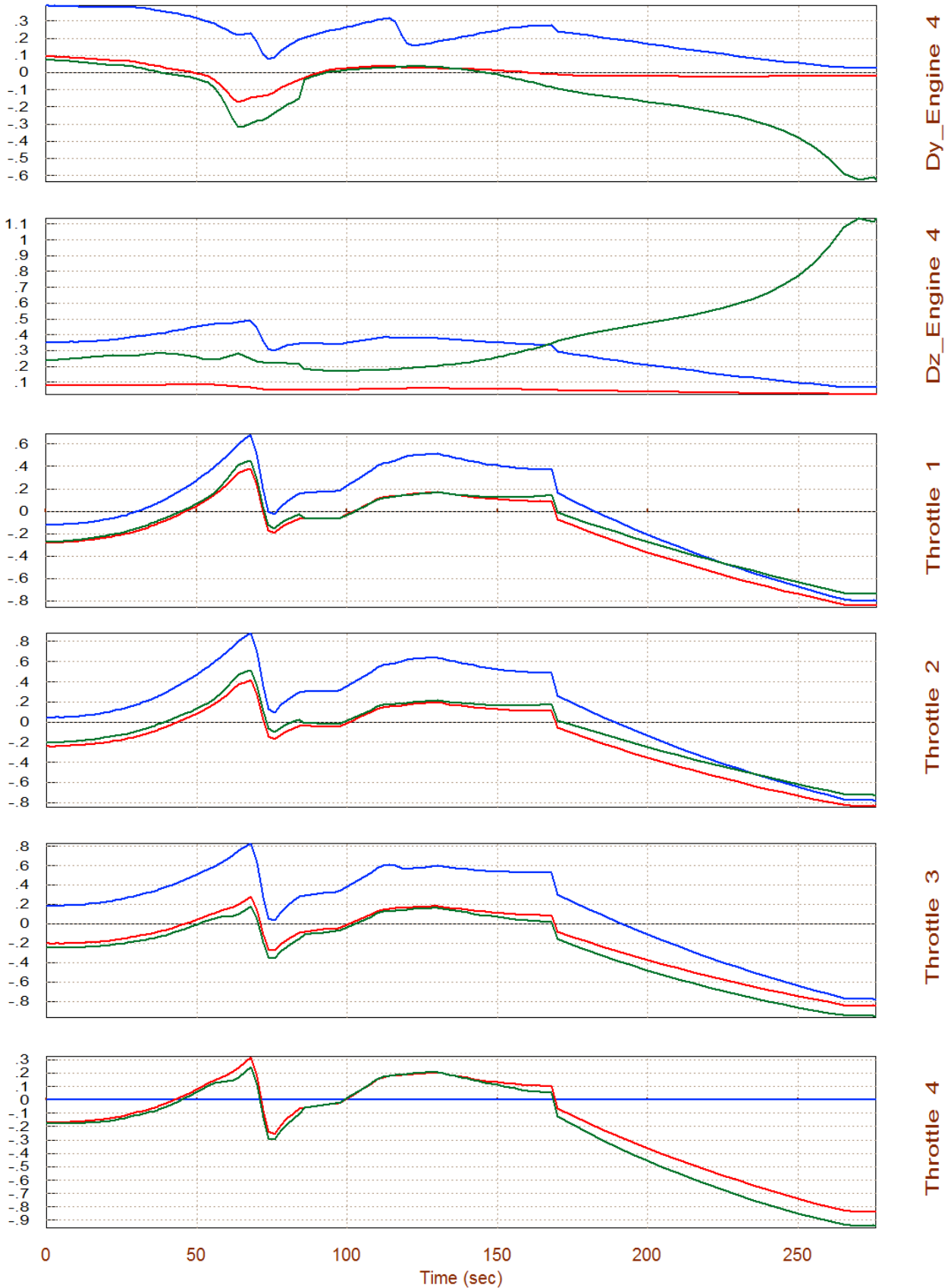
Surface & Engine Deflections/ Throttle, Reusable Space-Plane Ascent Trajectory-2



Surface & Engine Deflections/ Throttle, Reusable Space-Plane Ascent Trajectory-2



Surface & Engine Deflections/ Throttle, Reusable Space-Plane Ascent Trajectory-2



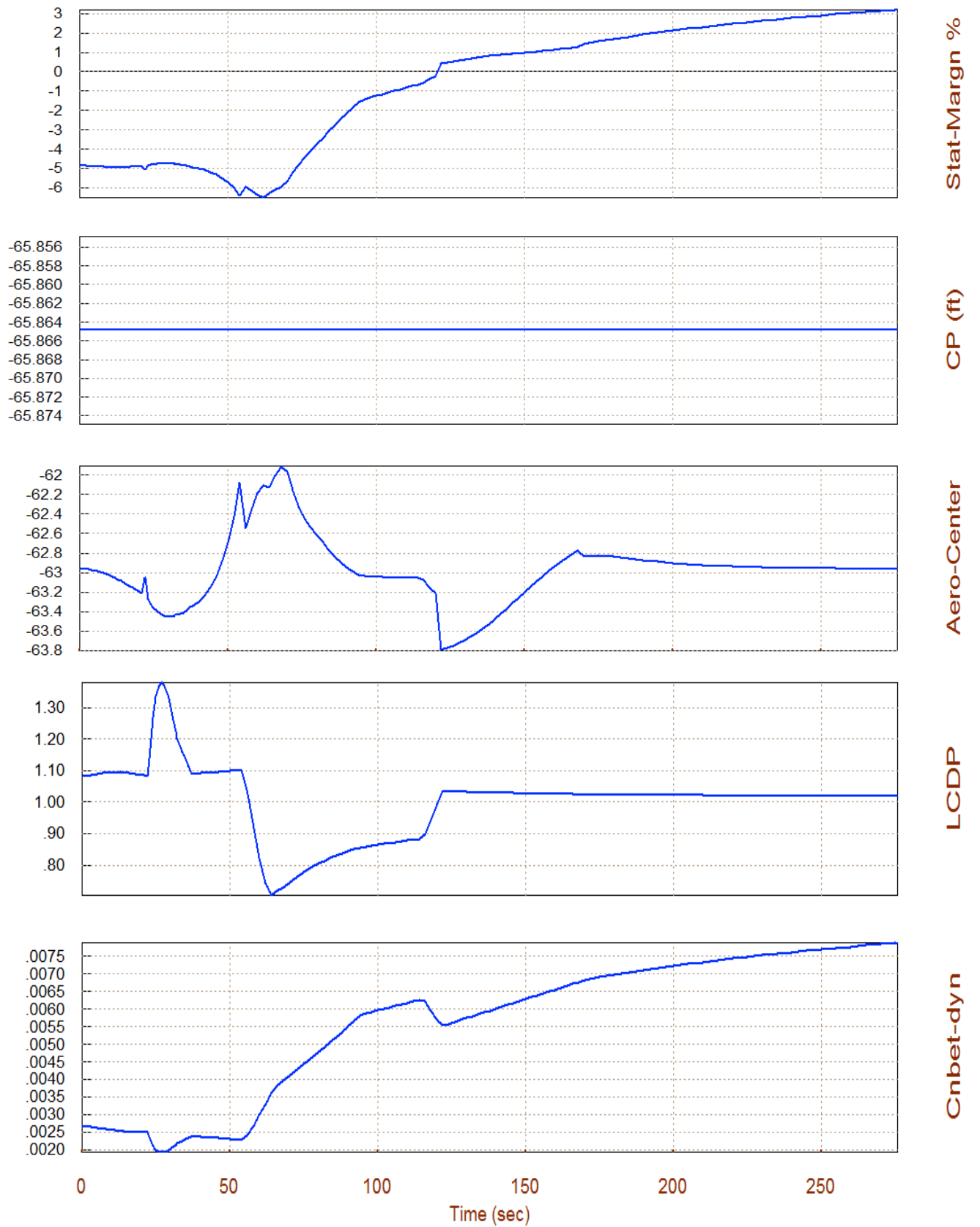
## 1.7 Static Performance Analysis Using Trajectory (B)

Let us now analyze the vehicle performance parameters along the second trajectory. The second trajectory is easier to analyze because it does not have the large pitch maneuver. Return to the main menu and select option-6 that plots the performance and stability parameters along the trajectory. Allow the program to calculate its own effector combination matrix, like before. This vehicle will be controlled along the four directions that we also trimmed, so the mixing matrix has four inputs that correspond to the four control/trim directions (three rotations plus x-acceleration), and 14 outputs that correspond to the 14 effectors (8 TVC, 4 throttling, and 2 aerosurfaces). In the disturbance definition dialog enter the maximum dispersion angles of attack and sideslip expected due to wind-shear  $\alpha_{\max}=4^\circ$ ,  $\beta_{\max}=4^\circ$ , and also the maximum airspeed variation  $V_{\max}$ .

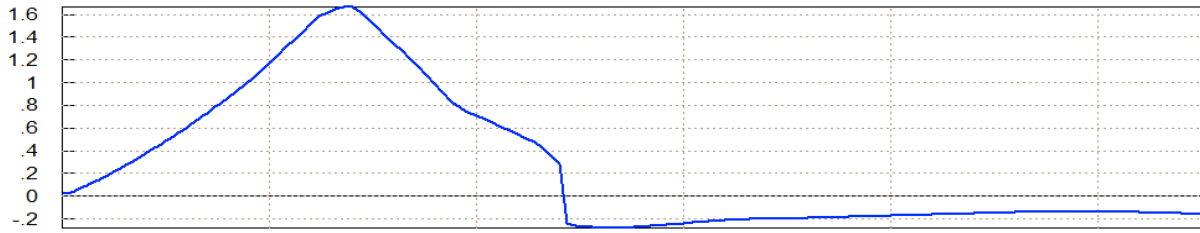
The static stability results of this launch vehicle along the second trajectory are similar to the results obtained from the first trajectory. It is unstable in pitch during the first 130 seconds and later it becomes stable. During the unstable period the shortest time to double amplitude  $T_2=0.62$  (sec) and occurs at 70 sec. In the lateral direction the vehicle is always stable with a positive  $C_n\beta$ -dynamic. The yaw  $T_2$ -inverse parameter is negative (stable), and the Dutch-roll resonance peaks to 1.4 (rad/sec) at 80 sec. The  $(Q_\alpha, Q_\beta)$  loading at  $\alpha_{\max}=4^\circ$ ,  $\beta_{\max}=4^\circ$  is also acceptable. It peaks at about 1000 (psf) in the high dynamic pressure region. The control effort is good in all four directions (roll, pitch, yaw, and x-acceleration), because it requires less than  $\pm 55\%$  of full control capability to counteract variations due to  $\alpha_{\max}=\pm 4^\circ$  and  $\beta_{\max}=\pm 4^\circ$ . This allows sufficient control authority for maneuvering and for reacting against wind-gust disturbances. It means that changes in the accelerations due to wind-shear disturbances are easily counteracted by the controls.

The LCDP ratio is positive and very close to one, which implies that the vehicle has good roll controllability and turn coordination through the entire ascent trajectory. It means that we can rely on the elevons and the TVC to control roll without any assistance from RCS jets. The  $C_n\beta$ -dynamic parameter is positive which implies that the vehicle also has good directional stability. This trajectory has a definite performance advantage in the LCDP in comparison with the previous trajectory. In the previous performance analysis the LCDP had changed signs a couple of times because of the large angle pitch maneuver. Small magnitudes and sign reversals in the LCDP are undesirable because it also implies reversals in the control law gains. In this trajectory, however, the LCDP ratio remains positive and very close to one (which implies good turn coordination) through the entire ascent trajectory. It means that we can rely on the flaps and the TVC to control roll along trajectory (b) without any assistance from RCS jets.

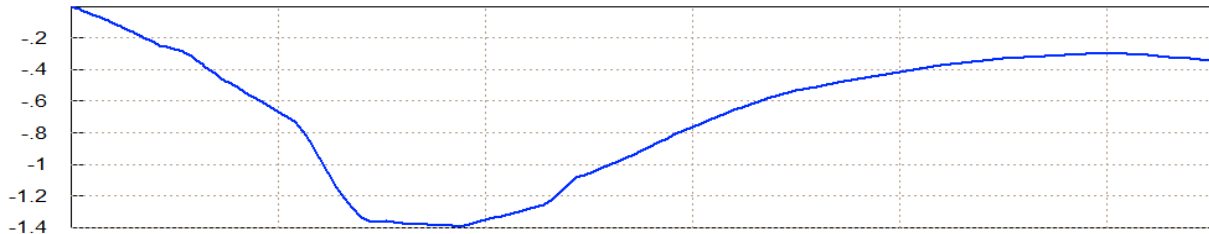
### Static Margin, Center of Pressure, Aero-Center (ft), Reusable Space-Plane Ascent



Short-Period ( $\omega$ ) / Time-to-Double-Ampl-Inverse (/sec),  $Q_{\alpha\beta}$  (deg-lb/ft<sup>2</sup>)

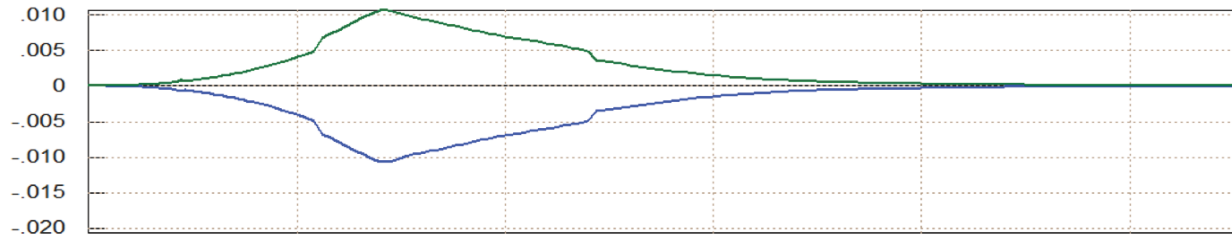


Ptch T2-inv



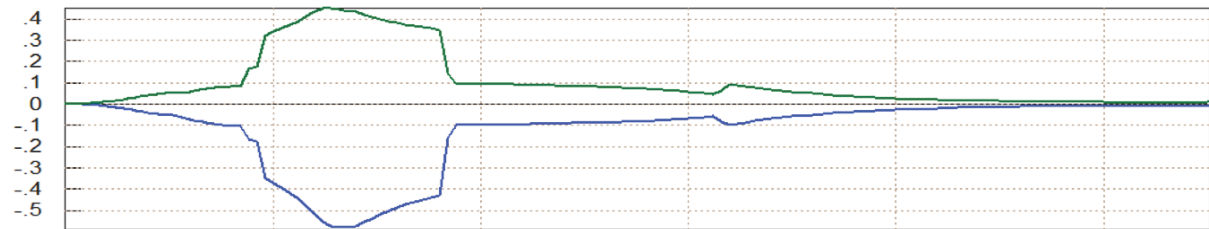
Yaw T2-inv

Translation Control Effort  $|dX/dX_{max}| < 1$  Against 4 (deg) of Alpha/Beta Variation

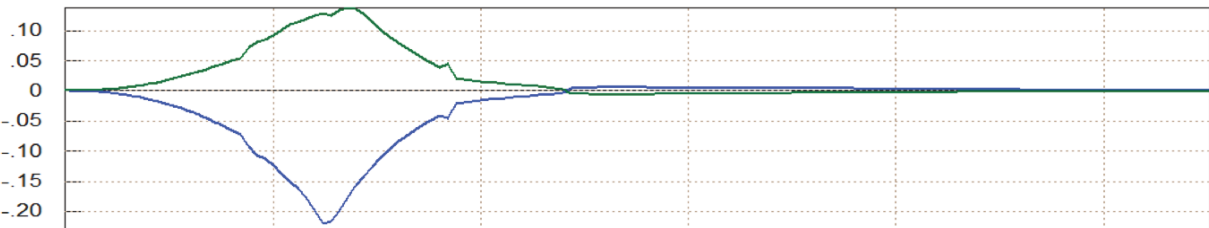


$dX_{dd}/dX_{max}$

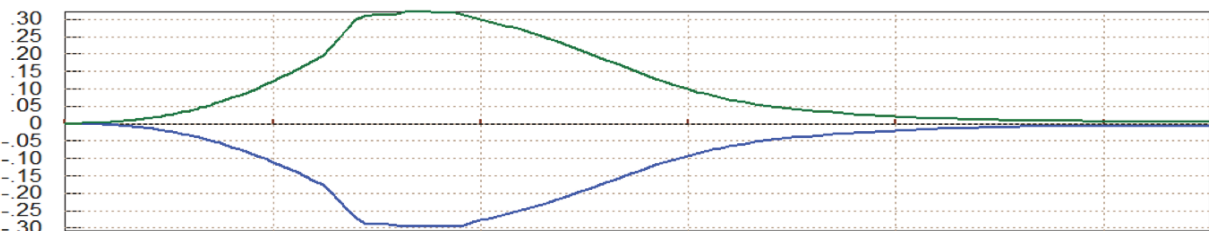
Rotation Control Authority  $|dQ/dQ_{max}| < 1$  for 4 (deg) of Alpha & Beta Variation



Roll Effort



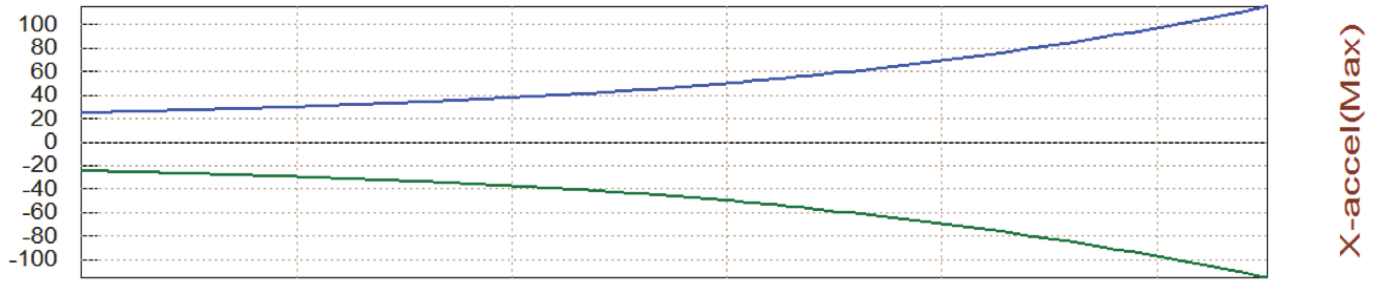
Pitch Effort



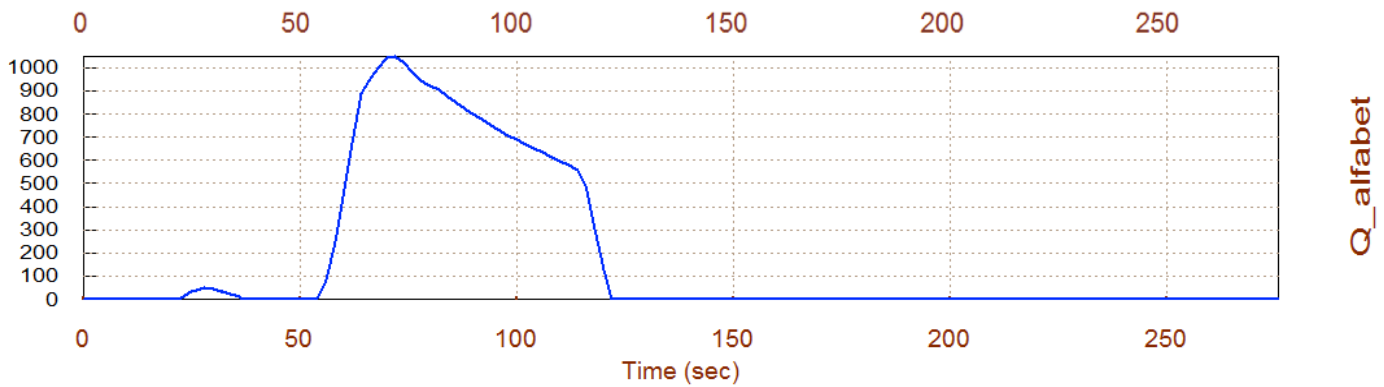
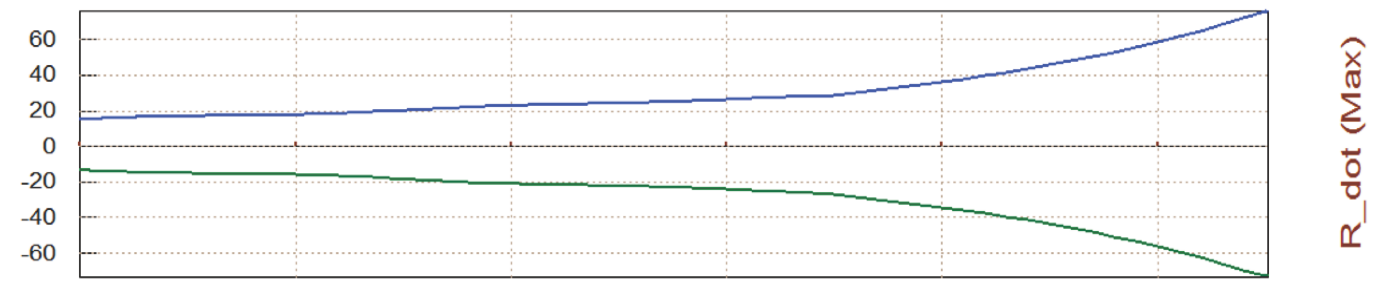
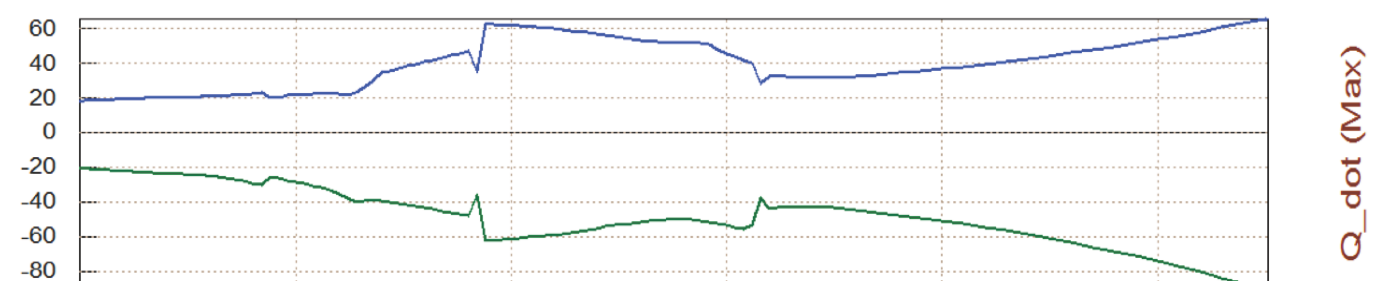
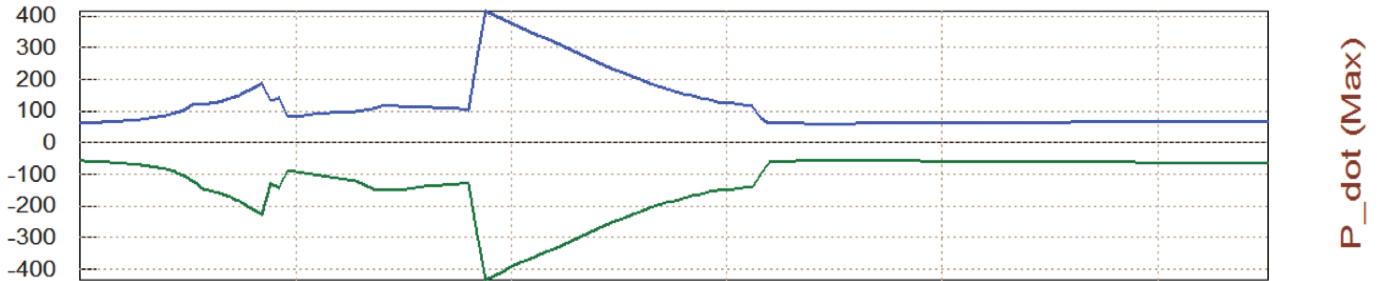
Yaw Effort

0 50 100 150 200 250  
Time (sec)

Max Linear Accelerations in (ft/sec<sup>2</sup>), at Maximum +ve and -ve Control Demands



Max Angular Accelerations (rad/sec<sup>2</sup>), at Maximum +ve and -ve Control Demands





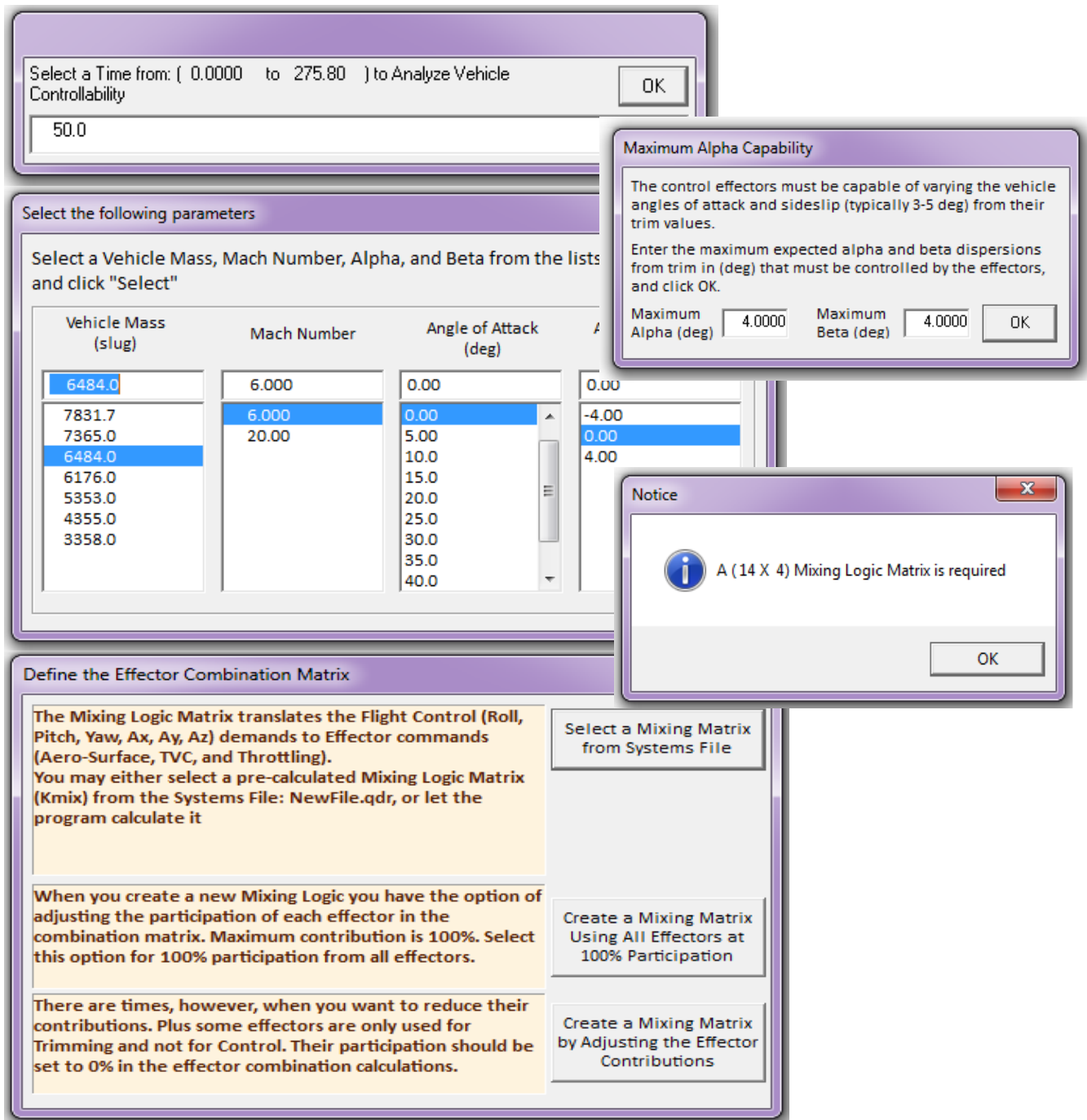
## 1.8 Vector Diagrams Analysis

Let us now examine the vehicle controllability against aerodynamic disturbances caused by alpha and beta dispersions from trim by using vector diagrams. The mixing logic matrix is shown below. It converts the control acceleration demands: roll, pitch, yaw, and axial accelerations to TVC and aerosurface deflections and to throttle commands. The deflections and thrust variations produce the accelerations in the required directions. Notice that the mixing matrix calculation is not exactly symmetric because of the YCG offset.

Acceleration Demands in 4 Directions				Effector Commands
dP	dQ	dR	dA <sub>x</sub>	
0.1465	-0.5698	0.0300	0.0003	Engine 1 Ptch Deflect dy1 (rad)
-0.1393	-0.4969	-0.0289	0.0003	Engine 2 Ptch Deflect dy2 (rad)
0.0071	-0.0552	0.0014	0.0000	Engine 3 Ptch Deflect dy3 (rad)
-0.0064	-0.0517	-0.0013	0.0000	Engine 4 Ptch Deflect dy4 (rad)
0.0101	-0.0015	-0.7207	-0.0002	Engine 1 Yaw Deflect dz1 (rad)
0.0101	-0.0015	-0.7207	-0.0002	Engine 2 Yaw Deflect dz2 (rad)
-0.0103	0.0028	-0.1007	-0.0000	Engine 3 Yaw Deflect dz3 (rad)
-0.0103	0.0028	-0.1007	-0.0000	Engine 4 Yaw Deflect dz4 (rad)
-0.0324	0.9381	0.9775	0.0292	Engine 3 Throttle Input dTh3/Th3
0.0063	0.9312	-0.9814	0.0286	Engine 4 Throttle Input dTh4/Th4
-0.0076	-0.9227	0.9943	0.0302	Engine 5 Throttle Input dTh5/Th5
0.0317	-0.9297	-0.9907	0.0296	Engine 6 Throttle Input dTh6/Th6
0.1680	-0.7860	-0.0353	-0.0002	Left Elevon Deflection (rad)
-0.2574	-0.6777	0.0220	-0.0001	Right Elevon Deflection (rad)

### The Mixing Logic Matrix

From the main menu select option-11 for plotting vector diagrams, assuming that the initialization files and the trim conditions have not changed. Choose a flight time between 0 and 276 sec, and from the following menus select the default values of Mass, Mach,  $\alpha$ , and  $\beta$  which correspond to the selected time. The aero disturbance must also be defined in terms of maximum alpha and beta dispersion angles from trim. The program requires a (14x4) effector mixing matrix that will combine the two aero-surfaces with the TVC and throttling engines. Since we have not selected a matrix yet, from the following dialog select the second option to allow the program to create a mixing logic matrix using 100% participation from all effectors.



The vector diagrams in Figure 1.8.1 show the maximum accelerations in the lateral directions produced by maximizing the lateral roll and yaw demands. The blue vectors are the moments and side accelerations produced at max  $\pm$  yaw demands  $\delta R_{\pm FCSMax}$ . The second figure shows that a +yaw demand produces negative side acceleration due to the negative ( $-\delta z$ ) TVC deflection. The green vectors are the moments and side accelerations produced at max  $\pm$  roll demands  $\delta P_{\pm FCSMax}$ . The solid line vectors denote the accelerations when the demands are maximum positive and the dashed vectors are due to max negative demands. The small red vectors represent the roll/yaw moments and accelerations generated by the  $\pm\alpha_{max}$  and  $\pm\beta_{max}$  dispersions from trim. The diagrams demonstrate that both roll and yaw directions are well controllable because the two control

vectors are more powerful than the red disturbance vectors. They are also very nicely decoupled from each other and are pointing in the demanded directions. If you right-click the mouse button it opens an info window showing the trimming conditions.

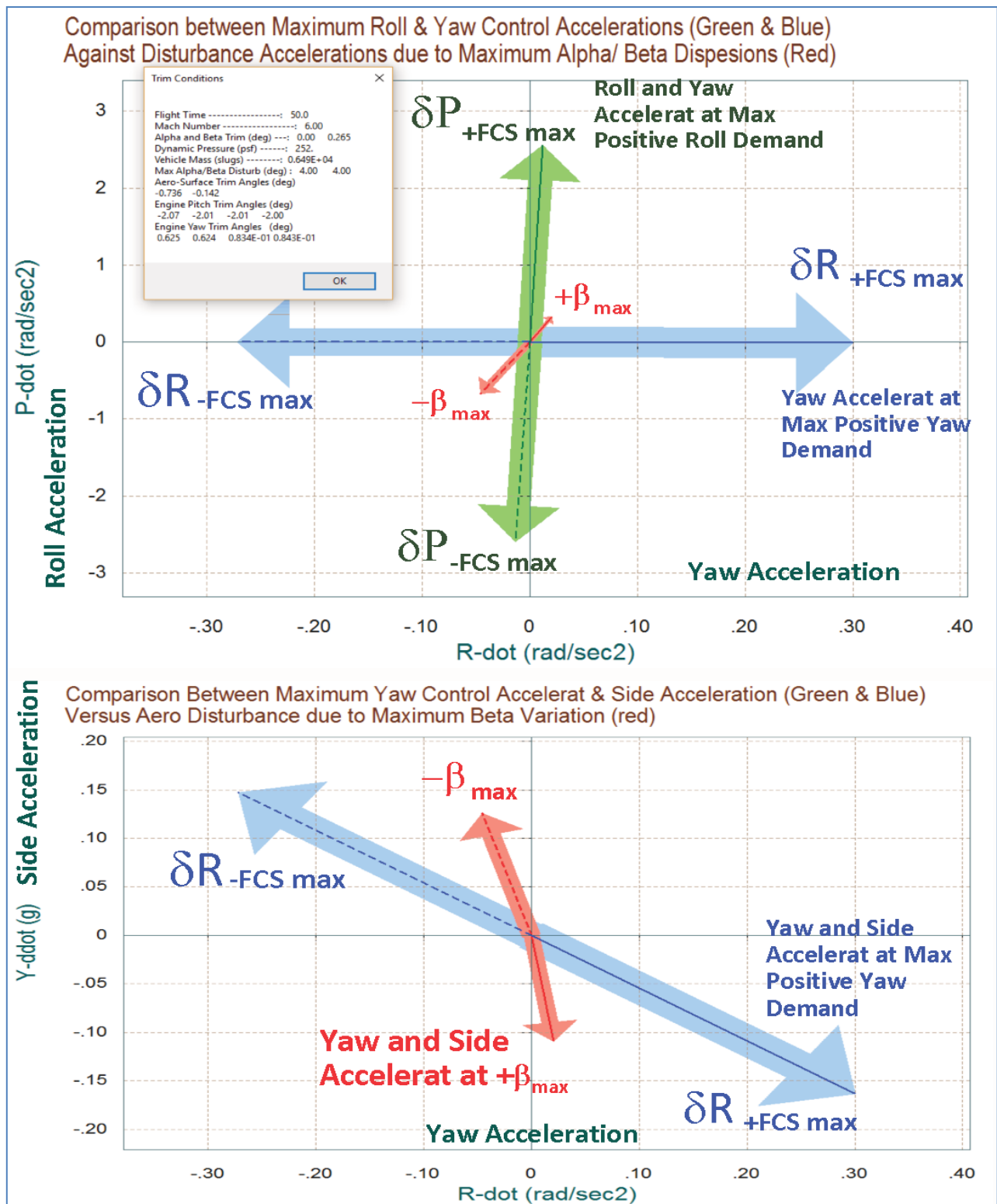


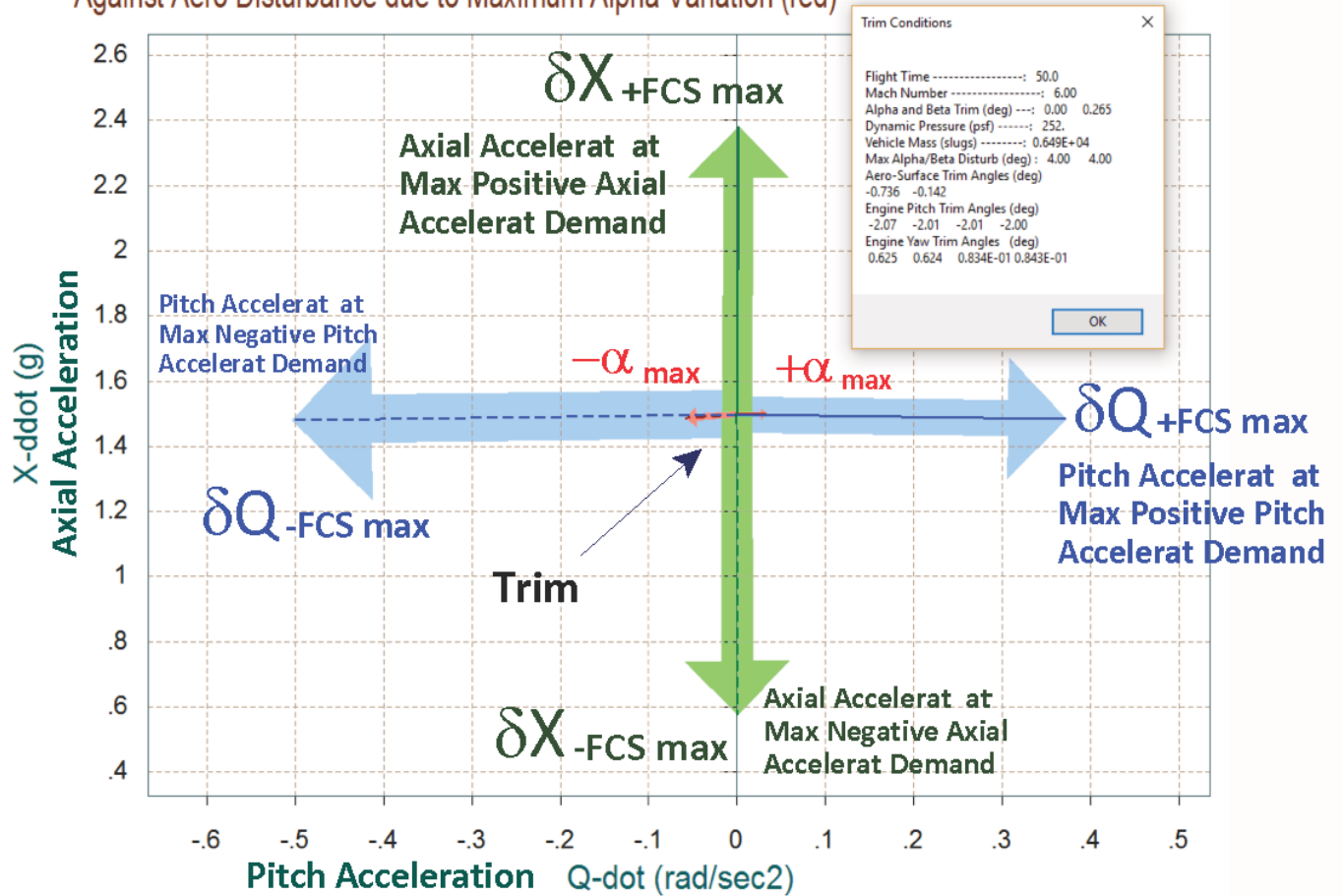
Figure 1.8.1 Maximum Acceleration Vector Diagrams in the Lateral Directions

The vector diagrams in Figure 1.8.2 show the maximum accelerations in the longitudinal directions produced by maximizing the pitch and axial acceleration demands. The pitch axis is controlled by gimbaling the first 4 TVC engines in pitch. The axial acceleration is controlled by throttling the last 4 engines. The pitch acceleration variations are in (rad/sec<sup>2</sup>) and the axial accelerations are in g. The horizontal blue vectors denote the accelerations at max  $\pm$ pitch demands  $\delta Q_{\pm FCSMax}$ , and the vertical green vectors represent the accelerations at max  $\pm$ axial acceleration demands  $\delta X_{\pm FCSMax}$ . The two control vectors are completely orthogonal and decoupled by the mixing-logic matrix. At trim the vehicle has 1.5 g x-acceleration and 0.15 g z-acceleration due to the engine steady thrusts. It can be varied between 0.6 g and 2.4 g by throttling the engines.

The vector diagram at the bottom shows the pitch and normal accelerations produced by maximizing the pitch demand  $\delta Q_{\pm FCSMax}$  in the positive (solid blue line) and in the negative (dashed blue line) directions. A pitch control demand requires negative pitch deflections in the 4 TVC engines which produce positive z-acceleration. The diagrams show that the pitch control is asymmetric. You can produce more negative pitch acceleration and more negative z-acceleration from trim than positive pitch and z-accelerations. This is because the TVC engines are already trimmed at -2.05°. They only have 3° of negative deflection remaining to hit the negative TVC limit in order to produce positive acceleration, but in the opposite direction they have 7° of rotation left towards the positive TVC limit to produce negative acceleration. The red vectors show the accelerations produced by the variations  $\pm\alpha_{max}$  and  $\pm\beta_{max}$  from trim. The alpha variation affects mainly the pitch and Z accelerations, not X. A positive dispersion  $\alpha_{max}$  produces positive pitching moment because the vehicle is unstable.

The vector diagrams in Figure 1.8.3 shows the effect of maximizing the axial control demand  $\delta X_{\pm FCSMax}$  on the X and Z translational accelerations. The blue control vectors are the same in both diagrams. It shows the trim x and z accelerations (1.5 and 0.15) g respectively, and the range of accelerations attained by  $\delta X_{\pm FCSMax}$ . The red disturbance vectors are different. The top diagram shows the accelerations produced by the  $\pm\alpha_{max}$  and  $\pm\beta_{max}$  dispersions from trim, and the bottom diagram shows the accelerations produced by the  $\pm V_{max}$  variations from  $V_0$ . Obviously, an increase in  $\alpha$  from  $\alpha_0$  produces negative z-acceleration, and an increase in air-speed at  $\alpha_0=0$  produces positive z-acceleration.

Comparison between Maximum Pitch and Axial X-Force Control Accelerat (Blue & Green) Against Aero Disturbance due to Maximum Alpha Variation (red)



Comparison between Maximum Pitch and Normal-Z Control Accelerations (Blue & Green) Against Aero Disturbance due to Maximum Alpha Variation (red)

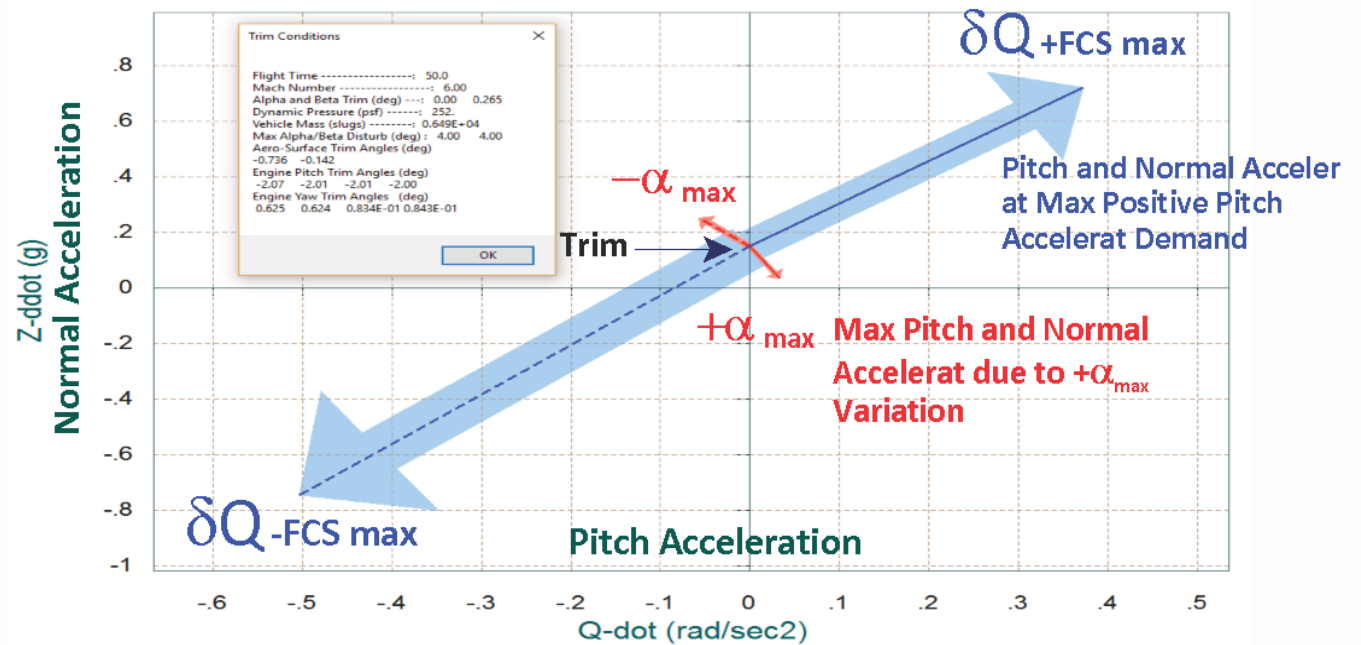
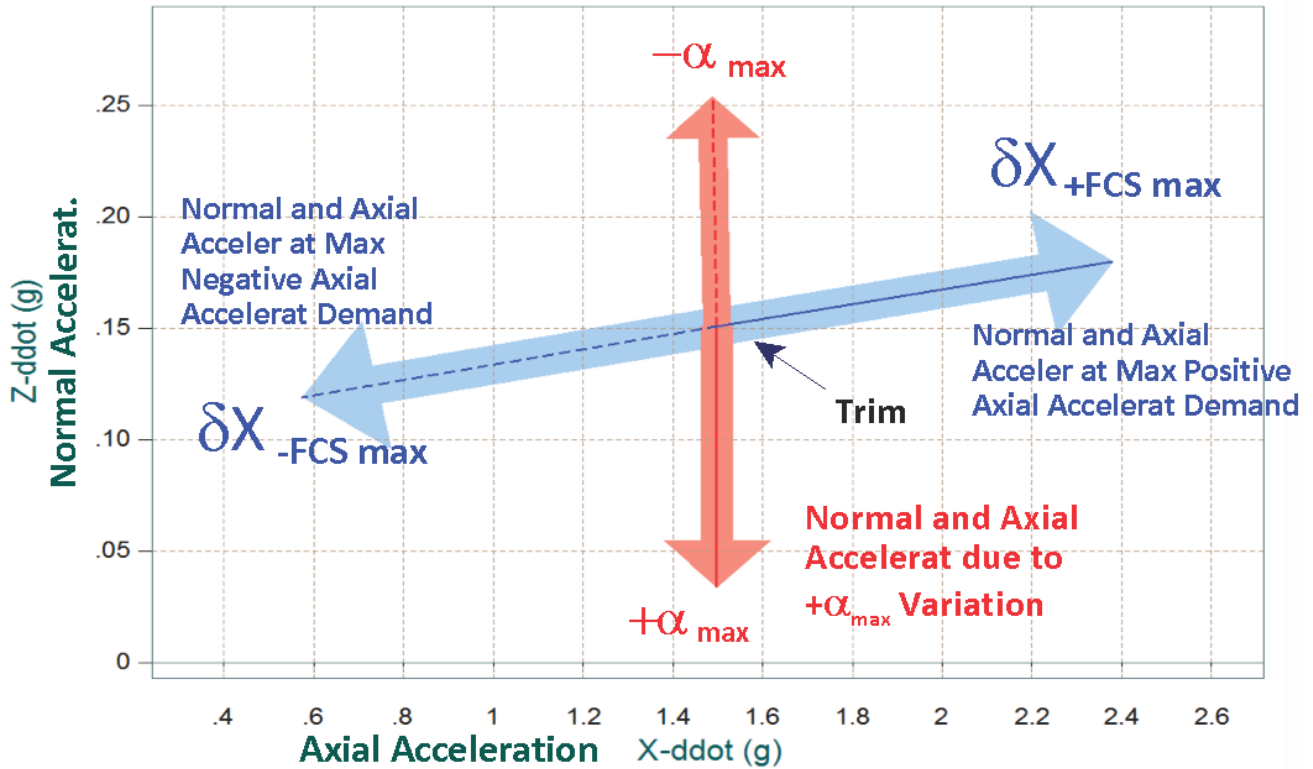


Figure 1.8.2 Maximum Acceleration Vector Diagrams in the Longitudinal Directions

Comparison between Maximum X and Z Control Accelerations (Blue & Green) against Disturbance Aero Forces due to Maximum Alpha/ Beta Dispersions (Red)



Comparison between Maximum X and Z Control Accelerations in g (Blue & Green) Against Aero Disturbance Forces due to Delta-Velocity Dispersion (Red)

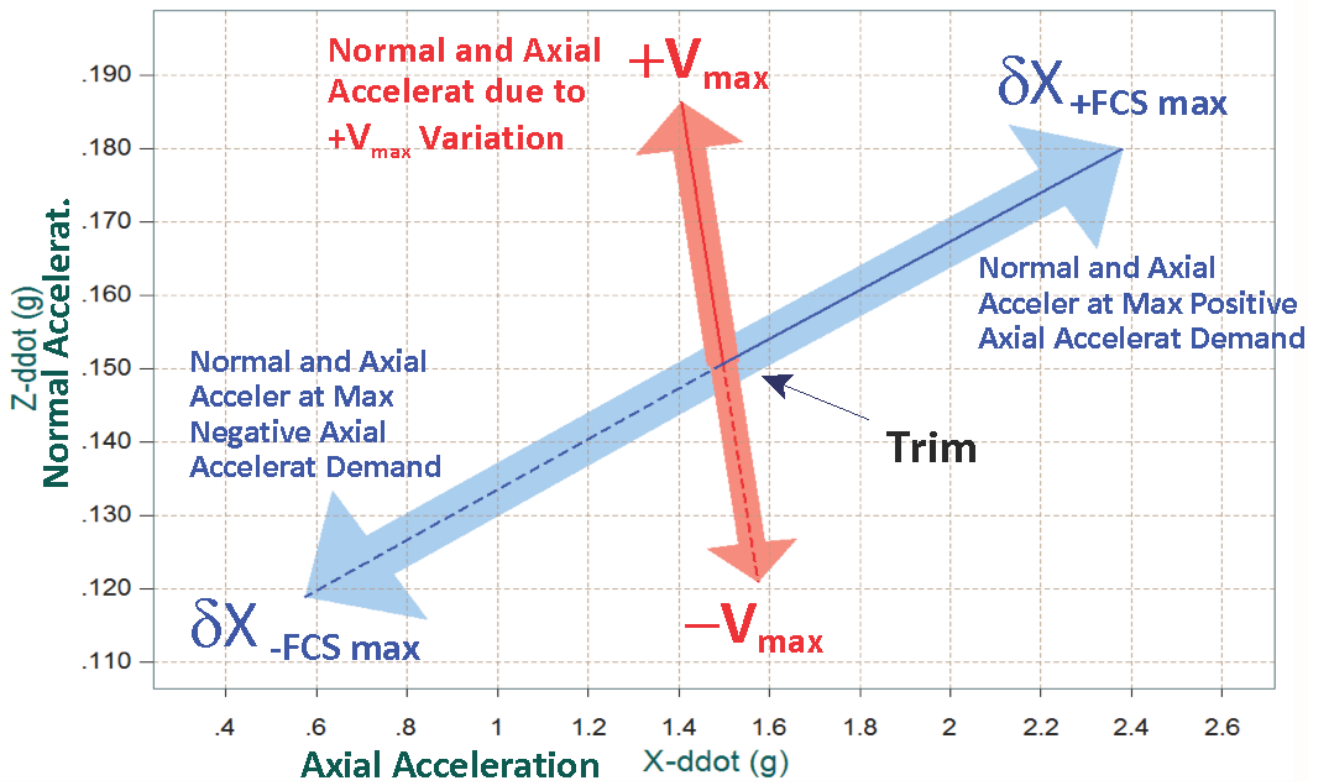
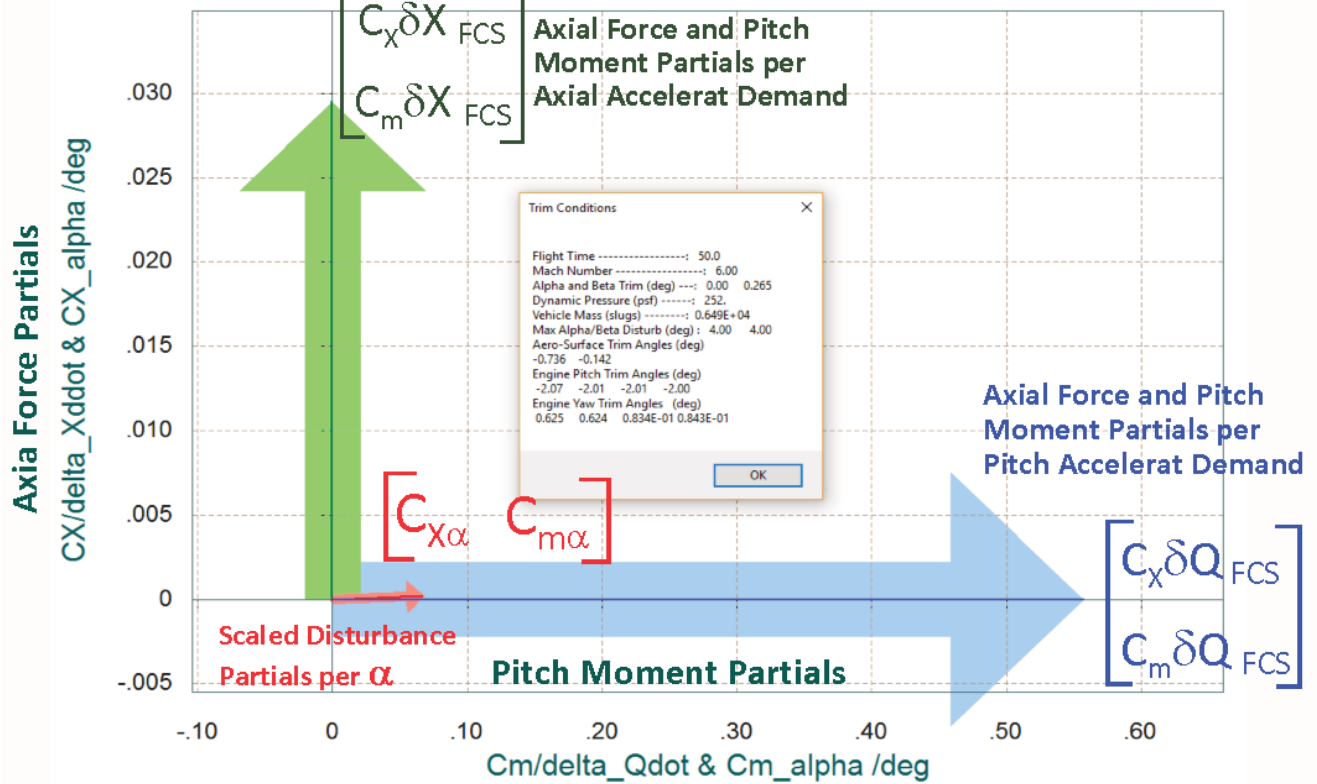


Figure 1.8.3 Maximum X and Z Accelerations Vector Diagrams Against  $\pm\alpha_{\max}$  and  $\pm V_{\max}$  Dispersions



Comparison Between Control Moment & Force Partial:  $\{C_m/\delta_Q \text{ \& } C_X/\delta_X\}$   
 (Blue and Green), Against Partial:  $\{C_m/\alpha \text{ \& } C_X/\alpha\}$  (Red Vectors)



Comparison Between Yaw & Roll Control Moment Partial  $\{C_n/\delta_R \text{ \& } C_l/\delta_P\}$   
 (Blue and Green Vectors) Against Partial:  $\{C_n/\beta \text{ \& } C_l/\beta\}$  (Red Vectors)

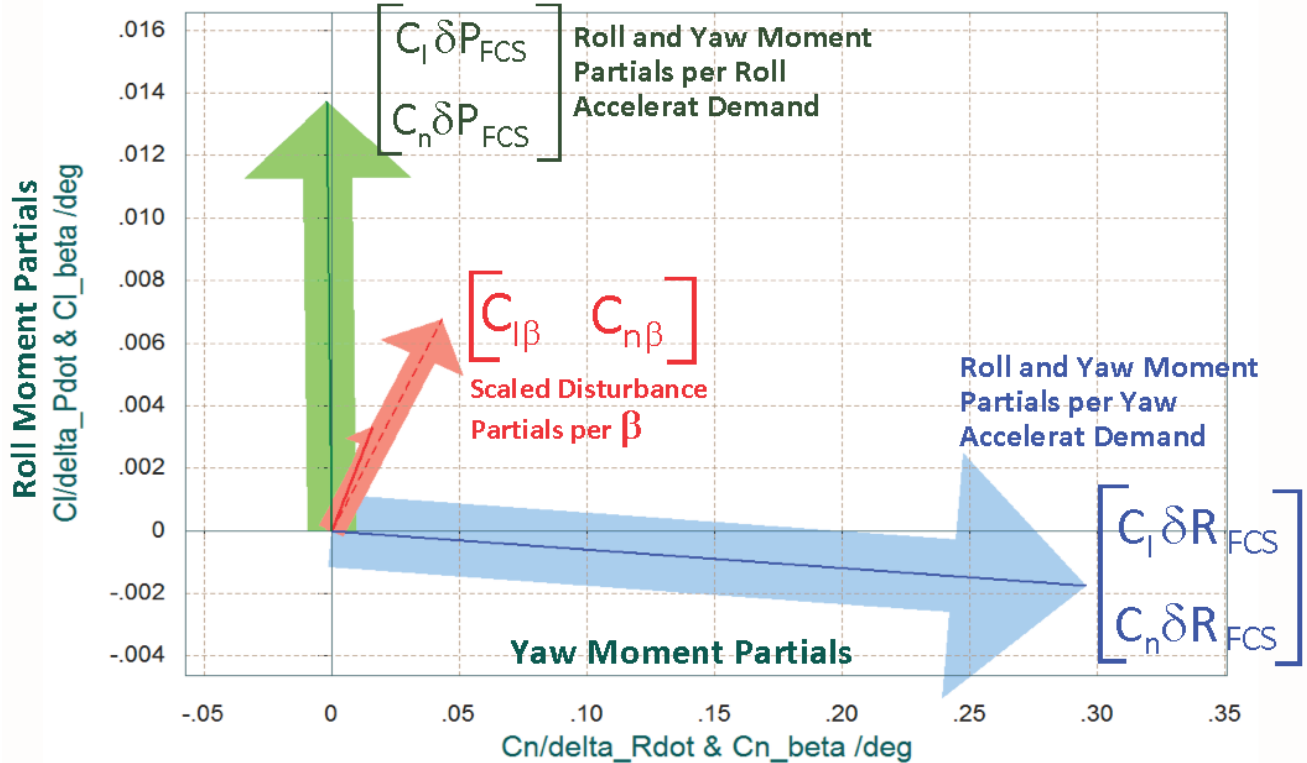


Figure 1.8.4 Vector Partial in the Longitudinal and Lateral Directions



The vector partials diagram in Figure 1.8.4 shows the moments and force per demand in the four control directions. The blue and green vectors in top diagram show the pitch moment and x-force partials per pitch ( $\delta Q_{FCS}$ ) and axial acceleration ( $\delta X_{FCS}$ ) demands respectively. They are orthogonal, perfectly decoupled, and pointing in the proper directions. The smaller red vector is the x-force and pitch moment partials per alpha ( $Cx\alpha$ ,  $Cm\alpha$ ) and it is small in comparison with the control partials. It is scaled to be made comparable with the control partials. The partial  $Cm\alpha$  is positive which means that the vehicle is unstable in pitch.

The bottom vector partial diagram in Figure 1.8.4 shows the roll and yaw moments per roll and yaw control demands, against the moment partials per beta variation ( $Cl\beta$ ,  $Cn\beta$ ). The green upward pointing vector is the moment partial per roll demand ( $Cl\delta P_{FCS}$ ,  $Cn\delta P_{FCS}$ ), and the blue vector toward the right is the moment partial per yaw demand ( $Cl\delta R_{FCS}$ ,  $Cn\delta R_{FCS}$ ). Both vectors are pointing in the proper directions with very small amount of cross-coupling. The red disturbance partials ( $Cl\beta$ ,  $Cn\beta$ ) are properly scaled in order to be made comparable to the blue and green control vectors, as described in equations (8.1 through 8.4). They are smaller in magnitude than the control partials which implies good static controllability. The vehicle is stable in yaw in this flight condition because  $Cn\beta$  is positive.

Figure 1.8.5 below shows the acceleration partials in the four control directions. The left diagram shows the pitch and axial accelerations per pitch demand (blue vector), and per axial or throttle control demand (green vector). The diagram on the right shows the roll and yaw accelerations per roll acceleration (green vector), and per yaw acceleration demand (blue vector). All four acceleration partials are perfectly orthogonal to each other. They are unit vectors pointing in the commanded directions. The decoupling between axes is accomplished by the mixing-logic matrix  $K_{mix}$ .

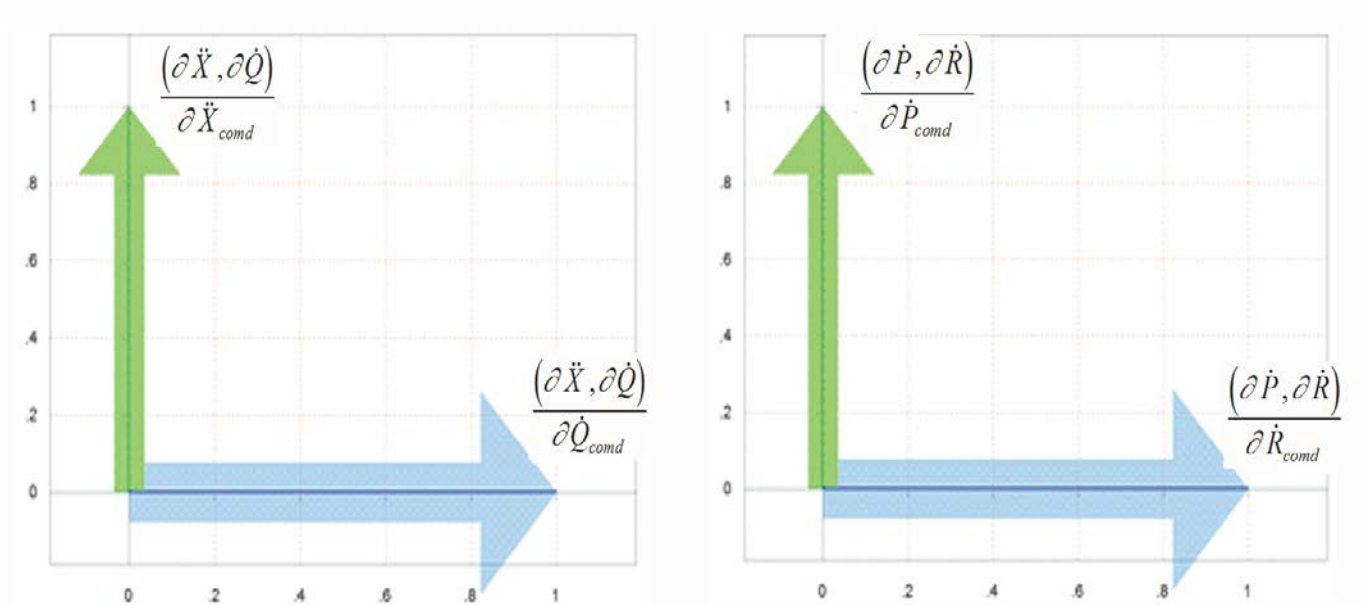


Figure 1.8.5 Acceleration Partial in the 4 Control Directions

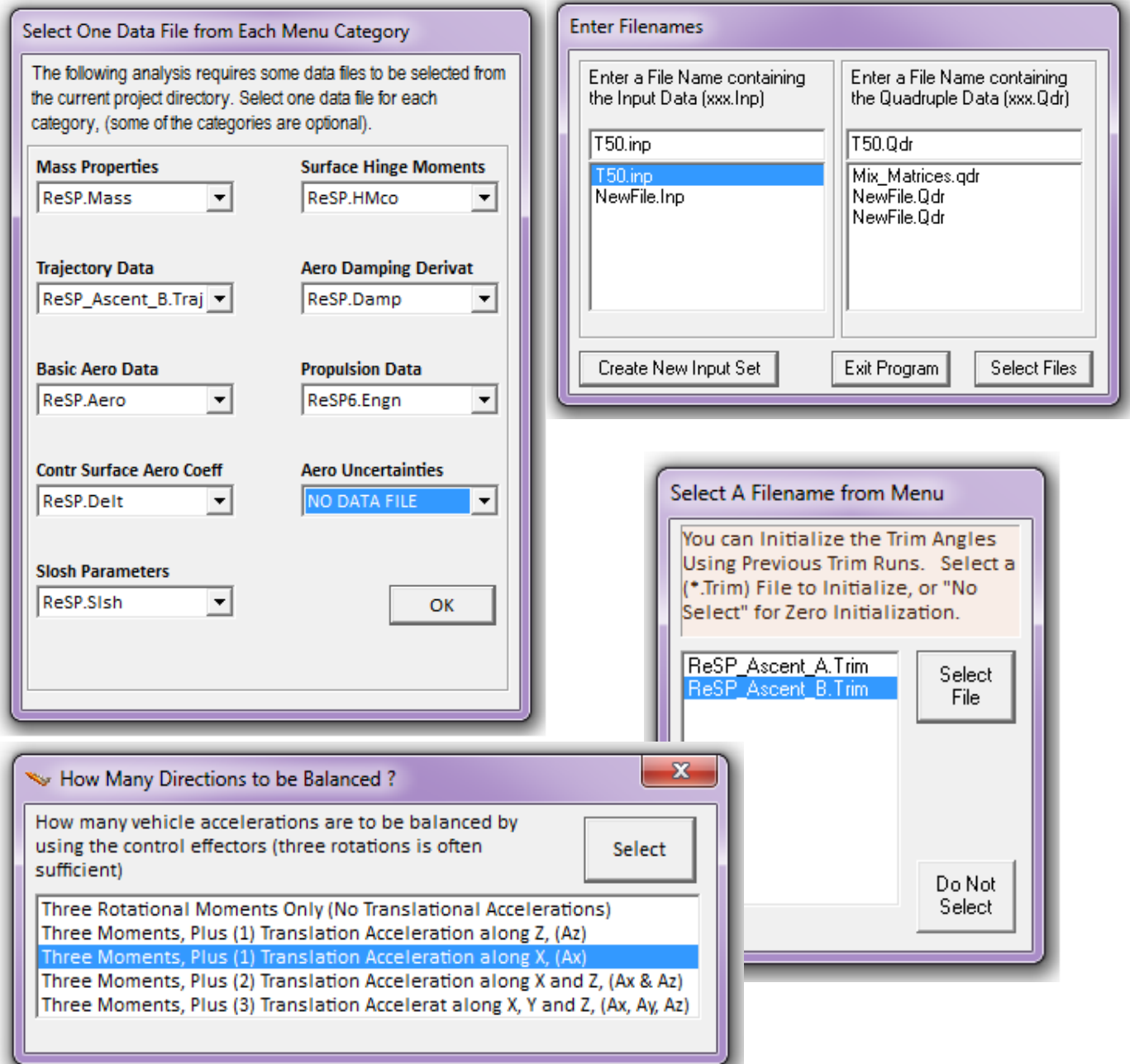
## 1.9 Creating an Ascent Dynamic Model

We now come to the most interesting part which is the control design. We will use the Flixan and Trim programs to generate a dynamic model for our launch vehicle at:  $t=50$  sec after lift-off, and in the next section we will design a flight control system and analyze it. This vehicle blends multiple types of effectors together and it makes a good example for demonstrating the capability of the program to create dynamic models of vehicles that use multiple types of effectors and the methodology for creating a mixing logic that optimally blends the effectors together for the purpose of maximizing controllability. The dynamic model will be created in file "T50.Qdr", and it has 14 control inputs and each input corresponds to an effector. That is: 8 gimbal deflection inputs for the four TVC engines (4 pitch and 4 yaw gimbals), 4 throttling inputs for the four engines that have a variable thrust, and 2 control surface inputs (deg). Among the 6 main engines; engines #1 and #2 are only gimbaling, engines #5 and #6 are only throttling, and engines #3 and #4 are both gimbaling and throttling. The engine nozzles are not mounted parallel to the vehicle x axis but they are tilted  $-2^\circ$  in pitch, which is also their trim positions. The max deflections of the first two engines are  $\pm 14^\circ$  in pitch and  $\pm 12^\circ$  yaw. Engines #3 and #4 are only able to gimbal  $\pm 5^\circ$  in pitch and yaw. The four throttling engines have a nominal thrust of 55,300 (lb) and they can vary their thrusts  $\pm 95\%$ . That is, from 9,200 (lb) when the throttle input is -1, to 101,400 (lb) when the throttle input is +1. Zero throttle control input corresponds to the nominal thrust. Inputs 13 and 14 correspond to the two Elevon deflections. The 15<sup>th</sup> input to the dynamic model is a wind-gust velocity disturbance in (feet/sec). The wind-gust direction relative to the vehicle is defined in the input data file "T50.inp" perpendicular to the vehicle x-axis in order to excite both pitch and lateral axes.

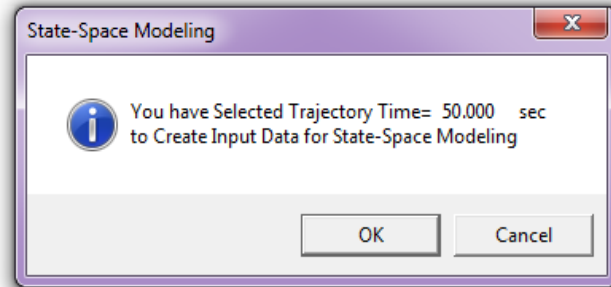
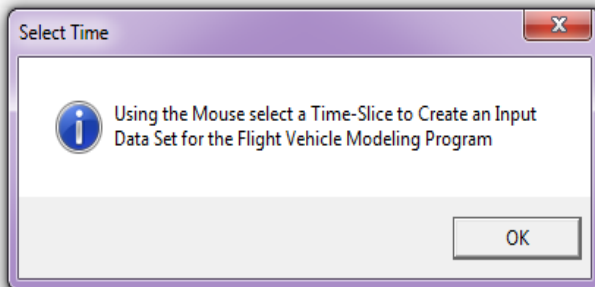
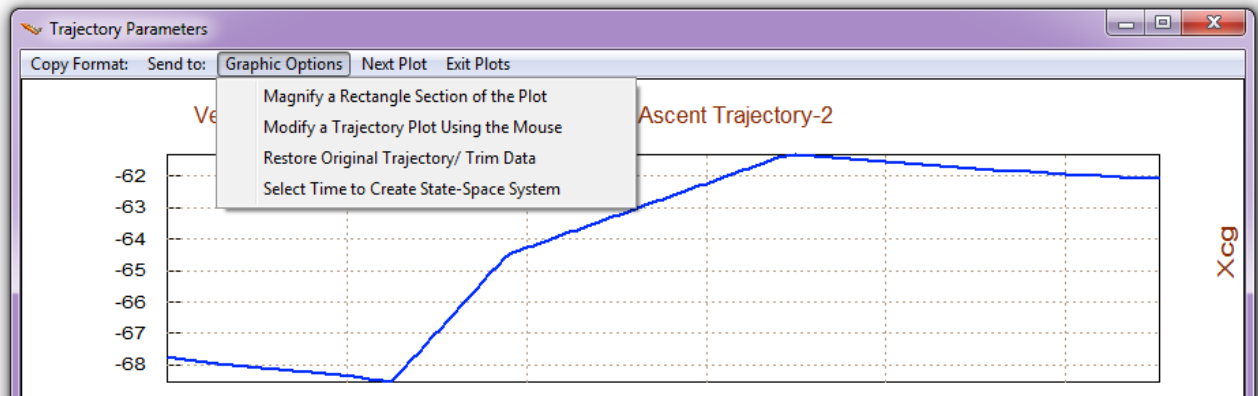
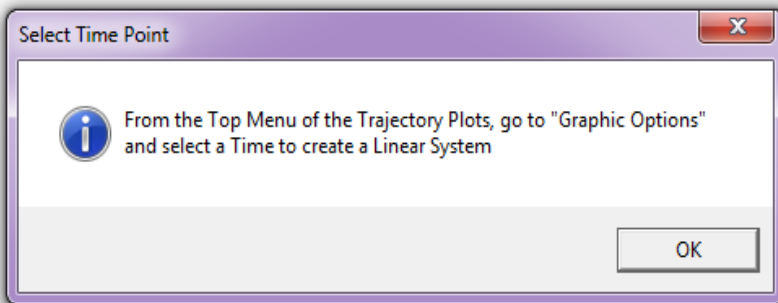
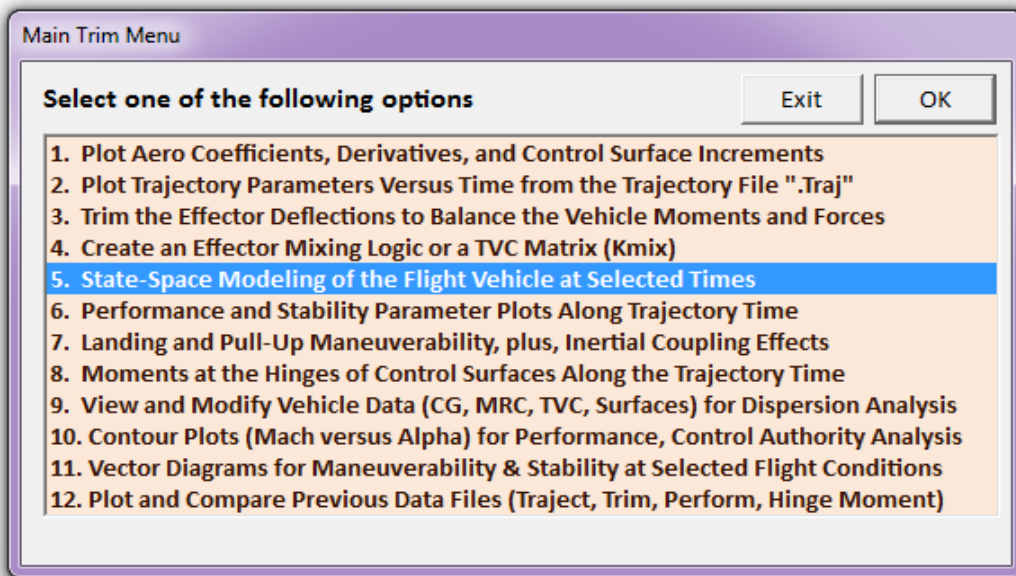
Since this vehicle has many effectors, the coordination between the effectors is taken care by the mixing logic matrix which is calculated by Flixan using the same vehicle model corresponding to this flight condition. The flight control system is expected to control the same four directions as in trimming, that is, roll, pitch, yaw, and axial velocity. The mixing logic matrix translates the four flight control system acceleration demands (roll, pitch, yaw, axial) to 14 effector commands. There is one additional feature that makes this vehicle configuration even more interesting. It includes two tanks containing sloshing propellants and Flixan will create dynamic models for the two slosh resonances. There is a liquid oxygen (LOX) tank located near the rear of the vehicle and it is much heavier than the liquid hydrogen (LH2) tank which is located closer to the front of the vehicle. The slosh parameters are automatically selected from the slosh file "*ReSP.Slsh*" where they are listed as a function of the vehicle mass. The program selects a set of parameters that correspond to the vehicle mass at the flight condition being analyzed, which is at  $t=50$  sec. The modeling procedure and analysis is identical for any other flight time.

So let us continue the analysis using the Trim program. Select the same files as before, and enter an input filename "*T50.inp*" in the filenames selection menu. This file will contain the vehicle input data plus other Flixan related data-sets. Enter also an output filename "*T50.Qdr*" to save the

vehicle systems and matrices. Re-trim the effectors using a mass properties file that has the  $Y_{CG}$  set at zero. It is better to use an unbiased model for the control design so that the gains do not depend on the  $Y_{CG}$  which may change. Do not select a trim initialization file, and trim along the 3 moments plus axial x-acceleration ( $A_x$ ), as shown. From the Trim main menu select option-5 to create a state-space dynamic model.



A dialog reminds the user how to select a flight time for the dynamic model, click "OK". From one of the trajectory plots go the top menu bar, and choose "Graphic Options", and then from the vertical pop-up menu click on "Select Time to Create State-Space System". Then using the mouse click at time= 50 sec, along the time axis. A dialog confirms the time selected for the linear model. Click "OK", otherwise, click "Cancel" and try again.



Flight Vehicle Parameters

**Vehicle System Title**  
Reusable Space-Plane Ascent Trajectory-2/ T= 50.0 sec

**Number of Vehicle Effectors**  
 Gimbal Engines or Jets. Include Tail-Wags-Dog? 6    
 Rotating Control Surfaces. Include Tail-Wags-Dog? 2    
 Reaction Wheels? 0 **Momentum Control Devices**  
 Single Gimbal CMGs? 0 Include a 3-axes Stabilized Double Gimbal CMG System?

**Number of Sensors**  
 Gyros 0  
 Accelerometer 3  
 Aero Vanes 0  
 External Torques 0

**Modeling Options (Flags)**  
 Output Rates in   Turn Coordination?    
 Aero-Elasticity Options   Attitude Angles

**Number of Modes**  
 Structure Bending 0  
 Fuel Sloshing: 2

Reaction Wheels | Single Gimbal CMGs | Double Gimbal CMG System | Slewing Appendages | Gyros | Accelerometer | Aero Sensors | Fuel Slosh | Flex Modes | User Notes  
 Mass Properties | Trajectory Data | Gust/ Aero Paramet. | Aero Force Coeffs | Aero Moment Coeffs | Control Surfaces | Gimbal Engines/ RCS | External Torques

**Trajectory Data**

Inertial Velocity/ Accelerat  
 Velocity (ft/sec) 549.0000  
 Accelerat ft/sec^2 17.50000

Body Sensed Accelerat.  
 Axial 48.20000  
 Ay 0.000000  
 Az -0.5000000  
 (ft/sec^2)

Dynamic Pressure, Q bar, (psf) 252.0000  
 Vehicle Altitude (H) in (feet) 11562.00  
 Mach Number 0.5100000

Angles of Attack, Sideslip in (deg), Rates  
 Alpha 0.000000 Alpha Dot 0.000000  
 Beta 0.000000 Beta Dot 0.000000

Euler Angles in (deg), Angular Rates (deg/sec)  
 Phi 0.000000 Po 0.000000  
 Theta 72.87000 Qo -1.050000  
 Psi 0.000000 Ro 0.000000

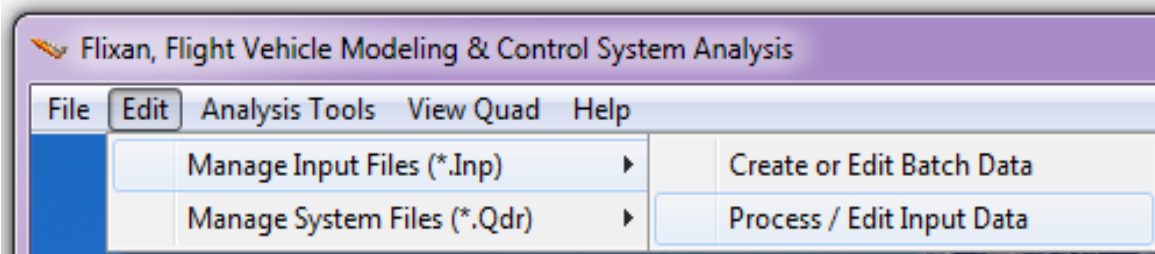
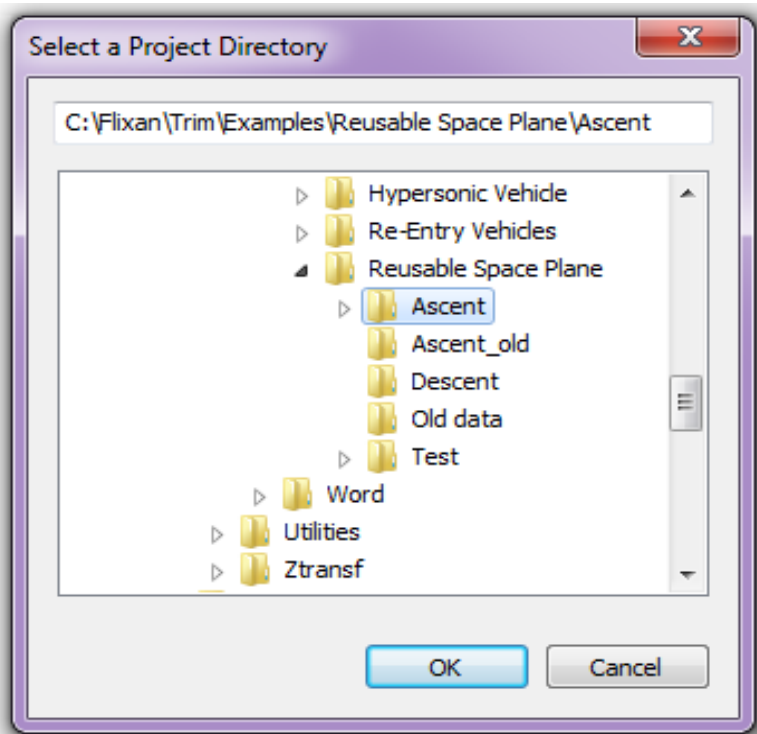
The program is now ready to create a dynamic model at the selected time. The above dialog from the FVMP shows the flight vehicle parameters prepared by Trim for the selected flight time and they are extracted from the data files. Notice that it located the two fuel sloshing modes from file "ReSP.Slsh". The user can modify some of the data or titles using this dialog before saving it. Click on "Update Data" after every modification. Do not run it yet because there is more work to be done and more data to be included in file "T50.inp". Instead, click on "Save in File" and the vehicle data will be saved in file "T50.inp", under the title "Reusable Space-Plane Ascent Trajectory-2/ T= 50.0 sec". The file "T50.inp" will eventually be processed by Flixan to generate the systems for control design and analysis using Matlab/ Simulink. However, in addition to the vehicle data the already prepared input file contains system interconnection and modification data sets which are related to this analysis and they will be processed by Flixan. The systems and matrices generated by Flixan will be saved in file "T50.Qdr".

Let us now take a look and see what is in file "T50.inp" before processing it in Flixan. This file has already been prepared and it contains several sets of data. Each set is processed by a Flixan utility which create systems and matrices that will be used for control analysis in the next section using Matlab.

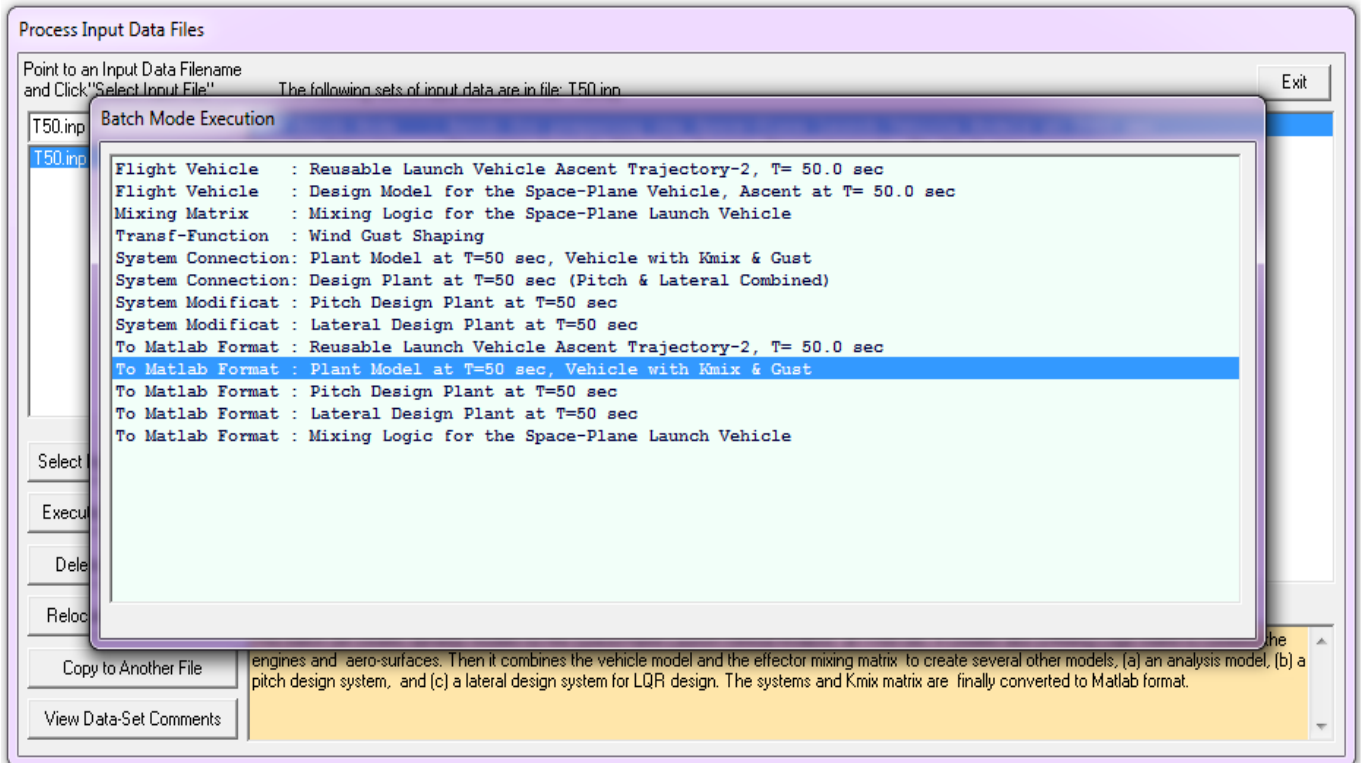
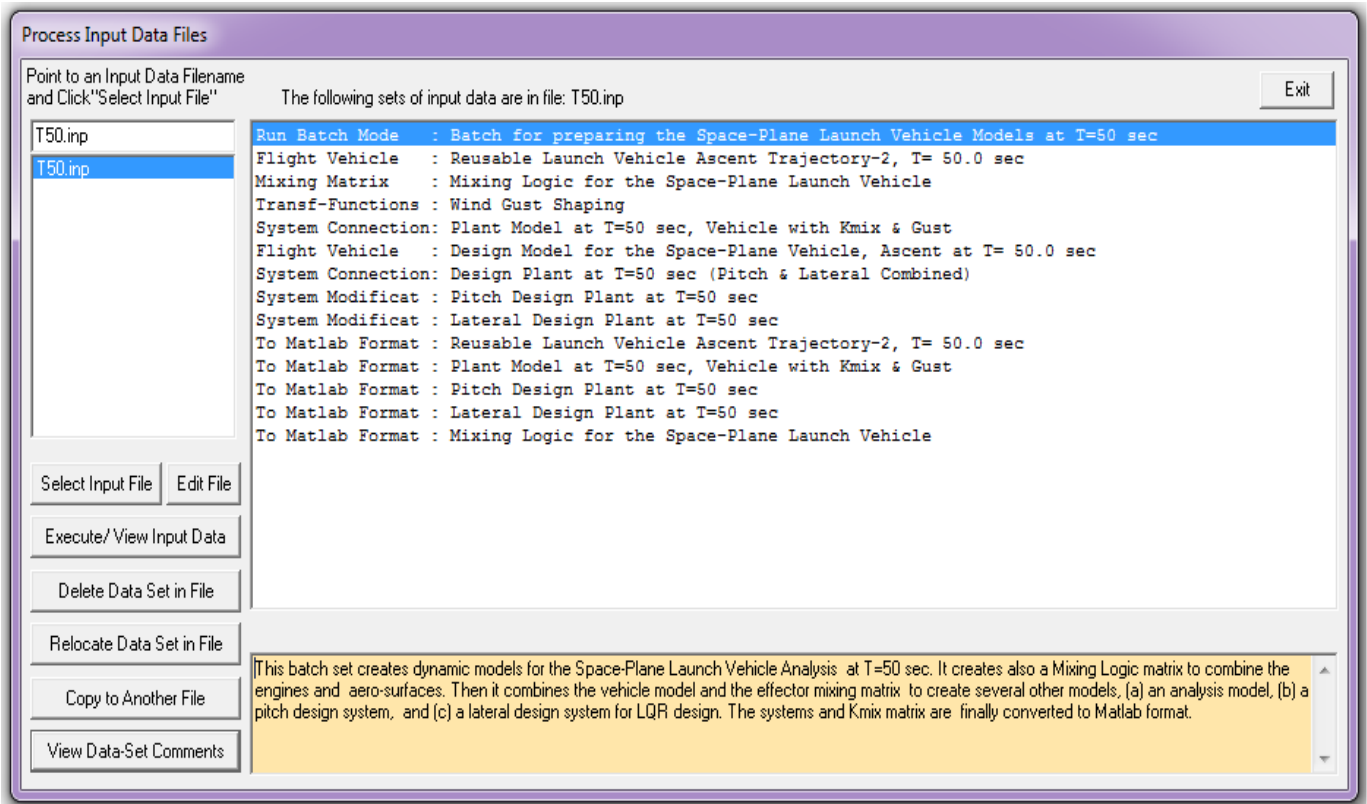
- The first set of data in this file is a batch set that processes all other data-sets in batch mode. This is faster because they are all processed together instead of processing each set interactively. Its title is "*Batch for preparing the Space-Plane Launch Vehicle Models at T=50 sec*".
- Below the batch there are two sets of flight vehicle data corresponding to the flight condition that we analyzed. The first one is the exact same case and title that we have just created "*Reusable Launch Vehicle Ascent Trajectory-2, T= 50.0 sec*". The second one is a slightly modified version that has the slosh modes and the accelerometers taken out. Its title is "*Design Model for the Space-Plane Vehicle, Ascent at T= 50.0 sec*" and it will be used to generate models for control design. It is more convenient to remove the slosh modes from the design model.
- The next data-set uses the Flixan mixing-logic algorithm to generate a (14 x 4) mixing logic matrix. The data-set title is "*Mixing Logic for the Space-Plane Launch Vehicle*" and the matrix name is Km<sub>mix\_50</sub>.
- There is also a wind-gust shaping transfer-function set "*Wind Gust Shaping*" for smoothing out the square gust impulse.
- The vehicle dynamic models have many control inputs and it is a good practice to combine the mixing logic matrix in series with the vehicle models at their input. This simplifies the plants by reducing the number of the control inputs to four. It also decouples the plant (not perfectly), and makes it easier to analyze and to design flight control laws. We have included, therefore, two system interconnection sets that combine the two vehicle models with the mixing matrix Km<sub>mix\_50</sub>. The title of the first one is "*Plant Model at T=50 sec, Vehicle with Km<sub>mix</sub> & Gust*" and it includes also the gust filter and the slosh modes. The title of the second one is "*Design Plant at T=50 sec (Pitch & Lateral Combined)*". The second one is simpler because it does not include the slosh resonances and the gust model. It will be split and used for pitch and lateral LQR designs.
- The next two sets are system modification data-sets used for separating the design plant in two separate sub-systems: (a) a pitch design plant "*Pitch Design Plant at T=50 sec*", and (b) a lateral design plant "*Lateral Design Plant at T=50 sec*".
- Finally, the input file T50.Inp also includes data-sets for converting the Flixan generated systems and matrix to Matlab m-functions for further analysis using Matlab. The first vehicle model will be used in the simulation model and it is saved in file "*vehicle.m*". The pitch and lateral design plants are saved in files "*vehi\_pitch.m*" and "*vehi\_later.m*" respectively. The effector mixing matrix is saved in file "*Kmix\_50.mat*". These files will be loaded into Matlab and used in the control design and analysis that will be described in the next section.

## Processing the Batch Using Flixan

The Flixan data-sets that will be used for the preparation of the vehicle models and the batch set for quick data processing have already been created in file "T50.Inp". This file has been moved to folder "C:\Flixan\Trim\Examples\Reusable Space Plane\Ascent\Analysis", where the control analysis will be performed using Matlab. The user must process this file in Flixan as follows. Start Flixan and select the project directory that contains the input data file. Then go to "Edit", "Manage Input Files" and then "Process/ Edit Input Data". When the following dialog appears, select the input data file "T50.Inp" form the left menu and click on "Select Input File". The menu on the right lists the titles of the data sets which are included in this file. On the left side of each title there is a short label defining the type of the data-set. It also identifies which program utility will process the data-set. On the top of the list there is a batch created to process the whole file. In order to process the batch, highlight the first line with title: "Batch for preparing the Space-Plane Launch Vehicle Models at T=50 sec", and click on "Execute/ View Input Data". Flixan will process the input file and save the systems and matrices in file "T50.Qdr". It will also create the matrices and system functions for Matlab analysis. The same process can be used to create models, design and analyze other flight condition along the trajectory.







## 1.10 Ascent Control Design, Analysis and Simulation

We will now use the dynamic models developed in section 1.9 to design control laws for our space-plane, analyze stability, and develop a simulation model in Matlab/Simulink. This vehicle is particularly interesting because it has fuel sloshing and it blends multiple types of effectors, such as: aero-surfaces, gimbaling TVC engines, and thrust varying engines. The analysis is performed in directory: "C:\Flixan\Trim\Examples\Reusable Space Plane\Ascent\Analysis". We will begin by designing the state-feedback control laws, performing stability analysis in the frequency domain, developing a simulation model, and simulating the system's response to flight control commands and also to wind-gusts.

### Flight Control Design

The flight control gains are calculated by the Matlab script file "*design.m*", shown below. It uses the pitch and lateral design systems in files "*vehi\_pitch.m*" and "*vehi\_later.m*", created in section 1.9, to generate state-feedback matrices *Kq.mat* and *Kpr.mat* for the pitch and lateral systems. The slosh resonances are not included in the design models. The pitch state-vector consists of:  $[\theta, q, \alpha, \theta\text{-integral}]$ , and the lateral state-vector consists of states:  $[\phi, p, \psi, r, \beta]$ .

```
% Pitch LQR Design for Space-Plane
[Ap,Bp,Cp,Dp]= vehi_pitch;           % Load the Pitch Design Model
[Api,Bpi,Cpi,Dpi]= linmod('Pitch_Design'); % Augment Pitch Simulink model
sys1=SS(Api,Bpi,Cpi,Dpi);
% Weights[thet, q, alpha, thet_int]
Q= diag( [5.5, 3.5, 0.01, 0.1]);    % Weights(thet,q,alpha,thet_int)
R=1;                                % Control Weights R=2
[Kq,S,E]= LQR(sys1,Q,R);
save Kq.mat Kq -ascii              % Save the LQR gains in Kq.mat

% LQR Lateral Design for Space-Plane
[Al,Bl,Cl,Dl]= vehi_later;          % Load the Lateral Design Model
sys2=SS(Al,Bl,Cl,Dl);
% states: [phi, p, psi,r, beta]
Q=diag([ 0.5, 0.2, 0.5, 0.2, 0.001]); % Control Weights
R=diag([1 1]*1);
[Kpr,S,E]= LQR(sys2,Q,R);
save Kpr.mat Kpr -ascii            % Save the LQR gains in Kpr.mat
```

### Simulation Model

The closed-loop simulation model is in file "*Sim\_Ascent.mdl*", shown in Figure (1.10.1). It is initialized by the Matlab file "*runsim.m*", shown below, which loads the system with title: "*Reusable Launch Vehicle Ascent Trajectory-2, T= 50.0 sec*", from file "*vehicle.m*", that was created in section 1.9. It also loads the mixing logic matrix and the state-feedback gain matrices.

```
% Initialize Simulation
r2d=180/pi; d2r=pi/180;
design
[Av,Bv,Cv,Dv]= vehicle;           % Calculate Gains using LQR Design
load Kmix_50 -ascii              % Load the Vehicle Analysis Model
load Kq -ascii                   % Load Mixing Logic Matrix
load Kpr -ascii                  % Load LQR Gains
```

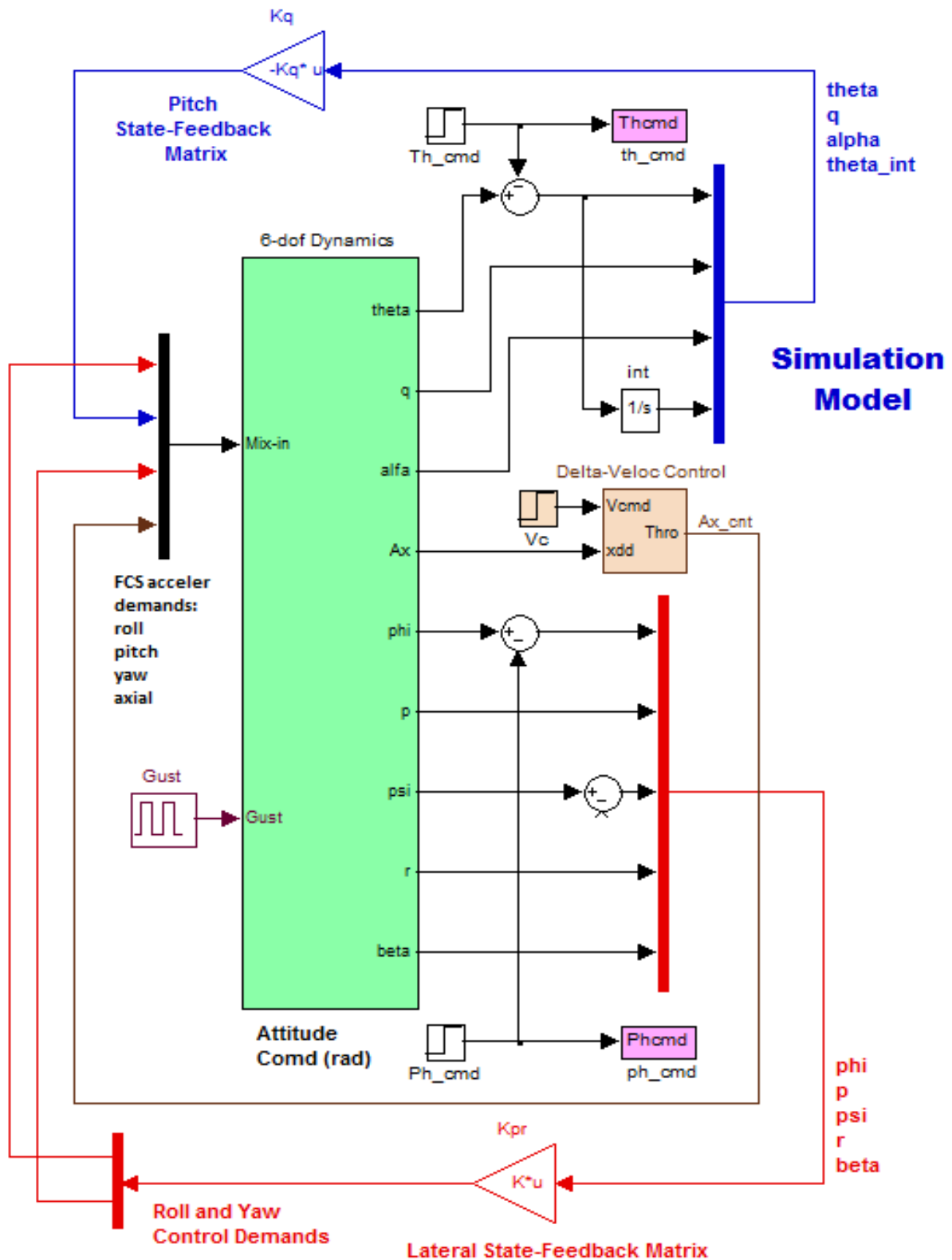


Figure 1.10.1a Space-Plane Simulation Model in File "Sim\_Ascnt.mdl".

## Pitch & Lateral Dynamics Reusable Space-Plane Launch Vehicle Ascent Trajectory-2, T= 50.0 sec

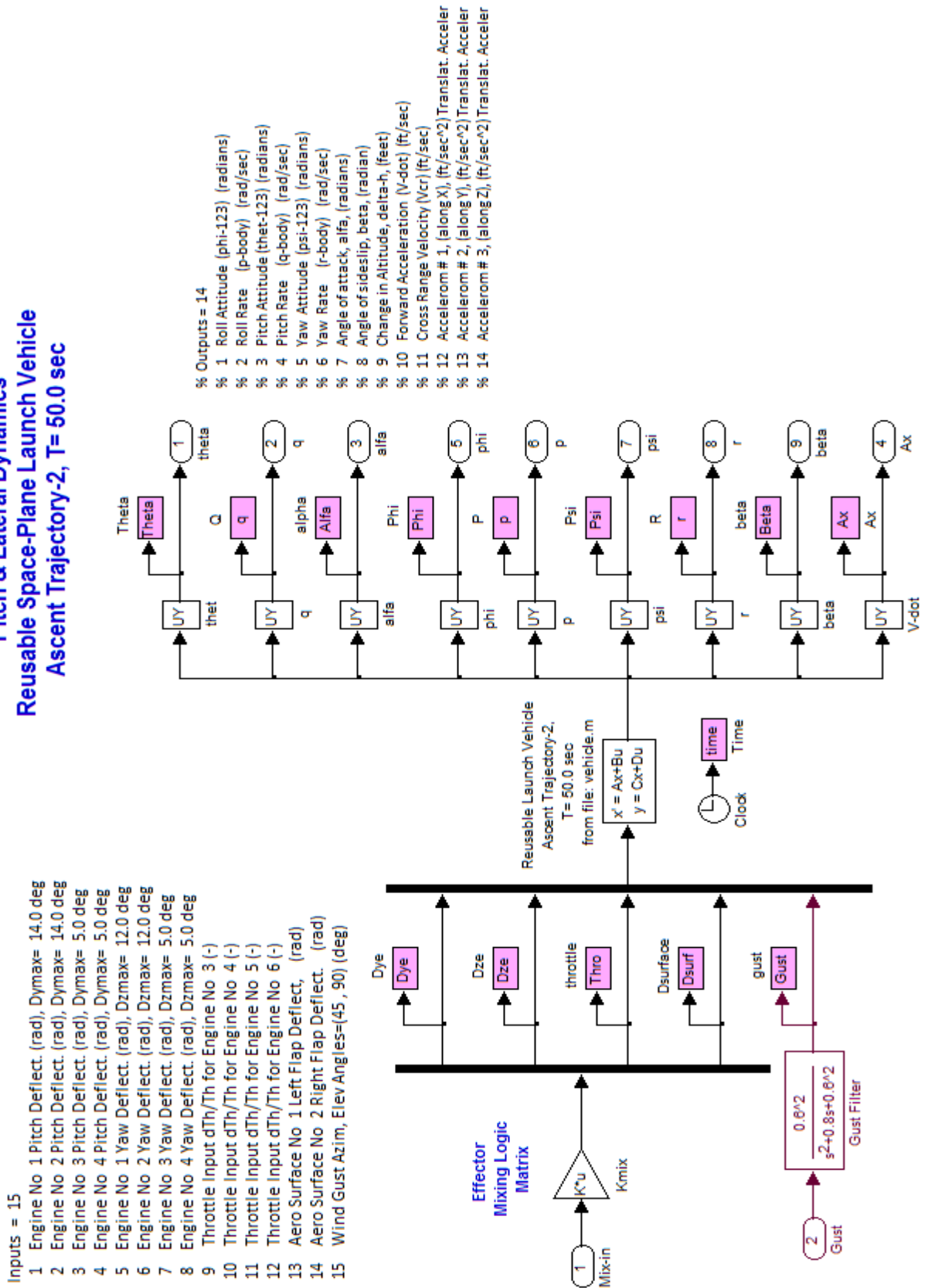
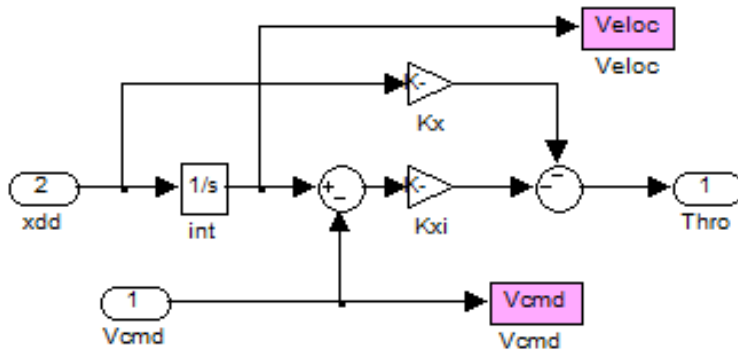


Figure 1.10.1b Space-Plane dynamics subsystem (green block in File "Sim\_Ascent.mdl") that uses state-space file "vehicle.m"

## Velocity Control System

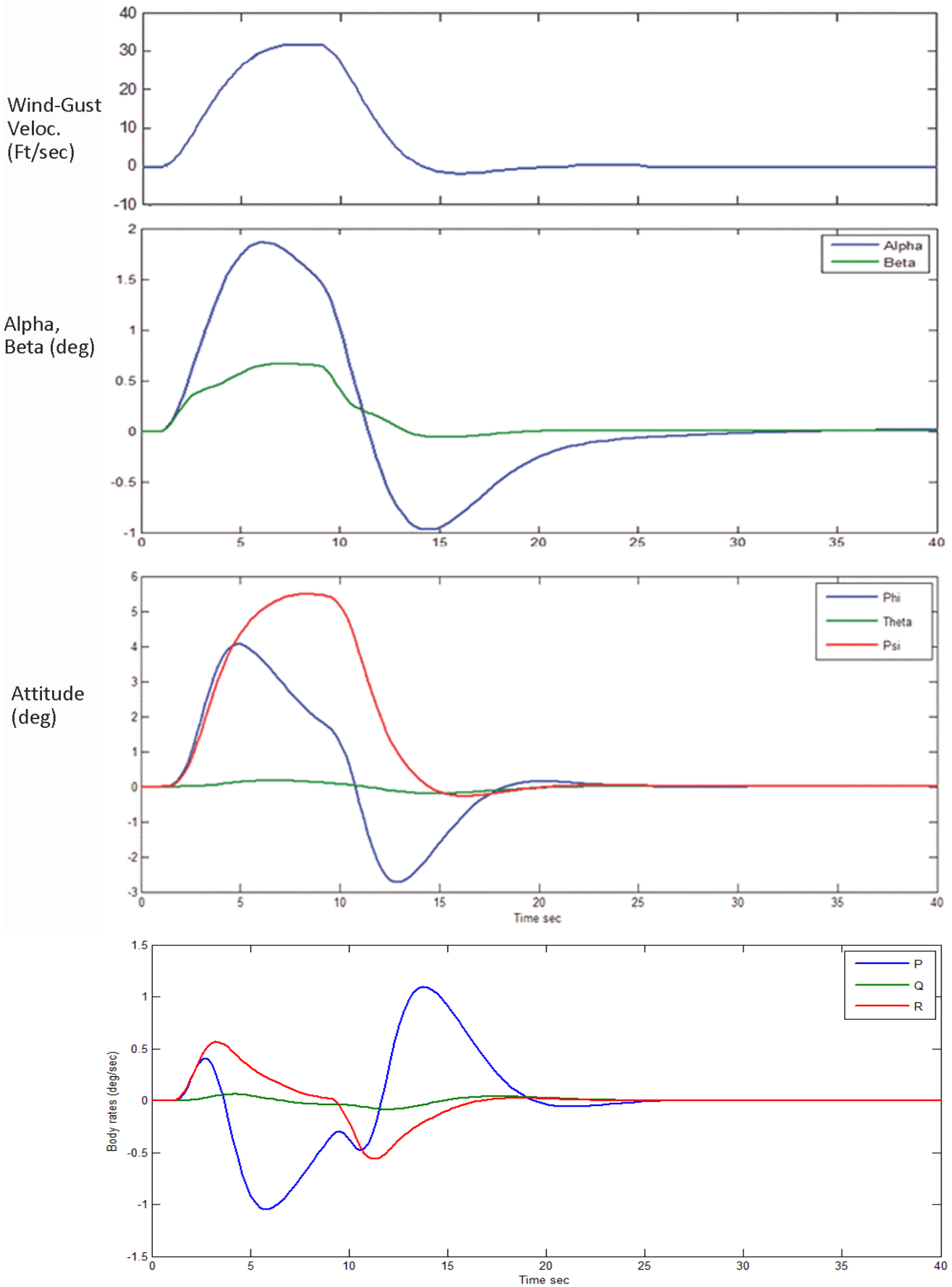


The simulation model has four control loops: roll, pitch, yaw attitude, plus a velocity control loop. The 4 control acceleration demands are converted to effector deflections and throttle commands by the mixing-logic matrix  $K_{mix\_50}$ . The model also has a wind-gust disturbance input which is excited by a square pulse and it is shaped into a more wind-gust shape by a low-pass filter. The wind-gust direction relative to the vehicle is defined in the input file. It is set perpendicular to the  $x$  axis and at  $45^\circ$  between the  $+z$  and the  $+y$  axes to excite both: pitch and yaw directions. The pitch state-feedback control loop via matrix  $K_q$  (blue) is seen in the upper section of Figure (1.10.1a). The lateral state-feedback control loop via matrix  $K_{pr}$  (red) is seen in the lower section of Figure (1.10.1a). There is also a velocity control loop that measures axial acceleration and velocity and controls velocity by the throttle control input (brown).

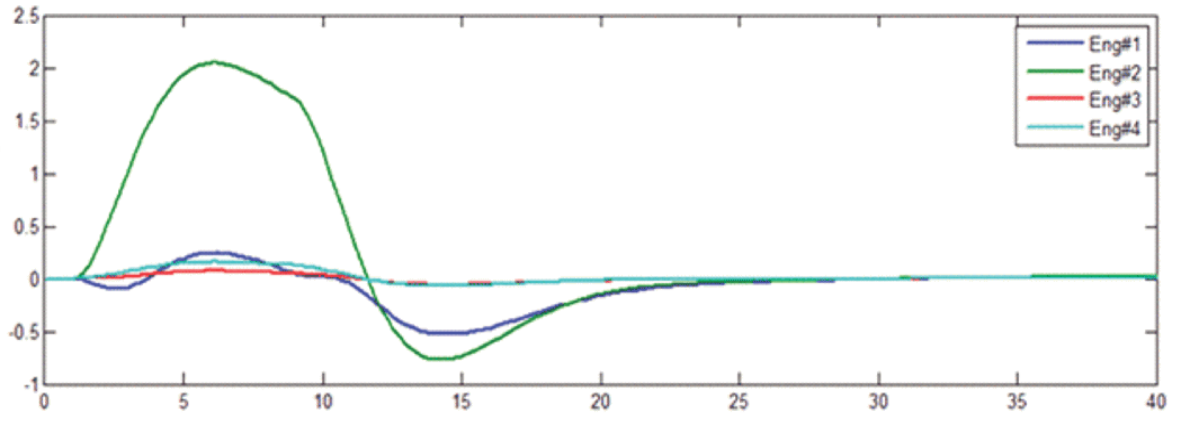
The simulation model receives 4 commands from guidance which are represented with step inputs in this simple model. There is a pitch attitude command ( $\theta_{cmd}$ ), a roll command ( $\phi_{cmd}$ ), and a delta-velocity command ( $\delta V_{cmd}$ ). It receives also a yaw command, but it is not shown. When the simulation is complete the file "pl.m" plots the data. We will perform 4 runs using this simulation model by commanding one input at a time: (a) a wind-gust excitation, (b) a pitch attitude step command, (c) a roll attitude step command, and (d) a velocity change step command. What is interesting in this simulation is to observe how the combination matrix allocates control to the various effectors in a coordinated action to achieve the desired guidance commands.

### Wind-Gust Simulation:

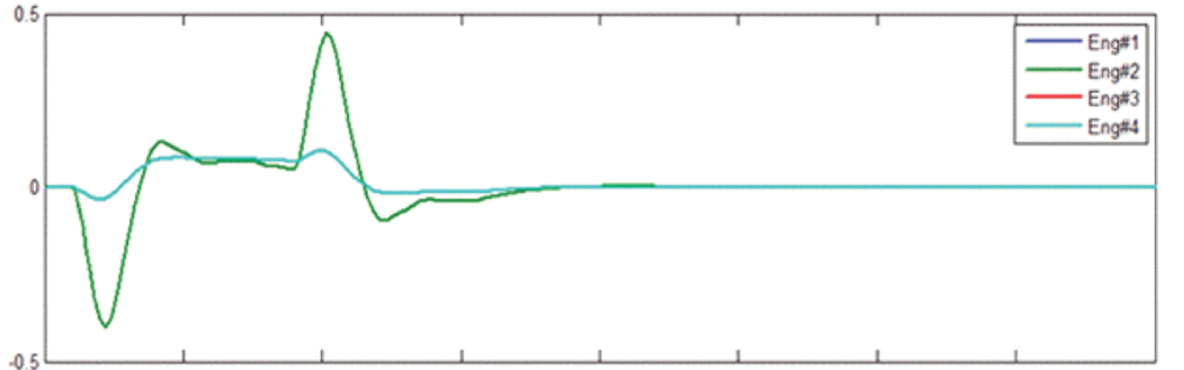
In the first simulation, shown below, we apply a square wind-gust velocity pulse of 30 (feet/sec) for a period of 4 sec. The pulse is smoothed by the low-pass filter that makes it look more like a real gust. The gust direction is perpendicular to the vehicle  $X$ -axis, exciting both the pitch and yaw directions and creating positive alpha and beta transients. It generates also a roll/ yaw transient. Notice how both: the TVC engines and aero-surfaces react against the gust disturbance as they stabilize the vehicle.



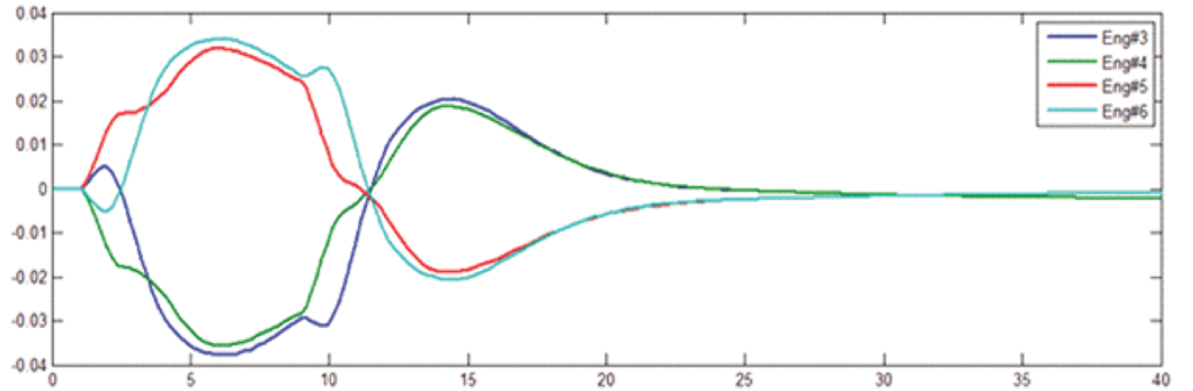
Pitch  
Engine  
Defl.  $\delta_y$   
(deg)



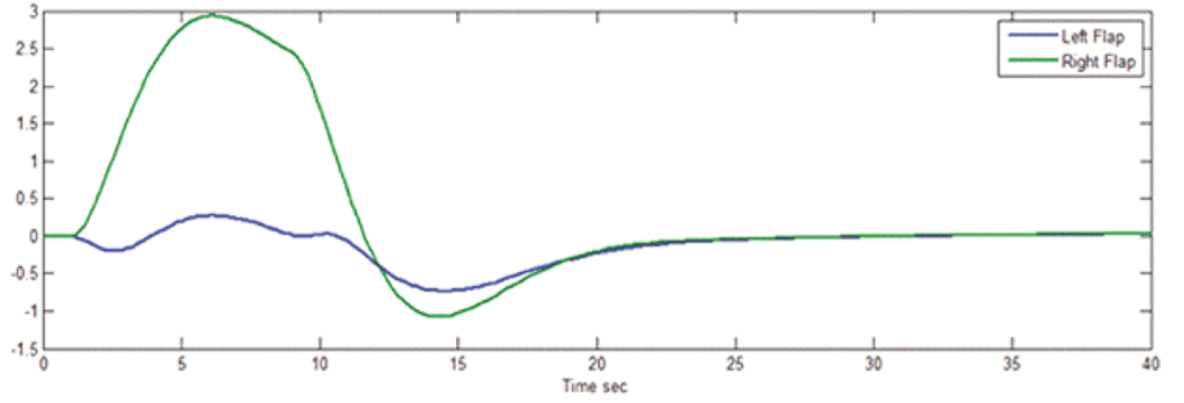
Yaw  
Engine  
Defl.  $\delta_z$   
(deg)



Engine  
Throttle  
(0 to  $\pm 1$ )

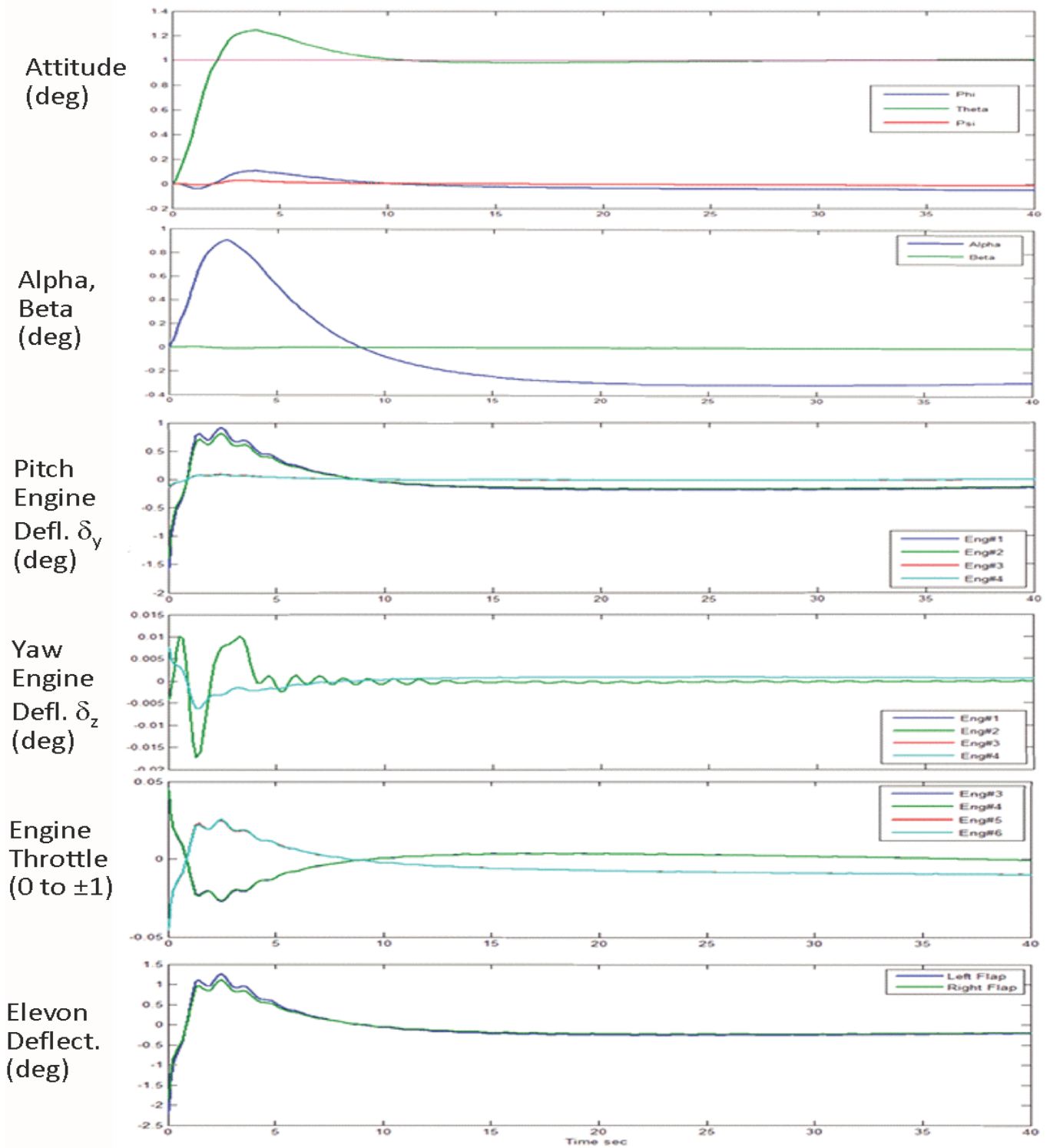


Elevon  
Deflect.  
(deg)





**Pitch Attitude Command Simulation:** The simulation model of Figure 1.10.1 is also used to calculate the system's response to a pitch attitude command  $\theta_{cmd} = 1$  (deg) with all other inputs set to zero. This maneuver is performed by deflecting the TVC engines and also the two aero-surfaces together in pitch. The small difference between the left and right elevon deflections is because of the  $Y_{CG}$  offset. The small oscillations are caused by the propellant sloshing and it is damping out.



## Change in Velocity Simulation

Let us now set all other commands to zero except for the velocity command, which is set to a step,  $\delta V_{cmd} = 10$  (ft/sec). It actually represents a change in velocity command from its current velocity. The velocity control system (beige block shown in detail below) reads the axial acceleration, integrates it, compares it with the velocity command and calculates the throttle command that produces an axial acceleration. The plot below shows the change in velocity and acceleration during the maneuver. The engine throttle commands momentarily jump from zero to 0.6 to begin the acceleration and they gradually decay back to zero as the commanded  $\delta V$  is achieved. The TVC engines and the two flaps rotate to counteract the pitching moment produced by the increase in thrust. The maneuver causes slosh excitation in the pitch direction which is eventually damping out.

## Velocity Control System

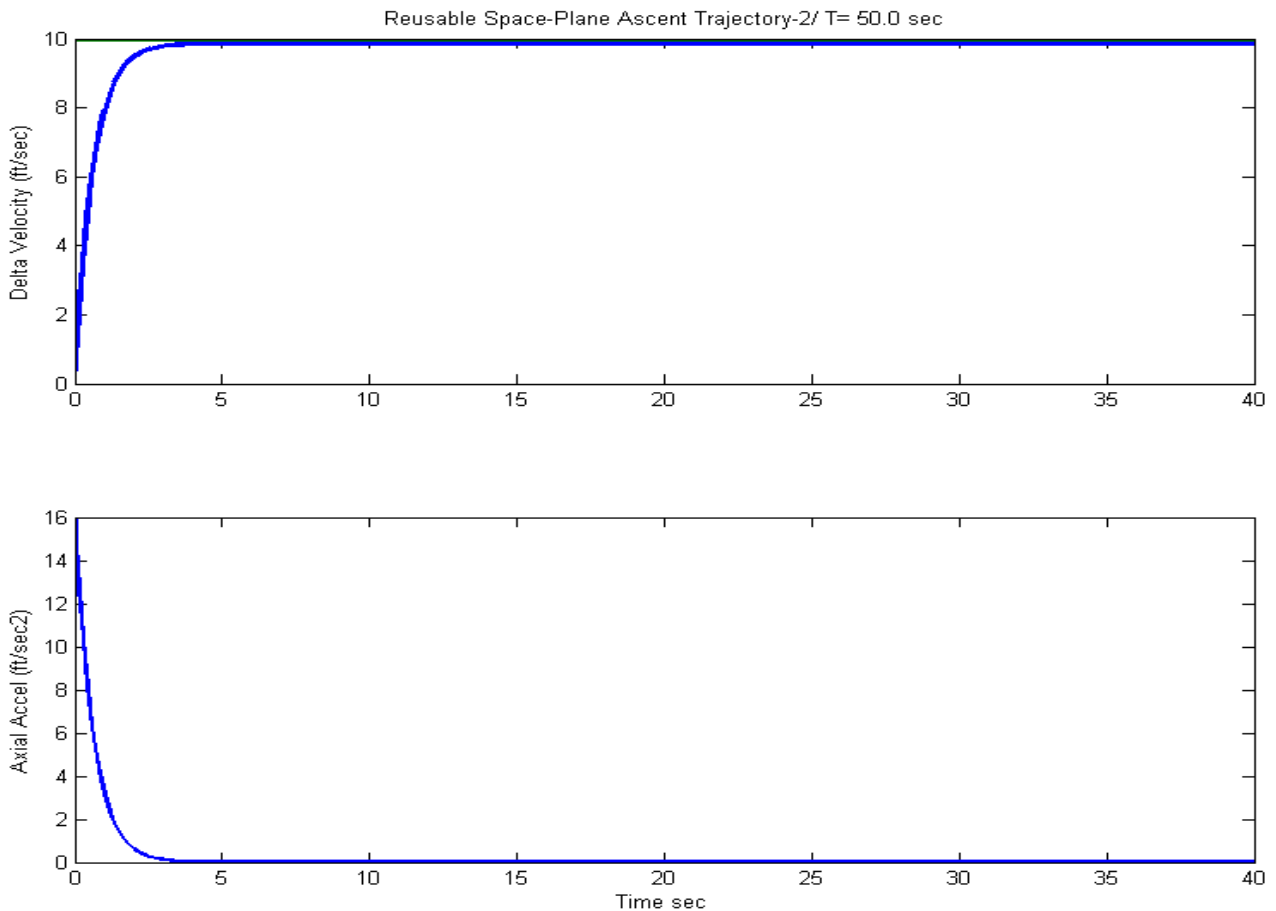
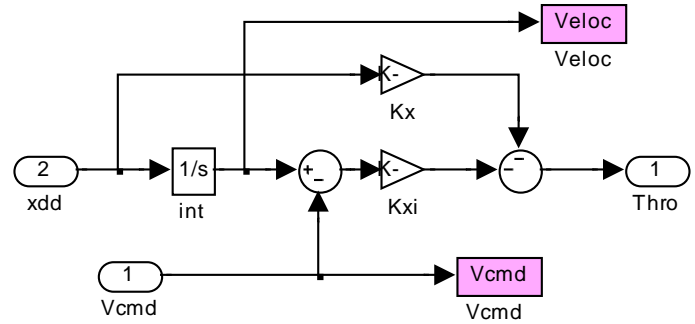
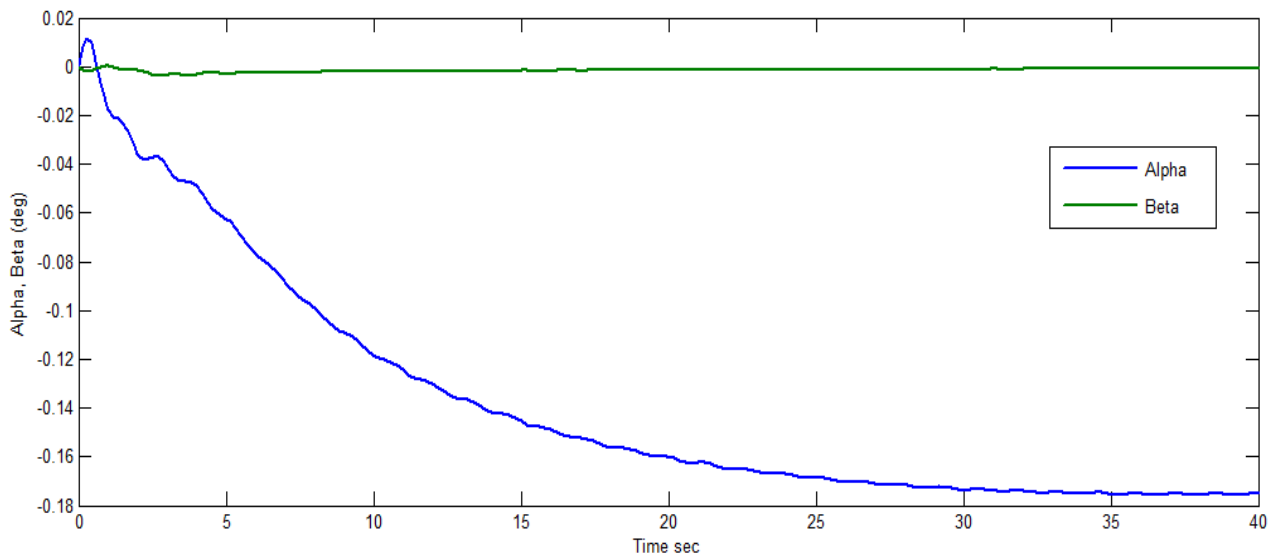
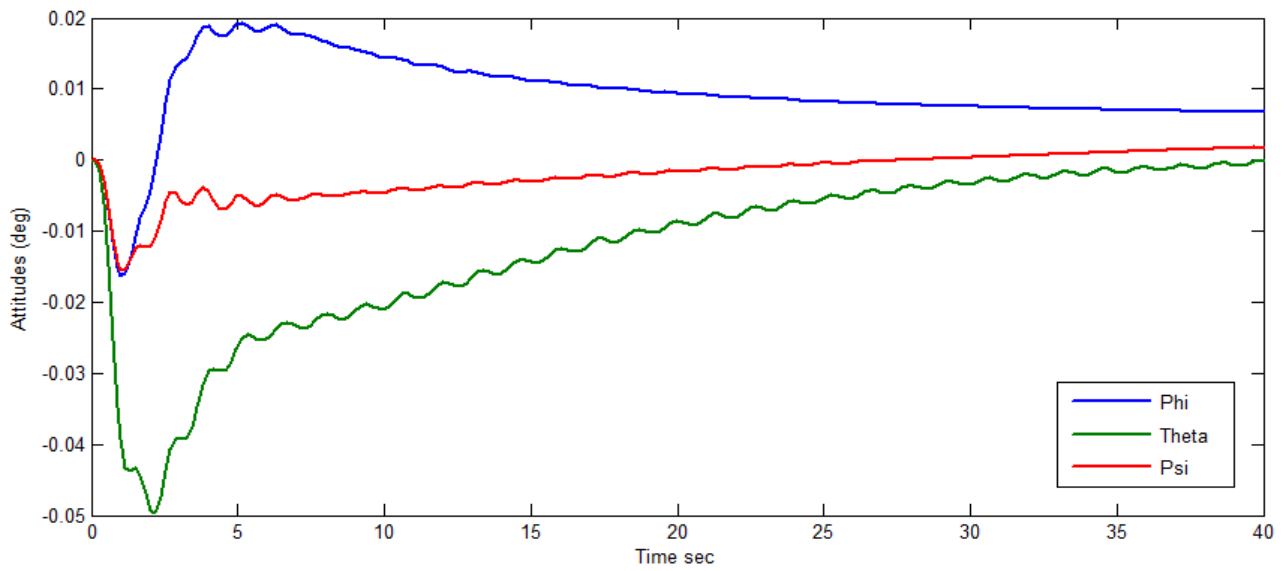
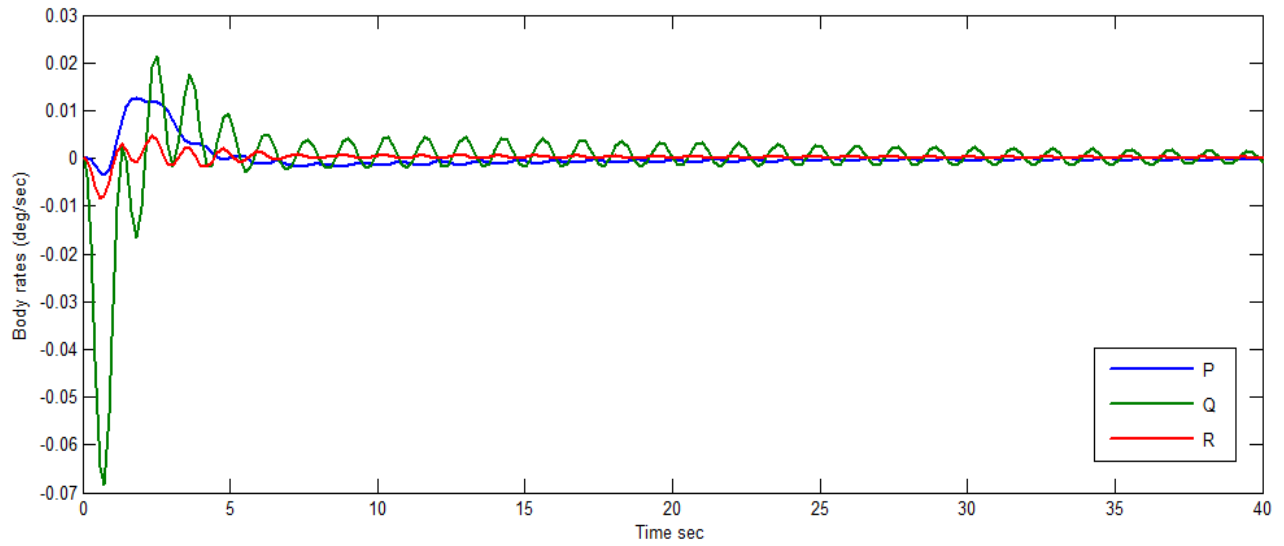
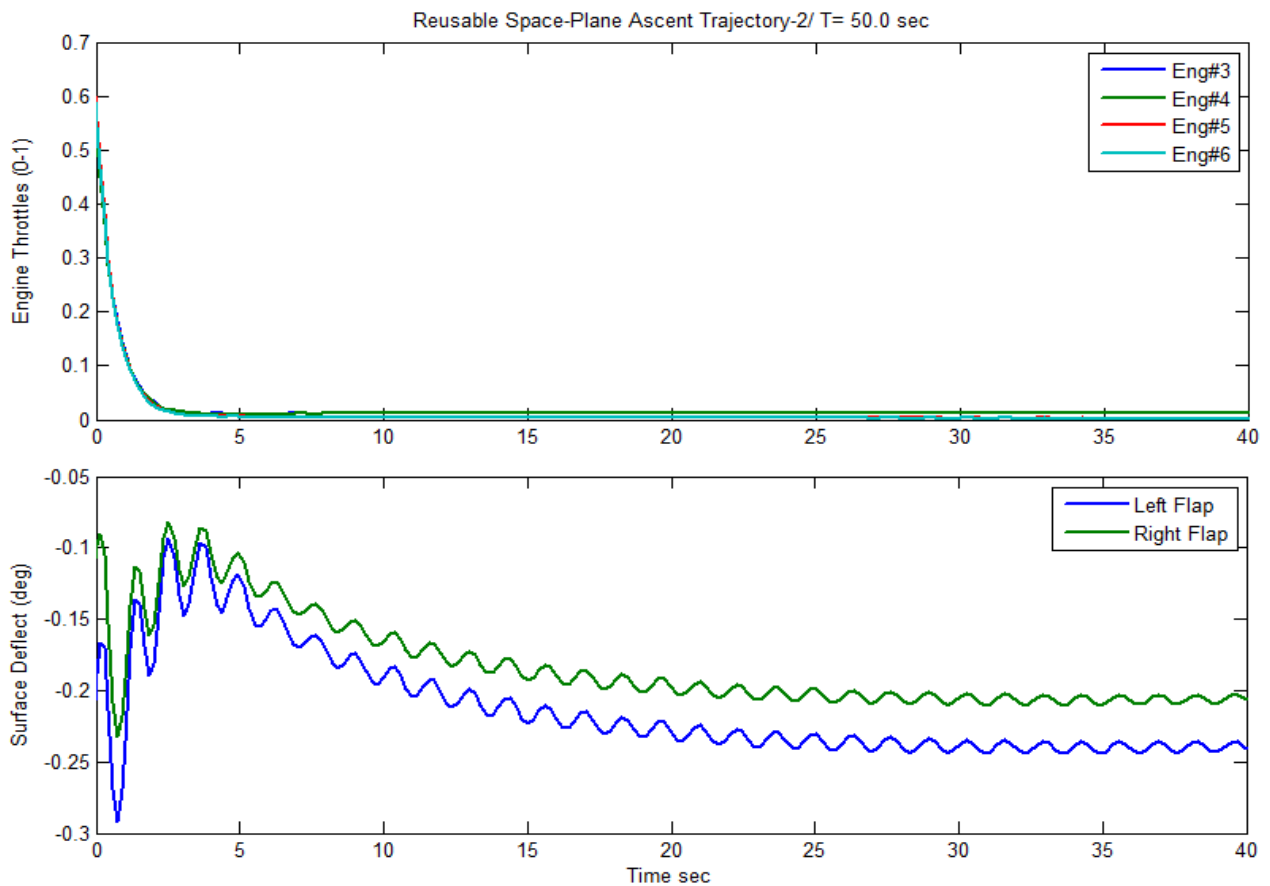
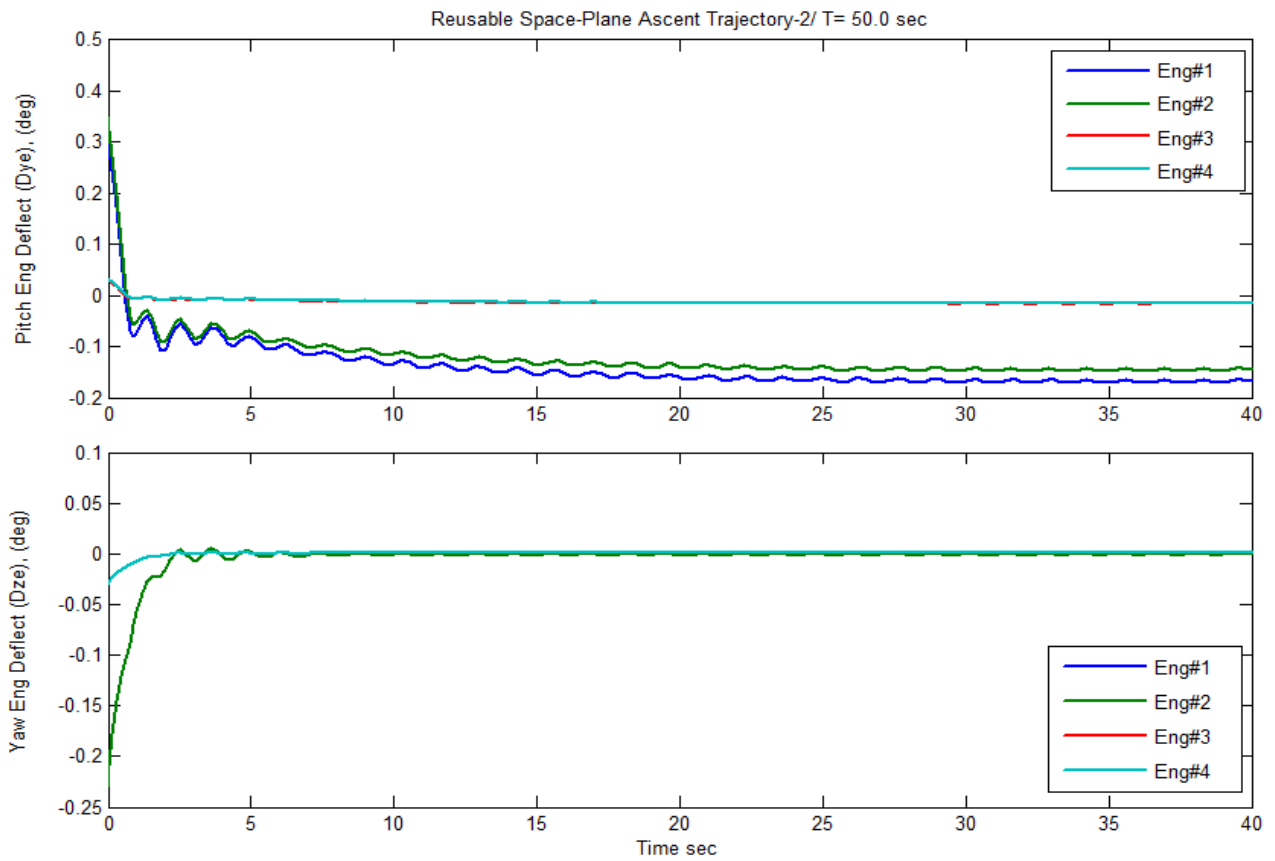


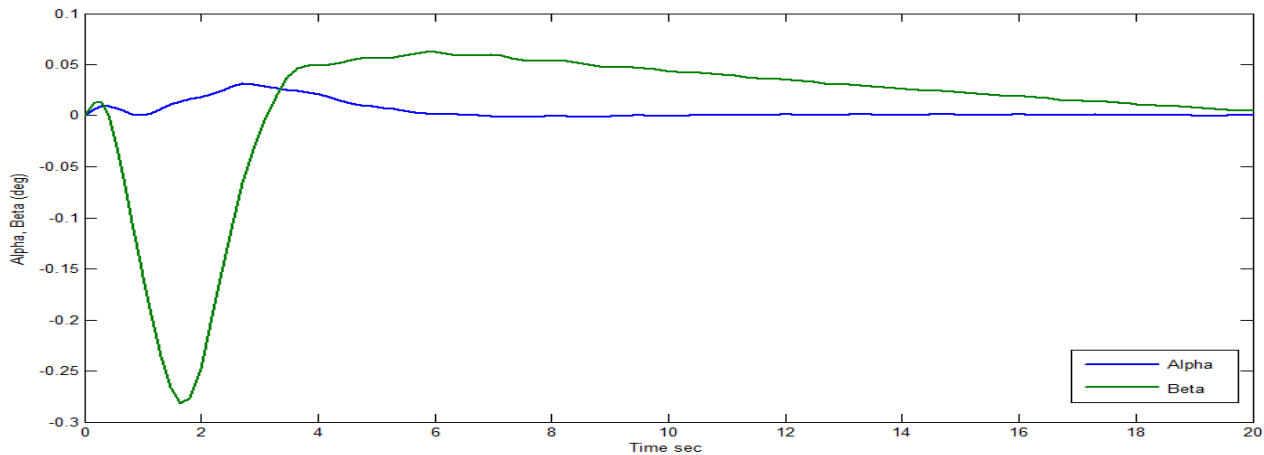
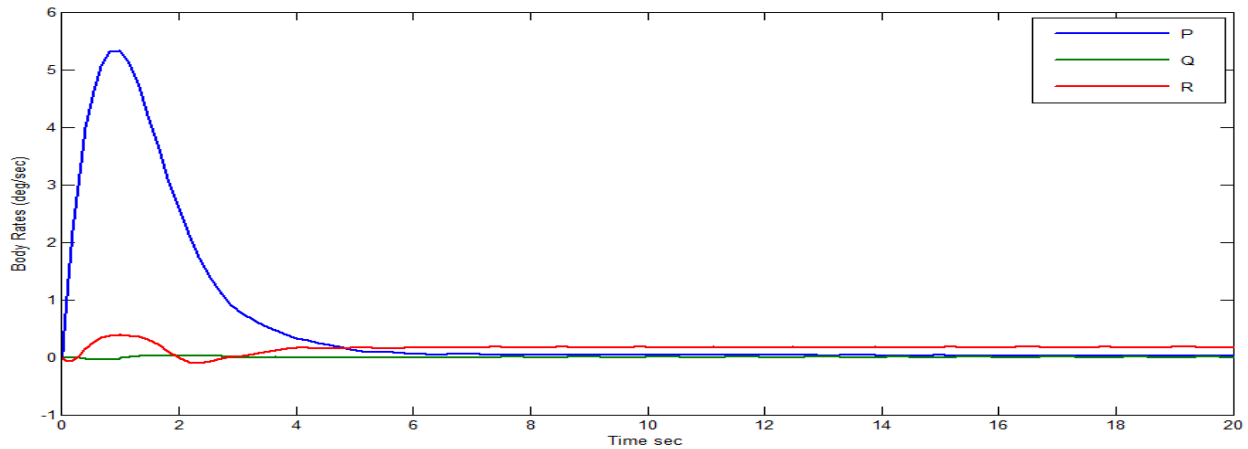
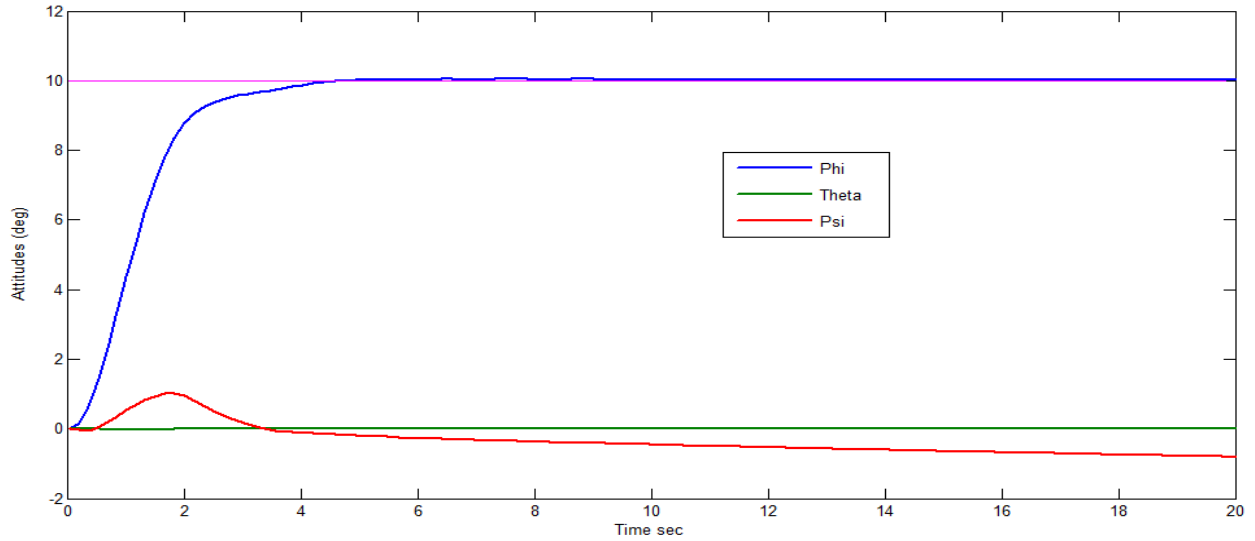
Figure 1.10.2 The Increase in Velocity causes a momentary increase in acceleration

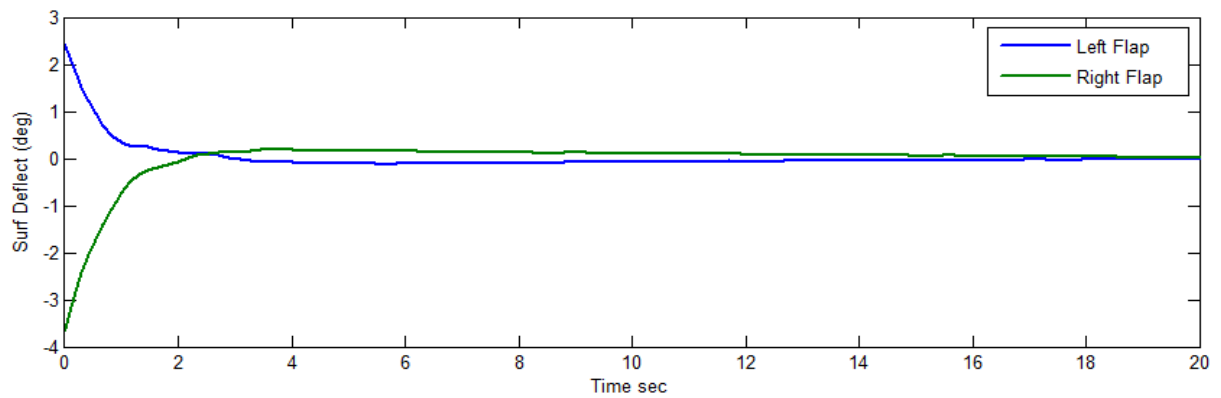
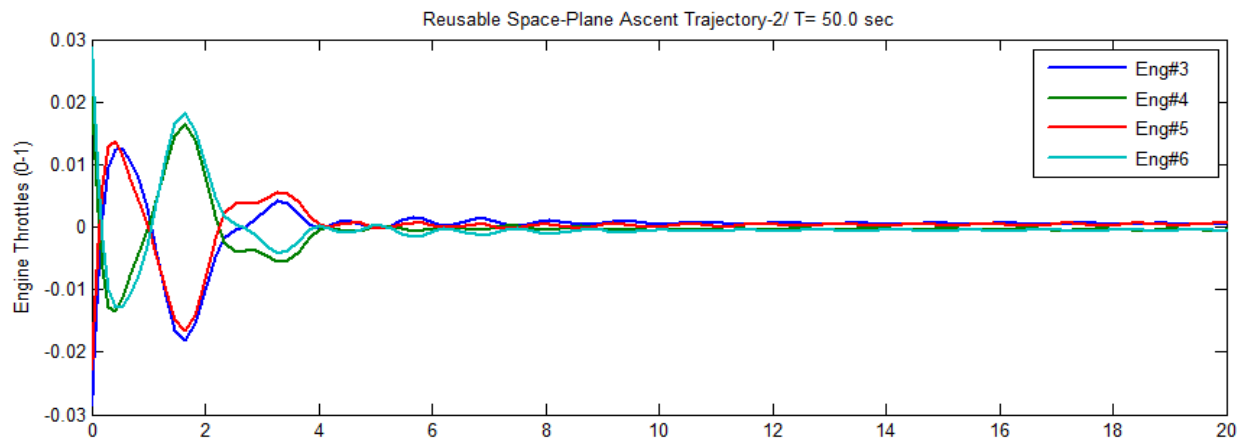
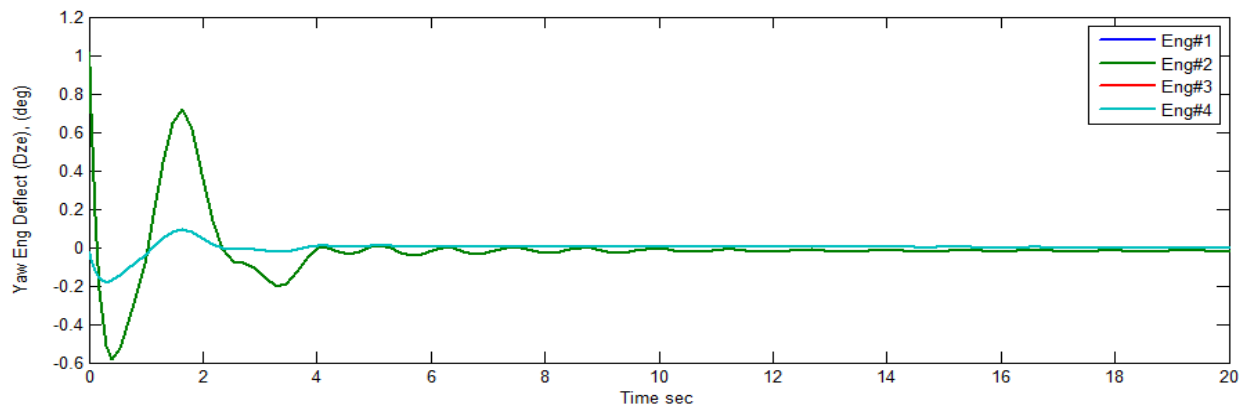
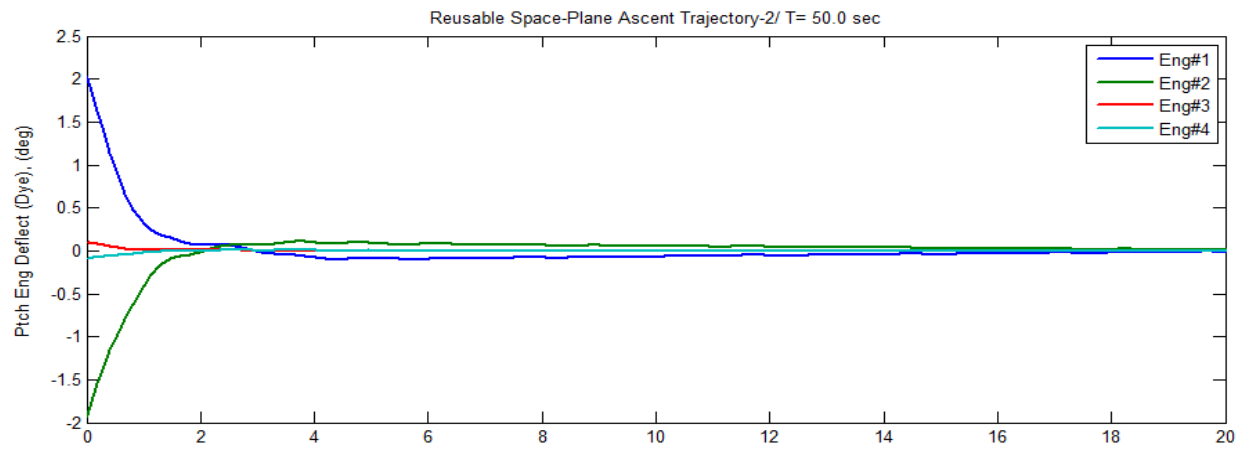




## Roll Command Simulation

Set the roll attitude command to a step input of  $\phi_{cmd} = 10^\circ$ , and all other commands set to zero. The commanded roll attitude is achieved by deflecting the two aero-surfaces differentially and also two of the TVC engines (eng#1 & eng#2) differentially in pitch. The differential deflections are not symmetric because of the YCG offset. The yaw TVC deflections are in the same direction.





## Stability Analysis

Figure 1.10.3 is a block diagram used for open-loop stability analysis. Since the system has four control loops, only one loop is opened at a time to calculate the frequency response across the opened connection, with the other three loops closed. The figure shows it configured in pitch (top) and yaw (bottom) analysis. The script file "*freq\_anal.m*" below performs stability analysis in the frequency domain. It uses the Simulink model "*Open\_Loop.mdl*", shown in figure (1.10.1a) for pitch, and figure (1.10.1b) for yaw analysis.

```
[Av,Bv,Cv,Dv]= vehicle; % Load the Vehicle Analysis Model
[Ao,Bo,Co,Do]= linmod('Open_Loop'); % Augment Pitch Simulink model
w=logspace(-3, 3, 10000); % Define Frequency Range
sys= SS(Ao,Bo,Co,Do); % Create SS System
figure(1); Nyquist(sys,w) % Plot Nichol's Chart
figure(2); Nichols(sys,w) % Plot Nichol's Chart
figure(3); Bode(sys,w); grid on % Plot Bode Plot
```

Figure 1.10.4 and 1.10.5 show the Bode and Nichols plots in the pitch and yaw axes, where the slosh resonances are more observable, and it highlights the phase and gain margins. The LOX mode corresponds to the heavier slosh mass and it is much stronger than the LH2. It is phase-stable in both pitch and yaw directions with plenty of margins and does not require baffles. The LH2 mode is also stable in yaw. In the pitch axis, however, the LH2 has a tendency towards instability (phase unstable) and when the damping coefficient is low the bubble in the Nichols chart will encircle the critical point and cause slosh oscillations when the loop is closed. The damping coefficient was increased in order to reach an acceptable gain margin. It means that baffles may have to be included in the LH2 tank in order to dampen out the fuel motion and to provide an acceptable stability margin.

However, simulation analysis may show that performance is acceptable without baffles simply because the LH2 slosh mass is small, and even if the LH2 resonance becomes unstable the amplitude of the oscillations will be limited by the radius of the tank and the disturbance effect on the vehicle may be a small limit-cycle that may not have any significant impact on performance.



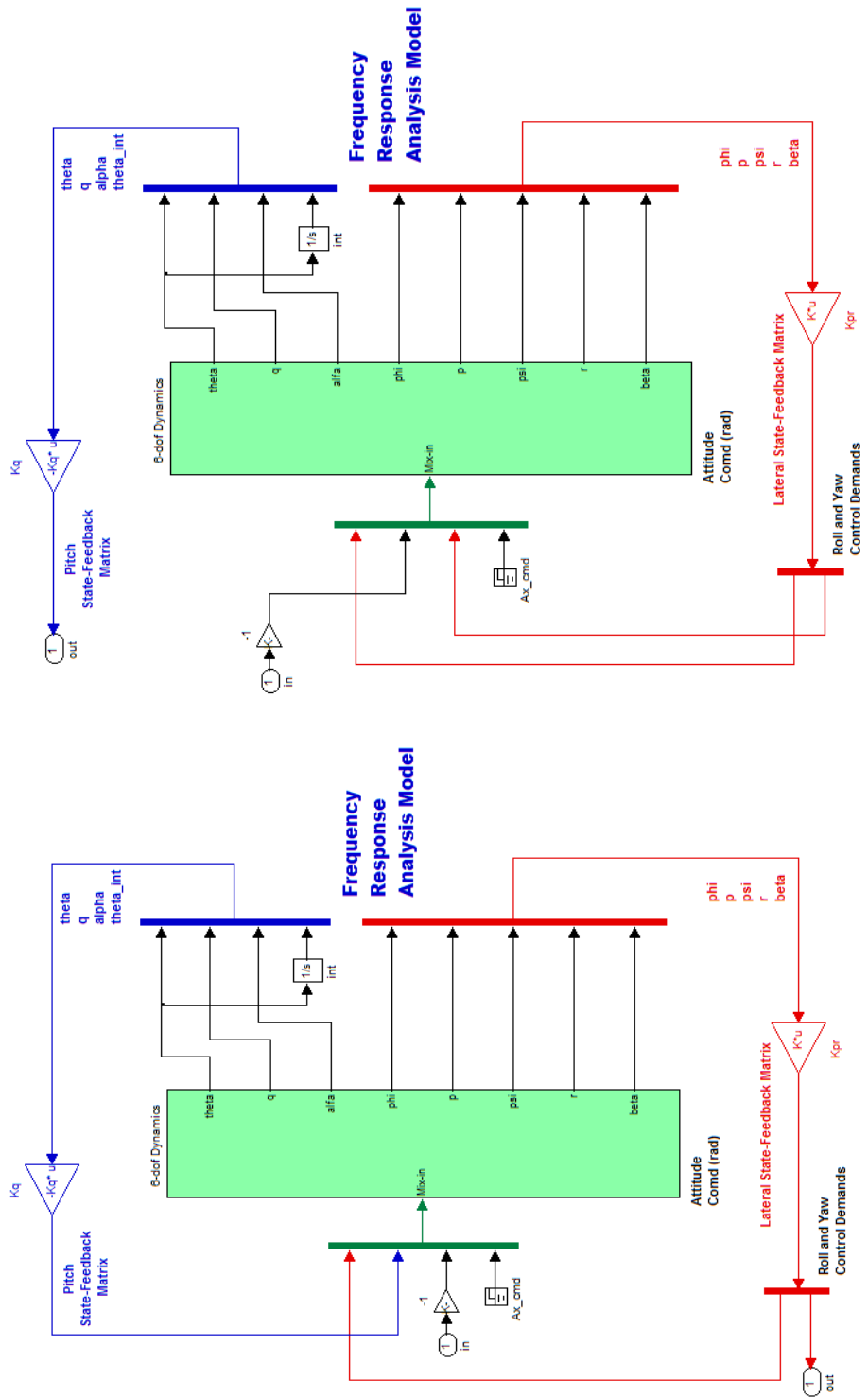


Figure 1.10.3 Simulink model "Open\_Loop.Mdl" shown for pitch and yaw frequency response stability analysis

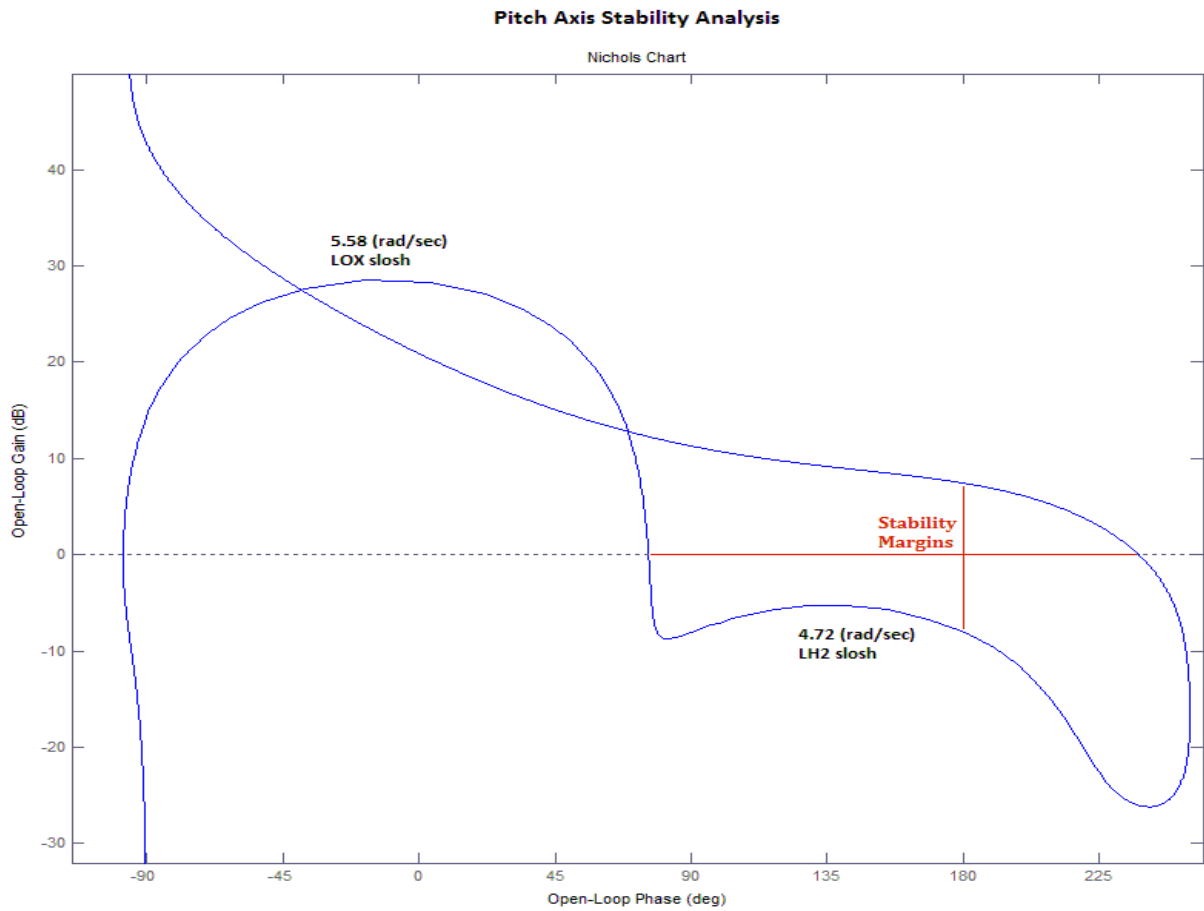
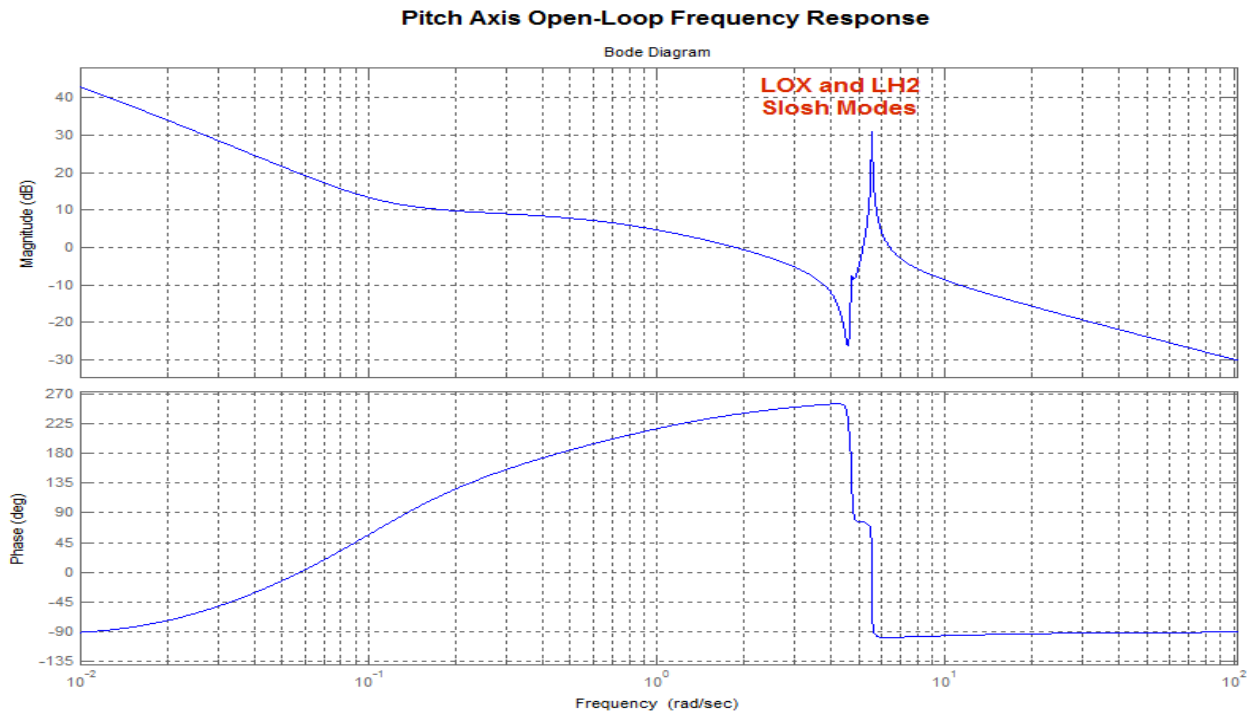


Figure 1.10.4 Pitch Axis Bode and Nichols Plots show stability margins and the two slosh resonances. Notice that the LH2 slosh mode has a tendency towards instability. It was stabilized by increasing the damping coefficient which implies that baffles are needed in the LH2 tank. The LOX mode is much stronger but it is phase stable and it does not require baffles.

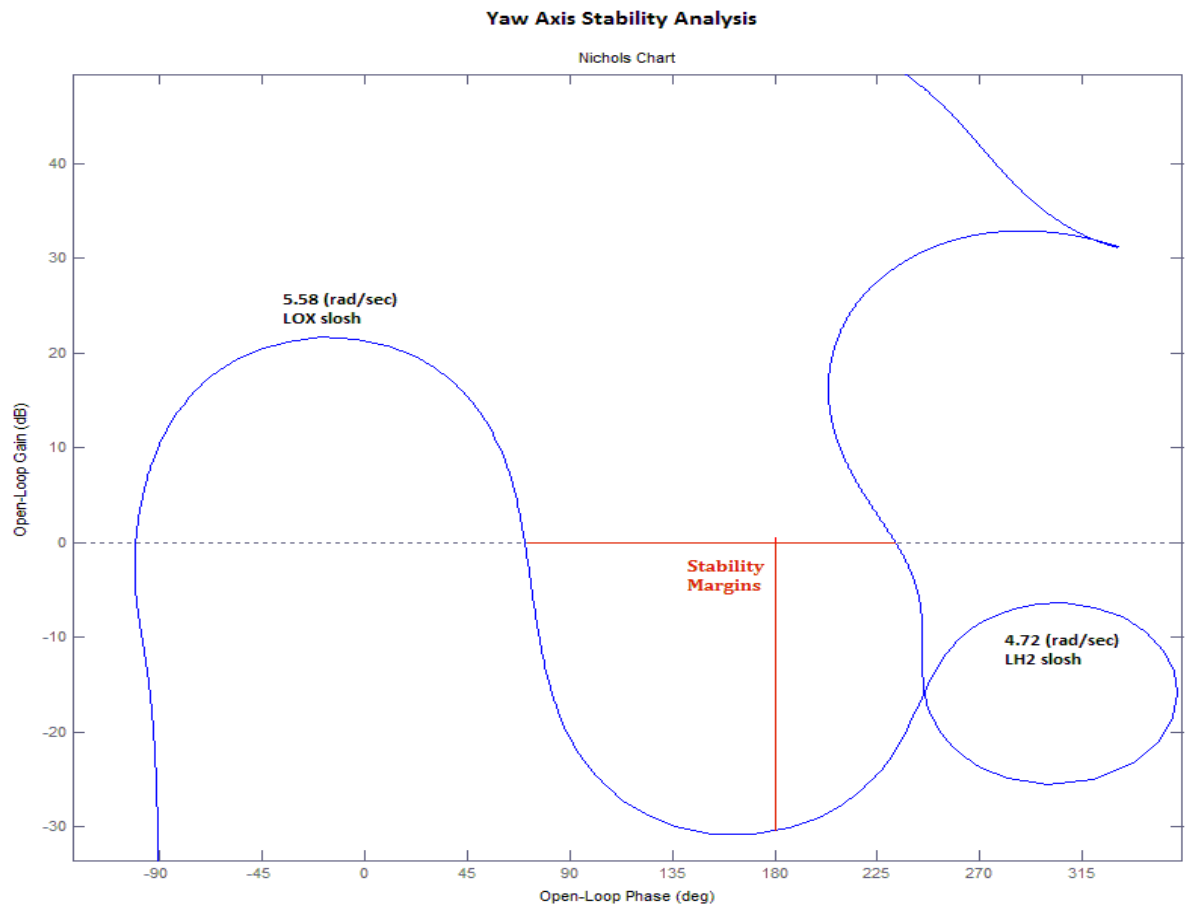
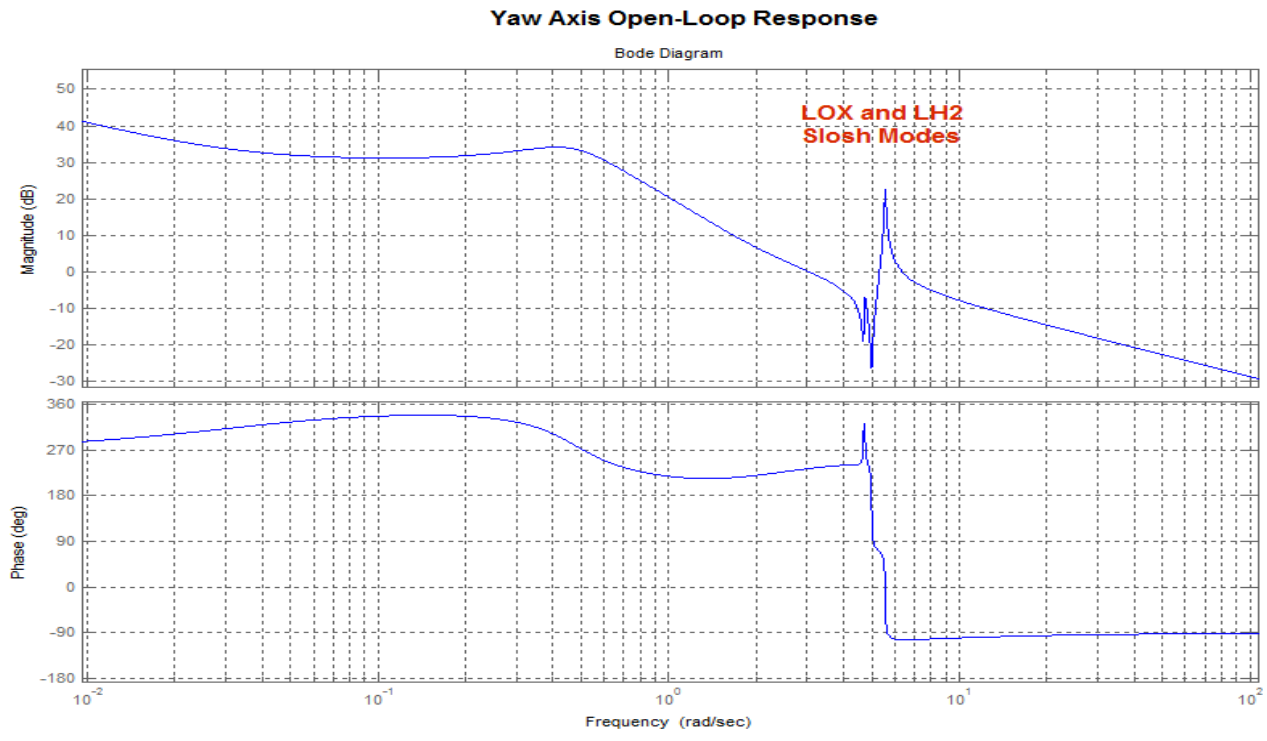


Figure 1.10.5 Yaw axis Bode and Nichols plots show stability margins and the two slosh resonances. Both slosh resonances are phase-stable in yaw.

## 2.0 Descent Phase

During descent the space-plane uses the two flaps for pitch and roll control and two side RCS jets for yaw control because it does not have a rudder. The jets provide 2,500 (lb) of thrust along the  $\pm y$  direction and they are located near the front of the vehicle for larger yawing moment. The analysis files during descent are in directory "C:\Flixan\Trim\Examples\Reusable Space Plane\Descent", and it includes the following files:

- The descent sections of the two trajectories are in files "RSP\_Entry-A.Traj" and "RSP\_Entry-B.Traj", where the thrust is now zero.
- The propulsion file: "RSP\_Entry.Engn" that contains only the information about the RCS jet. A pair of RCS jets, mounted back-to-back, which are firing in the  $\pm y$  direction are implemented in the Trim program as a single thruster that generates either: positive, negative or zero thrust in the y direction. The throttle parameter for the jet is set to one, which means that the actual jet thrust can be varied between zero and  $\pm 2,500$  (lb). The thrust during trimming is a continuous signal (not a bang-bang). It is adjusted by the throttle control input which varies between zero and  $\pm 1$ . The jet is rotated at a yaw angle  $Dz=90^\circ$  from the -x direction and a positive throttle input generates a +y force and a positive yaw. The engine mass, inertia, and moment arm are irrelevant in this case because the thruster is fixed and it does not gimbal.
- The mass properties, the basic aero, the damping derivatives, and the hinge moment coefficient files are the same as in ascent. The aerosurface coefficients file is "ReSP.Delt".

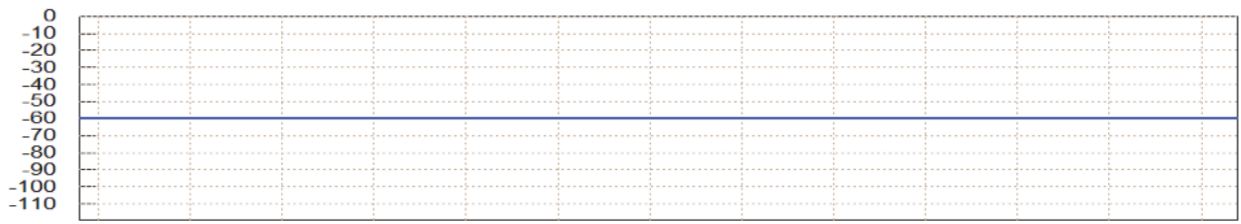
## 2.0 Trajectory (A) Analysis

Let us take a look at the first trajectory. You must first select the filename "RSP\_Entry-A.Traj" from the filename selection menu, and from the Trim menu we select option-2 for plotting it, as shown below. This is the descent phase of the first trajectory whose ascent part was analyzed in the ascent analysis. The CG location is constant because there is no significant fuel depletion. The angle of attack is initially high and it is gradually reduced to  $5^\circ$ . The flight path angle  $\gamma$  is mostly negative because the vehicle is descending. The peak velocity is in the beginning of the descent flight,  $V=4,500$  (ft/sec) at Mach 4. The max dynamic pressure is about 200 (psf) towards the lower end of the flight.

Select One Data File from Each Menu Category

The following analysis requires some data files to be selected from the current project directory. Select one data file for each category, (some of the categories are optional).

<b>Mass Properties</b> ReSP.Mass	<b>Surface Hinge Moments</b> ReSP.HMco
<b>Trajectory Data</b> RSP_Entry-A.Traj	<b>Aero Damping Derivat</b> ReSP.Damp
<b>Basic Aero Data</b> ReSP.Aero	<b>Propulsion Data</b> RSP_Entry.Engn
<b>Contr Surface Aero Coeff</b> ReSP.Delt	<b>Aero Uncertainties</b> NO DATA FILE
<b>Slosh Parameters</b> NO DATA FILE	<b>OK</b>



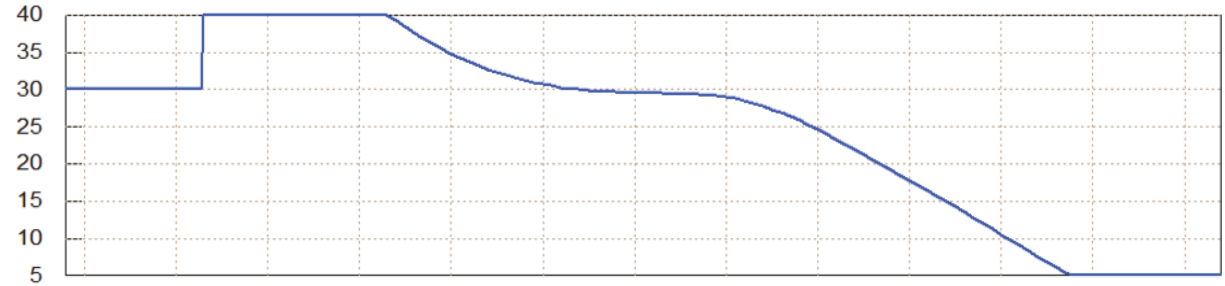
Xcg



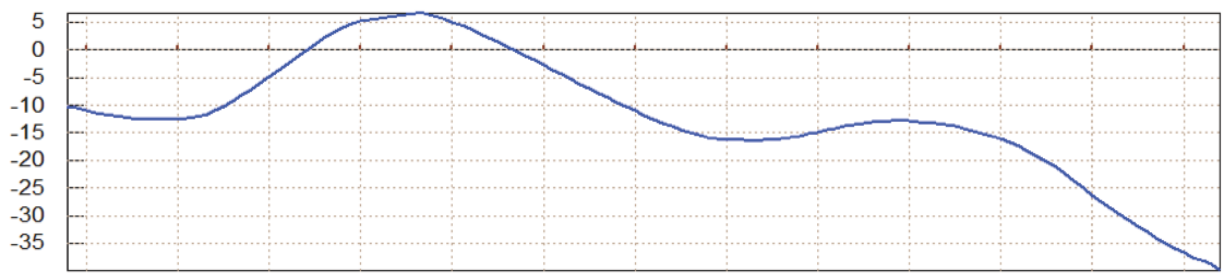
Ycg



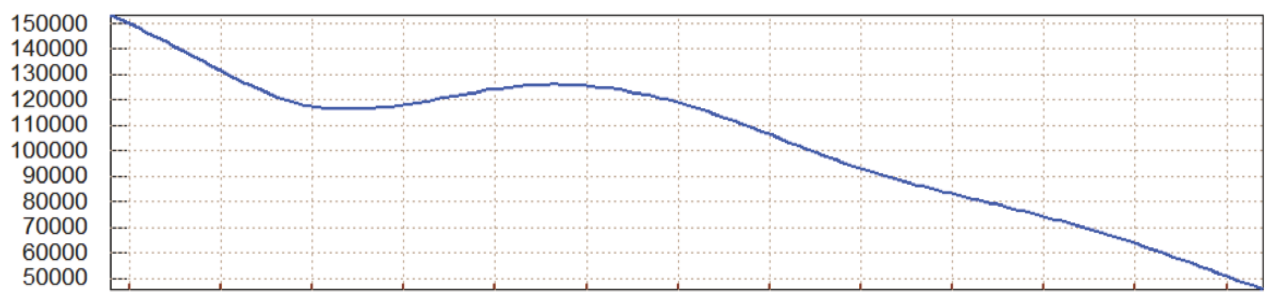
Zcg



Alpha

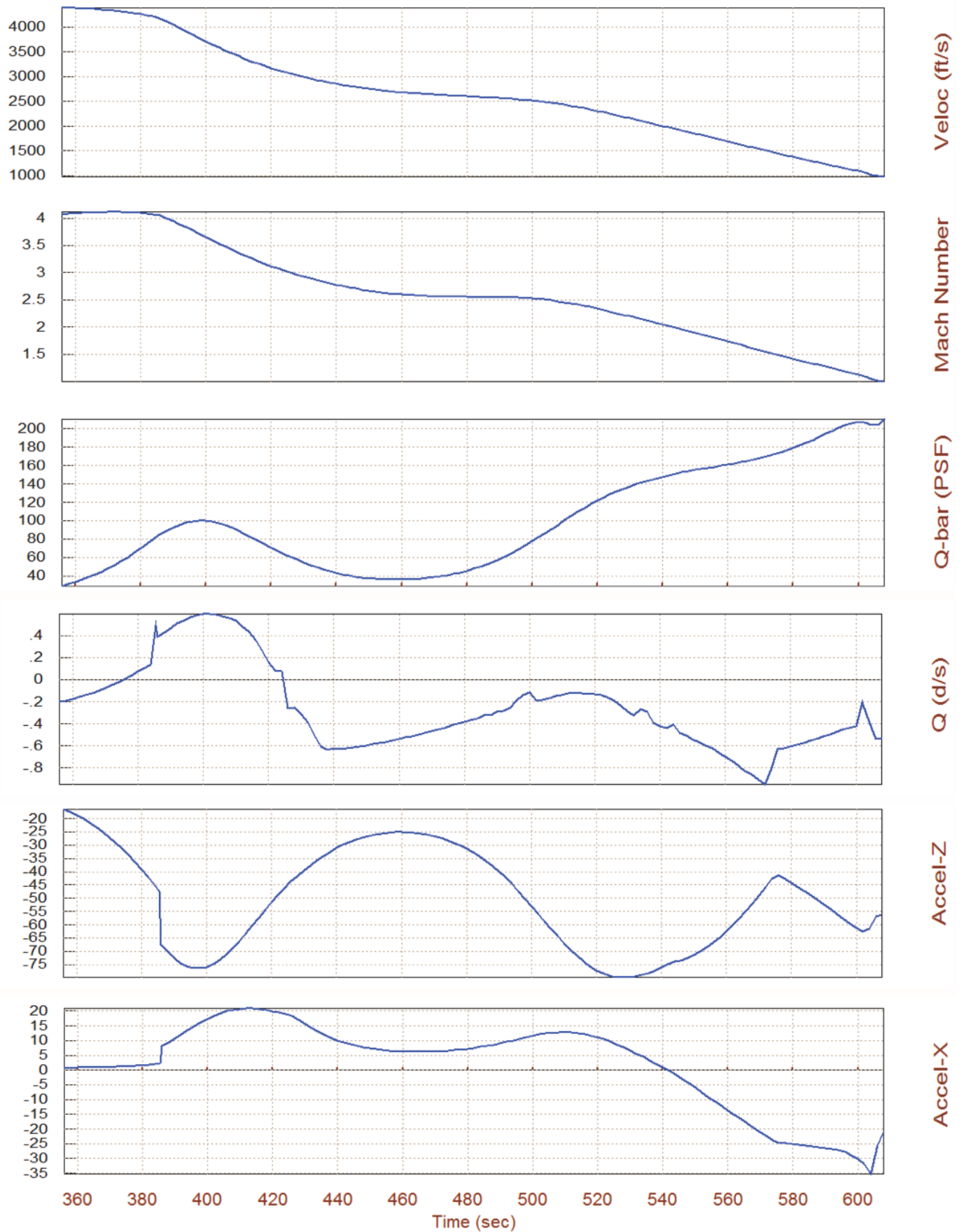


Gamma



Altitud (ft)

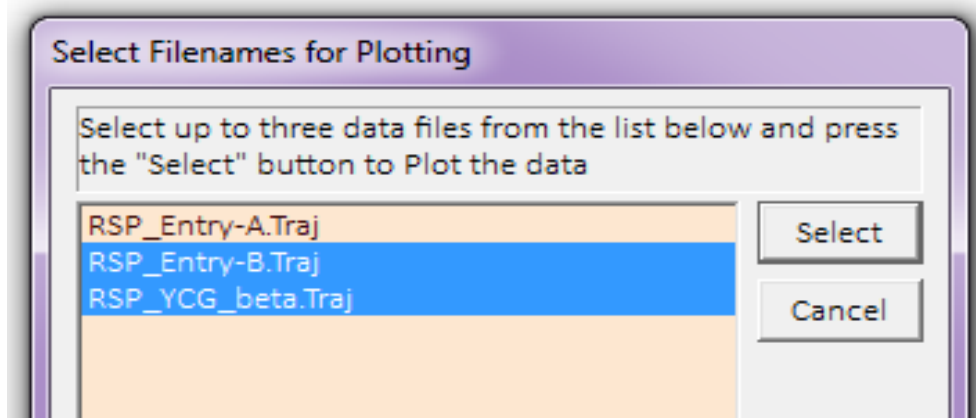
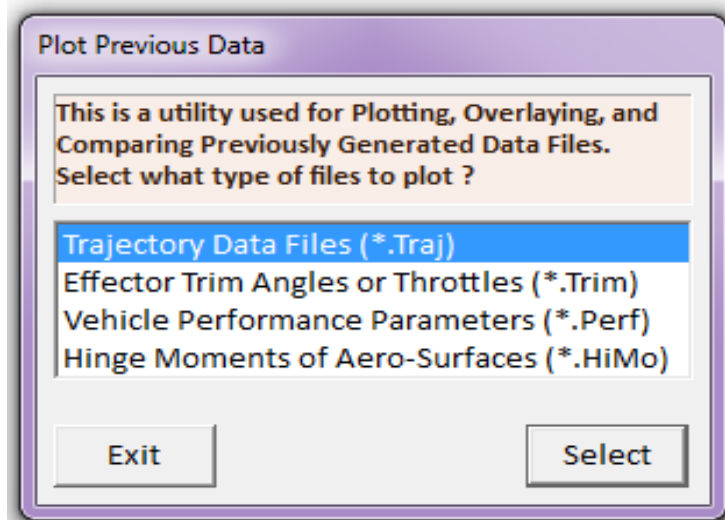
Time (sec)



## 2.2 Trajectory (B) analysis with Dispersions

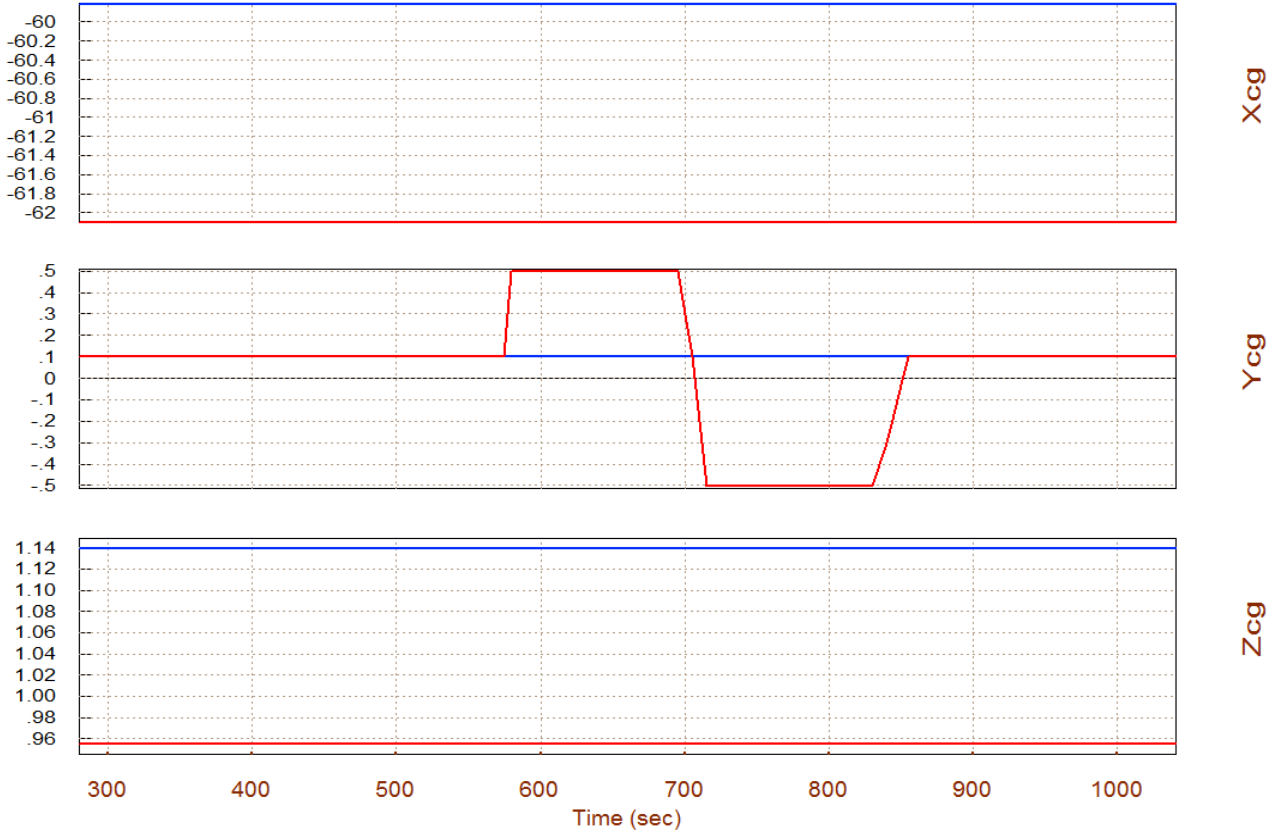
The descent phase of the second trajectory is in file "*RSP\_Entry-B.Traj*". The ascent segment of this trajectory was analyzed in the ascent section. This trajectory was modified slightly from its original version by introducing a few dispersions for analysis, a variation in beta and a CG offset. A trajectory is modified by going to "*Graphic Options*", then "*Modify a Trajectory Plot*", and selecting a variable to modify from the menu. However, we are not showing the details because this process was demonstrated in previous examples. The modified trajectory was saved and then renamed in a separate file "*RSP\_YCG\_beta.Traj*".

Before analyzing them let us plot the two trajectories together and show their differences. From the Trim menu select option -12, and from the menu that selects the file type select the trajectory type (.Traj). Then from the trajectory filenames menu select the two trajectory files, as shown. The blue curves represent the original and the red curves are the modified trajectory. The Ycg was moved from its original position +0.1 to 0.5 (ft) for a period of 120 sec, and then to -0.5 (ft) for another 120 sec, and then back to its original position. The Xcg and the Zcg were also modified. An angle of sideslip dispersion doublet was also introduced at t=400 sec. It was changed from  $\beta=0^\circ$  to  $\beta=2^\circ$ , then to  $\beta=-2^\circ$ , and then back to  $\beta=0^\circ$ . The  $\beta$ -disturbance and the CG variation occur at different times in order to analyze the effects separately.

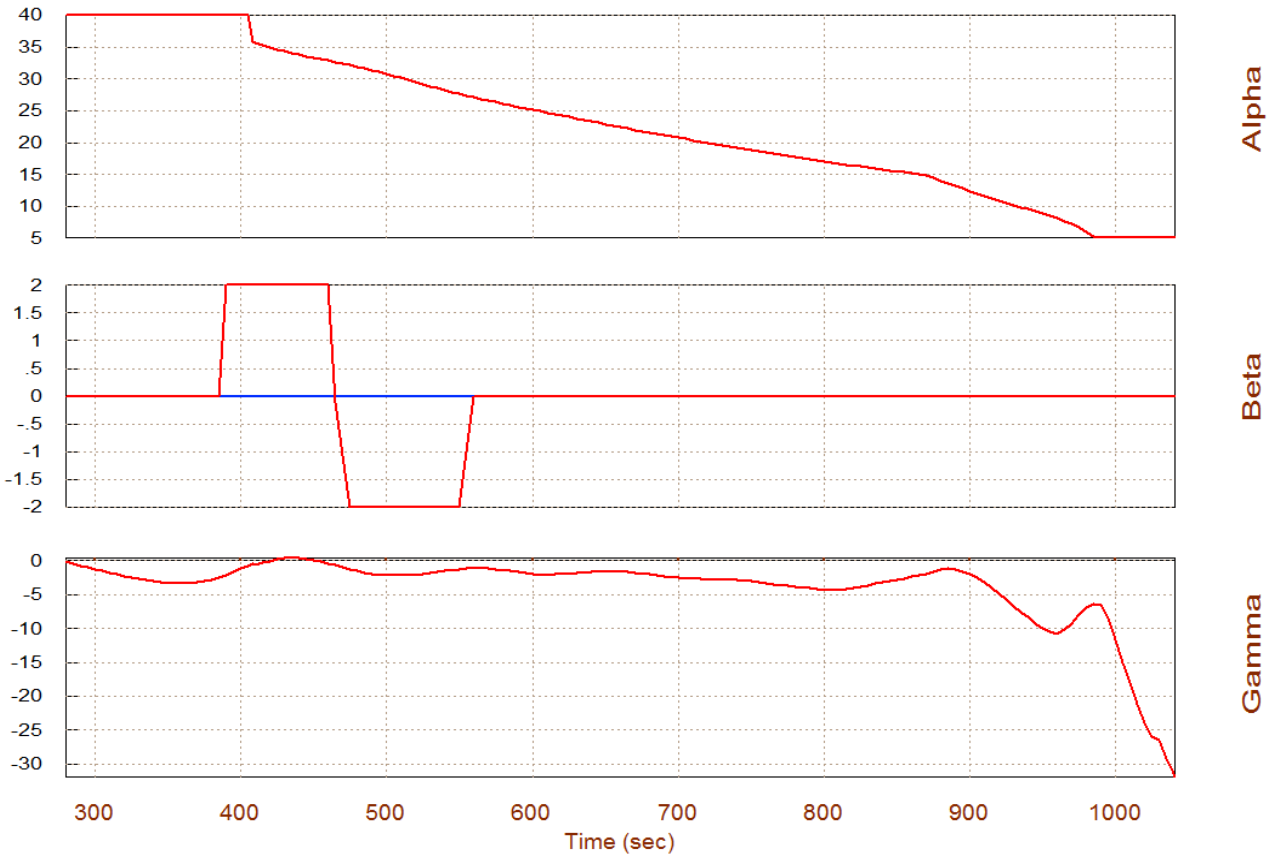




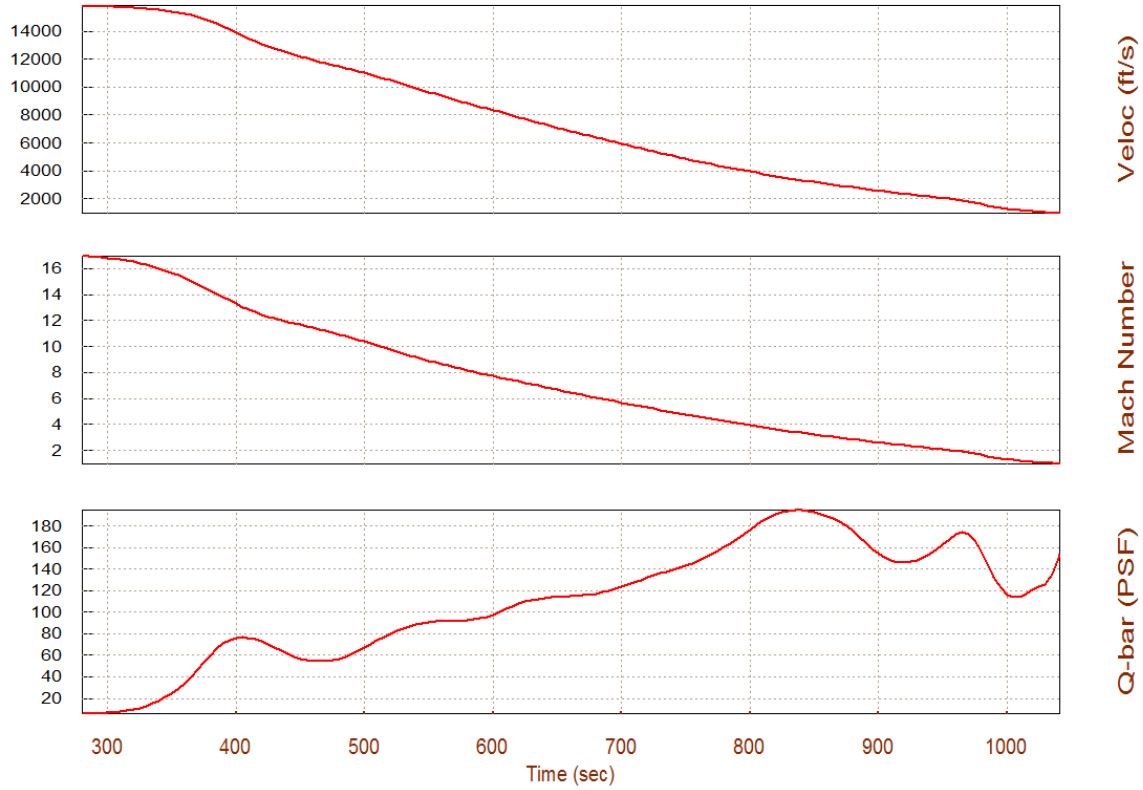
Vehicle CG in (feet), User Modified Trajectory ...



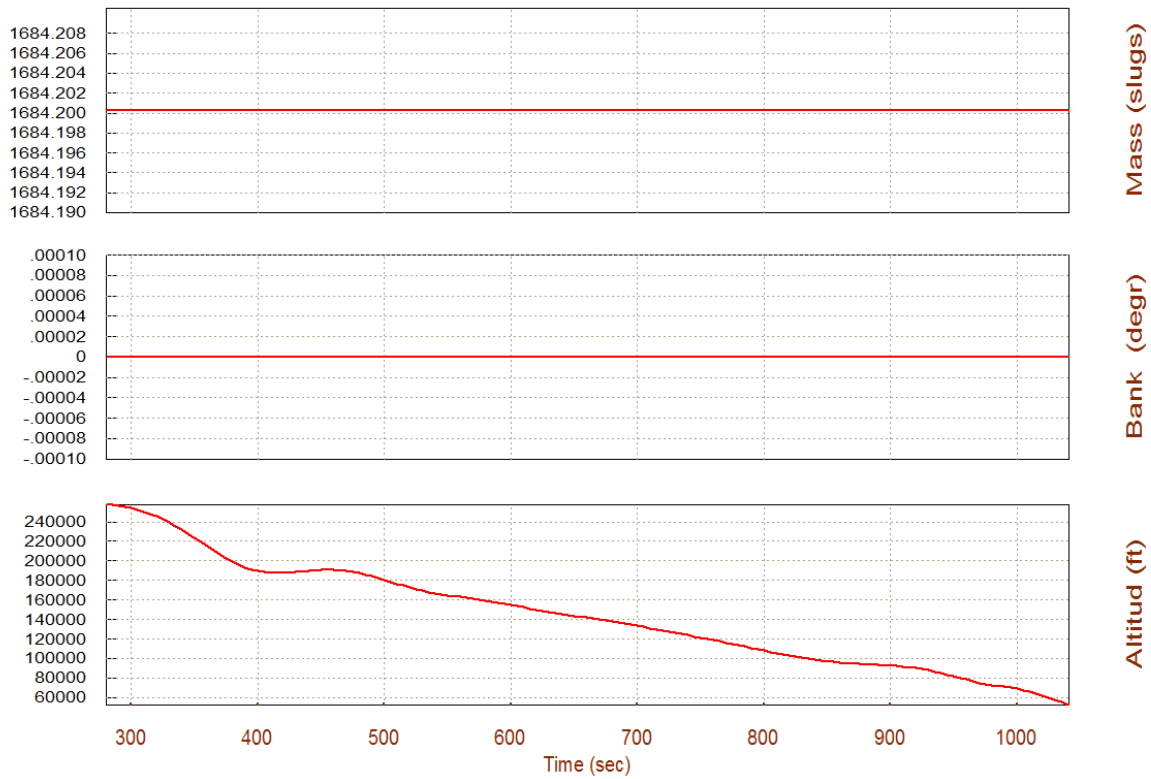
Angles of Attack/Sideslip/Flight Path (deg), User Modified Trajectory



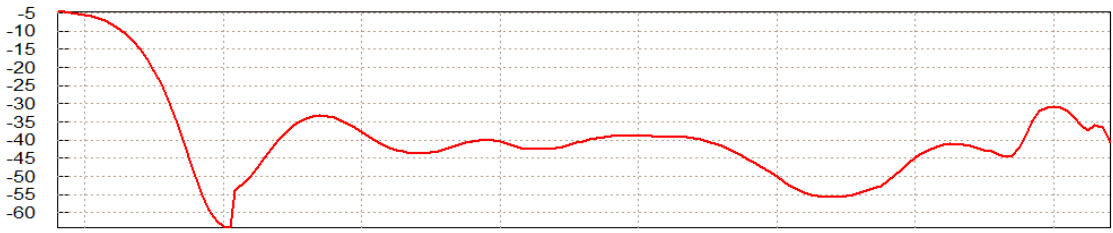
Velocity, Dynamic Pressure, User Modified Trajectory ...



Vehicle Altitude, Mass, Bank Angle, User Modified Trajectory ...



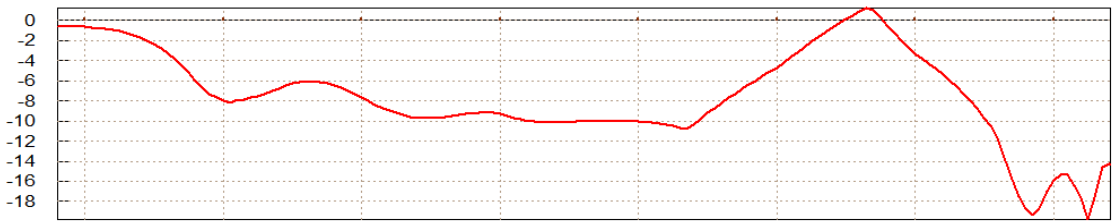
Sensed Acceleration in (ft/sec<sup>2</sup>), User Modified Trajectory ...



Accel-Z



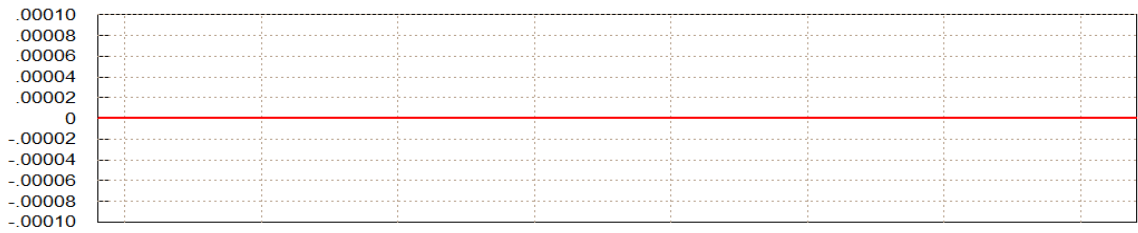
Accel-Y



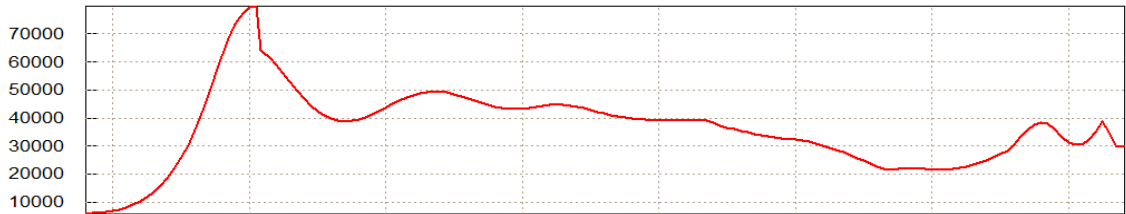
Accel-X

Time (sec)

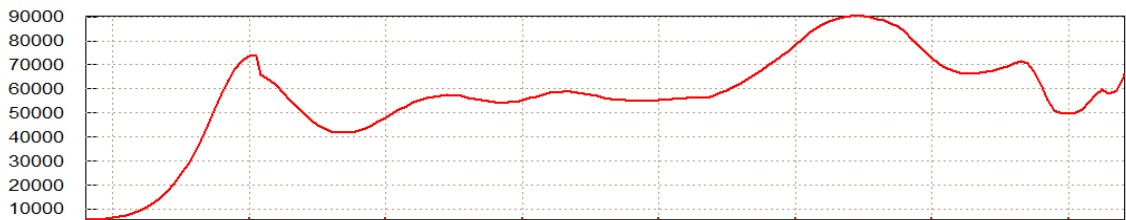
Aero Lift/Drag Forces, Eng. Thrust in (lb), User Modified Trajectory ...



Thrust Force



Drag Force

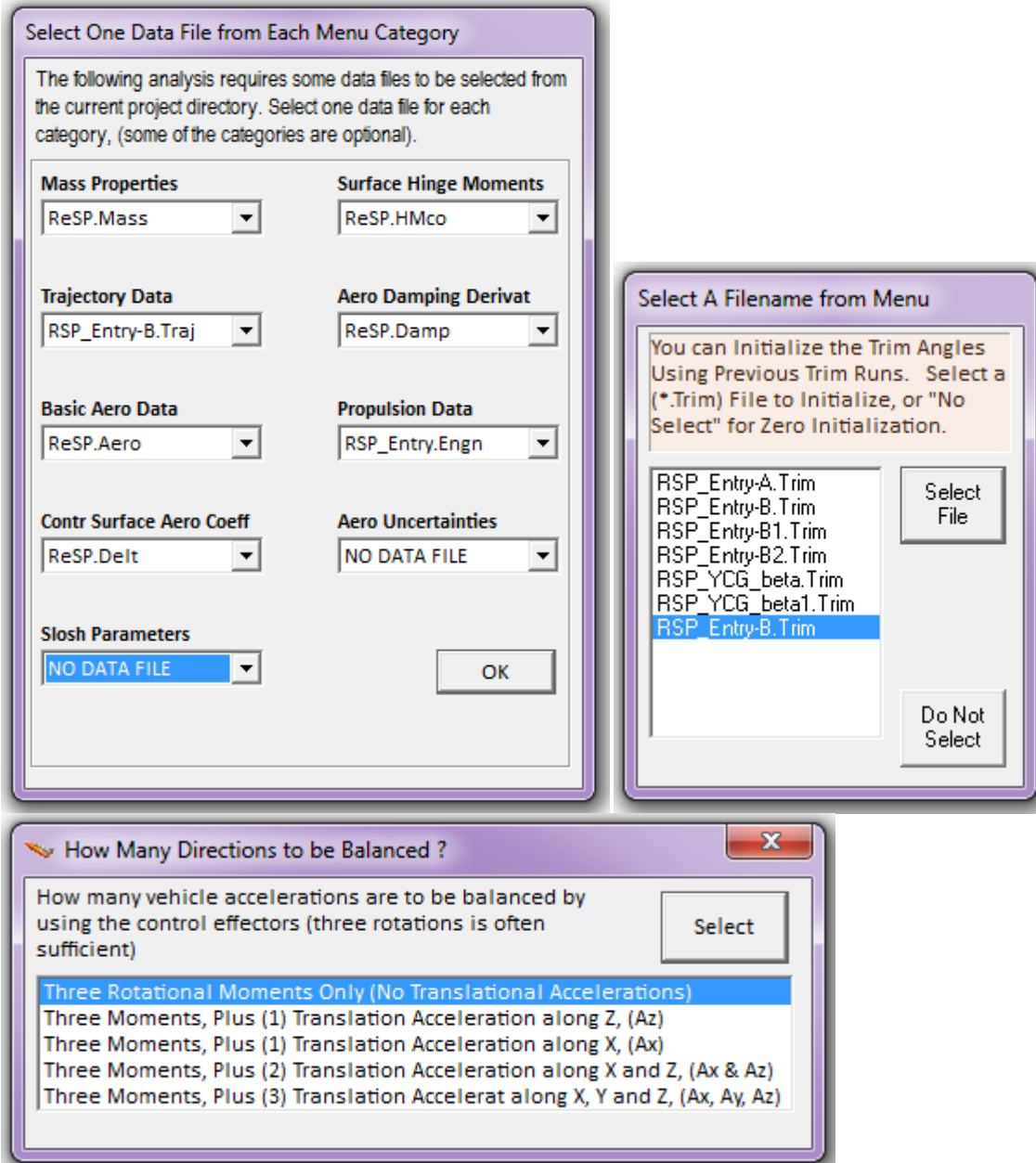


Lift Force

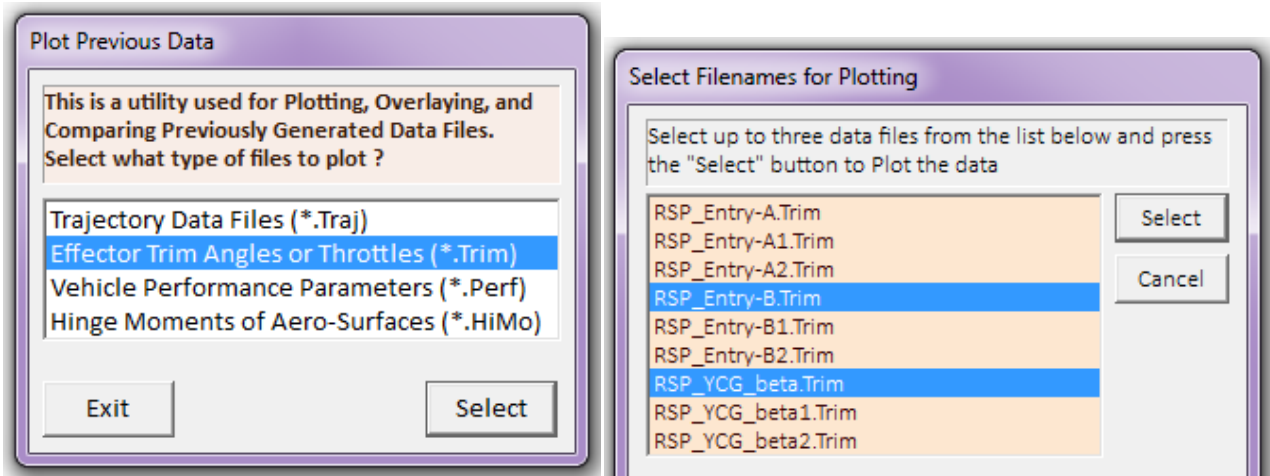
Time (sec)

## 2.3 Trimming with Trajectory (b) and the Dispersion Trajectory

Now we return to the Trim main menu and select option (3) to trim the vehicle, first along the two trajectories separately, the original trajectory (b) and then using the modified trajectory. Since we do not have the effector capability to trim along any translations we must trim only along the 3 moments.



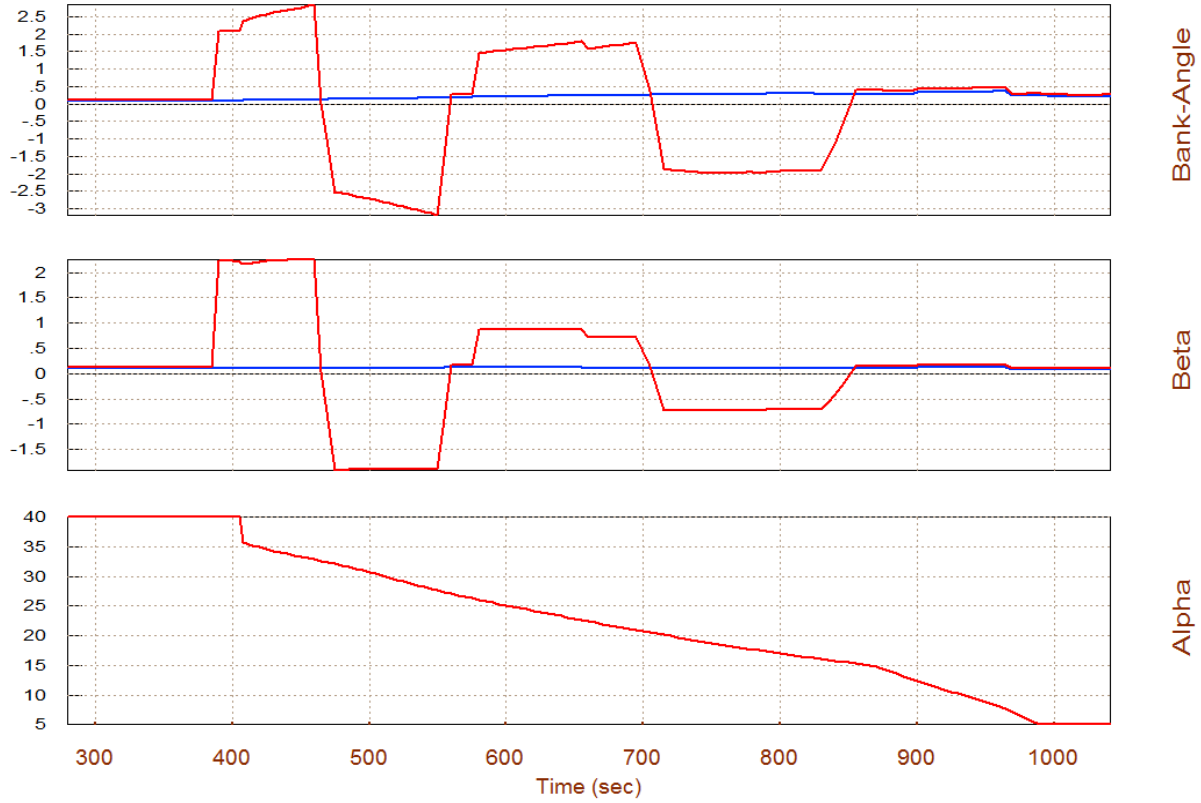
The generated trim files for the original and modified trajectories are: "*RSP\_Entry-B.Trim*" and "*RSP\_YCG\_beta.Trim*" respectively. After trimming, return to the Trim menu and select option (12) to plot these two files together and compare the trim results. The trim variables from the original trajectory are shown in blue and those from the modified trajectory are in red.



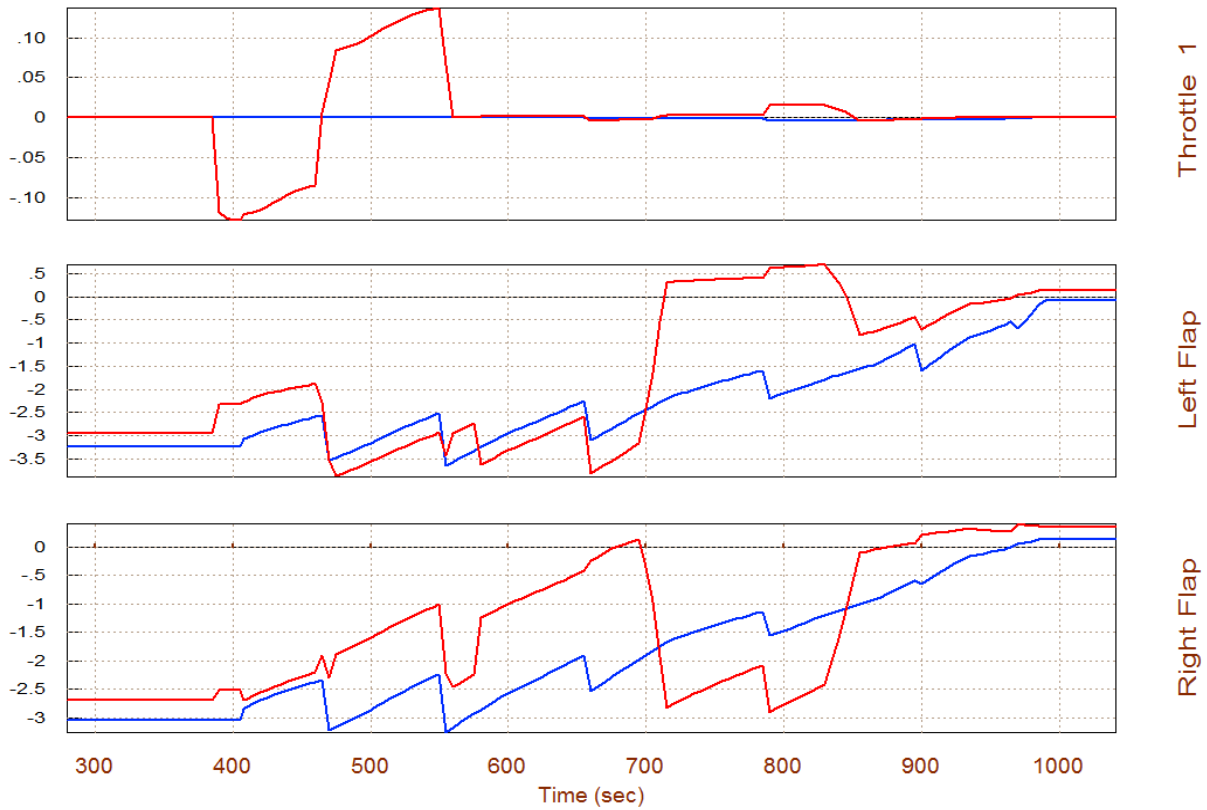
Notice how the sideslip beta and the bank angle are significantly affected by the dispersions: (a) the 2° beta dispersion doublet in the period between 390-560 (sec), and (b) the ±Ycg shift doublet in the period between 570-850 (sec).

- The bank angle was initially small less than 0.4° due to the original Ycg=0.1 (ft) shift. The beta disturbance causes the vehicle to bank +2.5° and then -3°.
- Similarly, the ±Ycg offset doublet causes the vehicle to bank ±2°, at different times as it changes sign.
- The throttle command for the yaw thruster (initially zero) becomes negative (-10%) at 390 (sec) to generate a negative yawing moment to trim against the positive yawing moment generated by the  $\beta=+2^\circ$ . Then it reverses when beta is reversed.
- The beta and Ycg variations also create differential trim deflections in the two flaps. They rotate in order to generate the rolling moments necessary to balance the moments caused the two dispersions. The differential flap deflections (red) is even more visible in ±Ycg shift region in comparison with the nominal trajectory trim results (blue).

Alpha, Beta, and Bank Angle (phi), Assuming Effectors are Trimmed (deg)

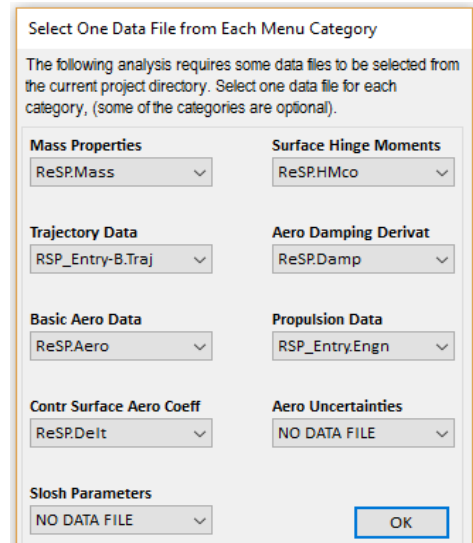
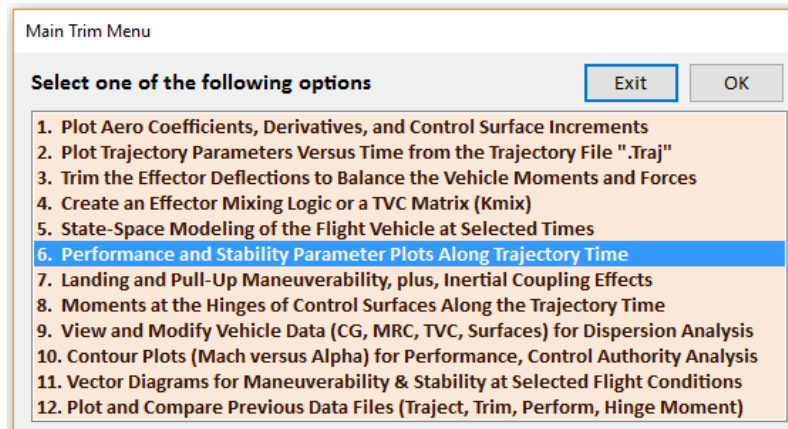


Surface & Engine Deflections/ Throttle, User Modified Trajectory ...

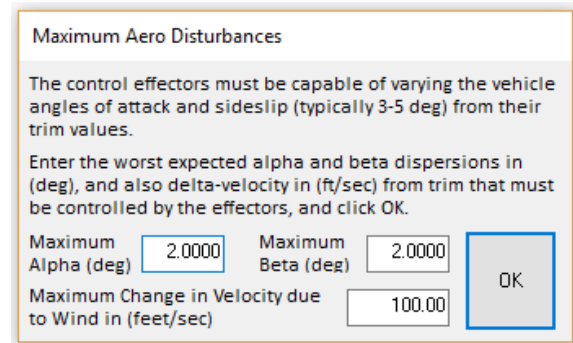


## 2.4 Performance Analysis along Trajectory (B)

Let us return to the Trim main menu and select option-6 to analyze the vehicle performance parameters along trajectory (b). We will assume max dispersions  $\alpha_{\max}=\pm 2^\circ$ ,  $\beta_{\max}=\pm 2^\circ$ ,  $V_{\max}=\pm 100$  (ft/sec), and shall allow the program calculate the (3x3) mixing-logic matrix that converts the roll, pitch and yaw demands to flap deflections and Y-jet throttle commands.



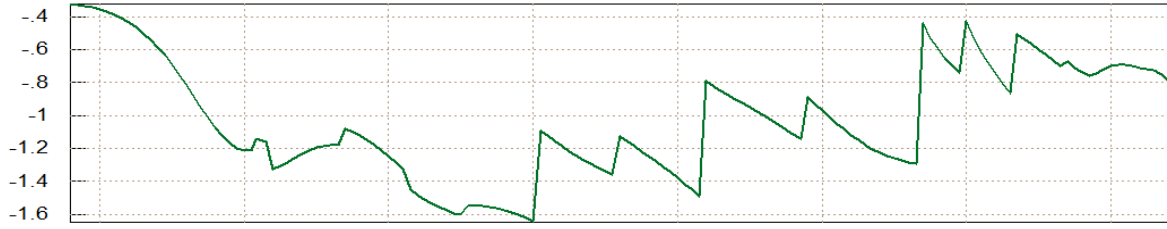
From the static margin and the pitch T2-inverse parameter we conclude that the vehicle is stable in pitch, throughout descent, with a short period resonance of 1.6 (rad/sec). The yaw axis is also stable because  $C_n\beta$ -dynamic  $> 0$ , and the yaw T2-inverse parameter is negative showing a Dutch-Roll resonance that peaks to 1.82 (rad/sec). The  $C_n\beta$ -dynamic becomes very small at  $t=900$  sec (barely acceptable), which causes the LCDP to increase at that time. The LCDP ratio is also small during the first 100 seconds (barely greater than 0.2).



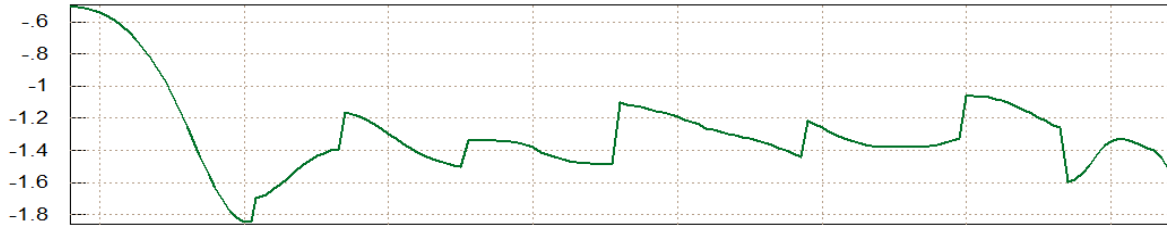
The control effort magnitude against  $\pm\alpha_{\max}$  and  $\pm\beta_{\max}$  dispersions is acceptable in all 3 control directions because it is less than 50%. It is worst in yaw because it relies on the reaction control jets since the vehicle has no rudder. The control authority against air-speed variations  $V_{\max}=\pm 100$  (feet/sec) is also more than adequate in all 3 directions. The bank angle ( $\phi$ ) is used for analyzing landing conditions with cross-wind. It occurs when the vehicle is landing with cross-wind defined by  $\beta_{\max}=2^\circ$ , and in this case  $\phi=6$  degrees, which is not bad. The blue line shows the additional bank that would occur due to an increase in wind-speed  $V_{\max}=100$  (feet/sec). The last plot shows the maximum positive and negative accelerations that can be produced by the effectors system in the 3 control directions before saturating. The yaw acceleration is constant because it is produced by the RCS jets, whereas the other two depend on the dynamic pressure and the aero coefficients.



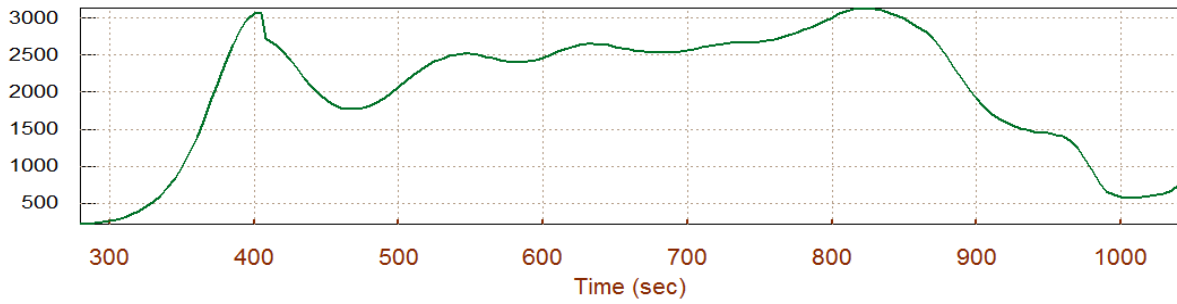
Short-Period (w/ Time-to-Double-Ampl-Inverse (/sec),  $Q_{\alpha\beta}$  (deg-lb/ft<sup>2</sup>))



Ptch T2-inv

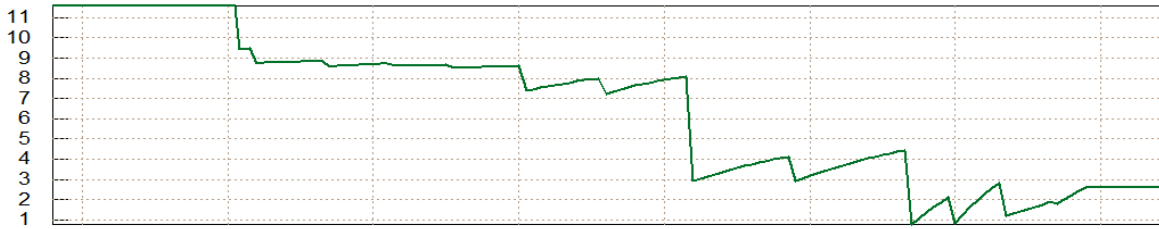


Yaw T2-inv

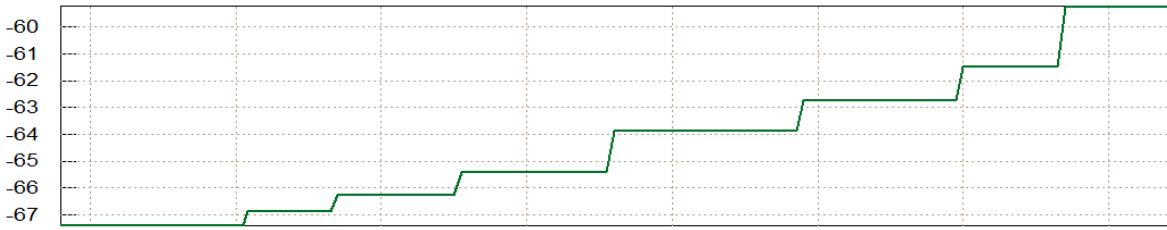


$Q_{\alpha\beta}$

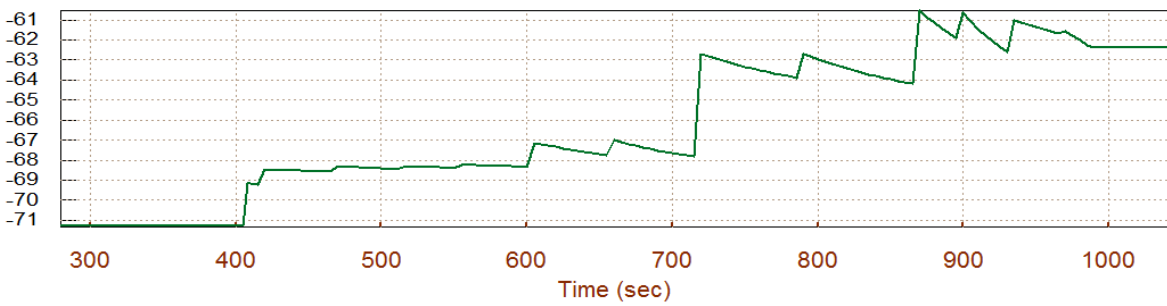
Static Margin, Center of Pressure, Aero-Center (ft) Reusable Rocket-Plane Descen



Stat-Margn %

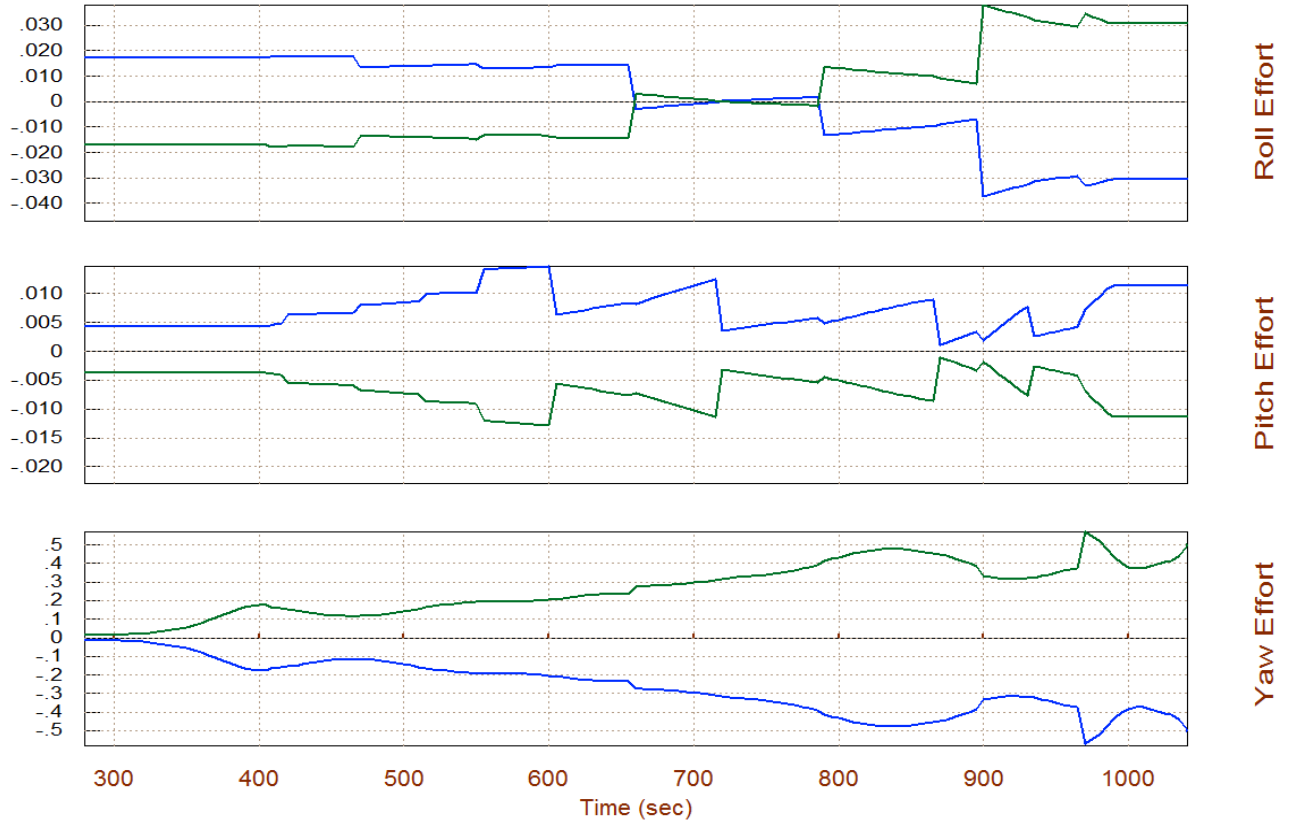


Centr-Press

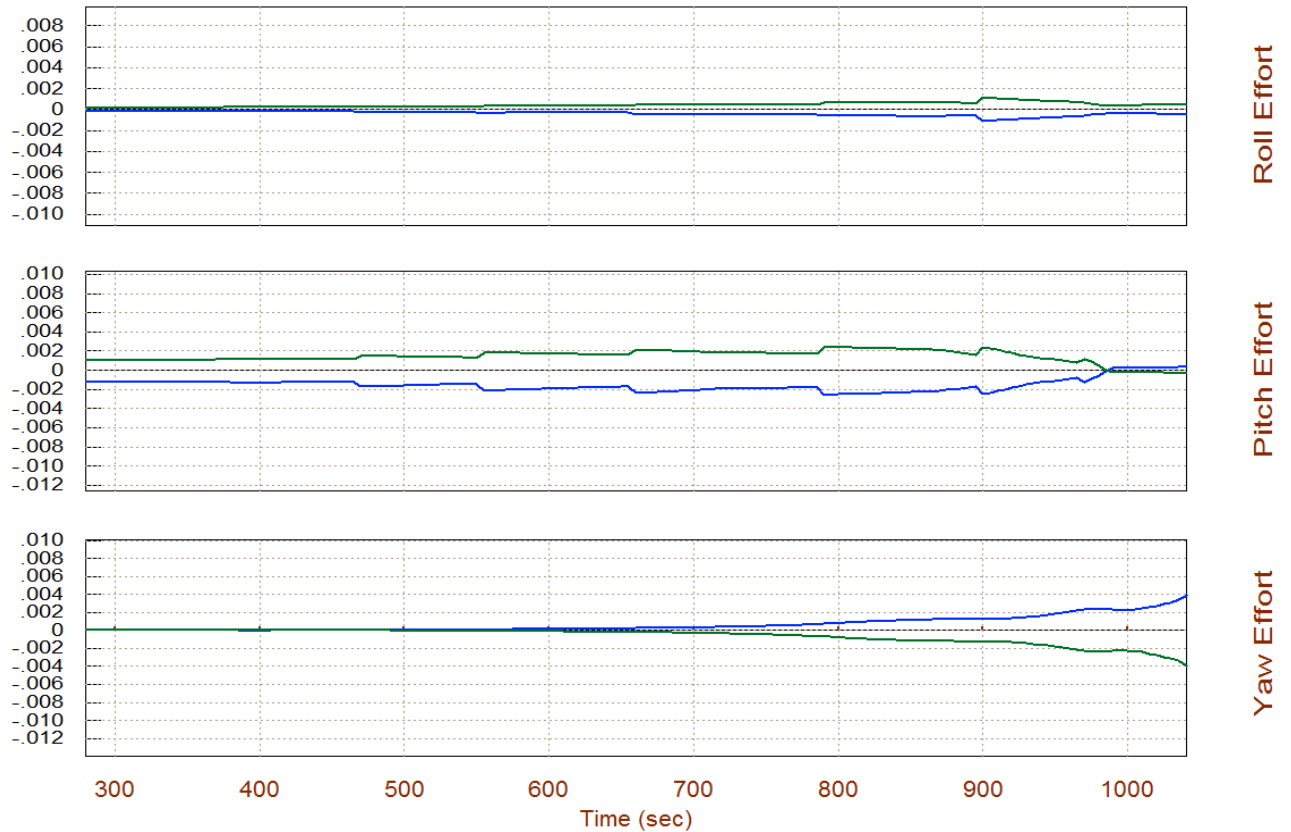


Aero-Center

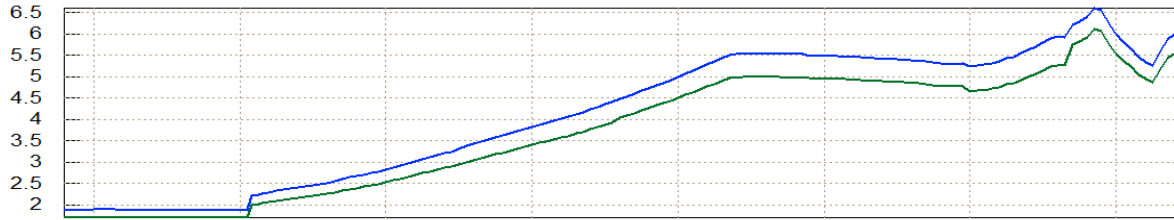
Rotation Control Authority  $|dQ/dQ_{max}| < 1$  for 2 (deg) of Alpha & Beta Variation



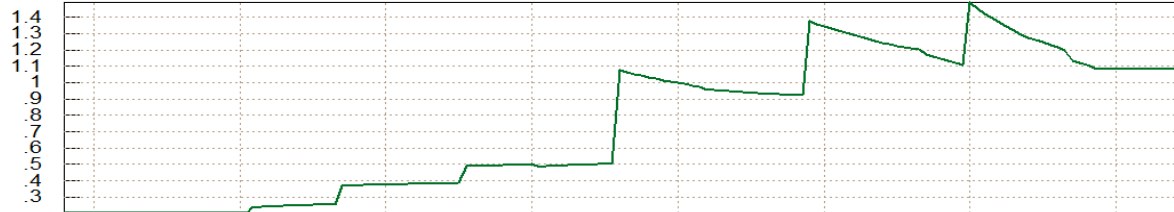
Rotational Control Authority  $|dQ/dQ_{max}| < 1$  Against +Vmax & -Vmax Veloc Variations



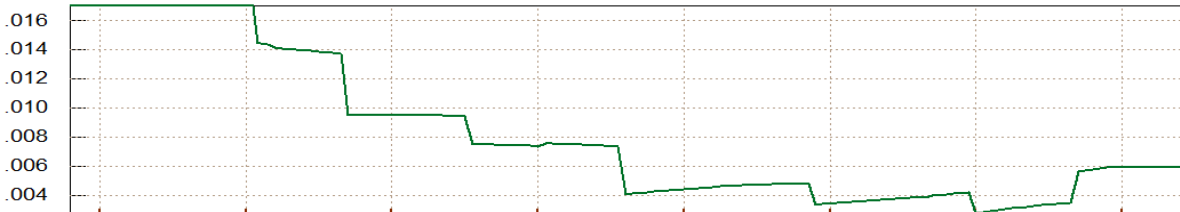
Bank Angle, LCDP Ratio, Cn\_beta\_dynam /deg, Reusable Rocket-Plane Descent Trajec



phi (deg)



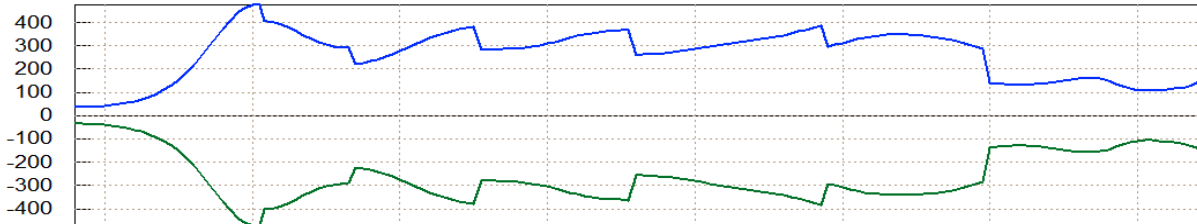
LCDP Ratio



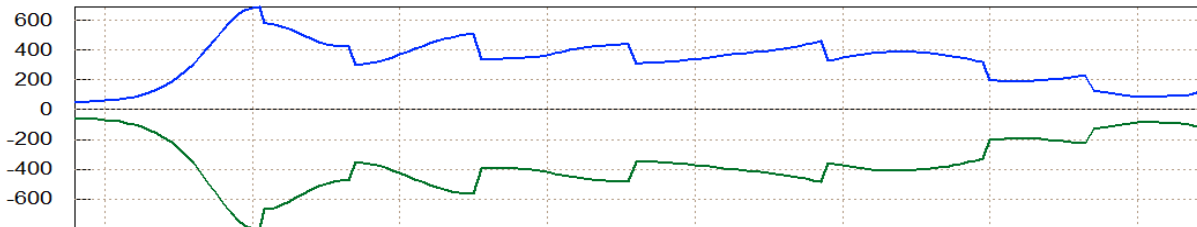
Cnbeta-dyn

Time (sec)

Max Angular Accelerations (rad/sec^2), at Maximum +ve and -ve Control Demands



P\_dot (Max)



Q\_dot (Max)



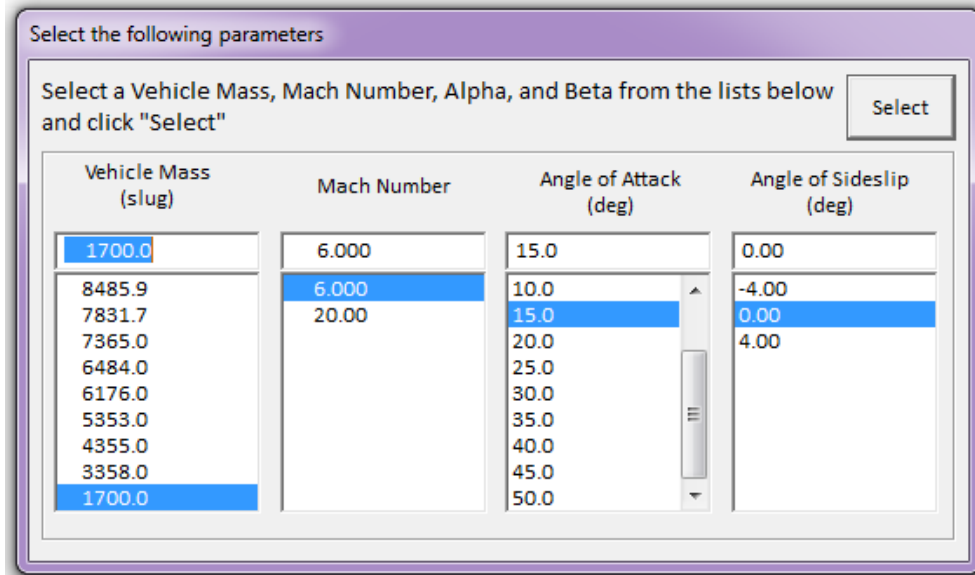
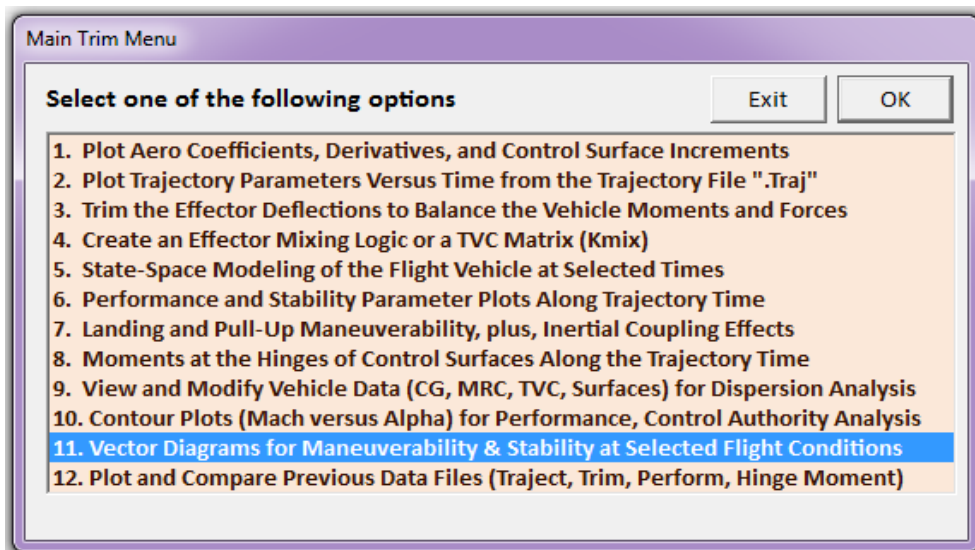
R\_dot (Max)

Time (sec)

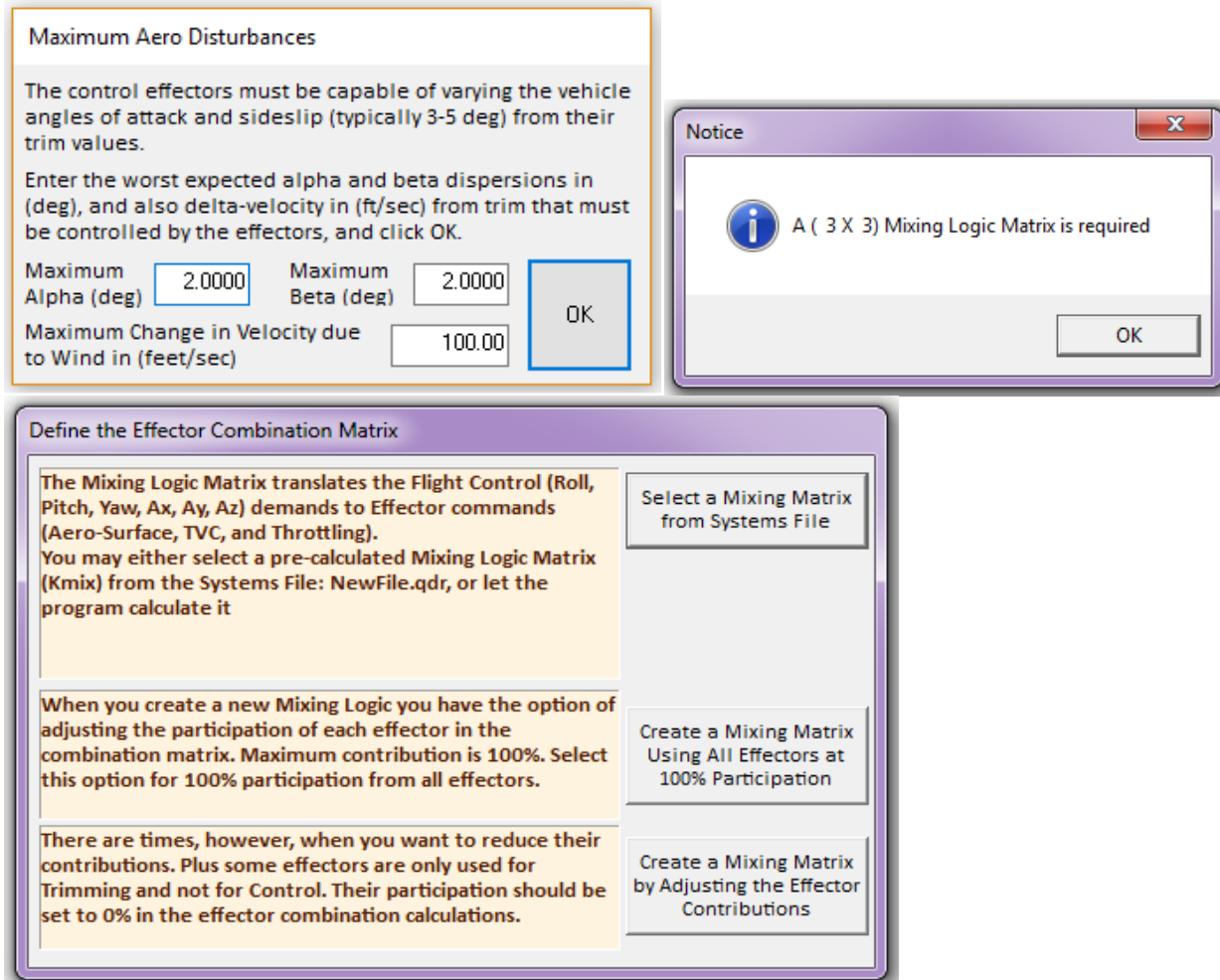
## 2.5 Vector Diagram Analysis

Vector diagrams are used for analyzing the vehicle controllability at a fixed flight condition. We compare the control authority of the aerosurfaces, in two directions, against the effects of a wind-shear disturbance on the vehicle that is defined in terms of dispersions  $\alpha_{\max}$  and  $\beta_{\max}$  in the angles of attack and sideslip from trim  $\alpha_0$  and  $\beta_0$ . It is also defined by variations in the air-speed  $V_{\max}$  along the velocity vector  $V_0$ . By comparing the control vectors against the disturbance vectors we can determine if the vehicle provides sufficient control authority to counteract the disturbance moments and forces. It also allows us to examine the directions of the controls against disturbances in two directions and evaluate static controllability.

From the Trim menu select option-11, and then a flight condition at  $t=800$  sec. The following dialog consists of menus used for selecting the vehicle mass, Mach number, alpha, and beta. Select the default values which correspond to this flight condition and click "Select".



The disturbances are defined by the maximum dispersions produced in alpha and beta from trim. In the next dialog enter the maximum variation angles ( $\alpha_{max}$  and  $\beta_{max}$ )=2°, and air-speed variation  $V_{max}$ , and allow the mixing-logic algorithm to calculate the (3x3) effector combination matrix using 100% participation from all 3 effectors (2 surfaces plus yaw RCS).



The vector diagrams in Figure 2.5.1 show the maximum roll and yaw accelerations  $\dot{P}$  and  $\dot{R}$  in (rad/sec<sup>2</sup>) and side acceleration  $\dot{y}$  achieved by maximizing the roll and yaw controls, at t=800 sec. The blue vectors show the maximum positive and negative yaw acceleration achieved by maximizing the yaw effector (which is mainly due to the yaw jets). The green vectors show the moments and accelerations achieved by maximizing the roll effectors (which is mainly due to the flaps). The yaw control is affecting the yaw direction, as expected, but the roll control is also coupling into yaw. The red vectors show the accelerations produced by the sideslip dispersion  $\beta_{max}=\pm 2^\circ$ . A positive beta causes positive yaw acceleration and negative side acceleration  $\dot{y}$ . The bottom diagram shows the variation in yaw ( $\dot{r}$ ) and side ( $\dot{y}$ ) accelerations as a result from maximizing the yaw control. A positive yaw demand due to RCS jet firing causes positive accelerations in both yaw ( $\dot{r}$ ) and in ( $\dot{y}$ ), as shown by the solid blue vector. The red vectors show the accelerations due to  $\beta_{max}=\pm 2^\circ$ . A positive beta produces negative side acceleration and positive yaw acceleration (solid red line). There is a small y-acceleration bias at trim due to  $\beta_0$  which is

caused by the 0.1 (feet) YCG offset. The yaw control dominates the disturbance acceleration in yaw, but not in  $\dot{y}$  because it is not controllable.

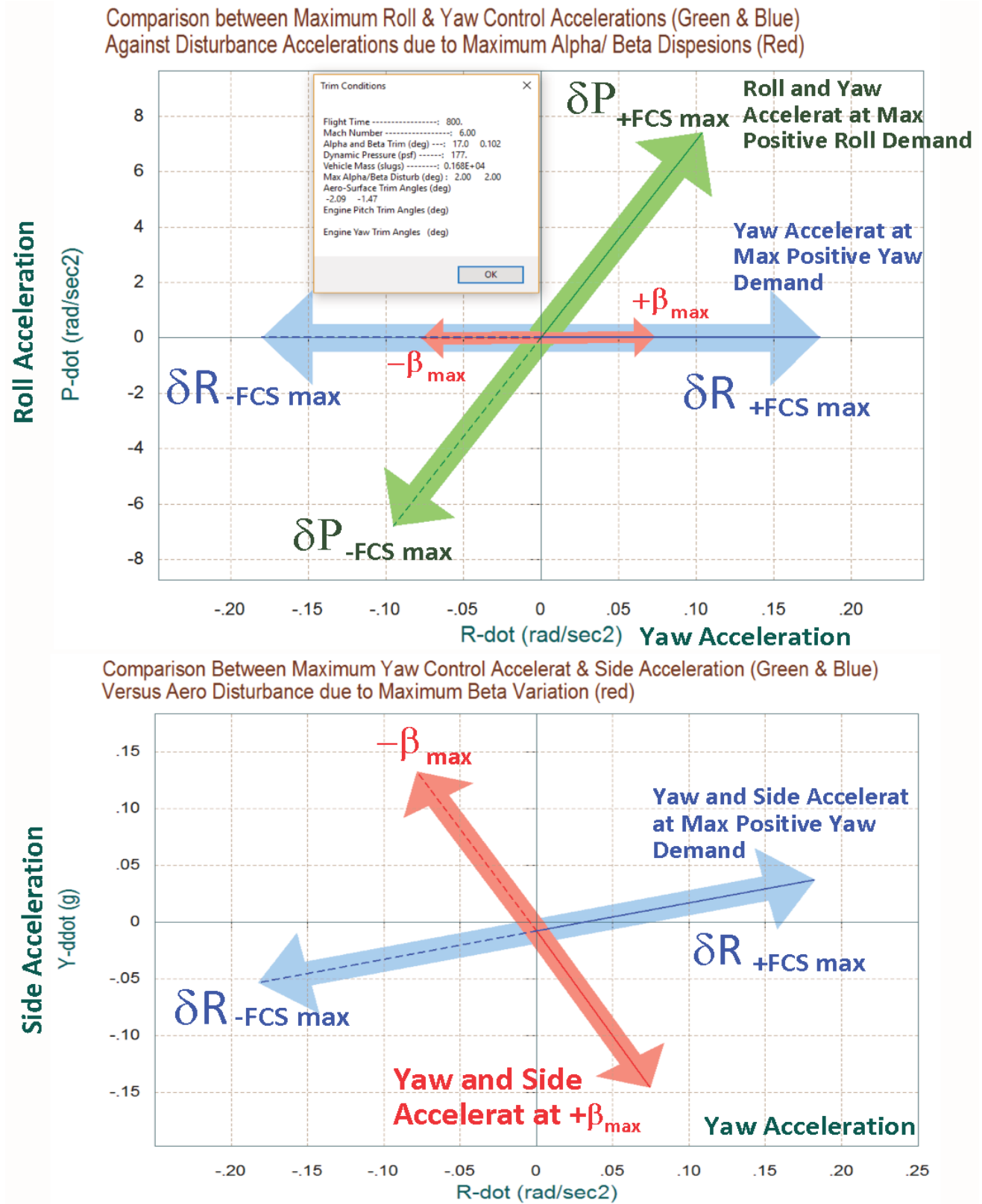


Figure 2.5.1 Roll, Yaw, and Side Accelerations at Maximum Roll and Yaw Demands

Figure 2.5.2 is a maximum accelerations vector diagram in the longitudinal (pitch, X, and Z) directions when the pitch control is maximized, in the extreme positive and extreme negative directions. Notice that at trim when the pitch control is zero the vehicle is trimmed at  $-0.9\text{ g}$  z-acceleration and at  $-0.588\text{ g}$  x-acceleration. Variation in the pitch control affects mostly the pitch direction. The pitch and Z accelerations produced when the pitch control is max negative, is bigger in magnitude (blue dashed vectors) than when the acceleration demand is max positive (blue solid vector). This asymmetry is due to the fact that the vehicle is statically stable and it is flying with a positive  $\alpha=15^\circ$ , and therefore, it is a lot easier to pitch down than it is to pitch up. The variation in wind direction due to  $\pm\alpha_{\max}$  affects mainly the x-acceleration asymmetrically. The drag force ( $C_A=-C_x$ ) is reduced (or acceleration along x is less negative), shown by the red solid vector pointing upwards, when  $\alpha_{\max}=+2^\circ$ .

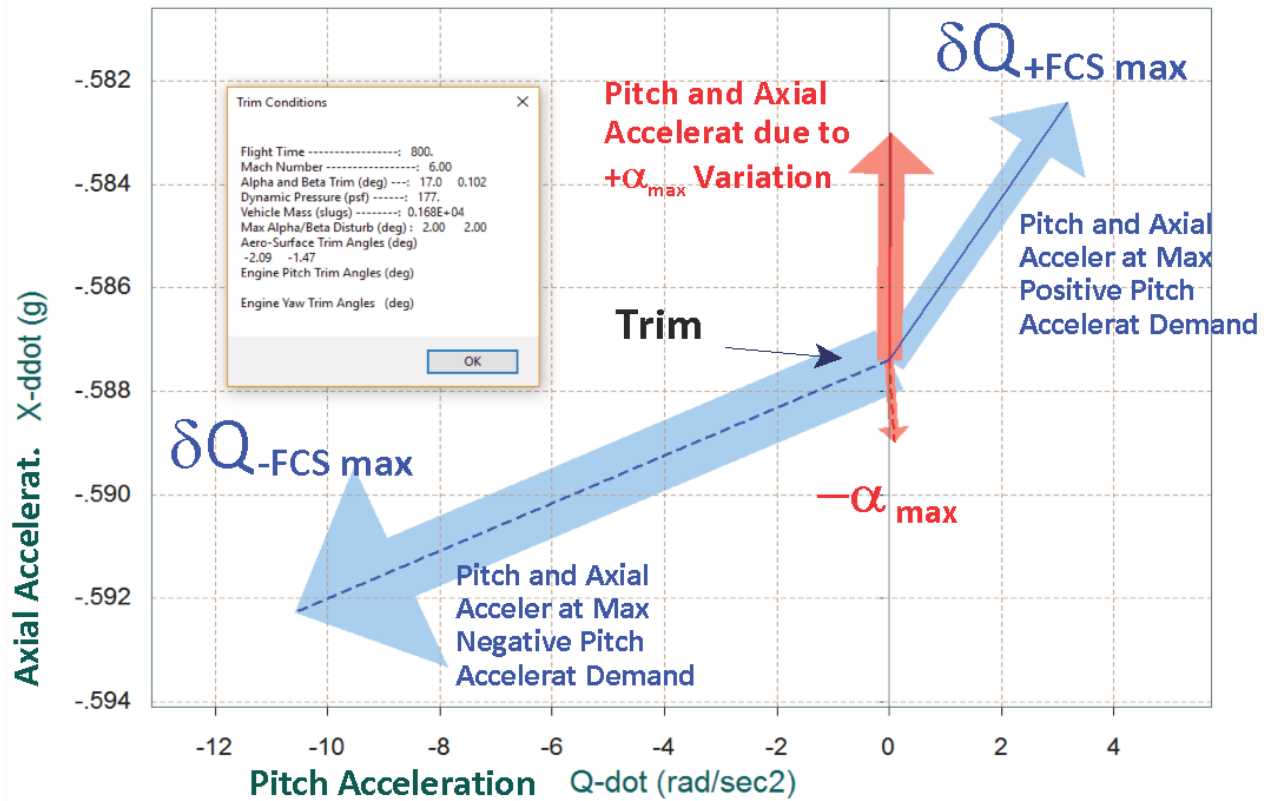
The vector diagrams in Figure 2.5.3 show the effects due to air-speed variations in the pitch and axial directions. The top diagram shows the maximum pitch and x accelerations produced when the pitch control is fully maximized, and obviously it is more effective when the demand is maximum negative, as already shown. The red vectors show the pitch and axial accelerations produced by the maximum airspeed variation  $V_{\max}=\pm 100$  (feet/sec). It shows that the airspeed variation has no effect in the pitch acceleration but it affects only the x-acceleration. A head-wind causes a negative x-acceleration (more drag), as expected. The airspeed variation effect is also shown in the partials diagram below. It strongly affects the x-force and acceleration but we cannot control it because the vehicle does not have a speed control effector.

The top diagram in figure 2.5.4 is a moment partials vector diagram showing the variation in roll and yaw moments per acceleration demands in roll and yaw in  $(\text{rad}/\text{sec}^2)$ . The blue vector is the moments partial  $\{C_n\delta R_{FCS}, C_l\delta R_{FCS}\}$  per yaw control demand and it is mainly in the yaw direction. The green vector is the moments partial  $\{C_n\delta P_{FCS}, C_l\delta P_{FCS}\}$  per roll control demand and it is pointing in the roll direction. They are almost orthogonal to each other. The red vector represents the moments per beta variation. It is mostly due to the  $C_n\beta$  partial. The  $C_l\beta$  partial is very small. It shows that the vehicle is directionally stable and the control authority is good because the control vectors dominate over the scaled partials due to beta.

The bottom diagram shows the partials of roll and yaw accelerations per acceleration demands. The green vector is accelerations per roll demand  $\{\dot{P}/\delta P_{FCS}, \dot{R}/\delta P_{FCS}\}$ , and the blue vector is accelerations per yaw demand  $\{\dot{P}/\delta R_{FCS}, \dot{R}/\delta R_{FCS}\}$ . The axis units are in  $(\text{rad}/\text{sec}^2)$  per  $(\text{rad}/\text{sec}^2)$ . Notice that they are unity vectors and are completely decoupled from each other, showing that the mixing-logic matrix perfectly diagonalizes the vehicle model.



Comparison between Maximum Pitch and Axial X-Force Control Accelerat (Blue & Green)  
Against Aero Disturbance due to Maximum Alpha Variation (red)



Comparison between Maximum Pitch and Normal-Z Control Accelerations (Blue & Green)  
Against Aero Disturbance due to Maximum Alpha Variation (red)

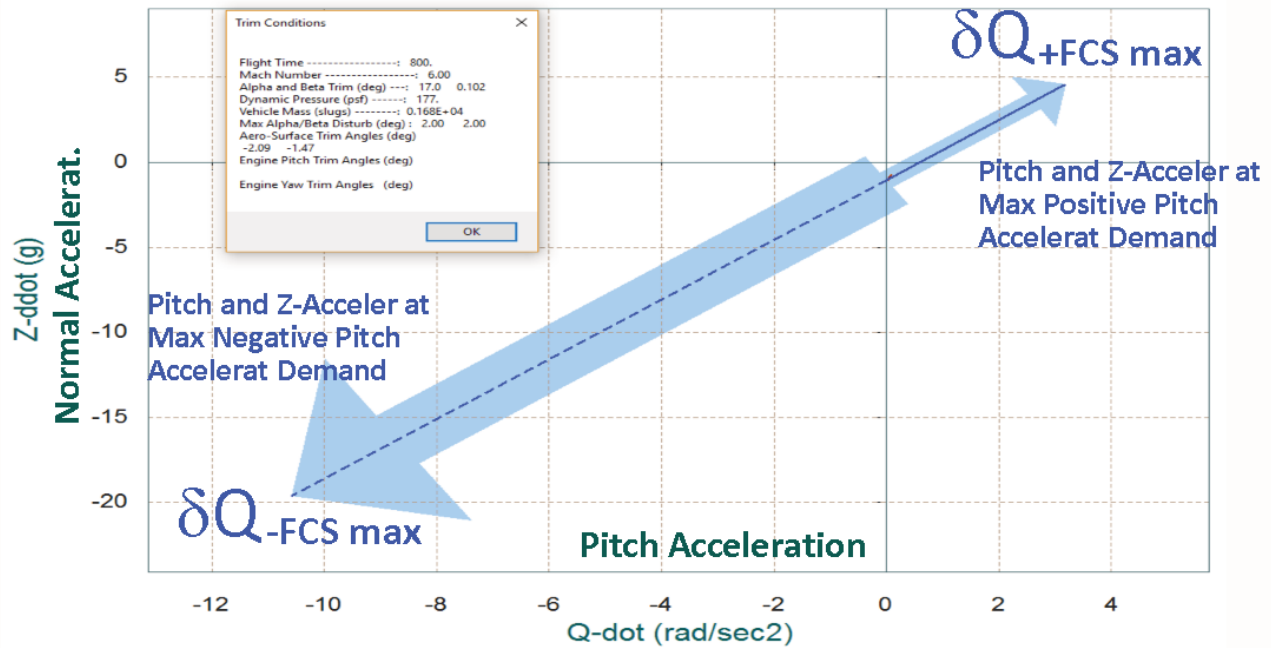
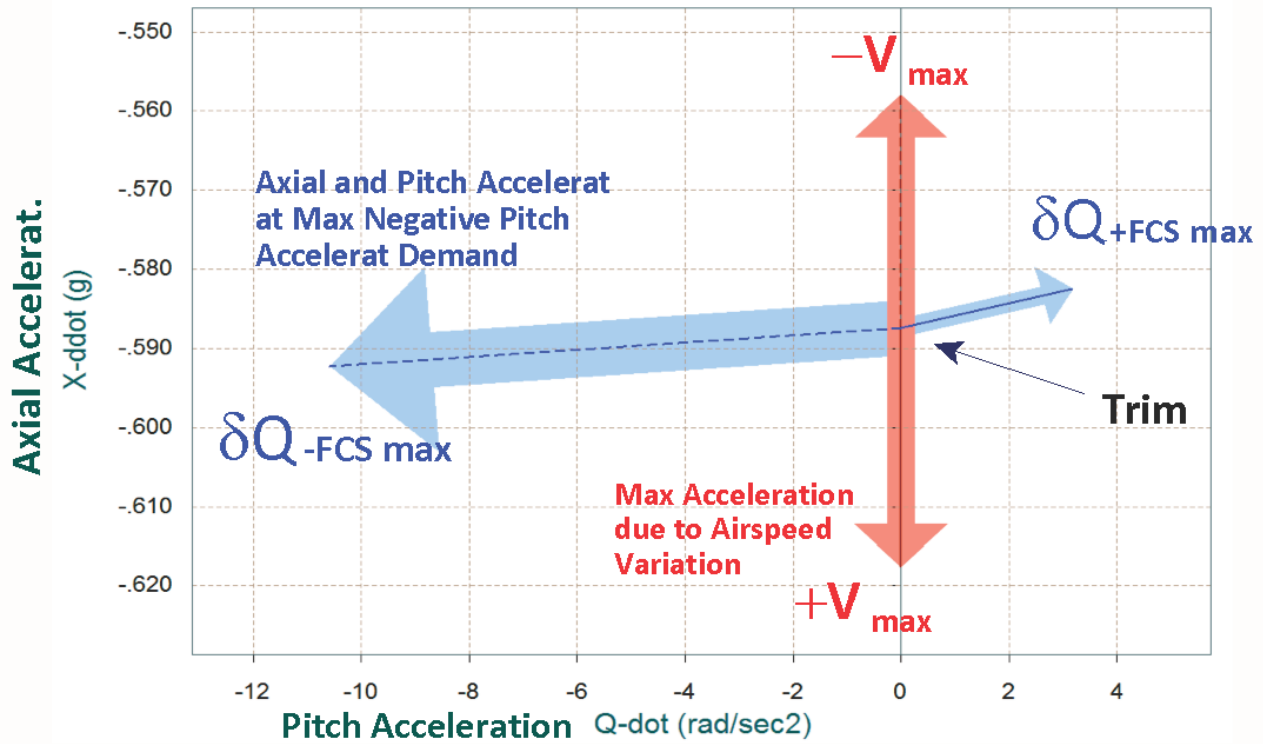


Figure 2.5.2 Pitch, X, and Z Accelerations at Maximum Pitch Demand



Comparison between Maximum Pitch and Axial X-Force Control Accelerat (Blue & Green) Against Aero Disturbance due to Delta-Velocity Dispersion (red)



Comparison Between Control Moment & X-Force Partial:  $\{C_m/\delta Q$  &  $C_X/\delta X\}$  (Blue and Green), Against Velocity Variat. Partial:  $\{C_m/\delta V$  &  $C_X/\delta V\}$  (Red)

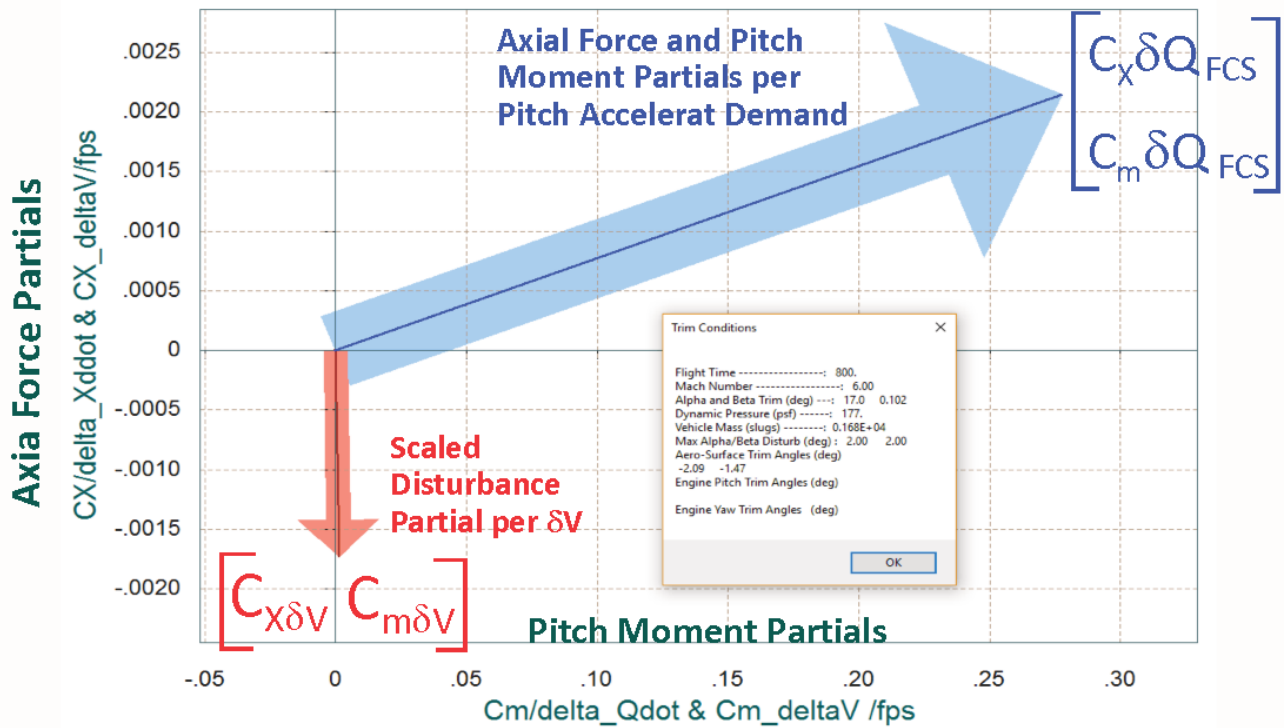
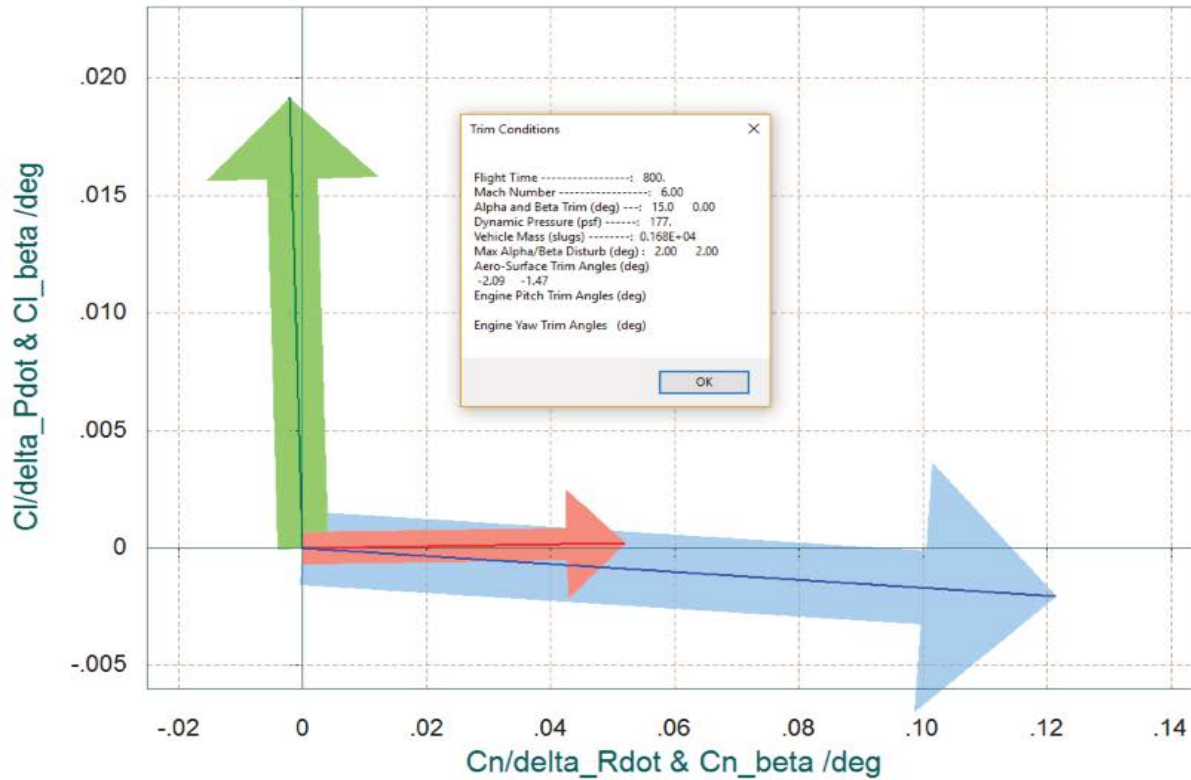


Figure 2.5.3 Air-Speed Variation Effects in the Pitch and Axial Directions

Comparison Between Yaw & Roll Control Moment Partial  $\{C_n/\delta_R \text{ and } C_l/\delta_P\}$  (Blue and Green Vectors) Against Partial:  $\{C_n/\beta \text{ and } C_l/\beta\}$  (Red Vectors)



Partials of Roll and Yaw Accelerations per Roll and Yaw Control Accelerat Demands  $(Rdot, Pdot)/Pdot\_Dem$  (green),  $(Rdot, Pdot)/Rdot\_Dem$  (blue),  $(rad/sec^2)/(rad/sec^2)$

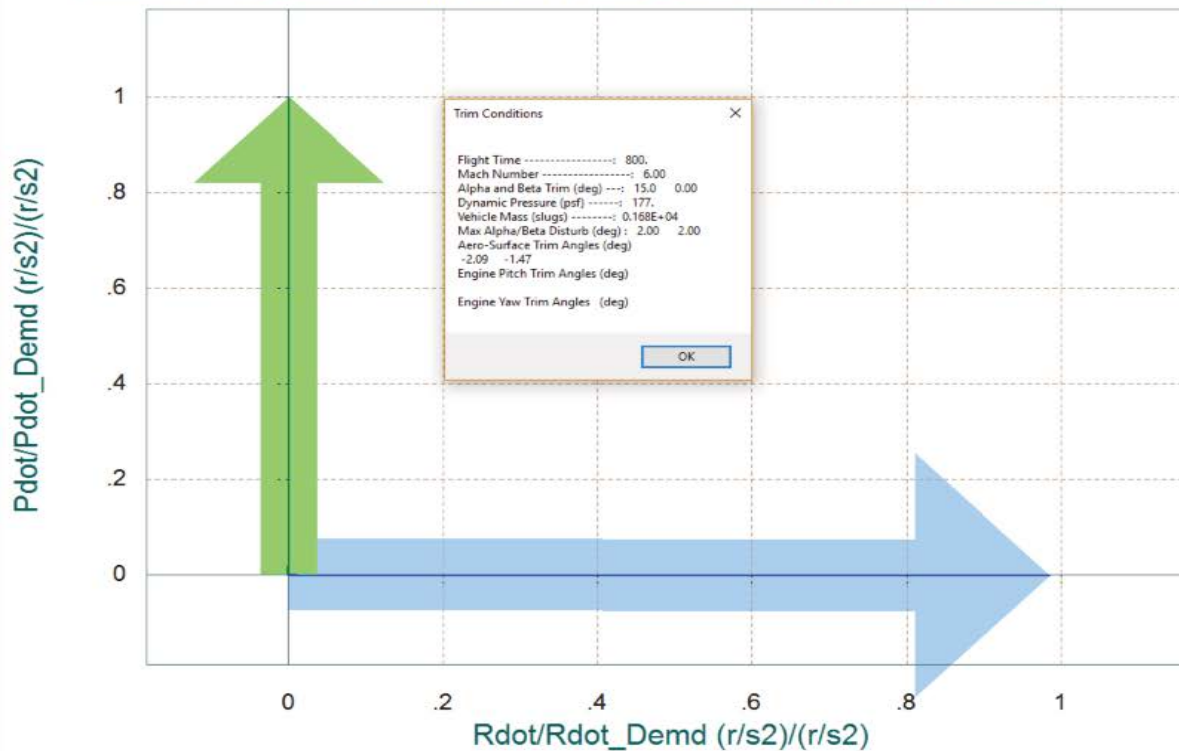


Figure 2.5.4 Moment and Acceleration Partial in the Roll and Yaw Directions

## 2.6 Creating a Dynamic Model for Descent

We will now create a dynamic model for our space-plane at a fixed flight condition during descent and in the next section we will design a flight control system and analyze the system. During descent the space-plane is using the two flaps and a pair of yaw jets. The two jets are represented by a single thruster that produces either positive or negative forces in the y direction. Its thrust is limited to  $\pm 2,500$  (lb).

Start the Trim program, select the same files as before and make "T800.inp" to be the Flixan input data file. This file contains the vehicle data plus other Flixan related model generation and data reduction data sets that will be discussed later. Re-trim the effectors to make sure that you are using a current trim condition. Do not select a trim initialization file, and trim only along the 3 moments (no translational trimming). From the Trim main menu select option (5) to create a state-space dynamic model. A dialog reminds the user how to select a flight time for the dynamic model, click "OK". From one of the trajectory plots go the top menu bar, and choose "Graphic Options", and then from the vertical pop-up menu click on "Select Time to Create State-Space System". Then using the mouse click at time  $t=800$  sec, along the x axis. Confirm that you have selected the correct time by clicking "OK", as shown below. Otherwise, click "Cancel" and try again.

Moments/ Products of Inertia (slg-ft <sup>2</sup> )		Location of Center of Gravity (ft)		Vehicle Mass in (slugs)	
Ixx	127176.6	Ixy	0.000000	Xcg	-59.81394
Iyy	997833.6	Ixz	13527.07	Ycg	0.1000000
Izz	819847.1	Iyz	0.000000	Zcg	1.139163
				Vehicle Mass in (slugs)	1684.200
				Accelerat. due to Gravity (ft/sec <sup>2</sup> )	32.17400
				Earth Radius (Re) in (feet)	0.2089600E+08

The program is now ready to create a dynamic model at the selected flight time. The above dialog shows the flight vehicle parameters prepared by Trim which are extracted from the data files. There are multiple tabs with data categories. The user can modify some of the data or titles using this dialog before saving it. Click on "Update Data" after any modifications. Do not run this file yet because there is more work to be done and more data to be included in T800.inp. Click on "Save in File" and the vehicle data will be saved in file "T800.inp", under the title "Reusable Space-Plane Descent Trajectory-2/ T= 800.0 sec". The file "T800.inp" will be processed by Flixan to generate the

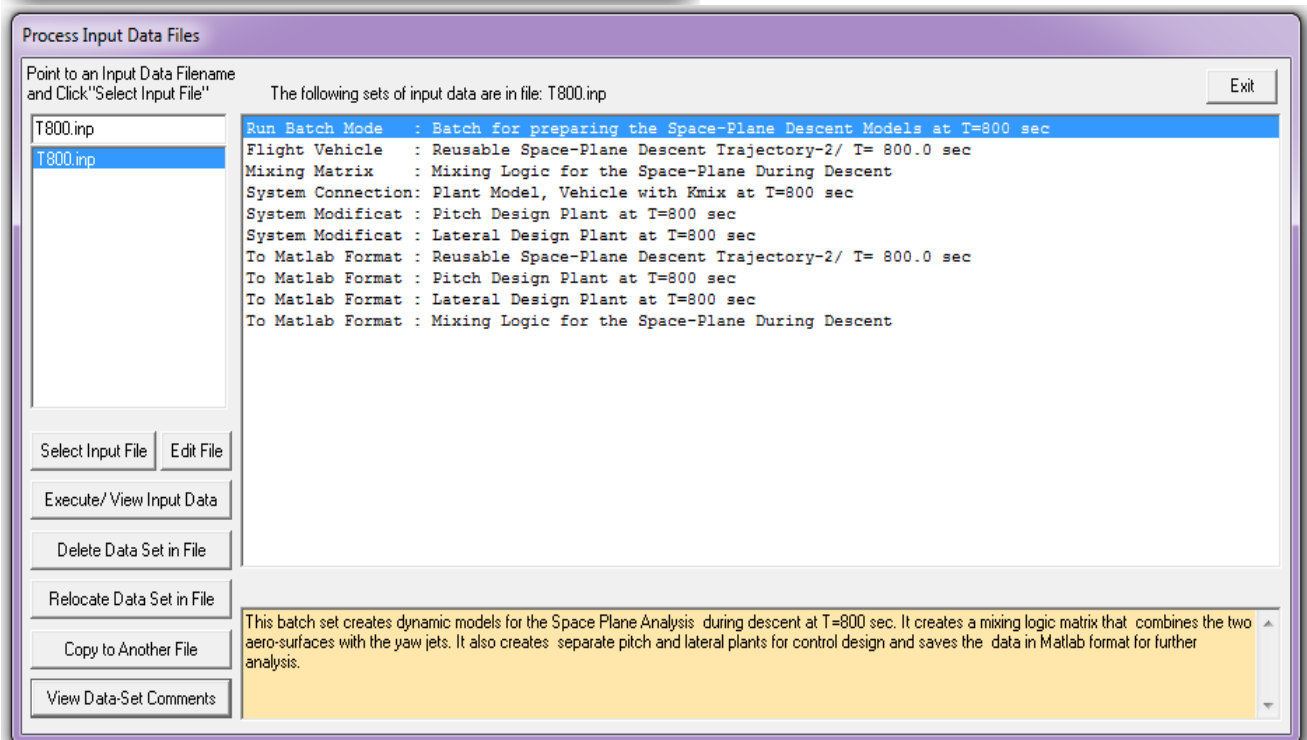
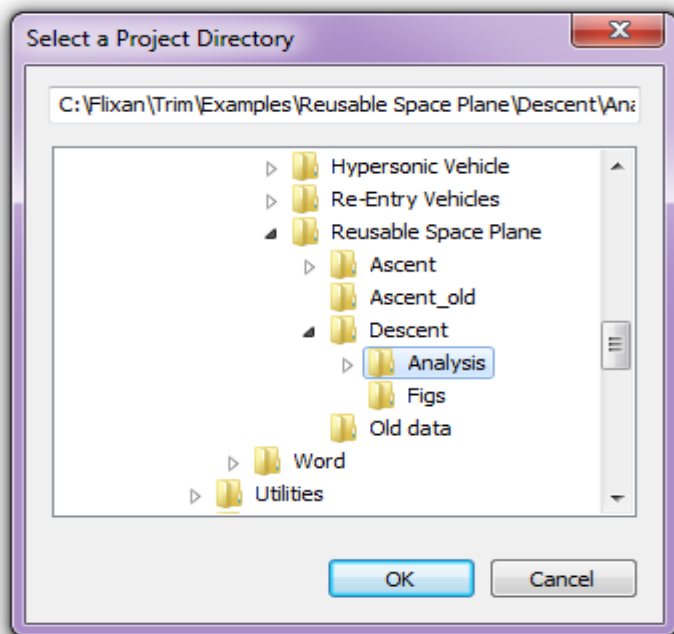
systems for control design and analysis using Matlab/ Simulink. In addition to the vehicle data the input file contains also system interconnection and modification data related to this analysis that will be processed by Flixan. The systems and matrices generated by Flixan will be saved in file "T800.Qdr".

Let us now take a look and see what is in file "T800.inp" before processing it in Flixan. The file contains several sets of data and each set corresponds to and it is processed by a Flixan utility.

- At the top of the file there is a batch set that can process all other data-sets in batch mode. This is faster because it processes them all together instead of interactively. Its title is "*Batch for preparing the Space-Plane Descent Models at T=800 sec*".
- Below the batch we see the flight vehicle data for the flight condition that we have just created. The first 3 inputs of the vehicle state-space model correspond to the 3 control inputs. That is, the yaw jet throttle input that varies between -1 to +1 representing a thrust variation between -2500 to +2500 (lb), and the two flap aero-surfaces. The 4<sup>th</sup> input is a wind-gust velocity disturbance in (ft/sec). The gust direction relative to the vehicle is defined in file T50.inp.
- The next data-set generates the (3x3) mixing logic matrix that converts the 3 (roll, pitch, yaw) rotational acceleration demands to flap deflections and yaw jet throttling commands. The data-set title is "*Mixing Logic for the Space-Plane During Descent*" and the matrix title is Kmix\_800.
- It is a good practice to include the mixing logic matrix in series with the vehicle model, at its input. This reduces the number of inputs, it decouples the plant, and makes it easier to analyze and to design flight control laws. The next set of data is a system interconnection set that combines the vehicle model with the mixing matrix. Its title is: "*Plant Model, Vehicle with Kmix at T=800 sec*". It will be split into pitch and lateral and used for LQR design.
- Next, we have two system modification data-sets for separating the previous plant in two separate sub-systems: (a) a pitch design plant "*Pitch Design Plant at T=800 sec*", and (b) a lateral design plant "*Lateral Design Plant at T=800 sec*", by extracting the corresponding inputs, states, and outputs.
- Finally, the Flixan input file also includes data-sets for converting the systems to Matlab m-functions for further analysis using Matlab. The original vehicle model is saved in file "*vehicle.m*". The pitch and lateral design plants are saved in files "*vehi\_pitch.m*" and "*vehi\_later.m*" respectively. The effector mixing matrix is saved in file "*Kmix\_800.mat*". These files will be loaded and analyzed in Matlab, as it will be described in the next section.

Let us now process this input file using Flixan. The preparation of the batch set and the system manipulation sets is not documented here because it is beyond the scope of this example and not difficult to follow. Plus these processes are documented in detail in Flixan. Start the Flixan program and select the same project folder. Then go to "Edit", "Manage Input Files (.Inp)", and "Process/

*Edit Input Data*", as shown below. The following dialog is for viewing and processing Flixan input files. It has two menus. The one on the left lists the input files in the project directory. There is only one in this case, select "T800.Inp" and click on "Select Input File". The menu on the right side shows the data-sets which are in file: "T800.Inp". They can be processed individually, but select the batch set on the top to process all of them together, and click on "Execute". Flixan will now process the input file and create the systems in file "T800.Qdr". It will also create the function files that will be loaded into Matlab. These files are transferred in subdirectory "Descent\Analysis" where we will perform the control design and analysis.



## 2.7 Descent Control Design and Analysis

We will now use the dynamic models developed in section 2.6 to design control laws for our space-plane during descent, analyze stability, and develop simulation models in Matlab/Simulink. The analysis is performed in directory: "C:\Flixan\Trim\Examples\ Reusable Space Plane\Descent\Analysis". We will begin by designing control laws, then perform stability analysis in the frequency domain, and eventually develop a simulation model.

### Flight Control Design

The flight control gains are calculated by the Matlab script file "*design.m*", shown below. It uses the pitch and lateral design systems in files "*vehi\_pitch.m*" and "*vehi\_later.m*", created in section 2.6, to generate state-feedback matrices *Kq.mat* and *Kpr.mat* for the pitch and lateral systems. The pitch state-vector consists of  $[\theta, q, \alpha, \alpha\text{-integral}]$ . The lateral state-vector consists of  $[\phi, p, \psi, r, \beta]$ .

```
% Pitch LQR Design for Space-Plane
[Ap,Bp,Cp,Dp]= vehi_pitch;                               % Load the Pitch Design Model
[Api,Bpi,Cpi,Dpi]= linmod('Pitch_Design');              % Augment Pitch Simulink model
sys1=SS(Api,Bpi,Cpi,Dpi);
% Weights[thet, q, alpha, alfa_int]
Q= diag( [0.01, 0.1, 5.0, 1.5]);                          %
Weights(thet,q,alpha,alfa_int)                          %
R=1;                                                       % Control Weights R=2
[Kq,S,E]= LQR(sys1,Q,R);                                  % Save the LQR gains in Kq.mat
save Kq.mat Kq -ascii

% LQR Lateral Design for Space-Plane
[Al,Bl,Cl,Dl]= vehi_later;                               % Load the Lateral Design Model
sys2=SS(Al,Bl,Cl,Dl);
% states: [phi, p, psi, r, beta]
Q=diag([ 3.5, 0.2, 3.5, 0.2, 0.00001]);                  % Control Weights
R=diag([1 1]*1);
[Kpr,S,E]= LQR(sys2,Q,R);                                % Save the LQR gains in Kpr.mat
save Kpr.mat Kpr -ascii                                  % Load Mixing Logic Matrix
load Kmix_800 -ascii
```

## Stability Analysis

The script file "*freq\_anal.m*" below performs stability analysis in the frequency domain. It uses the Simulink model "*Open\_Loop.mdl*", shown in figure 2.7.1 for yaw analysis.

```
[Av,Bv,Cv,Dv]= vehicle; % Load the Vehicle Analysis Model
[Ao,Bo,Co,Do]= linmod('Open_Loop'); % Augment Pitch Simulink model
w=logspace(-3, 3, 10000); % Define Frequency Range
sys= SS(Ao,Bo,Co,Do); % Create SS System
figure(1); Nyquist(sys,w) % Plot Nichol's Chart
figure(2); Nichols(sys,w) % Plot Nichol's Chart
figure(3); Bode(sys,w); grid on % Plot Bode Plot
```

Figure 2.7.2 shows the open-loop frequency response Bode plots of the pitch and yaw axes. The short period and Dutch-Roll resonances are close to those predicted by the performance analysis parameters in Section 2.4. The Nyquist diagrams in Figure (2.7.3) show stable margins in both pitch and yaw.

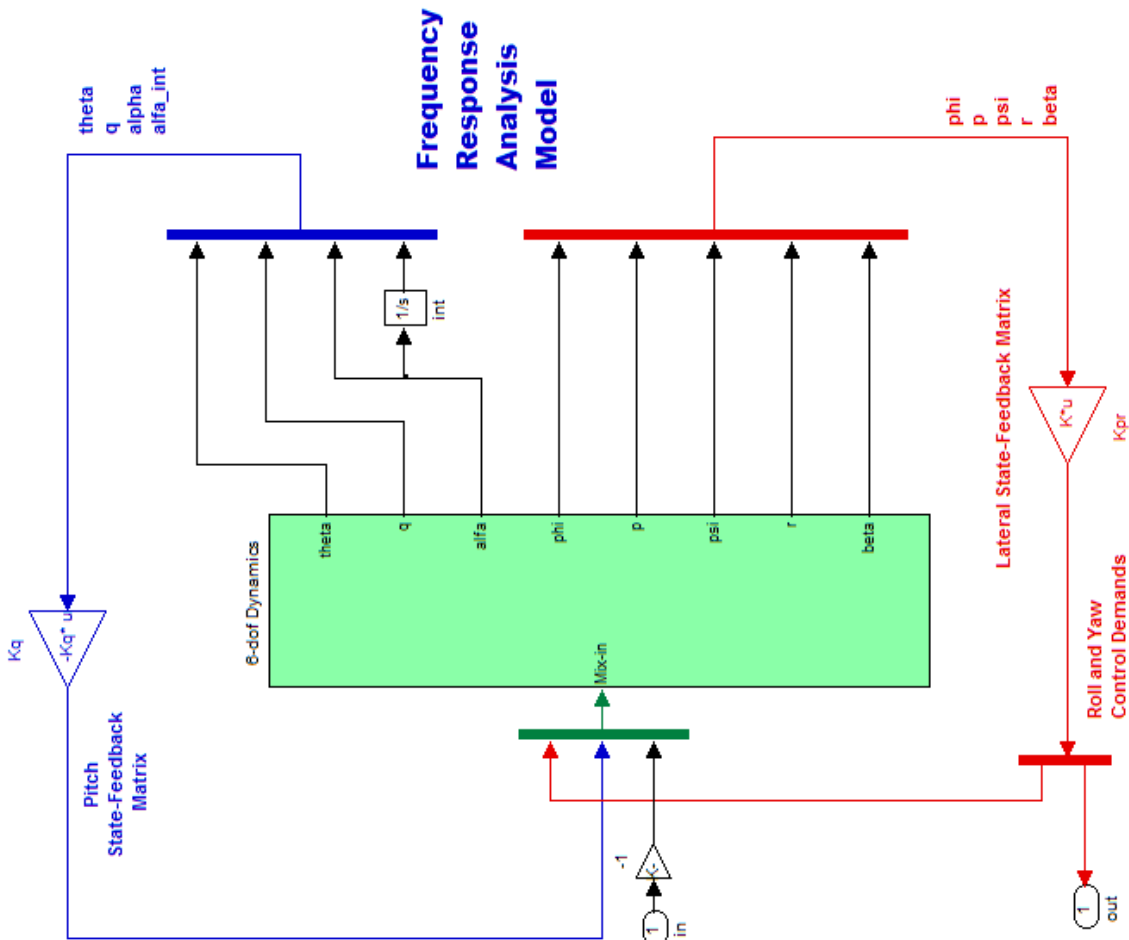
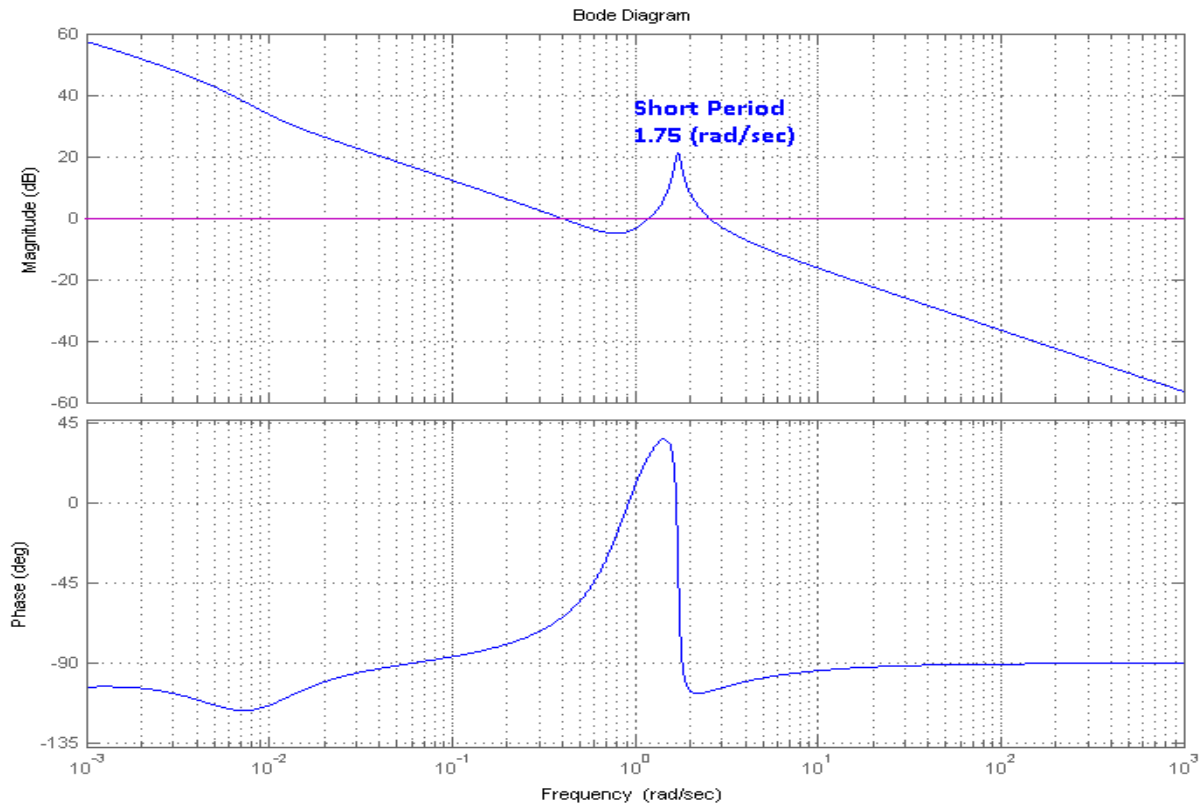


Figure 1.7.1 Open-Loop Frequency Response Analysis Simulink Model "Open\_Loop.Mdl".





**Yaw Axis Open-Loop Frequency Response**

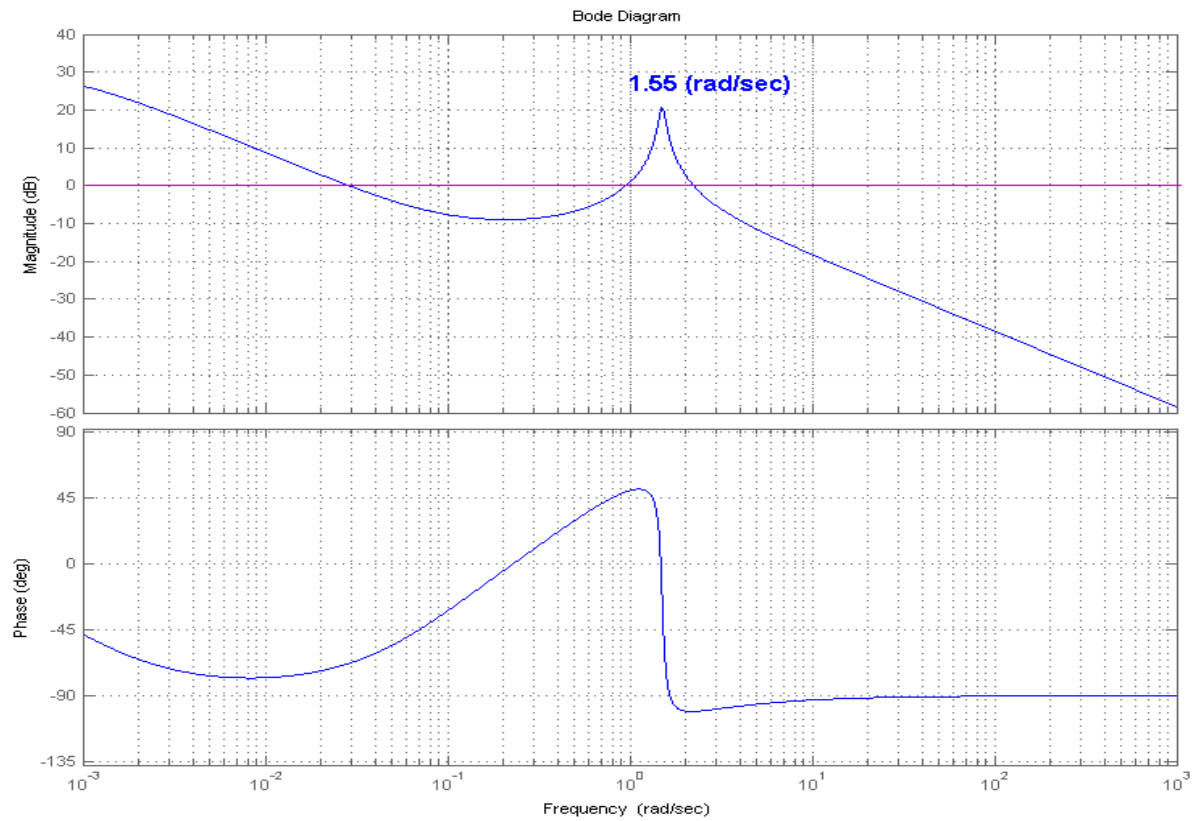


Figure 2.7.2 Open-Loop Bode Plots for the Pitch and Yaw Axes.



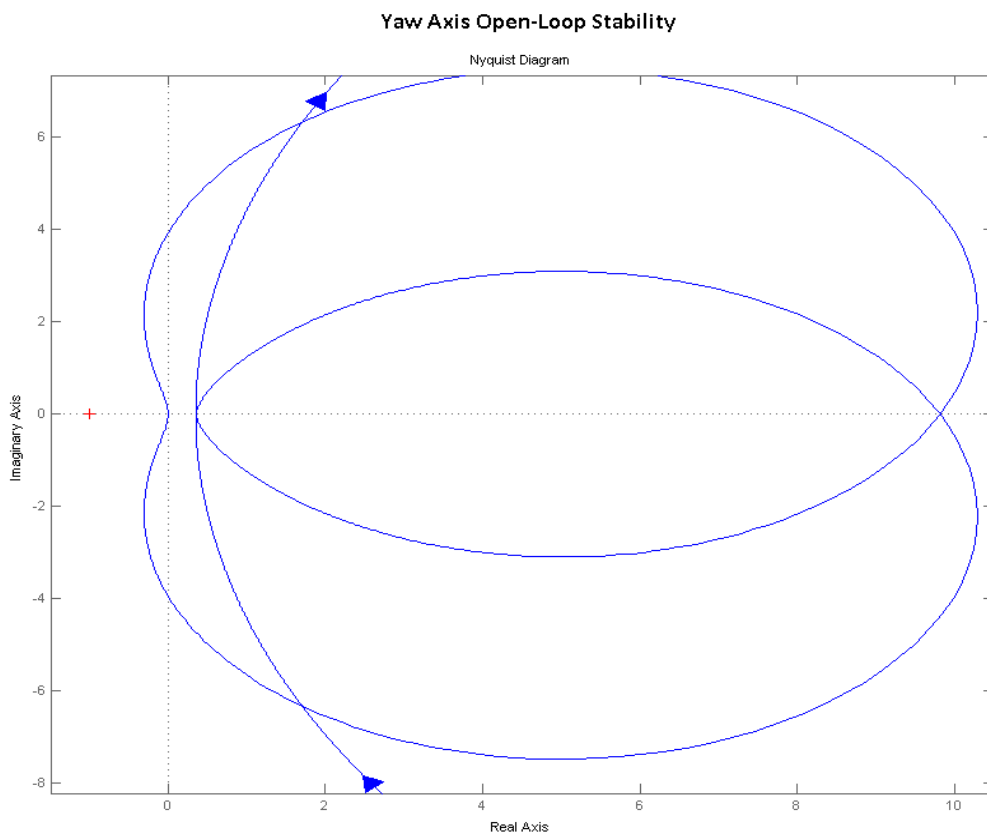
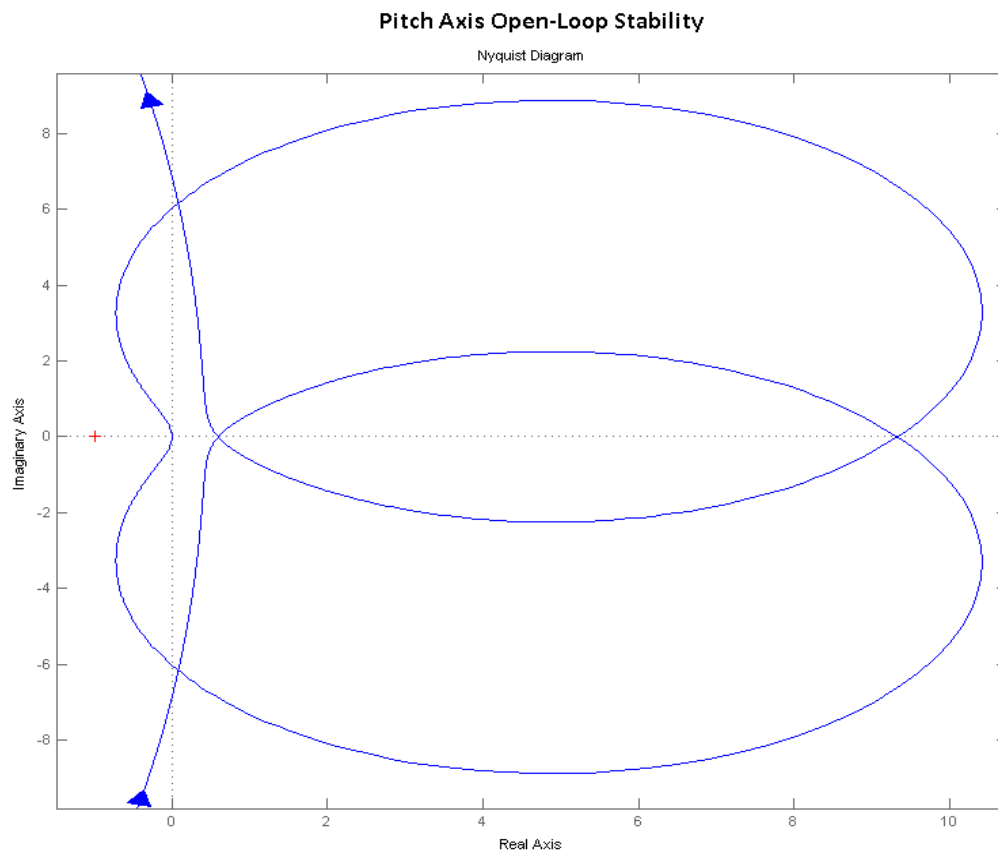


Figure 2.7.3 Nyquist Diagrams show that the Pitch and Yaw Axes are Stable

## Simulation

The closed-loop simulation model is in file "Sim\_6dof.mdl", shown in Figure (2.7.4 a&b). It is initialized by the Matlab file "runsim.m", shown below, and uses the system in file "vehicle.m", with title: "Reusable Space-Plane Descent Trajectory-2/ T= 800.0 sec" that was created in section 2.6. The file "runsim.m" also loads the mixing logic matrix and the two gain matrices.

```

% Initialize Simulation
r2d=180/pi; d2r=pi/180;
design
[Av,Bv,Cv,Dv]= vehicle;
load Kmix_800 -ascii
load Kq -ascii
load Kpr -ascii

% Calculate Gains using LQR Design
% Load the Vehicle Analysis Model
% Load Mixing Logic Matrix
% Load LQR Gains
    
```

The simulation model has three flight control loops: roll, pitch, and yaw. The pitch state-feedback control law via matrix  $K_q$  is seen in the upper section (blue), and the roll/yaw state-feedback control law via matrix  $K_{pr}$  is seen in the lower section (red). The model receives 3 FCS commands from guidance. In this case they are represented with step inputs. There is an alpha command ( $\alpha_{cmd}$ ), a roll command ( $\phi_{cmd}$ ), and a yaw command (not shown). There is also a wind-gust input pulse that is shaped into a more realistic gust by a low-pass filter. The wind-gust direction is defined in the input file and it is set at 45° to excite both: the pitch and yaw axes. When the simulation is complete the file "pl.m" plots the data. We are showing two cases that were made using this simulation model: (a) a roll attitude step command, and (b) an alpha command in combination with a wind-gust disturbance.

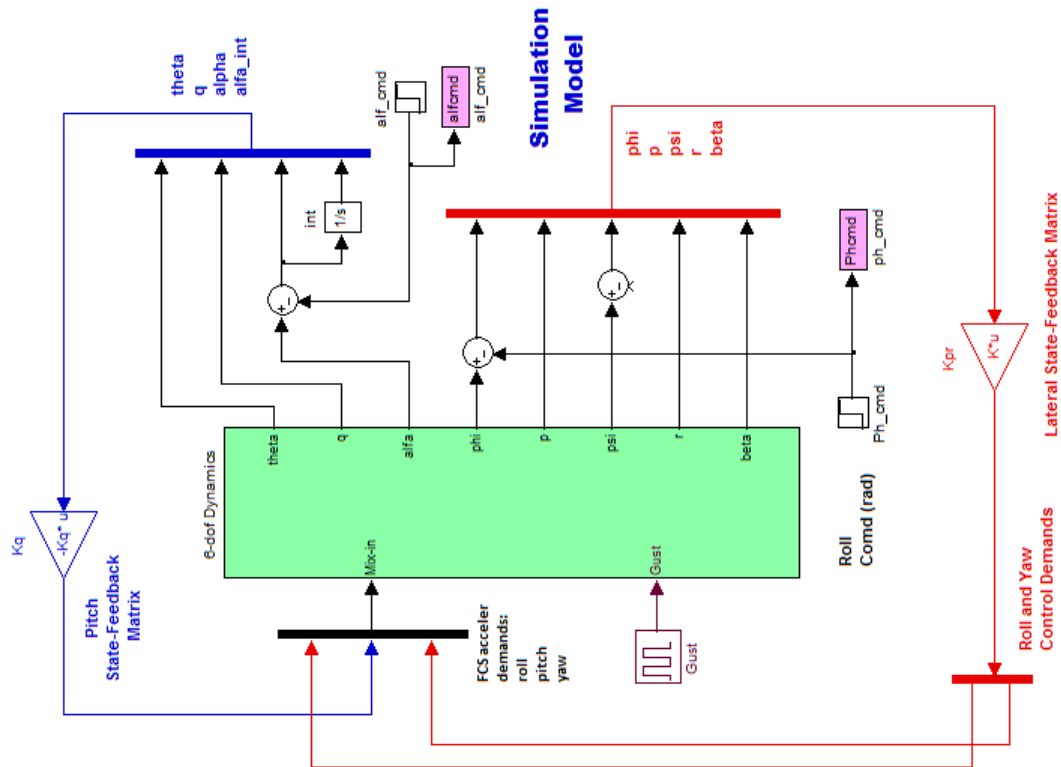


Figure 2.7.4a Descent Simulation Model "Sim\_6dof.mdl".

## Pitch & Lateral Dynamics Reusable Space-Plane Descent Trajectory-2/ T= 800.0 sec

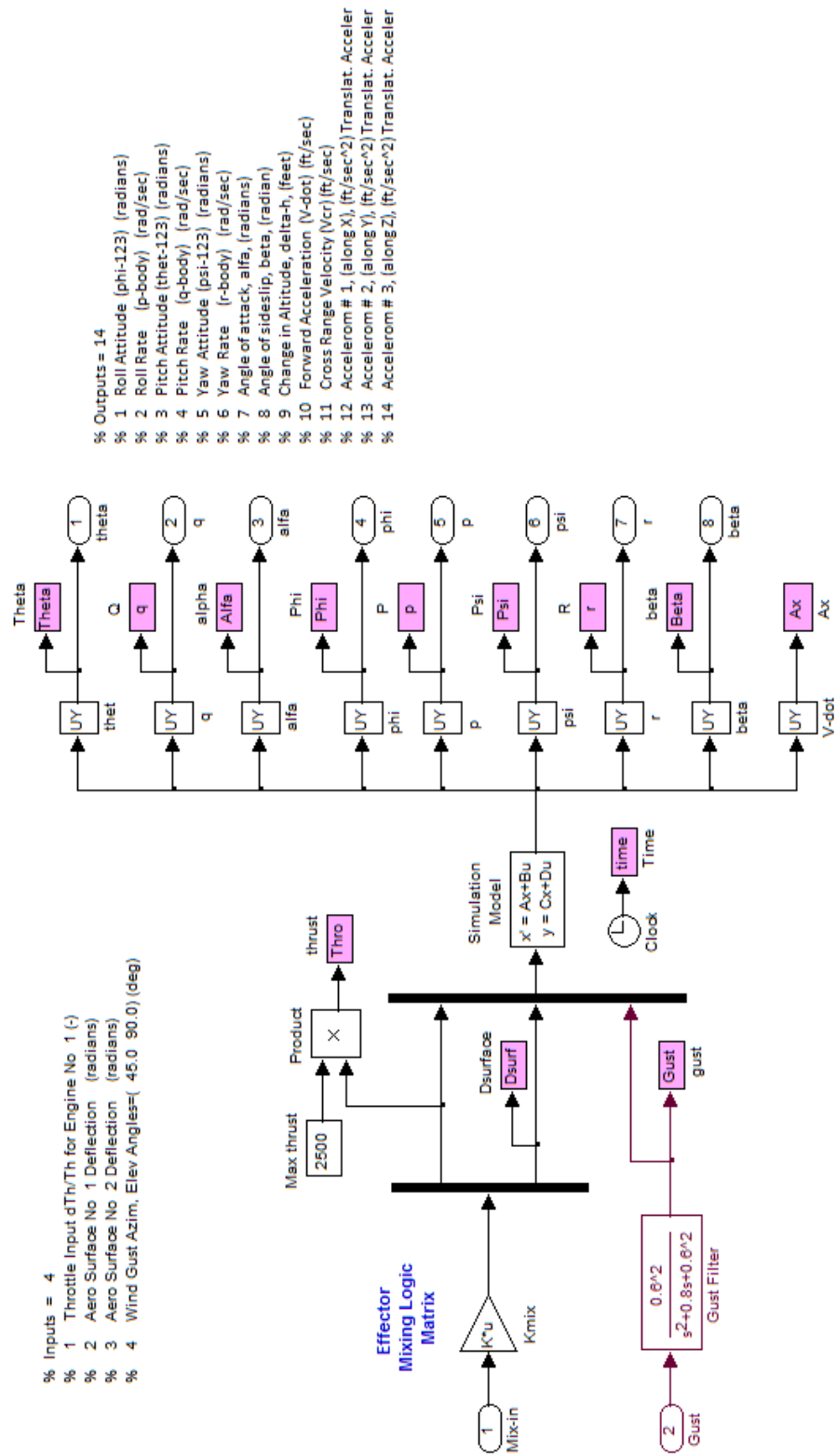
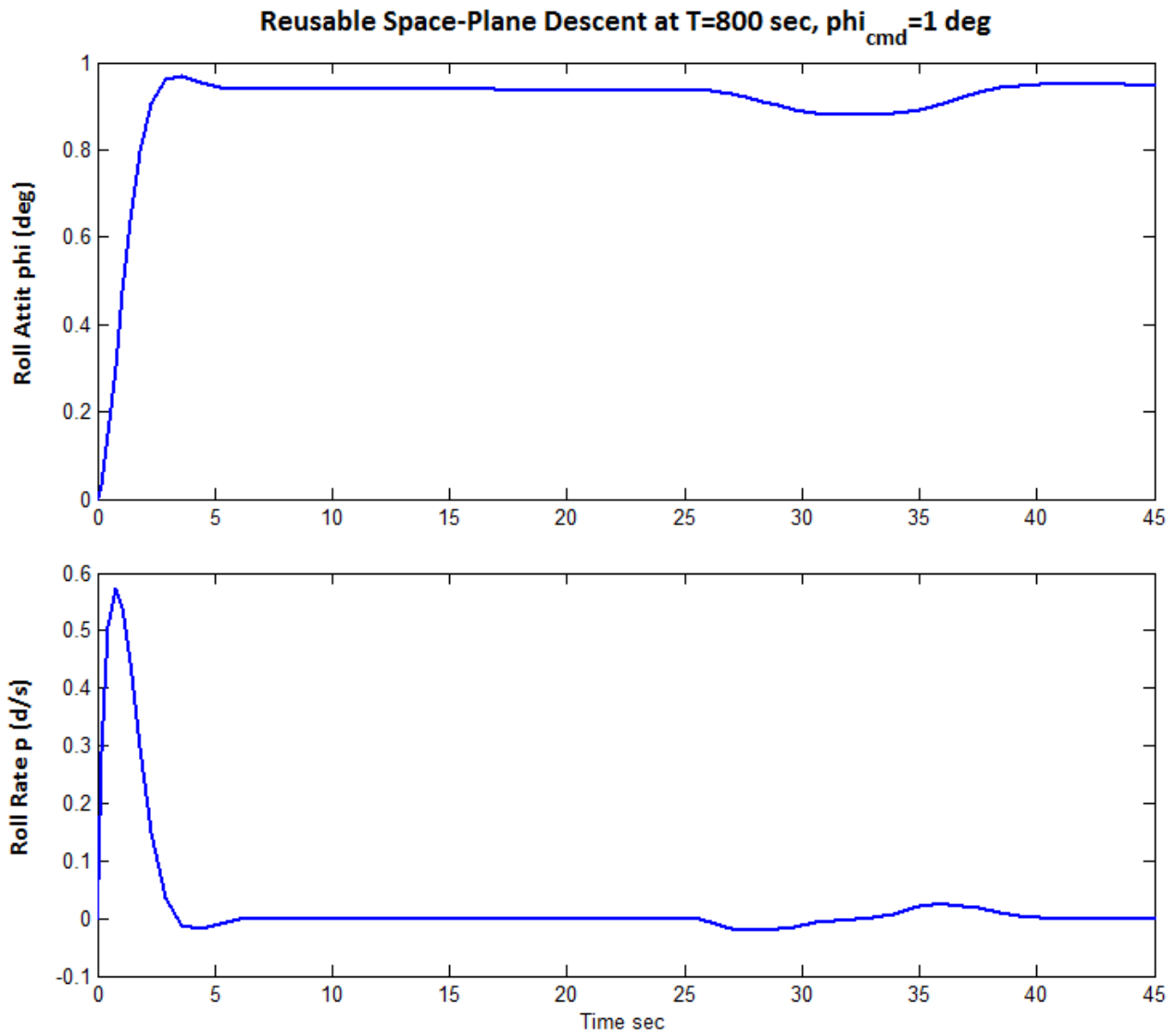


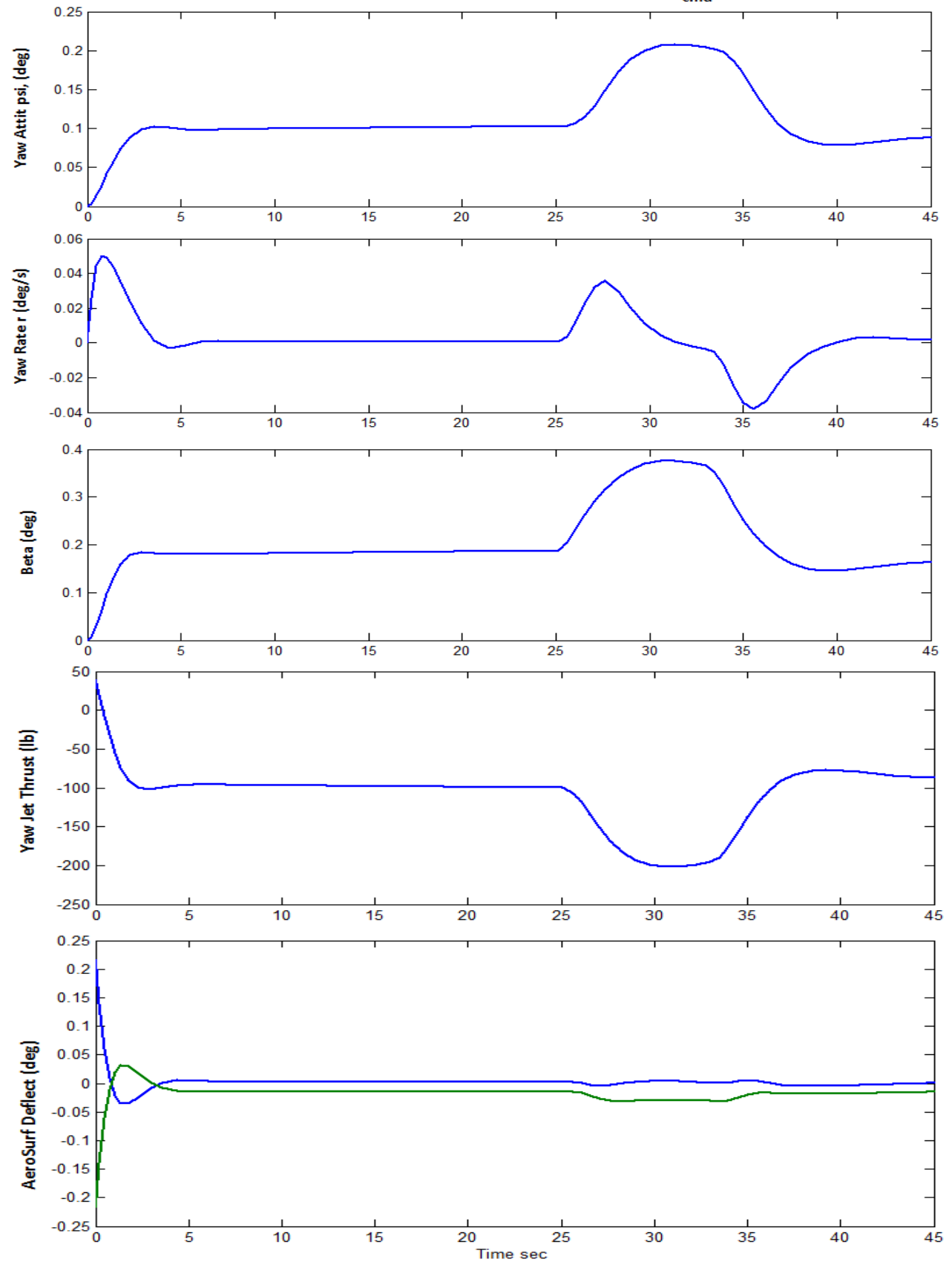
Figure 2.7.4b Descent Simulation Model. Vehicle Dynamics Subsystem

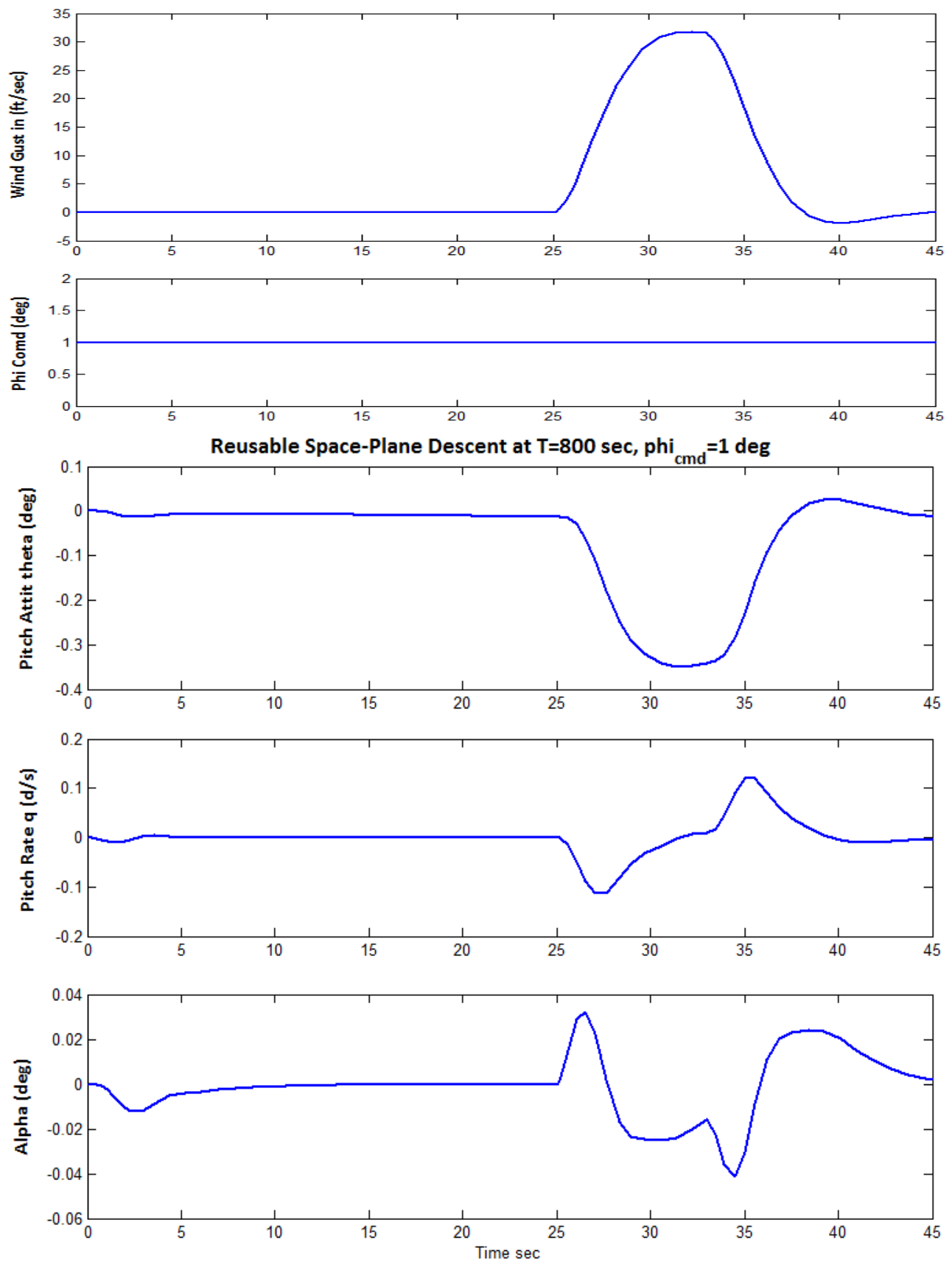
## Simulation with a Phi-Command

In this case we apply 1 (deg) phi-command and a wind-gust disturbance pulse with a peak of 30 (feet/sec) is applied 25 seconds later. The alpha-command is set to zero. The plot below shows that the roll angle approaches 1° with a small steady-state error and a transient due to gust. This error can be eliminated with a phi-integrator, but it is not necessary because it is taken care by the closed guidance loop. The Phi-command causes a small steady-state beta. The maneuver is achieved by pivoting the left and right flaps differentially and firing the yaw thruster with a steady -100 (lb) thrust. The thrust is used to counteract the positive moment due to beta and also against the wind-gust impulse.



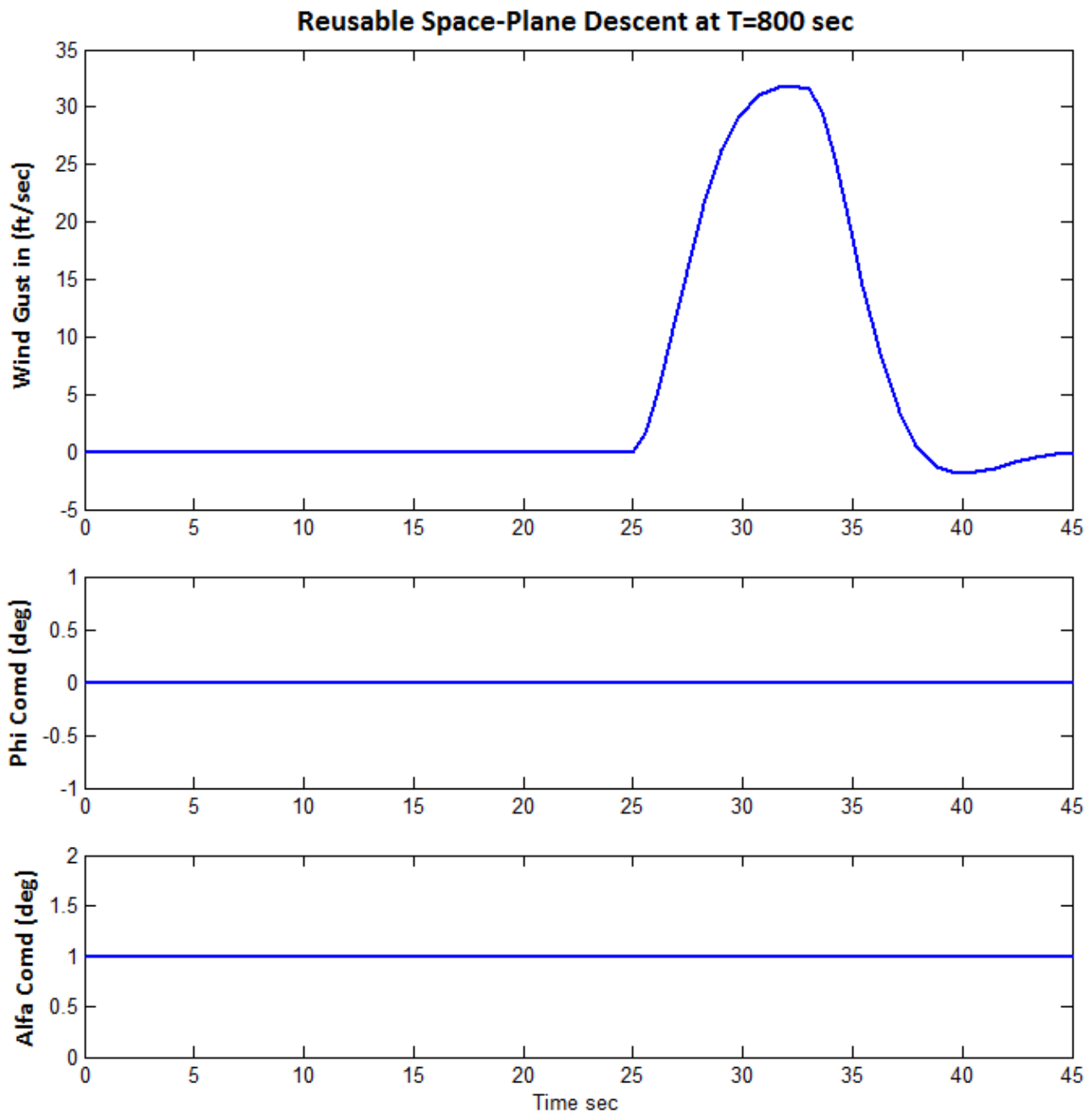
Reusable Space-Plane Descent at T=800 sec,  $\phi_{cmd}=1$  deg



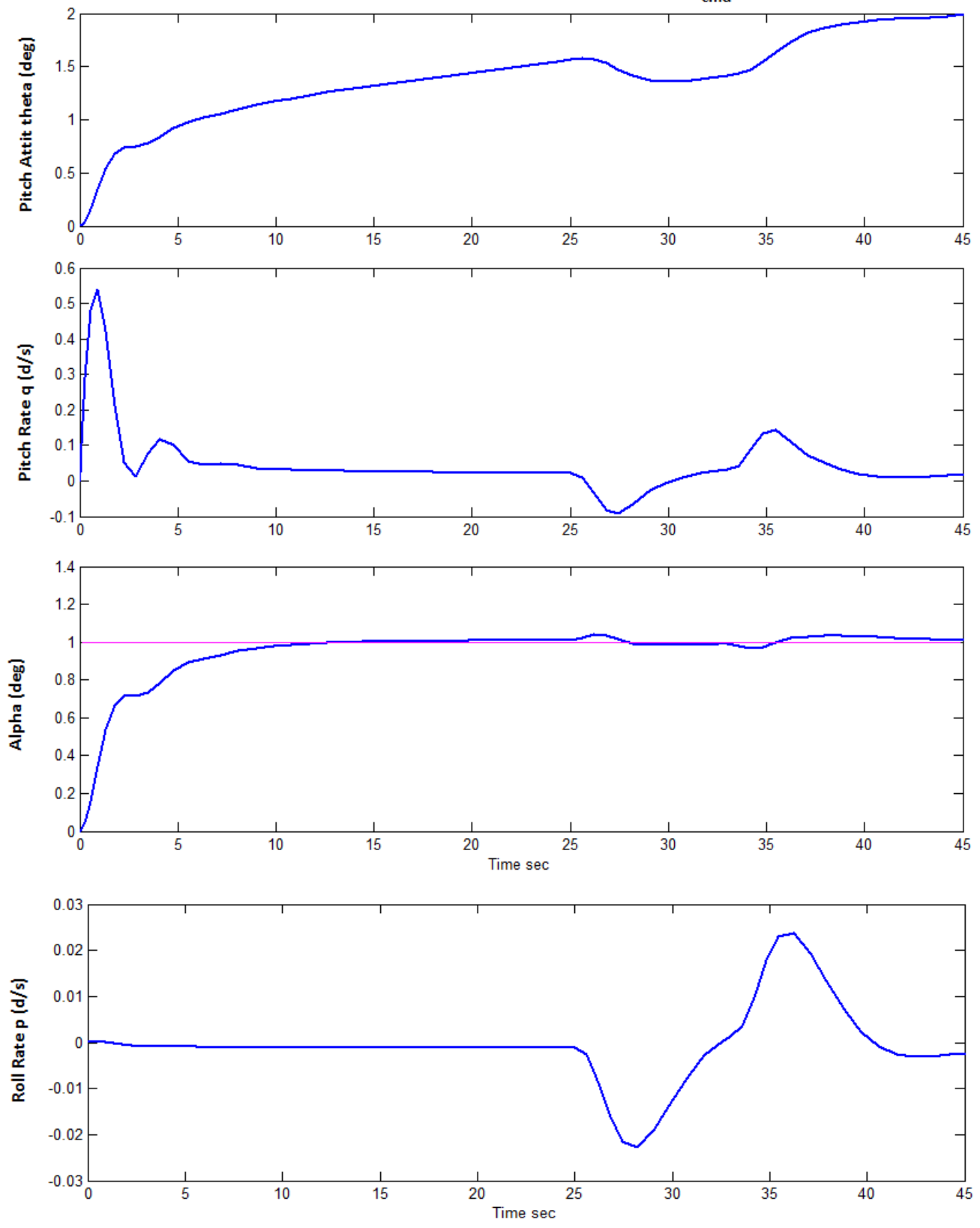


## Simulation with an Alpha-Command plus Wind-Gust

In this case we apply a step alpha command at  $t=0$ , and a wind-gust disturbance pulse with a peak of 30 (feet/sec) is applied 25 seconds later. The angle of attack catches up with the command. The deflections of the left and right flaps are negative during the alpha maneuver in order to generate a positive pitch-up moment. Their deflections are not symmetrical because of the  $Y_{CG}$  offset. The gust causes transients in beta, pitch and roll attitude. The yaw thruster reacts against the gust disturbance by generating a negative side-force.



Reusable Space-Plane Descent at T=800 sec,  $\alpha_{cmd}=1$  deg





### Reusable Space-Plane Descent at T=800 sec, $\alpha_{cmd}=1$ deg

