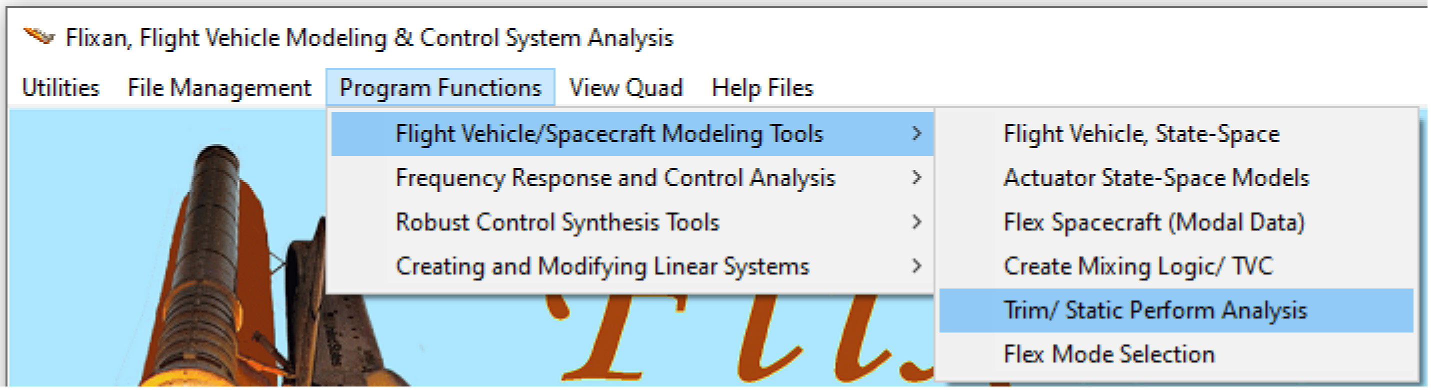
Classic Launch Vehicle Design

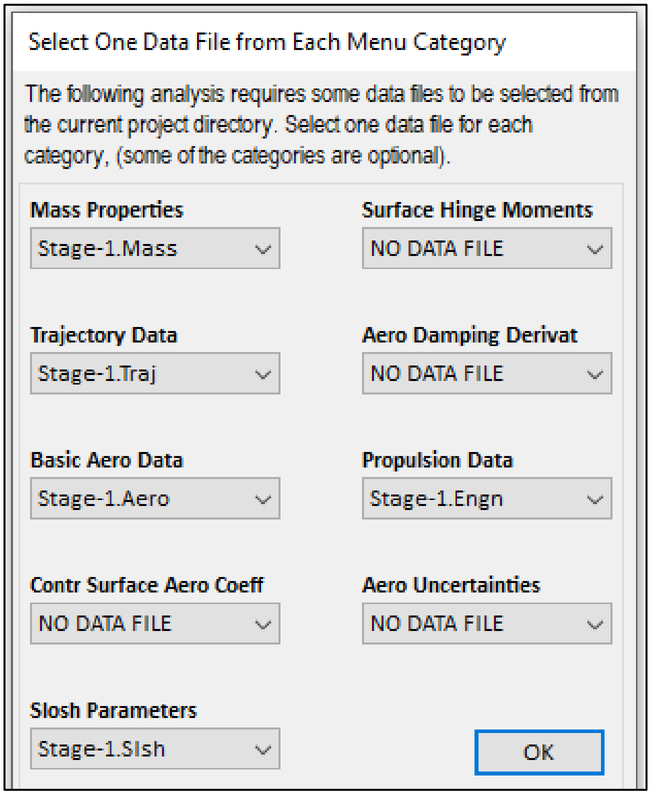
Introduction

In this example we will analyze a two-stages launch vehicle similar to the one shown in the picture, from lift-off to orbit insertion. We begin in Section-1 with a static analysis which is based on mass properties, trajectory and aero data to analyze the static stability and performance qualities of the vehicle, prior to design. In Section-2 we will design the first-stage control system and analyze the system stability at different flight conditions, including: lift-off, max-Q, and pre-separation. The Flixan program will be used to create the vehicle models including propellant sloshing and structural flexibility. In Section-3 we will design the control system during second stage and perform similar analysis including slosh and flexibility. In Section-4 we will analyze the system robustness to parameter uncertainties using μ-analysis. In Section-5 we introduce non-linear propellant sloshing models to analyze the effects of instabilities that were observed when using linear slosh models. In Section-6 we have included a 6DoF simulation model of the vehicle, from lift-off to orbit insertion, including detailed descriptions of the vehicle subsystems and the control system.

1.1 First Stage Static Analysis

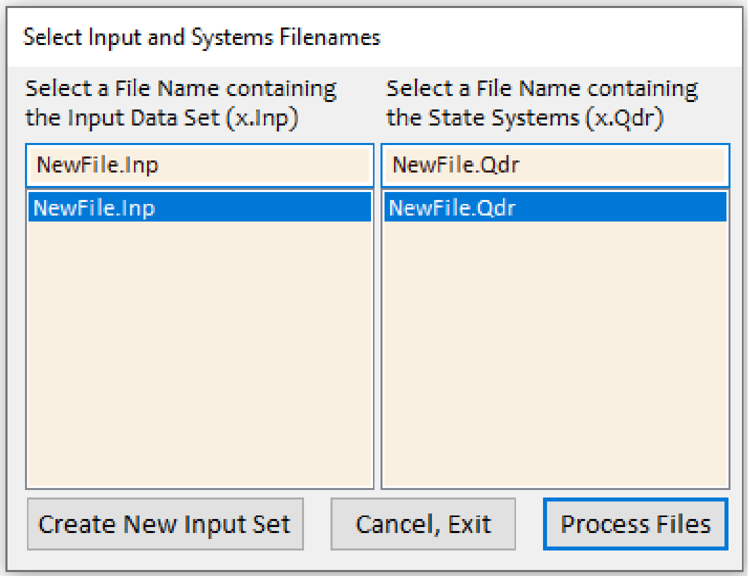
Static analysis is used to analyze the vehicle quality in the static sense. If the vehicle has acceptable static qualities, like good controllability from the TVC engines against external disturbances, or that it's not too aerodynamically unstable and the actuators won't be able to catch-up with the divergence. The files for 1st stage static analysis are in folder "*Examples\23-Classic Launch Vehicle Design & Simulation\1-Static Analysis\Stage-1*". It includes the aerodynamic data (.aero), the engine data (.engn), the vehicle mass properties (.mass), the first stage trajectory file (.traj), and the slosh parameters file (.slsh). We start the Flixan program and from the main menu we select "*Program Functions*" and then "*Trim/ Static Performance Analysis*". From the files selection menu, we select the first stage files, click OK, and from the next menu select the default input and system filenames, and click on "*Process Files*".




Select One Data File from Each Menu Category

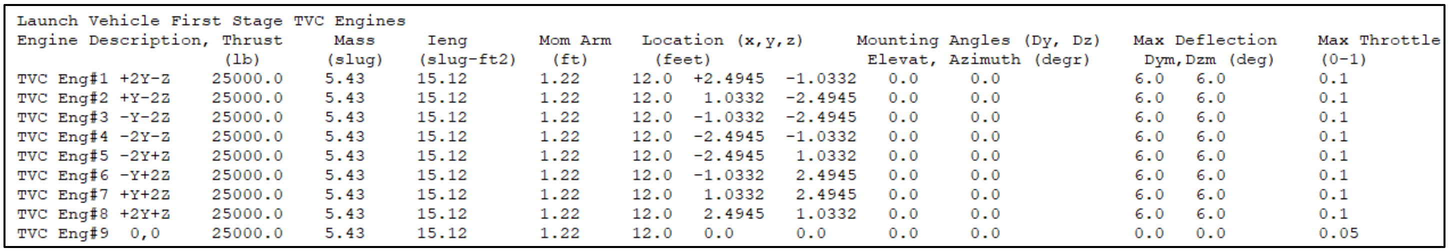
The following analysis requires some data files to be selected from the current project directory. Select one data file for each category, (some of the categories are optional).

Mass Properties: Stage-1.Mass	**Surface Hinge Moments**: NO DATA FILE
Trajectory Data: Stage-1.Traj	**Aero Damping Derivat**: NO DATA FILE
Basic Aero Data: Stage-1.Aero	**Propulsion Data**: Stage-1.Engn
Contr Surface Aero Coeff: NO DATA FILE	**Aero Uncertainties**: NO DATA FILE
Slosh Parameters: Stage-1.Slsh	


Select Input and Systems Filenames

Select a File Name containing the Input Data Set (x.Inp)	Select a File Name containing the State Systems (x.Qdr)
NewFile.Inp	NewFile.Qdr
NewFile.Inp	NewFile.Qdr

The engines file "*Stage-1.Engn*" includes 9 engines. The 8 peripheral engines are TVC and they gimbal. The 9th engine at the center is fixed. It neither gimbals nor throttles. However, in this file we do allow a little room for throttling in order to match the thrust during trimming along the trajectory. The engines file assumes a fixed thrust but the actual thrust varies along the 1st stage trajectory file, we therefore allow the engines to throttle slightly for trim. But we ignore the throttling in the Flixan derived vehicle models.



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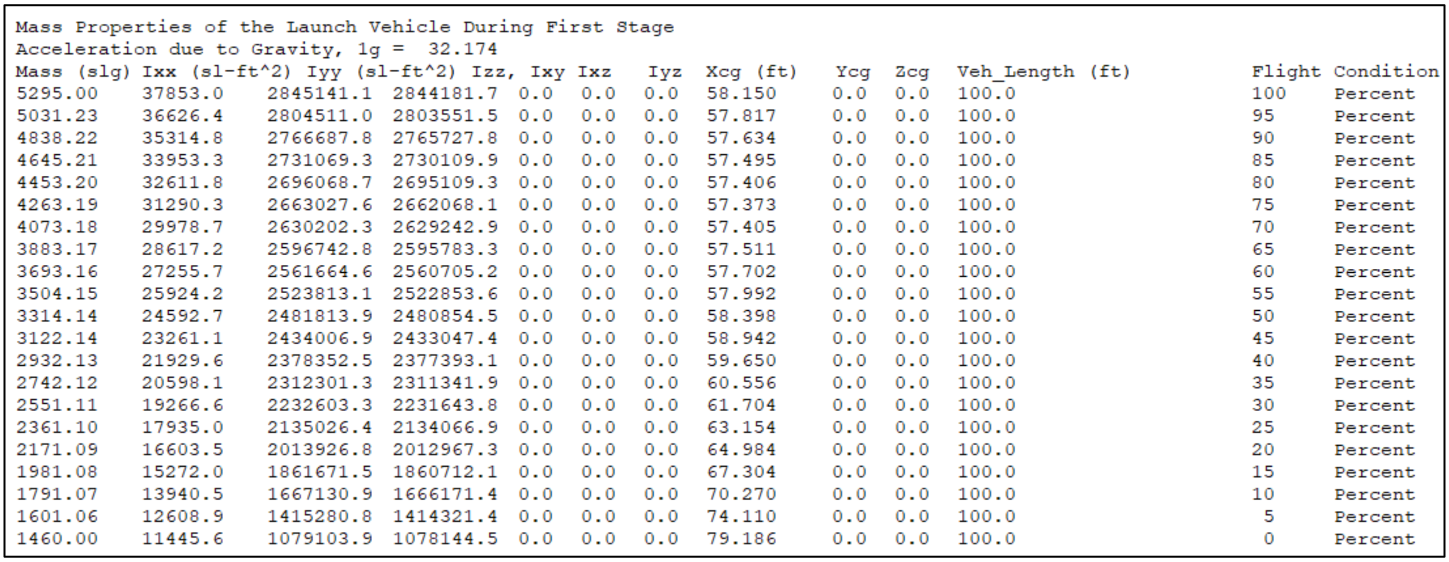
Launch Vehicle First Stage TVC Engines
Engine Description, Thrust     Mass     Ieng        Mom Arm    Location (x,y,z)        Mounting Angles (Dy, Dz)    Max Deflection    Max Throttle
                    (lb)       (slug)   (slug-ft2)   (ft)       (feet)                   Elevat, Azimuth (degr)     Dym,Dzm (deg)     (0-1)
TVC Eng#1 +2Y-Z     25000.0    5.43     15.12        1.22      12.0  +2.4945  -1.0332   0.0      0.0               6.0    6.0        0.1
TVC Eng#2 +Y-2Z     25000.0    5.43     15.12        1.22      12.0   1.0332  -2.4945   0.0      0.0               6.0    6.0        0.1
TVC Eng#3 -Y-2Z     25000.0    5.43     15.12        1.22      12.0  -1.0332  -2.4945   0.0      0.0               6.0    6.0        0.1
TVC Eng#4 -2Y-Z     25000.0    5.43     15.12        1.22      12.0  -2.4945  -1.0332   0.0      0.0               6.0    6.0        0.1
TVC Eng#5 -2Y+Z     25000.0    5.43     15.12        1.22      12.0  -2.4945   1.0332   0.0      0.0               6.0    6.0        0.1
TVC Eng#6 -Y+2Z     25000.0    5.43     15.12        1.22      12.0  -1.0332   2.4945   0.0      0.0               6.0    6.0        0.1
TVC Eng#7 +Y+2Z     25000.0    5.43     15.12        1.22      12.0   1.0332   2.4945   0.0      0.0               6.0    6.0        0.1
TVC Eng#8 +2Y+Z     25000.0    5.43     15.12        1.22      12.0   2.4945   1.0332   0.0      0.0               6.0    6.0        0.1
TVC Eng#9  0,0      25000.0    5.43     15.12        1.22      12.0   0.0      0.0      0.0      0.0               0.0    0.0        0.05
```

First Stage Engines File "Stage-1.Engn"



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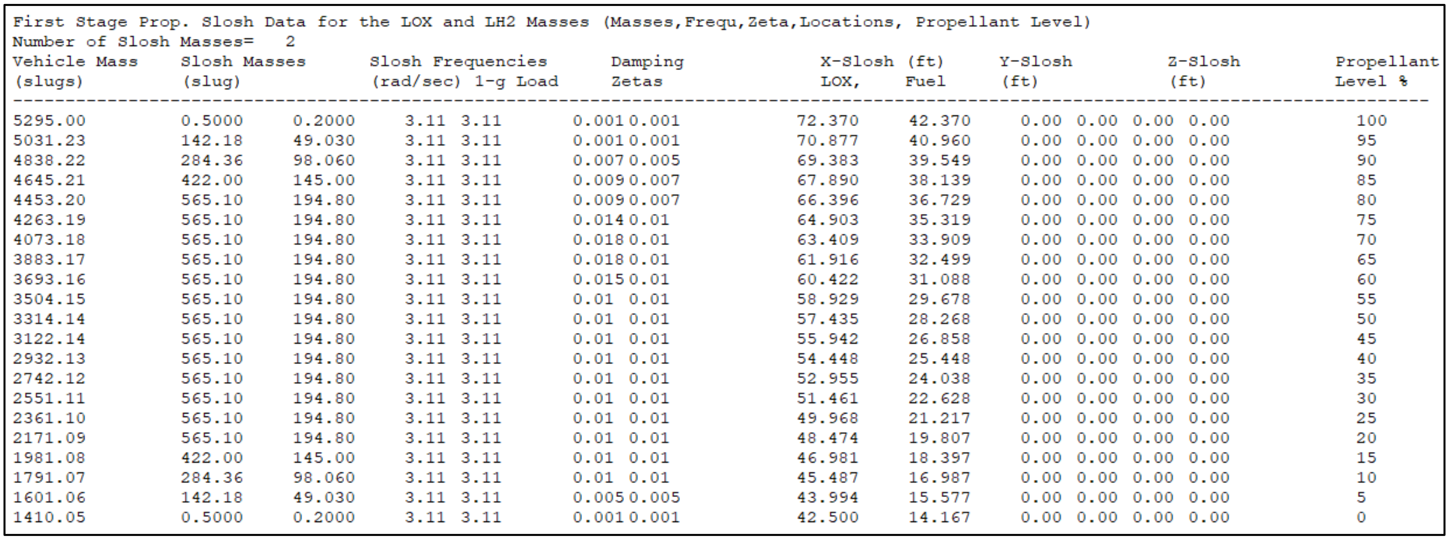
Mass Properties of the Launch Vehicle During First Stage
Acceleration due to Gravity, 1g =  32.174
Mass (slg) Ixx (sl-ft^2) Iyy (sl-ft^2) Izz, Ixy Ixz   Iyz  Xcg (ft)   Ycg  Zcg  Veh_Length (ft)         Flight Condition
5295.00    37853.0     2845141.1 2844181.7 0.0  0.0   0.0  58.150     0.0  0.0  100.0                   100    Percent
5031.23    36626.4     2804511.0 2803551.5 0.0  0.0   0.0  57.817     0.0  0.0  100.0                   95     Percent
4838.22    35314.8     2766687.8 2765727.8 0.0  0.0   0.0  57.634     0.0  0.0  100.0                   90     Percent
4645.21    33953.3     2731069.3 2730109.9 0.0  0.0   0.0  57.495     0.0  0.0  100.0                   85     Percent
4453.20    32611.8     2696068.7 2695109.3 0.0  0.0   0.0  57.406     0.0  0.0  100.0                   80     Percent
4263.19    31290.3     2663027.6 2662068.1 0.0  0.0   0.0  57.373     0.0  0.0  100.0                   75     Percent
4073.18    29978.7     2630202.3 2629242.9 0.0  0.0   0.0  57.405     0.0  0.0  100.0                   70     Percent
3883.17    28617.2     2596742.8 2595783.3 0.0  0.0   0.0  57.511     0.0  0.0  100.0                   65     Percent
3693.16    27255.7     2561664.6 2560705.2 0.0  0.0   0.0  57.702     0.0  0.0  100.0                   60     Percent
3504.15    25924.2     2523813.1 2522853.6 0.0  0.0   0.0  57.992     0.0  0.0  100.0                   55     Percent
3314.14    24592.7     2481813.9 2480854.5 0.0  0.0   0.0  58.398     0.0  0.0  100.0                   50     Percent
3122.14    23261.1     2434006.9 2433047.4 0.0  0.0   0.0  58.942     0.0  0.0  100.0                   45     Percent
2932.13    21929.6     2378352.5 2377393.1 0.0  0.0   0.0  59.650     0.0  0.0  100.0                   40     Percent
2742.12    20598.1     2312301.3 2311341.9 0.0  0.0   0.0  60.556     0.0  0.0  100.0                   35     Percent
2551.11    19266.6     2232603.3 2231643.8 0.0  0.0   0.0  61.704     0.0  0.0  100.0                   30     Percent
2361.10    17935.0     2135026.4 2134066.9 0.0  0.0   0.0  63.154     0.0  0.0  100.0                   25     Percent
2171.09    16603.5     2013926.8 2012967.3 0.0  0.0   0.0  64.984     0.0  0.0  100.0                   20     Percent
1981.08    15272.0     1861671.5 1860712.1 0.0  0.0   0.0  67.304     0.0  0.0  100.0                   15     Percent
1791.07    13940.5     1667130.9 1666171.4 0.0  0.0   0.0  70.270     0.0  0.0  100.0                   10     Percent
1601.06    12608.9     1415280.8 1414321.4 0.0  0.0   0.0  74.110     0.0  0.0  100.0                    5     Percent
1460.00    11445.6     1079103.9 1078144.5 0.0  0.0   0.0  79.186     0.0  0.0  100.0                    0     Percent
```

Mass Properties File "Stage-1.Mass"



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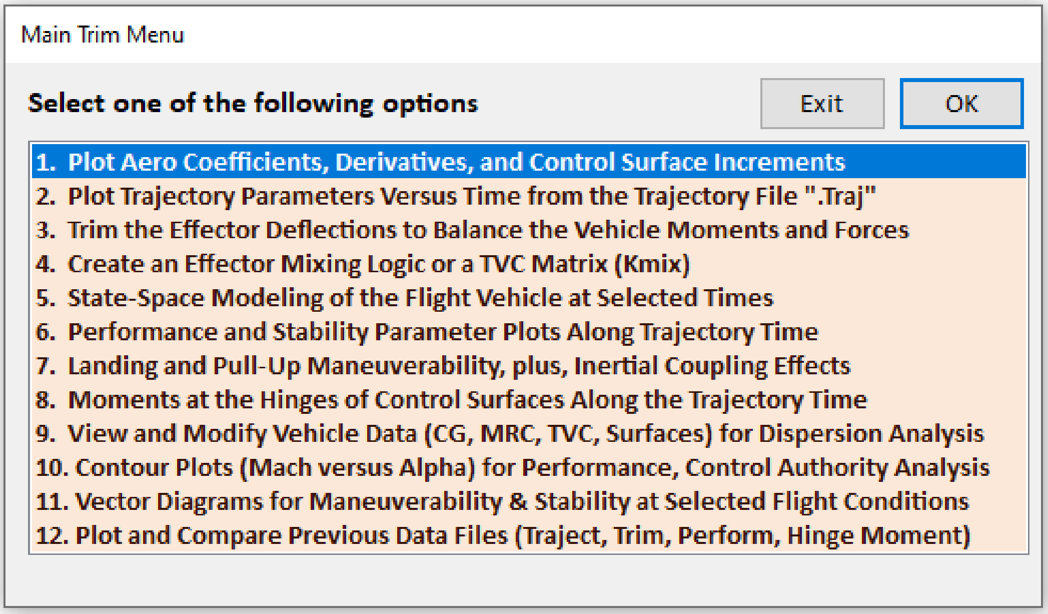
First Stage Prop. Slosh Data for the LOX and LH2 Masses (Masses,Frequ,Zeta,Locations, Propellant Level)
Number of Slosh Masses=   2
Vehicle Mass     Slosh Masses         Slosh Frequencies       Damping               X-Slosh (ft)       Y-Slosh              Z-Slosh              Propellant
(slugs)          (slug)               (rad/sec) 1-g Load      Zetas                 LOX,     Fuel      (ft)                 (ft)                 Level %
-------------------------------------------------------------------------------------------------------------------------------------------------------
5295.00          0.5000     0.2000       3.11  3.11        0.001 0.001           72.370     42.370     0.00  0.00  0.00  0.00               100
5031.23          142.18     49.030       3.11  3.11        0.001 0.001           70.877     40.960     0.00  0.00  0.00  0.00               95
4838.22          284.36     98.060       3.11  3.11        0.007 0.005           69.383     39.549     0.00  0.00  0.00  0.00               90
4645.21          422.00     145.00       3.11  3.11        0.009 0.007           67.890     38.139     0.00  0.00  0.00  0.00               85
4453.20          565.10     194.80       3.11  3.11        0.009 0.007           66.396     36.729     0.00  0.00  0.00  0.00               80
4263.19          565.10     194.80       3.11  3.11        0.014 0.01            64.903     35.319     0.00  0.00  0.00  0.00               75
4073.18          565.10     194.80       3.11  3.11        0.018 0.01            63.409     33.909     0.00  0.00  0.00  0.00               70
3883.17          565.10     194.80       3.11  3.11        0.018 0.01            61.916     32.499     0.00  0.00  0.00  0.00               65
3693.16          565.10     194.80       3.11  3.11        0.015 0.01            60.422     31.088     0.00  0.00  0.00  0.00               60
3504.15          565.10     194.80       3.11  3.11        0.01  0.01            58.929     29.678     0.00  0.00  0.00  0.00               55
3314.14          565.10     194.80       3.11  3.11        0.01  0.01            57.435     28.268     0.00  0.00  0.00  0.00               50
3122.14          565.10     194.80       3.11  3.11        0.01  0.01            55.942     26.858     0.00  0.00  0.00  0.00               45
2932.13          565.10     194.80       3.11  3.11        0.01  0.01            54.448     25.448     0.00  0.00  0.00  0.00               40
2742.12          565.10     194.80       3.11  3.11        0.01  0.01            52.955     24.038     0.00  0.00  0.00  0.00               35
2551.11          565.10     194.80       3.11  3.11        0.01  0.01            51.461     22.628     0.00  0.00  0.00  0.00               30
2361.10          565.10     194.80       3.11  3.11        0.01  0.01            49.968     21.217     0.00  0.00  0.00  0.00               25
2171.09          565.10     194.80       3.11  3.11        0.01  0.01            48.474     19.807     0.00  0.00  0.00  0.00               20
1981.08          422.00     145.00       3.11  3.11        0.01  0.01            46.981     18.397     0.00  0.00  0.00  0.00               15
1791.07          284.36     98.060       3.11  3.11        0.01  0.01            45.487     16.987     0.00  0.00  0.00  0.00               10
1601.06          142.18     49.030       3.11  3.11        0.005 0.005           43.994     15.577     0.00  0.00  0.00  0.00               5
1410.05          0.5000     0.2000       3.11  3.11        0.001 0.001           42.500     14.167     0.00  0.00  0.00  0.00               0
```

Slosh Data File "Stage-1.Slsh"

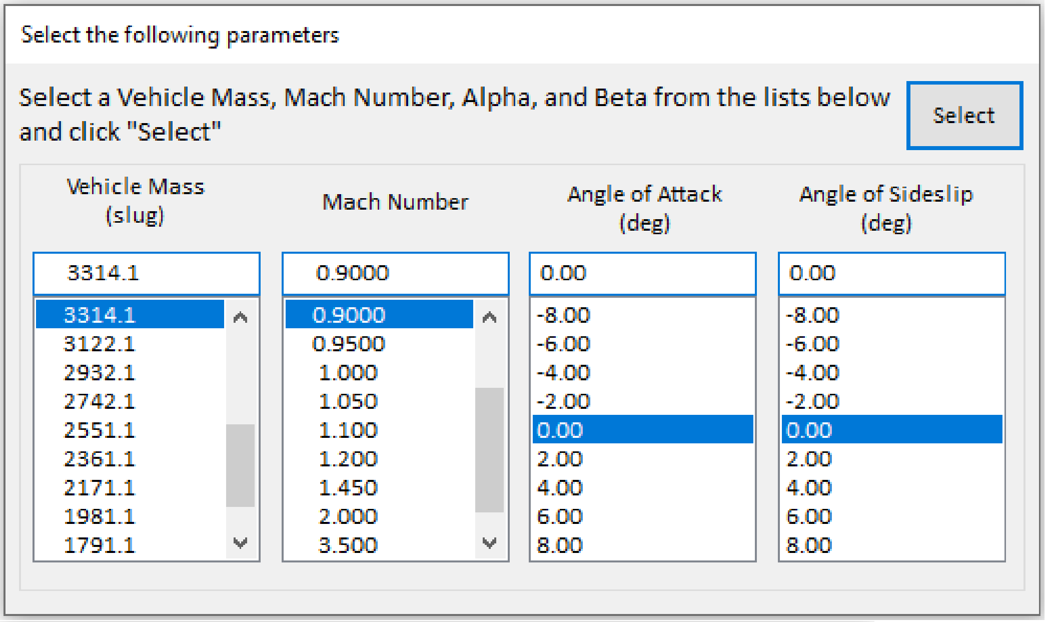
The mass properties file includes the vehicle moments of inertia and CG location as a function of vehicle mass at different propellant fill levels. The slosh file includes the 2 propellant slosh masses, 2 damping coefficients, and the two x, y, z locations as a function of the vehicle mass. The slosh mass location only varies along the x axis, y and z locations are zero.

The Trim program main menu comes up and the first step is to take a look at the aero data, we therefore select the first option in the menu. From the next menu choose a flight condition in terms of Mach number, angles of attack and sideslip, and vehicle mass and click "Select". Then choose to plot the pitch and lateral aero coefficients, click OK, and the program plots the aero force and moment coefficients versus α and β, at 5 nearby Mach numbers for comparison. Similarly plot the pitch and lateral aero derivatives.



Main Trim Menu

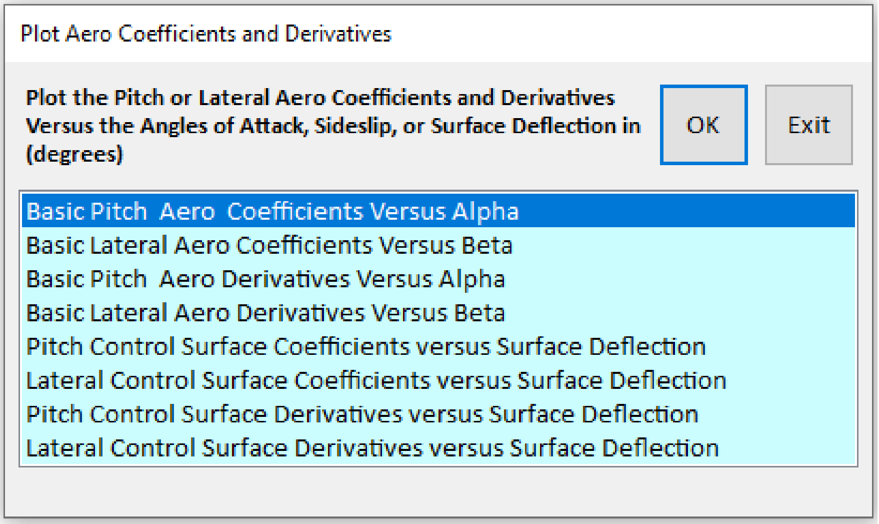
Select one of the following options



Select the following parameters

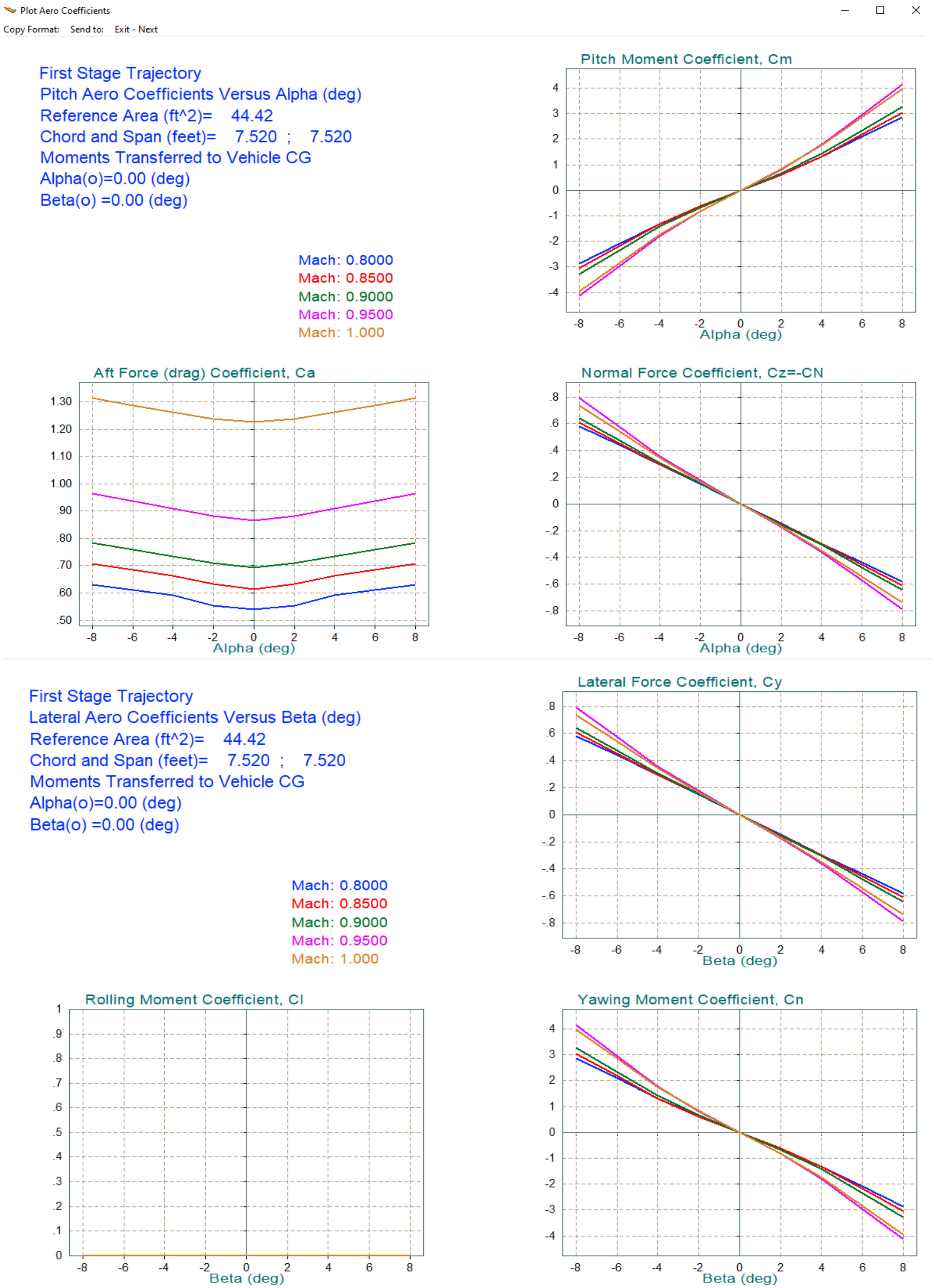
Select a Vehicle Mass, Mach Number, Alpha, and Beta from the lists below and click "Select"

Vehicle Mass (slug)	Mach Number	Angle of Attack (deg)	Angle of Sideslip (deg)
3314.1	0.9000	0.00	0.00



Plot Aero Coefficients and Derivatives

Plot the Pitch or Lateral Aero Coefficients and Derivatives Versus the Angles of Attack, Sideslip, or Surface Deflection in (degrees)



First Stage Trajectory
Pitch Aero Coefficients Versus Alpha (deg)
Reference Area (ft^2)= 44.42
Chord and Span (feet)= 7.520 ; 7.520
Moments Transferred to Vehicle CG
Alpha(o)=0.00 (deg)
Beta(o) =0.00 (deg)

Mach: 0.8000
Mach: 0.8500
Mach: 0.9000
Mach: 0.9500
Mach: 1.000

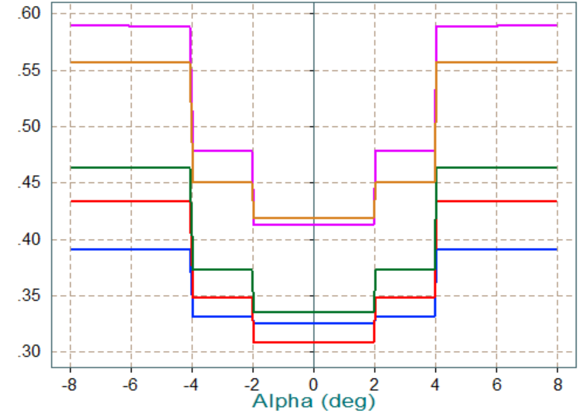
First Stage Trajectory
Lateral Aero Coefficients Versus Beta (deg)
Reference Area (ft^2)= 44.42
Chord and Span (feet)= 7.520 ; 7.520
Moments Transferred to Vehicle CG
Alpha(o)=0.00 (deg)
Beta(o) =0.00 (deg)

Mach: 0.8000
Mach: 0.8500
Mach: 0.9000
Mach: 0.9500
Mach: 1.000

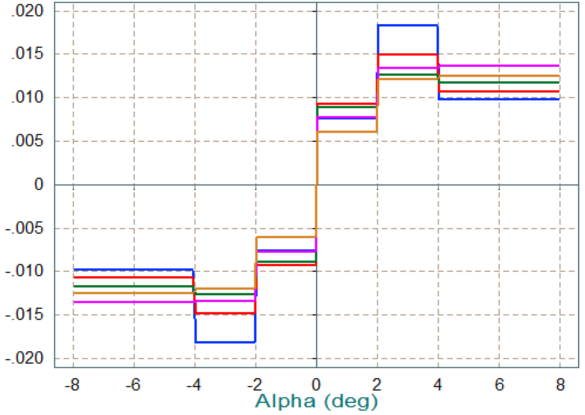
First Stage Trajectory
Pitch Aero Derivatives Versus Alpha (deg)
Reference Area (ft^2)= 44.42
Chord and Span (feet)= 7.520 ; 7.520
Moments Transferred to Vehicle CG
Alpha(o)=0.00 (deg)
Beta(o) =0.00 (deg)

Mach: 0.8000
Mach: 0.8500
Mach: 0.9000
Mach: 0.9500
Mach: 1.000

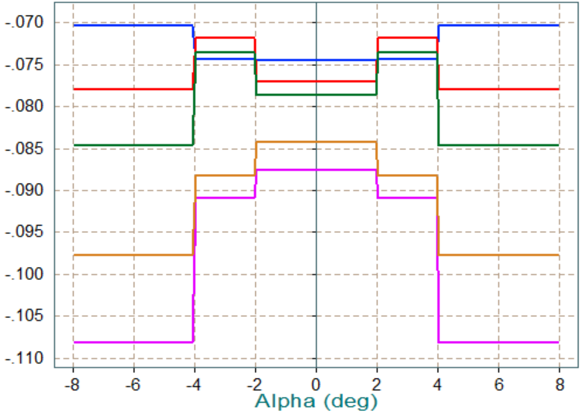
Pitch Moment Derivative, Cm_alpha



Aft Force Derivative, Ca_alpha

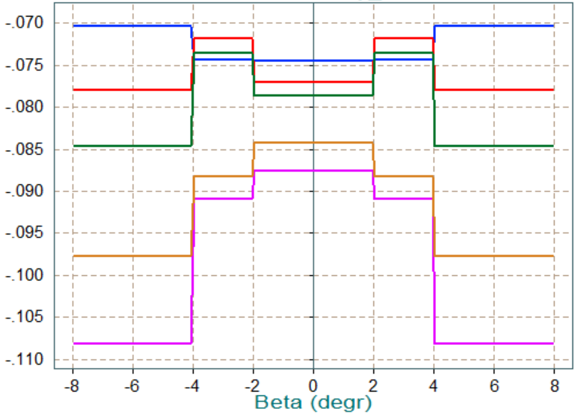


Normal Force Derivative, Cz_alpha



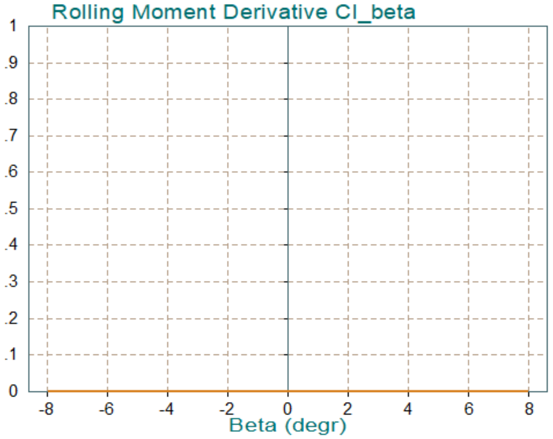
First Stage Trajectory
Lateral Aero Derivatives Versus Beta (deg)
Reference Area (ft^2)= 44.42
Chord and Span (feet)= 7.520 ; 7.520
Moments Transferred to Vehicle CG
Alpha(o)=0.00 (deg)
Beta(o) =0.00 (deg)

Lateral Force Derivative, Cy_beta

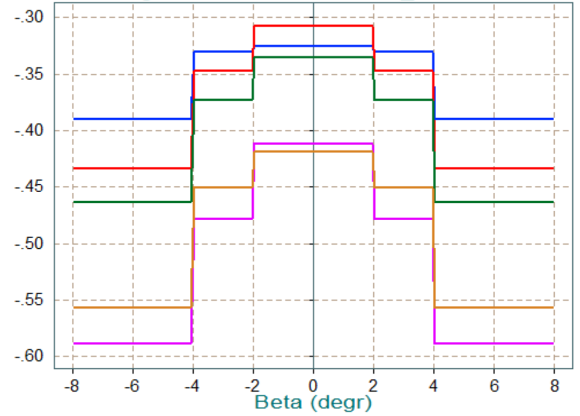


Mach: 0.8000
Mach: 0.8500
Mach: 0.9000
Mach: 0.9500
Mach: 1.000

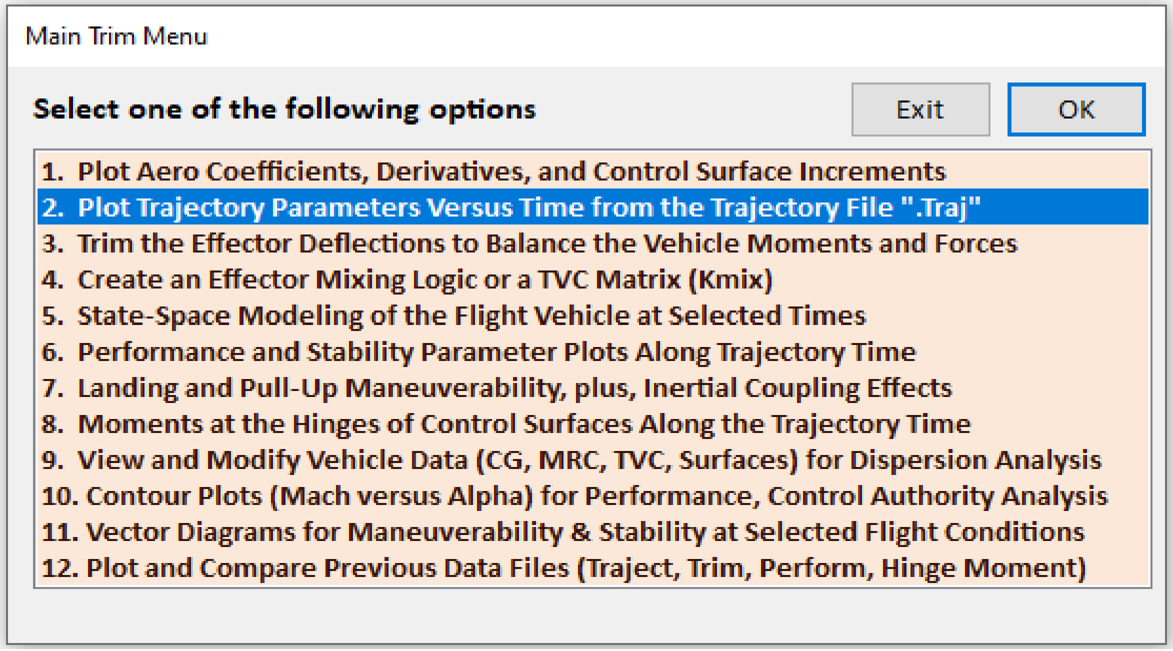
Rolling Moment Derivative Cl_beta



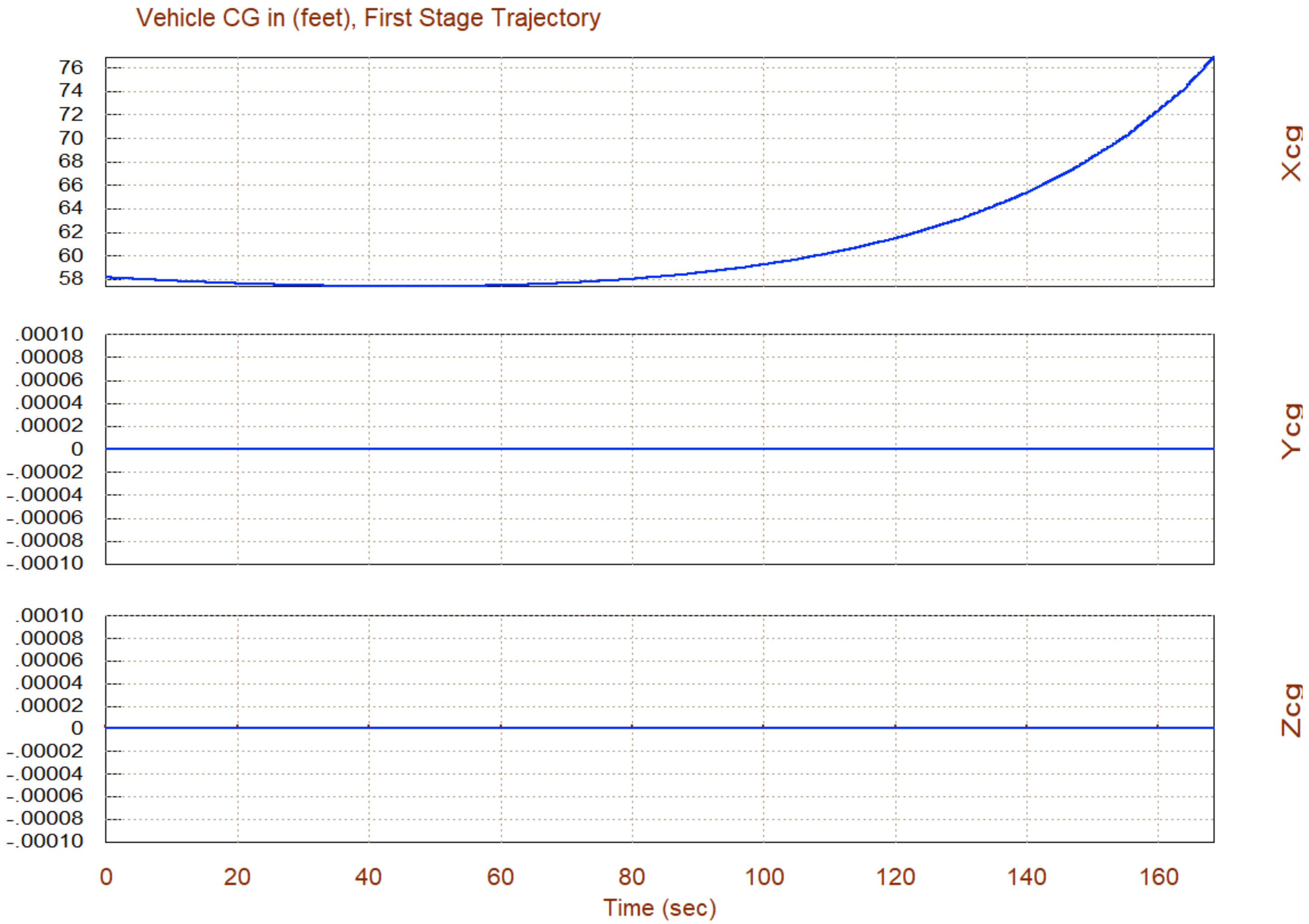
Yawing Moment Derivative, Cn_beta



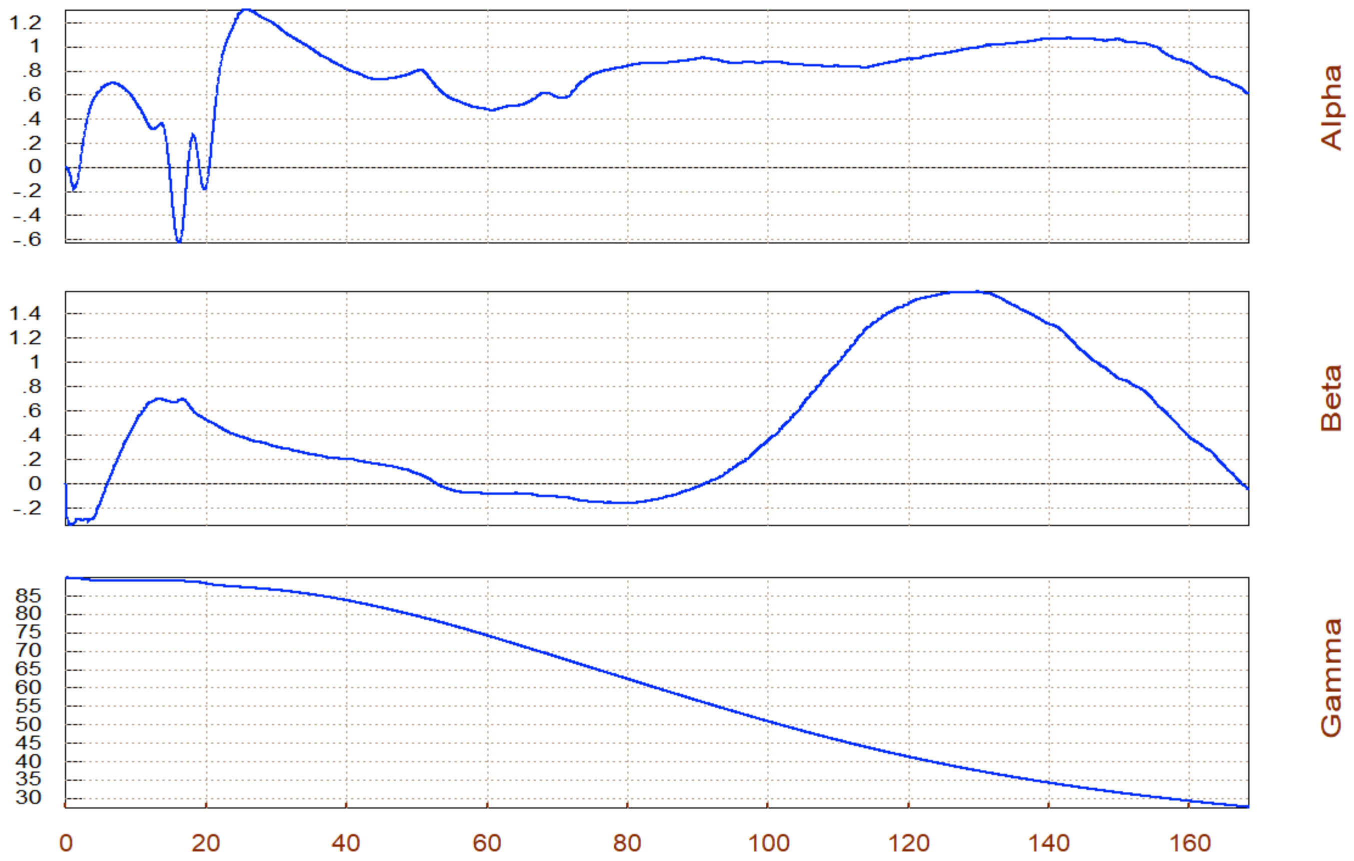
Return to the Trim program main menu and select the second item to plot the first stage trajectory data versus time. This is the data from file "*Stage-1.Traj*". The CG is not in the trajectory file but it gets calculated versus time from the mass properties.



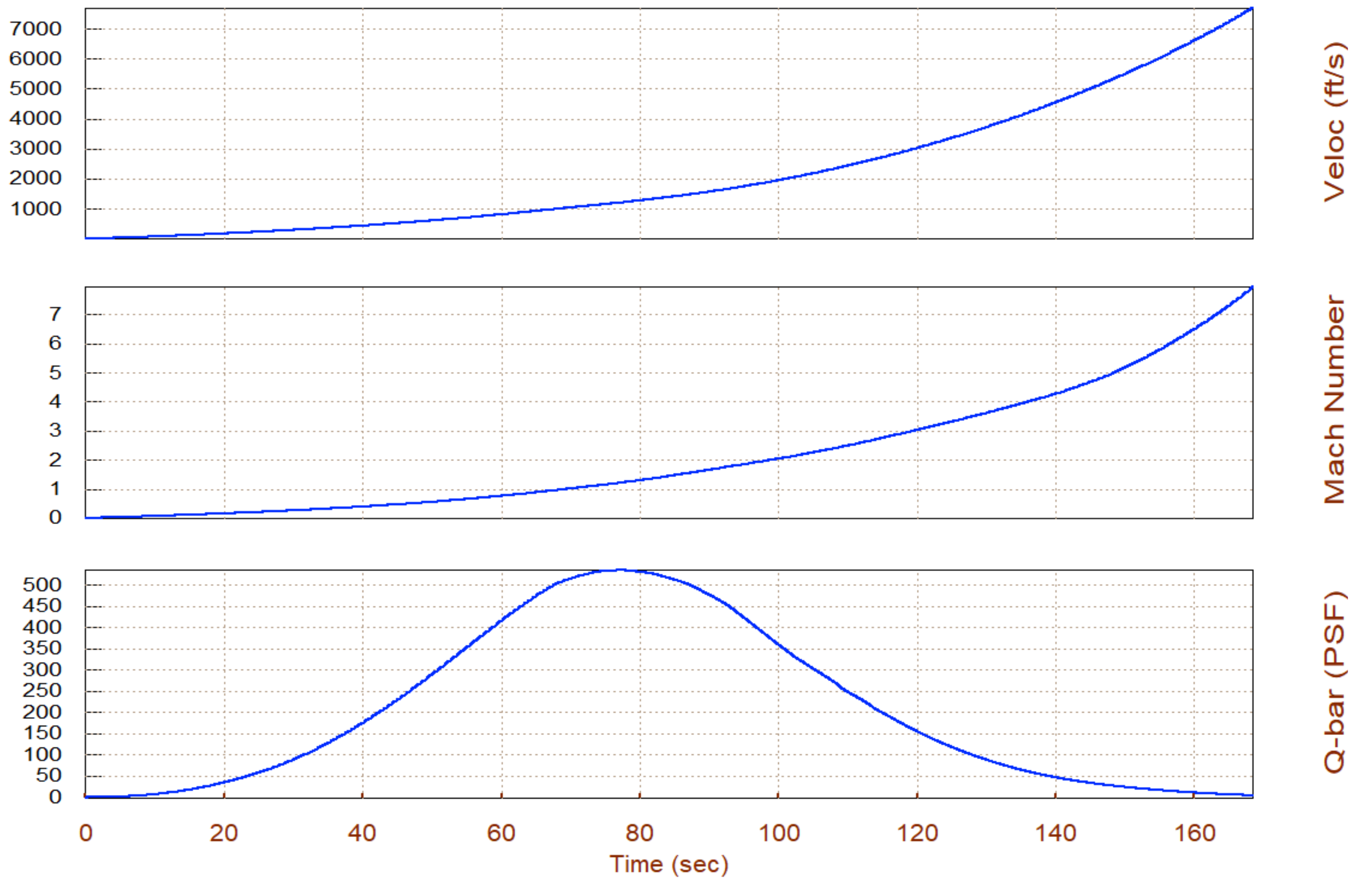
1. Plot Aero Coefficients, Derivatives, and Control Surface Increments
2. Plot Trajectory Parameters Versus Time from the Trajectory File ".Traj"
3. Trim the Effector Deflections to Balance the Vehicle Moments and Forces
4. Create an Effector Mixing Logic or a TVC Matrix (Kmix)
5. State-Space Modeling of the Flight Vehicle at Selected Times
6. Performance and Stability Parameter Plots Along Trajectory Time
7. Landing and Pull-Up Maneuverability, plus, Inertial Coupling Effects
8. Moments at the Hinges of Control Surfaces Along the Trajectory Time
9. View and Modify Vehicle Data (CG, MRC, TVC, Surfaces) for Dispersion Analysis
10. Contour Plots (Mach versus Alpha) for Performance, Control Authority Analysis
11. Vector Diagrams for Maneuverability & Stability at Selected Flight Conditions
12. Plot and Compare Previous Data Files (Traject, Trim, Perform, Hinge Moment)



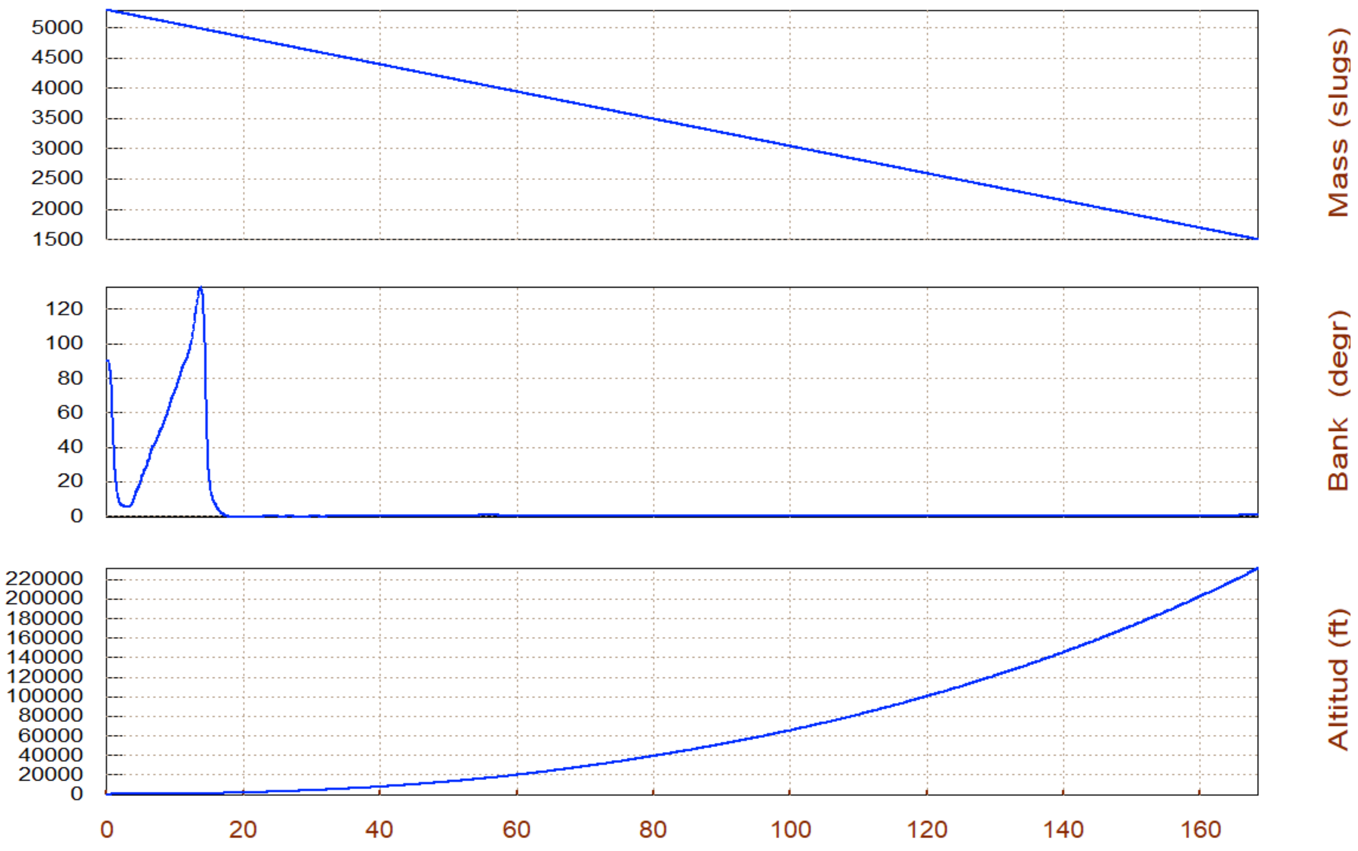
Angles of Attack/Sideslip/Flight Path (deg), First Stage Trajectory



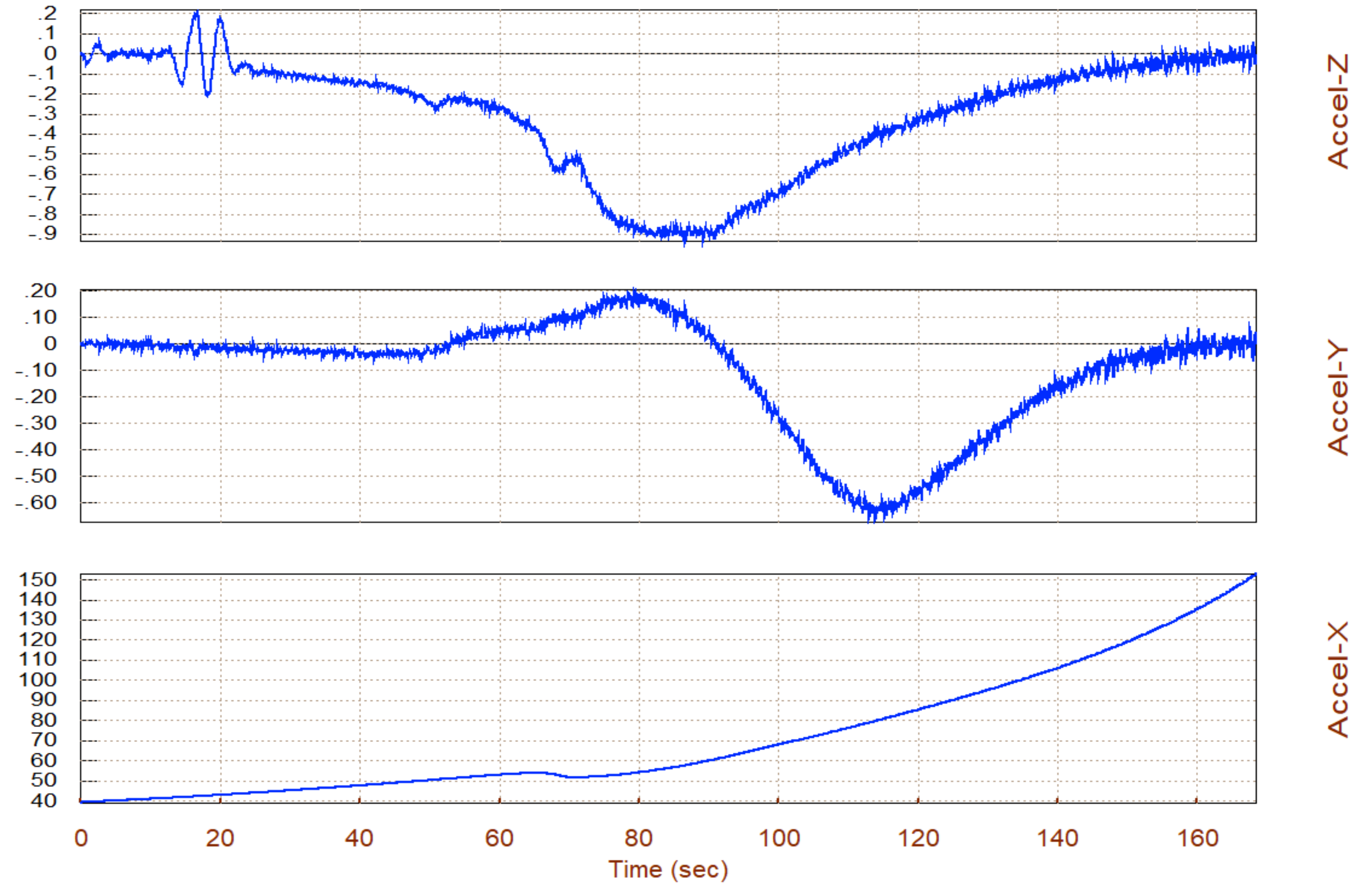
Velocity, Dynamic Pressure, First Stage Trajectory

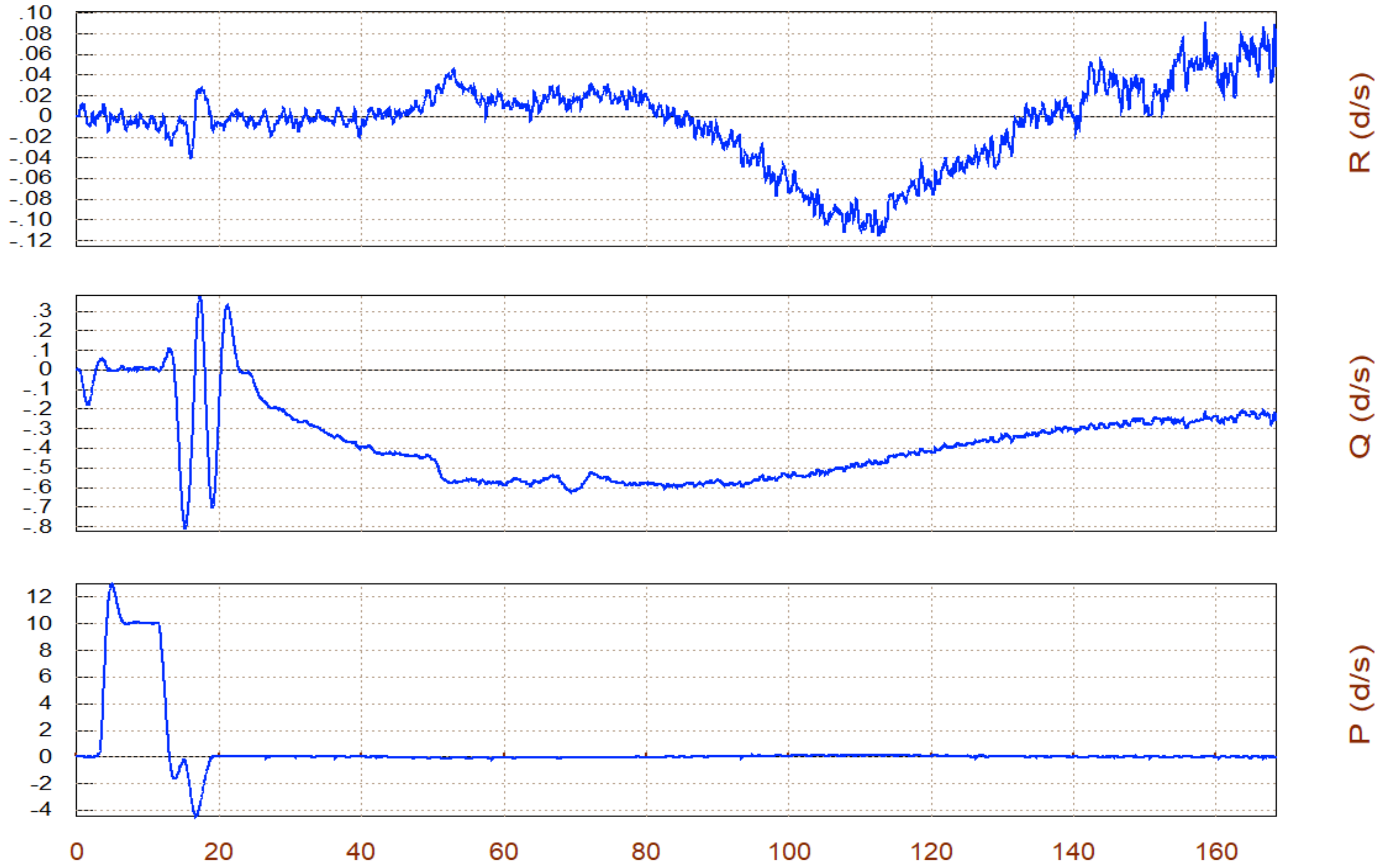


Vehicle Altitude, Mass, Bank Angle, First Stage Trajectory

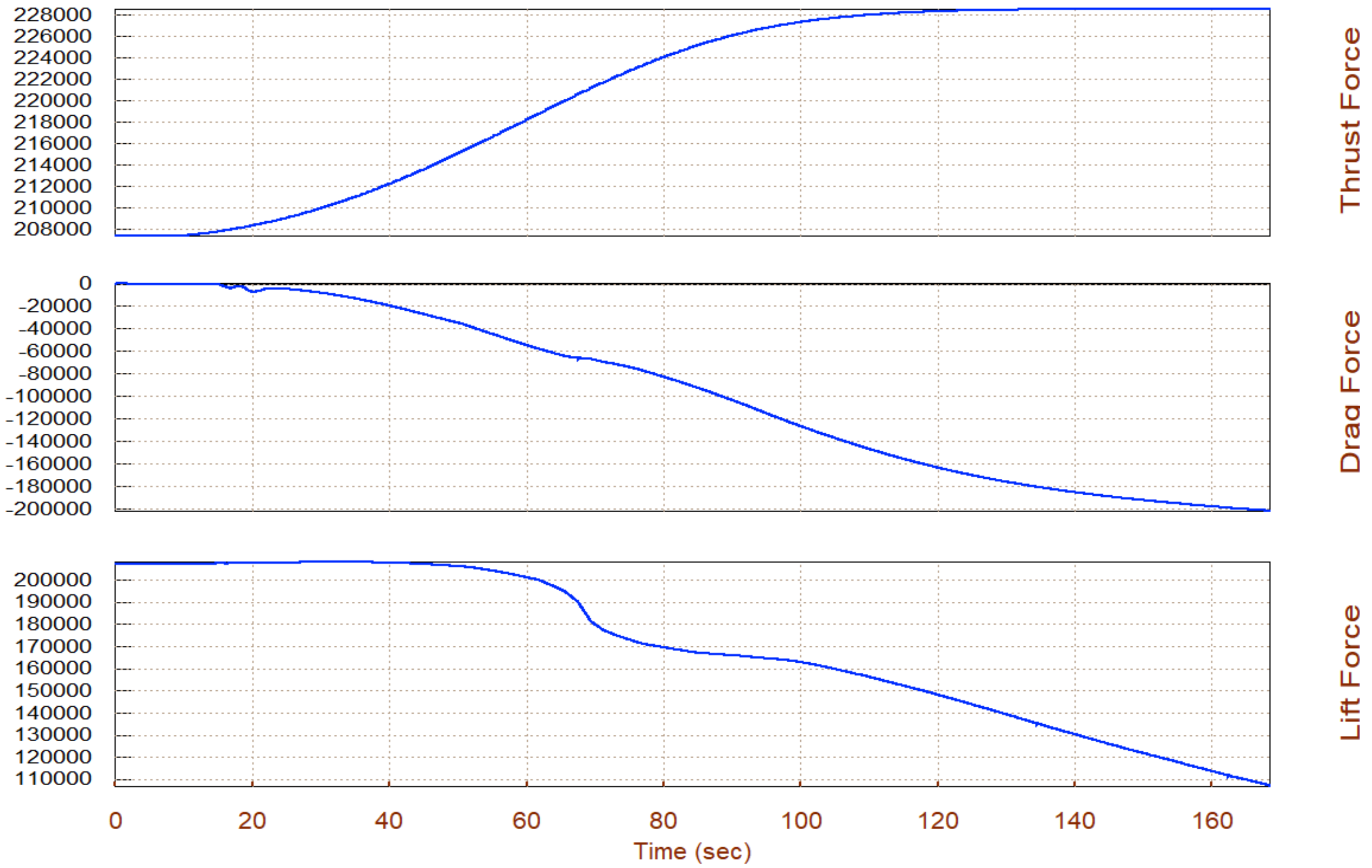


Sensed Acceleration in (ft/sec^2), First Stage Trajectory



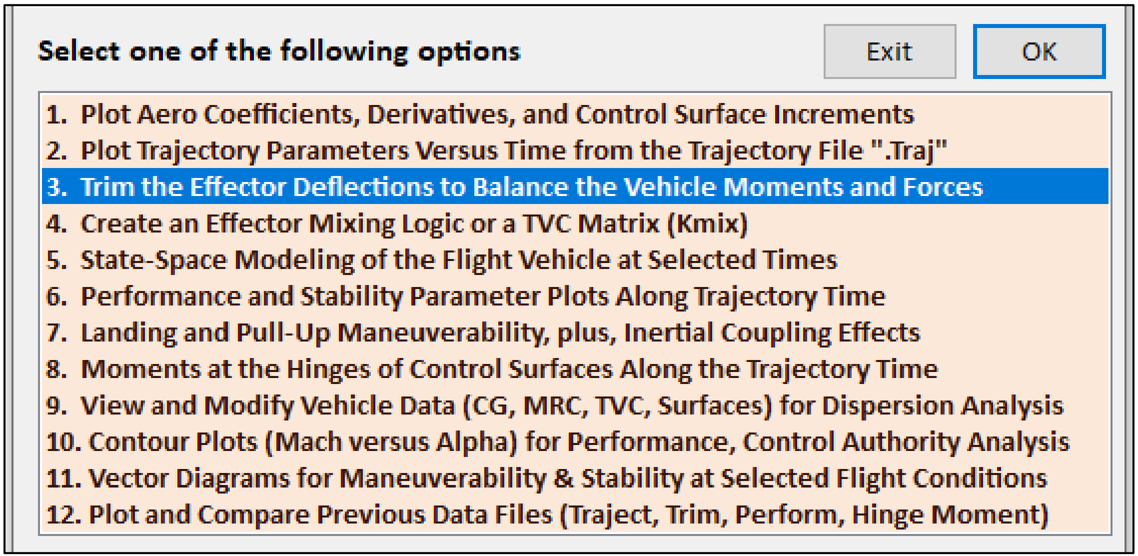


Angular Rates (rad/sec), First Stage Trajectory



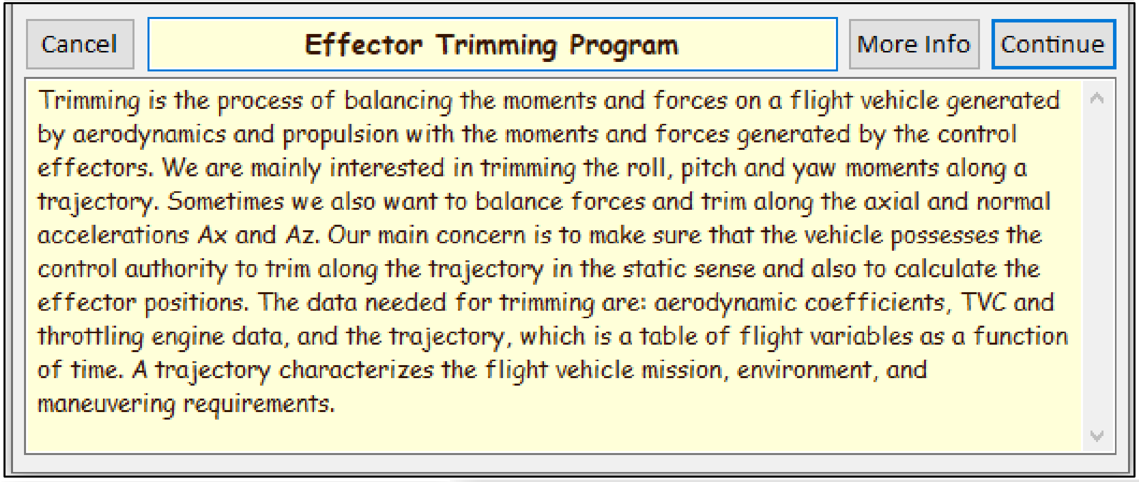
Aero Lift/Drag Forces, Eng. Thrust in (lb), First Stage Trajectory

The next step is to select the 3rd option from the Trim menu which is to trim the TVC engines along the trajectory. That is, to calculate gimbal deflections and thrust variations required in order to balance the moments and the axial acceleration defined in the trajectory file. In the next menu you may choose an already existing trim file to initialize your trim process. If it is the first time, there won't be any previous trim file and you can click on "*Do Not Select*" a previously created trim file. In the next menu select to balance the 3 moments along the (α, β) angles defined in the trajectory, including to also match the acceleration along the vehicle x-axis.



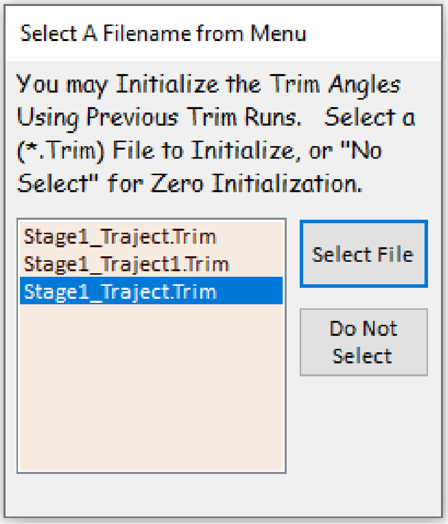
Select one of the following options

1. Plot Aero Coefficients, Derivatives, and Control Surface Increments
2. Plot Trajectory Parameters Versus Time from the Trajectory File ".Traj"
3. Trim the Effector Deflections to Balance the Vehicle Moments and Forces
4. Create an Effector Mixing Logic or a TVC Matrix (Kmix)
5. State-Space Modeling of the Flight Vehicle at Selected Times
6. Performance and Stability Parameter Plots Along Trajectory Time
7. Landing and Pull-Up Maneuverability, plus, Inertial Coupling Effects
8. Moments at the Hinges of Control Surfaces Along the Trajectory Time
9. View and Modify Vehicle Data (CG, MRC, TVC, Surfaces) for Dispersion Analysis
10. Contour Plots (Mach versus Alpha) for Performance, Control Authority Analysis
11. Vector Diagrams for Maneuverability & Stability at Selected Flight Conditions
12. Plot and Compare Previous Data Files (Traject, Trim, Perform, Hinge Moment)



Effector Trimming Program

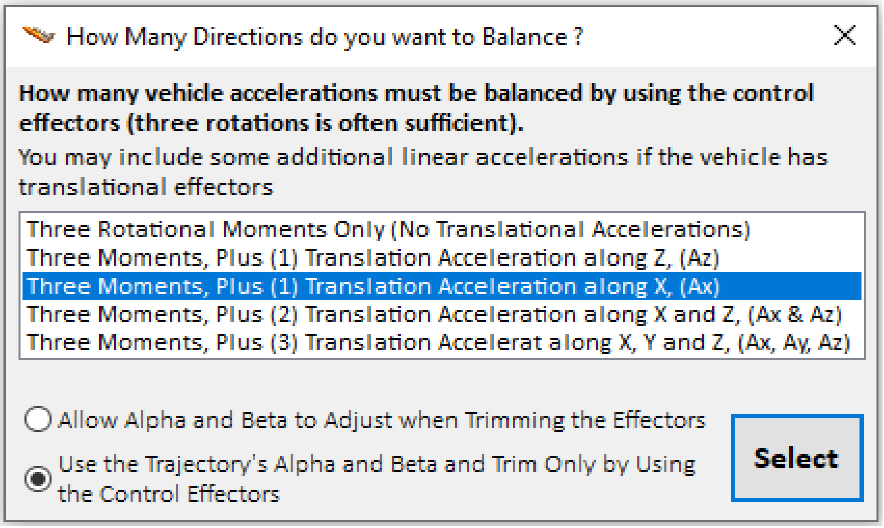
Trimming is the process of balancing the moments and forces on a flight vehicle generated by aerodynamics and propulsion with the moments and forces generated by the control effectors. We are mainly interested in trimming the roll, pitch and yaw moments along a trajectory. Sometimes we also want to balance forces and trim along the axial and normal accelerations Ax and Az. Our main concern is to make sure that the vehicle possesses the control authority to trim along the trajectory in the static sense and also to calculate the effector positions. The data needed for trimming are: aerodynamic coefficients, TVC and throttling engine data, and the trajectory, which is a table of flight variables as a function of time. A trajectory characterizes the flight vehicle mission, environment, and maneuvering requirements.



Select A Filename from Menu

You may Initialize the Trim Angles Using Previous Trim Runs. Select a (*.Trim) File to Initialize, or "No Select" for Zero Initialization.

Stage1_Traject.Trim
Stage1_Traject1.Trim
Stage1_Traject.Trim



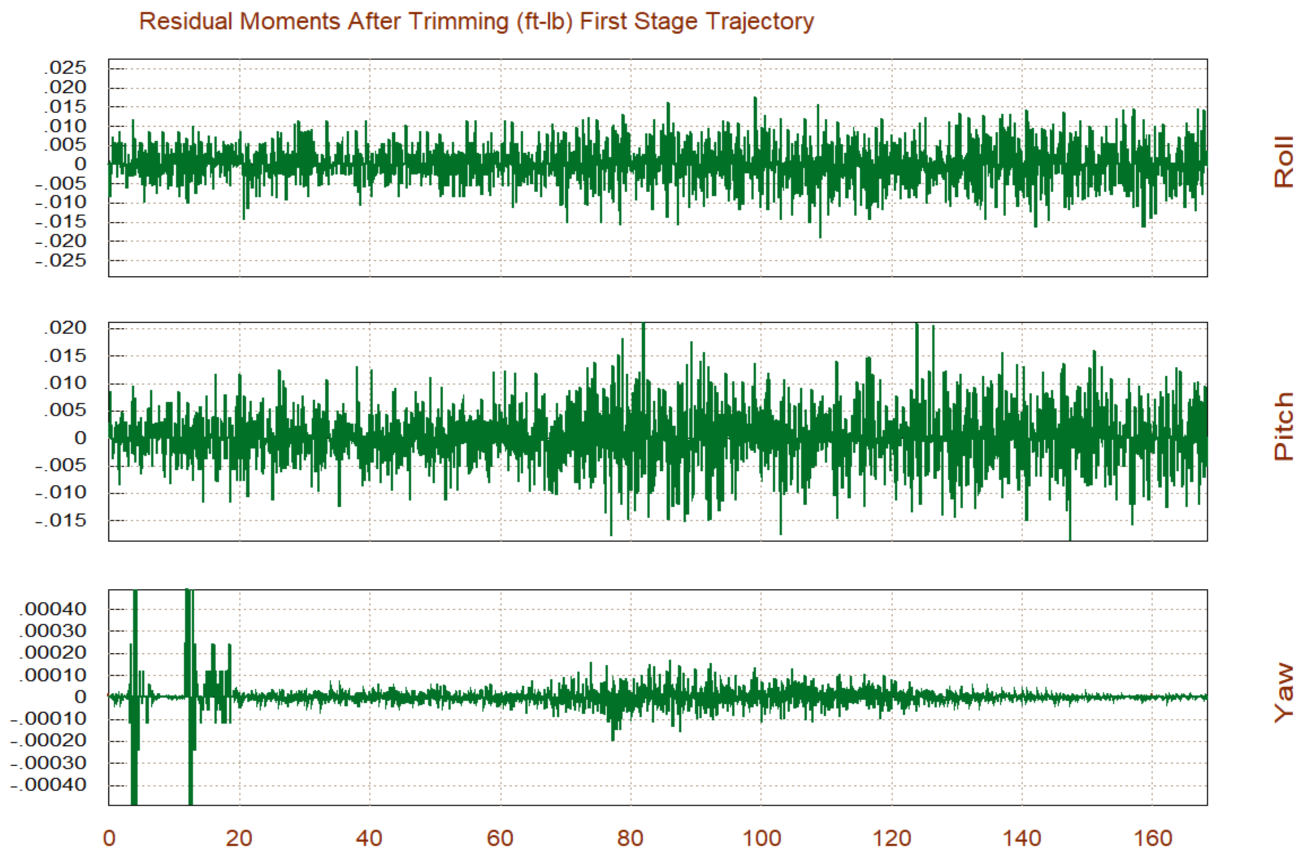
How Many Directions do you want to Balance ?

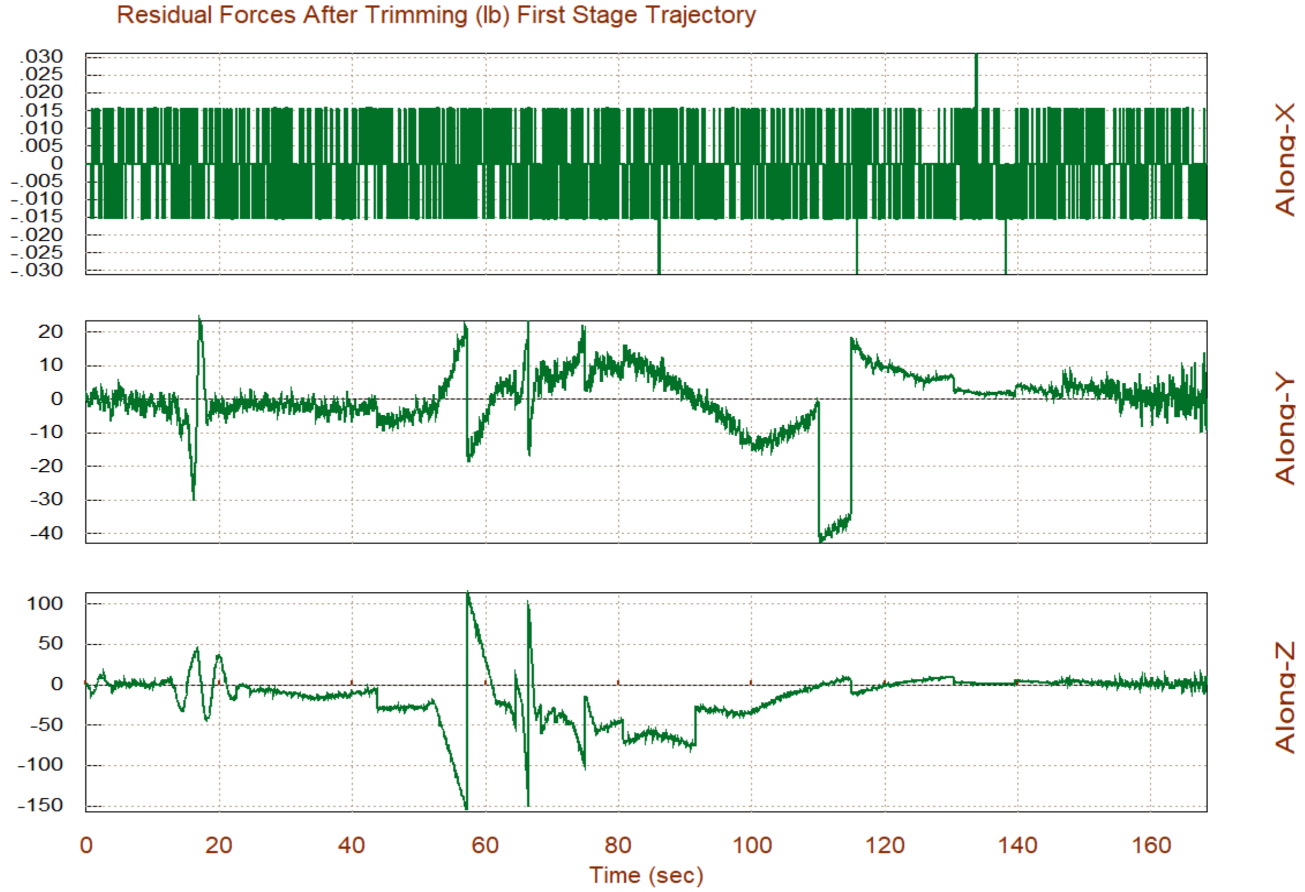
How many vehicle accelerations must be balanced by using the control effectors (three rotations is often sufficient).

You may include some additional linear accelerations if the vehicle has translational effectors

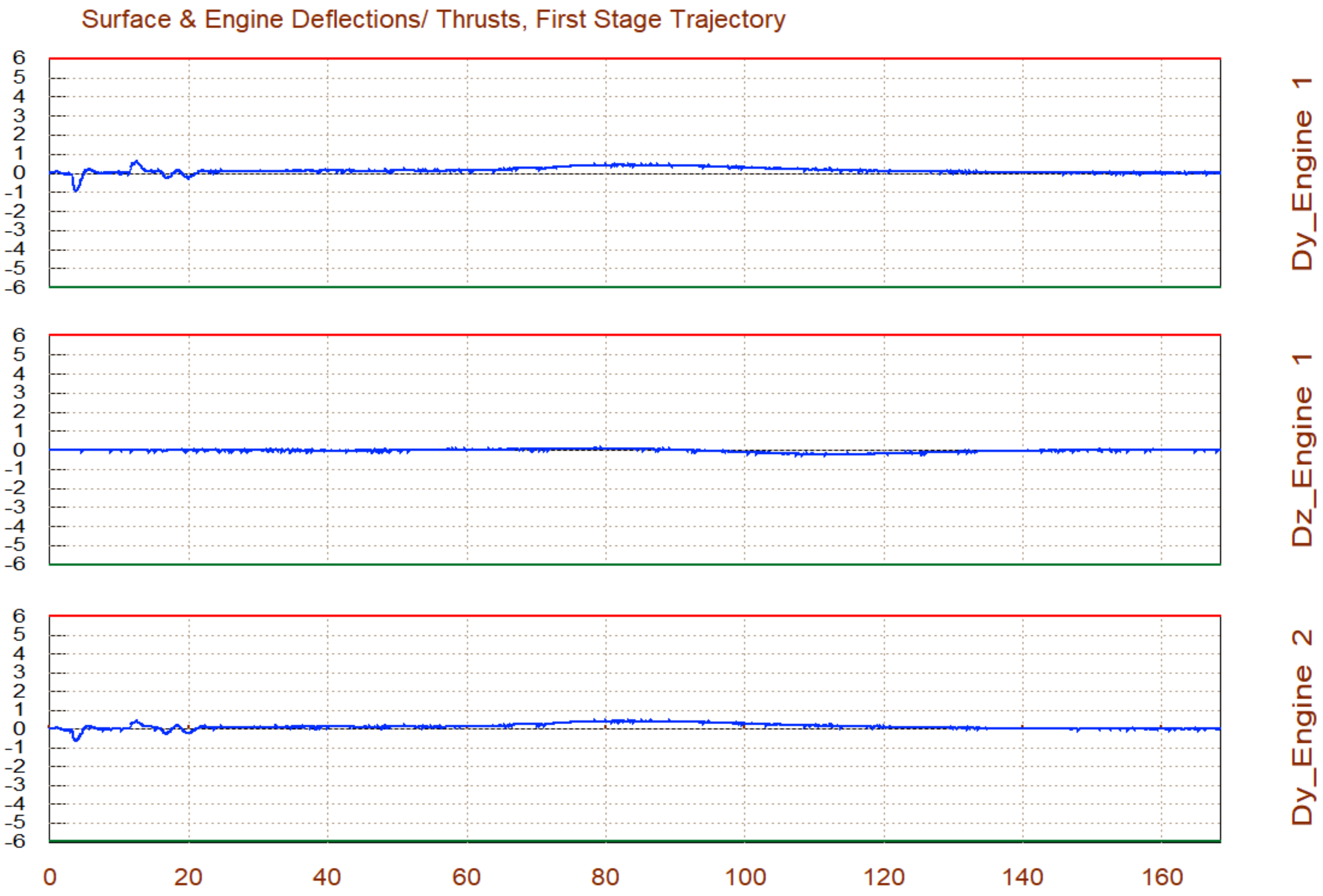
Three Rotational Moments Only (No Translational Accelerations)
Three Moments, Plus (1) Translation Acceleration along Z, (Az)
Three Moments, Plus (1) Translation Acceleration along X, (Ax)
Three Moments, Plus (2) Translation Acceleration along X and Z, (Ax & Az)
Three Moments, Plus (3) Translation Accelerat along X, Y and Z, (Ax, Ay, Az)

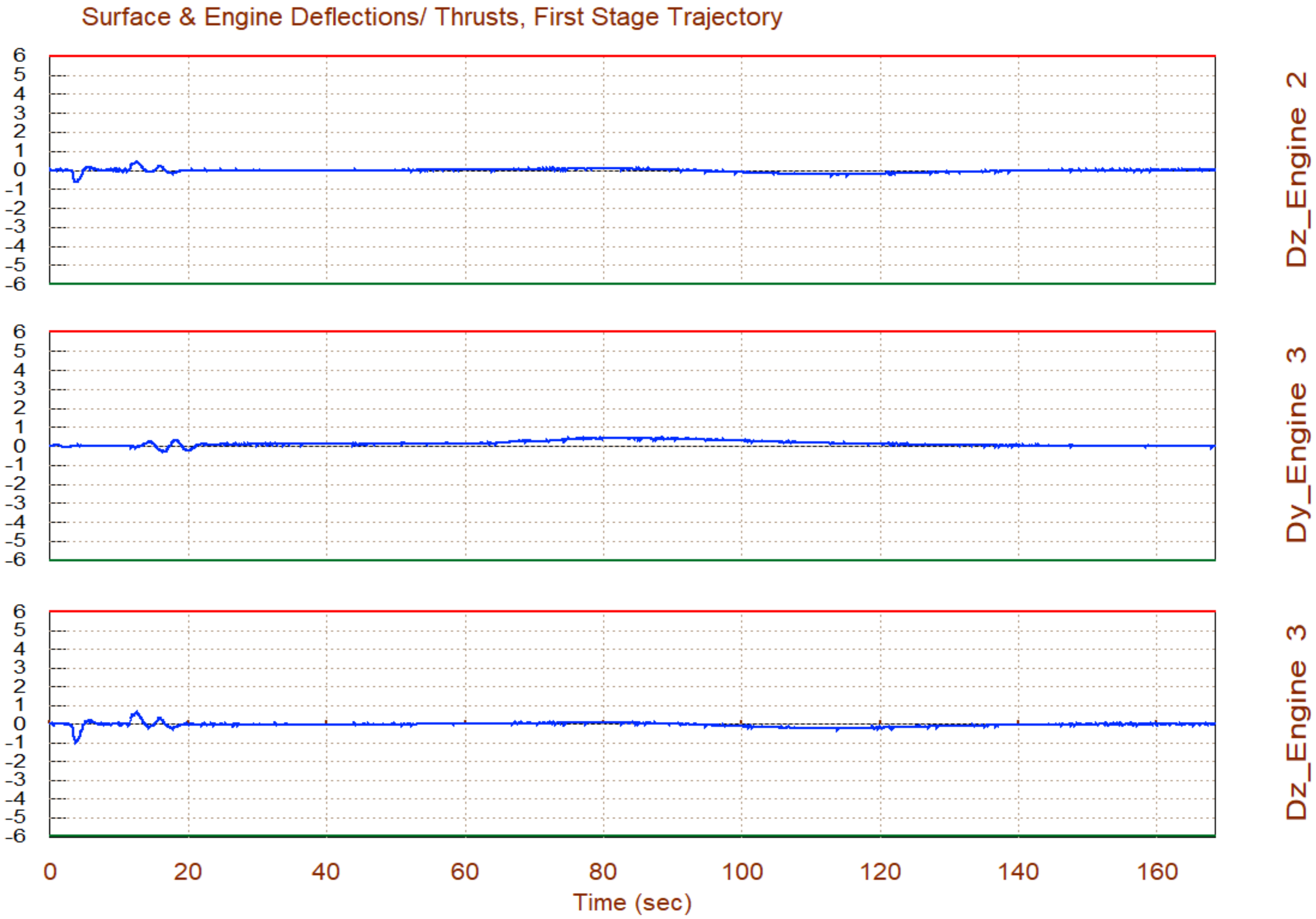
○ Allow Alpha and Beta to Adjust when Trimming the Effectors
● Use the Trajectory's Alpha and Beta and Trim Only by Using the Control Effectors



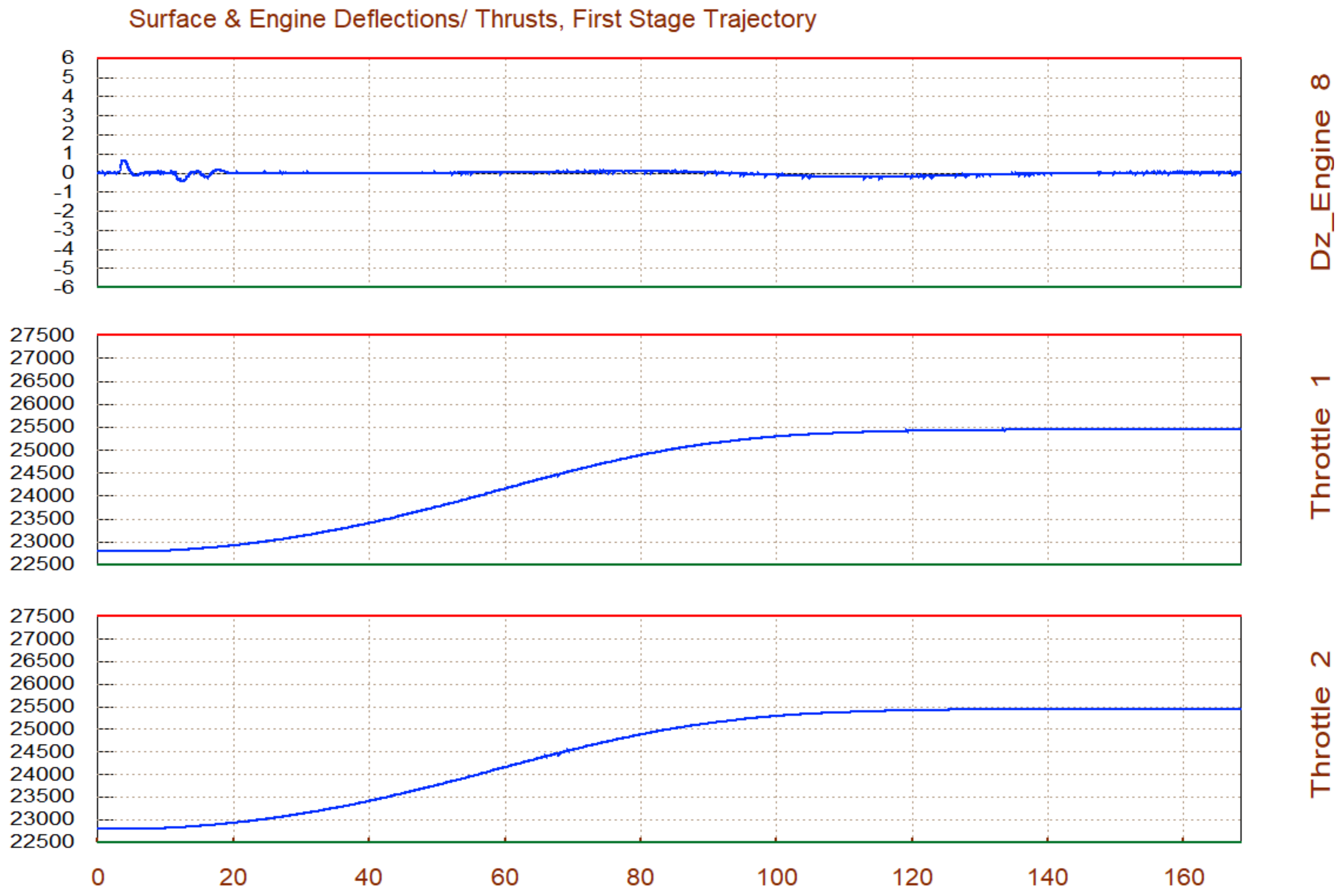


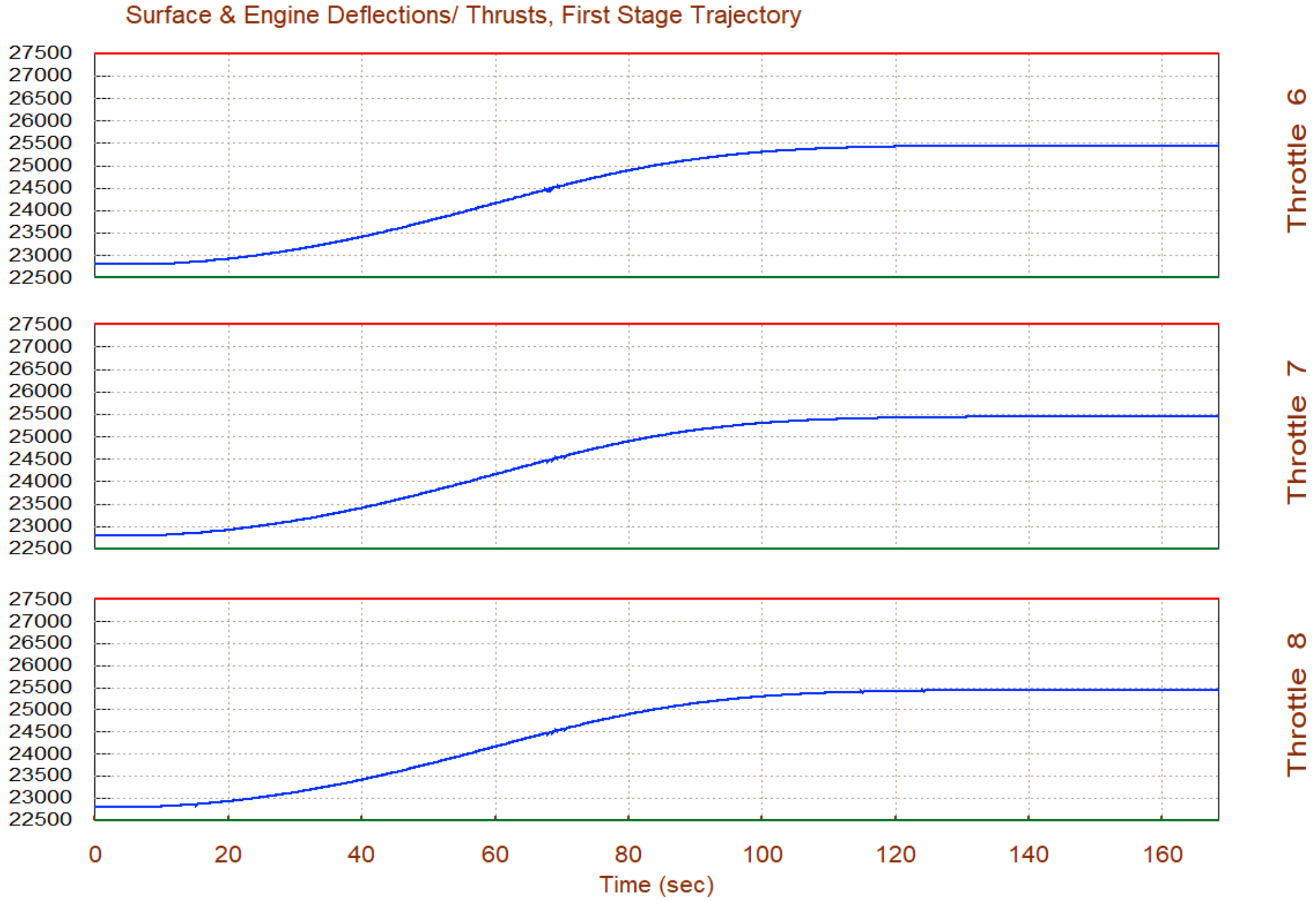
The above plots show the 3 residual moments and residual x-acceleration, which are zero as expected meaning that we were able to trim along the 4 desired directions by gimbaling and throttling the engines. As we said earlier, we allowed a little throttling in order to match the trajectory’s x-acceleration but not enough to produce differential throttling against the aero moments. This is done primarily by the TVC.





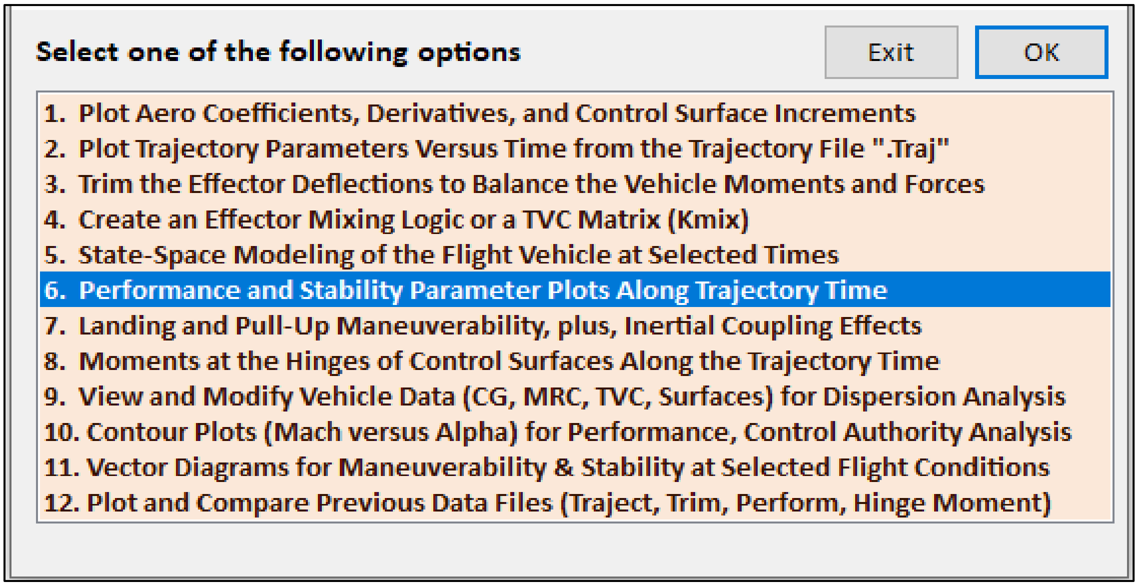
The 8 peripheral engines are gimbaling in order to balance the aero moments.

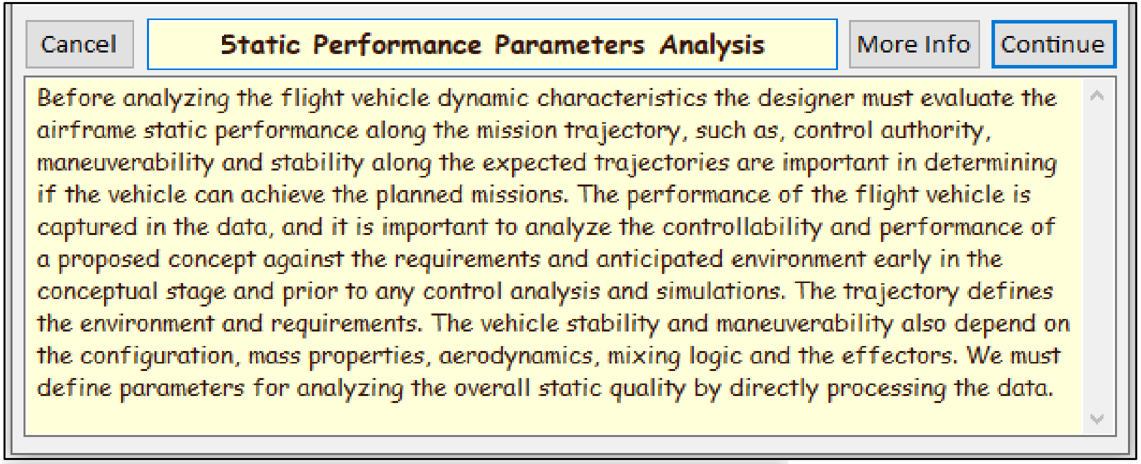




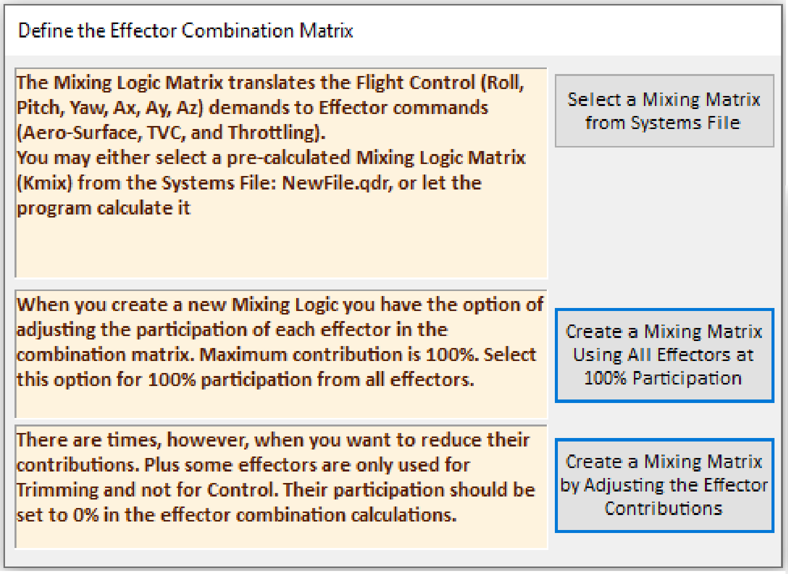
The engines are also allowed to throttle in order to match the trajectory’s x-acceleration.

The next step is to analyze the vehicle static performance along the trajectory. That is, static stability and controllability analysis. This is option-6 from the Trim menu. First, we need a (25x4) mixing matrix to combine the 25 controls (including throttling for now) with the 4 degrees-of-freedom that we are controlling (including axial acceleration control for now although it is not included in the design). We will temporarily allow the program to create an effector combination logic only for this analysis by allowing a full participation from all effectors. Next, we select a maximum 4° alpha/ beta dispersions and 30 (ft/sec) wind-gust variations, as worst possible disturbances along the trajectory.





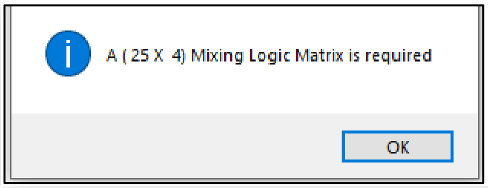
Before analyzing the flight vehicle dynamic characteristics the designer must evaluate the airframe static performance along the mission trajectory, such as, control authority, maneuverability and stability along the expected trajectories are important in determining if the vehicle can achieve the planned missions. The performance of the flight vehicle is captured in the data, and it is important to analyze the controllability and performance of a proposed concept against the requirements and anticipated environment early in the conceptual stage and prior to any control analysis and simulations. The trajectory defines the environment and requirements. The vehicle stability and maneuverability also depend on the configuration, mass properties, aerodynamics, mixing logic and the effectors. We must define parameters for analyzing the overall static quality by directly processing the data.



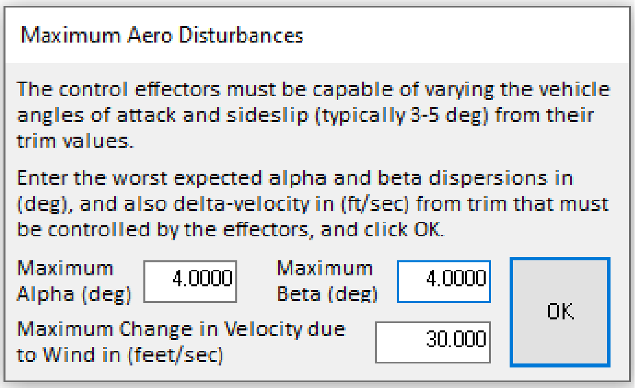
The Mixing Logic Matrix translates the Flight Control (Roll, Pitch, Yaw, Ax, Ay, Az) demands to Effector commands (Aero-Surface, TVC, and Throttling).
You may either select a pre-calculated Mixing Logic Matrix (Kmix) from the Systems File: NewFile.qdr, or let the program calculate it

When you create a new Mixing Logic you have the option of adjusting the participation of each effector in the combination matrix. Maximum contribution is 100%. Select this option for 100% participation from all effectors.

There are times, however, when you want to reduce their contributions. Plus some effectors are only used for Trimming and not for Control. Their participation should be set to 0% in the effector combination calculations.



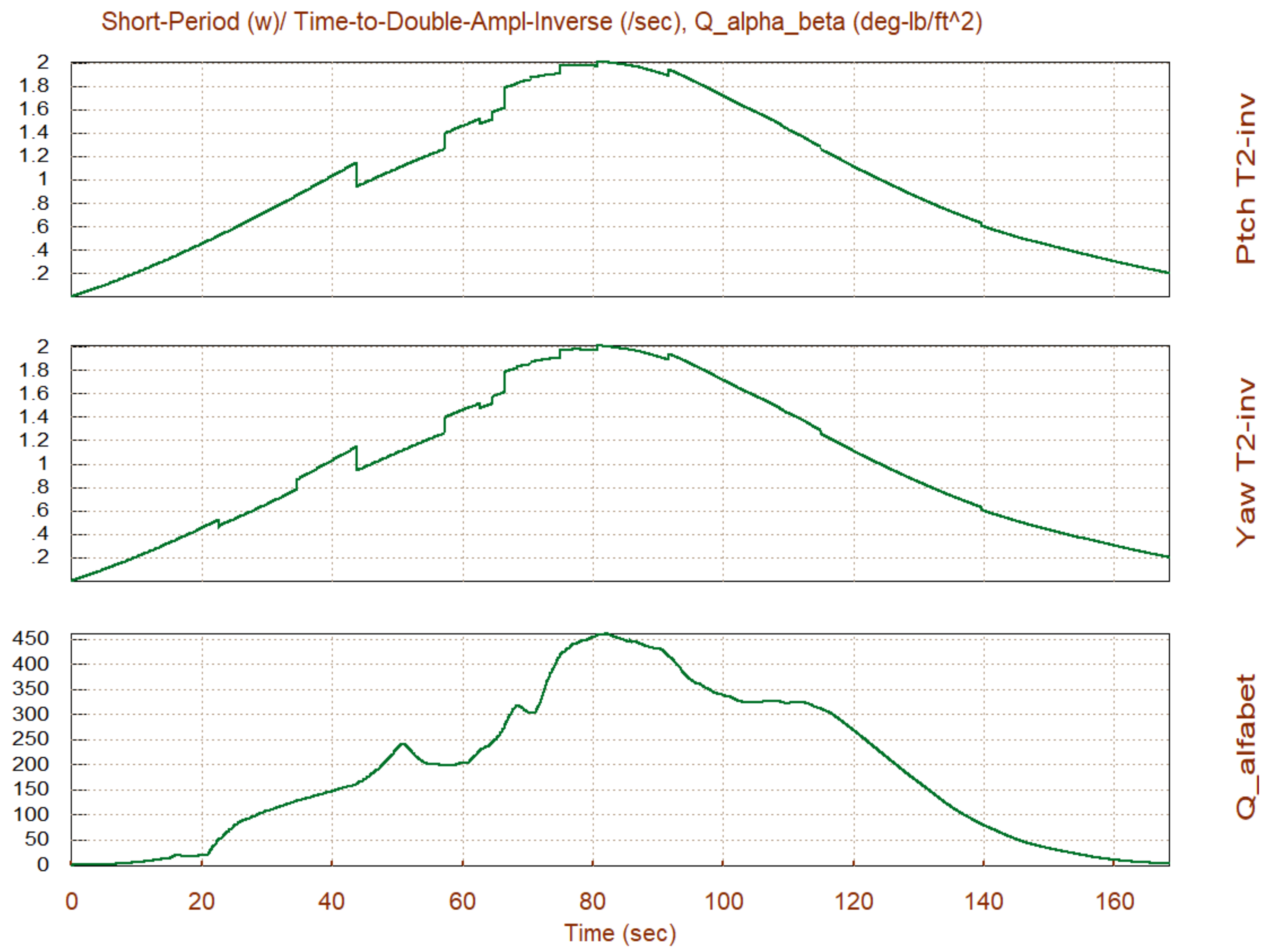
A (25 X 4) Mixing Logic Matrix is required



The control effectors must be capable of varying the vehicle angles of attack and sideslip (typically 3-5 deg) from their trim values.

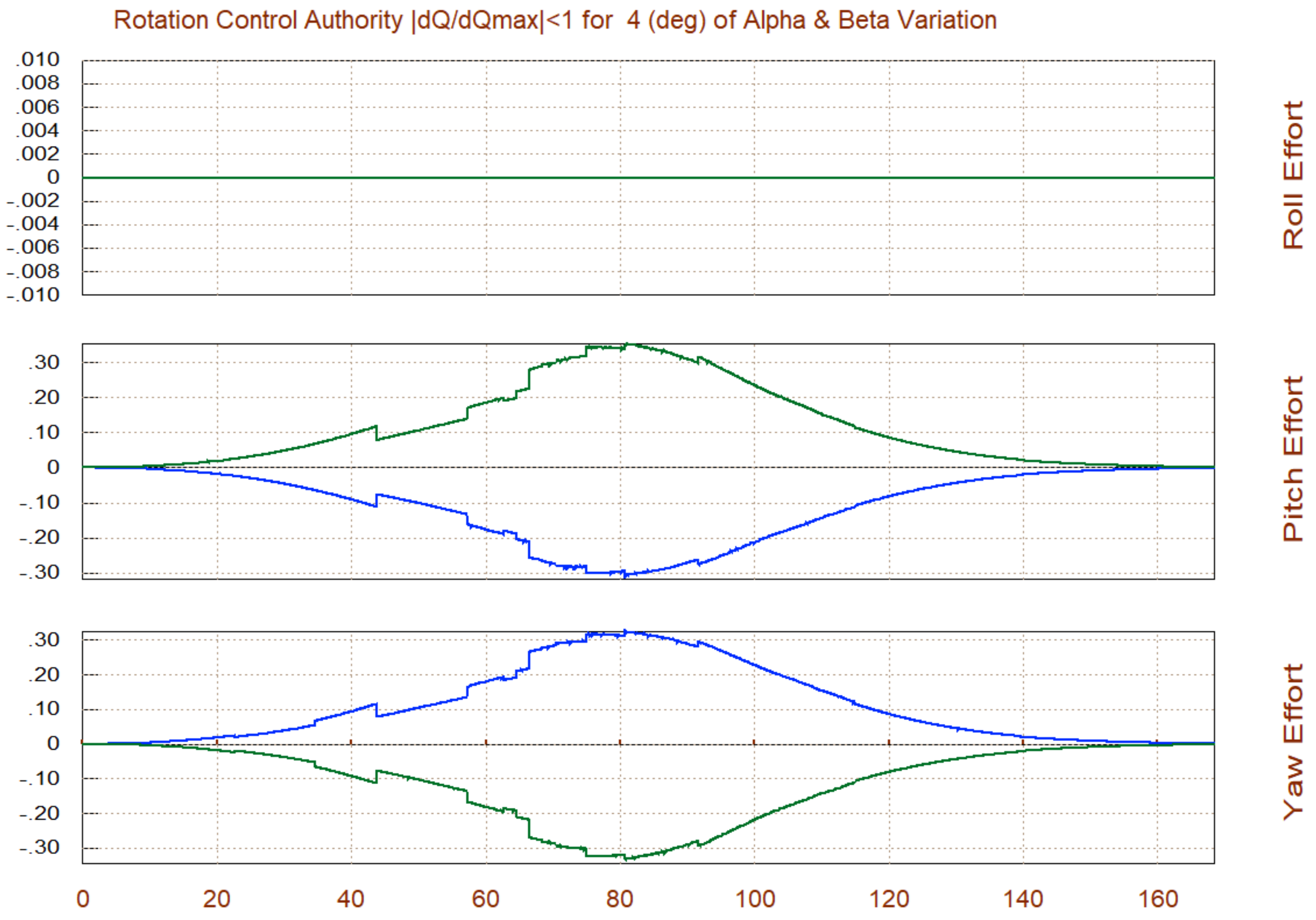
Enter the worst expected alpha and beta dispersions in (deg), and also delta-velocity in (ft/sec) from trim that must be controlled by the effectors, and click OK.

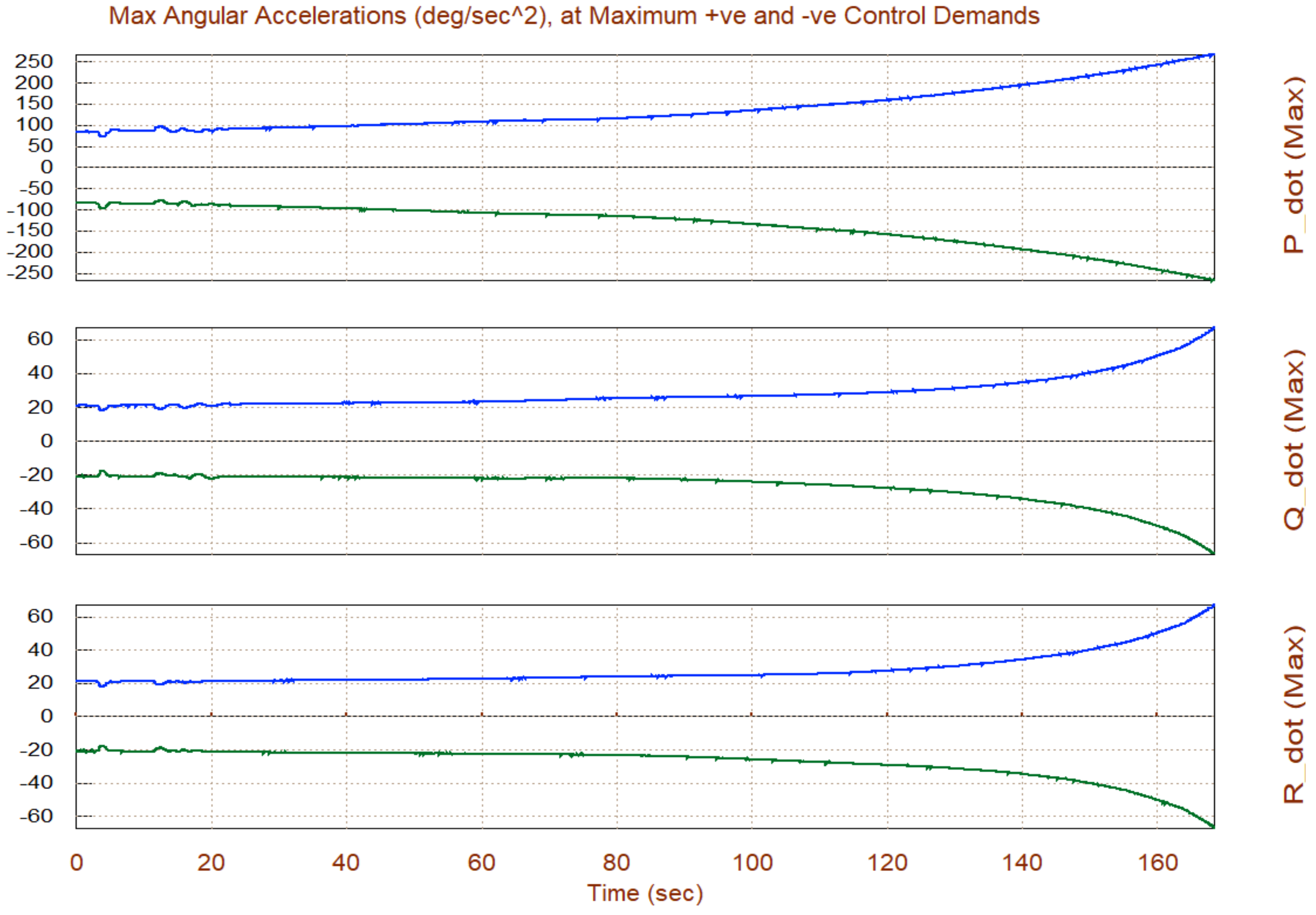
Maximum Alpha (deg) 4.0000
Maximum Beta (deg) 4.0000
Maximum Change in Velocity due to Wind in (feet/sec) 30.000



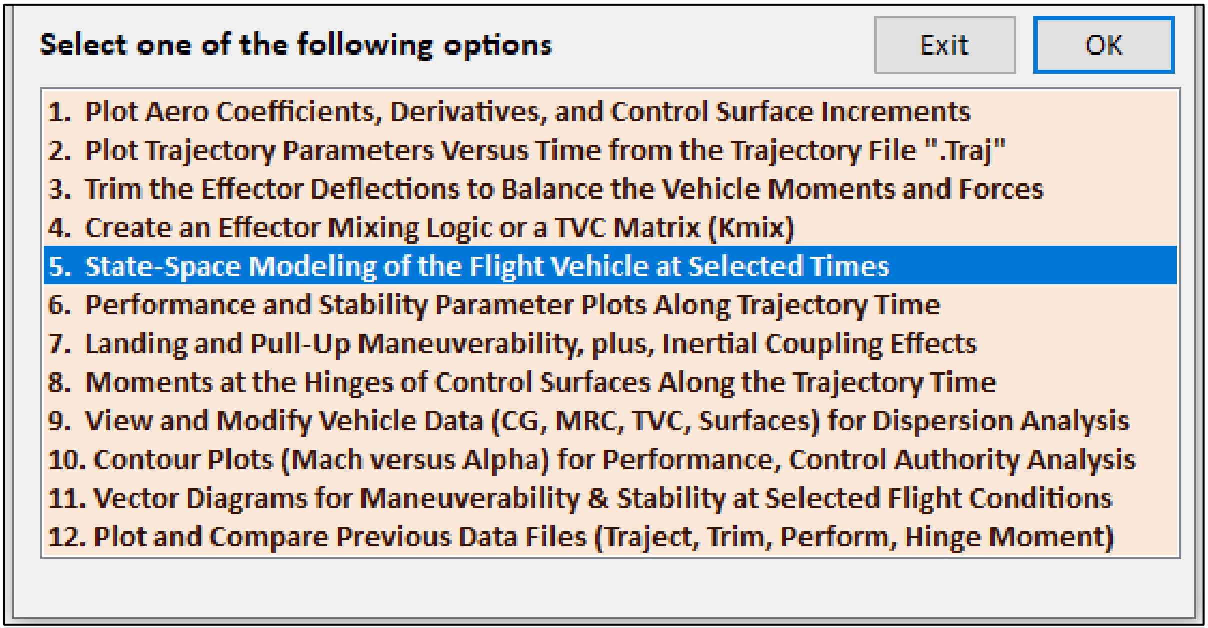
The above plot shows the static stability in pitch and yaw, which are the same because it is a cylindrical vehicle. It shows that the vehicle is statically unstable, as most launch vehicles are, and that it has a time-to-double amplitude inverse of 2 sec^{-1}, or a minimum T2= 0.5 sec, which is marginally acceptable for a launch vehicle, and it is occurring at Max-Q. It means that without controls, at Max-Q, it takes half a second for the angle of attack to grow from 1° to 2°. The third plot is used to evaluate the worst-case lateral loading due to angle of attack dispersions along the trajectory. It shows the Q-alpha and Q-beta combined, assuming that there are 4° α_{max} and β_{max} maximum dispersions along the entire trajectory. That is, in addition to the nominal α_0 and β_0 values. The worst loading is 450 (psf-deg) also occurring at max-Q.

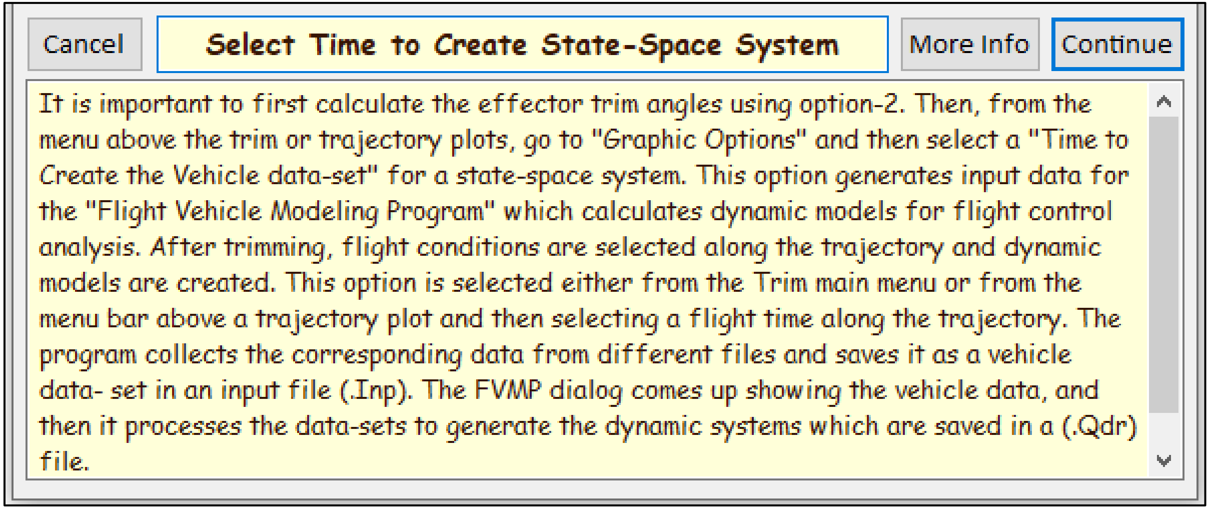
The next plot analyzes the authority of the control system to counteract the disturbance moment generated due to a ±4° dispersion in alpha or beta. It shows that only 32% of the max available control authority is needed at Max-Q to counteract the disturbance due to α_{max} = ±4°. The blue lines correspond to α_{max} and β_{max} having positive +4° values and the green lines correspond to α_{max} or β_{max} being negative, -4°. The control effort in the roll axis is zero because there is no aero disturbance to counteract in roll. The last plot shows the maximum angular accelerations produced when the controls are maximized in both positive and in negative directions. The max acceleration increases with time because the vehicle weight is reduced, and there is more acceleration in roll because the moment of inertia is smaller.


Rotation Control Authority |dQ/dQmax|<1 for 4 (deg) of Alpha & Beta Variation
Roll Effort
.010
.008
.006
.004
.002
0
-.002
-.004
-.006
-.008
-.010
Pitch Effort
.30
.20
.10
0
-.10
-.20
-.30
Yaw Effort
.30
.20
.10
0
-.10
-.20
-.30
0
20
40
60
80
100
120
140
160

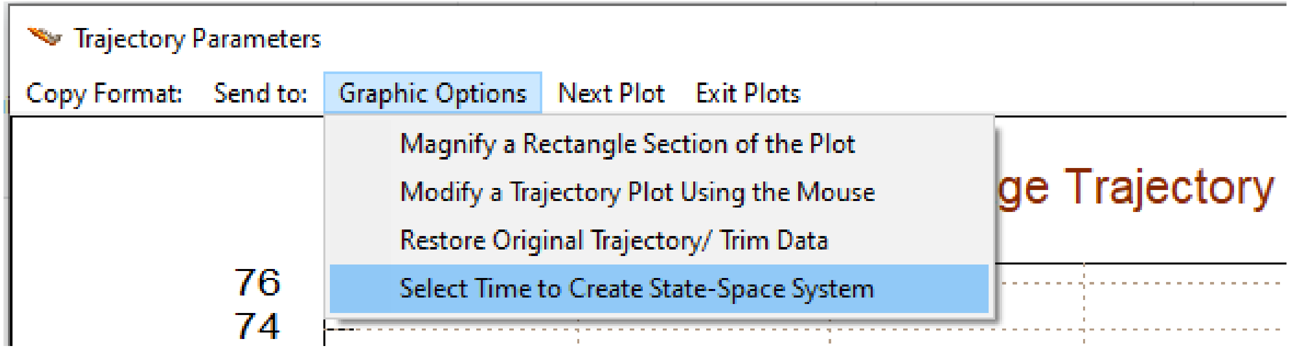

Max Angular Accelerations (deg/sec^2), at Maximum +ve and -ve Control Demands
P_dot (Max)
250
200
150
100
50
0
-50
-100
-150
-200
-250
Q_dot (Max)
60
40
20
0
-20
-40
-60
R_dot (Max)
60
40
20
0
-20
-40
-60
0
20
40
60
80
100
120
140
160
Time (sec)

One of the features of the Trim program is the ability to create linear systems along the trajectory provided. It is selected from option-5 which plots the trajectory data versus time. Then go to the menu bar which is above any of the trajectory plots, click on “*Graphic Options*” and select “*Select Time to Create State-Space System*”.

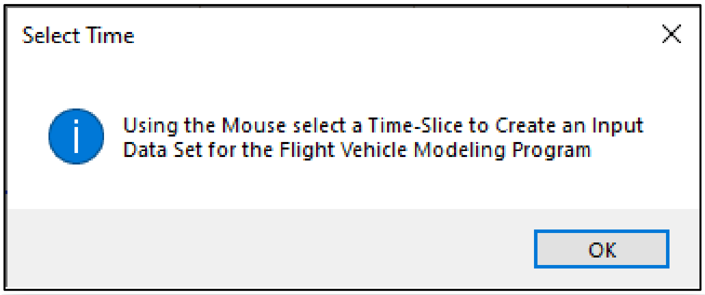




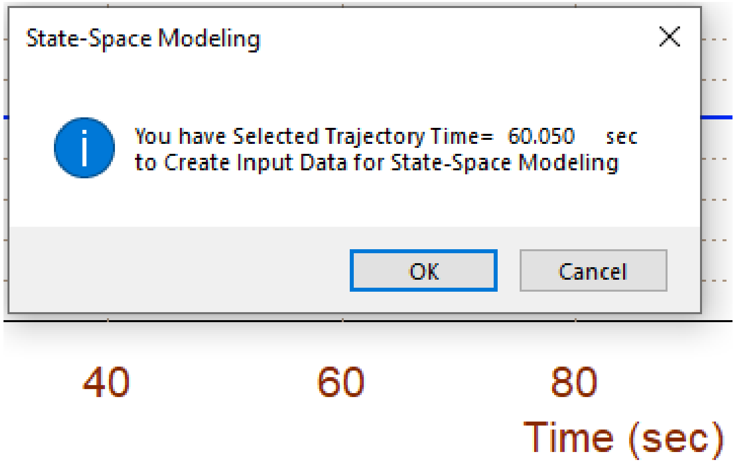
It is important to first calculate the effector trim angles using option-2. Then, from the menu above the trim or trajectory plots, go to "Graphic Options" and then select a "Time to Create the Vehicle data-set" for a state-space system. This option generates input data for the "Flight Vehicle Modeling Program" which calculates dynamic models for flight control analysis. After trimming, flight conditions are selected along the trajectory and dynamic models are created. This option is selected either from the Trim main menu or from the menu bar above a trajectory plot and then selecting a flight time along the trajectory. The program collects the corresponding data from different files and saves it as a vehicle data- set in an input file (.Inp). The FVMP dialog comes up showing the vehicle data, and then it processes the data-sets to generate the dynamic systems which are saved in a (.Qdr) file.



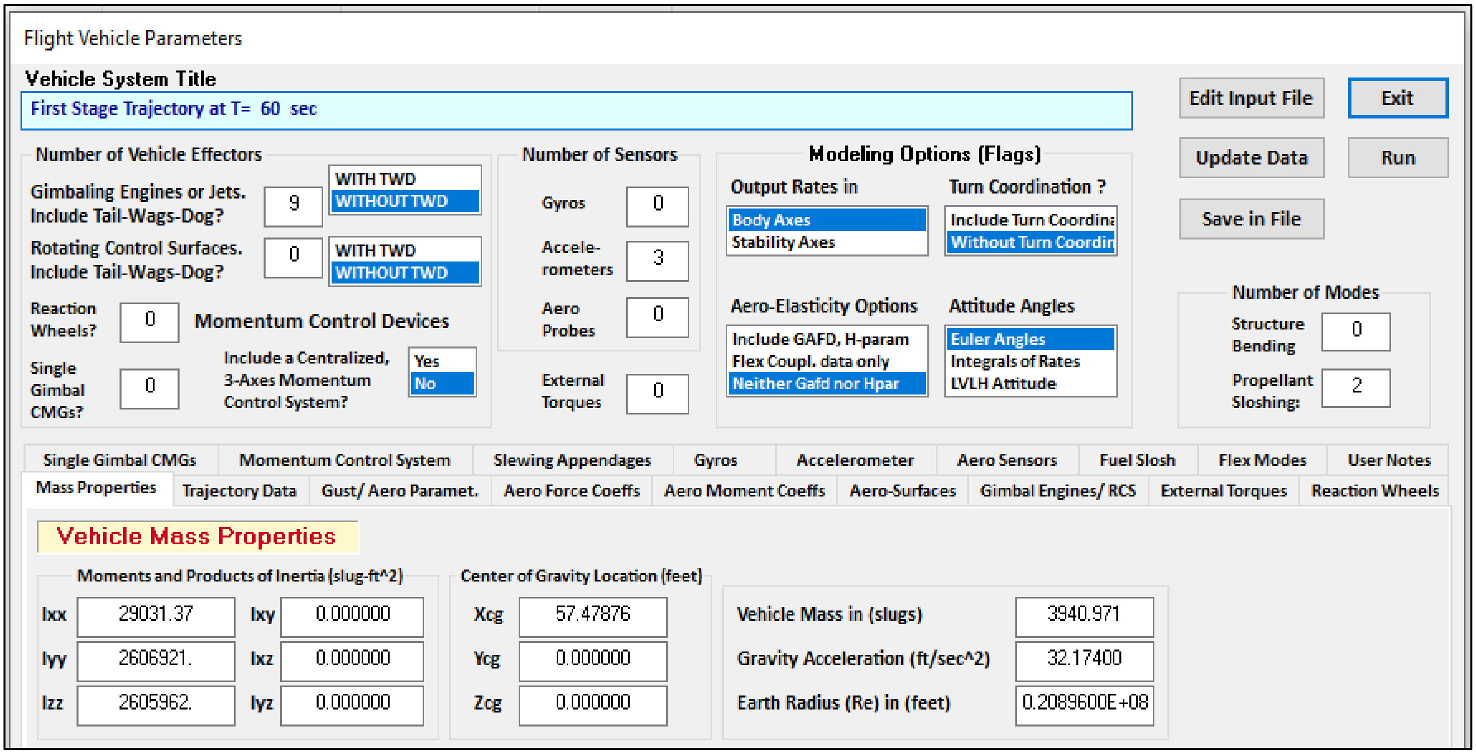
Then using the mouse, move the cursor along any of the plots, select a time point to create the linear model and click at the selected time point, let's say at 60 sec. The program will find the nearest trajectory time point and ask for a confirmation. If you accept the selected time the program will display the flight vehicle modeling program dialog where you can inspect the vehicle parameters, enter a title, and modify some of the parameters, as necessary, before saving it. The state-space model can be created by clicking on the "Run" button to process the vehicle parameters. But we are not do that right now. Instead, we will save the data in the input file (.Inp) that was selected in the beginning of the program, make the necessary additions, if any, and we will process the it later. We therefore click on "Save in File" for now, to save the flight vehicle dataset in the input file.


Select Time

Using the Mouse select a Time-Slice to Create an Input Data Set for the Flight Vehicle Modeling Program


State-Space Modeling

You have Selected Trajectory Time= 60.050 sec to Create Input Data for State-Space Modeling


Flight Vehicle Parameters

Vehicle System Title: First Stage Trajectory at T= 60 sec

Number of Vehicle Effectors
Gimbaling Engines or Jets. Include Tail-Wags-Dog? 9 (WITH TWD / WITHOUT TWD: WITHOUT TWD)
Rotating Control Surfaces. Include Tail-Wags-Dog? 0 (WITH TWD / WITHOUT TWD: WITHOUT TWD)
Reaction Wheels? 0
Single Gimbal CMGs? 0
Momentum Control Devices
Include a Centralized, 3-Axes Momentum Control System? (Yes / No: No)

Number of Sensors
Gyros 0
Accele-rometers 3
Aero Probes 0
External Torques 0

Modeling Options (Flags)
Output Rates in: Body Axes / Stability Axes (Body Axes)
Turn Coordination ?: Include Turn Coordina / Without Turn Coordin (Without Turn Coordin)
Aero-Elasticity Options: Include GAFD, H-param / Flex Coupl. data only / Neither Gafd nor Hpar (Neither Gafd nor Hpar)
Attitude Angles: Euler Angles / Integrals of Rates / LVLH Attitude (Euler Angles)

Number of Modes
Structure Bending 0
Propellant Sloshing: 2

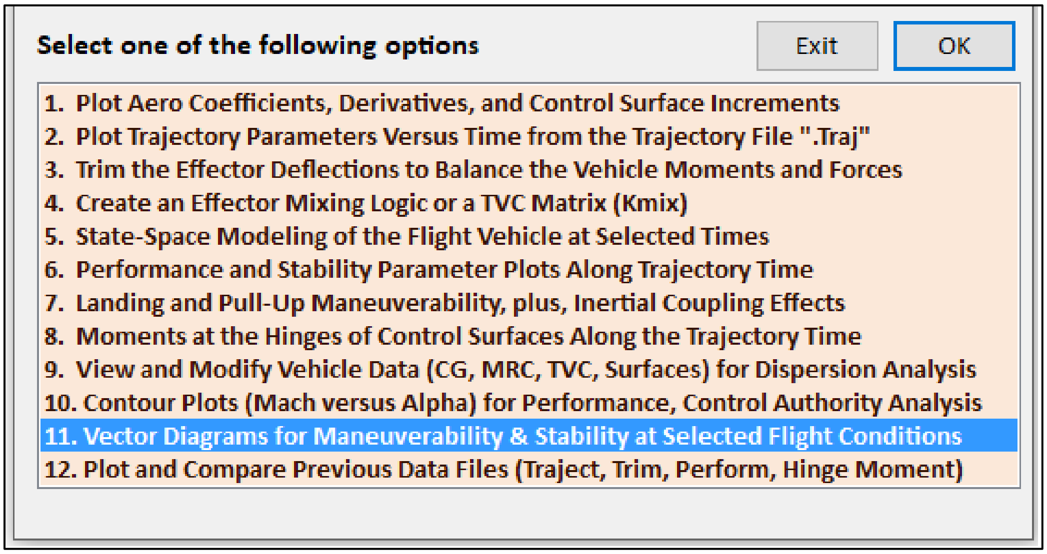
Vehicle Mass Properties

Moments and Products of Inertia (slug-ft^2)
Ixx 29031.37
Iyy 2606921.
Izz 2605962.
Ixy 0.000000
Ixz 0.000000
Iyz 0.000000

Center of Gravity Location (feet)
Xcg 57.47876
Ycg 0.000000
Zcg 0.000000

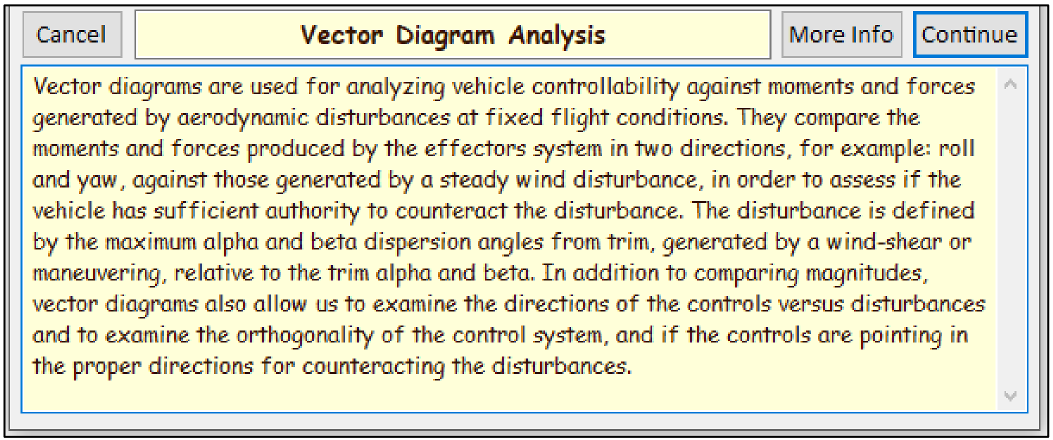
Vehicle Mass in (slugs) 3940.971
Gravity Acceleration (ft/sec^2) 32.17400
Earth Radius (Re) in (feet) 0.2089600E+08

Another useful option for launch vehicle design is the vector diagrams that demonstrate the effector TVC controllability using 2-direction vector diagrams. Select option-11, and in the dialog below enter the flight time to analyze the controllability. From the following menus the user selects the flight condition in terms of α & β, Mach number and vehicle weight. The default values correspond to the selected flight time.



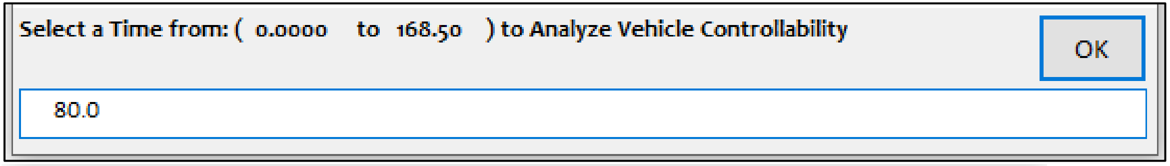
Select one of the following options

1. Plot Aero Coefficients, Derivatives, and Control Surface Increments
2. Plot Trajectory Parameters Versus Time from the Trajectory File ".Traj"
3. Trim the Effector Deflections to Balance the Vehicle Moments and Forces
4. Create an Effector Mixing Logic or a TVC Matrix (Kmix)
5. State-Space Modeling of the Flight Vehicle at Selected Times
6. Performance and Stability Parameter Plots Along Trajectory Time
7. Landing and Pull-Up Maneuverability, plus, Inertial Coupling Effects
8. Moments at the Hinges of Control Surfaces Along the Trajectory Time
9. View and Modify Vehicle Data (CG, MRC, TVC, Surfaces) for Dispersion Analysis
10. Contour Plots (Mach versus Alpha) for Performance, Control Authority Analysis
11. Vector Diagrams for Maneuverability & Stability at Selected Flight Conditions
12. Plot and Compare Previous Data Files (Traject, Trim, Perform, Hinge Moment)



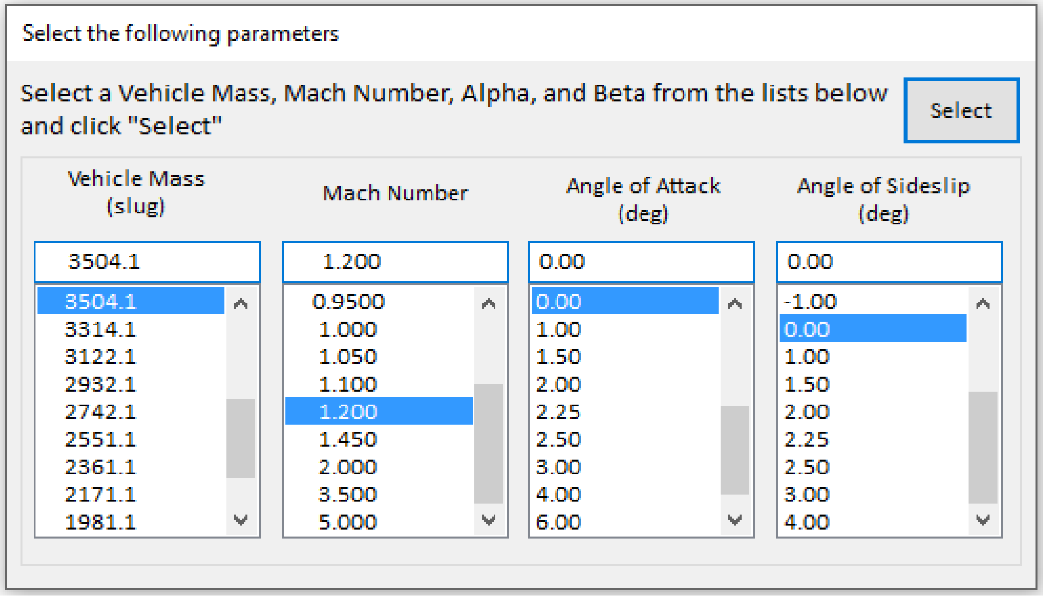
Vector Diagram Analysis

Vector diagrams are used for analyzing vehicle controllability against moments and forces generated by aerodynamic disturbances at fixed flight conditions. They compare the moments and forces produced by the effectors system in two directions, for example: roll and yaw, against those generated by a steady wind disturbance, in order to assess if the vehicle has sufficient authority to counteract the disturbance. The disturbance is defined by the maximum alpha and beta dispersion angles from trim, generated by a wind-shear or maneuvering, relative to the trim alpha and beta. In addition to comparing magnitudes, vector diagrams also allow us to examine the directions of the controls versus disturbances and to examine the orthogonality of the control system, and if the controls are pointing in the proper directions for counteracting the disturbances.



Select a Time from: (0.0000 to 168.50) to Analyze Vehicle Controllability

80.0

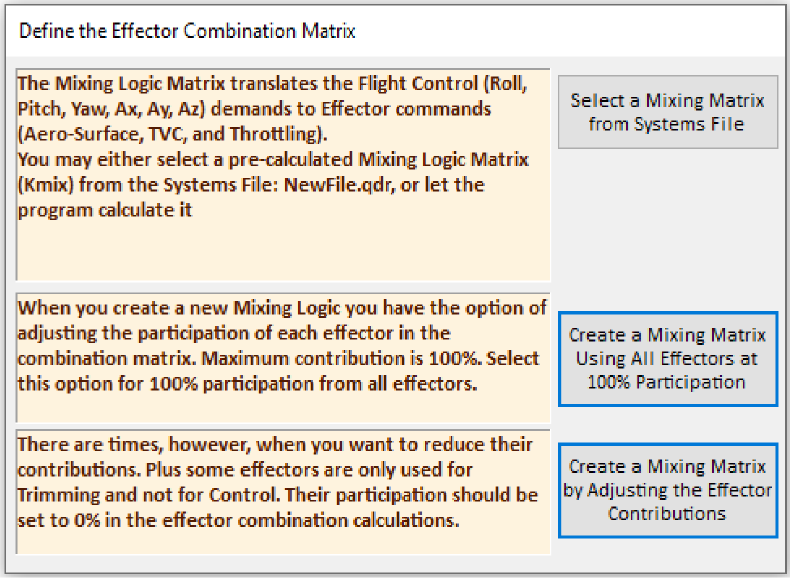


Select the following parameters

Select a Vehicle Mass, Mach Number, Alpha, and Beta from the lists below and click "Select"

Vehicle Mass (slug)	Mach Number	Angle of Attack (deg)	Angle of Sideslip (deg)
3504.1	1.200	0.00	0.00

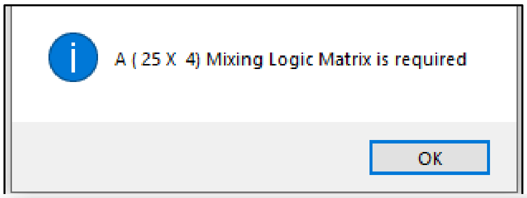
Next, the user is requested to select or to allow the program to create an effector combination matrix and to define the external disturbances. We select a maximum 4° alpha/ beta dispersions and 30 (ft/sec) wind-gust variations as worst possible disturbances along the trajectory.

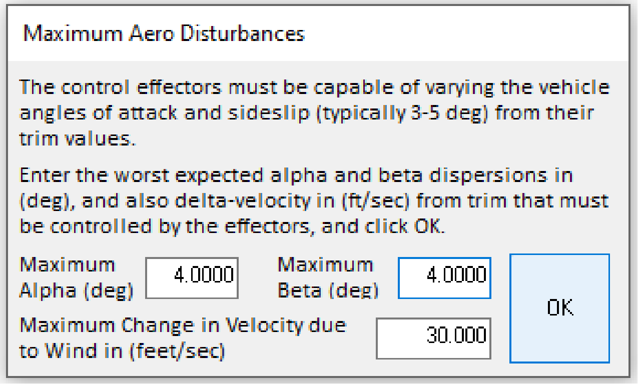

Define the Effector Combination Matrix

The Mixing Logic Matrix translates the Flight Control (Roll, Pitch, Yaw, Ax, Ay, Az) demands to Effector commands (Aero-Surface, TVC, and Throttling).
You may either select a pre-calculated Mixing Logic Matrix (Kmix) from the Systems File: NewFile.qdr, or let the program calculate it

When you create a new Mixing Logic you have the option of adjusting the participation of each effector in the combination matrix. Maximum contribution is 100%. Select this option for 100% participation from all effectors.

There are times, however, when you want to reduce their contributions. Plus some effectors are only used for Trimming and not for Control. Their participation should be set to 0% in the effector combination calculations.


A (25 X 4) Mixing Logic Matrix is required


Maximum Aero Disturbances

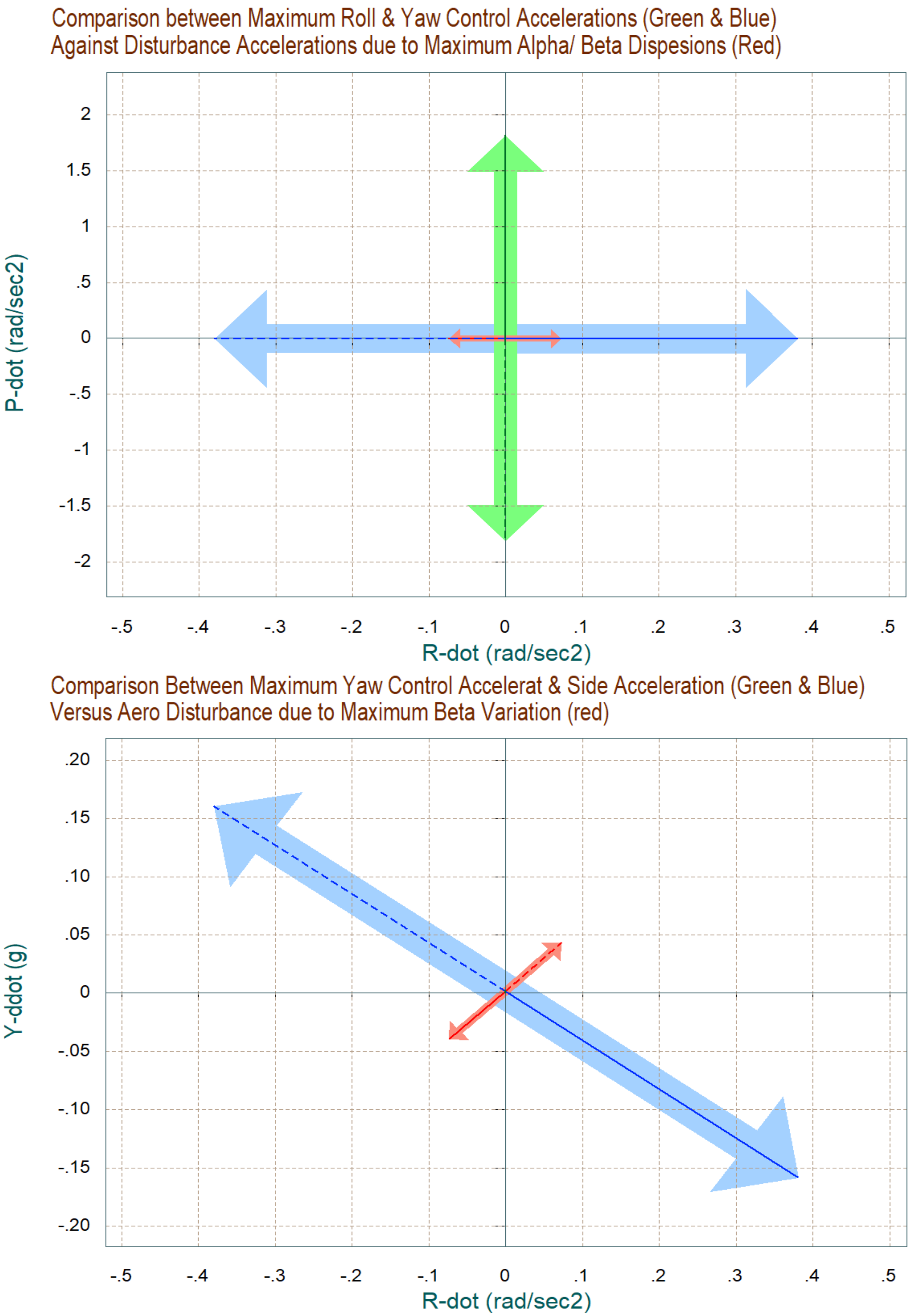
The control effectors must be capable of varying the vehicle angles of attack and sideslip (typically 3-5 deg) from their trim values.

Enter the worst expected alpha and beta dispersions in (deg), and also delta-velocity in (ft/sec) from trim that must be controlled by the effectors, and click OK.

Maximum Alpha (deg) 4.0000
Maximum Beta (deg) 4.0000
Maximum Change in Velocity due to Wind in (feet/sec) 30.000

The green and blue vectors in Figure 1.1a show the maximum roll and yaw accelerations that can be obtained by maximizing the roll and yaw TVC controls. They are orthogonal and pointing along the expected directions, without any cross-coupling between the axes. There is more acceleration available in roll which is consistent with the previous results. Figure 1.1b shows also the side acceleration $\ddot{y}$. Positive yaw control produces negative side acceleration. The red vectors show the acceleration effects due to β_{max} = ±4° disturbance which produces negative yaw and $\ddot{y}$ accelerations but it does not couple into roll. The disturbance vector is significantly smaller than the yaw control vector, which means that the vehicle has sufficient control authority to counteract the β_{max} dispersion.

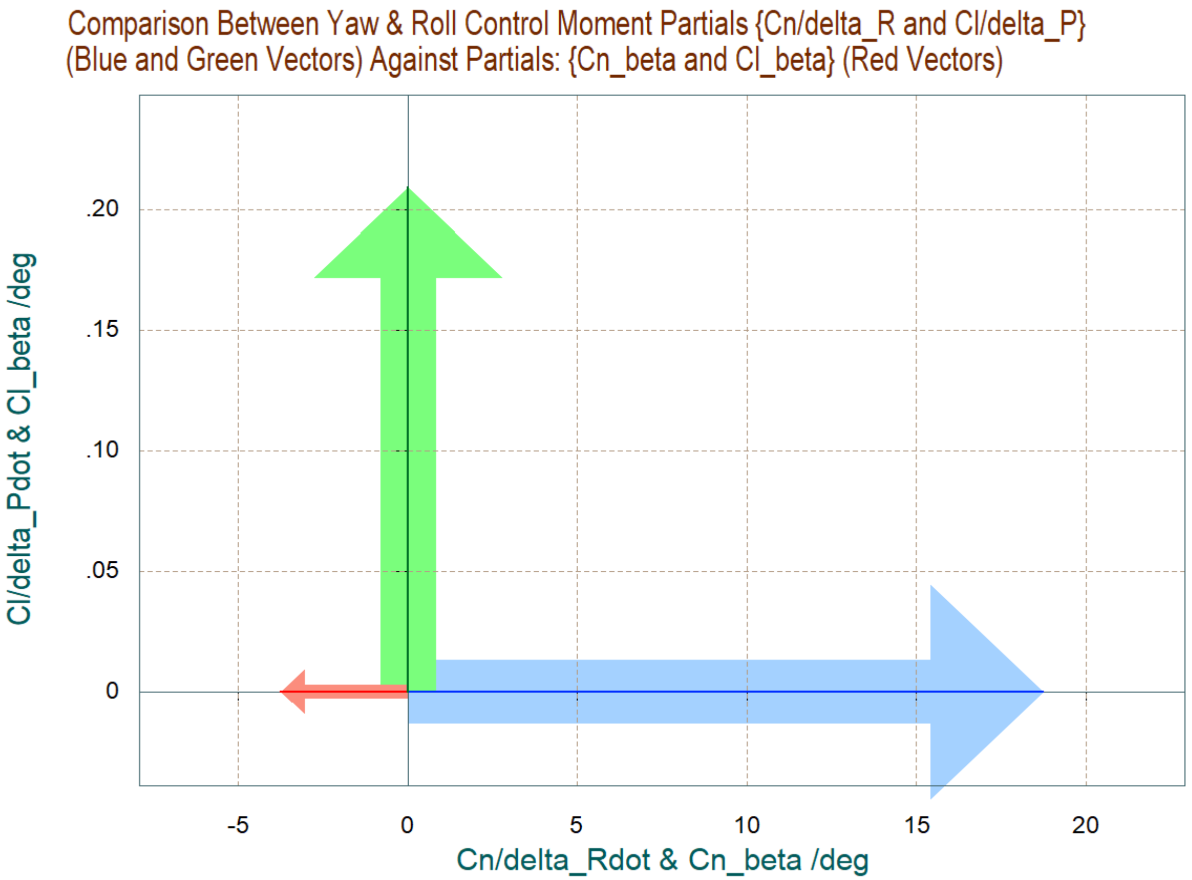
The green and blue vectors in Figure 1.2a show the roll and yaw control moment partials. That is, the moment per roll control deflection and the moment per yaw control deflection. The two vectors are orthogonal and pointing in the corresponding directions, no cross-coupling between roll and yaw. There is more yaw moment produced per yaw control deflection because the yaw inertia is a lot bigger and the wind disturbance affects only yaw. Figure 1.2b includes the side-force partial CY per yaw control deflection, which is negative. The red vectors in Figure 1.2 (a & b) are the moment and CY force partials per sideslip angle β. It is in the negative yaw direction, that is, positive beta produces negative yaw moment. It also has negative Cyβ and Cnβ which means that the vehicle is unstable in yaw. The red vector however is much smaller in magnitude than the blue control partial which means good yaw controllability.



Comparison between Maximum Roll & Yaw Control Accelerations (Green & Blue) Against Disturbance Accelerations due to Maximum Alpha/ Beta Dispesions (Red)

Comparison Between Maximum Yaw Control Accelerat & Side Acceleration (Green & Blue) Versus Aero Disturbance due to Maximum Beta Variation (red)

Figure 1.1 Roll and Yaw Acceleration Control Authority


Comparison Between Yaw & Roll Control Moment Partials {Cn/delta_R and Cl/delta_P} (Blue and Green Vectors) Against Partials: {Cn_beta and Cl_beta} (Red Vectors)

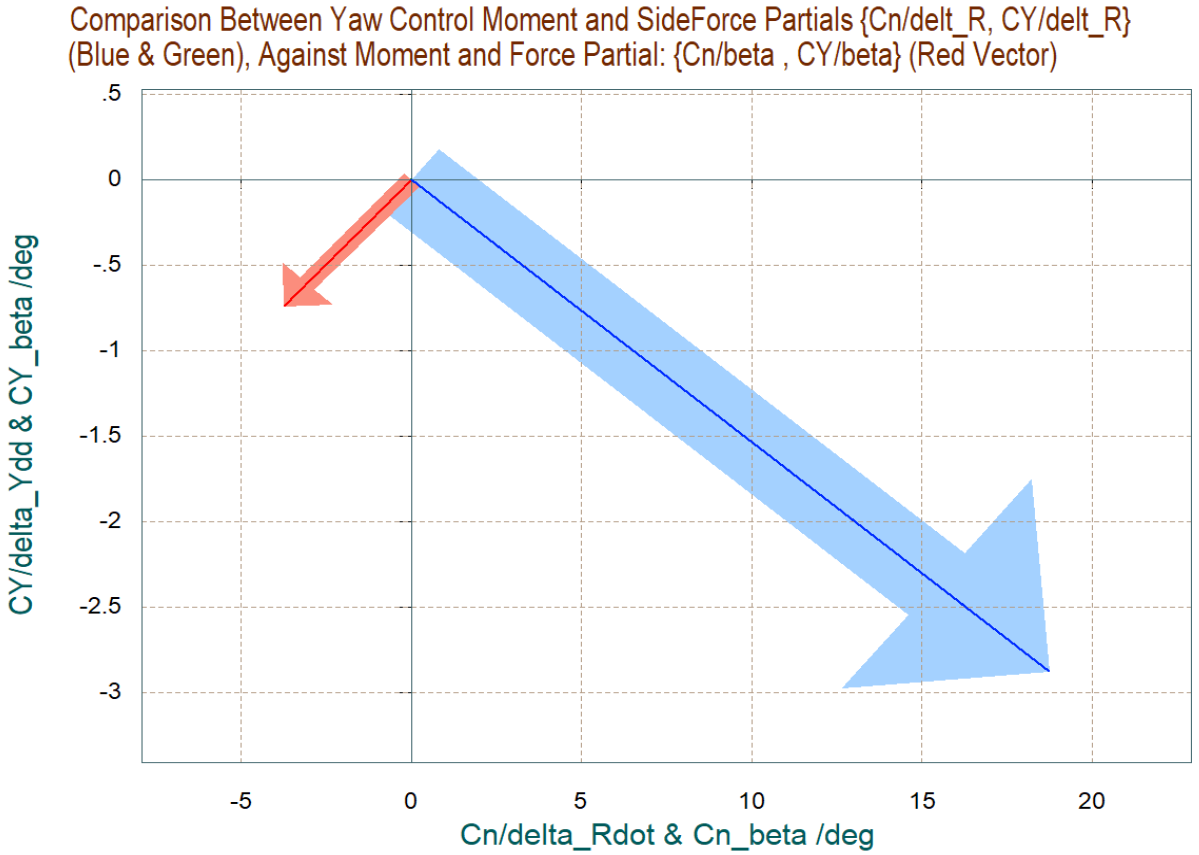

Comparison Between Yaw Control Moment and SideForce Partials {Cn/delt_R, CY/delt_R} (Blue & Green), Against Moment and Force Partial: {Cn/beta , CY/beta} (Red Vector)

Figure 1.2 Roll and Yaw Control Moment Partials Versus Beta Partials

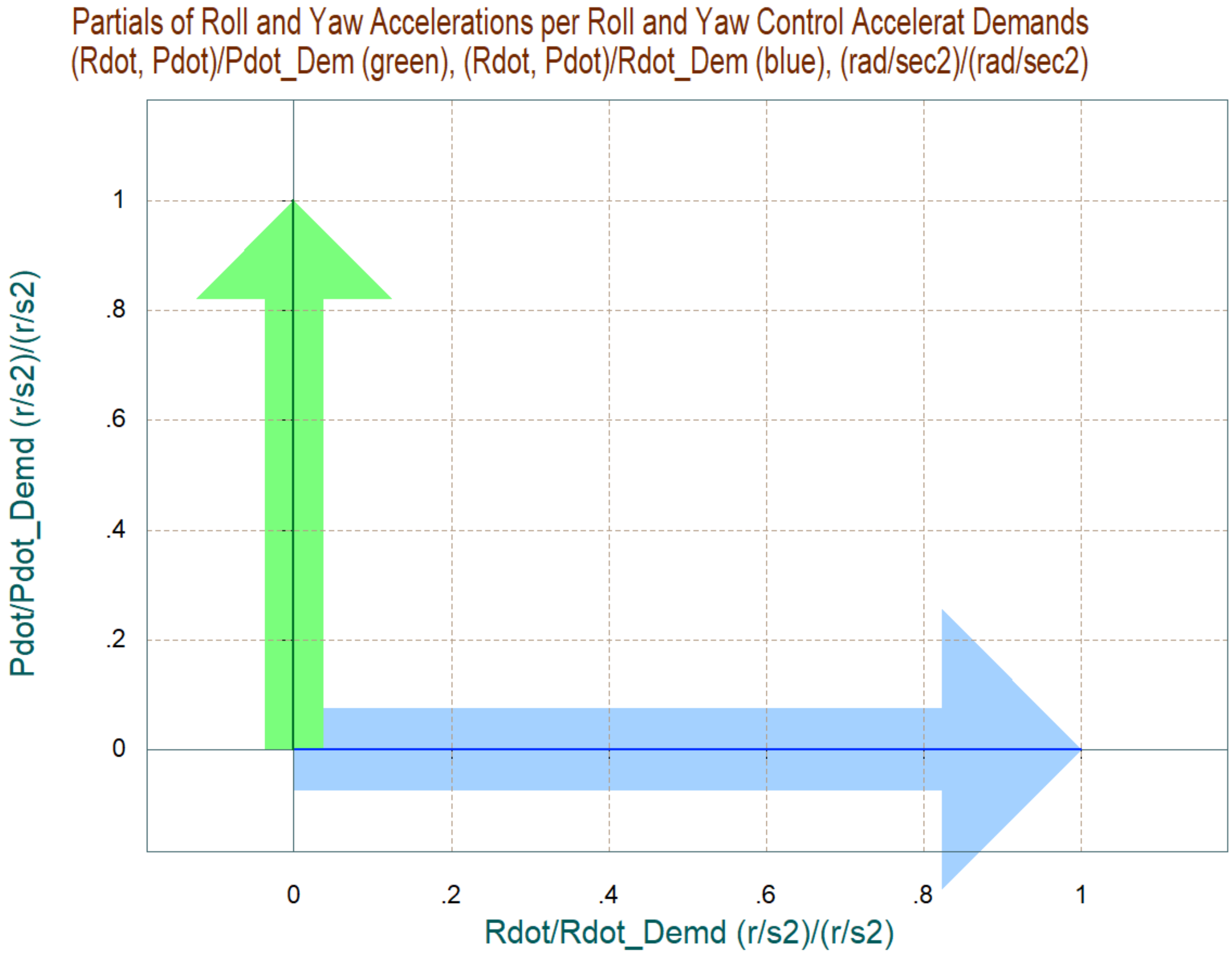


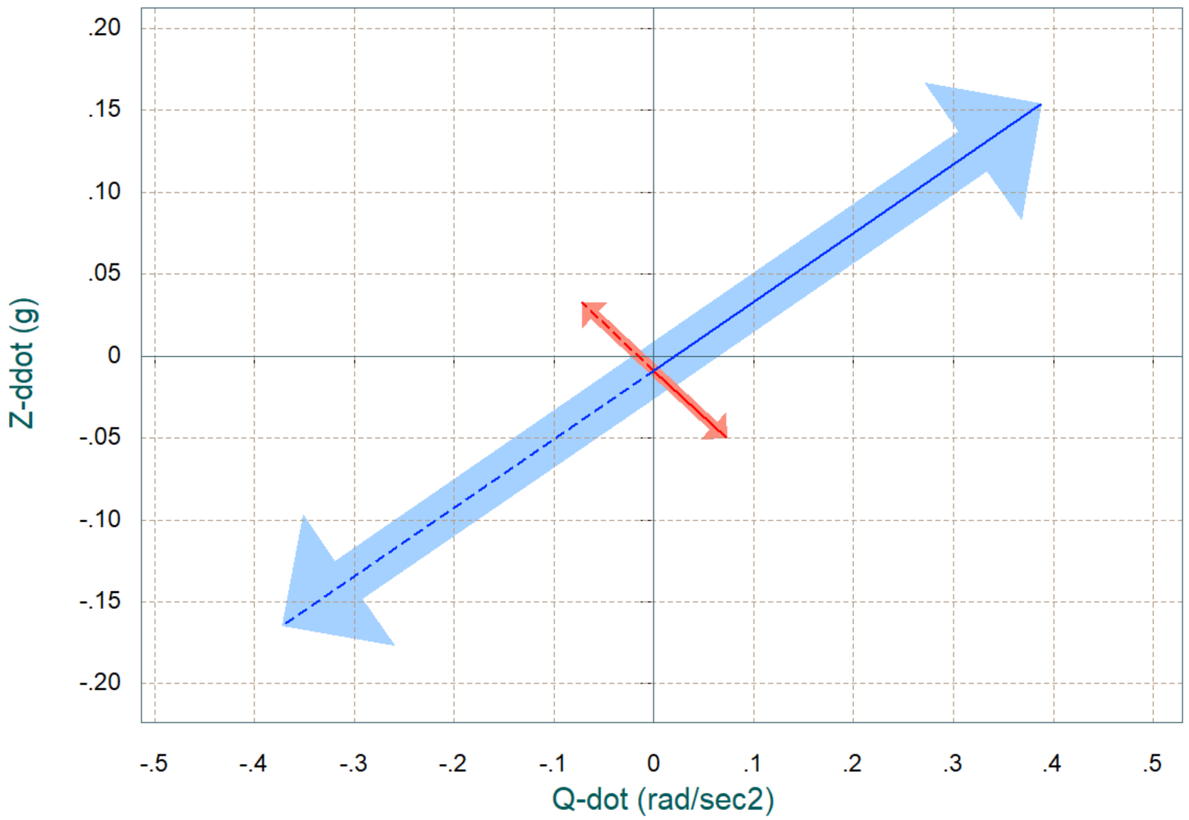
Figure 1.3 Roll and Yaw Accelerations per Acceleration Commands

Figure 1.3 shows the accelerations in roll and yaw per acceleration demands that can be achieved in roll and yaw by using the previously computed TVC effector combination matrix. It looks like an ideal situation where the accelerations achieved are identical to the commanded accelerations, unit vectors and orthogonal to each other.

Figure 1.4a shows the maximum pitch and normal accelerations produced by maximizing the pitch TVC control in both positive and negative directions. We only have one control in this diagram which is pitch TVC. There is no second vector because we do not control the z-acceleration. It shows that the pitch TVC also produces z-acceleration. The red vector shows the acceleration effect from α_{max} = 4° disturbance which produces positive pitch and negative $\ddot{z}$ accelerations. The disturbance vector is a lot smaller than the pitch control vector, which means that the vehicle has sufficient control authority.

Figure 1.4b shows the moment partials. The blue vector is the pitch moment and normal force CZ per pitch control deflection, and the red vector represents the pitch moment and normal force partial per angle of attack α. $Cz\alpha$ is negative and $Cm\alpha$ is positive which indicates that the vehicle is statically unstable also in pitch. The disturbance partial however red vector is a lot smaller than the pitch control partial blue vector which means that the vehicle can be controlled.

Comparison between Maximum Pitch and Normal-Z Control Accelerations (Blue & Green) Against Aero Disturbance due to Maximum Alpha Variation (red)



Comparison Between Control Moment and Normal Force Partials {Cm/delt_Q & CZ/delt_Z} (Blue & Green) Against Moment/ Force Partials {Cm/alpha & CZ/alpha} (Red Vectors)

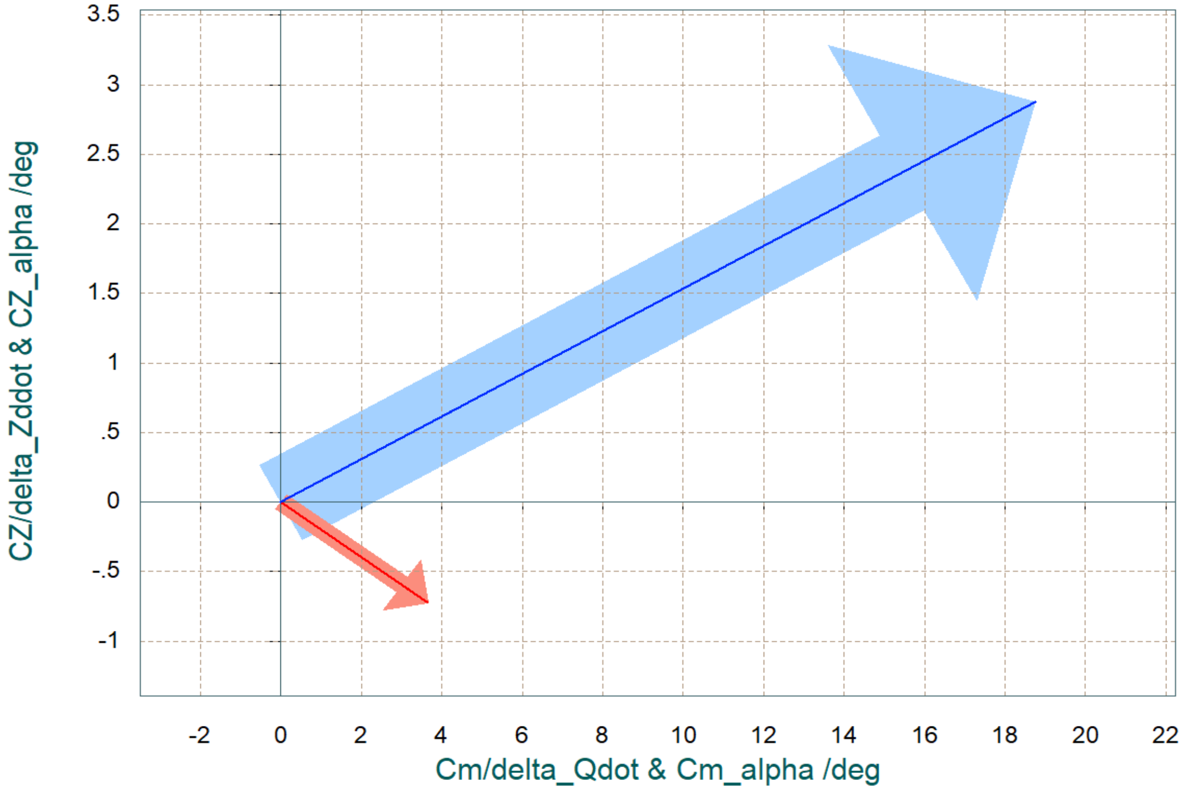
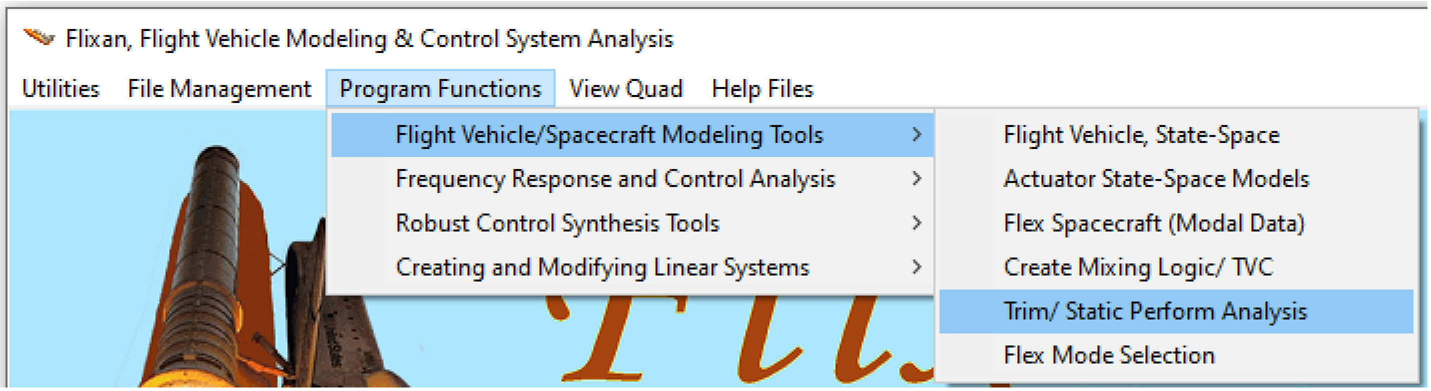


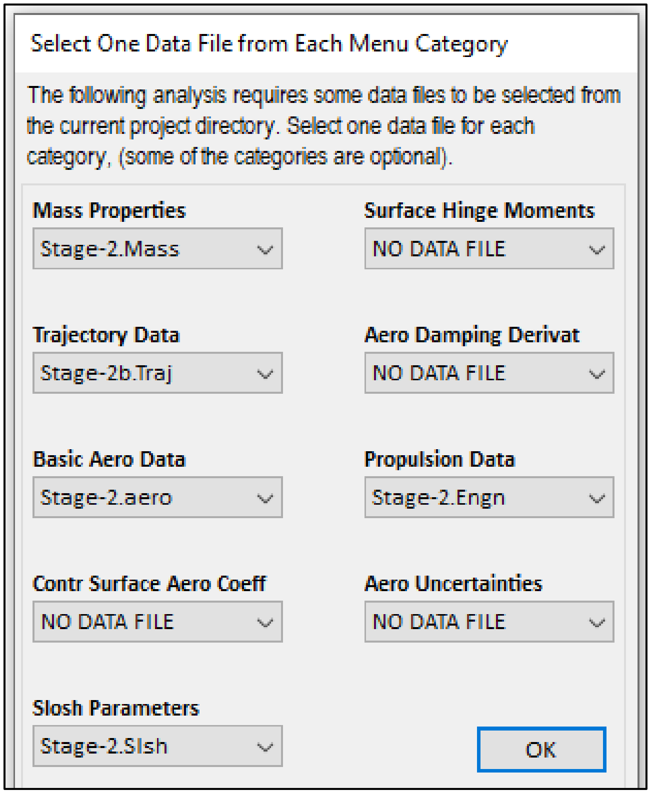
Figure 1.4 Pitch Controllability Versus Pitch Disturbance

1.2 Second Stage Static Analysis

During second stage the vehicle has only one TVC engine and roll control is accomplished by using 4 bidirectional RCS jet pairs located around the main engine. The aerodynamic effects are small during 2nd stage and static controllability will be analyzed relative to external disturbance torques in roll, pitch and yaw which are produced mostly due to thrust vector misalignment with the vehicle x-axis.

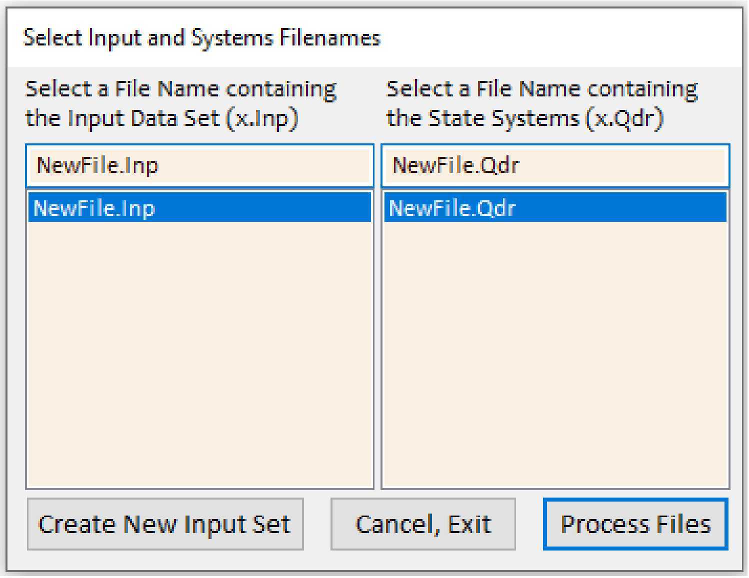
The files for 2nd stage static analysis are in folder "*Examples\23-Classic Launch Vehicle Design & Simulation\1-Static Analysis\Stage-2*". It includes the aerodynamic data "Stage-2.aero" which are at much lower dynamic pressure now, the engine data "Stage-2.engn" which includes the TVC engine and the 4 RCS jets, the 2nd stage vehicle mass properties "Stage-2.mass", the 2nd stage trajectory file "Stage-2b.Traj" which includes also the 3 disturbance moments on the right side, and the slosh parameters file "Stage-2.slsh". We start the Flixan program as before and select the Trim option "*Trim/ Static Performance Analysis*", as shown. From the files selection menu, select the second stage files, click OK, and from the next menu select the default input and system filenames, and click on "*Process Files*".




Select One Data File from Each Menu Category

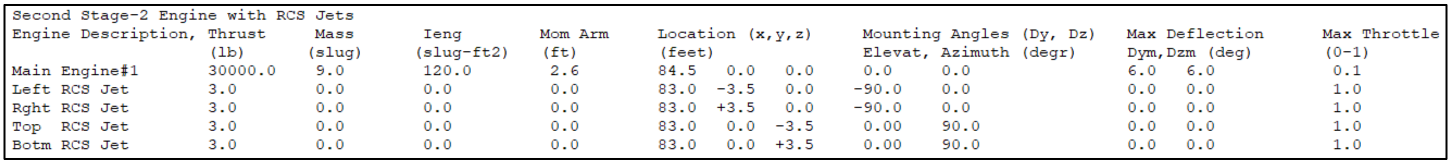
The following analysis requires some data files to be selected from the current project directory. Select one data file for each category, (some of the categories are optional).

Mass Properties: Stage-2.Mass
Surface Hinge Moments: NO DATA FILE
Trajectory Data: Stage-2b.Traj
Aero Damping Derivat: NO DATA FILE
Basic Aero Data: Stage-2.aero
Propulsion Data: Stage-2.Engn
Contr Surface Aero Coeff: NO DATA FILE
Aero Uncertainties: NO DATA FILE
Slosh Parameters: Stage-2.Slsh


Select Input and Systems Filenames

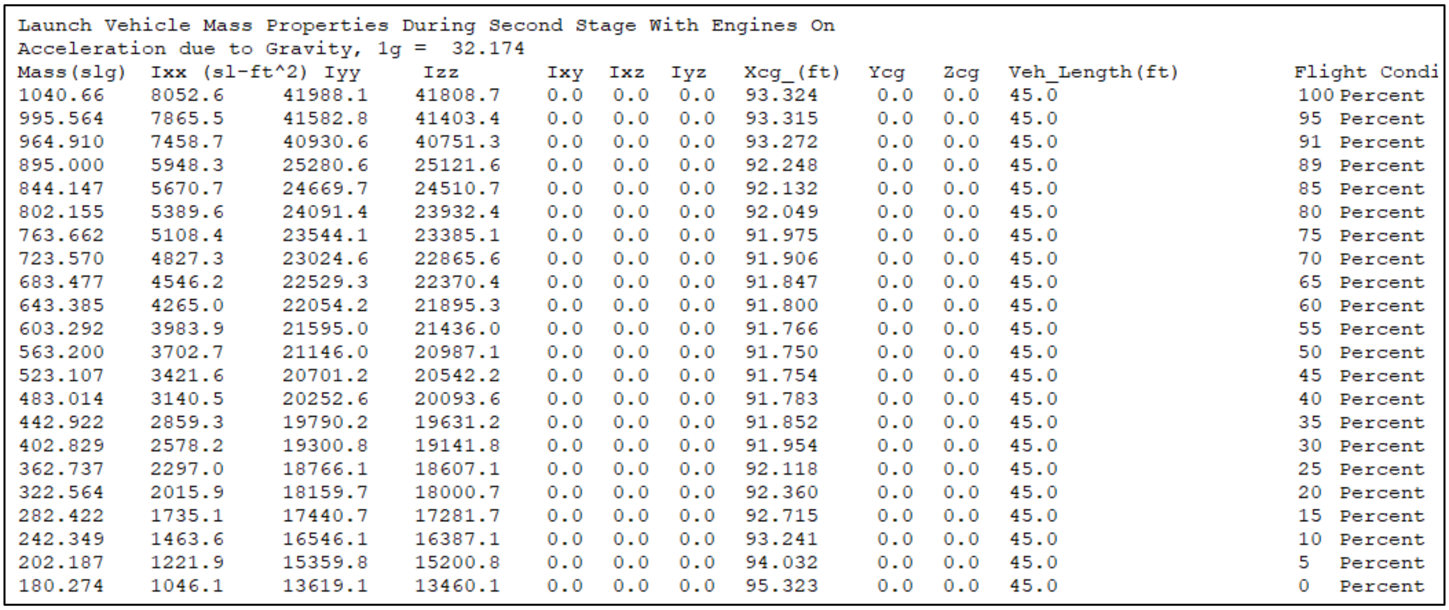
Select a File Name containing the Input Data Set (x.Inp): NewFile.Inp
Select a File Name containing the State Systems (x.Qdr): NewFile.Qdr

The engines file "*Stage-2.Engn*" includes the main TVC engine and the 4 RCS thrusters. Note, that the thrusters are bidirectional and they can produce ±3 (lbf) of thrust, which in essence each thruster consists of two back-to-back jets. The thrusters are not gimbaling but only throttling between zero and ±3 (lbf). The main engine is 30,000 (lbf) and it is gimbaling in pitch and yaw. However, like in first stage, we do allow a little room for throttling in order to match the acceleration when trimming along the trajectory, but we are ignore the throttling in the Flixan derived vehicle models. The mass properties and slosh data files are shown below. Their format is similar to the first stage.



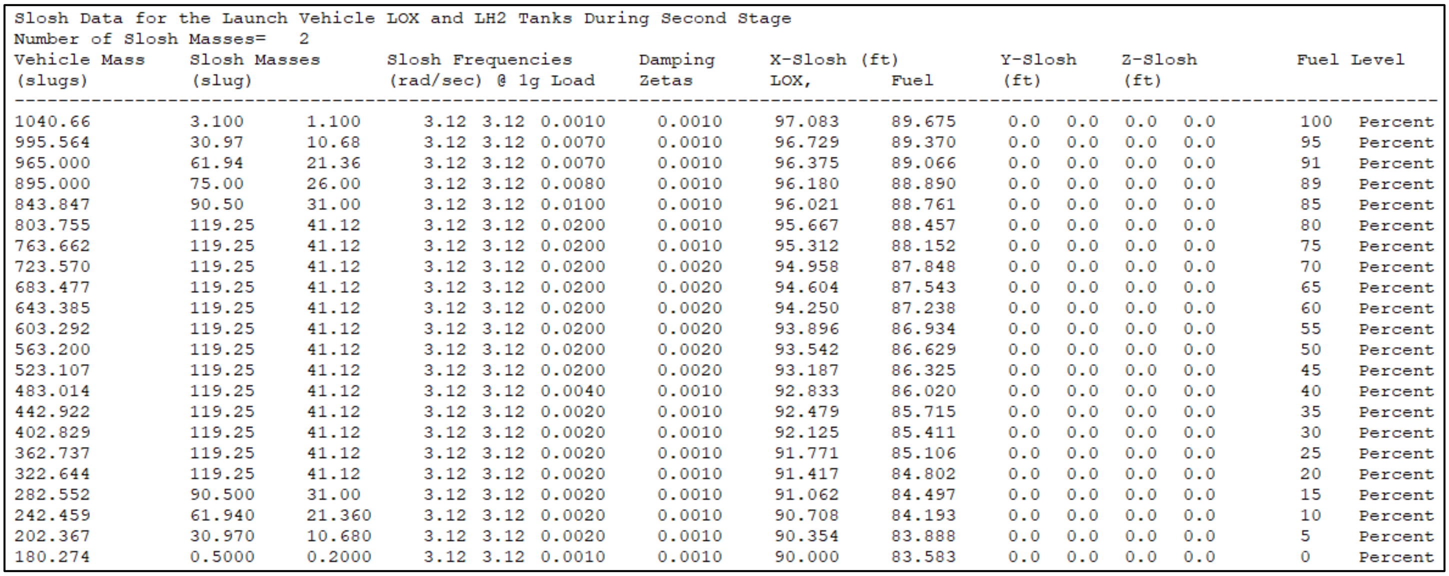
```
Second Stage-2 Engine with RCS Jets
Engine Description, Thrust     Mass      Ieng         Mom Arm     Location (x,y,z)        Mounting Angles (Dy, Dz)   Max Deflection      Max Throttle
                    (lb)       (slug)    (slug-ft2)   (ft)        (feet)                  Elevat, Azimuth (degr)     Dym,Dzm (deg)       (0-1)
Main Engine#1       30000.0    9.0       120.0         2.6        84.5   0.0   0.0         0.0     0.0               6.0   6.0           0.1
Left RCS Jet        3.0        0.0       0.0           0.0        83.0  -3.5   0.0        -90.0    0.0               0.0   0.0           1.0
Rght RCS Jet        3.0        0.0       0.0           0.0        83.0  +3.5   0.0        -90.0    0.0               0.0   0.0           1.0
Top  RCS Jet        3.0        0.0       0.0           0.0        83.0   0.0  -3.5         0.00    90.0              0.0   0.0           1.0
Botm RCS Jet        3.0        0.0       0.0           0.0        83.0   0.0  +3.5         0.00    90.0              0.0   0.0           1.0
```

Second Stage Engines File



```
Launch Vehicle Mass Properties During Second Stage With Engines On
Acceleration due to Gravity, 1g =  32.174
Mass(slg)  Ixx (sl-ft^2) Iyy      Izz       Ixy  Ixz  Iyz  Xcg_(ft)  Ycg  Zcg  Veh_Length(ft)       Flight Condi
1040.66    8052.6     41988.1   41808.7   0.0  0.0  0.0  93.324    0.0  0.0  45.0                 100 Percent
995.564    7865.5     41582.8   41403.4   0.0  0.0  0.0  93.315    0.0  0.0  45.0                 95  Percent
964.910    7458.7     40930.6   40751.3   0.0  0.0  0.0  93.272    0.0  0.0  45.0                 91  Percent
895.000    5948.3     25280.6   25121.6   0.0  0.0  0.0  92.248    0.0  0.0  45.0                 89  Percent
844.147    5670.7     24669.7   24510.7   0.0  0.0  0.0  92.132    0.0  0.0  45.0                 85  Percent
802.155    5389.6     24091.4   23932.4   0.0  0.0  0.0  92.049    0.0  0.0  45.0                 80  Percent
763.662    5108.4     23544.1   23385.1   0.0  0.0  0.0  91.975    0.0  0.0  45.0                 75  Percent
723.570    4827.3     23024.6   22865.6   0.0  0.0  0.0  91.906    0.0  0.0  45.0                 70  Percent
683.477    4546.2     22529.3   22370.4   0.0  0.0  0.0  91.847    0.0  0.0  45.0                 65  Percent
643.385    4265.0     22054.2   21895.3   0.0  0.0  0.0  91.800    0.0  0.0  45.0                 60  Percent
603.292    3983.9     21595.0   21436.0   0.0  0.0  0.0  91.766    0.0  0.0  45.0                 55  Percent
563.200    3702.7     21146.0   20987.1   0.0  0.0  0.0  91.750    0.0  0.0  45.0                 50  Percent
523.107    3421.6     20701.2   20542.2   0.0  0.0  0.0  91.754    0.0  0.0  45.0                 45  Percent
483.014    3140.5     20252.6   20093.6   0.0  0.0  0.0  91.783    0.0  0.0  45.0                 40  Percent
442.922    2859.3     19790.2   19631.2   0.0  0.0  0.0  91.852    0.0  0.0  45.0                 35  Percent
402.829    2578.2     19300.8   19141.8   0.0  0.0  0.0  91.954    0.0  0.0  45.0                 30  Percent
362.737    2297.0     18766.1   18607.1   0.0  0.0  0.0  92.118    0.0  0.0  45.0                 25  Percent
322.564    2015.9     18159.7   18000.7   0.0  0.0  0.0  92.360    0.0  0.0  45.0                 20  Percent
282.422    1735.1     17440.7   17281.7   0.0  0.0  0.0  92.715    0.0  0.0  45.0                 15  Percent
242.349    1463.6     16546.1   16387.1   0.0  0.0  0.0  93.241    0.0  0.0  45.0                 10  Percent
202.187    1221.9     15359.8   15200.8   0.0  0.0  0.0  94.032    0.0  0.0  45.0                 5   Percent
180.274    1046.1     13619.1   13460.1   0.0  0.0  0.0  95.323    0.0  0.0  45.0                 0   Percent
```

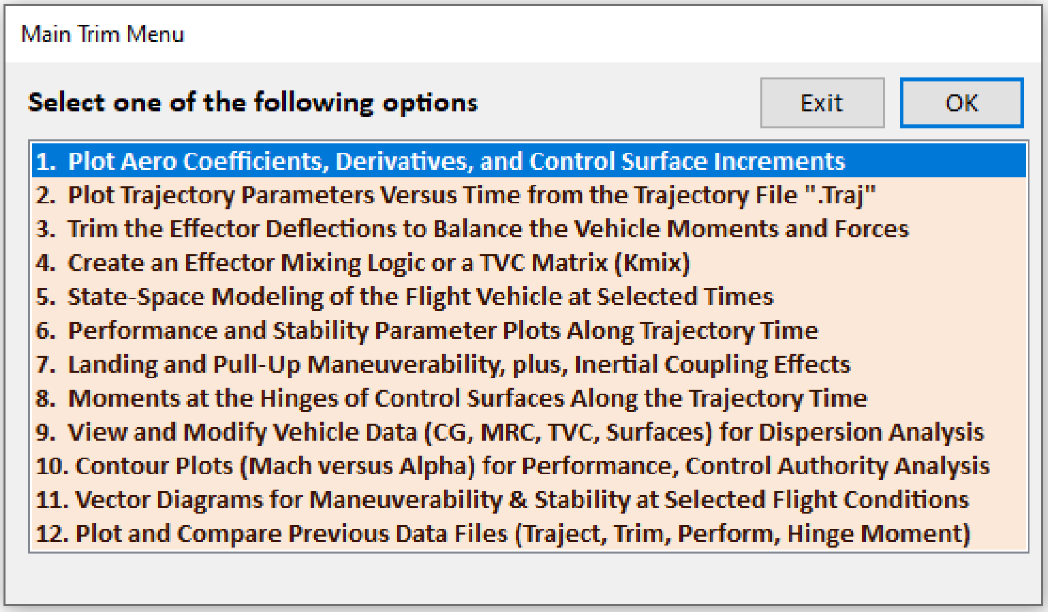
Second Stage Mass Properties



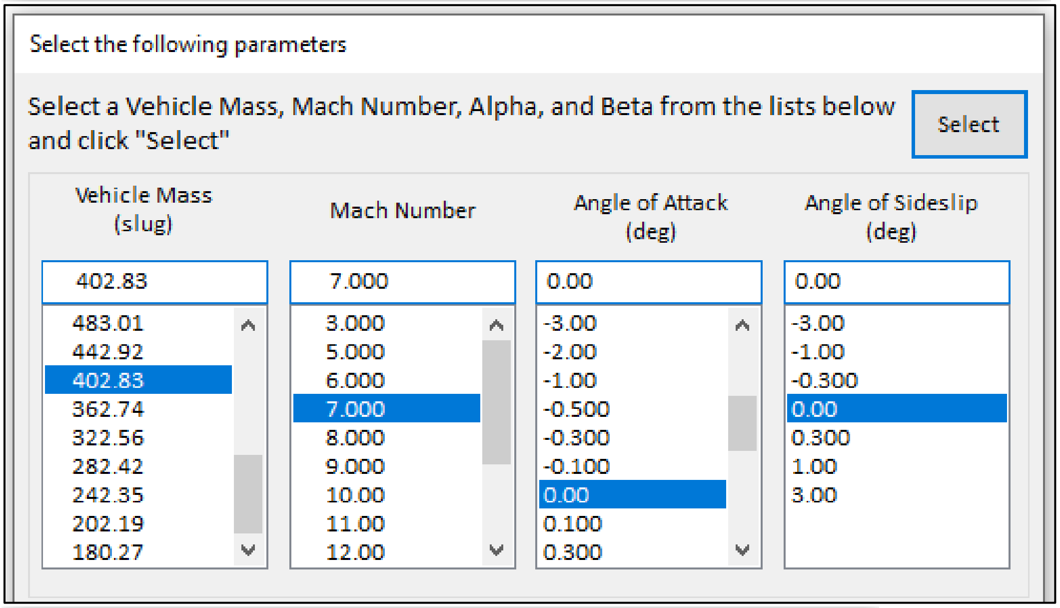
```
Slosh Data for the Launch Vehicle LOX and LH2 Tanks During Second Stage
Number of Slosh Masses=   2
Vehicle Mass     Slosh Masses         Slosh Frequencies       Damping      X-Slosh (ft)          Y-Slosh       Z-Slosh         Fuel Level
(slugs)          (slug)               (rad/sec) @ 1g Load     Zetas        LOX,        Fuel      (ft)          (ft)
-----------------------------------------------------------------------------------------------------------------------------------------
1040.66          3.100      1.100        3.12  3.12  0.0010      0.0010      97.083      89.675     0.0   0.0    0.0   0.0         100   Percent
995.564          30.97      10.68        3.12  3.12  0.0070      0.0010      96.729      89.370     0.0   0.0    0.0   0.0         95    Percent
965.000          61.94      21.36        3.12  3.12  0.0070      0.0010      96.375      89.066     0.0   0.0    0.0   0.0         91    Percent
895.000          75.00      26.00        3.12  3.12  0.0080      0.0010      96.180      88.890     0.0   0.0    0.0   0.0         89    Percent
843.847          90.50      31.00        3.12  3.12  0.0100      0.0010      96.021      88.761     0.0   0.0    0.0   0.0         85    Percent
803.755          119.25     41.12        3.12  3.12  0.0200      0.0010      95.667      88.457     0.0   0.0    0.0   0.0         80    Percent
763.662          119.25     41.12        3.12  3.12  0.0200      0.0010      95.312      88.152     0.0   0.0    0.0   0.0         75    Percent
723.570          119.25     41.12        3.12  3.12  0.0200      0.0020      94.958      87.848     0.0   0.0    0.0   0.0         70    Percent
683.477          119.25     41.12        3.12  3.12  0.0200      0.0020      94.604      87.543     0.0   0.0    0.0   0.0         65    Percent
643.385          119.25     41.12        3.12  3.12  0.0200      0.0020      94.250      87.238     0.0   0.0    0.0   0.0         60    Percent
603.292          119.25     41.12        3.12  3.12  0.0200      0.0020      93.896      86.934     0.0   0.0    0.0   0.0         55    Percent
563.200          119.25     41.12        3.12  3.12  0.0200      0.0020      93.542      86.629     0.0   0.0    0.0   0.0         50    Percent
523.107          119.25     41.12        3.12  3.12  0.0200      0.0020      93.187      86.325     0.0   0.0    0.0   0.0         45    Percent
483.014          119.25     41.12        3.12  3.12  0.0040      0.0010      92.833      86.020     0.0   0.0    0.0   0.0         40    Percent
442.922          119.25     41.12        3.12  3.12  0.0020      0.0010      92.479      85.715     0.0   0.0    0.0   0.0         35    Percent
402.829          119.25     41.12        3.12  3.12  0.0020      0.0010      92.125      85.411     0.0   0.0    0.0   0.0         30    Percent
362.737          119.25     41.12        3.12  3.12  0.0020      0.0010      91.771      85.106     0.0   0.0    0.0   0.0         25    Percent
322.644          119.25     41.12        3.12  3.12  0.0020      0.0010      91.417      84.802     0.0   0.0    0.0   0.0         20    Percent
282.552          90.500     31.00        3.12  3.12  0.0020      0.0010      91.062      84.497     0.0   0.0    0.0   0.0         15    Percent
242.459          61.940     21.360       3.12  3.12  0.0020      0.0010      90.708      84.193     0.0   0.0    0.0   0.0         10    Percent
202.367          30.970     10.680       3.12  3.12  0.0020      0.0010      90.354      83.888     0.0   0.0    0.0   0.0         5     Percent
180.274          0.5000     0.2000       3.12  3.12  0.0010      0.0010      90.000      83.583     0.0   0.0    0.0   0.0         0     Percent
```

Second Stage Slosh Parameters

From the Trim program main menu we select the first option to take a look at the aero data and from the next menu we choose a flight condition in terms of Mach number, angles of attack, sideslip, and vehicle mass.

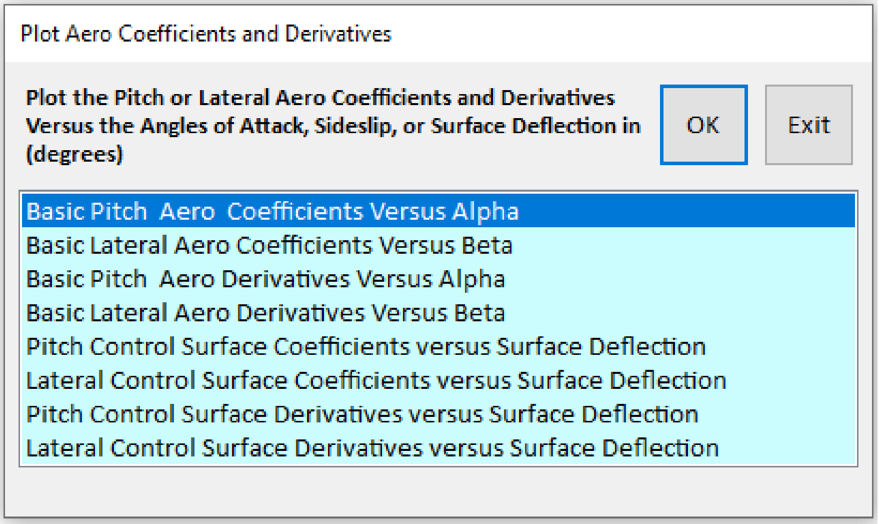

Main Trim Menu

Select one of the following options


Select the following parameters

Select a Vehicle Mass, Mach Number, Alpha, and Beta from the lists below and click "Select"

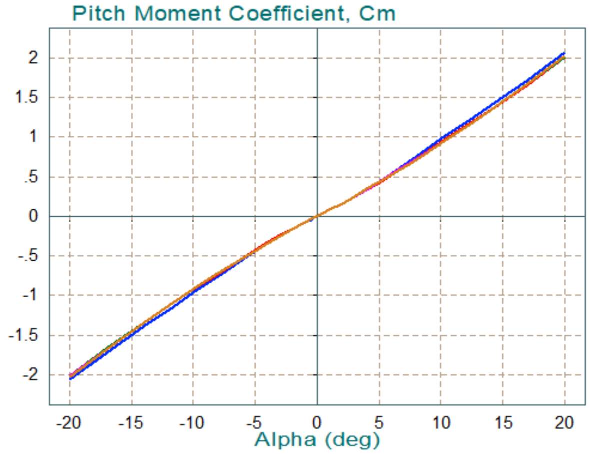
Vehicle Mass (slug)	Mach Number	Angle of Attack (deg)	Angle of Sideslip (deg)
402.83	7.000	0.00	0.00

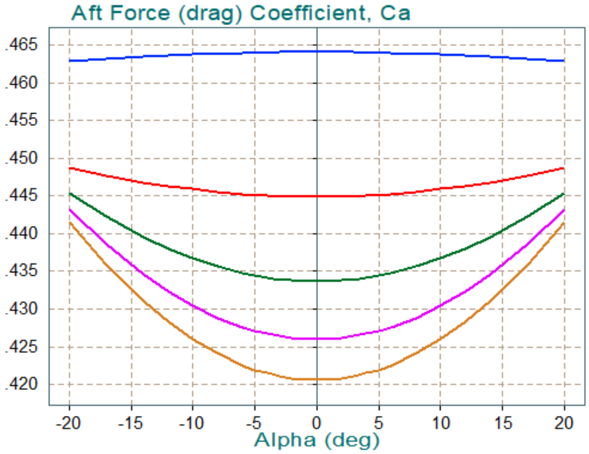

Plot Aero Coefficients and Derivatives

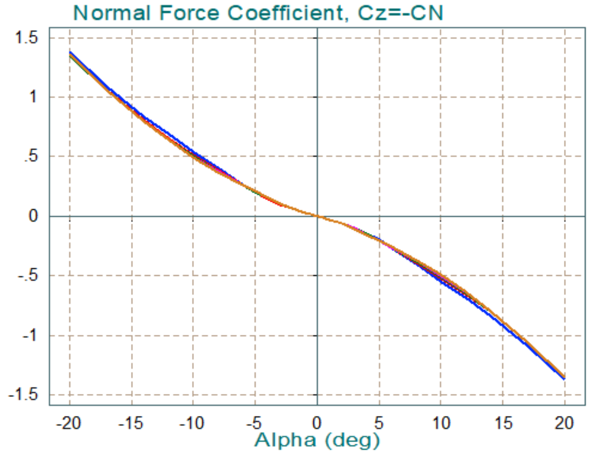
Plot the Pitch or Lateral Aero Coefficients and Derivatives Versus the Angles of Attack, Sideslip, or Surface Deflection in (degrees)

Launch Vehicle Stage-2 Trajectory
Pitch Aero Coefficients Versus Alpha (deg)
Reference Area (ft^2)= 38.50
Chord and Span (feet)= 7.200 ; 7.200
Moments Transferred to Vehicle CG
Alpha(o)=0.00 (deg)
Beta(o) =0.00 (deg)

Mach: 5.000
Mach: 6.000
Mach: 7.000
Mach: 8.000
Mach: 9.000

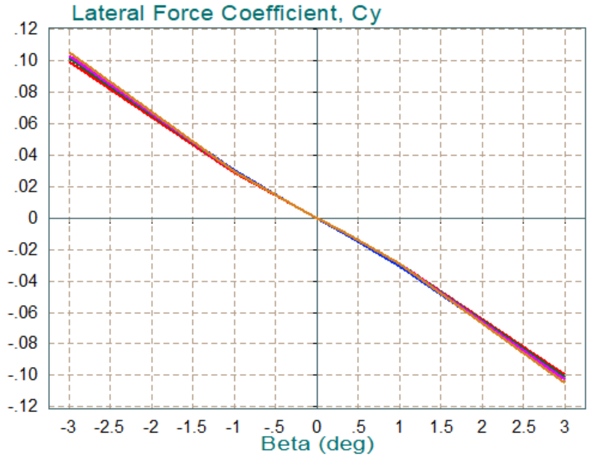

Pitch Moment Coefficient, Cm

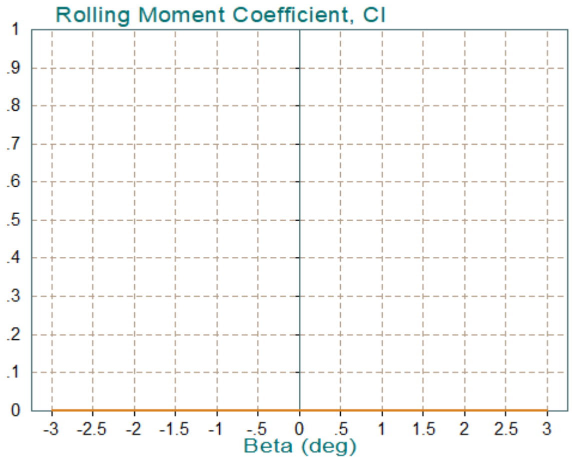

Aft Force (drag) Coefficient, Ca

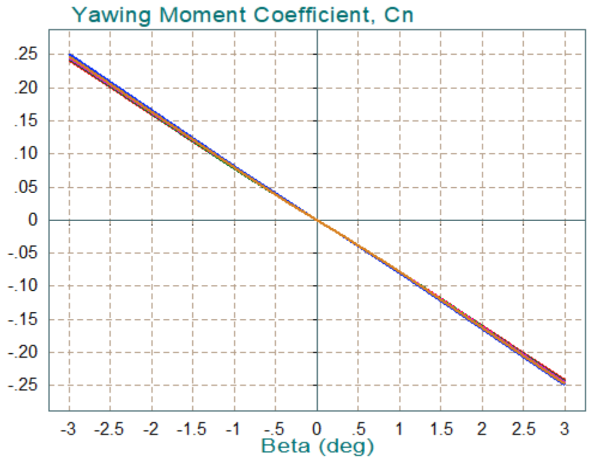

Normal Force Coefficient, Cz=-CN

Launch Vehicle Stage-2 Trajectory
Lateral Aero Coefficients Versus Beta (deg)
Reference Area (ft^2)= 38.50
Chord and Span (feet)= 7.200 ; 7.200
Moments Transferred to Vehicle CG
Alpha(o)=0.00 (deg)
Beta(o) =0.00 (deg)

Mach: 5.000
Mach: 6.000
Mach: 7.000
Mach: 8.000
Mach: 9.000

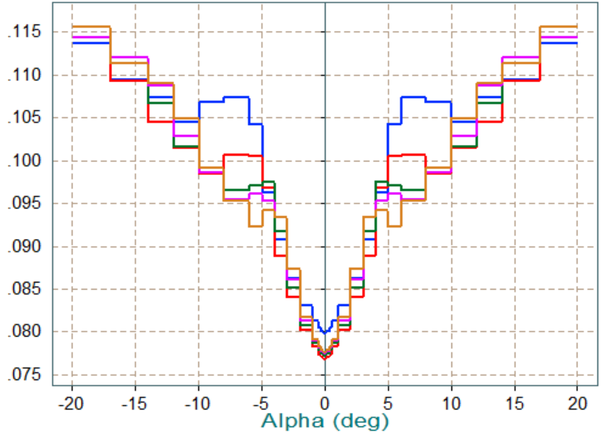

Lateral Force Coefficient, Cy


Rolling Moment Coefficient, Cl


Yawing Moment Coefficient, Cn

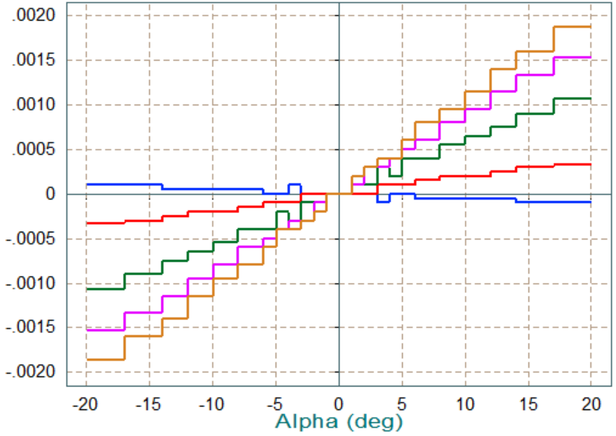
Launch Vehicle Stage-2 Trajectory
Pitch Aero Derivatives Versus Alpha (deg)
Reference Area (ft^2)= 38.50
Chord and Span (feet)= 7.200 ; 7.200
Moments Transferred to Vehicle CG
Alpha(o)=0.00 (deg)
Beta(o) =0.00 (deg)

Pitch Moment Derivative, Cm_alpha

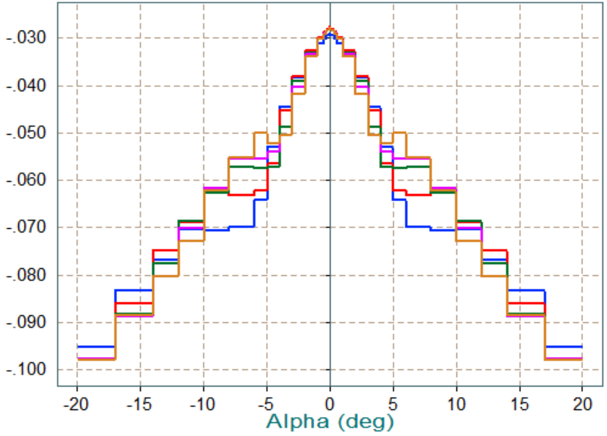


Mach: 5.000
Mach: 6.000
Mach: 7.000
Mach: 8.000
Mach: 9.000

Aft Force Derivative, Ca_alpha

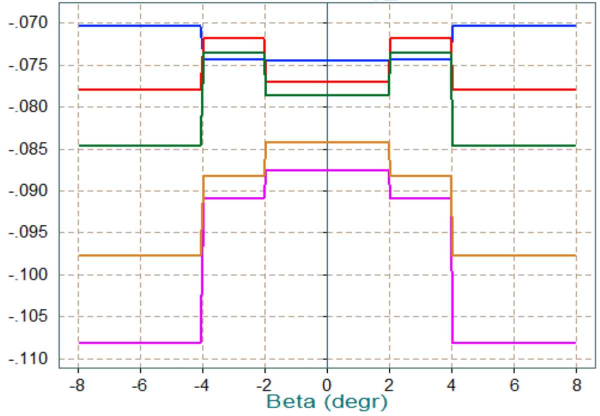


Normal Force Derivative, Cz_alpha



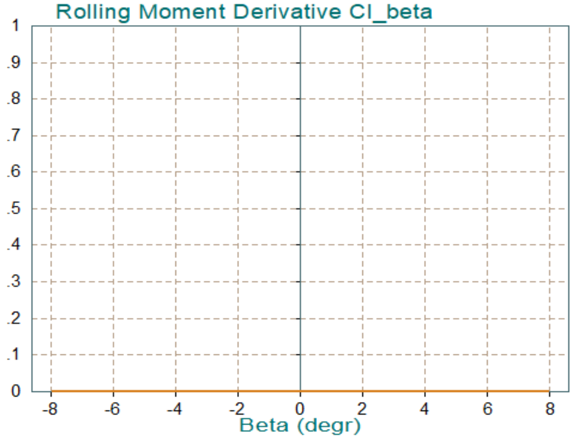
First Stage Trajectory
Lateral Aero Derivatives Versus Beta (deg)
Reference Area (ft^2)= 44.42
Chord and Span (feet)= 7.520 ; 7.520
Moments Transferred to Vehicle CG
Alpha(o)=0.00 (deg)
Beta(o) =0.00 (deg)

Lateral Force Derivative, Cy_beta

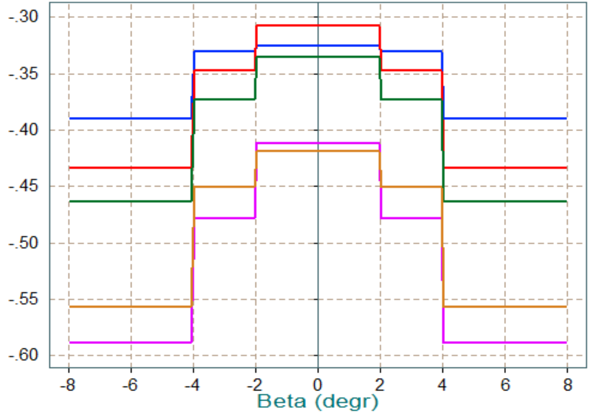


Mach: 0.8000
Mach: 0.8500
Mach: 0.9000
Mach: 0.9500
Mach: 1.000

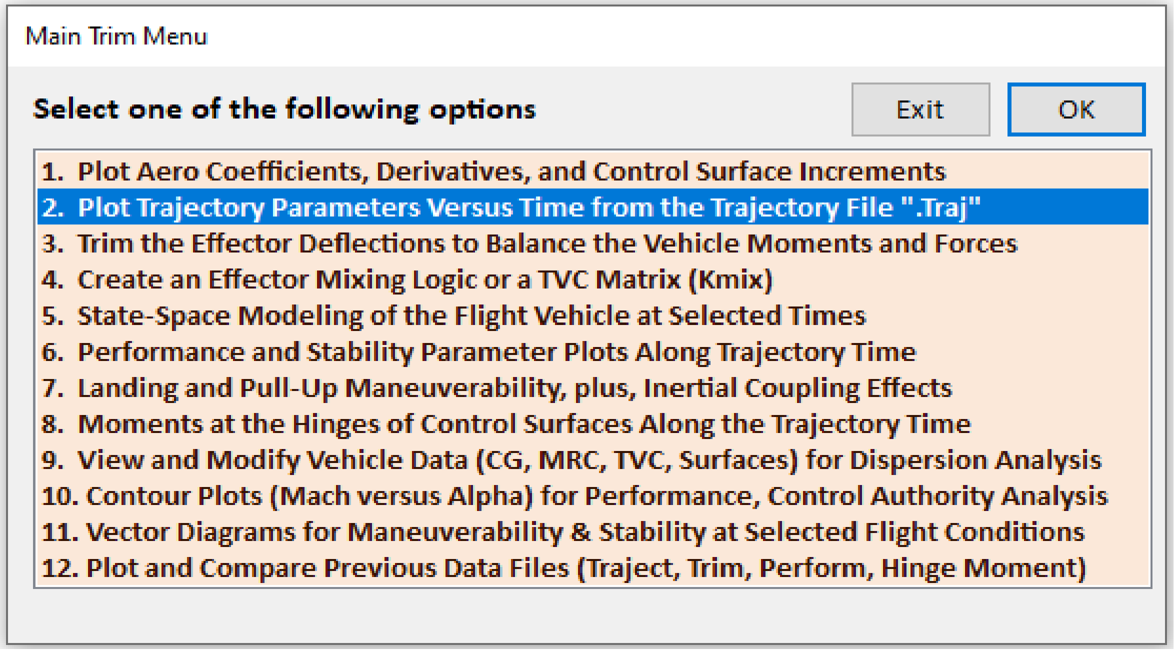
Rolling Moment Derivative Cl_beta



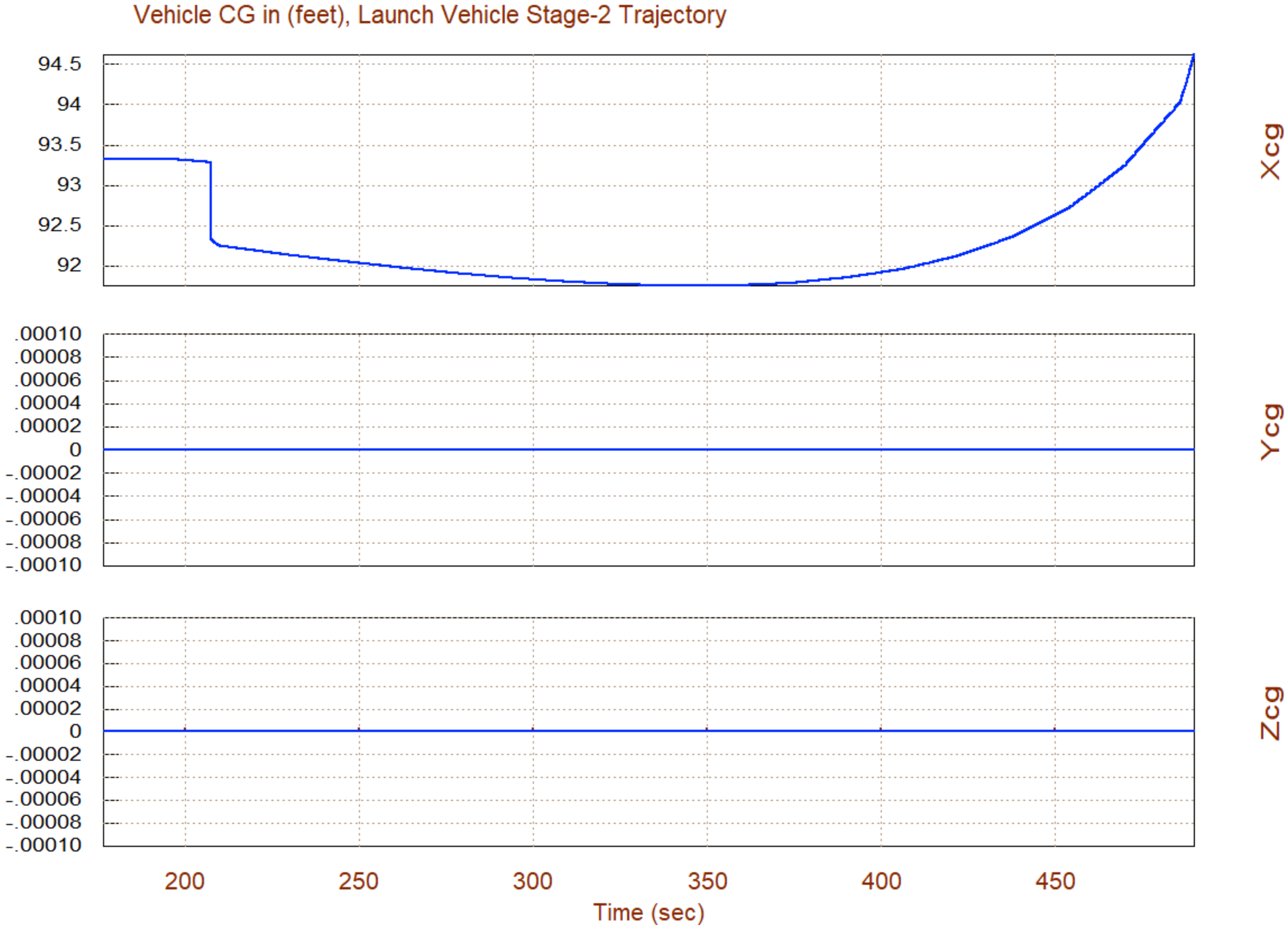
Yawing Moment Derivative, Cn_beta

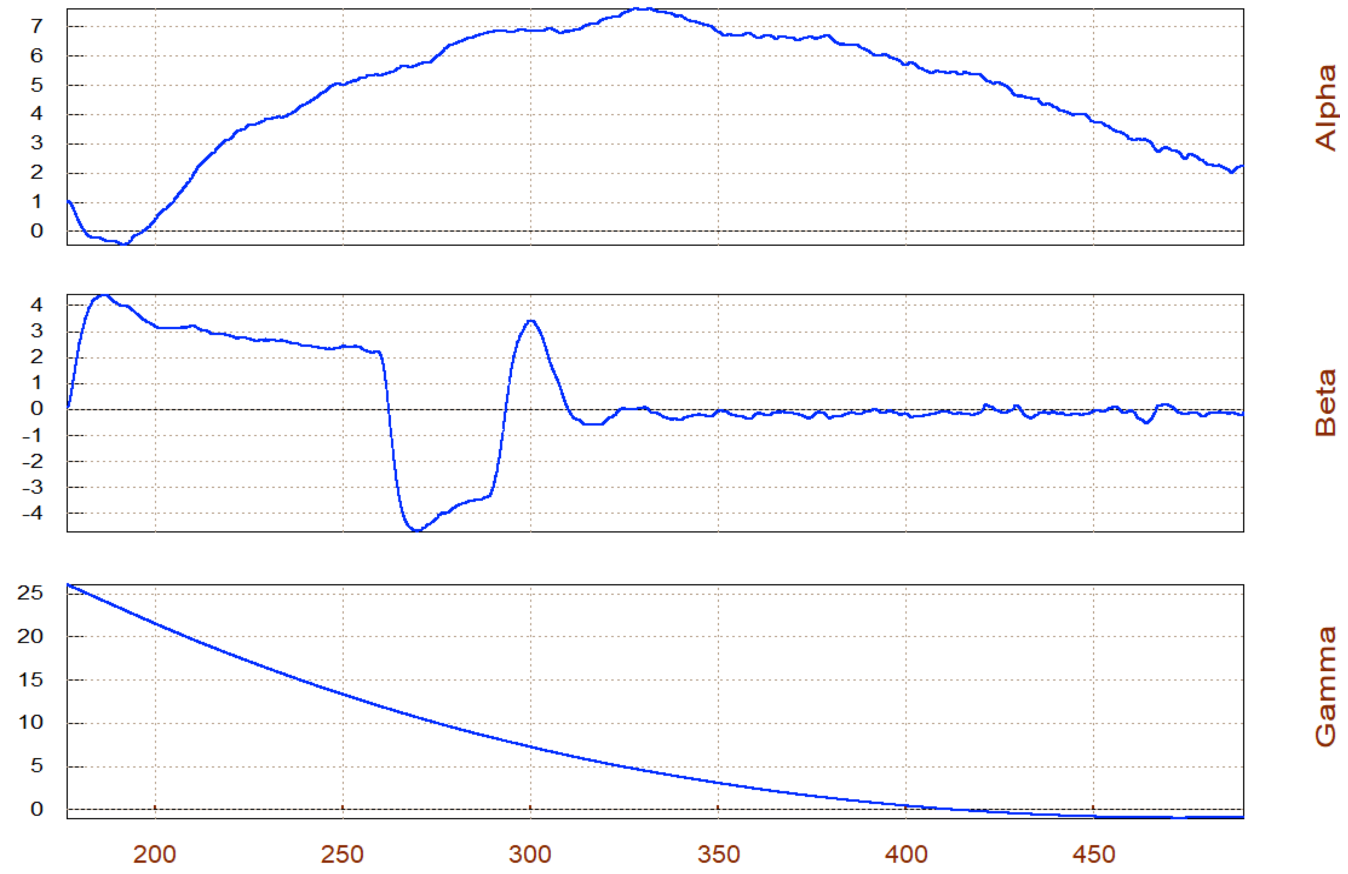


Return to the main menu and select the second item to plot the first stage trajectory data versus time. This is the data from file "*Stage-2b.Traj*". It includes the disturbance torques due to the thrust vector misalignment. The CG is not in the trajectory file but it is calculated versus time from the mass properties. The X_{CG} transient at t=206 sec is caused by the fairing separation.

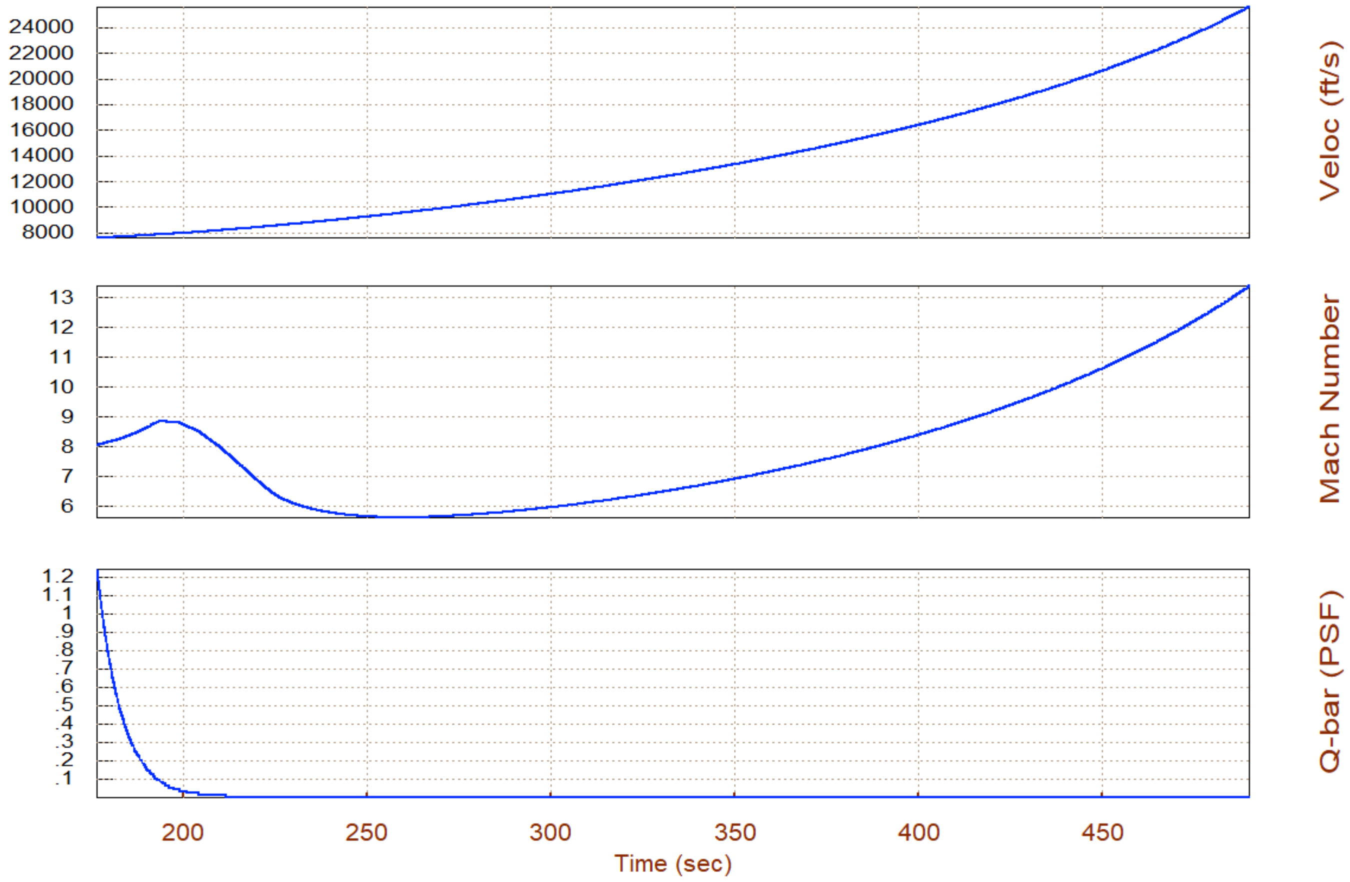


1. Plot Aero Coefficients, Derivatives, and Control Surface Increments
2. Plot Trajectory Parameters Versus Time from the Trajectory File ".Traj"
3. Trim the Effector Deflections to Balance the Vehicle Moments and Forces
4. Create an Effector Mixing Logic or a TVC Matrix (Kmix)
5. State-Space Modeling of the Flight Vehicle at Selected Times
6. Performance and Stability Parameter Plots Along Trajectory Time
7. Landing and Pull-Up Maneuverability, plus, Inertial Coupling Effects
8. Moments at the Hinges of Control Surfaces Along the Trajectory Time
9. View and Modify Vehicle Data (CG, MRC, TVC, Surfaces) for Dispersion Analysis
10. Contour Plots (Mach versus Alpha) for Performance, Control Authority Analysis
11. Vector Diagrams for Maneuverability & Stability at Selected Flight Conditions
12. Plot and Compare Previous Data Files (Traject, Trim, Perform, Hinge Moment)

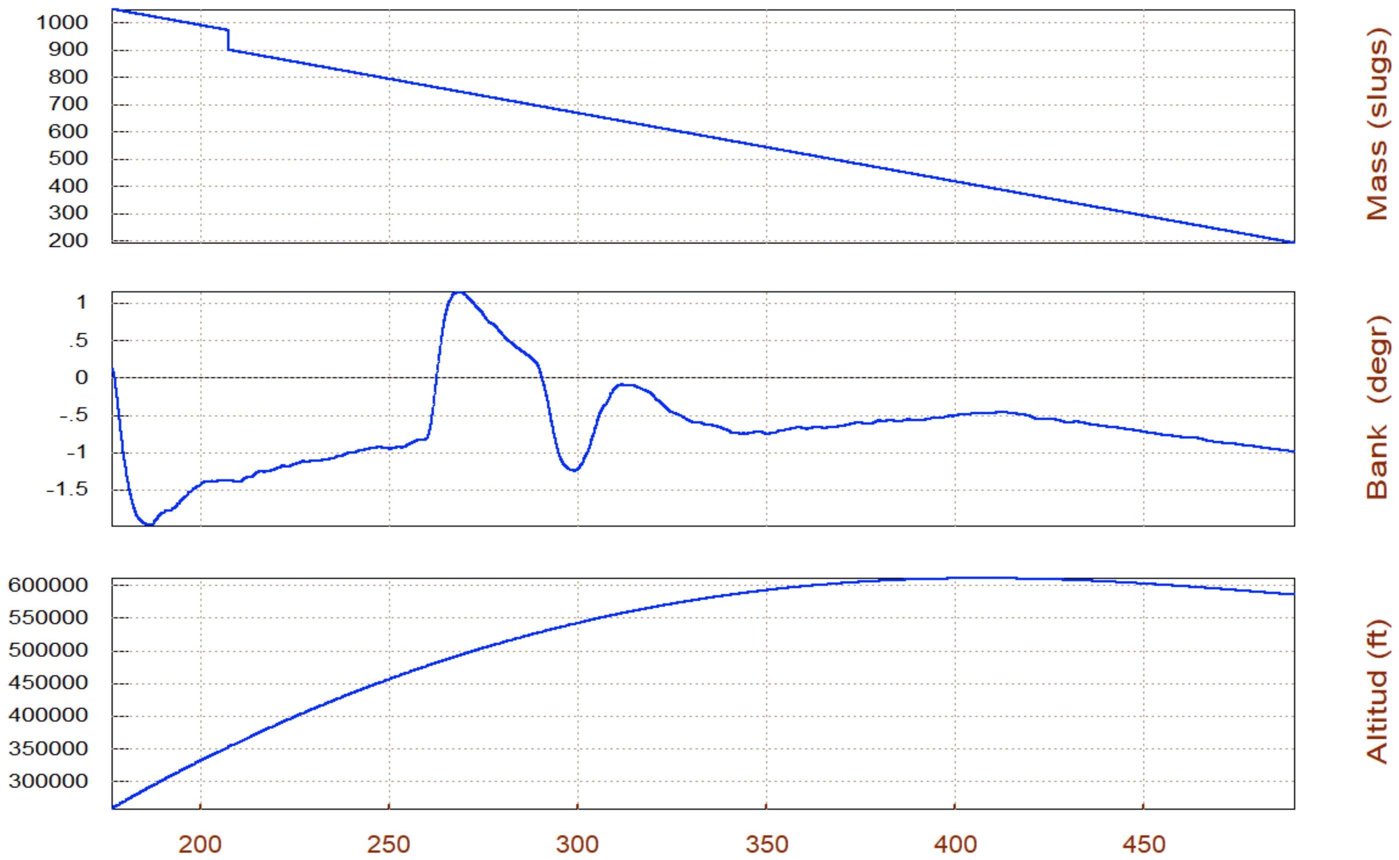


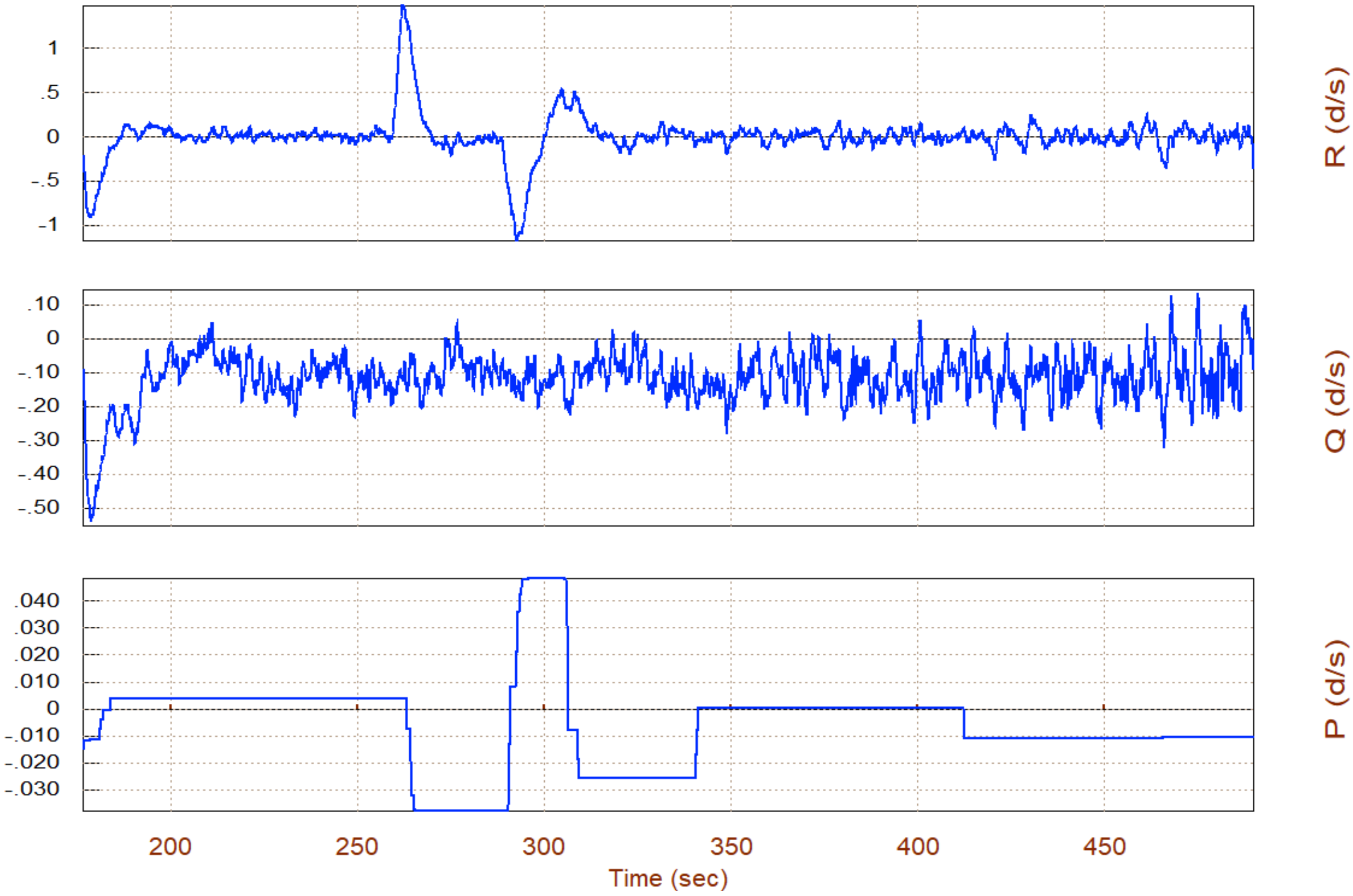


Angles of Attack/Sideslip/Flight Path (deg), Launch Vehicle Stage-2 Tr

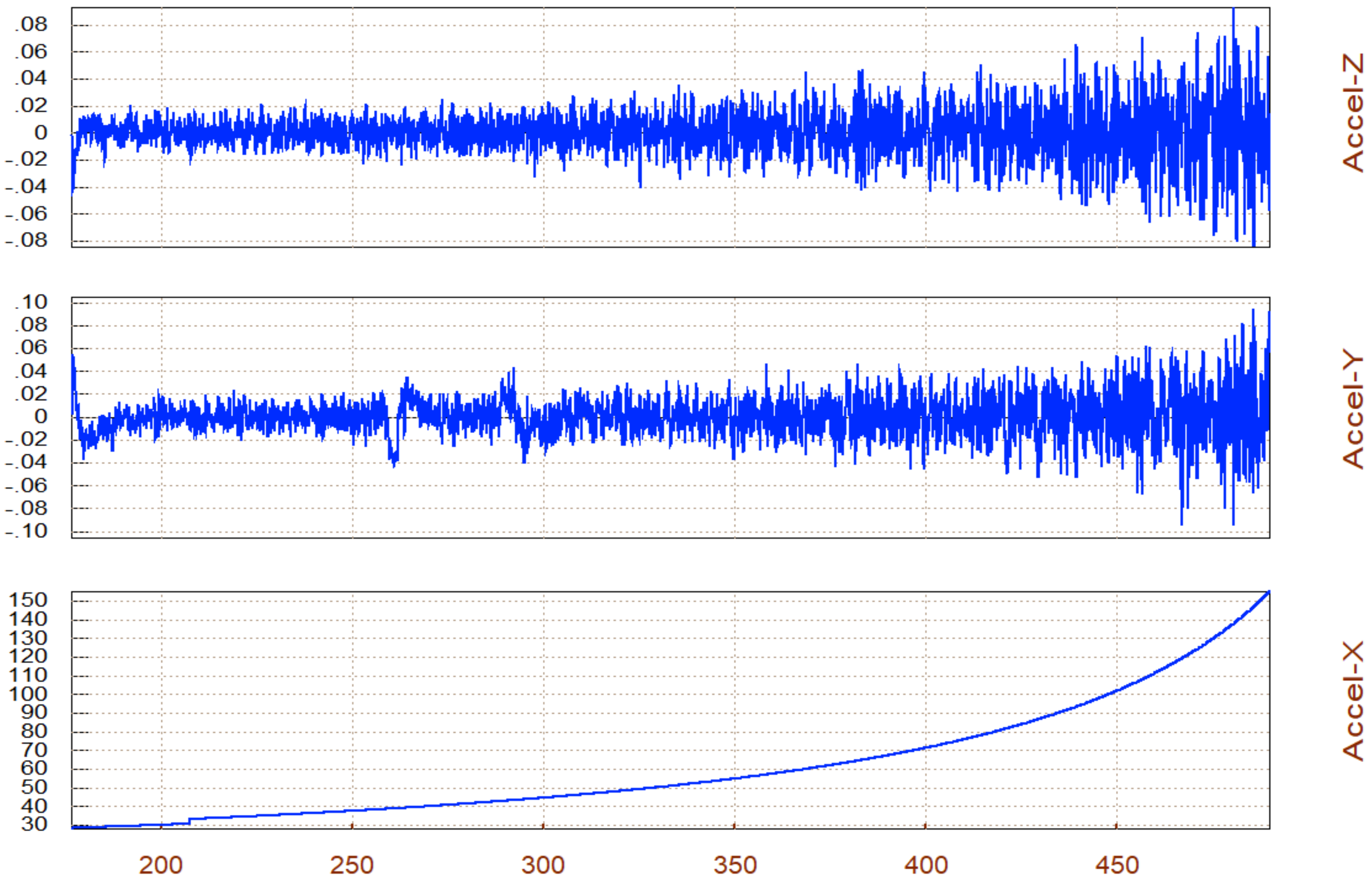


Velocity, Dynamic Pressure, Launch Vehicle Stage-2 Trajectory

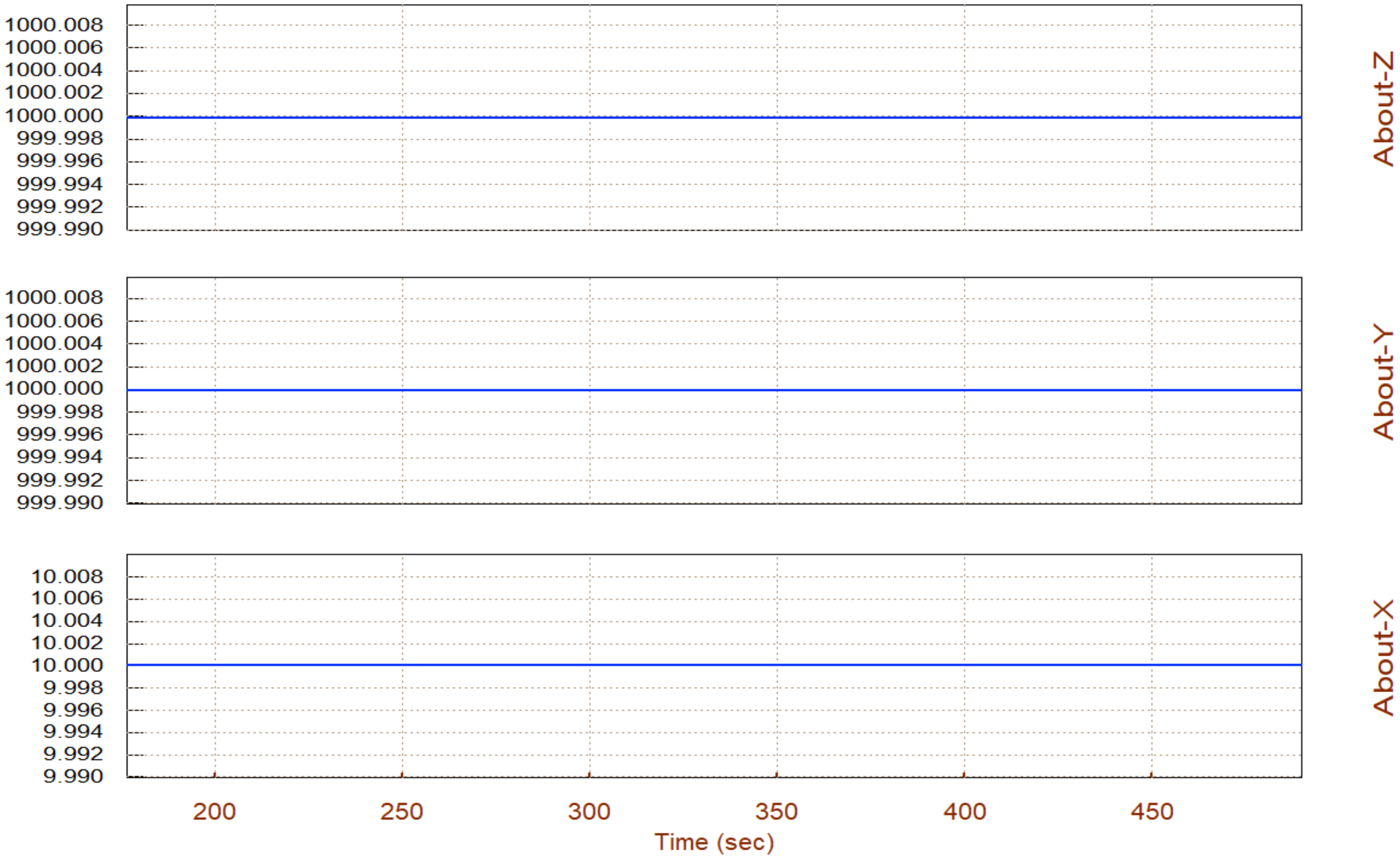

Vehicle Altitude, Mass, Bank Angle, Launch Vehicle Stage-2 Trajectory


Angular Rates (rad/sec), Launch Vehicle Stage-2 Trajectory

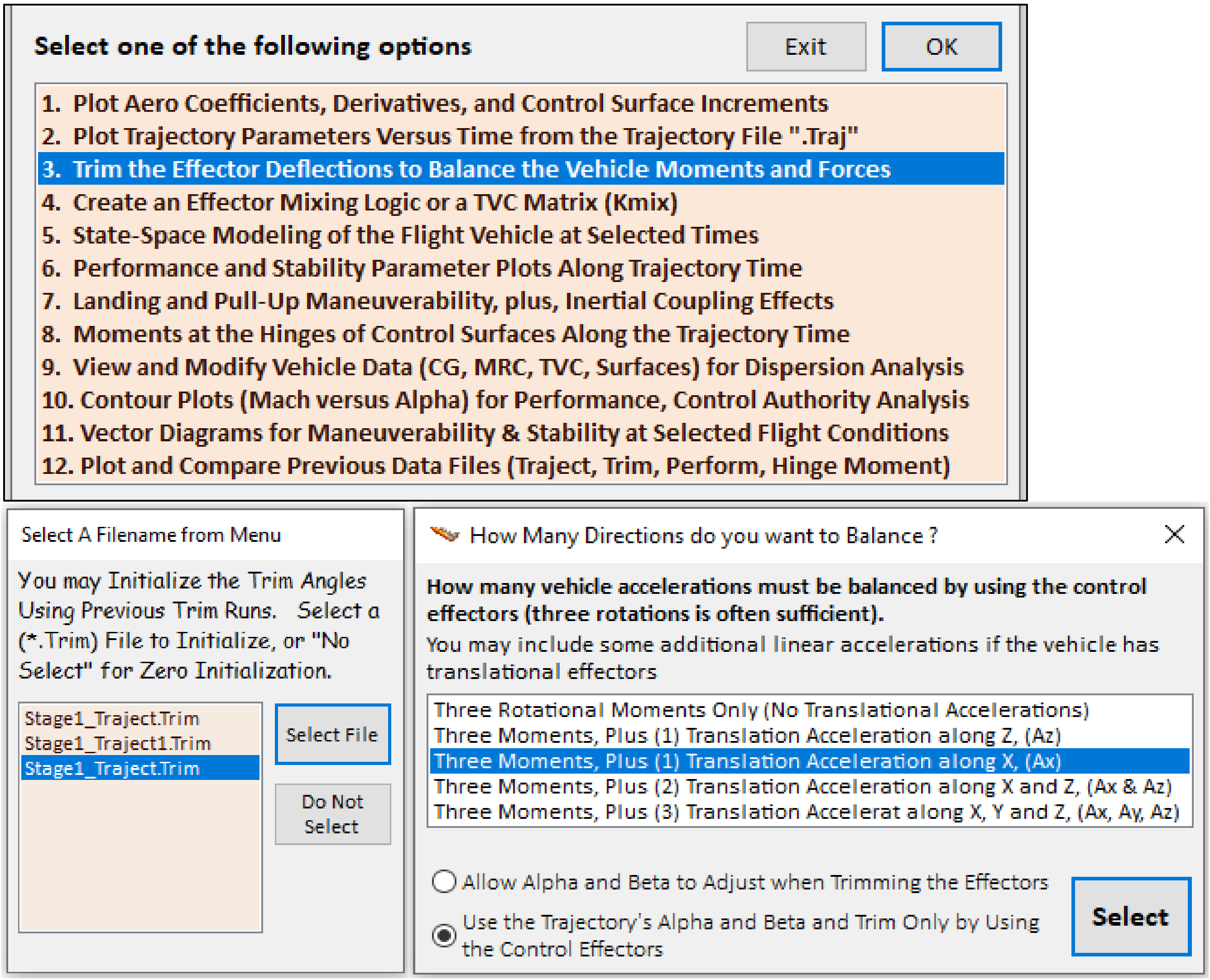
Sensed Acceleration in (ft/sec^2), Launch Vehicle Stage-2 Trajectory



External Disturbance Torques (ft-lb), TimeMassAltiAlpha Beta Gamm



The next step is to trim the TVC engine and the RCS along the trajectory in order to balance the external moments and axial acceleration. That is, the aerodynamic torques which are small and they only occur in the beginning of the 2nd stage, end the external torques which are constant and they are included in the trajectory file. Select the 3rd option from the Trim menu, then click onto "*Do Not Select*" a previously created trim file, and from the next menu select to balance the 3 moments along the (α, β) angles, including to also the x-acceleration from the trajectory.



Select one of the following options

1. Plot Aero Coefficients, Derivatives, and Control Surface Increments
2. Plot Trajectory Parameters Versus Time from the Trajectory File ".Traj"
3. Trim the Effector Deflections to Balance the Vehicle Moments and Forces
4. Create an Effector Mixing Logic or a TVC Matrix (Kmix)
5. State-Space Modeling of the Flight Vehicle at Selected Times
6. Performance and Stability Parameter Plots Along Trajectory Time
7. Landing and Pull-Up Maneuverability, plus, Inertial Coupling Effects
8. Moments at the Hinges of Control Surfaces Along the Trajectory Time
9. View and Modify Vehicle Data (CG, MRC, TVC, Surfaces) for Dispersion Analysis
10. Contour Plots (Mach versus Alpha) for Performance, Control Authority Analysis
11. Vector Diagrams for Maneuverability & Stability at Selected Flight Conditions
12. Plot and Compare Previous Data Files (Traject, Trim, Perform, Hinge Moment)

Select A Filename from Menu

You may Initialize the Trim Angles Using Previous Trim Runs. Select a (*.Trim) File to Initialize, or "No Select" for Zero Initialization.

Stage1_Traject.Trim
Stage1_Traject1.Trim
Stage1_Traject.Trim

How Many Directions do you want to Balance ?

How many vehicle accelerations must be balanced by using the control effectors (three rotations is often sufficient).

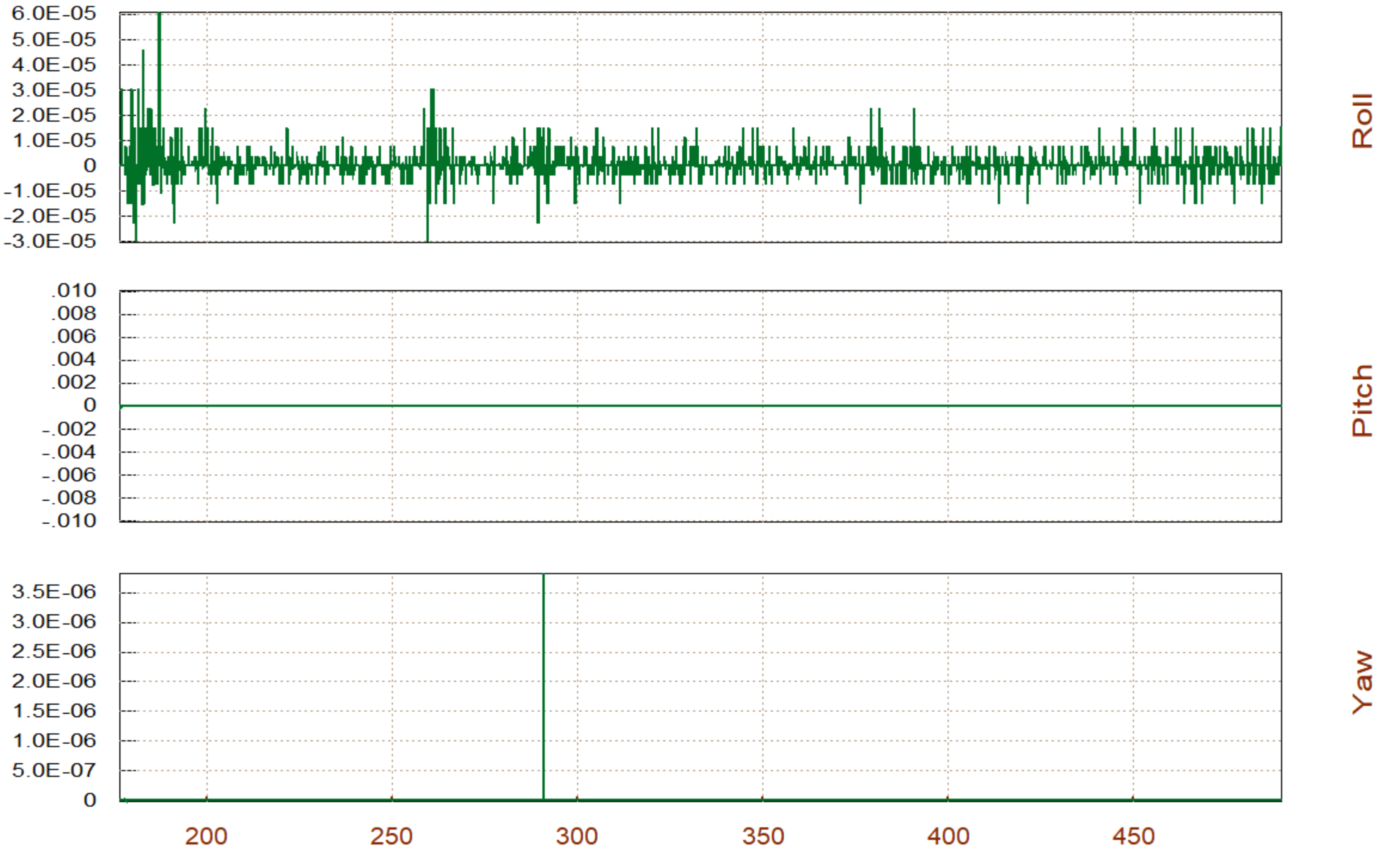
You may include some additional linear accelerations if the vehicle has translational effectors

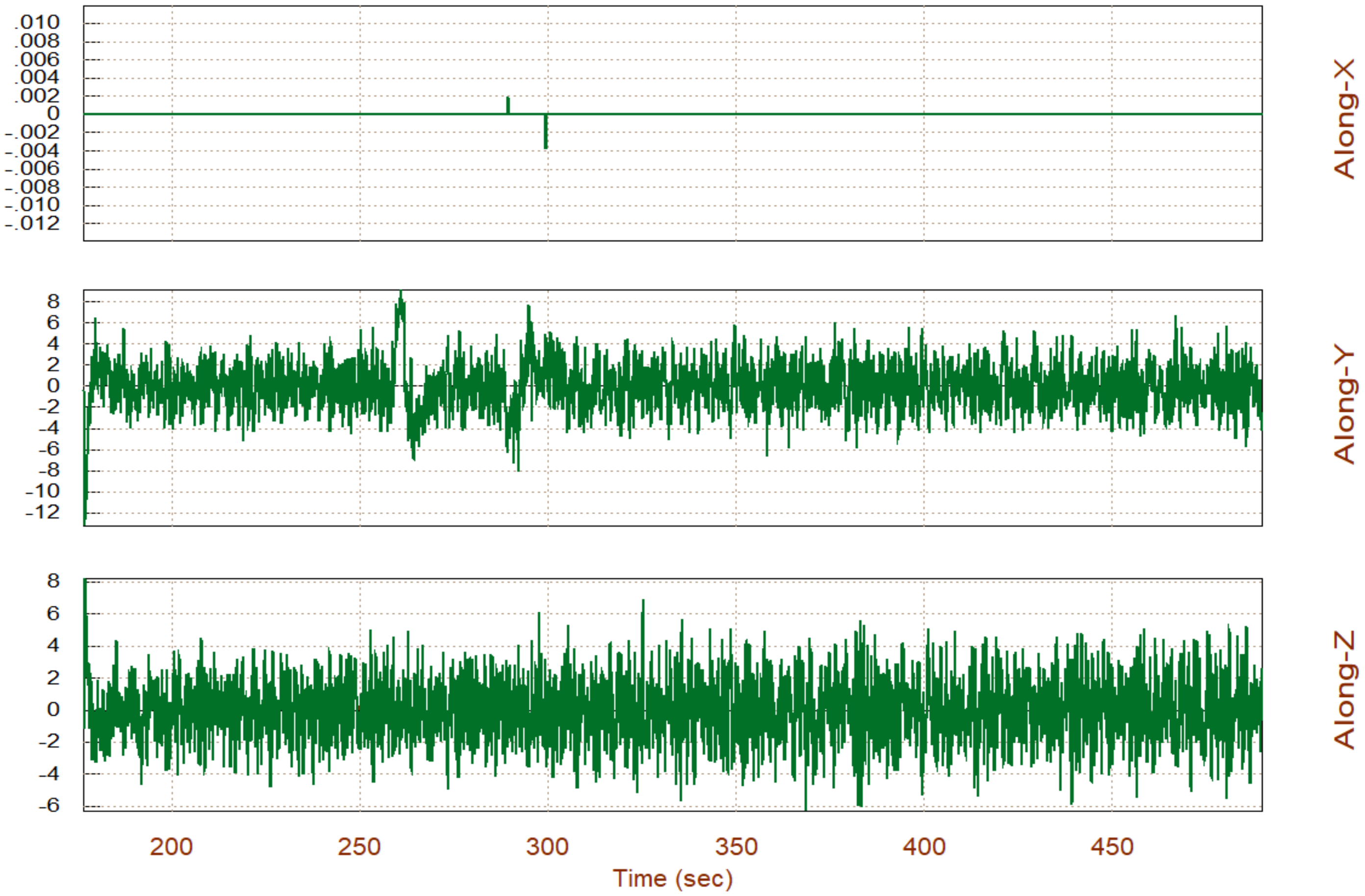
Three Rotational Moments Only (No Translational Accelerations)
Three Moments, Plus (1) Translation Acceleration along Z, (Az)
Three Moments, Plus (1) Translation Acceleration along X, (Ax)
Three Moments, Plus (2) Translation Acceleration along X and Z, (Ax & Az)
Three Moments, Plus (3) Translation Accelerat along X, Y and Z, (Ax, Ay, Az)

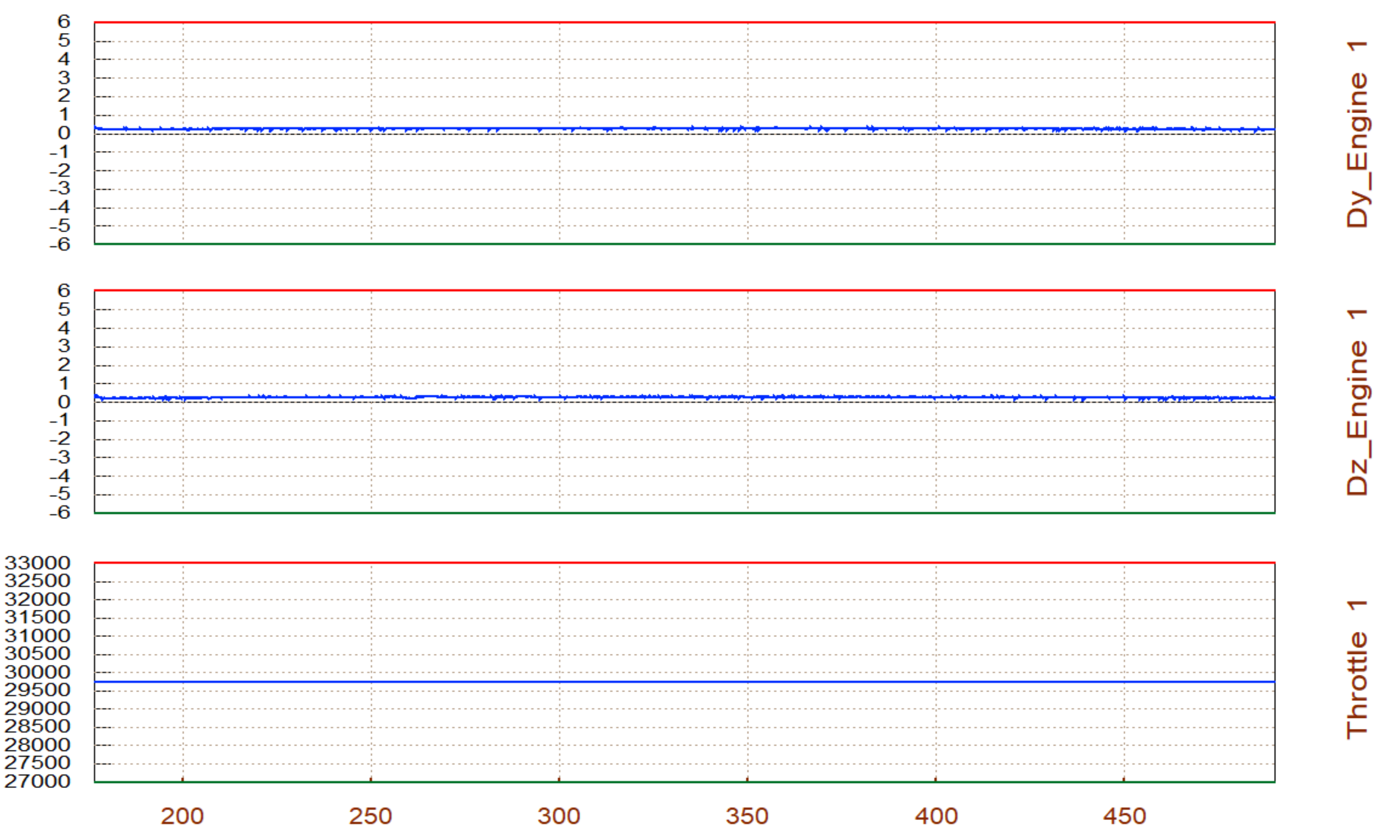
○ Allow Alpha and Beta to Adjust when Trimming the Effectors
● Use the Trajectory's Alpha and Beta and Trim Only by Using the Control Effectors

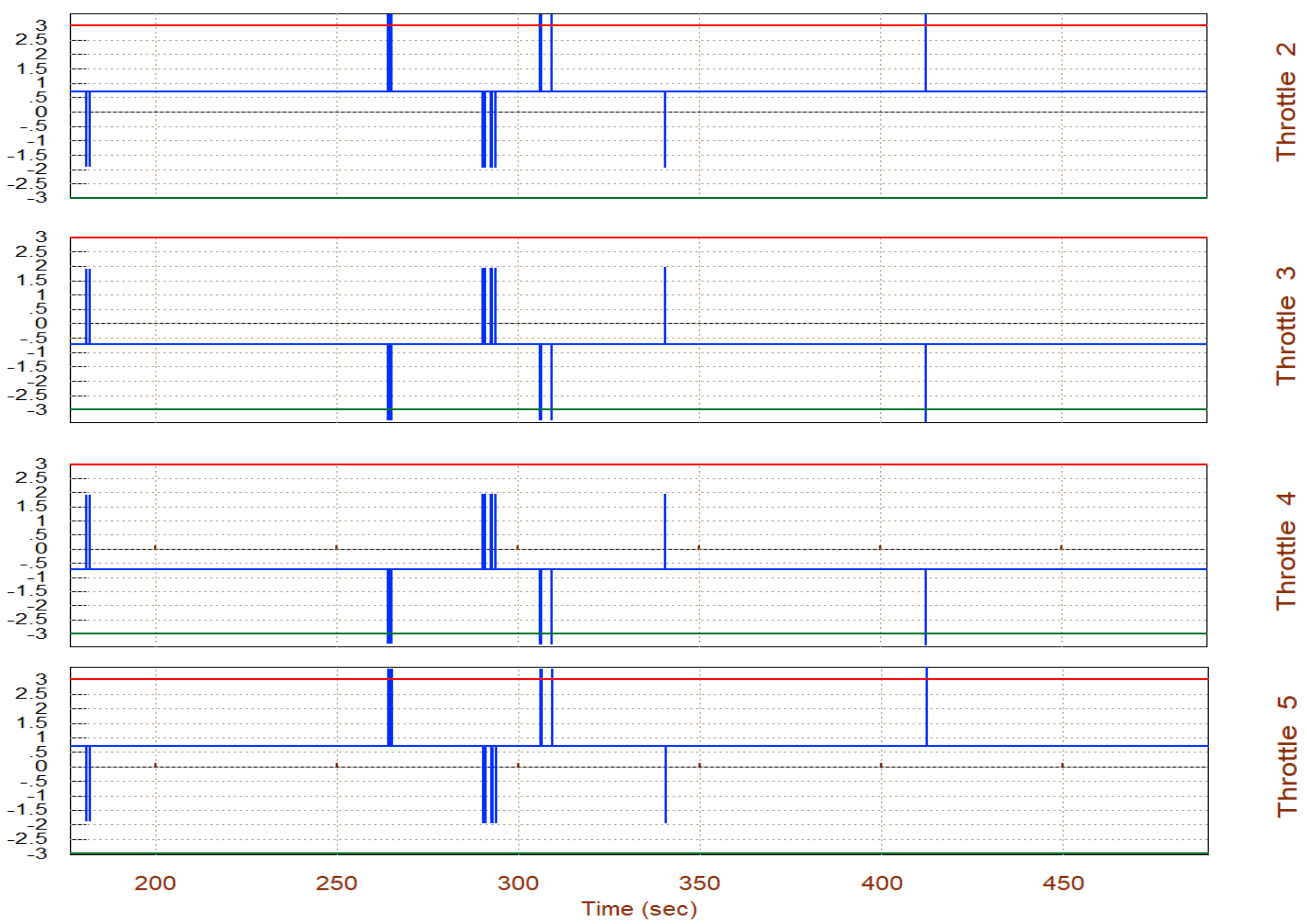
The next two plots show the 3 residual moments and residual x-acceleration which are zero as expected. It means that we were able to trim along the selected 4 trim directions by gimbaling and throttling the engine and the RCS jets. A little throttling of the TVC engine was introduced in the engine data, in order to match the trajectory's x-acceleration. We couldn't use the RCS jets to trim along x because they are only throttling along y and z for roll control.

The next plot shows the TVC engine pitch and yaw gimbal deflections and thrust. The small deflections are needed to balance the 1,000 (ft-lb) external torques in pitch and yaw. The main engine thrust-1 value is adjusted by throttling to a steady value of 30,000 (lbf) in order to match the x-acceleration. The next plot shows the 4 RCS thrusts, numbers (2,3,4,5). In addition to the discrete pulses used to trim along the trajectory, their thrusts are biased in order to produce the roll torque necessary to counteract the external 10 (ft-lb) roll disturbance.

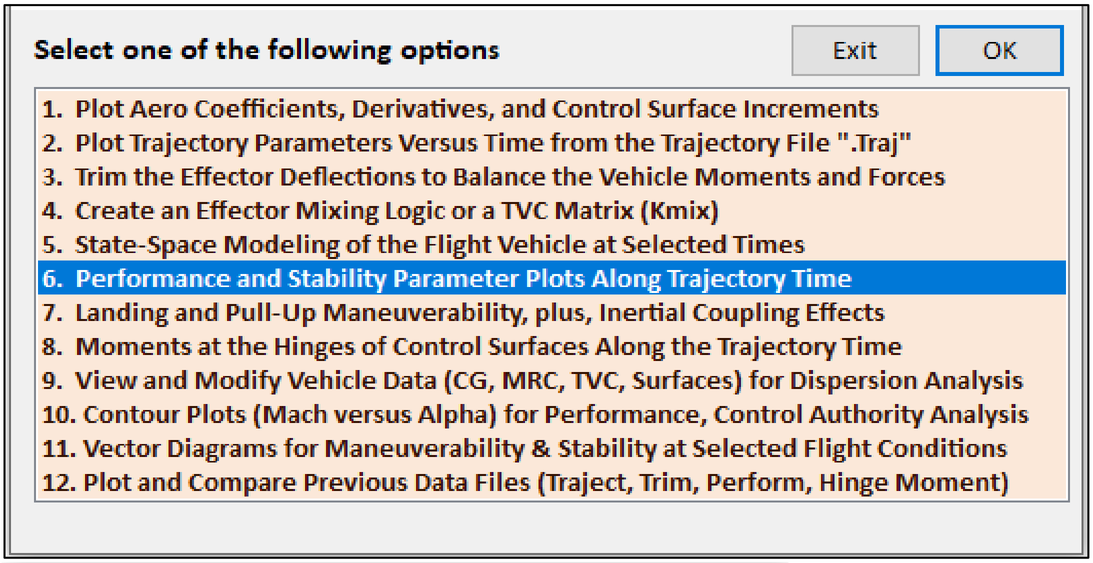

Residual Moments After Trimming (ft-lb) Launch Vehicle Stage-2 Trajectory


Residual Forces After Trimming (lb) Launch Vehicle Stage-2 Trajectory


Surface & Engine Deflections/ Thrusts, TimeMassAltiAlpha Beta GammaRollVr


Surface & Engine Deflections/ Thrusts, TimeMassAltiAlpha Beta GammaRollVr

The next step is to analyze the vehicle static performance along the trajectory. A (7x4) mixing matrix is calculated to combine the 7 controls (including throttling) with the 4 degrees-of-freedom to be controlled (including the axial acceleration for now). We will allow the program to create an effector combination logic by allowing a full participation from all effectors. Then, we select a maximum 4° alpha/ beta dispersions and 30 (ft/sec) wind-gust variations, as worst possible disturbances along the trajectory.



Select one of the following options

1. Plot Aero Coefficients, Derivatives, and Control Surface Increments
2. Plot Trajectory Parameters Versus Time from the Trajectory File ".Traj"
3. Trim the Effector Deflections to Balance the Vehicle Moments and Forces
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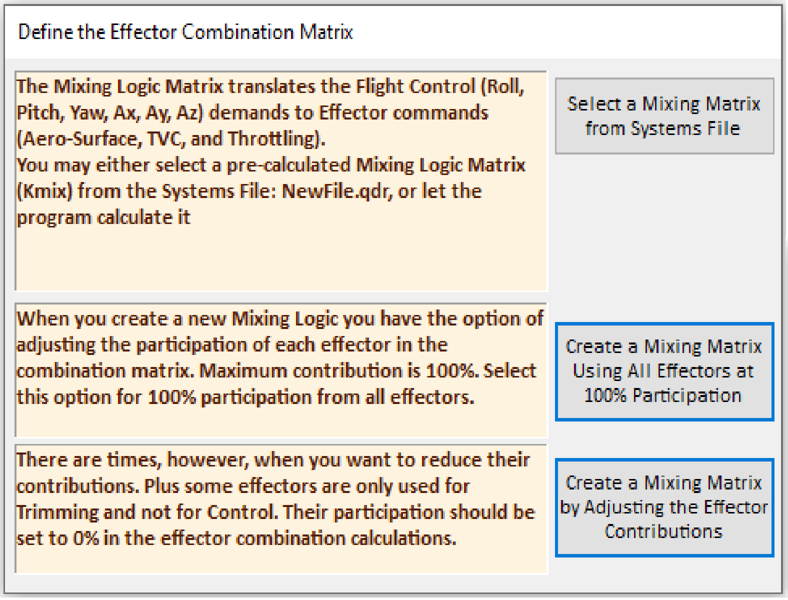
11. Vector Diagrams for Maneuverability & Stability at Selected Flight Conditions
12. Plot and Compare Previous Data Files (Traject, Trim, Perform, Hinge Moment)

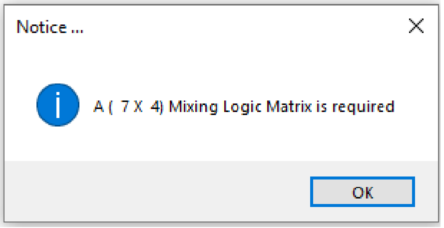


Define the Effector Combination Matrix

The Mixing Logic Matrix translates the Flight Control (Roll, Pitch, Yaw, Ax, Ay, Az) demands to Effector commands (Aero-Surface, TVC, and Throttling).
You may either select a pre-calculated Mixing Logic Matrix (Kmix) from the Systems File: NewFile.qdr, or let the program calculate it

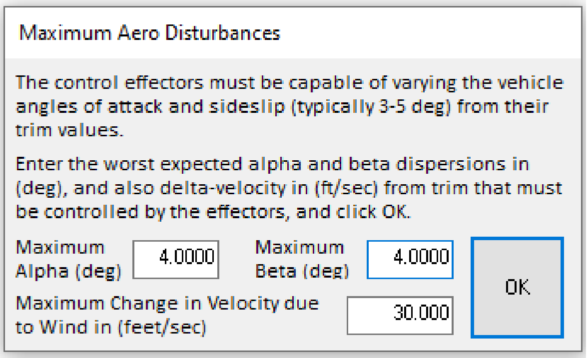
When you create a new Mixing Logic you have the option of adjusting the participation of each effector in the combination matrix. Maximum contribution is 100%. Select this option for 100% participation from all effectors.

There are times, however, when you want to reduce their contributions. Plus some effectors are only used for Trimming and not for Control. Their participation should be set to 0% in the effector combination calculations.



Notice ...

A (7 X 4) Mixing Logic Matrix is required



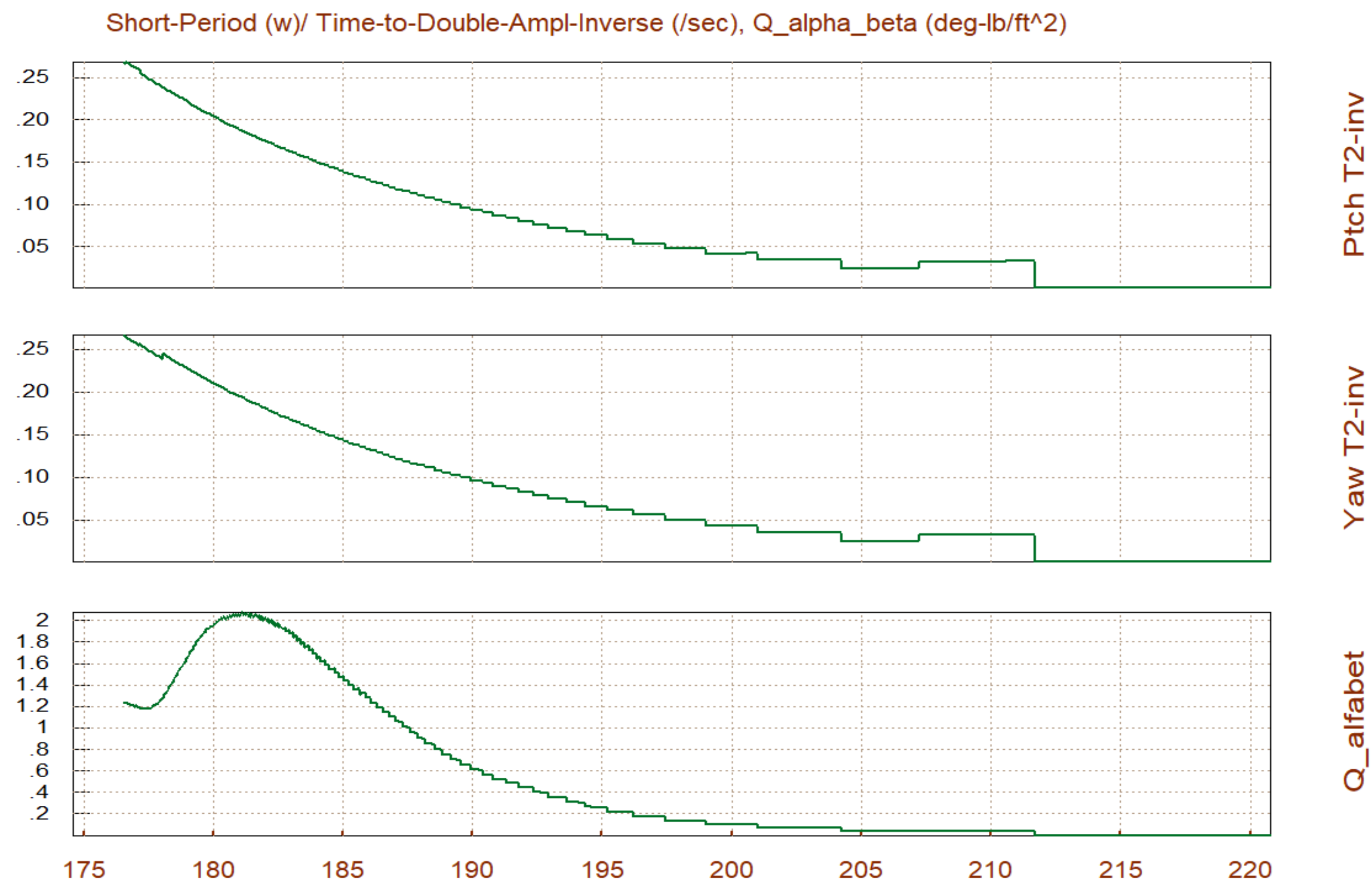
Maximum Aero Disturbances

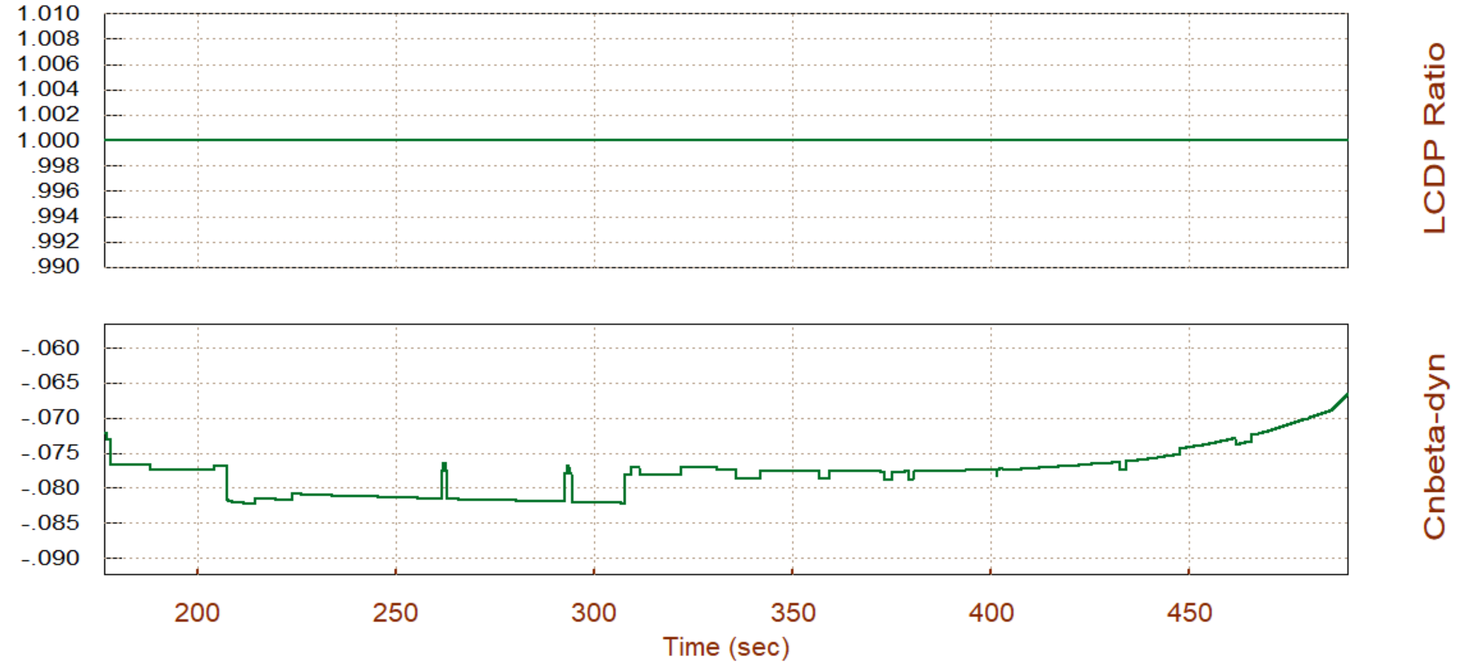
The control effectors must be capable of varying the vehicle angles of attack and sideslip (typically 3-5 deg) from their trim values.

Enter the worst expected alpha and beta dispersions in (deg), and also delta-velocity in (ft/sec) from trim that must be controlled by the effectors, and click OK.

Maximum Alpha (deg) 4.0000
Maximum Beta (deg) 4.0000
Maximum Change in Velocity due to Wind in (feet/sec) 30.000

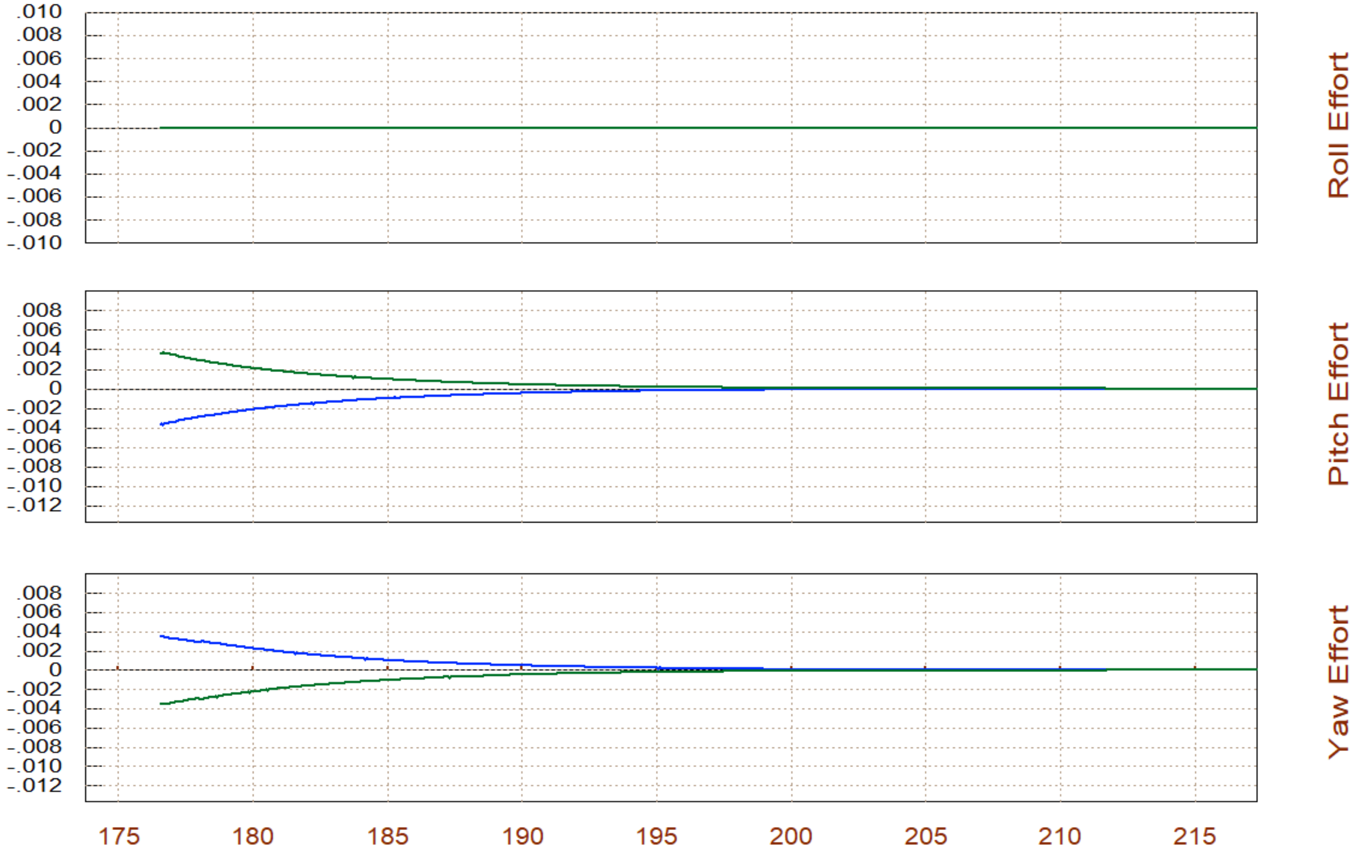
The next plot shows the T2-inverse parameters, which measure static stability in pitch and yaw. They are the same because it is a cylindrical vehicle. The vehicle is statically unstable but the instability is very slow. The time-to-double amplitude after stage-1 separation is 4 sec and it keeps getting longer as the dynamic pressure and the T2-inverse parameter approach zero. The third plot shows the “Q-alpha/ Q-beta” parameter which is used to analyze the lateral loading caused by angle of attack dispersions. It is assumed that α_{max} and β_{max} dispersions are 4° along the entire trajectory. The worst-case loading in this case is only 2 (psf-deg) early 2nd stage. The control authority to counteract the disturbance moment due to ±4° dispersions in α_{max} and β_{max} is also very good. The control effort considering max deflection availability is very small.



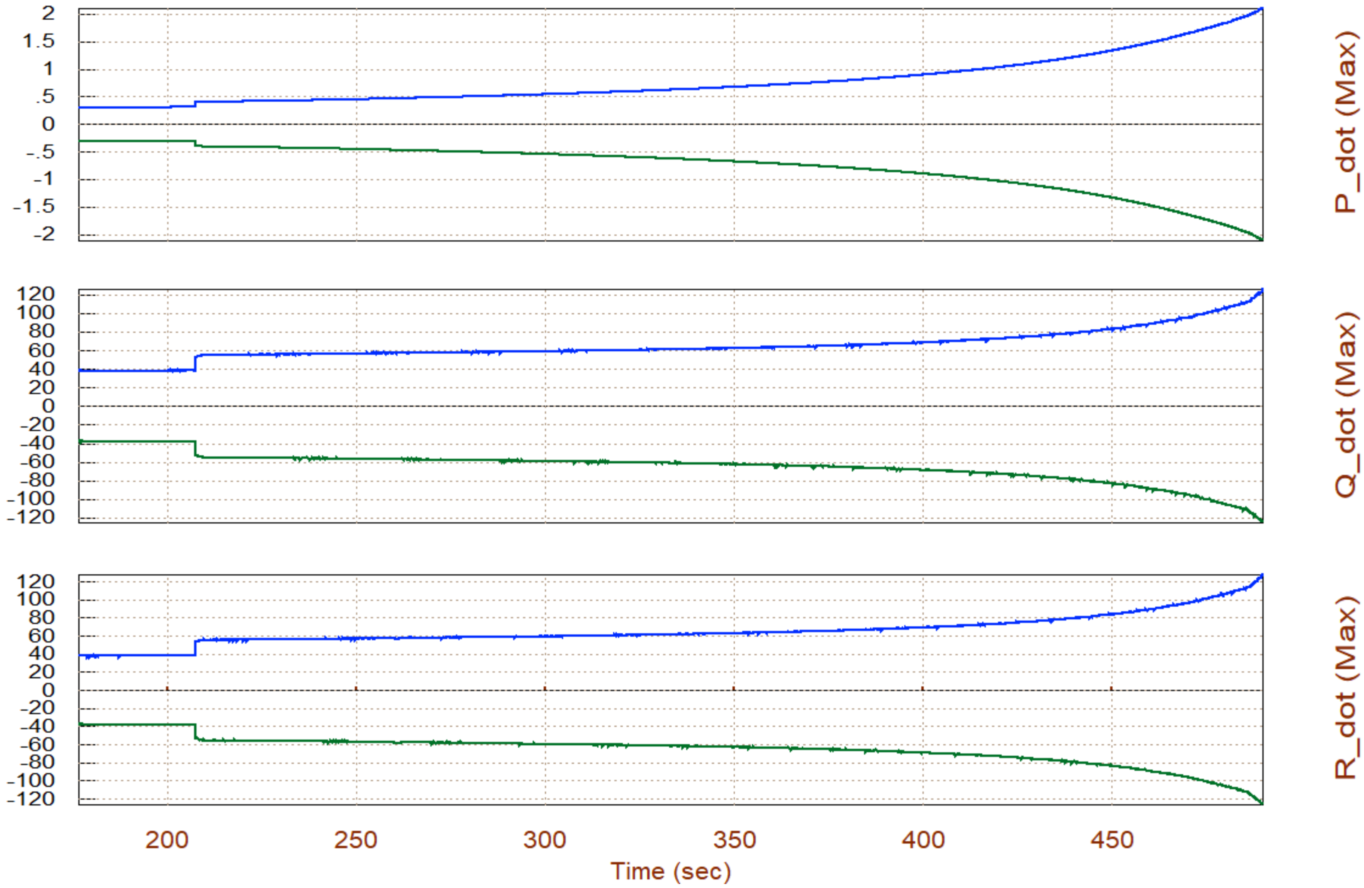


The LCDP and the Cnβ-dynamic are parameters which are not typically used in launch vehicles, but in this case the LCDP is equal to one, which is ideal. The Cnβ-dynamic is negative indicating open-loop instability. The last plot shows the maximum angular accelerations produced when the controls are maximized in both positive and in negative directions. The max acceleration increases with time because the vehicle weight is reduced, and there is more acceleration in roll because the moment of inertia is smaller.

Rotation Control Authority |dQ/dQmax|<1 for 4 (deg) of Alpha & Beta Variation



Max Angular Accelerations (deg/sec^2), at Maximum +ve and -ve Control Demands



2.1 First Stage Control Design

During first stage the vehicle is powered by 9 engines shown in Figure 2.1. The 8 peripheral engines are gimbaling in both pitch and yaw. The one at the center is fixed. The total engines thrust is approximately 210,000 (lbf) but it varies during 1st stage as a function of altitude. We will use the Flixan program to create rigid vehicle models during 1st stage at 8 flight conditions or "time-slices" and then use these models to design state-feedback control laws. Later in the analysis section the control laws will be adjusted in order to include the propellant sloshing and structural flexibility.

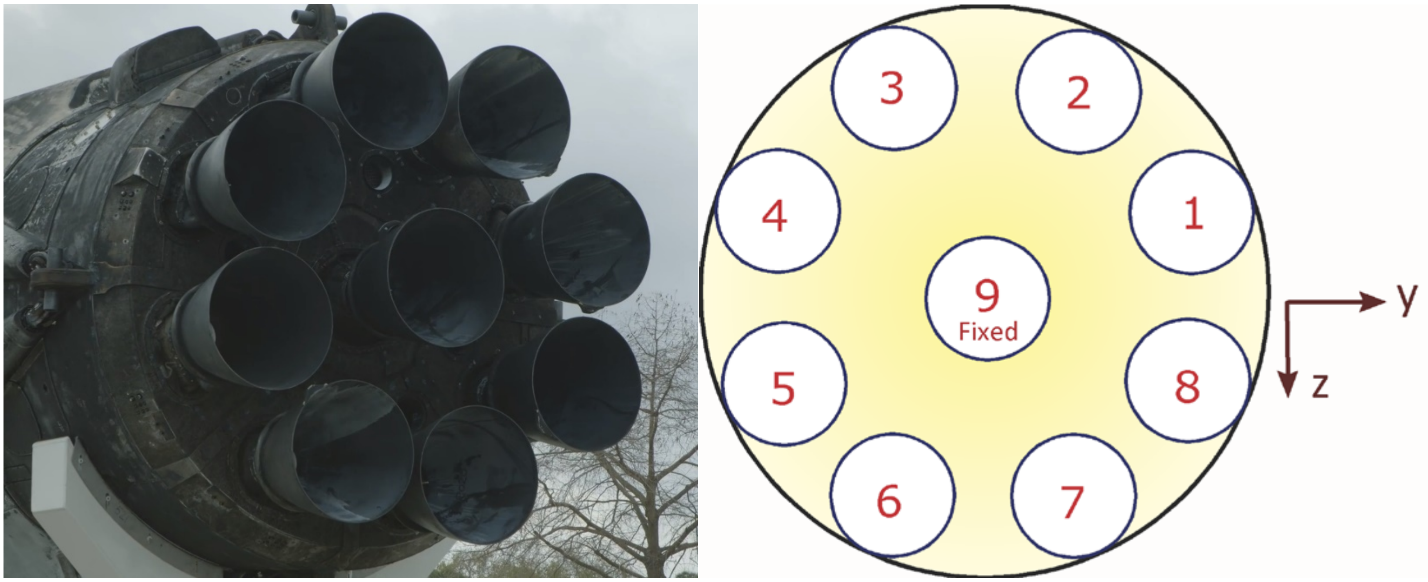


Figure 2.1 TVC Engines

In addition to the vehicle system, the Flixan program also calculates the TVC matrix at each flight condition. This mixing logic matrix combines the 8 engine gimbals together to achieve the accelerations demanded by the flight control system. The vehicle models are then combined with the TVC matrix and separated into pitch and lateral design systems which are used to create the pitch and lateral LQR state-feedback controllers. Two different control structures are implemented in the LQR design to satisfy requirements for different phases along the 1st stage trajectory.

1. A low dynamic pressure structure that emphasizes mostly in controlling the vehicle attitude but not ignoring the aerodynamic loads on the vehicle, and
2. A high dynamic pressure structure that emphasizes mostly in aerodynamic load-relief but still trying to maintain the commanded attitude.

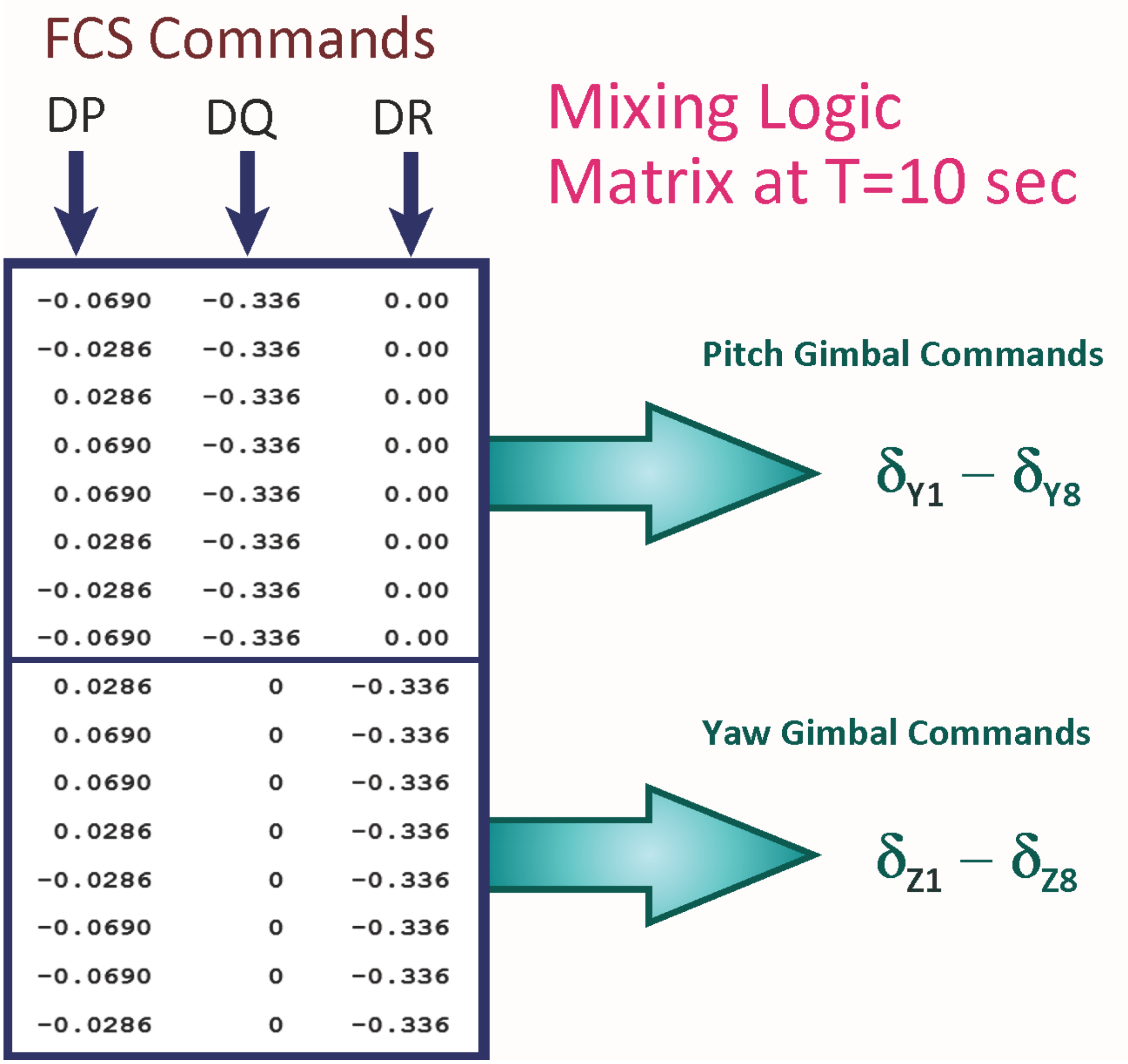
The first structure that emphasizes on attitude control uses feedback from the vehicle attitude (θ), rate (q), angle of attack (α) and θ-integral. The second structure that emphasizes on load-relief uses feedback from the vehicle attitude (θ), rate (q), angle of attack (α) and α-integral.

2.1.1 Control Design at T= 10 sec

We begin with a low Q-bar case, 10 seconds after lift-off. The design files are in directory "*Examples**23-Classic Launch Vehicle Design & Simulation**2-Control Gains Design**1st Stage**T10".*

Vehicle Input File

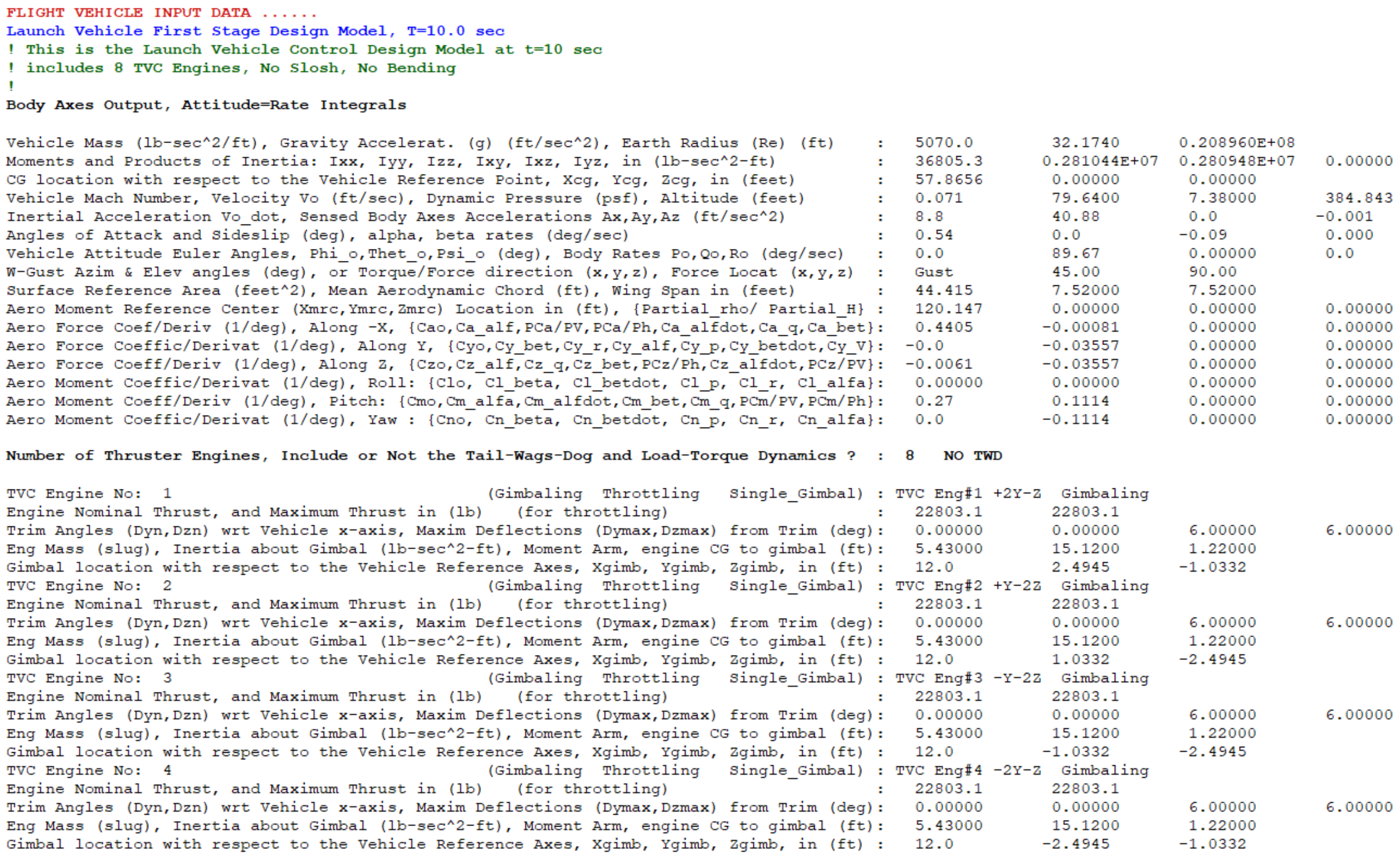
The Flixan input file is “*Rig_Vehi_T10.Inp*”. It begins with a batch set “*Batch for Launch Vehicle Stage-1 Control Design at T=10 sec*” which enables fast execution of the entire input file in batch mode. It includes the vehicle dataset “*Launch Vehicle First Stage Design Model, T=10.0 sec*” that creates the vehicle system with 8 TVC engines of 22,803 (lbf) thrust each, gimbaling in pitch and yaw, no throttling. The fixed engine is not included. There is a mixing logic dataset “*Vehicle Mixing Logic*” that creates the TVC matrix, shown in Figure 2.1.2, that converts the roll, pitch and yaw flight control acceleration demands to 8 pitch and 8 yaw gimbal deflections commands. The vehicle and TVC matrix are combined together to create 2 systems: the “*Vehicle Design Model with TVC*” and the “*Vehicle Analysis Model with TVC*”. The “*Augmented Pitch Design Model*” is created from the vehicle design model by selecting the 3 pitch states (θ, q, α) and including a 4th state, the θ-integral. Similarly, the “*Augmented Lateral Design Model*” is created from the vehicle design model by selecting the 5 lateral states (ϕ, p, ψ, r, β) and including a 6th state, the ψ-integral. By including attitude integrals in the LQR design models we improve command tracking because it eliminates the steady-state errors. The roll attitude does not need a trim integrator because the disturbances in roll are small. The LQR design matrices Qc and Rc are preserved in the systems file “*Rig_Vehi_T10.Qdr*”.



DP	DQ	DR
-0.0690	-0.336	0.00
-0.0286	-0.336	0.00
0.0286	-0.336	0.00
0.0690	-0.336	0.00
0.0690	-0.336	0.00
0.0286	-0.336	0.00
-0.0286	-0.336	0.00
-0.0690	-0.336	0.00
0.0286	0	-0.336
0.0690	0	-0.336
0.0690	0	-0.336
0.0286	0	-0.336
-0.0286	0	-0.336
-0.0690	0	-0.336
-0.0690	0	-0.336
-0.0286	0	-0.336

Figure 2.1.2 TVC Matrix

```
BATCH MODE INSTRUCTIONS ...............
Batch for Launch Vehicle Stage-1 Control Design at T=10 sec
! This batch set creates Pitch and Lateral State-Space models for a
! Classical Launch Vehicle and performs LQR Control Design
!
! --------------   LQR Matrices Qc,Rc for Pitch and Lateral
Retain Matrix    : Pitch State Weight Matrix Qc (4x4)
Retain Matrix    : Pitch Control Weight Matrix Rc
Retain Matrix    : Lateral State Weight Matrix Qc (6x6)
Retain Matrix    : Lateral Control Weight Matrix Rc (2x2)
!
! --------------   Create Vehicle Models
Flight Vehicle   : Launch Vehicle First Stage Design Model, T=10.0 sec
Mixing Matrix    : Vehicle Mixing Logic
Transf-Functions : Integrator
System Connection: Vehicle Design Model with TVC
System Connection: Vehicle Analysis Model with TVC
!
! --------------   Pitch LQR Design on Augmented Model
System Modificat : Pitch Design Model
System Connection: Augmented Pitch Design Model
LQR Control Des  : Pitch LQR Control Design
!
! --------------   Lateral LQR Design on Augmented Model
System Modificat : Lateral Design Model
System Connection: Augmented Lateral Design Model
LQR Control Des  : Lateral LQR Control Design
!
! --------------   Export Systems & TVC to Matlab
To Matlab Format : Vehicle Analysis Model with TVC
To Matlab Format : Vehicle Mixing Logic
To Matlab Format : Pitch LQR State-Feedback Controller
To Matlab Format : Lateral LQR State-Feedback Controller
To Matlab Format : Augmented Pitch Design Model
To Matlab Format : Augmented Lateral Design Model
------------------------------------------------------------------------------
MIXING LOGIC MATRIX DATA ..........    (Matrix Title, Name, Vehicle Title, Control Directions)
Vehicle Mixing Logic
! Generates the Thrust Vector Control Matrix for the Launch Vehicle at T=10 sec
! This vehicle has 8 Gimbaling Engines.
TVC
Launch Vehicle First Stage Design Model, T=10.0 sec
P-dot Roll  Acceleration About X Axis
Q-dot Pitch Acceleration About Y Axis
R-dot Yaw   Acceleration About Z Axis
-----------------------------------------------------------------------------------------------------------
SYSTEM OF TRANSFER FUNCTIONS ...
Integrator
Continuous
TF. Block #  1  (1/s)                                            Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
-------------------------------------------------------------------------------
Block #, from Input #, Gain
  1  1    1.00000
...........................
Outpt #, from Block #, Gain
  1  1    1.00000
...........................
-----------------------------------------------------------------------------------------------------------
```

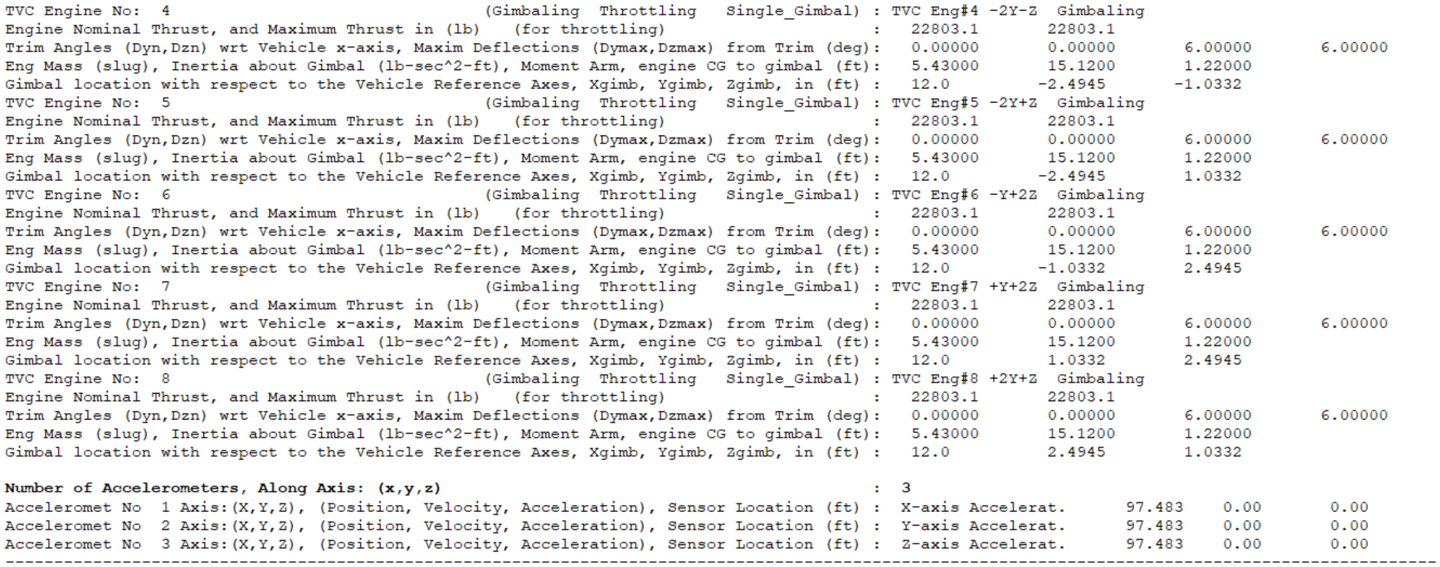


```
FLIGHT VEHICLE INPUT DATA ......
Launch Vehicle First Stage Design Model, T=10.0 sec
! This is the Launch Vehicle Control Design Model at t=10 sec
! includes 8 TVC Engines, No Slosh, No Bending
!
Body Axes Output, Attitude=Rate Integrals

Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft)    :  5070.0       32.1740      0.208960E+08
Moments and Products of Inertia: Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)          :  36805.3     0.281044E+07  0.280948E+07   0.00000
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)        :  57.8656      0.00000      0.00000
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)       :  0.071        79.6400      7.38000        384.843
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)         :  8.8          40.88        0.0           -0.001
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                         :  0.54         0.0         -0.09           0.000
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)   :  0.0          89.67        0.00000        0.0
W-Gust Azim & Elev angles (deg), or Torque/Force direction (x,y,z), Force Locat (x,y,z)  :  Gust         45.00        90.00
Surface Reference Area (feet^2), Mean Aerodynamic Chord (ft), Wing Span in (feet)        :  44.415       7.52000      7.52000
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial_rho/ Partial_H} :  120.147      0.00000      0.00000        0.00000
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:  0.4405      -0.00081      0.00000        0.00000
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}: -0.0         -0.03557      0.00000        0.00000
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}: -0.0061      -0.03557      0.00000        0.00000
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:  0.00000      0.00000      0.00000        0.00000
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}:  0.27         0.1114       0.00000        0.00000
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}:  0.0         -0.1114       0.00000        0.00000

Number of Thruster Engines, Include or Not the Tail-Wags-Dog and Load-Torque Dynamics ?  :  8   NO TWD

TVC Engine No:  1                              (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#1 +2Y-Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                     :  22803.1      22803.1
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000        6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000      15.1200      1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0         2.4945      -1.0332
TVC Engine No:  2                              (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#2 +Y-2Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                     :  22803.1      22803.1
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000        6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000      15.1200      1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0         1.0332      -2.4945
TVC Engine No:  3                              (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#3 -Y-2Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                     :  22803.1      22803.1
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000        6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000      15.1200      1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0        -1.0332      -2.4945
TVC Engine No:  4                              (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#4 -2Y-Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                     :  22803.1      22803.1
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000        6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000      15.1200      1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0        -2.4945      -1.0332
```



```
TVC Engine No:  4                              (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#4 -2Y-Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                     :  22803.1      22803.1
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000        6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000      15.1200      1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0        -2.4945      -1.0332
TVC Engine No:  5                              (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#5 -2Y+Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                     :  22803.1      22803.1
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000        6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000      15.1200      1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0        -2.4945       1.0332
TVC Engine No:  6                              (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#6 -Y+2Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                     :  22803.1      22803.1
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000        6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000      15.1200      1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0        -1.0332       2.4945
TVC Engine No:  7                              (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#7 +Y+2Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                     :  22803.1      22803.1
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000        6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000      15.1200      1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0         1.0332       2.4945
TVC Engine No:  8                              (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#8 +2Y+Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                     :  22803.1      22803.1
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000        6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000      15.1200      1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0         2.4945       1.0332

Number of Accelerometers, Along Axis: (x,y,z)                                             :  3
Acceleromet No  1 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Location (ft) :  X-axis Accelerat.     97.483    0.00       0.00
Acceleromet No  2 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Location (ft) :  Y-axis Accelerat.     97.483    0.00       0.00
Acceleromet No  3 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Location (ft) :  Z-axis Accelerat.     97.483    0.00       0.00
----------------------------------------------------------------------------------------------------------------------------------------
```

The flight vehicle model includes only the 8 gimbaling engines. The 9th engine which does not gimbal is not included in the vehicle system because it is not used for control. The TWD flag is set to “No TWD” because the TWD dynamics are not included in the design model and the TWD parameters in the engines are ignored. The 3 accelerometers are not used in this case and they could be taken out. The next two datasets create systems by combining the vehicle with the TVC matrix at the input to create the design model. The vehicle design model is then split into pitch and lateral systems for the LQR design, as shown below.

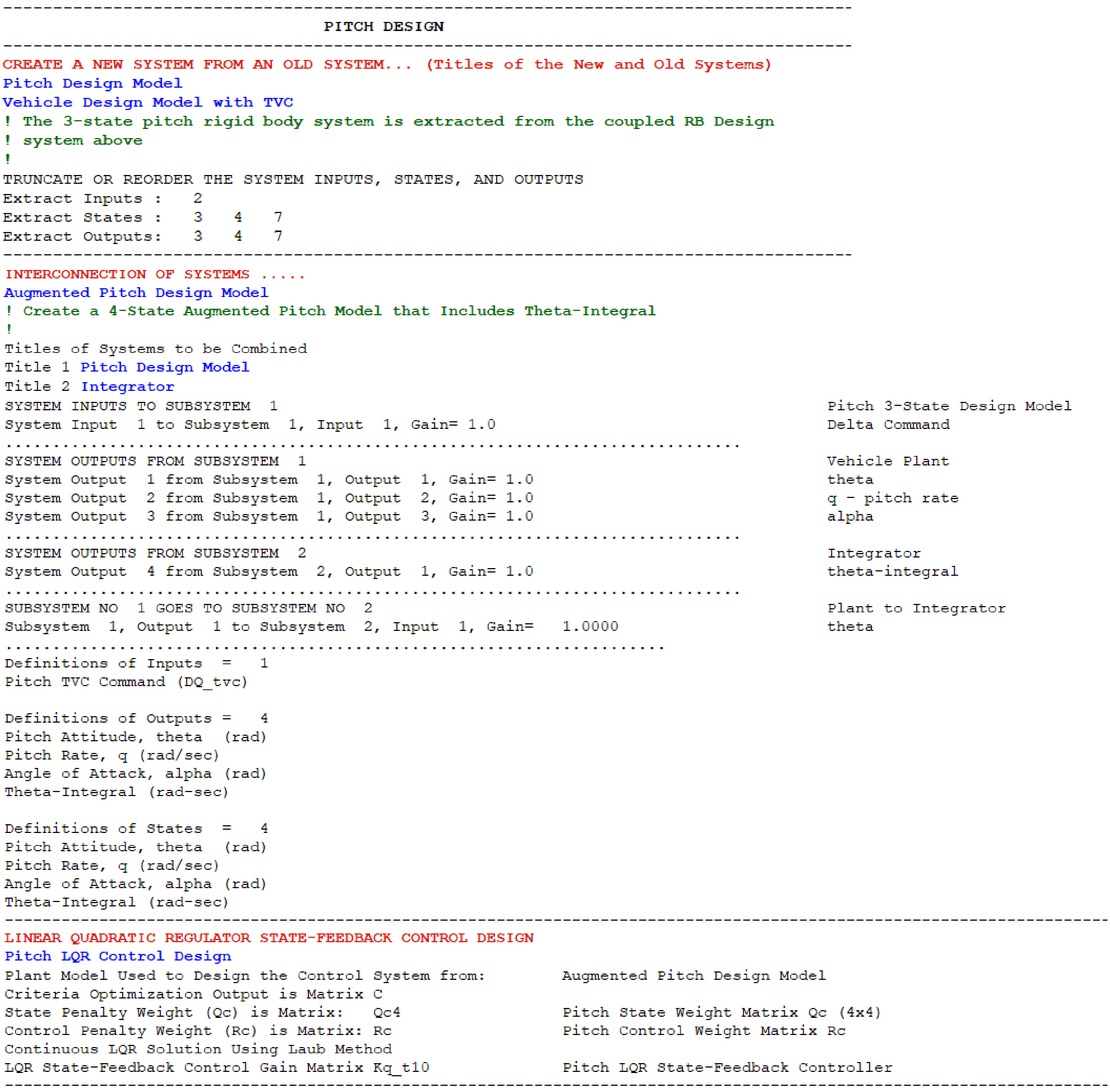
```
INTERCONNECTION OF SYSTEMS .....
Vehicle Design Model with TVC
! Combines the Vehicle Model with the TVC Matrix
!
Titles of Systems to be Combined
Title 1 Launch Vehicle First Stage Design Model, T=10.0 sec
SYSTEM INPUTS TO SUBSYSTEM  1                                                                   TVC to Vehicle
Via Matrix +TVC                                                                                 roll, pitch, yaw
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                                                Vehicle Outputs
Via Matrix +I8                                                                                  First 8 Outputs
..............................................................................
Definitions of Inputs  =  3
Roll  TVC Demand from Flight Control
Pitch TVC Demand from Flight Control
Yaw   TVC Demand from Flight Control

Definitions of Outputs =  8
Roll Attitude  (phi)     (radians)
Roll Rate      (p-body)  (rad/sec)
Pitch Attitude (theta)   (radians)
Pitch Rate     (q-body)  (rad/sec)
Yaw  Attitude  (psi)     (radians)
Yaw  Rate      (r-body)  (rad/sec)
Angle of attack, alfa,   (radians)
Angle of sideslip, beta, (radian)

Definitions of States  = 10
Roll Attitude  (phi)     (radians)
Roll Rate      (p-body)  (rad/sec)
Pitch Attitude (theta)   (radians)
Pitch Rate     (q-body)  (rad/sec)
Yaw  Attitude  (psi)     (radians)
Yaw  Rate      (r-body)  (rad/sec)
Angle of attack, alfa,   (radians)
Angle of sideslip, beta, (radian)
Change in Altitude (delta-h)  (feet)
Change in Velocity (delta-V)  (ft/sec)
------------------------------------------------------------------------------------------------------------------
INTERCONNECTION OF SYSTEMS .....
Vehicle Analysis Model with TVC
! Combines the Vehicle Model with the TVC Matrix
! It includes the Wind-Gust Input
!
Titles of Systems to be Combined
Title 1 Launch Vehicle First Stage Design Model, T=10.0 sec
SYSTEM INPUTS TO SUBSYSTEM  1                                                                  TVC to Vehicle
Via Matrix +TVC                                                                                roll, pitch, yaw Demands
Via Matrix +I01                                                                                Gust Input
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                                               Vehicle Outputs
Via Matrix +I8                                                                                 First 8 Vehi Outputs
..............................................................................
Definitions of Inputs  =  4
Roll  TVC Demand from Flight Control
Pitch TVC Demand from Flight Control
Yaw   TVC Demand from Flight Control
Wind-Gust Velocity in (ft/sec)

Definitions of Outputs =  8
Roll Attitude  (phi)     (radians)
Roll Rate      (p-body)  (rad/sec)
Pitch Attitude (theta)   (radians)
Pitch Rate     (q-body)  (rad/sec)
Yaw  Attitude  (psi)     (radians)
Yaw  Rate      (r-body)  (rad/sec)
Angle of attack, alfa,   (radians)
Angle of sideslip, beta, (radian)

Definitions of States  = 10
Roll Attitude  (phi)     (radians)
Roll Rate      (p-body)  (rad/sec)
Pitch Attitude (theta)   (radians)
Pitch Rate     (q-body)  (rad/sec)
Yaw  Attitude  (psi)     (radians)
Yaw  Rate      (r-body)  (rad/sec)
Angle of attack, alfa,   (radians)
Angle of sideslip, beta, (radian)
Change in Altitude (delta-h)  (feet)
Change in Velocity (delta-V)  (ft/sec)
------------------------------------------------------------------------------------------------------------------------
```

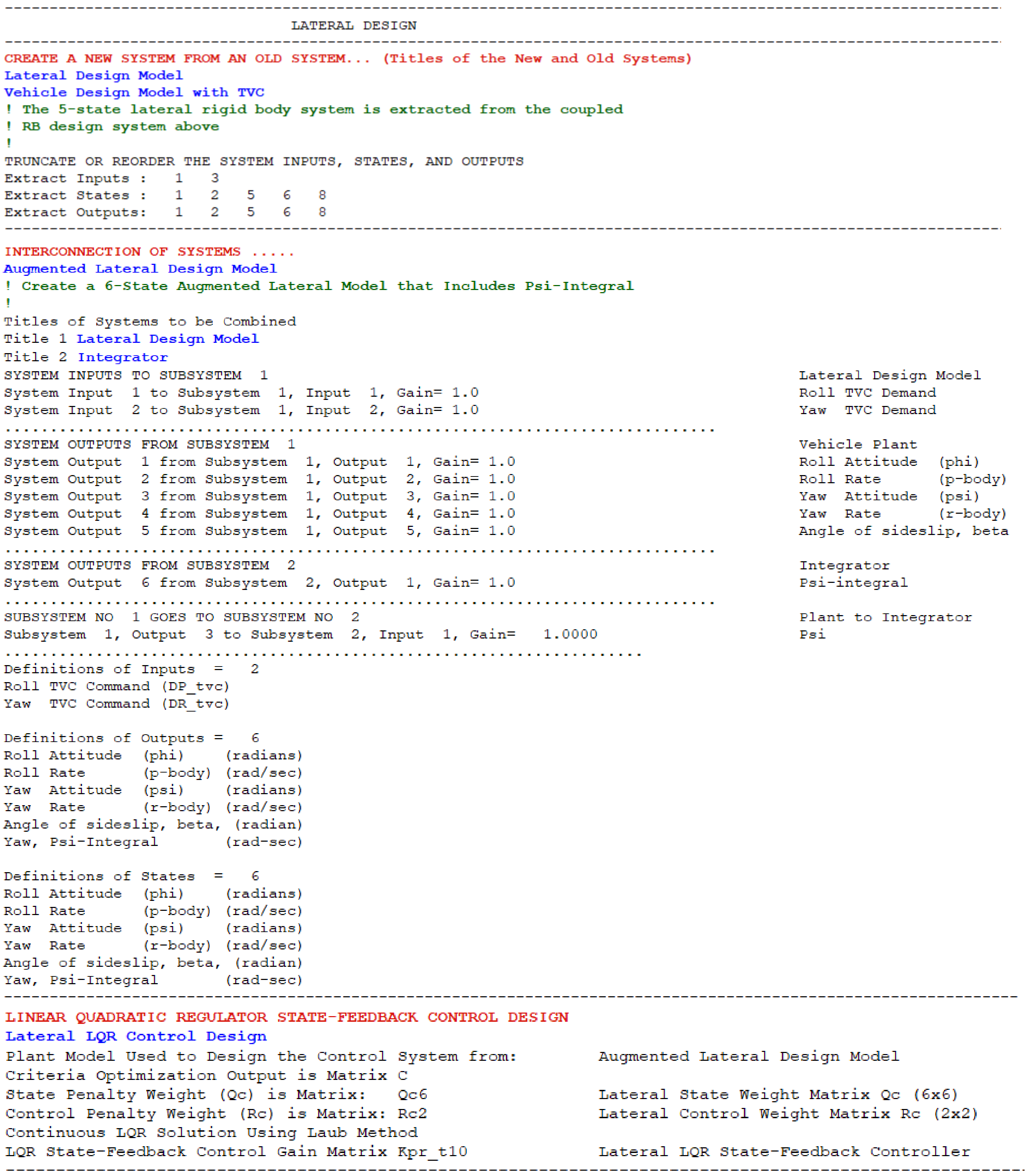


```
-----------------------------------------------------------------------------------
                                PITCH DESIGN
-----------------------------------------------------------------------------------
CREATE A NEW SYSTEM FROM AN OLD SYSTEM... (Titles of the New and Old Systems)
Pitch Design Model
Vehicle Design Model with TVC
! The 3-state pitch rigid body system is extracted from the coupled RB Design
! system above
!
TRUNCATE OR REORDER THE SYSTEM INPUTS, STATES, AND OUTPUTS
Extract Inputs :    2
Extract States :    3    4    7
Extract Outputs:    3    4    7
-----------------------------------------------------------------------------------
INTERCONNECTION OF SYSTEMS .....
Augmented Pitch Design Model
! Create a 4-State Augmented Pitch Model that Includes Theta-Integral
!
Titles of Systems to be Combined
Title 1 Pitch Design Model
Title 2 Integrator
SYSTEM INPUTS TO SUBSYSTEM  1                                              Pitch 3-State Design Model
System Input  1 to Subsystem  1, Input  1, Gain= 1.0                       Delta Command
........................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                           Vehicle Plant
System Output  1 from Subsystem  1, Output  1, Gain= 1.0                   theta
System Output  2 from Subsystem  1, Output  2, Gain= 1.0                   q - pitch rate
System Output  3 from Subsystem  1, Output  3, Gain= 1.0                   alpha
........................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                                           Integrator
System Output  4 from Subsystem  2, Output  1, Gain= 1.0                   theta-integral
........................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2                                    Plant to Integrator
Subsystem  1, Output  1 to Subsystem  2, Input  1, Gain=   1.0000          theta
.................................................................
Definitions of Inputs  =   1
Pitch TVC Command (DQ_tvc)

Definitions of Outputs =   4
Pitch Attitude, theta  (rad)
Pitch Rate, q (rad/sec)
Angle of Attack, alpha (rad)
Theta-Integral (rad-sec)

Definitions of States  =   4
Pitch Attitude, theta  (rad)
Pitch Rate, q (rad/sec)
Angle of Attack, alpha (rad)
Theta-Integral (rad-sec)
---------------------------------------------------------------------------------------------
LINEAR QUADRATIC REGULATOR STATE-FEEDBACK CONTROL DESIGN
Pitch LQR Control Design
Plant Model Used to Design the Control System from:       Augmented Pitch Design Model
Criteria Optimization Output is Matrix C
State Penalty Weight (Qc) is Matrix:   Qc4                 Pitch State Weight Matrix Qc (4x4)
Control Penalty Weight (Rc) is Matrix: Rc                  Pitch Control Weight Matrix Rc
Continuous LQR Solution Using Laub Method
LQR State-Feedback Control Gain Matrix Kq_t10              Pitch LQR State-Feedback Controller
---------------------------------------------------------------------------------------------
```

In this early first stage flight condition the dynamic pressure is small but not trivial either. Our emphasis is in maintaining good attitude control and, therefore, in addition to (θ, q, α) we introduce a 4th state in the pitch LQR design model for state-feedback, that is, θ-integral for attitude trim. The angle of attack α is also included in the design model because it should also be optimized in order to prevent big alphas in response to gusts. The trade-off between control and state penalization in the LQR optimization is defined in the Qc4 and Rc matrices which are included in file "*Rig_Vehi_T10.Qdr*". The derived (1x4) state-feedback gain Kq_t10 is also saved in the systems file with the title "*Pitch LQR State-Feedback Controller*" **.**

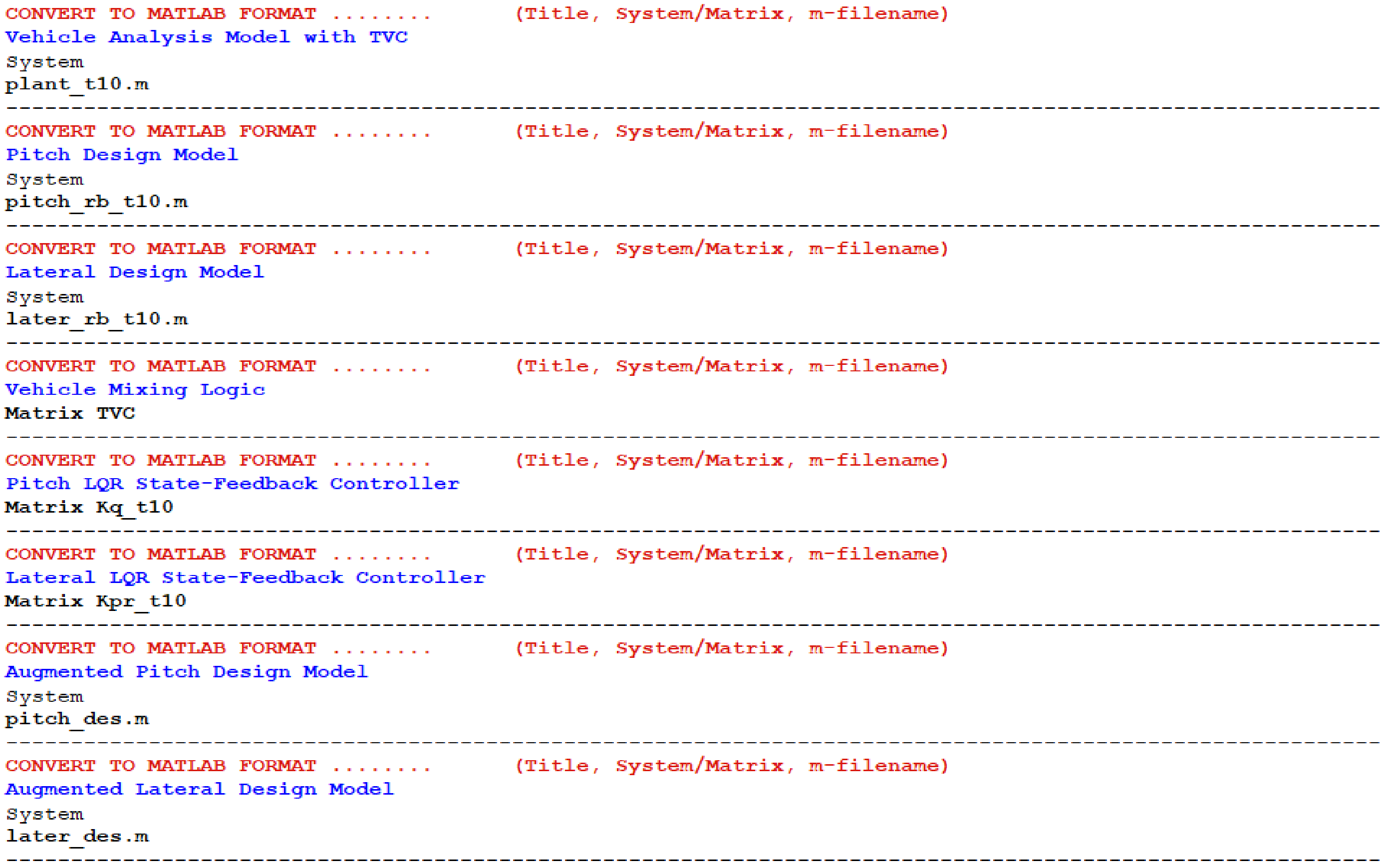


```
----------------------------------------------------------------------------------------
                              LATERAL DESIGN
----------------------------------------------------------------------------------------
CREATE A NEW SYSTEM FROM AN OLD SYSTEM... (Titles of the New and Old Systems)
Lateral Design Model
Vehicle Design Model with TVC
! The 5-state lateral rigid body system is extracted from the coupled
! RB design system above
!
TRUNCATE OR REORDER THE SYSTEM INPUTS, STATES, AND OUTPUTS
Extract Inputs :    1   3
Extract States :    1   2   5   6   8
Extract Outputs:    1   2   5   6   8
----------------------------------------------------------------------------------------
INTERCONNECTION OF SYSTEMS .....
Augmented Lateral Design Model
! Create a 6-State Augmented Lateral Model that Includes Psi-Integral
!
Titles of Systems to be Combined
Title 1 Lateral Design Model
Title 2 Integrator
SYSTEM INPUTS TO SUBSYSTEM  1                                   Lateral Design Model
System Input  1 to Subsystem  1, Input  1, Gain= 1.0            Roll TVC Demand
System Input  2 to Subsystem  1, Input  2, Gain= 1.0            Yaw  TVC Demand
..............................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                Vehicle Plant
System Output  1 from Subsystem  1, Output  1, Gain= 1.0        Roll Attitude  (phi)
System Output  2 from Subsystem  1, Output  2, Gain= 1.0        Roll Rate      (p-body)
System Output  3 from Subsystem  1, Output  3, Gain= 1.0        Yaw  Attitude  (psi)
System Output  4 from Subsystem  1, Output  4, Gain= 1.0        Yaw  Rate      (r-body)
System Output  5 from Subsystem  1, Output  5, Gain= 1.0        Angle of sideslip, beta
..............................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                                Integrator
System Output  6 from Subsystem  2, Output  1, Gain= 1.0        Psi-integral
..............................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2                         Plant to Integrator
Subsystem  1, Output  3 to Subsystem  2, Input  1, Gain=   1.0000  Psi
........................................................
Definitions of Inputs  =   2
Roll TVC Command (DP_tvc)
Yaw  TVC Command (DR_tvc)

Definitions of Outputs =   6
Roll Attitude  (phi)     (radians)
Roll Rate      (p-body)  (rad/sec)
Yaw  Attitude  (psi)     (radians)
Yaw  Rate      (r-body)  (rad/sec)
Angle of sideslip, beta, (radian)
Yaw, Psi-Integral        (rad-sec)

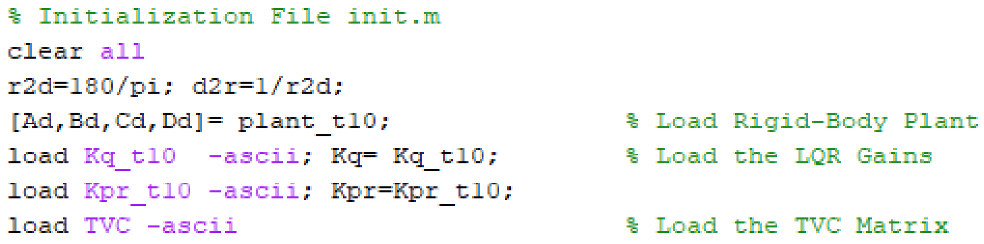
Definitions of States  =   6
Roll Attitude  (phi)     (radians)
Roll Rate      (p-body)  (rad/sec)
Yaw  Attitude  (psi)     (radians)
Yaw  Rate      (r-body)  (rad/sec)
Angle of sideslip, beta, (radian)
Yaw, Psi-Integral        (rad-sec)
----------------------------------------------------------------------------------------
LINEAR QUADRATIC REGULATOR STATE-FEEDBACK CONTROL DESIGN
Lateral LQR Control Design
Plant Model Used to Design the Control System from:     Augmented Lateral Design Model
Criteria Optimization Output is Matrix C
State Penalty Weight (Qc) is Matrix:    Qc6             Lateral State Weight Matrix Qc (6x6)
Control Penalty Weight (Rc) is Matrix: Rc2              Lateral Control Weight Matrix Rc (2x2)
Continuous LQR Solution Using Laub Method
LQR State-Feedback Control Gain Matrix Kpr_t10          Lateral LQR State-Feedback Controller
----------------------------------------------------------------------------------------
```

Similarly, the lateral design model has 5 states (ϕ, p, ψ, r, β) and we introduce a 6th state, ψ-integral for attitude trim. The calculated controller is a (2x6) state-feedback gain matrix Kpr_t10 and it is also saved in the systems file with the title "*Lateral LQR State-Feedback Controller*" .



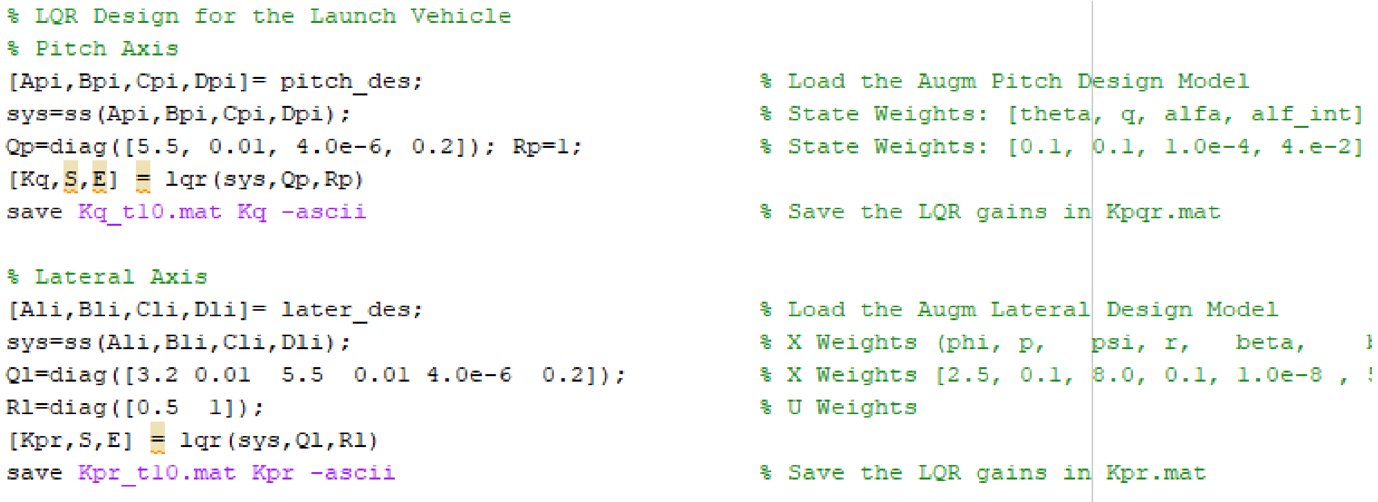
```
CONVERT TO MATLAB FORMAT ........       (Title, System/Matrix, m-filename)
Vehicle Analysis Model with TVC
System
plant_t10.m
----------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........       (Title, System/Matrix, m-filename)
Pitch Design Model
System
pitch_rb_t10.m
----------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........       (Title, System/Matrix, m-filename)
Lateral Design Model
System
later_rb_t10.m
----------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........       (Title, System/Matrix, m-filename)
Vehicle Mixing Logic
Matrix TVC
----------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........       (Title, System/Matrix, m-filename)
Pitch LQR State-Feedback Controller
Matrix Kq_t10
----------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........       (Title, System/Matrix, m-filename)
Lateral LQR State-Feedback Controller
Matrix Kpr_t10
----------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........       (Title, System/Matrix, m-filename)
Augmented Pitch Design Model
System
pitch_des.m
----------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........       (Title, System/Matrix, m-filename)
Augmented Lateral Design Model
System
later_des.m
----------------------------------------------------------------------------------------------
```

The systems and matrices are also saved as m-files to be loaded into Matlab by the initialization file "init.m".



```
% Initialization File init.m
clear all
r2d=180/pi; d2r=1/r2d;
[Ad,Bd,Cd,Dd]= plant_t10;                 % Load Rigid-Body Plant
load Kq_t10  -ascii; Kq= Kq_t10;          % Load the LQR Gains
load Kpr_t10 -ascii; Kpr=Kpr_t10;
load TVC -ascii                           % Load the TVC Matrix
```

The pitch and lateral LQR design can also be performed in Matlab by loading the pitch and lateral design systems and running the Matlab script file "LQR_Design.m", as shown below.



```
% LQR Design for the Launch Vehicle
% Pitch Axis
[Api,Bpi,Cpi,Dpi]= pitch_des;                       % Load the Augm Pitch Design Model
sys=ss(Api,Bpi,Cpi,Dpi);                            % State Weights: [theta, q, alfa, alf_int]
Qp=diag([5.5, 0.01, 4.0e-6, 0.2]); Rp=1;            % State Weights: [0.1, 0.1, 1.0e-4, 4.e-2]
[Kq,S,E] = lqr(sys,Qp,Rp)
save Kq_t10.mat Kq -ascii                           % Save the LQR gains in Kpqr.mat

% Lateral Axis
[Ali,Bli,Cli,Dli]= later_des;                       % Load the Augm Lateral Design Model
sys=ss(Ali,Bli,Cli,Dli);                            % X Weights (phi, p,   psi, r,   beta,
Ql=diag([3.2 0.01  5.5  0.01 4.0e-6  0.2]);         % X Weights [2.5, 0.1, 8.0, 0.1, 1.0e-8 ,
Rl=diag([0.5  1]);                                  % U Weights
[Kpr,S,E] = lqr(sys,Ql,Rl)
save Kpr_t10.mat Kpr -ascii                         % Save the LQR gains in Kpr.mat
```

Stability Analysis at T=10 sec

The quality of the LQR control system is first analyzed in the frequency domain using the coupled rigid-body analysis system in file “plant_t10.m”. The Simulink model “Open_RB.slx” is used to calculate the Bode and Nichols plots by running the Matlab script “freq.m. It is shown in Figure 2.1.3 configured for pitch open-loop analysis with the pitch loop opened and the lateral loops closed. Low-pass filters are included in the 3 control loops.

```
% Preliminary Stability Analysis freq.m
init;
[A1,B1,C1,D1]= linmod('Open_RB');       % Linearize Open-Loop Simulink model
sys= ss(A1,B1,C1,D1);                   % Create SS System
w=logspace(-3, 3, 24000);               % Define Frequ Range
figure(1); nichols(sys,w)               % Plot Nichol's Chart
figure(2); bode(sys,w)                  % Plot Bode
```

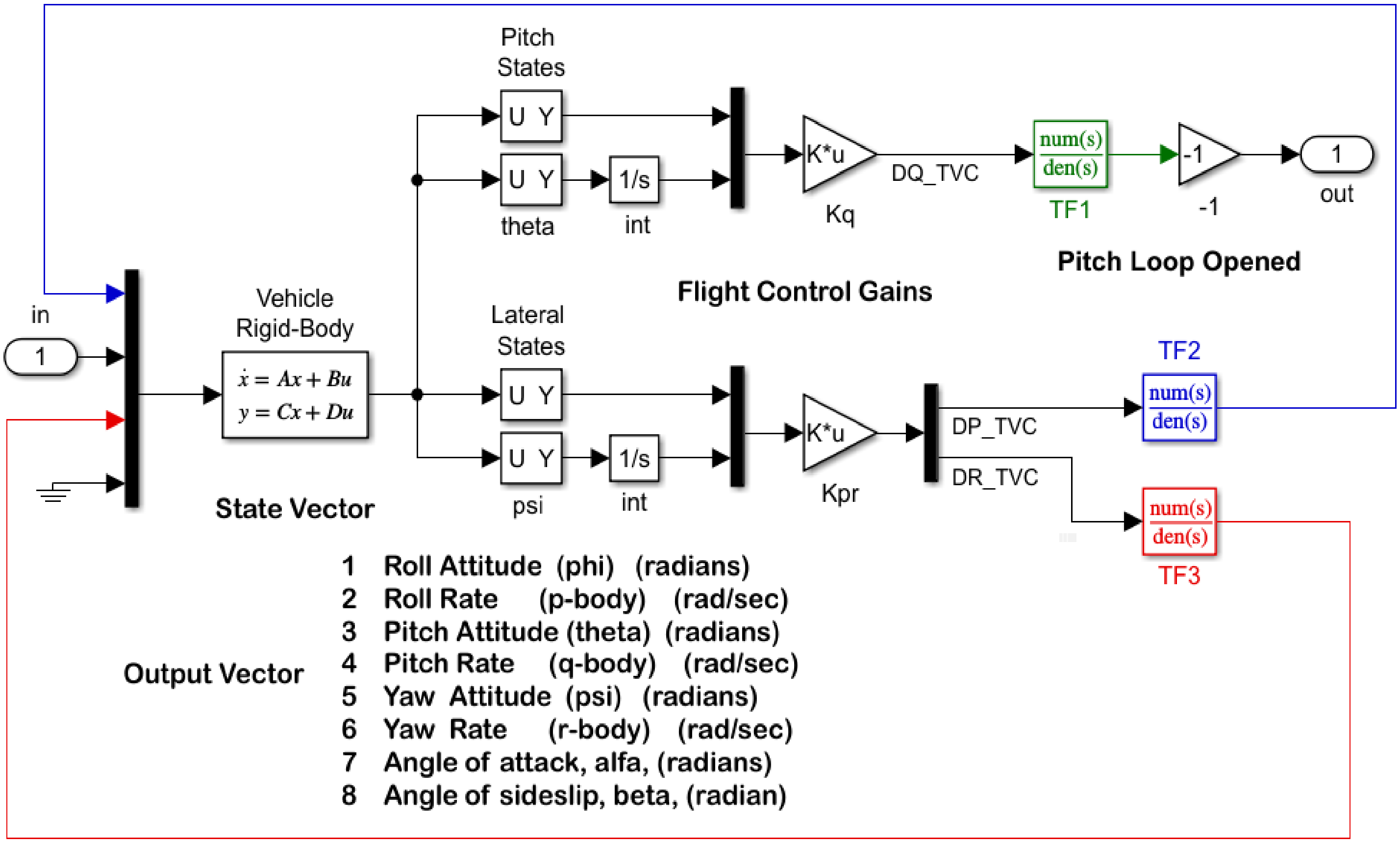


Figure 2.1.3 Preliminary Open-Loop Stability Analysis Model “Open_RB.Slx”

Figure 2.1.4 shows the Bode and Nichols plots in pitch calculated from the open-loop model. The yaw axis is identical. The control bandwidth is 2.5 (rad/sec) determined by the cross-over frequency. The phase and gain margins are more than sufficient for now without slosh and flexibility.

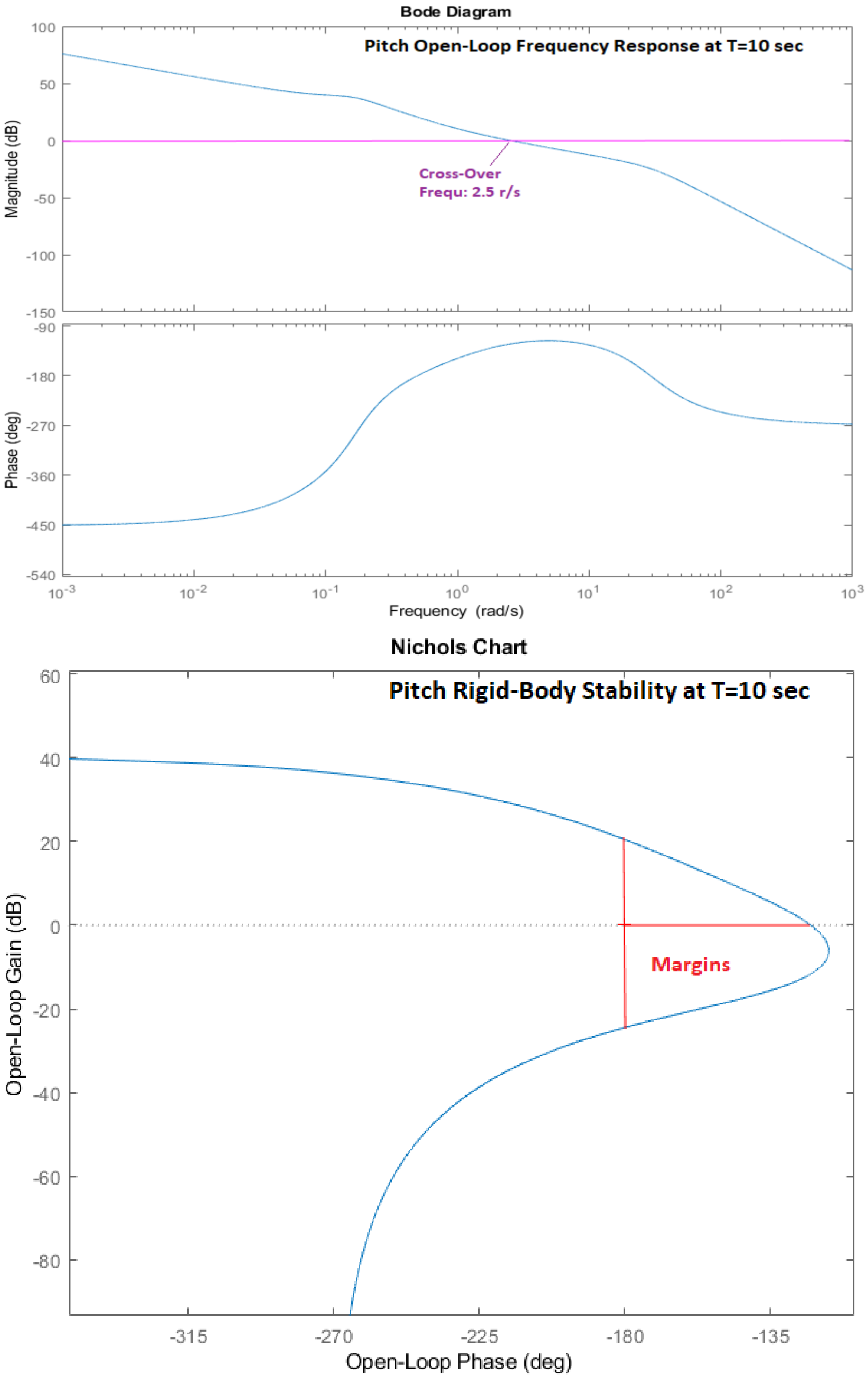


Figure 2.1.1 Pitch Bode and Nichols Plots Showing Plenty of Gain and Phase Margins

Simulation

A similar Simulink model with all 3 loops closed, “*Sim_RB.Slx*” in Figure 2.1.5, is used to analyze the system’s response to attitude commands and to wind-gust disturbances. It includes the rigid-body analysis system from file “plant_t10.m” that includes the TVC matrix. The LQR derived state-feedback gains Kq and Kpr are included in the control loops.

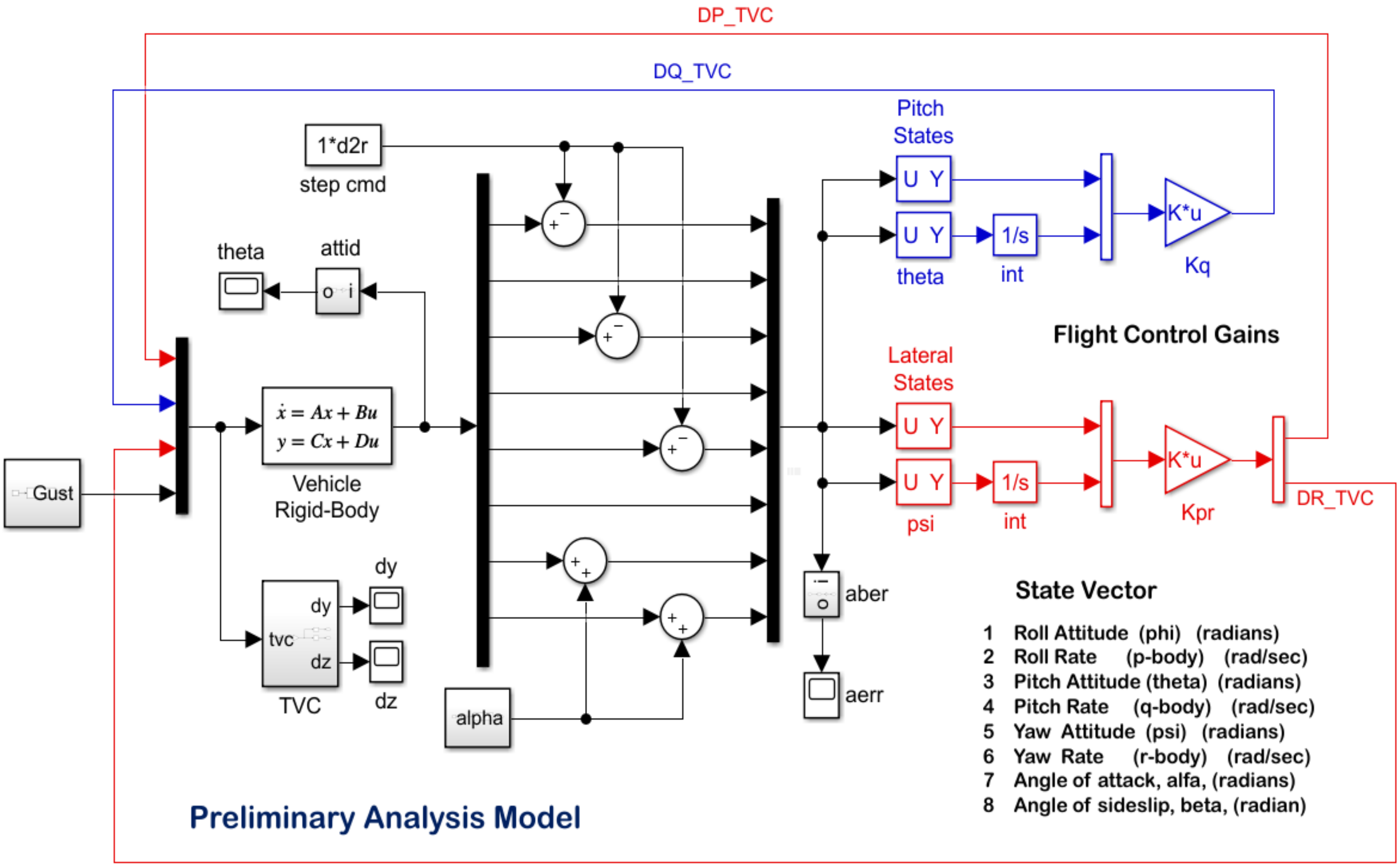


Figure 2.1.2 Rigid-Body Simulation Model “Sim_RB.Slx”

Figure 2.1.6 shows the closed loop system response to 1 (deg) attitude commands in roll, pitch and yaw axes simultaneously. The pitch and yaw responses are identical and they have slightly more overshoot than roll because of the attitude trim integrators. The initial gimbal deflections for the maneuver are 1°.

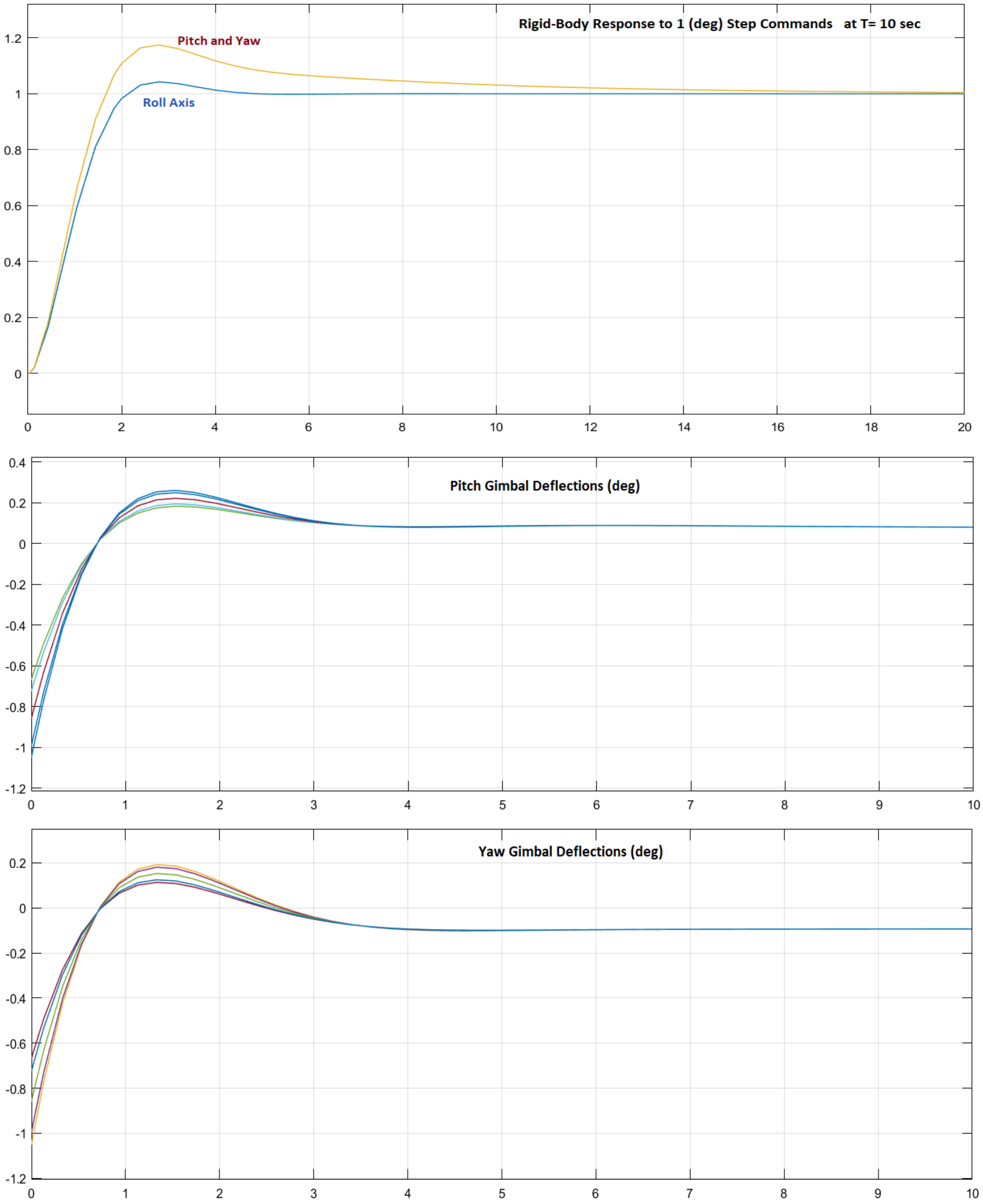
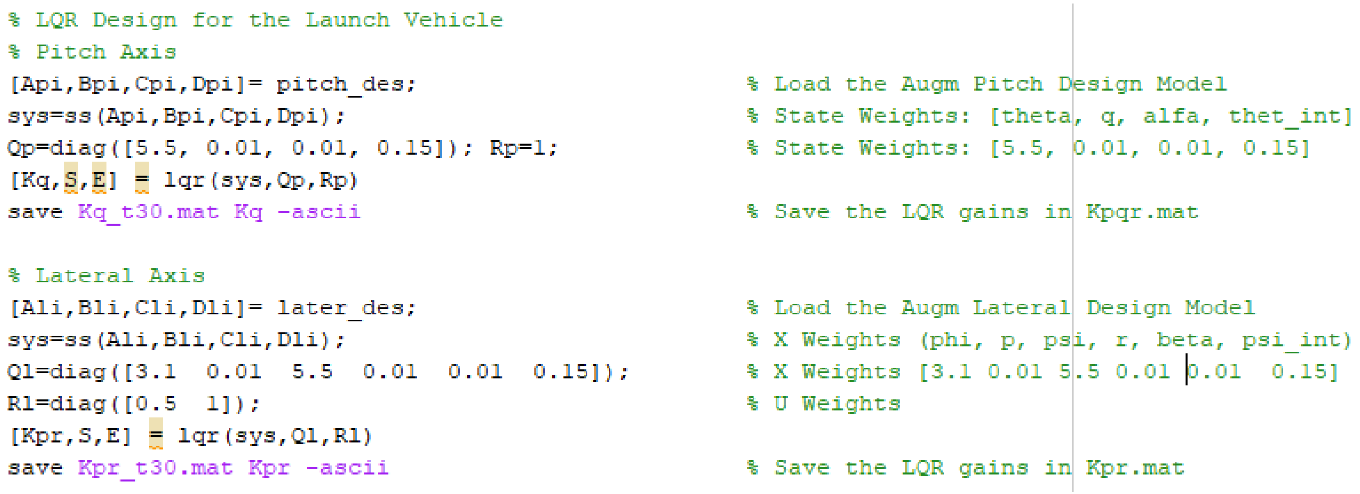


Figure 2.1.6 Attitude Response to 1° Commands and the Pitch and Yaw Gimbal Deflection in Response to Commands

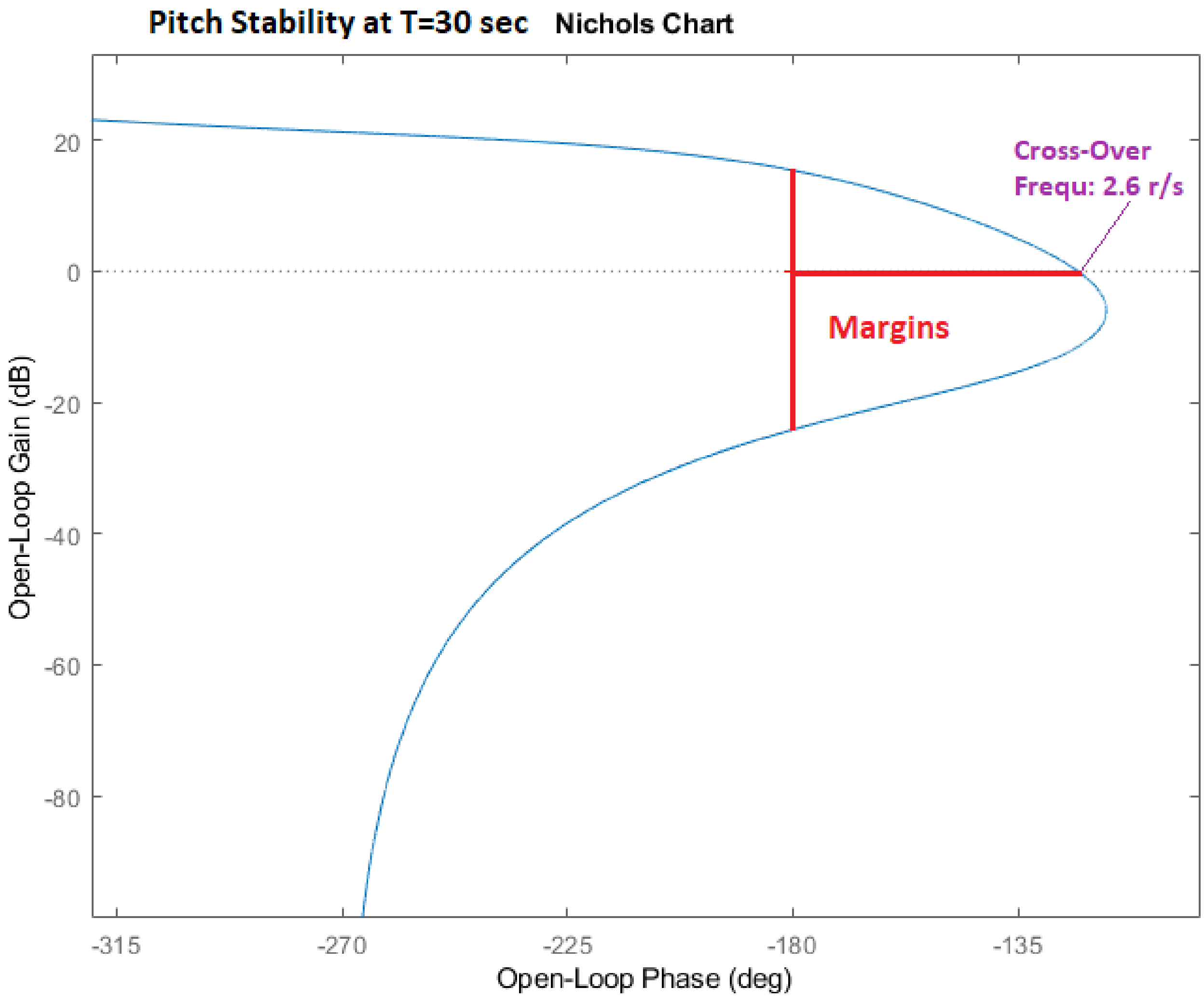
2.1.2 Control Design at T= 30 sec

At 30 sec after lift-off the dynamic pressure increases to 90 (lbf/ft^2). The control emphasis is still in tracking attitude commands from guidance and not yet in relieving the aerodynamic loading. The integral feedback is still applied in pitch and yaw attitude trimming. We are beginning, however, to increase the LQR penalties on the alpha and beta states. The roll and yaw axes are treated as a coupled lateral system producing a (2x6) state-feedback matrix Kpr_t30. However, in the analysis section the lateral control law will be separated because the roll/yaw coupling is small. The work files are in directory "*2-Control Gains Design\1st Stage\T30".*



```
% LQR Design for the Launch Vehicle
% Pitch Axis
[Api,Bpi,Cpi,Dpi]= pitch_des;                       % Load the Augm Pitch Design Model
sys=ss(Api,Bpi,Cpi,Dpi);                            % State Weights: [theta, q, alfa, thet_int]
Qp=diag([5.5, 0.01, 0.01, 0.15]); Rp=1;             % State Weights: [5.5, 0.01, 0.01, 0.15]
[Kq,S,E] = lqr(sys,Qp,Rp)
save Kq_t30.mat Kq -ascii                           % Save the LQR gains in Kpqr.mat

% Lateral Axis
[Ali,Bli,Cli,Dli]= later_des;                       % Load the Augm Lateral Design Model
sys=ss(Ali,Bli,Cli,Dli);                            % X Weights (phi, p, psi, r, beta, psi_int)
Ql=diag([3.1  0.01  5.5  0.01  0.01  0.15]);        % X Weights [3.1 0.01 5.5 0.01 0.01  0.15]
Rl=diag([0.5  1]);                                  % U Weights
[Kpr,S,E] = lqr(sys,Ql,Rl)
save Kpr_t30.mat Kpr -ascii                         % Save the LQR gains in Kpr.mat
```



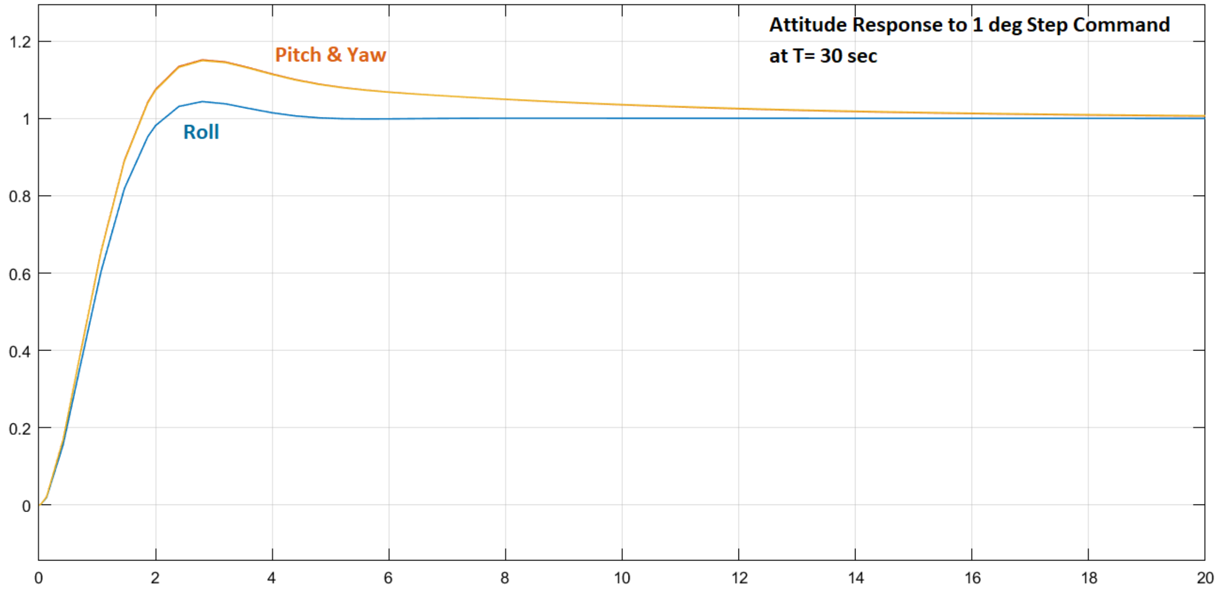


Figure 2.1.3 The Response to Step Attitude Commands is Very Good Because the Dynamic Pressure is Still Small at T30

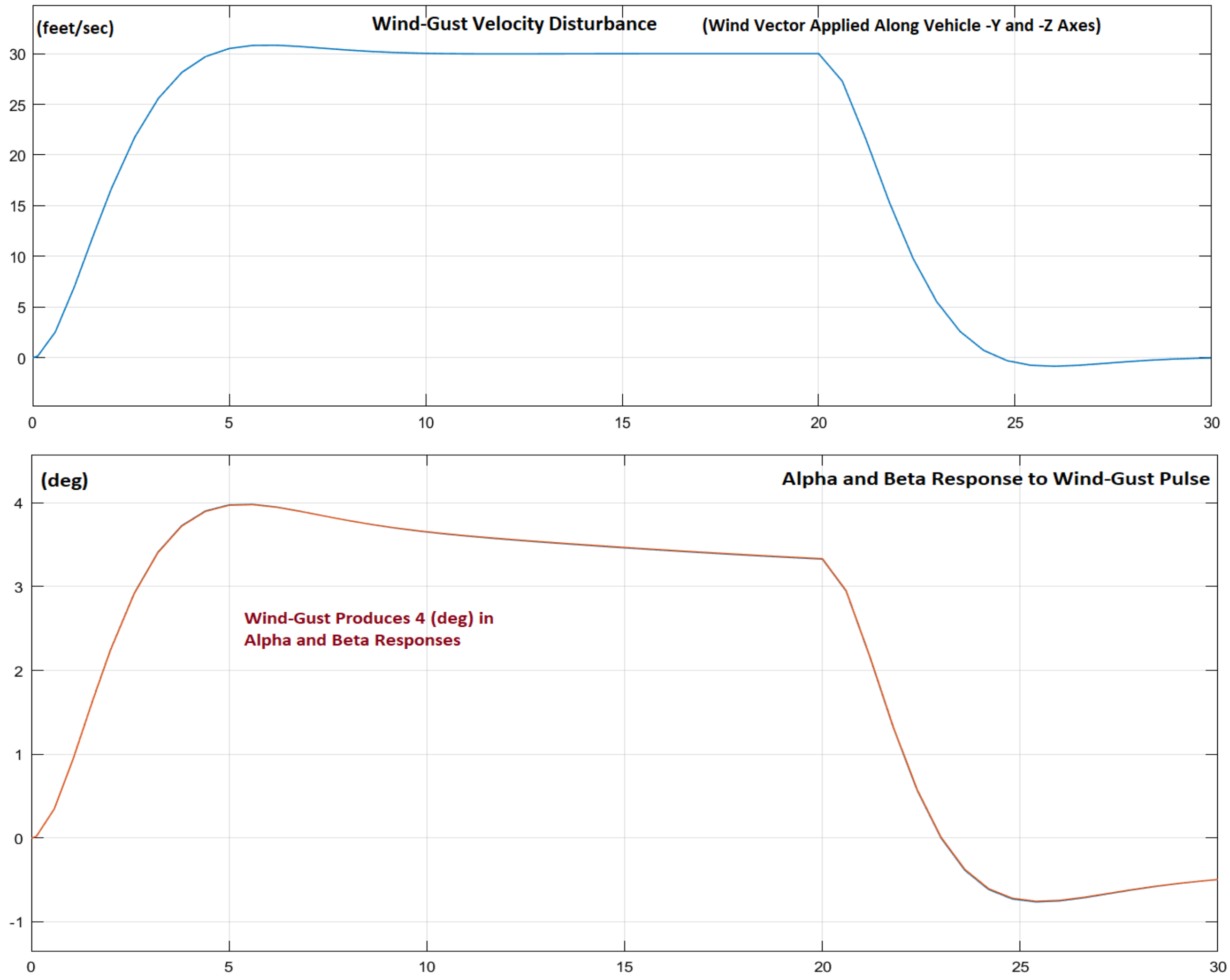
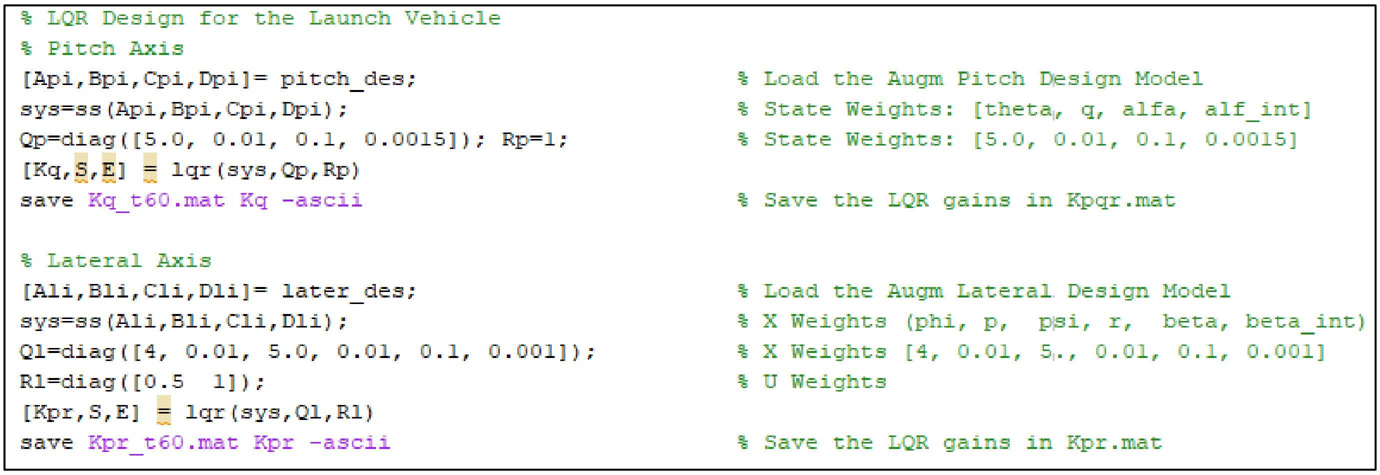


Figure 2.1.4 The Alpha and Beta Incidence Angles due to the Wind-Gust Pulse are Big in this Time-Slice because there is not much of a Load-relief action yet

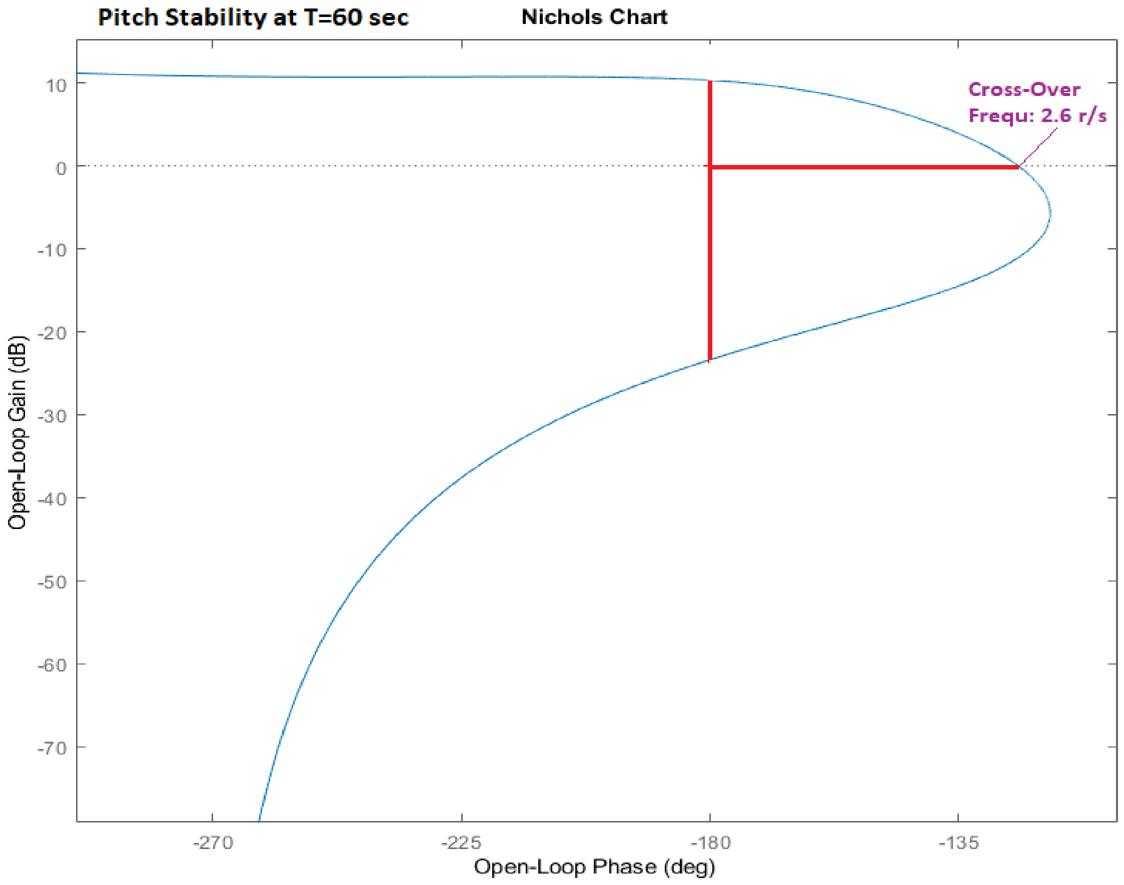
2.1.3 Control Design at T= 60 sec

At 60 sec after lift-off the dynamic pressure is increased to 416 (lbf/ft^2) and it is approaching towards Max-Q. We are now introducing load-relief feedback from (α & β) states and from (α & β) integrals. We are no longer feeding back (θ & ψ) integrals and the attitude command following is therefore expected to deteriorate a little in comparison with the low-Q cases. The pitch and lateral design models now include (α & β) integrals instead of (θ & ψ) integrals. The alpha and beta weights in the LQR optimization are further increased from the previous cases but the weights on (α & β) integral are small. The pitch controller is (1x4) state-feedback gain matrix Kq_t60 from states: θ, q, α, α-integral. The roll and yaw axes are still treated as a coupled lateral system for now, creating a (2x6) state-feedback controller matrix Kpr_t60 from states: ϕ, p, ψ, r, β, β-integral. The work files are in directory: "*\2-Control Gains Design\1st Stage\T60*".



```
% LQR Design for the Launch Vehicle
% Pitch Axis
[Api,Bpi,Cpi,Dpi]= pitch_des;                       % Load the Augm Pitch Design Model
sys=ss(Api,Bpi,Cpi,Dpi);                            % State Weights: [theta, q, alfa, alf_int]
Qp=diag([5.0, 0.01, 0.1, 0.0015]); Rp=1;            % State Weights: [5.0, 0.01, 0.1, 0.0015]
[Kq,S,E] = lqr(sys,Qp,Rp)
save Kq_t60.mat Kq -ascii                           % Save the LQR gains in Kpqr.mat

% Lateral Axis
[Ali,Bli,Cli,Dli]= later_des;                       % Load the Augm Lateral Design Model
sys=ss(Ali,Bli,Cli,Dli);                            % X Weights (phi, p,  psi, r,  beta, beta_int)
Ql=diag([4, 0.01, 5.0, 0.01, 0.1, 0.001]);          % X Weights [4, 0.01, 5., 0.01, 0.1, 0.001]
Rl=diag([0.5  1]);                                  % U Weights
[Kpr,S,E] = lqr(sys,Ql,Rl)
save Kpr_t60.mat Kpr -ascii                         % Save the LQR gains in Kpr.mat
```



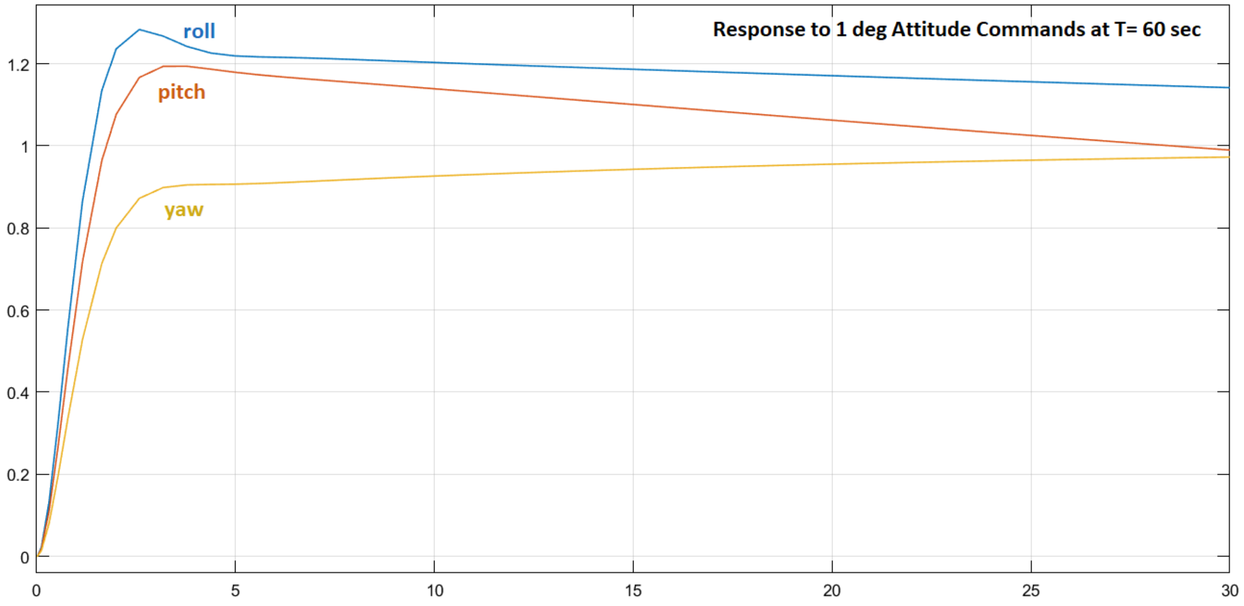


Figure 2.1.5 The Response to 1° Step Attitude Commands is beginning to deteriorate at T=60 sec due to the Load-Relief

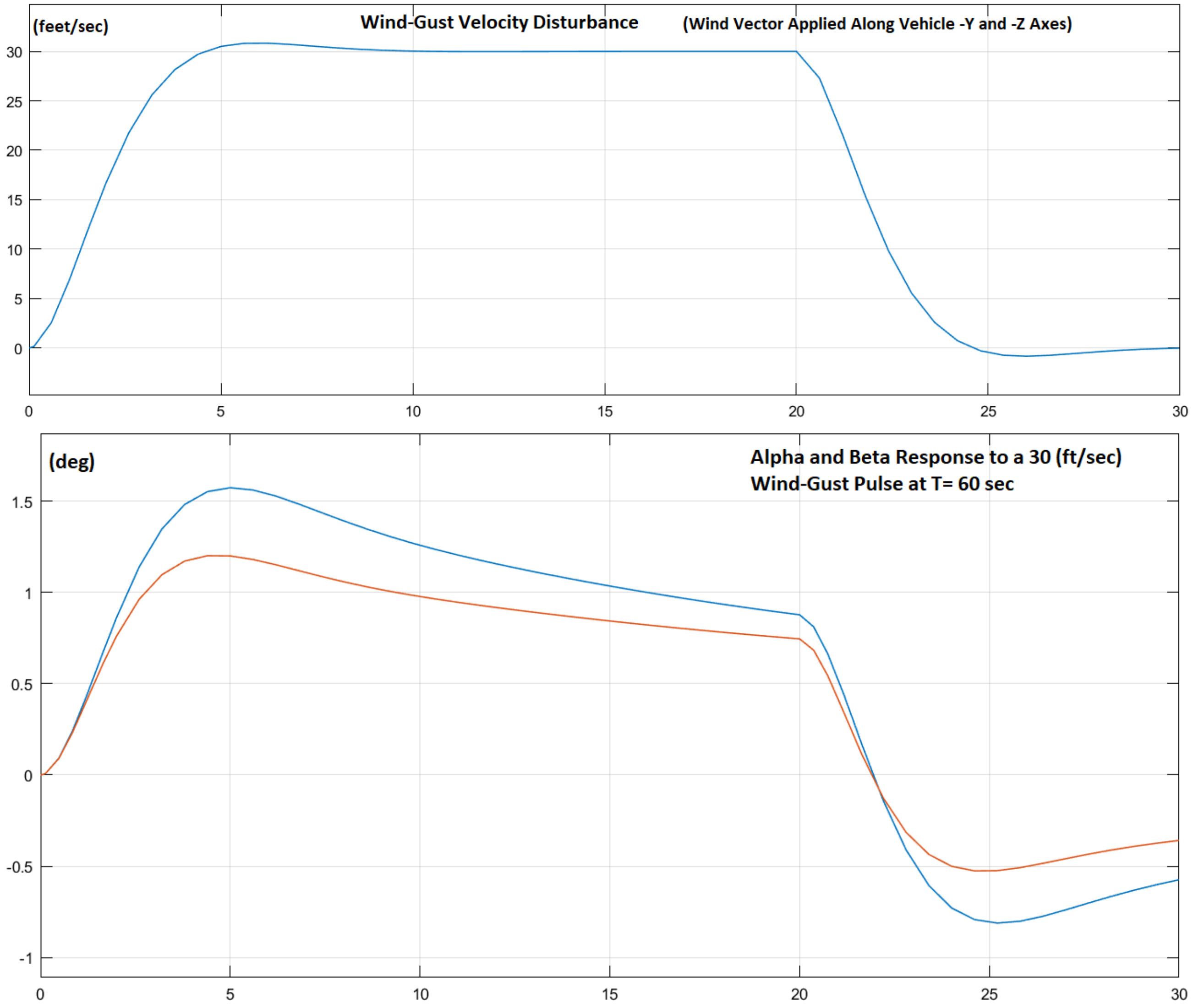


Figure 2.1.6 The Alpha and Beta Incidence Angles Caused by the Wind-Gust Pulse are getting Smaller at T60 due to the Load-Relief action

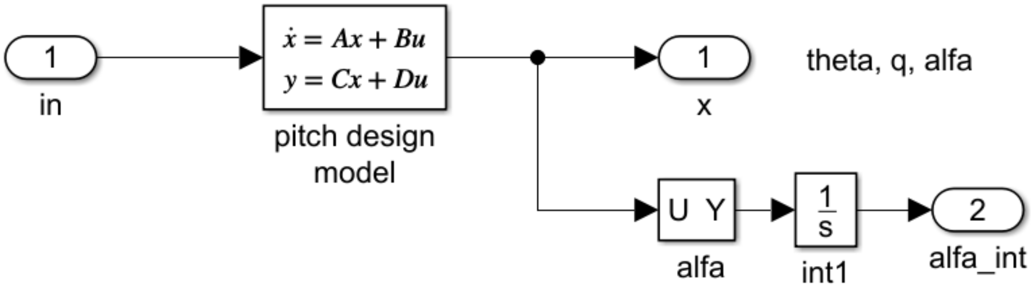
2.1.4 Control Design at Max-Q, T= 80 sec

Max-Q occurs 80 seconds after lift-off when the dynamic pressure is 530 (lbf/ft^2). The control emphasis during the high Q phase is to relieve the aerodynamic loads rather than tracking the guidance commands which are small during this high loading period. The attitude gains are de-emphasized and the gains from the angle of attack, sideslip and (α,β)-integral are increased in the LQR design. The design files in this case are in directory "*\Examples\23-Classic Launch Vehicle Design & Simulation\2-Control Gains Design\1st Stage\T80".*

Vehicle Input File

The Flixan input file is "*Rig_Vehi_T80.Inp*". It includes a batch set "*Batch for Launch Vehicle Stage-1 Control Design at T=80 sec*", the vehicle dataset "*Launch Vehicle First Stage Design Model, T=80.0 sec*", the TVC mixing-logic, the design and analysis models which include the TVC matrix, the pitch and lateral design models, and the LQR design datasets that calculate the pitch and lateral state-feedback gains Kq_t80 and Kpr_t80. Unlike the low Q-bar cases, the "*Augmented Pitch Design Model*" is now created from the vehicle design model by selecting the 3 pitch states (θ, q, α) and including the α-integral as a 4th state. Similarly, the "*Augmented Lateral Design Model*" is created from the vehicle design model by selecting the 5 lateral states (ϕ, p, ψ, r, β) and including the β-integral as 6th state. This type of state-feedback reduces the aerodynamic loading on the vehicle, especially in the frequency range between 0.5 – 1 (rad/sec), where the gust disturbances are stronger, while still maintaining a substantial amount of command following.

Augmented Pitch Design Model with Alpha Integral



Augmented Lateral Design Model with Beta Integral

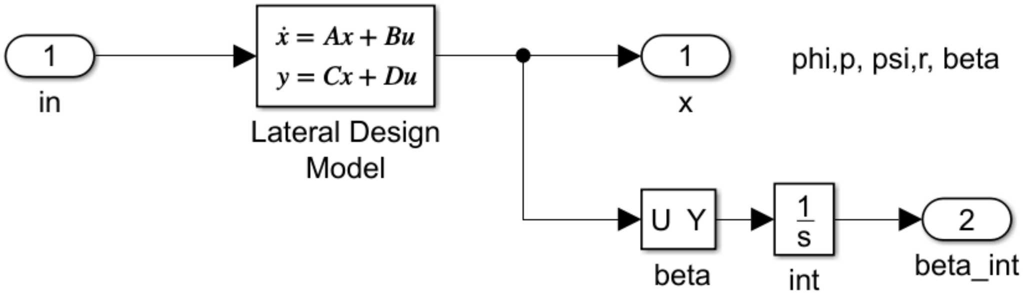
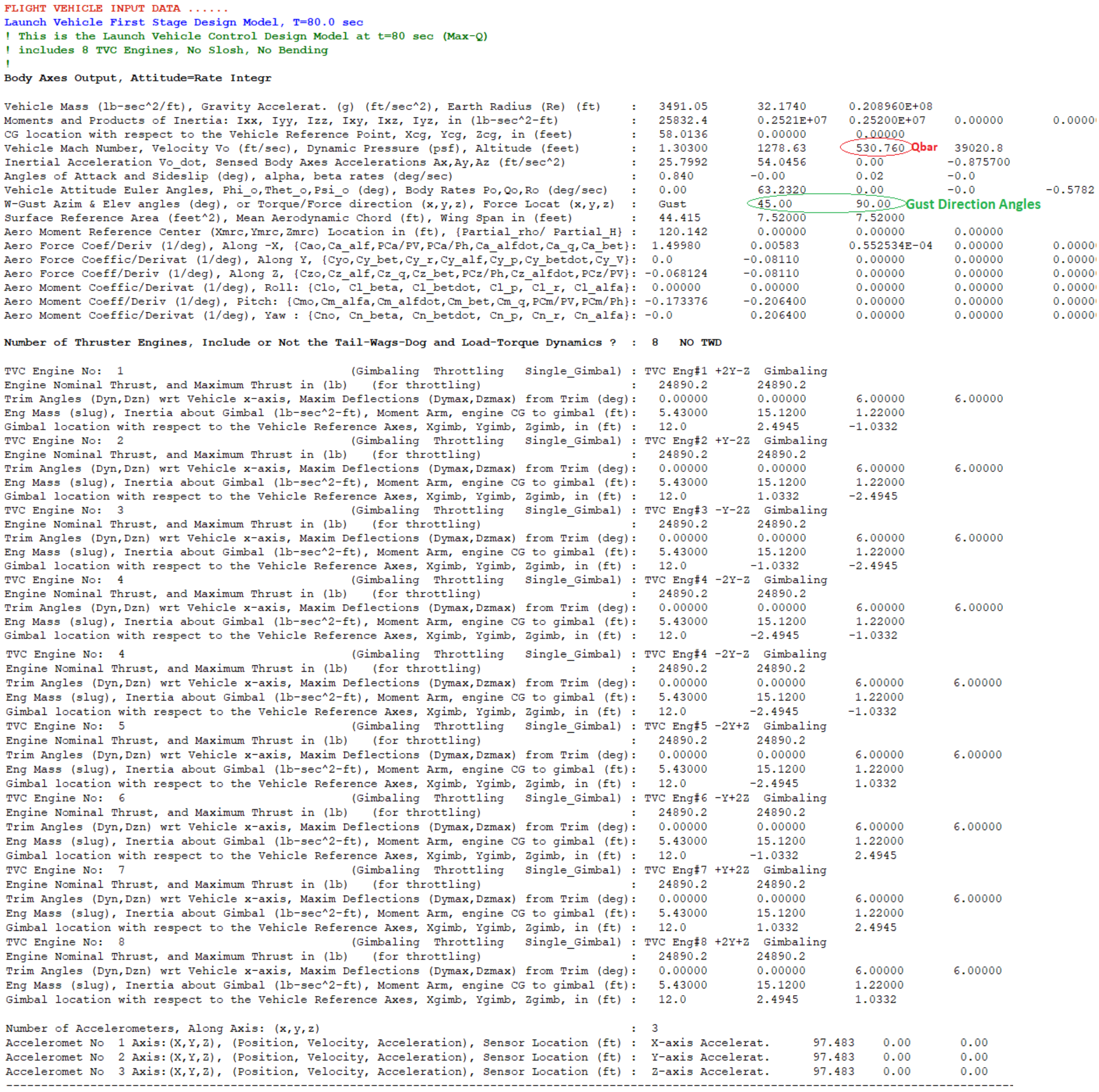


Figure 2.1.7 Pitch and Lateral Models Used for LQR Control Design During High Dynamic Pressures

Flixan Input File: Rig_Vehi_T80.Inp

```
BATCH MODE INSTRUCTIONS ...............
Batch for Launch Vehicle Stage-1 Control Design at T=80 sec
! This batch set creates Pitch and Lateral State-Space models for a
! Classical Launch Vehicle and performs LQR Control Design
!
! --------------   LQR Matrices Qc,Rc for Pitch and Lateral
Retain Matrix     : Pitch State Weight Matrix Qc (4x4)
Retain Matrix     : Pitch Control Weight Matrix Rc
Retain Matrix     : Lateral State Weight Matrix Qc (6x6)
Retain Matrix     : Lateral Control Weight Matrix Rc (2x2)
!
! --------------   Create Vehicle Models
Flight Vehicle    : Launch Vehicle First Stage Design Model, T=80.0 sec
Mixing Matrix     : Vehicle Mixing Logic
Transf-Functions  : Integrator
System Connection: Vehicle Design Model with TVC
System Connection: Vehicle Analysis Model with TVC
!
! --------------   Pitch LQR Design on Augmented Model
System Modificat  : Pitch Design Model
System Connection: Augmented Pitch Design Model
LQR Control Des   : Pitch LQR Control Design
!
! --------------   Lateral LQR Design on Augmented Model
System Modificat  : Lateral Design Model
System Connection: Augmented Lateral Design Model
LQR Control Des   : Lateral LQR Control Design
!
! --------------   Export Systems & TVC to Matlab
To Matlab Format  : Vehicle Analysis Model with TVC
To Matlab Format  : Vehicle Mixing Logic
To Matlab Format  : Pitch LQR State-Feedback Controller
To Matlab Format  : Lateral LQR State-Feedback Controller
To Matlab Format  : Augmented Pitch Design Model
To Matlab Format  : Augmented Lateral Design Model
---------------------------------------------------------------------------------------------
MIXING LOGIC MATRIX DATA ..........     (Matrix Title, Name, Vehicle Title, Control Directions)
Vehicle Mixing Logic
! Generates the Thrust Vector Control Matrix for the Launch Vehicle at T=80 sec
! This vehicle has 8 Gimbaling Engines.
TVC
Launch Vehicle First Stage Design Model, T=80.0 sec
P-dot Roll  Acceleration About X Axis
Q-dot Pitch Acceleration About Y Axis
R-dot Yaw   Acceleration About Z Axis
---------------------------------------------------------------------------------------------
SYSTEM OF TRANSFER FUNCTIONS ...
Integrator
Continuous
TF. Block #   1   (1/s)                                   Order of Numer, Denom=  0  1
Numer 0.0          1.0
Denom 1.0          0.0
-----------------------------------------------------------------------
Block #, from Input #, Gain
  1  1     1.00000
...........................
Outpt #, from Block #, Gain
  1  1     1.00000
...........................
---------------------------------------------------------------------------------------------
```

Integrator Used to
Create Alpha-Integral



```
FLIGHT VEHICLE INPUT DATA ......
Launch Vehicle First Stage Design Model, T=80.0 sec
! This is the Launch Vehicle Control Design Model at t=80 sec (Max-Q)
! includes 8 TVC Engines, No Slosh, No Bending
!
Body Axes Output, Attitude=Rate Integr

Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft)    :   3491.05      32.1740      0.208960E+08
Moments and Products of Inertia: Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)          :   25832.4      0.2521E+07   0.25200E+07    0.00000       0.0000
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)        :   58.0136      0.00000      0.00000
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)       :   1.30300      1278.63      530.760        39020.8
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)         :   25.7992      54.0456      0.00          -0.875700
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                         :   0.840       -0.00         0.02          -0.0
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)   :   0.00         63.2320      0.00          -0.0          -0.5782
W-Gust Azim & Elev angles (deg), or Torque/Force direction (x,y,z), Force Locat (x,y,z)  :   Gust         45.00        90.00
Surface Reference Area (feet^2), Mean Aerodynamic Chord (ft), Wing Span in (feet)        :   44.415       7.52000      7.52000
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial_rho/ Partial_H} :   120.142      0.00000      0.00000       0.00000
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:  1.49980      0.00583      0.552534E-04  0.00000       0.0000
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}:  0.0         -0.08110      0.00000       0.00000       0.0000
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}: -0.068124    -0.08110      0.00000       0.00000       0.0000
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:  0.00000      0.00000      0.00000       0.00000       0.0000
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}: -0.173376    -0.206400      0.00000       0.00000       0.0000
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}: -0.0          0.206400      0.00000       0.00000       0.0000

Number of Thruster Engines, Include or Not the Tail-Wags-Dog and Load-Torque Dynamics ?  :  8   NO TWD

TVC Engine No:  1                          (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#1 +2Y-Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)              :  24890.2      24890.2
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000       6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000      15.1200      1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0         2.4945      -1.0332
TVC Engine No:  2                          (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#2 +Y-2Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)              :  24890.2      24890.2
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000       6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000      15.1200      1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0         1.0332      -2.4945
TVC Engine No:  3                          (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#3 -Y-2Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)              :  24890.2      24890.2
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000       6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000      15.1200      1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0        -1.0332      -2.4945
TVC Engine No:  4                          (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#4 -2Y-Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)              :  24890.2      24890.2
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000       6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000      15.1200      1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0        -2.4945      -1.0332
TVC Engine No:  4                          (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#4 -2Y-Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)              :  24890.2      24890.2
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000       6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000      15.1200      1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0        -2.4945      -1.0332
TVC Engine No:  5                          (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#5 -2Y+Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)              :  24890.2      24890.2
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000       6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000      15.1200      1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0        -2.4945       1.0332
TVC Engine No:  6                          (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#6 -Y+2Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)              :  24890.2      24890.2
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000       6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000      15.1200      1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0        -1.0332       2.4945
TVC Engine No:  7                          (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#7 +Y+2Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)              :  24890.2      24890.2
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000       6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000      15.1200      1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0         1.0332       2.4945
TVC Engine No:  8                          (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#8 +2Y+Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)              :  24890.2      24890.2
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000      6.00000       6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000      15.1200      1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0         2.4945       1.0332

Number of Accelerometers, Along Axis: (x,y,z)                                        :  3
Acceleromet No  1 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Location (ft) :  X-axis Accelerat.     97.483    0.00      0.00
Acceleromet No  2 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Location (ft) :  Y-axis Accelerat.     97.483    0.00      0.00
Acceleromet No  3 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Location (ft) :  Z-axis Accelerat.     97.483    0.00      0.00
-----------------------------------------------------------------------------------------------------------------------------------------
```

The direction of the wind-gust vector is defined by the azimuth and elevation angles (45° and 90°) respectively in this case. It means, that the wind vector is perpendicular to the vehicle x-axis and coming towards the vehicle at 45° between the -Y and -Z axes.

Include the TVC Matrix in Front of the Vehicle Model to Create the Coupled (Roll, Pitch, Yaw) Design Model

```
INTERCONNECTION OF SYSTEMS .....
Vehicle Design Model with TVC
! Combines the Vehicle Model with the TVC Matrix
!
Titles of Systems to be Combined
Title 1 Launch Vehicle First Stage Design Model, T=80.0 sec
SYSTEM INPUTS TO SUBSYSTEM  1                                                                   TVC to Vehicle
Via Matrix +TVC                                                                                 roll, pitch, yaw
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                                                Vehicle Outputs
Via Matrix +I8                                                                                  First 8 Outputs
..............................................................................
Definitions of Inputs  =  3
Roll  TVC Demand from Flight Control
Pitch TVC Demand from Flight Control
Yaw   TVC Demand from Flight Control
```

The Inputs to the Design Model are the 3 Flight Control Demands (DQ, DP, DR) to the TVC Matrix

```
Definitions of Outputs =  8
Roll Attitude   (phi)     (radians)
Roll Rate       (p-body)  (rad/sec)
Pitch Attitude  (theta)   (radians)
Pitch Rate      (q-body)  (rad/sec)
Yaw  Attitude   (psi)     (radians)
Yaw  Rate       (r-body)  (rad/sec)
Angle of attack, alfa,   (radians)
Angle of sideslip, beta,  (radian)

Definitions of States  = 10
Roll Attitude   (phi)     (radians)
Roll Rate       (p-body)  (rad/sec)
Pitch Attitude  (theta)   (radians)
Pitch Rate      (q-body)  (rad/sec)
Yaw  Attitude   (psi)     (radians)
Yaw  Rate       (r-body)  (rad/sec)
Angle of attack, alfa,   (radians)
Angle of sideslip, beta,  (radian)
Change in Altitude (delta-h)  (feet)
Change in Velocity (delta-V)  (ft/sec)
```

The Last 2 States are Not Used and they will be taken out of the Pitch Design Model

```
----------------------------------------------------------------------------------------------------------------
INTERCONNECTION OF SYSTEMS .....
Vehicle Analysis Model with TVC
! Combines the Vehicle Model with the TVC Matrix
! It includes the Wind-Gust Input
!
Titles of Systems to be Combined
Title 1 Launch Vehicle First Stage Design Model, T=80.0 sec
SYSTEM INPUTS TO SUBSYSTEM  1                                                                   TVC to Vehicle
Via Matrix +TVC                                                                                 roll, pitch, yaw Demands
Via Matrix +I01                                                                                 Gust Input
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                                                Vehicle Outputs
Via Matrix +I8                                                                                  First 8 Vehi Outputs
..............................................................................
Definitions of Inputs  =  4
Roll  TVC Demand from Flight Control
Pitch TVC Demand from Flight Control
Yaw   TVC Demand from Flight Control
Wind-Gust Velocity in (ft/sec)
```

This will become the Analysis and Simulation Model plant_t80.m

Via Matrix +TVC — First 3 Inputs DP, DQ, DR Connect to the first 16 Vehicle Gimbal Inputs (dy, dz)

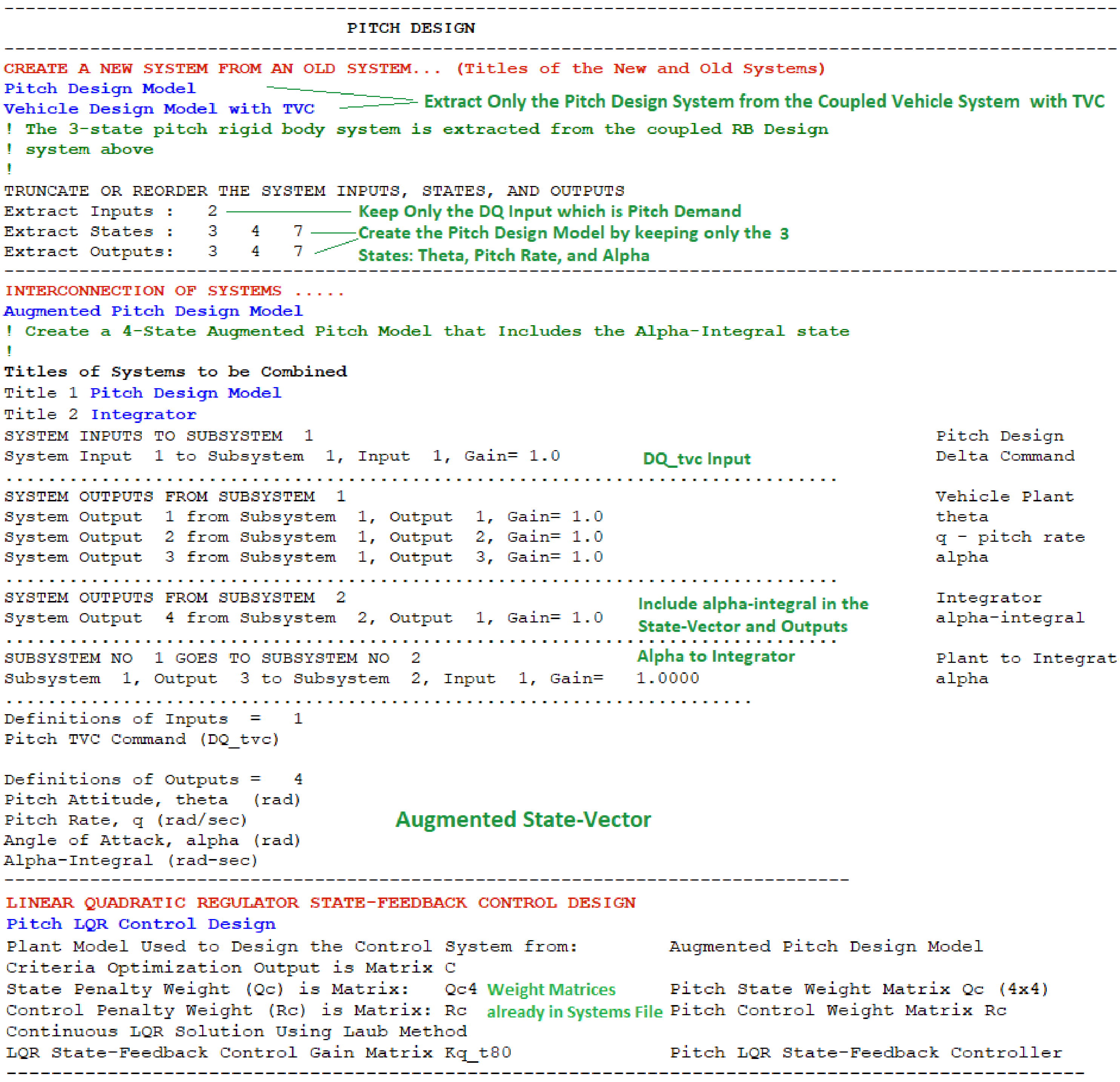
Via Matrix +I01 — The 4th Input goes to Vehicle Input #17 which is the Wind-Gust Velocity

It Includes the Wind-Gust Velocity Input
Its Direction is Desined in the Vehicle Input Data

```
Definitions of Outputs =  8
Roll Attitude   (phi)     (radians)
Roll Rate       (p-body)  (rad/sec)
Pitch Attitude  (theta)   (radians)
Pitch Rate      (q-body)  (rad/sec)
Yaw  Attitude   (psi)     (radians)
Yaw  Rate       (r-body)  (rad/sec)
Angle of attack, alfa,   (radians)
Angle of sideslip, beta,  (radian)

Definitions of States  = 10
Roll Attitude   (phi)     (radians)
Roll Rate       (p-body)  (rad/sec)
Pitch Attitude  (theta)   (radians)
Pitch Rate      (q-body)  (rad/sec)
Yaw  Attitude   (psi)     (radians)
Yaw  Rate       (r-body)  (rad/sec)
Angle of attack, alfa,   (radians)
Angle of sideslip, beta,  (radian)
Change in Altitude (delta-h)  (feet)
Change in Velocity (delta-V)  (ft/sec)
------------------------------------------------------------------------------------------------------------------------
```

the dH and dV States are kept



```
-----------------------------------------------------------------------------------------
                            PITCH DESIGN
-----------------------------------------------------------------------------------------
CREATE A NEW SYSTEM FROM AN OLD SYSTEM... (Titles of the New and Old Systems)
Pitch Design Model
Vehicle Design Model with TVC        Extract Only the Pitch Design System from the Coupled Vehicle System  with TVC
! The 3-state pitch rigid body system is extracted from the coupled RB Design
! system above
!
TRUNCATE OR REORDER THE SYSTEM INPUTS, STATES, AND OUTPUTS
Extract Inputs :   2           Keep Only the DQ Input which is Pitch Demand
Extract States :   3   4   7   Create the Pitch Design Model by keeping only the 3
Extract Outputs:   3   4   7   States: Theta, Pitch Rate, and Alpha
-----------------------------------------------------------------------------------------
INTERCONNECTION OF SYSTEMS .....
Augmented Pitch Design Model
! Create a 4-State Augmented Pitch Model that Includes the Alpha-Integral state
!
Titles of Systems to be Combined
Title 1 Pitch Design Model
Title 2 Integrator
SYSTEM INPUTS TO SUBSYSTEM  1                                            Pitch Design
System Input  1 to Subsystem  1, Input  1, Gain= 1.0      DQ_tvc Input   Delta Command
.........................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                         Vehicle Plant
System Output  1 from Subsystem  1, Output  1, Gain= 1.0                 theta
System Output  2 from Subsystem  1, Output  2, Gain= 1.0                 q - pitch rate
System Output  3 from Subsystem  1, Output  3, Gain= 1.0                 alpha
.........................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                   Include alpha-integral in the   Integrator
System Output  4 from Subsystem  2, Output  1, Gain= 1.0  State-Vector and Outputs  alpha-integral
.........................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2            Alpha to Integrator   Plant to Integrat
Subsystem  1, Output  3 to Subsystem  2, Input  1, Gain=  1.0000         alpha
.................................................................
Definitions of Inputs  =   1
Pitch TVC Command (DQ_tvc)

Definitions of Outputs =   4
Pitch Attitude, theta  (rad)
Pitch Rate, q (rad/sec)              Augmented State-Vector
Angle of Attack, alpha (rad)
Alpha-Integral (rad-sec)
-------------------------------------------------------------------------
LINEAR QUADRATIC REGULATOR STATE-FEEDBACK CONTROL DESIGN
Pitch LQR Control Design
Plant Model Used to Design the Control System from:        Augmented Pitch Design Model
Criteria Optimization Output is Matrix C
State Penalty Weight (Qc) is Matrix:   Qc4 Weight Matrices      Pitch State Weight Matrix Qc (4x4)
Control Penalty Weight (Rc) is Matrix: Rc  already in Systems File Pitch Control Weight Matrix Rc
Continuous LQR Solution Using Laub Method
LQR State-Feedback Control Gain Matrix Kq_t80                 Pitch LQR State-Feedback Controller
-------------------------------------------------------------------------------------
```

```
--------------------------------------------------------------------------------------------------------
                               LATERAL DESIGN
--------------------------------------------------------------------------------------------------------
CREATE A NEW SYSTEM FROM AN OLD SYSTEM... (Titles of the New and Old Systems)
Lateral Design Model
Vehicle Design Model with TVC       Extract Only the Lateral Design System from the Coupled Vehicle System with TVC
! The 5-state lateral rigid body system is extracted from the coupled
! RB design system above
!
TRUNCATE OR REORDER THE SYSTEM INPUTS, STATES, AND OUTPUTS
Extract Inputs :    1    3 ------------Keep only the DP and DR Inputs which are the Roll and Yaw Demands
Extract States :    1    2    5    6    8   Create the Lateral Design System by keeping only 5
Extract Outputs:    1    2    5    6    8   States: Phi, P, Psi, R, and Beta
--------------------------------------------------------------------------------------------------------
INTERCONNECTION OF SYSTEMS .....
Augmented Lateral Design Model
! Create a 6-State Augmented Lateral Model that Includes the Beta-Integral state
!
Titles of Systems to be Combined
Title 1 Lateral Design Model
Title 2 Integrator
SYSTEM INPUTS TO SUBSYSTEM  1                                                      Lateral Design Model
System Input  1 to Subsystem  1, Input  1, Gain= 1.0      DP_tvc Input             Roll TVC Demand
System Input  2 to Subsystem  1, Input  2, Gain= 1.0      DR_tvc Input             Yaw  TVC Demand
...................................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                                   Vehicle Plant
System Output  1 from Subsystem  1, Output  1, Gain= 1.0                           Roll Attitude  (phi)
System Output  2 from Subsystem  1, Output  2, Gain= 1.0                           Roll Rate      (p-body)
System Output  3 from Subsystem  1, Output  3, Gain= 1.0                           Yaw  Attitude  (psi)
System Output  4 from Subsystem  1, Output  4, Gain= 1.0                           Yaw  Rate      (r-body)
System Output  5 from Subsystem  1, Output  5, Gain= 1.0                           Angle of sideslip, beta
...................................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                Include beta-integral in the       Integrator
System Output  6 from Subsystem  2, Output  1, Gain= 1.0   State-Vector and Outputs  Beta-integral
...................................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2                  Beta to Integrator        Plant to Integrator
Subsystem  1, Output  5 to Subsystem  2, Input  1, Gain=   1.0000                  Beta
.........................................................................
Definitions of Inputs  =   2
Roll TVC Command (DP_tvc)
Yaw  TVC Command (DR_tvc)

Definitions of Outputs =   6
Roll Attitude  (phi)     (radians)
Roll Rate      (p-body)  (rad/sec)
Yaw  Attitude  (psi)     (radians)           Augmented Lateral State-Vector
Yaw  Rate      (r-body)  (rad/sec)
Angle of sideslip, beta, (radian)
Beta-Integral            (rad-sec)
--------------------------------------------------------------------------------------------------------
LINEAR QUADRATIC REGULATOR STATE-FEEDBACK CONTROL DESIGN
Lateral LQR Control Design
Plant Model Used to Design the Control System from:        Augmented Lateral Design Model
Criteria Optimization Output is Matrix C
State Penalty Weight (Qc) is Matrix:   Qc6  Weight Matrices   Lateral State Weight Matrix Qc (6x6)
Control Penalty Weight (Rc) is Matrix: Rc2  Included in Systems Lateral Control Weight Matrix Rc (2x2)
Continuous LQR Solution Using Laub Method
LQR State-Feedback Control Gain Matrix Kpr_t80              Lateral LQR State-Feedback Controller
--------------------------------------------------------------------------------------------------------
```

```
CONVERT TO MATLAB FORMAT ........       (Title, System/Matrix, m-filename)
Vehicle Analysis Model with TVC
System                                  Vehicle System Used in Simulations and Stability Analysis
plant_t80.m
----------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........       (Title, System/Matrix, m-filename)
Pitch Design Model
System                                  Intermediate Pitch Design System
pitch_rb_t80.m
----------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........       (Title, System/Matrix, m-filename)
Lateral Design Model
System                                  Intermediate Lateral Design System
later_rb_t80.m
----------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........       (Title, System/Matrix, m-filename)
Vehicle Mixing Logic
Matrix TVC                              TVC Matrix at T=80 sec
----------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........       (Title, System/Matrix, m-filename)
Pitch LQR State-Feedback Controller
Matrix Kq_t80                           Pitch LQR State-Feedback Controller
----------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........       (Title, System/Matrix, m-filename)
Lateral LQR State-Feedback Controller
Matrix Kpr_t80                          Lateral LQR State-Feedback Controller
----------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........       (Title, System/Matrix, m-filename)
Augmented Pitch Design Model
System                                  Pitch Design System Used in the LQR Algorithm
pitch_des.m
----------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........       (Title, System/Matrix, m-filename)
Augmented Lateral Design Model
System                                  Lateral Design System Used in the LQR Algorithm
later_des.m
----------------------------------------------------------------------------------------------------
```

Analysis

The vehicle systems and matrices are converted to Matlab m-files format, and the file “init.m” loads the LQR state-feedback matrices Kq_t80 and Kpr_t80 and the TVC matrix into Matlab, and the file “freq.m” calculates the Bode and Nichols plots using the open-loop Simulink model “Open_RB.Slx”. The stability analysis model and the simulation model are shown in Figure 2.1.12. They both use the “*Vehicle Analysis Model with TVC*” loaded from file “*plant_t80*”. The bandwidth is a little higher at Max-Q because the vehicle must be able to respond fast against wind disturbances. In Figure 2.1.14, the α and β response to the 33 (ft/sec) wind pulse is much smaller in comparison to previous cases. The strong load-relief however degrades the command following performance at High-Q although it is still acceptable during this phase.

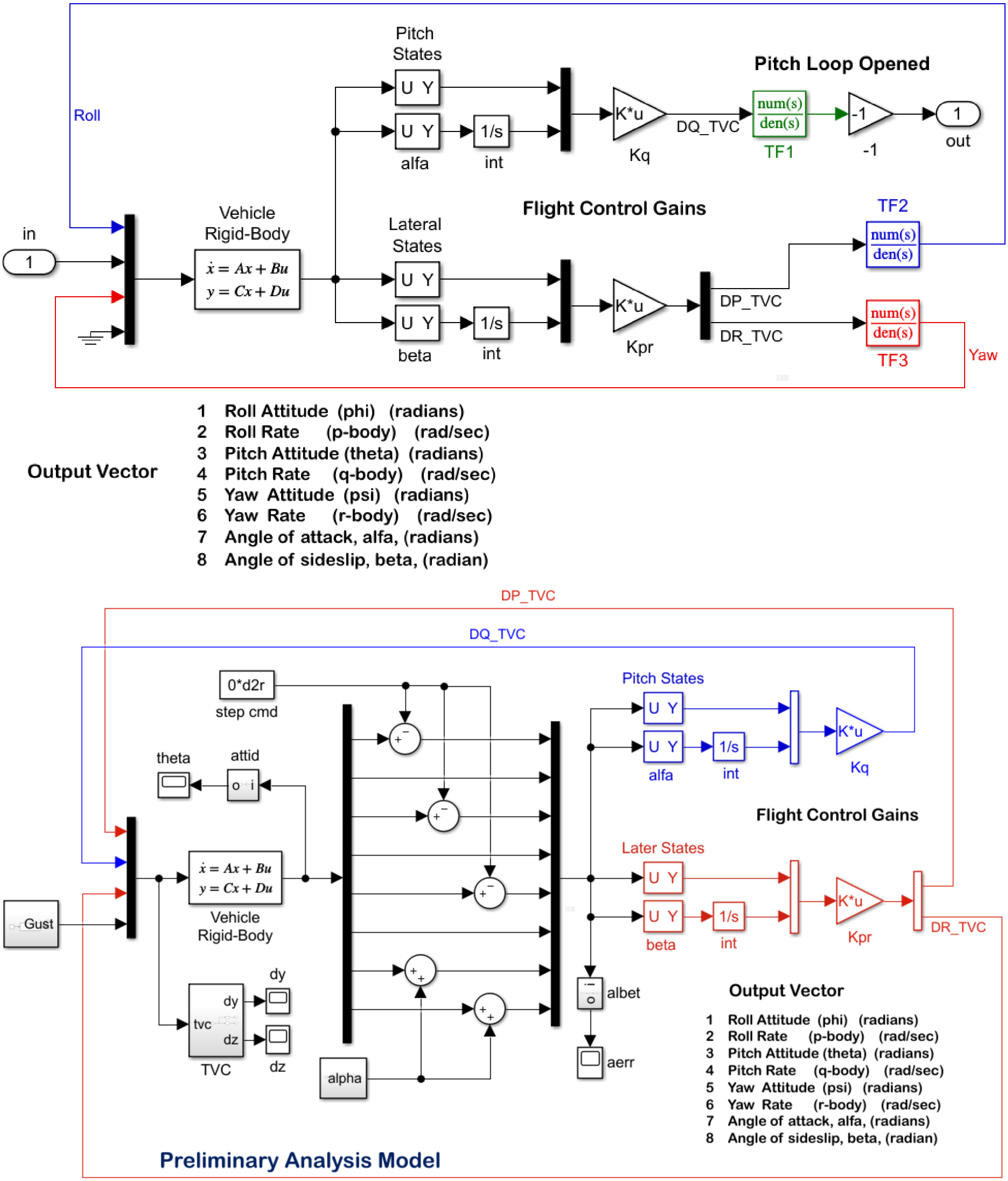


Figure 8 Open Loop Stability Analysis Model "Open_RB.slx" and the Closed-Loop Simulation Model "Sim_RB.slx". They are both now using State-Feedback from $\alpha-$Integral and $\beta-$Integral

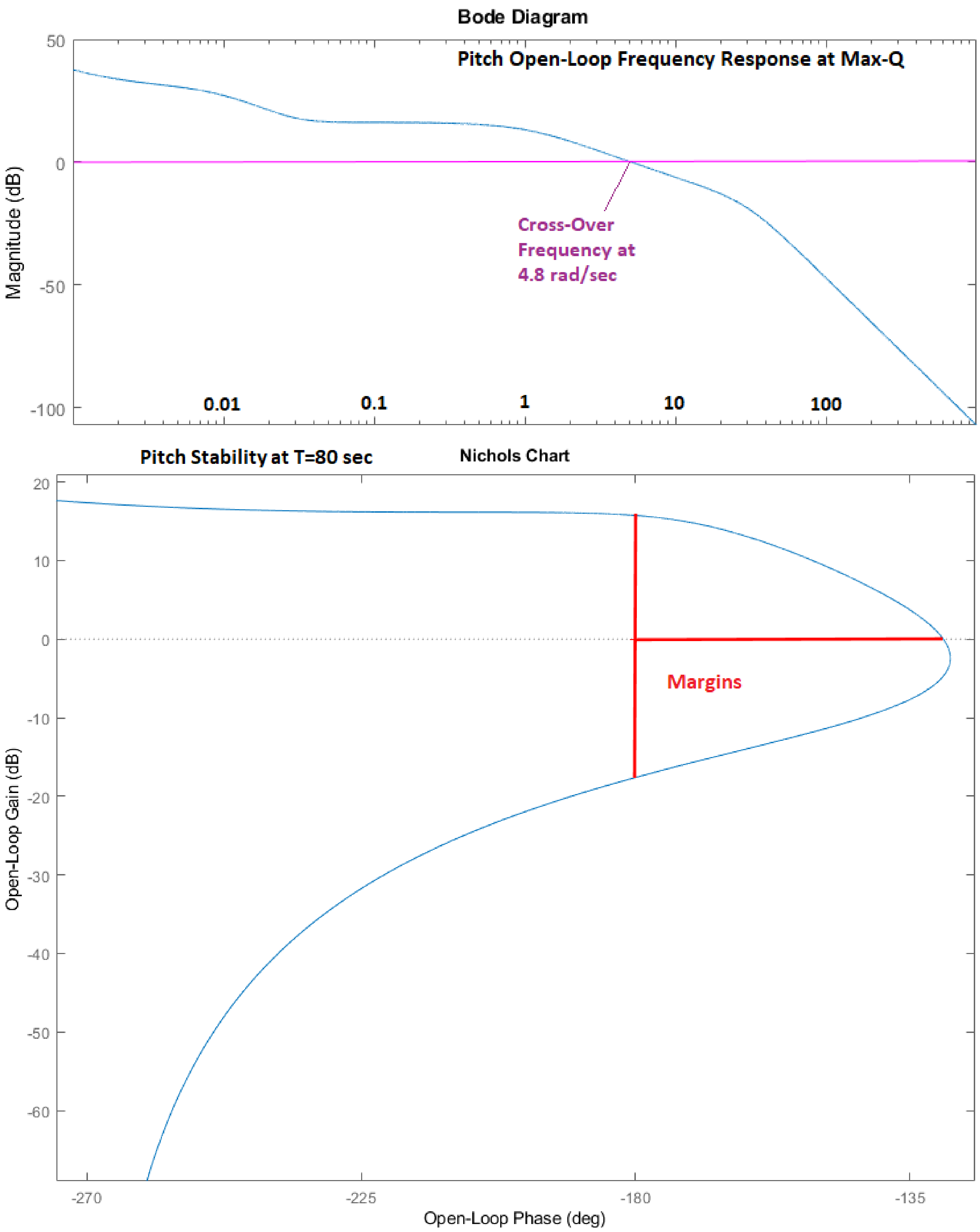


Figure 2.1.9 Bode and Nichols Plots at Max-Q

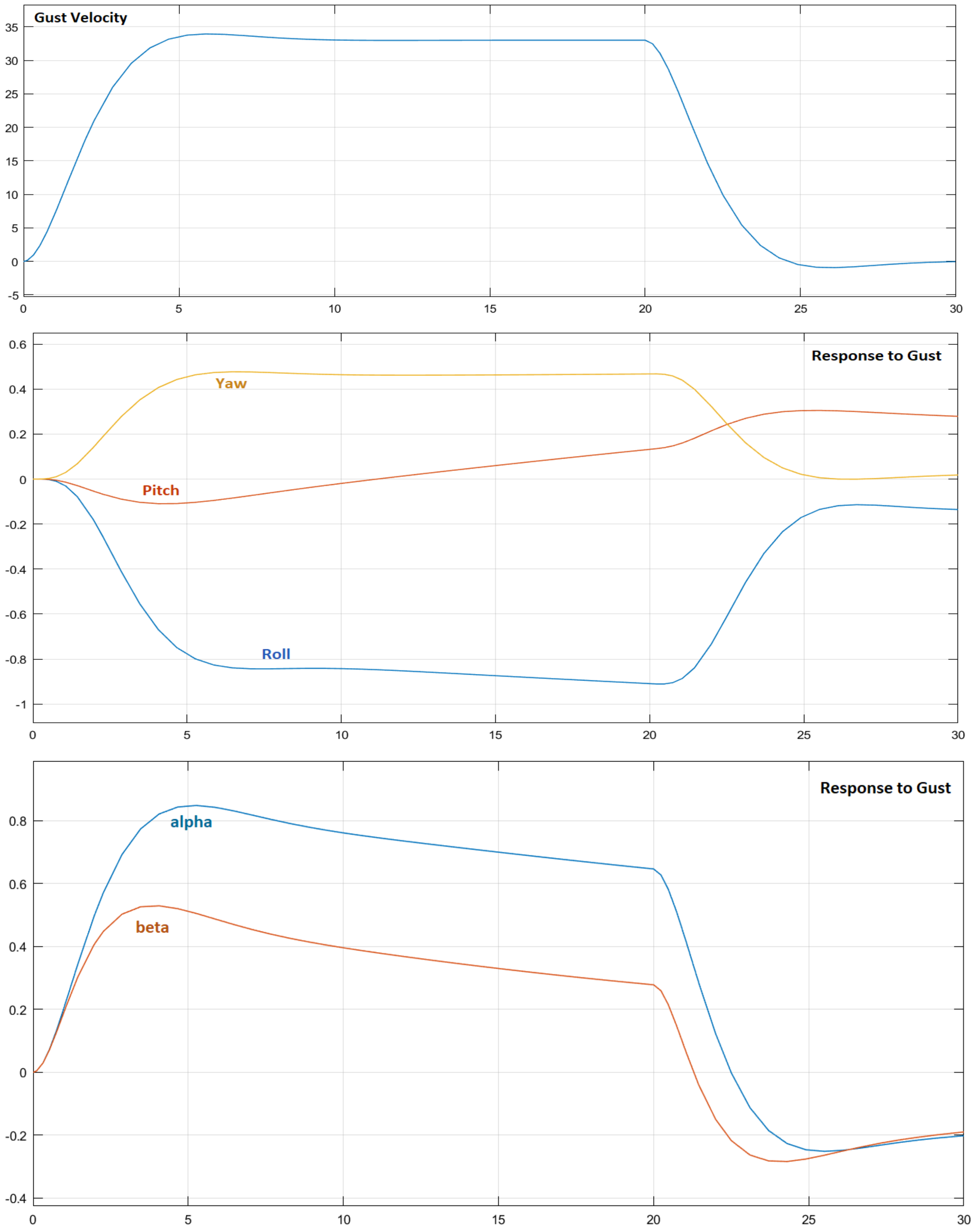


Figure 2.1.10 System Response to Wind-Gust Pulse along both Y and Z Directions. It Excites the Vehicle Attitude. The responses in α *&* β *are Small in comparison to previous cases due to the Increased Load-Relief*

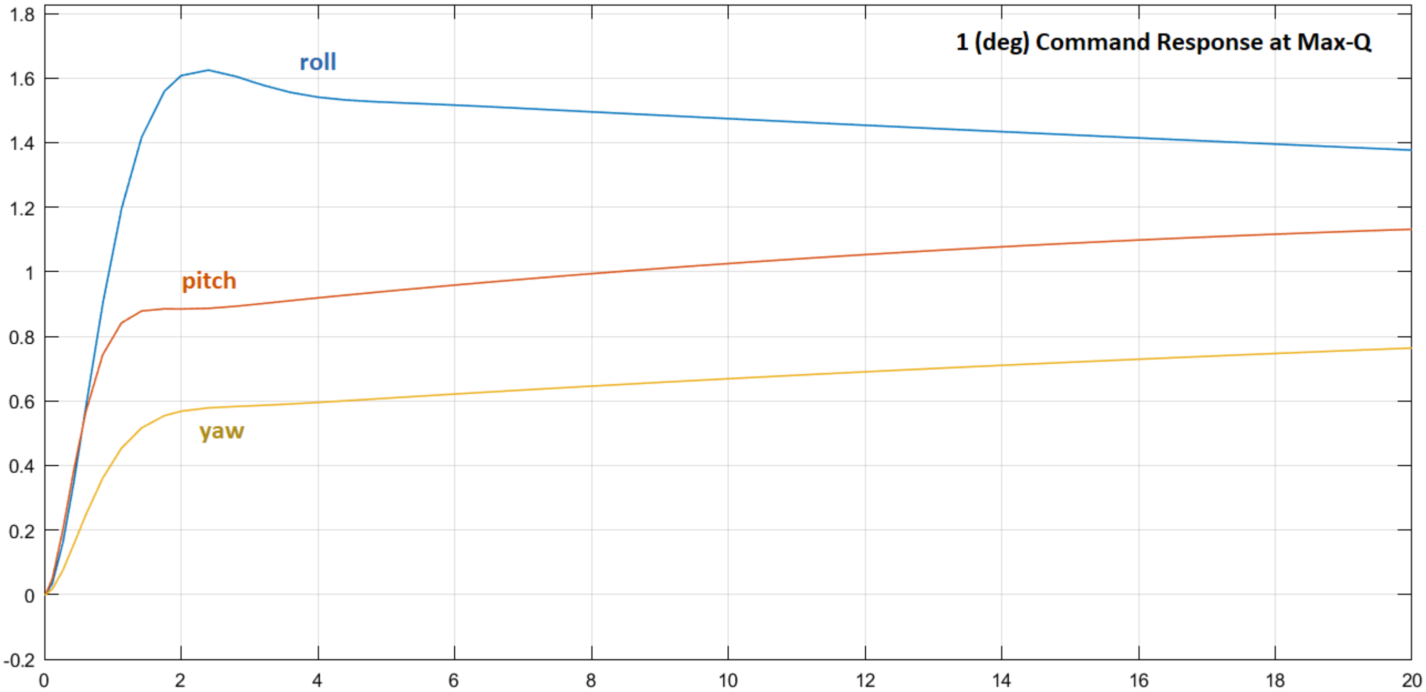
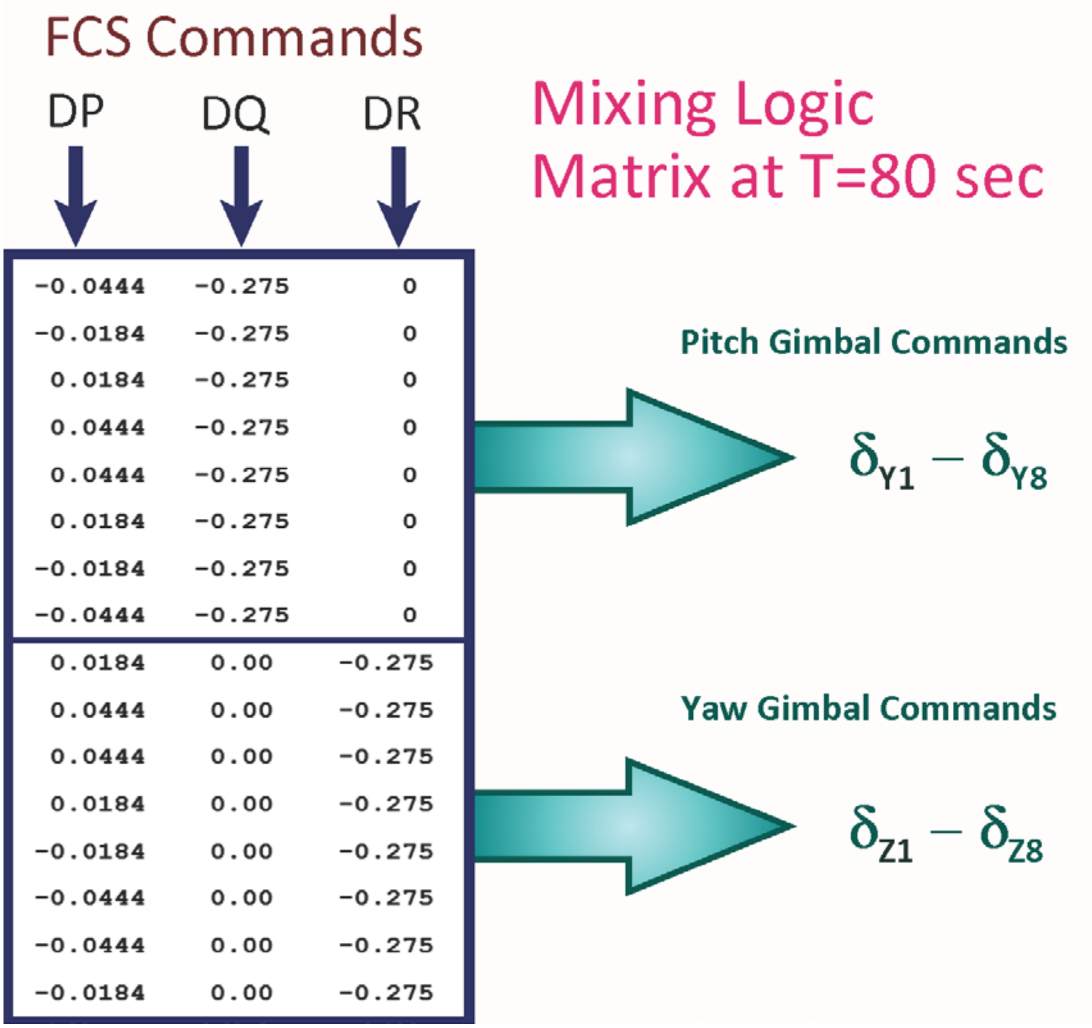


Figure 2.1.11 Attitude Response to 1° Commands in Roll, Pitch and Yaw Simultaneously. The Response to the Commands is Not Great at Max-Q because of the Load-Relief Function and the System is Not Commanded During Max-Q



FCS Commands

Mixing Logic Matrix at T=80 sec

DP	DQ	DR
-0.0444	-0.275	0
-0.0184	-0.275	0
0.0184	-0.275	0
0.0444	-0.275	0
0.0444	-0.275	0
0.0184	-0.275	0
-0.0184	-0.275	0
-0.0444	-0.275	0
0.0184	0.00	-0.275
0.0444	0.00	-0.275
0.0444	0.00	-0.275
0.0184	0.00	-0.275
-0.0184	0.00	-0.275
-0.0444	0.00	-0.275
-0.0444	0.00	-0.275
-0.0184	0.00	-0.275

Pitch Gimbal Commands

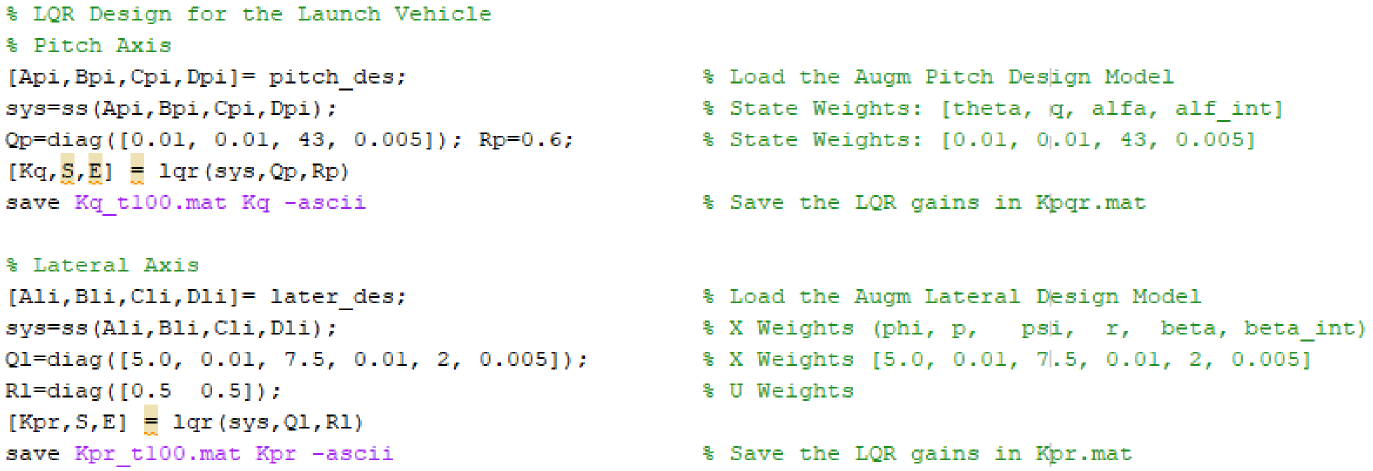
$\delta_{Y1} - \delta_{Y8}$

Yaw Gimbal Commands

$\delta_{Z1} - \delta_{Z8}$

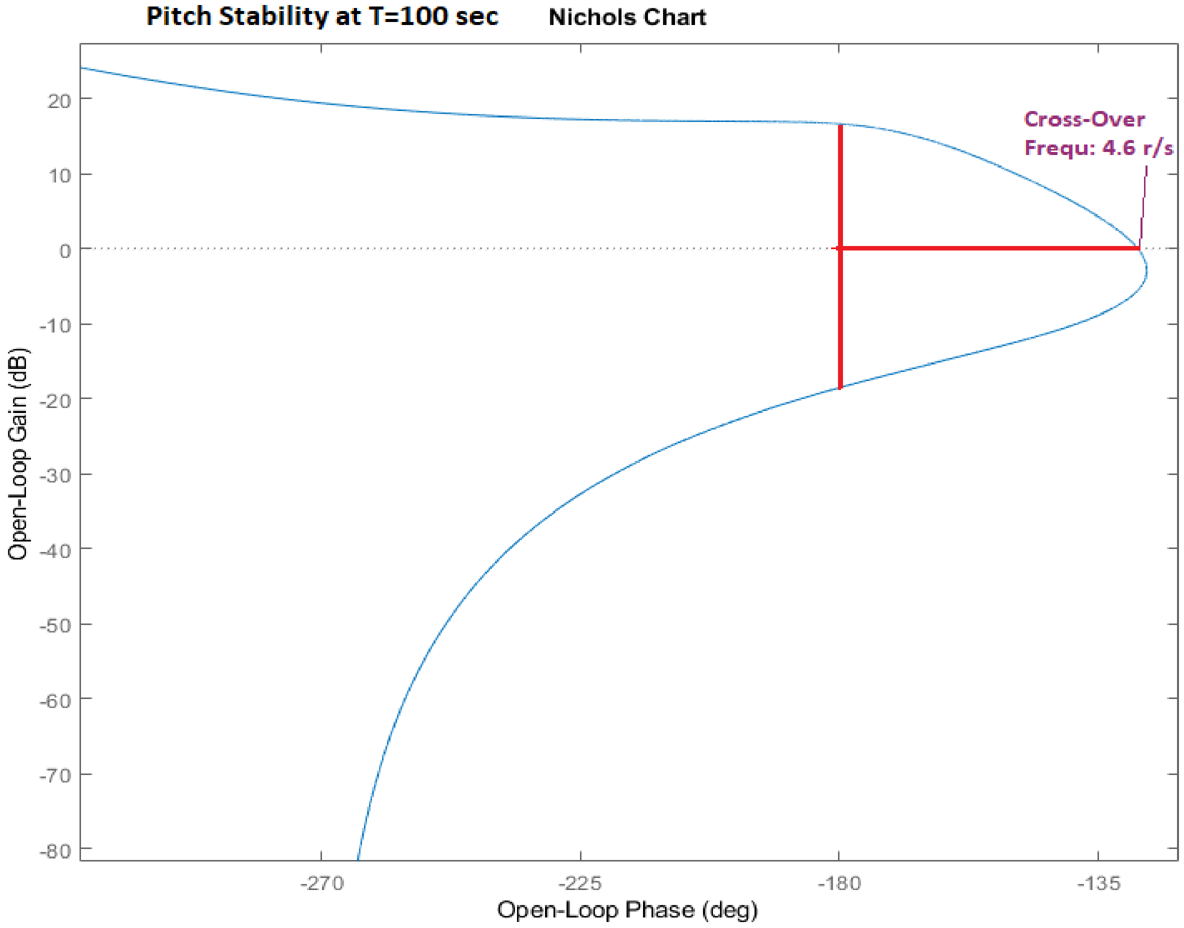
2.1.5 Control Design at T= 100 sec

At T=100 sec the dynamic pressure is reduced to 360 (lbf/ft^2) which is still high and the load-relief is still active, applying feedback from (α & β) states and from (α & β) integrals. The attitude command following is therefore still degraded because it is trying to operate against the load-relief. The pitch and lateral design models include (α & β) integrals. The alpha and beta state weights are further increased in the LQR design from the previous cases and the (α & β) integral weights are small. The pitch controller is (1x4) state-feedback gain matrix Kq_t100 from states: θ, q, α, α-integral. The roll and yaw axes are again treated as a coupled lateral system for now, creating a (2x6) state-feedback controller matrix Kpr_t100 from states: ϕ, p, ψ, r, β, β-integral.



```
% LQR Design for the Launch Vehicle
% Pitch Axis
[Api,Bpi,Cpi,Dpi]= pitch_des;                          % Load the Augm Pitch Design Model
sys=ss(Api,Bpi,Cpi,Dpi);                               % State Weights: [theta, q, alfa, alf_int]
Qp=diag([0.01, 0.01, 43, 0.005]); Rp=0.6;              % State Weights: [0.01, 0.01, 43, 0.005]
[Kq,S,E] = lqr(sys,Qp,Rp)
save Kq_t100.mat Kq -ascii                             % Save the LQR gains in Kpqr.mat

% Lateral Axis
[Ali,Bli,Cli,Dli]= later_des;                          % Load the Augm Lateral Design Model
sys=ss(Ali,Bli,Cli,Dli);                               % X Weights (phi, p,   psi,  r,  beta, beta_int)
Ql=diag([5.0, 0.01, 7.5, 0.01, 2, 0.005]);             % X Weights [5.0, 0.01, 7.5, 0.01, 2, 0.005]
Rl=diag([0.5  0.5]);                                   % U Weights
[Kpr,S,E] = lqr(sys,Ql,Rl)
save Kpr_t100.mat Kpr -ascii                           % Save the LQR gains in Kpr.mat
```



Pitch Stability at T=100 sec Nichols Chart

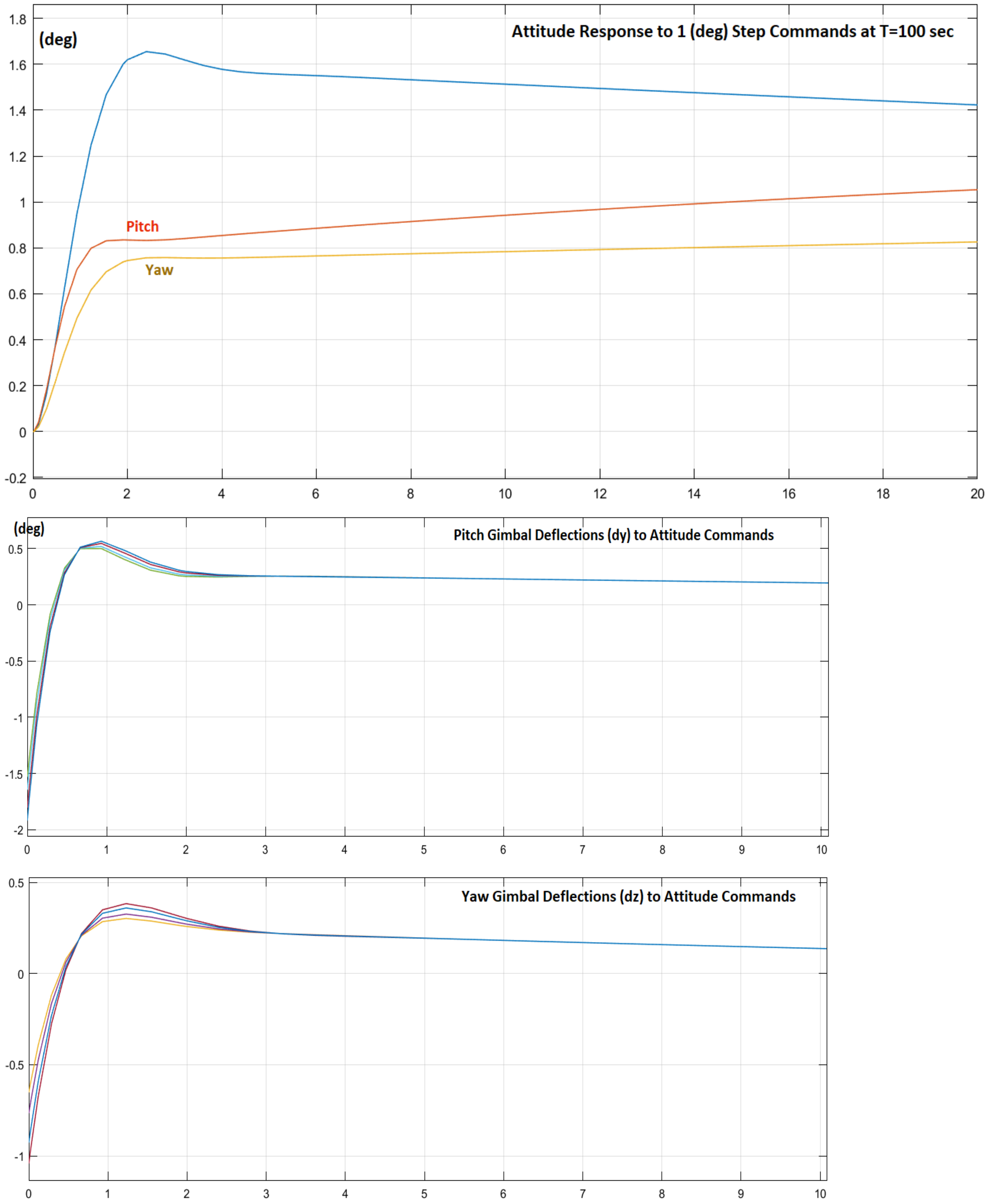


Figure 2.1.12 Attitude and Gimbal Responses to 1° Attitude Commands in Roll, Pitch & Yaw

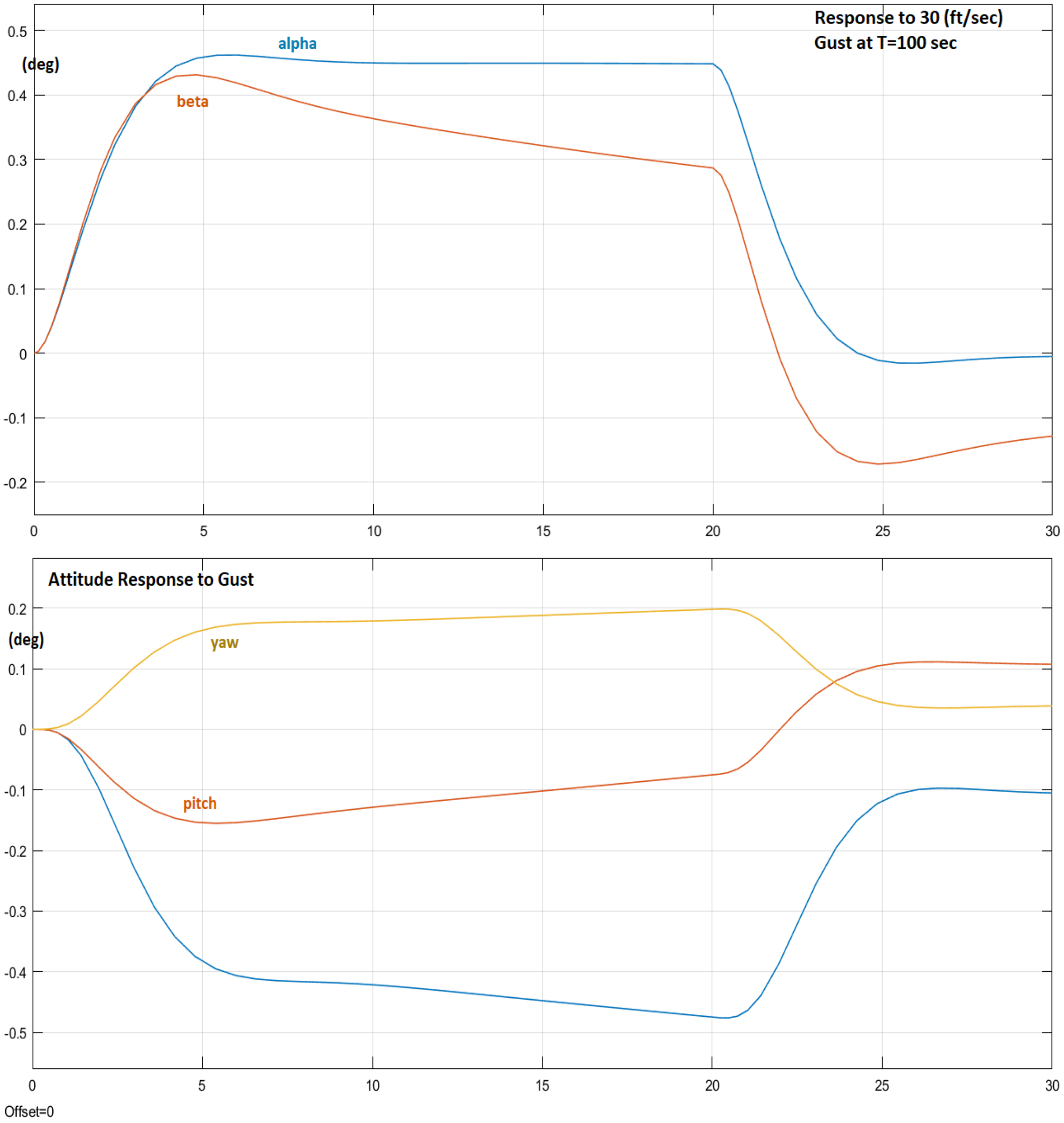
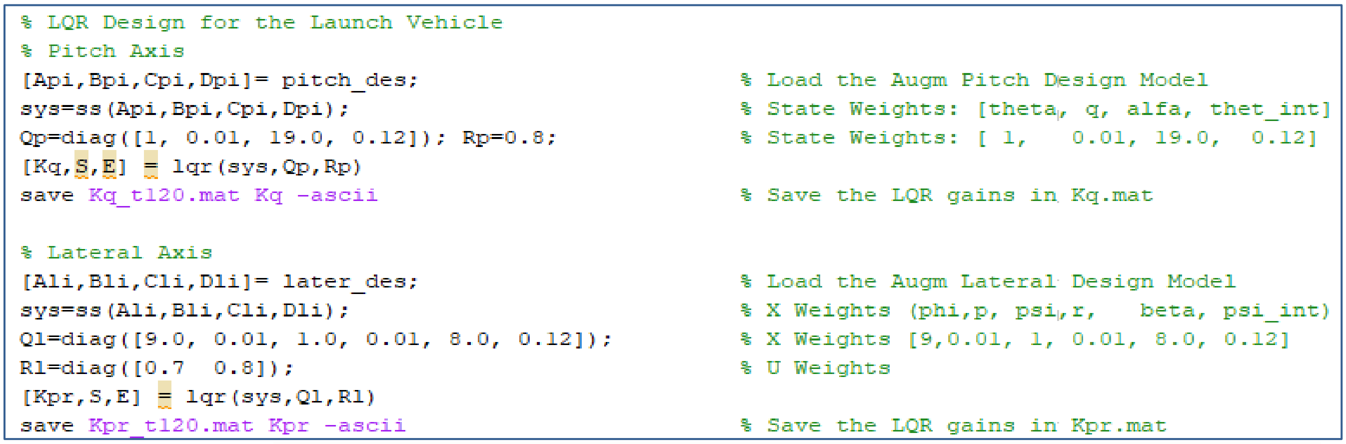


Figure 2.1.13 Vehicle Response to a 30 (feet/sec) Wind-Gust Pulse. The Load-Relief Minimizes the Effect of the Wind-Gust on the Alpha and Beta Incidence Angles but it Reduces the Command-Following Performance at T=100 sec.

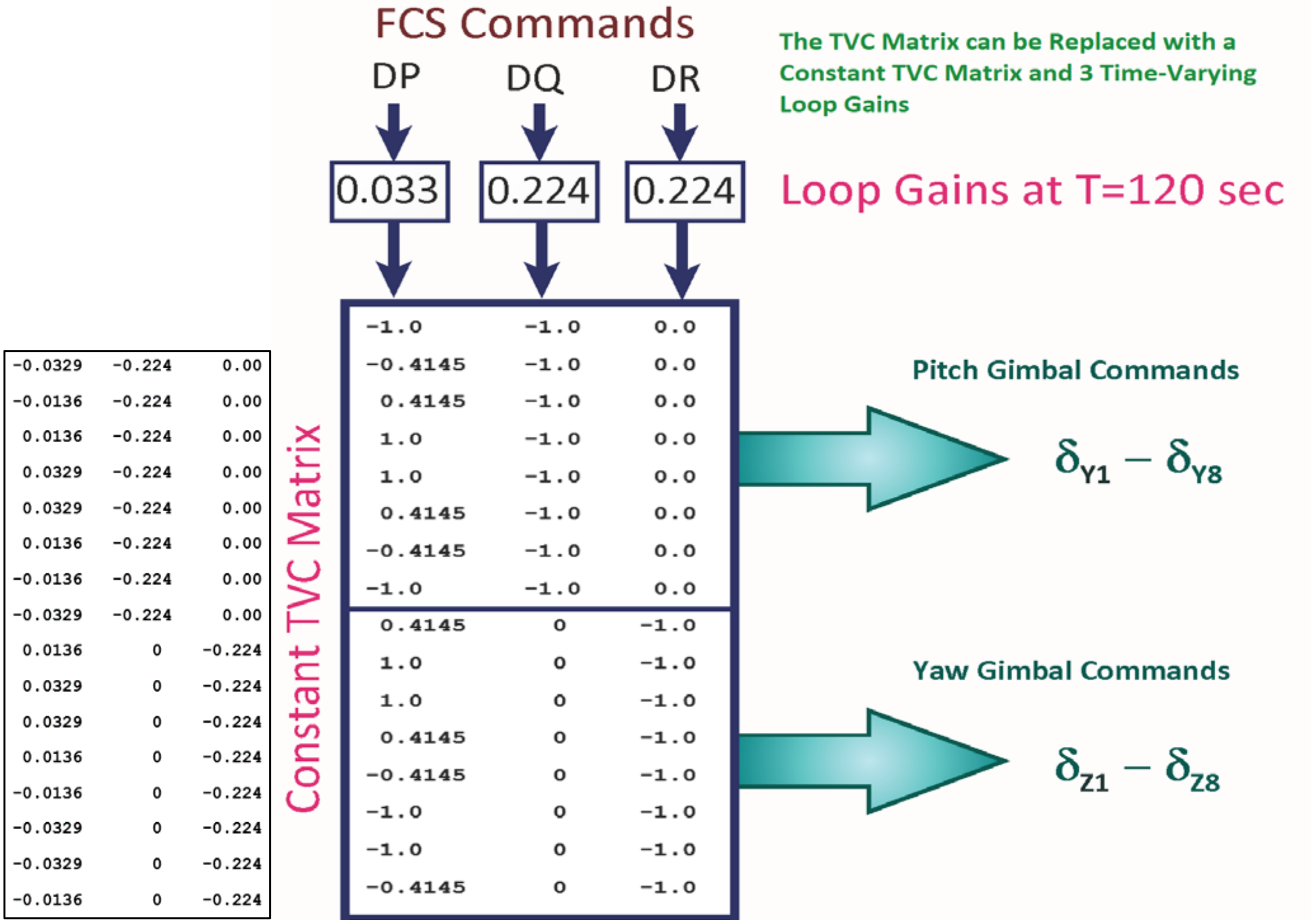
2.1.6 Control Design at T= 120 sec

At T=120 sec the vehicle has passed the High-Q region but the dynamic pressure is still big enough, 155 (lbf/ft^2), and the control system requires a significant amount of feedback from (α & β) states but not (α & β)-integrals. The control emphasis is now in tracking attitude commands and we will go back and use (θ & ψ)-integral feedback for trimming the pitch and yaw attitude errors. The LQR weights on the α & β states are still significant and the weights on (θ & ψ)-integral states are small because we are still experiencing a substantial amount of Q-bar. The pitch controller is (1x4) state-feedback gain matrix Kq_t120 from states: θ, q, α, θ-integral. The roll and yaw axes are treated as a coupled lateral system, creating a (2x6) state-feedback controller matrix Kpr_t120 from states: ϕ, p, ψ, r, β, and ψ-integral. In the analysis section the lateral controller will be separated because the roll/yaw coupling is small. DP will be a function of (ϕ, p) and DR will be a function of (ψ, r, β, and ψ-integral).



```
% LQR Design for the Launch Vehicle
% Pitch Axis
[Api,Bpi,Cpi,Dpi]= pitch_des;                  % Load the Augm Pitch Design Model
sys=ss(Api,Bpi,Cpi,Dpi);                       % State Weights: [theta, q, alfa, thet_int]
Qp=diag([1, 0.01, 19.0, 0.12]); Rp=0.8;        % State Weights: [ 1,    0.01, 19.0,   0.12]
[Kq,S,E] = lqr(sys,Qp,Rp)
save Kq_t120.mat Kq -ascii                     % Save the LQR gains in Kq.mat

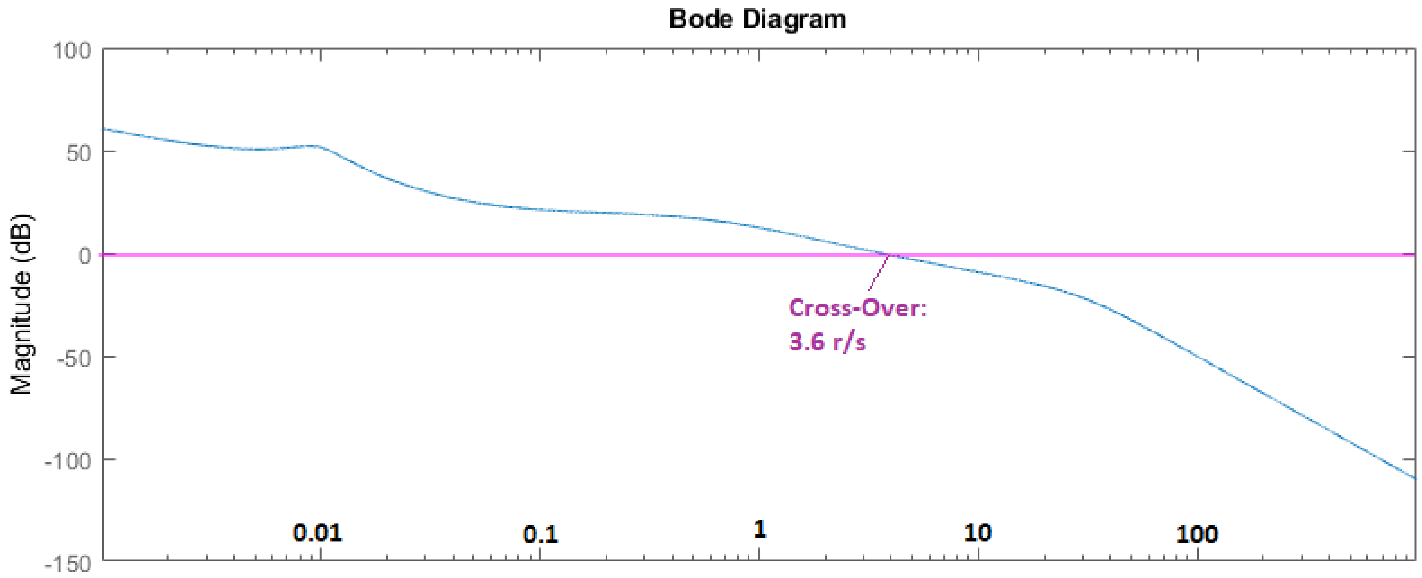
% Lateral Axis
[Ali,Bli,Cli,Dli]= later_des;                  % Load the Augm Lateral Design Model
sys=ss(Ali,Bli,Cli,Dli);                       % X Weights (phi,p, psi,r,    beta, psi_int)
Ql=diag([9.0, 0.01, 1.0, 0.01, 8.0, 0.12]);    % X Weights [9,0.01, 1, 0.01, 8.0, 0.12]
Rl=diag([0.7  0.8]);                           % U Weights
[Kpr,S,E] = lqr(sys,Ql,Rl)
save Kpr_t120.mat Kpr -ascii                   % Save the LQR gains in Kpr.mat
```

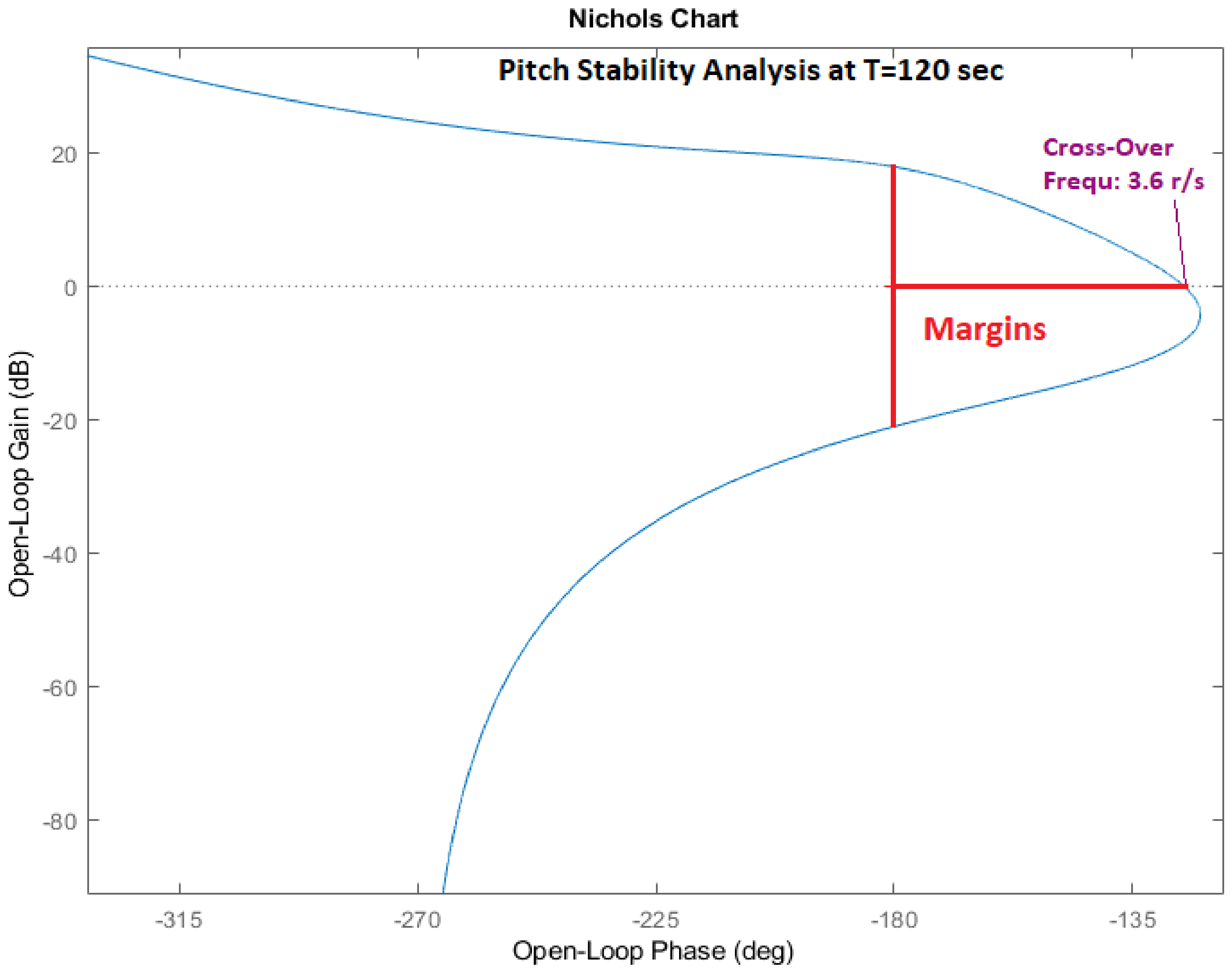


-0.0329	-0.224	0.00
-0.0136	-0.224	0.00
0.0136	-0.224	0.00
0.0329	-0.224	0.00
0.0329	-0.224	0.00
0.0136	-0.224	0.00
-0.0136	-0.224	0.00
-0.0329	-0.224	0.00
0.0136	0	-0.224
0.0329	0	-0.224
0.0329	0	-0.224
0.0136	0	-0.224
-0.0136	0	-0.224
-0.0329	0	-0.224
-0.0329	0	-0.224
-0.0136	0	-0.224

-1.0	-1.0	0.0
-0.4145	-1.0	0.0
0.4145	-1.0	0.0
1.0	-1.0	0.0
1.0	-1.0	0.0
0.4145	-1.0	0.0
-0.4145	-1.0	0.0
-1.0	-1.0	0.0
0.4145	0	-1.0
1.0	0	-1.0
1.0	0	-1.0
0.4145	0	-1.0
-0.4145	0	-1.0
-1.0	0	-1.0
-1.0	0	-1.0
-0.4145	0	-1.0

Notice, in 6DOF simulations it is convenient to keep a constant TVC matrix instead of varying all the TVC elements as a function of mass. For the entire first stage the TVC matrix is saved as a fixed matrix that is pre-multiplied with 3 time-varying loop gains, see Figure. Each axis column is multiplied by a time-varying loop gain calculated at each time-slice. The pitch and yaw loop gains are the same in this case because the vehicle is a cylinder.





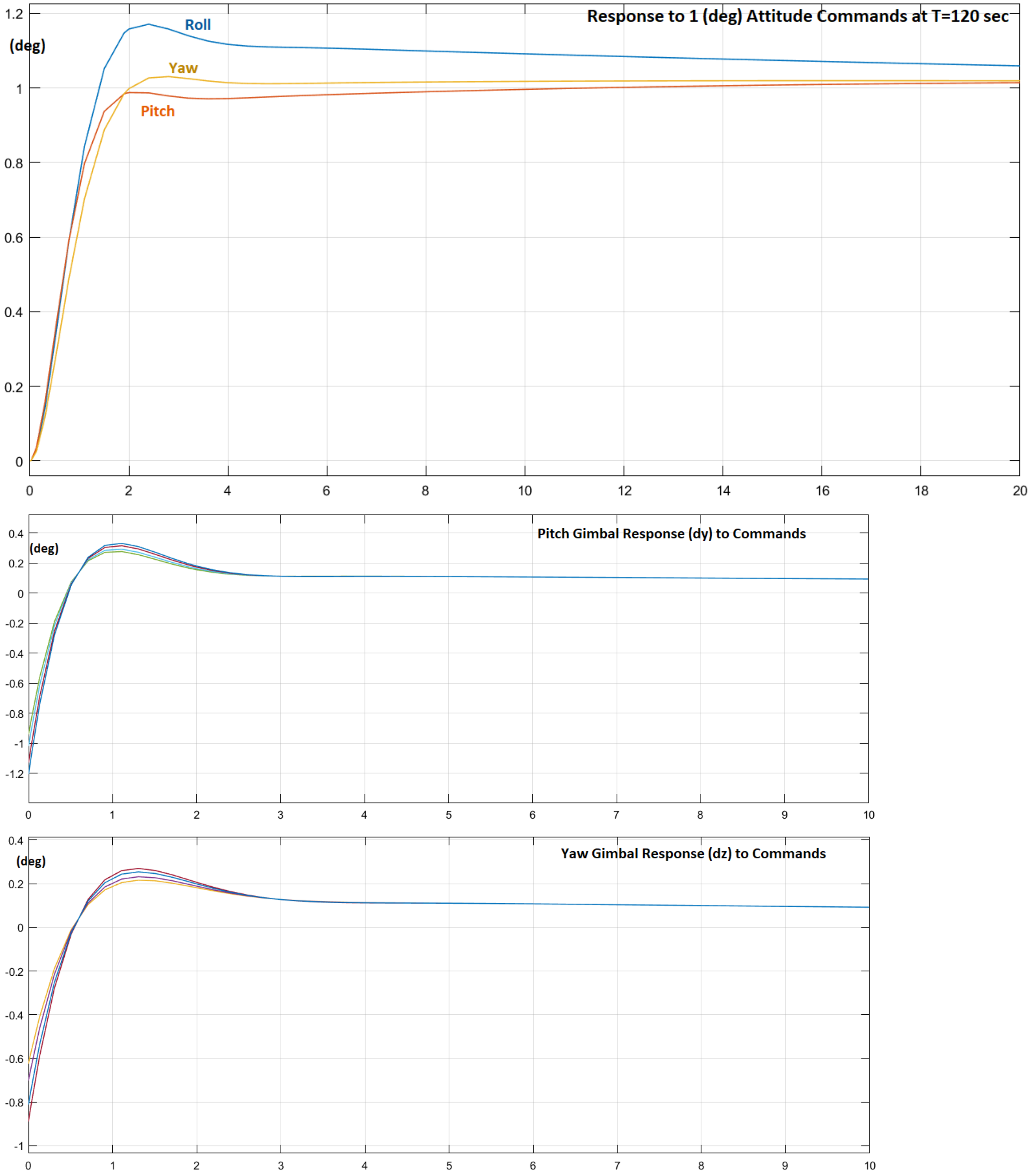


Figure 2.1.14 Response to 1° Step Attitude Commands in Roll, Pitch and Yaw Together

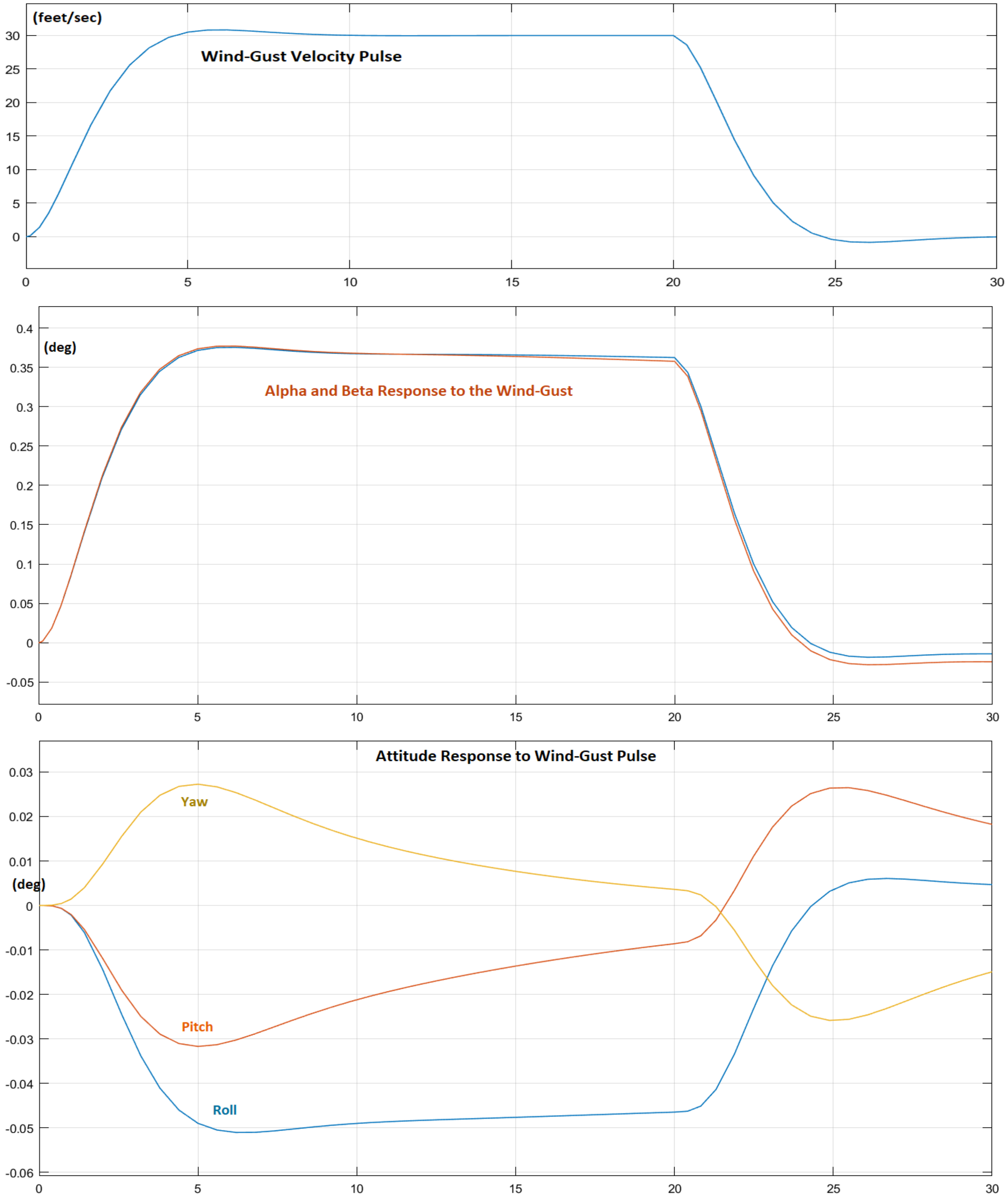
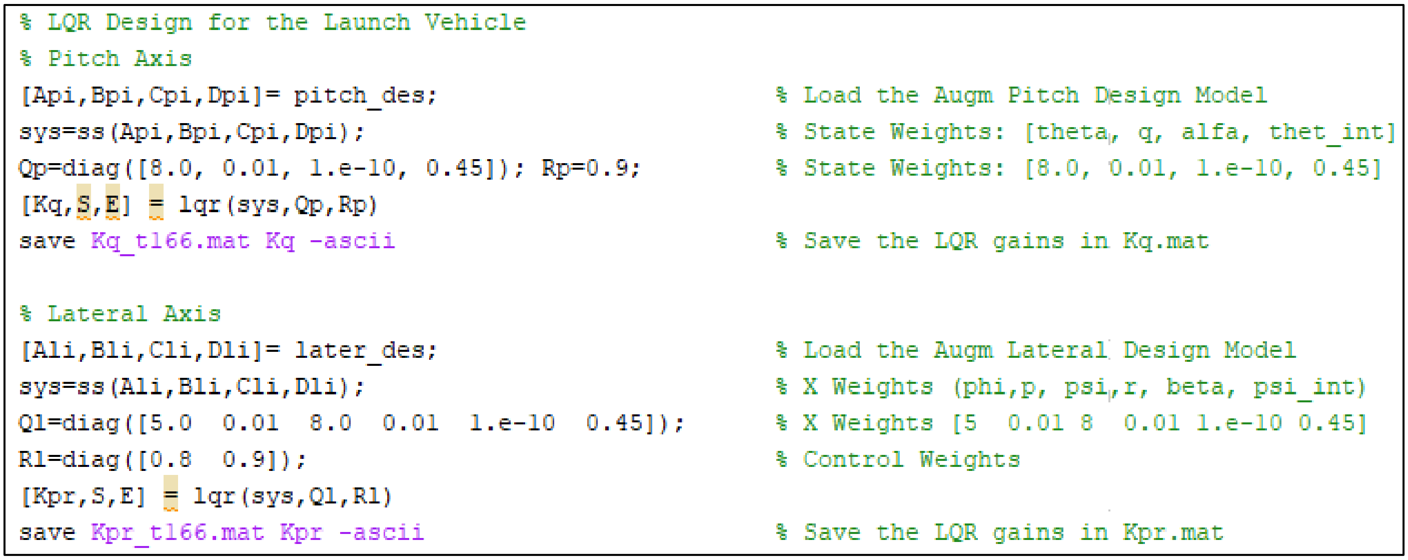


Figure 2.1.15 System Response to a 30 (ft/sec) Wind-Gust. The Attitude Errors due to Gust are Very Small

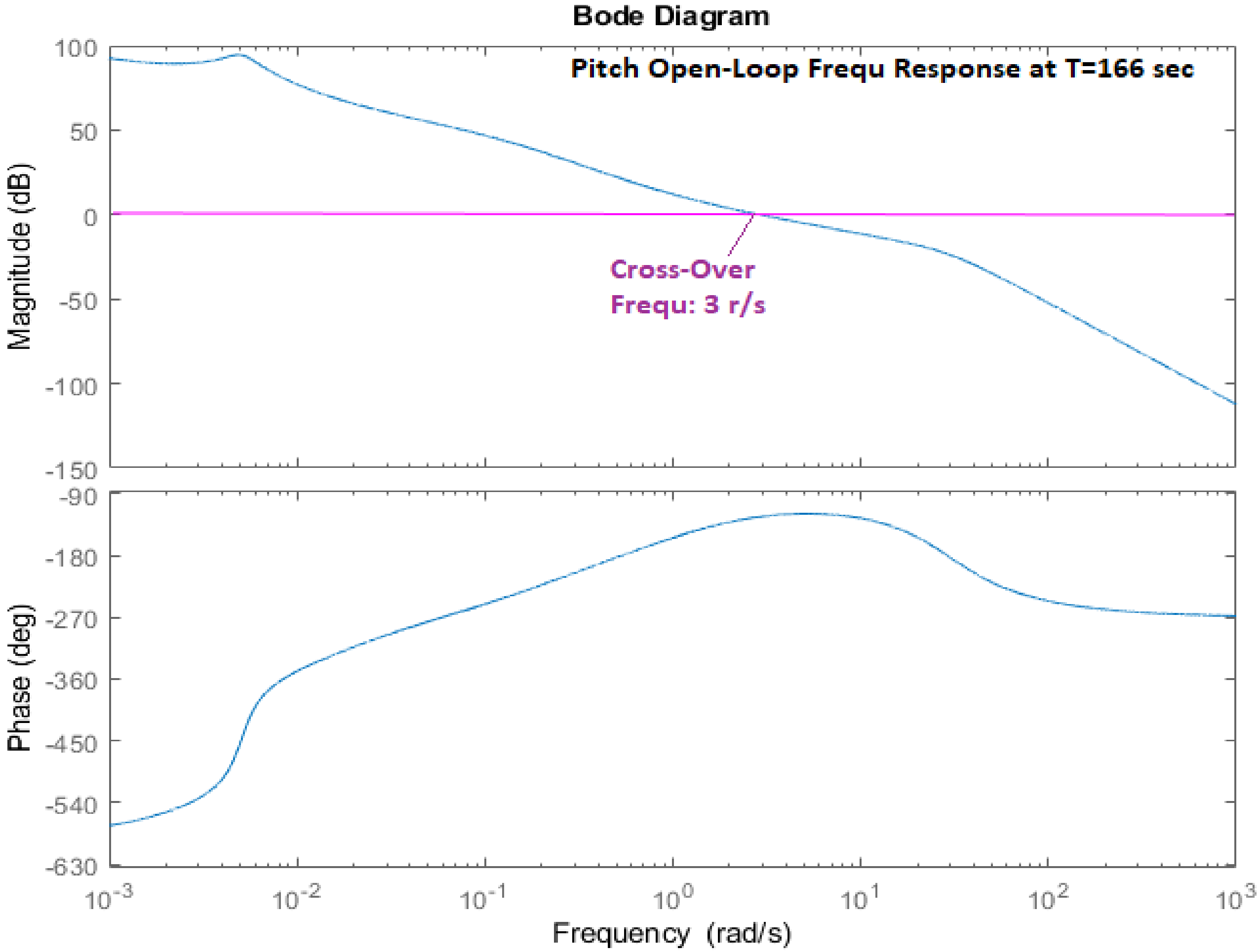
2.1.7 Control Design at Pre-Separation, T= 166 sec

At T=166 sec the dynamic pressure is very low, only 6 (lbf/ft^2) and we do not need load-relief. The control system is fully in attitude command tracking mode and are applying (θ & ψ)-integral feedback for trimming the pitch and yaw attitude errors. The LQR weights on the α & β states are very small now and the (θ & ψ)-integral weights have been increased for better attitude trimming. The control system bandwidth is 3 (rad/sec), not as high as in the High-Q region. The work files are in directory "*\2-Control Gains Design\1st Stage\T166".*



```
% LQR Design for the Launch Vehicle
% Pitch Axis
[Api,Bpi,Cpi,Dpi]= pitch_des;                        % Load the Augm Pitch Design Model
sys=ss(Api,Bpi,Cpi,Dpi);                             % State Weights: [theta, q, alfa, thet_int]
Qp=diag([8.0, 0.01, 1.e-10, 0.45]); Rp=0.9;          % State Weights: [8.0, 0.01, 1.e-10, 0.45]
[Kq,S,E] = lqr(sys,Qp,Rp)
save Kq_t166.mat Kq -ascii                           % Save the LQR gains in Kq.mat

% Lateral Axis
[Ali,Bli,Cli,Dli]= later_des;                        % Load the Augm Lateral Design Model
sys=ss(Ali,Bli,Cli,Dli);                             % X Weights (phi,p, psi,r, beta, psi_int)
Ql=diag([5.0  0.01  8.0  0.01  1.e-10  0.45]);       % X Weights [5  0.01 8  0.01 1.e-10 0.45]
Rl=diag([0.8  0.9]);                                 % Control Weights
[Kpr,S,E] = lqr(sys,Ql,Rl)
save Kpr_t166.mat Kpr -ascii                         % Save the LQR gains in Kpr.mat
```



The pitch controller is (1x4) state-feedback gain matrix Kq_t166. The pitch control demand DQ is calculated as a function of states: θ, q, α, θ-integral. The roll and yaw axes are again treated as a coupled lateral system, for now, creating a (2x6) state-feedback controller matrix Kpr_t166 from states: φ, p, ψ, r, β, and ψ-integral. Later, in the analysis section the lateral controller will be separated because the roll/yaw coupling is small. The roll control demand DP will be a function of (φ and p), and the yaw demand DR will be a function of (ψ, r, β, and ψ-integral).

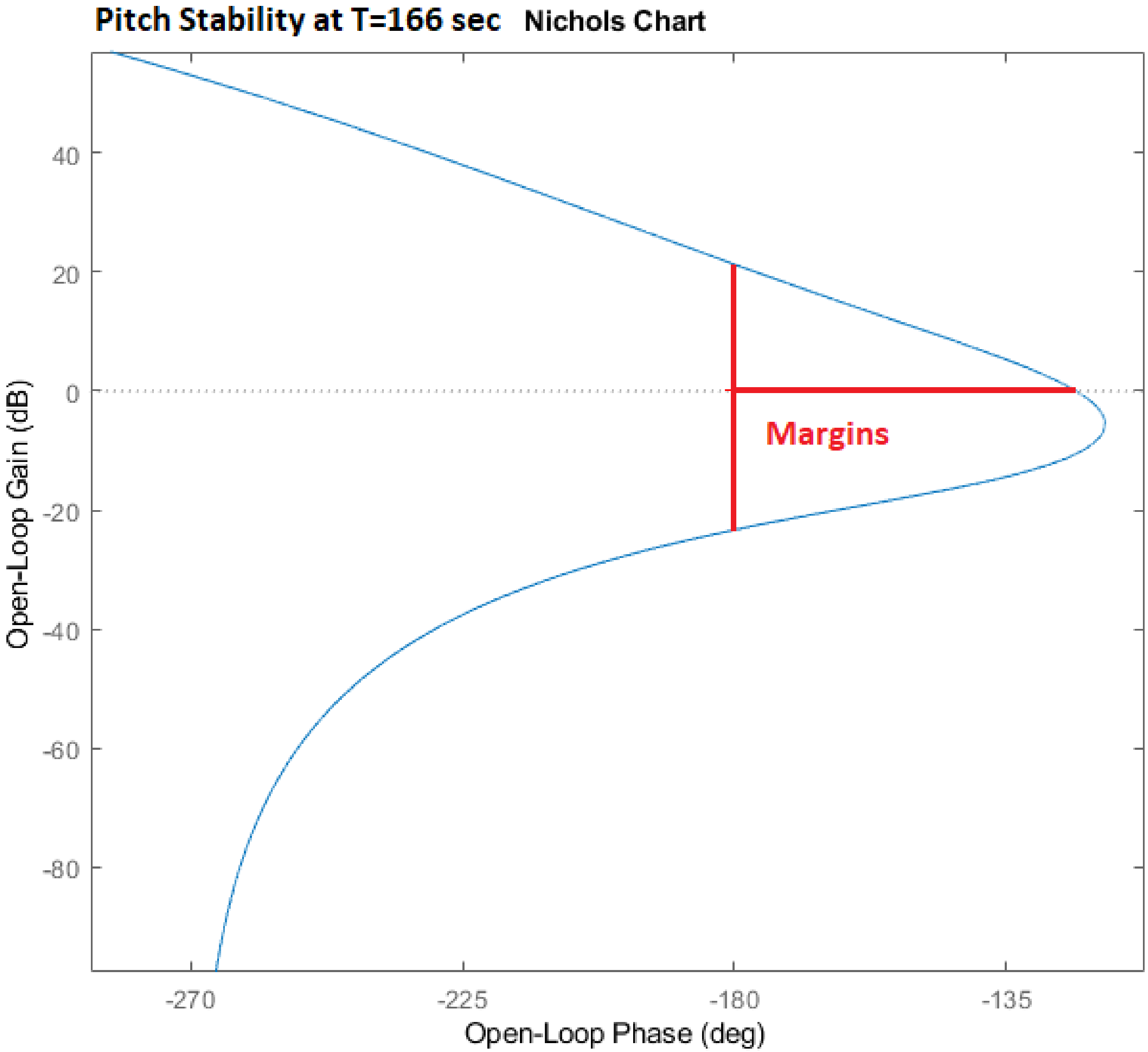


Figure 2.1.16 Pitch and Yaw Stability Margins at T=166 sec

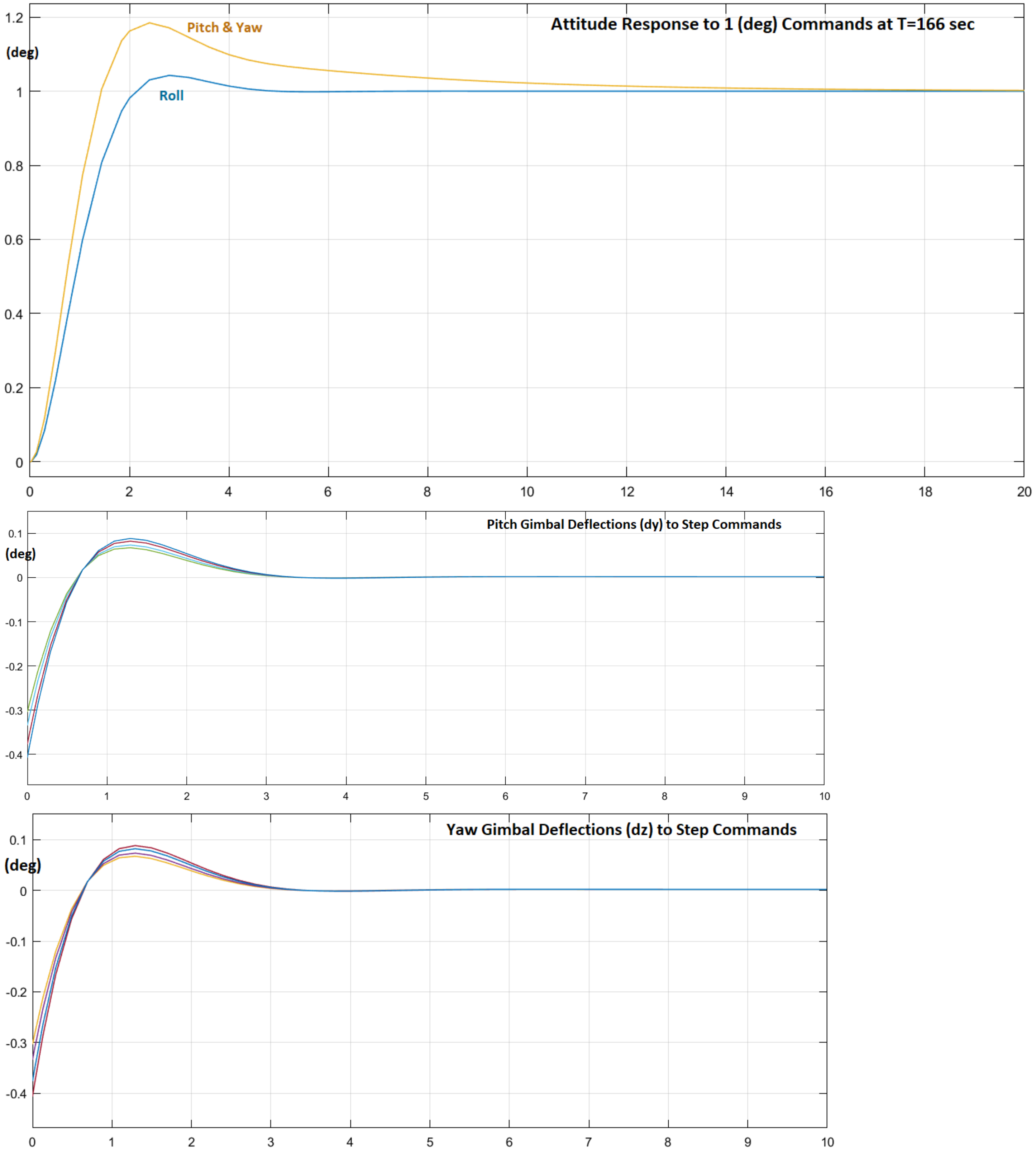


Figure 2.1.17 The Response to Step Attitude Commands is Very Good at Pre-Separation Because the Dynamic Pressure is Small and there is No Load-Relief

2.2 First Stage Control Analysis with Slosh and Flexibility

We will now update the 1st stage vehicle models created for the 8 time-slices: T10, T30, T60, T80, T100, T120, T140, and T166 to analyze the system stability and performance in those flight conditions, including propellant sloshing and structural flexibility. The 8 analysis folders are in directory "\23-Classic Launch Vehicle Design & Simulation\ 3-Stability Analysis with Flex & Slosh\1st Stage". Each analysis folder includes two approaches for creating the vehicle models and the corresponding work files are placed in separate subfolders. The subfolder "*Matlab Analysis*" uses the Flixan program to generate the vehicle, mixing-logic, and actuator systems and they are combined together in Matlab using Simulink models for the open and closed loop analysis. The subfolder "*Flixan Analysis*" generates all systems, matrices and the combined systems using the Flixan program. The analyses of the 8 flight conditions are similar and we will describe some of them in detail. Specifically, at T10 we will describe the Matlab analysis process in detail, and at T80 we will describe the Flixan analysis process. For the remaining time-slice cases we will just present the stability analysis and performance results.

The vehicle datasets are the same as the ones used in the control design, except that they now include slosh parameters and structural flexibility modes. The direction of the wind-gust is defined by the azimuth and elevation angles which are 45° and 90° respectively. It means that the wind velocity is coming towards the vehicle perpendicular to the x-axis and at 45 between the -Y and -Z axes. The flex modes are already preselected and included at the bottom of the input files. The state-feedback gains are copied from the corresponding control design sections. However, the lateral roll/ yaw coupled LQR controller Kpr from the design section is now separated. It is replaced by a yaw state-feedback gain which is identical to pitch, and a PD controller for roll control that does not change very much during flight. The variation in vehicle mass properties is taken care by a varying TVC matrix which in the 6DOF is implemented as a fixed TVC matrix with 3 time-varying loop gains.

As already described in the control design section, the state-feedback gain Kq is using (θ and ψ) attitude integral feedback at low-Q and it switches to (α and β) integral feedback at high dynamic pressures. Some of the state-feedback gains were slightly adjusted to accommodate flexibility and low-pass filters are included. Also, launch vehicles in general do not have an aero-data probe to measure the angles of attack and sideslip which are needed for the LQR state-feedback. In this case α and β are estimated from the normal and lateral accelerometers as we shall describe in detail.

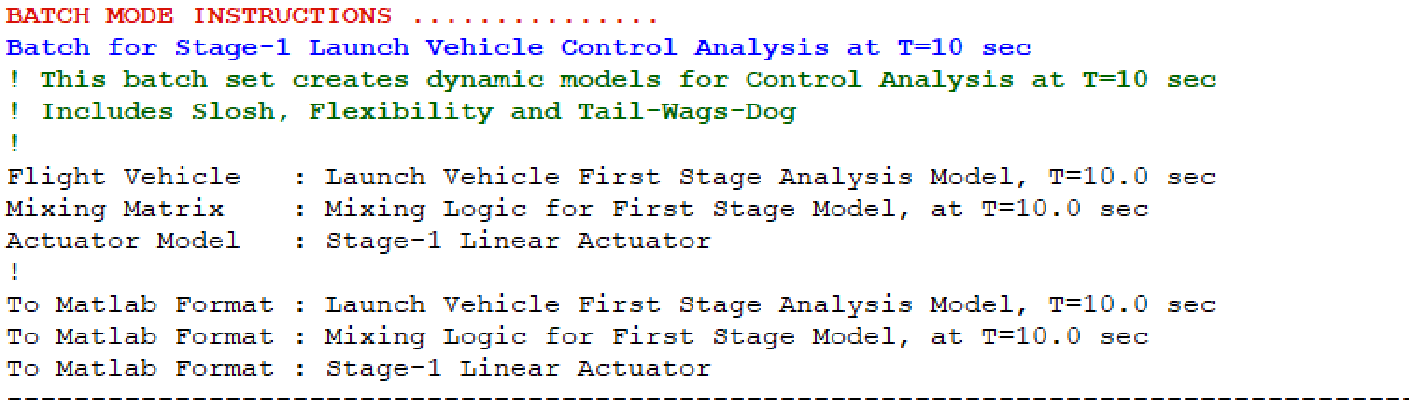
2.2.1 Control Analysis at T= 10 sec

The analysis files for this flight condition are in "*Examples\23-Classic Launch Vehicle Design & Simulation\3-Stability Analysis with Flex & Slosh\1st Stage 1st Stage\T10\ Matlab Analysis*". The input file is "*Flex_Vehi_T10a.inp*". The equivalent Flixan analysis files are in "*\1st Stage 1st Stage\T10\Flixan Analysis*" and the input file is "*Flex_Vehi_T10b.inp*". The LQR state-feedback gain is Kq_t10= [2.75, 2.3, 0.02, 0.44] from pitch states: θ, q, α, and θ-integral respectively. It is also used for yaw control from states: ψ, r, β, and ψ-integral. The roll control PD gain is just, [2, 2.25] from the states: ϕ and p respectively.

The title of the vehicle dataset is "*Launch Vehicle First Stage Analysis Model, T=10.0 sec*" and it includes the 8 gimbaling engines, propellant sloshing parameters for the LOX and LH2 tanks, and the flex modes. The slosh parameters consist of the 2 slosh masses, slosh frequencies for each mass along y and z at 1g, the damping coefficients along y and z, and the x, y, z locations of the 2 slosh masses. The LOX and LH2 slosh frequencies are both 3.11 rad/sec at 1g acceleration. They are scaled by the program proportionally to the square root of the vehicle acceleration. Although the 2 propellant frequencies are defined to be the same at 1g, they change slightly under closed-loop control. The vehicle dataset also includes rate gyros for attitude control and Ny & Nz accelerometers for load-relief.

The modal data are created from finite element models at different propellant levels. At T10 when both tanks are full, the modal data file is "*Stage1_100%.Mod*" and the nodes ID file is "*Stage1_100%.Nod*". They are used by the Flixan mode selection process to select a set of structural modes that will be combined with the rigid vehicle dataset to create the flex vehicle state-space system for the analysis. The mode selection process is described in Section x. The selected set of modes is already included at the bottom of the vehicle file "*Flex_Vehi_T10a.inp*" with the title "*First Stage Flex Modes at 100% Full Tanks*". The number of modes to be included from the selected set of modes is shown at the bottom of the vehicle dataset.

The input file also includes a dataset that creates a linear actuator model for first stage. The parameters are a little different from the second stage actuator, like inertia, friction, etc. A non-linear Simulink actuator model will be used in the simulation. It includes Coulomb friction at the gimbal, position and rate limits. A batch set "*Batch for Stage-1 Launch Vehicle Control Analysis at T=10 sec*" is included at the top of the input file that creates the vehicle, TVC and actuator models in batch and exports the state-space systems into Matlab format.



```
BATCH MODE INSTRUCTIONS ...............
Batch for Stage-1 Launch Vehicle Control Analysis at T=10 sec
! This batch set creates dynamic models for Control Analysis at T=10 sec
! Includes Slosh, Flexibility and Tail-Wags-Dog
!
Flight Vehicle    : Launch Vehicle First Stage Analysis Model, T=10.0 sec
Mixing Matrix     : Mixing Logic for First Stage Model, at T=10.0 sec
Actuator Model    : Stage-1 Linear Actuator
!
To Matlab Format : Launch Vehicle First Stage Analysis Model, T=10.0 sec
To Matlab Format : Mixing Logic for First Stage Model, at T=10.0 sec
To Matlab Format : Stage-1 Linear Actuator
----------------------------------------------------------------------------
```

```
FLIGHT VEHICLE INPUT DATA ......
Launch Vehicle First Stage Analysis Model, T=10.0 sec
! This is a Launch Vehicle Control Analysis Model at t=10 sec with 8 TVC Engines.
! The model includes two slosh modes for the LOX and LH2 tanks at 100% Propellant level.
! The LOX tank requires baffles and the damping coefficient was increased to 0.012 .
! The Flight Control Sensors include 3 Rate Gyros (p,q,r) and 2 Accelerometers (Ny,Nz).
! The model also includes 33 Structural Modes Selected between the TVC and the Nav Base
!
Body Axes Output,Attitude=Rate Integral

Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft)    :  5070.0         32.1740       0.208960E+08
Moments and Products of Inertia: Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)          :  36805.3       0.281044E+07  0.280948E+07   0.00000
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)        :  57.8656        0.00000       0.00000
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)       :  0.071          79.6400       7.38000        384.843
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)         :  8.8            40.88         0.0           -0.001
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                         :  0.54           0.0          -0.09           0.000
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)   :  0.0            89.67         0.00000        0.0
W-Gust Azim & Elev angles (deg), or Torque/Force direction (x,y,z), Force Locat (x,y,z)  :  Gust           45.00         90.00
Surface Reference Area (feet^2), Mean Aerodynamic Chord (ft), Wing Span in (feet)        :  44.415         7.52000       7.52000
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial_rho/ Partial_H} :  120.147        0.00000       0.00000        0.00000
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:  0.4405        -0.00081       0.00000        0.00000
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}: -0.0           -0.03557       0.00000        0.00000
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}: -0.0061        -0.03557       0.00000        0.00000
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:  0.00000        0.00000       0.00000        0.00000
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}:  0.27           0.1114        0.00000        0.00000
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}:  0.0           -0.1114        0.00000        0.00000

Number of Thruster Engines, Include or Not the Tail-Wags-Dog and Load-Torque Dynamics ?  :  8   WITH TWD

TVC Engine No:  1                          (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#1 +2Y-Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                 :  22803.1        22803.1
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000        0.00000       6.00000        6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000        15.1200       1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0           2.4945       -1.0332
TVC Engine No:  2                          (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#2 +Y-2Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                 :  22803.1        22803.1
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000        0.00000       6.00000        6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000        15.1200       1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0           1.0332       -2.4945
TVC Engine No:  3                          (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#3 -Y-2Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                 :  22803.1        22803.1
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000        0.00000       6.00000        6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000        15.1200       1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0          -1.0332       -2.4945
TVC Engine No:  4                          (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#4 -2Y-Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                 :  22803.1        22803.1
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000        0.00000       6.00000        6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000        15.1200       1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0          -2.4945       -1.0332
TVC Engine No:  5                          (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#5 -2Y+Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                 :  22803.1        22803.1
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000        0.00000       6.00000        6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000        15.1200       1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0          -2.4945        1.0332
TVC Engine No:  6                          (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#6 -Y+2Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                 :  22803.1        22803.1
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000        0.00000       6.00000        6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000        15.1200       1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0          -1.0332        2.4945
TVC Engine No:  7                          (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#7 +Y+2Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                 :  22803.1        22803.1
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000        0.00000       6.00000        6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000        15.1200       1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0           1.0332        2.4945
TVC Engine No:  8                          (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#8 +2Y+Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                 :  22803.1        22803.1
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000        0.00000       6.00000        6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000        15.1200       1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0           2.4945        1.0332

Number of Gyros, (Attitude and Rate)                                                     :  3
Gyro No  1 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Location in (feet) :  Roll   Rate           97.438    0.000      0.000
Gyro No  2 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Location in (feet) :  Pitch  Rate           97.438    0.000      0.000
Gyro No  3 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Location in (feet) :  Yaw    Rate           97.438    0.000      0.000

Number of Accelerometers, Along Axis: (x,y,z)                                            :  2
Acceleromet No  2 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Location (ft) :  Y-axis Accelerat.     97.438    0.000      0.000
Acceleromet No  3 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Location (ft) :  Z-axis Accelerat.     97.438    0.000      0.000

Number of Slosh Modes                                                                    :  2
LOX Mass  (slug), Frequency 1g (Wy,Wz) (rad/s), Damp (zeta-y-z), Locat.{Xsl,Ysl,Zsl} (ft):  121.51   3.11   3.11   0.005  0.005  71.09   0.0  0.0
LH2 Mass  (slug), Frequency 1g (Wy,Wz) (rad/s), Damp (zeta-y-z), Locat.{Xsl,Ysl,Zsl} (ft):  41.906   3.11   3.11   0.001  0.001  41.16   0.0  0.0

Number of Bending Modes                                                                  : 31
First Stage Flex Modes at 100% Full Tanks
---------------------------------------------------------------------------------------------------------------------------------------------
```

```
MIXING LOGIC MATRIX DATA ..........     (Matrix Title, Name, Vehicle Title, Control Directions)
Mixing Logic for First Stage Model, at T=10.0 sec
! Thrust Vector Control Matrix at t=10 sec
! This multi-engine vehicle has 8 Gimbaling Engines.
TVC
Launch Vehicle First Stage Analysis Model, T=10.0 sec
P-dot Roll   Acceleration About X Axis
Q-dot Pitch Acceleration About Y Axis
R-dot Yaw    Acceleration About Z Axis
------------------------------------------------------------------------------------------------------
ACTUATOR INPUT DATA ............ SIMPLE GENERIC MODEL B
Stage-1 Linear Actuator
Symbol Parameter Description                        (Units)       Value
C(s)   Order of Pade Delay (0,1,2)                    ( - )         1, -0.001, 0.001
Ka     Gain of Amplifier ...................... (amps/volt)   28.0
Wsv    Bandwidth of the Linear Servo Actuator .   (rad/sec)     70.0
Kact   Actuator Stiffness (Piston+Oil+Electric)    (lb/ft)       2.4e+6
Klod   Stiffness at Surface or Nozzle Connection   (lb/ft)       1.2e+9
Kbck   Stiffness at Vehicle Backup Structure ..    (lb/ft)       7.0e+7
R      Moment Arm between Actuator Rod & Gimbal    (feet)        0.667
Jl     Load Inertia about the Gimbal .......... (ft-lb-s^2)   15.12
Kg     Load Gimbal Bearing Spring Constant      (ft-lb/rad)   0.0
Bg     Load Gimbal Bearing Viscous Damping .... (ft-lb-sec)   550.0
------------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Mixing Logic for First Stage Model, at T=10.0 sec
Matrix TVC
------------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Launch Vehicle First Stage Analysis Model, T=10.0 sec
System
flex_vehicle.m
------------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Stage-1 Linear Actuator
System
actuator.m
------------------------------------------------------------------------------------------------------
SELECTED MODAL DATA AND LOCATIONS FOR : 100% Full
First Stage Flex Modes at 100% Full Tanks
! Flex Modes, First Stage 100% Full Tanks from Files: Stg1_100%.Mod, Stg1_100%.Nod
! Sensors are at the Top of LOX Tank
! The Modes were selected between the TVC and the IMU Location

MODE#  1/  1, Frequency (rad/sec), Damping (zeta), Generalized Mass=    19.2         0.50000E-02    12.000
DEFINITION OF LOCATIONS (NODES)            phi along X   phi along Y   phi along Z   sigm about X  sigm about Y  sigm about Z

                              Node ID#     Modal Data at the  8 Engines, (x,y,z)...
S1 Engine No:1 +Y-Z               1151    -0.11835D-01  -0.11684D+00   0.66210D-02   0.16324D-05   0.27168D-03   0.48347D-02
S1 Engine No:2 +Y-Z               1152    -0.54460D-02  -0.11683D+00   0.66203D-02  -0.26923D-06   0.27091D-03   0.48442D-02
S1 Engine No:3 -Y-Z               1153     0.41356D-02  -0.11683D+00   0.66217D-02  -0.68227D-07   0.27704D-03   0.48445D-02
S1 Engine No:4 -Y-Z               1154     0.11292D-01  -0.11684D+00   0.66208D-02   0.15055D-05   0.27746D-03   0.48349D-02
S1 Engine No:5 -Y+Z               1155     0.11835D-01  -0.11684D+00   0.66208D-02  -0.16901D-05   0.27175D-03   0.48347D-02
S1 Engine No:6 -Y+Z               1156     0.54463D-02  -0.11683D+00   0.66202D-02   0.47088D-06   0.27080D-03   0.48442D-02
S1 Engine No:7 +Y+Z               1157    -0.41355D-02  -0.11683D+00   0.66220D-02   0.16997D-06   0.27713D-03   0.48445D-02
S1 Engine No:8 +Y+Z               1158    -0.11292D-01  -0.11684D+00   0.66211D-02  -0.17161D-05   0.27732D-03   0.48350D-02

                              Node ID#     Modal Data at the  3 Gyros ...
Stage-2 Tank Top, IMU Locat.     40015     0.41474D-07  -0.54406D-01   0.30829D-02   0.48748D-08  -0.23545D-03  -0.41551D-02
Stage-2 Tank Top, IMU Locat.     40015     0.41474D-07  -0.54406D-01   0.30829D-02   0.48748D-08  -0.23545D-03  -0.41551D-02
Stage-2 Tank Top, IMU Locat.     40015     0.41474D-07  -0.54406D-01   0.30829D-02   0.48748D-08  -0.23545D-03  -0.41551D-02

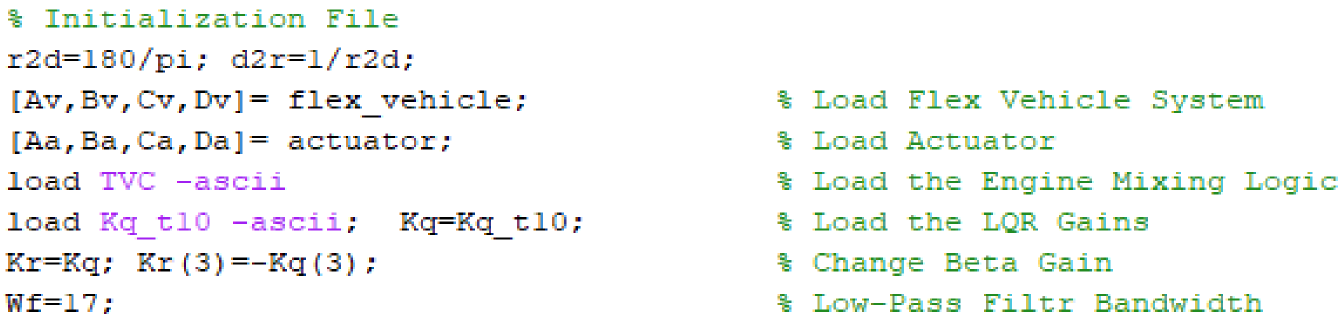
                              Node ID#     Modal Data at the  2 Accelerometers, along (x,y,z)...
Stage-2 Tank Top, IMU Locat.     40015     0.41474D-07  -0.54406D-01   0.30829D-02
Stage-2 Tank Top, IMU Locat.     40015     0.41474D-07  -0.54406D-01   0.30829D-02

                              Node ID#     Modal Data at the  2 Slosh Masses...
LOX  Slosh Mass Locat.             601     0.57960D-07   0.31849D-01  -0.18050D-02   0.48010D-08  -0.12250D-03  -0.21616D-02
Fuel Slosh Mass Locat.             600     0.88999D-07   0.13928D-01  -0.78942D-03   0.43838D-08   0.18448D-03   0.32552D-02

                              Node ID#     Modal Data at the Disturbance Point
S1 Engine No:9 center             1159     0.22434D-04  -0.11741D+00   0.66533D-02  -0.16100D-04   0.29930D-03   0.52819D-02

MODE#  2/  2, Frequency (rad/sec), Damping (zeta), Generalized Mass=    19.2         0.50000E-02    12.000
```

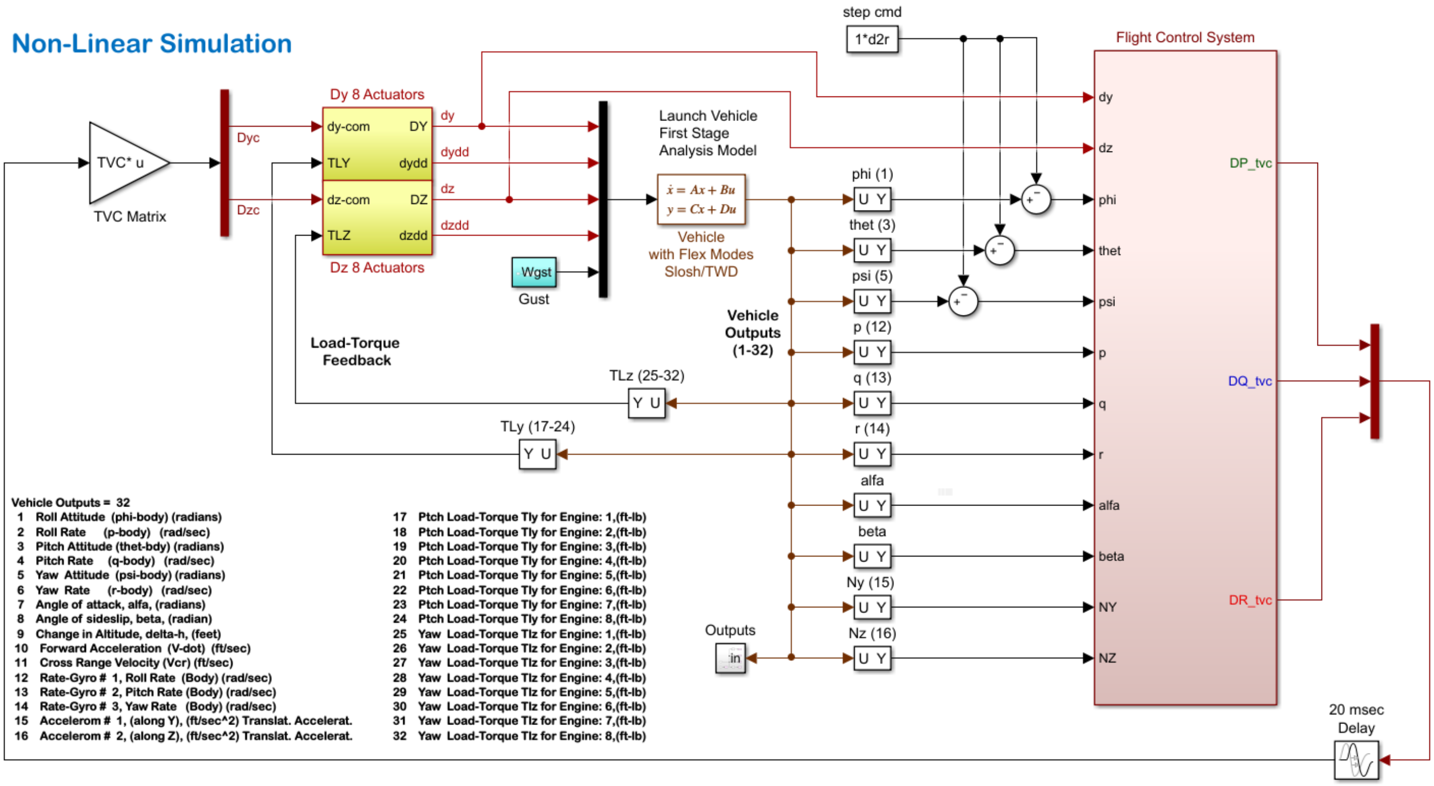
The initialization file “init.m” loads the Flixan generated vehicle “flex_vehicle”, and the actuator systems into Matlab. It also loads the TVC matrix and the LQR control gain Kq_t10. It also includes the bandwidth of the low-pass filter which varies with flight conditions.



```
% Initialization File
r2d=180/pi; d2r=1/r2d;
[Av,Bv,Cv,Dv]= flex_vehicle;                    % Load Flex Vehicle System
[Aa,Ba,Ca,Da]= actuator;                        % Load Actuator
load TVC -ascii                                 % Load the Engine Mixing Logic
load Kq_t10 -ascii;  Kq=Kq_t10;                 % Load the LQR Gains
Kr=Kq; Kr(3)=-Kq(3);                            % Change Beta Gain
Wf=17;                                          % Low-Pass Filtr Bandwidth
```

Simulation Model

The 1st stage simulation model “*Sim_Flex.slx*” is shown in Figure 2.2.1. It includes the flexible vehicle first stage model from file “flex_vehicle.m”, the control system which is shown in detain in Figure 2.2.2, the TVC matrix, and the non-linear TVC actuators, shown in Figure 2.2.4. The estimators in Figure 2.2.3 estimate alpha and beta from the normal and lateral accelerometer measurements, the pitch and yaw rates (q & r), and the pitch and yaw gimbal deflections (dy & dz). The (α & β) estimates are used in the state-feedback controller for load-relief, instead of the real (α & β).



Vehicle Outputs = 32
1 Roll Attitude (phi-body) (radians)
2 Roll Rate (p-body) (rad/sec)
3 Pitch Attitude (thet-bdy) (radians)
4 Pitch Rate (q-body) (rad/sec)
5 Yaw Attitude (psi-body) (radians)
6 Yaw Rate (r-body) (rad/sec)
7 Angle of attack, alfa, (radians)
8 Angle of sideslip, beta, (radian)
9 Change in Altitude, delta-h, (feet)
10 Forward Acceleration (V-dot) (ft/sec)
11 Cross Range Velocity (Vcr) (ft/sec)
12 Rate-Gyro # 1, Roll Rate (Body) (rad/sec)
13 Rate-Gyro # 2, Pitch Rate (Body) (rad/sec)
14 Rate-Gyro # 3, Yaw Rate (Body) (rad/sec)
15 Accelerom # 1, (along Y), (ft/sec^2) Translat. Accelerat.
16 Accelerom # 2, (along Z), (ft/sec^2) Translat. Accelerat.
17 Ptch Load-Torque Tly for Engine: 1,(ft-lb)
18 Ptch Load-Torque Tly for Engine: 2,(ft-lb)
19 Ptch Load-Torque Tly for Engine: 3,(ft-lb)
20 Ptch Load-Torque Tly for Engine: 4,(ft-lb)
21 Ptch Load-Torque Tly for Engine: 5,(ft-lb)
22 Ptch Load-Torque Tly for Engine: 6,(ft-lb)
23 Ptch Load-Torque Tly for Engine: 7,(ft-lb)
24 Ptch Load-Torque Tly for Engine: 8,(ft-lb)
25 Yaw Load-Torque Tlz for Engine: 1,(ft-lb)
26 Yaw Load-Torque Tlz for Engine: 2,(ft-lb)
27 Yaw Load-Torque Tlz for Engine: 3,(ft-lb)
28 Yaw Load-Torque Tlz for Engine: 4,(ft-lb)
29 Yaw Load-Torque Tlz for Engine: 5,(ft-lb)
30 Yaw Load-Torque Tlz for Engine: 6,(ft-lb)
31 Yaw Load-Torque Tlz for Engine: 7,(ft-lb)
32 Yaw Load-Torque Tlz for Engine: 8,(ft-lb)

Figure 2.2.18 First Stage Simulation Model “Sim_Flex.slx”

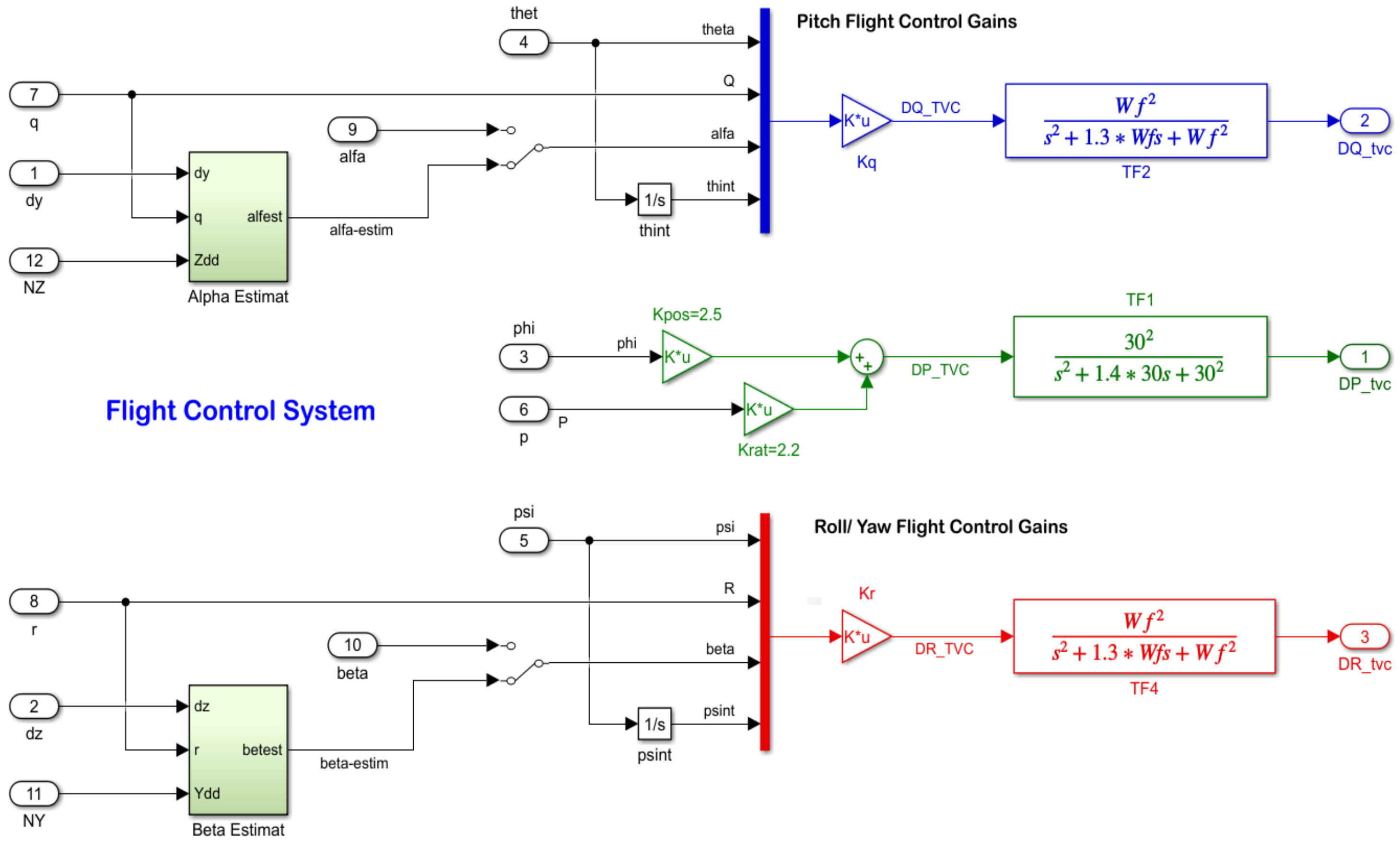


Figure 2.2.19 Pitch, Yaw and Roll Flight Control System

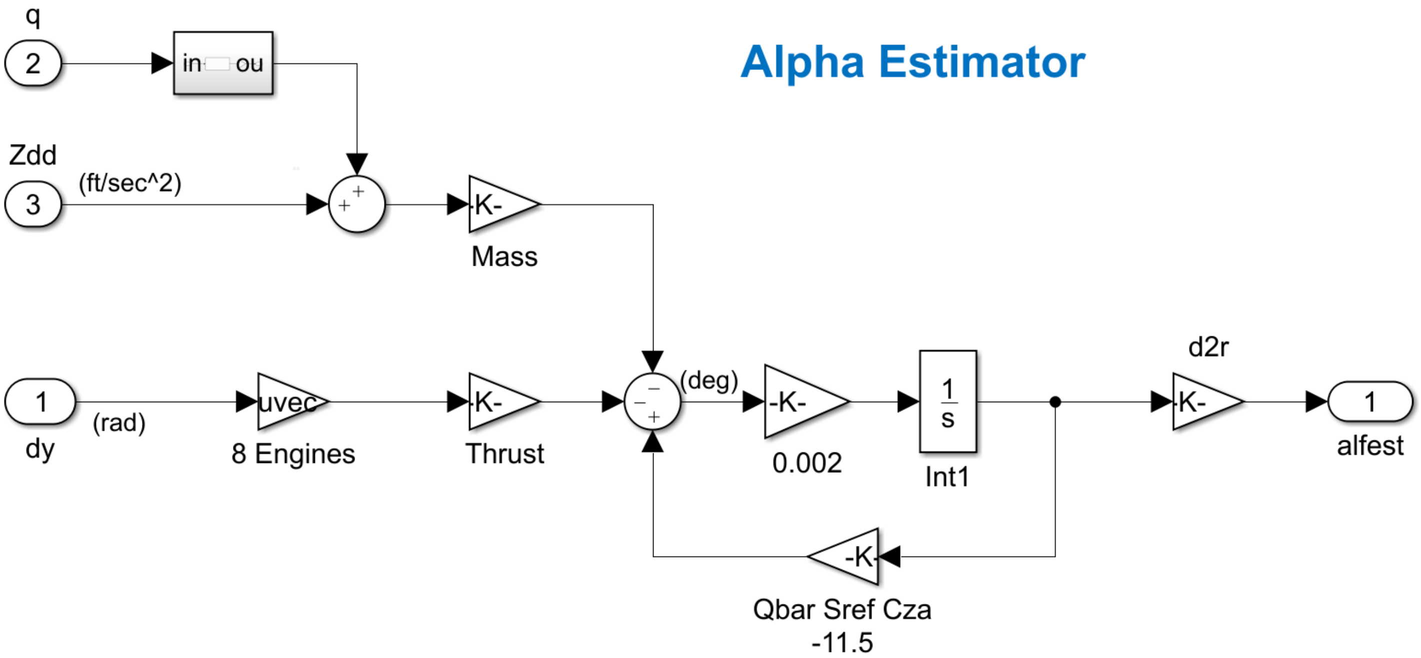
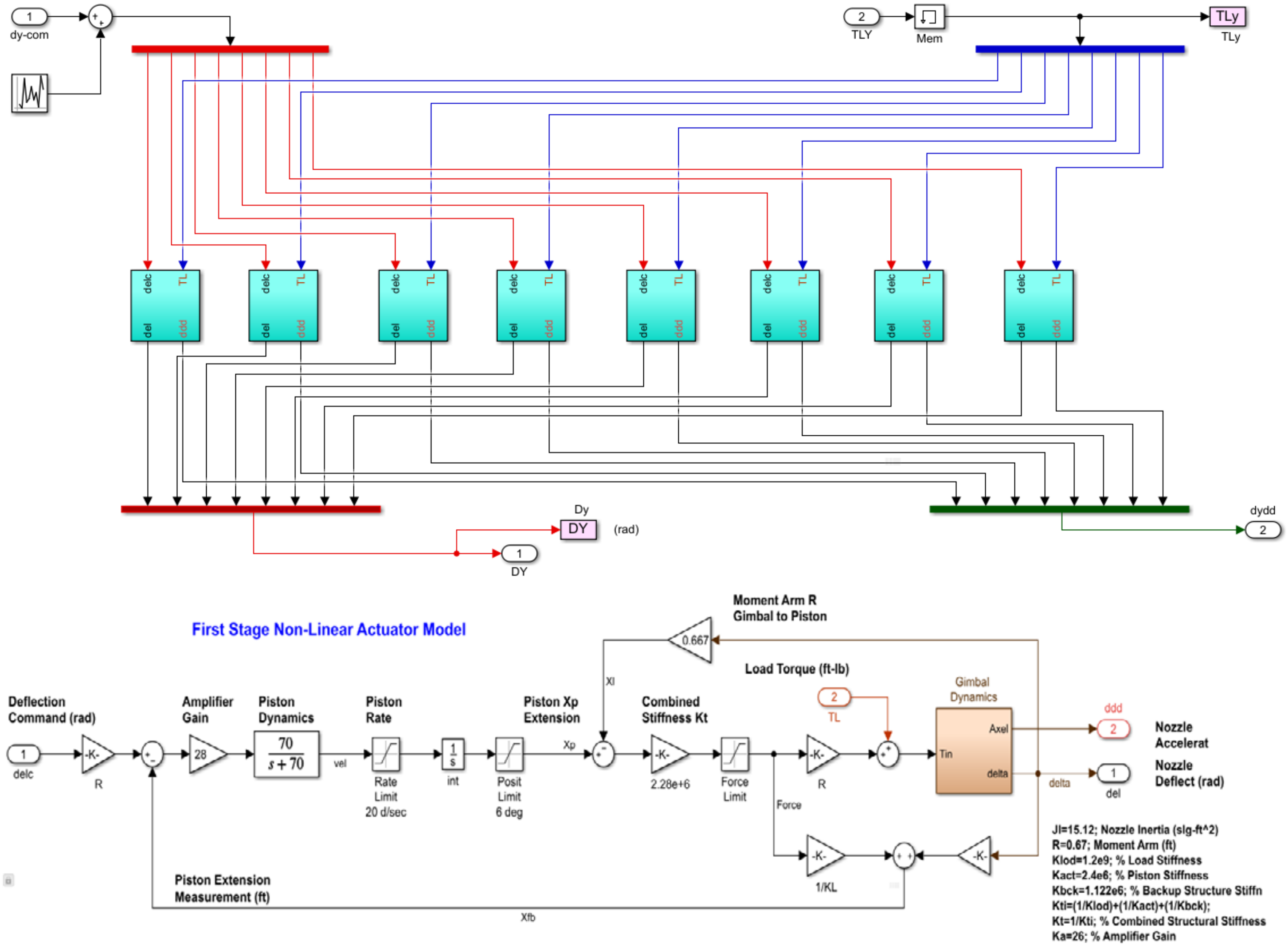


Figure 2.2.20 Alpha Estimator Used for Load-Relief

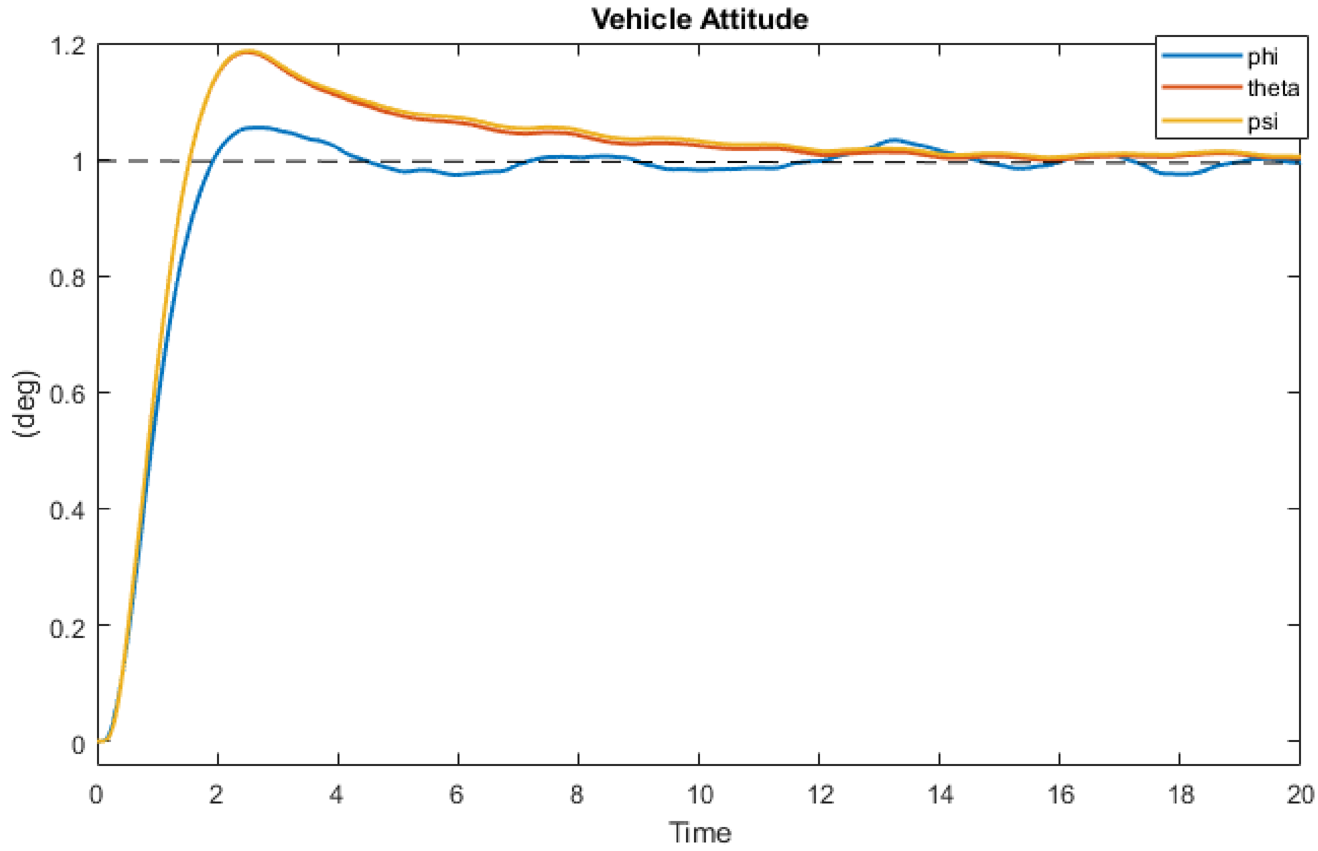


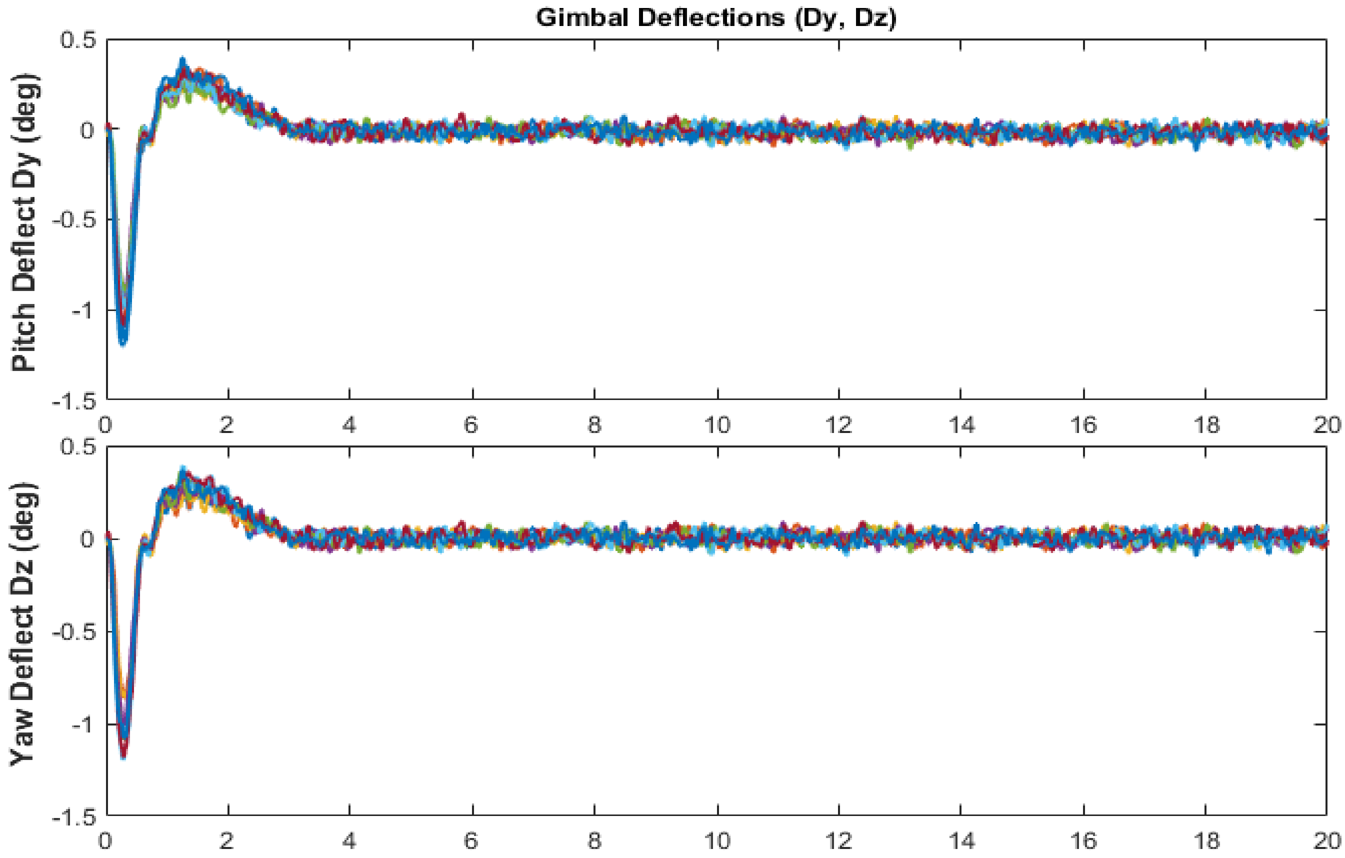
```
Jl=15.12; Nozzle Inertia (slg-ft^2)
R=0.67; Moment Arm (ft)
Klod=1.2e9; % Load Stiffness
Kact=2.4e6; % Piston Stiffness
Kbck=1.122e6; % Backup Structure Stiffn
Kti=(1/Klod)+(1/Kact)+(1/Kbck);
Kt=1/Kti; % Combined Structural Stiffness
Ka=26; % Amplifier Gain
```

Figure 2.2.21 Non-Linear Actuator Subsystem

Simulation Results

Figure 2.2.5 shows the system's response to the simultaneously applied 1° commands in roll pitch and yaw. The noise in the actuator inputs represents gimbal measurement noise and it produces jitter at the gimbal deflections. The Coulomb friction in the non-linear actuator models also causes small attitude disturbances, especially in roll. Figure 2.2.6 shows the system's response to a 30 (ft/sec) wind-gust.


Vehicle Attitude
phi
theta
psi
(deg)
Time
0
0.2
0.4
0.6
0.8
1
1.2
0
2
4
6
8
10
12
14
16
18
20


Gimbal Deflections (Dy, Dz)
Pitch Deflect Dy (deg)
Yaw Deflect Dz (deg)
0.5
0
-0.5
-1
-1.5
0
2
4
6
8
10
12
14
16
18
20

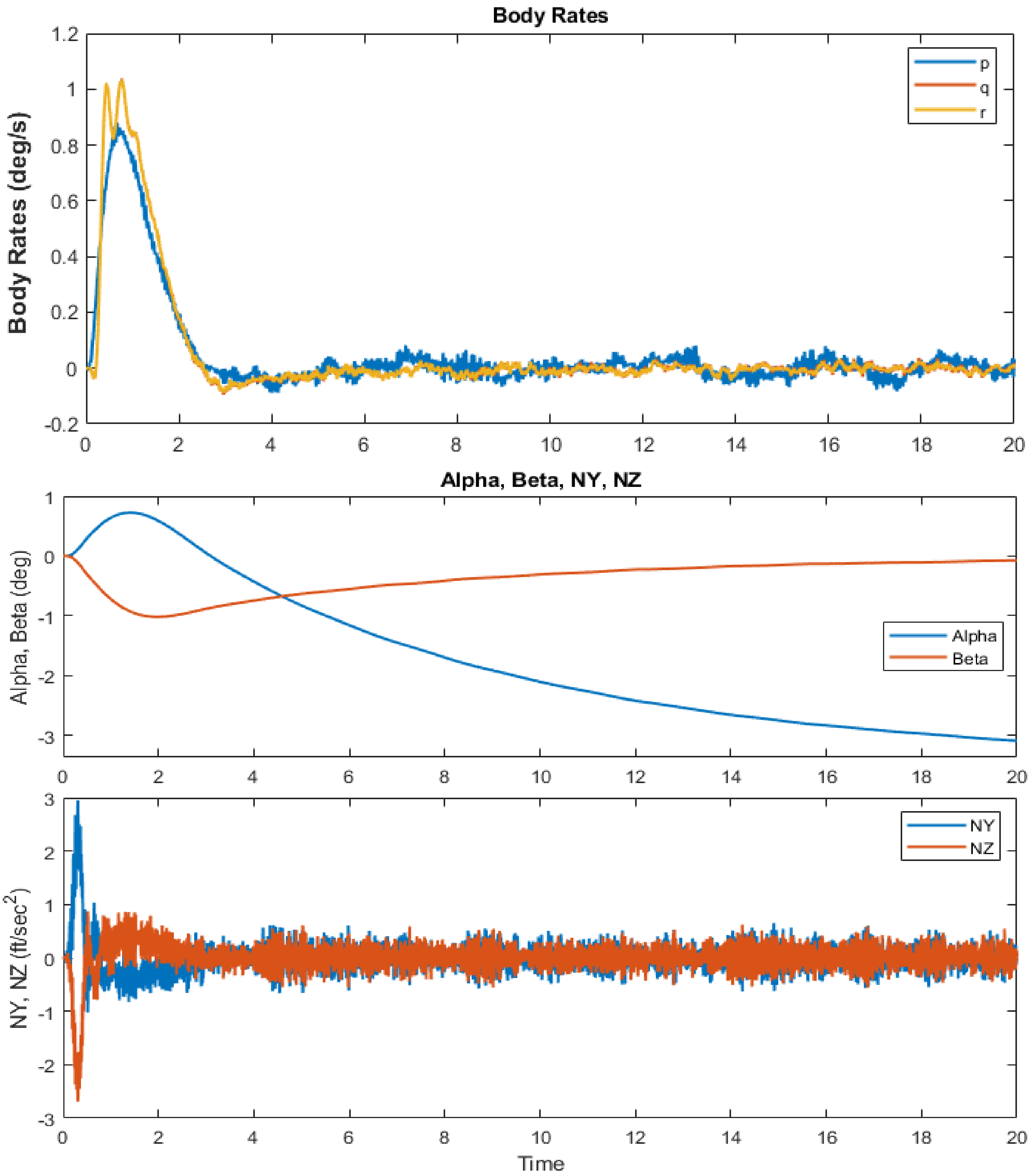
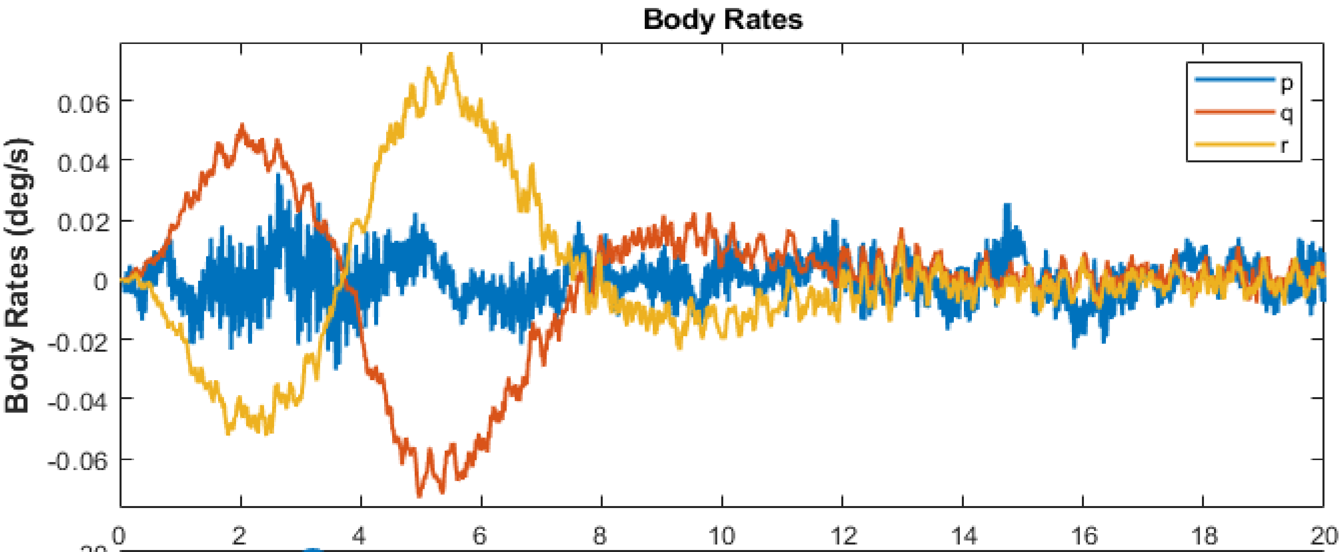
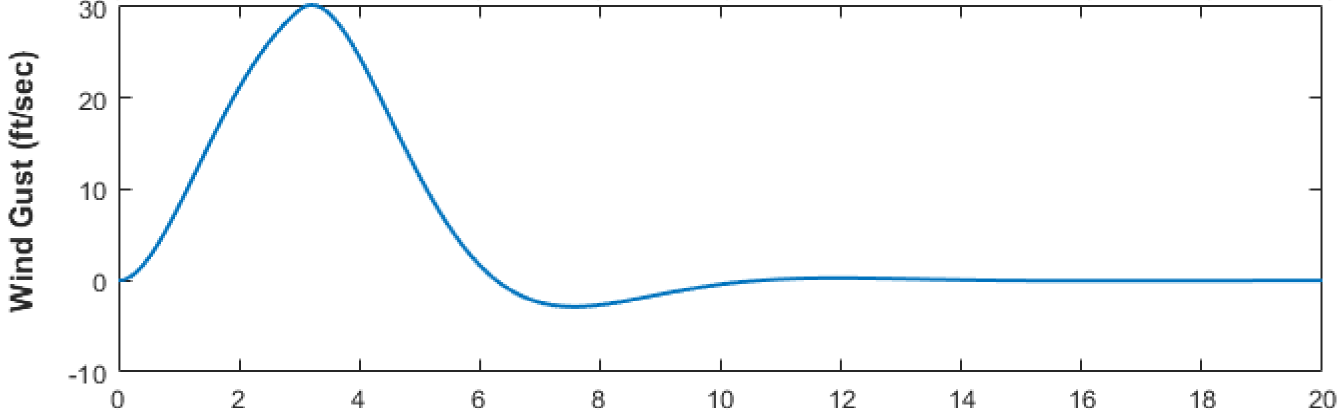


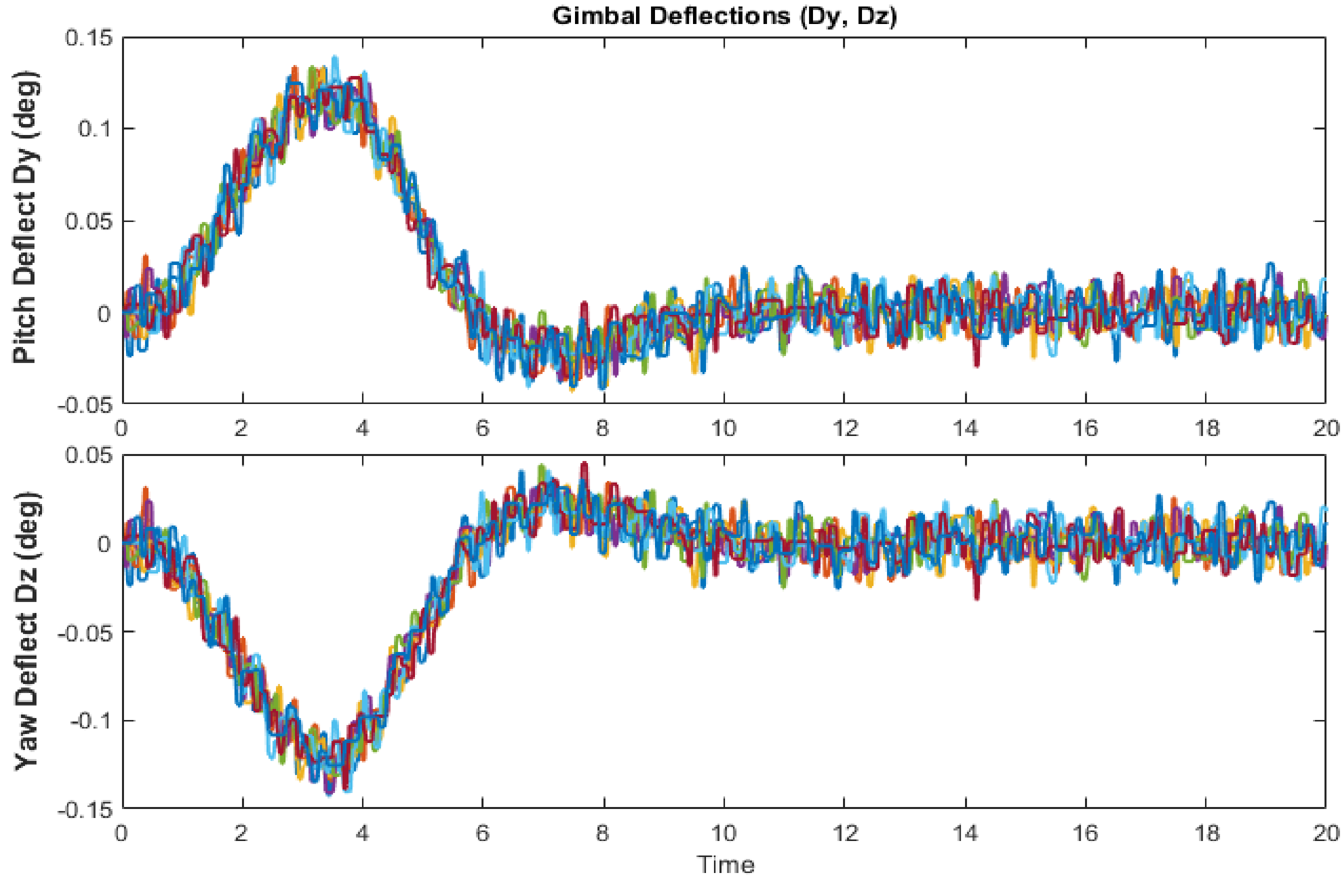
Figure 2.2.22 System Response to 1° Attitude Commands

Figure 2.2.6 shows the system's response to a 30 (ft/sec) wind-gust velocity pulse which is applied perpendicular to the vehicle and towards the -Y and -Z axes as defined in the vehicle input data-set. The jitter is caused by the actuator measurement noise.



Body Rates





Gimbal Deflections (Dy, Dz)

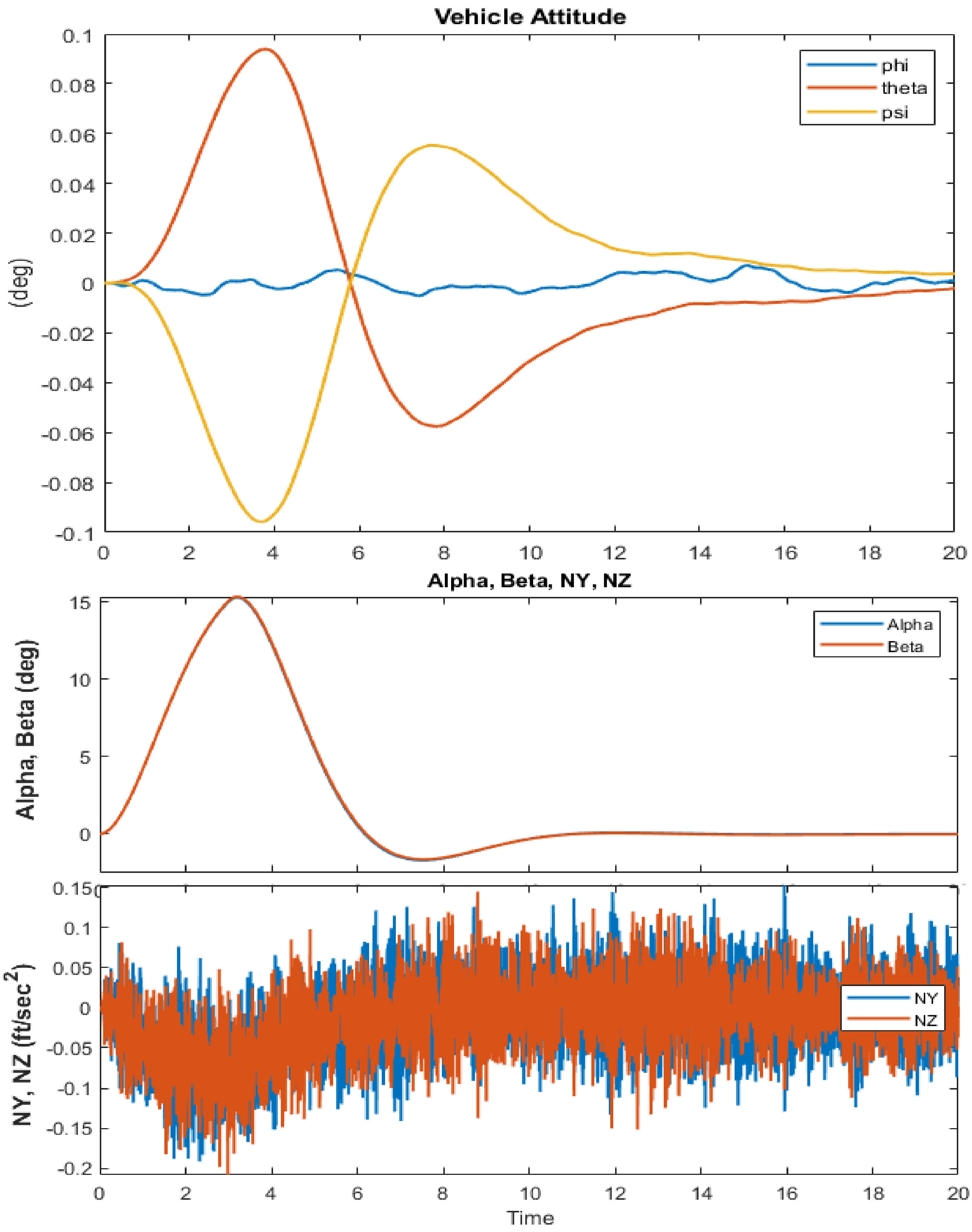
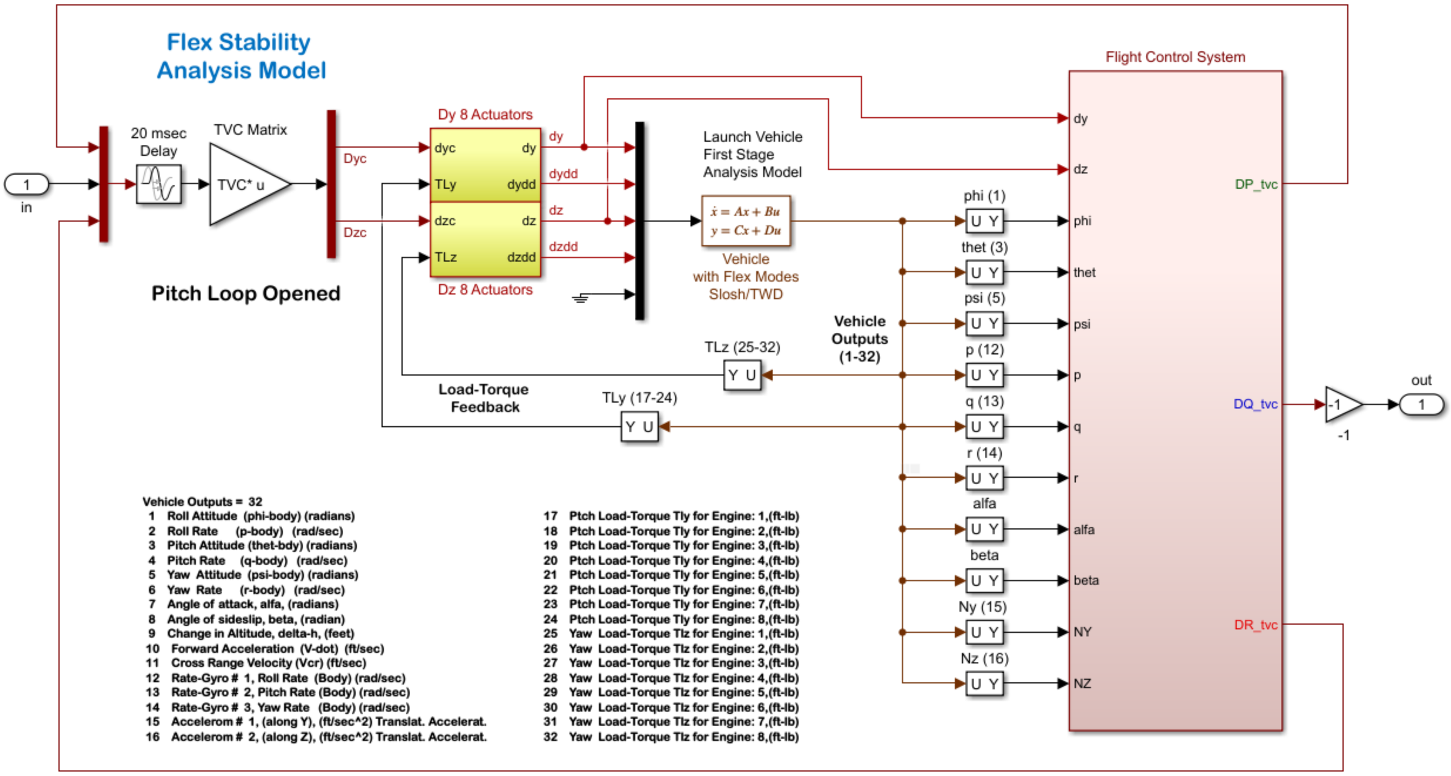


Figure 2.2.23 System's Response to the Wind-Gust Pulse. The attitude errors are small. The (α, β) angles are big because the vehicle speed is still small at T=10 sec

Stability Analysis

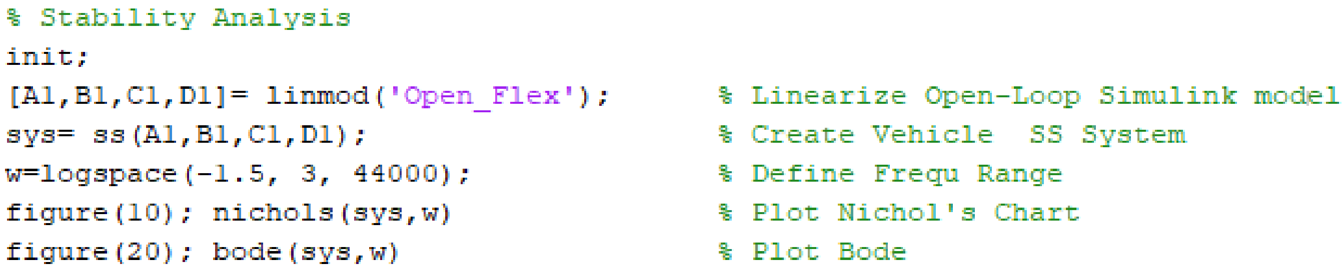
The Simulink model "*Open_Flex.Slx*" is used to analyze the system stability in roll, pitch and yaw. It is shown in Figure 2.2.7 configured for pitch open-loop analysis with the roll and yaw loops closed. The Flixan derived linear actuator system is used in this model. It consists of the same elements as the simulation model, except for the actuator which is now the linear system.



Vehicle Outputs = 32
1 Roll Attitude (phi-body) (radians)
2 Roll Rate (p-body) (rad/sec)
3 Pitch Attitude (thet-bdy) (radians)
4 Pitch Rate (q-body) (rad/sec)
5 Yaw Attitude (psi-body) (radians)
6 Yaw Rate (r-body) (rad/sec)
7 Angle of attack, alfa, (radians)
8 Angle of sideslip, beta, (radian)
9 Change in Altitude, delta-h, (feet)
10 Forward Acceleration (V-dot) (ft/sec)
11 Cross Range Velocity (Vcr) (ft/sec)
12 Rate-Gyro # 1, Roll Rate (Body) (rad/sec)
13 Rate-Gyro # 2, Pitch Rate (Body) (rad/sec)
14 Rate-Gyro # 3, Yaw Rate (Body) (rad/sec)
15 Accelerom # 1, (along Y), (ft/sec^2) Translat. Accelerat.
16 Accelerom # 2, (along Z), (ft/sec^2) Translat. Accelerat.
17 Ptch Load-Torque Tly for Engine: 1,(ft-lb)
18 Ptch Load-Torque Tly for Engine: 2,(ft-lb)
19 Ptch Load-Torque Tly for Engine: 3,(ft-lb)
20 Ptch Load-Torque Tly for Engine: 4,(ft-lb)
21 Ptch Load-Torque Tly for Engine: 5,(ft-lb)
22 Ptch Load-Torque Tly for Engine: 6,(ft-lb)
23 Ptch Load-Torque Tly for Engine: 7,(ft-lb)
24 Ptch Load-Torque Tly for Engine: 8,(ft-lb)
25 Yaw Load-Torque Tlz for Engine: 1,(ft-lb)
26 Yaw Load-Torque Tlz for Engine: 2,(ft-lb)
27 Yaw Load-Torque Tlz for Engine: 3,(ft-lb)
28 Yaw Load-Torque Tlz for Engine: 4,(ft-lb)
29 Yaw Load-Torque Tlz for Engine: 5,(ft-lb)
30 Yaw Load-Torque Tlz for Engine: 6,(ft-lb)
31 Yaw Load-Torque Tlz for Engine: 7,(ft-lb)
32 Yaw Load-Torque Tlz for Engine: 8,(ft-lb)

Figure 2.2.24 Simulink Model "Open_Flex.Slx" Used for Stability Analysis

The Bode and Nichols plots are calculated from this model using the script file "freq.m". The slosh modes are small because the tanks still have a lot of propellant. Slosh does not affect the roll axis. In pitch and yaw, the LH2 mode is phase-stable with ζ=0.001. The LOX mode with ζ=0.005 is opening towards the critical + point but it still has enough margin. The flex modes are sufficiently attenuated with the low-pass filters. The first bending mode at 19 (rad/sec) is very strong but phase-stable. The mode is perfectly phased by the low-pass filter with its peak in the Nichols plot at +22 (dB) between two critical points. This type of a design provides active control and attenuation of the first bending mode via negative feedback.



```
% Stability Analysis
init;
[A1,B1,C1,D1]= linmod('Open_Flex');          % Linearize Open-Loop Simulink model
sys= ss(A1,B1,C1,D1);                        % Create Vehicle  SS System
w=logspace(-1.5, 3, 44000);                  % Define Frequ Range
figure(10); nichols(sys,w)                   % Plot Nichol's Chart
figure(20); bode(sys,w)                      % Plot Bode
```

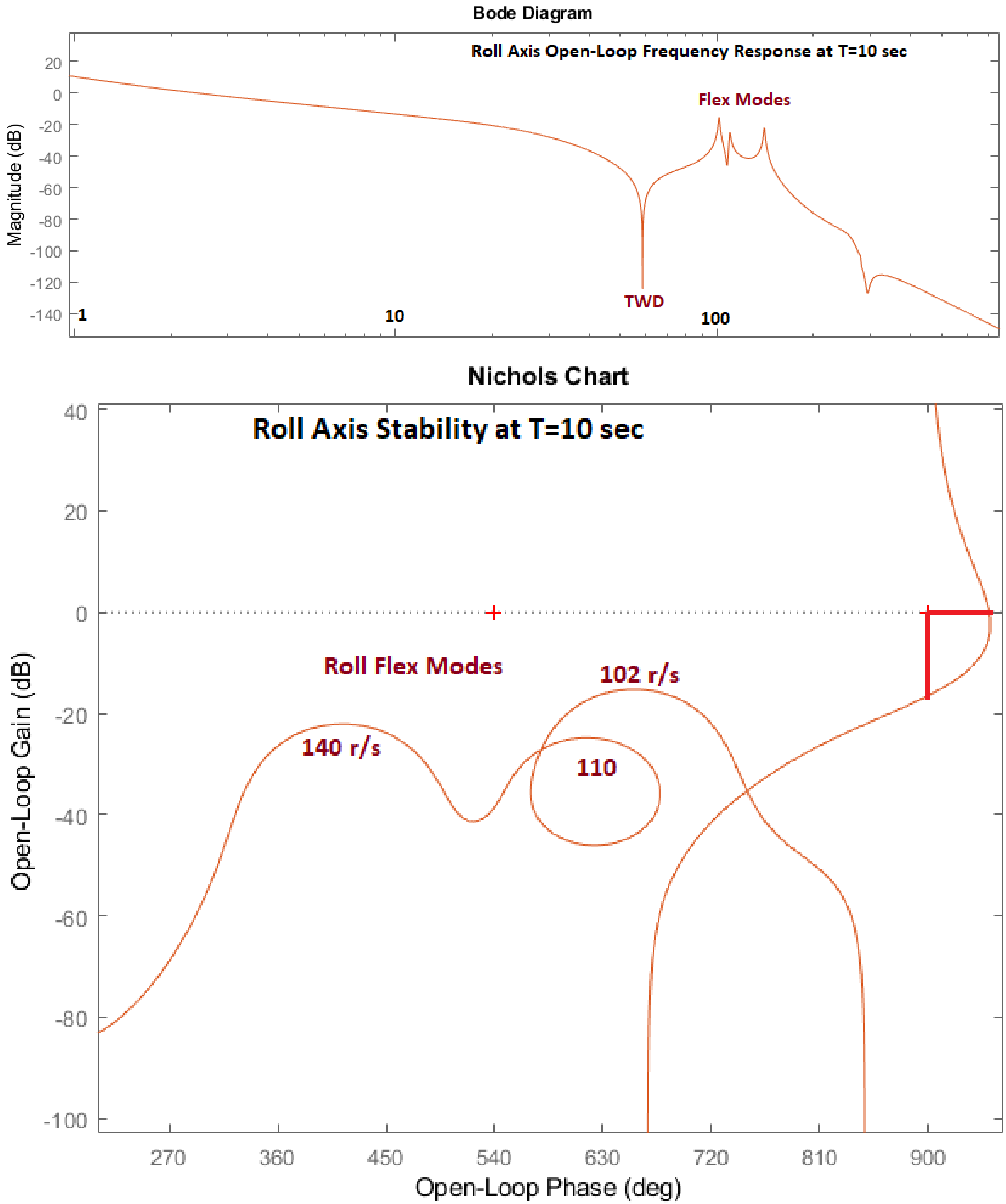


Figure 2.2.25 Roll Axis Stability Analysis at T= 10 sec

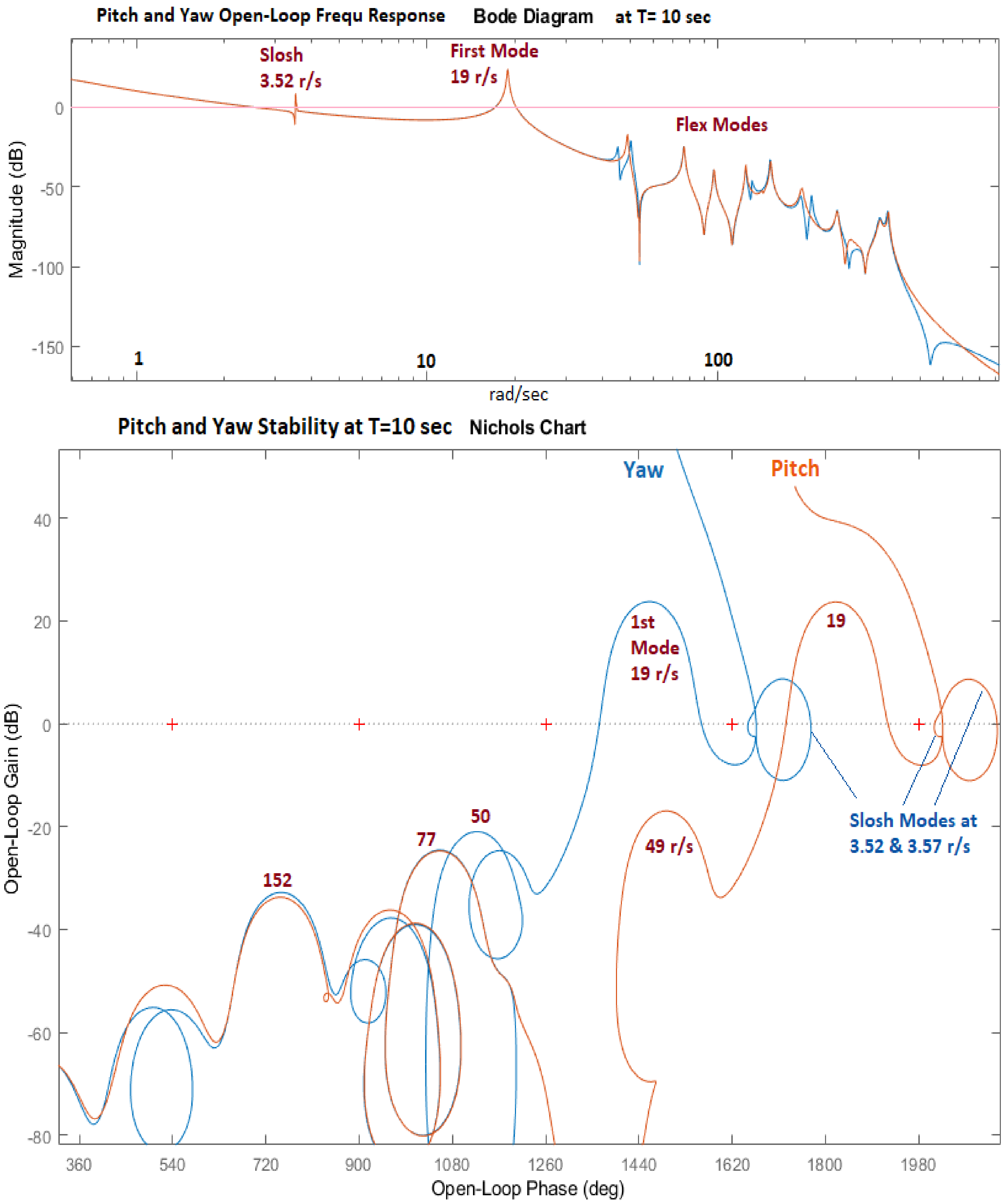
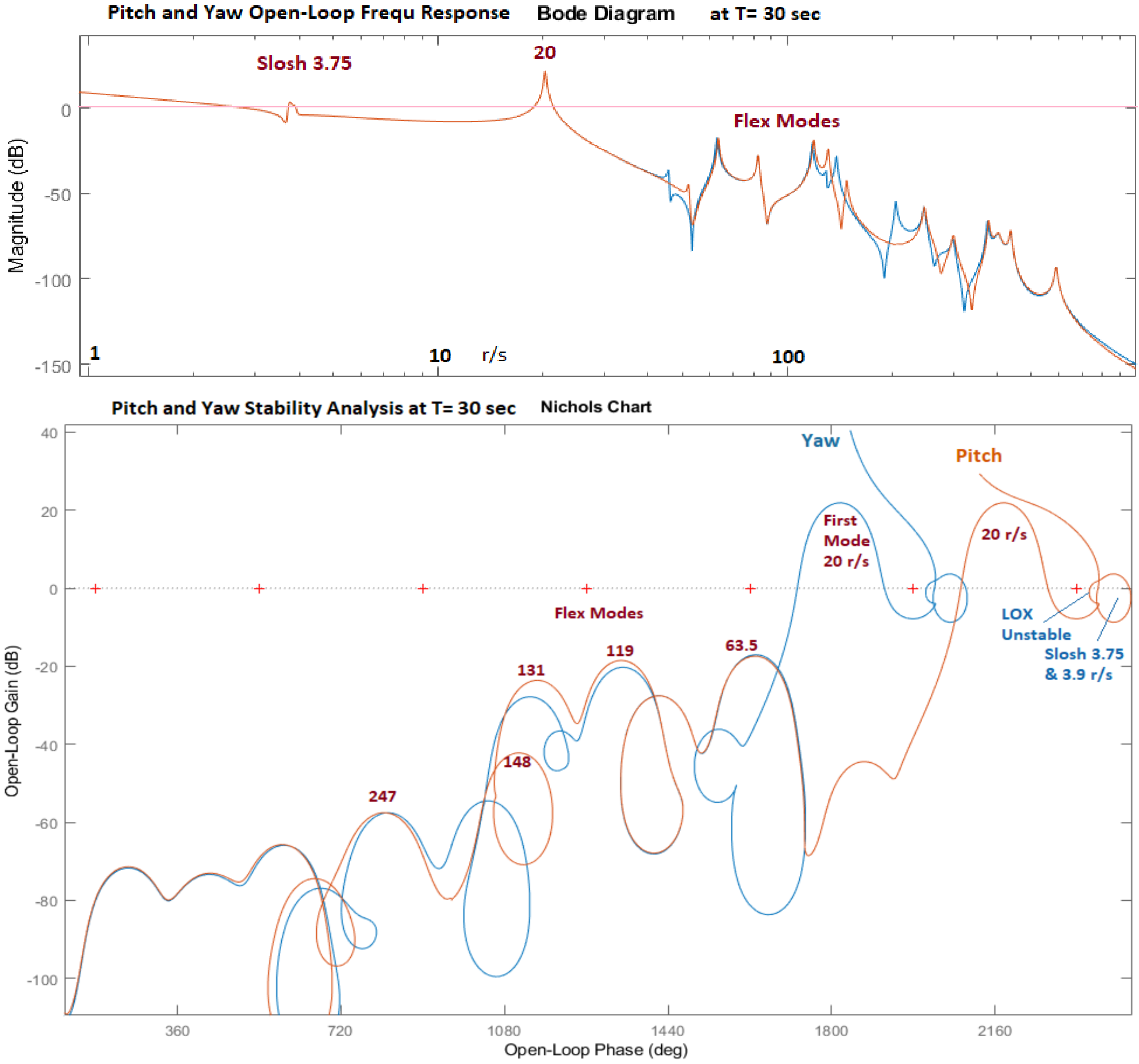


Figure 2.2.26 Pitch and Yaw Stability Analysis Plots are Very Similar

2.2.2 Control Analysis at T= 30 sec

At T= 30 sec the tanks are more depleted and the propellant sloshing effect is stronger because the slosh masses are bigger and the slosh forces against the tank walls are creating a bigger disturbance on the vehicle. To make things worse, the location of the LOX mass happens to be between the vehicle center of rotation and the CG which makes it phase unstable, meaning that the slosh mode in the Nichols chart is opening towards the critical -1 point (+) instead of opening away from the critical point, like a minimum phase system, similar to the LH2 tank. In this case, the easiest way to stabilize the slosh mode is to increase the damping coefficients to ζ=0.015 for LOX and ζ=0.01 for the LH2 tank. This is accomplished by including baffles inside the tanks which dampen the propellant sloshing.

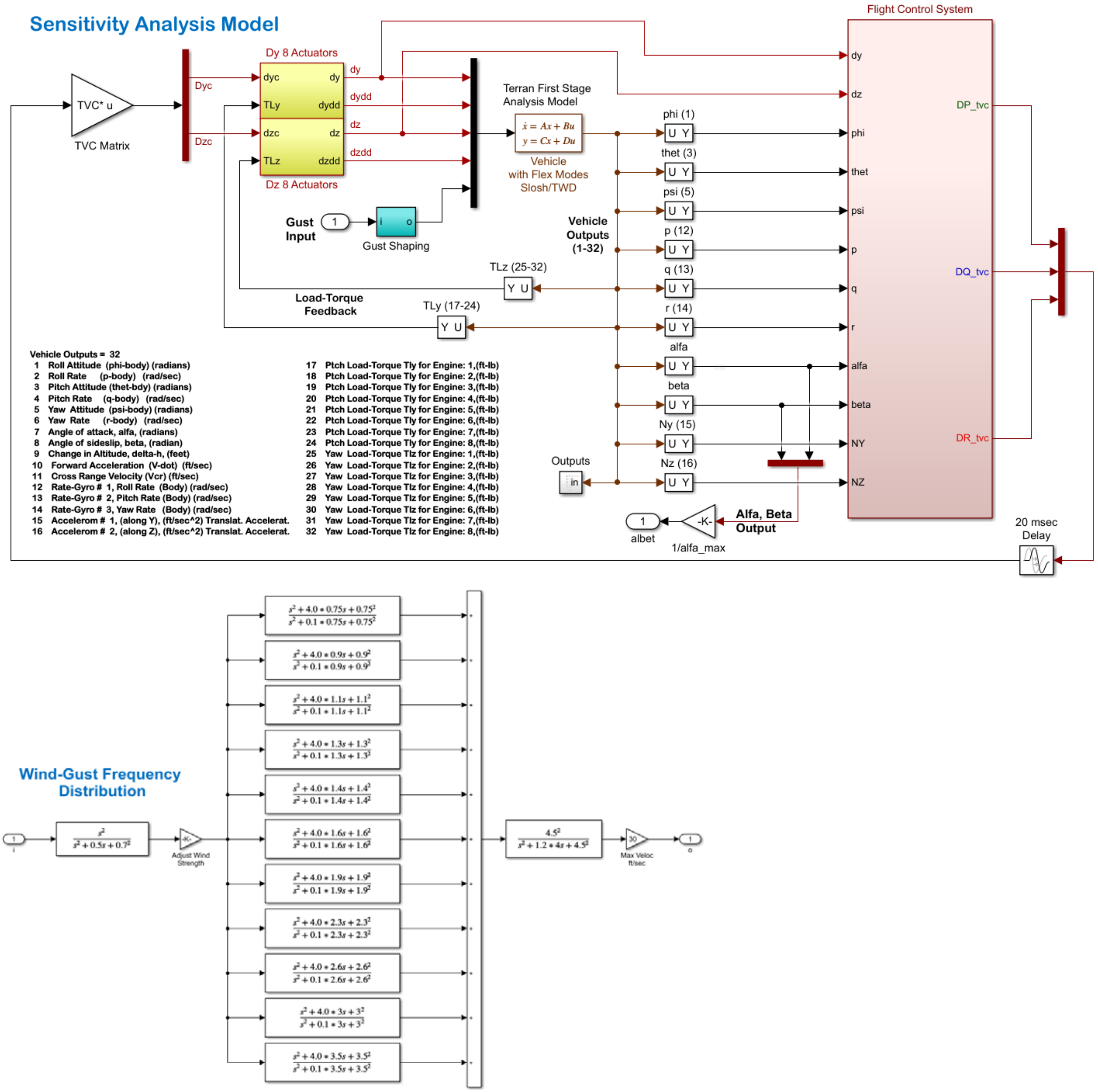

Pitch and Yaw Open-Loop Frequ Response Bode Diagram at T= 30 sec

Pitch and Yaw Stability Analysis at T= 30 sec Nichols Chart

Figure 2.2.27 The LH2 Slosh Mode is Phase-Stable, the LOX Mode is Opening Towards Instability

Sensitivity Analysis to Gust Disturbances

The closed-loop Simulink model “*Sensitiv_Flex.Slx*” in Figure 2.2.11 is used for analyzing the vehicle sensitivity to gusts. It is located in folder “*3-Stability Analysis with Flex & Slosh\1st Stage\T30\Matlab Analysis*”. The gust input is shaped by transfer functions to immitate the frequency characteristics of the wind-gust spectral density. The outputs are α and β angles divided by the max allowable α_{max} and β_{max} which can be as high as 7° in that time period.

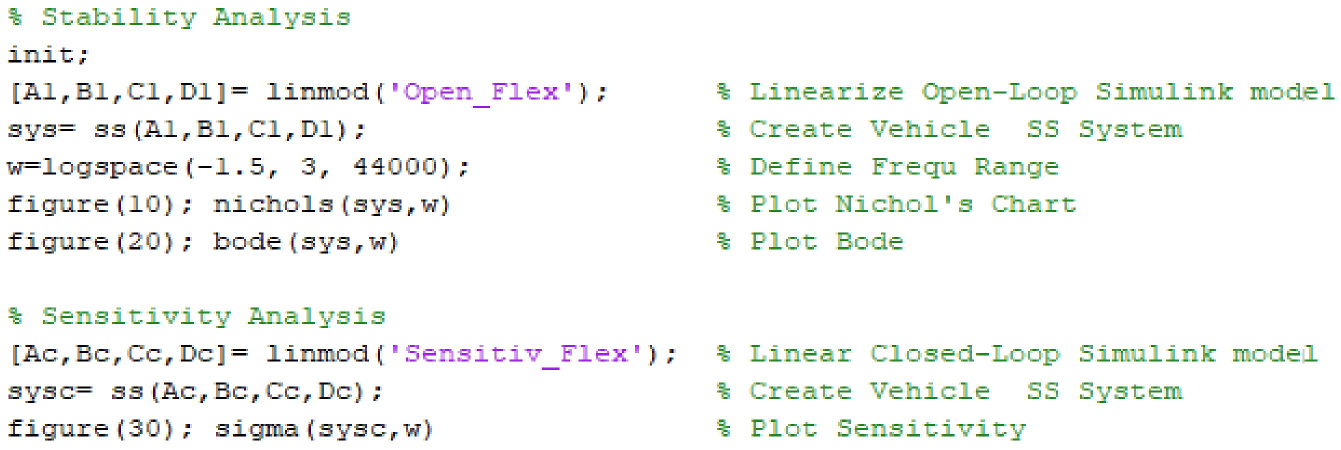


Vehicle Outputs = 32

1	Roll Attitude (phi-body) (radians)	17	Ptch Load-Torque Tly for Engine: 1,(ft-lb)
2	Roll Rate (p-body) (rad/sec)	18	Ptch Load-Torque Tly for Engine: 2,(ft-lb)
3	Pitch Attitude (thet-bdy) (radians)	19	Ptch Load-Torque Tly for Engine: 3,(ft-lb)
4	Pitch Rate (q-body) (rad/sec)	20	Ptch Load-Torque Tly for Engine: 4,(ft-lb)
5	Yaw Attitude (psi-body) (radians)	21	Ptch Load-Torque Tly for Engine: 5,(ft-lb)
6	Yaw Rate (r-body) (rad/sec)	22	Ptch Load-Torque Tly for Engine: 6,(ft-lb)
7	Angle of attack, alfa, (radians)	23	Ptch Load-Torque Tly for Engine: 7,(ft-lb)
8	Angle of sideslip, beta, (radian)	24	Ptch Load-Torque Tly for Engine: 8,(ft-lb)
9	Change in Altitude, delta-h, (feet)	25	Yaw Load-Torque Tlz for Engine: 1,(ft-lb)
10	Forward Acceleration (V-dot) (ft/sec)	26	Yaw Load-Torque Tlz for Engine: 2,(ft-lb)
11	Cross Range Velocity (Vcr) (ft/sec)	27	Yaw Load-Torque Tlz for Engine: 3,(ft-lb)
12	Rate-Gyro # 1, Roll Rate (Body) (rad/sec)	28	Yaw Load-Torque Tlz for Engine: 4,(ft-lb)
13	Rate-Gyro # 2, Pitch Rate (Body) (rad/sec)	29	Yaw Load-Torque Tlz for Engine: 5,(ft-lb)
14	Rate-Gyro # 3, Yaw Rate (Body) (rad/sec)	30	Yaw Load-Torque Tlz for Engine: 6,(ft-lb)
15	Accelerom # 1, (along Y), (ft/sec^2) Translat. Accelerat.	31	Yaw Load-Torque Tlz for Engine: 7,(ft-lb)
16	Accelerom # 2, (along Z), (ft/sec^2) Translat. Accelerat.	32	Yaw Load-Torque Tlz for Engine: 8,(ft-lb)

Figure 2.2.28 Sensitivity Analysis Model “Sensitiv_Flex.slx”

The system sensitivity is analyzed in the frequency domain by calculating the Singular Values frequency response between the shaped gust input (W_{gust}) and the normalized (α and β) outputs. According to theory the system satisfies the sensitivity to gust requirements if the Singular Values frequency response is less than 1 at all frequencies. The frequency response file “freq.m”, shown below, calculates the Bode and Nichols plots from the open-loop model “Open_Flex.Slx”. It also calculates the Sensitivity plot using the open-loop model “Sensitiv_Flex.Slx” which is shown in Figure 2.2.12.



```
% Stability Analysis
init;
[A1,B1,C1,D1]= linmod('Open_Flex');      % Linearize Open-Loop Simulink model
sys= ss(A1,B1,C1,D1);                    % Create Vehicle  SS System
w=logspace(-1.5, 3, 44000);              % Define Frequ Range
figure(10); nichols(sys,w)               % Plot Nichol's Chart
figure(20); bode(sys,w)                  % Plot Bode

% Sensitivity Analysis
[Ac,Bc,Cc,Dc]= linmod('Sensitiv_Flex');  % Linear Closed-Loop Simulink model
sysc= ss(Ac,Bc,Cc,Dc);                   % Create Vehicle  SS System
figure(30); sigma(sysc,w)                % Plot Sensitivity
```

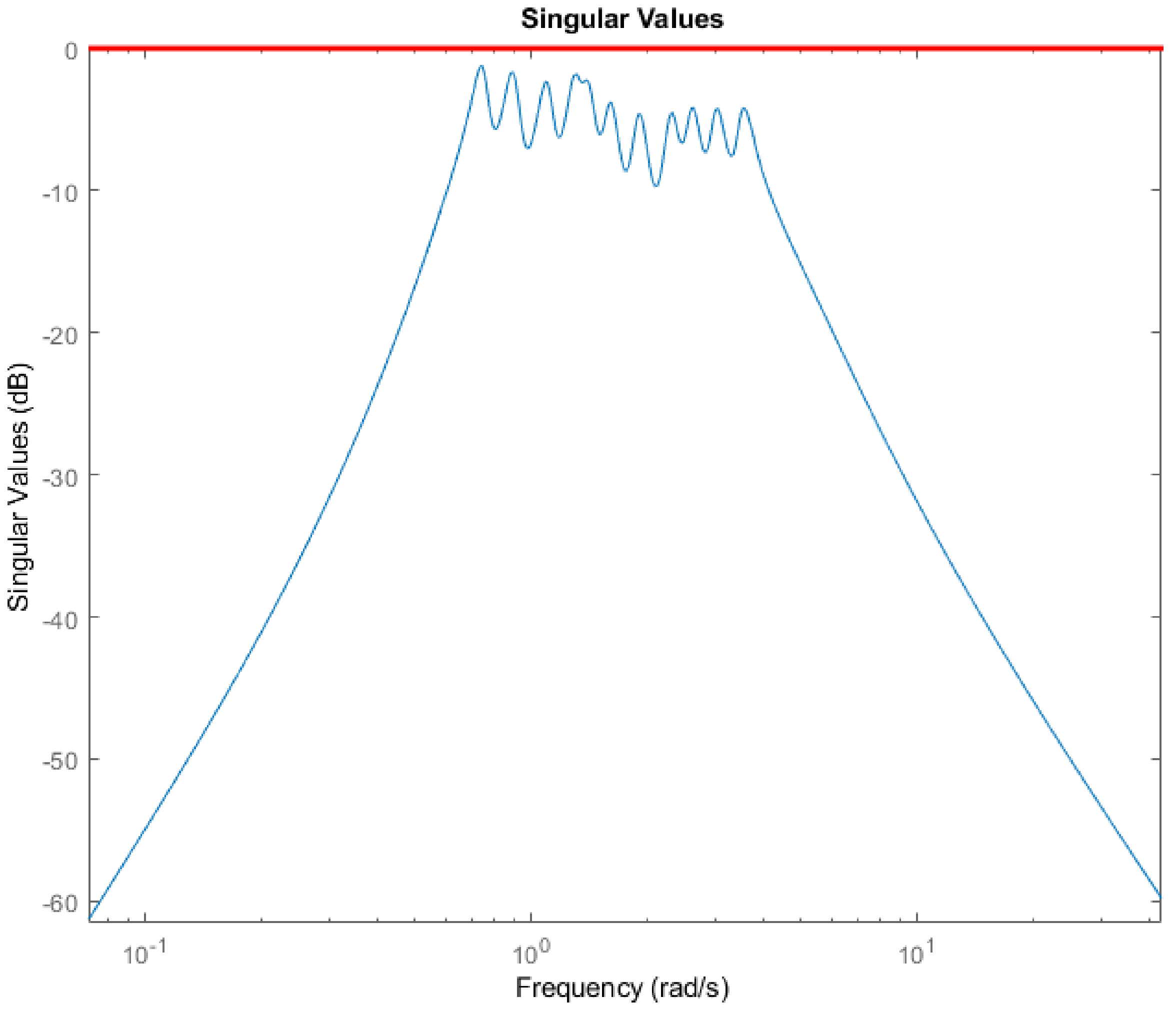


Figure 2.2.29 Sensitivity Analysis Plot is Less than 1 at All Frequencies

2.2.3 Control Analysis at T= 60 sec

At T= 60 sec the dynamic pressure increases to 416 (lbf/ft^2) and we are beginning to introduce the load-relief effect from the estimated α and β, and from (α, β)-integrals. Sloshing is also stronger and the instability of the LOX tank is more powerful to the point that the LOX damping was increased using baffles to ζ=0.018. In this time-slice we used the Flixan program to create the models in directory "*23-Classic Launch Vehicle Design & Simulation\3-Stability Analysis with Flex & Slosh\1st Stage\T60\Flixan Analysis*". Figures 2.2.30 and 2.2.31 show the system stability in pitch and yaw using Nyquist, Bode and Nichols plots generated using the Flixan program. The sensitivity to wind-gusts in Figure 2.2.32 is satisfied, assuming that α and β do not exceed 3°. With the increased dynamic pressure and the load-relief becoming more active, the step responses to guidance commands in Figure 2.2.33 are beginning to deteriorate, as expected, but they are still satisfactory. The structural flexibility and sloshing effects are visible in the responses. The actuator model is linear and the actuator noise is not included in this case.

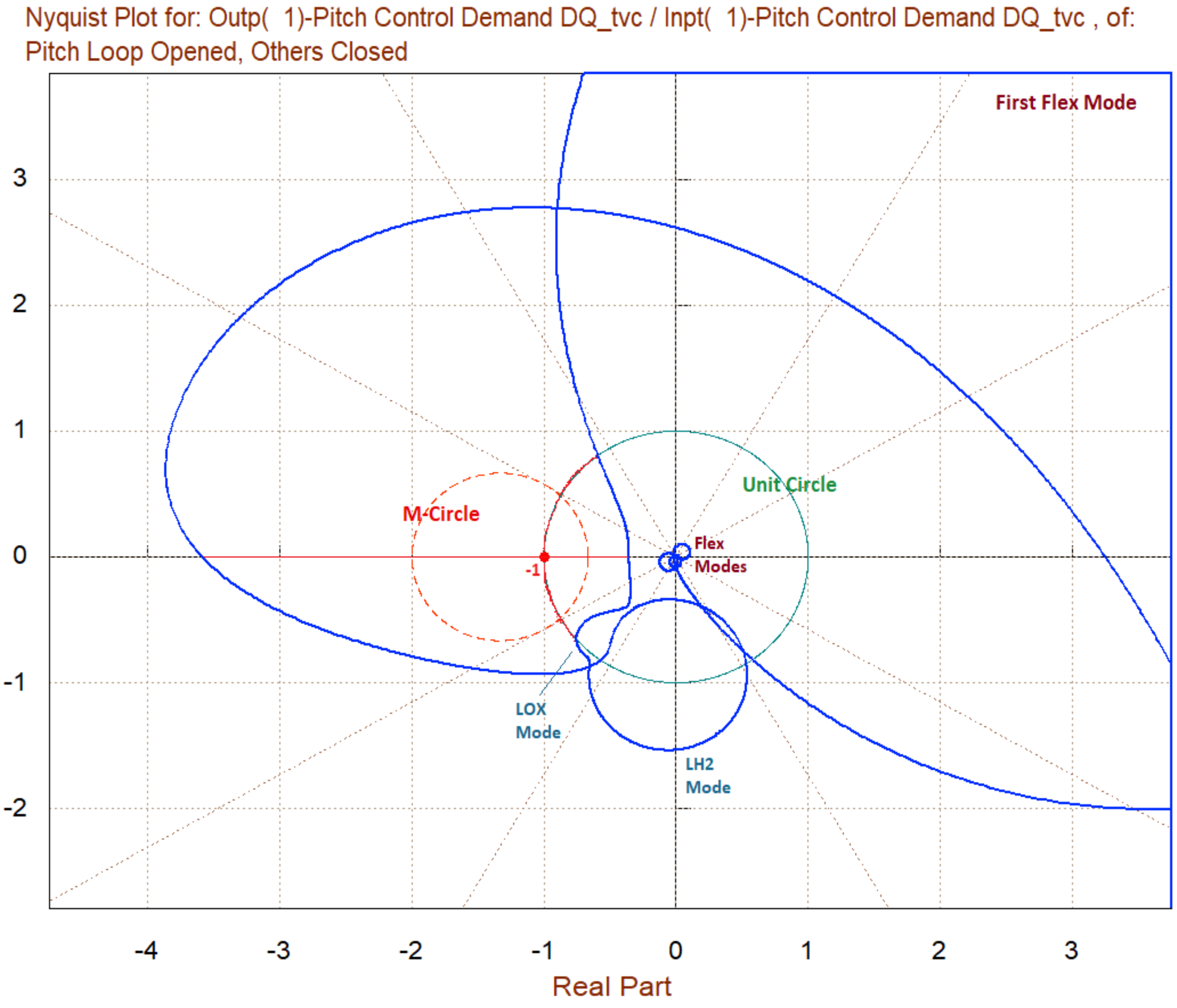
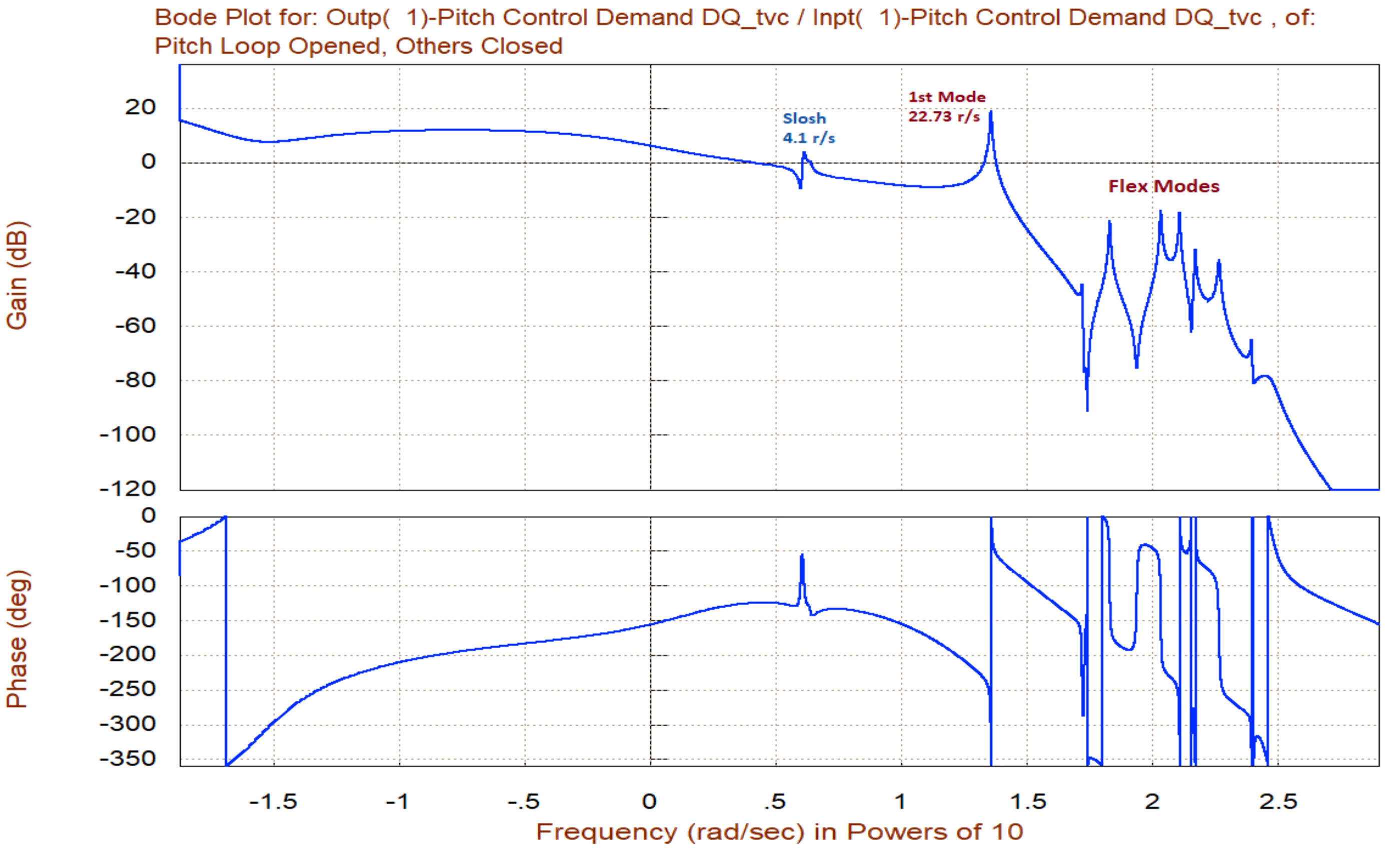
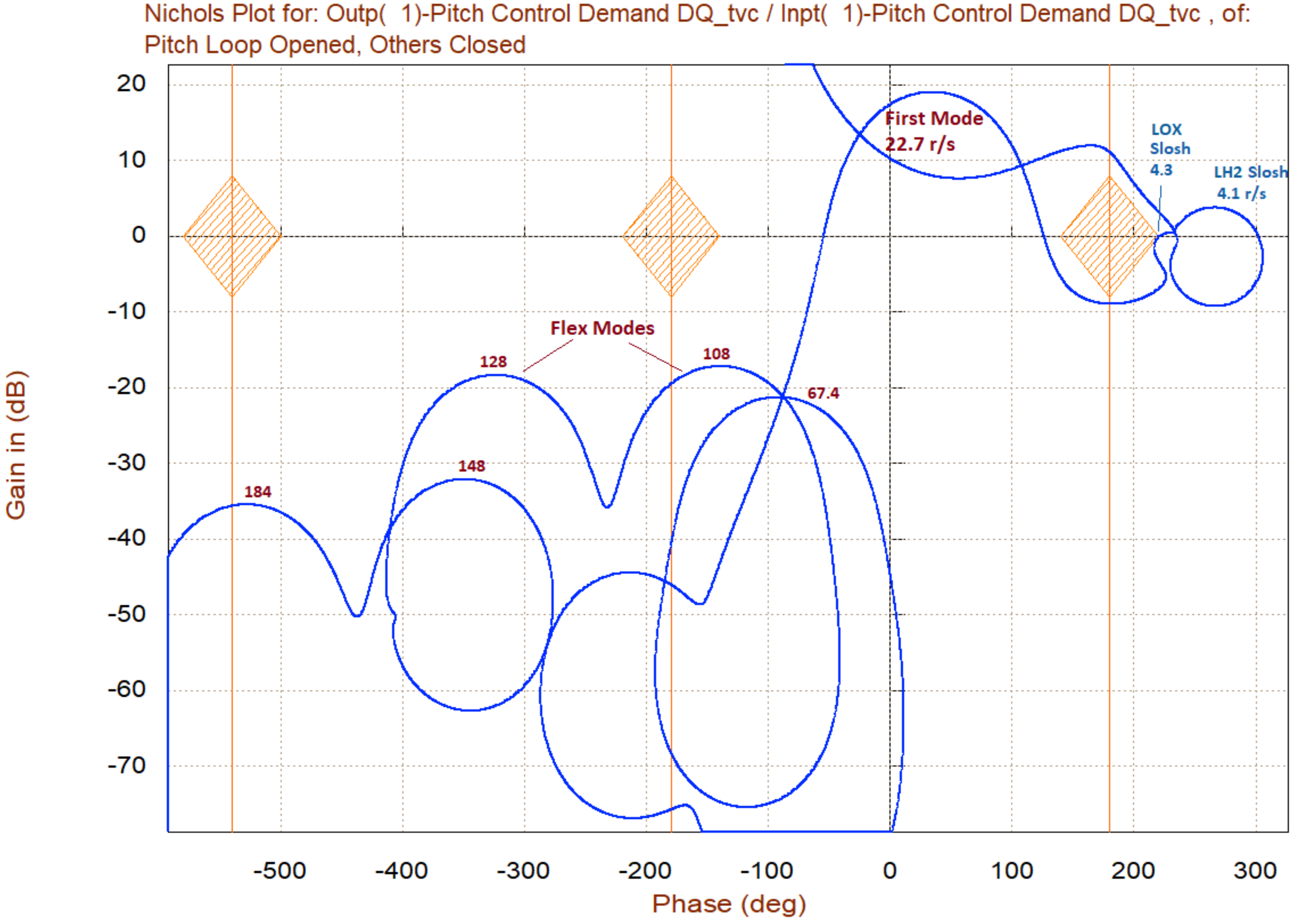
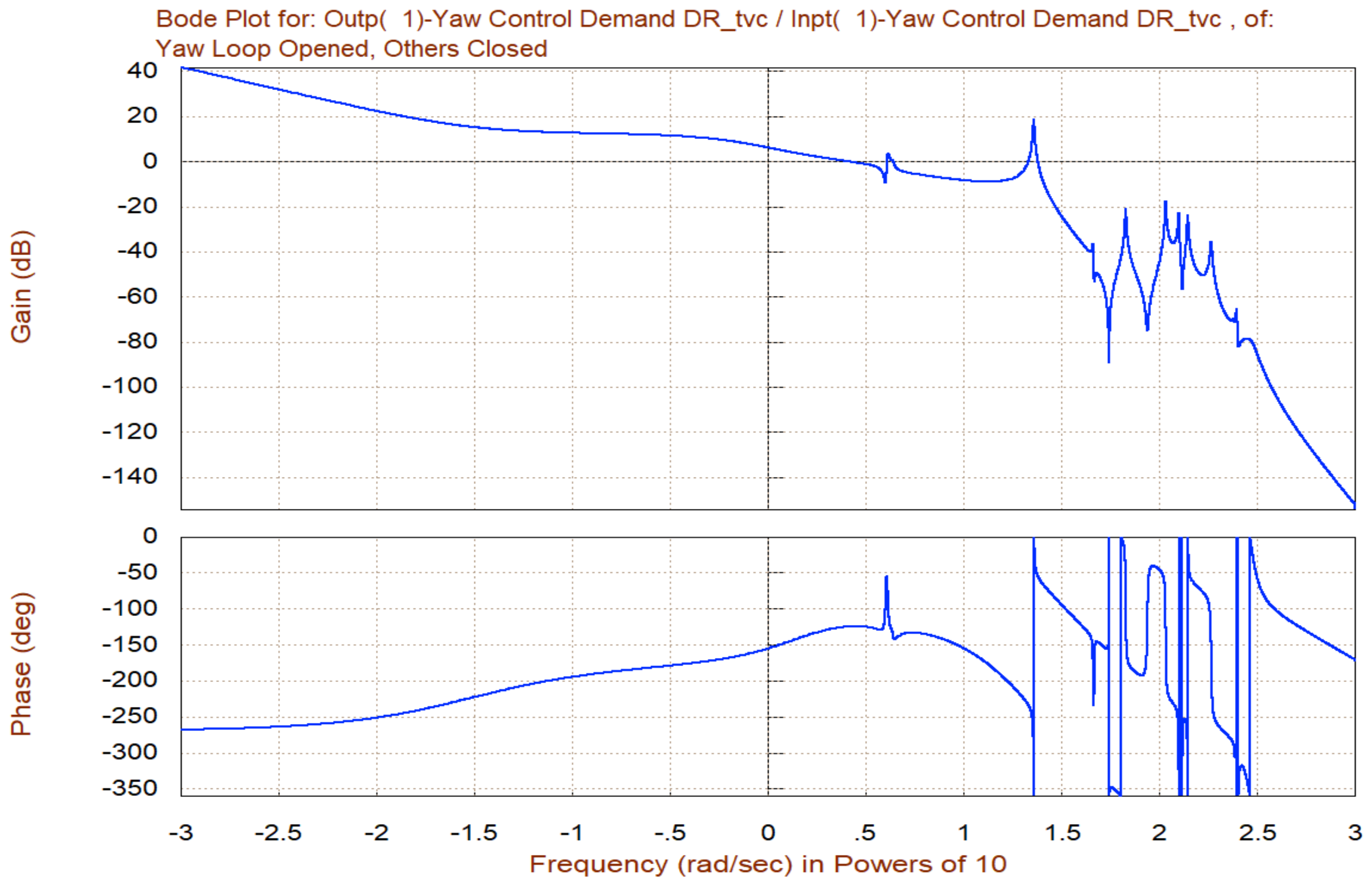


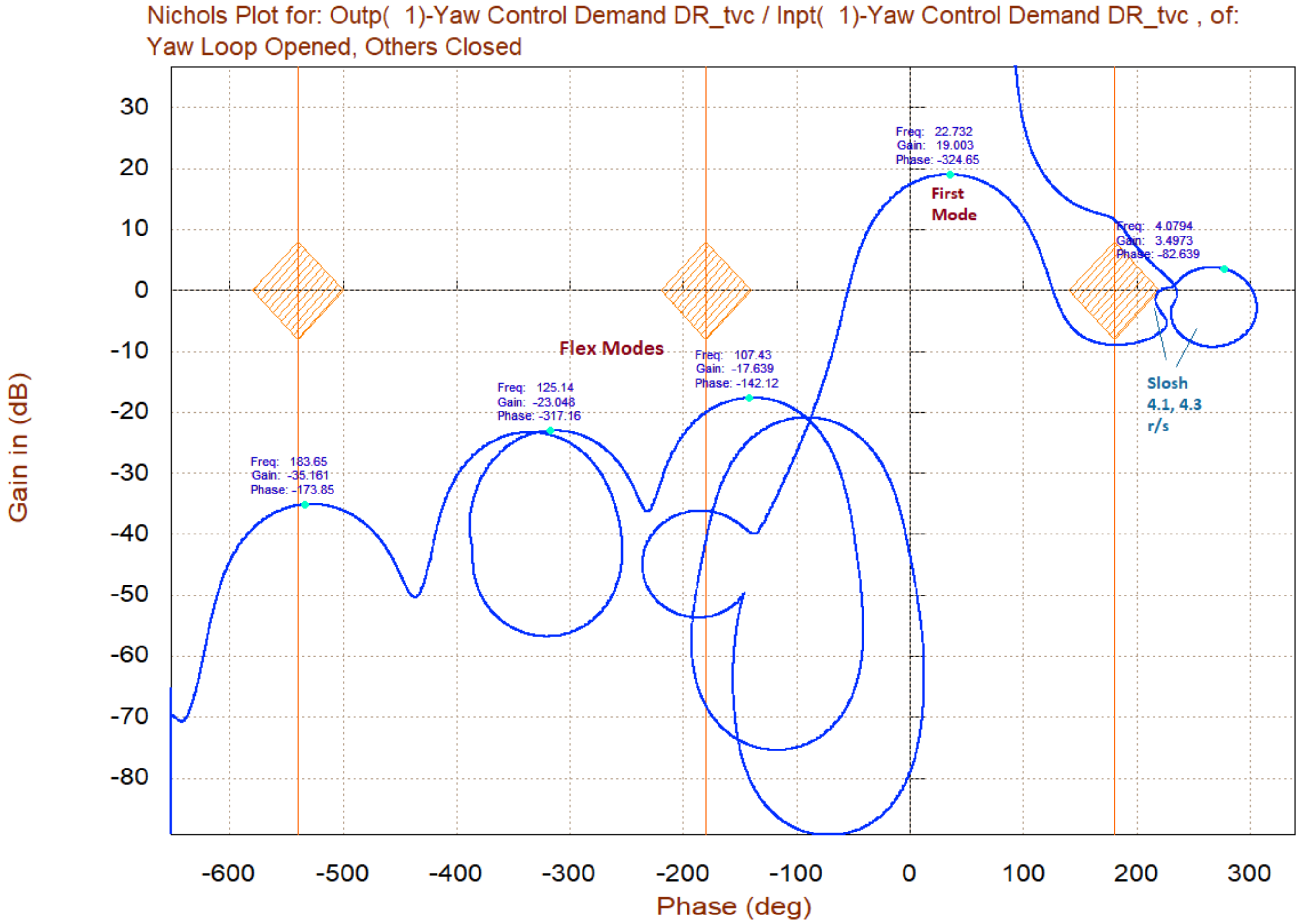
Figure 2.2.30 Pitch Axis Nyquist, Bode and Nichols Plots


Bode Plot for: Outp(1)-Pitch Control Demand DQ_tvc / Inpt(1)-Pitch Control Demand DQ_tvc , of:
Pitch Loop Opened, Others Closed
Gain (dB)
20
0
-20
-40
-60
-80
-100
-120
Slosh
4.1 r/s
1st Mode
22.73 r/s
Flex Modes
Phase (deg)
0
-50
-100
-150
-200
-250
-300
-350
-1.5
-1
-.5
0
.5
1
1.5
2
2.5
Frequency (rad/sec) in Powers of 10


Nichols Plot for: Outp(1)-Pitch Control Demand DQ_tvc / Inpt(1)-Pitch Control Demand DQ_tvc , of:
Pitch Loop Opened, Others Closed
Gain in (dB)
20
10
0
-10
-20
-30
-40
-50
-60
-70
First Mode
22.7 r/s
LOX
Slosh
4.3
LH2 Slosh
4.1 r/s
Flex Modes
128
108
67.4
148
184
-500
-400
-300
-200
-100
0
100
200
300
Phase (deg)



Bode Plot for: Outp(1)-Yaw Control Demand DR_tvc / Inpt(1)-Yaw Control Demand DR_tvc , of: Yaw Loop Opened, Others Closed



Nichols Plot for: Outp(1)-Yaw Control Demand DR_tvc / Inpt(1)-Yaw Control Demand DR_tvc , of: Yaw Loop Opened, Others Closed

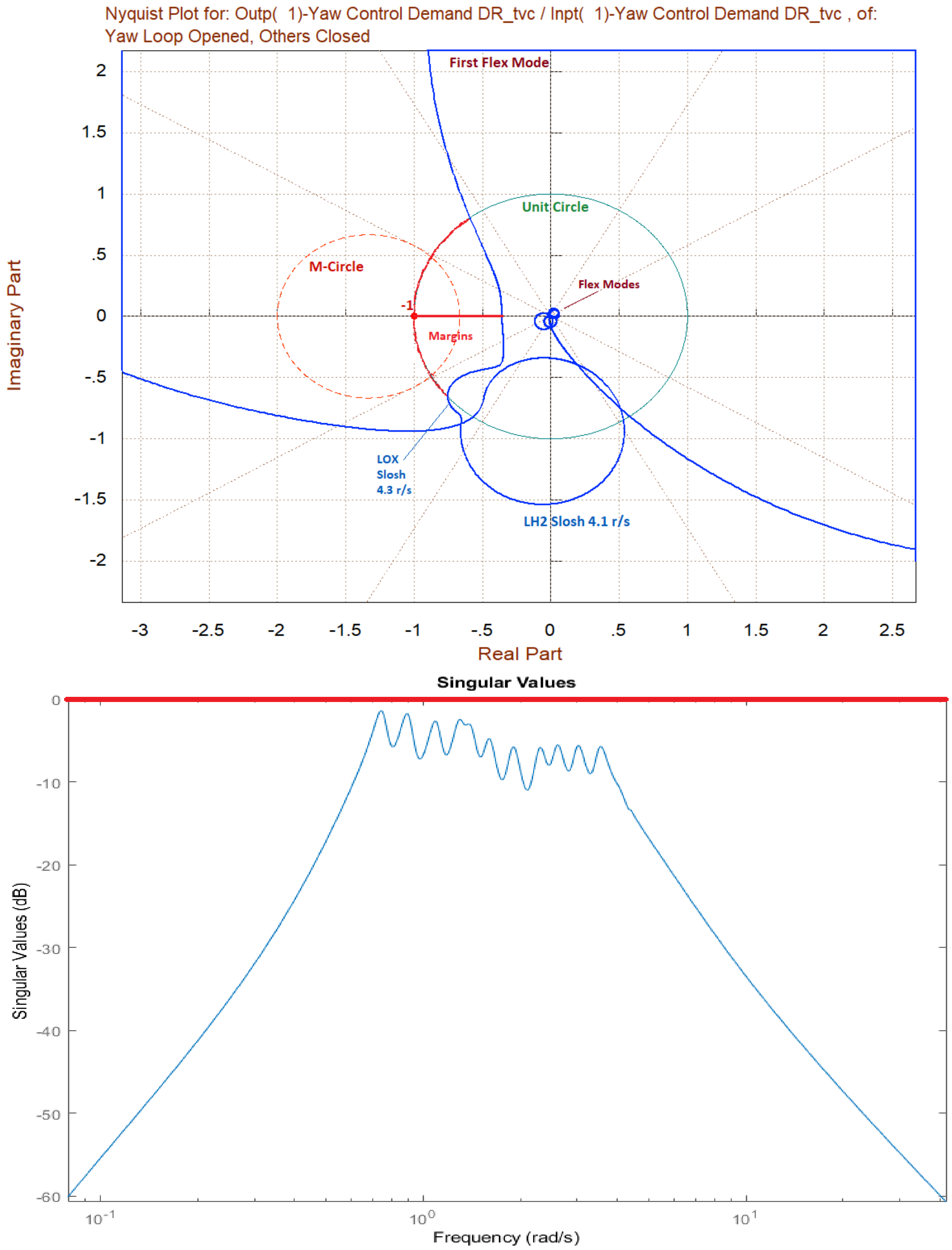
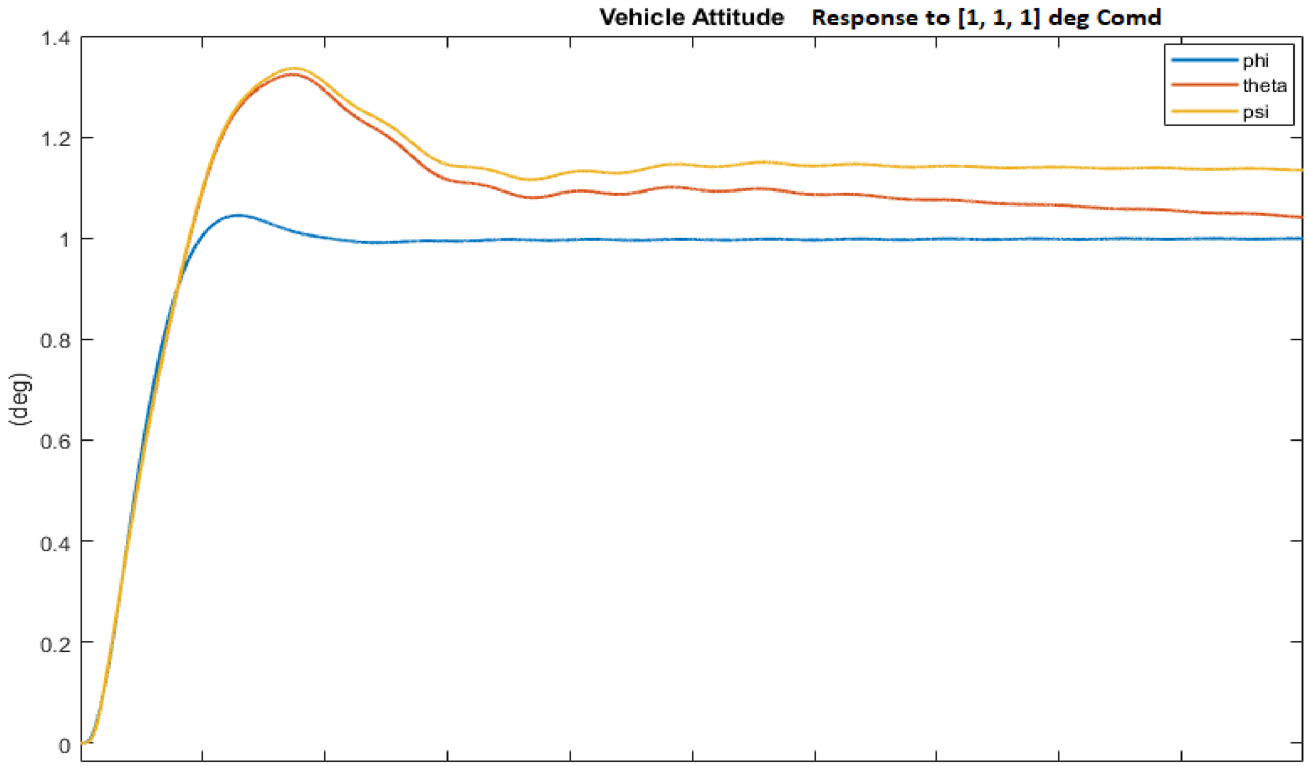
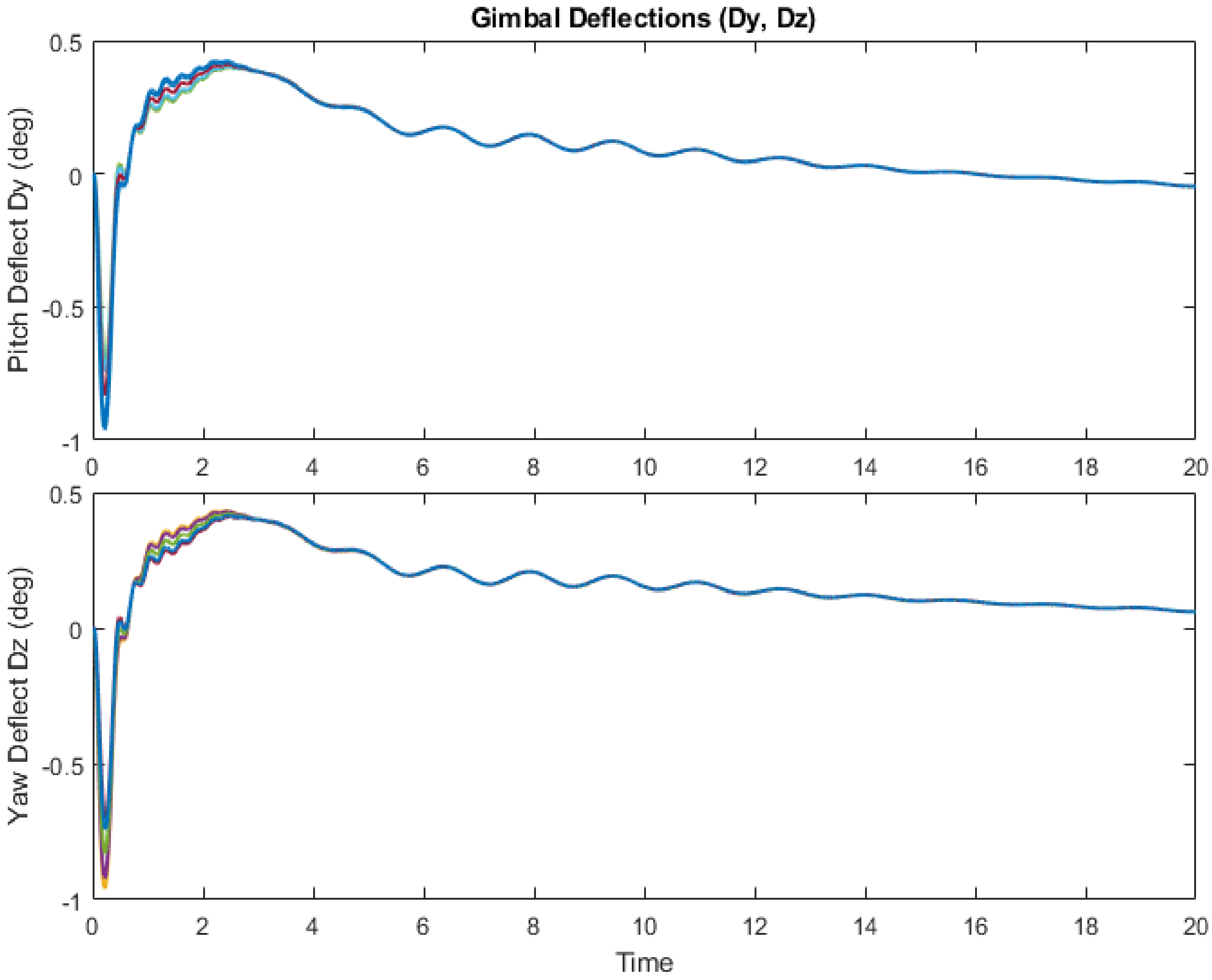


Figure 2.2.32 Sensitivity to Wind-Gusts Assuming that alpha and beta dispersions are less than 3 (deg)



Vehicle Attitude Response to [1, 1, 1] deg Comd



Gimbal Deflections (Dy, Dz)

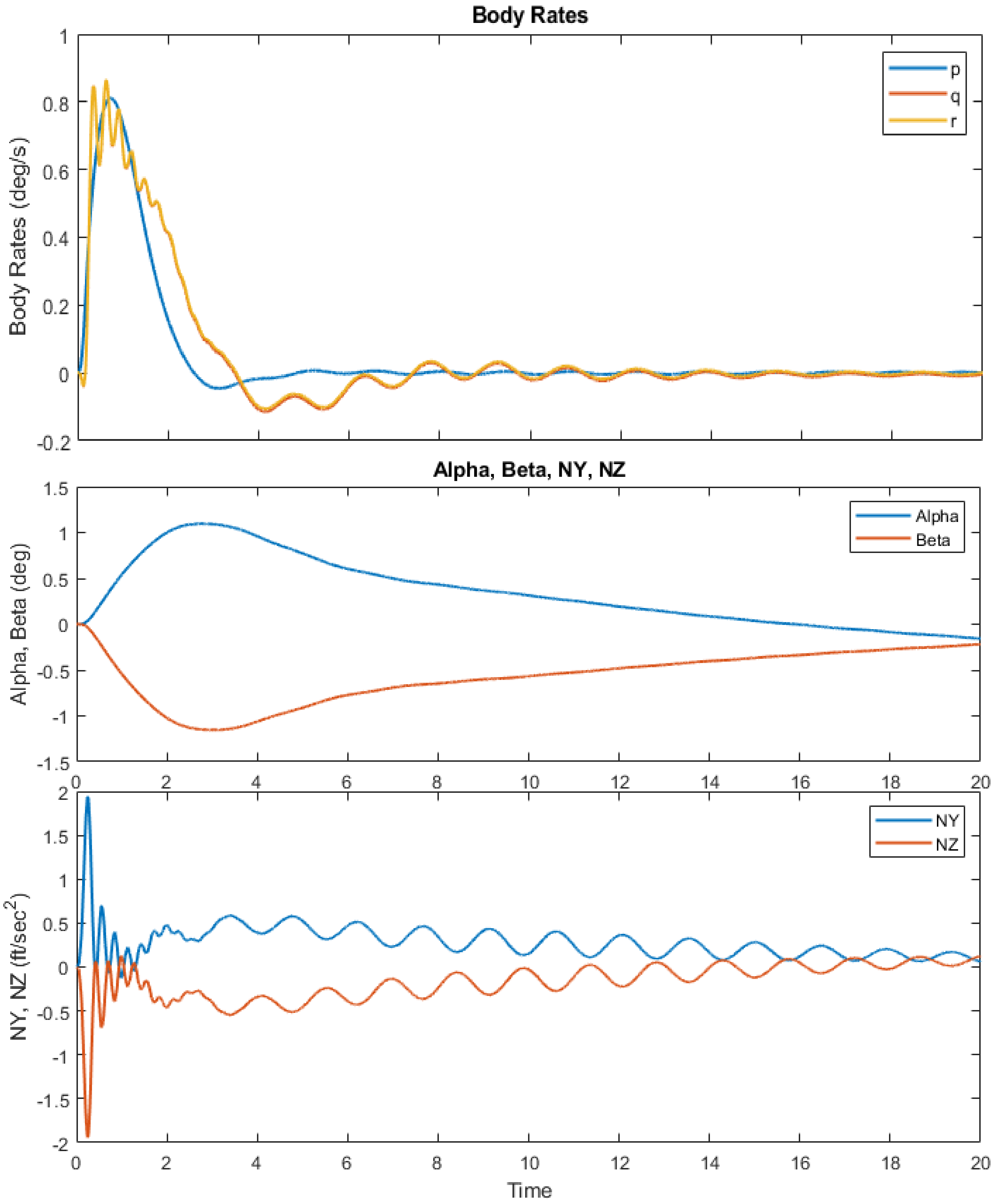


Figure 2.2.33 System's Response to Attitude Commands Created from the Flixan Generated "Closed-Loop System" Included in Simulink Model "Sim_Flex3.slx"

2.2.4 Control Analysis at Max-Q, T= 80 sec

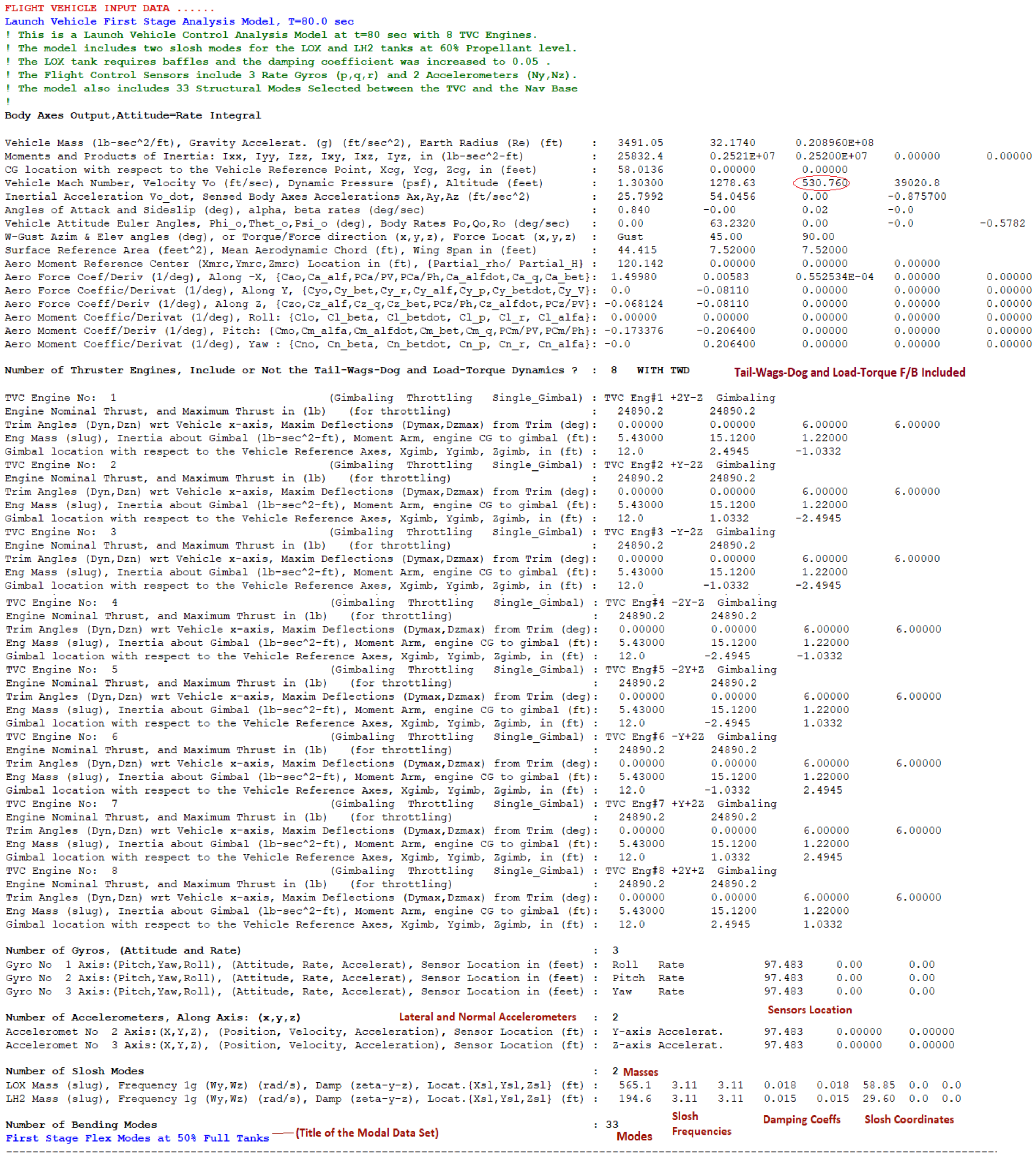
We will now describe the Max-Q modeling and analysis which was implemented mostly using the Flixan program. The work files are in folder: "*Examples\ 23-Classic Launch Vehicle Design & Simulation\3-Stability Analysis with Flex & Slosh\1st Stage\T80\Flixan Analysis*". The input file is "*Flex_Vehi_T80b.inp*". There is also a Matlab implementation input file "*Flex_Vehi_T80a.inp*" under T80. The TVC and the LQR state-feedback were calculated in the design section and they are slightly adjusted to improve stability margins. The state-feedback gain is Kq_t80= [6.0, 2.8, 3.3, -0.2] corresponding to pitch states: θ, q, α, and α-integral respectively. It is also used for yaw control from states: ψ, r, β, and β-integral. The roll control PD gain is just, [2, 2.25] from the states: ϕ and p respectively. The title of the vehicle dataset is "*Launch Vehicle First Stage Analysis Model, T=80.0 sec*" and it includes the 8 gimbaling engines, propellant sloshing parameters for the LOX and LH2 tanks, and the flex modes.

The slosh parameters consist of the 2 slosh masses, the 2 slosh frequencies along y and z, the 2 damping coefficients along y and z, and the x, y, z locations of the 2 slosh masses. The LOX and LH2 slosh frequencies are both 3.11 rad/sec calculated at 1g. They are scaled by the program proportionally with the square root of the vehicle acceleration. The vehicle dataset also includes rate gyros for attitude control and Ny & Nz accelerations for load-relief. The modal data at T80 are created from a finite elements model where the tanks are 50% full. The modal data and the nodes ID files are "*Stage1_50%.Mod*" and "*Stage1_50%.Nod*". A data set of preselected flex modes is included at the bottom of file: "*Flex_Vehi_T80b.inp*" with a title: "*First Stage Flex Modes at 50% Full Tanks*". The modes will be combined with the rigid vehicle dataset to create the flex vehicle state-space system for the analysis. The number of modes to be included in the model and the title of the selected modal data are included at the bottom of the vehicle data. A batch set "*Batch for Stage-1 Launch Vehicle Control Analysis at T=80 sec*" is included at the top of the input file that creates the vehicle, TVC, actuator and flight control systems, and combines them together in Flixan to create stability analysis and simulation systems.

Input File

The Flixan input file: Flex_Vehi_T80b.inp is shown below. It begins by creating the flex vehicle system at T80, the actuator and the TVC matrix. Then it combines 8 actuator systems in parallel to create a "*System of 8 Actuators*". One of the combined actuator systems will be used to drive the pitch gimbals and an identical system will drive the yaw gimbals. The vehicle and the two combination systems of 8 actuators (a total of 16 actuators) are combined together to create the system "*Plant Model at T=80 sec, Vehicle/ Actuators*". Then, the alpha and beta estimators, the low-pass filters, and integrators are implemented using transfer-functions. The flight control system is then implemented by combining the alpha/ beta estimators, the low-pass filters and the integrators that produce (α, β)-integrals for state-feedback. Finally, the flight control system and the plant model are combined together in two configurations to create two systems, an "*Open-Loop System*" to be used for stability analysis, and a "*Closed-Loop System*" to be used in simulations. The open-loop system has all 3 loops opened and it is reconfigured into 3 additional systems by opening one loop at a time and closing the other two, as identified by their titles, for frequency response analysis.

```
BATCH MODE INSTRUCTIONS ...............
Batch for Stage-1 Launch Vehicle Control Analysis at T=80 sec
! This batch set creates dynamic models for Control Analysis at T=80 sec
! Includes Slosh, Flexibility and Tail-Wags-Dog
!
! --------------   Create Vehicle, TVC and Actuator Models  ------------
Flight Vehicle    : Launch Vehicle First Stage Analysis Model, T=80.0 sec
Mixing Matrix     : Mixing Logic for First Stage Model, at T=80.0 sec
Actuator Model    : Stage-1 Linear Actuator
System Connection: System of 8 TVC Actuators
!
! --------------   Build the Flight Control System  ---------------------
Transf-Function   : Integrator
Transf-Function   : Alpha Estimator
Transf-Function   : Beta Estimator
Transf-Function   : Low Pass Filters
System Connection: Flight Control System
!
!  -------------   Create Plant Models  ---------------------------------
System Connection: Plant Model at T=80 sec, Vehicle/ Actuators
System Connection: Closed-Loop System
System Connection: Open-Loop System
System Connection: Roll Loop Opened, Others Closed
System Connection: Pitch Loop Opened, Others Closed
System Connection: Yaw Loop Opened, Others Closed
!
!  -------------   Export to Matlab  ------------------------------------
To Matlab Format : Launch Vehicle First Stage Analysis Model, T=80.0 sec
To Matlab Format : Plant Model at T=80 sec, Vehicle/ Actuators
To Matlab Format : Mixing Logic for First Stage Model, at T=80.0 sec
To Matlab Format : Stage-1 Linear Actuator
To Matlab Format : Flight Control System
To Matlab Format : Closed-Loop System
To Matlab Format : Open-Loop System
To Matlab Format : Roll Loop Opened, Others Closed
To Matlab Format : Pitch Loop Opened, Others Closed
To Matlab Format : Yaw Loop Opened, Others Closed
-----------------------------------------------------------------------------------------------
MIXING LOGIC MATRIX DATA ..........     (Matrix Title, Name, Vehicle Title, Control Directions)
Mixing Logic for First Stage Model, at T=80.0 sec
! Thrust Vector Control Matrix at t=80 sec
! This multi-engine vehicle has 8 Gimbaling Engines.
TVC
Launch Vehicle First Stage Analysis Model, T=80.0 sec
P-dot Roll  Acceleration About X Axis
Q-dot Pitch Acceleration About Y Axis
R-dot Yaw   Acceleration About Z Axis
-----------------------------------------------------------------------------------------------
ACTUATOR INPUT DATA ............ SIMPLE GENERIC MODEL B
Stage-1 Linear Actuator
Symbol Parameter Description                       (Units)       Value
C(s)   Order of Pade Delay (0,1,2)                   ( - )       1, -0.001, 0.001
Ka     Gain of Amplifier ...................... (amps/volt)    28.0
Wsv    Bandwidth of the Linear Servo Actuator .  (rad/sec)     70.0
Kact   Actuator Stiffness (Piston+Oil+Electric)   (lb/ft)      2.4e+6
Klod   Stiffness at Surface or Nozzle Connection  (lb/ft)      1.2e+9
Kbck   Stiffness at Vehicle Backup Structure ..   (lb/ft)      7.0e+7
R      Moment Arm between Actuator Rod & Gimbal   (feet)       0.667
Jl     Load Inertia about the Gimbal .......... (ft-lb-s^2)    15.12
Kg     Load Gimbal Bearing Spring Constant      (ft-lb/rad)    0.0
Bg     Load Gimbal Bearing Viscous Damping .... (ft-lb-sec)    550.0
-----------------------------------------------------------------------------------------------
```



```
FLIGHT VEHICLE INPUT DATA ......
Launch Vehicle First Stage Analysis Model, T=80.0 sec
! This is a Launch Vehicle Control Analysis Model at t=80 sec with 8 TVC Engines.
! The model includes two slosh modes for the LOX and LH2 tanks at 60% Propellant level.
! The LOX tank requires baffles and the damping coefficient was increased to 0.05 .
! The Flight Control Sensors include 3 Rate Gyros (p,q,r) and 2 Accelerometers (Ny,Nz).
! The model also includes 33 Structural Modes Selected between the TVC and the Nav Base
!
Body Axes Output,Attitude=Rate Integral

Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft)     :  3491.05       32.1740      0.208960E+08
Moments and Products of Inertia: Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)           :  25832.4       0.2521E+07   0.25200E+07    0.00000        0.00000
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)         :  58.0136       0.00000      0.00000
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)        :  1.30300       1278.63      530.760        39020.8
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)          :  25.7992       54.0456      0.00          -0.875700
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                          :  0.840        -0.00         0.02          -0.0
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)    :  0.00          63.2320      0.00          -0.0           -0.5782
W-Gust Azim & Elev angles (deg), or Torque/Force direction (x,y,z), Force Locat (x,y,z)   :  Gust          45.00        90.00
Surface Reference Area (feet^2), Mean Aerodynamic Chord (ft), Wing Span in (feet)         :  44.415        7.52000      7.52000
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial_rho/ Partial_H} :  120.142       0.00000      0.00000        0.00000
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:  1.49980       0.00583      0.552534E-04   0.00000        0.00000
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}:  0.0          -0.08110      0.00000        0.00000        0.00000
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}: -0.068124     -0.08110      0.00000        0.00000        0.00000
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:  0.00000       0.00000      0.00000        0.00000        0.00000
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}: -0.173376     -0.206400      0.00000        0.00000        0.00000
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}: -0.0           0.206400      0.00000        0.00000        0.00000

Number of Thruster Engines, Include or Not the Tail-Wags-Dog and Load-Torque Dynamics ?  :  8   WITH TWD     Tail-Wags-Dog and Load-Torque F/B Included

TVC Engine No:  1                             (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#1 +2Y-Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)                  :  24890.2       24890.2
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000       0.00000      6.00000        6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000       15.1200      1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0          2.4945      -1.0332
TVC Engine No:  2                             (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#2 +Y-2Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)                  :  24890.2       24890.2
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000       0.00000      6.00000        6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000       15.1200      1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0          1.0332      -2.4945
TVC Engine No:  3                             (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#3 -Y-2Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)                  :  24890.2       24890.2
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000       0.00000      6.00000        6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000       15.1200      1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0         -1.0332      -2.4945
TVC Engine No:  4                             (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#4 -2Y-Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)                  :  24890.2       24890.2
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000       0.00000      6.00000        6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000       15.1200      1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0         -2.4945      -1.0332
TVC Engine No:  5                             (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#5 -2Y+Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)                  :  24890.2       24890.2
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000       0.00000      6.00000        6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000       15.1200      1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0         -2.4945       1.0332
TVC Engine No:  6                             (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#6 -Y+2Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)                  :  24890.2       24890.2
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000       0.00000      6.00000        6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000       15.1200      1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0         -1.0332       2.4945
TVC Engine No:  7                             (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#7 +Y+2Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)                  :  24890.2       24890.2
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000       0.00000      6.00000        6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000       15.1200      1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0          1.0332       2.4945
TVC Engine No:  8                             (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#8 +2Y+Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)                  :  24890.2       24890.2
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000       0.00000      6.00000        6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  5.43000       15.1200      1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  12.0          2.4945       1.0332

Number of Gyros, (Attitude and Rate)                                                     :  3
Gyro No  1 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Location in (feet) :  Roll   Rate          97.483    0.00      0.00
Gyro No  2 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Location in (feet) :  Pitch  Rate          97.483    0.00      0.00
Gyro No  3 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Location in (feet) :  Yaw    Rate          97.483    0.00      0.00
                                                                                                              Sensors Location
Number of Accelerometers, Along Axis: (x,y,z)          Lateral and Normal Accelerometers :  2
Acceleromet No  2 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Location (ft) :  Y-axis Accelerat.     97.483    0.00000   0.00000
Acceleromet No  3 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Location (ft) :  Z-axis Accelerat.     97.483    0.00000   0.00000

Number of Slosh Modes                                                                    :  2 Masses
LOX Mass (slug), Frequency 1g (Wy,Wz) (rad/s), Damp (zeta-y-z), Locat.{Xsl,Ysl,Zsl} (ft) :   565.1   3.11   3.11   0.018   0.018  58.85  0.0  0.0
LH2 Mass (slug), Frequency 1g (Wy,Wz) (rad/s), Damp (zeta-y-z), Locat.{Xsl,Ysl,Zsl} (ft) :   194.6   3.11   3.11   0.015   0.015  29.60  0.0  0.0
                                                                                                  Slosh
Number of Bending Modes                                                                  : 33             Damping Coeffs  Slosh Coordinates
First Stage Flex Modes at 50% Full Tanks —— (Title of the Modal Data Set)                   Modes  Frequencies
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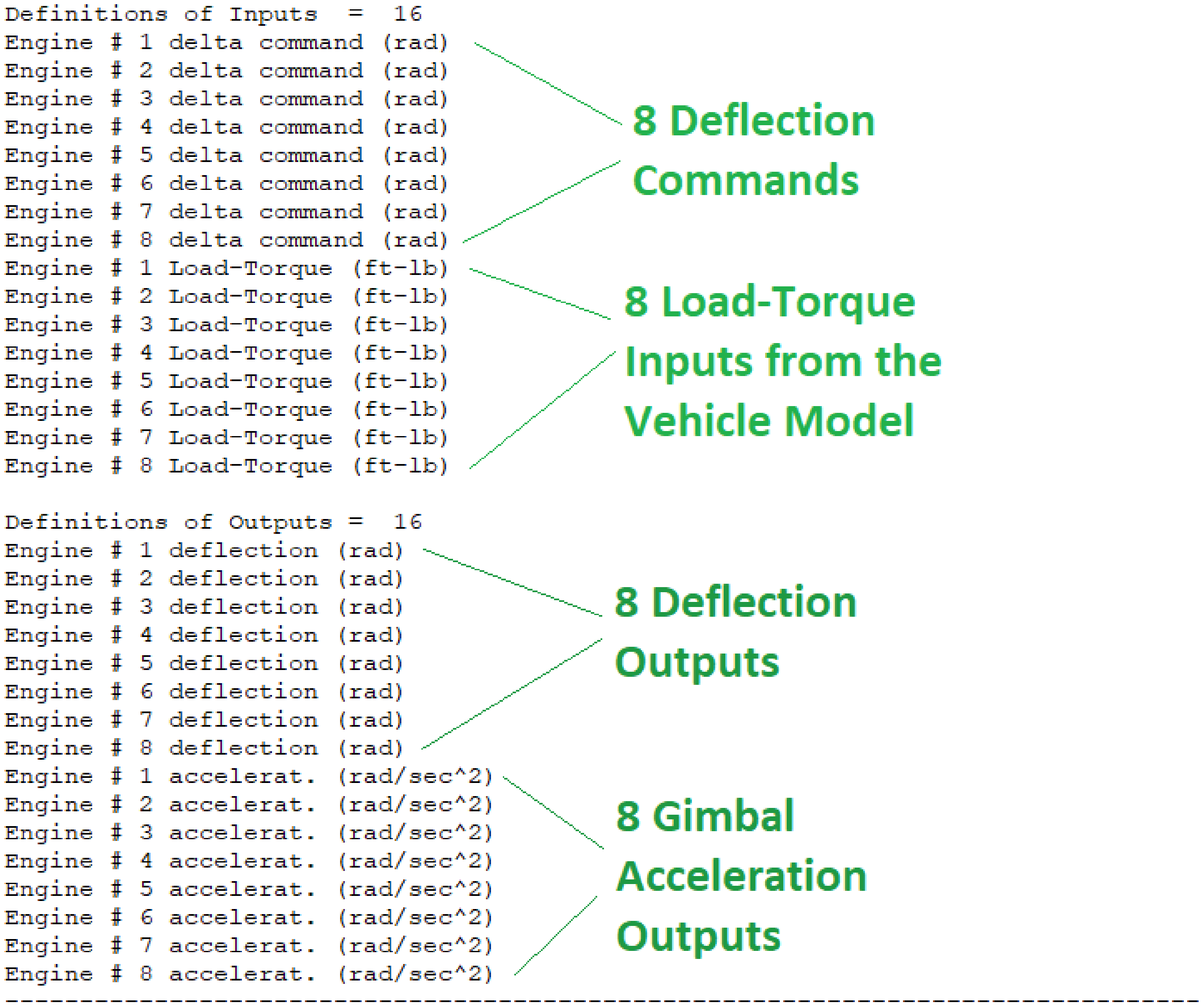
Combine 8 Engine Actuatos Together into a System of Actuators to Drive the 8 Engines in Pitch and in Yaw

```
INTERCONNECTION OF SYSTEMS .....
System of 8 TVC Actuators
! Combination of 8 TVC Actuators in Parallel
Titles of Systems to be Combined
Title 1 Stage-1 Linear Actuator
Title 2 Stage-1 Linear Actuator
Title 3 Stage-1 Linear Actuator
Title 4 Stage-1 Linear Actuator
Title 5 Stage-1 Linear Actuator
Title 6 Stage-1 Linear Actuator
Title 7 Stage-1 Linear Actuator
Title 8 Stage-1 Linear Actuator
SYSTEM INPUTS TO SUBSYSTEM  1                                                           Engine # 1
System Input  1 to Subsystem  1, Input  1, Gain= 1.00000                                Command Input
System Input  9 to Subsystem  1, Input  2, Gain= 1.00000                                Load-Torque
...............................................................................
SYSTEM INPUTS TO SUBSYSTEM  2                                                           Engine # 2
System Input  2 to Subsystem  2, Input  1, Gain= 1.00000                                Command Input
System Input 10 to Subsystem  2, Input  2, Gain= 1.00000                                Load-Torque
...............................................................................
SYSTEM INPUTS TO SUBSYSTEM  3                                                           Engine # 3
System Input  3 to Subsystem  3, Input  1, Gain= 1.00000                                Command Input
System Input 11 to Subsystem  3, Input  2, Gain= 1.00000                                Load-Torque
...............................................................................
SYSTEM INPUTS TO SUBSYSTEM  4                                                           Engine # 4
System Input  4 to Subsystem  4, Input  1, Gain= 1.00000                                Command Input
System Input 12 to Subsystem  4, Input  2, Gain= 1.00000                                Load-Torque
...............................................................................
SYSTEM INPUTS TO SUBSYSTEM  5                                                           Engine # 5
System Input  5 to Subsystem  5, Input  1, Gain= 1.00000                                Command Input
System Input 13 to Subsystem  5, Input  2, Gain= 1.00000                                Load-Torque
...............................................................................
SYSTEM INPUTS TO SUBSYSTEM  6                                                           Engine # 6
System Input  6 to Subsystem  6, Input  1, Gain= 1.00000                                Command Input
System Input 14 to Subsystem  6, Input  2, Gain= 1.00000                                Load-Torque
...............................................................................
SYSTEM INPUTS TO SUBSYSTEM  7                                                           Engine # 7
System Input  7 to Subsystem  7, Input  1, Gain= 1.00000                                Command Input
System Input 15 to Subsystem  7, Input  2, Gain= 1.00000                                Load-Torque
...............................................................................
SYSTEM INPUTS TO SUBSYSTEM  8                                                           Engine # 8
System Input  8 to Subsystem  8, Input  1, Gain= 1.00000                                Command Input
System Input 16 to Subsystem  8, Input  2, Gain= 1.00000                                Load-Torque
...............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                                        Engine # 1
System Output  1 from Subsystem  1, Output  1, Gain= 1.00000                            Deflection
System Output  9 from Subsystem  1, Output  3, Gain= 1.00000                            Acceleration
...............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                                                        Engine # 2
System Output  2 from Subsystem  2, Output  1, Gain= 1.00000                            Deflection
System Output 10 from Subsystem  2, Output  3, Gain= 1.00000                            Acceleration
...............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  3                                                        Engine # 3
System Output  3 from Subsystem  3, Output  1, Gain= 1.00000                            Deflection
System Output 11 from Subsystem  3, Output  3, Gain= 1.00000                            Acceleration
...............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  4                                                        Engine # 4
System Output  4 from Subsystem  4, Output  1, Gain= 1.00000                            Deflection
System Output 12 from Subsystem  4, Output  3, Gain= 1.00000                            Acceleration
...............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  5                                                        Engine # 5
System Output  5 from Subsystem  5, Output  1, Gain= 1.00000                            Deflection
System Output 13 from Subsystem  5, Output  3, Gain= 1.00000                            Acceleration
...............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  6                                                        Engine # 6
System Output  6 from Subsystem  6, Output  1, Gain= 1.00000                            Deflection
System Output 14 from Subsystem  6, Output  3, Gain= 1.00000                            Acceleration
...............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  7                                                        Engine # 7
System Output  7 from Subsystem  7, Output  1, Gain= 1.00000                            Deflection
System Output 15 from Subsystem  7, Output  3, Gain= 1.00000                            Acceleration
...............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  8                                                        Engine # 8
System Output  8 from Subsystem  8, Output  1, Gain= 1.00000                            Deflection
System Output 16 from Subsystem  8, Output  3, Gain= 1.00000                            Acceleration
...............................................................................
```

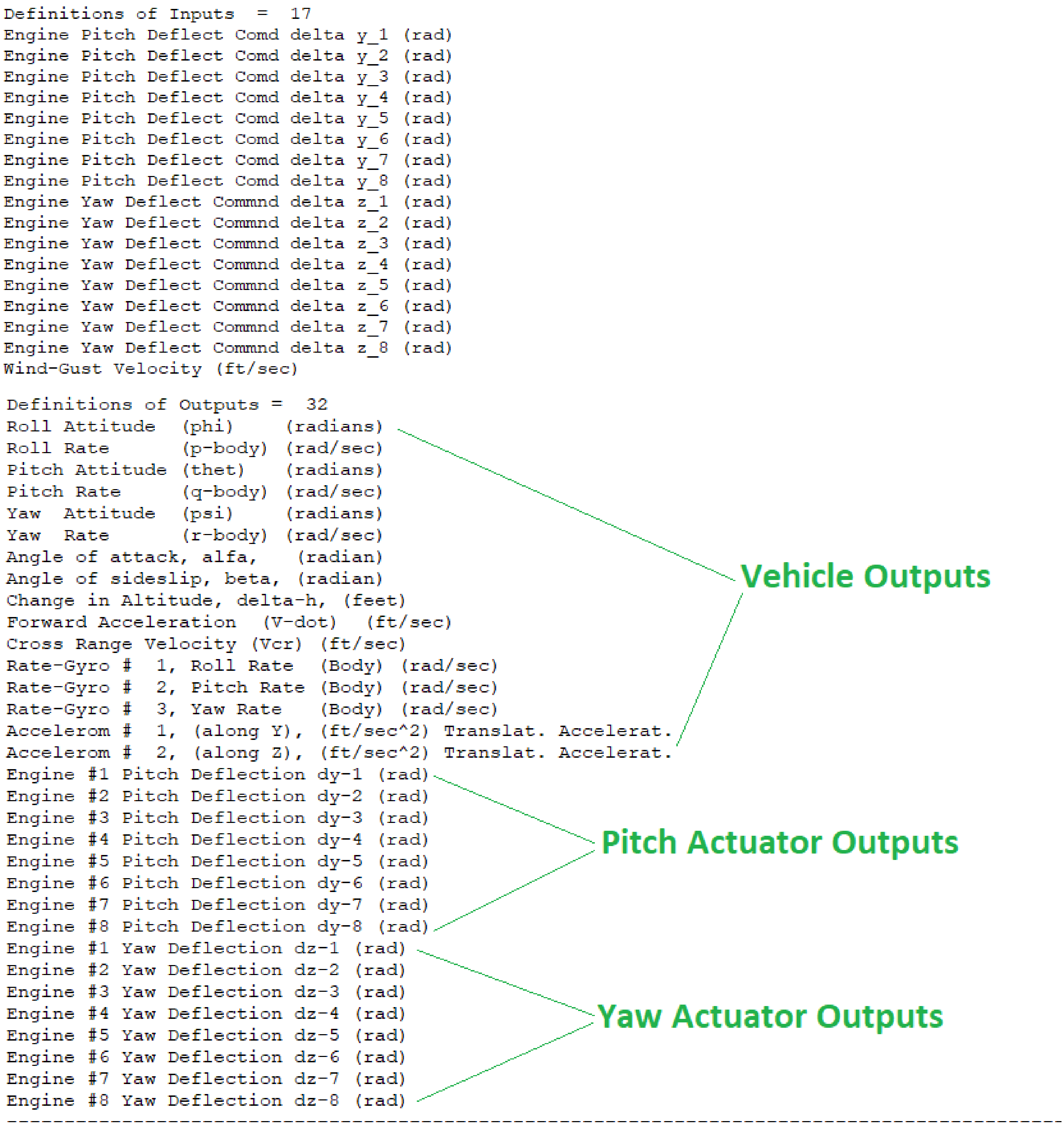


```
Definitions of Inputs  =  16
Engine # 1 delta command (rad)
Engine # 2 delta command (rad)
Engine # 3 delta command (rad)
Engine # 4 delta command (rad)
Engine # 5 delta command (rad)
Engine # 6 delta command (rad)
Engine # 7 delta command (rad)
Engine # 8 delta command (rad)
Engine # 1 Load-Torque (ft-lb)
Engine # 2 Load-Torque (ft-lb)
Engine # 3 Load-Torque (ft-lb)
Engine # 4 Load-Torque (ft-lb)
Engine # 5 Load-Torque (ft-lb)
Engine # 6 Load-Torque (ft-lb)
Engine # 7 Load-Torque (ft-lb)
Engine # 8 Load-Torque (ft-lb)

Definitions of Outputs =  16
Engine # 1 deflection (rad)
Engine # 2 deflection (rad)
Engine # 3 deflection (rad)
Engine # 4 deflection (rad)
Engine # 5 deflection (rad)
Engine # 6 deflection (rad)
Engine # 7 deflection (rad)
Engine # 8 deflection (rad)
Engine # 1 accelerat. (rad/sec^2)
Engine # 2 accelerat. (rad/sec^2)
Engine # 3 accelerat. (rad/sec^2)
Engine # 4 accelerat. (rad/sec^2)
Engine # 5 accelerat. (rad/sec^2)
Engine # 6 accelerat. (rad/sec^2)
Engine # 7 accelerat. (rad/sec^2)
Engine # 8 accelerat. (rad/sec^2)
--------------------------------------------------------------------------------
```

Combine the Vehicle System, the 8 Pitch, and the 8 Yaw Gimbal Actuators into a Single Plant System

```
INTERCONNECTION OF SYSTEMS .....
Plant Model at T=80 sec, Vehicle/ Actuators
! Combines the Vehicle with the 8 Pitch and 8 Yaw Actuators
!
Titles of Systems to be Combined
Title 1 Launch Vehicle First Stage Analysis Model, T=80.0 sec
Title 2 System of 8 TVC Actuators
Title 3 System of 8 TVC Actuators
SYSTEM INPUTS TO SUBSYSTEM  2                                                                    To Pitch Actuators (1-8)
System Input  1 to Subsystem  2, Input  1, Gain= 1.00000
System Input  2 to Subsystem  2, Input  2, Gain= 1.00000
System Input  3 to Subsystem  2, Input  3, Gain= 1.00000
System Input  4 to Subsystem  2, Input  4, Gain= 1.00000
System Input  5 to Subsystem  2, Input  5, Gain= 1.00000
System Input  6 to Subsystem  2, Input  6, Gain= 1.00000
System Input  7 to Subsystem  2, Input  7, Gain= 1.00000
System Input  8 to Subsystem  2, Input  8, Gain= 1.00000
..............................................................................
SYSTEM INPUTS TO SUBSYSTEM  3                                                                    To Yaw Actuators (1-8)
System Input  9 to Subsystem  3, Input  1, Gain= 1.00000
System Input 10 to Subsystem  3, Input  2, Gain= 1.00000
System Input 11 to Subsystem  3, Input  3, Gain= 1.00000
System Input 12 to Subsystem  3, Input  4, Gain= 1.00000
System Input 13 to Subsystem  3, Input  5, Gain= 1.00000
System Input 14 to Subsystem  3, Input  6, Gain= 1.00000
System Input 15 to Subsystem  3, Input  7, Gain= 1.00000
System Input 16 to Subsystem  3, Input  8, Gain= 1.00000
..............................................................................
SYSTEM INPUTS TO SUBSYSTEM  1
System Input 17 to Subsystem  1, Input 33, Gain= 1.0                                             Wind-Gust
..............................................................................
SUBSYSTEM NO  2 GOES TO SUBSYSTEM NO  1                                                          Pitch Actuat to Vehicle
Subsystem  2, Output  1 to Subsystem  1, Input  1, Gain= 1.0                                     delta_y1  Pitch Engine Deflects
Subsystem  2, Output  2 to Subsystem  1, Input  2, Gain= 1.0                                     delta_y2
Subsystem  2, Output  3 to Subsystem  1, Input  3, Gain= 1.0                                     delta_y3
Subsystem  2, Output  4 to Subsystem  1, Input  4, Gain= 1.0                                     delta_y4
Subsystem  2, Output  5 to Subsystem  1, Input  5, Gain= 1.0                                     delta_y5
Subsystem  2, Output  6 to Subsystem  1, Input  6, Gain= 1.0                                     delta_y6
Subsystem  2, Output  7 to Subsystem  1, Input  7, Gain= 1.0                                     delta_y7
Subsystem  2, Output  8 to Subsystem  1, Input  8, Gain= 1.0                                     delta_y8
Subsystem  2, Output  9 to Subsystem  1, Input  9, Gain= 1.0                                     dy1_dd    Engine Accels
Subsystem  2, Output 10 to Subsystem  1, Input 10, Gain= 1.0                                     dy2_dd
Subsystem  2, Output 11 to Subsystem  1, Input 11, Gain= 1.0                                     dy3_dd
Subsystem  2, Output 12 to Subsystem  1, Input 12, Gain= 1.0                                     dy4_dd
Subsystem  2, Output 13 to Subsystem  1, Input 13, Gain= 1.0                                     dy5_dd
Subsystem  2, Output 14 to Subsystem  1, Input 14, Gain= 1.0                                     dy6_dd
Subsystem  2, Output 15 to Subsystem  1, Input 15, Gain= 1.0                                     dy7_dd
Subsystem  2, Output 16 to Subsystem  1, Input 16, Gain= 1.0                                     dy8_dd
..............................................................................
SUBSYSTEM NO  3 GOES TO SUBSYSTEM NO  1                                                          Yaw Actuat to Vehicle
Subsystem  3, Output  1 to Subsystem  1, Input 17, Gain= 1.0                                     delta_z1  Yaw Engine Deflects
Subsystem  3, Output  2 to Subsystem  1, Input 18, Gain= 1.0                                     delta_z2
Subsystem  3, Output  3 to Subsystem  1, Input 19, Gain= 1.0                                     delta_z3
Subsystem  3, Output  4 to Subsystem  1, Input 20, Gain= 1.0                                     delta_z4
Subsystem  3, Output  5 to Subsystem  1, Input 21, Gain= 1.0                                     delta_z5
Subsystem  3, Output  6 to Subsystem  1, Input 22, Gain= 1.0                                     delta_z6
Subsystem  3, Output  7 to Subsystem  1, Input 23, Gain= 1.0                                     delta_z7
Subsystem  3, Output  8 to Subsystem  1, Input 24, Gain= 1.0                                     delta_z8
Subsystem  3, Output  9 to Subsystem  1, Input 25, Gain= 1.0                                     dz1_dd    Yaw Engine Accels
Subsystem  3, Output 10 to Subsystem  1, Input 26, Gain= 1.0                                     dz2_dd
Subsystem  3, Output 11 to Subsystem  1, Input 27, Gain= 1.0                                     dz3_dd
Subsystem  3, Output 12 to Subsystem  1, Input 28, Gain= 1.0                                     dz4_dd
Subsystem  3, Output 13 to Subsystem  1, Input 29, Gain= 1.0                                     dz5_dd
Subsystem  3, Output 14 to Subsystem  1, Input 30, Gain= 1.0                                     dz6_dd
Subsystem  3, Output 15 to Subsystem  1, Input 31, Gain= 1.0                                     dz7_dd
Subsystem  3, Output 16 to Subsystem  1, Input 32, Gain= 1.0                                     dz8_dd
..............................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2                                                          Load-Torq TLY to Pitch Actuat
Subsystem  1, Output 17 to Subsystem  2, Input  9, Gain= 1.0                                     TL_y1
Subsystem  1, Output 18 to Subsystem  2, Input 10, Gain= 1.0                                     TL_y2
Subsystem  1, Output 19 to Subsystem  2, Input 11, Gain= 1.0                                     TL_y3
Subsystem  1, Output 20 to Subsystem  2, Input 12, Gain= 1.0                                     TL_y4
Subsystem  1, Output 21 to Subsystem  2, Input 13, Gain= 1.0                                     TL_y5
Subsystem  1, Output 22 to Subsystem  2, Input 14, Gain= 1.0                                     TL_y6
Subsystem  1, Output 23 to Subsystem  2, Input 15, Gain= 1.0                                     TL_y7
Subsystem  1, Output 24 to Subsystem  2, Input 16, Gain= 1.0                                     TL_y8
..............................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  3                                                          Load-Torq TLZ to Yaw Actuators
Subsystem  1, Output 25 to Subsystem  3, Input  9, Gain= 1.0                                     TL_z1
Subsystem  1, Output 26 to Subsystem  3, Input 10, Gain= 1.0                                     TL_z2
Subsystem  1, Output 27 to Subsystem  3, Input 11, Gain= 1.0                                     TL_z3
Subsystem  1, Output 28 to Subsystem  3, Input 12, Gain= 1.0                                     TL_z4
Subsystem  1, Output 29 to Subsystem  3, Input 13, Gain= 1.0                                     TL_z5
Subsystem  1, Output 30 to Subsystem  3, Input 14, Gain= 1.0                                     TL_z6
Subsystem  1, Output 31 to Subsystem  3, Input 15, Gain= 1.0                                     TL_z7
Subsystem  1, Output 32 to Subsystem  3, Input 16, Gain= 1.0                                     TL_z8
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                                                 Vehicle Outputs
Via Matrix +I16
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                                                                 Pitch Gimbal Deflects
System Output 17 from Subsystem  2, Output  1, Gain= 1.0                                         dy-1
System Output 18 from Subsystem  2, Output  2, Gain= 1.0                                         dy-2
System Output 19 from Subsystem  2, Output  3, Gain= 1.0                                         dy-3
System Output 20 from Subsystem  2, Output  4, Gain= 1.0                                         dy-4
System Output 21 from Subsystem  2, Output  5, Gain= 1.0                                         dy-5
System Output 22 from Subsystem  2, Output  6, Gain= 1.0                                         dy-6
System Output 23 from Subsystem  2, Output  7, Gain= 1.0                                         dy-7
System Output 24 from Subsystem  2, Output  8, Gain= 1.0                                         dy-8
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  3                                                                 Yaw Gimbal Deflects
System Output 25 from Subsystem  3, Output  1, Gain= 1.0                                         dz-1
System Output 26 from Subsystem  3, Output  2, Gain= 1.0                                         dz-2
System Output 27 from Subsystem  3, Output  3, Gain= 1.0                                         dz-3
System Output 28 from Subsystem  3, Output  4, Gain= 1.0                                         dz-4
System Output 29 from Subsystem  3, Output  5, Gain= 1.0                                         dz-5
System Output 30 from Subsystem  3, Output  6, Gain= 1.0                                         dz-6
System Output 31 from Subsystem  3, Output  7, Gain= 1.0                                         dz-7
System Output 32 from Subsystem  3, Output  8, Gain= 1.0                                         dz-8
..............................................................................
```



```
Definitions of Inputs  =  17
Engine Pitch Deflect Comd delta y_1 (rad)
Engine Pitch Deflect Comd delta y_2 (rad)
Engine Pitch Deflect Comd delta y_3 (rad)
Engine Pitch Deflect Comd delta y_4 (rad)
Engine Pitch Deflect Comd delta y_5 (rad)
Engine Pitch Deflect Comd delta y_6 (rad)
Engine Pitch Deflect Comd delta y_7 (rad)
Engine Pitch Deflect Comd delta y_8 (rad)
Engine Yaw Deflect Commnd delta z_1 (rad)
Engine Yaw Deflect Commnd delta z_2 (rad)
Engine Yaw Deflect Commnd delta z_3 (rad)
Engine Yaw Deflect Commnd delta z_4 (rad)
Engine Yaw Deflect Commnd delta z_5 (rad)
Engine Yaw Deflect Commnd delta z_6 (rad)
Engine Yaw Deflect Commnd delta z_7 (rad)
Engine Yaw Deflect Commnd delta z_8 (rad)
Wind-Gust Velocity (ft/sec)

Definitions of Outputs =  32
Roll Attitude   (phi)     (radians)
Roll Rate       (p-body)  (rad/sec)
Pitch Attitude  (thet)    (radians)
Pitch Rate      (q-body)  (rad/sec)
Yaw  Attitude   (psi)     (radians)
Yaw  Rate       (r-body)  (rad/sec)
Angle of attack, alfa,    (radian)
Angle of sideslip, beta,  (radian)
Change in Altitude, delta-h, (feet)
Forward Acceleration   (V-dot)   (ft/sec)
Cross Range Velocity (Vcr)  (ft/sec)
Rate-Gyro #  1, Roll Rate   (Body)  (rad/sec)
Rate-Gyro #  2, Pitch Rate  (Body)  (rad/sec)
Rate-Gyro #  3, Yaw Rate    (Body)  (rad/sec)
Accelerom #  1, (along Y),  (ft/sec^2) Translat. Accelerat.
Accelerom #  2, (along Z),  (ft/sec^2) Translat. Accelerat.
Engine #1 Pitch Deflection dy-1  (rad)
Engine #2 Pitch Deflection dy-2  (rad)
Engine #3 Pitch Deflection dy-3  (rad)
Engine #4 Pitch Deflection dy-4  (rad)
Engine #5 Pitch Deflection dy-5  (rad)
Engine #6 Pitch Deflection dy-6  (rad)
Engine #7 Pitch Deflection dy-7  (rad)
Engine #8 Pitch Deflection dy-8  (rad)
Engine #1 Yaw Deflection dz-1  (rad)
Engine #2 Yaw Deflection dz-2  (rad)
Engine #3 Yaw Deflection dz-3  (rad)
Engine #4 Yaw Deflection dz-4  (rad)
Engine #5 Yaw Deflection dz-5  (rad)
Engine #6 Yaw Deflection dz-6  (rad)
Engine #7 Yaw Deflection dz-7  (rad)
Engine #8 Yaw Deflection dz-8  (rad)
-------------------------------------------------------------------------------------
```

```
SYSTEM OF TRANSFER FUNCTIONS ...
Alpha Estimator
Continuous
TF. Block #  1 Nz-Different.                                  Order of Numer, Denom=  1  1
Numer 1.0         0.0
Denom 0.015       1.0
TF. Block #  2 Alfa Integrator                                Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
TF. Block #  3 Slsh-Notch at 4.38 (rad)                       Order of Numer, Denom=  2  2
Numer 1.0         0.0704      19.1844
Denom 1.0         1.408       19.1844
```

Includes a Notch Filter to Attenuate the LOX Slosh Mode

```
..........................................................
Block #, from Input #, Gain
 2       1     -24890.0                                        Eng #1 Dy
 2       2     -24890.0                                        Eng #2 Dy
 2       3     -24890.0                                        Eng #3 Dy
 2       4     -24890.0                                        Eng #4 Dy
 2       5     -24890.0                                        Eng #5 Dy
 2       6     -24890.0                                        Eng #6 Dy
 2       7     -24890.0                                        Eng #7 Dy
 2       8     -24890.0                                        Eng #8 Dy
 1       9      19.0                                           Pitch Rate q
 2      10     -3491.0                                         Zdd, V.Mass
...........................
Block #, from Block #, Gain
 2       1     -3491.0                                         Vehi Mass
 2       3     -1912.0                                         Qbar.Sref.Cza
 3       2      0.00035                                        Estimator Gain
...........................
Outpt #, from Block #, Gain
 1       3      0.0175                                         Alpha Estimate (rad)
............................
Definitions of Inputs  =  10
Engine # 1 DY Deflection in (rad)
Engine # 2 DY Deflection in (rad)
Engine # 3 DY Deflection in (rad)
Engine # 4 DY Deflection in (rad)
Engine # 5 DY Deflection in (rad)
Engine # 6 DY Deflection in (rad)
Engine # 7 DY Deflection in (rad)
Engine # 8 DY Deflection in (rad)
Vehicle Pitch Rate      (rad/sec)
Normal  Accelerat. Nz (ft/sec^2)

Definitions of Outputs =   1
Alpha Estimate in (rad)
```

Inputs to the Alpha Estimator are: 8 Pitch Gimbal Deflects (dy), Pitch Rate (q) and Normal Accelerom. (Nz)

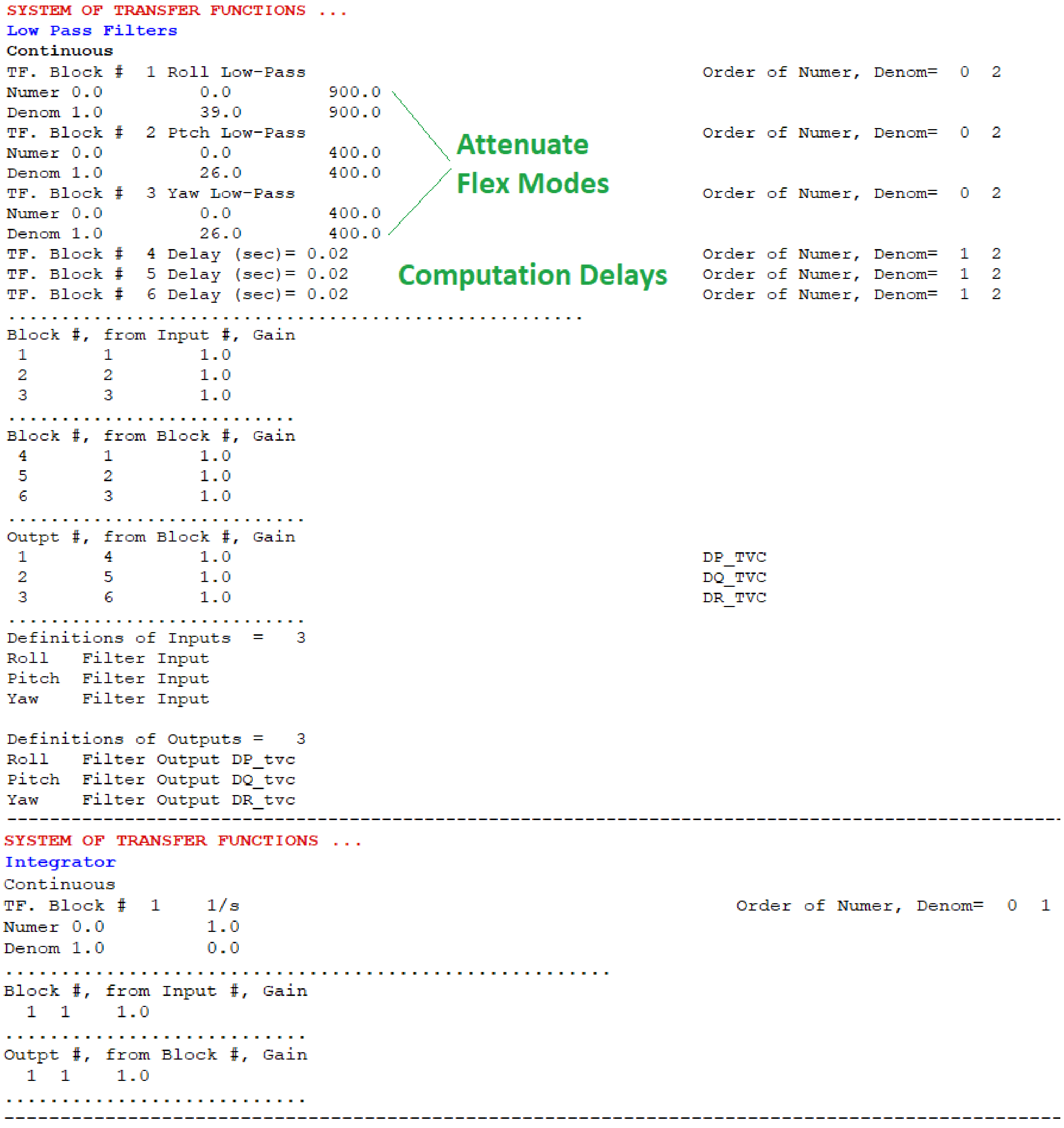
```
-------------------------------------------------------------------------------------------
SYSTEM OF TRANSFER FUNCTIONS ...
Beta Estimator
Continuous
TF. Block #  1 Ny-Different.                                  Order of Numer, Denom=  1  1
Numer 1.0         0.0
Denom 0.015       1.0
TF. Block #  2 Beta Integrator                                Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
TF. Block #  3 Slsh-Notch at 4.38 (rad)                       Order of Numer, Denom=  2  2
Numer 1.0         0.0704      19.1844
Denom 1.0         1.408       19.1844
```

Includes Notch Filter to Attenuate the LOX Slosh Mode

```
..........................................................
Block #, from Input #, Gain
 2       1      24890.0                                        Eng #1 Dz
 2       2      24890.0                                        Eng #2 Dz
 2       3      24890.0                                        Eng #3 Dz
 2       4      24890.0                                        Eng #4 Dz
 2       5      24890.0                                        Eng #5 Dz
 2       6      24890.0                                        Eng #6 Dz
 2       7      24890.0                                        Eng #7 Dz
 2       8      24890.0                                        Eng #8 Dz
 1       9      19.0                                           Yaw Rate r
 2      10     -3491.0                                         Ydd, V.Mass
...........................
Block #, from Block #, Gain
 2       1     +3491.0                                         Vehi Mass
 2       3     -1912.0                                         Qbar.Sref.Cyb
 3       2      0.00035                                        Estimator Gain
...........................
Outpt #, from Block #, Gain
 1       3      0.0175                                         Alpha Estimate (rad)
............................
Definitions of Inputs  =  10
Engine # 1 DZ Deflection in (rad)
Engine # 2 DZ Deflection in (rad)
Engine # 3 DZ Deflection in (rad)
Engine # 4 DZ Deflection in (rad)
Engine # 5 DZ Deflection in (rad)
Engine # 6 DZ Deflection in (rad)
Engine # 7 DZ Deflection in (rad)
Engine # 8 DZ Deflection in (rad)
Vehicle Yaw Rate      (rad/sec)
Lateral  Accelerat. Ny (ft/sec^2)

Definitions of Outputs =   1
Beta Estimate in (rad)
-------------------------------------------------------------------------------------------
```

The Inputs to Beta Estimator are Yaw Gimbal Deflections (dz), Yaw Rate (r) and Lateral Accelerometer (Ny)



```
SYSTEM OF TRANSFER FUNCTIONS ...
Low Pass Filters
Continuous
TF. Block #  1 Roll Low-Pass                                    Order of Numer, Denom=  0  2
Numer 0.0         0.0         900.0
Denom 1.0         39.0        900.0
TF. Block #  2 Ptch Low-Pass                                    Order of Numer, Denom=  0  2
Numer 0.0         0.0         400.0
Denom 1.0         26.0        400.0
TF. Block #  3 Yaw Low-Pass                                     Order of Numer, Denom=  0  2
Numer 0.0         0.0         400.0
Denom 1.0         26.0        400.0
TF. Block #  4 Delay (sec)= 0.02                                Order of Numer, Denom=  1  2
TF. Block #  5 Delay (sec)= 0.02                                Order of Numer, Denom=  1  2
TF. Block #  6 Delay (sec)= 0.02                                Order of Numer, Denom=  1  2
......................................................
Block #, from Input #, Gain
 1        1         1.0
 2        2         1.0
 3        3         1.0
...........................
Block #, from Block #, Gain
 4        1         1.0
 5        2         1.0
 6        3         1.0
............................
Outpt #, from Block #, Gain
 1        4         1.0                                         DP_TVC
 2        5         1.0                                         DQ_TVC
 3        6         1.0                                         DR_TVC
............................
Definitions of Inputs  =   3
Roll   Filter Input
Pitch  Filter Input
Yaw    Filter Input

Definitions of Outputs =   3
Roll   Filter Output DP_tvc
Pitch  Filter Output DQ_tvc
Yaw    Filter Output DR_tvc
----------------------------------------------------------------------------------------------------
SYSTEM OF TRANSFER FUNCTIONS ...
Integrator
Continuous
TF. Block #  1    1/s                                              Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
.........................................................
Block #, from Input #, Gain
  1  1    1.0
............................
Outpt #, from Block #, Gain
  1  1    1.0
............................
----------------------------------------------------------------------------------------------------
```

```
INTERCONNECTION OF SYSTEMS .....
Flight Control System
! Combine the Filters, Gains and Alpha/Beta Estimators
!
Titles of Systems to be Combined
Title 1 Alpha Estimator
Title 2 Beta Estimator
Title 3 Low Pass Filters
Title 4 Integrator
Title 5 Integrator
SYSTEM INPUTS TO SUBSYSTEM  1                                                     to Alfa Estimator
System Input  1 to Subsystem  1, Input  1, Gain= 1.00000                          Pitch Gimbal Deflects DY (1-8)
System Input  2 to Subsystem  1, Input  2, Gain= 1.00000
System Input  3 to Subsystem  1, Input  3, Gain= 1.00000
System Input  4 to Subsystem  1, Input  4, Gain= 1.00000
System Input  5 to Subsystem  1, Input  5, Gain= 1.00000     Inputs to Alpha Estimator
System Input  6 to Subsystem  1, Input  6, Gain= 1.00000
System Input  7 to Subsystem  1, Input  7, Gain= 1.00000
System Input  8 to Subsystem  1, Input  8, Gain= 1.00000
System Input 21 to Subsystem  1, Input  9, Gain= 1.00000                          Pitch Rate Q
System Input 23 to Subsystem  1, Input 10, Gain= 1.00000                          NZ Accelerat.
...............................................................................
SYSTEM INPUTS TO SUBSYSTEM  2                                                     to Beta Estimator
System Input  9 to Subsystem  2, Input  1, Gain= 1.00000                          Yaw Gimbal Deflects DZ (1-8)
System Input 10 to Subsystem  2, Input  2, Gain= 1.00000
System Input 11 to Subsystem  2, Input  3, Gain= 1.00000
System Input 12 to Subsystem  2, Input  4, Gain= 1.00000
System Input 13 to Subsystem  2, Input  5, Gain= 1.00000     Inputs to Beta Estimator
System Input 14 to Subsystem  2, Input  6, Gain= 1.00000
System Input 15 to Subsystem  2, Input  7, Gain= 1.00000
System Input 16 to Subsystem  2, Input  8, Gain= 1.00000
System Input 22 to Subsystem  2, Input  9, Gain= 1.00000                          Yaw Rate R
System Input 24 to Subsystem  2, Input 10, Gain= 1.00000                          NY Accelerat.
...............................................................................
SYSTEM INPUTS TO SUBSYSTEM  3                                                     to Low-Pass Filters
System Input 17 to Subsystem  3, Input  1, Gain= 2.5                              KL_phi
System Input 18 to Subsystem  3, Input  2, Gain= 6.0                              KM_theta
System Input 19 to Subsystem  3, Input  3, Gain= 6.0                              KN_psi
System Input 20 to Subsystem  3, Input  1, Gain= 2.2         Inputs to Filters    KL_p
System Input 21 to Subsystem  3, Input  2, Gain= 2.8                              KM_q
System Input 22 to Subsystem  3, Input  3, Gain= 2.8                              KN_r
...............................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  3                                           Alfa Estim to Filters
Subsystem  1, Output  1 to Subsystem  3, Input  2, Gain= 3.3                      KM_alfa
...............................................................................
SUBSYSTEM NO  2 GOES TO SUBSYSTEM NO  3                                           Beta Estim to Filters
Subsystem  2, Output  1 to Subsystem  3, Input  3, Gain= -3.3                     KN_beta
...............................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  4                                           Alfa Estim to Integrator
Subsystem  1, Output  1 to Subsystem  4, Input  1, Gain= 1.0                      alfin
...............................................................................
SUBSYSTEM NO  2 GOES TO SUBSYSTEM NO  5                                           Beta Estim to Integrator
Subsystem  2, Output  1 to Subsystem  5, Input  1, Gain= 1.0                      betin
...............................................................................
SUBSYSTEM NO  4 GOES TO SUBSYSTEM NO  3                                           alfin to Low-Pass
Subsystem  4, Output  1 to Subsystem  3, Input  2, Gain= -0.2                     KM_alfint
...............................................................................
SUBSYSTEM NO  5 GOES TO SUBSYSTEM NO  3                                           alfin to Low-Pass
Subsystem  5, Output  1 to Subsystem  3, Input  3, Gain=  0.2                     KN_betint
...............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  3                                                  from Low-Pass Filters
System Output  1 from Subsystem  3, Output  1, Gain= -1.0   Flight Control Demands  DP_tvc
System Output  2 from Subsystem  3, Output  2, Gain= -1.0   to TVC Matrix           DQ_tvc
System Output  3 from Subsystem  3, Output  3, Gain= -1.0                           DR_tvc
...............................................................................
Definitions of Inputs  =  24
Engine Pitch Deflect Comd delta y_1 (rad)
Engine Pitch Deflect Comd delta y_2 (rad)
Engine Pitch Deflect Comd delta y_3 (rad)
Engine Pitch Deflect Comd delta y_4 (rad)
Engine Pitch Deflect Comd delta y_5 (rad)
Engine Pitch Deflect Comd delta y_6 (rad)
Engine Pitch Deflect Comd delta y_7 (rad)
Engine Pitch Deflect Comd delta y_8 (rad)
Engine Yaw Deflect Commnd delta z_1 (rad)
Engine Yaw Deflect Commnd delta z_2 (rad)
Engine Yaw Deflect Commnd delta z_3 (rad)
Engine Yaw Deflect Commnd delta z_4 (rad)
Engine Yaw Deflect Commnd delta z_5 (rad)
Engine Yaw Deflect Commnd delta z_6 (rad)
Engine Yaw Deflect Commnd delta z_7 (rad)
Engine Yaw Deflect Commnd delta z_8 (rad)
Roll  Attitude Error (rad)
Pitch Attitude Error (rad)
Yaw   Attitude Error (rad)
Roll  Rate (rad/sec)
Pitch Rate (rad/sec)
Yaw   Rate (rad/sec)
Normal  Acceleration NZ (ft/sec^2)
Lateral Acceleration NY (ft/sec^2)

Definitions of Outputs =   3
Roll  FCS Demand DP_tvc
Pitch FCS Demand DQ_tvc
Yaw   FCS Demand DR_tvc
-------------------------------------------------------------------------------------
```

This System is Used for Simulations

```
INTERCONNECTION OF SYSTEMS .....
Closed-Loop System
! Combines the Plant Model with the Flight Control System in
! Closed-Loop form
!
Titles of Systems to be Combined
Title 1 Flight Control System
Title 2 Plant Model at T=80 sec, Vehicle/ Actuators
SYSTEM INPUTS TO SUBSYSTEM  1                                                              To FCS
System Input  1 to Subsystem  1, Input 17, Gain= -1.0                                      Roll Command
System Input  2 to Subsystem  1, Input 18, Gain= -1.0                                      Ptch Command
System Input  3 to Subsystem  1, Input 19, Gain= -1.0                                      Yaw  Command
```

Attitude Commands from Guidance

```
..............................................................................
SYSTEM INPUTS TO SUBSYSTEM  2                                                              To Vehi
System Input  4 to Subsystem  2, Input 17, Gain=  1.0                                      Wind Gust
```

Wind-Gust Input

```
..............................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  1                                                    FCS
Via Matrix +TVC
..............................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2                                                    FCS to Vehi
Via Matrix +TVC                                                                            DY(8), DZ(8)
..............................................................................
SUBSYSTEM NO  2 GOES TO SUBSYSTEM NO  1                                                    Vehi to FCS
Subsystem  2, Output  1 to Subsystem  1, Input 17, Gain= 1.0                               Roll Attit f/b
Subsystem  2, Output  3 to Subsystem  1, Input 18, Gain= 1.0                               Ptch Attit f/b
Subsystem  2, Output  5 to Subsystem  1, Input 19, Gain= 1.0                               Yaw  Attit f/b
Subsystem  2, Output 12 to Subsystem  1, Input 20, Gain= 1.0                               Roll Rate f/b
Subsystem  2, Output 13 to Subsystem  1, Input 21, Gain= 1.0                               Ptch Rate f/b
Subsystem  2, Output 14 to Subsystem  1, Input 22, Gain= 1.0                               Yaw  Rate f/b
Subsystem  2, Output 16 to Subsystem  1, Input 23, Gain= 1.0                               Nz f/b
Subsystem  2, Output 15 to Subsystem  1, Input 24, Gain= 1.0                               Ny f/b
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                                                           Vehicle Outputs
Via Matrix +I32
```

Outputs from the Plant Model

```
..............................................................................
Definitions of Inputs  =   4
Roll  Attitude Command (rad)
Pitch Attitude Command (rad)
Yaw   Attitude Command (rad)
Wind-Gust Velocity (feet/sec)
```

3 Attitude Commands, and
1 Wind-Gust Velocity Input

```
Definitions of Outputs =  32
Roll Attitude  (phi)    (radians)
Roll Rate      (p-body) (rad/sec)
Pitch Attitude (thet)   (radians)
Pitch Rate     (q-body) (rad/sec)
Yaw  Attitude  (psi)    (radians)
Yaw  Rate      (r-body) (rad/sec)
Angle of attack, alfa,   (radian)
Angle of sideslip, beta, (radian)
Change in Altitude, delta-h, (feet)
Forward Acceleration  (V-dot)  (ft/sec)
Cross Range Velocity (Vcr)  (ft/sec)
Rate-Gyro #  1, Roll Rate  (Body) (rad/sec)
Rate-Gyro #  2, Pitch Rate (Body) (rad/sec)
Rate-Gyro #  3, Yaw Rate   (Body) (rad/sec)
Accelerom #  1, (along Y), (ft/sec^2) Translat. Accelerat.
Accelerom #  2, (along Z), (ft/sec^2) Translat. Accelerat.
Engine #1 Pitch Deflection dy-1 (rad)
Engine #2 Pitch Deflection dy-2 (rad)
Engine #3 Pitch Deflection dy-3 (rad)
Engine #4 Pitch Deflection dy-4 (rad)
Engine #5 Pitch Deflection dy-5 (rad)
Engine #6 Pitch Deflection dy-6 (rad)
Engine #7 Pitch Deflection dy-7 (rad)
Engine #8 Pitch Deflection dy-8 (rad)
Engine #1 Yaw Deflection dz-1 (rad)
Engine #2 Yaw Deflection dz-2 (rad)
Engine #3 Yaw Deflection dz-3 (rad)
Engine #4 Yaw Deflection dz-4 (rad)
Engine #5 Yaw Deflection dz-5 (rad)
Engine #6 Yaw Deflection dz-6 (rad)
Engine #7 Yaw Deflection dz-7 (rad)
Engine #8 Yaw Deflection dz-8 (rad)
--------------------------------------------------------------------------------------------
```

32 Outputs from the
Plant Model

This System will be Used for Frequency Response Stability Analysis

```
INTERCONNECTION OF SYSTEMS .....
Open-Loop System
! Combines the Plant Model with the Flight Control System in
! Open-Loop config
!
Titles of Systems to be Combined
Title 1 Flight Control System
Title 2 Plant Model at T=80 sec, Vehicle/ Actuators
SYSTEM INPUTS TO SUBSYSTEM  1                                                  To FCS
Via Matrix +TVC                                                                DP, DQ, DR
.............................................................................
SYSTEM INPUTS TO SUBSYSTEM  2                                                  To Vehicle
Via Matrix +TVC                                                                DP, DQ, DR
.............................................................................
SUBSYSTEM NO  2 GOES TO SUBSYSTEM NO  1                                        Vehi to FCS
Subsystem  2, Output  1 to Subsystem  1, Input 17, Gain= 1.0                   Roll Attit f/b
Subsystem  2, Output  3 to Subsystem  1, Input 18, Gain= 1.0                   Ptch Attit f/b
Subsystem  2, Output  5 to Subsystem  1, Input 19, Gain= 1.0                   Yaw  Attit f/b
Subsystem  2, Output 12 to Subsystem  1, Input 20, Gain= 1.0                   Roll Rate f/b
Subsystem  2, Output 13 to Subsystem  1, Input 21, Gain= 1.0                   Ptch Rate f/b
Subsystem  2, Output 14 to Subsystem  1, Input 22, Gain= 1.0                   Yaw  Rate f/b
Subsystem  2, Output 16 to Subsystem  1, Input 23, Gain= 1.0                   Nz f/b
Subsystem  2, Output 15 to Subsystem  1, Input 24, Gain= 1.0                   Ny f/b
.............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                               Vehicle Outputs
Via Matrix +I03
.............................................................................
Definitions of Inputs  =   3
Roll  Control Demand DP_tvc
Pitch Control Demand DQ_tvc
Yaw   Control Demand DR_tvc

Definitions of Outputs =   3
Roll  Control Demand DP_tvc
Pitch Control Demand DQ_tvc
Yaw   Control Demand DR_tvc
-----------------------------------------------------------------------------------------------------------------------------
```

One Loop Opened, The Other Two Closed

Used for Roll Stability Analysis

```
INTERCONNECTION OF SYSTEMS .....
Roll Loop Opened, Others Closed
! Roll Loop Opened for Roll Stability Analysis.
! The other two loops are closed
!
Titles of Systems to be Combined
Title 1 Open-Loop System
SYSTEM INPUTS TO SUBSYSTEM  1                                                To Open-Loop System
System Input  1 to Subsystem  1, Input  1, Gain= 1.0                         Roll Input
.............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                             from Open-Loop System
System Output  1 from Subsystem  1, Output  1, Gain= -1.0                    DP_tvc
.............................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  1                                      Vehi to FCS
Subsystem  1, Output  2 to Subsystem  1, Input  2, Gain= 1.0                 Pitch Closed
Subsystem  1, Output  3 to Subsystem  1, Input  3, Gain= 1.0                 Yaw Closed
.............................................................................
Definitions of Inputs  =   1
Roll Control Demand DP_tvc

Definitions of Outputs =   1
Roll Control Demand DP_tvc
-----------------------------------------------------------------------------------------------------------------------------
```

Used for Pitch Stability Analysis

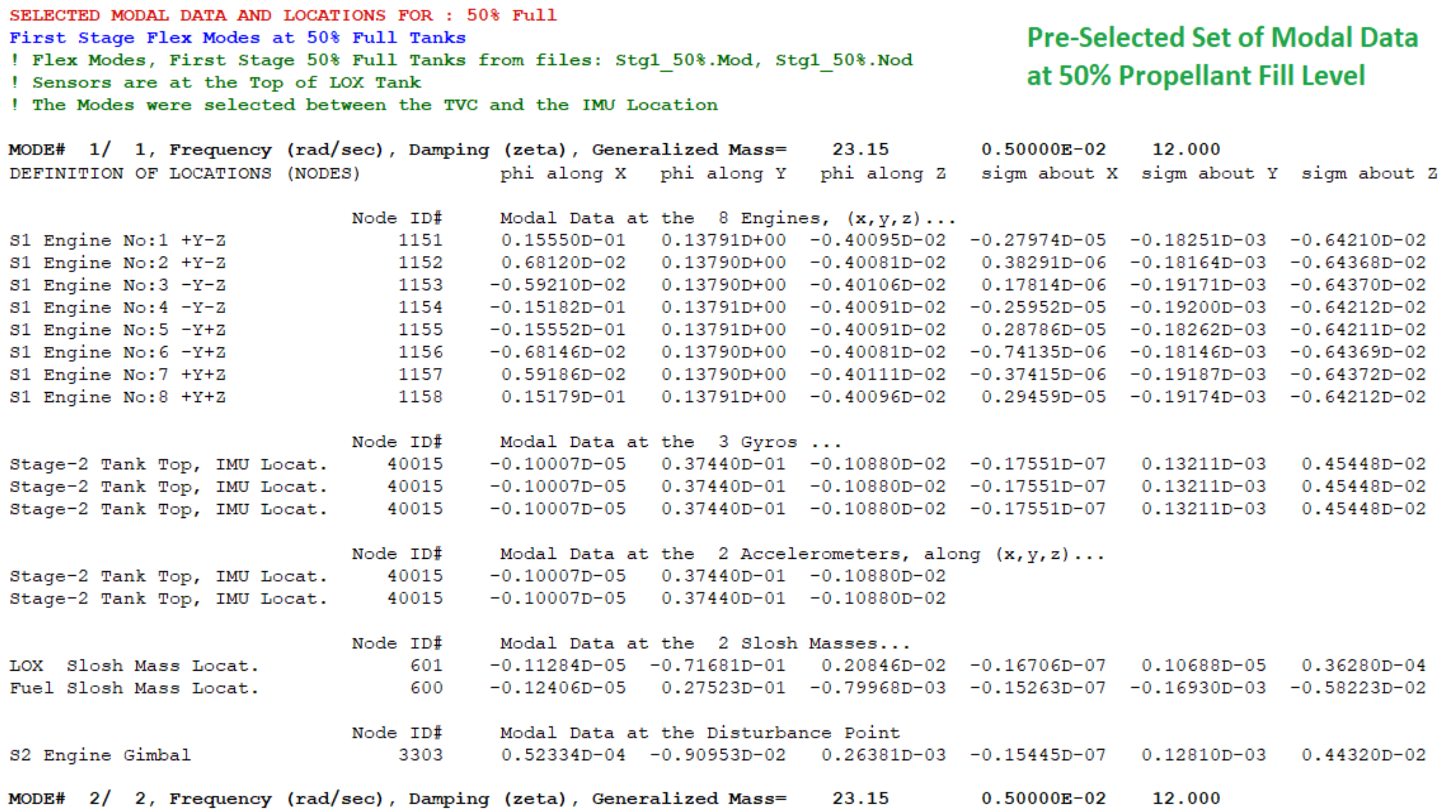
```
INTERCONNECTION OF SYSTEMS .....
Pitch Loop Opened, Others Closed
! Pitch Loop Opened for Pitch Stability Analysis.
! The other two loops are closed
!
Titles of Systems to be Combined
Title 1 Open-Loop System
SYSTEM INPUTS TO SUBSYSTEM  1                                                To Open-Loop System
System Input  1 to Subsystem  1, Input  2, Gain= 1.0                         Pitch Input
.............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                             from Open-Loop System
System Output  1 from Subsystem  1, Output  2, Gain= -1.0                    DQ_tvc
.............................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  1                                      Vehi to FCS
Subsystem  1, Output  1 to Subsystem  1, Input  1, Gain= 1.0                 Roll Closed
Subsystem  1, Output  3 to Subsystem  1, Input  3, Gain= 1.0                 Yaw  Closed
.............................................................................
Definitions of Inputs  =   1
Pitch Control Demand DQ_tvc

Definitions of Outputs =   1
Pitch Control Demand DQ_tvc
-----------------------------------------------------------------------------------------------------------------------------
```

Used for Yaw Stability Analysis

```
INTERCONNECTION OF SYSTEMS .....
Yaw Loop Opened, Others Closed
! Yaw Loop Opened for Yaw Stability Analysis.
! The other two loops are closed
!
Titles of Systems to be Combined
Title 1 Open-Loop System
SYSTEM INPUTS TO SUBSYSTEM  1                                              To Open-Loop System
System Input  1 to Subsystem  1, Input  3, Gain= 1.0                       Yaw Input
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                           from Open-Loop System
System Output  1 from Subsystem  1, Output  3, Gain= -1.0                  DR_tvc
..............................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  1                                    Vehi to FCS
Subsystem  1, Output  1 to Subsystem  1, Input  1, Gain= 1.0               Roll  Closed
Subsystem  1, Output  2 to Subsystem  1, Input  2, Gain= 1.0               Pitch Closed
..............................................................................
Definitions of Inputs  =   1
Yaw Control Demand DR_tvc

Definitions of Outputs =   1
Yaw Control Demand DR_tvc
-----------------------------------------------------------------------------------------------
                            CONVERT TO MATLAB DATA
-----------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........     (Title, System/Matrix, m-filename)
Mixing Logic for First Stage Model, at T=80.0 sec
Matrix TVC
-----------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........     (Title, System/Matrix, m-filename)
Launch Vehicle First Stage Analysis Model, T=80.0 sec
System
flex_vehicle.m
-----------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........     (Title, System/Matrix, m-filename)
Stage-1 Linear Actuator
System
actuator.m
-----------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........     (Title, System/Matrix, m-filename)
Flight Control System
System
fcs.m
-----------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........     (Title, System/Matrix, m-filename)
Plant Model at T=80 sec, Vehicle/ Actuators
System
plant.m
-----------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........     (Title, System/Matrix, m-filename)
Closed-Loop System
System
closed_loop.m
-----------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........     (Title, System/Matrix, m-filename)
Open-Loop System
System
open_loop.m
-----------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........     (Title, System/Matrix, m-filename)
Roll Loop Opened, Others Closed
System
roll_open.m
-----------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........     (Title, System/Matrix, m-filename)
Pitch Loop Opened, Others Closed
System
pitch_open.m
-----------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........     (Title, System/Matrix, m-filename)
Yaw Loop Opened, Others Closed
System
yaw_open.m
-----------------------------------------------------------------------------------------------
```



Pre-Selected Set of Modal Data at 50% Propellant Fill Level

```
SELECTED MODAL DATA AND LOCATIONS FOR : 50% Full
First Stage Flex Modes at 50% Full Tanks
! Flex Modes, First Stage 50% Full Tanks from files: Stg1_50%.Mod, Stg1_50%.Nod
! Sensors are at the Top of LOX Tank
! The Modes were selected between the TVC and the IMU Location

MODE#  1/  1, Frequency (rad/sec), Damping (zeta), Generalized Mass=    23.15        0.50000E-02    12.000
DEFINITION OF LOCATIONS (NODES)                 phi along X   phi along Y   phi along Z   sigm about X  sigm about Y  sigm about Z

                             Node ID#     Modal Data at the  8 Engines, (x,y,z)...
S1 Engine No:1 +Y-Z              1151     0.15550D-01   0.13791D+00  -0.40095D-02  -0.27974D-05  -0.18251D-03  -0.64210D-02
S1 Engine No:2 +Y-Z              1152     0.68120D-02   0.13790D+00  -0.40081D-02   0.38291D-06  -0.18164D-03  -0.64368D-02
S1 Engine No:3 -Y-Z              1153    -0.59210D-02   0.13790D+00  -0.40106D-02   0.17814D-06  -0.19171D-03  -0.64370D-02
S1 Engine No:4 -Y-Z              1154    -0.15182D-01   0.13791D+00  -0.40091D-02  -0.25952D-05  -0.19200D-03  -0.64212D-02
S1 Engine No:5 -Y+Z              1155    -0.15552D-01   0.13791D+00  -0.40091D-02   0.28786D-05  -0.18262D-03  -0.64211D-02
S1 Engine No:6 -Y+Z              1156    -0.68146D-02   0.13790D+00  -0.40081D-02  -0.74135D-06  -0.18146D-03  -0.64369D-02
S1 Engine No:7 +Y+Z              1157     0.59186D-02   0.13790D+00  -0.40111D-02  -0.37415D-06  -0.19187D-03  -0.64372D-02
S1 Engine No:8 +Y+Z              1158     0.15179D-01   0.13791D+00  -0.40096D-02   0.29459D-05  -0.19174D-03  -0.64212D-02

                             Node ID#     Modal Data at the  3 Gyros ...
Stage-2 Tank Top, IMU Locat.    40015    -0.10007D-05   0.37440D-01  -0.10880D-02  -0.17551D-07   0.13211D-03   0.45448D-02
Stage-2 Tank Top, IMU Locat.    40015    -0.10007D-05   0.37440D-01  -0.10880D-02  -0.17551D-07   0.13211D-03   0.45448D-02
Stage-2 Tank Top, IMU Locat.    40015    -0.10007D-05   0.37440D-01  -0.10880D-02  -0.17551D-07   0.13211D-03   0.45448D-02

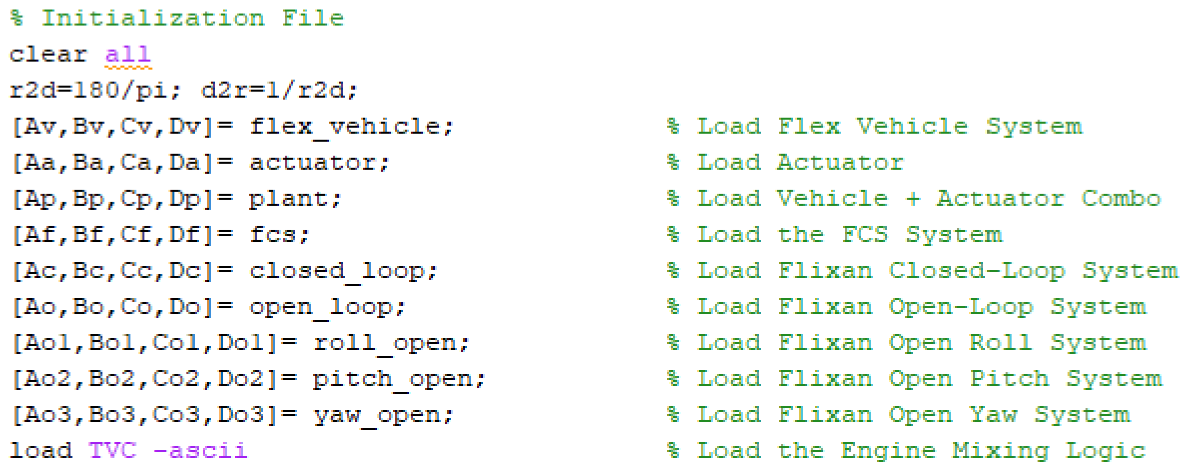
                             Node ID#     Modal Data at the  2 Accelerometers, along (x,y,z)...
Stage-2 Tank Top, IMU Locat.    40015    -0.10007D-05   0.37440D-01  -0.10880D-02
Stage-2 Tank Top, IMU Locat.    40015    -0.10007D-05   0.37440D-01  -0.10880D-02

                             Node ID#     Modal Data at the  2 Slosh Masses...
LOX  Slosh Mass Locat.            601    -0.11284D-05  -0.71681D-01   0.20846D-02  -0.16706D-07   0.10688D-05   0.36280D-04
Fuel Slosh Mass Locat.            600    -0.12406D-05   0.27523D-01  -0.79968D-03  -0.15263D-07  -0.16930D-03  -0.58223D-02

                             Node ID#     Modal Data at the Disturbance Point
S2 Engine Gimbal                 3303     0.52334D-04  -0.90953D-02   0.26381D-03  -0.15445D-07   0.12810D-03   0.44320D-02

MODE#  2/  2, Frequency (rad/sec), Damping (zeta), Generalized Mass=    23.15        0.50000E-02    12.000
```

The following initialization file “init.m” loads the systems and TVC matrix to Matlab for the simulations. The flight control load-relief system includes feedback from (a, b)-integral. Slosh filters are included in the (a, b)-estimators to improve the margin of the LOX mode.



```
% Initialization File
clear all
r2d=180/pi; d2r=1/r2d;
[Av,Bv,Cv,Dv]= flex_vehicle;             % Load Flex Vehicle System
[Aa,Ba,Ca,Da]= actuator;                 % Load Actuator
[Ap,Bp,Cp,Dp]= plant;                    % Load Vehicle + Actuator Combo
[Af,Bf,Cf,Df]= fcs;                      % Load the FCS System
[Ac,Bc,Cc,Dc]= closed_loop;              % Load Flixan Closed-Loop System
[Ao,Bo,Co,Do]= open_loop;                % Load Flixan Open-Loop System
[Ao1,Bo1,Co1,Do1]= roll_open;            % Load Flixan Open Roll System
[Ao2,Bo2,Co2,Do2]= pitch_open;           % Load Flixan Open Pitch System
[Ao3,Bo3,Co3,Do3]= yaw_open;             % Load Flixan Open Yaw System
load TVC -ascii                          % Load the Engine Mixing Logic
```

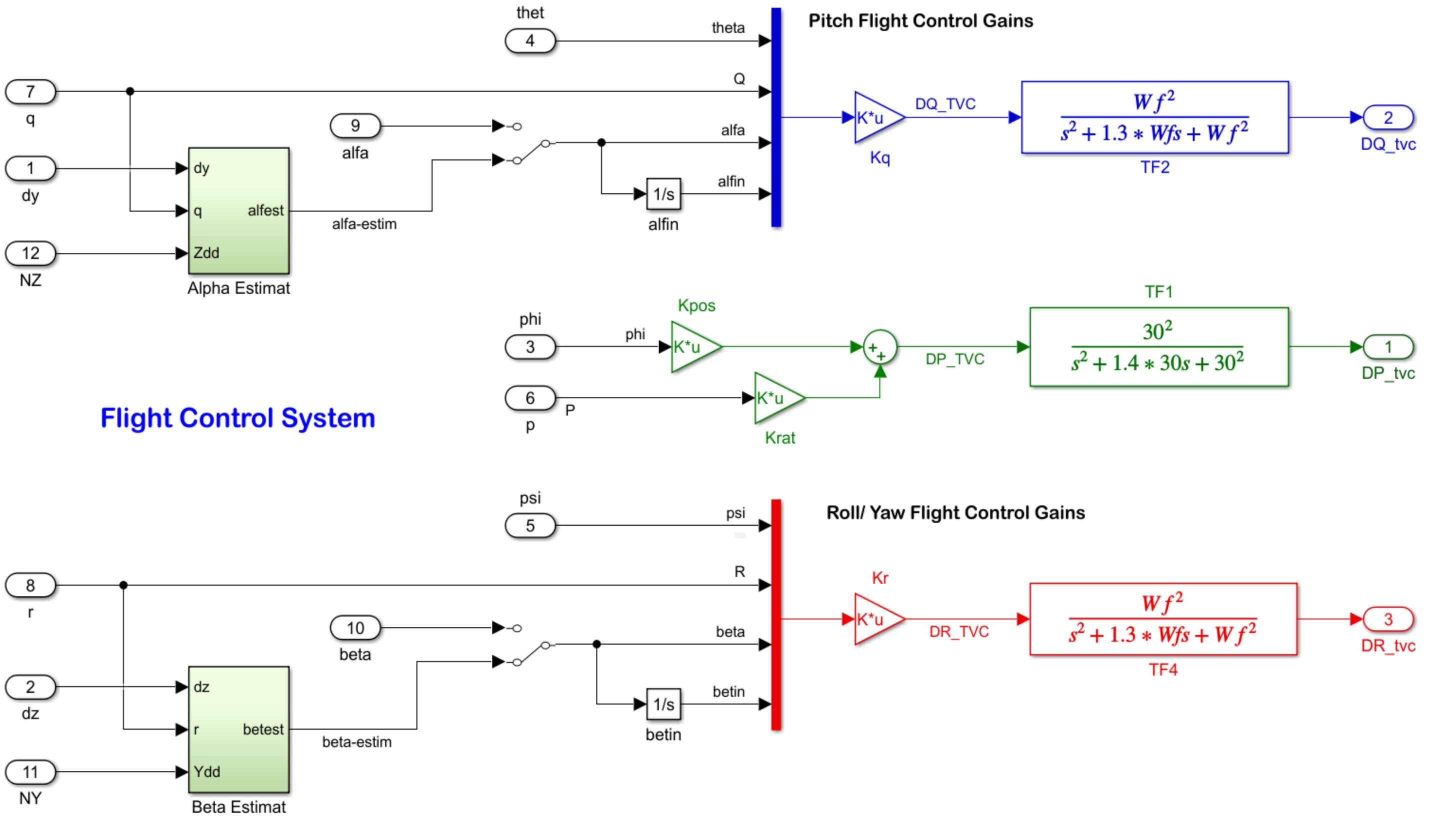


Figure 2.2.34 Flight Control System During High Q-bar. The State-Feedback is from θ, q, a, and a-Integral States

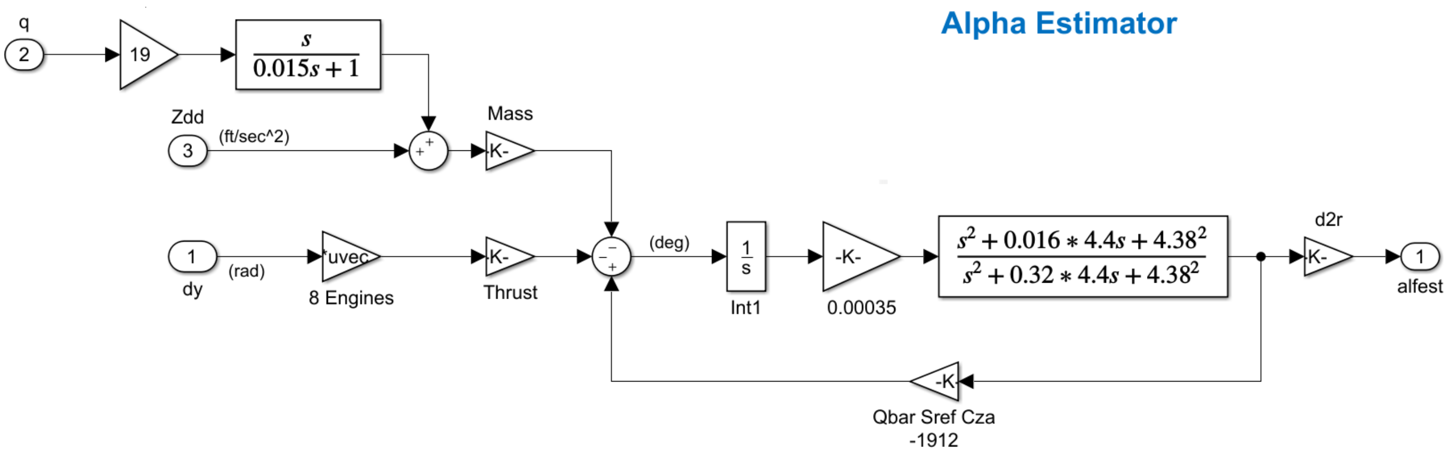


Figure 2.2.35 The Alpha and Beta Estimators Include a Notch Filter to Improve the LOX Slosh Margin which is Phase Unstable

Figures (2.2.36-38) show the pitch, yaw and roll stability analysis results at max-Q. They are created using the Flixan generated open-loop model by opening one loop at a time. The Flixan frequency response analysis program was used to calculate the Nichols, Bode and Nyquist diagrams. For good margins in the Nichols plots, the G(jω) locus must avoid intersecting the 40 (deg), 8 (dB) diamond shaped area. Sensitivity to wind-gusts is also satisfied, se Figure 2.2.39, assuming that α and β do not exceed 3°.

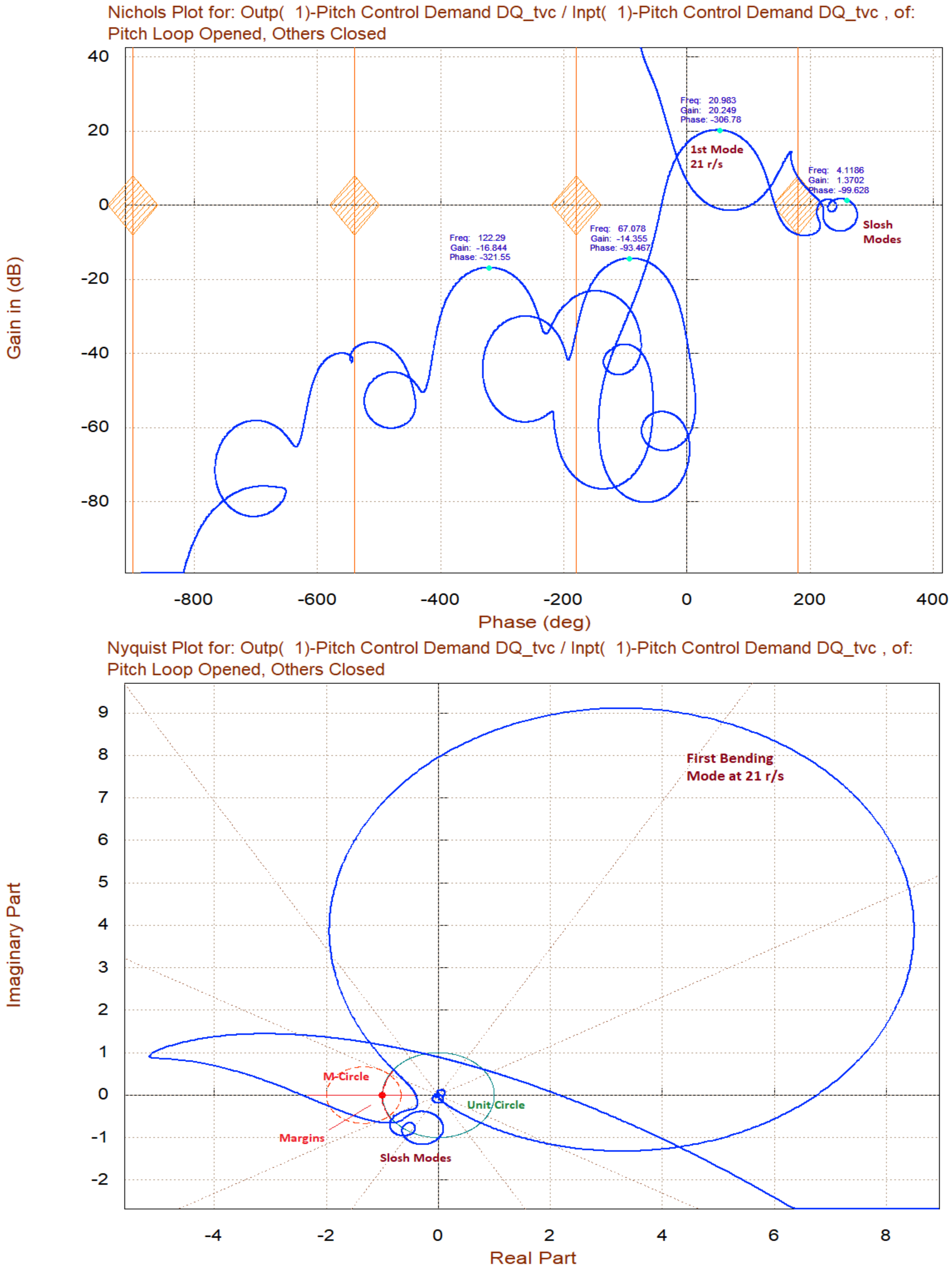


Figure 2.2.36 Pitch Stability Analysis Using Nichols and Nyquist Plots

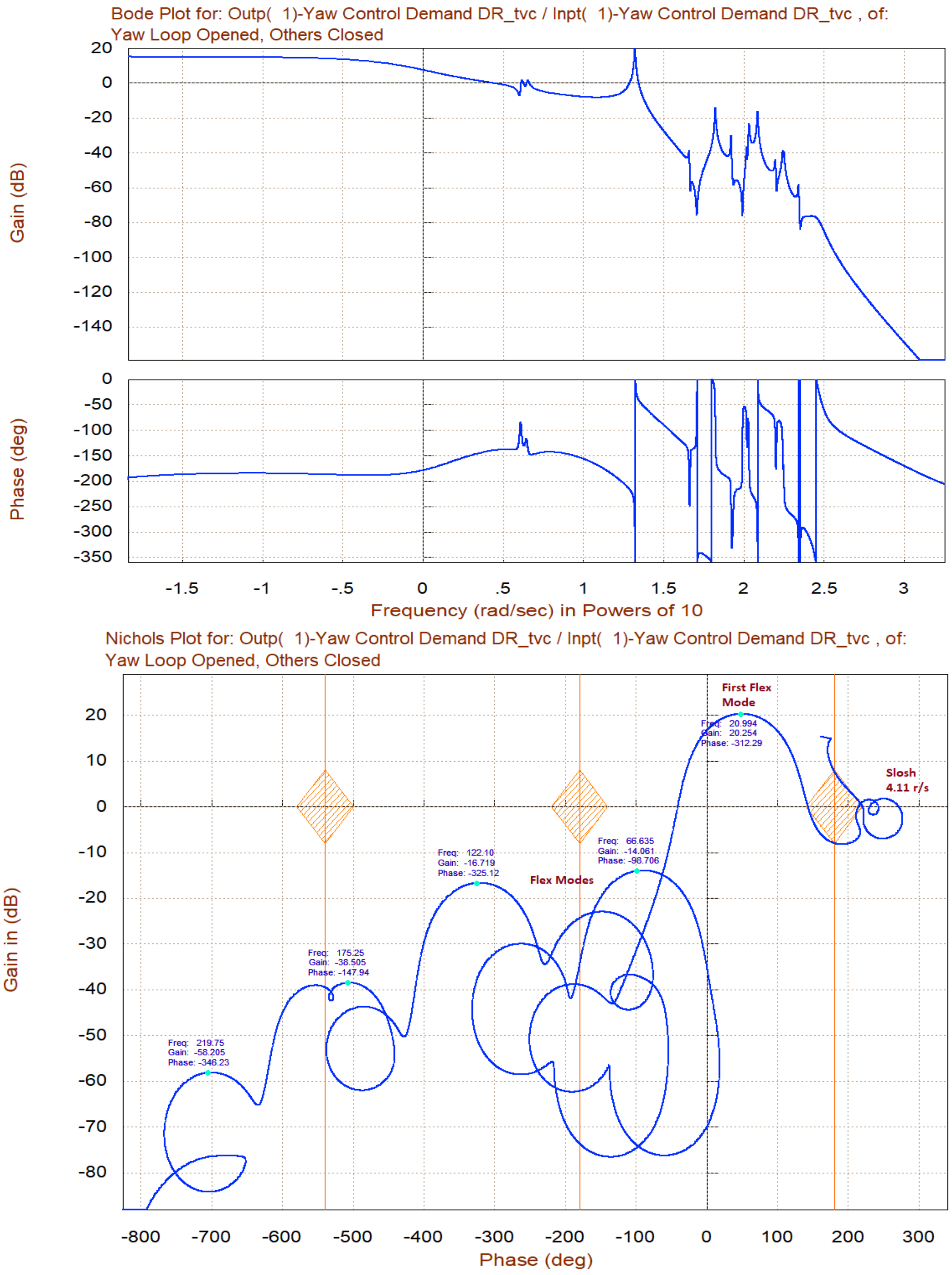


Figure 2.2.37 Yaw Stability Analysis Using Bode and Nichols Plots

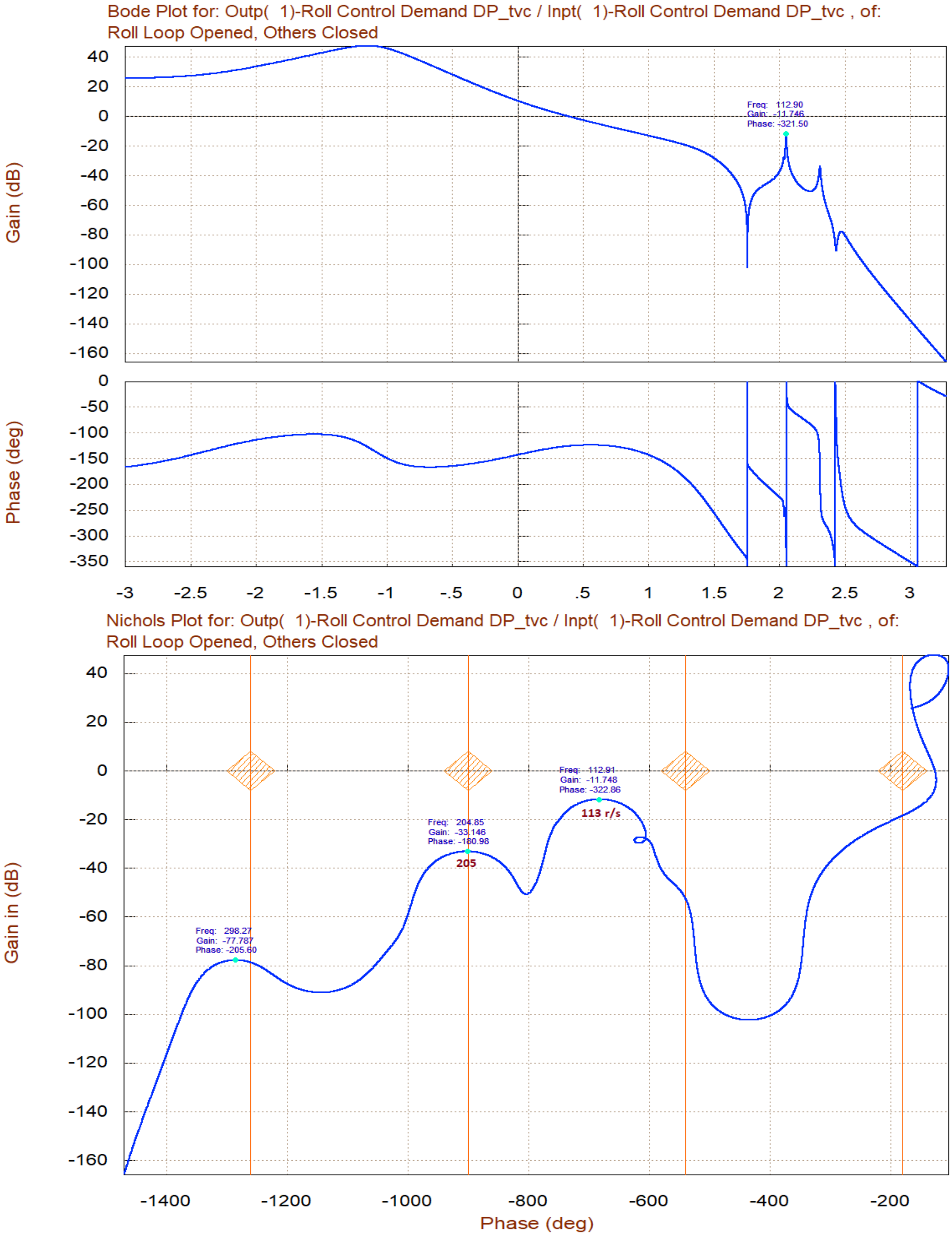
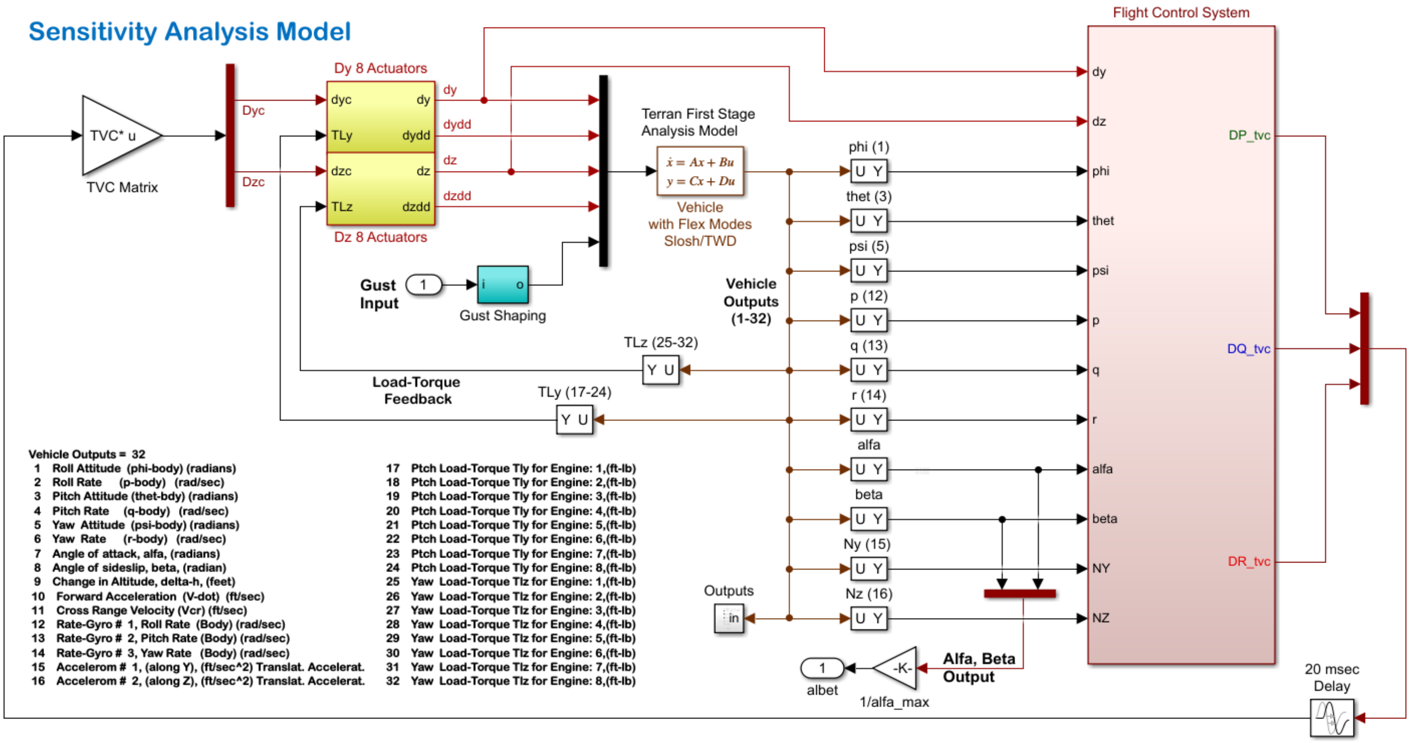
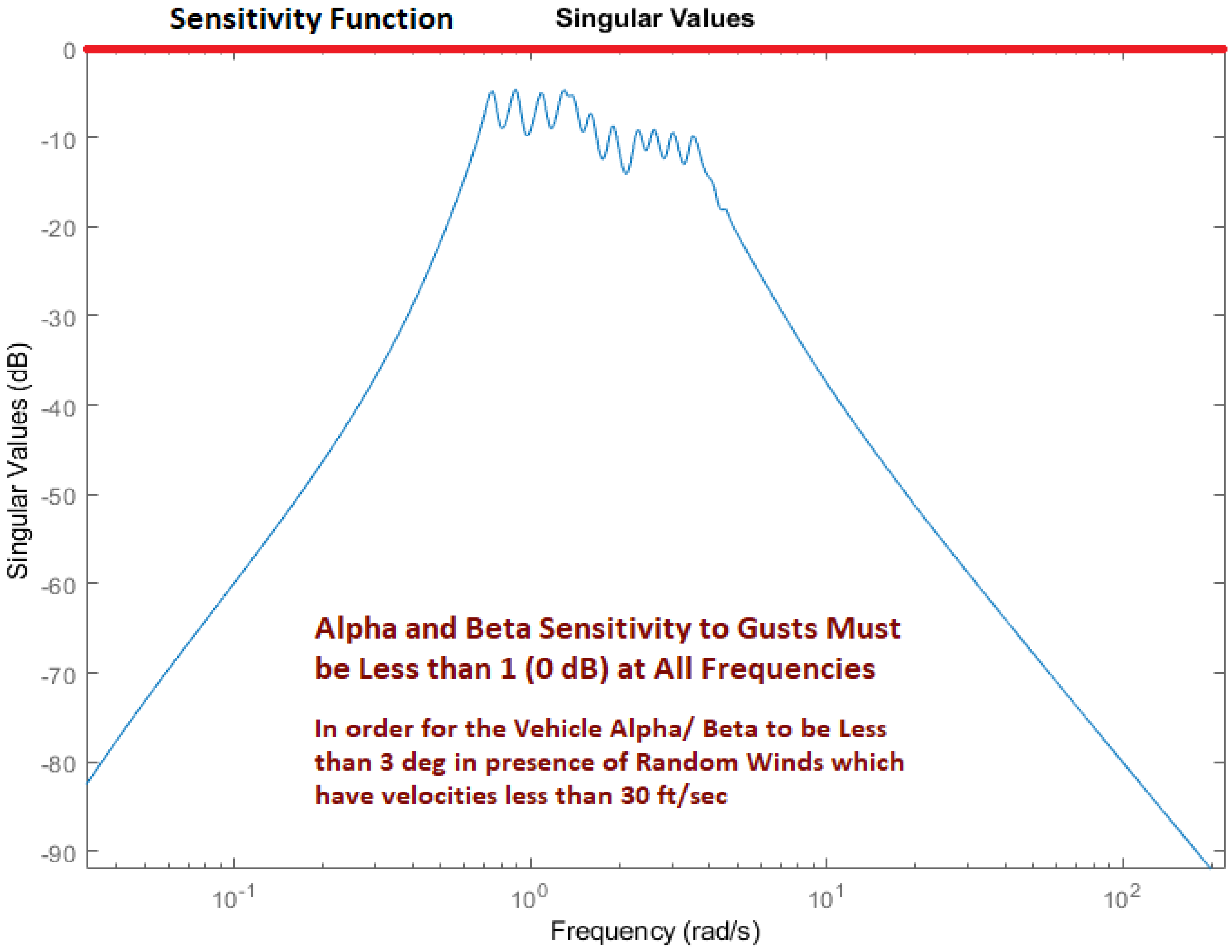


Figure 2.2.38 Roll Stability Analysis Using Bode and Nichols Plots



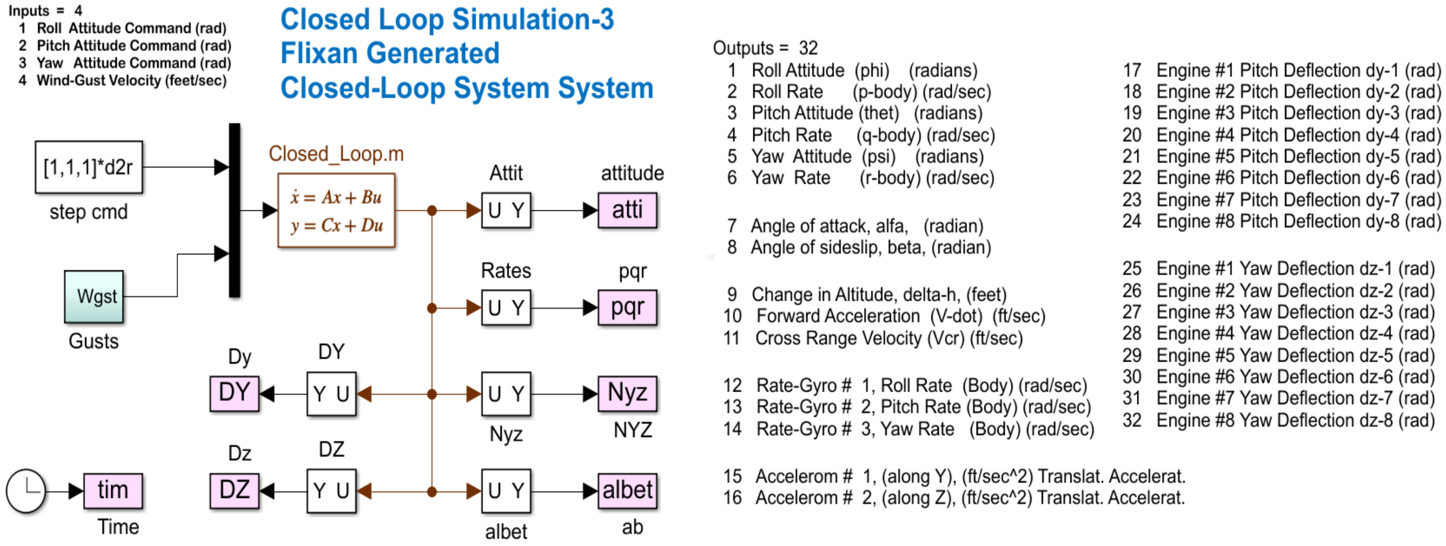


Alpha and Beta Sensitivity to Gusts Must be Less than 1 (0 dB) at All Frequencies

In order for the Vehicle Alpha/ Beta to be Less than 3 deg in presence of Random Winds which have velocities less than 30 ft/sec

Figure 2.2.39 Sensitivity to Wind-Gusts Assuming that alpha and beta dispersions are less than 3 (deg)

Two simulation models are created in Simulink that use the Flixan generated systems. The simulations are performed using Simulink and the results are identical. The model "*Sim_Flex3.slx*" in Figure 2.2.40 uses the already combined "Closed-Loop System" from Flixan which is loaded from file "closed_loop". The model "*Sim_Flex2.slx*" in Figure 2.2.41 combines the "*Flight Control System*" and "*Plant Model at T=80 sec, Vehicle/ Actuators*" systems from files "fcs" and "plant" respectively in Simulink. The inputs to the simulation models are either attitude commands or wind-gust velocity disturbances.



Inputs = 4
1 Roll Attitude Command (rad)
2 Pitch Attitude Command (rad)
3 Yaw Attitude Command (rad)
4 Wind-Gust Velocity (feet/sec)

Outputs = 32
1 Roll Attitude (phi) (radians)
2 Roll Rate (p-body) (rad/sec)
3 Pitch Attitude (thet) (radians)
4 Pitch Rate (q-body) (rad/sec)
5 Yaw Attitude (psi) (radians)
6 Yaw Rate (r-body) (rad/sec)

7 Angle of attack, alfa, (radian)
8 Angle of sideslip, beta, (radian)

9 Change in Altitude, delta-h, (feet)
10 Forward Acceleration (V-dot) (ft/sec)
11 Cross Range Velocity (Vcr) (ft/sec)

12 Rate-Gyro # 1, Roll Rate (Body) (rad/sec)
13 Rate-Gyro # 2, Pitch Rate (Body) (rad/sec)
14 Rate-Gyro # 3, Yaw Rate (Body) (rad/sec)

15 Accelerom # 1, (along Y), (ft/sec^2) Translat. Accelerat.
16 Accelerom # 2, (along Z), (ft/sec^2) Translat. Accelerat.

17 Engine #1 Pitch Deflection dy-1 (rad)
18 Engine #2 Pitch Deflection dy-2 (rad)
19 Engine #3 Pitch Deflection dy-3 (rad)
20 Engine #4 Pitch Deflection dy-4 (rad)
21 Engine #5 Pitch Deflection dy-5 (rad)
22 Engine #6 Pitch Deflection dy-6 (rad)
23 Engine #7 Pitch Deflection dy-7 (rad)
24 Engine #8 Pitch Deflection dy-8 (rad)

25 Engine #1 Yaw Deflection dz-1 (rad)
26 Engine #2 Yaw Deflection dz-2 (rad)
27 Engine #3 Yaw Deflection dz-3 (rad)
28 Engine #4 Yaw Deflection dz-4 (rad)
29 Engine #5 Yaw Deflection dz-5 (rad)
30 Engine #6 Yaw Deflection dz-6 (rad)
31 Engine #7 Yaw Deflection dz-7 (rad)
32 Engine #8 Yaw Deflection dz-8 (rad)

Figure 2.2.40 Closed-Loop Simulation Model "Sim_Flex3.slx"

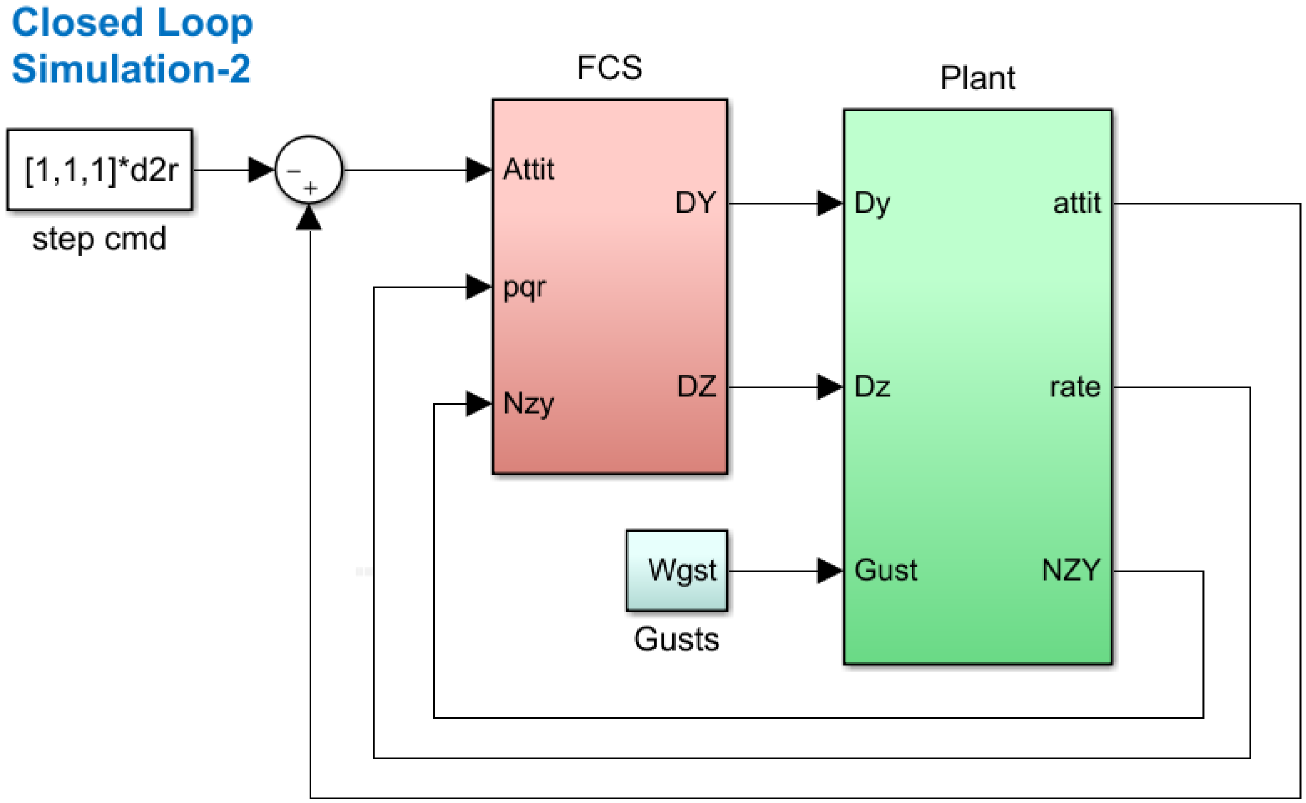


Figure 2.2.41 Closed-Loop Simulation Model "Sim_Flex2.slx"

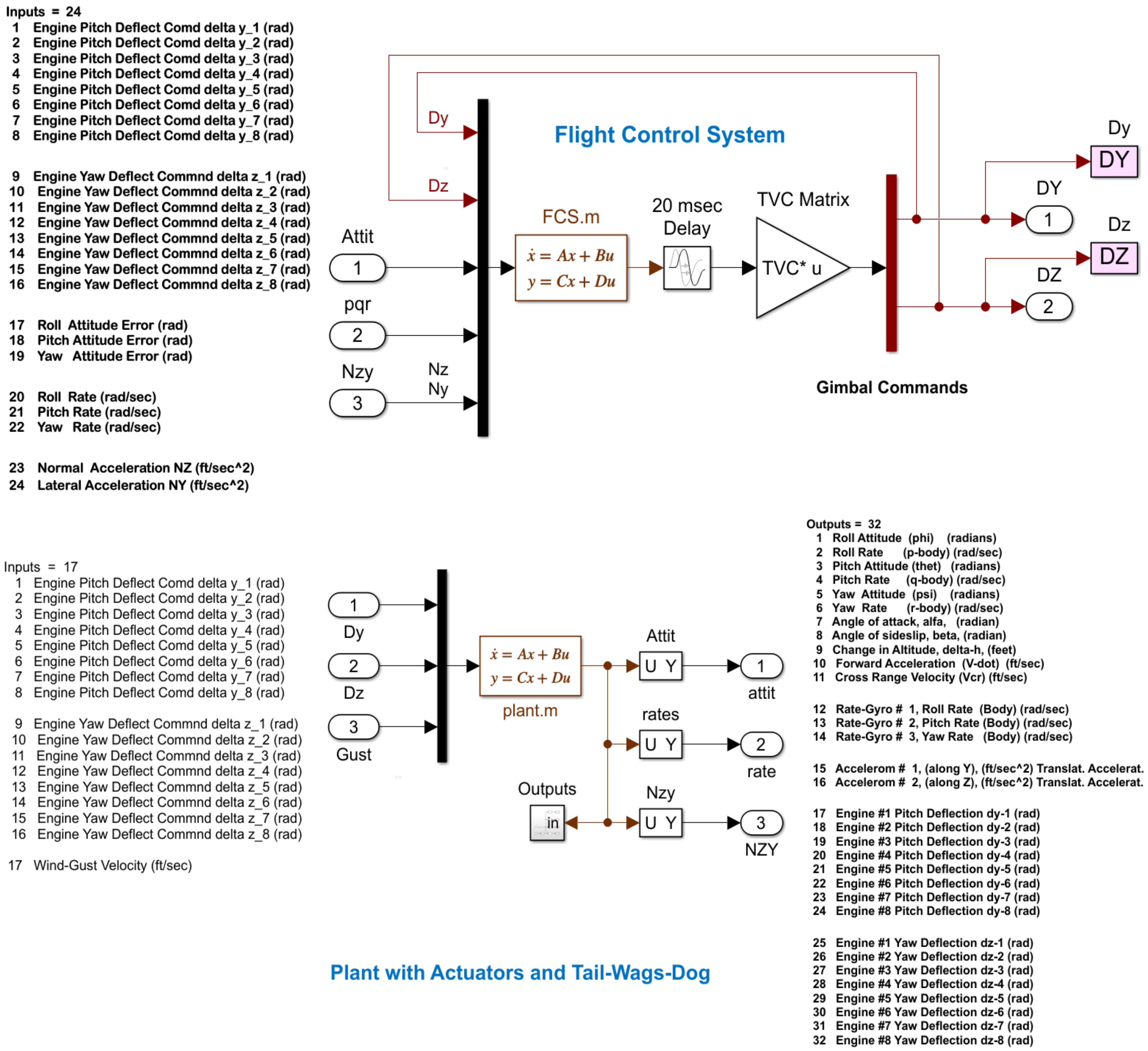
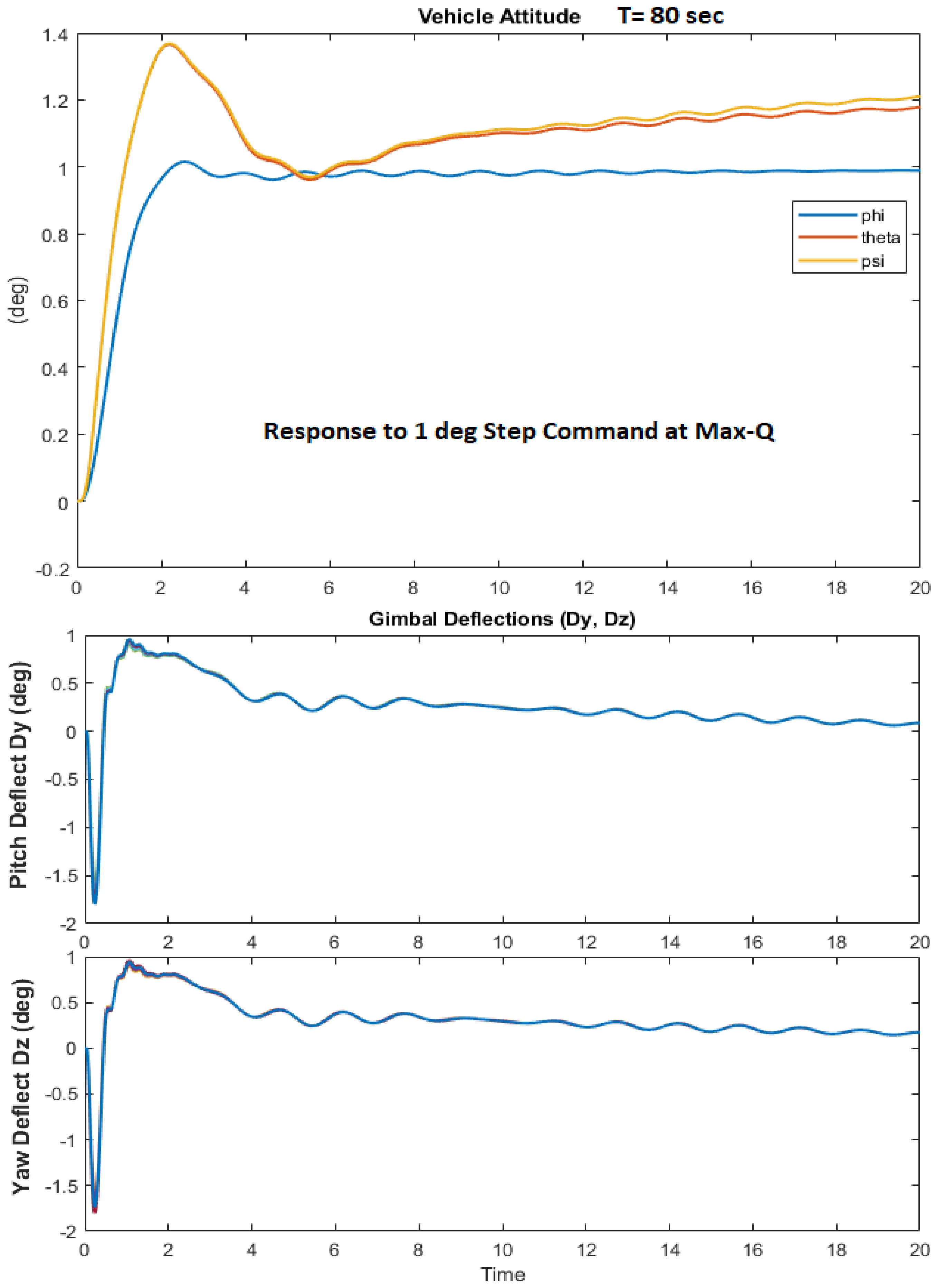


Figure 2.2.42 Flight Control and Plant Subsystems

Figure 2.2.42 shows the system's response to simultaneously applied roll, pitch and yaw unit step commands. At Max-Q the response to commands is not expected to be so great because it is counteracted by the load-relief, but it's not bad either.

Figure 2.2.43 shows the system's response to random gusts which have wind velocity peaks less than 30 (feet/sec) and are applied perpendicular to the vehicle, as defined by the two angles in the vehicle input data. It demonstrates the ability of the load-relief system to maintain reasonable the Q-alpha-beta loads less than 550 (psf-deg) since the (α, β) dispersions are about 1 (deg) and the gimbal deflections are less than 0.8 (deg).


Vehicle Attitude T= 80 sec
(deg)
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
2
4
6
8
10
12
14
16
18
20
phi
theta
psi
Response to 1 deg Step Command at Max-Q
Gimbal Deflections (Dy, Dz)
Pitch Deflect Dy (deg)
1
0.5
0
-0.5
-1
-1.5
-2
Yaw Deflect Dz (deg)
Time

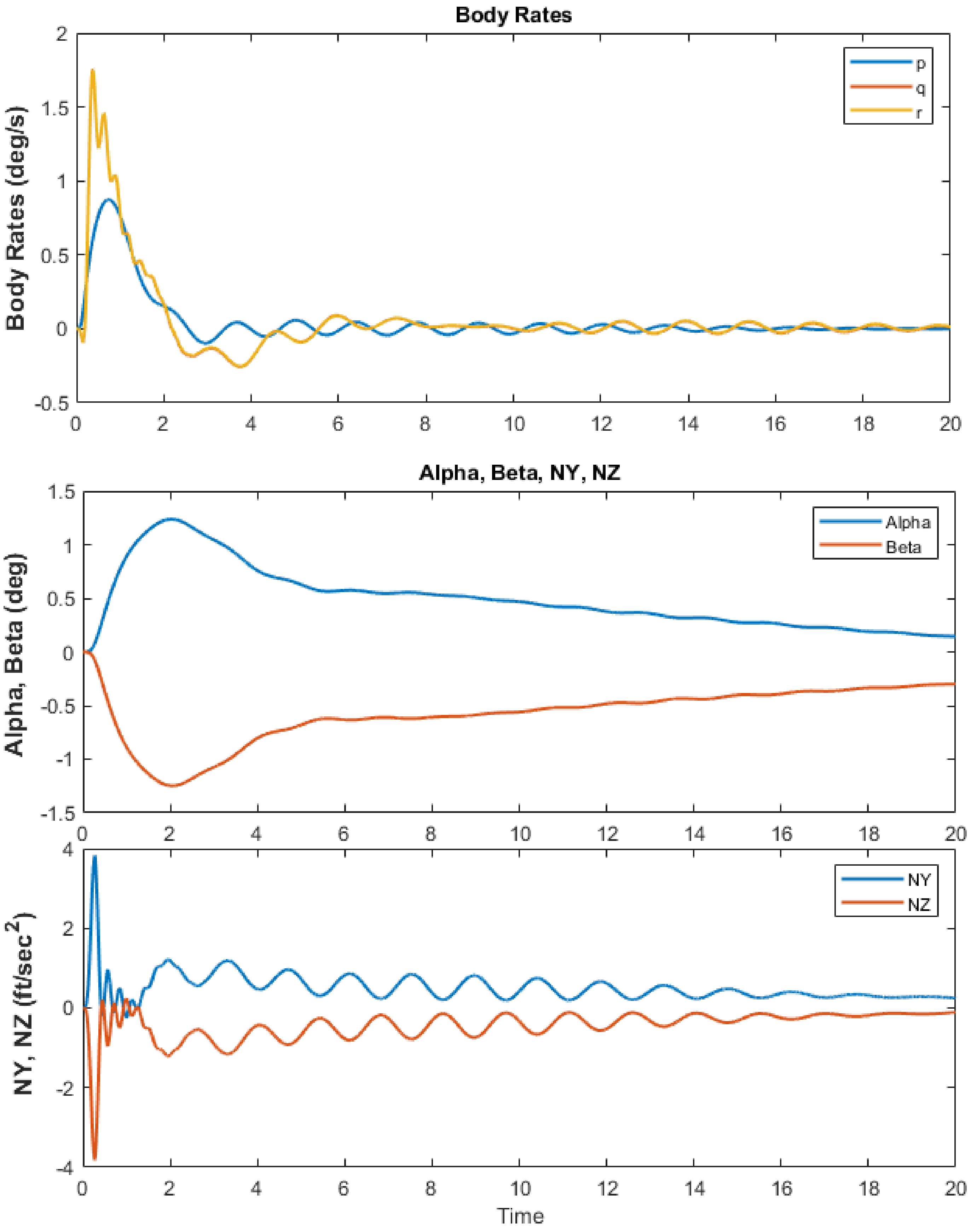
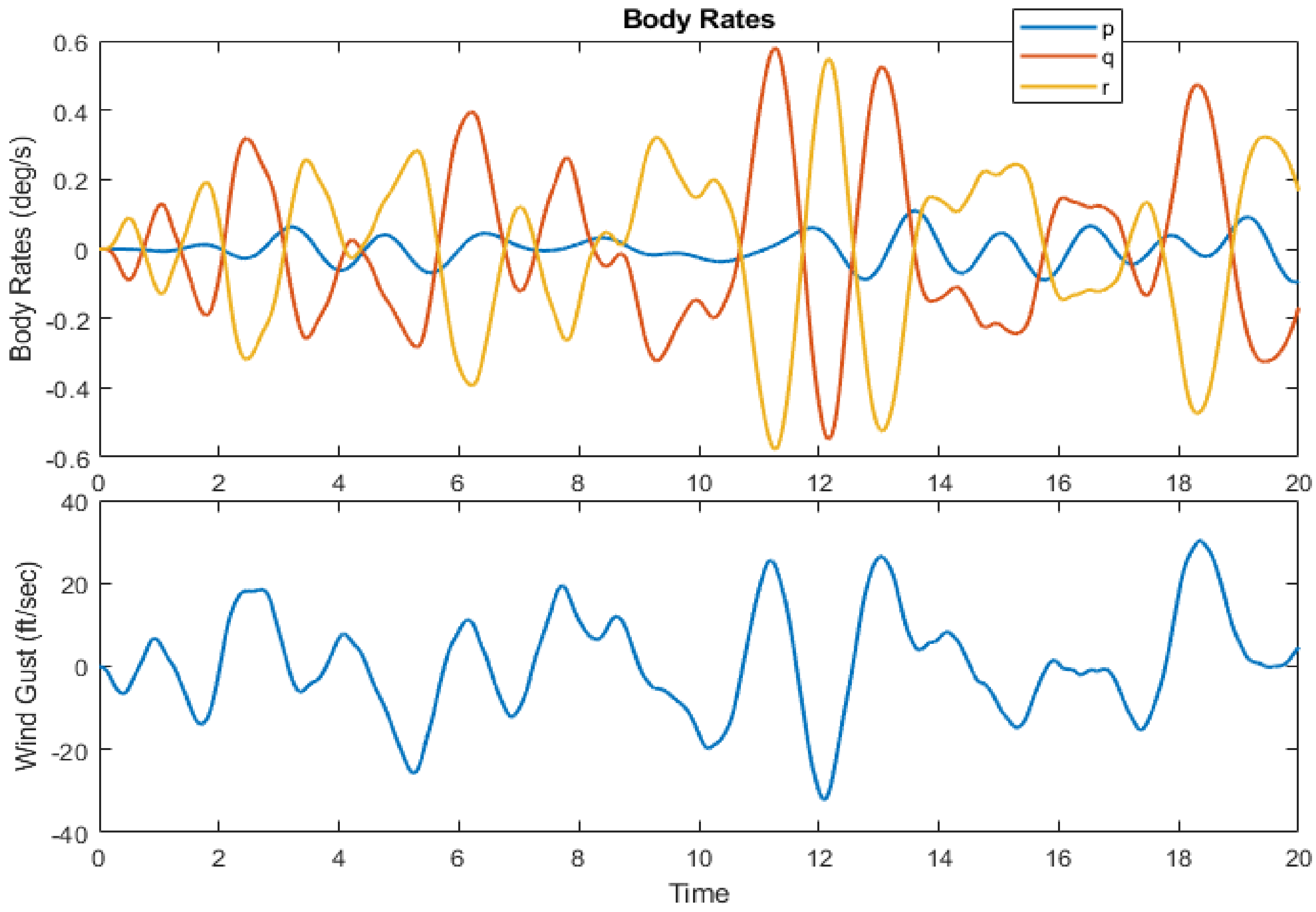
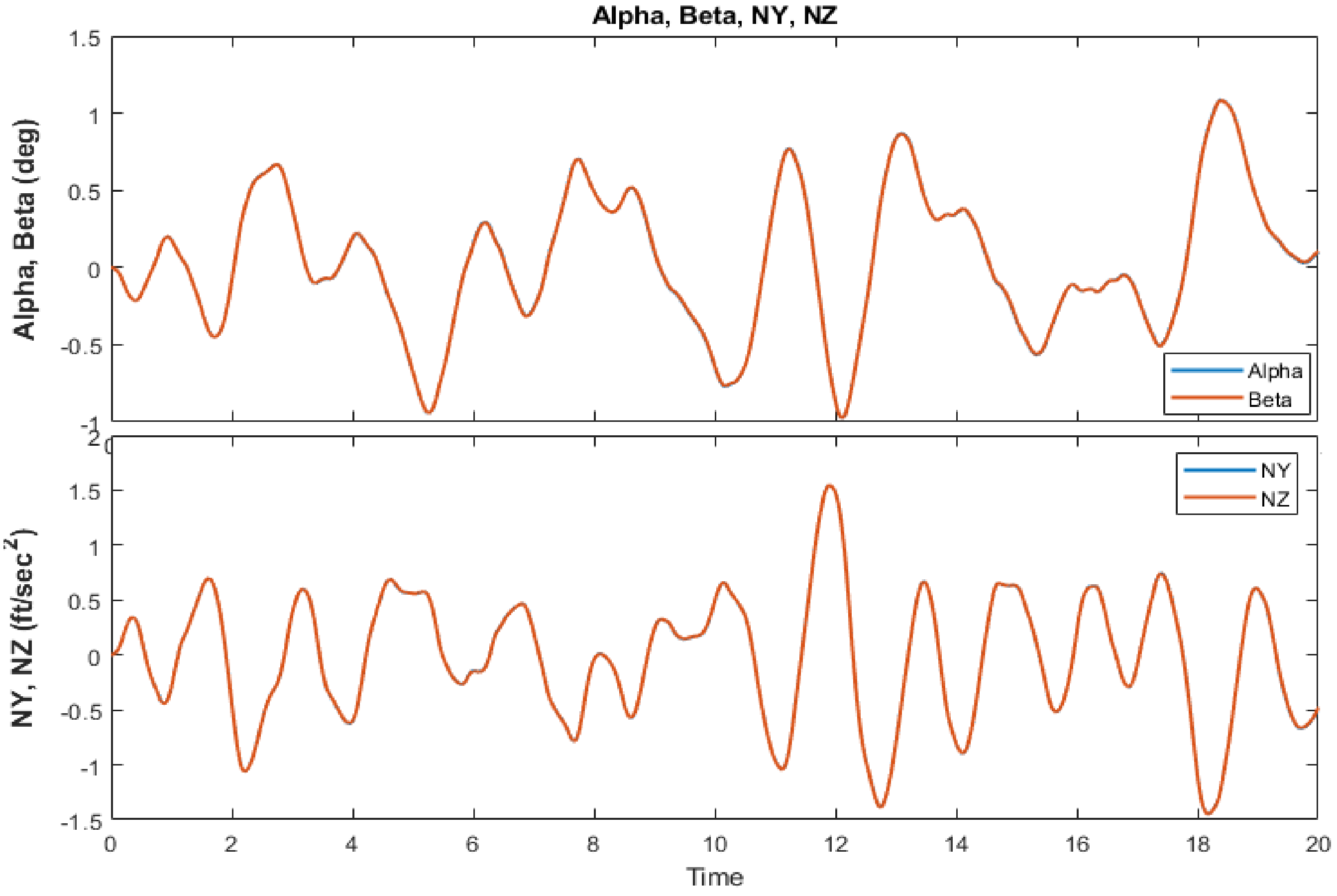


Figure 2.2.42 System Response to 1° Attitude Commands in Roll, Pitch and Yaw



Body Rates
p
q
r
Body Rates (deg/s)
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0
2
4
6
8
10
12
14
16
18
20
Wind Gust (ft/sec)
40
20
0
-20
-40
Time



Alpha, Beta, NY, NZ
Alpha, Beta (deg)
1.5
1
0.5
0
-0.5
-1
Alpha
Beta
NY, NZ (ft/sec^2)
2
1.5
1
0.5
0
-0.5
-1
-1.5
NY
NZ
0
2
4
6
8
10
12
14
16
18
20
Time

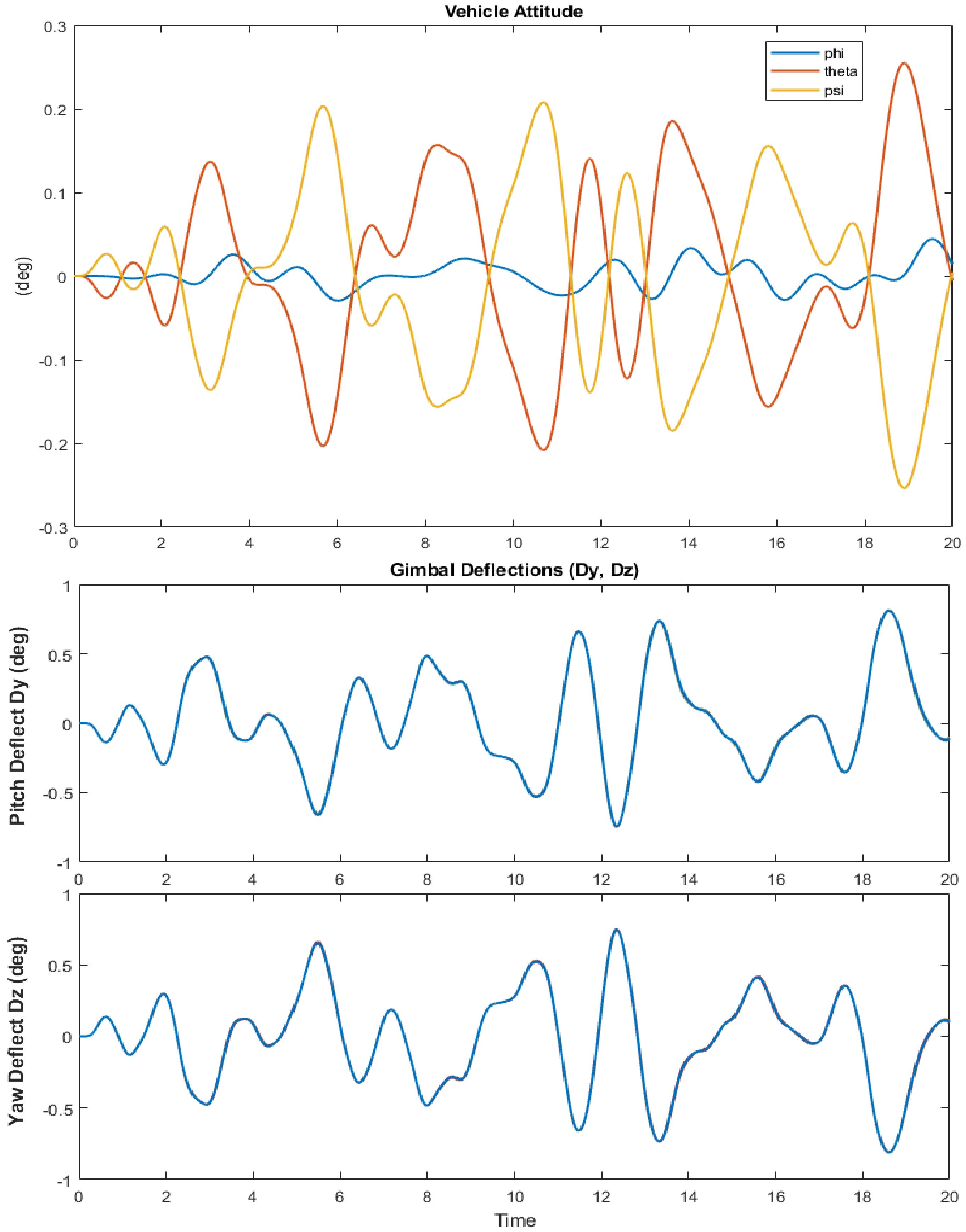
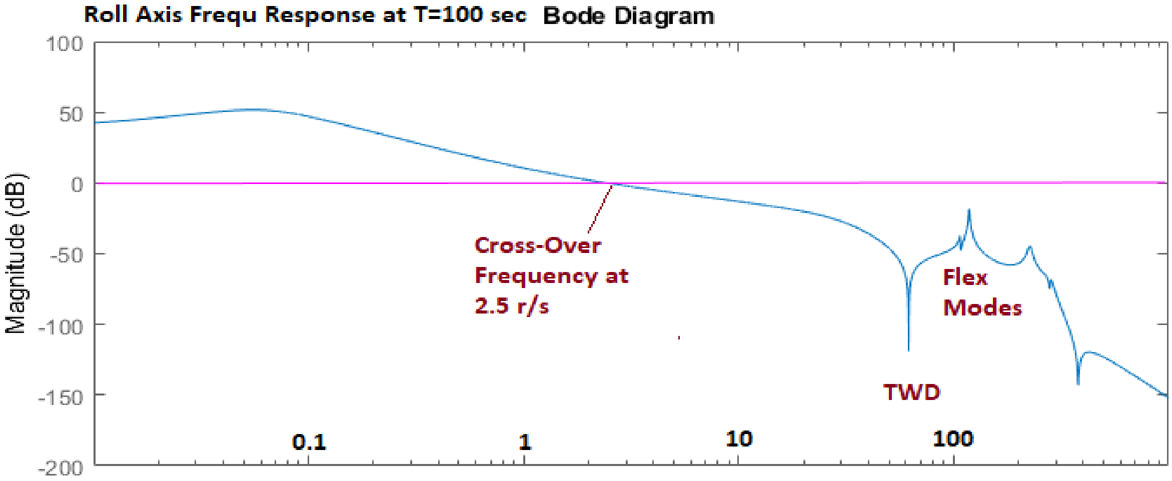
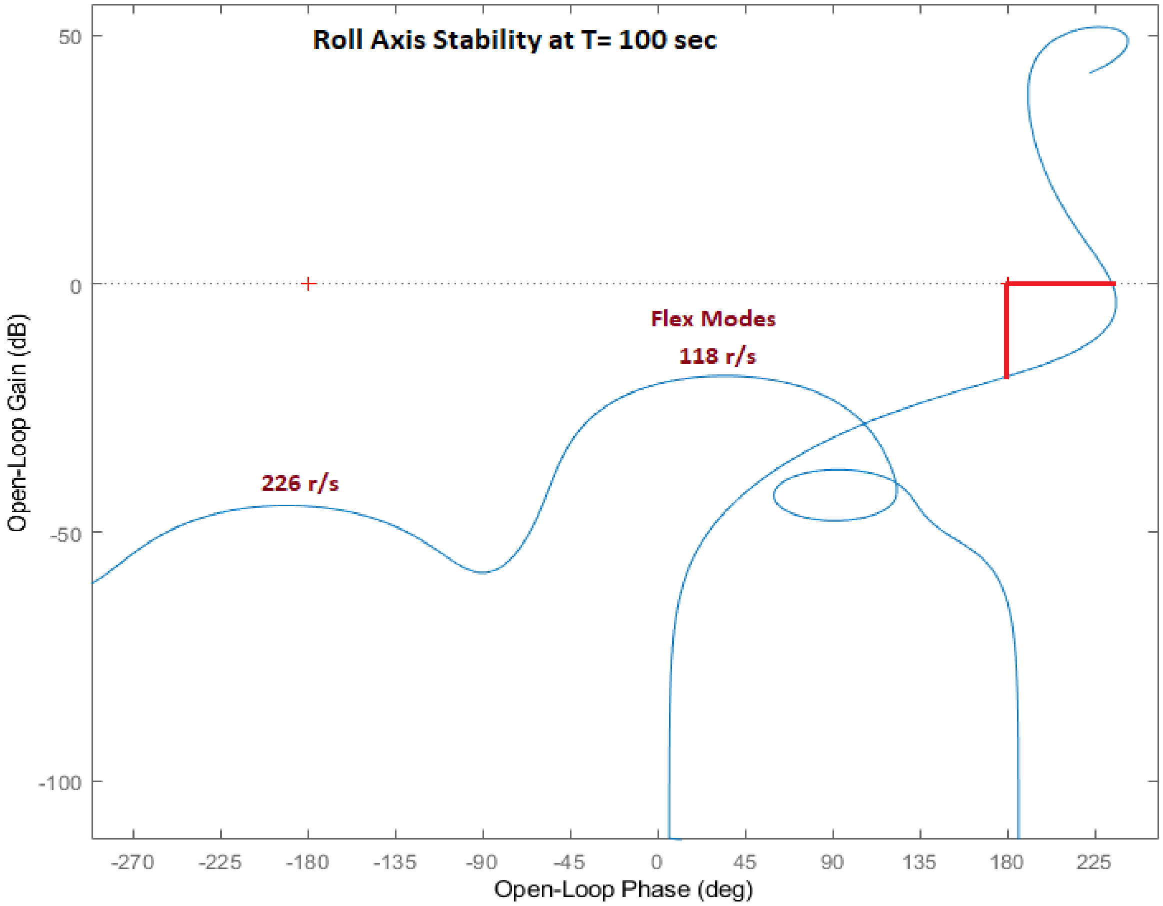


Figure 2.2.30 System Response to Random Wind-Gusts with Peaks Under 30 (ft/sec)

2.2.5 Control Analysis at T= 100 sec

At T=100 sec the dynamic pressure is 360 (lbf/ft^2) which is still high and the load-relief action from the estimated α and β, and from (α, β)-integrals is significant. Sloshing is also strong and both LOX and LH2 modes are now phase-stable. However, the slosh damping was set to ζ=0.015 with baffles, because the LOX mode was stabilized with a filter and its stability is not very reliable due to LOX frequency uncertainty. The analysis files are in directory "*3-Stability Analysis with Flex & Slosh\1st Stage\T100*". The following figures show the system stability in roll, pitch and yaw using Bode and Nichols plots. With the dynamic pressure reduced, the step responses to guidance commands are a little better than Max-Q, even though the load-relief is still active. Structural flexibility and sloshing are visible in the responses.





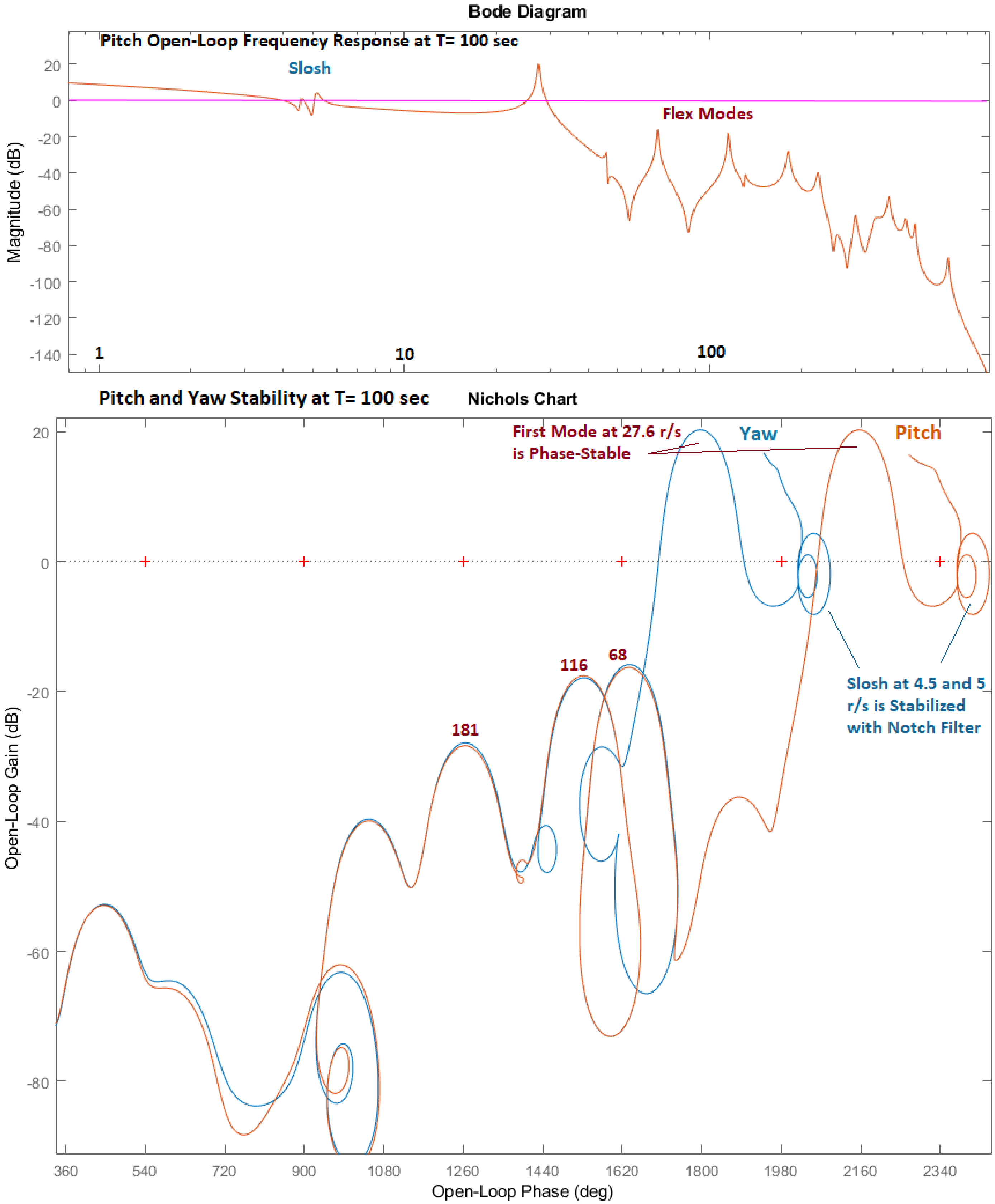
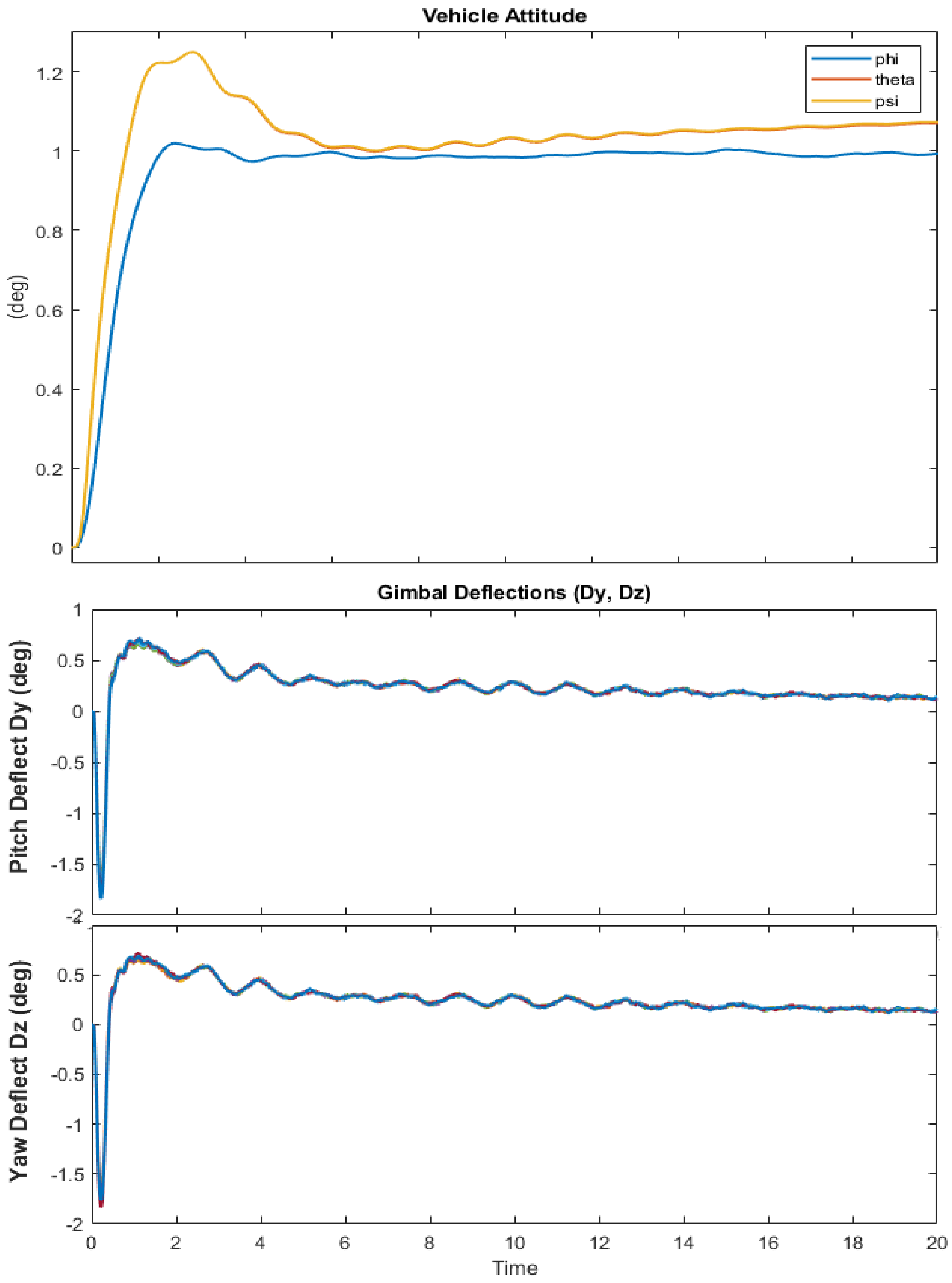


Figure 2.2.31 Roll, Pitch and Yaw Stability Analysis at T=100 sec. Slosh is now Phase-Stable. The First Bending Mode is also Phase-Stable. Its Frequency went up to 27.6 (rad/sec)


Vehicle Attitude
phi
theta
psi
(deg)
1.2
1
0.8
0.6
0.4
0.2
0
Gimbal Deflections (Dy, Dz)
Pitch Deflect Dy (deg)
1
0.5
0
-0.5
-1
-1.5
-2
Yaw Deflect Dz (deg)
0.5
0
-0.5
-1
-1.5
-2
0
2
4
6
8
10
12
14
16
18
20
Time

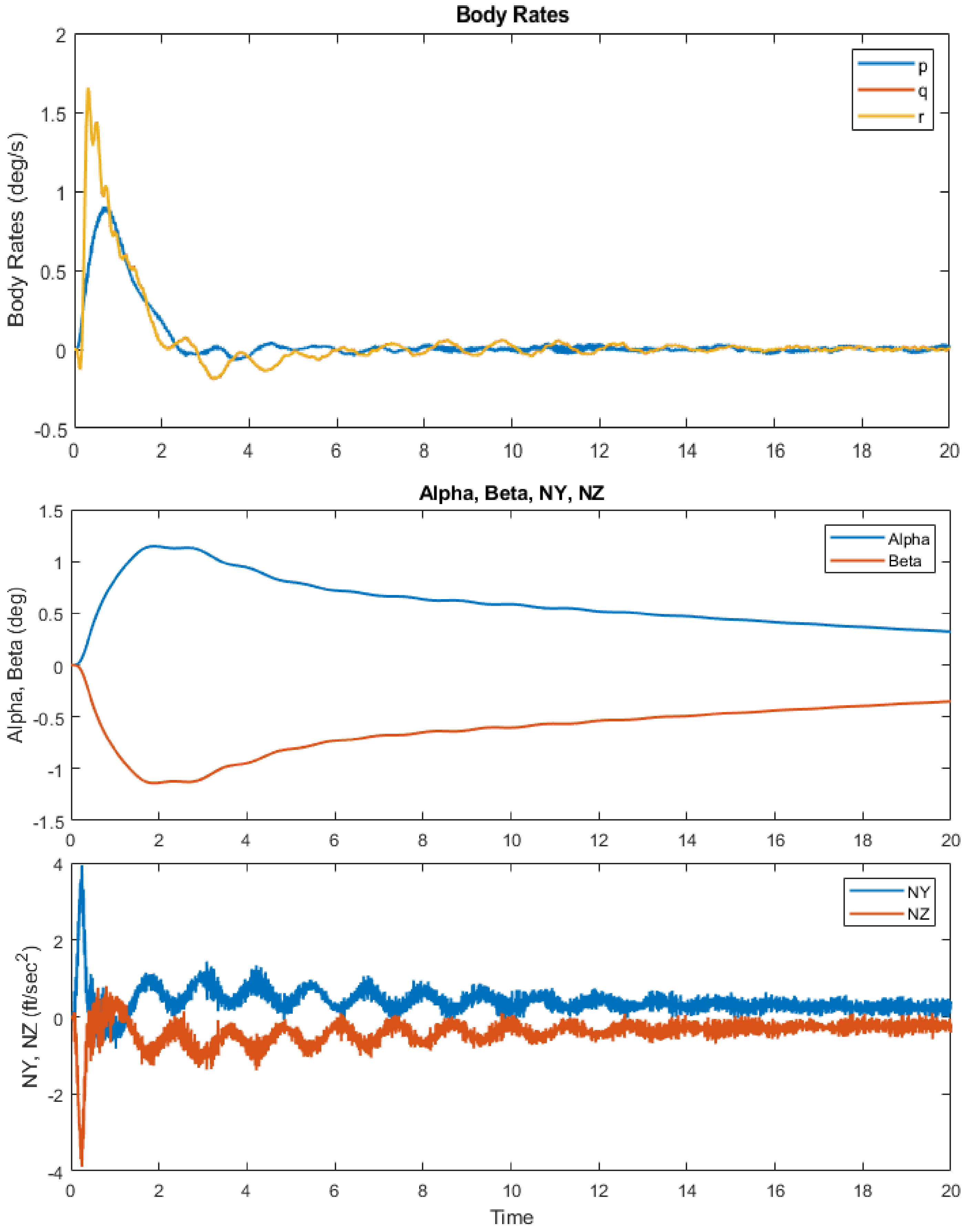


Figure 2.2.32 System Responses to 1° Attitude Commands

2.2.6 Control Analysis at T= 120 sec

At T=120 sec the dynamic pressure has dropped to 155 (lbf/ft^2) and the load-relief feedback from estimated α and β is significantly reduced, but we are no longer using (α, β)-integrals. We are now back to attitude trimming all the way to staging, by using feedback from attitude (θ, ψ)-integrals. The analysis files are in directory "*3-Stability Analysis with Flex & Slosh\1st Stage\T120*". Figure 2.2.46 shows the system stability in pitch and yaw. The LOX and LH2 slosh modes are phase-stable and their damping coefficients are now reduced to ζ=0.01 using fewer baffles. The low-pass filters bandwidths are increased as we approach staging in order to adjust the phase-margin.

Figure 2.2.47 shows the system's responses to 1° commands in roll, pitch and yaw. With the dynamic pressure reduced and the attitude trim-integrators active, the step responses to attitude commands have been improved. This time, however, we are using the non-linear actuator model with Coulomb friction which is implemented in the "Matlab Analysis" folder and it includes noise in the actuator position measurement which causes some jitter in the responses, especially in the roll attitude. Figure 2.2.48 shows a similar response generated from the linear "Flixan Analysis" closed-loop model.

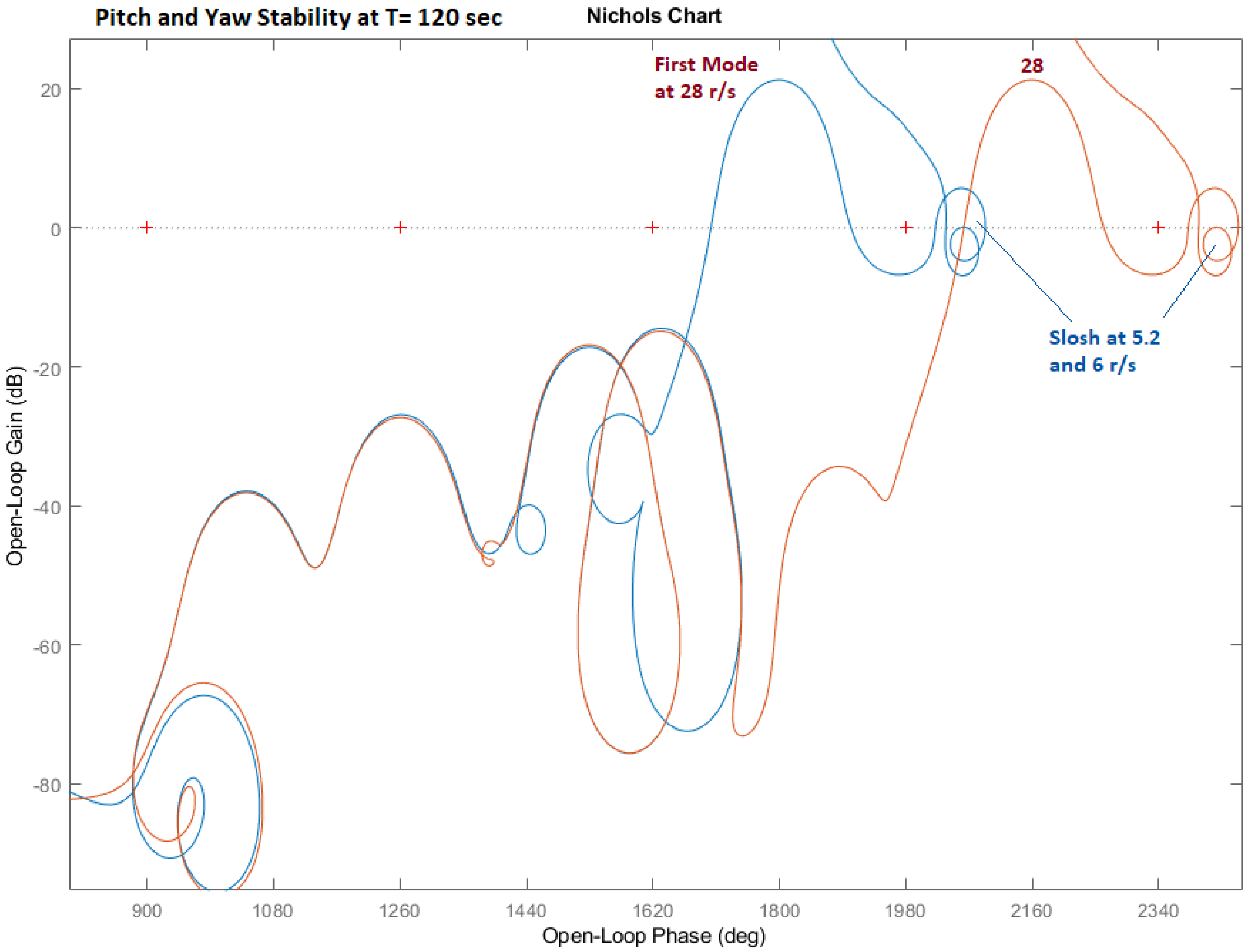
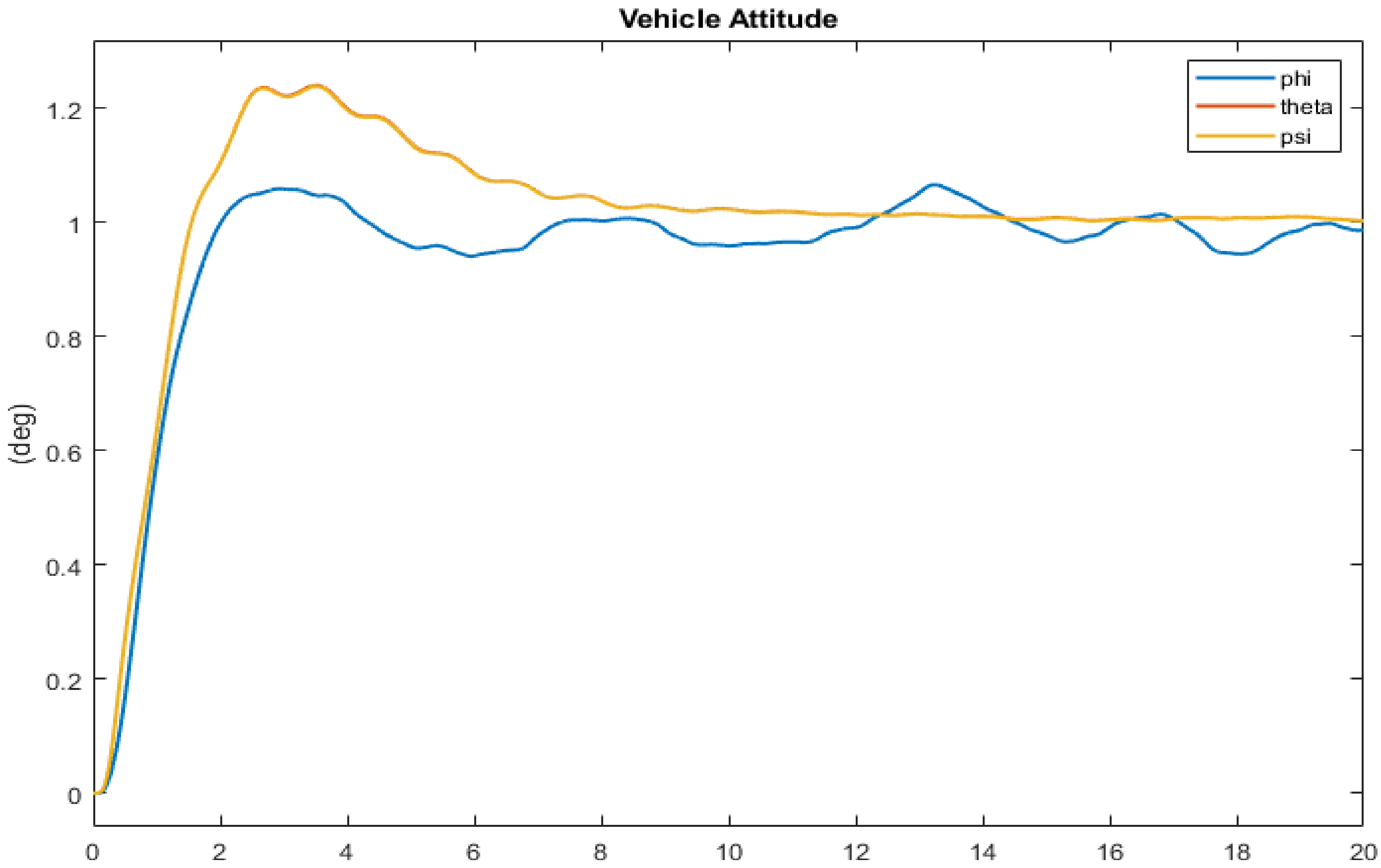
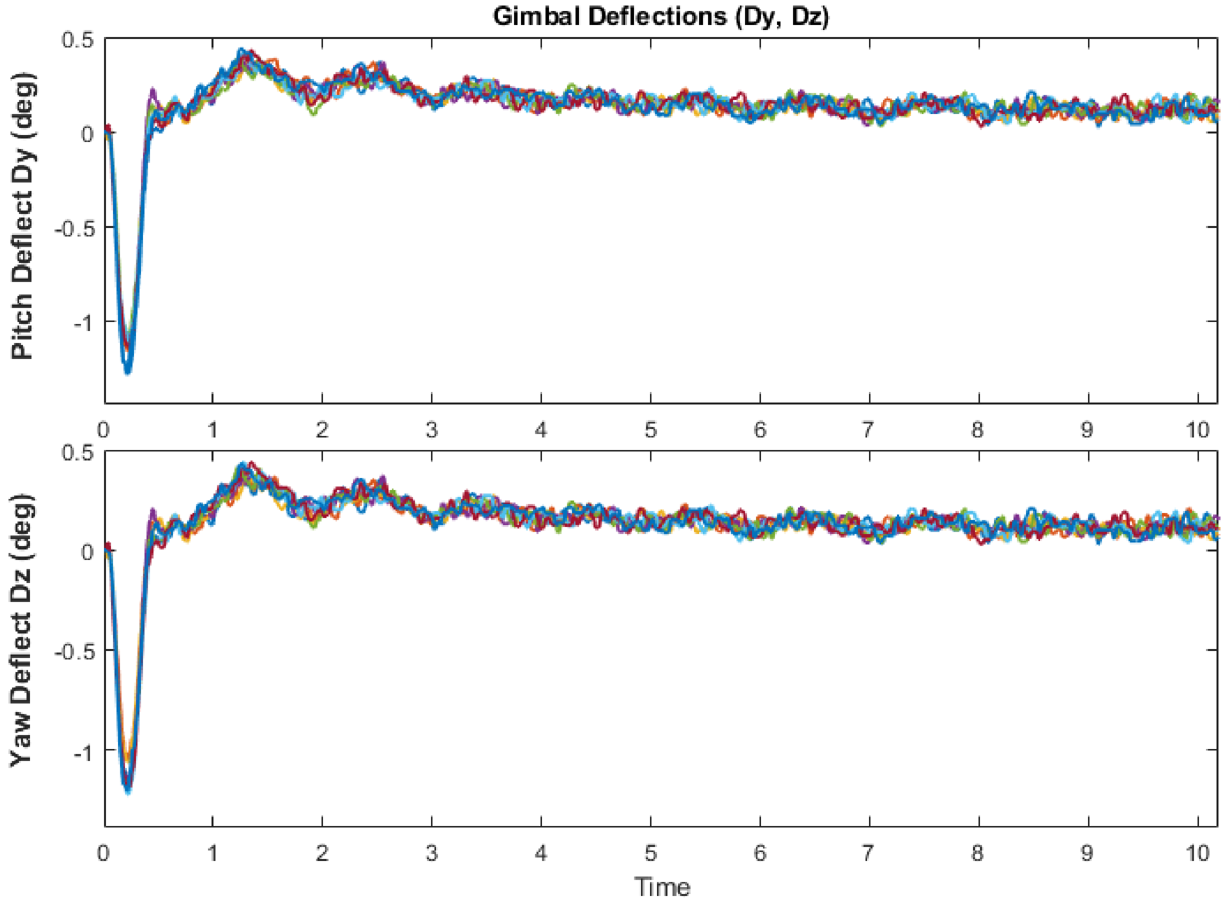


Figure 2.2.33 Pitch and Yaw Stability at T=120. Slosh Modes are Phase-Stable and the First Bending Mode is also Phase-Stable. Its Frequency is now 28 (rad/sec)


Vehicle Attitude
phi
theta
psi
(deg)
1.2
1
0.8
0.6
0.4
0.2
0
0
2
4
6
8
10
12
14
16
18
20


Gimbal Deflections (Dy, Dz)
Pitch Deflect Dy (deg)
0.5
0
-0.5
-1
0
1
2
3
4
5
6
7
8
9
10
Yaw Deflect Dz (deg)
0.5
0
-0.5
-1
0
1
2
3
4
5
6
7
8
9
10
Time

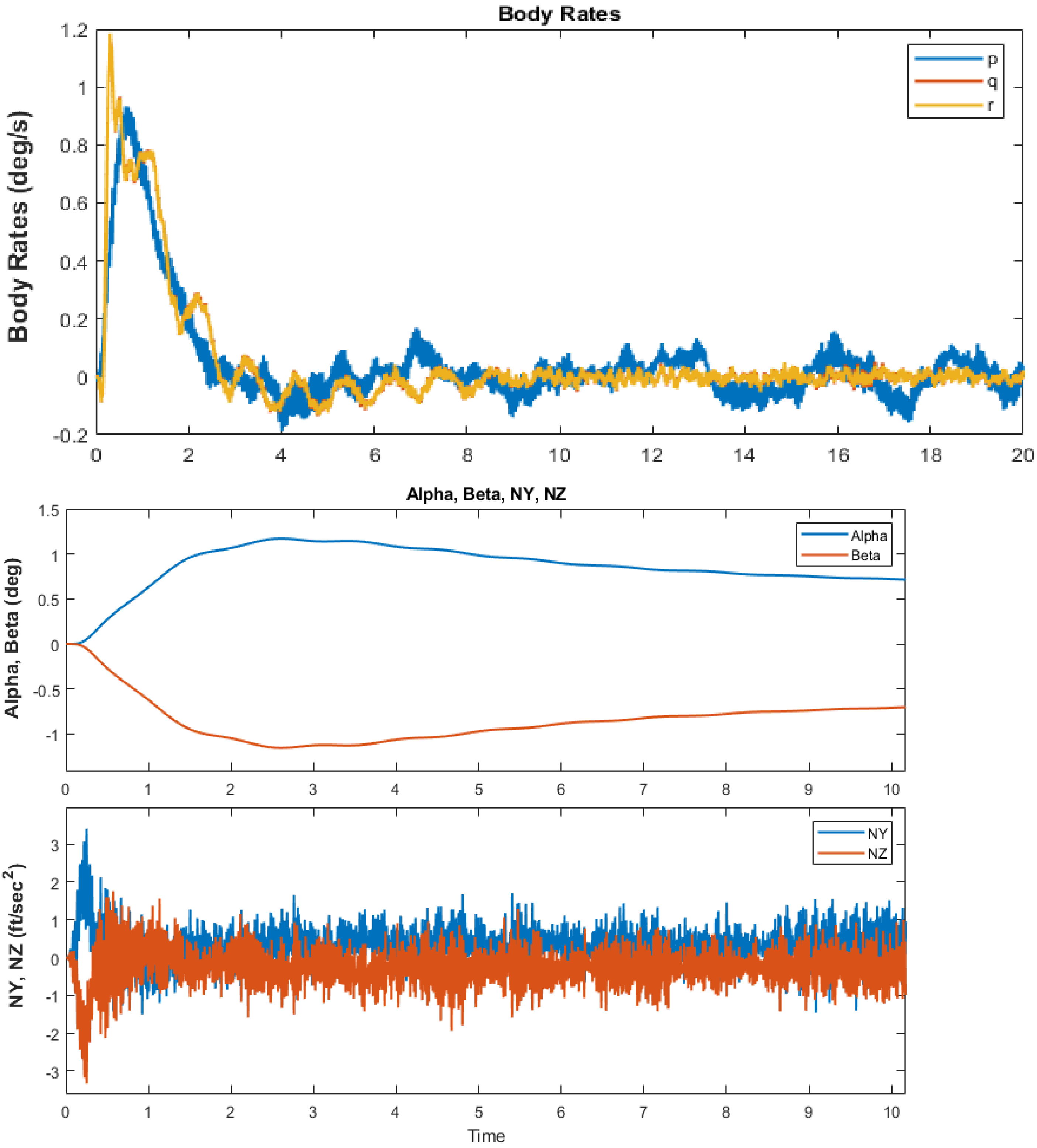
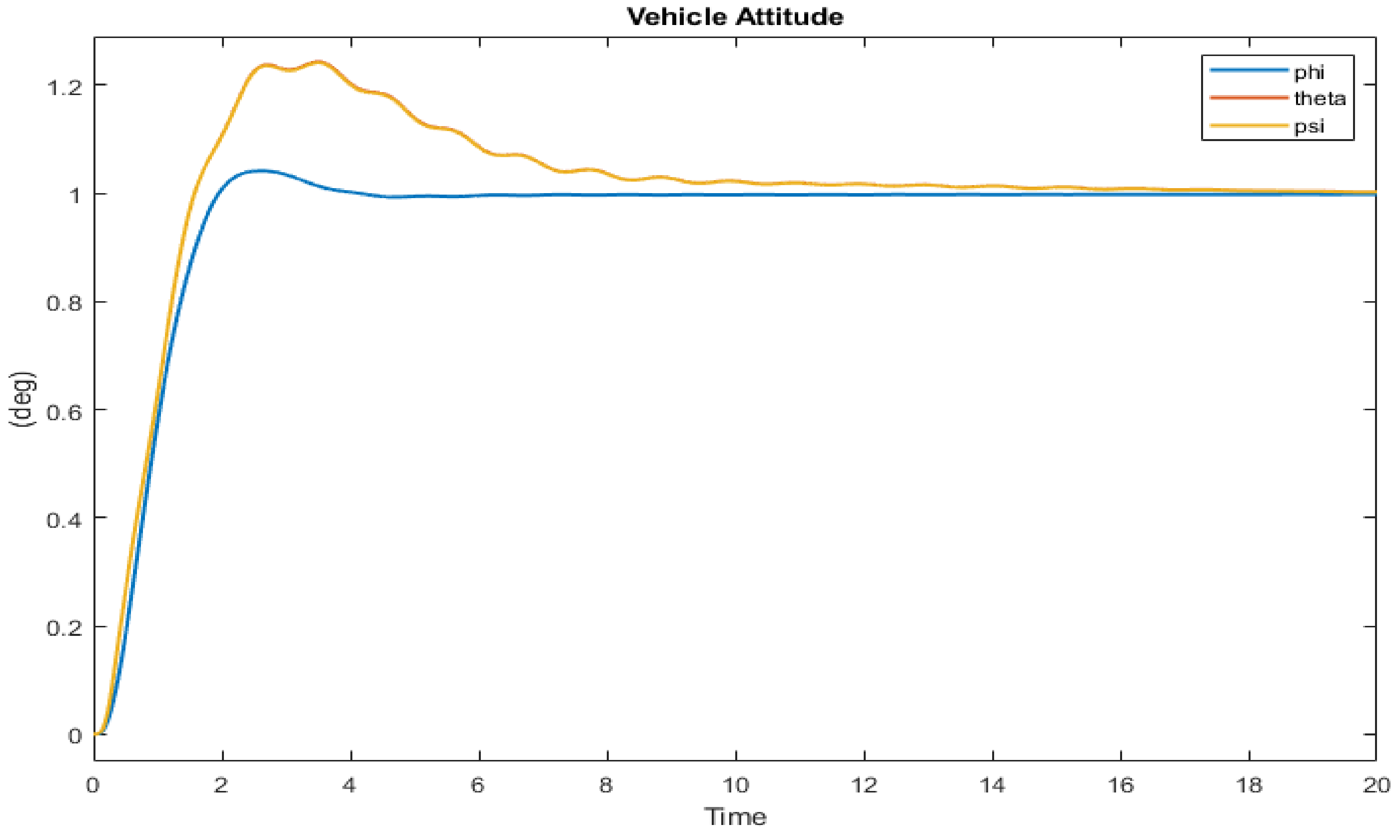
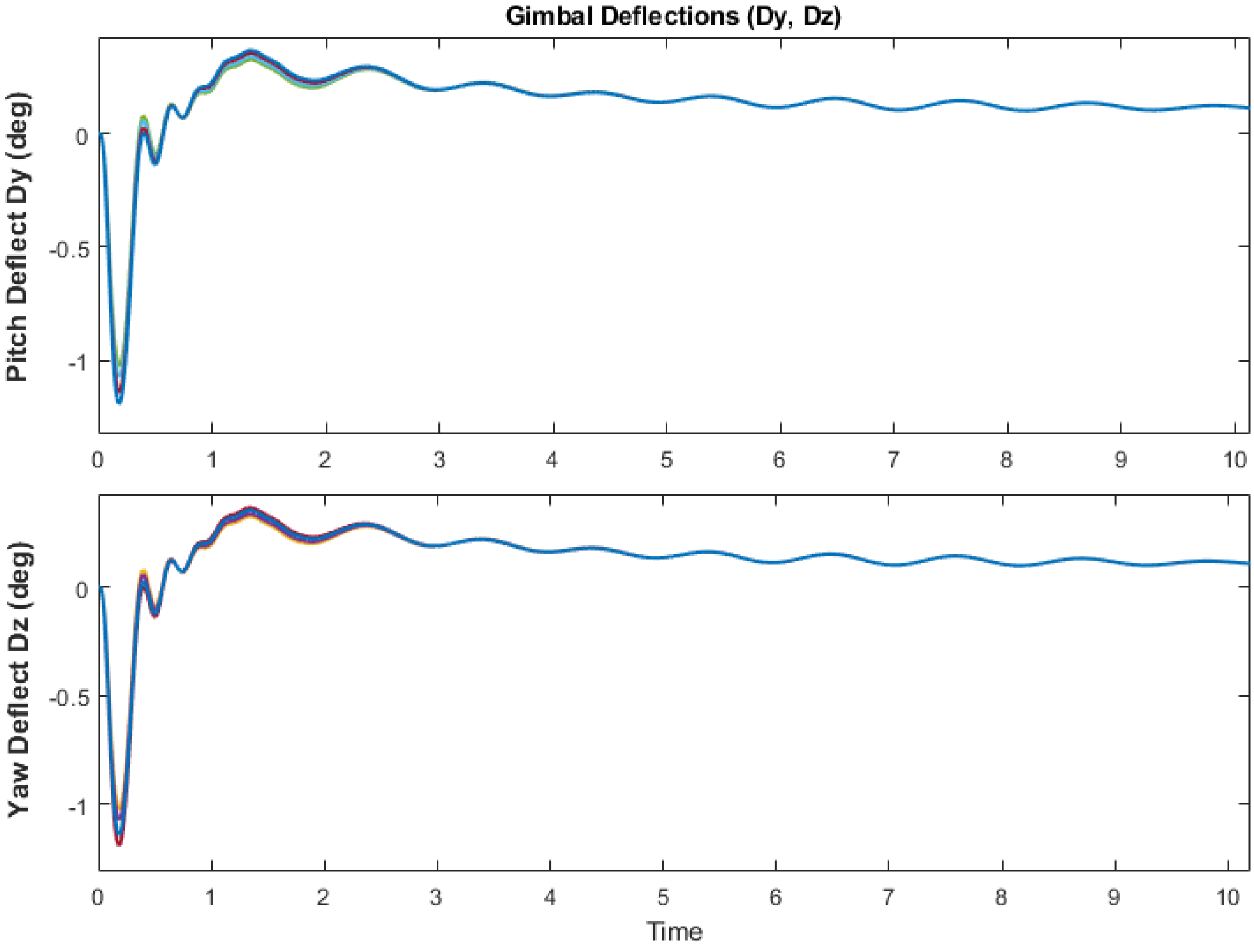


Figure 2.2.34 Vehicle responses to 1° Attitude Commands Using the Non-Linear Actuator Model


Vehicle Attitude


Gimbal Deflections (Dy, Dz)

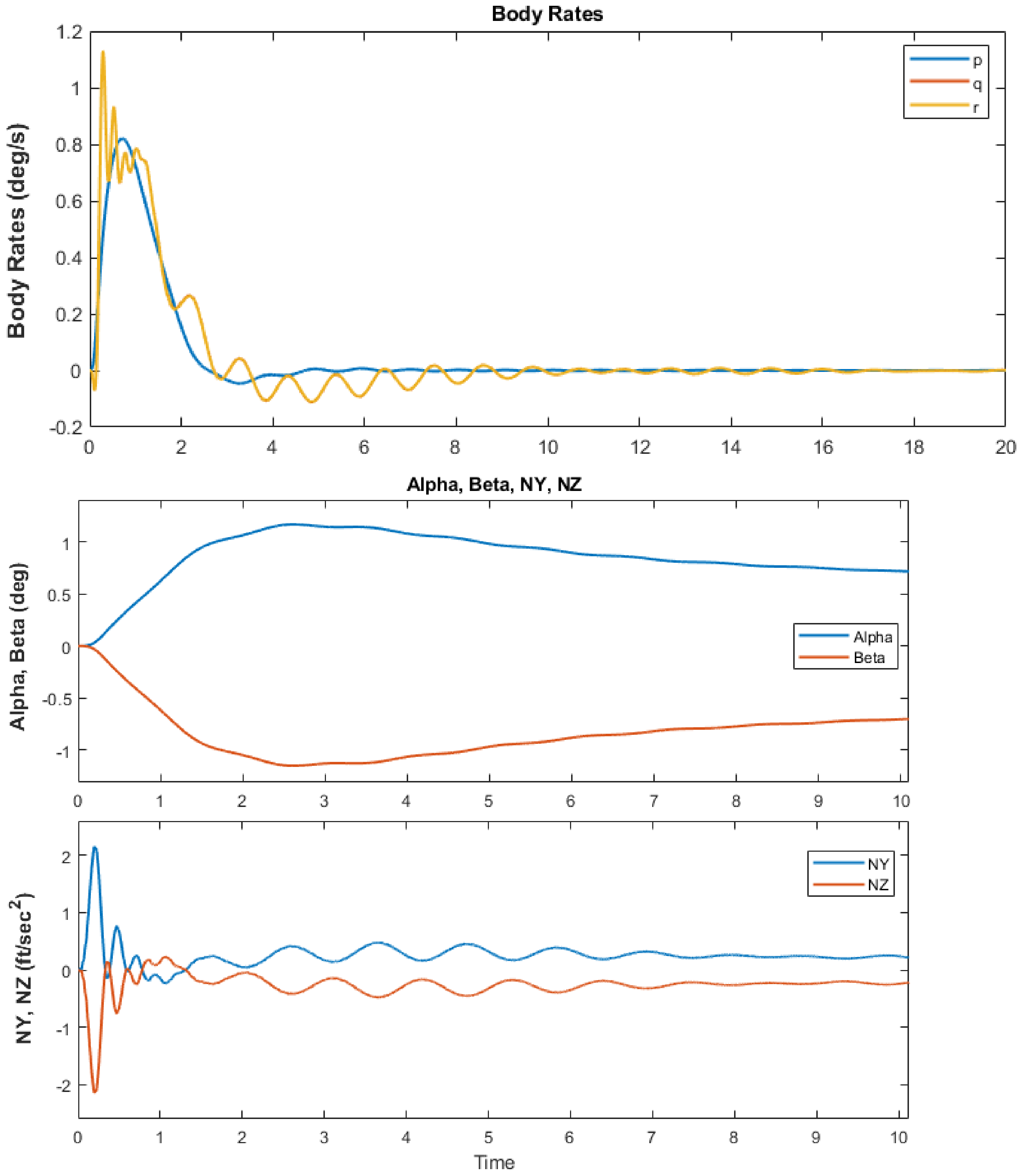
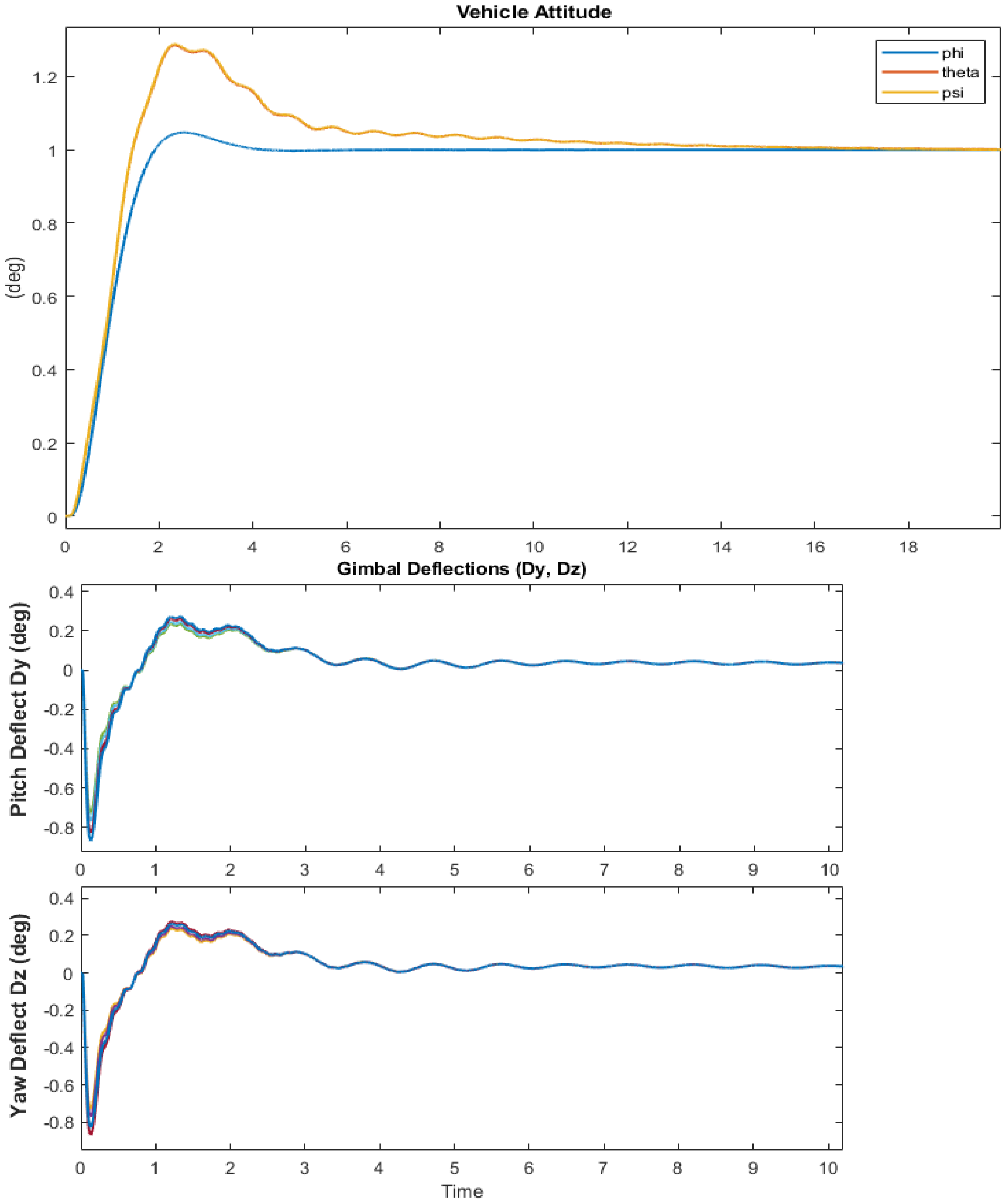


Figure 2.2.35 Vehicle responses to 1° Attitude Commands Using the Linear Actuator Flixan Analysis Model

2.2.7 Control Analysis at T= 140 sec

At T=140 sec the dynamic pressure is only 47 (lbf/ft^2) and the load-relief gain from the estimated α and β is further reduced and the attitude trimming from (θ, ψ)-integrals is increased which further improves the command tracking performance. Figure 2.2.49 shows the system's responses to 1° commands in roll, pitch and yaw obtained from the linear closed-loop model. The analysis files are in directory "*3-Stability Analysis with Flex & Slosh\1st Stage\T140*".



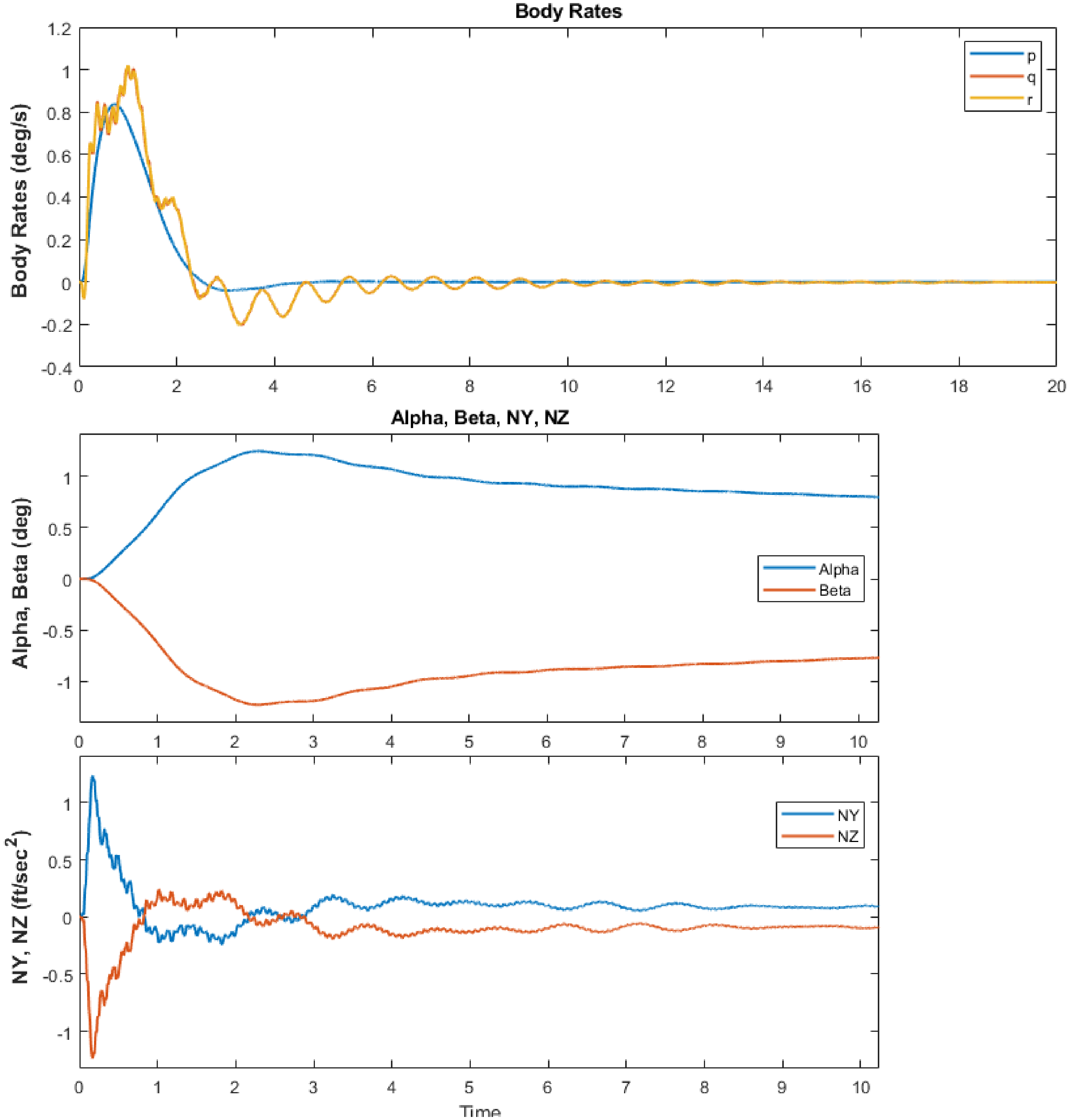


Figure 2.2.36 Vehicle Responses to 1° Attitude Commands Using the Linear Actuator Flixan Analysis Model

Figure 2.2.50 shows the system stability in pitch and yaw. The LOX and LH2 slosh modes are phase-stable with damping coefficients ζ=0.01. They were calculated using Flixan and Matlab programs and are showing identical results. The phasing is a little different because the delays are a little different. The low-pass filter bandwidths continue to increase as we approach staging.

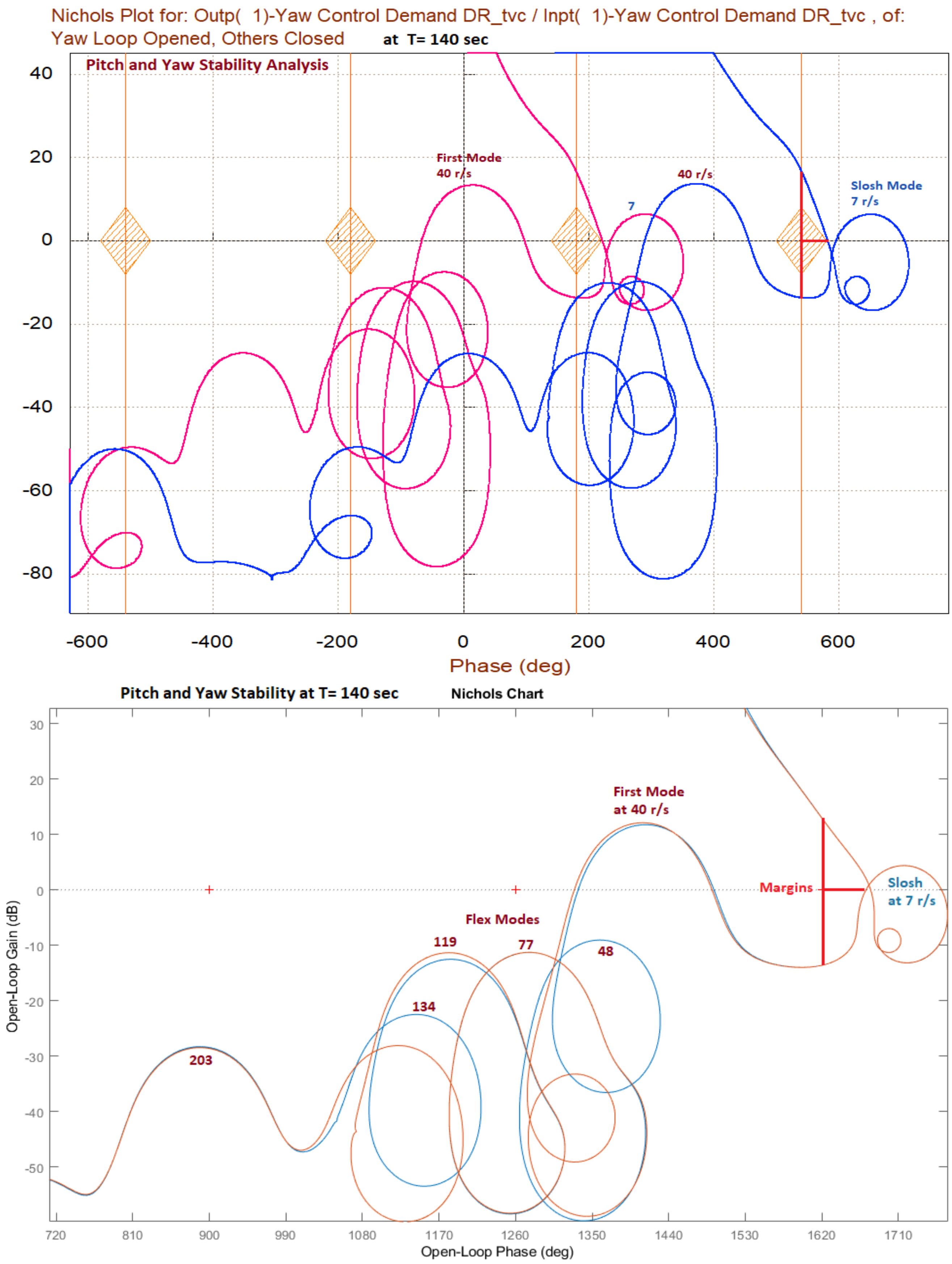


Figure 2.2.37 Pitch and Yaw Stability at T=140. Slosh Modes and the First Bending Mode are Phase-Stable. The Flex Mode Frequency is now Increased to 40 (rad/sec)

2.2.8 Control Analysis at Pre-Separation, T= 166 sec

Separation occurs at T=166 sec where the dynamic pressure is very low, only 6 (lbf/ft^2) and the load-relief gain from the estimated α and β is reduced even further and the attitude trimming is more emphasized to improve the command following performance. The frequency response analysis Figures (2.2.51-54) obtained by the Flixan program demonstrate the system stability in Roll, Pitch and Yaw using Bode, Nyquist and Nichols diagrams.

Figure 2.2.55 shows the system's responses to 1° commands in roll, pitch and yaw. They are generated using the non-linear simulation which includes the non-linear actuator, implemented in the "Matlab Analysis" folder and it includes Coulomb friction and the actuator position measurement error which causes jitter and small dispersions in the attitude responses. The analysis files are in directory "*3-Stability Analysis with Flex & Slosh\1st Stage\T166*".

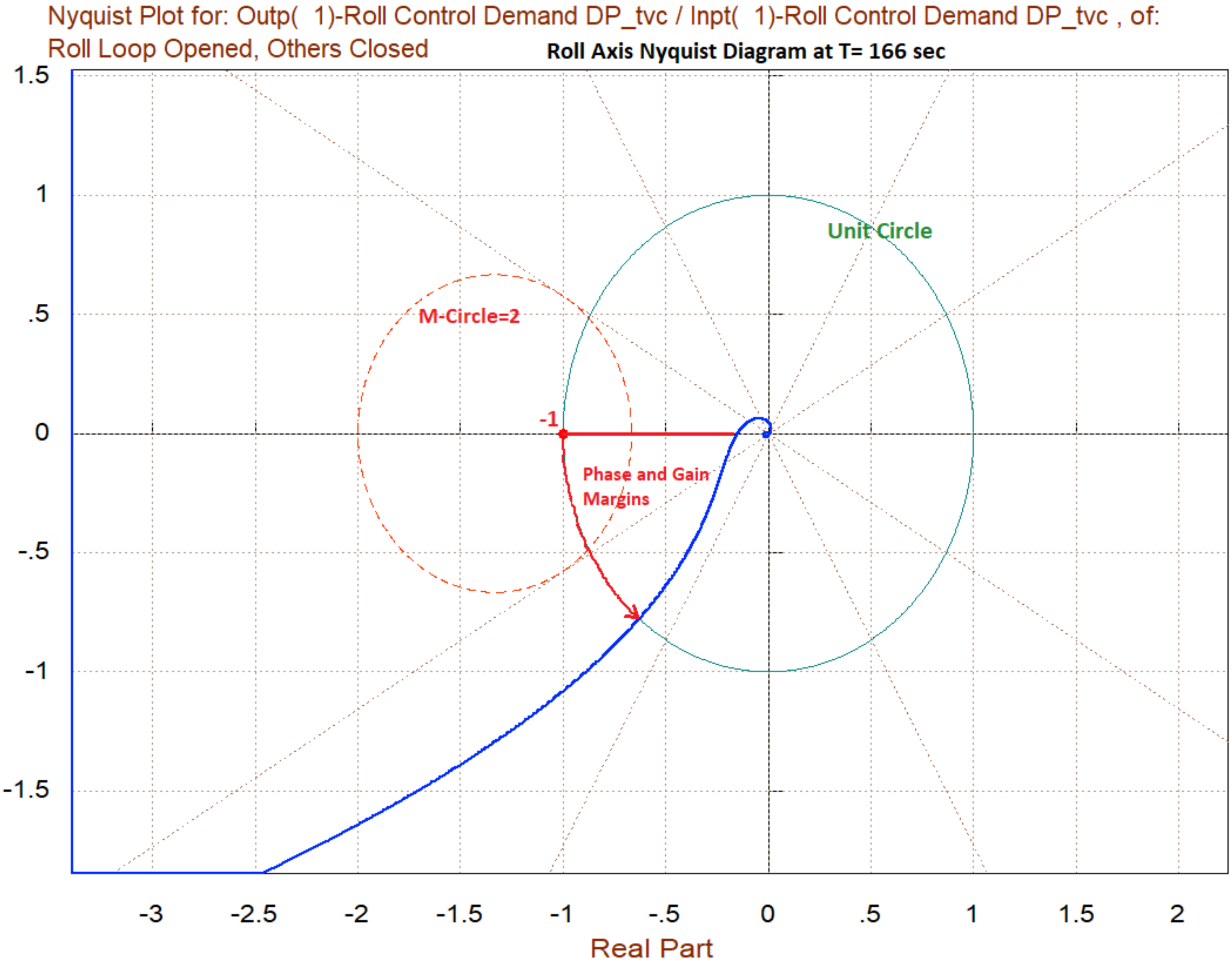


Figure 2.2.38 Roll Axis Nyquist Diagram Showing the Phase and Gain Margins

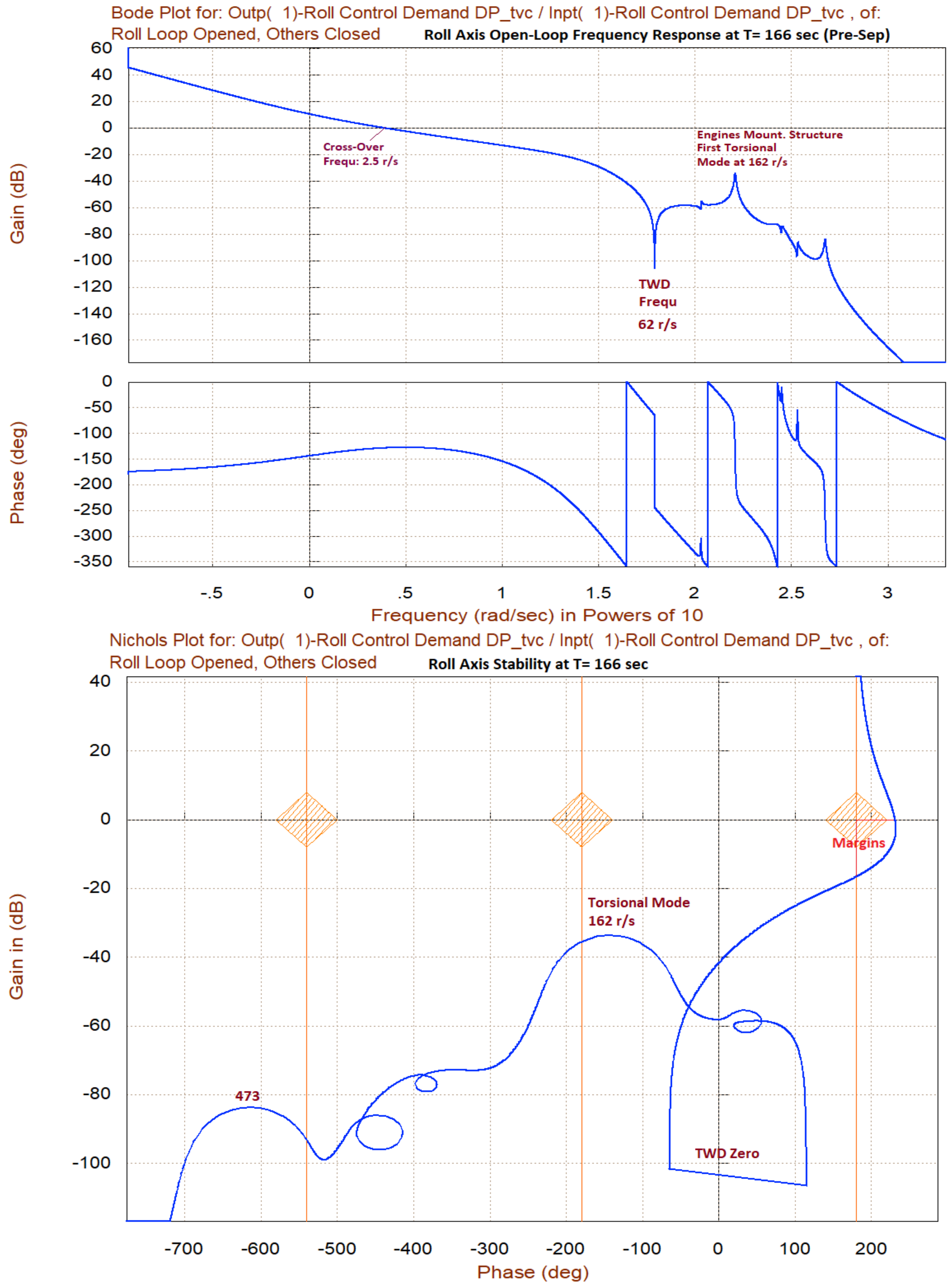


Figure 2.2.39 Roll Axis Stability Analysis Using Bode and Nichols Plots

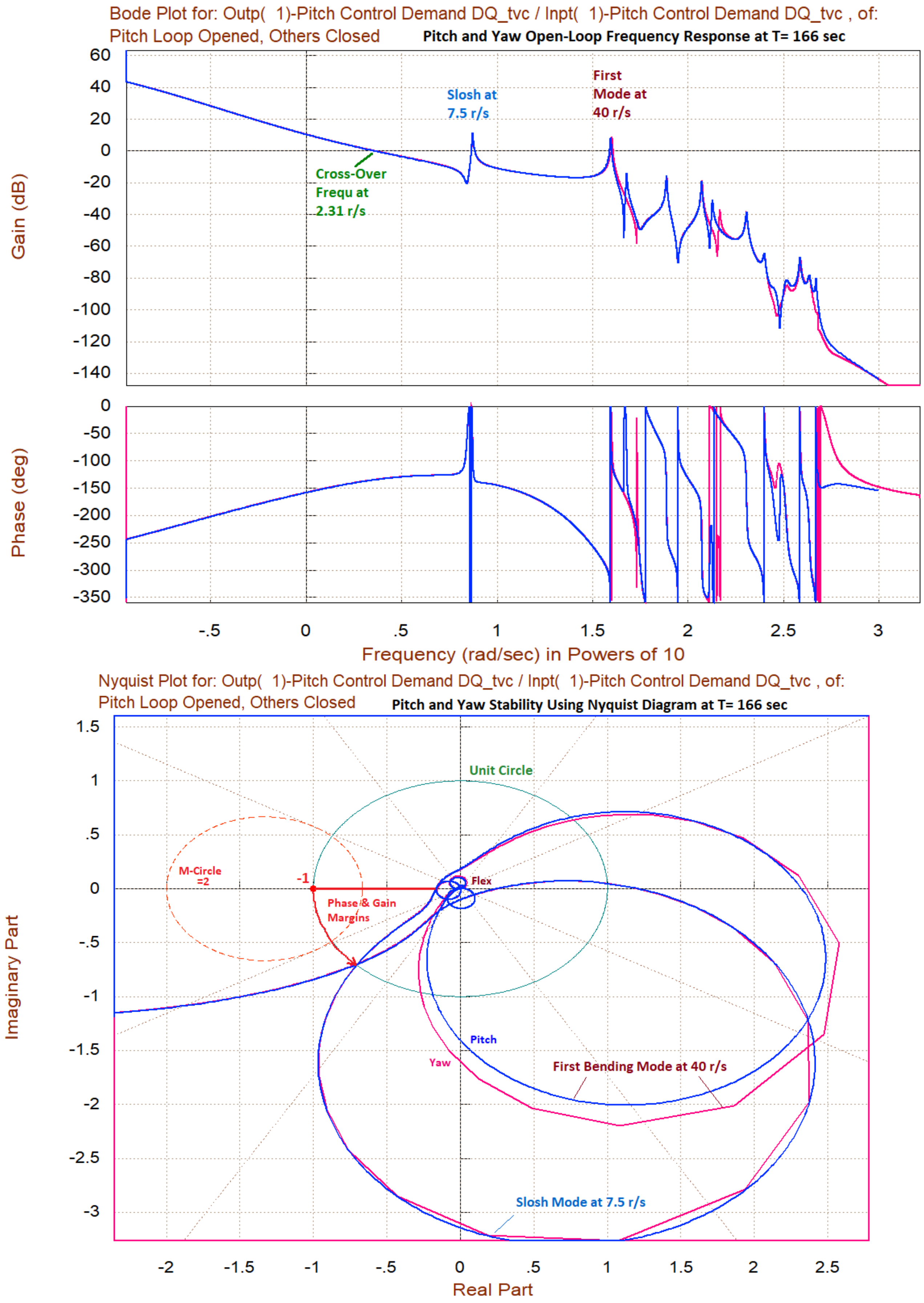


Figure 2.2.40 Pitch and Yaw Axes Stability in Bode and Nyquist

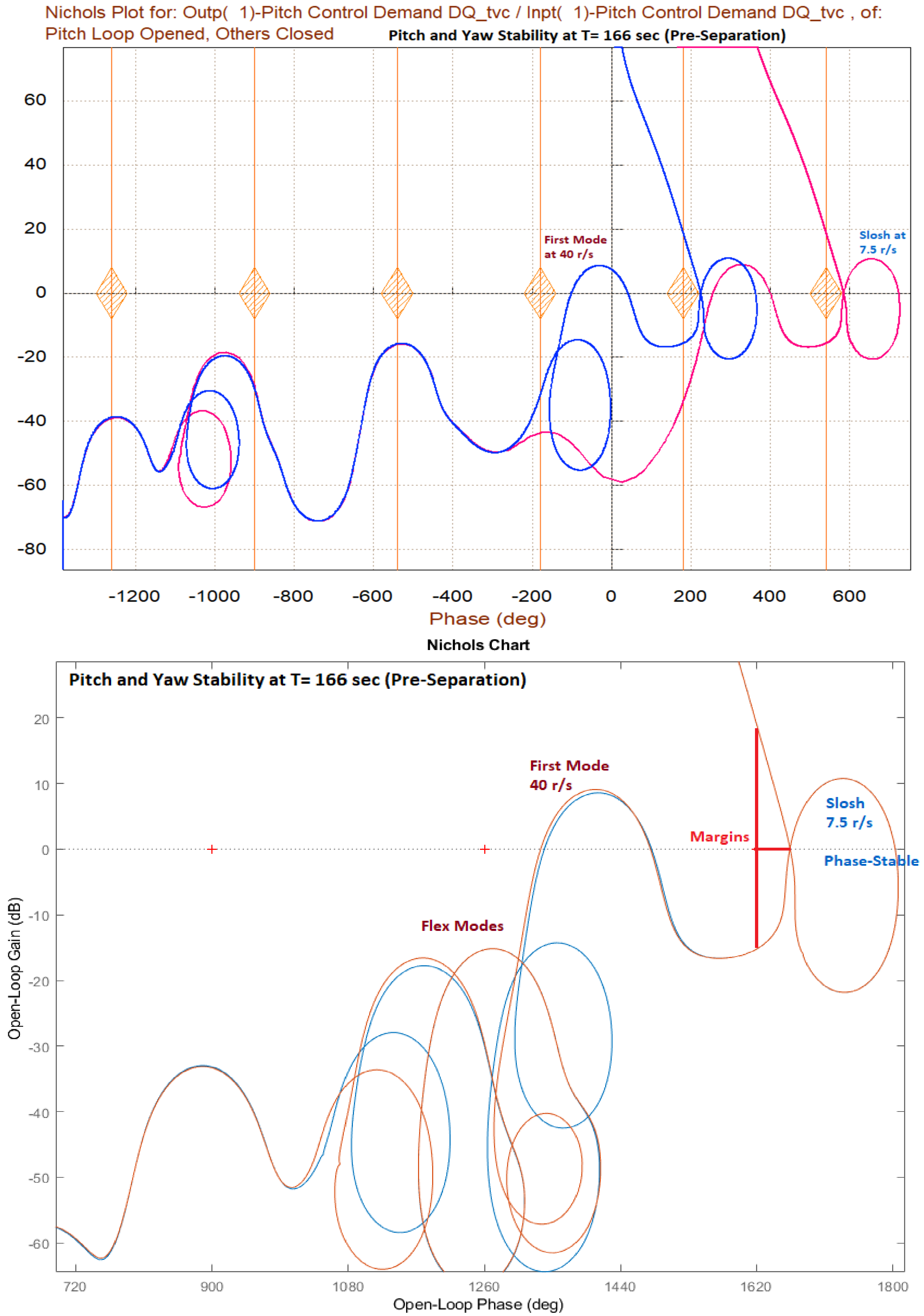
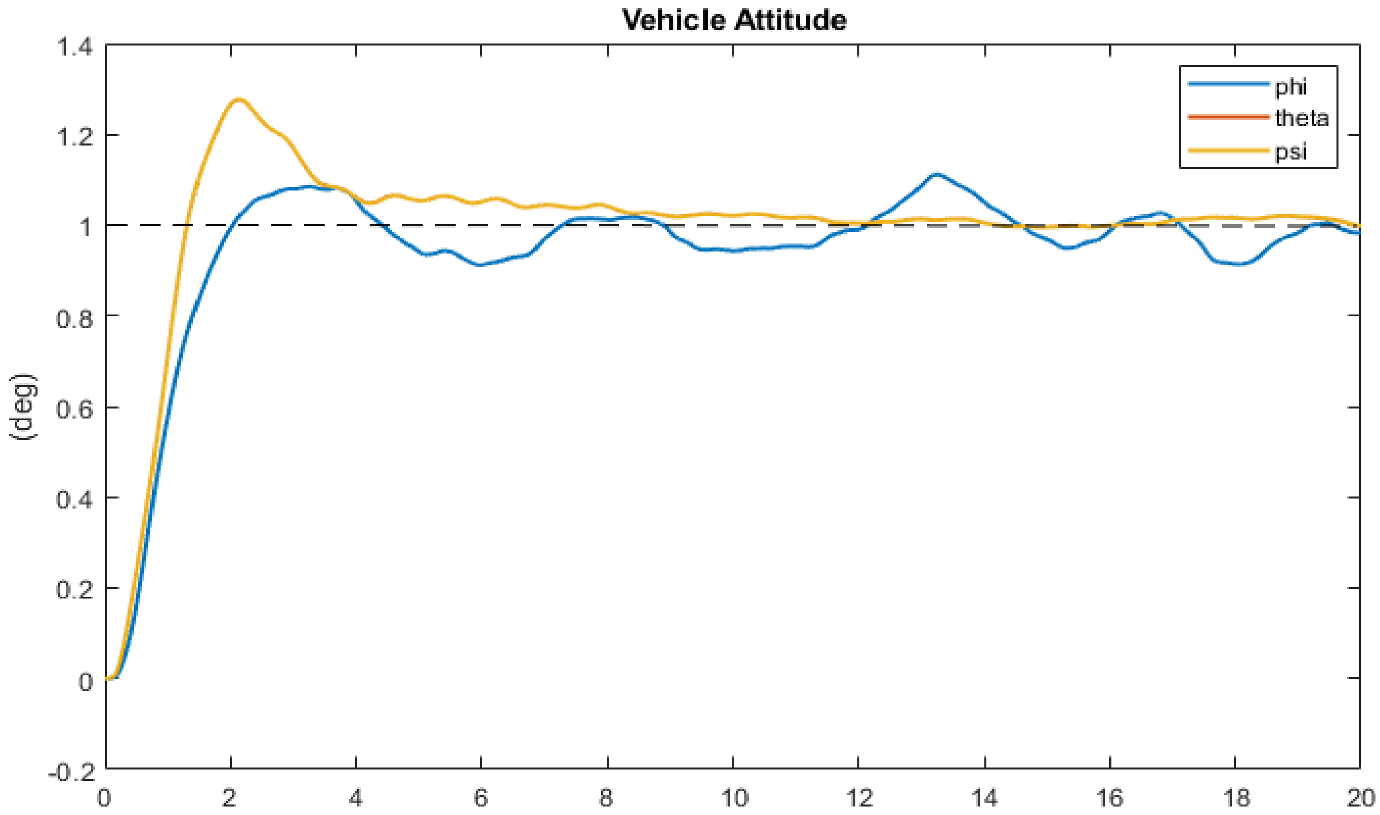
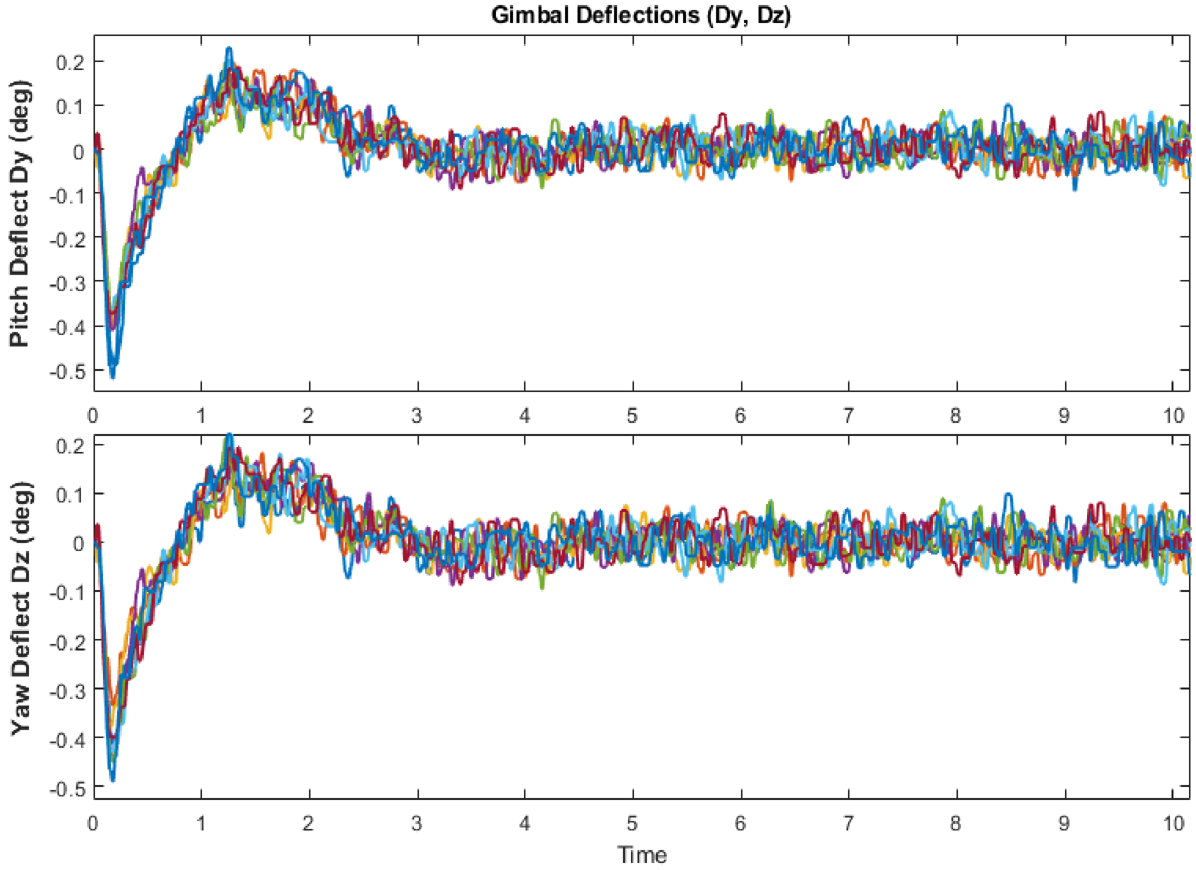


Figure 2.2.41 Pitch and Yaw Axes Stability Using Nichols Generated from Flixan and Matlab Programs


Vehicle Attitude
phi
theta
psi
(deg)
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
2
4
6
8
10
12
14
16
18
20


Gimbal Deflections (Dy, Dz)
Pitch Deflect Dy (deg)
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
1
2
3
4
5
6
7
8
9
10
Yaw Deflect Dz (deg)
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
1
2
3
4
5
6
7
8
9
10
Time

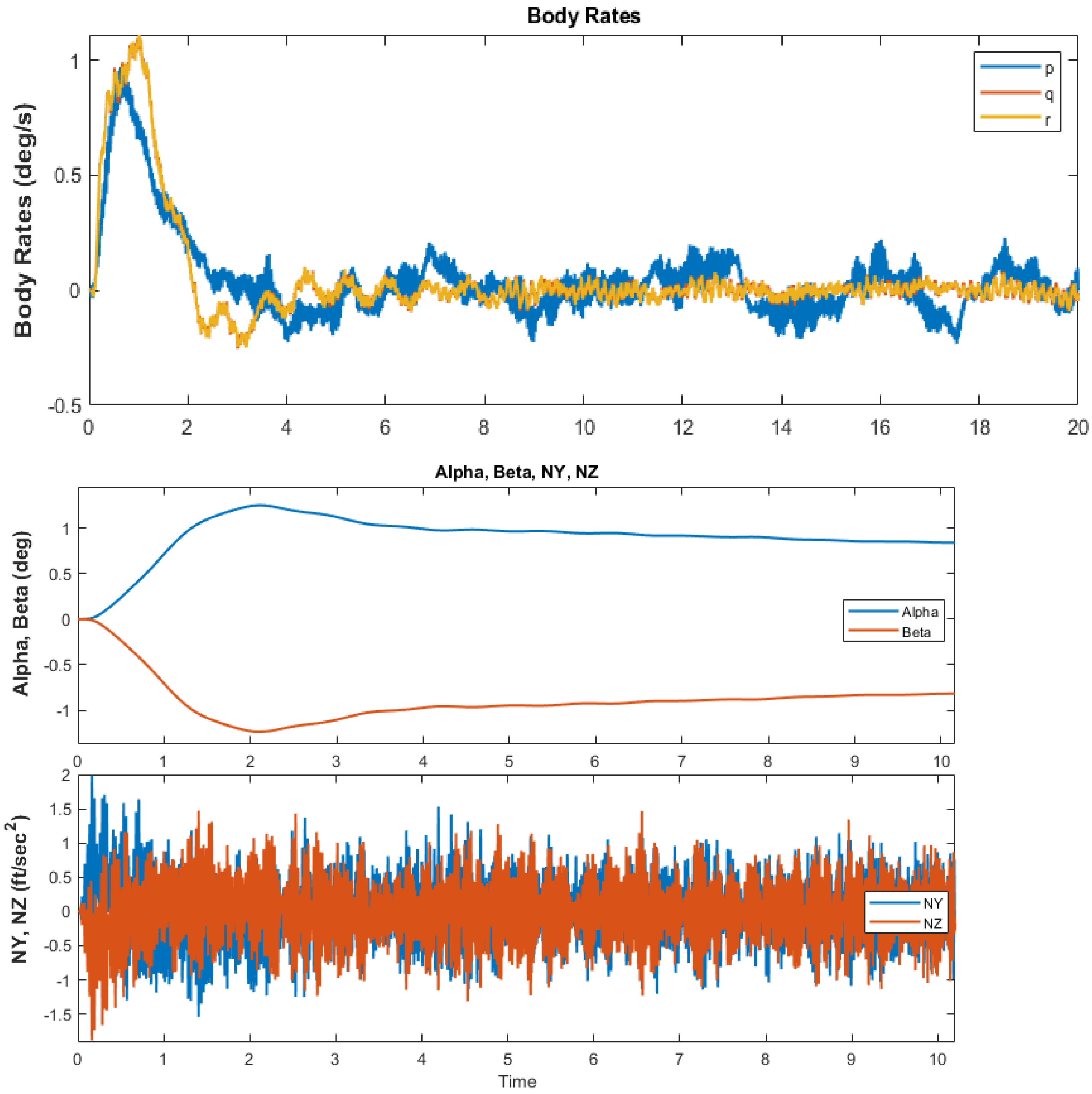


Figure 2.2.42 Vehicle responses to 1° Attitude Commands Using the Non-Linear Actuator Model

3.1 Second Stage Control Design

During second stage the vehicle is using a single TVC engine of 29,750 (lbf) thrust for pitch and yaw control and 8 RCS jets for roll control. The 8 jets are located in a circle around the TVC structure and they are implemented in the Flixan model as 4 bidirectional ±3 (lbf) thrusters mounted with the positive thrust directions as shown in Figure 3.1. In other words, each bidirectional thruster in the Flixan model represents two back-to-back firing jets.

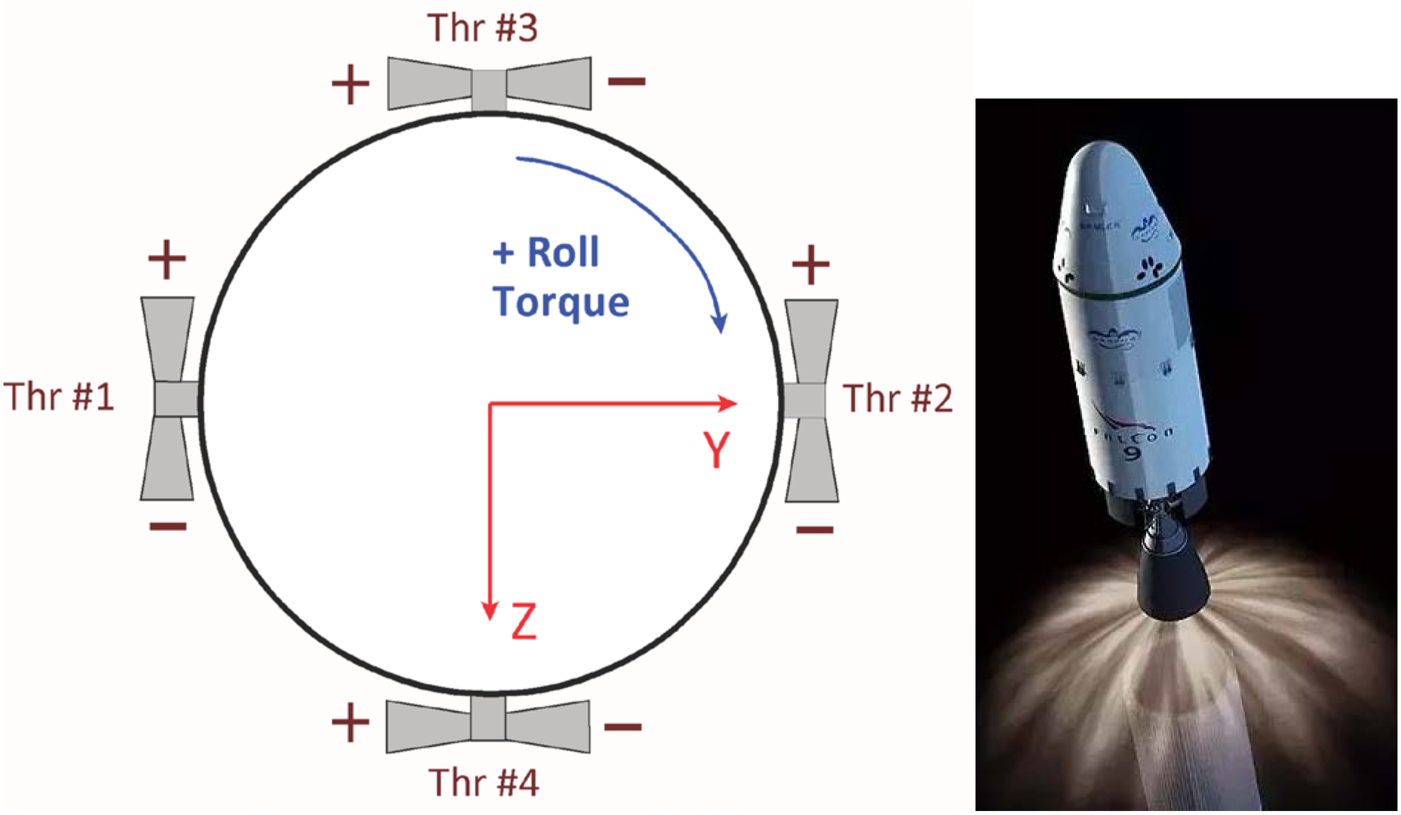


Figure 3.1 Reaction Control Thrusters

In addition to the vehicle system, the Flixan program also calculates the mixing-logic matrix that combines the TVC pitch and yaw gimbals with the 4 bi-directional thrusters to achieve the accelerations demanded by the flight control system. Note that in this design phase we are assuming that the thrusters are analog and they can provide a continuous linear thrust that will allow us to use the LQR method for lateral (roll/ yaw) control. Later we will change the roll control into a more realistic, on-off phase-plane system. We will therefore decouple the rigid vehicle system into separate pitch and lateral design models that will be used to create two separate LQR controllers.

3.1.1 Control Design at T= 180 sec

We will design and analyze the 2^{nd} stage in 5 flight conditions beginning with the post-separation case at T= 180 sec. The design files for this case are saved in directory "*Examples\23-Classic Launch Vehicle Design & Simulation\2-Control Gains Design\2nd Stage\T180*". The Flixan derived mixing-logic matrix in Figure 3.1.1 converts the 3 control demands to effector commands.

The inputs are roll, pitch and yaw demands, and it has 6 outputs: 2 gimbal deflection commands for the TVC engine in pitch and yaw (δy, δz) and 4 throttle commands for the 4 thrusters (δ_{Thr1} to δ_{Thr4}). The 3 (lbf) Jet forces are included in the vehicle input data.

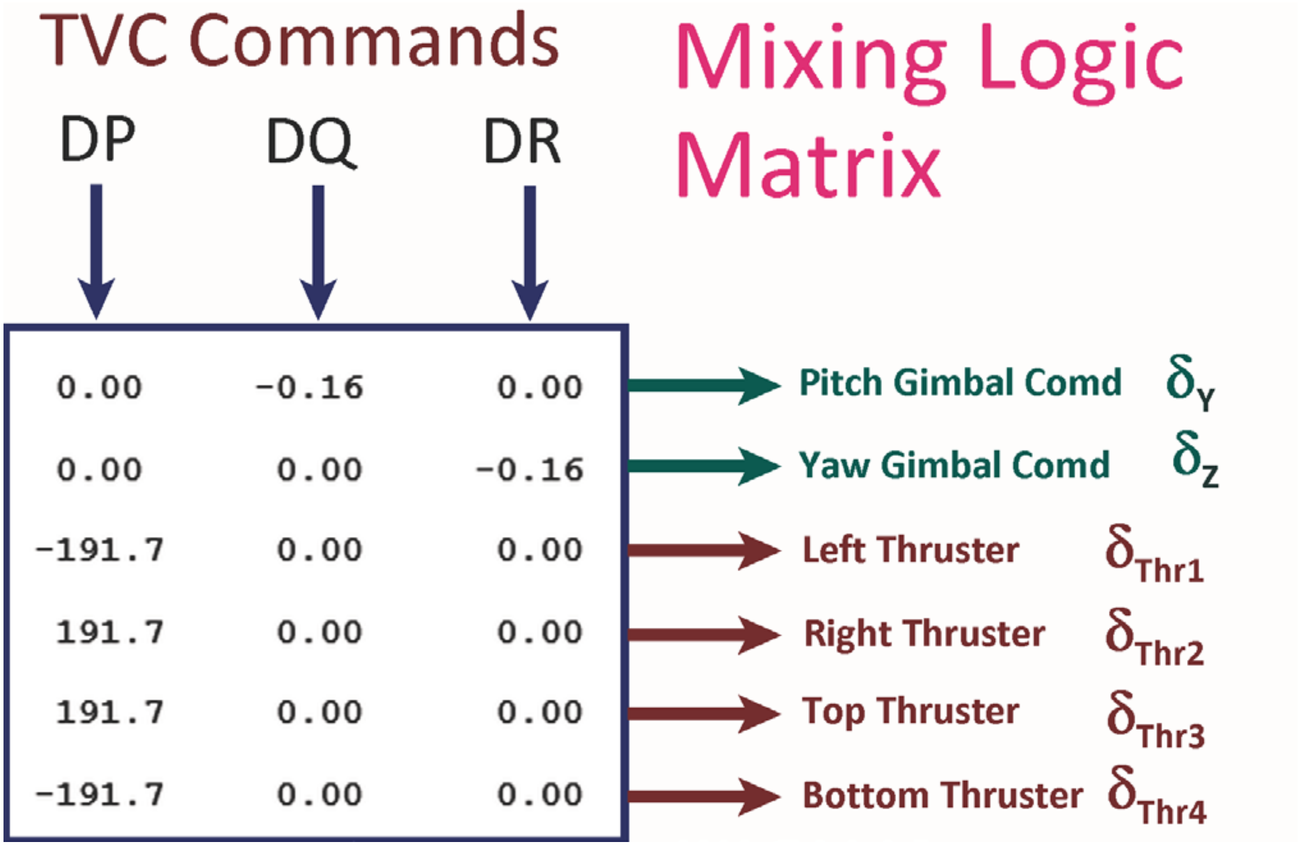


Figure 3.1.1 Mixing Logic Matrix

The Vehicle Input File

The vehicle input file is "*Rig_Vehi_T180.Inp*" in subdirectory "*2-Control Gains Design\2nd Stage\T180*". It begins with a batch set "*Batch for Calculating the Vehicle files during Stage-2*" which enables fast execution of the input file in batch mode. It also preserves the LQR design matrices Qc and Rc in the systems file "*Rig_Vehi_T180.Qdr*". The vehicle dataset is "*Launch Vehicle Second Stage Design Model at T=180 sec*" which includes a 29,750 (lbf) thrust engine that is gimbaling in pitch and yaw, and the 4 thrusters shown in Figure 3.1. The + nozzles of the left and right bidirectional thrusters are pointing upwards along -z axis, (Δy=-90° rotation relative to -x axis), and the + nozzles of the top and bottom bi-directional thrusters are pointing towards the -y axis (Δz=90° rotation relative to -x axis). The input file includes also a mixing logic dataset "*Mixing Logic Matrix for Second Stage at t=180 sec*" that creates matrix Kmix, shown in Figure 3.1.1. It converts the roll, pitch, yaw acceleration demands to 2 gimbal deflections and 4 throttle commands. Note that, since the 3 (lbf) jet thrusts are included in the vehicle engine data, the roll RCS control system bandwidth should be adjusted so that the throttle commands do not exceed ±1, which implies that the analog thrust magnitude does not exceed the max thruster capability, but this will change in the analysis phase when we will implement the phase-plane roll RCS system with bang-bang thrusters.

```
BATCH MODE INSTRUCTIONS ...............
Batch for Calculating the Vehicle files during Stage-2
! This batch set creates state-space systems for the Launch Vehicle during second stage
! and the Mixing Logic Matrix. Then it extracts the longitudinal and lateral subsystems,
! adds attitude trim integrators, and performs LQR control design.
!
!                   Keep the Previous LQR Design Matrices
Retain Matrix     : Pitch State Weight Matrix Qc (3x3)
Retain Matrix     : Pitch Control Weight Matrix Rc
Retain Matrix     : Lateral State Weight Matrix Qc (5x5)
Retain Matrix     : Lateral Control Weight Matrix Rc (2x2)
!
!                   Create the Vehicle and the Mixing Logic Matrix
Flight Vehicle    : Launch Vehicle Second Stage Design Model at T=180 sec
Mixing Matrix     : Mixing Logic Matrix for Second Stage at t=180 sec
System Connection: Vehicle and Mixing Logic Combined System
Transf-Functions : Integrator
!
!                   Perform Pitch LQR Design
System Modificat : Stage-2 Pitch Design Model
System Connection: Augmented Pitch Design Model
LQR Control Des   : Pitch LQR Control Design
!
!                   Perform Lateral LQR Design
System Modificat : Stage-2 Lateral Design Model
System Connection: Augmented Lateral Design Model
LQR Control Des   : Lateral LQR Control Design
!
!                   Convert Data to Matlab Format
To Matlab Format : Launch Vehicle Second Stage Design Model at T=180 sec
To Matlab Format : Mixing Logic Matrix for Second Stage at t=180 sec
To Matlab Format : Augmented Pitch Design Model
To Matlab Format : Augmented Lateral Design Model
To Matlab Format : Pitch LQR State-Feedback Controller
To Matlab Format : Lateral LQR State-Feedback Controller
--------------------------------------------------------------------------------------------------
```

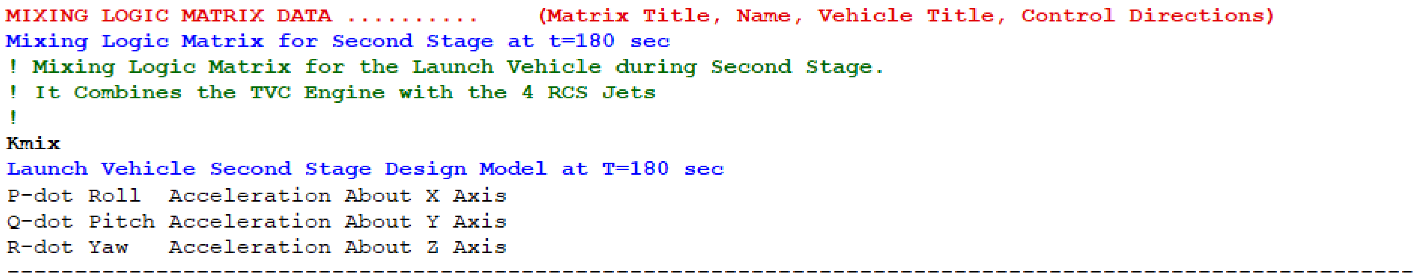
```
FLIGHT VEHICLE INPUT DATA ......
Launch Vehicle Second Stage Design Model at T=180 sec
! This dataset creates the Design Vehicle Model during Second Stage, consisting of
! the main engine and four bi-directional RCS jets.
!
Body Axes Output, Attitude=Rate Integral

Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft)    :  1040.75      32.1740      0.208960E+08
Moments and Products of Inertia: Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)          :  8052.60      41988.1       41808.7      0.00000      0.00000
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)        :  93.3240      0.00000      0.00000
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)       :  8.16600      7642.13      0.730000     269948.0
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)         :  15.3018      28.5736       0.0         -0.0
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                         :  0.0000       0.0           0.0          0.0
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)   :  0.000        25.5860       0.00000     -0.0         -0.436000
W-Gust Azim & Elev angles (deg), or Torque/Force direction (x,y,z), Force Locat (x,y,z)  :  Gust         45.000        90.00
Surface Reference Area (feet^2), Mean Aerodynamic Chord (ft), Wing Span in (feet)        :  38.5000      7.20000       7.20000
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial_rho/ Partial_H} :  116.800      0.00000       0.00000      0.00000
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:  0.4251       0.00000     -0.141175E-04  0.00000      0.00000
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}: -0.091747    -0.036         0.00000      0.00000      0.00000
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}: -0.0067816   -0.036         0.00000      0.00000      0.00000
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:  0.00000      0.00000       0.00000      0.00000      0.00000
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}: -0.004773    -0.04          0.00000      0.00000      0.00000
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}:  0.0          0.04          0.00000      0.00000      0.00000

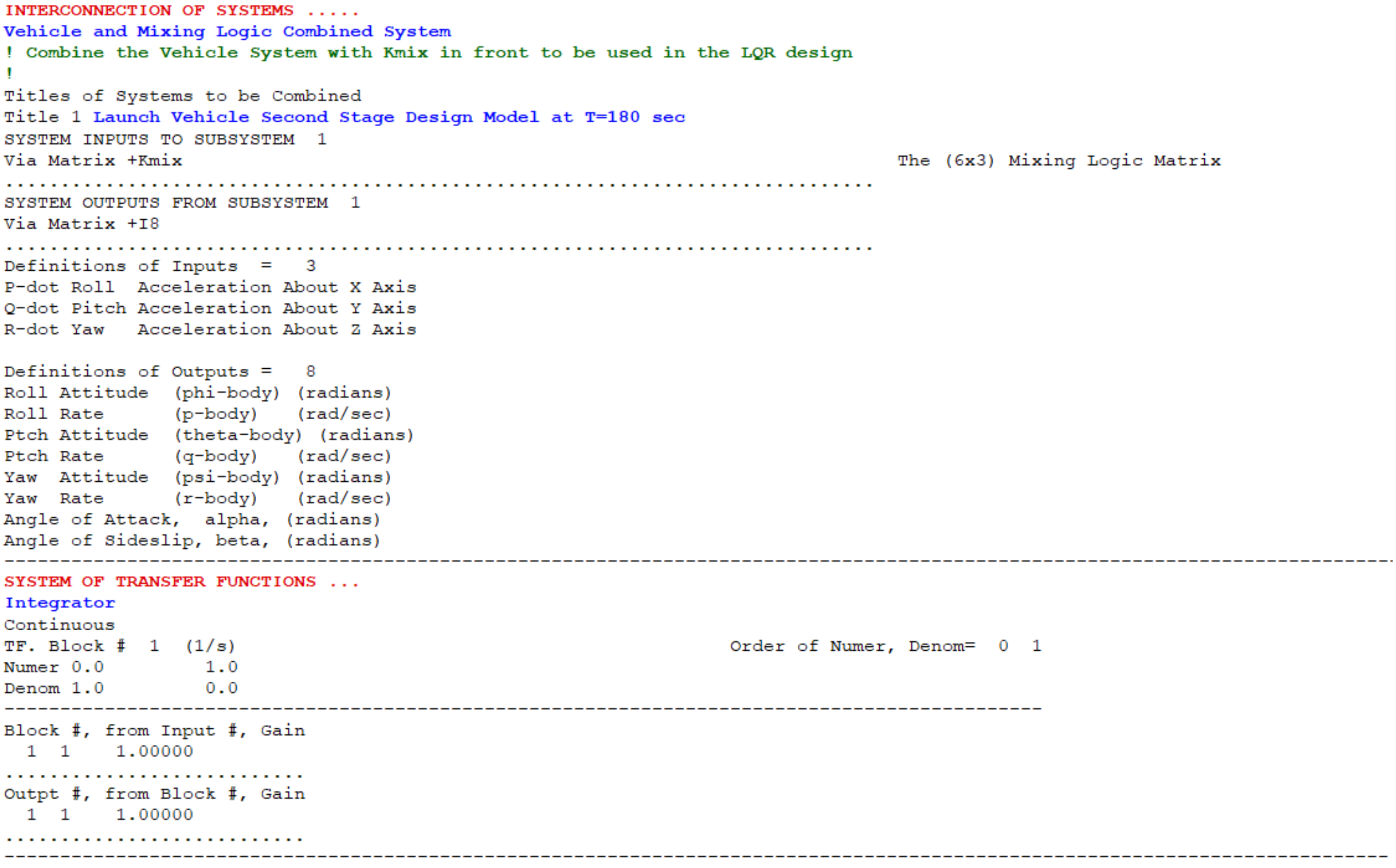
Number of Thruster Engines, Include or Not the Tail-Wags-Dog and Load-Torque Dynamics ?  :  5   NO TWD

TVC Engine No:  1                         (Gimbaling  Throttling   Single_Gimbal) : Main Engine      Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)             :  29750.0      29750.0
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000      0.00000       6.00000      6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  9.00000      120.000       2.60000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  84.5000      0.00000       0.00000
TVC Engine No:  2                         (Gimbaling  Throttling   Single_Gimbal) : Left RCS Jet     Throttling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)             :  0.00000      3.0
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg): -90.00        0.00000       0.00000      0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  0.00000      0.00000       0.00000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  83.00       -3.50          0.00000
TVC Engine No:  3                         (Gimbaling  Throttling   Single_Gimbal) : Rght RCS Jet     Throttling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)             :  0.00000      3.0
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg): -90.00        0.00000       0.00000      0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  0.00000      0.00000       0.00000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  83.00        3.50          0.00000
TVC Engine No:  4                         (Gimbaling  Throttling   Single_Gimbal) : Top  RCS Jet     Throttling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)             :  0.00000      3.0
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.0000       90.000        0.00000      0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  0.00000      0.00000       0.00000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  83.00        0.00000      -3.50
TVC Engine No:  5                         (Gimbaling  Throttling   Single_Gimbal) : Botm RCS Jet     Throttling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)             :  0.00000      3.0
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.0000       90.000        0.00000      0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  0.00000      0.00000       0.00000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  83.00        0.00000       3.50
---------------------------------------------------------------------------------------------------------------------------
```

The following dataset combines the vehicle system with the mixing-logic matrix Kmix to create the vehicle design model “*Vehicle and Mixing Logic Combined System*” whose inputs are the 3 FCS commands (DP, DQ, DR).



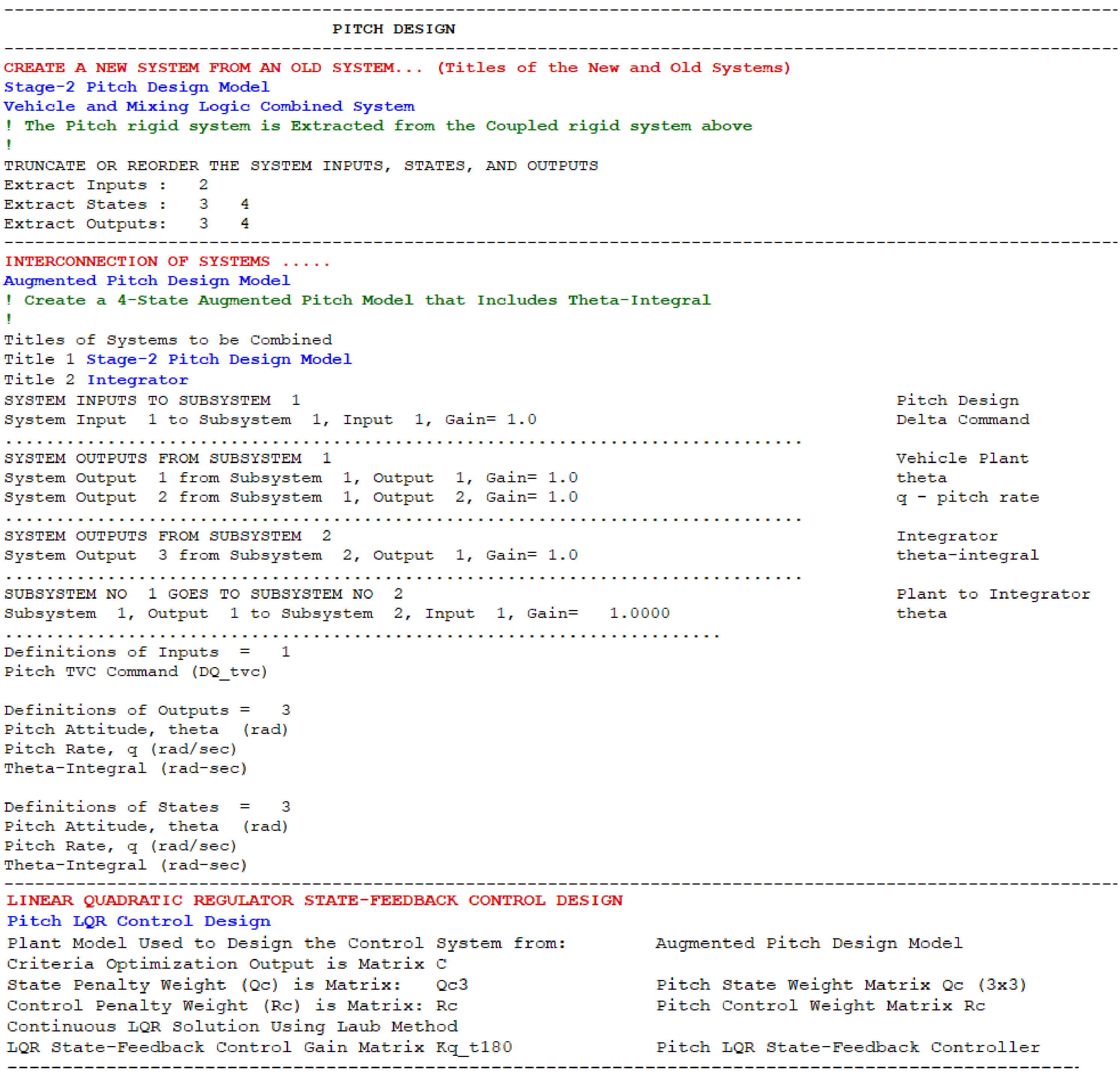
```
MIXING LOGIC MATRIX DATA ..........     (Matrix Title, Name, Vehicle Title, Control Directions)
Mixing Logic Matrix for Second Stage at t=180 sec
! Mixing Logic Matrix for the Launch Vehicle during Second Stage.
! It Combines the TVC Engine with the 4 RCS Jets
!
Kmix
Launch Vehicle Second Stage Design Model at T=180 sec
P-dot Roll  Acceleration About X Axis
Q-dot Pitch Acceleration About Y Axis
R-dot Yaw   Acceleration About Z Axis
-----------------------------------------------------------------------------------------------
```



```
INTERCONNECTION OF SYSTEMS .....
Vehicle and Mixing Logic Combined System
! Combine the Vehicle System with Kmix in front to be used in the LQR design
!
Titles of Systems to be Combined
Title 1 Launch Vehicle Second Stage Design Model at T=180 sec
SYSTEM INPUTS TO SUBSYSTEM  1
Via Matrix +Kmix                                                           The (6x3) Mixing Logic Matrix
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1
Via Matrix +I8
..............................................................................
Definitions of Inputs  =   3
P-dot Roll  Acceleration About X Axis
Q-dot Pitch Acceleration About Y Axis
R-dot Yaw   Acceleration About Z Axis

Definitions of Outputs =   8
Roll Attitude  (phi-body)  (radians)
Roll Rate      (p-body)    (rad/sec)
Ptch Attitude  (theta-body) (radians)
Ptch Rate      (q-body)    (rad/sec)
Yaw  Attitude  (psi-body)  (radians)
Yaw  Rate      (r-body)    (rad/sec)
Angle of Attack,  alpha, (radians)
Angle of Sideslip, beta, (radians)
----------------------------------------------------------------------------------------------------------
SYSTEM OF TRANSFER FUNCTIONS ...
Integrator
Continuous
TF. Block #  1  (1/s)                                  Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
----------------------------------------------------------------------------------------
Block #, from Input #, Gain
  1  1    1.00000
...........................
Outpt #, from Block #, Gain
  1  1    1.00000
...........................
----------------------------------------------------------------------------------------------------------
```

The following 3 datasets extract the pitch design system from the combined pitch and lateral system, augment it to include a third state, the theta-integral state that will enable the FCS to better track pitch attitude commands from guidance, and the third dataset uses the LQR method to design the pitch state-feedback controller Kq_t180 from states: theta, q, and theta-integral. The matrices Qc3 and Rc in the systems file “Rig_Vehi_T180.Qdr” are the state and control penalty weights in the LQR optimization. Notice that the alpha state is not used for feedback during 2nd stage because the aerodynamic effects are negligible and it doesn’t need to be optimized, by not even including it in the design model.



```
-------------------------------------------------------------------------------------------------------------
                                PITCH DESIGN
-------------------------------------------------------------------------------------------------------------
CREATE A NEW SYSTEM FROM AN OLD SYSTEM... (Titles of the New and Old Systems)
Stage-2 Pitch Design Model
Vehicle and Mixing Logic Combined System
! The Pitch rigid system is Extracted from the Coupled rigid system above
!
TRUNCATE OR REORDER THE SYSTEM INPUTS, STATES, AND OUTPUTS
Extract Inputs :   2
Extract States :   3   4
Extract Outputs:   3   4
-------------------------------------------------------------------------------------------------------------
INTERCONNECTION OF SYSTEMS .....
Augmented Pitch Design Model
! Create a 4-State Augmented Pitch Model that Includes Theta-Integral
!
Titles of Systems to be Combined
Title 1 Stage-2 Pitch Design Model
Title 2 Integrator
SYSTEM INPUTS TO SUBSYSTEM  1                                                     Pitch Design
System Input  1 to Subsystem  1, Input  1, Gain= 1.0                              Delta Command
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                                  Vehicle Plant
System Output  1 from Subsystem  1, Output  1, Gain= 1.0                          theta
System Output  2 from Subsystem  1, Output  2, Gain= 1.0                          q - pitch rate
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                                                  Integrator
System Output  3 from Subsystem  2, Output  1, Gain= 1.0                          theta-integral
..............................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2                                           Plant to Integrator
Subsystem  1, Output  1 to Subsystem  2, Input  1, Gain=   1.0000                 theta
......................................................................
Definitions of Inputs  =   1
Pitch TVC Command (DQ_tvc)

Definitions of Outputs =   3
Pitch Attitude, theta  (rad)
Pitch Rate, q (rad/sec)
Theta-Integral (rad-sec)

Definitions of States  =   3
Pitch Attitude, theta  (rad)
Pitch Rate, q (rad/sec)
Theta-Integral (rad-sec)
-------------------------------------------------------------------------------------------------------------
LINEAR QUADRATIC REGULATOR STATE-FEEDBACK CONTROL DESIGN
Pitch LQR Control Design
Plant Model Used to Design the Control System from:      Augmented Pitch Design Model
Criteria Optimization Output is Matrix C
State Penalty Weight (Qc) is Matrix:   Qc3               Pitch State Weight Matrix Qc (3x3)
Control Penalty Weight (Rc) is Matrix: Rc                Pitch Control Weight Matrix Rc
Continuous LQR Solution Using Laub Method
LQR State-Feedback Control Gain Matrix Kq_t180           Pitch LQR State-Feedback Controller
---------------------------------------------------------------------------------------------------------
```

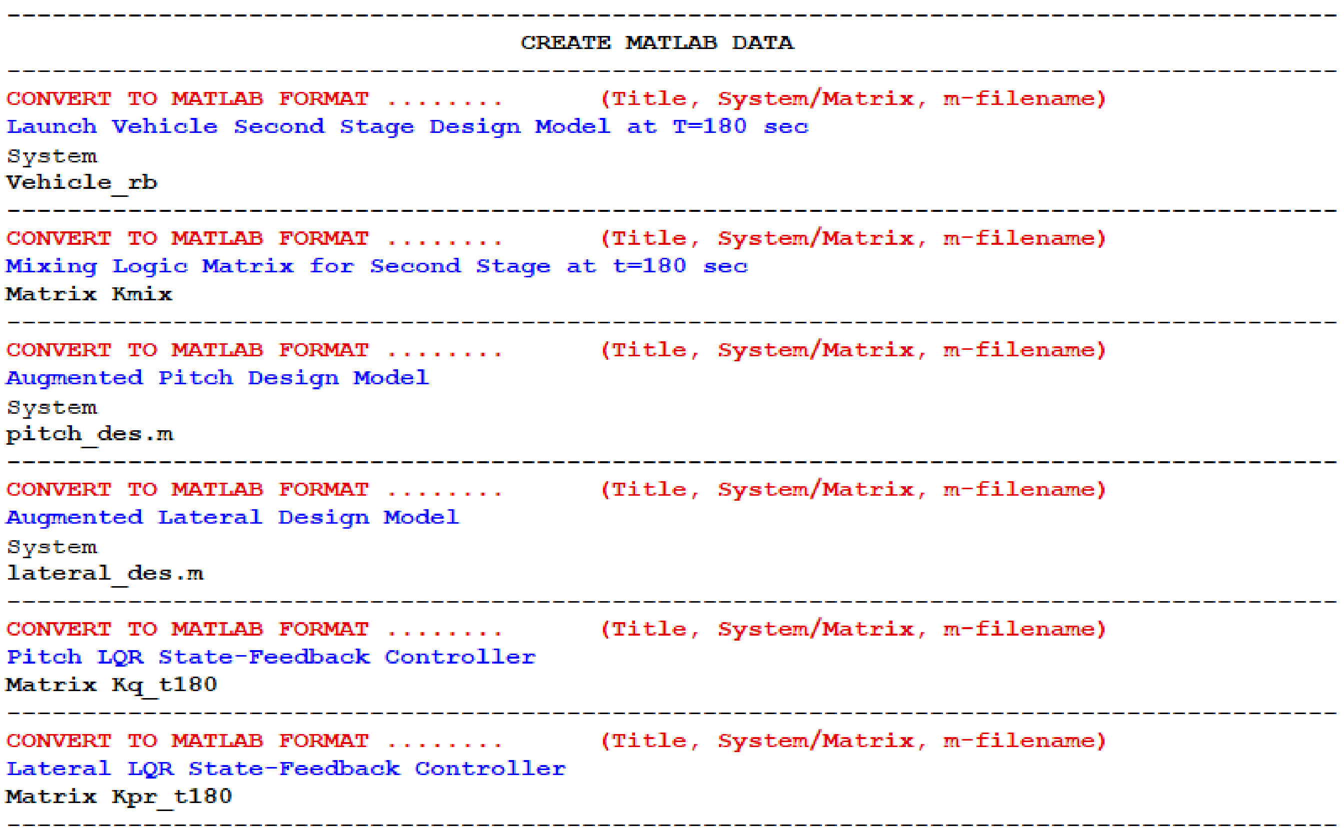
A similar process is repeated for the lateral design where the roll and yaw axes are combined and augmented to create the lateral design model “Augmented Lateral Design Model” that includes states: [ϕ, p, ψ, r, and ψ-integral]. The lateral LQR controller is a (2x5) state-feedback matrix Kpr_t180 that feeds back roll and yaw commands (DP, DR) from the 5 states. Qc5 and Rc2 are the state and control penalty weight matrices in the LQR optimization. Notice, there is a Matlab script “*LQR_Design.m*” that loads the pitch and lateral design plants and performs the LQR design in Matlab, and the results are identical to the Flixan process.

```
---------------------------------------------------------------------------------------------------------
                                LATERAL DESIGN
---------------------------------------------------------------------------------------------------------
CREATE A NEW SYSTEM FROM AN OLD SYSTEM... (Titles of the New and Old Systems)
Stage-2 Lateral Design Model
Vehicle and Mixing Logic Combined System
! The Lateral rigid body system is Extracted from the Coupled RB system above
!
TRUNCATE OR REORDER THE SYSTEM INPUTS, STATES, AND OUTPUTS
Extract Inputs :   1   3
Extract States :   1   2   5   6
Extract Outputs:   1   2   5   6
---------------------------------------------------------------------------------------------------------
INTERCONNECTION OF SYSTEMS .....
Augmented Lateral Design Model
! Create a 5-State Augmented Lateral Model that Includes Psi-Integral
!
Titles of Systems to be Combined
Title 1 Stage-2 Lateral Design Model
Title 2 Integrator
SYSTEM INPUTS TO SUBSYSTEM  1                                                  Lateral Design Model
System Input  1 to Subsystem  1, Input  1, Gain= 1.0                           Roll TVC Demand
System Input  2 to Subsystem  1, Input  2, Gain= 1.0                           Yaw  TVC Demand
.............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                               Vehicle Plant
System Output  1 from Subsystem  1, Output  1, Gain= 1.0                       Roll Attitude  (phi)
System Output  2 from Subsystem  1, Output  2, Gain= 1.0                       Roll Rate      (p-body)
System Output  3 from Subsystem  1, Output  3, Gain= 1.0                       Yaw  Attitude  (psi)
System Output  4 from Subsystem  1, Output  4, Gain= 1.0                       Yaw  Rate      (r-body)
.............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                                               Integrator
System Output  5 from Subsystem  2, Output  1, Gain= 1.0                       Psi-integral
.............................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2                                        Plant to Integrator
Subsystem  1, Output  3 to Subsystem  2, Input  1, Gain=   1.0000              Psi
.....................................................................
Definitions of Inputs  =   2
Roll TVC Command (DP_tvc)
Yaw  TVC Command (DR_tvc)

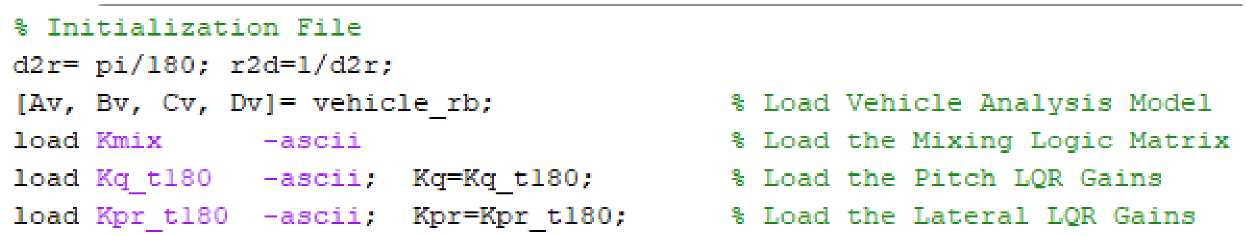
Definitions of Outputs =   5
Roll Attitude  (phi)     (radians)
Roll Rate      (p-body)  (rad/sec)
Yaw  Attitude  (psi)     (radians)
Yaw  Rate      (r-body)  (rad/sec)
Yaw, Psi-Integral        (rad-sec)

Definitions of States  =   5
Roll Attitude  (phi)     (radians)
Roll Rate      (p-body)  (rad/sec)
Yaw  Attitude  (psi)     (radians)
Yaw  Rate      (r-body)  (rad/sec)
Yaw, Psi-Integral        (rad-sec)
--------------------------------------------------------------------------------------------------------
LINEAR QUADRATIC REGULATOR STATE-FEEDBACK CONTROL DESIGN
Lateral LQR Control Design
Plant Model Used to Design the Control System from:        Augmented Lateral Design Model
Criteria Optimization Output is Matrix C
State Penalty Weight (Qc) is Matrix:    Qc5                Lateral State Weight Matrix Qc (5x5)
Control Penalty Weight (Rc)  is Matrix: Rc2                Lateral Control Weight Matrix Rc (2x2)
Continuous LQR Solution Using Laub Method
LQR State-Feedback Control Gain Matrix Kpr_t180            Lateral LQR State-Feedback Controller
--------------------------------------------------------------------------------------------------------
```

Matlab conversion datasets are included for the vehicle system “vehicle_rb”, the mixing-logic matrix “Kmix”, the two augmented design models “pitch_des” and “lateral_des”, and the two state-feedback matrices “Kq_t180” and “Kpr_t180”. They are loaded into Matlab and will be used to analyze the rigid vehicle stability and its response to step commands.



```
------------------------------------------------------------------------------------------
                                       CREATE MATLAB DATA
------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Launch Vehicle Second Stage Design Model at T=180 sec
System
Vehicle_rb
------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Mixing Logic Matrix for Second Stage at t=180 sec
Matrix Kmix
------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Augmented Pitch Design Model
System
pitch_des.m
------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Augmented Lateral Design Model
System
lateral_des.m
------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Pitch LQR State-Feedback Controller
Matrix Kq_t180
------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Lateral LQR State-Feedback Controller
Matrix Kpr_t180
------------------------------------------------------------------------------------------
```

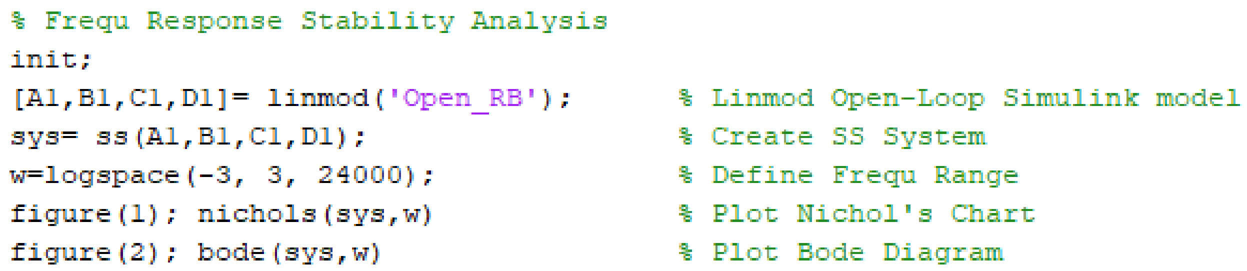


```
% Initialization File
d2r= pi/180; r2d=1/d2r;
[Av, Bv, Cv, Dv]= vehicle_rb;                % Load Vehicle Analysis Model
load Kmix      -ascii                        % Load the Mixing Logic Matrix
load Kq_t180   -ascii;  Kq=Kq_t180;          % Load the Pitch LQR Gains
load Kpr_t180  -ascii;  Kpr=Kpr_t180;        % Load the Lateral LQR Gains
```

Figure 2 Initialization File in Matlab

Frequency Response Analysis

The open-loop Simulink model "Open_RB.slx" in Figure 3.1.2a is used by the script "freq.m" to calculate the open-loop system frequency response and to plot the Bode and Nichols plots in 3.1.2b.



```
% Frequ Response Stability Analysis
init;
[A1,B1,C1,D1]= linmod('Open_RB');        % Linmod Open-Loop Simulink model
sys= ss(A1,B1,C1,D1);                    % Create SS System
w=logspace(-3, 3, 24000);                % Define Frequ Range
figure(1); nichols(sys,w)                % Plot Nichol's Chart
figure(2); bode(sys,w)                   % Plot Bode Diagram
```

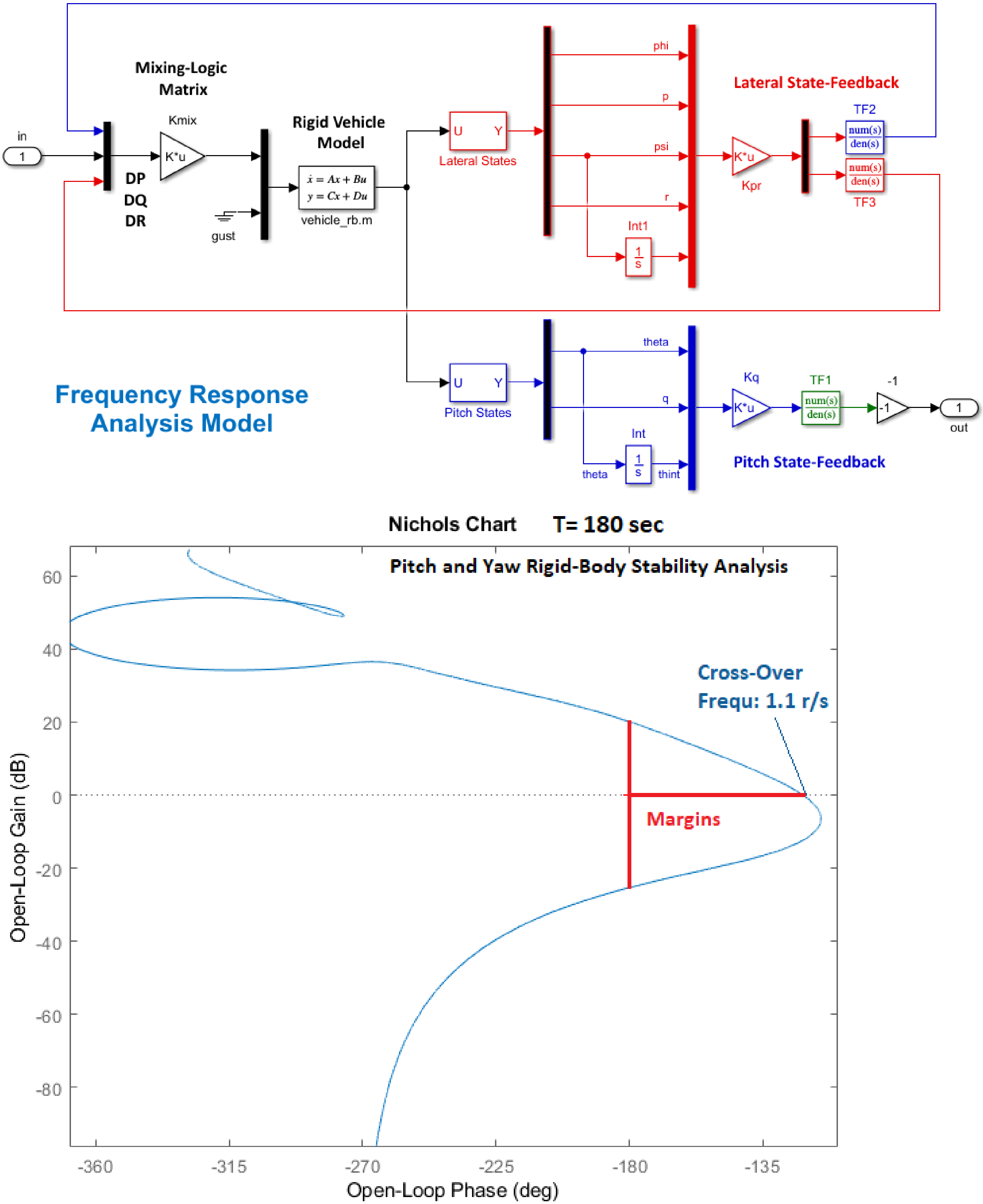


Figure 3.1.2 Open-Loop Analysis Model "Open_RB" and Nichols Plot Showing Phase and Gain Margins

Rigid-Body Simulation

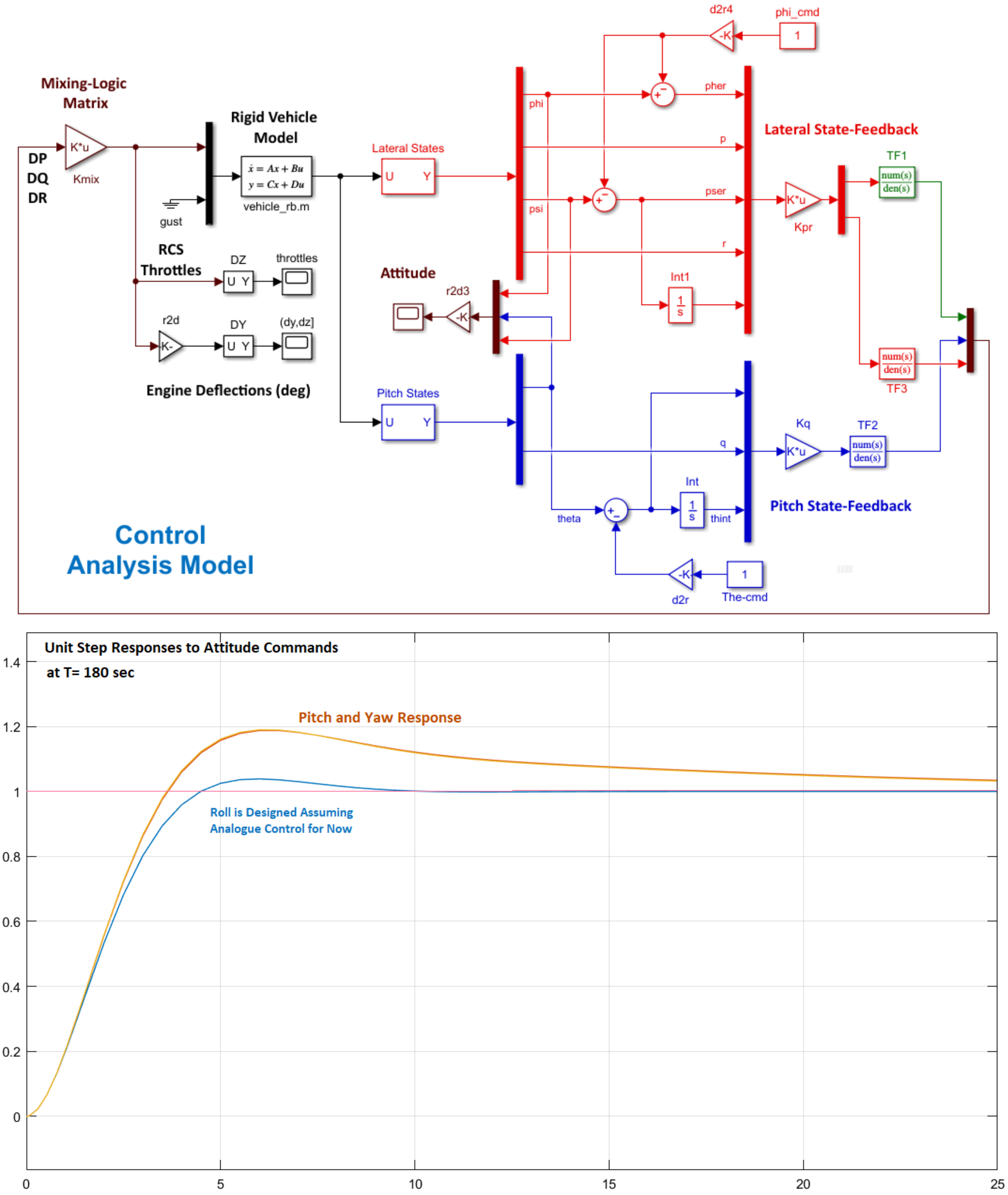


Figure 3.1.3 Simulink Model "RB_Sim.slx" is used to Analyze the Closed-Loop System's Step Response to Attitude Commands

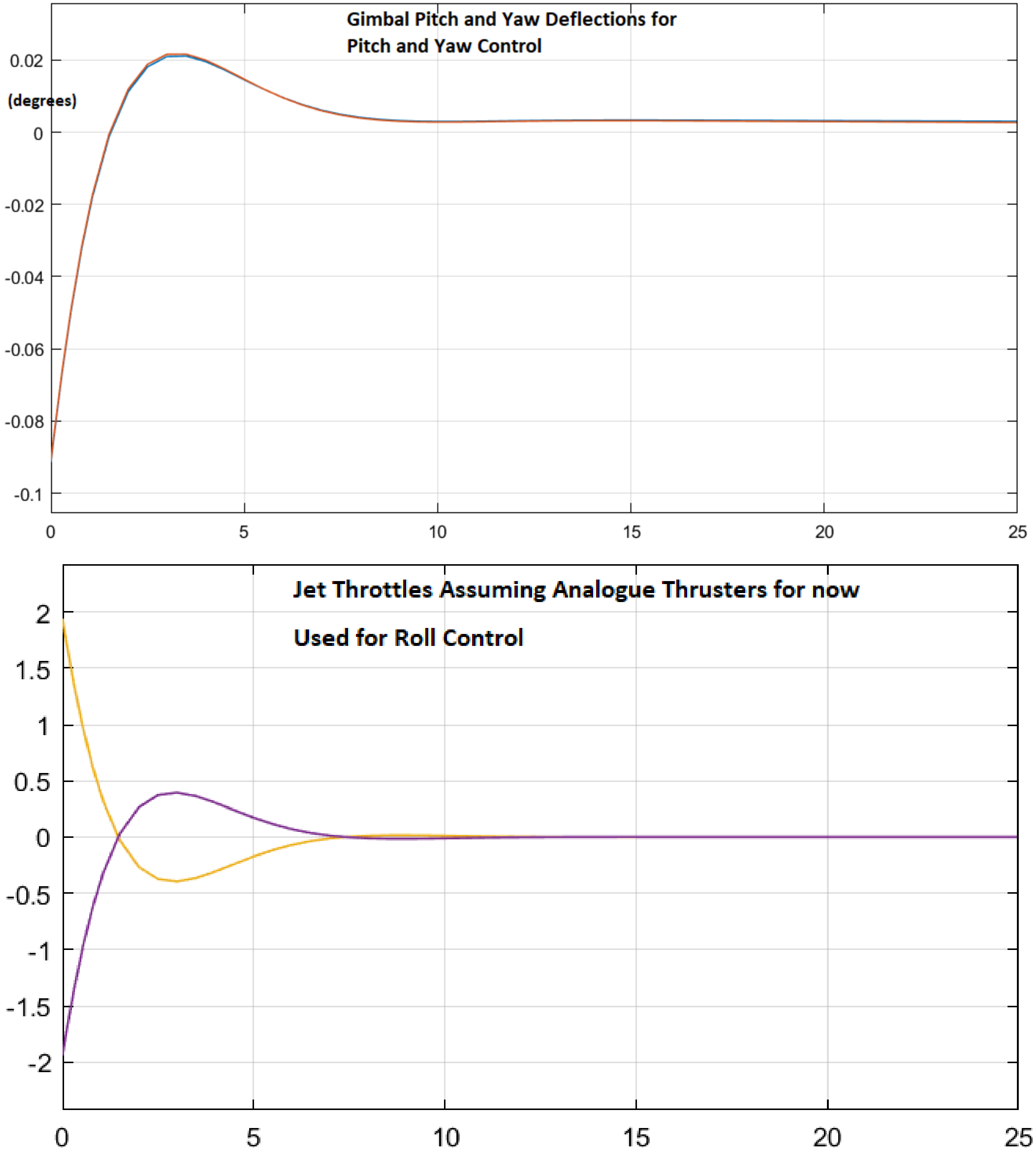
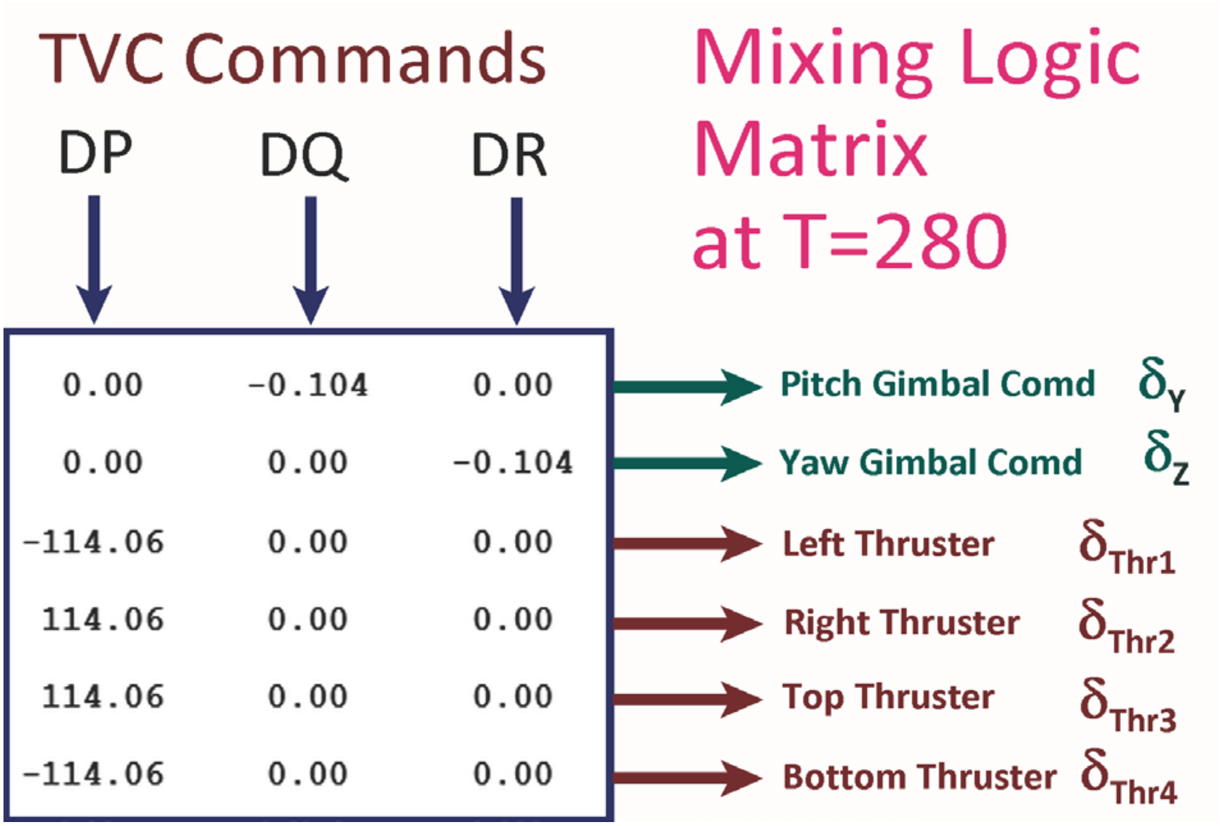


Figure 3.1.4 Gimbal and Throttle Responses to the 1° Step Commands in Roll, Pitch and Yaw Simultaneously

The Simulink model "RB_Sim.slx" in Figure 3.1.3 is used to analyze the rigid system's response to 1° attitude step commands in all 3 axes. The TVC engine is used for pitch and yaw control. The thrusters fire differentially for roll control. The roll LQR parameters should be readjusted to reduce the roll bandwidth and to prevent the throttles from exceeding ±1 but we don't care about roll in this point because it will be replaced in the analysis phase.

3.1.2 Control Design at T= 280 sec

At T=280 sec the design process is similar. The values of coefficients of the mixing logic matrix Kmix have changed because the vehicle mass properties have also changed. The bandwidth of the pitch and yaw LQR state-feedback controller is slightly higher at 1.3 r/s and the gain of the attitude trim integrator was also increased. The work files are in directory *"…\2-Control Gains Design\2nd Stage\T280"*. The pitch and lateral LQR state-feedback gains are in files Kq_t280 and Kpr_t280.



$$\begin{array}{ccc} DP & DQ & DR \\ 0.00 & -0.104 & 0.00 \\ 0.00 & 0.00 & -0.104 \\ -114.06 & 0.00 & 0.00 \\ 114.06 & 0.00 & 0.00 \\ 114.06 & 0.00 & 0.00 \\ -114.06 & 0.00 & 0.00 \end{array}$$

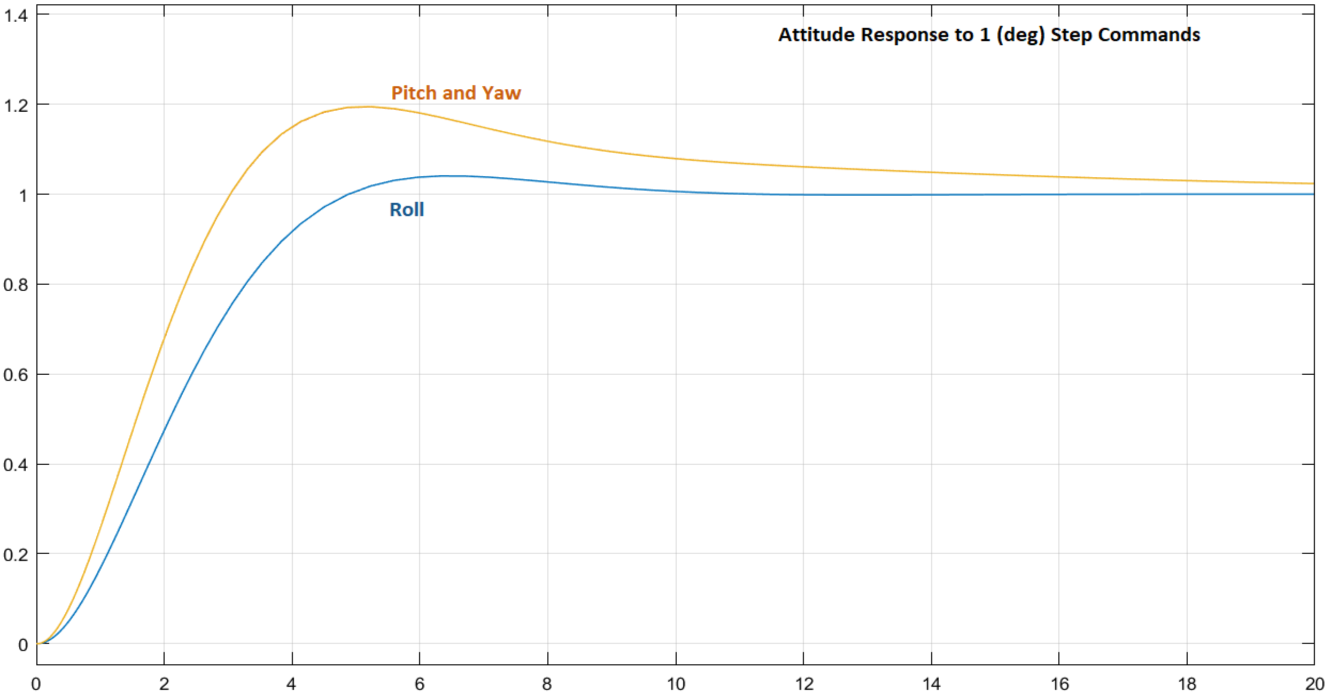


Figure 3.1.5 Attitude Response to 1 (deg) Step Commands in Roll, Pitch and Yaw

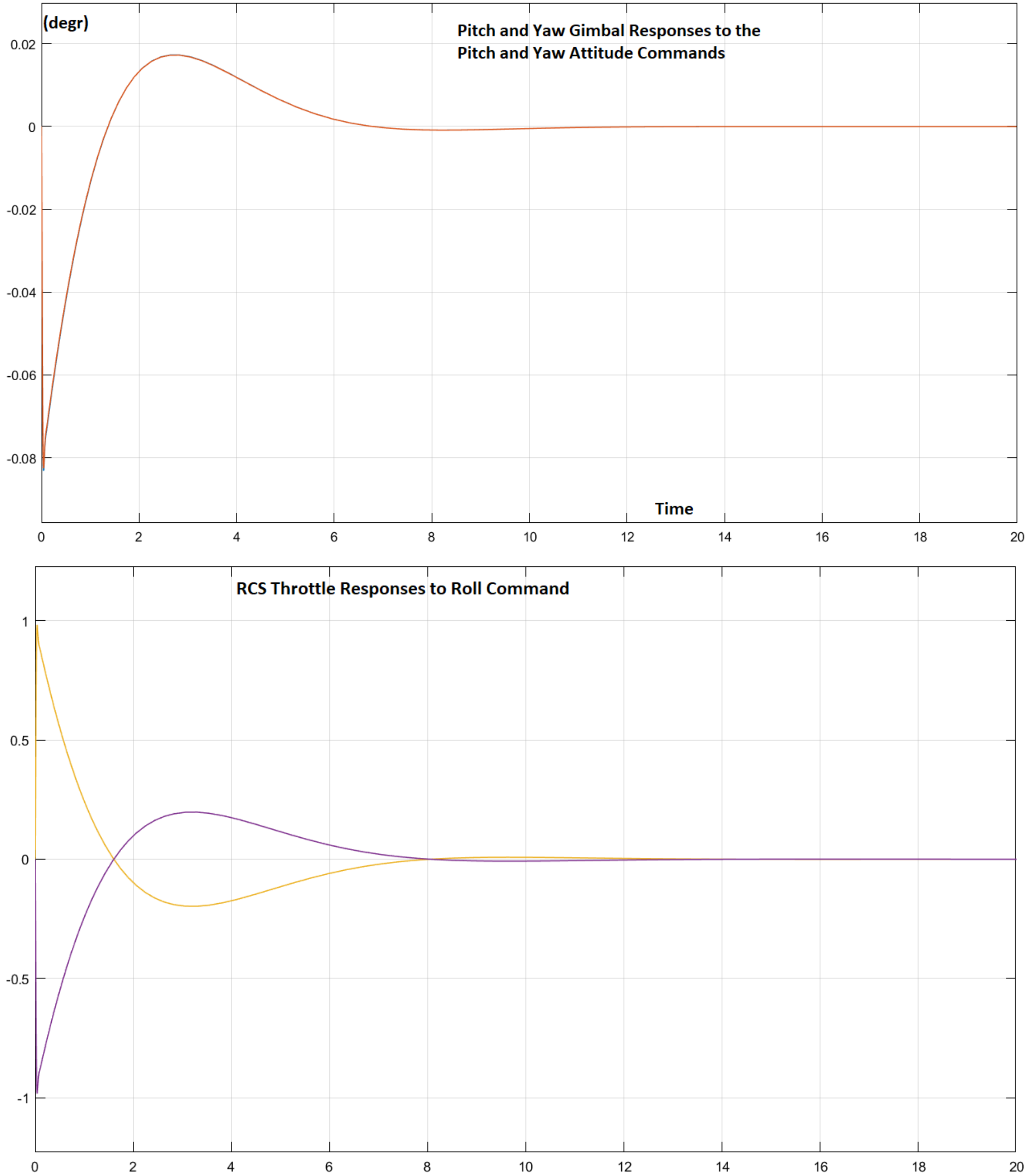


Figure 3.1.6 TVC Pitch and Yaw Gimbal Responses to Pitch and Yaw Attitude Commands (top). Also, the Throttle Responses of the RCS Jets (bottom). This time the Roll Gains were adjusted to keep the Magnitudes of the Throttle Commands to the thrusters less than 1.

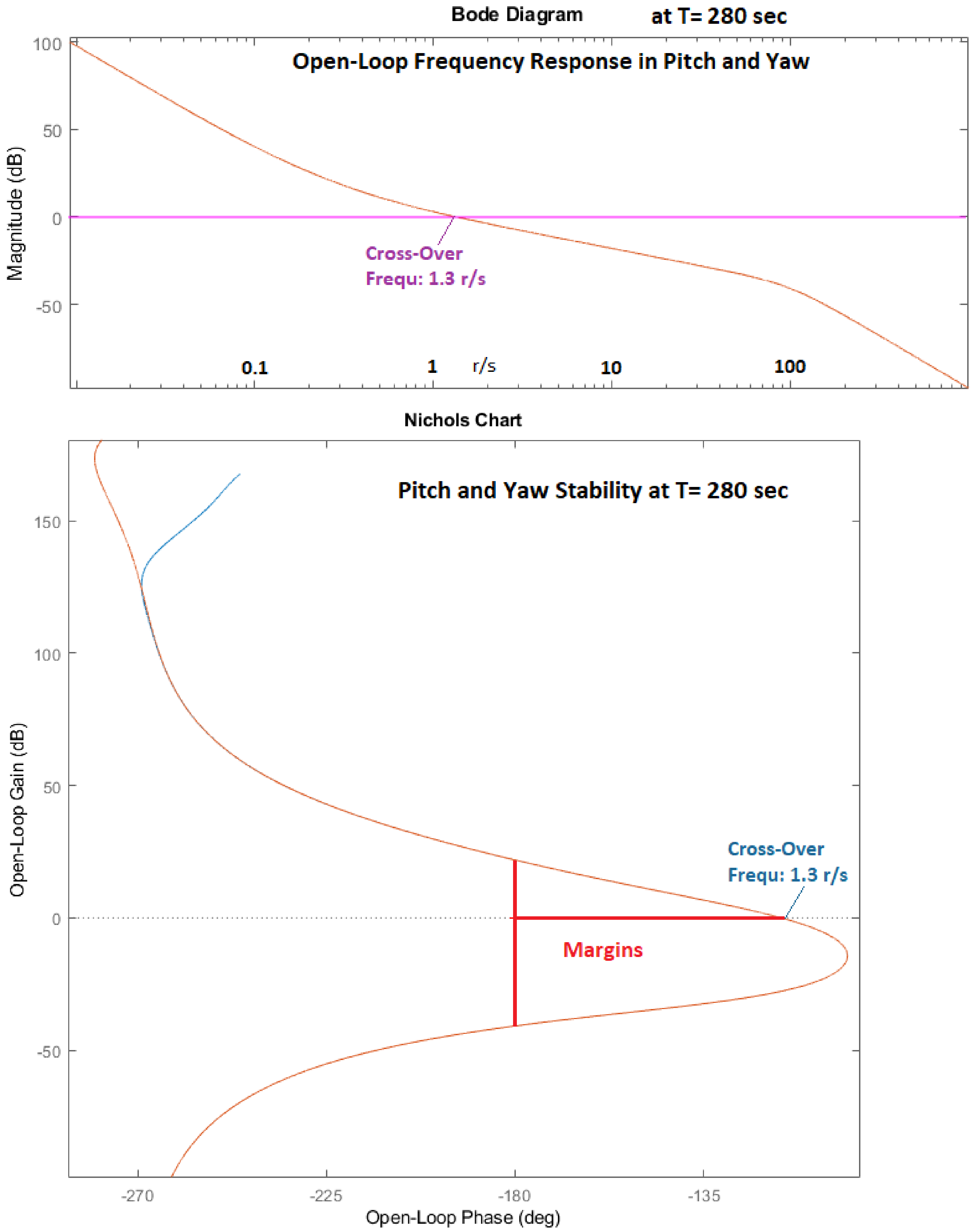


Figure 3.1.7 Pitch and Yaw Stability at T= 280 sec

3.1.3 Control Design at T= 490 sec

At T=490 sec the control design process does not change much because there is no aero. The values of coefficients of the mixing logic matrix Kmix are even smaller because the vehicle moments of inertia are reduced. The state-feedback gains however do not change much between flight conditions, only the mixing logic gains change. The LQR design plant includes the mixing logic matrix and it is almost the same during the entire 2nd stage. The mass properties variations are affecting the Kmix gains and not in the state-feedback. The bandwidth of the pitch and yaw LQR state-feedback controller however is a little higher this time, at 1.4 r/s and the attitude trim integrator gain was also increased. The work files are in directory *"…\2-Control Gains Design\2nd Stage\T490"*. The pitch and lateral LQR state-feedback gains are in files Kq_t490 and Kpr_t490 and they will also be used in the analysis model with flexibility and propellant sloshing in directory *"…\3-Stability Analysis with Flex & Slosh\2nd Stage\T490"*.

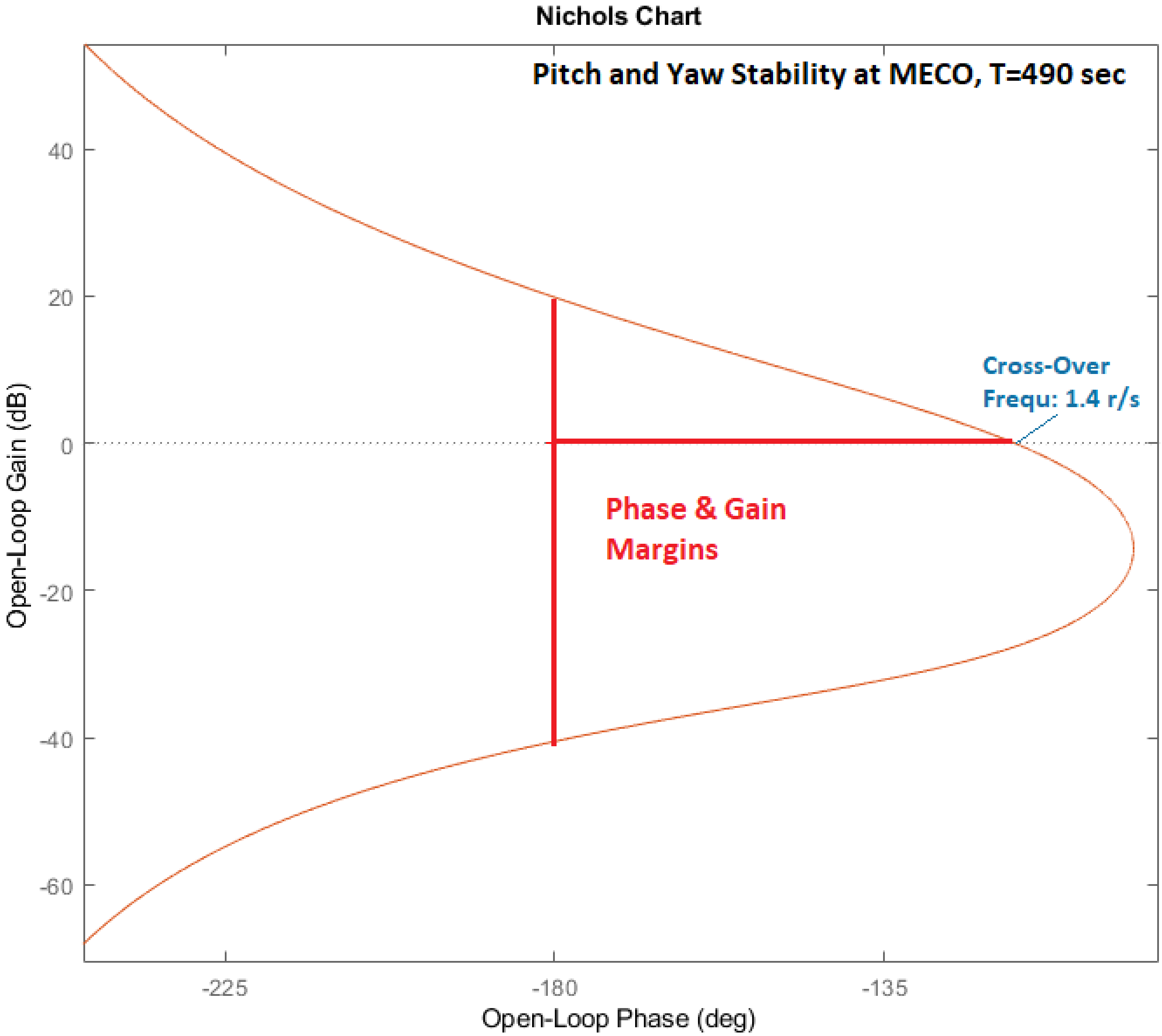
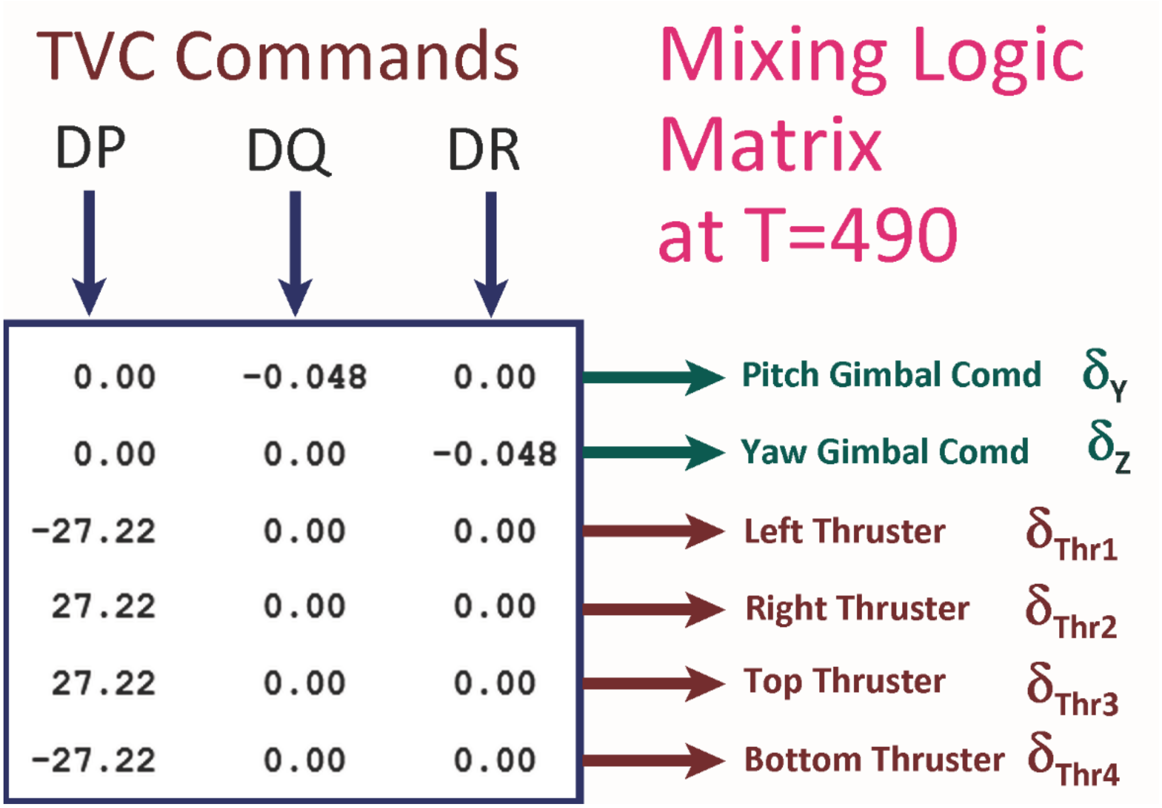


Figure 3.1.8 Pitch and Yaw Stability Margins at T= 490 sec



DP	DQ	DR
0.00	-0.048	0.00
0.00	0.00	-0.048
-27.22	0.00	0.00
27.22	0.00	0.00
27.22	0.00	0.00
-27.22	0.00	0.00

Figure 3.1.9 TVC/ RCS Mixing Logic Matrix at T= 490 sec

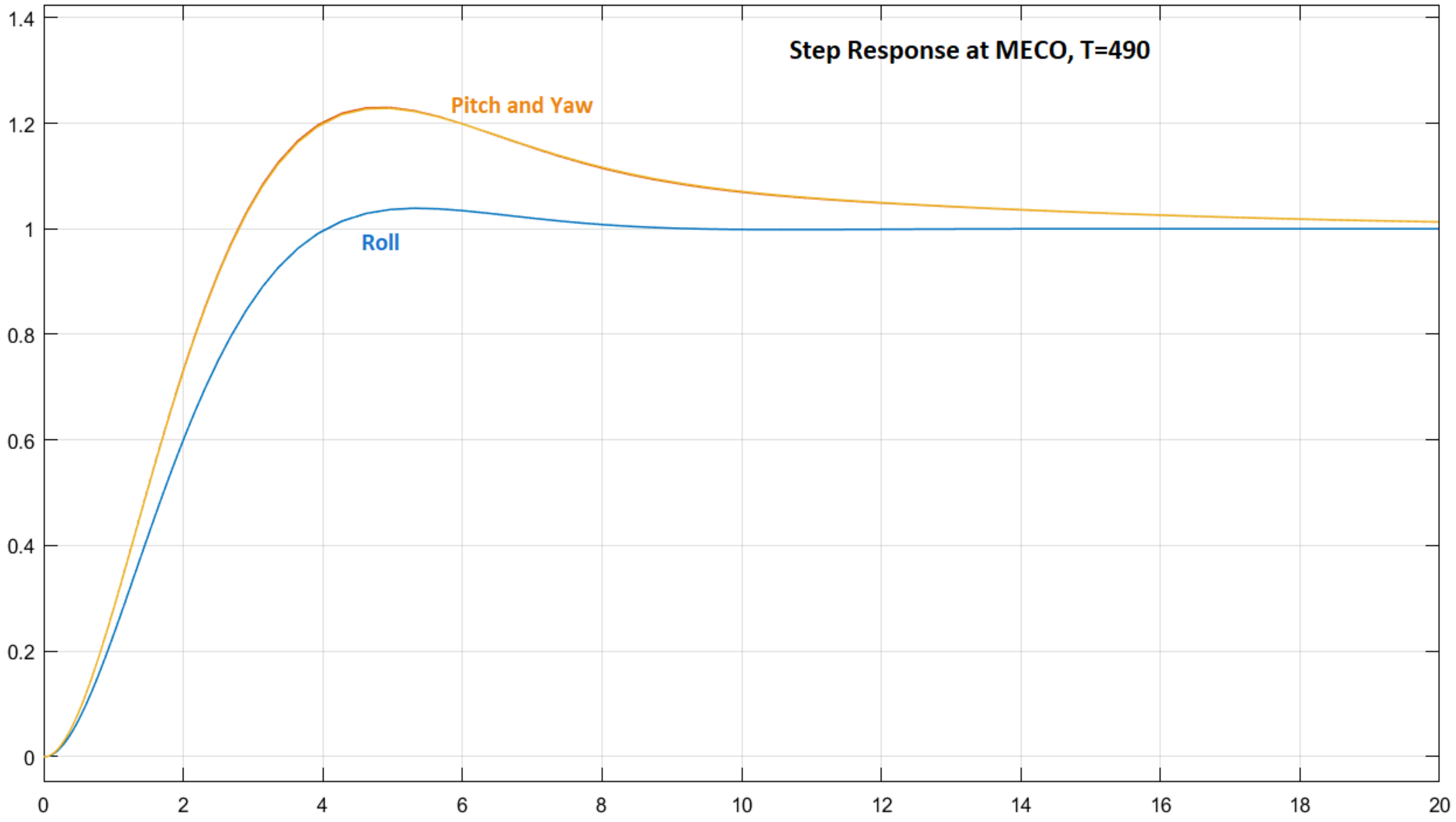


Figure 3.1.10 Attitude Response to 1 (deg) Step Commands in Roll, Pitch and Yaw

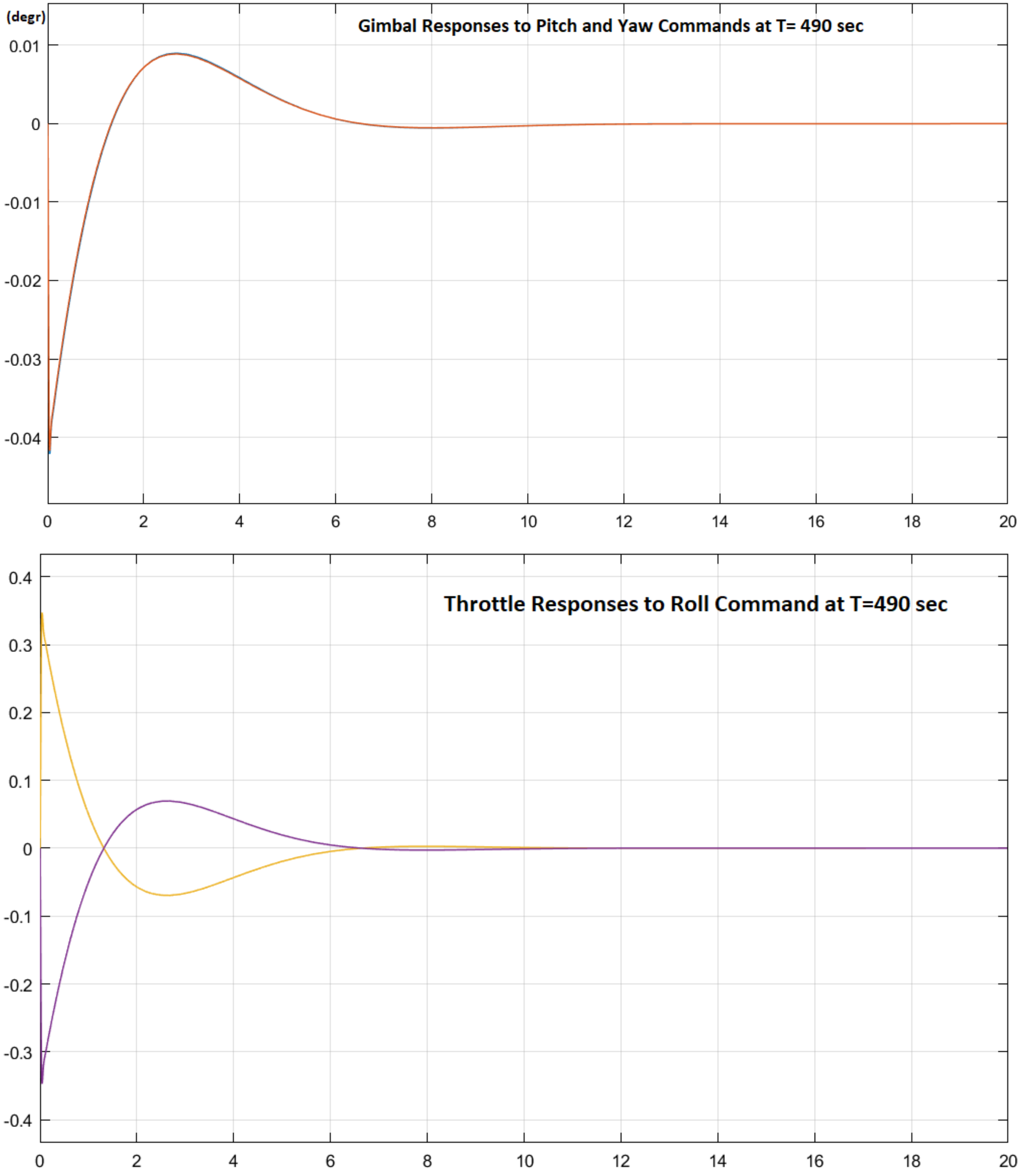


Figure 3.1.11 TVC Pitch and Yaw Gimbal Deflections to Pitch and Yaw Attitude Commands (top). Also, the Throttle Responses of the RCS Jets (bottom) which control roll. The Throttles this time are much smaller than ±1 which is the max capability because the roll inertia is smaller and requires less roll control. The Gimbal Deflections are also smaller.

3.2 Second Stage Control Analysis with Slosh and Flexibility

We will now update the vehicle models created for the 5 time-slices: T180, T280, T350, T420, and T490, to analyze the system performance in those flight conditions, including propellant sloshing and structural flexibility. The 5 analysis folders are in directory “23-Classic Launch Vehicle Design & Simulation\3-Stability Analysis with Flex & Slosh\2nd Stage”. The analyses of the 5 flight conditions are very similar and we will describe in detail only the first one. The vehicle datasets now include the slosh parameters and the flex modes which consist of preselected modes at different fuel weights and they included at the bottom of the input files. The lateral roll/ yaw coupled LQR control system from the design section will now be replaced by a yaw system which is identical to pitch, and a phase-plane RCS system for roll control with a jet-selection logic that activates the 4 bi-directional jets in “on-off” mode rather than continuous (analog) thrust forces.

3.2.1 Analysis at Post-Separation, T= 180 sec

The analysis files for this flight condition are in “*Examples\23-Classic Launch Vehicle Design & Simulation\3-Stability Analysis with Flex & Slosh\2nd Stage\T180*”. The input file is “*Flex_Vehi_T180.inp*”. The LQR state-feedback gains Kq_t180= [0.57, 1.08, 0.035] will be used for both pitch and yaw control, feeding back: attitude, rate, and attitude integral. The modal data and node ID files at T180 are “*Stage2_100%.Mod*” and “*Stage2_100%.Nod*” respectively. They are created from finite element models when both tanks are full. They are used by the Flixan mode selection process to select and scale a set of structural modes that will be combined with the rigid vehicle data and create the flex vehicle state-space system for the analysis. The mode-selection process for the T180 case is described in detail in Section 3.3. The selected set of modes is included in file “*Flex_Vehi_T180.inp*” under the title “*Vehicle Second Stage Flex Modes with 100% Full Tanks*”.

The title of the vehicle data is “*Launch Vehicle Second Stage Analysis Model, T=180 sec*”. It includes the gimbaling engine and the 4 bidirectional thrusters, ±3 (lbf) each, which in fact they are 8 jets as shown in Figure 3.1. The data include propellant sloshing parameters for the LOX and LH2 tanks, such as: the 2 slosh masses which are small in this post-separation time-slice, slosh frequencies for each mass along y and z at 1g, the damping coefficients along y and z, and the x, y, z locations of the 2 slosh masses. The LOX and LH2 slosh frequencies are both 3.12 rad/sec at 1g acceleration. They are scaled by the program proportionally to the square root of the vehicle acceleration. Although the 2 propellant frequencies are defined to be the same at 1g, they change slightly under closed-loop control.

The number of modes to be included from the selected set of modes is also shown at the bottom of the vehicle dataset. The modal data set typically includes more modes than needed for the analysis. The input file also includes a dataset that creates a linear actuator model for second stage. The parameters are a little different from 1st stage, such as, inertia, friction, etc. A non-linear actuator Simulink model will be used in the simulation. The non-linearities are due to Coulomb friction at the gimbal, position and rate limits. A batch set is included at the top of the input file that can process the vehicle and actuator data faster in batch mode and it exports the state-space systems in Matlab format.

```
BATCH MODE INSTRUCTIONS ...............
Batch for calculating the Launch Vehicle systems during Second Stage
! This batch set creates state-space systems for the Flexible Launch Vehicle during
! Second Stage and also for the 2nd Stage Actuator.
!
Flight Vehicle   : Launch Vehicle Second Stage Analysis Model, T=180 sec
Actuator Model   : Stage-2 Linear Actuator
!
To Matlab Format : Launch Vehicle Second Stage Analysis Model, T=180 sec
To Matlab Format : Stage-2 Linear Actuator
----------------------------------------------------------------------------------------------------------------------------
FLIGHT VEHICLE INPUT DATA ......
Launch Vehicle Second Stage Analysis Model, T=180 sec
! This Model is for the Launch Vehicle's Second Stage consisting of the Main Engine
! and four bi-directional RCS jets. It includes Slosh and Structural Flexibility
!
Body Axes Output, Attitude=Rate Integral

Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft)    :  1040.75      32.1740      0.208960E+08
Moments and Products of Inertia: Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)          :  8052.60      41988.1       41808.7      0.00000      0.00000
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)        :  93.3240      0.00000       0.00000
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)       :  8.16600      7642.13      0.730000      269948.0
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)         :  15.3018      28.5736    -0.136000E-01 -0.900000E-03
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                         :  0.0000       0.0          0.0          0.0
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)   :  0.000        25.5860      0.00000      -0.0          -0.436000
W-Gust Azim & Elev angles (deg), or Torque/Force direction (x,y,z), Force Locat (x,y,z)  :  Gust        45.00000     90.00000
Surface Reference Area (feet^2), Mean Aerodynamic Chord (ft), Wing Span in (feet)        :  38.5000      7.20000      7.20000
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial_rho/ Partial_H} :  116.800      0.00000      0.00000      0.00000
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:  0.4251       0.00000    -0.141175E-04  0.00000      0.00000
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}: -0.091747    -0.036        0.00000      0.00000      0.00000
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}: -0.0067816   -0.036        0.00000      0.00000      0.00000
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:  0.00000      0.00000      0.00000      0.00000      0.00000
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}: -0.004773    -0.04         0.00000      0.00000      0.00000
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}:  0.0          0.04         0.00000      0.00000      0.00000

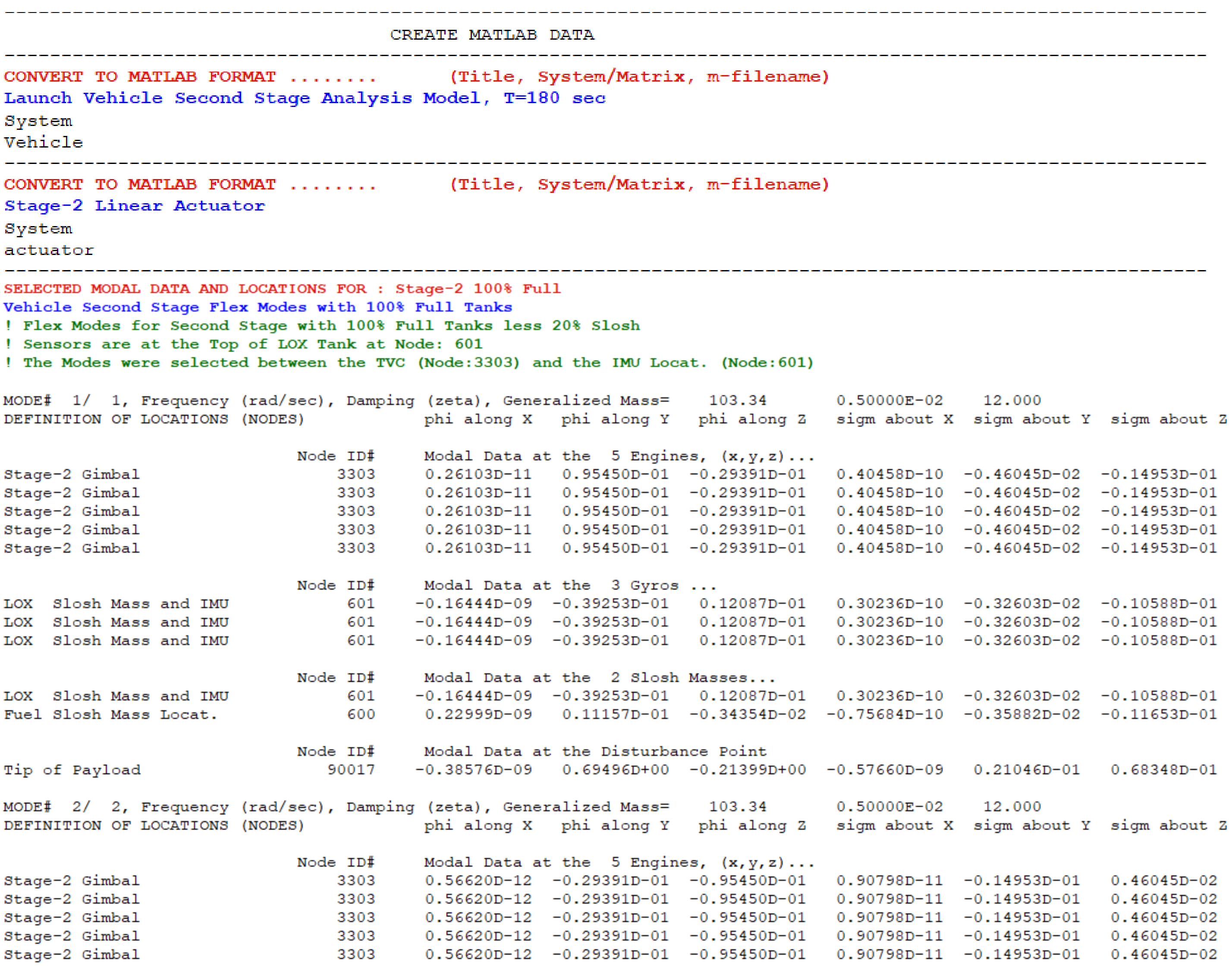
Number of Thruster Engines, Include or Not the Tail-Wags-Dog and Load-Torque Dynamics ?  :  5   WITH TWD

TVC Engine No:  1                              (Gimbaling  Throttling   Single_Gimbal) : Main Engine      Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                   :  29750.0       29750.0
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000       0.00000       6.00000       6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  9.00000       120.000       2.60000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  84.5000       0.00000       0.00000
TVC Engine No:  2                              (Gimbaling  Throttling   Single_Gimbal) : Left RCS Jet     Throttling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                   :  0.00000       3.0
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg): -90.00         0.00000       0.00000       0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  0.00000       0.00000       0.00000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  83.00        -3.50         0.00000
TVC Engine No:  3                              (Gimbaling  Throttling   Single_Gimbal) : Rght RCS Jet     Throttling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                   :  0.00000       3.0
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg): -90.00         0.00000       0.00000       0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  0.00000       0.00000       0.00000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  83.00         3.50         0.00000
TVC Engine No:  4                              (Gimbaling  Throttling   Single_Gimbal) : Top  RCS Jet     Throttling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                   :  0.00000       3.0
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.0000        90.000        0.00000       0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  0.00000       0.00000       0.00000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  83.00         0.00000      -3.50
TVC Engine No:  5                              (Gimbaling  Throttling   Single_Gimbal) : Botm RCS Jet     Throttling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                   :  0.00000       3.0
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.0000        90.000        0.00000       0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  0.00000       0.00000       0.00000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  83.00         0.00000       3.50

Number of Gyros, (Attitude and Rate)                                                     :  3
Gyro No  1 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Location in (feet) :  Roll   Rate            97.438     0.00       0.00
Gyro No  2 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Location in (feet) :  Pitch  Rate            97.438     0.00       0.00
Gyro No  3 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Location in (feet) :  Yaw    Rate            97.438     0.00       0.00

Number of Slosh Modes                                                                    :  2
LOX Mass (slug), Frequenc 1g (Wy,Wz) (rad/s), Damp (zeta-y-z), Locat.{Xsl,Ysl,Zsl} (ft)  :  3.1    3.12  3.12   0.001  0.001   97.08  0.0  0.0
LH2 Mass (slug), Frequenc 1g (Wy,Wz) (rad/s), Damp (zeta-y-z), Locat.{Xsl,Ysl,Zsl} (ft)  :  1.2    3.12  3.12   0.001  0.001   89.65  0.0  0.0

Number of Bending Modes                                                                  : 10
Vehicle Second Stage Flex Modes with 100% Full Tanks
----------------------------------------------------------------------------------------------------------------------------
ACTUATOR INPUT DATA ............ SIMPLE GENERIC MODEL B
Stage-2 Linear Actuator
Symbol Parameter Description                   (Units)       Value
C(s)   Order of Pade Delay (0,1,2)              ( - )         1, -0.001, 0.001
Ka     Gain of Amplifier ..................... (amps/volt)    8.0
Wsv    Bandwidth of the Linear Servo Actuator .  (rad/sec)    50.0
Kact   Actuator Stiffness (Piston+Oil+Electric)   (lb/ft)    2400000.0
Klod   Stiffness at Surface or Nozzle Connection  (lb/ft)    1.0e+09
Kbck   Stiffness at Vehicle Backup Structure ..   (lb/ft)    1130000.0
R      Moment Arm between Actuator Rod & Gimbal    (feet)     0.72
Jl     Load Inertia about the Gimbal ......... (ft-lb-s^2)    120.0
Kg     Load Gimbal Bearing Spring Constant     (ft-lb/rad)    0.0
Bg     Load Gimbal Bearing Viscous Damping .... (ft-lb-sec)   3500.0
----------------------------------------------------------------------------------------------------------------------------
```



```
------------------------------------------------------------------------------------------------
                                  CREATE MATLAB DATA
------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Launch Vehicle Second Stage Analysis Model, T=180 sec
System
Vehicle
------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Stage-2 Linear Actuator
System
actuator
------------------------------------------------------------------------------------------------
SELECTED MODAL DATA AND LOCATIONS FOR : Stage-2 100% Full
Vehicle Second Stage Flex Modes with 100% Full Tanks
! Flex Modes for Second Stage with 100% Full Tanks less 20% Slosh
! Sensors are at the Top of LOX Tank at Node: 601
! The Modes were selected between the TVC (Node:3303) and the IMU Locat. (Node:601)

MODE#  1/  1, Frequency (rad/sec), Damping (zeta), Generalized Mass=    103.34       0.50000E-02    12.000
DEFINITION OF LOCATIONS (NODES)               phi along X    phi along Y    phi along Z    sigm about X   sigm about Y   sigm about Z

                                 Node ID#     Modal Data at the  5 Engines, (x,y,z)...
Stage-2 Gimbal                     3303       0.26103D-11    0.95450D-01   -0.29391D-01    0.40458D-10   -0.46045D-02   -0.14953D-01
Stage-2 Gimbal                     3303       0.26103D-11    0.95450D-01   -0.29391D-01    0.40458D-10   -0.46045D-02   -0.14953D-01
Stage-2 Gimbal                     3303       0.26103D-11    0.95450D-01   -0.29391D-01    0.40458D-10   -0.46045D-02   -0.14953D-01
Stage-2 Gimbal                     3303       0.26103D-11    0.95450D-01   -0.29391D-01    0.40458D-10   -0.46045D-02   -0.14953D-01
Stage-2 Gimbal                     3303       0.26103D-11    0.95450D-01   -0.29391D-01    0.40458D-10   -0.46045D-02   -0.14953D-01

                                 Node ID#     Modal Data at the  3 Gyros ...
LOX  Slosh Mass and IMU             601      -0.16444D-09   -0.39253D-01    0.12087D-01    0.30236D-10   -0.32603D-02   -0.10588D-01
LOX  Slosh Mass and IMU             601      -0.16444D-09   -0.39253D-01    0.12087D-01    0.30236D-10   -0.32603D-02   -0.10588D-01
LOX  Slosh Mass and IMU             601      -0.16444D-09   -0.39253D-01    0.12087D-01    0.30236D-10   -0.32603D-02   -0.10588D-01

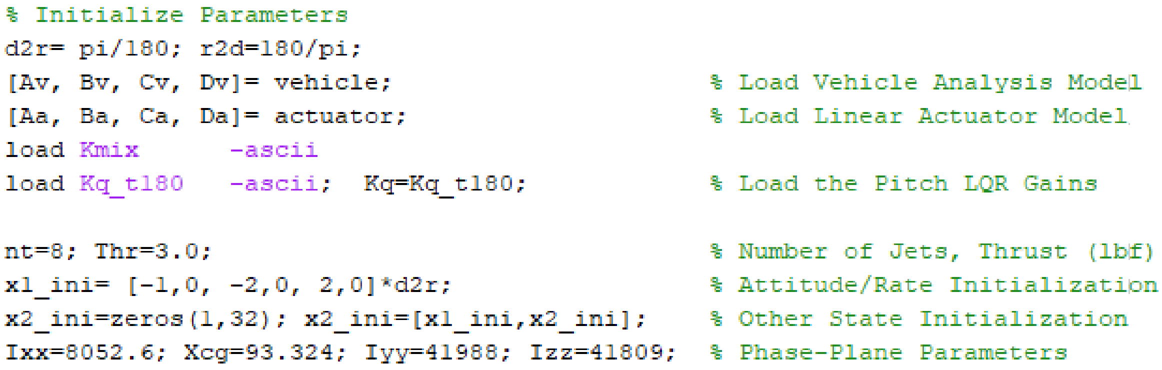
                                 Node ID#     Modal Data at the  2 Slosh Masses...
LOX  Slosh Mass and IMU             601      -0.16444D-09   -0.39253D-01    0.12087D-01    0.30236D-10   -0.32603D-02   -0.10588D-01
Fuel Slosh Mass Locat.              600       0.22999D-09    0.11157D-01   -0.34354D-02   -0.75684D-10   -0.35882D-02   -0.11653D-01

                                 Node ID#     Modal Data at the Disturbance Point
Tip of Payload                    90017      -0.38576D-09    0.69496D+00   -0.21399D+00   -0.57660D-09    0.21046D-01    0.68348D-01

MODE#  2/  2, Frequency (rad/sec), Damping (zeta), Generalized Mass=    103.34       0.50000E-02    12.000
DEFINITION OF LOCATIONS (NODES)               phi along X    phi along Y    phi along Z    sigm about X   sigm about Y   sigm about Z

                                 Node ID#     Modal Data at the  5 Engines, (x,y,z)...
Stage-2 Gimbal                     3303       0.56620D-12   -0.29391D-01   -0.95450D-01    0.90798D-11   -0.14953D-01    0.46045D-02
Stage-2 Gimbal                     3303       0.56620D-12   -0.29391D-01   -0.95450D-01    0.90798D-11   -0.14953D-01    0.46045D-02
Stage-2 Gimbal                     3303       0.56620D-12   -0.29391D-01   -0.95450D-01    0.90798D-11   -0.14953D-01    0.46045D-02
Stage-2 Gimbal                     3303       0.56620D-12   -0.29391D-01   -0.95450D-01    0.90798D-11   -0.14953D-01    0.46045D-02
Stage-2 Gimbal                     3303       0.56620D-12   -0.29391D-01   -0.95450D-01    0.90798D-11   -0.14953D-01    0.46045D-02
```

The following initialization file “init.m” loads the Flixan generated vehicle and actuator systems, the mixing-logic matrix, and the LQR control gains into Matlab. Also, the vehicle mass properties which are needed in the RCS phase-plane logic.



```
% Initialize Parameters
d2r= pi/180; r2d=180/pi;
[Av, Bv, Cv, Dv]= vehicle;                   % Load Vehicle Analysis Model
[Aa, Ba, Ca, Da]= actuator;                  % Load Linear Actuator Model
load Kmix      -ascii
load Kq_t180   -ascii;  Kq=Kq_t180;          % Load the Pitch LQR Gains

nt=8; Thr=3.0;                               % Number of Jets, Thrust (lbf)
x1_ini= [-1,0, -2,0, 2,0]*d2r;               % Attitude/Rate Initialization
x2_ini=zeros(1,32); x2_ini=[x1_ini,x2_ini];  % Other State Initialization
Ixx=8052.6; Xcg=93.324; Iyy=41988; Izz=41809; % Phase-Plane Parameters
```

Simulation Model

The simulation model is "*NonLinear_Sim.slx*" shown in Figure 3.2.1. It consists of the vehicle second stage model which is shown in detail in Figure 3.2.3, the non-linear actuators is in Figure 3.2.4, the TVC flight control system is in Figure 3.2.2, and the phase-plane RCS for roll control is in Figure 3.2.5. The vehicle attitude is initialized at a non-zero initial state [-1, 2, -2] and it is commanded to go to [+5, -5, +5] (deg) in roll, pitch and yaw, respectively.

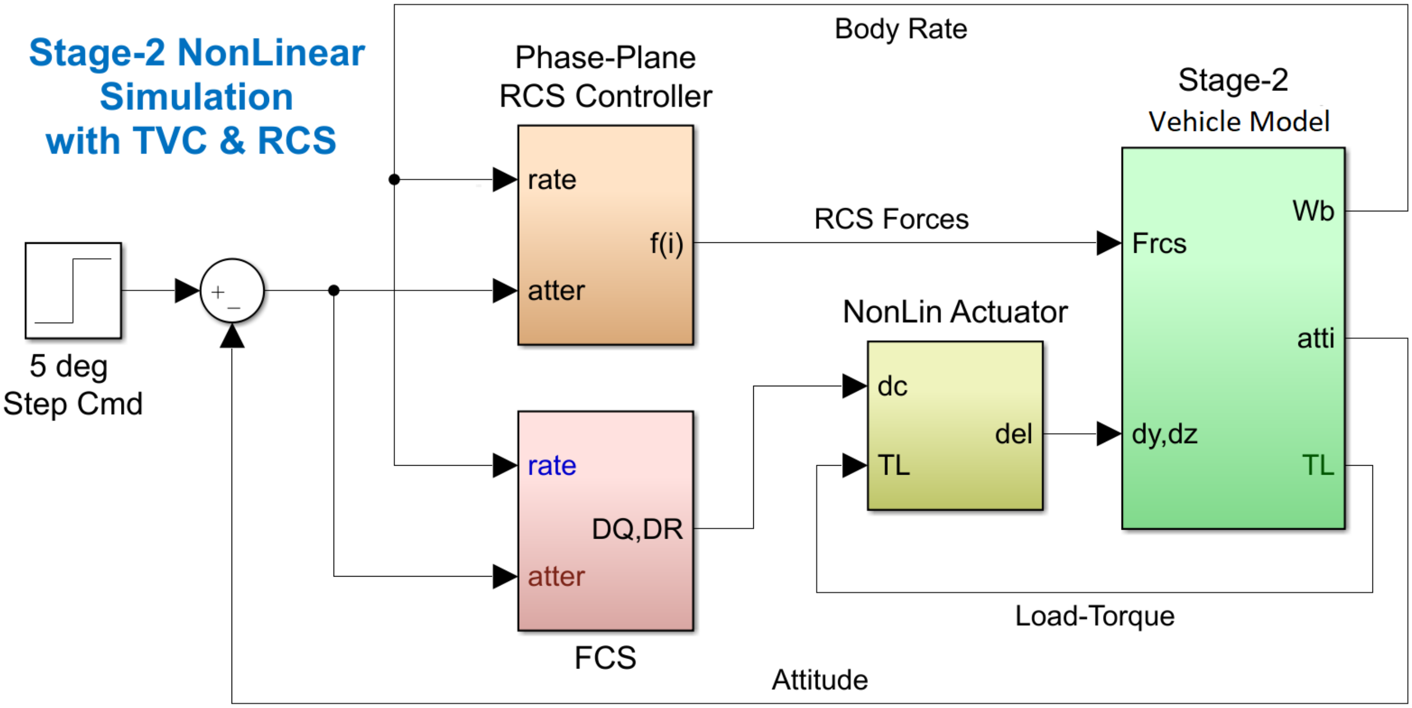


Figure 3.2.1 Second Stage Simulation Model "NonLinear_Sim.slx"

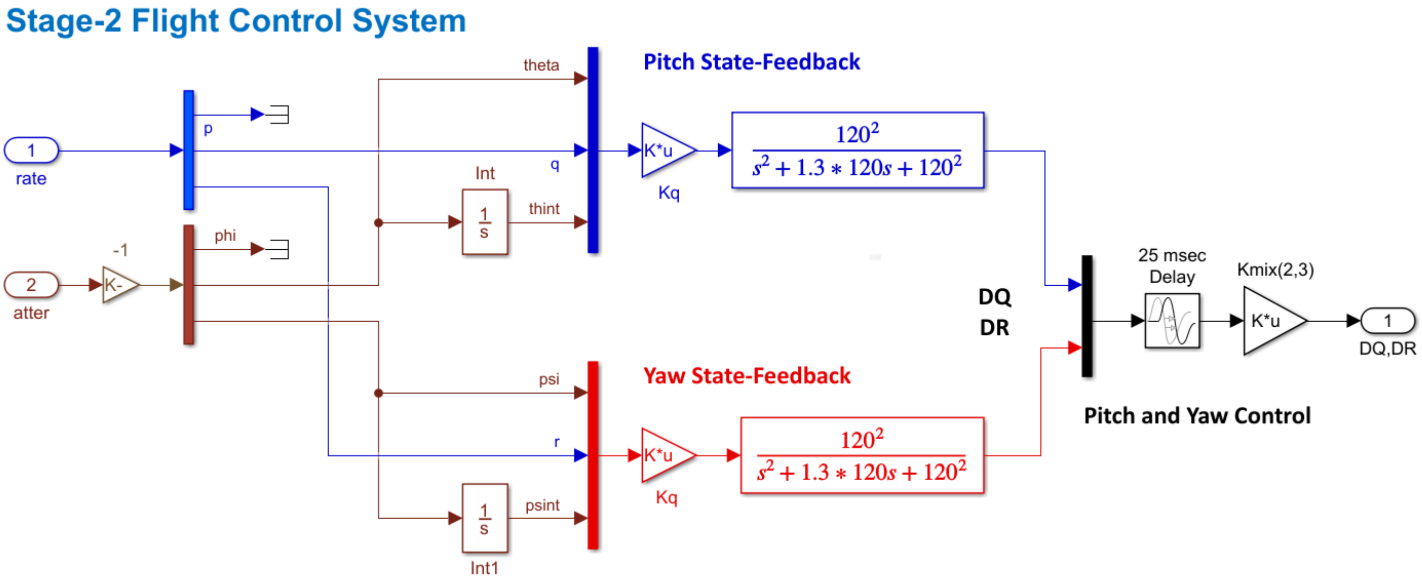
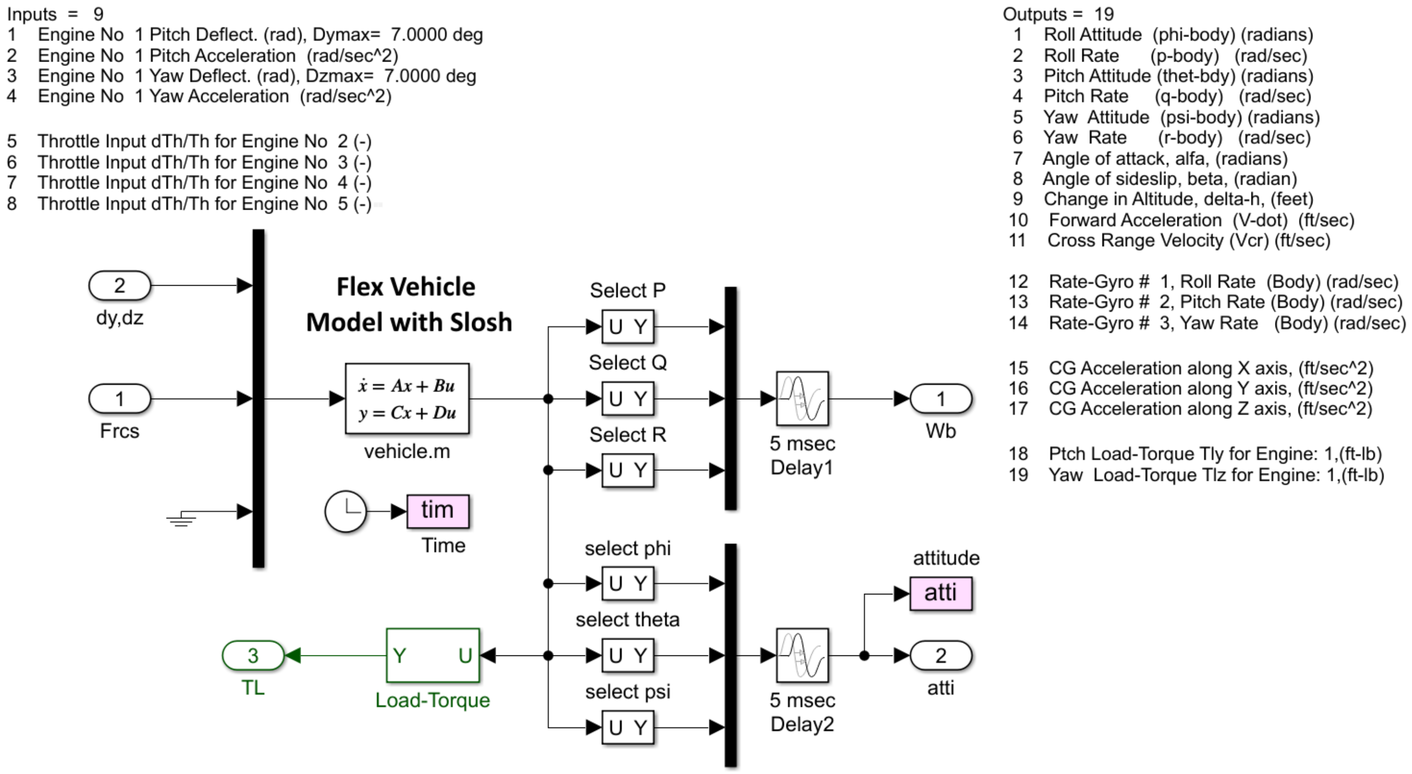


Figure 3.2.2 Pitch and Yaw State-Feedback Control System



Inputs = 9
1 Engine No 1 Pitch Deflect. (rad), Dymax= 7.0000 deg
2 Engine No 1 Pitch Acceleration (rad/sec^2)
3 Engine No 1 Yaw Deflect. (rad), Dzmax= 7.0000 deg
4 Engine No 1 Yaw Acceleration (rad/sec^2)

5 Throttle Input dTh/Th for Engine No 2 (-)
6 Throttle Input dTh/Th for Engine No 3 (-)
7 Throttle Input dTh/Th for Engine No 4 (-)
8 Throttle Input dTh/Th for Engine No 5 (-)

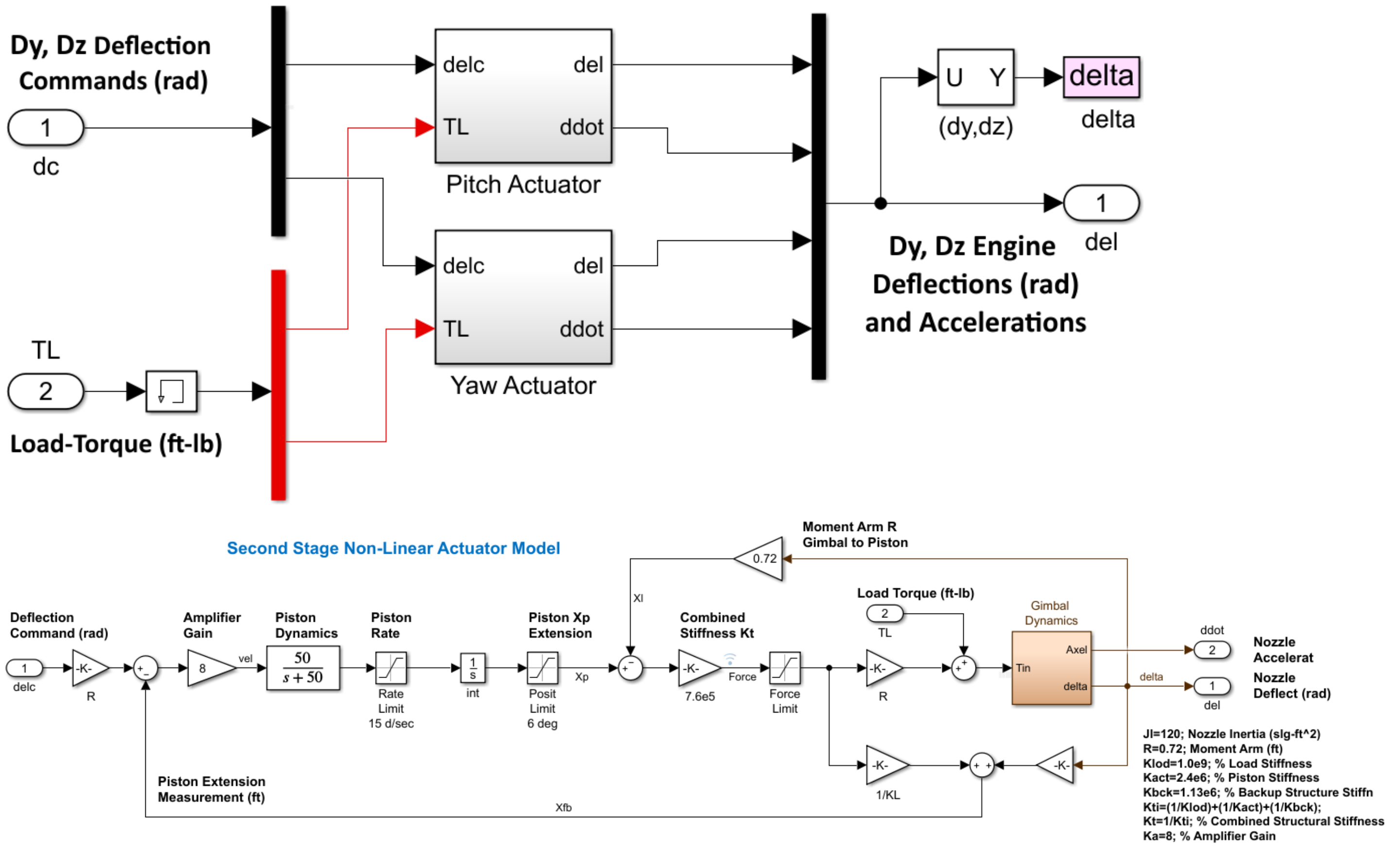
Outputs = 19
1 Roll Attitude (phi-body) (radians)
2 Roll Rate (p-body) (rad/sec)
3 Pitch Attitude (thet-bdy) (radians)
4 Pitch Rate (q-body) (rad/sec)
5 Yaw Attitude (psi-body) (radians)
6 Yaw Rate (r-body) (rad/sec)
7 Angle of attack, alfa, (radians)
8 Angle of sideslip, beta, (radian)
9 Change in Altitude, delta-h, (feet)
10 Forward Acceleration (V-dot) (ft/sec)
11 Cross Range Velocity (Vcr) (ft/sec)

12 Rate-Gyro # 1, Roll Rate (Body) (rad/sec)
13 Rate-Gyro # 2, Pitch Rate (Body) (rad/sec)
14 Rate-Gyro # 3, Yaw Rate (Body) (rad/sec)

15 CG Acceleration along X axis, (ft/sec^2)
16 CG Acceleration along Y axis, (ft/sec^2)
17 CG Acceleration along Z axis, (ft/sec^2)

18 Ptch Load-Torque Tly for Engine: 1,(ft-lb)
19 Yaw Load-Torque Tlz for Engine: 1,(ft-lb)

Figure 3.2.3 Second Stage Vehicle Subsystem with Slosh and Flexibility



```
Jl=120; Nozzle Inertia (slg-ft^2)
R=0.72; Moment Arm (ft)
Klod=1.0e9; % Load Stiffness
Kact=2.4e6; % Piston Stiffness
Kbck=1.13e6; % Backup Structure Stiffn
Kti=(1/Klod)+(1/Kact)+(1/Kbck);
Kt=1/Kti; % Combined Structural Stiffness
Ka=8; % Amplifier Gain
```

Figure 3.2.4 Non-Linear Actuator Subsystem

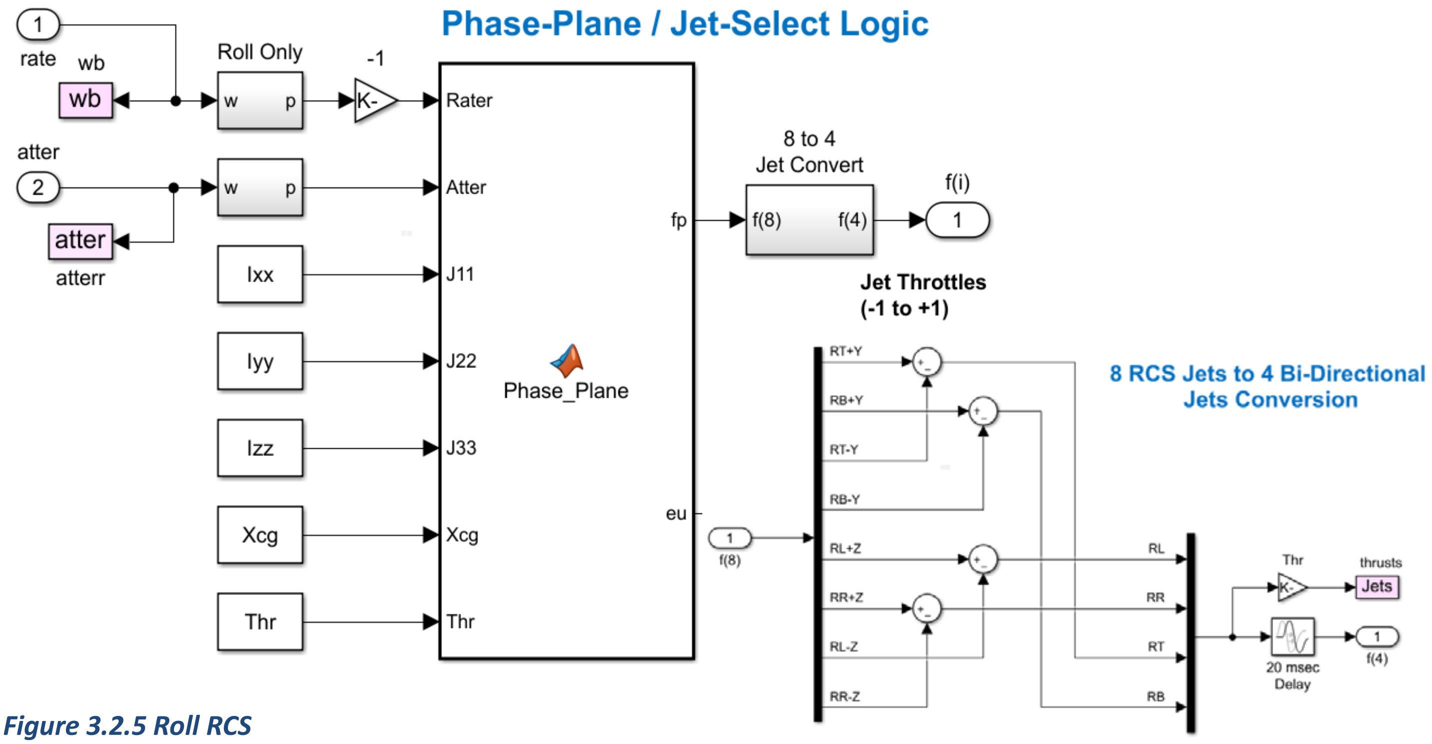


Figure 3.2.5 Roll RCS

Simulation Results

Figure 3.2.6 shows the system’s response to 5°commands in all 3 axes. Pitch and Yaw are controlled by the single TVC engine. Roll is controlled by the phase-plane bang-bang reaction control system.

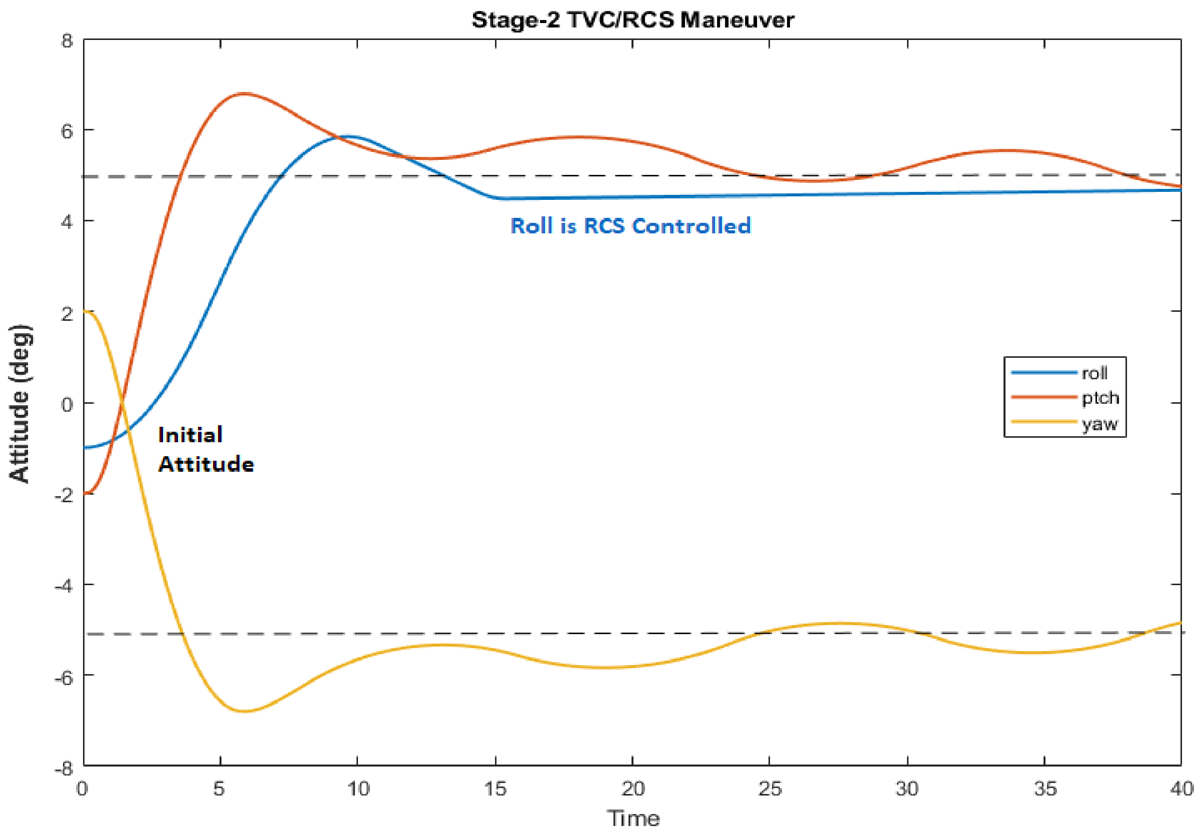


Figure 3.2.6 Attitude Response to ±5° Commands

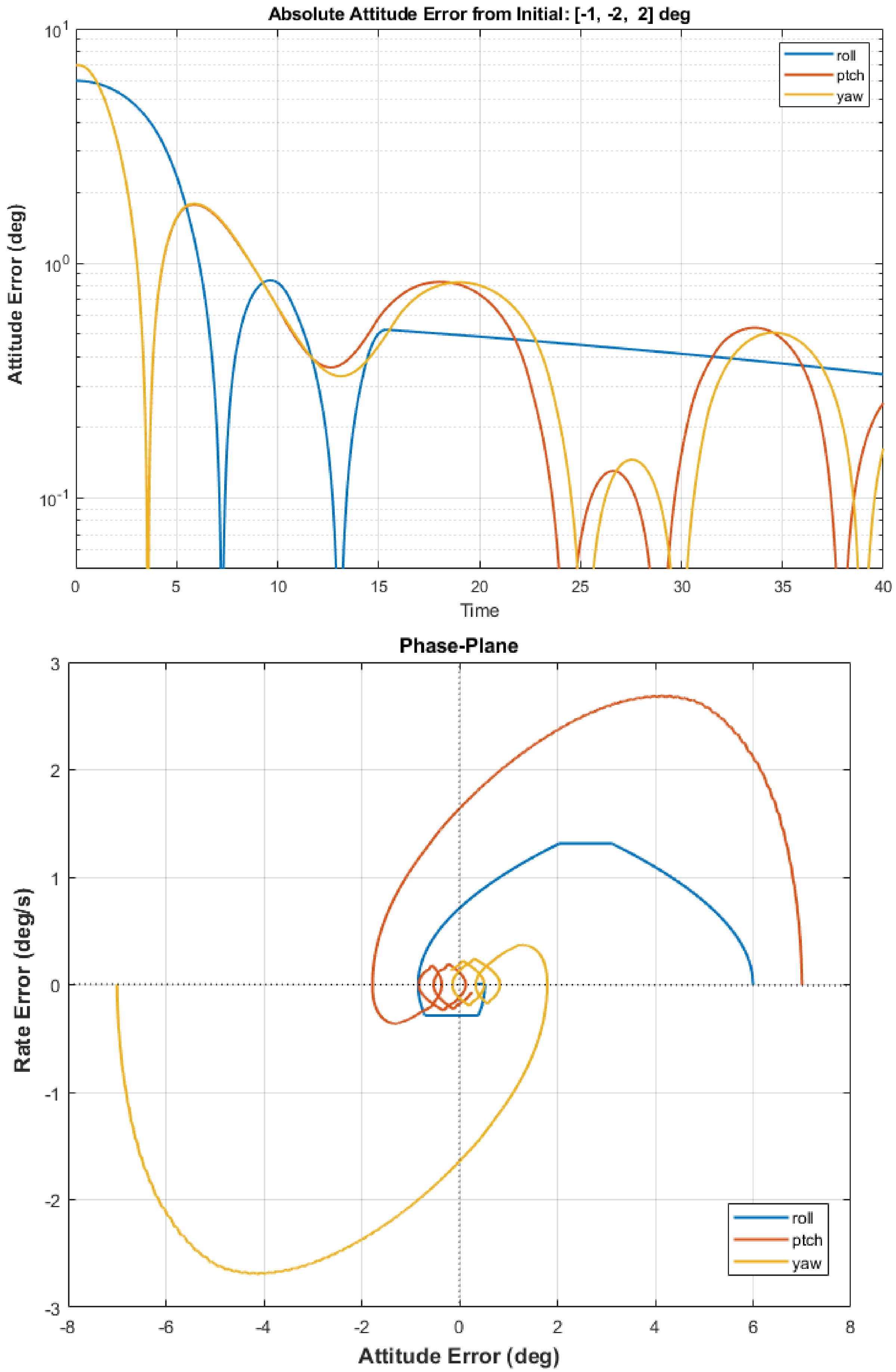


Figure 3.2.7 Attitude Error versus Time and Attitude Error versus Rate

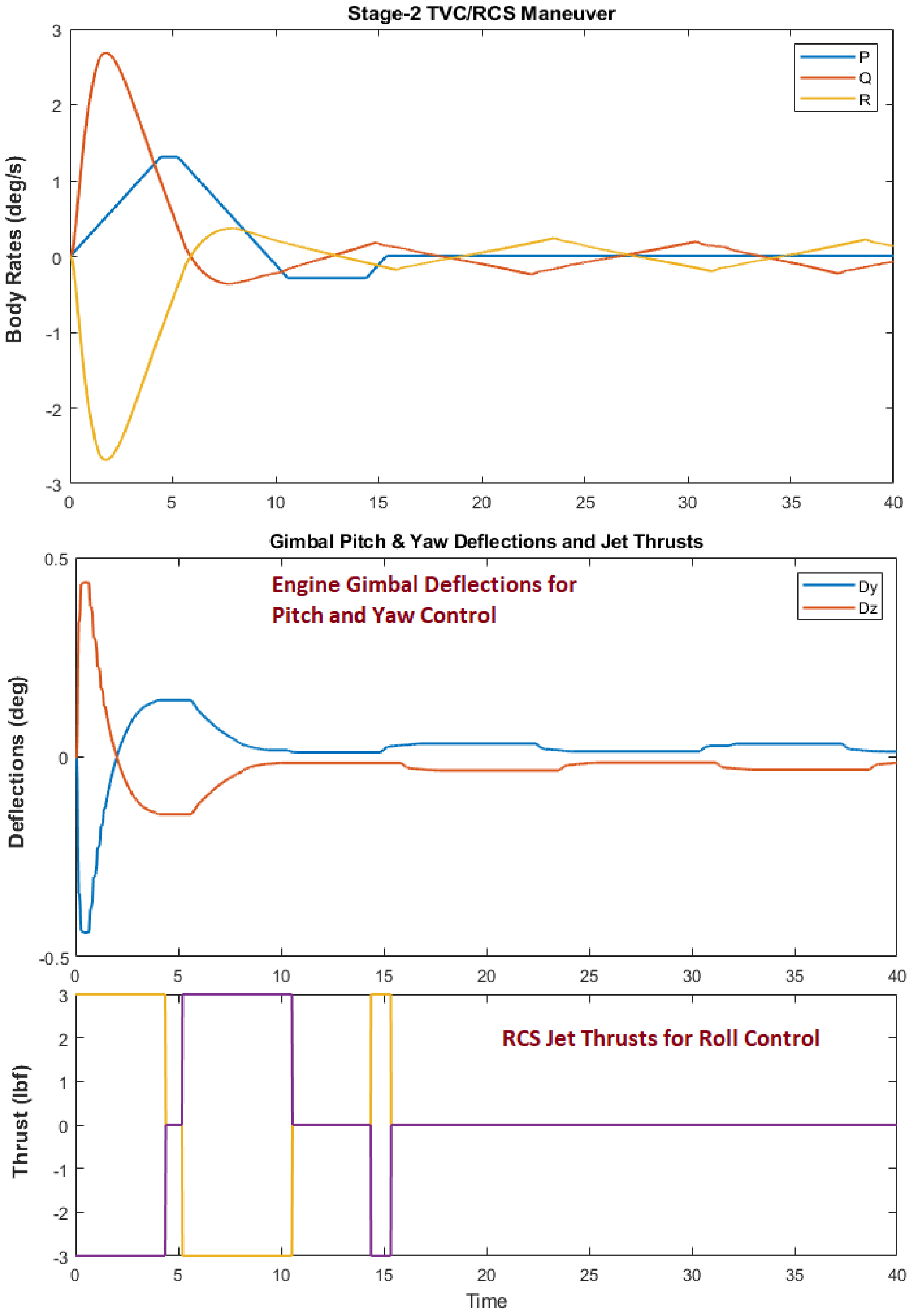


Figure 3.2.8 Body Rates, Gimbal Deflections and RCS Thrusts. The Coulomb Friction at the Gimbal Causes a Low Amplitude Periodic Oscillation in Pitch and Yaw

Stability Analysis at Post-Separation, T= 180 sec

The Simulink model “*Open_Loop.Slx*” is used to analyze the system stability in pitch and yaw. It is shown in Figure 3.2.9 configured for pitch open-loop analysis with yaw loop closed. We don’t worry much about roll because it is a very low bandwidth non-linear system. The Flixan derived linear actuator system is used in this model. The slosh modes in Figure 3.2.10 are small because the propellant tanks are almost full at post-separation.

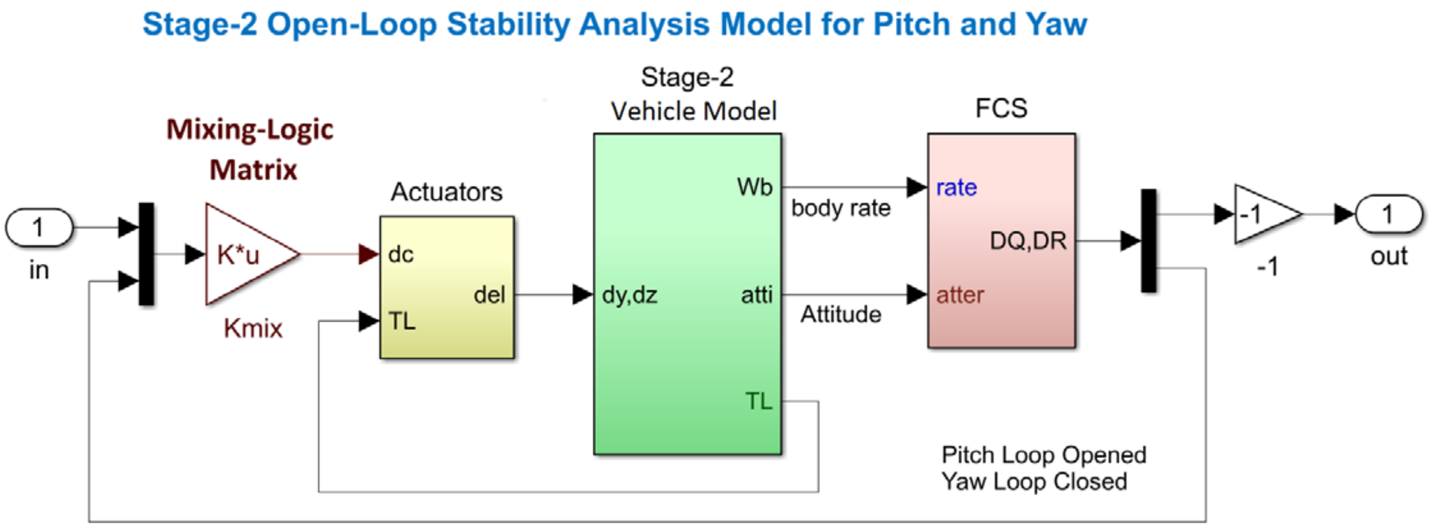
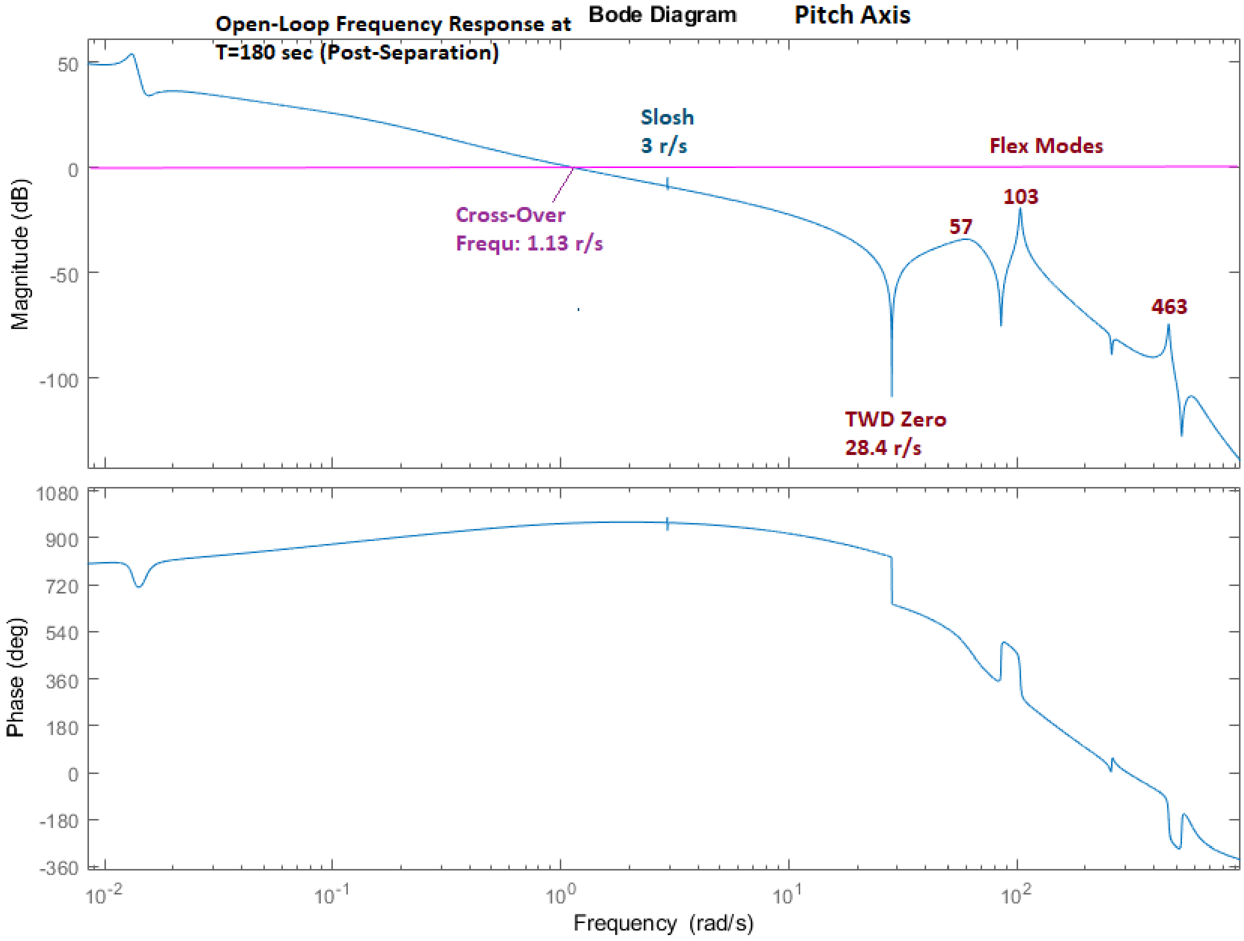


Figure 3.2.9 Simulink Model “Open_Loop.Slx” Used for Pitch and Yaw Stability Analysis



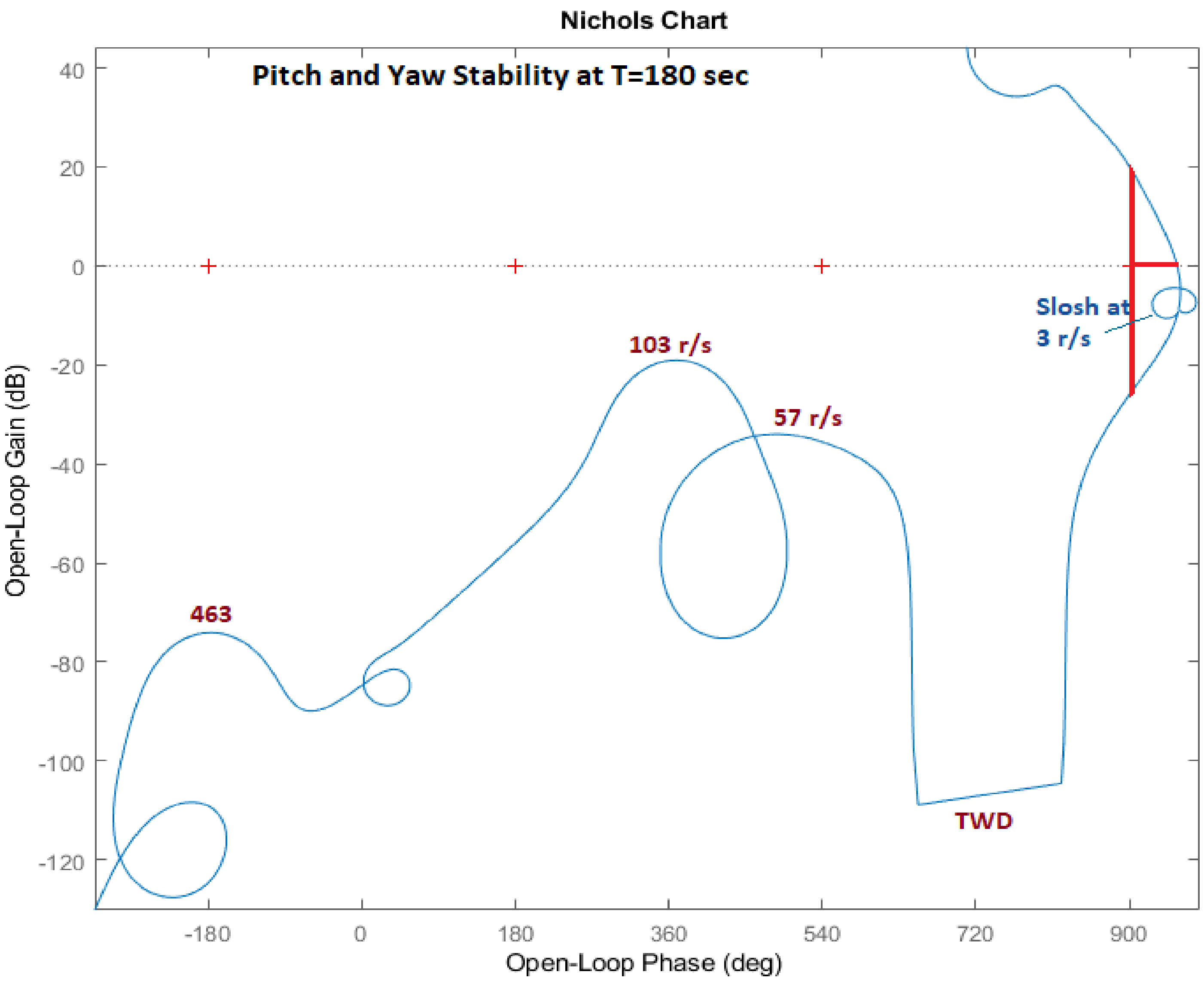


Figure 3.2.10 Pitch and Yaw Stability Analysis at Post-Separation, T= 180 sec. It Shows that the Flex Modes are Attenuated and the Slosh Modes are Very Small at this time.

3.2.2 Control Analysis at T= 280 sec

At T= 280 sec the tanks are more depleted and the propellant sloshing effect becomes stronger because the slosh masses are bigger and the slosh forces against the tank walls are creating a bigger disturbance on the vehicle. To make things worse, the x-location of the LOX mass happens to be between the vehicle center of rotation and the X_{CG} which makes the phase of the slosh mode opening towards the critical -1 point in the Nichols chart, Figure 3.2.11, which is pointing towards instability. The LH2 tank mode is fine, opening away from the + point towards the stable direction. In this case, the easiest way to stabilize the LOX mode is to increase the damping coefficient from ζ=0.002 to ζ=0.03 which is accomplished by including baffles inside the tank which dampens the sloshing effect. Figure 3.2.11 compares the system stability when using two different values of LOX damping coefficient.

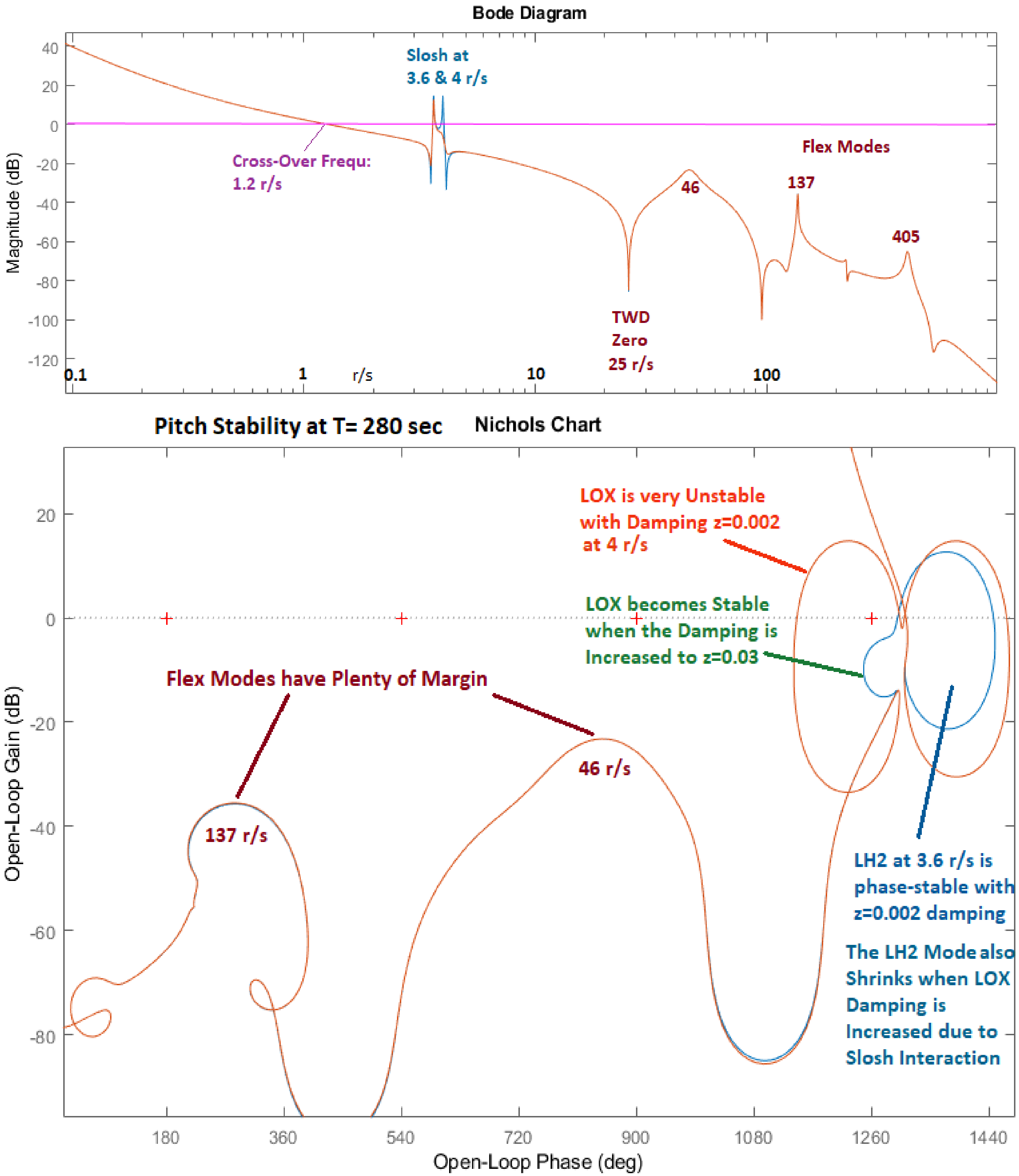


Figure 3.2.11 Stability at T= 280 sec is Strongly affected by the LOX Damping. The LOX Slosh Mode is Phase Unstable at Low Damping (Orange Curve). It can be Stabilized by Increasing the Damping Coefficient to ζ= 0.03 (Blue Curve), which means, adding Baffles inside the Tank. The LH2 Slosh Mode is Phase Stable. The Magnitude of the LH2 Mode is also affected by increasing Damping in the LOX Mode because their Frequencies are very close and they interact.

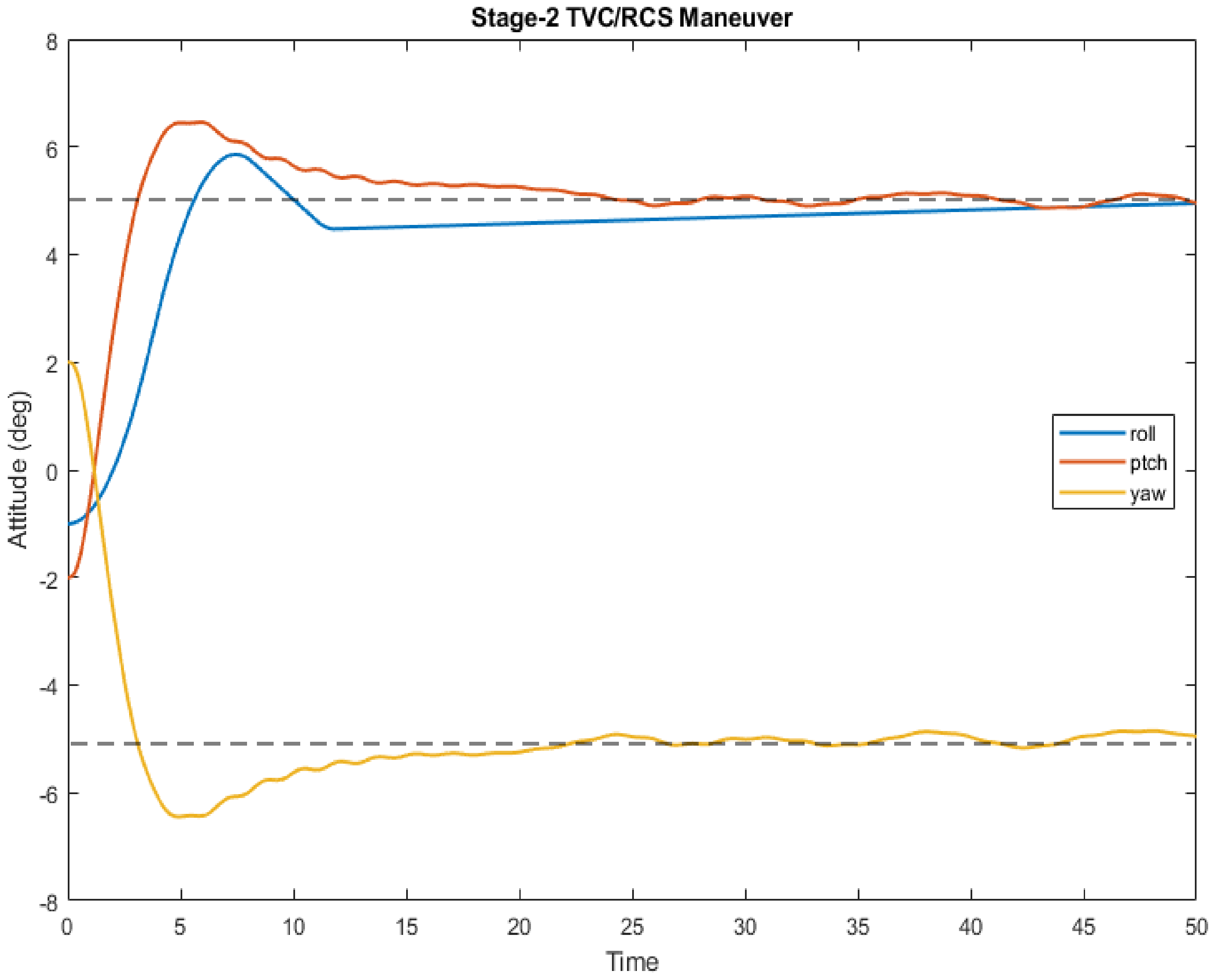
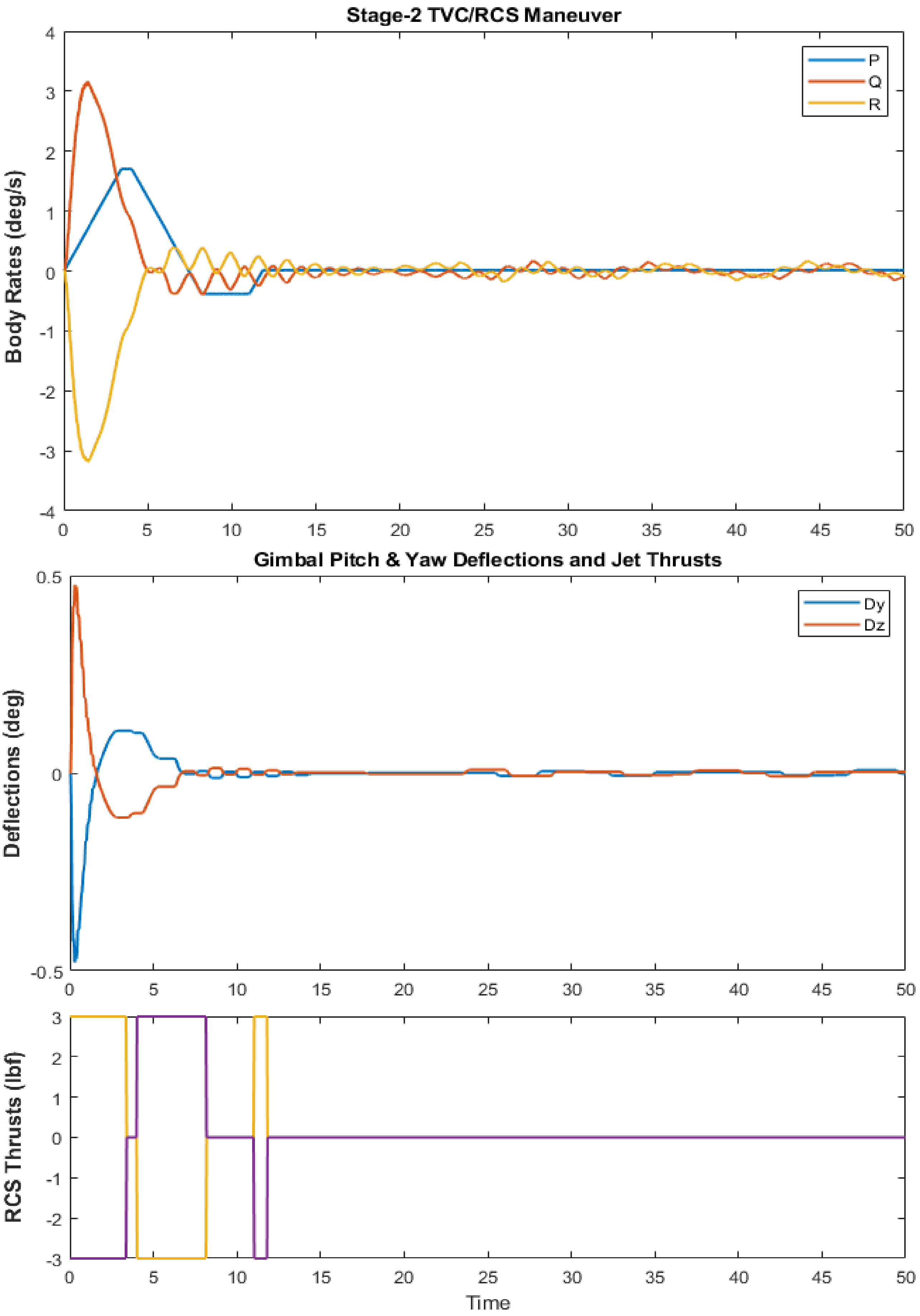
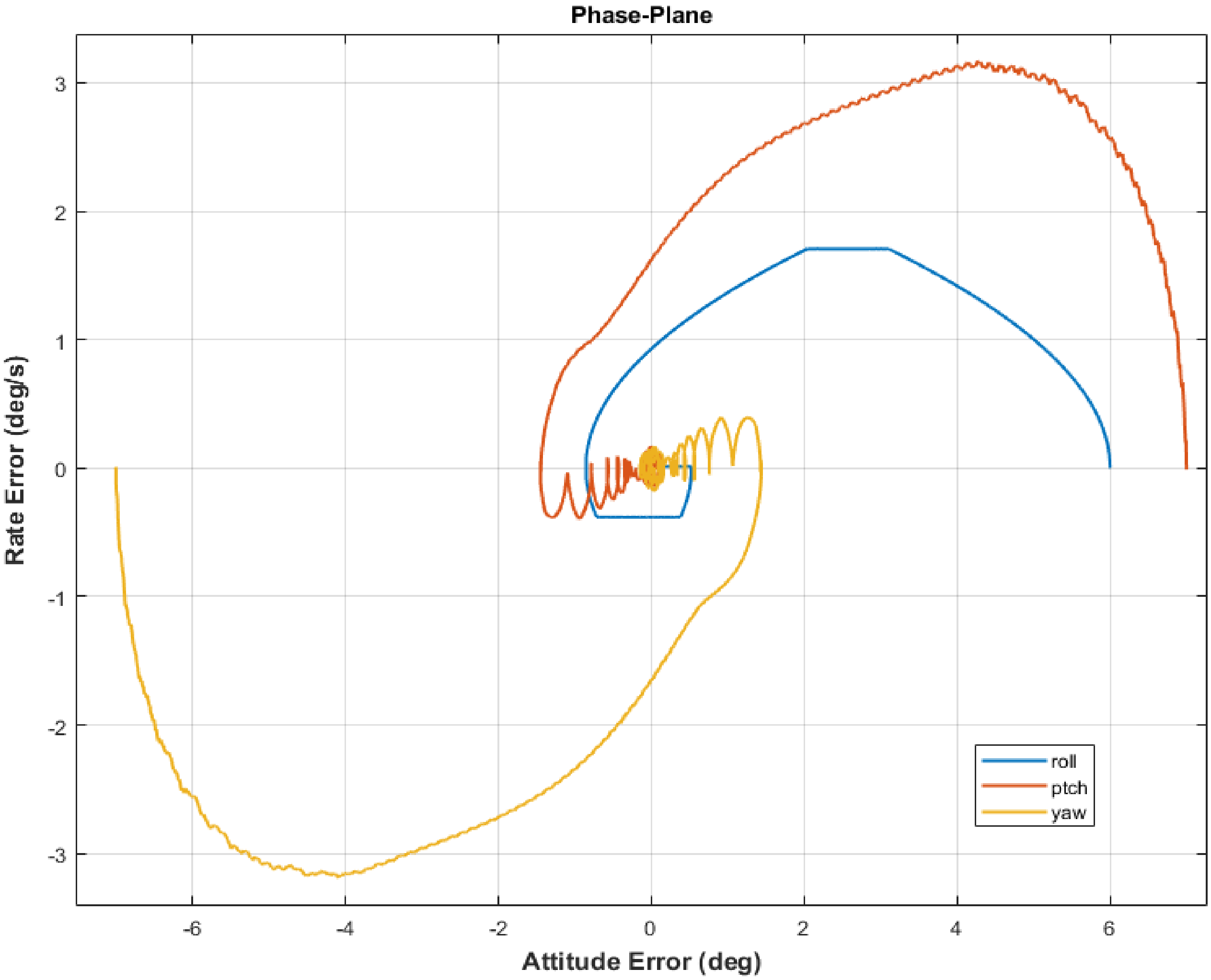
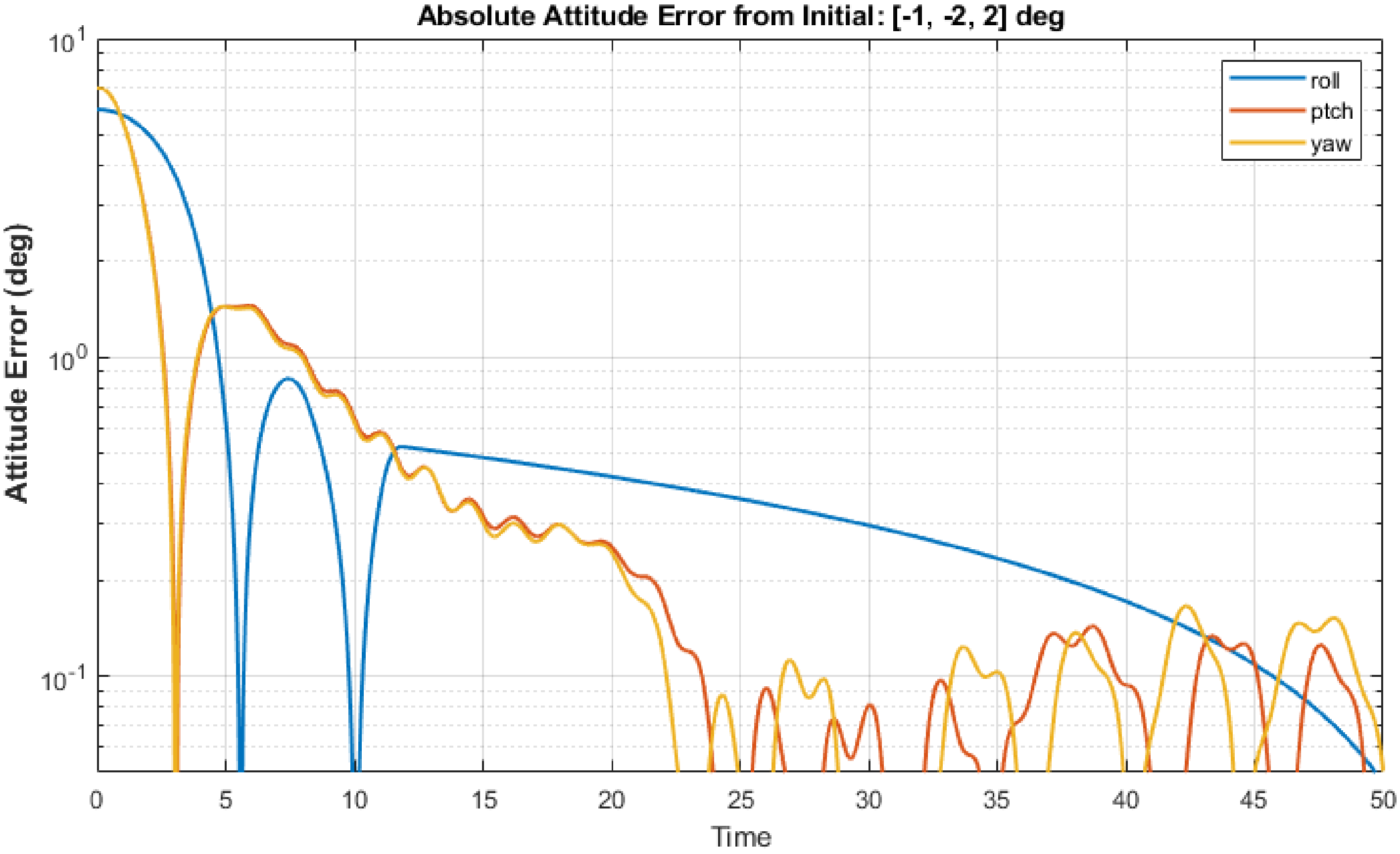


Figure 3.2.12 The Above Simulation Results Show the System Response to a [+5, +5, -5] Attitude Command in Roll, Pitch and Yaw Respectively from an Initial Attitude [-1, -2, +2]. The Unstable LOX Mode has been Stabilized by Increasing the Damping. The effects of Sloshing and Bending are visible in the Body Rates


Stage-2 TVC/RCS Maneuver
Body Rates (deg/s)
4
3
2
1
0
-1
-2
-3
-4
0
5
10
15
20
25
30
35
40
45
50
P
Q
R
Gimbal Pitch & Yaw Deflections and Jet Thrusts
Deflections (deg)
0.5
0
-0.5
Dy
Dz
RCS Thrusts (lbf)
3
2
1
0
-1
-2
-3
Time


Phase-Plane
Rate Error (deg/s)
Attitude Error (deg)
3
2
1
0
-1
-2
-3
-6
-4
-2
0
2
4
6
roll
ptch
yaw


Absolute Attitude Error from Initial: [-1, -2, 2] deg
Attitude Error (deg)
Time
10^1
10^0
10^{-1}
0
5
10
15
20
25
30
35
40
45
50
roll
ptch
yaw

3.2.3 Control Analysis at T= 350 sec

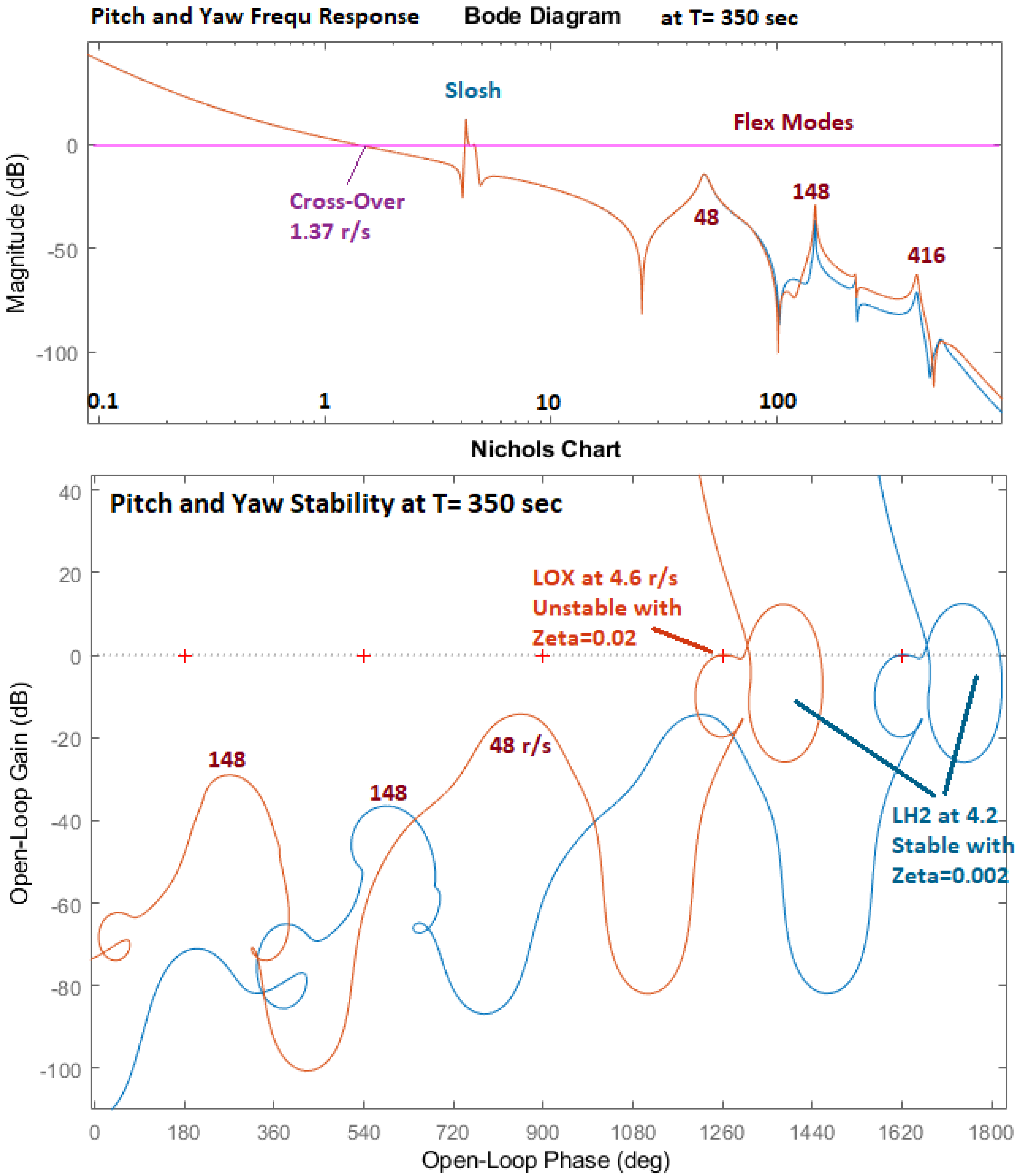
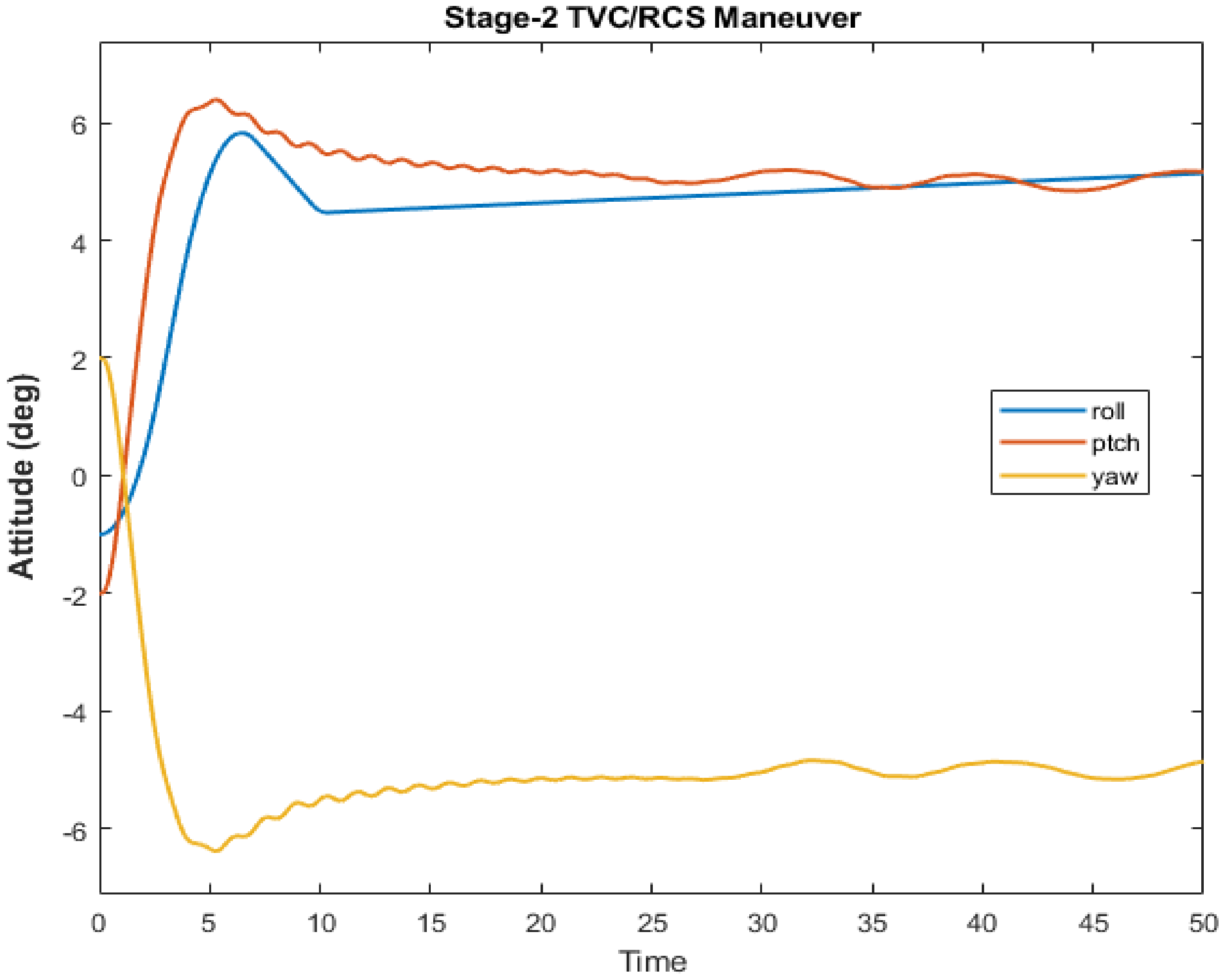
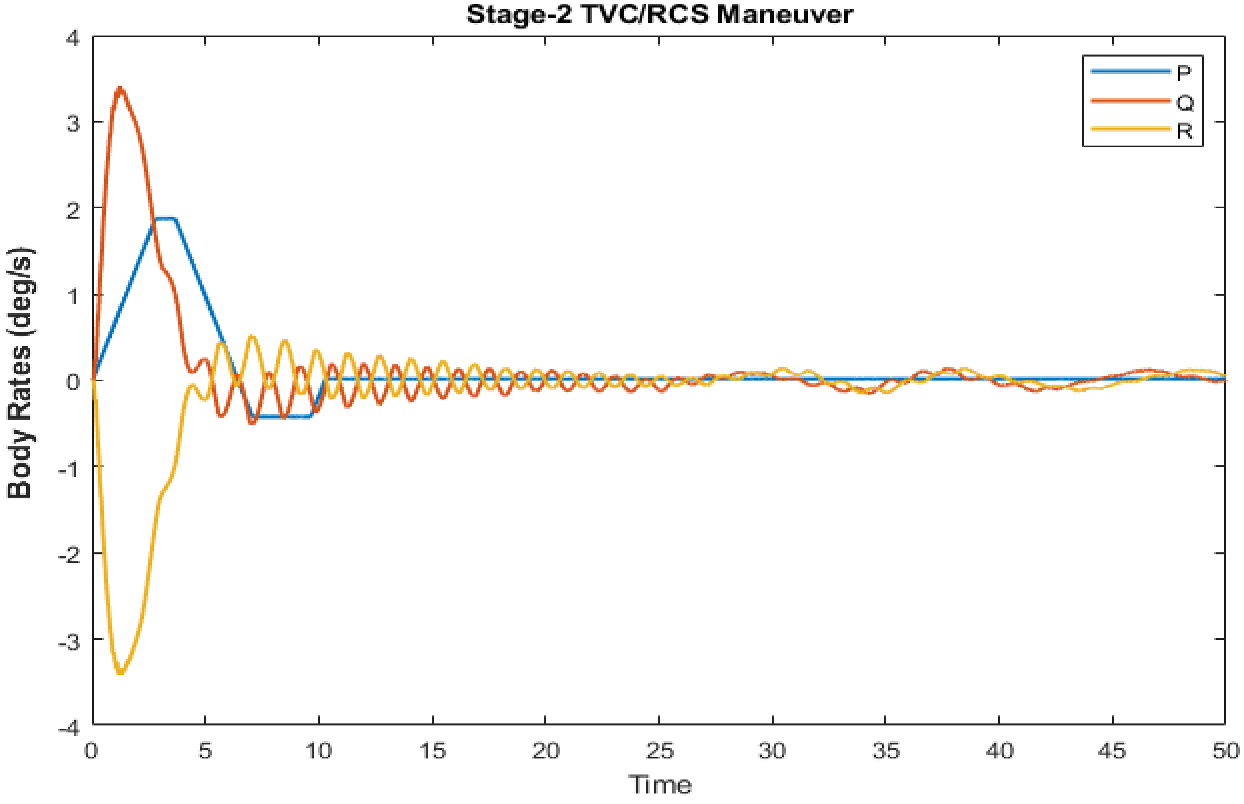
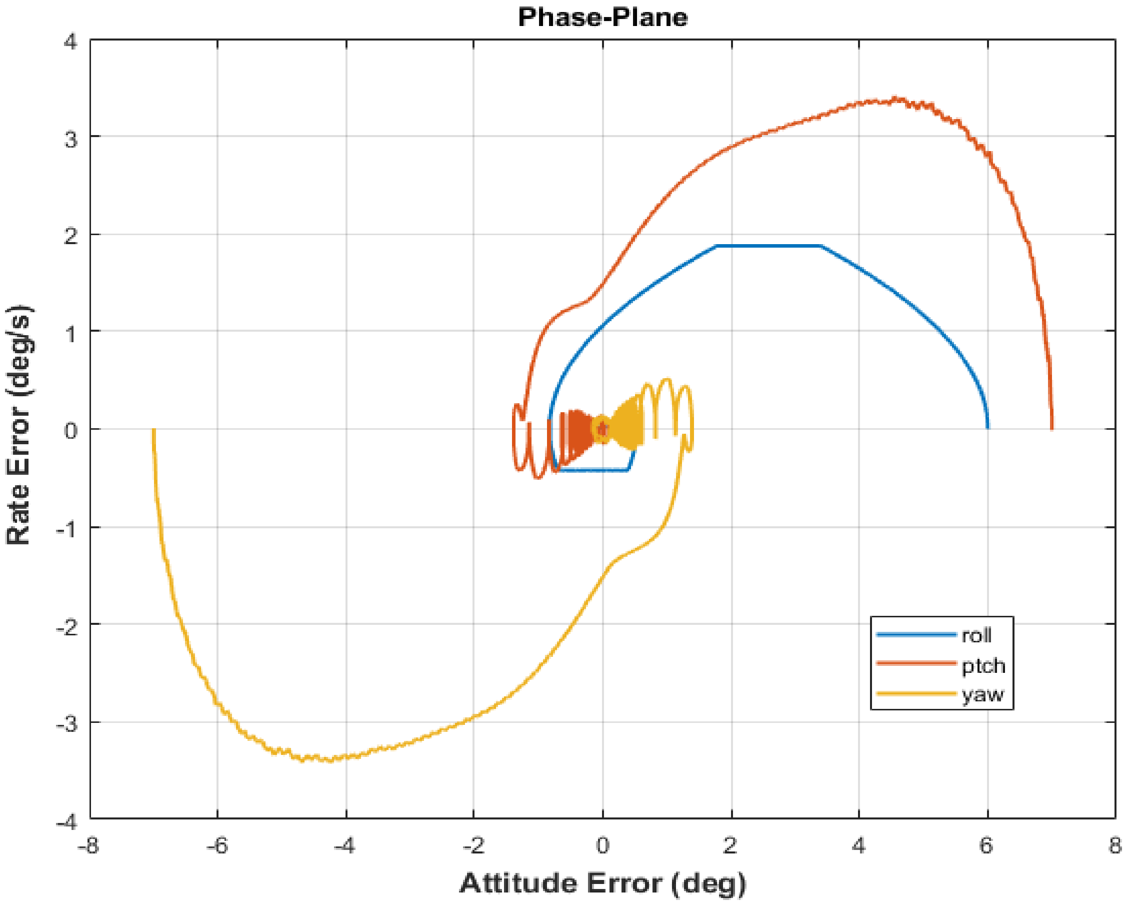
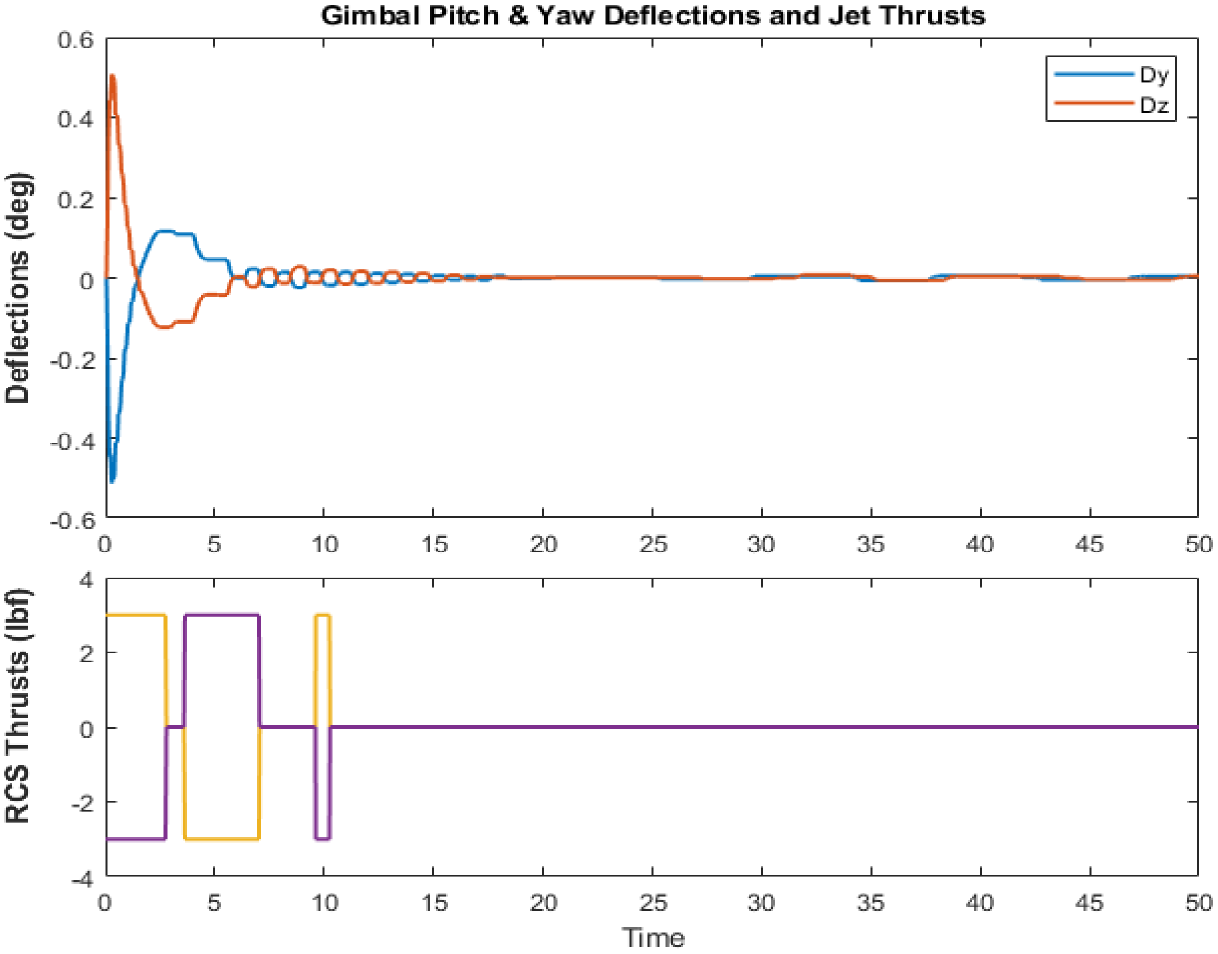


Figure 3.2.13 Pitch and Yaw Stability at T= 350 sec. LH2 is Phase-Stable with Low Damping. LOX is Phase-Unstable and Requires Additional Damping in order to be Passively Stabilized


Stage-2 TVC/RCS Maneuver
Attitude (deg)
Time
roll
ptch
yaw
6
4
2
0
-2
-4
-6
0
5
10
15
20
25
30
35
40
45
50


Stage-2 TVC/RCS Maneuver
Body Rates (deg/s)
Time
P
Q
R
4
3
2
1
0
-1
-2
-3
-4
0
5
10
15
20
25
30
35
40
45
50


Phase-Plane
Rate Error (deg/s)
Attitude Error (deg)
4
3
2
1
0
-1
-2
-3
-4
-8
-6
-4
-2
0
2
4
6
8
roll
ptch
yaw


Gimbal Pitch & Yaw Deflections and Jet Thrusts
Deflections (deg)
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
Dy
Dz
0
5
10
15
20
25
30
35
40
45
50
RCS Thrusts (lbf)
4
2
0
-2
-4
0
5
10
15
20
25
30
35
40
45
50
Time

3.2.4 Control Analysis at T= 420 sec

At T= 420 the LOX mode is still phase unstable (opening towards the + critical point) but the instability is somewhat reduced. With a damping coefficient ζ=0.02 the mode has enough margin. The pitch/ yaw system bandwidth is slightly increased to 1.35 r/s. In Figure 3.2.14, the engine mounting structural mode at 48.5 r/s is becoming stronger now as the vehicle gets lighter, but there is still enough gain margin. The transient responses to commands in the next figures, they also look good.

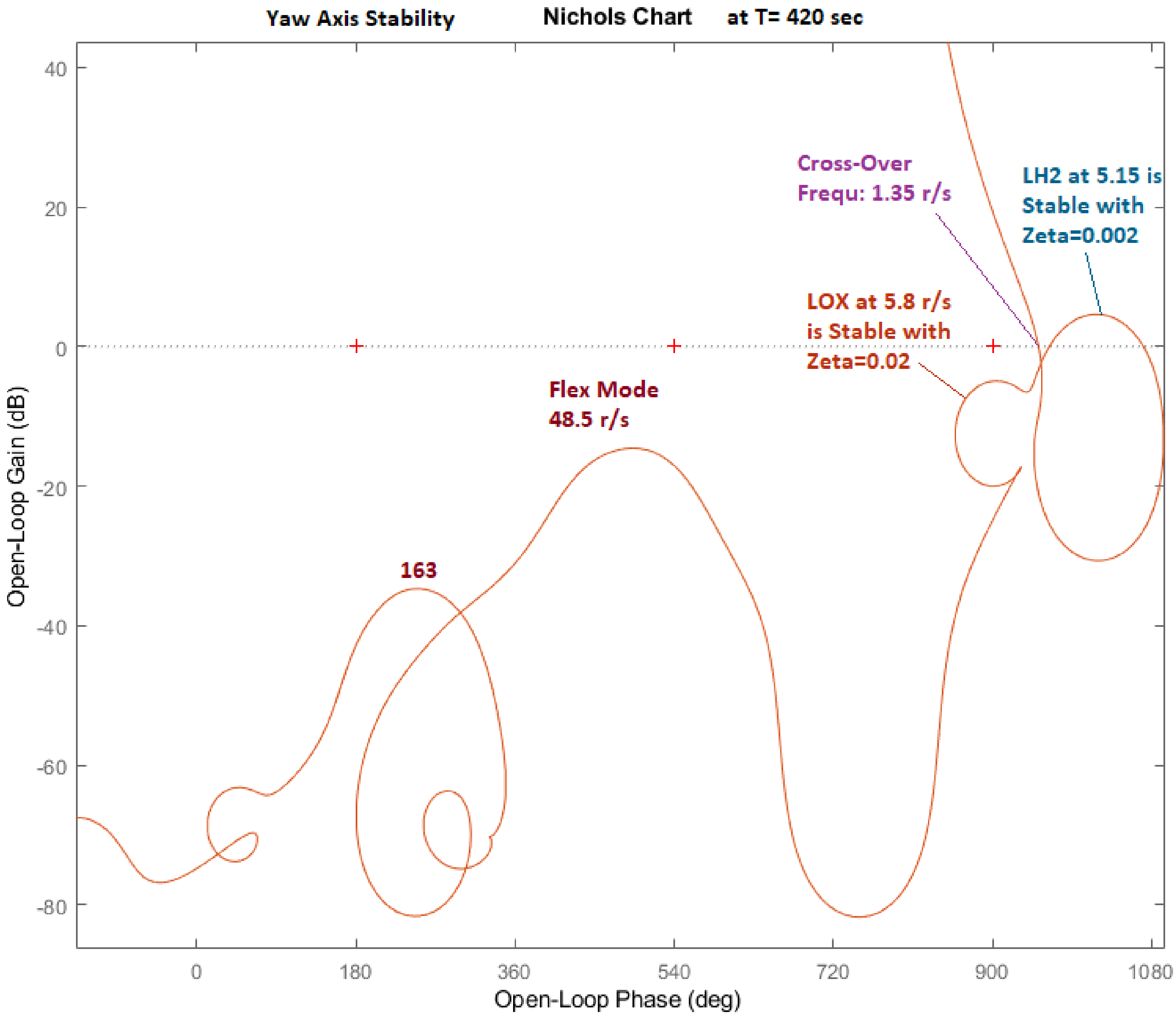
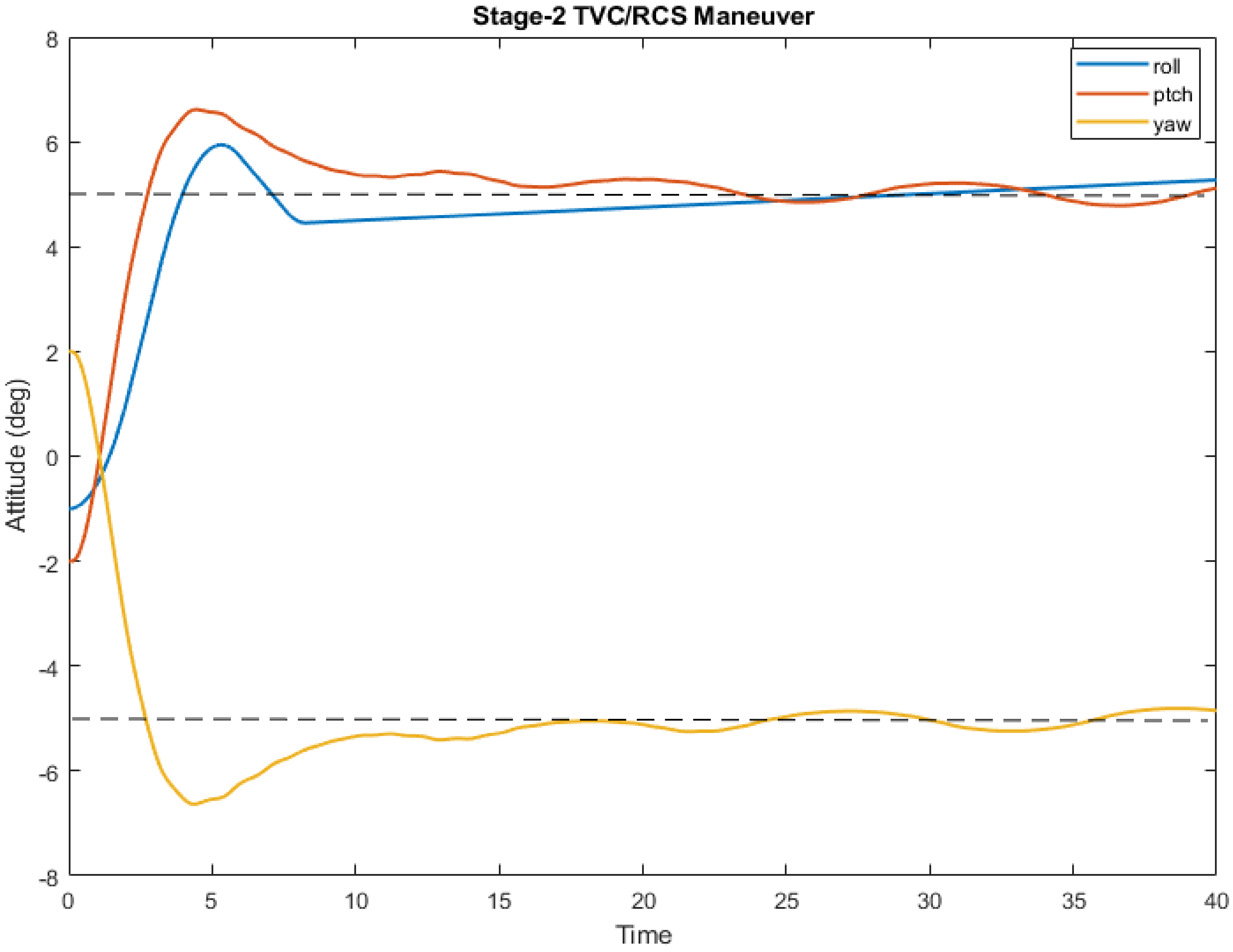
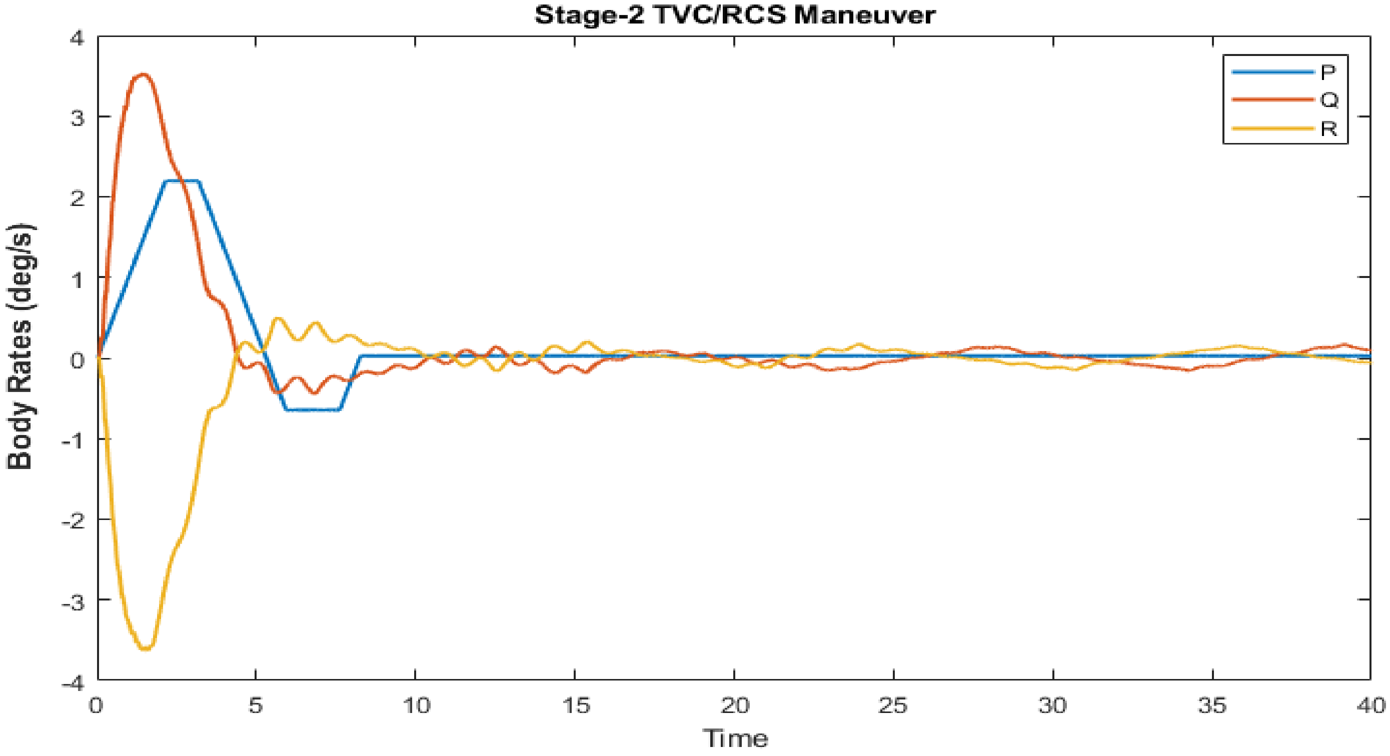
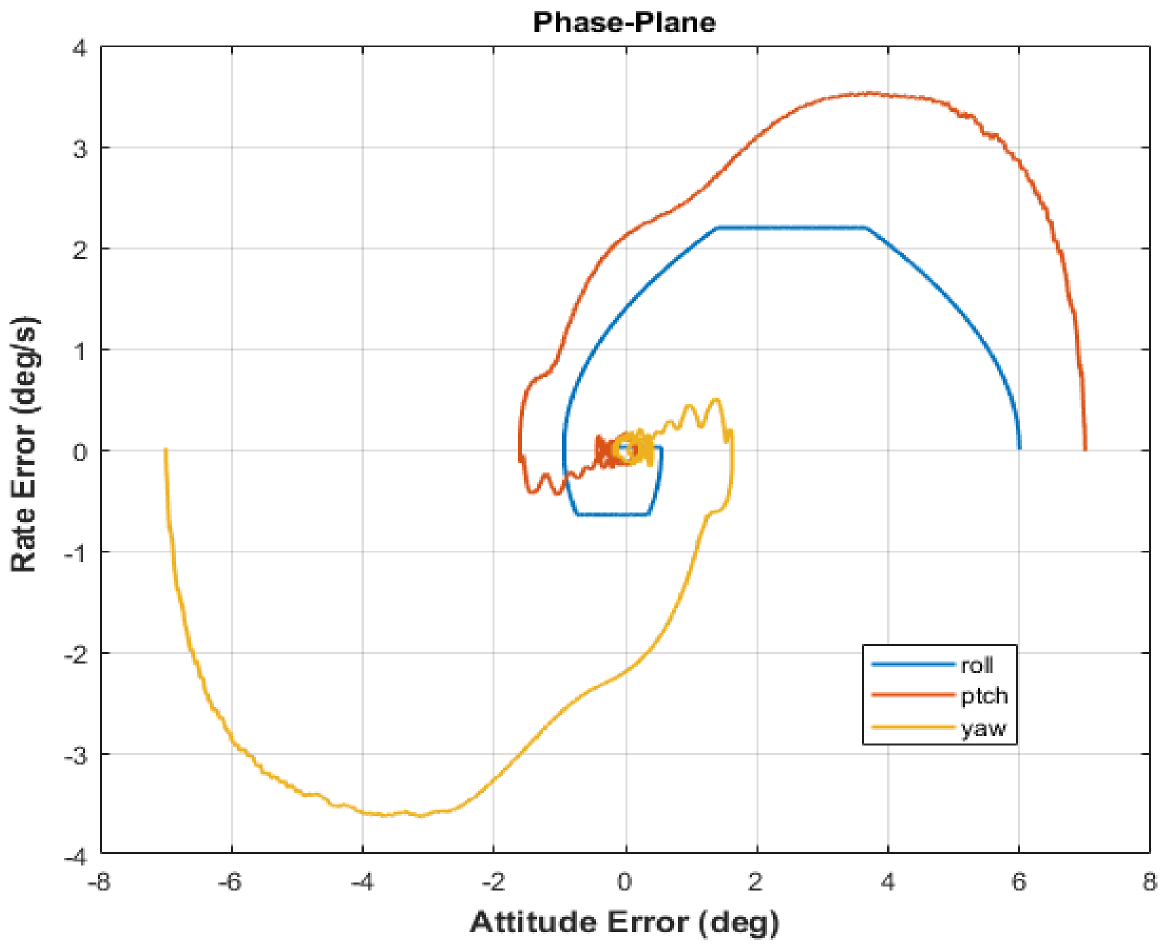
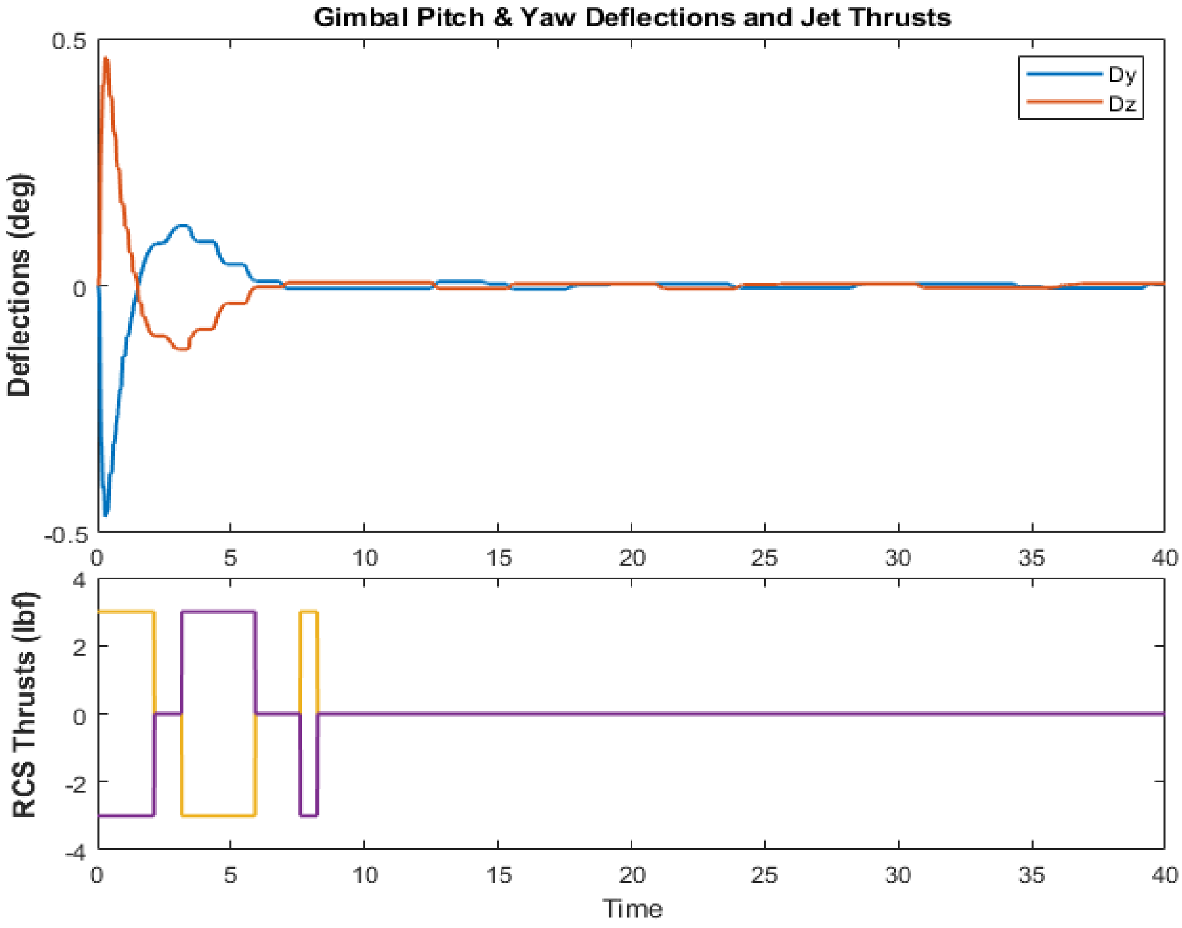


Figure 3.2.14 Pitch and yaw TVC Stability at T= 350 sec.


Stage-2 TVC/RCS Maneuver
Attitude (deg)
Time
roll
ptch
yaw
8
6
4
2
0
-2
-4
-6
-8
0
5
10
15
20
25
30
35
40


Stage-2 TVC/RCS Maneuver
Body Rates (deg/s)
Time
P
Q
R
4
3
2
1
0
-1
-2
-3
-4
0
5
10
15
20
25
30
35
40


Phase-Plane
Rate Error (deg/s)
Attitude Error (deg)
4
3
2
1
0
-1
-2
-3
-4
-8
-6
-4
-2
0
2
4
6
8
roll
ptch
yaw


Gimbal Pitch & Yaw Deflections and Jet Thrusts
Deflections (deg)
0.5
0
-0.5
Dy
Dz
RCS Thrusts (lbf)
4
2
0
-2
-4
0
5
10
15
20
25
30
35
40
Time

3.2.5 Control Analysis at the End of 2nd Stage, T= 490 sec

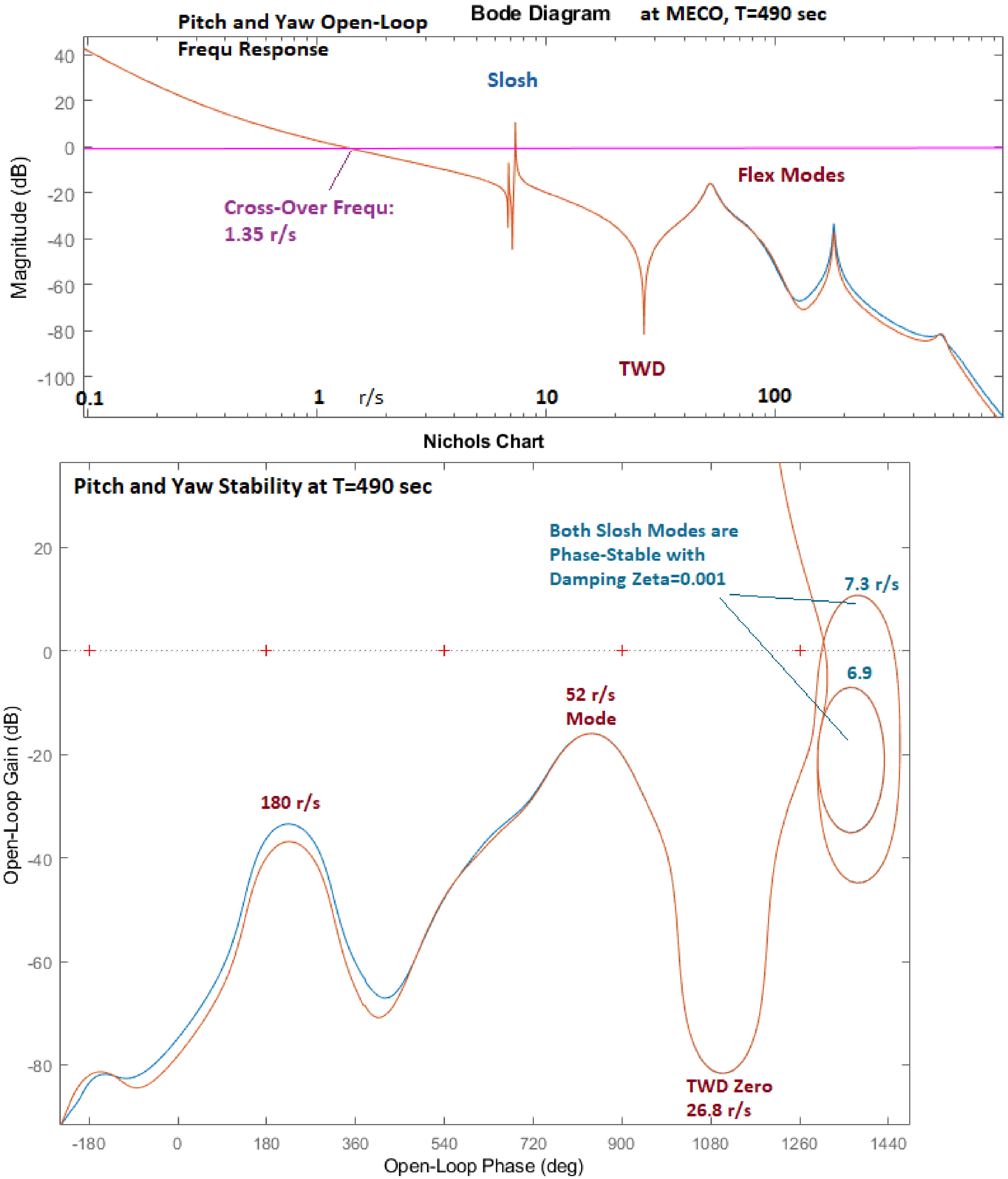
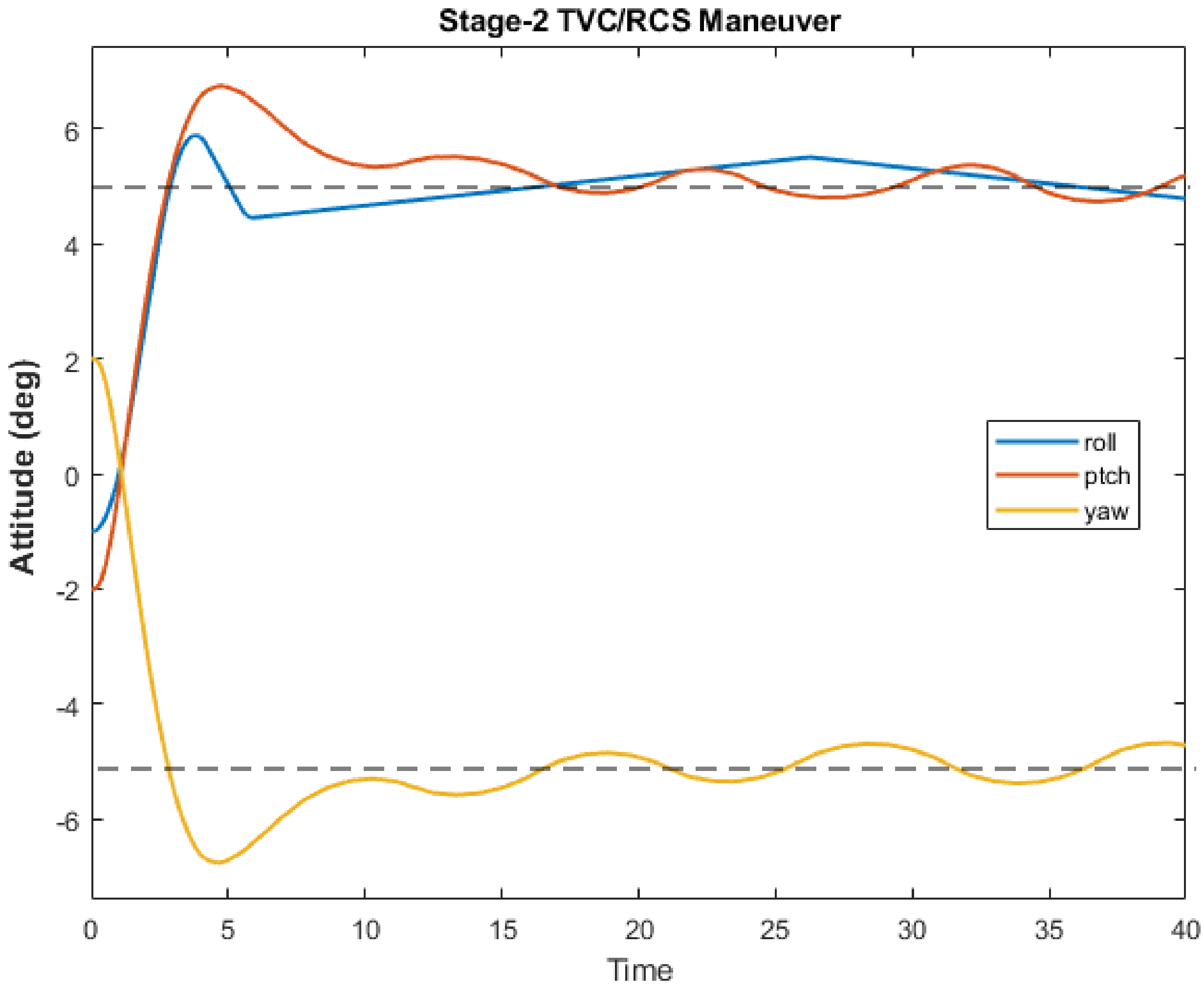
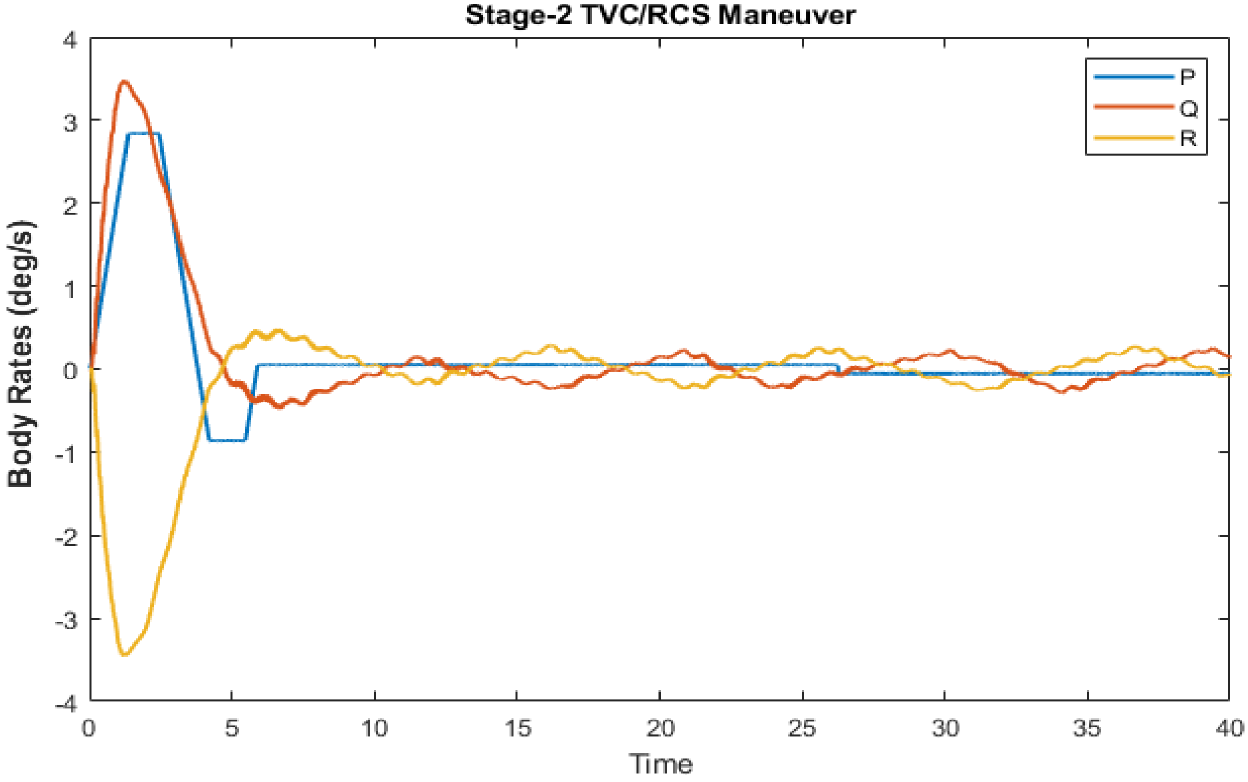
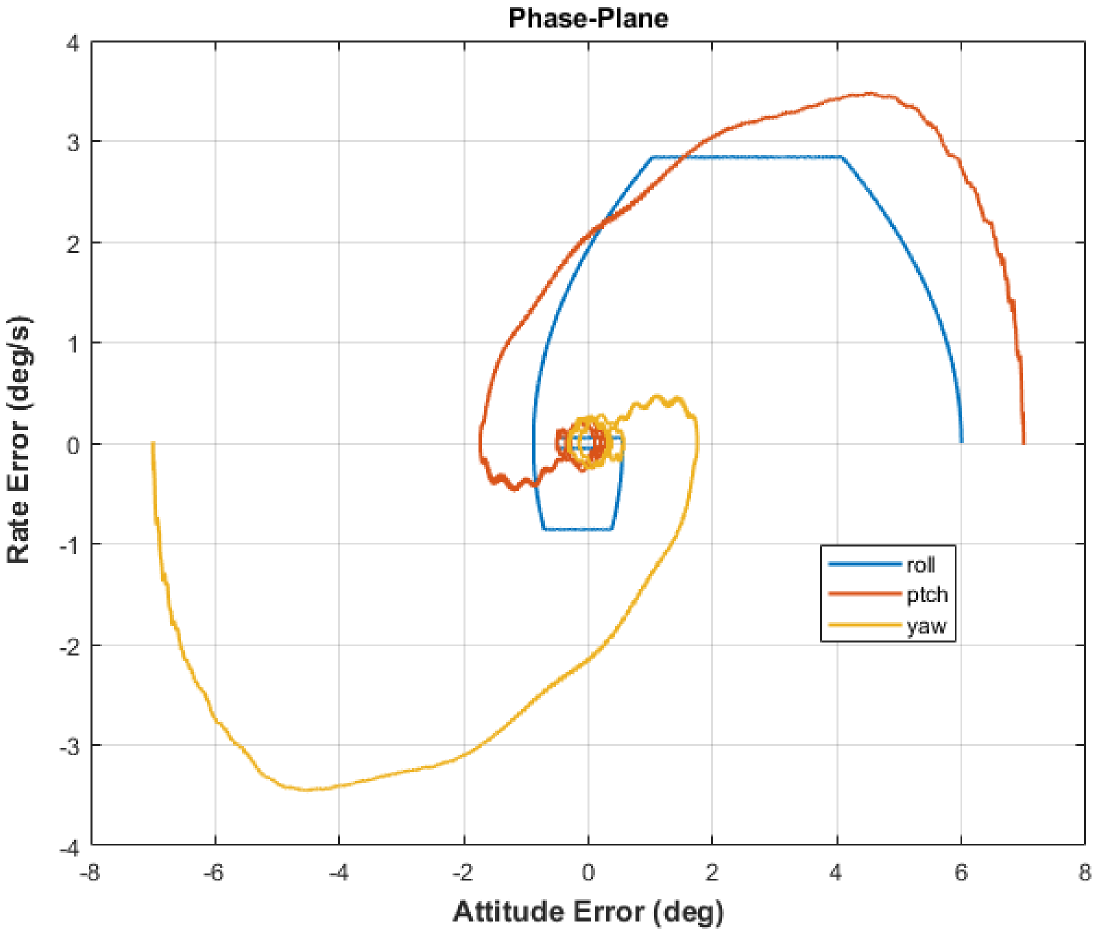
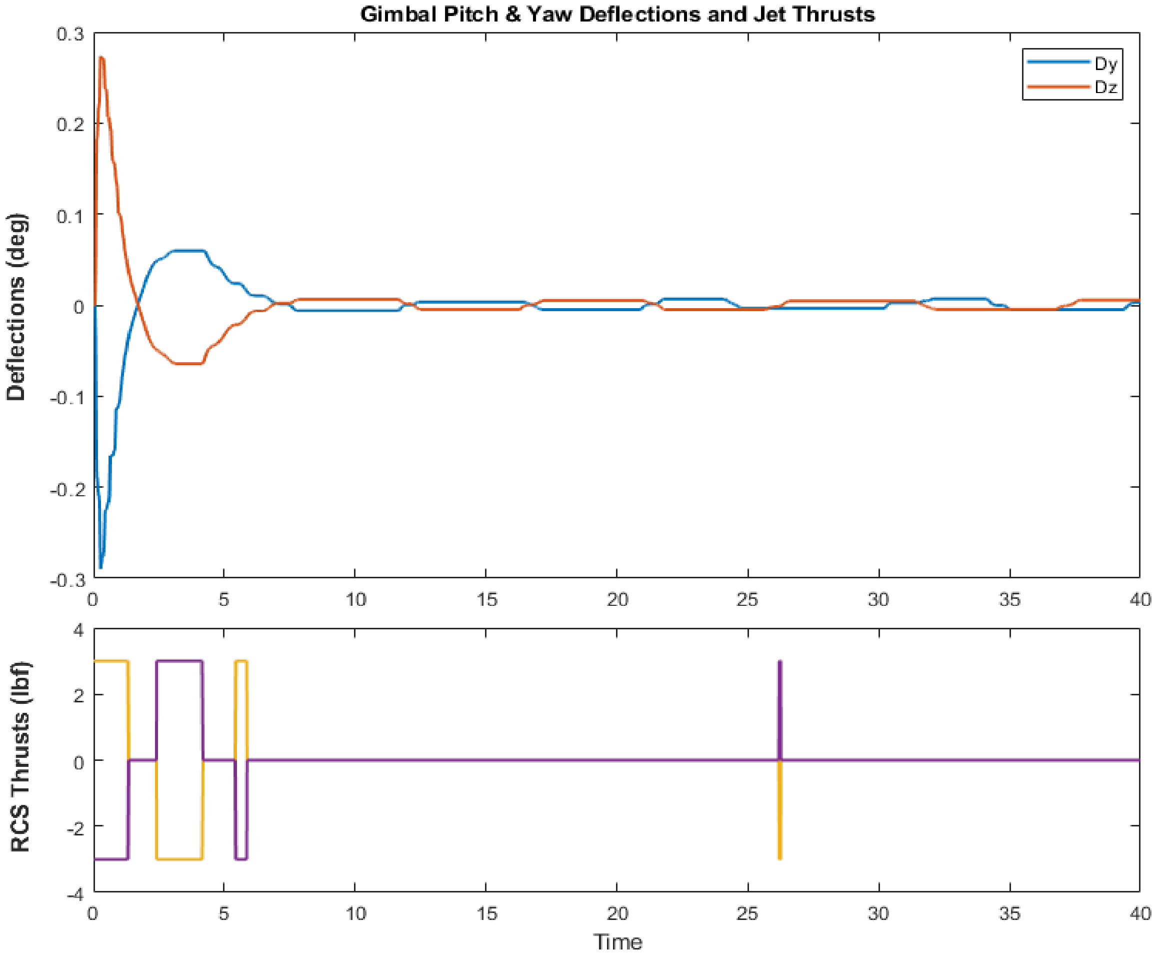


Figure 3.2.15 Pitch and Yaw Stability at T= 490 sec. Both LOX and LH2 slosh modes are phase-stable at a low ζ=0.001 damping. The location of the LOX mass is no longer between the CG and the Center of Rotation. The Slosh Frequencies are increased because the Vehicle Acceleration is higher as it loses mass. The Slosh disturbance on the vehicle, however, is increased because the vehicle is lighter


Stage-2 TVC/RCS Maneuver
Attitude (deg)
Time
6
4
2
0
-2
-4
-6
0
5
10
15
20
25
30
35
40
roll
ptch
yaw


Stage-2 TVC/RCS Maneuver
Body Rates (deg/s)
Time
4
3
2
1
0
-1
-2
-3
-4
0
5
10
15
20
25
30
35
40
P
Q
R

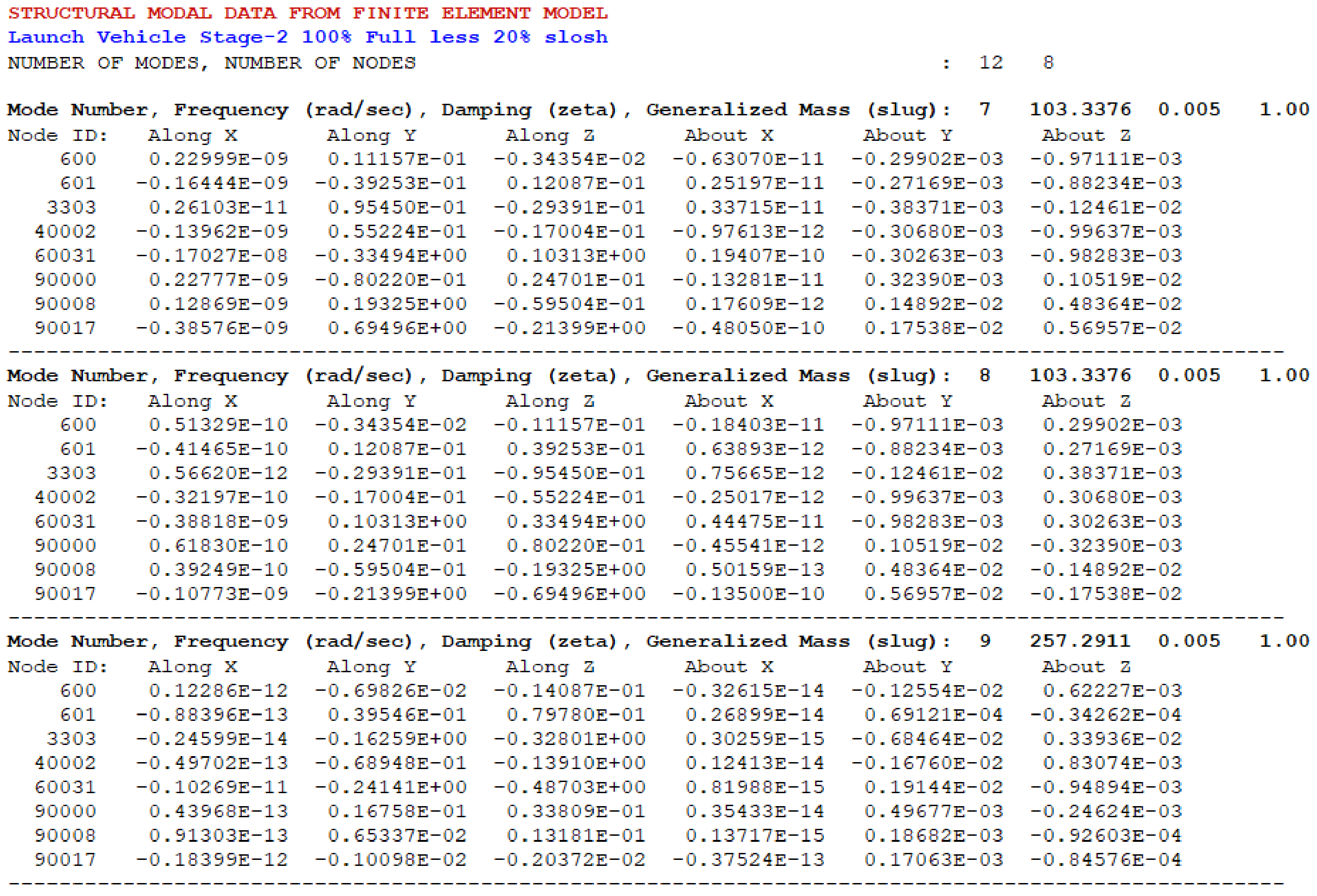

Phase-Plane
Rate Error (deg/s)
Attitude Error (deg)
4
3
2
1
0
-1
-2
-3
-4
-8
-6
-4
-2
0
2
4
6
8
roll
ptch
yaw


Gimbal Pitch & Yaw Deflections and Jet Thrusts
Deflections (deg)
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
Dy
Dz
0
5
10
15
20
25
30
35
40
RCS Thrusts (lbf)
4
2
0
-2
-4
Time

3.3 Mode Selection

The file that contains the finite element structural modes at T180 is “Stage2_100%.Mod” in folder: “*23-Classic Launch Vehicle Design & Simulation\3-Stability Analysis with Flex & Slosh\2nd Stage\T180*”. It is a Nastran output that contains the mode shapes and slopes for 12 modes, at 8 vehicle locations (nodes). The modal data file contains frames of data at each mode frequency. Each frame consists of the mode frequency in (rad/sec), the modal damping coefficient (ζ) which are all set to ζ=0.005, the generalized mass (all set to 1 in this case), followed by the mode shapes and slopes at the 8 vehicle locations (3 translations x, y, z, and 3 rotations about x, y, z). The modal data filenames must have an extension (Mod). The data is in Nastran units is not compatible with the vehicle data.

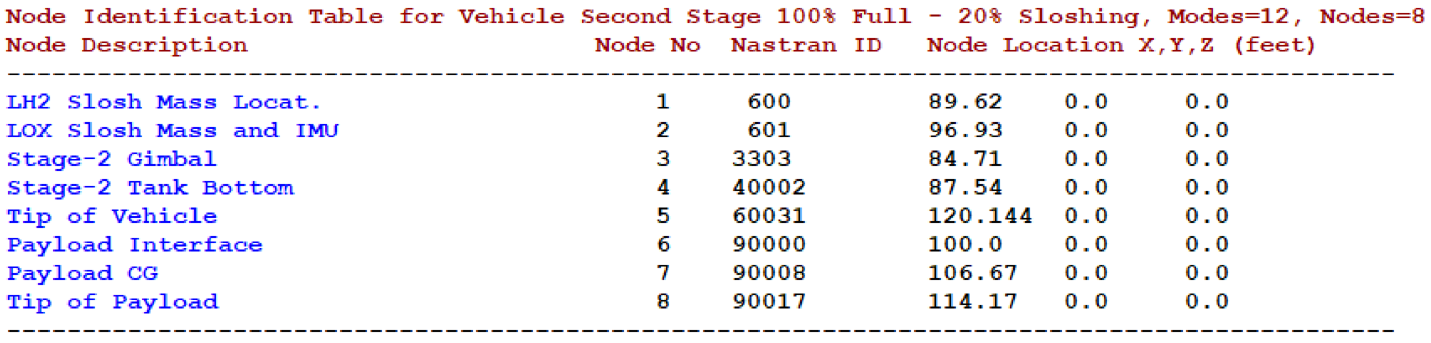
The locations which are important for flight control analysis are the engine gimbal, the sensor location, and the locations of the two slosh masses. The vehicle locations are defined in a separate nodes file “*Stage2_100%.Nod*”, also known as map, that contains a description for each node, the node numbers (in this case 1 to 8), a node identification number (that is the node number used in the Nastran model), and the location of each node in vehicle coordinates which is only used for reference. The node ID filenames must have an extension (Nod).



```
STRUCTURAL MODAL DATA FROM FINITE ELEMENT MODEL
Launch Vehicle Stage-2 100% Full less 20% slosh
NUMBER OF MODES, NUMBER OF NODES                                         :  12   8

Mode Number, Frequency (rad/sec), Damping (zeta), Generalized Mass (slug):  7   103.3376  0.005   1.00
Node ID:    Along X       Along Y       Along Z       About X       About Y       About Z
    600    0.22999E-09   0.11157E-01  -0.34354E-02  -0.63070E-11  -0.29902E-03  -0.97111E-03
    601   -0.16444E-09  -0.39253E-01   0.12087E-01   0.25197E-11  -0.27169E-03  -0.88234E-03
   3303    0.26103E-11   0.95450E-01  -0.29391E-01   0.33715E-11  -0.38371E-03  -0.12461E-02
  40002   -0.13962E-09   0.55224E-01  -0.17004E-01  -0.97613E-12  -0.30680E-03  -0.99637E-03
  60031   -0.17027E-08  -0.33494E+00   0.10313E+00   0.19407E-10  -0.30263E-03  -0.98283E-03
  90000    0.22777E-09  -0.80220E-01   0.24701E-01  -0.13281E-11   0.32390E-03   0.10519E-02
  90008    0.12869E-09   0.19325E+00  -0.59504E-01   0.17609E-12   0.14892E-02   0.48364E-02
  90017   -0.38576E-09   0.69496E+00  -0.21399E+00  -0.48050E-10   0.17538E-02   0.56957E-02
---------------------------------------------------------------------------------------------------
Mode Number, Frequency (rad/sec), Damping (zeta), Generalized Mass (slug):  8   103.3376  0.005   1.00
Node ID:    Along X       Along Y       Along Z       About X       About Y       About Z
    600    0.51329E-10  -0.34354E-02  -0.11157E-01  -0.18403E-11  -0.97111E-03   0.29902E-03
    601   -0.41465E-10   0.12087E-01   0.39253E-01   0.63893E-12  -0.88234E-03   0.27169E-03
   3303    0.56620E-12  -0.29391E-01  -0.95450E-01   0.75665E-12  -0.12461E-02   0.38371E-03
  40002   -0.32197E-10  -0.17004E-01  -0.55224E-01  -0.25017E-12  -0.99637E-03   0.30680E-03
  60031   -0.38818E-09   0.10313E+00   0.33494E+00   0.44475E-11  -0.98283E-03   0.30263E-03
  90000    0.61830E-10   0.24701E-01   0.80220E-01  -0.45541E-12   0.10519E-02  -0.32390E-03
  90008    0.39249E-10  -0.59504E-01  -0.19325E+00   0.50159E-13   0.48364E-02  -0.14892E-02
  90017   -0.10773E-09  -0.21399E+00  -0.69496E+00  -0.13500E-10   0.56957E-02  -0.17538E-02
---------------------------------------------------------------------------------------------------
Mode Number, Frequency (rad/sec), Damping (zeta), Generalized Mass (slug):  9   257.2911  0.005   1.00
Node ID:    Along X       Along Y       Along Z       About X       About Y       About Z
    600    0.12286E-12  -0.69826E-02  -0.14087E-01  -0.32615E-14  -0.12554E-02   0.62227E-03
    601   -0.88396E-13   0.39546E-01   0.79780E-01   0.26899E-14   0.69121E-04  -0.34262E-04
   3303   -0.24599E-14  -0.16259E+00  -0.32801E+00   0.30259E-15  -0.68464E-02   0.33936E-02
  40002   -0.49702E-13  -0.68948E-01  -0.13910E+00   0.12413E-14  -0.16760E-02   0.83074E-03
  60031   -0.10269E-11  -0.24141E+00  -0.48703E+00   0.81988E-15   0.19144E-02  -0.94894E-03
  90000    0.43968E-13   0.16758E-01   0.33809E-01   0.35433E-14   0.49677E-03  -0.24624E-03
  90008    0.91303E-13   0.65337E-02   0.13181E-01   0.13717E-15   0.18682E-03  -0.92603E-04
  90017   -0.18399E-12  -0.10098E-02  -0.20372E-02  -0.37524E-13   0.17063E-03  -0.84576E-04
---------------------------------------------------------------------------------------------------
```

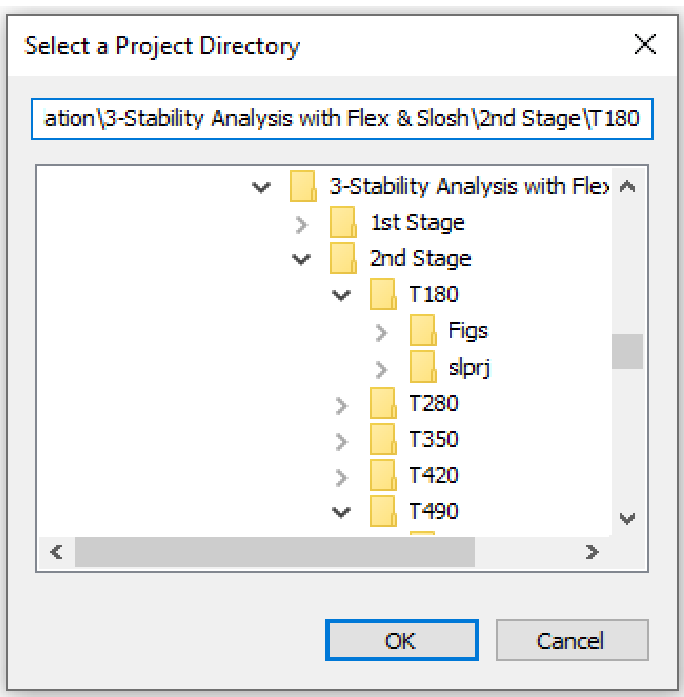
Figure 3.2.16 Modes File “Stage2_100%.Mod”



```
Node Identification Table for Vehicle Second Stage 100% Full - 20% Sloshing, Modes=12, Nodes=8
Node Description                       Node No   Nastran ID   Node Location X,Y,Z (feet)
-------------------------------------------------------------------------------------------
LH2 Slosh Mass Locat.                     1         600         89.62    0.0     0.0
LOX Slosh Mass and IMU                    2         601         96.93    0.0     0.0
Stage-2 Gimbal                            3        3303         84.71    0.0     0.0
Stage-2 Tank Bottom                       4       40002         87.54    0.0     0.0
Tip of Vehicle                            5       60031        120.144   0.0     0.0
Payload Interface                         6       90000        100.0     0.0     0.0
Payload CG                                7       90008        106.67    0.0     0.0
Tip of Payload                            8       90017        114.17    0.0     0.0
-------------------------------------------------------------------------------------------
```

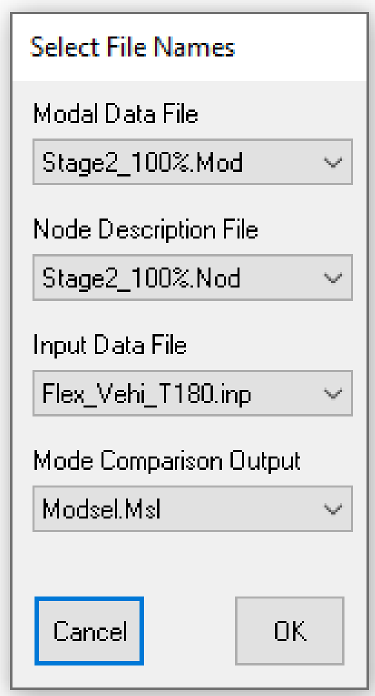
Figure 3.2.17 Nodes Identification File "Stage2_100%.Nod"

The mode selection is a Flixan process by which a smaller number of modes (12 dominant modes in this case) are selected to be included in the vehicle input file and be combined with the rigid vehicle data. After selecting the project directory, the Flex Mode Selection program is selected from the Flixan main menu. It begins with a small filenames selection menu where the user selects the modal data and the node files, the flight vehicle input data file, and an output filename (Modsel.Msl by default) that will include the relative mode strength at the completion of the mode selection.



Select a Project Directory

ation\3-Stability Analysis with Flex & Slosh\2nd Stage\T180



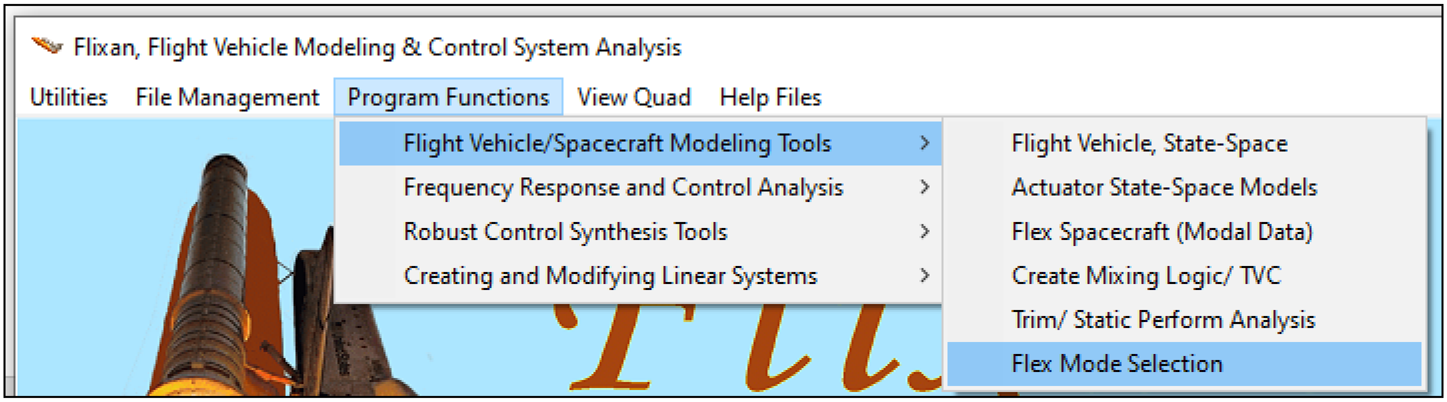
Select File Names

Modal Data File: Stage2_100%.Mod

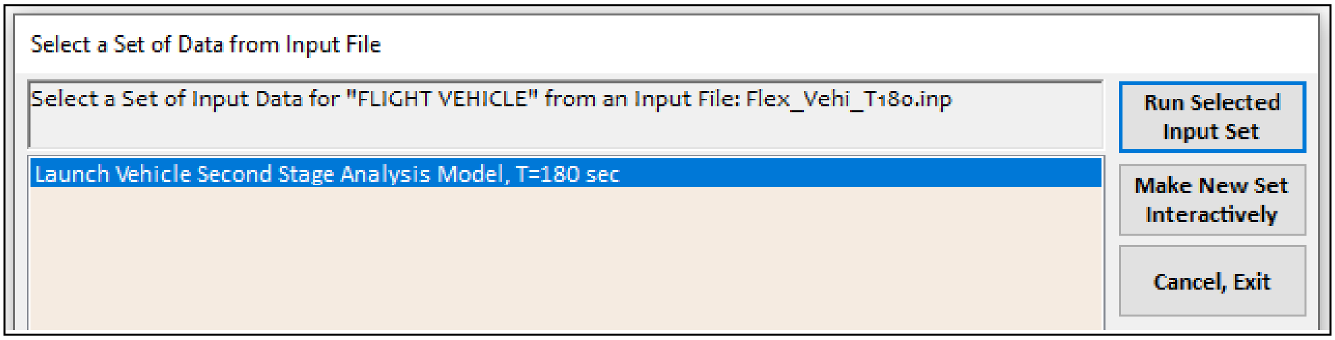
Node Description File: Stage2_100%.Nod

Input Data File: Flex_Vehi_T180.inp

Mode Comparison Output: Modsel.Msl

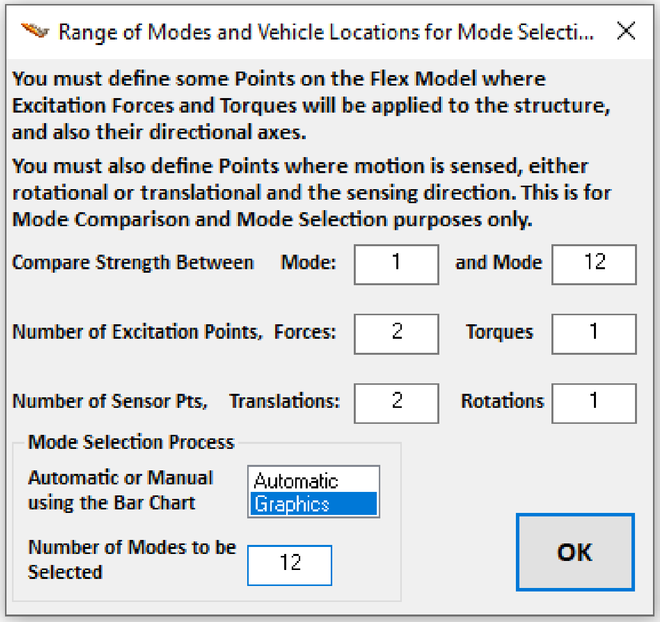


The next step is to locate the flight vehicle input dataset from file “*Flex_Vehi_T180.Inp*”. The vehicle input data will be combined with the selected modes to create the vehicle state-space system. It is also used by the mode selection program to extract information regarding the TVC, RCS, gyros, and slosh parameters which are defined in yhe vehicle dataset. Select the vehicle title “*Launch Vehicle Second Stage Analysis Model, T=180 sec*” and click on the “*Run Selected Input Set*” button. The mode shapes must be scaled during this process in order for the units to be compatible with the rigid-body data.


Select a Set of Data from Input File

Select a Set of Input Data for "FLIGHT VEHICLE" from an Input File: Flex_Vehi_T18o.inp

Launch Vehicle Second Stage Analysis Model, T=180 sec


Range of Modes and Vehicle Locations for Mode Selecti...

You must define some Points on the Flex Model where Excitation Forces and Torques will be applied to the structure, and also their directional axes.

You must also define Points where motion is sensed, either rotational or translational and the sensing direction. This is for Mode Comparison and Mode Selection purposes only.

Compare Strength Between Mode: 1 and Mode 12

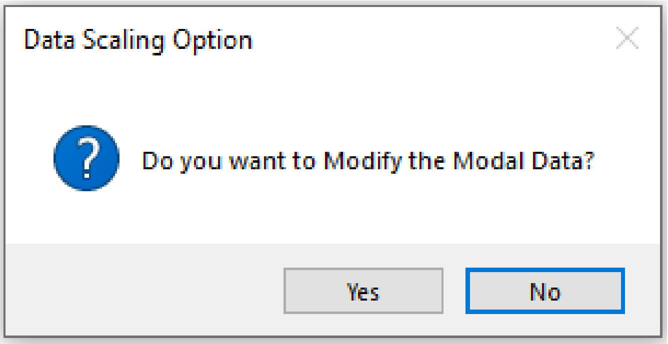
Number of Excitation Points, Forces: 2 Torques 1

Number of Sensor Pts, Translations: 2 Rotations 1

Mode Selection Process

Automatic or Manual using the Bar Chart: Automatic / Graphics (selected: Graphics)

Number of Modes to be Selected 12

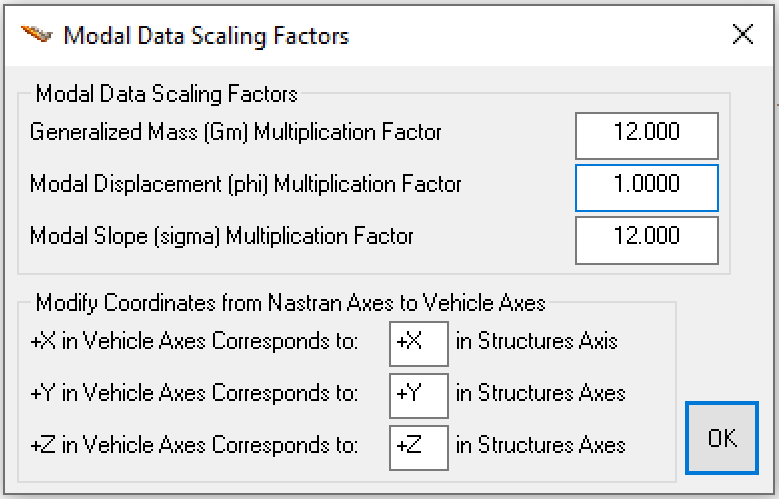

Data Scaling Option

Do you want to Modify the Modal Data?

The above dialog is used to define the following:

1. The range of modes to be compared. In this case it is from 1 to 12, all flex modes.
2. The number of excitation force and torque points used in the mode selection process. 2 forces and 1 torque in this case. These nodes are only used for mode strength comparison purposes and they are not necessarily vehicle actuator locations.
3. The number of translational and rotational sensor points used in the mode selection process. 2 translational and 1 rotational in this case. These nodes are only used for mode strength comparison purposes and they are not necessarily vehicle sensor locations.
4. The program has two Mode Selection options: (a) a number of modes are either selected automatically based on their relative modal strength or (b) they are selected manually using a graphics bar chart that plots the relative mode strength versus the mode number. We more often prefer the manual graphic selection using the bar chart.

The program will ask you if you want to modify and rescale the modal data. Answer “Yes”. The dialog below is used for scaling and modifying the data. The default units from a Nastran model are often different from the units of the vehicle model and we must convert the modal data to units which are compatible with the vehicle data. The directions of the coordinate axes may also be different between the two models but in this case the axes are in the same directions. The generalized mass is often defined in “snails” (lb-sec^2/inch), and it must be multiplied by 12 to be converted to “slugs” (lb-sec^2/feet). Similarly, the generalized modal rotations (slopes σ) are often defined in (rad/inch) in the Nastran model and they must be multiplied by 12 in order to be converted to (rad/foot). The generalized modal displacements (ϕ) don’t need any scaling to be converted from (in/inch) to (ft/feet).


Modal Data Scaling Factors

Modal Data Scaling Factors

Generalized Mass (Gm) Multiplication Factor: 12.000

Modal Displacement (phi) Multiplication Factor: 1.0000

Modal Slope (sigma) Multiplication Factor: 12.000

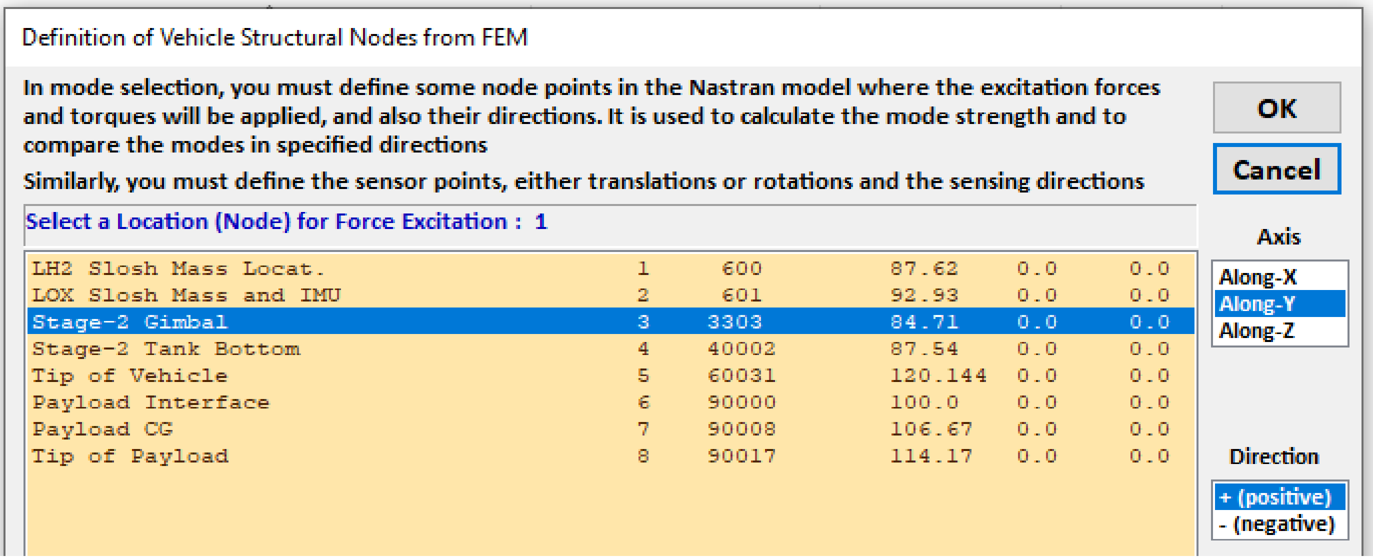
Modify Coordinates from Nastran Axes to Vehicle Axes

+X in Vehicle Axes Corresponds to: +X in Structures Axis

+Y in Vehicle Axes Corresponds to: +Y in Structures Axes

+Z in Vehicle Axes Corresponds to: +Z in Structures Axes

The next step is to identify the excitation and sensor points on the structure that will be used to calculate and to compare the mode strengths in the selection process. In the previous dialog we defined 2 excitation forces and 1 torque. We also defined 2 translational and 1 rotational measurement points. We must now define the nodes of those excitation and sensor points. They will be used only for mode selection purposes and they are not always the same locations as the TVC gimbal and the flight control sensors, that will be defined later. The mode selection program provides interactive menus which are used to define the locations of the excitation and sensor points. It displays the nodes map in a menu form that allows the user to select the excitation and sensor locations in the structure model and also the directions of the excitation and measurement. Each selection defines a node number and the program reads the corresponding mode shapes at all frequencies. From the mode shapes and from the directions of the excitations and sensors the program calculates the modal strength at each frequency. In the following case, the program wants to define a node for the force excitation #1. We select the 3rd node that corresponds to the TVC gimbal, and the force direction is along +Y. Click “OK” to continue. We do the same for the second force excitation along +Z, and the torque excitation which is in +roll. We must also define locations for the 3 sensors. They are all in the IMU node #2, measuring along Y, Z, and roll.

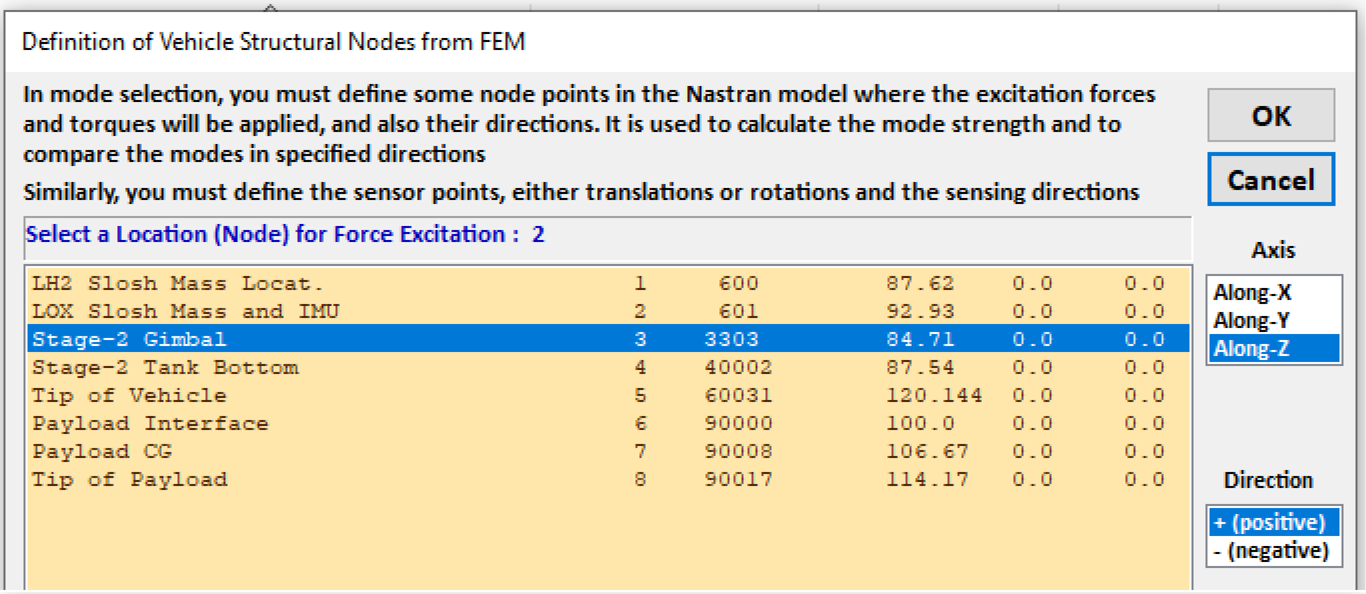

Definition of Vehicle Structural Nodes from FEM

In mode selection, you must define some node points in the Nastran model where the excitation forces and torques will be applied, and also their directions. It is used to calculate the mode strength and to compare the modes in specified directions

Similarly, you must define the sensor points, either translations or rotations and the sensing directions

Select a Location (Node) for Force Excitation : 1

LH2 Slosh Mass Locat.	1	600	87.62	0.0	0.0
LOX Slosh Mass and IMU	2	601	92.93	0.0	0.0
Stage-2 Gimbal	3	3303	84.71	0.0	0.0
Stage-2 Tank Bottom	4	40002	87.54	0.0	0.0
Tip of Vehicle	5	60031	120.144	0.0	0.0
Payload Interface	6	90000	100.0	0.0	0.0
Payload CG	7	90008	106.67	0.0	0.0
Tip of Payload	8	90017	114.17	0.0	0.0



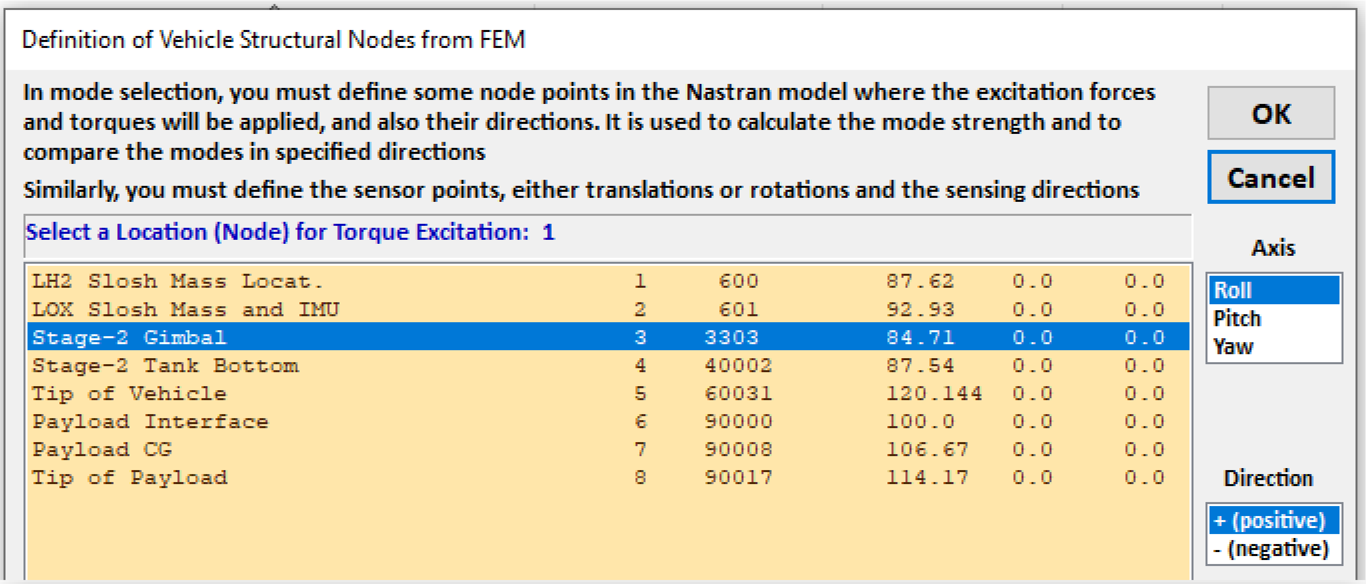
Definition of Vehicle Structural Nodes from FEM

In mode selection, you must define some node points in the Nastran model where the excitation forces and torques will be applied, and also their directions. It is used to calculate the mode strength and to compare the modes in specified directions

Similarly, you must define the sensor points, either translations or rotations and the sensing directions

Select a Location (Node) for Force Excitation : 2

LH2 Slosh Mass Locat.	1	600	87.62	0.0	0.0
LOX Slosh Mass and IMU	2	601	92.93	0.0	0.0
Stage-2 Gimbal	3	3303	84.71	0.0	0.0
Stage-2 Tank Bottom	4	40002	87.54	0.0	0.0
Tip of Vehicle	5	60031	120.144	0.0	0.0
Payload Interface	6	90000	100.0	0.0	0.0
Payload CG	7	90008	106.67	0.0	0.0
Tip of Payload	8	90017	114.17	0.0	0.0



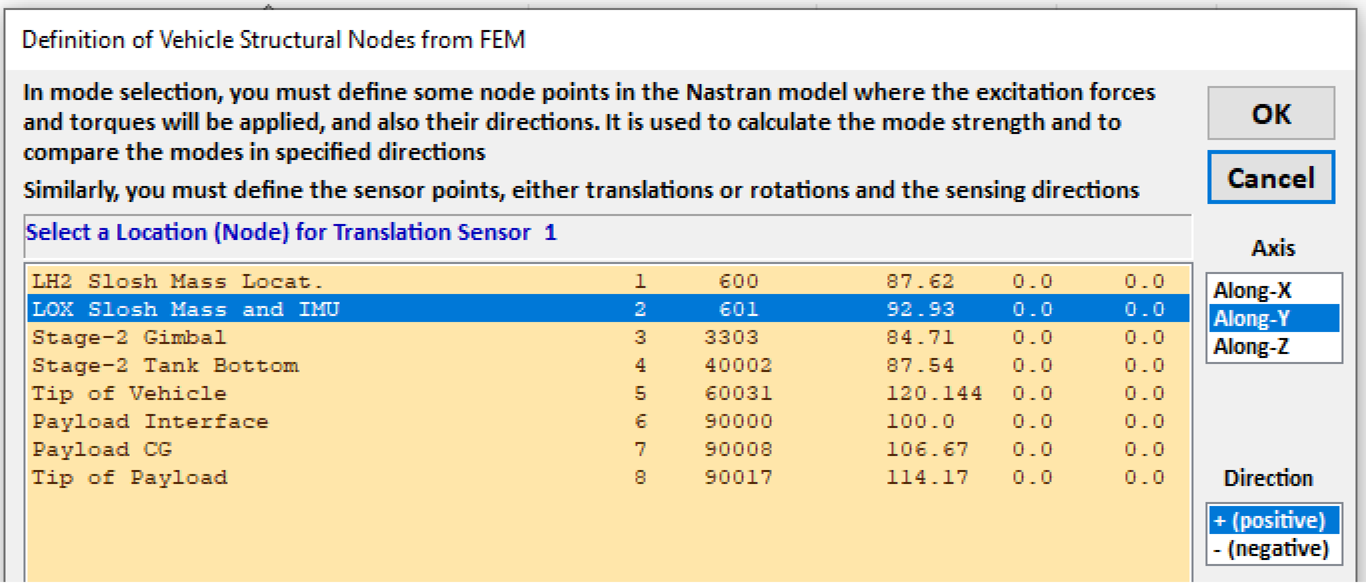
Definition of Vehicle Structural Nodes from FEM

In mode selection, you must define some node points in the Nastran model where the excitation forces and torques will be applied, and also their directions. It is used to calculate the mode strength and to compare the modes in specified directions

Similarly, you must define the sensor points, either translations or rotations and the sensing directions

Select a Location (Node) for Torque Excitation: 1

LH2 Slosh Mass Locat.	1	600	87.62	0.0	0.0
LOX Slosh Mass and IMU	2	601	92.93	0.0	0.0
Stage-2 Gimbal	3	3303	84.71	0.0	0.0
Stage-2 Tank Bottom	4	40002	87.54	0.0	0.0
Tip of Vehicle	5	60031	120.144	0.0	0.0
Payload Interface	6	90000	100.0	0.0	0.0
Payload CG	7	90008	106.67	0.0	0.0
Tip of Payload	8	90017	114.17	0.0	0.0



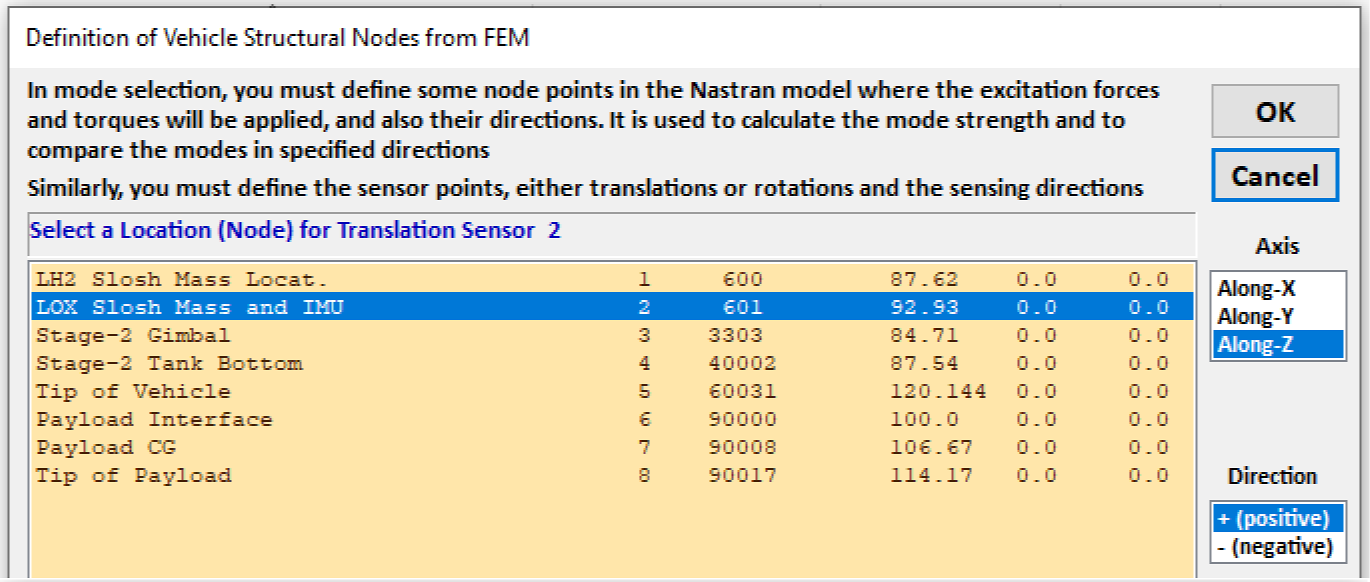
Definition of Vehicle Structural Nodes from FEM

In mode selection, you must define some node points in the Nastran model where the excitation forces and torques will be applied, and also their directions. It is used to calculate the mode strength and to compare the modes in specified directions

Similarly, you must define the sensor points, either translations or rotations and the sensing directions

Select a Location (Node) for Translation Sensor 1

LH2 Slosh Mass Locat.	1	600	87.62	0.0	0.0
LOX Slosh Mass and IMU	2	601	92.93	0.0	0.0
Stage-2 Gimbal	3	3303	84.71	0.0	0.0
Stage-2 Tank Bottom	4	40002	87.54	0.0	0.0
Tip of Vehicle	5	60031	120.144	0.0	0.0
Payload Interface	6	90000	100.0	0.0	0.0
Payload CG	7	90008	106.67	0.0	0.0
Tip of Payload	8	90017	114.17	0.0	0.0



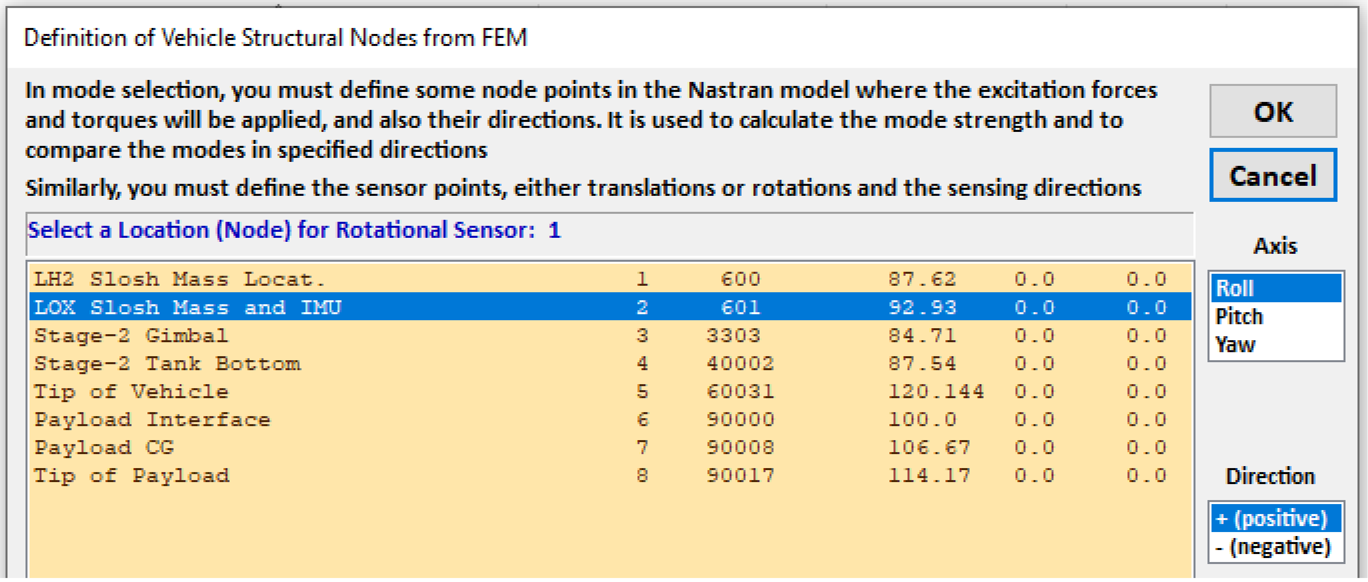
Definition of Vehicle Structural Nodes from FEM

In mode selection, you must define some node points in the Nastran model where the excitation forces and torques will be applied, and also their directions. It is used to calculate the mode strength and to compare the modes in specified directions

Similarly, you must define the sensor points, either translations or rotations and the sensing directions

Select a Location (Node) for Translation Sensor 2

```
LH2 Slosh Mass Locat.                1     600      87.62    0.0     0.0
LOX Slosh Mass and IMU               2     601      92.93    0.0     0.0
Stage-2 Gimbal                       3    3303      84.71    0.0     0.0
Stage-2 Tank Bottom                  4   40002      87.54    0.0     0.0
Tip of Vehicle                       5   60031     120.144   0.0     0.0
Payload Interface                    6   90000     100.0     0.0     0.0
Payload CG                           7   90008     106.67    0.0     0.0
Tip of Payload                       8   90017     114.17    0.0     0.0
```



Definition of Vehicle Structural Nodes from FEM

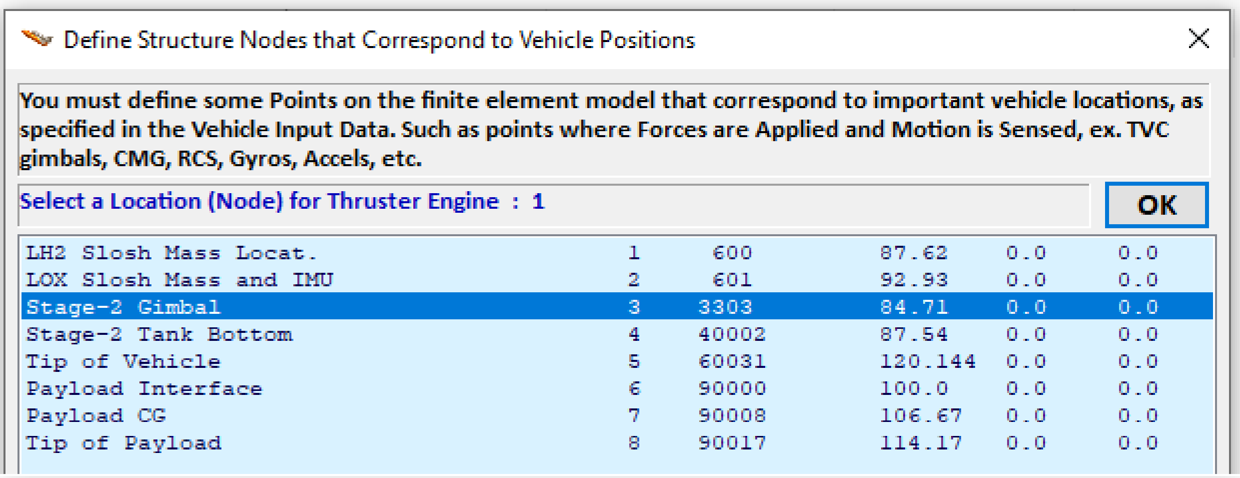
In mode selection, you must define some node points in the Nastran model where the excitation forces and torques will be applied, and also their directions. It is used to calculate the mode strength and to compare the modes in specified directions

Similarly, you must define the sensor points, either translations or rotations and the sensing directions

Select a Location (Node) for Rotational Sensor: 1

```
LH2 Slosh Mass Locat.                1     600      87.62    0.0     0.0
LOX Slosh Mass and IMU               2     601      92.93    0.0     0.0
Stage-2 Gimbal                       3    3303      84.71    0.0     0.0
Stage-2 Tank Bottom                  4   40002      87.54    0.0     0.0
Tip of Vehicle                       5   60031     120.144   0.0     0.0
Payload Interface                    6   90000     100.0     0.0     0.0
Payload CG                           7   90008     106.67    0.0     0.0
Tip of Payload                       8   90017     114.17    0.0     0.0
```

The mode selection process is not complete yet because the program needs more information from the user before it can calculate the reduced set of modes in the input data file. The selected modes dataset will consist of the dominant mode frequencies and the mode shapes only at the locations that correspond to the locations defined in the vehicle input data, such as, the engine gimbal, RCS, sensors, and slosh masses. This is why we need the vehicle dataset in the mode selection, in order to match the vehicle locations with structural nodes from the FEM. A similar node identification process will be repeated where the user must select structure nodes that correspond to important vehicle locations which are defined in the vehicle input data and they are not necessarily the same as the points that were used in the mode strength comparison. The program uses menus similar to the previous menus and it will ask the user to identify locations for the TVC engine, the 4 RCS jets, the 2 slosh masses, the IMU sensors, and a disturbance point. These vehicle node selection menus are in blue color to be differentiated from the previous yellow menus which are used for mode strength calculations.

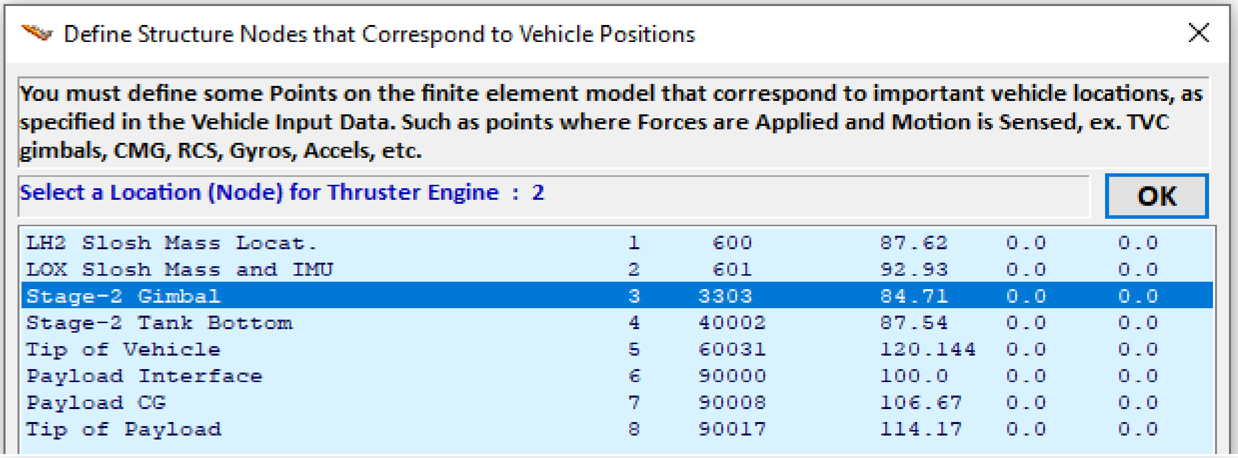


Define Structure Nodes that Correspond to Vehicle Positions

You must define some Points on the finite element model that correspond to important vehicle locations, as specified in the Vehicle Input Data. Such as points where Forces are Applied and Motion is Sensed, ex. TVC gimbals, CMG, RCS, Gyros, Accels, etc.

Select a Location (Node) for Thruster Engine : 1

```
LH2 Slosh Mass Locat.                1    600      87.62    0.0     0.0
LOX Slosh Mass and IMU               2    601      92.93    0.0     0.0
Stage-2 Gimbal                       3    3303     84.71    0.0     0.0
Stage-2 Tank Bottom                  4    40002    87.54    0.0     0.0
Tip of Vehicle                       5    60031    120.144  0.0     0.0
Payload Interface                    6    90000    100.0    0.0     0.0
Payload CG                           7    90008    106.67   0.0     0.0
Tip of Payload                       8    90017    114.17   0.0     0.0
```

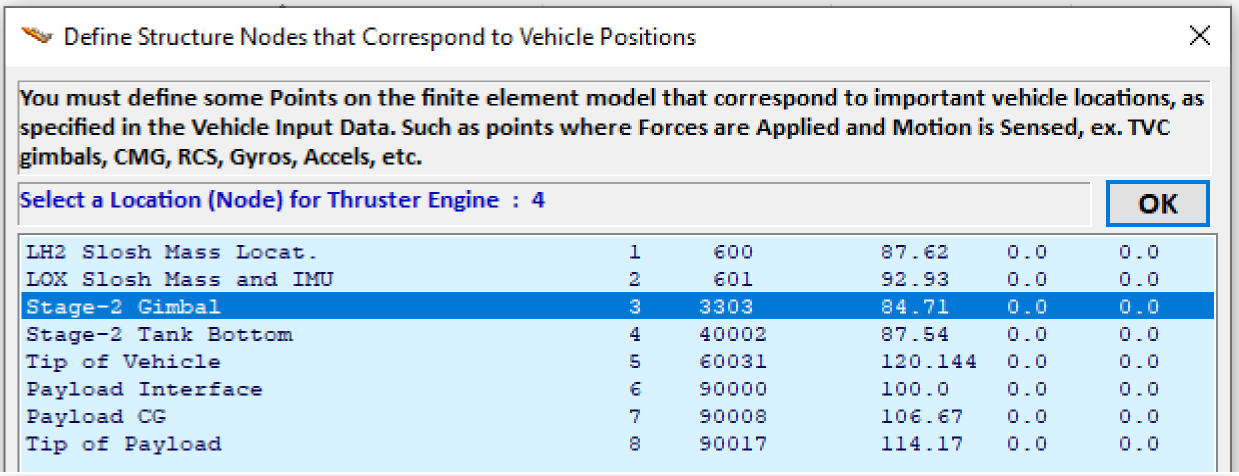


Define Structure Nodes that Correspond to Vehicle Positions

You must define some Points on the finite element model that correspond to important vehicle locations, as specified in the Vehicle Input Data. Such as points where Forces are Applied and Motion is Sensed, ex. TVC gimbals, CMG, RCS, Gyros, Accels, etc.

Select a Location (Node) for Thruster Engine : 2

```
LH2 Slosh Mass Locat.                1    600      87.62    0.0     0.0
LOX Slosh Mass and IMU               2    601      92.93    0.0     0.0
Stage-2 Gimbal                       3    3303     84.71    0.0     0.0
Stage-2 Tank Bottom                  4    40002    87.54    0.0     0.0
Tip of Vehicle                       5    60031    120.144  0.0     0.0
Payload Interface                    6    90000    100.0    0.0     0.0
Payload CG                           7    90008    106.67   0.0     0.0
Tip of Payload                       8    90017    114.17   0.0     0.0
```

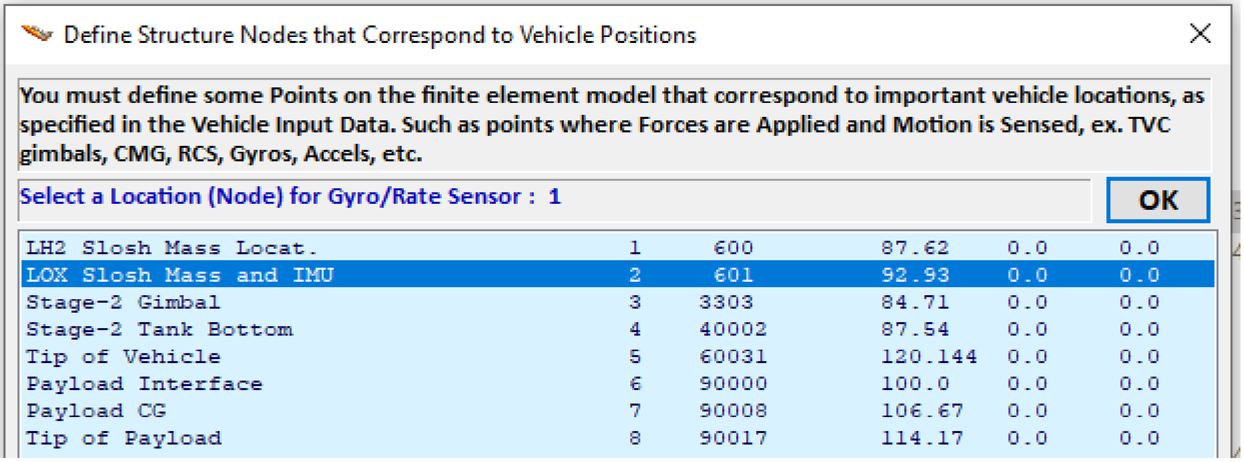


Define Structure Nodes that Correspond to Vehicle Positions

You must define some Points on the finite element model that correspond to important vehicle locations, as specified in the Vehicle Input Data. Such as points where Forces are Applied and Motion is Sensed, ex. TVC gimbals, CMG, RCS, Gyros, Accels, etc.

Select a Location (Node) for Thruster Engine : 4

```
LH2 Slosh Mass Locat.                1    600      87.62    0.0     0.0
LOX Slosh Mass and IMU               2    601      92.93    0.0     0.0
Stage-2 Gimbal                       3    3303     84.71    0.0     0.0
Stage-2 Tank Bottom                  4    40002    87.54    0.0     0.0
Tip of Vehicle                       5    60031    120.144  0.0     0.0
Payload Interface                    6    90000    100.0    0.0     0.0
Payload CG                           7    90008    106.67   0.0     0.0
Tip of Payload                       8    90017    114.17   0.0     0.0
```

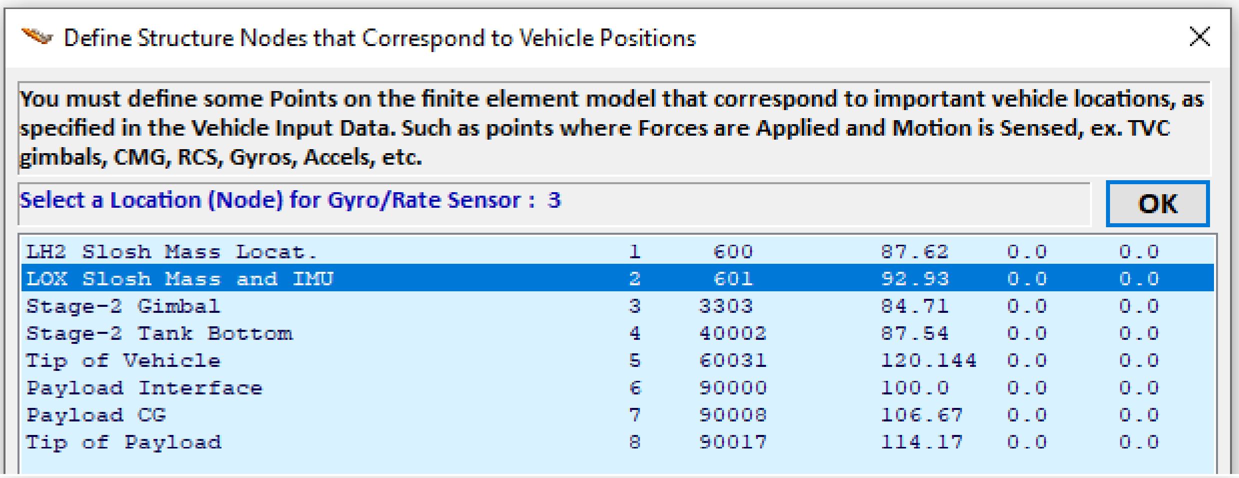


Define Structure Nodes that Correspond to Vehicle Positions

You must define some Points on the finite element model that correspond to important vehicle locations, as specified in the Vehicle Input Data. Such as points where Forces are Applied and Motion is Sensed, ex. TVC gimbals, CMG, RCS, Gyros, Accels, etc.

Select a Location (Node) for Gyro/Rate Sensor : 1

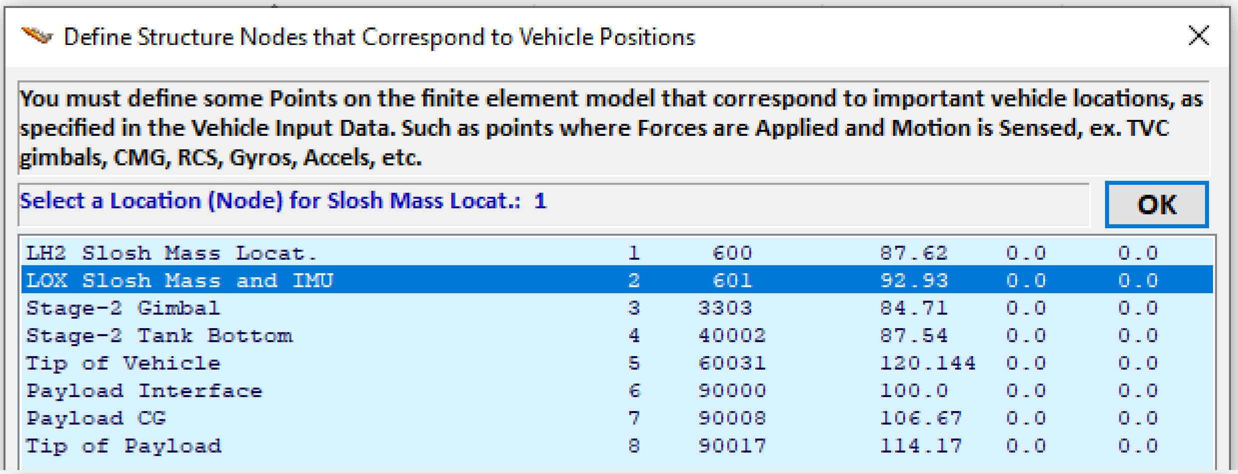
```
LH2 Slosh Mass Locat.                1    600      87.62    0.0     0.0
LOX Slosh Mass and IMU               2    601      92.93    0.0     0.0
Stage-2 Gimbal                       3    3303     84.71    0.0     0.0
Stage-2 Tank Bottom                  4    40002    87.54    0.0     0.0
Tip of Vehicle                       5    60031    120.144  0.0     0.0
Payload Interface                    6    90000    100.0    0.0     0.0
Payload CG                           7    90008    106.67   0.0     0.0
Tip of Payload                       8    90017    114.17   0.0     0.0
```



You must define some Points on the finite element model that correspond to important vehicle locations, as specified in the Vehicle Input Data. Such as points where Forces are Applied and Motion is Sensed, ex. TVC gimbals, CMG, RCS, Gyros, Accels, etc.

Select a Location (Node) for Gyro/Rate Sensor : 3

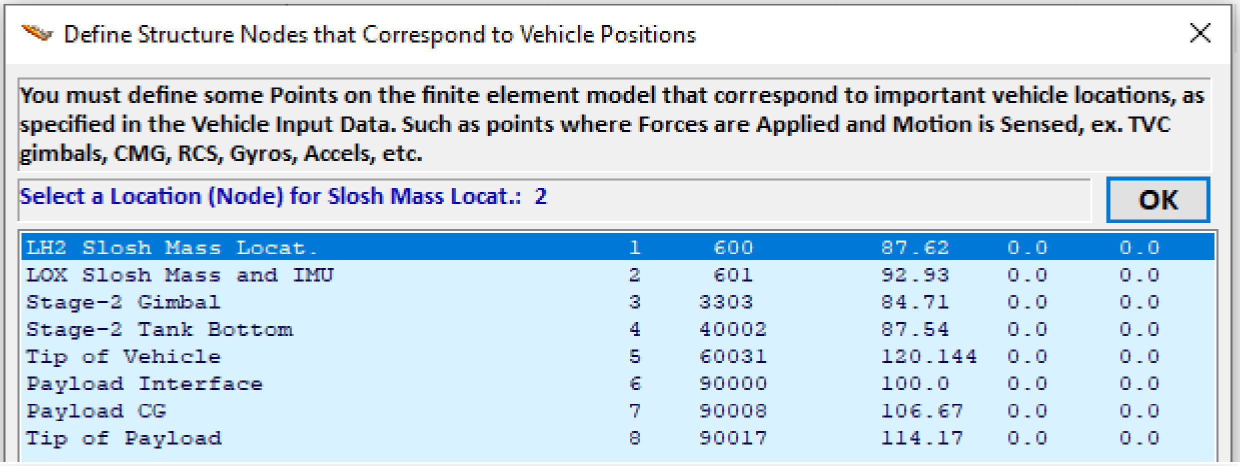
LH2 Slosh Mass Locat.	1	600	87.62	0.0	0.0
LOX Slosh Mass and IMU	2	601	92.93	0.0	0.0
Stage-2 Gimbal	3	3303	84.71	0.0	0.0
Stage-2 Tank Bottom	4	40002	87.54	0.0	0.0
Tip of Vehicle	5	60031	120.144	0.0	0.0
Payload Interface	6	90000	100.0	0.0	0.0
Payload CG	7	90008	106.67	0.0	0.0
Tip of Payload	8	90017	114.17	0.0	0.0



You must define some Points on the finite element model that correspond to important vehicle locations, as specified in the Vehicle Input Data. Such as points where Forces are Applied and Motion is Sensed, ex. TVC gimbals, CMG, RCS, Gyros, Accels, etc.

Select a Location (Node) for Slosh Mass Locat.: 1

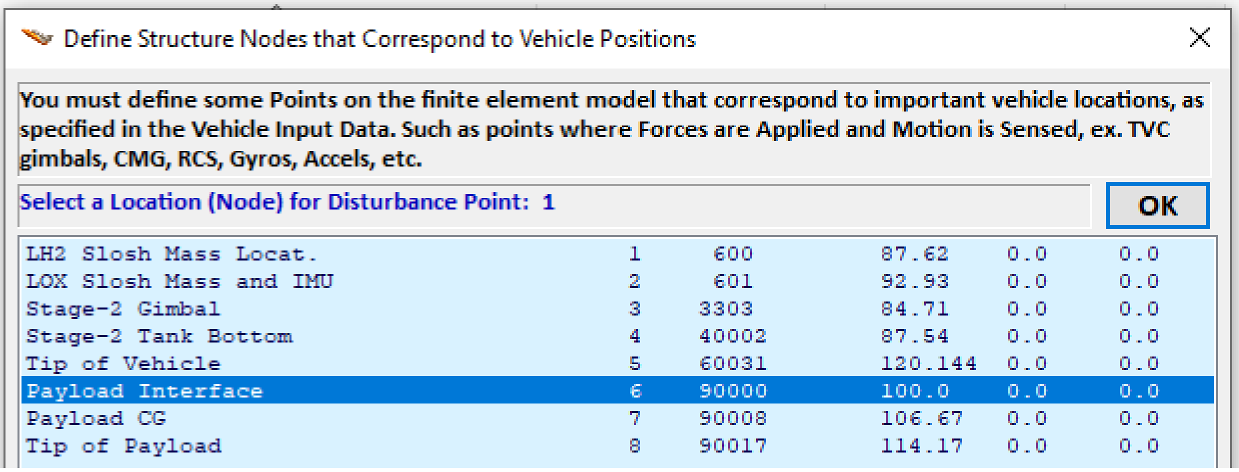
LH2 Slosh Mass Locat.	1	600	87.62	0.0	0.0
LOX Slosh Mass and IMU	2	601	92.93	0.0	0.0
Stage-2 Gimbal	3	3303	84.71	0.0	0.0
Stage-2 Tank Bottom	4	40002	87.54	0.0	0.0
Tip of Vehicle	5	60031	120.144	0.0	0.0
Payload Interface	6	90000	100.0	0.0	0.0
Payload CG	7	90008	106.67	0.0	0.0
Tip of Payload	8	90017	114.17	0.0	0.0



You must define some Points on the finite element model that correspond to important vehicle locations, as specified in the Vehicle Input Data. Such as points where Forces are Applied and Motion is Sensed, ex. TVC gimbals, CMG, RCS, Gyros, Accels, etc.

Select a Location (Node) for Slosh Mass Locat.: 2

LH2 Slosh Mass Locat.	1	600	87.62	0.0	0.0
LOX Slosh Mass and IMU	2	601	92.93	0.0	0.0
Stage-2 Gimbal	3	3303	84.71	0.0	0.0
Stage-2 Tank Bottom	4	40002	87.54	0.0	0.0
Tip of Vehicle	5	60031	120.144	0.0	0.0
Payload Interface	6	90000	100.0	0.0	0.0
Payload CG	7	90008	106.67	0.0	0.0
Tip of Payload	8	90017	114.17	0.0	0.0

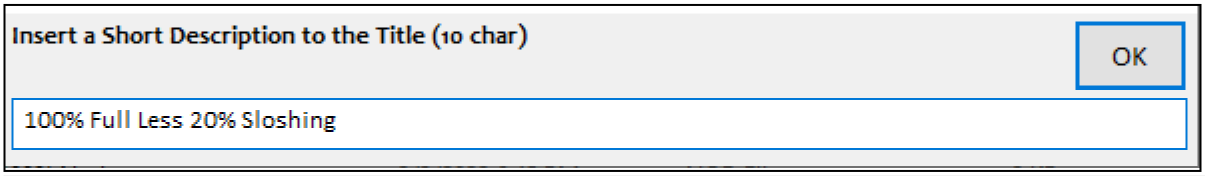


You must define some Points on the finite element model that correspond to important vehicle locations, as specified in the Vehicle Input Data. Such as points where Forces are Applied and Motion is Sensed, ex. TVC gimbals, CMG, RCS, Gyros, Accels, etc.

Select a Location (Node) for Disturbance Point: 1

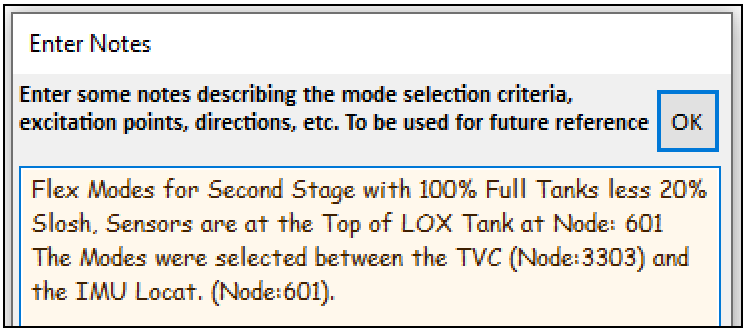
LH2 Slosh Mass Locat.	1	600	87.62	0.0	0.0
LOX Slosh Mass and IMU	2	601	92.93	0.0	0.0
Stage-2 Gimbal	3	3303	84.71	0.0	0.0
Stage-2 Tank Bottom	4	40002	87.54	0.0	0.0
Tip of Vehicle	5	60031	120.144	0.0	0.0
Payload Interface	6	90000	100.0	0.0	0.0
Payload CG	7	90008	106.67	0.0	0.0
Tip of Payload	8	90017	114.17	0.0	0.0

You must also enter a label that will be inserted in the title of the modal dataset to identify the dataset. At this point the mode strength comparison is complete. The program saves the relative mode strength for each mode in file "*Modsel.Dat*" and expects the user to manually select the dominant modes from the following bar-chart. Each mode appears as a vertical red bar of the mode strength plotted against the mode numbers, and the height of each bar is logarithmically proportional to its modal strength. The strong modes appear tall and the weak modes are short. The user can manually select some of the strongest modes by pointing the mouse cursor on a bar and clicking the mouse. The modes change color from red to green when they are selected. 9 flex modes were selected in this case. When you finish selecting the modes press the "*Enter*" button on the keyboard and it will save the selected modal data set in the input file (.Inp). Additional notes or comments can be entered in the field below that will provide information about the mode selection process.


Insert a Short Description to the Title (10 char)

100% Full Less 20% Sloshing

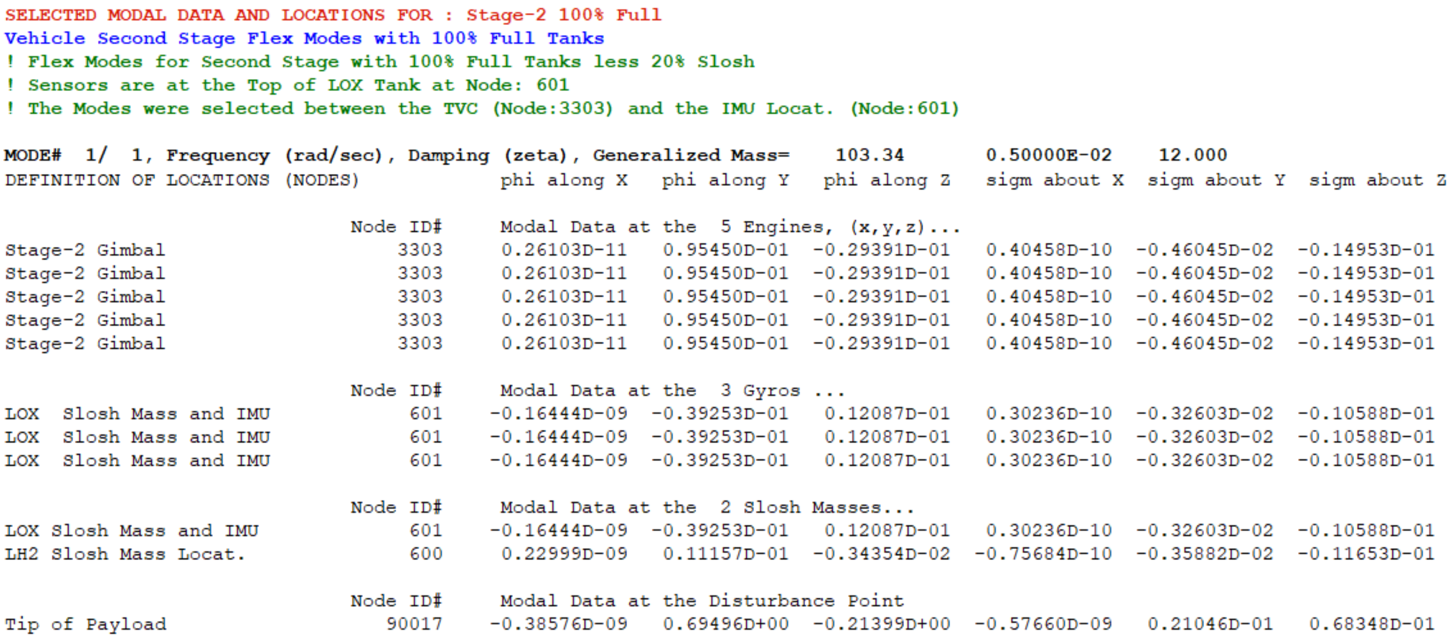
The selected and scaled set of mode frequencies and shapes are finally saved in the input file under the specified title and ready to be processed by the vehicle modeling program. The title can be changed to better define the selected modes. The title of the selected modes must also be included in the last line of the vehicle input data (below the line that specifies the number of flex modes) in order for the flight vehicle modeling program to associate the modes with the vehicle data.



Enter some notes describing the mode selection criteria, excitation points, directions, etc. To be used for future reference

Flex Modes for Second Stage with 100% Full Tanks less 20% Slosh, Sensors are at the Top of LOX Tank at Node: 601 The Modes were selected between the TVC (Node:3303) and the IMU Locat. (Node:601).

The selected set of modes is shown below (only the first mode). The top red line is the id line that identifies the dataset as modal data. The second line (blue) is the title of the modal data set "*Vehicle Second Stage Flex Modes with 100% Full Tanks*". The third set of lines (green) are the comments added by the user. The scaled modal data set consists of frames of data for each mode that include: the mode frequency in (rad/sec), damping coefficient zeta, and the generalized modal mass. Each frame also includes the mode shapes and slopes (three translations along x, y, z, and three rotations about x, y, z) at the vehicle locations defined. The data is already properly scaled and ready to be processed by the vehicle modeling program together with the rigid vehicle data



```
SELECTED MODAL DATA AND LOCATIONS FOR : Stage-2 100% Full
Vehicle Second Stage Flex Modes with 100% Full Tanks
! Flex Modes for Second Stage with 100% Full Tanks less 20% Slosh
! Sensors are at the Top of LOX Tank at Node: 601
! The Modes were selected between the TVC (Node:3303) and the IMU Locat. (Node:601)

MODE#  1/  1, Frequency (rad/sec), Damping (zeta), Generalized Mass=    103.34      0.50000E-02    12.000
DEFINITION OF LOCATIONS (NODES)           phi along X   phi along Y   phi along Z   sigm about X  sigm about Y  sigm about Z

                              Node ID#     Modal Data at the  5 Engines, (x,y,z)...
Stage-2 Gimbal                   3303     0.26103D-11   0.95450D-01  -0.29391D-01   0.40458D-10  -0.46045D-02  -0.14953D-01
Stage-2 Gimbal                   3303     0.26103D-11   0.95450D-01  -0.29391D-01   0.40458D-10  -0.46045D-02  -0.14953D-01
Stage-2 Gimbal                   3303     0.26103D-11   0.95450D-01  -0.29391D-01   0.40458D-10  -0.46045D-02  -0.14953D-01
Stage-2 Gimbal                   3303     0.26103D-11   0.95450D-01  -0.29391D-01   0.40458D-10  -0.46045D-02  -0.14953D-01
Stage-2 Gimbal                   3303     0.26103D-11   0.95450D-01  -0.29391D-01   0.40458D-10  -0.46045D-02  -0.14953D-01

                              Node ID#     Modal Data at the  3 Gyros ...
LOX  Slosh Mass and IMU           601    -0.16444D-09  -0.39253D-01   0.12087D-01   0.30236D-10  -0.32603D-02  -0.10588D-01
LOX  Slosh Mass and IMU           601    -0.16444D-09  -0.39253D-01   0.12087D-01   0.30236D-10  -0.32603D-02  -0.10588D-01
LOX  Slosh Mass and IMU           601    -0.16444D-09  -0.39253D-01   0.12087D-01   0.30236D-10  -0.32603D-02  -0.10588D-01

                              Node ID#     Modal Data at the  2 Slosh Masses...
LOX Slosh Mass and IMU            601    -0.16444D-09  -0.39253D-01   0.12087D-01   0.30236D-10  -0.32603D-02  -0.10588D-01
LH2 Slosh Mass Locat.             600     0.22999D-09   0.11157D-01  -0.34354D-02  -0.75684D-10  -0.35882D-02  -0.11653D-01

                              Node ID#     Modal Data at the Disturbance Point
Tip of Payload                  90017    -0.38576D-09   0.69496D+00  -0.21399D+00  -0.57660D-09   0.21046D-01   0.68348D-01
```

4. Analyzing the Control System Performance and Robustness to Uncertainties

Robustness is the ability of the control system to tolerate external disturbances and also variations or uncertainties in vehicle parameters. In this section we will create dynamic models that can be used to analyze the system robustness to parameter uncertainties and also sensitivity to wind-gusts. Sensitivity is defined as the ability of the control system to counteract disturbances, and in this case, it is the effects of wind-gusts on the angles of attack and sideslip which represent lateral loads. The Singular Value (SV) plots are used to analyze the system's sensitivity between certain inputs and outputs with the control loop closed. We will also analyze the system's robustness to internal parameter variations. That is, how much of parameter variations is a system able to tolerate before it becomes unstable, or stops performing properly? Parameter uncertainties can be seen as imprecise knowledge of the plant parameters, such as: mass, moments of inertia, aerodynamic coefficients and derivatives, dynamic pressure, center of gravity, thrust variations, slosh and flex parameters, etc. The uncertainties in a model are defined in terms of variations of the actual plant parameters, relative to their nominal values. These uncertainties are called *"Structured"*, in contrast with the *"Unstructured"* uncertainties which are described in the frequency domain in terms of maximum amplitude error in the transfer function model.

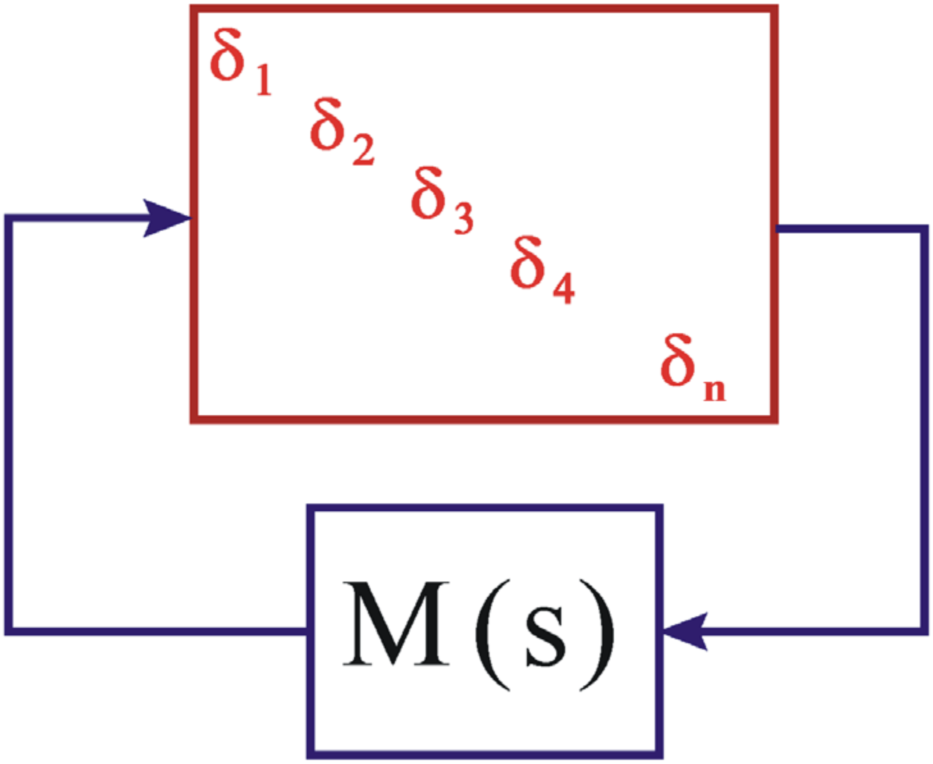


Figure 4.1 The Uncertainties are extracted from the plant M(s) and are placed in a diagonal Δ block

The following method is used by the Flixan program to create the vehicle state-space systems for analyzing the control system robustness to parameter uncertainties, where the magnitude of the variation is known. The variation of each parameter is "pulled out" of the uncertain plant model and it is placed inside a diagonal block Δ that contains only the uncertainties. The remaining plant is assumed to be known (best guess).

The Δ block is attached to the known plant M(s) by means of (n) input/ output "wires", where (n) is the number of plant uncertainties, as shown in Figure 1. In essence, if M(s) is the plant model representing the flight vehicle, we are creating (n) additional inputs and outputs to M(s) that connect with the uncertainties block Δ, which is a block diagonal matrix $\Delta = \text{diag}(\delta_1, \delta_2, \delta_3, \ldots \delta_n)$. The individual elements δ_i of the matrix block Δ may be scalars or small matrix blocks and each element represents a real uncertainty in the plant. The magnitude of each δ_i represents the maximum variation of the corresponding parameter above or below its nominal value p_i. Note that M(s) in addition to the vehicle dynamics it also includes the control system in closed-loop form and M(s) is assumed to be closed-loop stable. So, the internal uncertainty Δ block is "pulled out" of the closed-loop plant M(s) and it is connected to M(s) via the additional inputs and outputs.

The stable closed-loop system M(s) in Figure 1 is defined to be robust to a set of parameter variations δ_i which are included in the Δ block if it remains stable in the presence of all possible variations of those parameters as long as the magnitude of each variation from its nominal value does not exceed the amount of the corresponding uncertainty δ_i. The control system Robustness and Performance are analyzed in the frequency domain, similar to sensitivity analysis, using the structured singular value (SSV) or μ–method. The following three types of criteria are used for analyzing the closed-loop system performance and robustness:

1. Nominal Performance: is the ability of the nominal closed-loop plant to satisfy the sensitivity requirements to winds or to commands without parameter variations
2. Robustness to Parameter Variations: is the ability of the control system to remain stable in presence of all parameter variations which are included in the block Δ, and
3. Robust Performance: is the ability of the control system to satisfy conditions (1) and (2) together. That is, satisfying robustness to parameter variations and simultaneously maintaining an acceptable sensitivity response to external disturbances or commands.

The augmented state-space system M(s), without the Δ block, is used to calculate robustness using μ-methods. To simplify the analysis, the inputs and outputs of the plant M(s) which are connected to the Δ block are scaled so that the individual elements of the diagonal uncertainty block Δ can now vary between +1 and -1. This simple scaling allows the gains of the parameter variations δ_i to be absorbed in M(s) and the magnitudes of the new uncertainties are now bounded to be less than 1. The value of $1/\mu(M)$ represents the magnitude of the smallest perturbation that will destabilize the normalized closed-loop system M(s). According to the small gain theorem, the closed-loop system is robust as long as $\mu(M)$ across the normalized block Δ is less than one at all frequencies.

We will now describe an algorithm which is included in the Flixan vehicle modeling program to extract the uncertainties and generate the dynamic model that can be used to analyze the control system robustness to parameter uncertainties. it does not change the vehicle model but it includes additional inputs and outputs that connect with the uncertainty parameter block Δ. The uncertain parameters and their magnitudes are defined in a data set in the input data file together with the vehicle input data. The augmented model is then used to analyze robustness in Matlab by computing the mu frequency response across the open I/O connections.

4.1 The Internal Feedback Loop (IFL) Structure

The IFL method allows internal parameter perturbations in a plant to be treated like external disturbances in the system by means of fictitious inputs and outputs. This representation allows us to use μ-tools for analyzing robustness to uncertainties, or to apply H_∞ and other robust methods to design control systems that can tolerate a certain amount of parameter variations. To utilize the IFL concept the system must be expressed in the following form, where [ΔA, ΔB, ΔC, ΔD] are variations in the state-space system matrices as a result of variation in one of the parameters.

$$\begin{bmatrix} \dot{x} \\ y \end{bmatrix} = \left\{ \begin{bmatrix} A & B \\ C & D \end{bmatrix} + \begin{bmatrix} \Delta A & \Delta B \\ \Delta C & \Delta D \end{bmatrix} \right\} \begin{bmatrix} x \\ u \end{bmatrix}$$

Suppose that they are (l) independently perturbed parameters: p_1, p_2, ... p_l with bounded parameter variations δp_i, where their magnitude $|\delta p_i| \leq 1$. The perturbation matrix ΔP= [ΔA, ΔB; ΔC, ΔD] can be decomposed with respect to each parameter variation as follows:

$$\Delta_i = -\sum_{i=1}^{l} \delta p_i \begin{pmatrix} \alpha_x^{(i)} \\ \alpha_y^{(i)} \end{pmatrix} \begin{pmatrix} \beta_x^{(i)} & \beta_u^{(i)} \end{pmatrix}$$

Where for each parameter p_i

$\alpha_x^{(i)}$ and $\alpha_y^{(i)}$	are column vectors
$\beta_x^{(i)}$, and $\beta_u^{(i)}$	are row vectors

The plant uncertainty matrix ΔP due to all perturbations can be written in the following form, where the perturbation block ΔP is assumed to have a rank-1 dependency with respect to each parameter p_i.

$$\Delta P = -\begin{pmatrix} M_x \\ M_y \end{pmatrix} \Delta \begin{pmatrix} N_x & N_u \end{pmatrix} = -M \Delta N$$

Where M_x and M_y are stacks of column vectors, and N_x and N_u are stacks of row vectors as shown below

$$M_x = \begin{bmatrix} \alpha_x^{(1)} & \alpha_x^{(2)} & .... & \alpha_x^{(l)} \end{bmatrix}; \qquad M_y = \begin{bmatrix} \alpha_y^{(1)} & \alpha_y^{(2)} & .... & \alpha_y^{(l)} \end{bmatrix}$$

$$N_x = \begin{bmatrix} \beta_x^{(1)} \\ \vdots \\ \beta_x^{(l)} \end{bmatrix}; \qquad N_u = \begin{bmatrix} \beta_u^{(1)} \\ \vdots \\ \beta_u^{(l)} \end{bmatrix} \qquad \text{and}$$

Where $\Delta = \text{diag}[\delta p_1, \delta p_2, \delta p_3, \dots \delta p_l]$ is the diagonal block of Figure-1 containing the uncertainties. Notice, that in order to simplify the implementation, the columns of matrices M_x and M_y and the rows of matrices N_x and N_u are scaled, so that the elements of the diagonal block Δ have unity upper bound. Now let us introduce two new variables (z_p and w_p) and rewrite the equations in the following system form in order to express it as a block diagram.

$$z_p = N_x x + N_u u \quad and \quad w_p = -\Delta z_p$$

The perturbed state-space system can be expressed by the following augmented representation which is the same as the original system in the upper left side, with some additional input and output vectors, an input and an output for each parameter uncertainty.

$$\begin{pmatrix} \dot{x} \\ y \\ z_p \end{pmatrix} = \begin{bmatrix} A & B & M_x \\ C & D & M_y \\ N_x & N_u & 0 \end{bmatrix} \begin{pmatrix} x \\ u \\ w_p \end{pmatrix}$$

If we further separate the plant inputs (u) into disturbances (w) and controls (u_c). That is: $u=[w, u_c]$, and if we also separate the plant outputs (y) into performance criteria (z) and control measurements (y_m), the above system is augmented as shown below.

$$\begin{pmatrix} \dot{x} \\ z \\ y_m \\ z_p \end{pmatrix} = \begin{bmatrix} A & B_1 & B_2 & M_x \\ C_1 & D_{11} & D_{12} & M_w \\ C_2 & D_{21} & D_{22} & M_{ym} \\ N_x & N_w & N_{uc} & 0 \end{bmatrix} \begin{pmatrix} x \\ w \\ u_c \\ w_p \end{pmatrix}$$

The above formulation is useful for μ-synthesis or robustness/ performance analysis using μ-methods. It is also shown in block diagram form in Figure 2. The uncertainties block Δ is connected to the plant by means of the inputs w_p and the outputs z_p. The columns in the M_x, M_w, and M_{ym} matrices and the rows in the N_x , N_w, and N_{uc} matrices are scaled by dividing with the square root of the corresponding singular value in order to allow the elements of the uncertainty block Δ to be normalized to unity.

The control system K(s) is designed to stabilize the nominal plant P(s). When the feedback loop is closed between y_m and u_c the control system is also expected to keep the plant stable despite all possible variations in the elements of the block Δ which are allowed to vary between -1 and +1. This property is defined as Robust Stability. In addition to robust stability the control system must also be able to satisfy “Nominal Performance” requirements. That is a bounded and well-behaved response between the disturbances w and the criteria z.

We can also analyze robustness to uncertainties and performance to disturbances together, a property known as "*Robust Performance*". In Figure 4.2 the plant P(s) has the control loop closed and also the uncertainty loop closed via the Δ-block. The closed-loop system satisfies the Robust Performance criterion when it remains stable, and it is also able to satisfy the required performance criteria between w and z despite the possible variations in the internal parameters represented in the normalized uncertainties block Δ, where the individual magnitudes δ_i do not exceed 1.

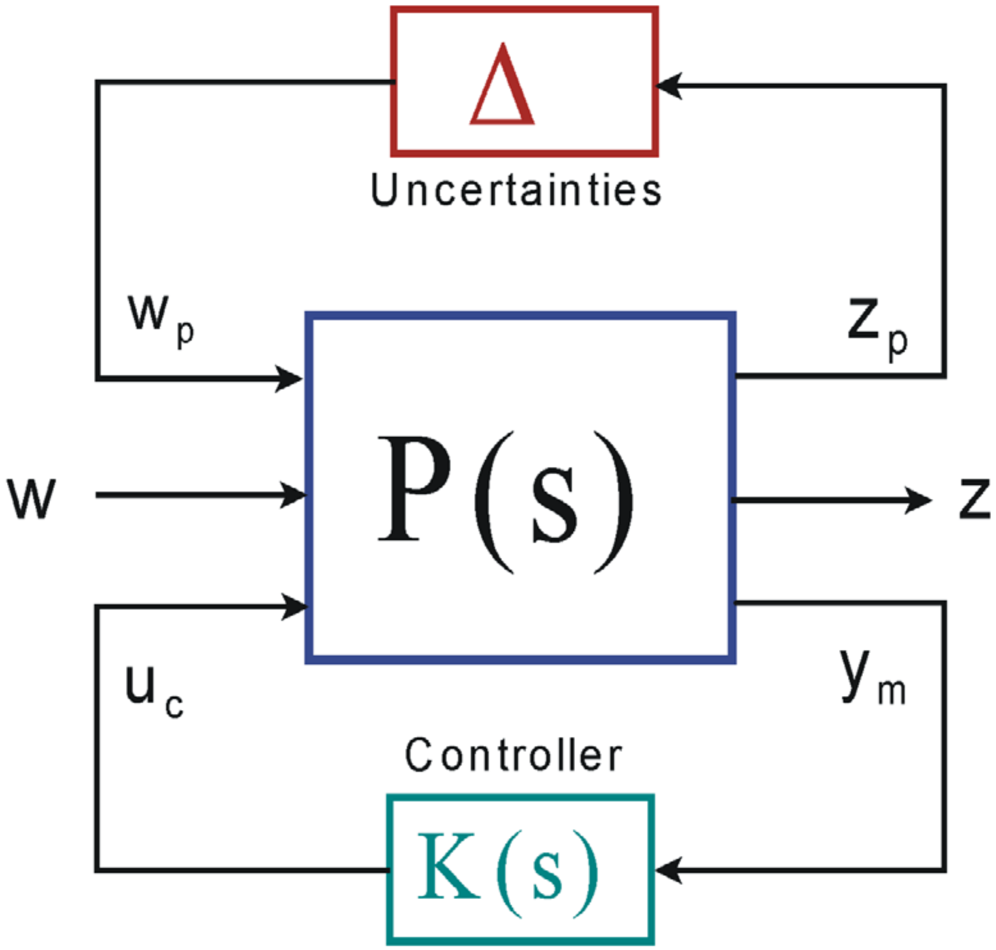


Figure 4.2 Robustness Analysis Block showing the Uncertainties IFL loop, the control feedback loop, the disturbances (w), and performance outputs (z)

This system can also be represented in matrix transfer function form as follows

$$\begin{pmatrix} z_p \\ z \\ y_m \end{pmatrix} = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \begin{pmatrix} w_p \\ w \\ u_c \end{pmatrix}$$ where:

$$w_p = -\Delta z_p \qquad and \qquad u_c = -K(s)\, y_m$$

After closing the loop with a stabilizing controller K(s) the closed loop system is represented with the following transfer function matrix

$$\begin{pmatrix} z_p \\ z \end{pmatrix} = \begin{bmatrix} T_{11}(s) & T_{12}(s) \\ T_{21}(s) & T_{22}(s) \end{bmatrix} \begin{pmatrix} w_p \\ w \end{pmatrix} \text{ and } \qquad w_p = -\Delta z_p$$

where

$$T_{11}(s) = G_{11} - G_{13}K\left(I + G_{33}K\right)^{-1} G_{31}; \qquad T_{12}(s) = G_{12} - G_{13}K\left(I + G_{33}K\right)^{-1} G_{32}$$

$$T_{21}(s) = G_{21} - G_{23}K\left(I + G_{33}K\right)^{-1} G_{31}; \qquad T_{22}(s) = G_{22} - G_{23}K\left(I + G_{33}K\right)^{-1} G_{32}$$

The above transfer functions are used to evaluate system robustness and performance of the closed loop system

Robust Stability: Stability robustness with respect to parameter uncertainty is determined by the transfer function $T_{11}(s)$. Smaller $\|T_{11}\|_\infty$ allows larger parameter uncertainty for closed loop stability. The closed loop system is considered to be robustly stable with respect to the parameter perturbations block Δ, where $\|\Delta\| \le 1$, when the $\mu\{T_{11}(\omega)\} < 1$ at all frequencies (ω).

Nominal Performance: Nominal performance is used to calculate the system's sensitivity to excitations and it is obtained from the transfer function $T_{22}(s)$. This transfer function must be scaled by multiplying its inputs with the max magnitude of the excitations and by dividing its outputs with the max allowable error. The system meets Nominal Performance when the scaled $\|T_{22}(\omega)\|_\infty < 1$ at all frequencies (ω). For example, maximum wind-gust velocity disturbance must not exceed the maximum allowable angle of attack dispersion.

Robust Performance: is achieved when the system meets the performance and robustness requirements together. This happens when the following condition is satisfied at all frequencies.

$$\mu \begin{bmatrix} T_{11}(s) & T_{12}(s) \\ T_{21}(s) & T_{22}(s) \end{bmatrix} < 1$$

4.2 Parameter Uncertainties Modeling Program

The parameter uncertainties modeling program in Flixan implements the IFL method of extracting the parameter uncertainties and creating an augmented state-space vehicle model that includes the additional fictitious inputs and outputs that connect with the normalized uncertainty block Δ, as shown in Figure 2 and described in the previous section. The program calls the flight vehicle modeling program that processes the vehicle data and generates state-space systems. In addition to the vehicle data, the program also reads the uncertainties data from the input data file (.Inp). The algorithm calls the vehicle modeling program multiple times. It begins by processing the nominal vehicle dataset and repeats the data processing for each parameter variation. It eventually generates the uncertainty state-space vehicle model, which is similar to the nominal model, but it includes the additional input/ output pairs that connect with the extracted uncertainties.

The parameter variations from their nominal values are included in a separate uncertainties dataset which is located in the input data file (.Inp) together with the vehicle data. The title of the dataset that includes uncertainties must also be included in the vehicle input data in order for the program to associate the variations with the actual vehicle parameters. The program reads and processes the uncertainties together with the vehicle data by calling the vehicle modeling program for each variation. The following process is used to calculate the uncertainty model:

1. The vehicle modeling program is called initially to process the nominal set of vehicle data and to create the "known" plant state-space model [A, B; C, D].

2. One and only one of the vehicle data parameters must be modified at a time, by either increasing or decreasing the parameter from its nominal value by the amount of the maximum expected variation (δp_1) and the modified vehicle data is reprocessed by the vehicle modeling program to create a new state-space system [A_1, B_1, C_1, D_1] that corresponds to parameter #1 variation. The matrix difference between the nominal and the perturbed state-space models is calculated:
$$\begin{bmatrix} \Delta A_1 & \Delta B_1 \\ \Delta C_1 & \Delta D_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} - \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

3. This matrix is decomposed using SVD to calculate the column vectors $\alpha_x^{(1)}$ and $\alpha_y^{(1)}$ and the row vectors $\beta_x^{(1)}$, and $\beta_u^{(1)}$, as shown in the equation.

4. The previous parameter is restored to its original value and another parameter #2 in the vehicle input data is modified by an amount δp_2 which represents the maximum variation of this parameter, as in step-2. Repeat steps 2 and 3 and calculate the vectors $\alpha_x^{(2)}$, $\alpha_y^{(2)}$, $\beta_x^{(2)}$, and $\beta_u^{(2)}$.

5. Select another parameter to perturb and repeat steps 2, and 3 until there are no more uncertain parameters to vary. Stack the row and column vectors as shown in the equations to create the stacks of column vectors: M_x and M_y and the stacks of row vectors: N_x and N_u.

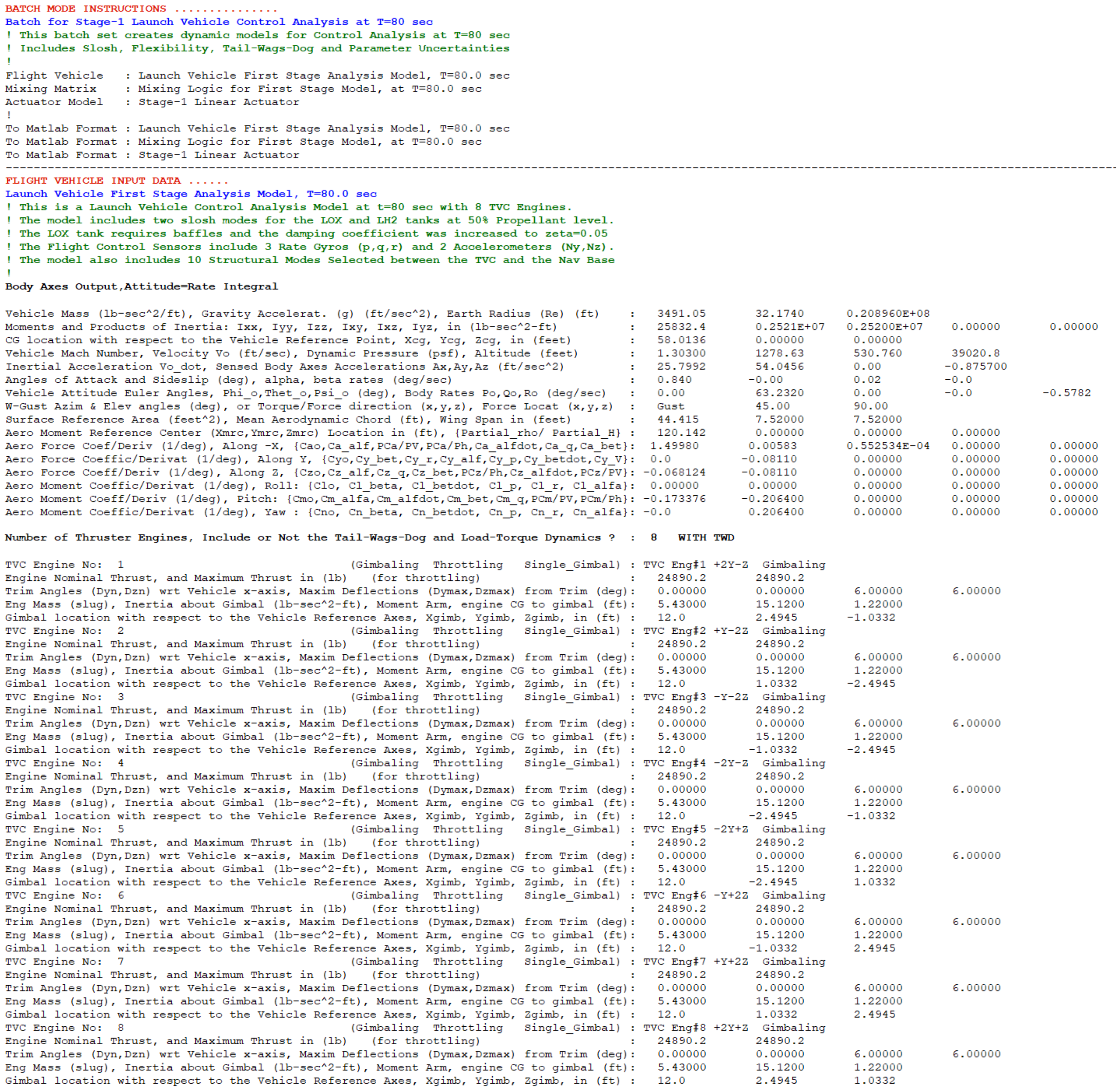
6. These matrices are then used to create the additional inputs and outputs in the state-space model. The columns of matrices M_x and M_y and the rows of matrices N_x and N_u are also scaled according to the magnitude of the uncertainties δp_i so that the interconnections correspond to a unity normalized Δ-block.

The uncertainty model is then used in combination with the flight control system to analyze the closed-loop system performance and robustness to uncertainties by calculating the μ-frequency response of the plant across the interconnections with the Δ block, as shown in Figure 2. That is, between w_p and z_p, with the control loop K(s) closed.

4.3 Max-Q Analysis Model

The Flixan input data file for this Max-Q robustness analysis is "*Flex_Vehi_T80.inp*". It includes the vehicle dataset at Max-Q, the uncertainties set, the mixing logic, the actuator and the previously selected modal data. It also includes data conversion datasets. The uncertainties dataset has a similar structure as the vehicle parameters dataset. It includes variations relative to the nominal vehicle parameters and it has a title. It includes: mass properties, aero data, slosh and flex parameter variations. The known elements that do not vary must be set to zero and only the maximum value of the parameters that vary are included.

The Flixan program identifies the parameter uncertainties dataset from the label: "*UNCERTAIN PARAMETER VARIATIONS FROM NOMINAL …*" which is located at the top of the dataset. There may be more than one uncertainties dataset, and each uncertainties dataset is identified by a separate title. The title of the uncertainties data set is "*Uncertainties for First Stage Max-Q*" and it is also included at the bottom of the vehicle data, same as the title of the modal data set "*First Stage Flex Modes at 50% Full Tanks*". The vehicle model "*Launch Vehicle First Stage Analysis Model, T=80.0 sec*", including slosh, flexibility and uncertainties, is saved in the systems file "Flex_Vehi_T80.Qdr" and in file "vehicle_t80.m" for Matlab analysis. An input/ output pair is created in the system for each uncertainty. That is, in addition to the usual inputs and outputs which are defined by the nominal vehicle dataset.

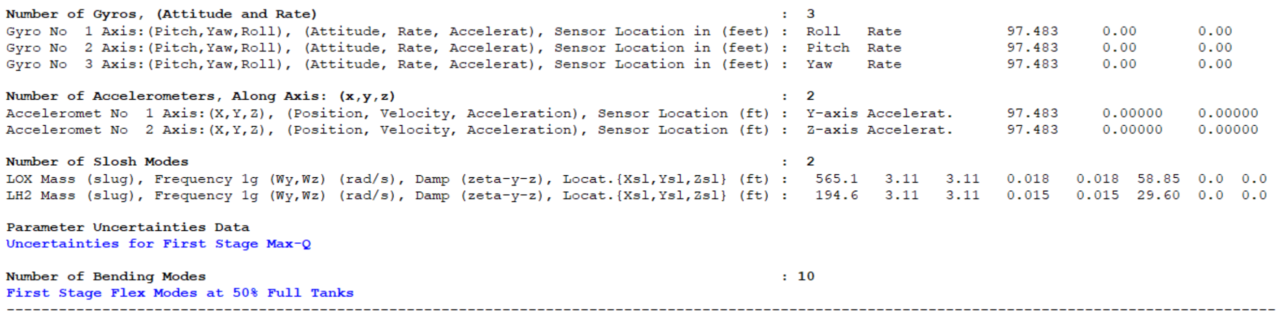


```
BATCH MODE INSTRUCTIONS ...............
Batch for Stage-1 Launch Vehicle Control Analysis at T=80 sec
! This batch set creates dynamic models for Control Analysis at T=80 sec
! Includes Slosh, Flexibility, Tail-Wags-Dog and Parameter Uncertainties
!
Flight Vehicle    : Launch Vehicle First Stage Analysis Model, T=80.0 sec
Mixing Matrix     : Mixing Logic for First Stage Model, at T=80.0 sec
Actuator Model    : Stage-1 Linear Actuator
!
To Matlab Format : Launch Vehicle First Stage Analysis Model, T=80.0 sec
To Matlab Format : Mixing Logic for First Stage Model, at T=80.0 sec
To Matlab Format : Stage-1 Linear Actuator
-----------------------------------------------------------------------------------------------------------------------------------------------------------------------
FLIGHT VEHICLE INPUT DATA ......
Launch Vehicle First Stage Analysis Model, T=80.0 sec
! This is a Launch Vehicle Control Analysis Model at t=80 sec with 8 TVC Engines.
! The model includes two slosh modes for the LOX and LH2 tanks at 50% Propellant level.
! The LOX tank requires baffles and the damping coefficient was increased to zeta=0.05
! The Flight Control Sensors include 3 Rate Gyros (p,q,r) and 2 Accelerometers (Ny,Nz).
! The model also includes 10 Structural Modes Selected between the TVC and the Nav Base
!
Body Axes Output,Attitude=Rate Integral

Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft)    :   3491.05       32.1740      0.208960E+08
Moments and Products of Inertia: Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)          :   25832.4       0.2521E+07   0.25200E+07    0.00000       0.00000
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)        :   58.0136       0.00000       0.00000
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)       :   1.30300       1278.63       530.760       39020.8
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)         :   25.7992       54.0456       0.00         -0.875700
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                         :   0.840        -0.00          0.02         -0.0
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)   :   0.00          63.2320       0.00         -0.0          -0.5782
W-Gust Azim & Elev angles (deg), or Torque/Force direction (x,y,z), Force Locat (x,y,z)  :   Gust          45.00         90.00
Surface Reference Area (feet^2), Mean Aerodynamic Chord (ft), Wing Span in (feet)        :   44.415        7.52000       7.52000
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial_rho/ Partial_H} :   120.142       0.00000       0.00000       0.00000
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:  1.49980       0.00583      0.552534E-04   0.00000       0.00000
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}:  0.0          -0.08110       0.00000       0.00000       0.00000
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}: -0.068124     -0.08110       0.00000       0.00000       0.00000
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:  0.00000       0.00000       0.00000       0.00000       0.00000
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}: -0.173376     -0.206400       0.00000       0.00000       0.00000
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}: -0.0           0.206400       0.00000       0.00000       0.00000

Number of Thruster Engines, Include or Not the Tail-Wags-Dog and Load-Torque Dynamics ?  :  8   WITH TWD

TVC Engine No:  1                                (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#1 +2Y-Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)                    :   24890.2       24890.2
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):   0.00000       0.00000       6.00000       6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):   5.43000       15.1200       1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :   12.0          2.4945       -1.0332
TVC Engine No:  2                                (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#2 +Y-2Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)                    :   24890.2       24890.2
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):   0.00000       0.00000       6.00000       6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):   5.43000       15.1200       1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :   12.0          1.0332       -2.4945
TVC Engine No:  3                                (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#3 -Y-2Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)                    :   24890.2       24890.2
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):   0.00000       0.00000       6.00000       6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):   5.43000       15.1200       1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :   12.0         -1.0332       -2.4945
TVC Engine No:  4                                (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#4 -2Y-Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)                    :   24890.2       24890.2
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):   0.00000       0.00000       6.00000       6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):   5.43000       15.1200       1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :   12.0         -2.4945       -1.0332
TVC Engine No:  5                                (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#5 -2Y+Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)                    :   24890.2       24890.2
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):   0.00000       0.00000       6.00000       6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):   5.43000       15.1200       1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :   12.0         -2.4945        1.0332
TVC Engine No:  6                                (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#6 -Y+2Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)                    :   24890.2       24890.2
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):   0.00000       0.00000       6.00000       6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):   5.43000       15.1200       1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :   12.0         -1.0332        2.4945
TVC Engine No:  7                                (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#7 +Y+2Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)                    :   24890.2       24890.2
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):   0.00000       0.00000       6.00000       6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):   5.43000       15.1200       1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :   12.0          1.0332        2.4945
TVC Engine No:  8                                (Gimbaling  Throttling   Single_Gimbal) : TVC Eng#8 +2Y+Z  Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)    (for throttling)                    :   24890.2       24890.2
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):   0.00000       0.00000       6.00000       6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):   5.43000       15.1200       1.22000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :   12.0          2.4945        1.0332
```



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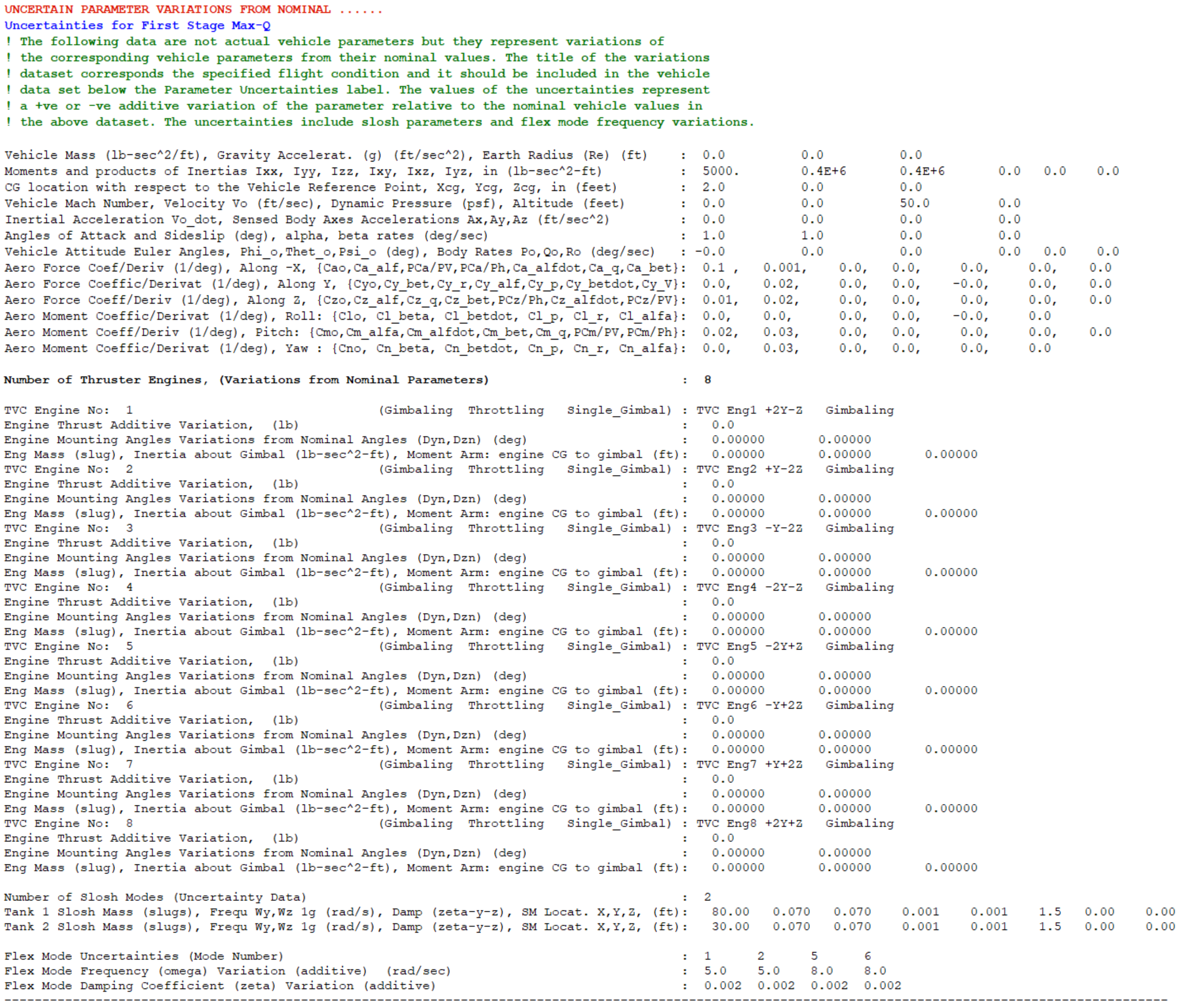
Number of Gyros, (Attitude and Rate)                                             :  3
Gyro No  1 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Location in (feet) :  Roll   Rate           97.483     0.00       0.00
Gyro No  2 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Location in (feet) :  Pitch  Rate           97.483     0.00       0.00
Gyro No  3 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Location in (feet) :  Yaw    Rate           97.483     0.00       0.00

Number of Accelerometers, Along Axis: (x,y,z)                                    :  2
Acceleromet No  1 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Location (ft) :  Y-axis Accelerat.      97.483     0.00000    0.00000
Acceleromet No  2 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Location (ft) :  Z-axis Accelerat.      97.483     0.00000    0.00000

Number of Slosh Modes                                                            :  2
LOX Mass (slug), Frequency 1g (Wy,Wz) (rad/s), Damp (zeta-y-z), Locat.{Xsl,Ysl,Zsl} (ft) :   565.1    3.11    3.11    0.018    0.018  58.85  0.0  0.0
LH2 Mass (slug), Frequency 1g (Wy,Wz) (rad/s), Damp (zeta-y-z), Locat.{Xsl,Ysl,Zsl} (ft) :   194.6    3.11    3.11    0.015    0.015  29.60  0.0  0.0

Parameter Uncertainties Data
Uncertainties for First Stage Max-Q

Number of Bending Modes                                                          : 10
First Stage Flex Modes at 50% Full Tanks
-------------------------------------------------------------------------------------------------------------------------------------
```



```
UNCERTAIN PARAMETER VARIATIONS FROM NOMINAL ......
Uncertainties for First Stage Max-Q
! The following data are not actual vehicle parameters but they represent variations of
! the corresponding vehicle parameters from their nominal values. The title of the variations
! dataset corresponds the specified flight condition and it should be included in the vehicle
! data set below the Parameter Uncertainties label. The values of the uncertainties represent
! a +ve or -ve additive variation of the parameter relative to the nominal vehicle values in
! the above dataset. The uncertainties include slosh parameters and flex mode frequency variations.

Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft)    :  0.0          0.0          0.0
Moments and products of Inertias Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)          :  5000.        0.4E+6       0.4E+6       0.0    0.0    0.0
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)        :  2.0          0.0          0.0
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)       :  0.0          0.0          50.0         0.0
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)         :  0.0          0.0          0.0          0.0
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                         :  1.0          1.0          0.0          0.0
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)   : -0.0          0.0          0.0          0.0    0.0    0.0
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:  0.1 ,    0.001,     0.0,   0.0,     0.0,      0.0,     0.0
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}:  0.0,     0.02,      0.0,   0.0,    -0.0,      0.0,     0.0
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}:  0.01,    0.02,      0.0,   0.0,     0.0,      0.0,     0.0
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:  0.0,     0.0,       0.0,   0.0,    -0.0,      0.0
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}:  0.02,    0.03,      0.0,   0.0,     0.0,      0.0,     0.0
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}:  0.0,     0.03,      0.0,   0.0,     0.0,      0.0

Number of Thruster Engines, (Variations from Nominal Parameters)                         :  8

TVC Engine No:  1                               (Gimbaling  Throttling   Single_Gimbal) : TVC Eng1 +2Y-Z   Gimbaling
Engine Thrust Additive Variation,  (lb)                                                 :   0.0
Engine Mounting Angles Variations from Nominal Angles (Dyn,Dzn) (deg)                   :   0.00000       0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm: engine CG to gimbal (ft):   0.00000       0.00000       0.00000
TVC Engine No:  2                               (Gimbaling  Throttling   Single_Gimbal) : TVC Eng2 +Y-2Z   Gimbaling
Engine Thrust Additive Variation,  (lb)                                                 :   0.0
Engine Mounting Angles Variations from Nominal Angles (Dyn,Dzn) (deg)                   :   0.00000       0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm: engine CG to gimbal (ft):   0.00000       0.00000       0.00000
TVC Engine No:  3                               (Gimbaling  Throttling   Single_Gimbal) : TVC Eng3 -Y-2Z   Gimbaling
Engine Thrust Additive Variation,  (lb)                                                 :   0.0
Engine Mounting Angles Variations from Nominal Angles (Dyn,Dzn) (deg)                   :   0.00000       0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm: engine CG to gimbal (ft):   0.00000       0.00000       0.00000
TVC Engine No:  4                               (Gimbaling  Throttling   Single_Gimbal) : TVC Eng4 -2Y-Z   Gimbaling
Engine Thrust Additive Variation,  (lb)                                                 :   0.0
Engine Mounting Angles Variations from Nominal Angles (Dyn,Dzn) (deg)                   :   0.00000       0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm: engine CG to gimbal (ft):   0.00000       0.00000       0.00000
TVC Engine No:  5                               (Gimbaling  Throttling   Single_Gimbal) : TVC Eng5 -2Y+Z   Gimbaling
Engine Thrust Additive Variation,  (lb)                                                 :   0.0
Engine Mounting Angles Variations from Nominal Angles (Dyn,Dzn) (deg)                   :   0.00000       0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm: engine CG to gimbal (ft):   0.00000       0.00000       0.00000
TVC Engine No:  6                               (Gimbaling  Throttling   Single_Gimbal) : TVC Eng6 -Y+2Z   Gimbaling
Engine Thrust Additive Variation,  (lb)                                                 :   0.0
Engine Mounting Angles Variations from Nominal Angles (Dyn,Dzn) (deg)                   :   0.00000       0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm: engine CG to gimbal (ft):   0.00000       0.00000       0.00000
TVC Engine No:  7                               (Gimbaling  Throttling   Single_Gimbal) : TVC Eng7 +Y+2Z   Gimbaling
Engine Thrust Additive Variation,  (lb)                                                 :   0.0
Engine Mounting Angles Variations from Nominal Angles (Dyn,Dzn) (deg)                   :   0.00000       0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm: engine CG to gimbal (ft):   0.00000       0.00000       0.00000
TVC Engine No:  8                               (Gimbaling  Throttling   Single_Gimbal) : TVC Eng8 +2Y+Z   Gimbaling
Engine Thrust Additive Variation,  (lb)                                                 :   0.0
Engine Mounting Angles Variations from Nominal Angles (Dyn,Dzn) (deg)                   :   0.00000       0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm: engine CG to gimbal (ft):   0.00000       0.00000       0.00000

Number of Slosh Modes (Uncertainty Data)                                                 :  2
Tank 1 Slosh Mass (slugs), Frequ Wy,Wz 1g (rad/s), Damp (zeta-y-z), SM Locat. X,Y,Z, (ft):   80.00   0.070   0.070    0.001    0.001    1.5   0.00    0.00
Tank 2 Slosh Mass (slugs), Frequ Wy,Wz 1g (rad/s), Damp (zeta-y-z), SM Locat. X,Y,Z, (ft):   30.00   0.070   0.070    0.001    0.001    1.5   0.00    0.00

Flex Mode Uncertainties (Mode Number)                                                    :  1      2      5      6
Flex Mode Frequency (omega) Variation (additive)  (rad/sec)                              :  5.0    5.0    8.0    8.0
Flex Mode Damping Coefficient (zeta) Variation (additive)                                :  0.002  0.002  0.002  0.002
------------------------------------------------------------------------------------------------------------------------------------------------------------
```

Three Simulink models will be created to analyze the following: (a) Nominal Performance in terms of angle of attack sensitivity to gusts assuming fixed vehicle parameters, (b) Robust Stability to parameter uncertainties, assuming known max variations, and (c) Robust Performance, which is, the control system's ability to simultaneously satisfy performance to gusts (a) and being robust to the uncertainties (b) simultaneously, which is more challenging than (a) and (b) alone.

```
MIXING LOGIC MATRIX DATA .........     (Matrix Title, Name, Vehicle Title, Control Directions)
Mixing Logic for First Stage Model, at T=80.0 sec
! Thrust Vector Control Matrix at t=80 sec
! This multi-engine vehicle has 8 Gimbaling Engines.
TVC
Launch Vehicle First Stage Analysis Model, T=80.0 sec
P-dot Roll  Acceleration About X Axis
Q-dot Pitch Acceleration About Y Axis
R-dot Yaw   Acceleration About Z Axis
-----------------------------------------------------------------------------------------------------------------------
ACTUATOR INPUT DATA ........... SIMPLE GENERIC MODEL B
Stage-1 Linear Actuator
Symbol Parameter Description                    (Units)       Value
C(s)   Order of Pade Delay (0,1,2)                ( - )       1, -0.001, 0.001
Ka     Gain of Amplifier ...................... (amps/volt)   28.0
Wsv    Bandwidth of the Linear Servo Actuator .  (rad/sec)    70.0
Kact   Actuator Stiffness (Piston+Oil+Electric)   (lb/ft)     2.4e+6
Klod   Stiffness at Surface or Nozzle Connection  (lb/ft)     1.2e+9
Kbck   Stiffness at Vehicle Backup Structure ..   (lb/ft)     7.0e+7
R      Moment Arm between Actuator Rod & Gimbal   (feet)      0.667
Jl     Load Inertia about the Gimbal .......... (ft-lb-s^2)   15.12
Kg     Load Gimbal Bearing Spring Constant      (ft-lb/rad)   0.0
Bg     Load Gimbal Bearing Viscous Damping .... (ft-lb-sec)   550.0
-----------------------------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Mixing Logic for First Stage Model, at T=80.0 sec
Matrix TVC
-----------------------------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Launch Vehicle First Stage Analysis Model, T=80.0 sec
System
vehicle_t80.m
-----------------------------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Stage-1 Linear Actuator
System
actuator.m
-----------------------------------------------------------------------------------------------------------------------
SELECTED MODAL DATA AND LOCATIONS FOR : 50% Full
First Stage Flex Modes at 50% Full Tanks
! Flex Modes, First Stage at 50% Full Tanks from files: Stg1_50%.Mod, Stg1_50%.Nod
! Sensors are at the Top of LOX Tank
! The Modes were selected between the TVC and the IMU Location

MODE#  1/  1, Frequency (rad/sec), Damping (zeta), Generalized Mass=     23.165       0.50000E-02    12.000
DEFINITION OF LOCATIONS (NODES)            phi along X   phi along Y   phi along Z    sigm about X  sigm about Y  sigm about Z

                              Node ID#     Modal Data at the  8 Engines, (x,y,z)...
S1 Engine No:1 +Y-Z               1151     0.15550D-01   0.13791D+00  -0.40095D-02  -0.27974D-05  -0.18251D-03  -0.64210D-02
S1 Engine No:2 +Y-Z               1152     0.68120D-02   0.13790D+00  -0.40081D-02   0.38291D-06  -0.18164D-03  -0.64368D-02
S1 Engine No:3 -Y-Z               1153    -0.59210D-02   0.13790D+00  -0.40106D-02   0.17814D-06  -0.19171D-03  -0.64370D-02
S1 Engine No:4 -Y-Z               1154    -0.15182D-01   0.13791D+00  -0.40091D-02  -0.25952D-05  -0.19200D-03  -0.64212D-02
S1 Engine No:5 -Y+Z               1155    -0.15552D-01   0.13791D+00  -0.40091D-02   0.28786D-05  -0.18262D-03  -0.64211D-02
S1 Engine No:6 -Y+Z               1156    -0.68146D-02   0.13790D+00  -0.40081D-02  -0.74135D-06  -0.18146D-03  -0.64369D-02
S1 Engine No:7 +Y+Z               1157     0.59186D-02   0.13790D+00  -0.40111D-02  -0.37415D-06  -0.19187D-03  -0.64372D-02
S1 Engine No:8 +Y+Z               1158     0.15179D-01   0.13791D+00  -0.40096D-02   0.29459D-05  -0.19174D-03  -0.64212D-02

                              Node ID#     Modal Data at the  3 Gyros ...
Stage-2 Tank Top, IMU Locat.     40015    -0.10007D-05   0.37440D-01  -0.10880D-02  -0.17551D-07   0.13211D-03   0.45448D-02
Stage-2 Tank Top, IMU Locat.     40015    -0.10007D-05   0.37440D-01  -0.10880D-02  -0.17551D-07   0.13211D-03   0.45448D-02
Stage-2 Tank Top, IMU Locat.     40015    -0.10007D-05   0.37440D-01  -0.10880D-02  -0.17551D-07   0.13211D-03   0.45448D-02

                              Node ID#     Modal Data at the  2 Accelerometers, along (x,y,z)...
Stage-2 Tank Top, IMU Locat.     40015    -0.10007D-05   0.37440D-01  -0.10880D-02
Stage-2 Tank Top, IMU Locat.     40015    -0.10007D-05   0.37440D-01  -0.10880D-02

                              Node ID#     Modal Data at the  2 Slosh Masses...
LOX  Slosh Mass Locat.             601    -0.11284D-05  -0.71681D-01   0.20846D-02  -0.16706D-07   0.10688D-05   0.36280D-04
Fuel Slosh Mass Locat.             600    -0.12406D-05   0.27523D-01  -0.79968D-03  -0.15263D-07  -0.16930D-03  -0.58223D-02

                              Node ID#     Modal Data at the Disturbance Point
S2 Engine Gimbal                  3303     0.52334D-04  -0.90953D-02   0.26381D-03  -0.15445D-07   0.12810D-03   0.44320D-02

MODE#  2/  2, Frequency (rad/sec), Damping (zeta), Generalized Mass=     23.168       0.50000E-02    12.000
DEFINITION OF LOCATIONS (NODES)            phi along X   phi along Y   phi along Z    sigm about X  sigm about Y  sigm about Z
```

4.4 Nominal Performance Analysis

The Simulink model "*Performance.slx*", shown in Figure 4.4, is used to analyze the (α, β) sensitivity to gusts at Max-Q. The performance requirement is for the angles of attack and sideslip α and β to be less than 4° at all frequencies. This is used for analyzing the lateral loading on the vehicle in the presence of wind-gust disturbances which are less than 25 (ft/sec) of magnitude, perpendicular to the vehicle x-axis.

We close the control loop to stabilize the vehicle and scale the plant input and output to normalize it. The input is multiplied by 25 (ft/sec) and output is divided by 0.068 (rad). Then we run the script file "*Run_Performance.m*" to calculate the SV response between inputs and outputs. The magnitude of the transfer function should, therefore, be less than one (or zero dB) at all frequencies, as it is shown below in Figure 4.3, in order to satisfy the performance requirement.

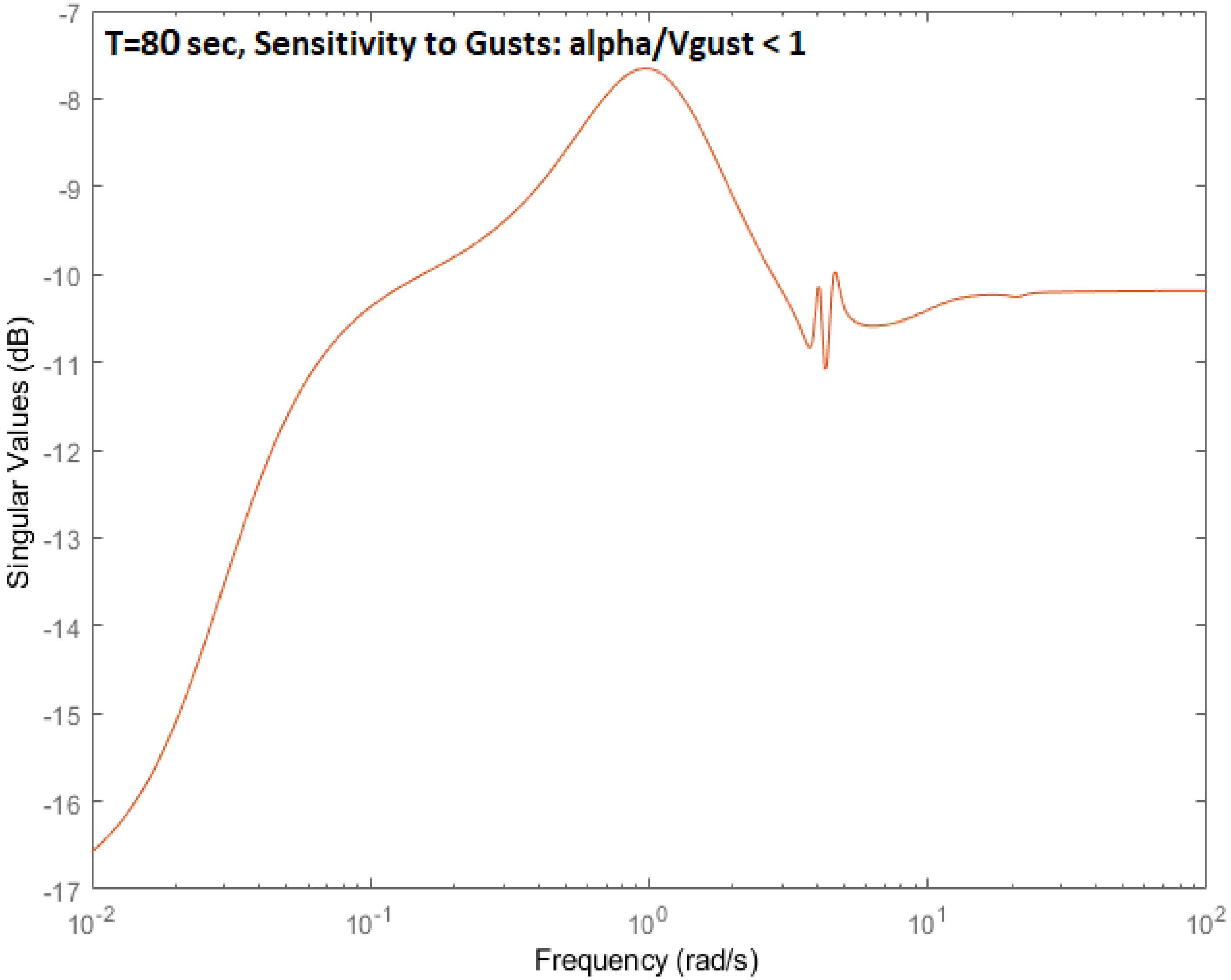
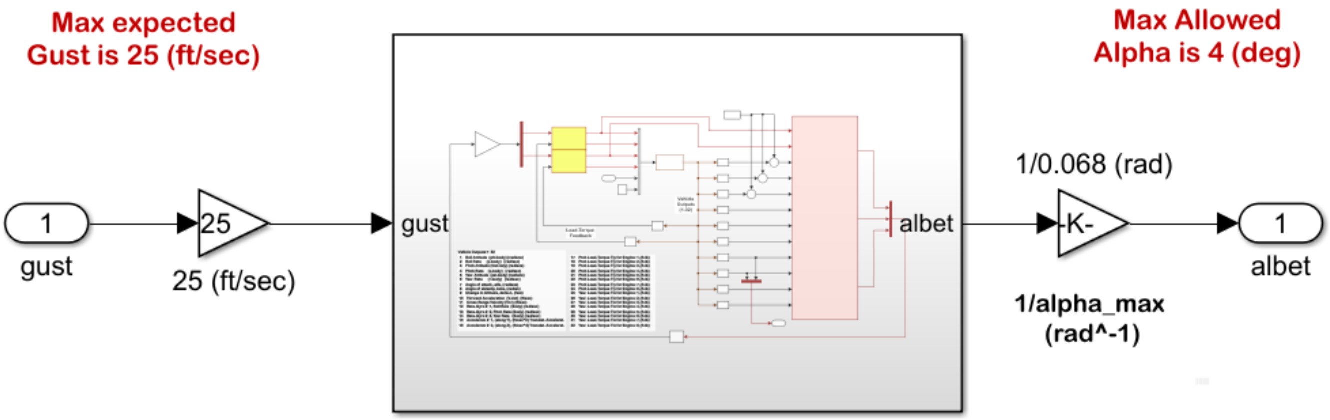


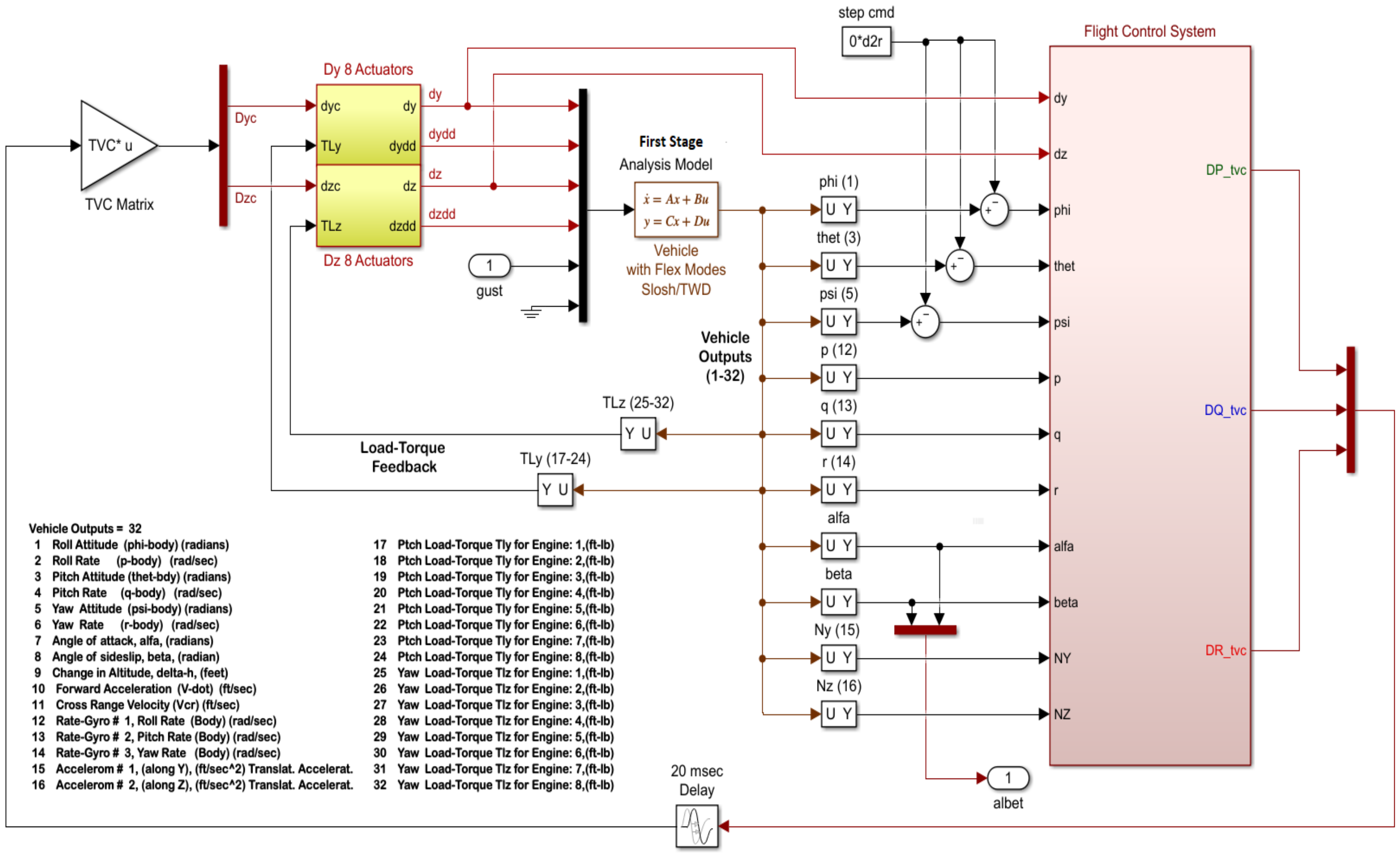
Figure 4.3 Nominal Performance is Satisfied when the Magnitude of the Sensitivity Curve is Less than 1 (0 dB) at all Frequencies

Performance to Gusts Analysis



Plant Must Be Stabilized First

This System Satisfies Performance Requirement when the Transfer Function Magnitude is Less than One at All Frequencies

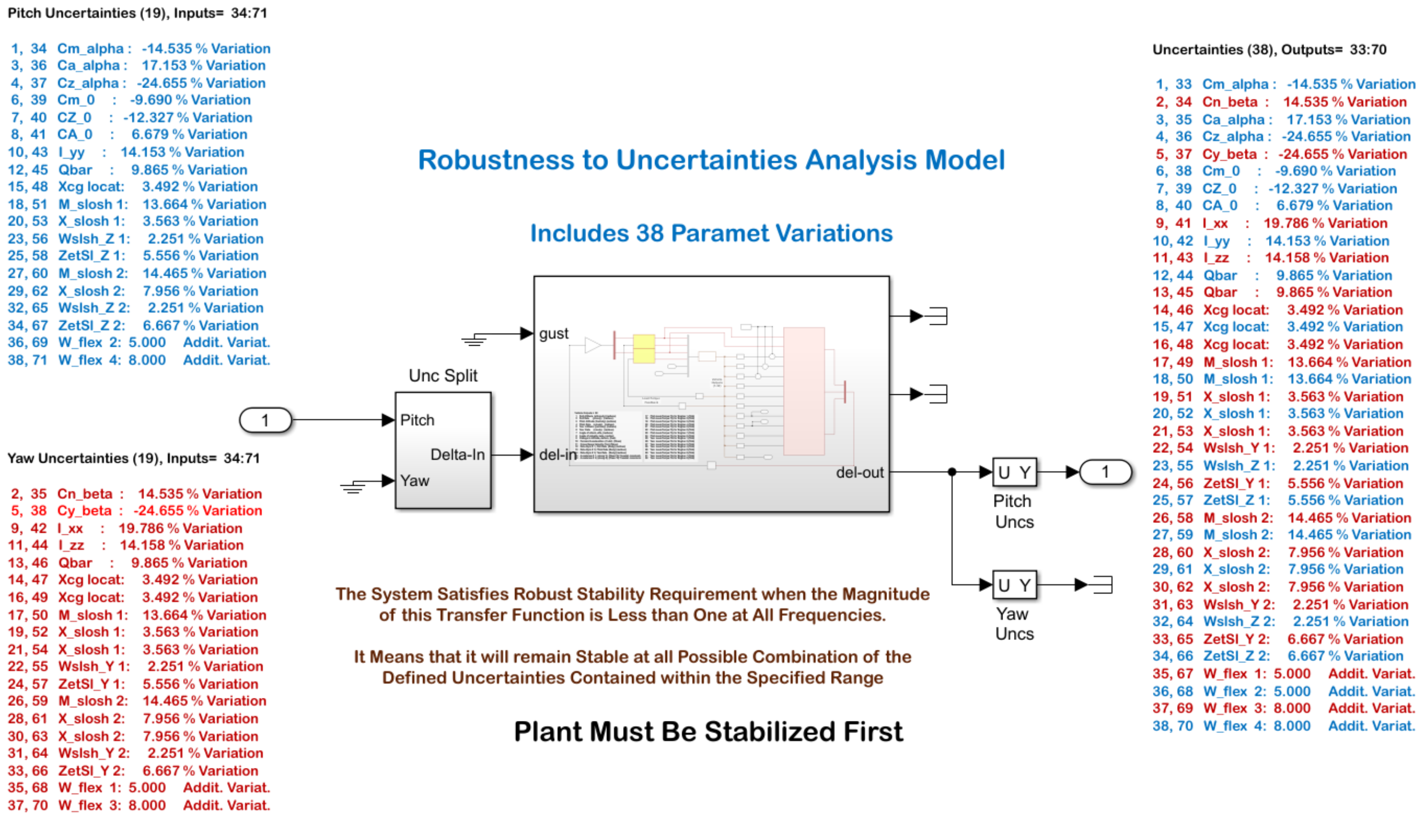


Vehicle Outputs = 32
1 Roll Attitude (phi-body) (radians)
2 Roll Rate (p-body) (rad/sec)
3 Pitch Attitude (thet-bdy) (radians)
4 Pitch Rate (q-body) (rad/sec)
5 Yaw Attitude (psi-body) (radians)
6 Yaw Rate (r-body) (rad/sec)
7 Angle of attack, alfa, (radians)
8 Angle of sideslip, beta, (radian)
9 Change in Altitude, delta-h, (feet)
10 Forward Acceleration (V-dot) (ft/sec)
11 Cross Range Velocity (Vcr) (ft/sec)
12 Rate-Gyro # 1, Roll Rate (Body) (rad/sec)
13 Rate-Gyro # 2, Pitch Rate (Body) (rad/sec)
14 Rate-Gyro # 3, Yaw Rate (Body) (rad/sec)
15 Accelerom # 1, (along Y), (ft/sec^2) Translat. Accelerat.
16 Accelerom # 2, (along Z), (ft/sec^2) Translat. Accelerat.
17 Ptch Load-Torque Tly for Engine: 1,(ft-lb)
18 Ptch Load-Torque Tly for Engine: 2,(ft-lb)
19 Ptch Load-Torque Tly for Engine: 3,(ft-lb)
20 Ptch Load-Torque Tly for Engine: 4,(ft-lb)
21 Ptch Load-Torque Tly for Engine: 5,(ft-lb)
22 Ptch Load-Torque Tly for Engine: 6,(ft-lb)
23 Ptch Load-Torque Tly for Engine: 7,(ft-lb)
24 Ptch Load-Torque Tly for Engine: 8,(ft-lb)
25 Yaw Load-Torque Tlz for Engine: 1,(ft-lb)
26 Yaw Load-Torque Tlz for Engine: 2,(ft-lb)
27 Yaw Load-Torque Tlz for Engine: 3,(ft-lb)
28 Yaw Load-Torque Tlz for Engine: 4,(ft-lb)
29 Yaw Load-Torque Tlz for Engine: 5,(ft-lb)
30 Yaw Load-Torque Tlz for Engine: 6,(ft-lb)
31 Yaw Load-Torque Tlz for Engine: 7,(ft-lb)
32 Yaw Load-Torque Tlz for Engine: 8,(ft-lb)

Figure 4.4 Performance Analysis Model “Performance.slx” Analyzes Sensitivity to Gusts

4.5 Analyzing Stability Robustness

The next step is to analyze robustness. That is, if the control system remains stable when the selected uncertain parameters vary from their nominal values. The Simulink model “Robustness.slx” will be used to check the control system robustness in presence of the selected parameter variations within the specified range. The system robustness is obtained by running the script file “*Run_Robustness.m*” which calculates the SSV frequency response or μ-plot of the closed-loop plant M(s) across the inputs and outputs w_p and z_p that connect with the variations block Δ, as shown in Figure 4.2. The closed-loop plant M(s) is already stabilized with a controller K(s). The system’s stability is robust to the uncertainties when the SSV plot is less than one at all frequencies. In order to properly apply the theory and to avoid being over-conservative, each parameter variation should correspond to a plant variation (ΔA, ΔB, ΔC, ΔD) that has a rank-1 dependency to the variation, producing therefore, a single δi element in the Δ block. But in this case some variations produce 2 or even 3 δi elements, for example, the CG variation, because they affect both pitch and lateral axes. This complicates the analysis slightly and in order to avoid being over-conservative we separate the pitch and lateral δi elements into separate groups and analyze pitch and lateral axes separately using their corresponding uncertainties. So, when the program produces 2 δi elements from one parameter perturbation, we separate them and place the δi element that is coupling with the pitch dynamics in the pitch subsystem and the other δi element that is coupling with the lateral dynamics in the lateral subsystem, as shown in Figure 4.5 which is configured here to analyze pitch robustness. For lateral analysis the figure must be modified to connect to the second (yaw) input and output. Figure 4.6 shows that the mu frequency responses of the pitch and lateral systems across the normalized Δ blocks are less than 1 at all frequencies which satisfies the robust stability requirement.



Robustness to Uncertainties Analysis Model

Includes 38 Paramet Variations

Pitch Uncertainties (19), Inputs= 34:71

1, 34	Cm_alpha :	-14.535 % Variation
3, 36	Ca_alpha :	17.153 % Variation
4, 37	Cz_alpha :	-24.655 % Variation
6, 39	Cm_0 :	-9.690 % Variation
7, 40	CZ_0 :	-12.327 % Variation
8, 41	CA_0 :	6.679 % Variation
10, 43	I_yy :	14.153 % Variation
12, 45	Qbar :	9.865 % Variation
15, 48	Xcg locat:	3.492 % Variation
18, 51	M_slosh 1:	13.664 % Variation
20, 53	X_slosh 1:	3.563 % Variation
23, 56	Wslsh_Z 1:	2.251 % Variation
25, 58	ZetSl_Z 1:	5.556 % Variation
27, 60	M_slosh 2:	14.465 % Variation
29, 62	X_slosh 2:	7.956 % Variation
32, 65	Wslsh_Z 2:	2.251 % Variation
34, 67	ZetSl_Z 2:	6.667 % Variation
36, 69	W_flex 2:	5.000 Addit. Variat.
38, 71	W_flex 4:	8.000 Addit. Variat.

Yaw Uncertainties (19), Inputs= 34:71

2, 35	Cn_beta :	14.535 % Variation
5, 38	Cy_beta :	-24.655 % Variation
9, 42	I_xx :	19.786 % Variation
11, 44	I_zz :	14.158 % Variation
13, 46	Qbar :	9.865 % Variation
14, 47	Xcg locat:	3.492 % Variation
16, 49	Xcg locat:	3.492 % Variation
17, 50	M_slosh 1:	13.664 % Variation
19, 52	X_slosh 1:	3.563 % Variation
21, 54	X_slosh 1:	3.563 % Variation
22, 55	Wslsh_Y 1:	2.251 % Variation
24, 57	ZetSl_Y 1:	5.556 % Variation
26, 59	M_slosh 2:	14.465 % Variation
28, 61	X_slosh 2:	7.956 % Variation
30, 63	X_slosh 2:	7.956 % Variation
31, 64	Wslsh_Y 2:	2.251 % Variation
33, 66	ZetSl_Y 2:	6.667 % Variation
35, 68	W_flex 1:	5.000 Addit. Variat.
37, 70	W_flex 3:	8.000 Addit. Variat.

Uncertainties (38), Outputs= 33:70

1, 33	Cm_alpha :	-14.535 % Variation
2, 34	Cn_beta :	14.535 % Variation
3, 35	Ca_alpha :	17.153 % Variation
4, 36	Cz_alpha :	-24.655 % Variation
5, 37	Cy_beta :	-24.655 % Variation
6, 38	Cm_0 :	-9.690 % Variation
7, 39	CZ_0 :	-12.327 % Variation
8, 40	CA_0 :	6.679 % Variation
9, 41	I_xx :	19.786 % Variation
10, 42	I_yy :	14.153 % Variation
11, 43	I_zz :	14.158 % Variation
12, 44	Qbar :	9.865 % Variation
13, 45	Qbar :	9.865 % Variation
14, 46	Xcg locat:	3.492 % Variation
15, 47	Xcg locat:	3.492 % Variation
16, 48	Xcg locat:	3.492 % Variation
17, 49	M_slosh 1:	13.664 % Variation
18, 50	M_slosh 1:	13.664 % Variation
19, 51	X_slosh 1:	3.563 % Variation
20, 52	X_slosh 1:	3.563 % Variation
21, 53	X_slosh 1:	3.563 % Variation
22, 54	Wslsh_Y 1:	2.251 % Variation
23, 55	Wslsh_Z 1:	2.251 % Variation
24, 56	ZetSl_Y 1:	5.556 % Variation
25, 57	ZetSl_Z 1:	5.556 % Variation
26, 58	M_slosh 2:	14.465 % Variation
27, 59	M_slosh 2:	14.465 % Variation
28, 60	X_slosh 2:	7.956 % Variation
29, 61	X_slosh 2:	7.956 % Variation
30, 62	X_slosh 2:	7.956 % Variation
31, 63	Wslsh_Y 2:	2.251 % Variation
32, 64	Wslsh_Z 2:	2.251 % Variation
33, 65	ZetSl_Y 2:	6.667 % Variation
34, 66	ZetSl_Z 2:	6.667 % Variation
35, 67	W_flex 1:	5.000 Addit. Variat.
36, 68	W_flex 2:	5.000 Addit. Variat.
37, 69	W_flex 3:	8.000 Addit. Variat.
38, 70	W_flex 4:	8.000 Addit. Variat.

The System Satisfies Robust Stability Requirement when the Magnitude of this Transfer Function is Less than One at All Frequencies.

It Means that it will remain Stable at all Possible Combination of the Defined Uncertainties Contained within the Specified Range

Plant Must Be Stabilized First

Figure 4.5 Robustness Analysis Simulink Model “Robustness.slx” Shown for Analyzing Pitch Robustness

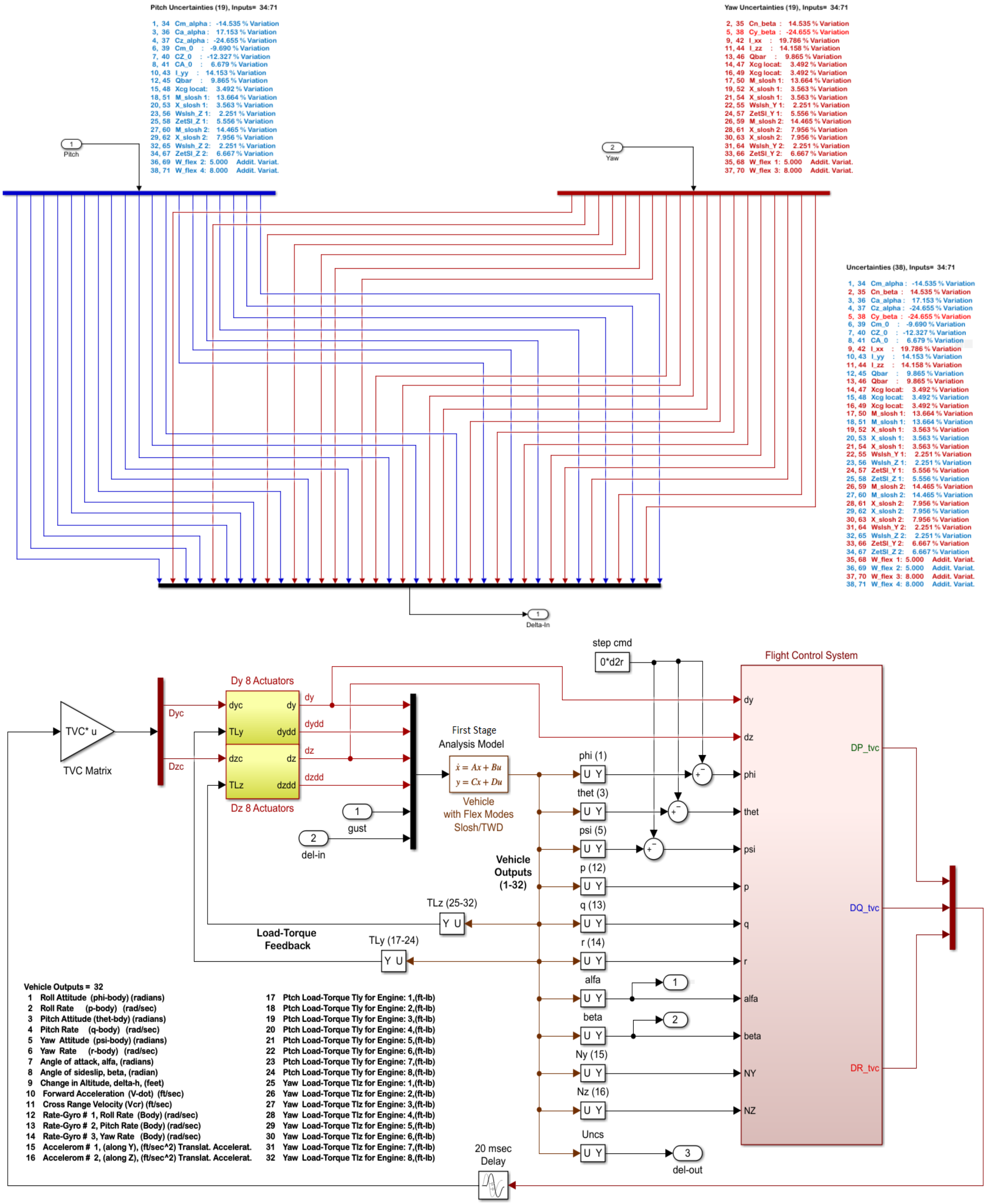


Figure 4.6 The Uncertainty Input and Output Connections are Separated into Pitch and Lateral and the Pitch and Lateral Robustness are Separately Analyzed Using Simulink Model "Robustness.slx"

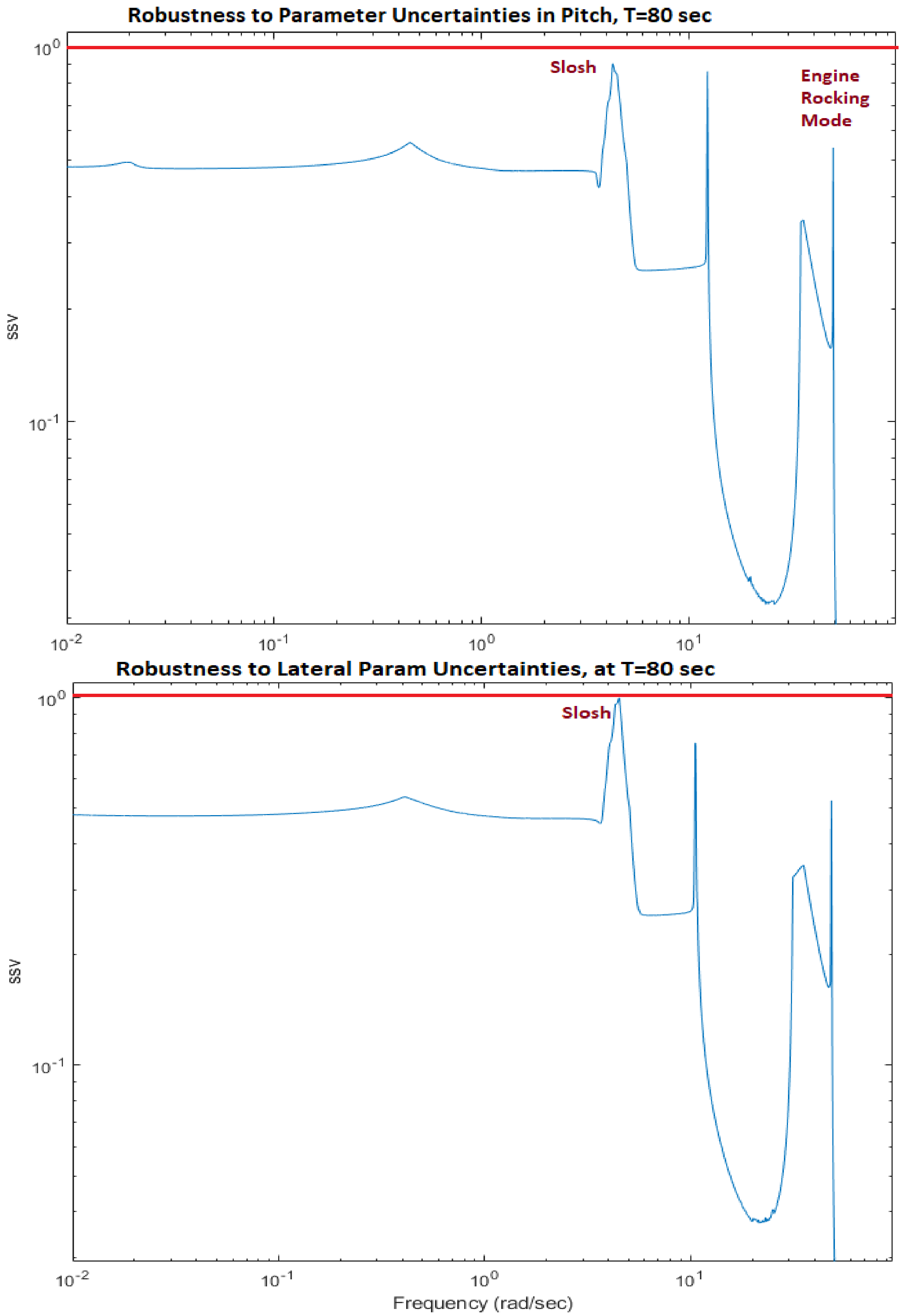
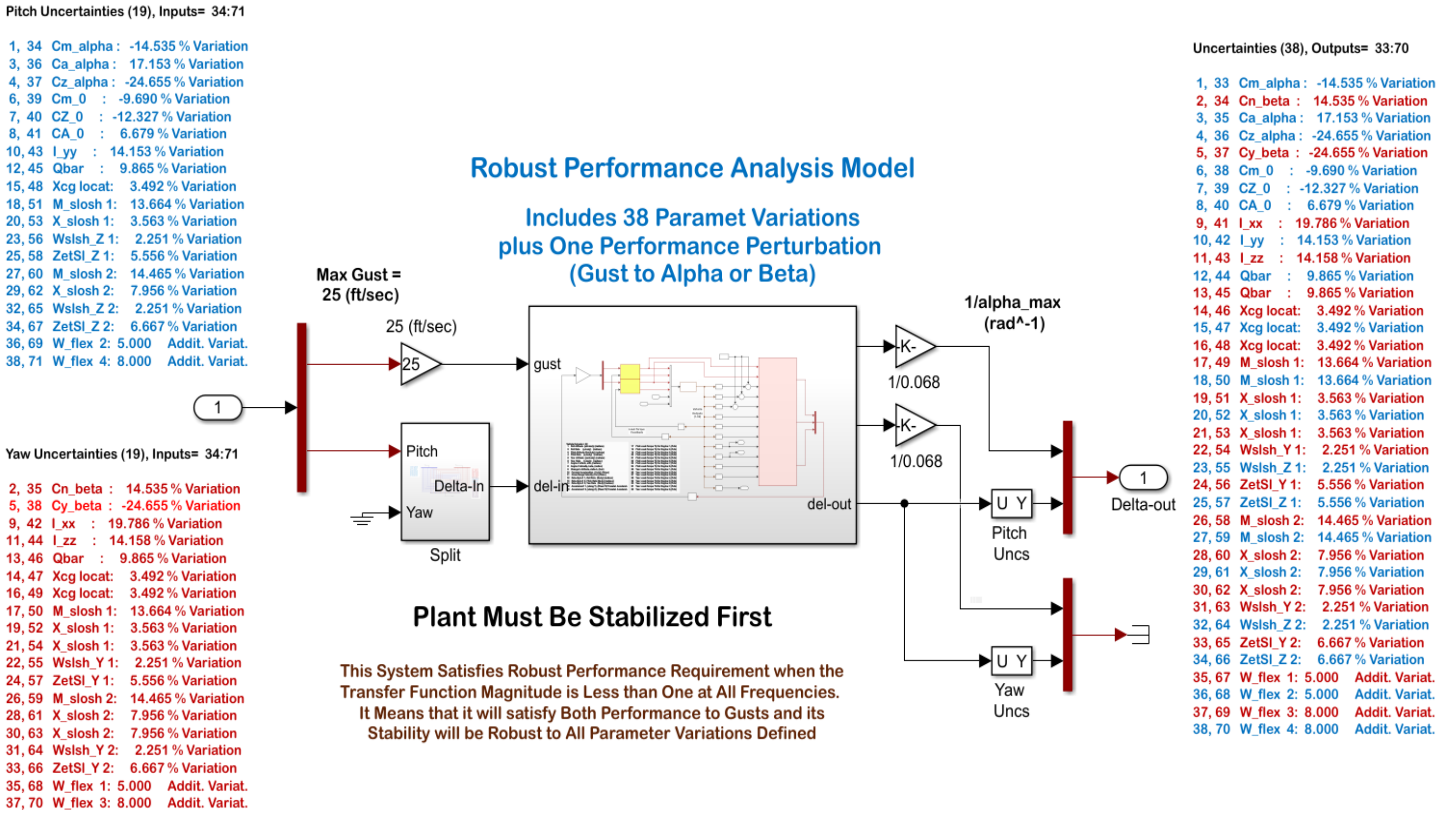


Figure 4.6 Robustness is Satisfied in the Pitch and Lateral Axes with some margin to spare. It shows sensitivity at the slosh frequency 4.1 (rad/sec), at 10 (rad/sec) possibly due to actuator limitations? There is also a spike at the engine rocking mode frequency at 50 (rad/sec).

4.6 Analyzing Robust Performance

The Simulink model "*Robust_Performance.slx*" is used to analyze the control system's Robust Performance, which is the ability to satisfy the normal and lateral aero loading due to wind gusts and to maintain system stability in the presence of all parameter variations within the specified limits. The Robust Performance analysis is similar to the robustness method, but it includes one additional input/output pair: the gust input to the vehicle and the alpha or beta output. The orientation of the gust input is defined by two angles in the vehicle dataset and it excites both pitch and lateral axes. The normalized sensitivity/ performance I/O pair (similar to the one used in the analysis model) is now included to the uncertainty I/O pairs to create the Robust Performance model. Robust Performance is obtained by calculating the SSV frequency response of the closed-loop plant M(s) across the (n+1) inputs and (n+1) outputs that connect with the n parameter variations block Δ, plus one additional sensitivity analysis I/O, as shown in Figure 4.7. It is calculated by running the script file "*Run_Robust_Performance.m*". The system satisfies Robustness and Performance together when this SSV plot is less than one at all frequencies.



Pitch Uncertainties (19), Inputs= 34:71

1, 34 Cm_alpha : -14.535 % Variation
3, 36 Ca_alpha : 17.153 % Variation
4, 37 Cz_alpha : -24.655 % Variation
6, 39 Cm_0 : -9.690 % Variation
7, 40 CZ_0 : -12.327 % Variation
8, 41 CA_0 : 6.679 % Variation
10, 43 I_yy : 14.153 % Variation
12, 45 Qbar : 9.865 % Variation
15, 48 Xcg locat: 3.492 % Variation
18, 51 M_slosh 1: 13.664 % Variation
20, 53 X_slosh 1: 3.563 % Variation
23, 56 Wslsh_Z 1: 2.251 % Variation
25, 58 ZetSl_Z 1: 5.556 % Variation
27, 60 M_slosh 2: 14.465 % Variation
29, 62 X_slosh 2: 7.956 % Variation
32, 65 Wslsh_Z 2: 2.251 % Variation
34, 67 ZetSl_Z 2: 6.667 % Variation
36, 69 W_flex 2: 5.000 Addit. Variat.
38, 71 W_flex 4: 8.000 Addit. Variat.

Yaw Uncertainties (19), Inputs= 34:71

2, 35 Cn_beta : 14.535 % Variation
5, 38 Cy_beta : -24.655 % Variation
9, 42 I_xx : 19.786 % Variation
11, 44 I_zz : 14.158 % Variation
13, 46 Qbar : 9.865 % Variation
14, 47 Xcg locat: 3.492 % Variation
16, 49 Xcg locat: 3.492 % Variation
17, 50 M_slosh 1: 13.664 % Variation
19, 52 X_slosh 1: 3.563 % Variation
21, 54 X_slosh 1: 3.563 % Variation
22, 55 Wslsh_Y 1: 2.251 % Variation
24, 57 ZetSl_Y 1: 5.556 % Variation
26, 59 M_slosh 2: 14.465 % Variation
28, 61 X_slosh 2: 7.956 % Variation
30, 63 X_slosh 2: 7.956 % Variation
31, 64 Wslsh_Y 2: 2.251 % Variation
33, 66 ZetSl_Y 2: 6.667 % Variation
35, 68 W_flex 1: 5.000 Addit. Variat.
37, 70 W_flex 3: 8.000 Addit. Variat.

Robust Performance Analysis Model

Includes 38 Paramet Variations
plus One Performance Perturbation
(Gust to Alpha or Beta)

Plant Must Be Stabilized First

This System Satisfies Robust Performance Requirement when the Transfer Function Magnitude is Less than One at All Frequencies. It Means that it will satisfy Both Performance to Gusts and its Stability will be Robust to All Parameter Variations Defined

Uncertainties (38), Outputs= 33:70

1, 33 Cm_alpha : -14.535 % Variation
2, 34 Cn_beta : 14.535 % Variation
3, 35 Ca_alpha : 17.153 % Variation
4, 36 Cz_alpha : -24.655 % Variation
5, 37 Cy_beta : -24.655 % Variation
6, 38 Cm_0 : -9.690 % Variation
7, 39 CZ_0 : -12.327 % Variation
8, 40 CA_0 : 6.679 % Variation
9, 41 I_xx : 19.786 % Variation
10, 42 I_yy : 14.153 % Variation
11, 43 I_zz : 14.158 % Variation
12, 44 Qbar : 9.865 % Variation
13, 45 Qbar : 9.865 % Variation
14, 46 Xcg locat: 3.492 % Variation
15, 47 Xcg locat: 3.492 % Variation
16, 48 Xcg locat: 3.492 % Variation
17, 49 M_slosh 1: 13.664 % Variation
18, 50 M_slosh 1: 13.664 % Variation
19, 51 X_slosh 1: 3.563 % Variation
20, 52 X_slosh 1: 3.563 % Variation
21, 53 X_slosh 1: 3.563 % Variation
22, 54 Wslsh_Y 1: 2.251 % Variation
23, 55 Wslsh_Z 1: 2.251 % Variation
24, 56 ZetSl_Y 1: 5.556 % Variation
25, 57 ZetSl_Z 1: 5.556 % Variation
26, 58 M_slosh 2: 14.465 % Variation
27, 59 M_slosh 2: 14.465 % Variation
28, 60 X_slosh 2: 7.956 % Variation
29, 61 X_slosh 2: 7.956 % Variation
30, 62 X_slosh 2: 7.956 % Variation
31, 63 Wslsh_Y 2: 2.251 % Variation
32, 64 Wslsh_Z 2: 2.251 % Variation
33, 65 ZetSl_Y 2: 6.667 % Variation
34, 66 ZetSl_Z 2: 6.667 % Variation
35, 67 W_flex 1: 5.000 Addit. Variat.
36, 68 W_flex 2: 5.000 Addit. Variat.
37, 69 W_flex 3: 8.000 Addit. Variat.
38, 70 W_flex 4: 8.000 Addit. Variat.

Figure 4.7 Robust Performance Analysis Simulink Model "Robust_Performance.slx" Configured for Analyzing the Pitch Axis

Conclusions

The analysis results show that the control system is robust to both, gusts and to parameter variations at Max-Q. However, it also indicates that there is some sensitivity to variations at 3 frequencies that we should try to improve, the slosh frequency at 4.1 (rad/sec), at 10 (rad/sec) which is near the cross-over (possibly needs a little higher actuator bandwidth?), and at the frequency of the engine rocking mode which has been attenuated with a filter.

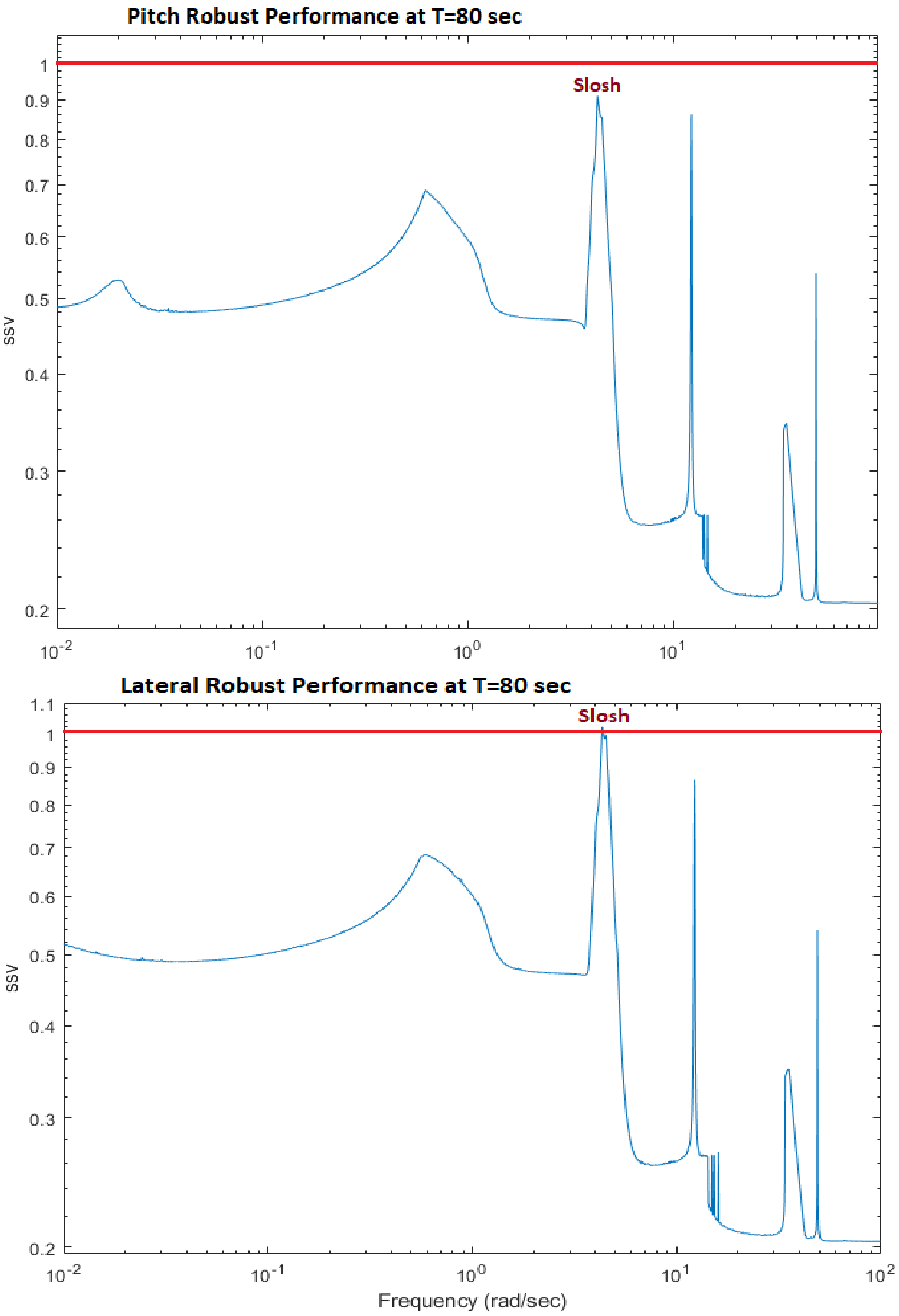


Figure 4.8 System Satisfies Robust Performance Requirement in Both Pitch and Lateral Axes

5. Analyzing the Unstable Slosh Effects During 2nd Stage

In Sections 2 and 3 the linear spring-mass analogy propellant slosh model was included in the Flixan vehicle systems and used for the stability analysis. The mass is initialized resting at the center of the tank. It is excited by the normal and lateral accelerations (a_z and a_y) at the tank centerline and it deflects relative to the tank centerline perpendicular to the vehicle axial acceleration vector Ax. The mass responds by applying reaction forces and torques back on the vehicle as it oscillates along the y and z axes. The analysis performed at around T=350 sec shows that the LOX mode is unstable under TVC control because the mode incircles the critical point +. The LH2 slosh node, however, is phase-stable. The LOX instability in this flight condition with an unbaffled tank of damping coefficient ζ= 0.002 is very unstable when analyzed using linear Nichols plots. It remains unstable even when the LOX damping coefficient is increased to ζ= 0.02. Linear stability is not the ideal criterion to determine the optimal damping coefficient and baffle requirements versus tank fill level. A better than a spring-mass model is needed to determine what is the minimum damping coefficient that the system will be able handle before the slosh oscillations degrade the vehicle performance. To do that we will need a non-linear simulation model that includes non-linear actuators and a more realistic slosh model, the spherical pendulum model.

The spherical pendulum model is a mass suspended by a string with the other end of the string attached to the tank centerline. Unlike the simple pendulum that only swings in a plane, the spherical pendulum can swing inside a sphere, along both y and z directions. The advantage of this model is that it limits the amplitude of the mass deflection from the tank center to the length of the pendulum, unlike the linear model where the mass deflection may exceed the tank radius and extend to infinity. The second advantage is that it includes the centripetal reaction forces against the tank walls which are generated by the angular velocity of the slosh mass when it spins around the tank. It allows us to analyze vortex type of dynamic instabilities in simulations when the mass develops a swirling motion and the centripetal disturbance forces from the spinning mass are coupling with the TVC control system and induce rotational TVC gimbaling that further aggravates the spinning motion of the mass. The linear spring-mass or linear pendulum models include only the reaction components of force due to the spring constant or pendulum deflection under gravity acceleration. They do not include the centripetal forces that can be generated when the mass is spinning around the tank. The non-linear spherical pendulum model allows us to analyze vortex type of slosh instabilities by initializing the simulation with a slosh mass angular rate in addition to pendulum angle deflection and observing if it converges to a reasonably small amplitude or if it couples with flexibility and the control system and it diverges further to produce unacceptable limit-cycles.

Figure 5.1 shows the pendulum slosh analogy of a tank that is filled at 60% level. In this tank we can assume that only 20% of the total propellant is sloshing near the surface of the liquid and the remaining 40% is rigidly attached at its center of mass on the tank centerline near the bottom. The length of the pendulum string is a little shorter (approx. 3/4) of the tank radius and it is attached at a point on the tank centerline a little below the liquid surface when it's at rest. Unlike the linear analysis models where the slosh modes are included in the Flixan vehicle system, we will now create two separate spherical pendulum slosh subsystems in Simulink that will be coupled with the vehicle. We must therefore remove the two linear slosh modes from the Flixan vehicle and create the capability to physically couple the vehicle with the non-linear slosh models.

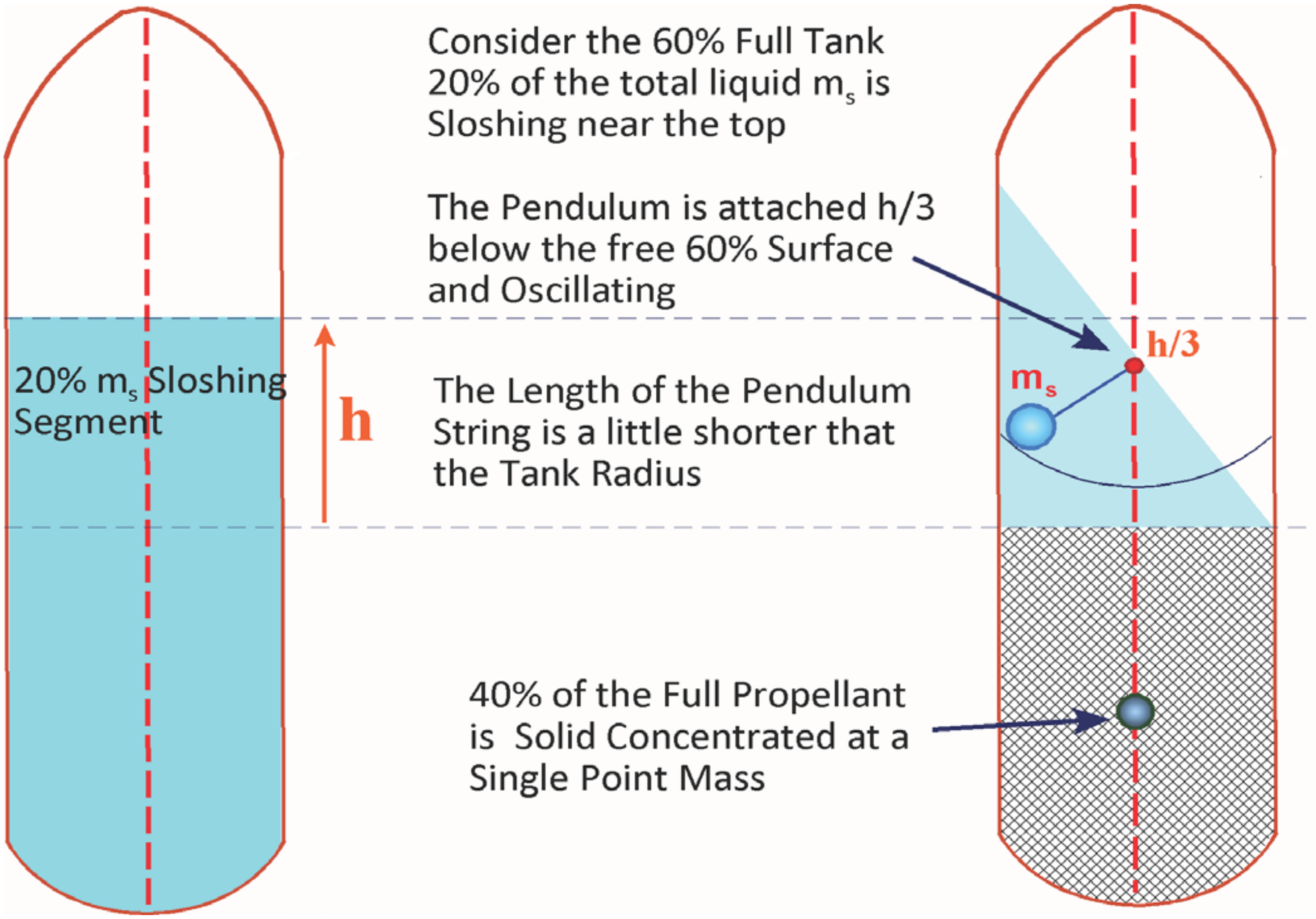


Figure 5.1 The Pendulum Slosh Model

The first step is to create a Flixan vehicle model with the capability to couple with the external slosh subsystems. The new vehicle model should include inputs that will receive the external forces and moments generated by the slosh masses. It must also include acceleration outputs at the 2 pendulum pivot points that will excite the pendulum mass deflections.

The second step is to couple this new vehicle model with the 2 spring-mass linear models which are wrapped externally around the vehicle in Simulink and make sure that we get the same results as in the previous linear model that included the slosh internally.

The third step is to replace the 2 external spring-mass models with the equivalent spherical pendulum models. Replace also the linear TVC actuators with non-linear TVC actuator models that include Coulomb friction at the gimbal and limits at the gimbal positions and rates. Finally, we will simulate the combined vehicle and pendulum slosh system initialized from different pendulum deflections and spin rates and adjusting the damping coefficient ζ from the low unbaffled value of ζ=0.002 to higher values, until a stable, acceptable and robust limit-cycle response is obtained. Let us begin by describing the linear spring-mass and the spherical pendulum slosh models.

5.1 Linear Slosh Model

The linearized spring-mass slosh model is described by the following 2nd order equations. The slosh mass displacements (z_s and y_s) are relative to tank centerline, and (a_z and a_y) are the normal and lateral vehicle accelerations at the slosh mass location. The accelerations include components due to vehicle rotational accelerations and local flexibility at the tank centerline.

$$\ddot{z}_s[s^2 + 2\zeta\omega_s s + \omega_s^2] = -a_z$$

$$\ddot{y}_s[s^2 + 2\zeta\omega_s s + \omega_s^2] = -a_y$$

The slosh frequency is $\omega_s = \sqrt{\frac{A_x}{l_p}}$ is calculated from an equivalent pendulum

The forces applied back to the vehicle by the slosh mass are:

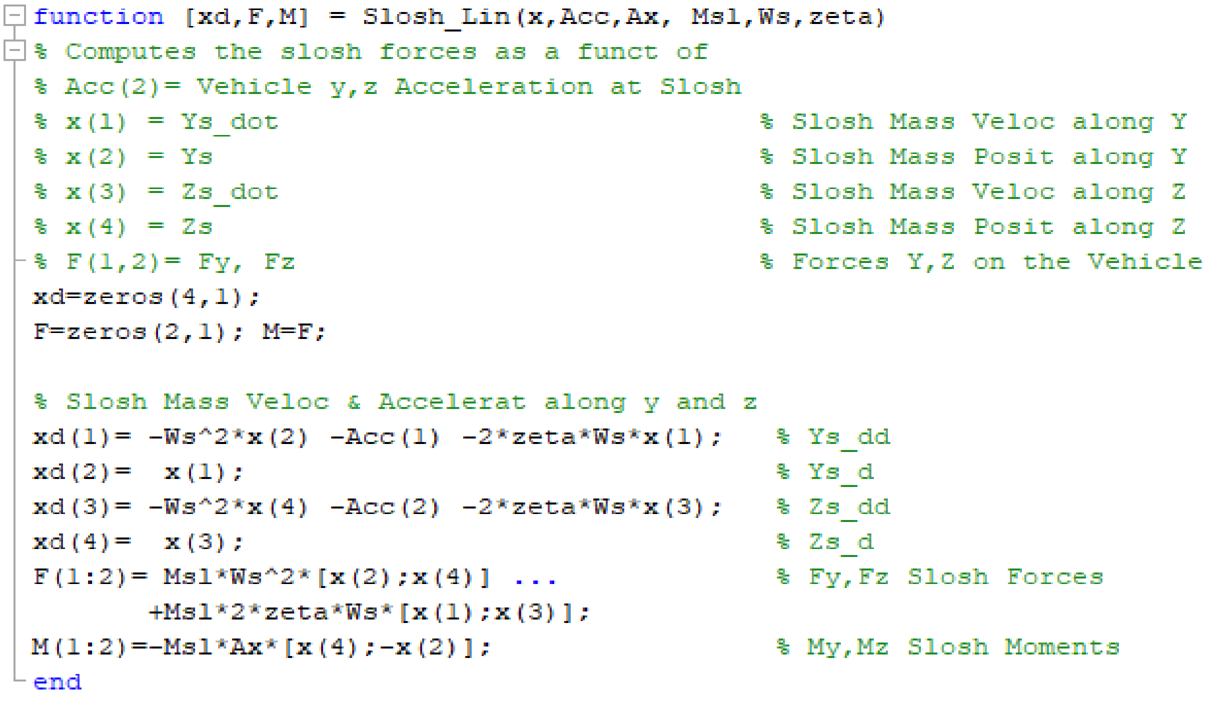
$$F_z = m_s(\omega_s^2 z_s + 2\zeta\omega_s\dot{z}_s)\,;\ F_y = m_s(\omega_s^2 y_s + 2\zeta\omega_s\dot{y}_s)$$

The vehicle pitch and yaw moments are:

$$M_Y = -F_Z l_x - m_s A_x z_s;\ N_Z = F_Y l_x + m_s A_x y_s$$

Ax	is the axial acceleration along the pendulum
l_p	is the length of the equivalent pendulum
l_x	is the moment arm between the slosh mass and the vehicle CG

They are implemented in the following Matlab function. Note, that only the second moment terms: $m_s A_x z_s$ and $m_s A_x y_s$ are included in this function because the moments due to (Forces x Moment Arms) are already included in the Flixan model.



```
function [xd,F,M] = Slosh_Lin(x,Acc,Ax, Msl,Ws,zeta)
% Computes the slosh forces as a funct of
% Acc(2)= Vehicle y,z Acceleration at Slosh
% x(1) = Ys_dot                             % Slosh Mass Veloc along Y
% x(2) = Ys                                 % Slosh Mass Posit along Y
% x(3) = Zs_dot                             % Slosh Mass Veloc along Z
% x(4) = Zs                                 % Slosh Mass Posit along Z
% F(1,2)= Fy, Fz                            % Forces Y,Z on the Vehicle
xd=zeros(4,1);
F=zeros(2,1); M=F;

% Slosh Mass Veloc & Accelerat along y and z
xd(1)= -Ws^2*x(2) -Acc(1) -2*zeta*Ws*x(1);  % Ys_dd
xd(2)=  x(1);                                % Ys_d
xd(3)= -Ws^2*x(4) -Acc(2) -2*zeta*Ws*x(3);  % Zs_dd
xd(4)=  x(3);                                % Zs_d
F(1:2)= Msl*Ws^2*[x(2);x(4)] ...             % Fy,Fz Slosh Forces
       +Msl*2*zeta*Ws*[x(1);x(3)];
M(1:2)=-Msl*Ax*[x(4);-x(2)];                 % My,Mz Slosh Moments
end
```

5.2 Spherical Pendulum Model

The spherical pendulum is the exact non-linear mathematical model of a mass suspended from a pivot by a non-elastic string that can rotate inside a sphere under the influence of accelerations applied at the pivot. The length of the string is l and it is attached at the tank centerline, see Figure 5.2. The slosh mass can swing in two directions along the y and z axes. It can also be seen as a mass rolling along the surface of the sphere. The mass displacement can be resolved by two rotations, a vertical rotation θ of the string along a longitude, and a horizontal rotation ϕ about the vehicle x-axis along a latitude circling around the tank centerline. The angle θ is measured from the vertical and it is always greater than zero and the angle ϕ is measured counterclockwise from the projection of l on the y-z plane.

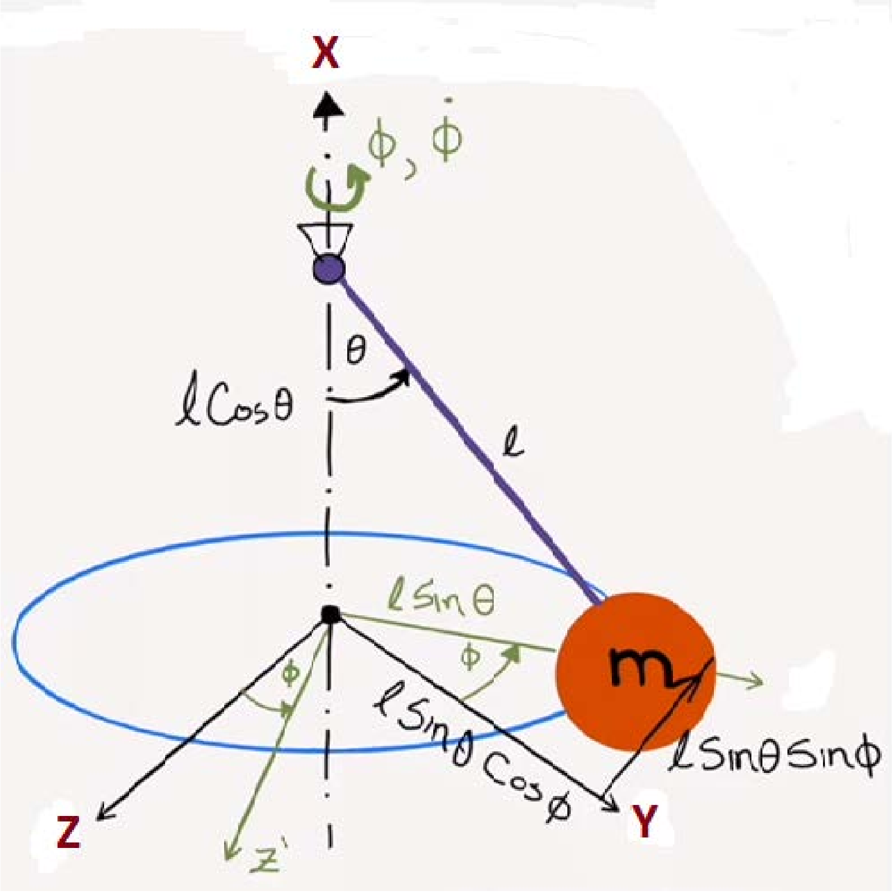


Figure 5.2 Spherical Pendulum

The mass is initialized at (θ_0 , ϕ_0) and its motion relative to the tank is excited by the accelerations A_y and A_z of the pivot point along the vehicle y and z axes. The pivot acceleration relative to the mass can be resolved into an axial acceleration A_ξ towards the mass and a tangential A_t of the pivot parallel to the mass, as shown in Figure 5.3. The pendulum motion is described by two moment equations: vertical and horizontal.

Vertical Moment Equation:

Equation 1 represents the moment along the vertical motion and calculates the pendulum angle θ. It is excited on the RHS by the axial component of the vehicle acceleration A_ξ towards the slosh mass which produces the vertical moment. The friction force Dv_θ is produced as the mass is sliding along the surface with velocity v_θ and produces an opposing torque, where D is the viscous friction coefficient.

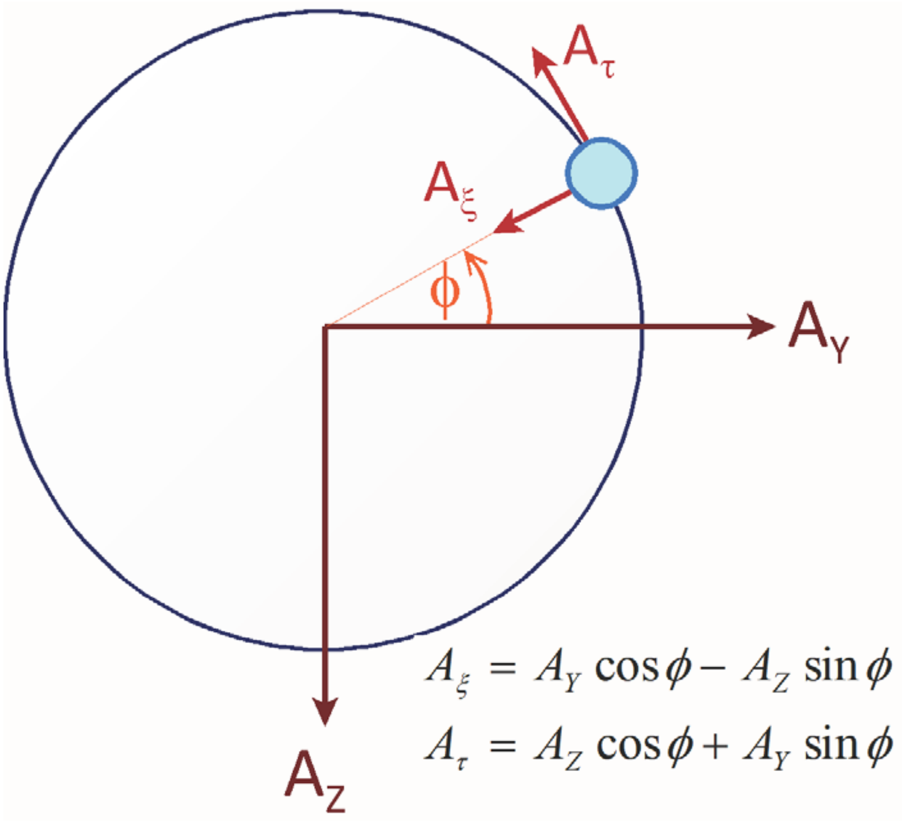
$$ml^2\ddot{\theta} - ml^2\dot{\phi}^2\cos\theta\sin\theta + mlA_X\sin\theta = -mlA_\xi\cos\theta - Dl^2\dot{\theta} \qquad (1)$$

$$\ddot{\theta} = +\dot{\phi}^2\cos\theta\sin\theta - \frac{A_X}{l}\sin\theta - \frac{A_\xi}{l}\cos\theta - \frac{D}{m}\dot{\theta} \qquad (2)$$

For small angles and without the lateral motion this equation reduces to

$$\ddot{\theta} + 2\zeta\omega\dot{\theta} + \omega^2\theta = -\frac{A_\xi}{l} \qquad (3)$$

Where the oscillation frequency: $\omega^2 = \frac{A_X}{l}$ and the viscous friction coefficient $D = 2\zeta\omega\, m$, where ζ is the damping coefficient. The coefficient D is selected to produce a ζ =0.01. The pendulum length l is a little smaller than the tank radius.



$$A_\xi = A_Y\cos\phi - A_Z\sin\phi$$
$$A_\tau = A_Z\cos\phi + A_Y\sin\phi$$

Figure 5.3 Top View. The Vehicle Normal and Lateral Accelerations are Resolved into Axial and Tangential Relative Accelerations

Lateral Moment Equation:

In the lateral direction the spin moment about x is calculated by equation 4, where the rotational angle ϕ is about the tank centerline x. It is excited by the torque produced by the relative tangential acceleration A_τ between the tank and the mass, which is perpendicular to the A_ξ acceleration. There is also a viscous friction force Dv_ϕ proportional to the horizontal velocity component producing a negative torque.

$$ml^2\ddot{\phi}\sin^2\theta + 2\,ml^2\dot{\theta}\dot{\phi}\;\cos\theta\sin\theta = +mlA_{\mathrm{t}}\sin\theta - D\dot{\phi}l^2\sin^2\theta \qquad (4)$$

$$\ddot{\phi} = -2\,\dot{\theta}\dot{\phi}\,\frac{\cos\theta}{\sin\theta} + \frac{A_{\mathrm{t}}}{l\sin\theta} - \frac{D}{m}\dot{\phi} \qquad (5)$$

Slosh Mass Kinematics Relative to Tank Centerline Attachment:

$$\begin{aligned} Y_s &= l\;\sin\theta\cos\phi \\ X_s &= -l\;\cos\theta \\ Z_s &= -l\;\sin\theta\sin\phi \end{aligned}$$

Slosh Mass Velocities:

$$\begin{aligned} \dot{Y}_s &= +l\dot{\theta}\cos\theta\cos\phi - \;l\,\dot{\phi}\sin\theta\sin\phi \\ \dot{Z}_s &= -l\dot{\theta}\cos\theta\sin\phi - \;l\,\dot{\phi}\sin\theta\cos\phi \end{aligned}$$

Slosh Mass Accelerations Relative to Tank:

$$\begin{aligned} \ddot{Y}_s/_l &= +\ddot{\theta}\cos\theta\cos\phi - \ddot{\phi}\sin\theta\sin\phi \;-\left(\dot{\theta}^2+\dot{\phi}^2\right)\sin\theta\cos\phi - 2\dot{\theta}\dot{\phi}\cos\theta\sin\phi \\ \ddot{Z}_s/_l &= -\ddot{\theta}\cos\theta\sin\phi - \ddot{\phi}\sin\theta\cos\phi + \left(\dot{\theta}^2+\dot{\phi}^2\right)\sin\theta\sin\phi - 2\dot{\theta}\dot{\phi}\cos\theta\cos\phi \end{aligned}$$

Slosh Forces on the Vehicle:

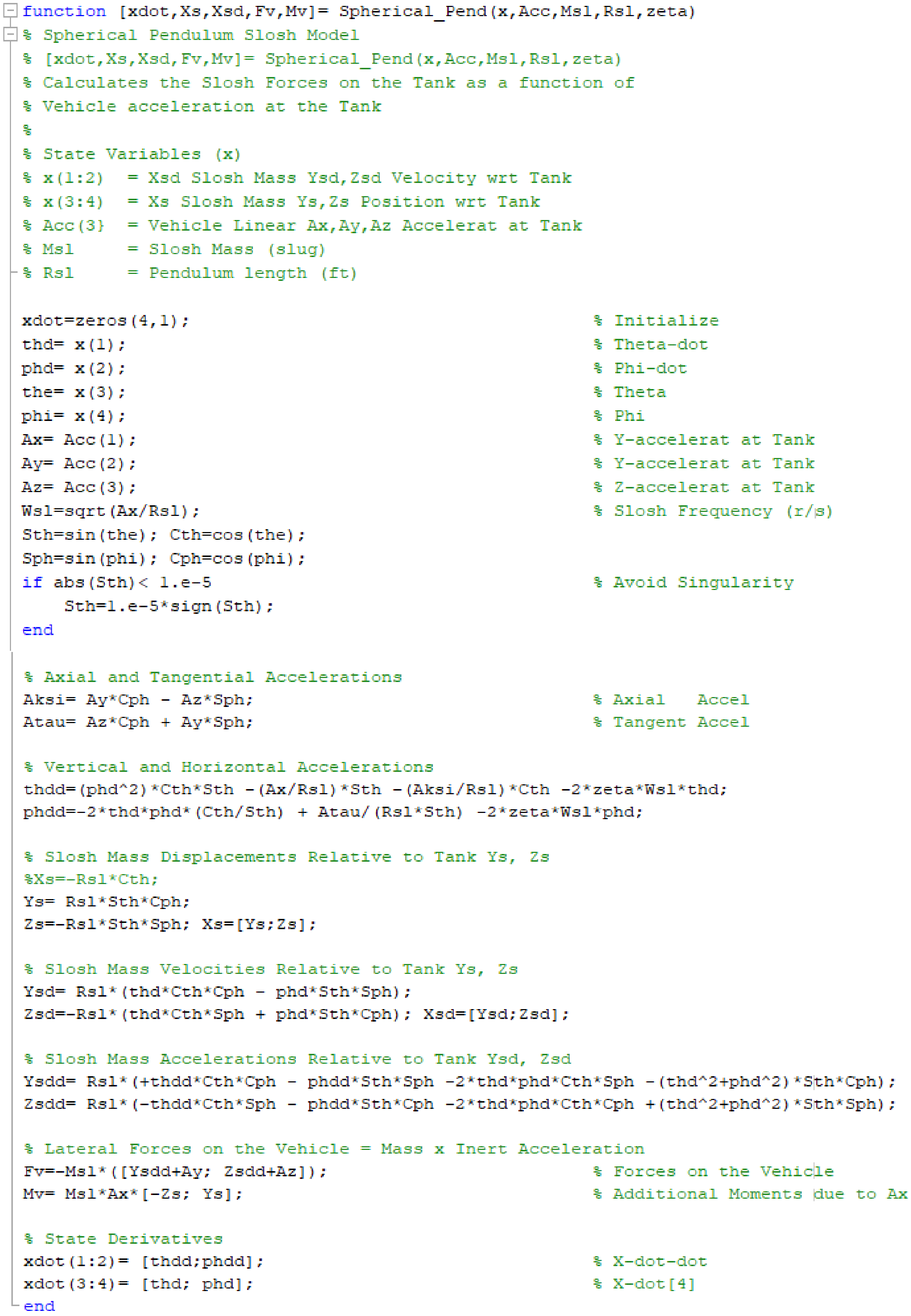
$$F_Y = m_s\left(\ddot{Y}_s + A_{Yt}\right) \qquad \text{Mass x Inertial Acceleration}$$

$$F_Z = m_s\left(\ddot{Z}_s + A_{Zt}\right)$$

Slosh Moments on the Vehicle:

$$\boxed{\begin{aligned} M_{YSL} &= -\sum_{i=1}^{Nsl}\left\{F_{Zsi}\,l_{sxi} + m_s(i)A_X z_{si}\right\} \\ N_{ZSL} &= +\sum_{i=1}^{Nsl}\left\{F_{Ysi}\,l_{sxi} + m_s(i)A_X y_{si}\right\} \end{aligned}}$$

Spherical Pendulum Slosh Equations Implementation in Matlab



```
function [xdot,Xs,Xsd,Fv,Mv]= Spherical_Pend(x,Acc,Msl,Rsl,zeta)
% Spherical Pendulum Slosh Model
% [xdot,Xs,Xsd,Fv,Mv]= Spherical_Pend(x,Acc,Msl,Rsl,zeta)
% Calculates the Slosh Forces on the Tank as a function of
% Vehicle acceleration at the Tank
%
% State Variables (x)
% x(1:2)  = Xsd Slosh Mass Ysd,Zsd Velocity wrt Tank
% x(3:4)  = Xs Slosh Mass Ys,Zs Position wrt Tank
% Acc(3}  = Vehicle Linear Ax,Ay,Az Accelerat at Tank
% Msl     = Slosh Mass (slug)
% Rsl     = Pendulum length (ft)

xdot=zeros(4,1);                                      % Initialize
thd= x(1);                                            % Theta-dot
phd= x(2);                                            % Phi-dot
the= x(3);                                            % Theta
phi= x(4);                                            % Phi
Ax= Acc(1);                                           % Y-accelerat at Tank
Ay= Acc(2);                                           % Y-accelerat at Tank
Az= Acc(3);                                           % Z-accelerat at Tank
Wsl=sqrt(Ax/Rsl);                                     % Slosh Frequency (r/s)
Sth=sin(the); Cth=cos(the);
Sph=sin(phi); Cph=cos(phi);
if abs(Sth)< 1.e-5                                    % Avoid Singularity
    Sth=1.e-5*sign(Sth);
end

% Axial and Tangential Accelerations
Aksi= Ay*Cph - Az*Sph;                                % Axial   Accel
Atau= Az*Cph + Ay*Sph;                                % Tangent Accel

% Vertical and Horizontal Accelerations
thdd=(phd^2)*Cth*Sth -(Ax/Rsl)*Sth -(Aksi/Rsl)*Cth -2*zeta*Wsl*thd;
phdd=-2*thd*phd*(Cth/Sth) + Atau/(Rsl*Sth) -2*zeta*Wsl*phd;

% Slosh Mass Displacements Relative to Tank Ys, Zs
%Xs=-Rsl*Cth;
Ys= Rsl*Sth*Cph;
Zs=-Rsl*Sth*Sph; Xs=[Ys;Zs];

% Slosh Mass Velocities Relative to Tank Ys, Zs
Ysd= Rsl*(thd*Cth*Cph - phd*Sth*Sph);
Zsd=-Rsl*(thd*Cth*Sph + phd*Sth*Cph); Xsd=[Ysd;Zsd];

% Slosh Mass Accelerations Relative to Tank Ysd, Zsd
Ysdd= Rsl*(+thdd*Cth*Cph - phdd*Sth*Sph -2*thd*phd*Cth*Sph -(thd^2+phd^2)*Sth*Cph);
Zsdd= Rsl*(-thdd*Cth*Sph - phdd*Sth*Cph -2*thd*phd*Cth*Cph +(thd^2+phd^2)*Sth*Sph);

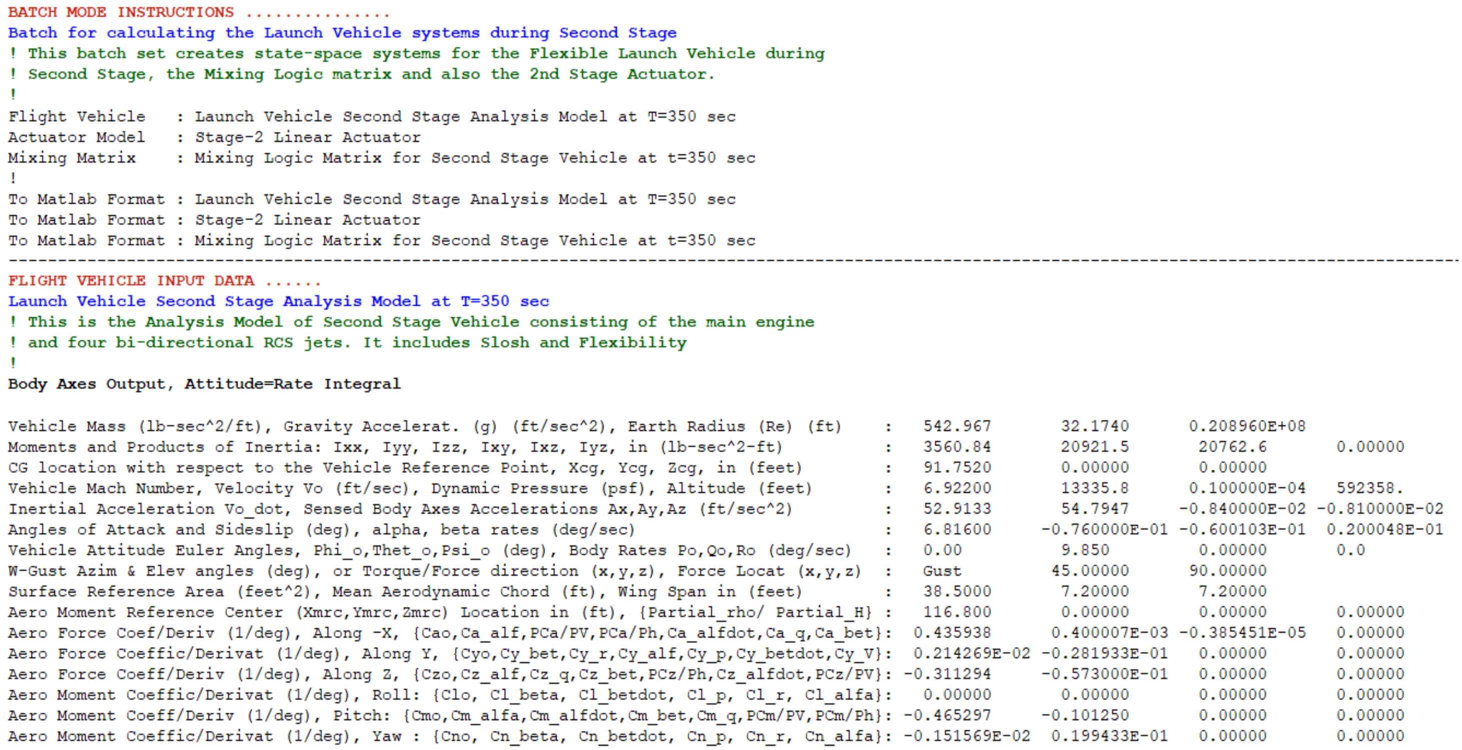
% Lateral Forces on the Vehicle = Mass x Inert Acceleration
Fv=-Msl*([Ysdd+Ay; Zsdd+Az]);                         % Forces on the Vehicle
Mv= Msl*Ax*[-Zs; Ys];                                 % Additional Moments due to Ax

% State Derivatives
xdot(1:2)= [thdd;phdd];                               % X-dot-dot
xdot(3:4)= [thd; phd];                                % X-dot[4]
end
```

5.3 The Flixan Vehicle Model with External Spring-Mass Model

This is an intermediate analysis that will help us test the vehicle model before coupling it with the spherical pendulum models. We will couple the new Flixan vehicle model that does not include the slosh modes with the two linear slosh models, externally via forces and accelerations, and compare results with the previous analysis. It allows us to validate the model coupling process.

The work files for this analysis are in directory "*Examples\23-Classic Launch Vehicle Design & Simulation\5-Non-Linear Slosh Analysis Using Spherical Pendulum)*". It includes the input file "*Flex_Vehi_T350.inp*" that was used for the 2nd stage stability analysis in Section 3.2.3 and it includes the two propellant modes internally. It also includes the input file "*Pend_Slosh_T350.inp*", shown below, that generates a vehicle model "*Launch Vehicle Second Stage Analysis Model at T=350 sec*" that does not include the slosh modes. It provides, however, the necessary inputs and outputs on the vehicle system that will allow us to attach the slosh subsystems externally. The vehicle system inputs are the Fy and Fz slosh forces which are applied at the tank centerline along y and z, at the slosh mass locations. Also, the pitch and yaw moments, My and Nz due to the slosh mass displacement y_s and z_s coupling with the vehicle axial acceleration Ax. The additional outputs are vehicle accelerations at the slosh mass locations a_y and a_z which include rotational acceleration effects and flexibility. The input file generates also a linear actuator model and the mixing logic matrix Kmix_t350.



```
BATCH MODE INSTRUCTIONS ...............
Batch for calculating the Launch Vehicle systems during Second Stage
! This batch set creates state-space systems for the Flexible Launch Vehicle during
! Second Stage, the Mixing Logic matrix and also the 2nd Stage Actuator.
!
Flight Vehicle    : Launch Vehicle Second Stage Analysis Model at T=350 sec
Actuator Model    : Stage-2 Linear Actuator
Mixing Matrix     : Mixing Logic Matrix for Second Stage Vehicle at t=350 sec
!
To Matlab Format : Launch Vehicle Second Stage Analysis Model at T=350 sec
To Matlab Format : Stage-2 Linear Actuator
To Matlab Format : Mixing Logic Matrix for Second Stage Vehicle at t=350 sec
----------------------------------------------------------------------------------------------------------------------------------------------------
FLIGHT VEHICLE INPUT DATA ......
Launch Vehicle Second Stage Analysis Model at T=350 sec
! This is the Analysis Model of Second Stage Vehicle consisting of the main engine
! and four bi-directional RCS jets. It includes Slosh and Flexibility
!
Body Axes Output, Attitude=Rate Integral

Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft)    :   542.967       32.1740      0.208960E+08
Moments and Products of Inertia: Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)          :   3560.84       20921.5       20762.6       0.00000
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)        :   91.7520       0.00000       0.00000
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)       :   6.92200       13335.8      0.100000E-04   592358.
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)         :   52.9133       54.7947     -0.840000E-02 -0.810000E-02
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                         :   6.81600     -0.760000E-01 -0.600103E-01  0.200048E-01
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)   :   0.00          9.850         0.00000       0.0
W-Gust Azim & Elev angles (deg), or Torque/Force direction (x,y,z), Force Locat (x,y,z)  :   Gust         45.00000      90.00000
Surface Reference Area (feet^2), Mean Aerodynamic Chord (ft), Wing Span in (feet)        :   38.5000       7.20000       7.20000
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial_rho/ Partial_H} :   116.800       0.00000       0.00000       0.00000
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:  0.435938      0.400007E-03 -0.385451E-05   0.00000
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}:  0.214269E-02 -0.281933E-01   0.00000       0.00000
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}: -0.311294     -0.573000E-01   0.00000       0.00000
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:   0.00000       0.00000       0.00000       0.00000
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}: -0.465297     -0.101250       0.00000       0.00000
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}: -0.151569E-02  0.199433E-01   0.00000       0.00000
```

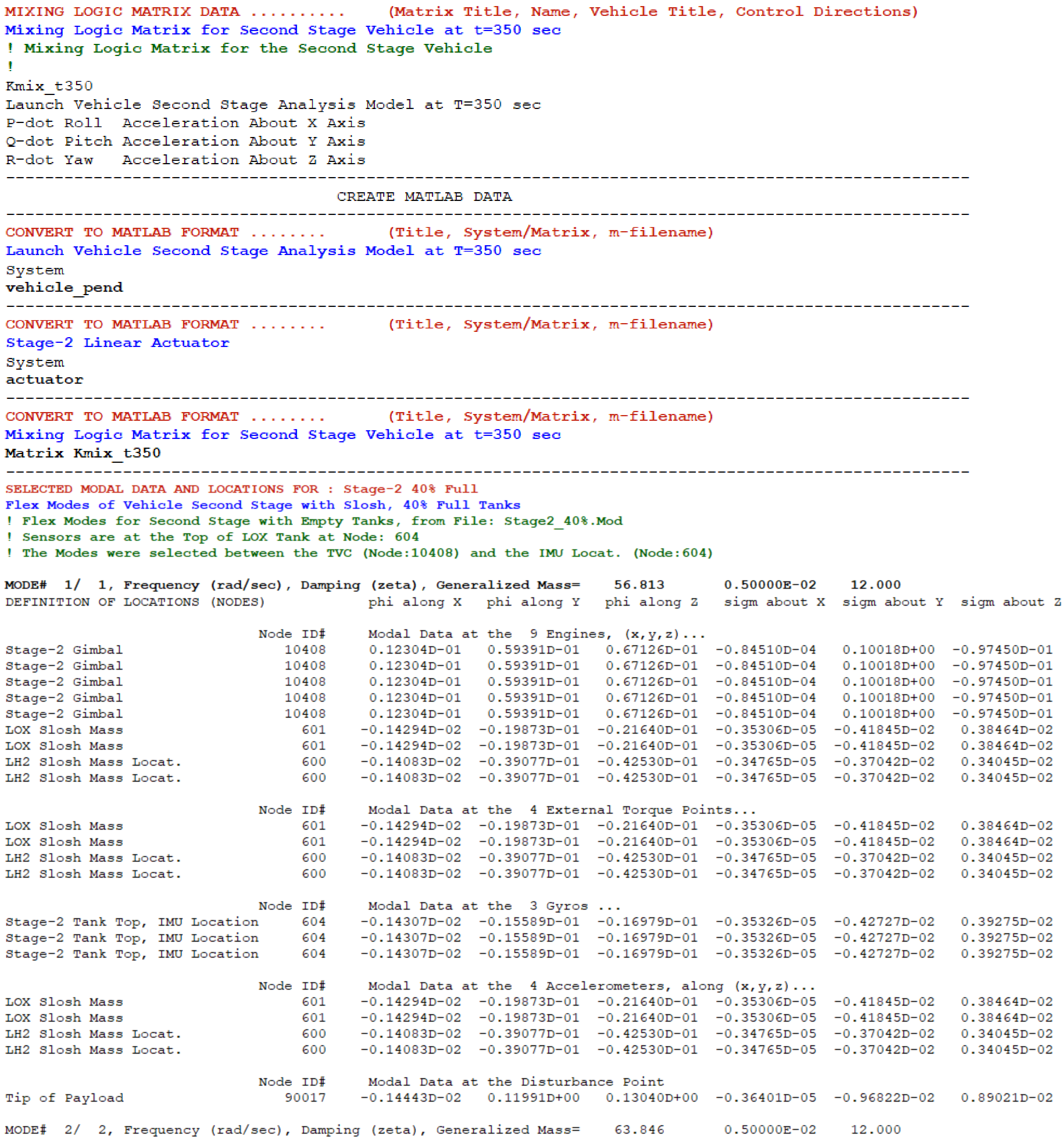
```
Number of Thruster Engines, Include or Not the Tail-Wags-Dog and Load-Torque Dynamics ?  :  9   WITH TWD

TVC Engine No:  1                                         (Gimbaling  Throttling   Single_Gimbal) : Main Engine#1    Gimbaling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                           :  29751.7         29751.7
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.00000         0.00000       6.00000       6.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  9.00000         120.000       2.60000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  84.5000         0.00000       0.00000
TVC Engine No:  2                                         (Gimbaling  Throttling   Single_Gimbal) : Left RCS Jet     Throttling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                           :  0.00000         3.0
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg): -90.00           0.00000       0.00000       0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  0.00000         0.00000       0.00000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  83.00          -3.50          0.00000
TVC Engine No:  3                                         (Gimbaling  Throttling   Single_Gimbal) : Rght RCS Jet     Throttling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                           :  0.00000         3.0
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg): -90.00           0.00000       0.00000       0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  0.00000         0.00000       0.00000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  83.00           3.50          0.00000
TVC Engine No:  4                                         (Gimbaling  Throttling   Single_Gimbal) : Top  RCS Jet     Throttling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                           :  0.00000         3.0
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.0000          90.000        0.00000       0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  0.00000         0.00000       0.00000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  83.00           0.00000      -3.50
TVC Engine No:  5                                         (Gimbaling  Throttling   Single_Gimbal) : Botm RCS Jet     Throttling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                           :  0.00000         3.0
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.0000          90.000        0.00000       0.00000
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  0.00000         0.00000       0.00000
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  83.00           0.00000       3.50
TVC Engine No:  6  LOX Force Inputs along Y and Z         (Gimbaling  Throttling   Single_Gimbal) : LOX Y-Force      Throttling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                           :  0.0             1.0
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.0             90.0          0.0           0.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  0.0             0.0           1.0
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  93.363          0.0           0.0
TVC Engine No:  7                                         (Gimbaling  Throttling   Single_Gimbal) : LOX Z-Force      Throttling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                           :  0.0             1.0
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg): -90.0             0.0           0.0           0.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  0.0             0.0           1.0
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  93.363          0.0           0.0
TVC Engine No:  8  LH2 Force Inputs along Y and Z         (Gimbaling  Throttling   Single_Gimbal) : LH2 Y-Force      Throttling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                           :  0.0             1.0
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg):  0.0             90.0          0.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  0.0             0.0           1.0
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  86.476          0.0           0.0
TVC Engine No:  9                                         (Gimbaling  Throttling   Single_Gimbal) : LH2 Z-Force      Throttling
Engine Nominal Thrust, and Maximum Thrust in (lb)   (for throttling)                           :  0.0             1.0
Trim Angles (Dyn,Dzn) wrt Vehicle x-axis, Maxim Deflections (Dymax,Dzmax) from Trim (deg): -90.0             0.0           0.0
Eng Mass (slug), Inertia about Gimbal (lb-sec^2-ft), Moment Arm, engine CG to gimbal (ft):  0.0             0.0           1.0
Gimbal location with respect to the Vehicle Reference Axes, Xgimb, Ygimb, Zgimb, in (ft) :  86.476          0.0           0.0

Number of External Torques on the Vehicle                                                  :  4
Torque No  1  Direction (x, y, z) LOX Pitch      Torque Inputs where we will               :  0.0        1.0        0.0
Torque No  2  Direction (x, y, z) LOX Yaw        apply the Pitch and Yaw Slosh             :  0.0        0.0        1.0
Torque No  3  Direction (x, y, z) LH2 Pitch      Torques for the 2 Tanks                   :  0.0        1.0        0.0
Torque No  4  Direction (x, y, z) LH2 Yaw                                                  :  0.0        0.0        1.0

Number of Gyros, (Attitude and Rate)                                                       :  3
Gyro No  1 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Location in (feet) :  Roll   Rate          97.438     0.00
Gyro No  2 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Location in (feet) :  Pitch  Rate          97.438     0.00
Gyro No  3 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Location in (feet) :  Yaw    Rate          97.438     0.00
                                                 Measuring the Pitch and yaw Vehicle
Number of Accelerometers, Along Axis: (x,y,z)    Accelerations at the 2 Tanks              :  4
Acceleromet No  1 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Location (ft) :  Y-axis Accelerat.    93.363     0.00
Acceleromet No  2 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Location (ft) :  Z-axis Accelerat.    93.363     0.00
Acceleromet No  3 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Location (ft) :  Y-axis Accelerat.    86.476     0.00
Acceleromet No  4 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Location (ft) :  Z-axis Accelerat.    86.476     0.00

Number of Bending Modes                                                                    :  10
Flex Modes of Vehicle Second Stage with Slosh, 40% Full Tanks
-----------------------------------------------------------------------------------------------------------------------
ACTUATOR INPUT DATA ............ SIMPLE GENERIC MODEL B
Stage-2 Linear Actuator
Symbol Parameter Description                  (Units)      Value
C(s)   Order of Pade Delay (0,1,2)             ( - )        1, -0.002, 0.002
Ka     Gain of Amplifier ..................... (amps/volt)  10.0
Wsv    Bandwidth of the Linear Servo Actuator .  (rad/sec)   60.0
Kact   Actuator Stiffness (Piston+Oil+Electric)  (lb/ft)     2.4e+6
Klod   Stiffness at Surface or Nozzle Connection (lb/ft)     1.0e+9
Kbck   Stiffness at Vehicle Backup Structure ..  (lb/ft)     1.8e+6
R      Moment Arm between Actuator Rod & Gimbal  (feet)      0.72
Jl     Load Inertia about the Gimbal .......... (ft-lb-s^2)  120.0
Kg     Load Gimbal Bearing Spring Constant      (ft-lb/rad)  0.0
Bg     Load Gimbal Bearing Viscous Damping .... (ft-lb-sec)  3000.0
----------------------------------------------------------------------------------------------------------------
```



```
MIXING LOGIC MATRIX DATA ..........    (Matrix Title, Name, Vehicle Title, Control Directions)
Mixing Logic Matrix for Second Stage Vehicle at t=350 sec
! Mixing Logic Matrix for the Second Stage Vehicle
!
Kmix_t350
Launch Vehicle Second Stage Analysis Model at T=350 sec
P-dot Roll   Acceleration About X Axis
Q-dot Pitch Acceleration About Y Axis
R-dot Yaw   Acceleration About Z Axis
---------------------------------------------------------------------------------------------------
                                  CREATE MATLAB DATA
---------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Launch Vehicle Second Stage Analysis Model at T=350 sec
System
vehicle_pend
---------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Stage-2 Linear Actuator
System
actuator
---------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Mixing Logic Matrix for Second Stage Vehicle at t=350 sec
Matrix Kmix_t350
---------------------------------------------------------------------------------------------------
SELECTED MODAL DATA AND LOCATIONS FOR : Stage-2 40% Full
Flex Modes of Vehicle Second Stage with Slosh, 40% Full Tanks
! Flex Modes for Second Stage with Empty Tanks, from File: Stage2_40%.Mod
! Sensors are at the Top of LOX Tank at Node: 604
! The Modes were selected between the TVC (Node:10408) and the IMU Locat. (Node:604)

MODE#  1/  1, Frequency (rad/sec), Damping (zeta), Generalized Mass=   56.813       0.50000E-02    12.000
DEFINITION OF LOCATIONS (NODES)                   phi along X   phi along Y   phi along Z   sigm about X  sigm about Y  sigm about Z

                                 Node ID#     Modal Data at the  9 Engines, (x,y,z)...
Stage-2 Gimbal                    10408       0.12304D-01   0.59391D-01   0.67126D-01  -0.84510D-04   0.10018D+00  -0.97450D-01
Stage-2 Gimbal                    10408       0.12304D-01   0.59391D-01   0.67126D-01  -0.84510D-04   0.10018D+00  -0.97450D-01
Stage-2 Gimbal                    10408       0.12304D-01   0.59391D-01   0.67126D-01  -0.84510D-04   0.10018D+00  -0.97450D-01
Stage-2 Gimbal                    10408       0.12304D-01   0.59391D-01   0.67126D-01  -0.84510D-04   0.10018D+00  -0.97450D-01
Stage-2 Gimbal                    10408       0.12304D-01   0.59391D-01   0.67126D-01  -0.84510D-04   0.10018D+00  -0.97450D-01
LOX Slosh Mass                      601      -0.14294D-02  -0.19873D-01  -0.21640D-01  -0.35306D-05  -0.41845D-02   0.38464D-02
LOX Slosh Mass                      601      -0.14294D-02  -0.19873D-01  -0.21640D-01  -0.35306D-05  -0.41845D-02   0.38464D-02
LH2 Slosh Mass Locat.               600      -0.14083D-02  -0.39077D-01  -0.42530D-01  -0.34765D-05  -0.37042D-02   0.34045D-02
LH2 Slosh Mass Locat.               600      -0.14083D-02  -0.39077D-01  -0.42530D-01  -0.34765D-05  -0.37042D-02   0.34045D-02

                                 Node ID#     Modal Data at the  4 External Torque Points...
LOX Slosh Mass                      601      -0.14294D-02  -0.19873D-01  -0.21640D-01  -0.35306D-05  -0.41845D-02   0.38464D-02
LOX Slosh Mass                      601      -0.14294D-02  -0.19873D-01  -0.21640D-01  -0.35306D-05  -0.41845D-02   0.38464D-02
LH2 Slosh Mass Locat.               600      -0.14083D-02  -0.39077D-01  -0.42530D-01  -0.34765D-05  -0.37042D-02   0.34045D-02
LH2 Slosh Mass Locat.               600      -0.14083D-02  -0.39077D-01  -0.42530D-01  -0.34765D-05  -0.37042D-02   0.34045D-02

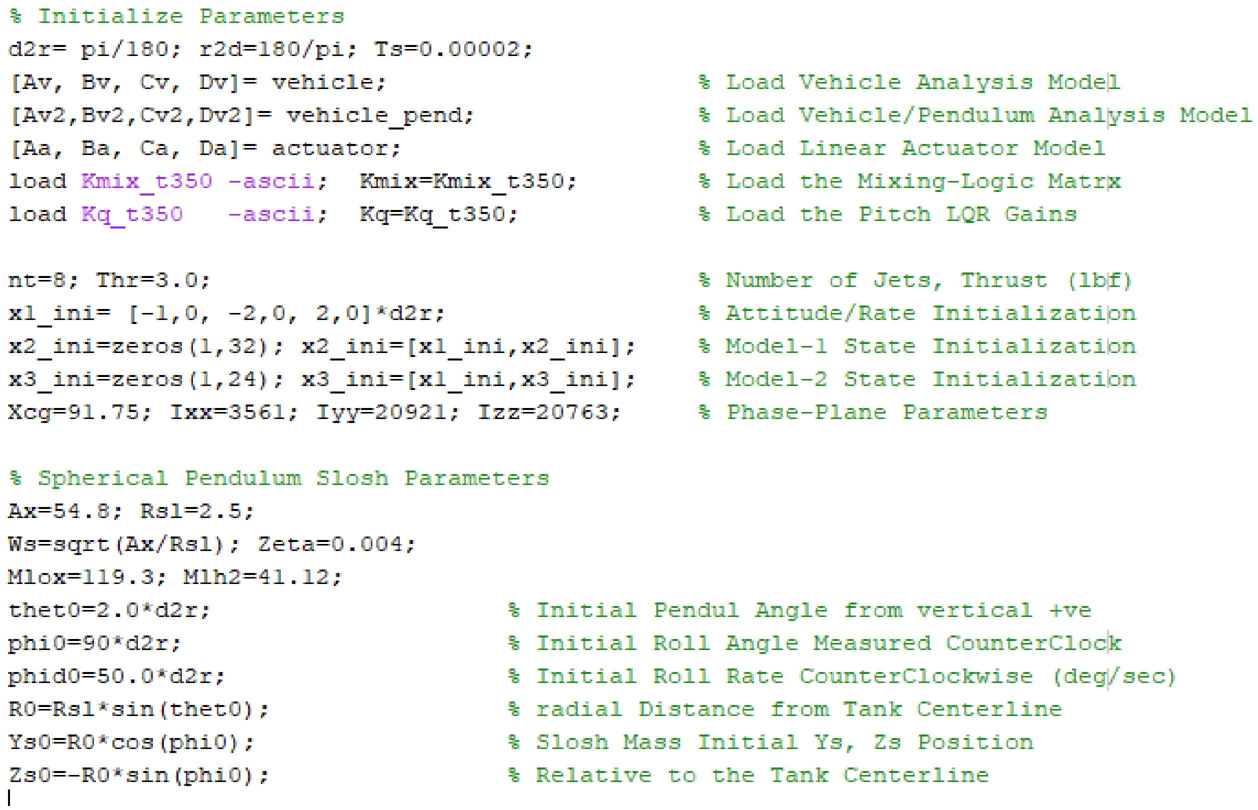
                                 Node ID#     Modal Data at the  3 Gyros ...
Stage-2 Tank Top, IMU Location      604      -0.14307D-02  -0.15589D-01  -0.16979D-01  -0.35326D-05  -0.42727D-02   0.39275D-02
Stage-2 Tank Top, IMU Location      604      -0.14307D-02  -0.15589D-01  -0.16979D-01  -0.35326D-05  -0.42727D-02   0.39275D-02
Stage-2 Tank Top, IMU Location      604      -0.14307D-02  -0.15589D-01  -0.16979D-01  -0.35326D-05  -0.42727D-02   0.39275D-02

                                 Node ID#     Modal Data at the  4 Accelerometers, along (x,y,z)...
LOX Slosh Mass                      601      -0.14294D-02  -0.19873D-01  -0.21640D-01  -0.35306D-05  -0.41845D-02   0.38464D-02
LOX Slosh Mass                      601      -0.14294D-02  -0.19873D-01  -0.21640D-01  -0.35306D-05  -0.41845D-02   0.38464D-02
LH2 Slosh Mass Locat.               600      -0.14083D-02  -0.39077D-01  -0.42530D-01  -0.34765D-05  -0.37042D-02   0.34045D-02
LH2 Slosh Mass Locat.               600      -0.14083D-02  -0.39077D-01  -0.42530D-01  -0.34765D-05  -0.37042D-02   0.34045D-02

                                 Node ID#     Modal Data at the Disturbance Point
Tip of Payload                    90017      -0.14443D-02   0.11991D+00   0.13040D+00  -0.36401D-05  -0.96822D-02   0.89021D-02

MODE#  2/  2, Frequency (rad/sec), Damping (zeta), Generalized Mass=   63.846       0.50000E-02    12.000
```

The following initialization file “init.m” loads the two vehicle systems into Matlab, that is, the previously used linear model “vehicle” that includes the slosh modes, and the recently described model “vehicle_pend” that is without the slosh. It also loads the linear actuator, the mixing logic, the LQR control gains and it initializes the vehicle attitude to [-1, -2, 2] (deg) in roll, pitch and yaw for the simulations. It also initializes the spherical pendulum parameters, such as the angles (θ_0 and ϕ_0), pendulum length, vehicle acceleration, damping coefficient ζ, etc. The slosh damping coefficient was initially set to ζ =0.002, same as the slosh in the original vehicle model.



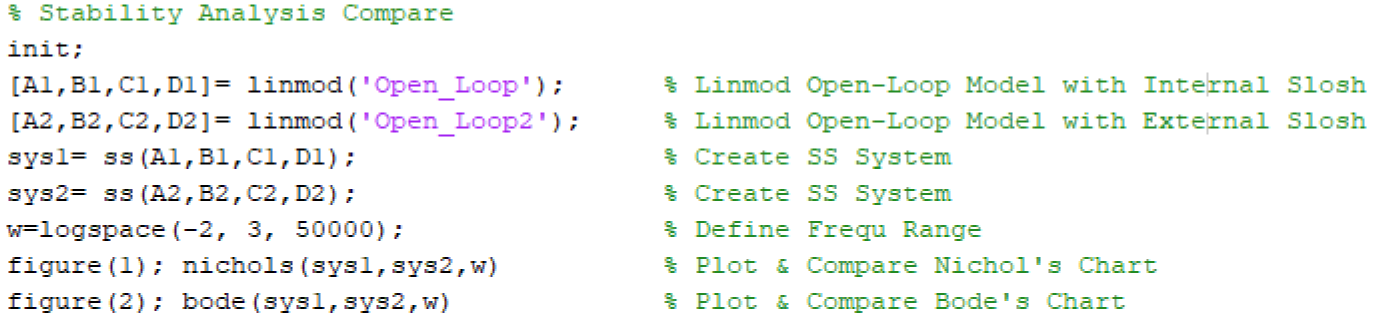
```
% Initialize Parameters
d2r= pi/180; r2d=180/pi; Ts=0.00002;
[Av, Bv, Cv, Dv]= vehicle;                      % Load Vehicle Analysis Model
[Av2,Bv2,Cv2,Dv2]= vehicle_pend;                % Load Vehicle/Pendulum Analysis Model
[Aa, Ba, Ca, Da]= actuator;                     % Load Linear Actuator Model
load Kmix_t350 -ascii;  Kmix=Kmix_t350;         % Load the Mixing-Logic Matrx
load Kq_t350   -ascii;  Kq=Kq_t350;             % Load the Pitch LQR Gains

nt=8; Thr=3.0;                                  % Number of Jets, Thrust (lbf)
x1_ini= [-1,0, -2,0, 2,0]*d2r;                  % Attitude/Rate Initialization
x2_ini=zeros(1,32); x2_ini=[x1_ini,x2_ini];     % Model-1 State Initialization
x3_ini=zeros(1,24); x3_ini=[x1_ini,x3_ini];     % Model-2 State Initialization
Xcg=91.75; Ixx=3561; Iyy=20921; Izz=20763;      % Phase-Plane Parameters

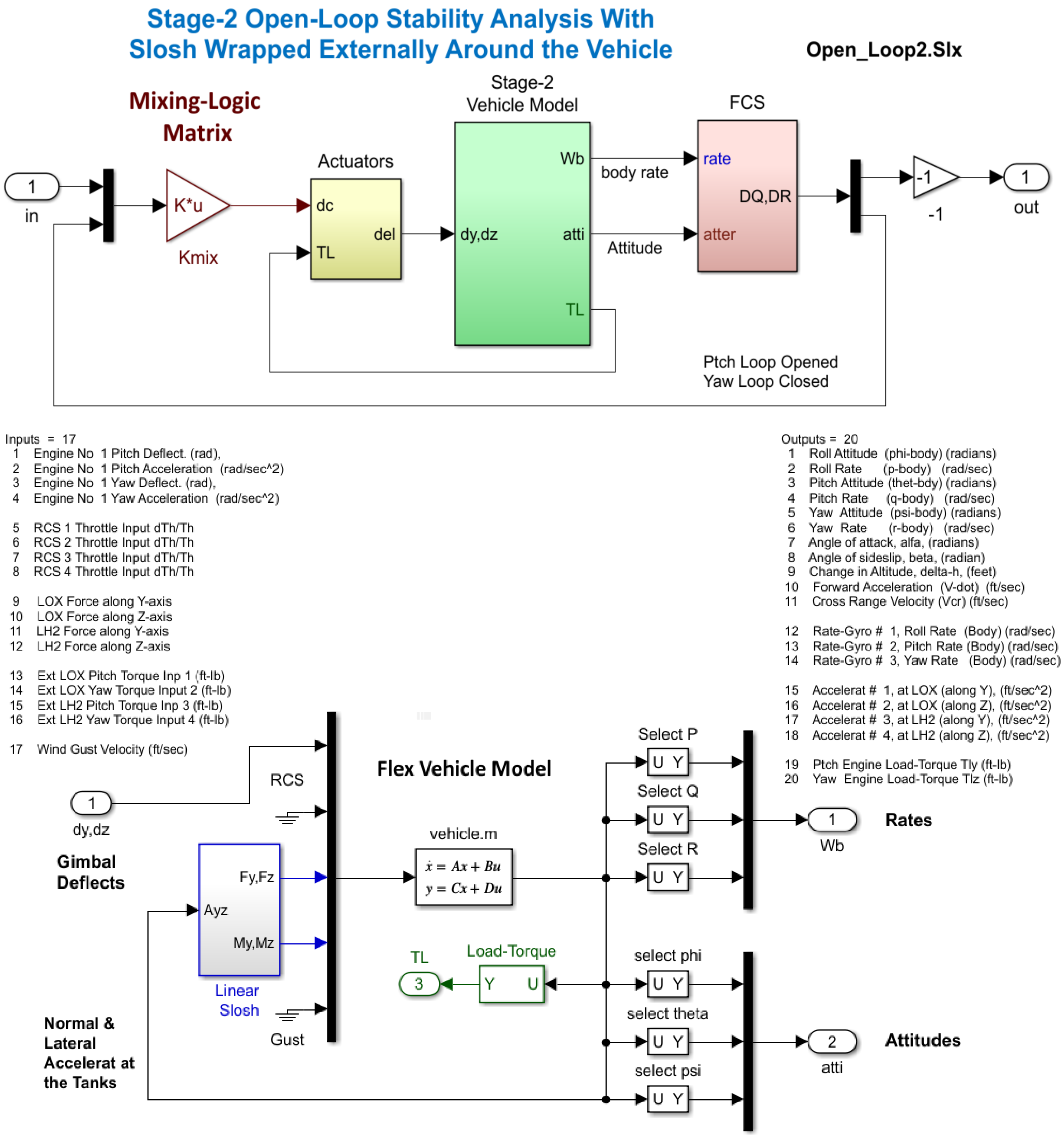
% Spherical Pendulum Slosh Parameters
Ax=54.8; Rsl=2.5;
Ws=sqrt(Ax/Rsl); Zeta=0.004;
M1ox=119.3; M1h2=41.12;
thet0=2.0*d2r;                      % Initial Pendul Angle from vertical +ve
phi0=90*d2r;                        % Initial Roll Angle Measured CounterClock
phid0=50.0*d2r;                     % Initial Roll Rate CounterClockwise (deg/sec)
R0=Rsl*sin(thet0);                  % radial Distance from Tank Centerline
Ys0=R0*cos(phi0);                   % Slosh Mass Initial Ys, Zs Position
Zs0=-R0*sin(phi0);                  % Relative to the Tank Centerline
```

Comparison of Linear Models, Internal versus External Sloshing

To validate the new vehicle model that includes the external spring-mass slosh model, we must compare its response with the previous model obtained in Section 3.2.3. We do it by calculating the open-loop frequency response of the new model and comparing the Nichols plots with those obtained from previous internal slosh implementation. The following frequency response script calculates and compares the Nichols plots from the 2 open-loop systems. The first one is obtained from the Simulink model "*Open_Loop.Slx*" which was created in Section 3.2.3 and the second model is "*Open_Loop2.Slx*", shown in Figure 5.4, and it includes the 2 external spring-mass models shown in detail in Figure 5.5.



```
% Stability Analysis Compare
init;
[A1,B1,C1,D1]= linmod('Open_Loop');      % Linmod Open-Loop Model with Internal Slosh
[A2,B2,C2,D2]= linmod('Open_Loop2');     % Linmod Open-Loop Model with External Slosh
sys1= ss(A1,B1,C1,D1);                   % Create SS System
sys2= ss(A2,B2,C2,D2);                   % Create SS System
w=logspace(-2, 3, 50000);                % Define Frequ Range
figure(1); nichols(sys1,sys2,w)          % Plot & Compare Nichol's Chart
figure(2); bode(sys1,sys2,w)             % Plot & Compare Bode's Chart
```



Inputs = 17

1	Engine No 1 Pitch Deflect. (rad),
2	Engine No 1 Pitch Acceleration (rad/sec^2)
3	Engine No 1 Yaw Deflect. (rad),
4	Engine No 1 Yaw Acceleration (rad/sec^2)
5	RCS 1 Throttle Input dTh/Th
6	RCS 2 Throttle Input dTh/Th
7	RCS 3 Throttle Input dTh/Th
8	RCS 4 Throttle Input dTh/Th
9	LOX Force along Y-axis
10	LOX Force along Z-axis
11	LH2 Force along Y-axis
12	LH2 Force along Z-axis
13	Ext LOX Pitch Torque Inp 1 (ft-lb)
14	Ext LOX Yaw Torque Input 2 (ft-lb)
15	Ext LH2 Pitch Torque Inp 3 (ft-lb)
16	Ext LH2 Yaw Torque Input 4 (ft-lb)
17	Wind Gust Velocity (ft/sec)

Outputs = 20

1	Roll Attitude (phi-body) (radians)
2	Roll Rate (p-body) (rad/sec)
3	Pitch Attitude (thet-bdy) (radians)
4	Pitch Rate (q-body) (rad/sec)
5	Yaw Attitude (psi-body) (radians)
6	Yaw Rate (r-body) (rad/sec)
7	Angle of attack, alfa, (radians)
8	Angle of sideslip, beta, (radian)
9	Change in Altitude, delta-h, (feet)
10	Forward Acceleration (V-dot) (ft/sec)
11	Cross Range Velocity (Vcr) (ft/sec)
12	Rate-Gyro # 1, Roll Rate (Body) (rad/sec)
13	Rate-Gyro # 2, Pitch Rate (Body) (rad/sec)
14	Rate-Gyro # 3, Yaw Rate (Body) (rad/sec)
15	Accelerat # 1, at LOX (along Y), (ft/sec^2)
16	Accelerat # 2, at LOX (along Z), (ft/sec^2)
17	Accelerat # 3, at LH2 (along Y), (ft/sec^2)
18	Accelerat # 4, at LH2 (along Z), (ft/sec^2)
19	Ptch Engine Load-Torque Tly (ft-lb)
20	Yaw Engine Load-Torque Tlz (ft-lb)

Figure 5.4 Open-Loop Model "Open_Loop2.Slx" for Calculating the Nichols Plot

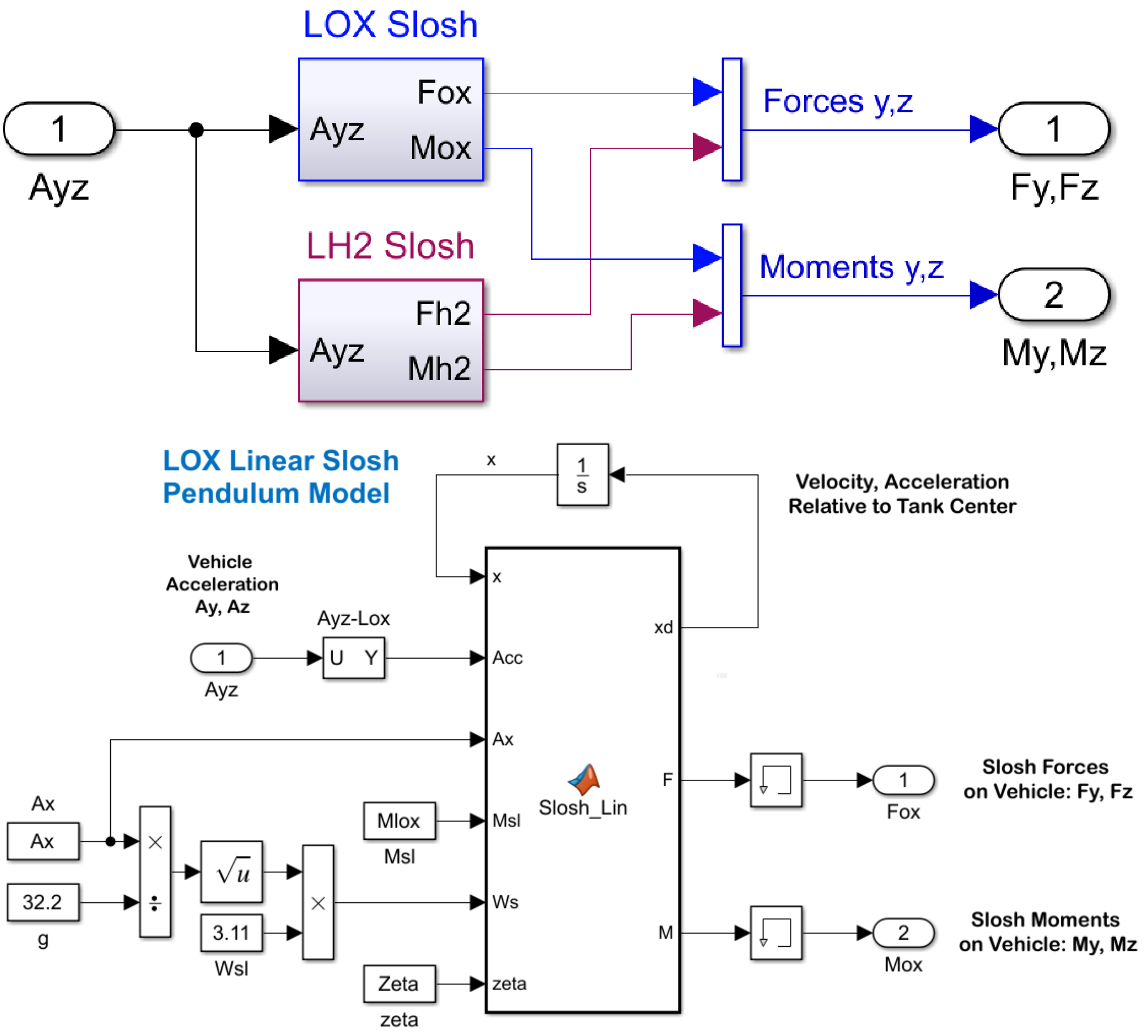


Figure 5.5 The 2 Spring-Mass Slosh Modes Externally Implemented in Model "Open_Loop2.Slx" Using Matlab Functions

Figure 5.6a compares the Nichols plots obtained from the two systems, the internal versus externally implemented spring-mass slosh dynamics, and the results are identical. It shows that with a ζ=0.002 in both tanks, the LOX propellant mode in the front of the vehicle is very unstable but the LH2 mode further back is phase-stable. Figure 5.6b shows that the instability is still significant even if we increase the damping to ζ=0.02 in both tanks. The closed-loop simulation model "*SpringMass_Sim.Slx*" in Figure 5.7 calculates the vehicle response to commands. Figure 5.8 shows that the divergence with the low damped slosh modes is unacceptable, which leads us to the need of a better simulation model in order to estimate an acceptable value for the damping coefficient. Having now validated the new vehicle model with the externally wrapped slosh, we can now remove the spring-mass systems and attach the two spherical pendulum models.

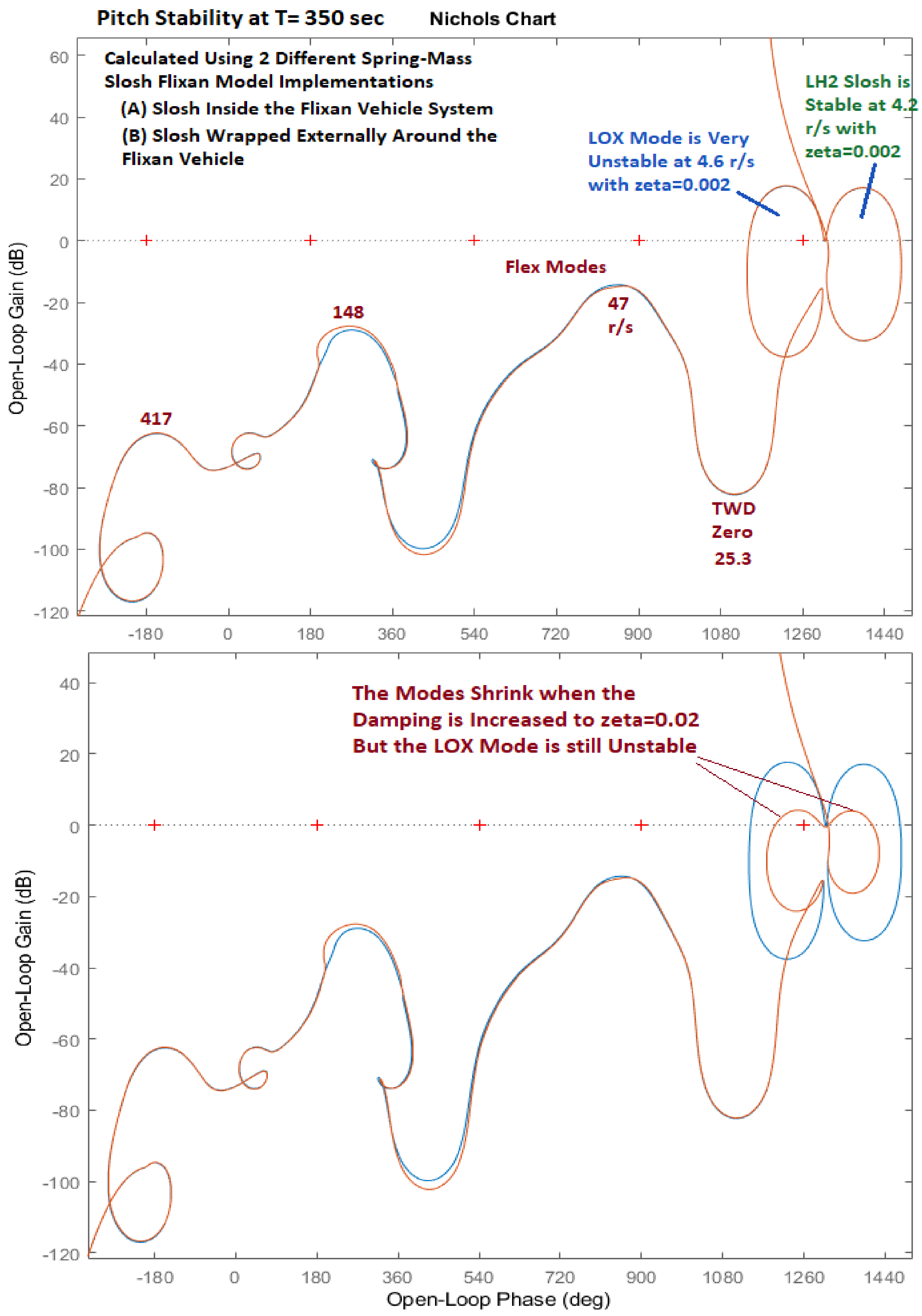
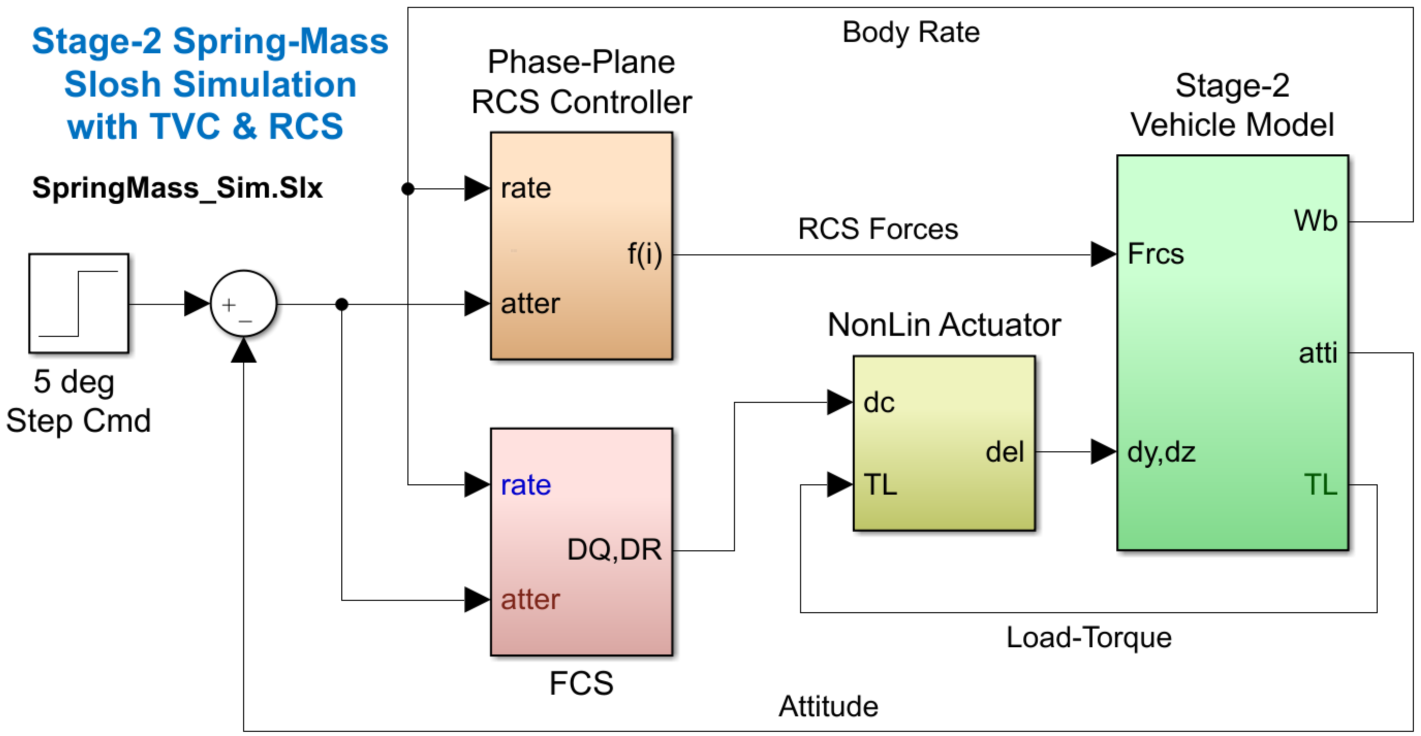


Figure 5.6 The LOX Slosh Instability is Strong even after Significantly Increasing the Damping Coefficient



Inputs = 9
1 Engine No 1 Pitch Deflect. (rad), Dymax= 7.0000 deg
2 Engine No 1 Pitch Acceleration (rad/sec^2)
3 Engine No 1 Yaw Deflect. (rad), Dzmax= 7.0000 deg
4 Engine No 1 Yaw Acceleration (rad/sec^2)

5 Throttle Input dTh/Th for Engine No 2 (-)
6 Throttle Input dTh/Th for Engine No 3 (-)
7 Throttle Input dTh/Th for Engine No 4 (-)
8 Throttle Input dTh/Th for Engine No 5 (-)

Outputs = 19
1 Roll Attitude (phi-body) (radians)
2 Roll Rate (p-body) (rad/sec)
3 Pitch Attitude (thet-bdy) (radians)
4 Pitch Rate (q-body) (rad/sec)
5 Yaw Attitude (psi-body) (radians)
6 Yaw Rate (r-body) (rad/sec)
7 Angle of attack, alfa, (radians)
8 Angle of sideslip, beta, (radian)
9 Change in Altitude, delta-h, (feet)
10 Forward Acceleration (V-dot) (ft/sec)
11 Cross Range Velocity (Vcr) (ft/sec)

12 Rate-Gyro # 1, Roll Rate (Body) (rad/sec)
13 Rate-Gyro # 2, Pitch Rate (Body) (rad/sec)
14 Rate-Gyro # 3, Yaw Rate (Body) (rad/sec)

15 CG Acceleration along X axis, (ft/sec^2)
16 CG Acceleration along Y axis, (ft/sec^2)
17 CG Acceleration along Z axis, (ft/sec^2)

18 Ptch Load-Torque Tly for Engine: 1,(ft-lb)
19 Yaw Load-Torque Tlz for Engine: 1,(ft-lb)

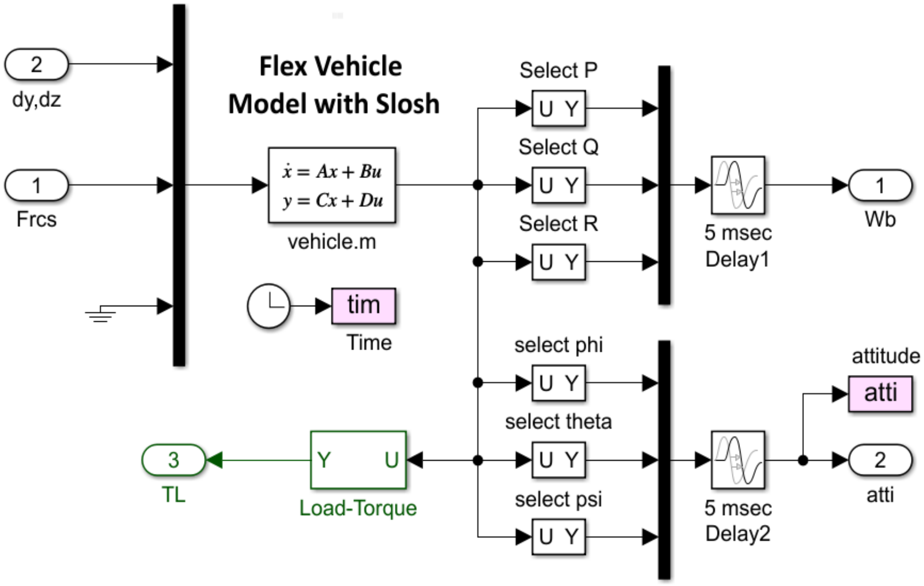


Figure 5.1 Simulation Model "SpringMass_Sim.slx" that Uses the vehicle System with Internal Spring-Mass Slosh Modes

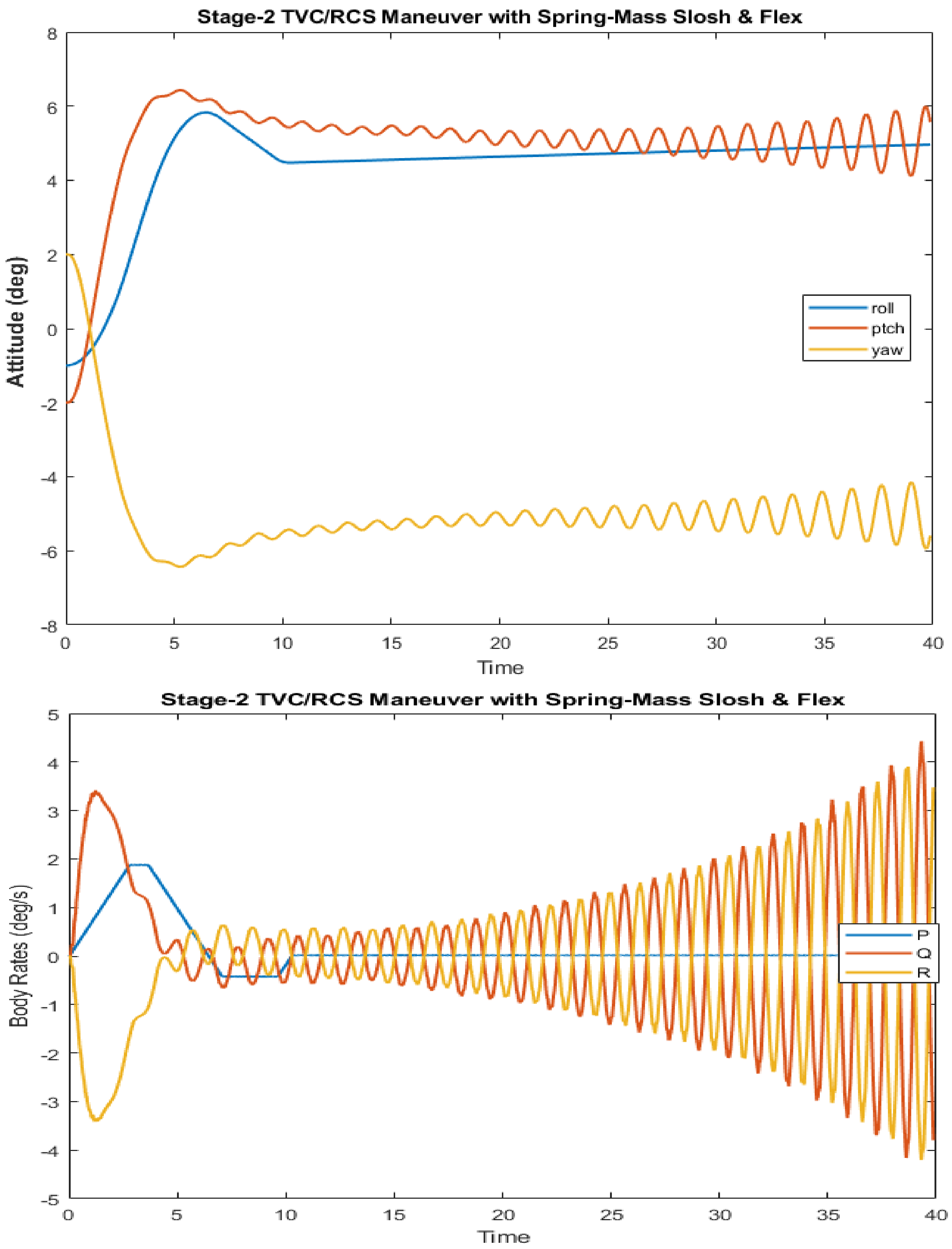
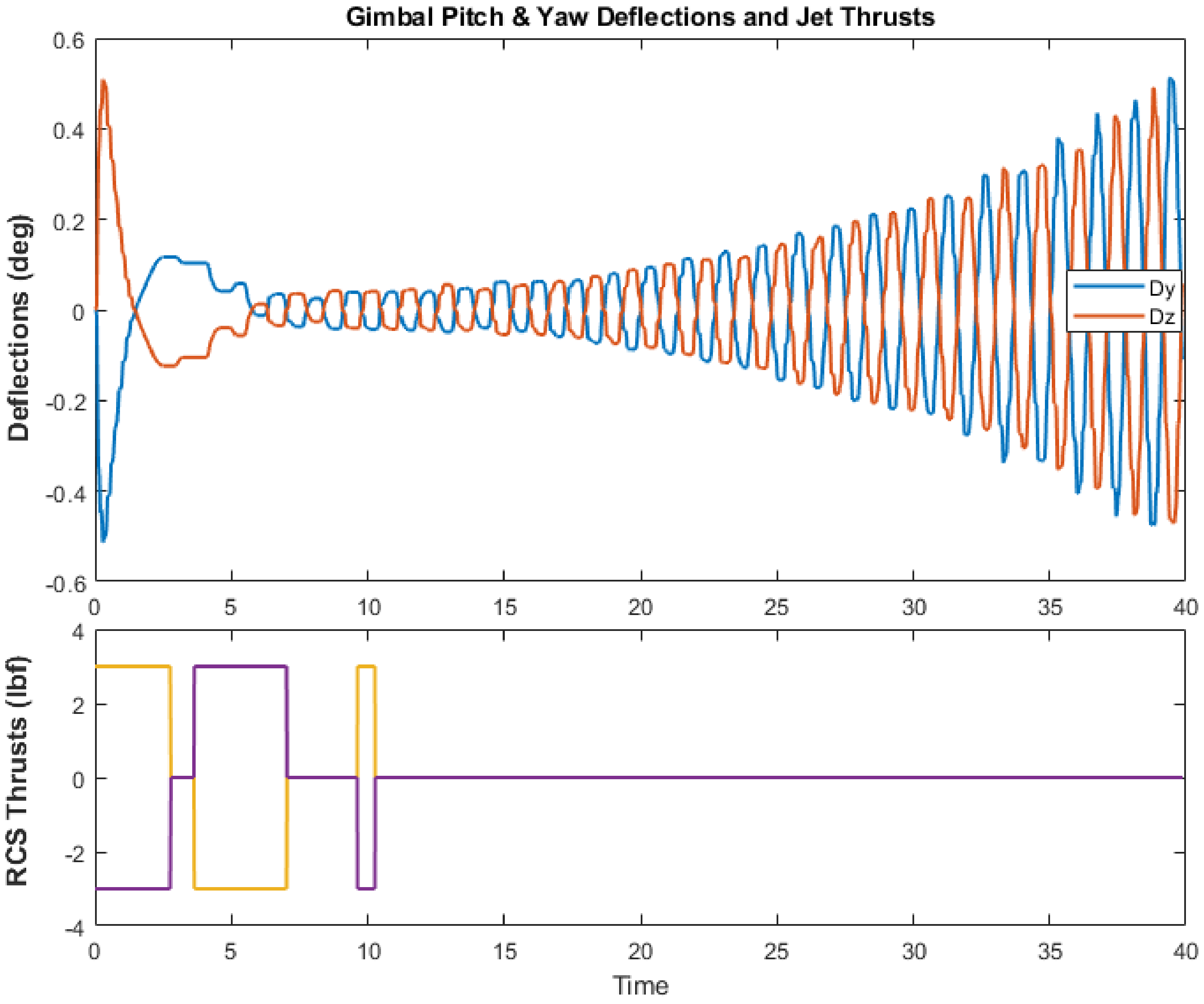
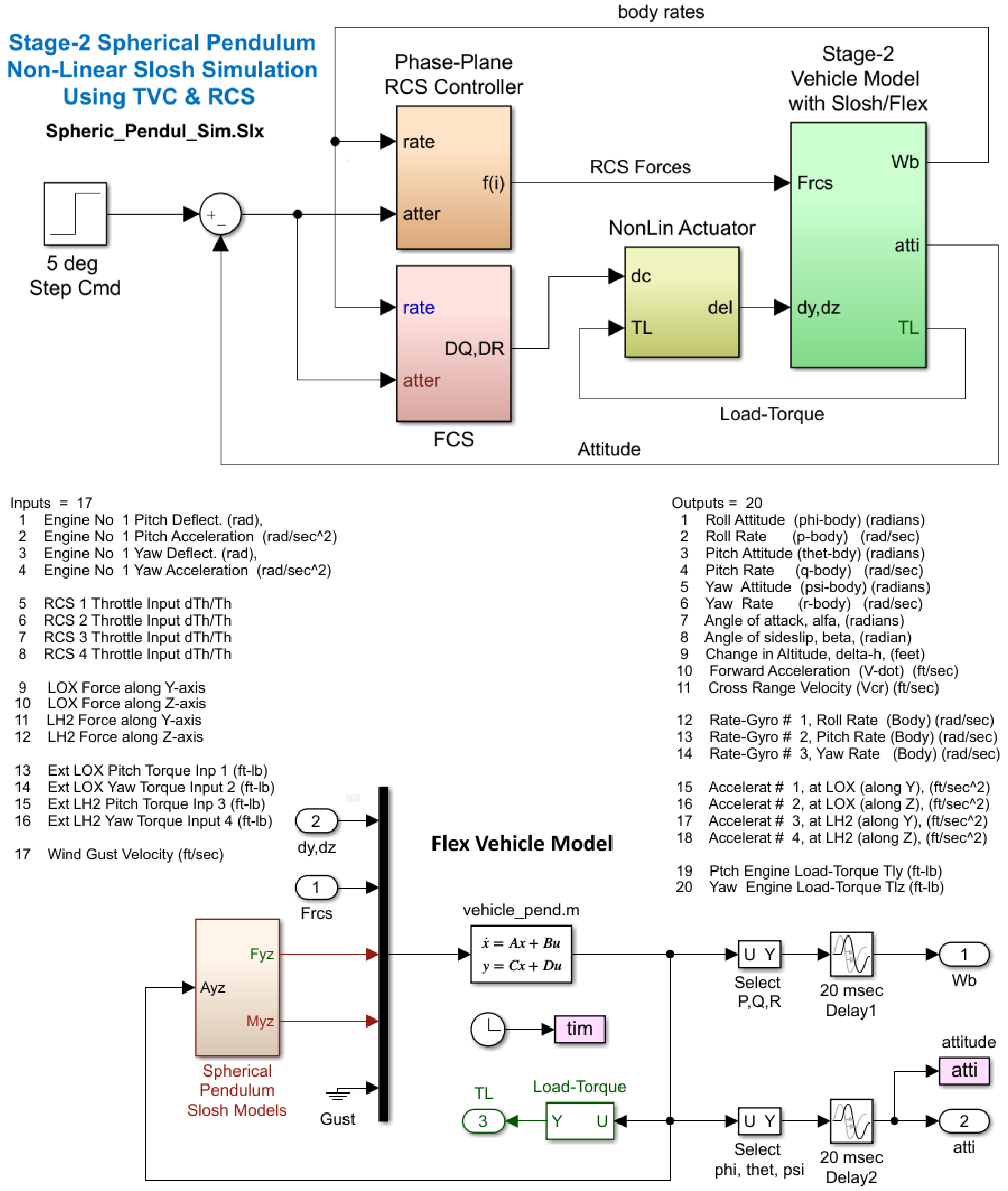


Figure 5.2 The Oscillatory Divergence due to Slosh Instability Using the Spring-Mass Model is Unacceptable



5.4 Non-Linear Slosh Simulation Using the Spherical Pendulum Model

The Simulink model "*Spheric_Pendul_Sim.Slx*" in Figure 5.9 includes the spherical pendulum slosh models. It is initialized from file "init.m" at selected pendulum angles (θ_0 and ϕ_0) and lateral angular spin rate $\dot{\phi}_0$. The response is strongly affected by the value of the damping coefficient ζ. The amplitude of the oscillations are also affected by the initial pendulum angle θ_0 and also the spin rate $\dot{\phi}_0$. The damping coefficient is adjusted in the simulations until we can obtain responses of acceptable limit-cycle amplitudes in the attitude, body rates and gimbal angles, which happens at a minimum ζ=0.01. This implies that some baffles are needed inside the tanks, especially in the LOX tank.



Inputs = 17
1 Engine No 1 Pitch Deflect. (rad),
2 Engine No 1 Pitch Acceleration (rad/sec^2)
3 Engine No 1 Yaw Deflect. (rad),
4 Engine No 1 Yaw Acceleration (rad/sec^2)

5 RCS 1 Throttle Input dTh/Th
6 RCS 2 Throttle Input dTh/Th
7 RCS 3 Throttle Input dTh/Th
8 RCS 4 Throttle Input dTh/Th

9 LOX Force along Y-axis
10 LOX Force along Z-axis
11 LH2 Force along Y-axis
12 LH2 Force along Z-axis

13 Ext LOX Pitch Torque Inp 1 (ft-lb)
14 Ext LOX Yaw Torque Input 2 (ft-lb)
15 Ext LH2 Pitch Torque Inp 3 (ft-lb)
16 Ext LH2 Yaw Torque Input 4 (ft-lb)

17 Wind Gust Velocity (ft/sec)

Outputs = 20
1 Roll Attitude (phi-body) (radians)
2 Roll Rate (p-body) (rad/sec)
3 Pitch Attitude (thet-bdy) (radians)
4 Pitch Rate (q-body) (rad/sec)
5 Yaw Attitude (psi-body) (radians)
6 Yaw Rate (r-body) (rad/sec)
7 Angle of attack, alfa, (radians)
8 Angle of sideslip, beta, (radian)
9 Change in Altitude, delta-h, (feet)
10 Forward Acceleration (V-dot) (ft/sec)
11 Cross Range Velocity (Vcr) (ft/sec)

12 Rate-Gyro # 1, Roll Rate (Body) (rad/sec)
13 Rate-Gyro # 2, Pitch Rate (Body) (rad/sec)
14 Rate-Gyro # 3, Yaw Rate (Body) (rad/sec)

15 Accelerat # 1, at LOX (along Y), (ft/sec^2)
16 Accelerat # 2, at LOX (along Z), (ft/sec^2)
17 Accelerat # 3, at LH2 (along Y), (ft/sec^2)
18 Accelerat # 4, at LH2 (along Z), (ft/sec^2)

19 Ptch Engine Load-Torque Tly (ft-lb)
20 Yaw Engine Load-Torque Tlz (ft-lb)

Figure 5.9 Spherical Pendulum Simulation Model "Spheric_Pendul_Sim.Slx" Showing the Vehicle Subsystem with the Pendulum Modes wrapped around it

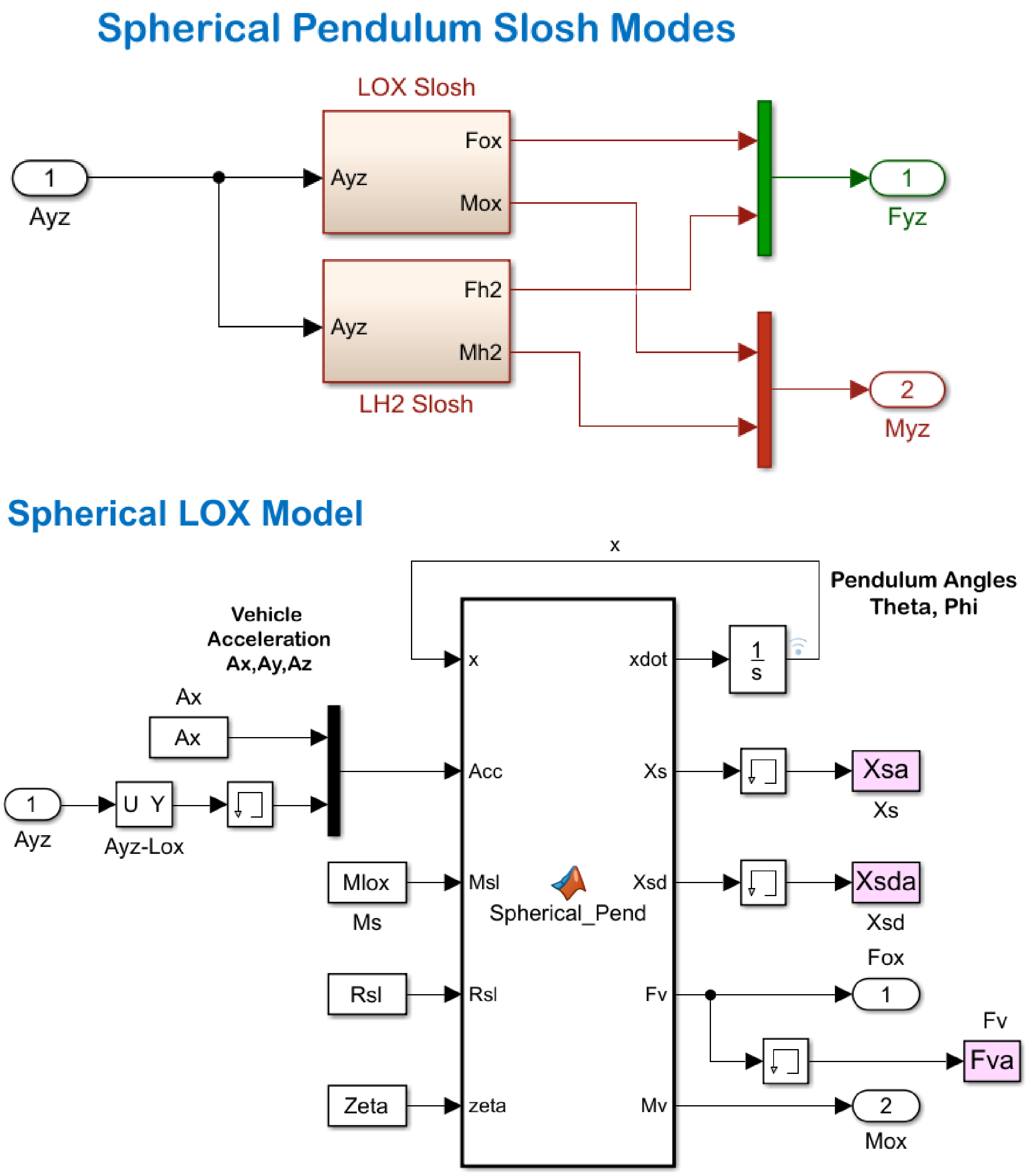
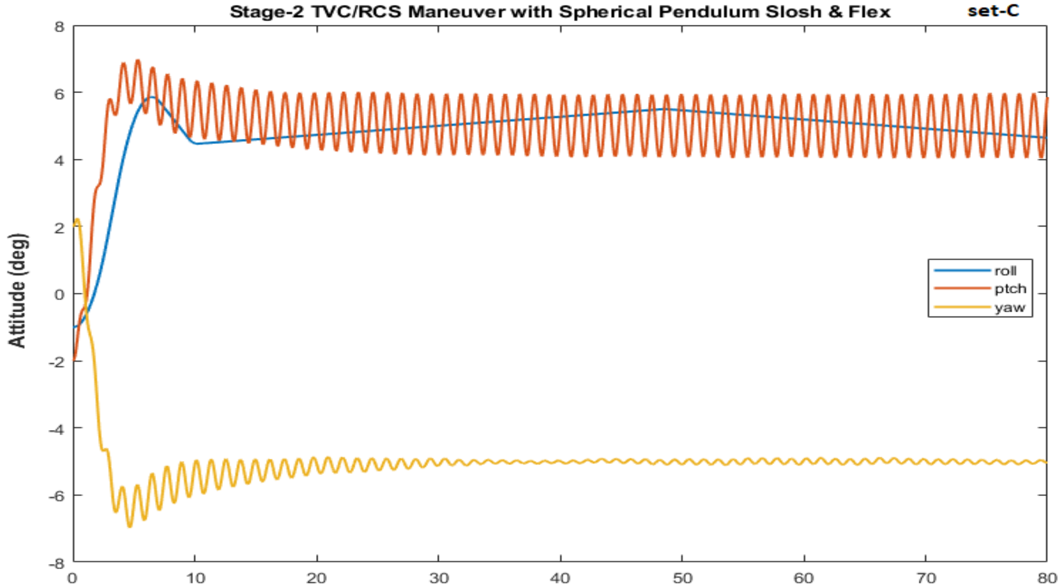


Figure 5.3 Spherical Pendulum LOX and LH2 Mode Subsystems Implemented in Matlab Functions

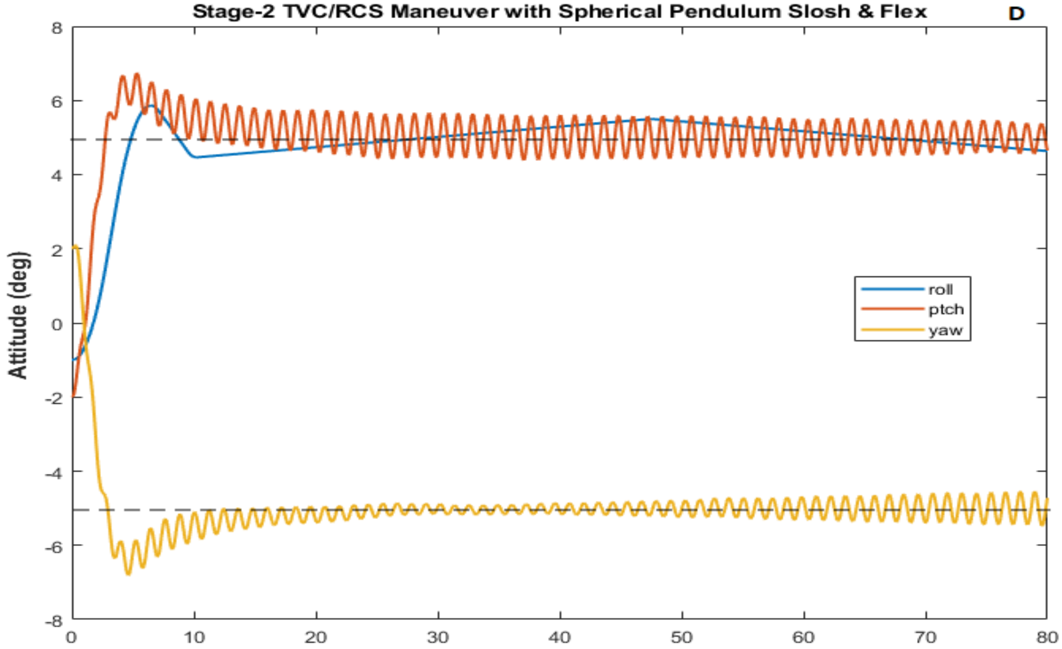
Simulation Results

Three sets of results are shown in Figure 5.11 (C, D, E) using a damping coefficient ζ=0.01. They are initialized from different pendulum angles and rates. The last one begins with a significant amount of slosh mass spin rate $\dot{\phi}_0$=200 (deg/sec). The results from the non-linear slosh model are almost acceptable even though linear analysis suggests strong LOX mode instability at ζ=0.01.

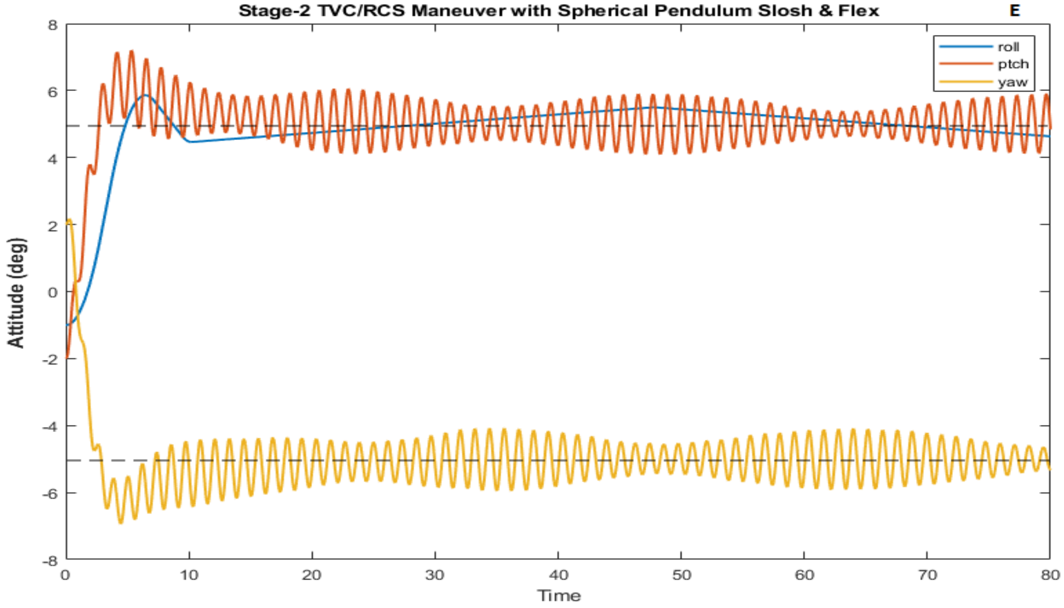
Stage-2 TVC/RCS Maneuver with Spherical Pendulum Slosh & Flex set-C

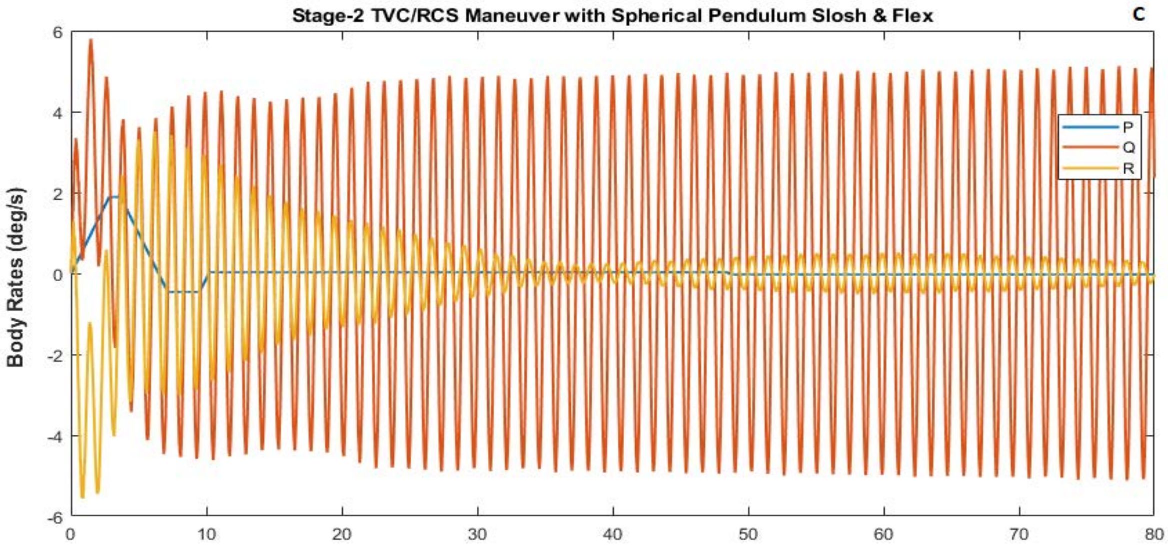


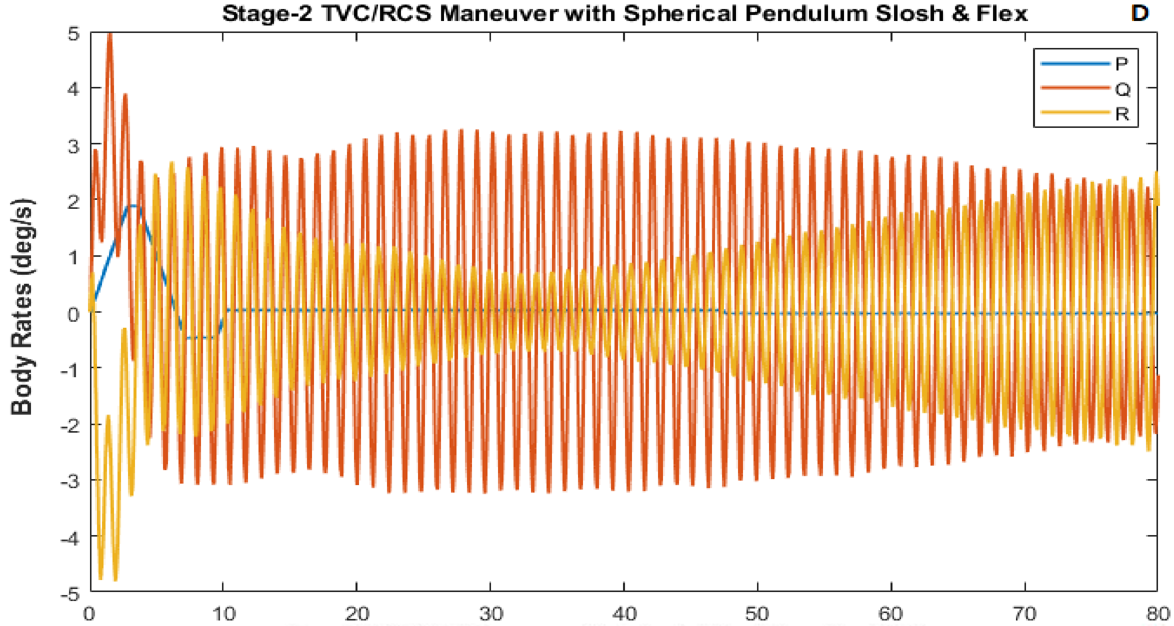
Stage-2 TVC/RCS Maneuver with Spherical Pendulum Slosh & Flex D

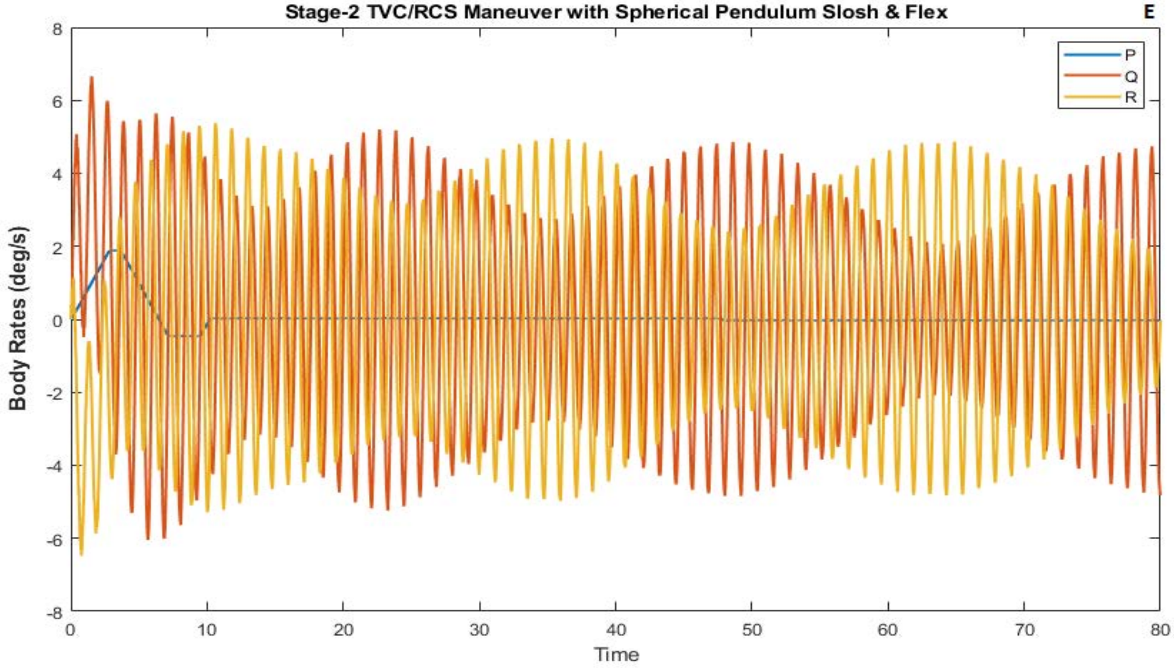


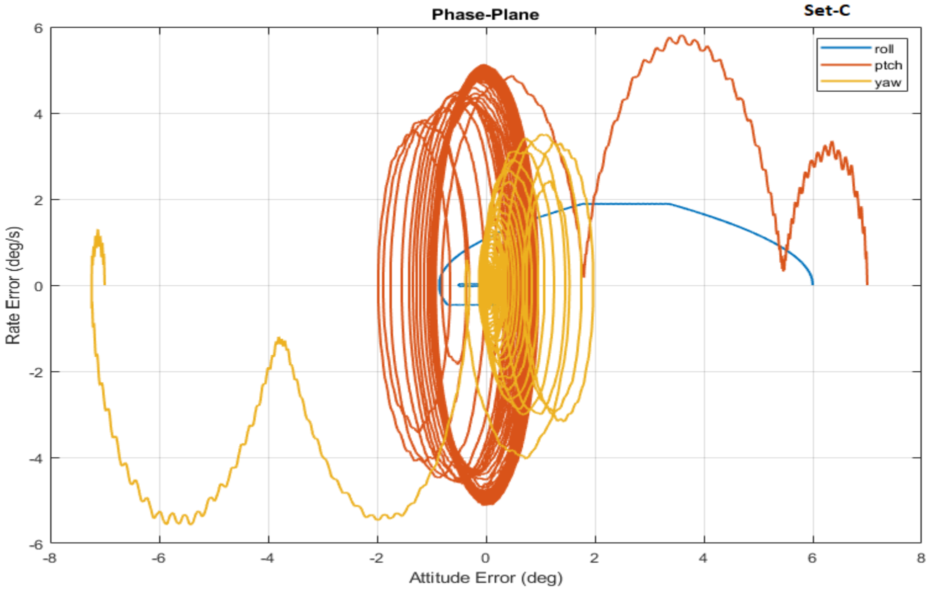
Stage-2 TVC/RCS Maneuver with Spherical Pendulum Slosh & Flex E

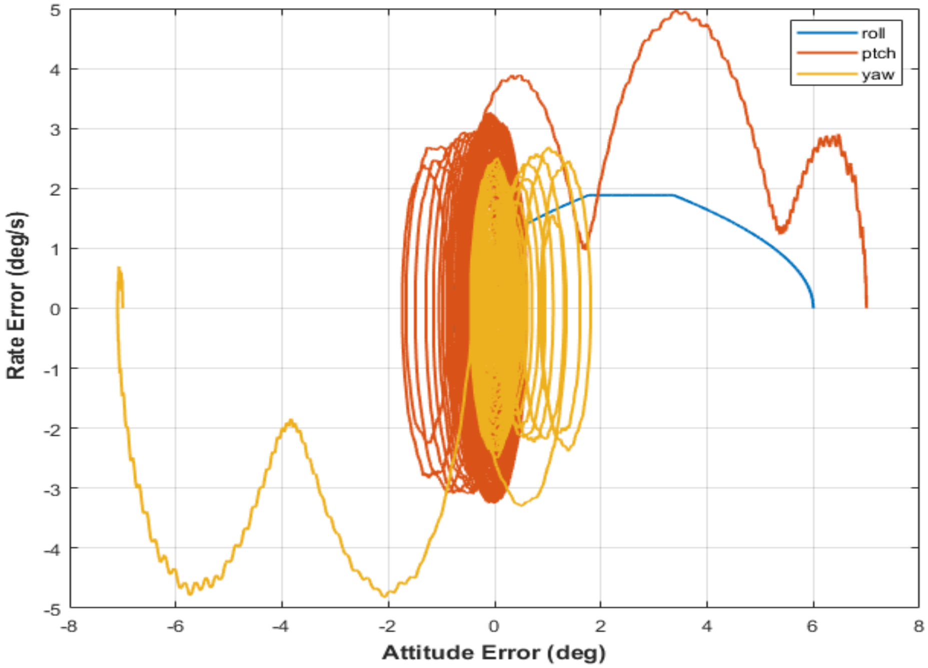


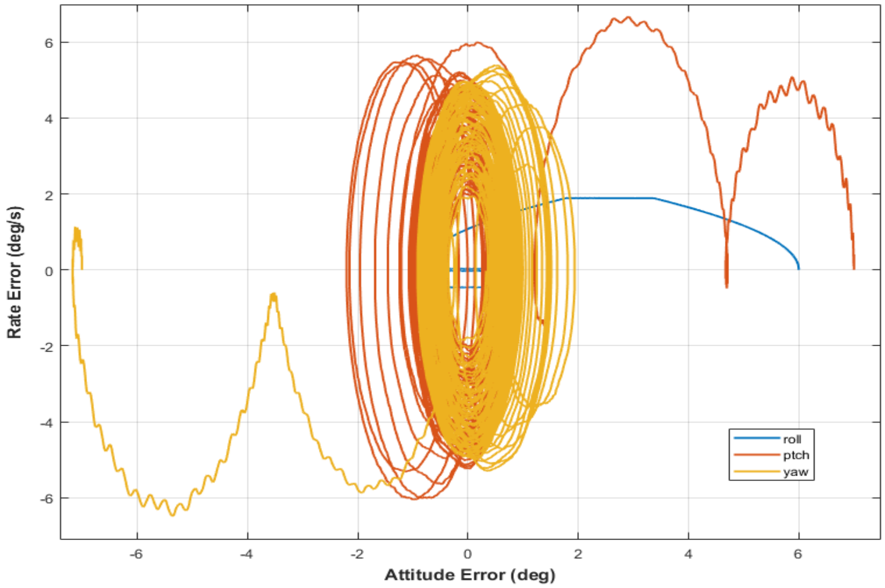

Stage-2 TVC/RCS Maneuver with Spherical Pendulum Slosh & Flex
C
Body Rates (deg/s)
6
4
2
0
-2
-4
-6
0
10
20
30
40
50
60
70
80
P
Q
R

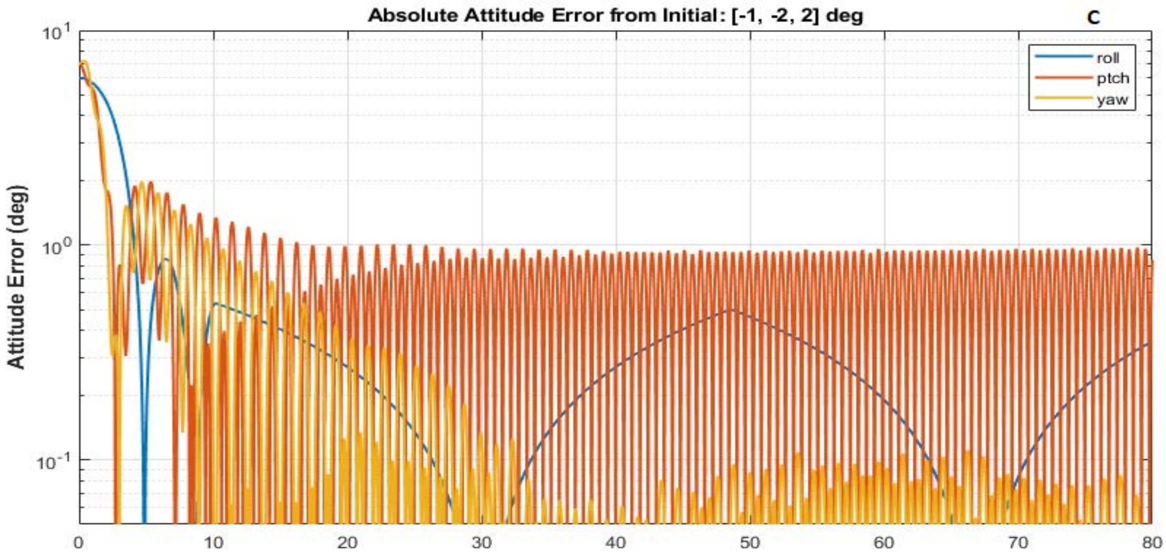

Stage-2 TVC/RCS Maneuver with Spherical Pendulum Slosh & Flex
D
Body Rates (deg/s)
5
4
3
2
1
0
-1
-2
-3
-4
-5
0
10
20
30
40
50
60
70
80
P
Q
R

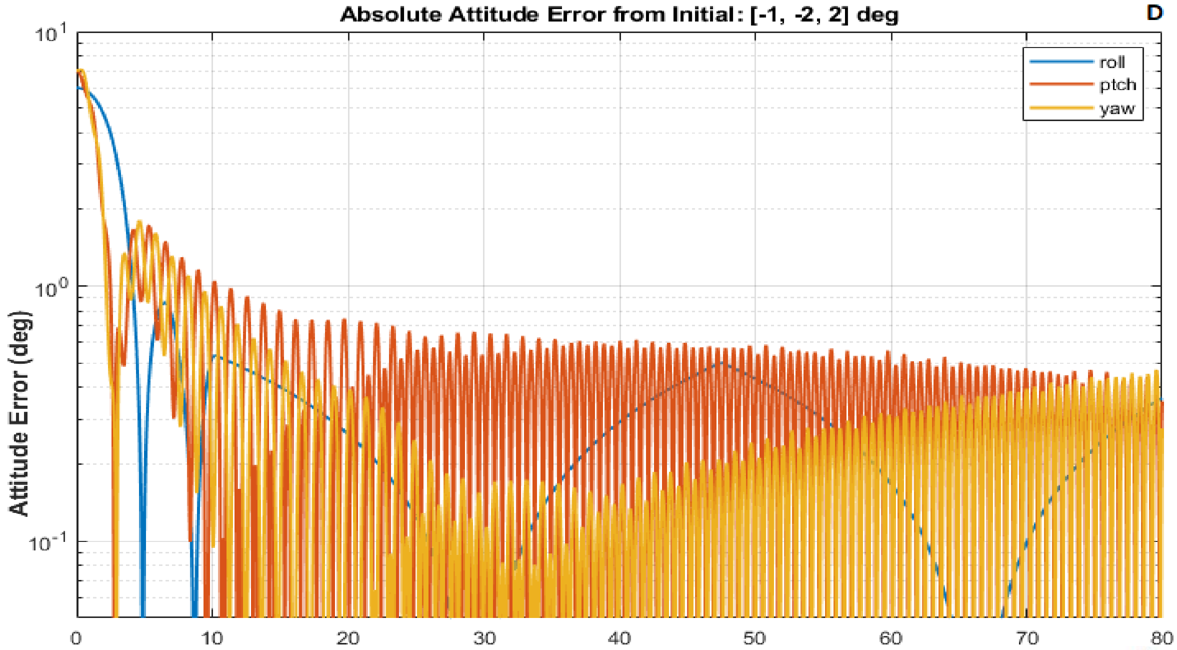

Stage-2 TVC/RCS Maneuver with Spherical Pendulum Slosh & Flex
E
Body Rates (deg/s)
8
6
4
2
0
-2
-4
-6
-8
0
10
20
30
40
50
60
70
80
Time
P
Q
R

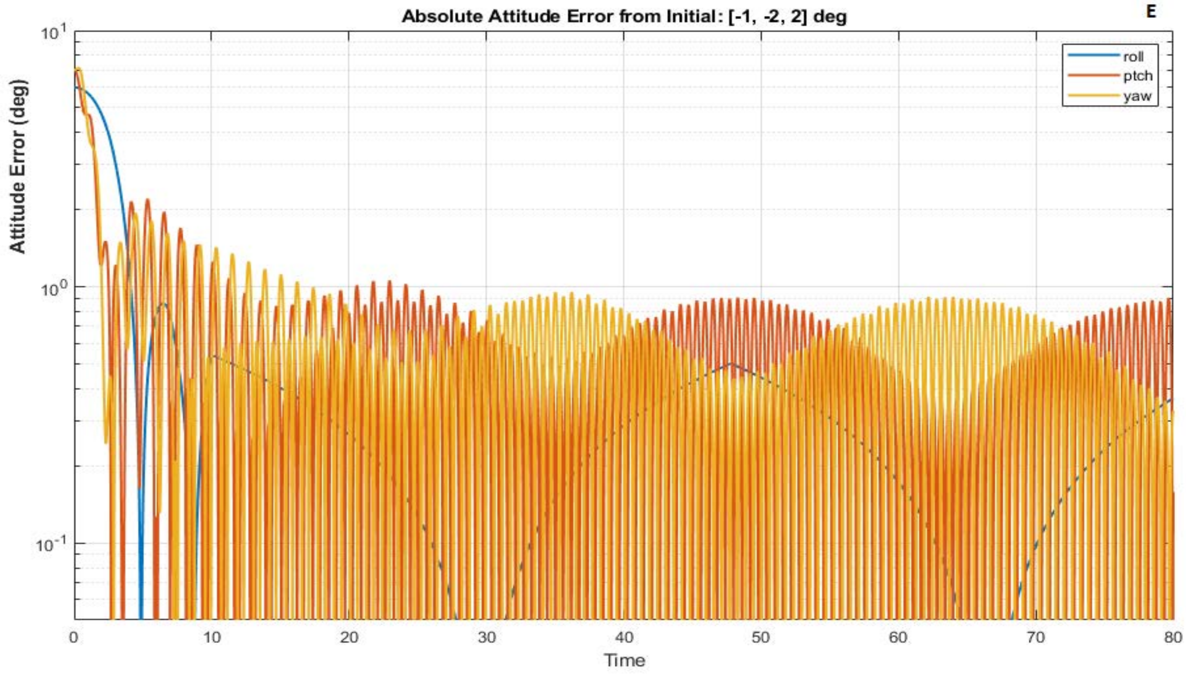

Phase-Plane
Set-C
roll
ptch
yaw
Rate Error (deg/s)
Attitude Error (deg)
6
4
2
0
-2
-4
-6
-8
-6
-4
-2
0
2
4
6
8

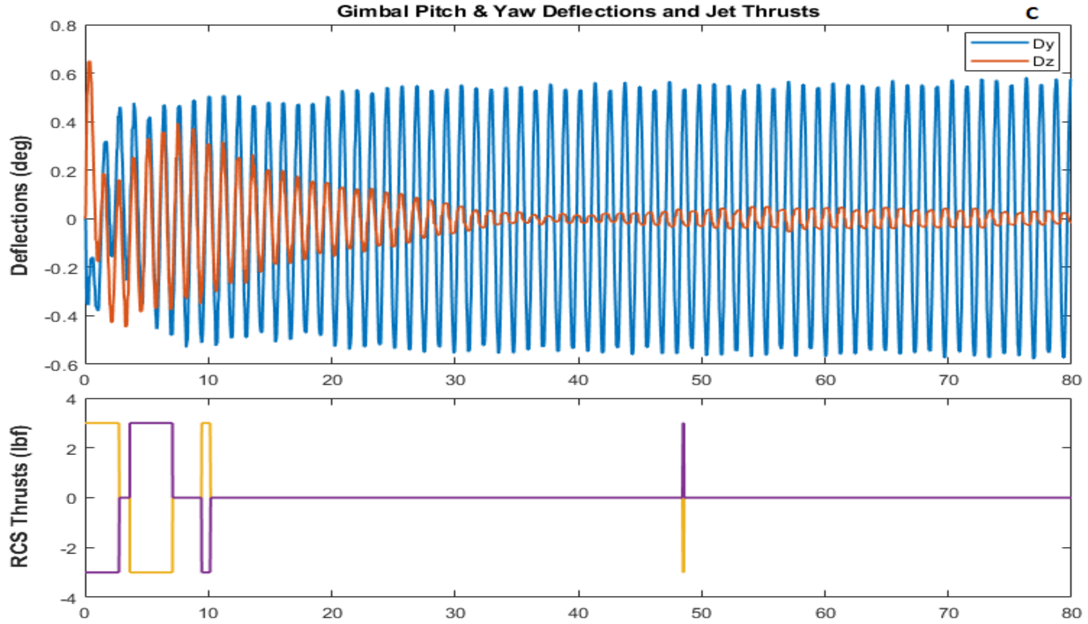

roll
ptch
yaw
Rate Error (deg/s)
Attitude Error (deg)
5
4
3
2
1
0
-1
-2
-3
-4
-5
-8
-6
-4
-2
0
2
4
6
8

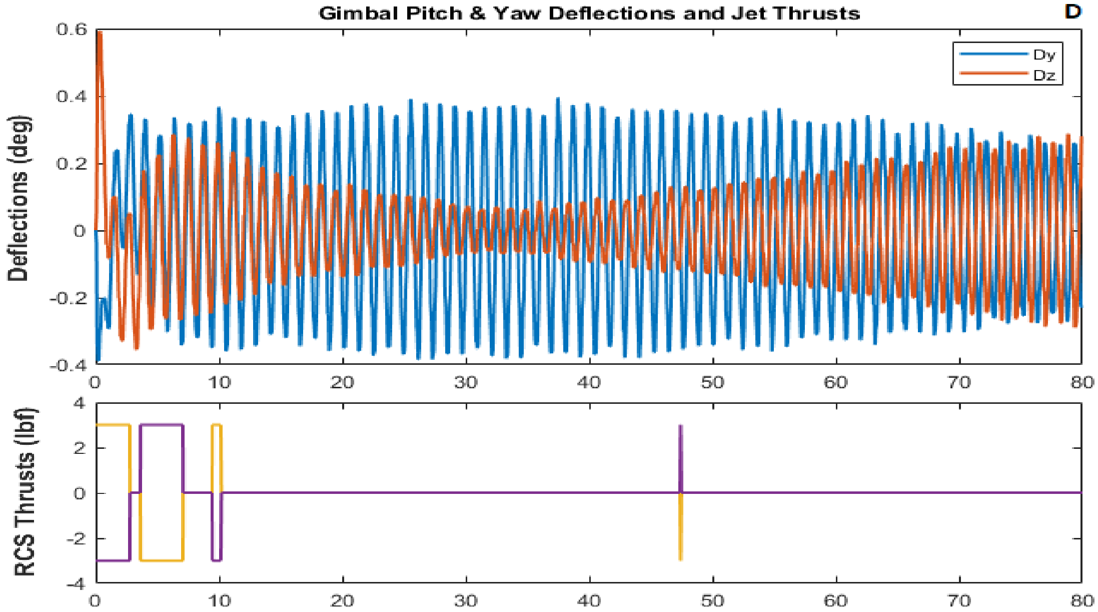

roll
ptch
yaw
Rate Error (deg/s)
Attitude Error (deg)
6
4
2
0
-2
-4
-6
-6
-4
-2
0
2
4
6

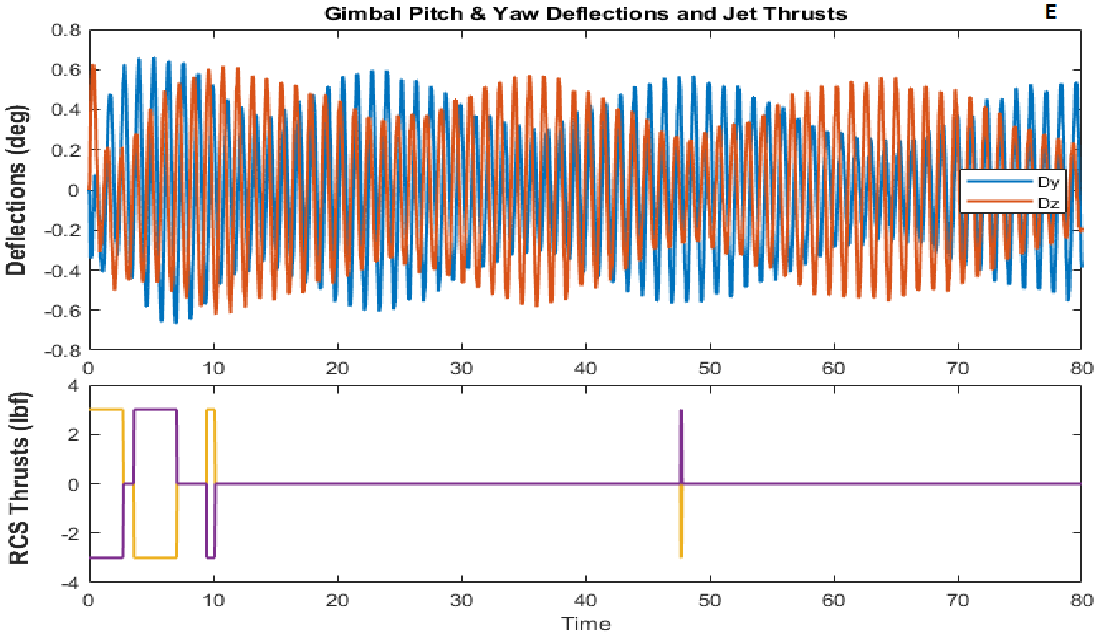

Absolute Attitude Error from Initial: [-1, -2, 2] deg
C
Attitude Error (deg)
10^1
10^0
10^{-1}
0
10
20
30
40
50
60
70
80
roll
ptch
yaw

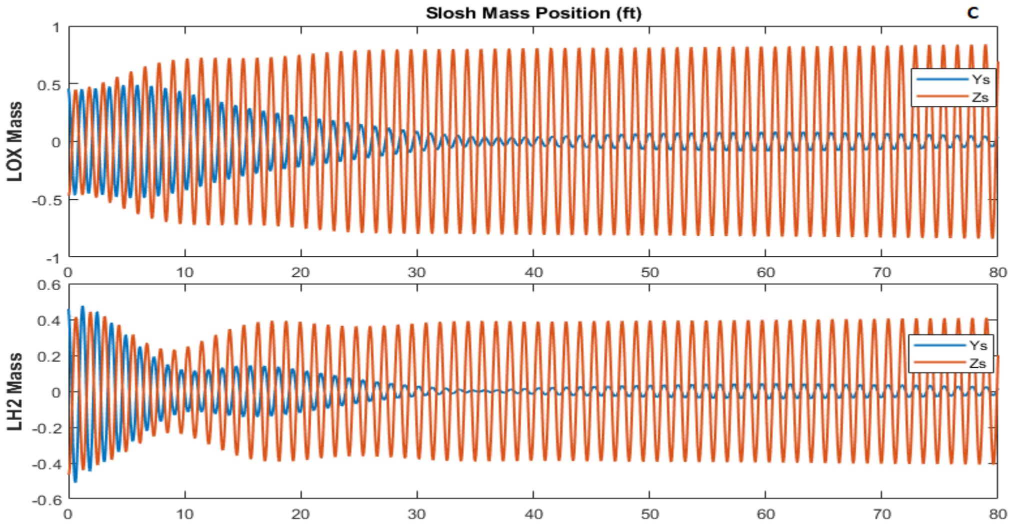

Absolute Attitude Error from Initial: [-1, -2, 2] deg
D
Attitude Error (deg)
10^1
10^0
10^{-1}
0
10
20
30
40
50
60
70
80
roll
ptch
yaw

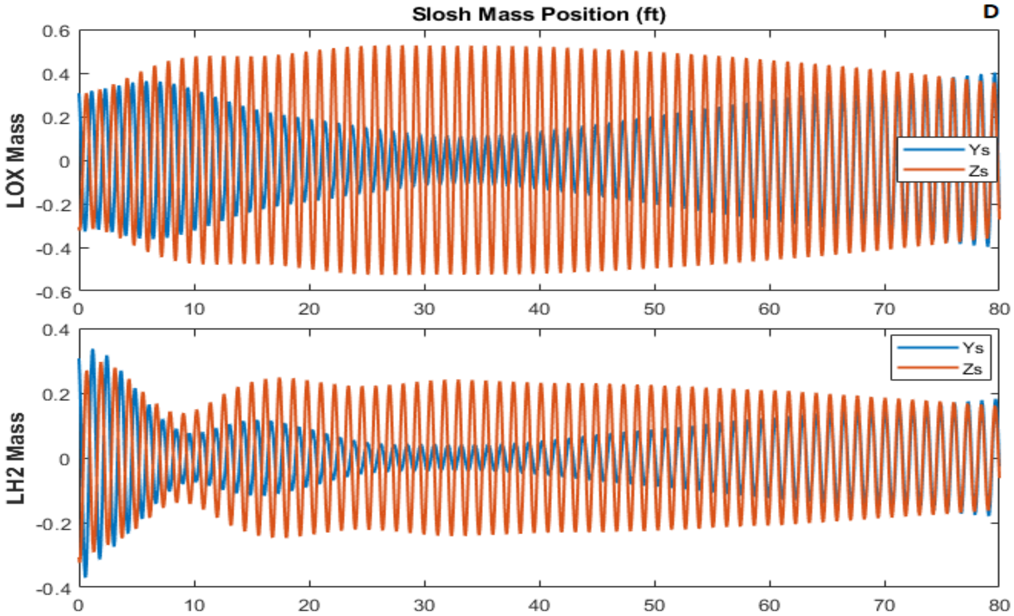

Absolute Attitude Error from Initial: [-1, -2, 2] deg
E
Attitude Error (deg)
10^1
10^0
10^{-1}
0
10
20
30
40
50
60
70
80
Time
roll
ptch
yaw

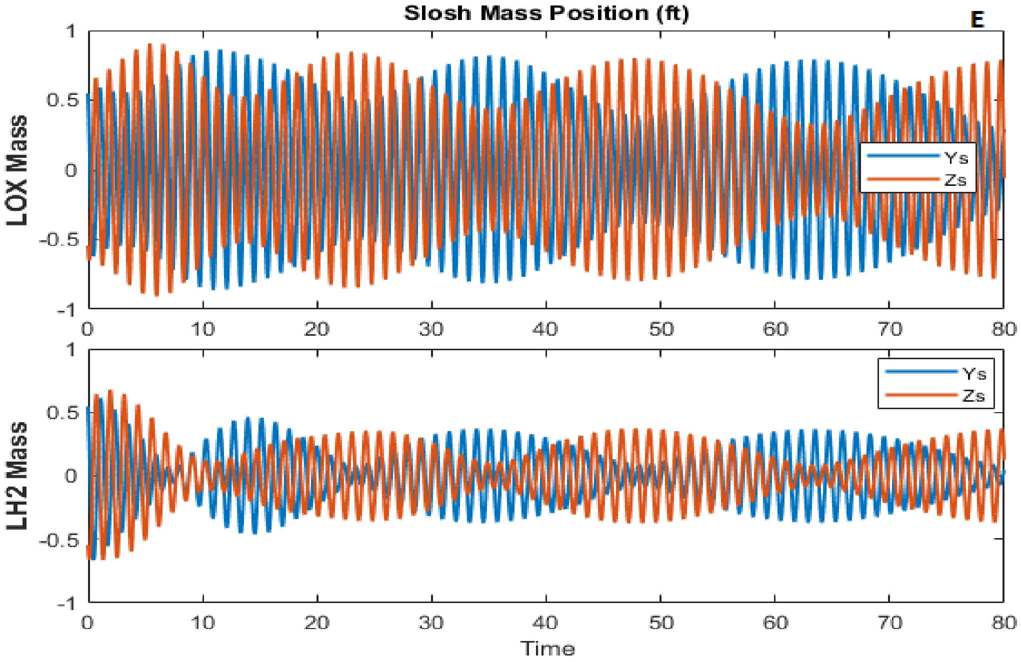

Gimbal Pitch & Yaw Deflections and Jet Thrusts
C
Dy
Dz
Deflections (deg)
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0
10
20
30
40
50
60
70
80
RCS Thrusts (lbf)
4
2
0
-2
-4
0
10
20
30
40
50
60
70
80

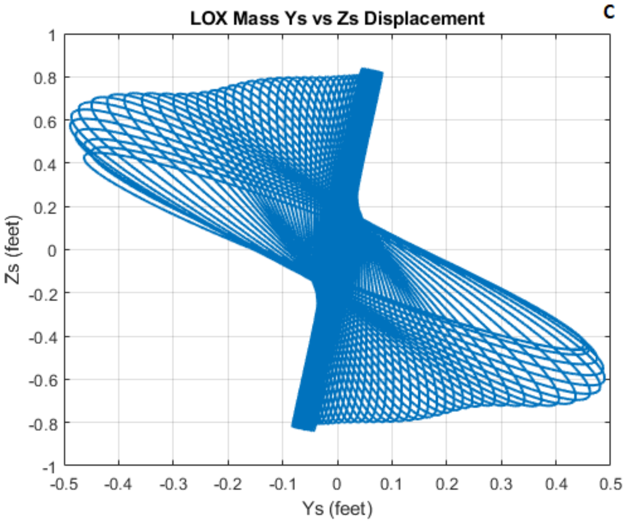

Gimbal Pitch & Yaw Deflections and Jet Thrusts
D
Dy
Dz
Deflections (deg)
0.6
0.4
0.2
0
-0.2
-0.4
0
10
20
30
40
50
60
70
80
RCS Thrusts (lbf)
4
2
0
-2
-4
0
10
20
30
40
50
60
70
80

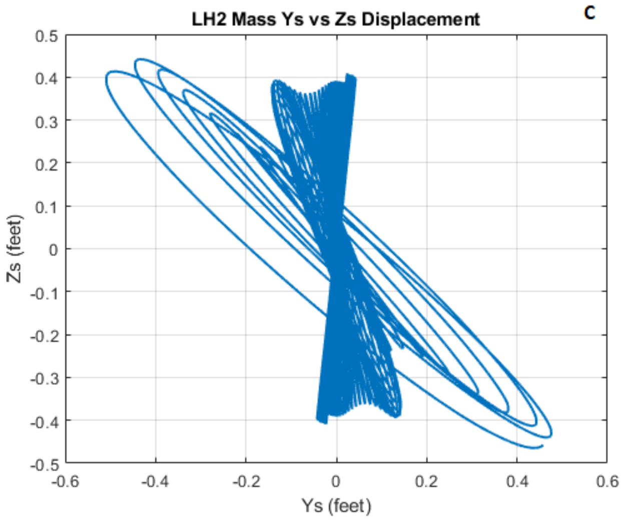

Gimbal Pitch & Yaw Deflections and Jet Thrusts
E
Dy
Dz
Deflections (deg)
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
10
20
30
40
50
60
70
80
RCS Thrusts (lbf)
4
2
0
-2
-4
0
10
20
30
40
50
60
70
80
Time

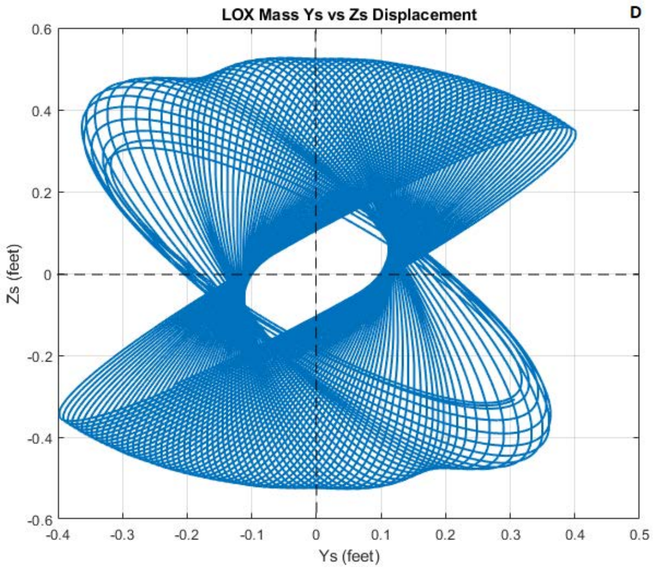

Slosh Mass Position (ft)
C
LOX Mass
LH2 Mass
Ys
Zs
1
0.5
0
-0.5
-1
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0
10
20
30
40
50
60
70
80

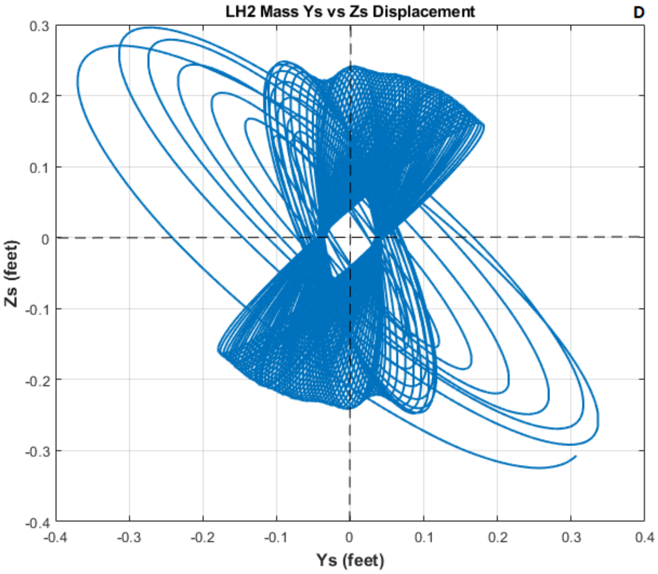

Slosh Mass Position (ft)
D
LOX Mass
LH2 Mass
Ys
Zs
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0.4
0.2
0
-0.2
-0.4
0
10
20
30
40
50
60
70
80

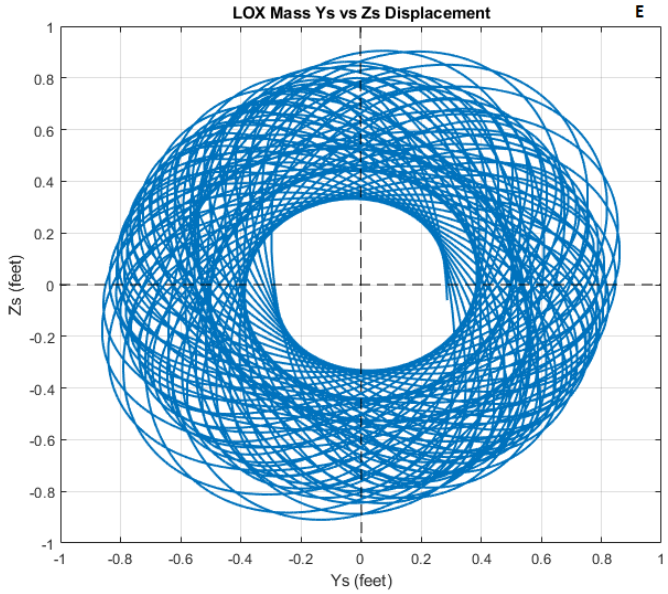

Slosh Mass Position (ft)
E
LOX Mass
LH2 Mass
Ys
Zs
1
0.5
0
-0.5
-1
0
10
20
30
40
50
60
70
80
Time

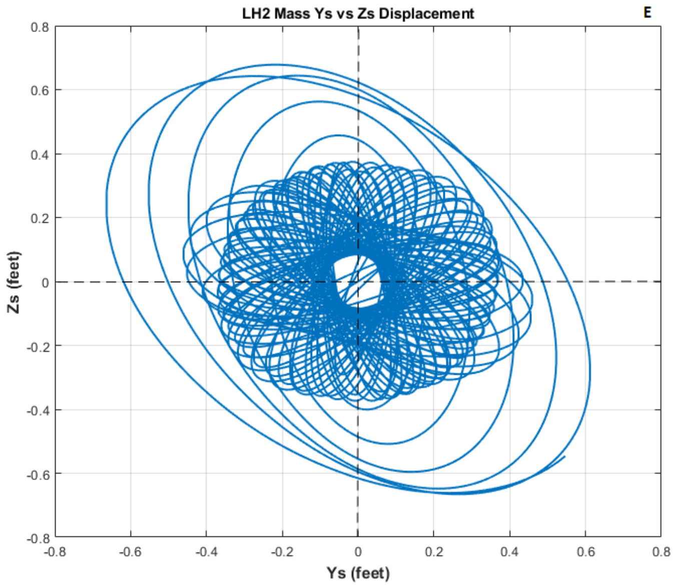

LOX Mass Ys vs Zs Displacement
C
Zs (feet)
Ys (feet)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5


LH2 Mass Ys vs Zs Displacement
C
Zs (feet)
Ys (feet)
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.4
-0.2
0
0.2
0.4
0.6


LOX Mass Ys vs Zs Displacement
D
Zs (feet)
Ys (feet)
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5


LH2 Mass Ys vs Zs Displacement
D
Zs (feet)
Ys (feet)
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4

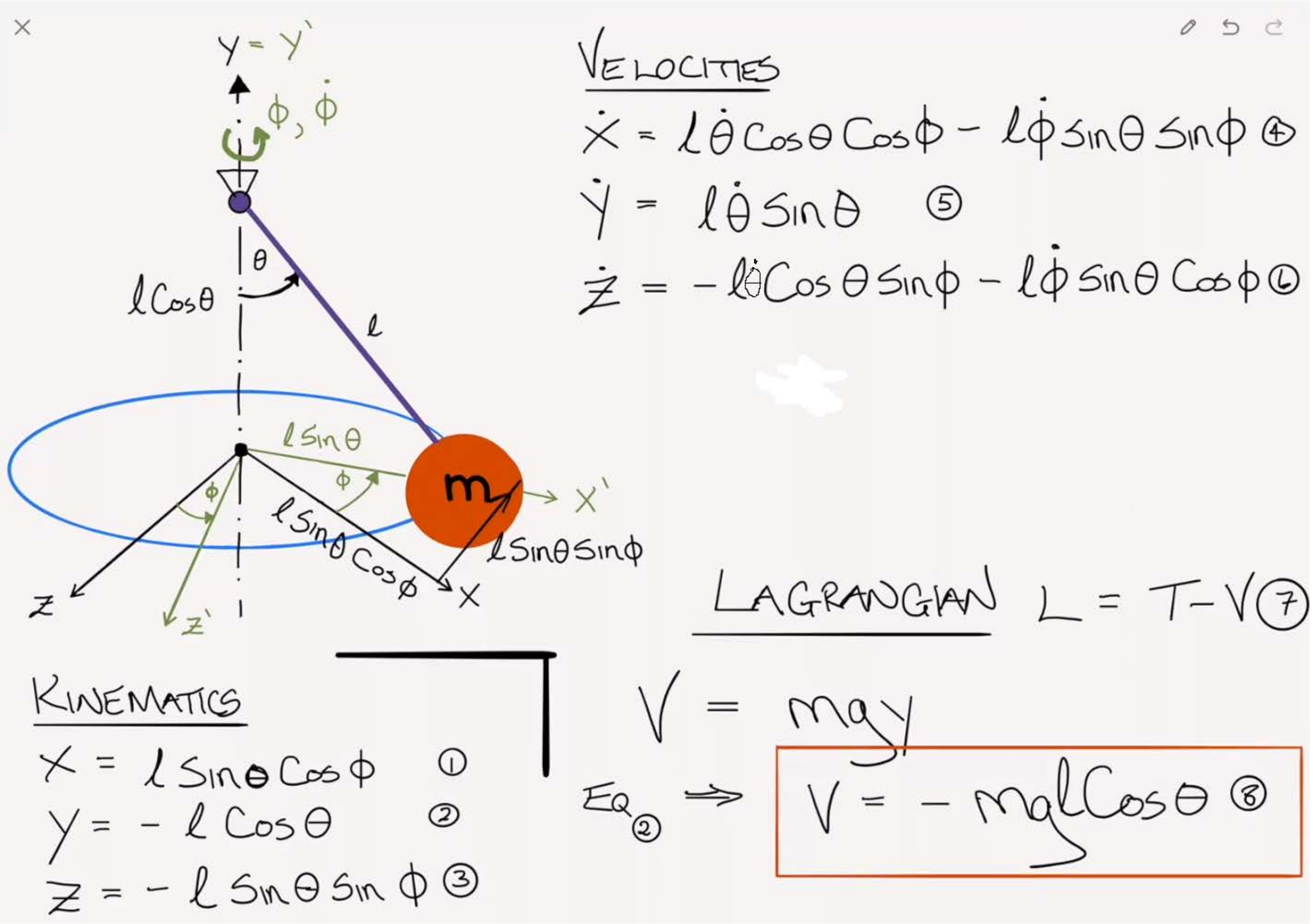

LOX Mass Ys vs Zs Displacement
E
Zs (feet)
Ys (feet)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1


LH2 Mass Ys vs Zs Displacement
E
Zs (feet)
Ys (feet)
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8

5.5 Derivation of Spherical Pendulum Equations

Downloaded from:

[(784) Equations of Motion for the Spherical Pendulum (2DOF) Using Lagrange's Equations - YouTube]



VELOCITIES

$$\dot{x} = l\dot{\theta}\cos\theta\cos\phi - l\dot{\phi}\sin\theta\sin\phi \quad ④$$

$$\dot{y} = l\dot{\theta}\sin\theta \quad ⑤$$

$$\dot{z} = -l\dot{\theta}\cos\theta\sin\phi - l\dot{\phi}\sin\theta\cos\phi \quad ⑥$$

KINEMATICS

$$x = l\sin\theta\cos\phi \quad ①$$

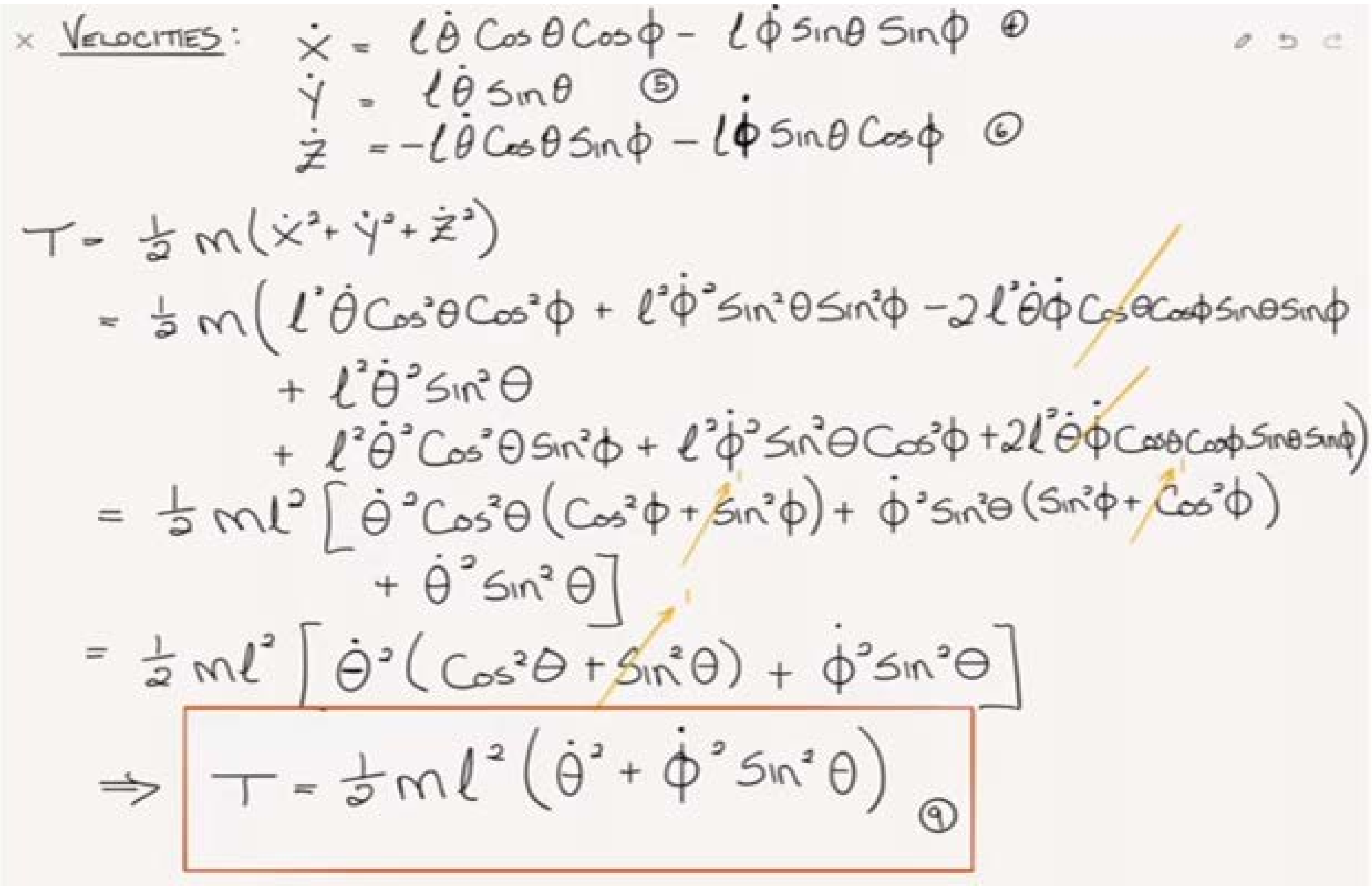
$$y = -l\cos\theta \quad ②$$

$$z = -l\sin\theta\sin\phi \quad ③$$

LAGRANGIAN $L = T - V$ ⑦

$$V = mgy$$

$$\text{EQ}_{②} \Rightarrow \boxed{V = -mgl\cos\theta \quad ⑧}$$



× VELOCITIES: $\dot{x} = l\dot{\theta}\cos\theta\cos\phi - l\dot{\phi}\sin\theta\sin\phi$ ④

$\dot{y} = l\dot{\theta}\sin\theta$ ⑤

$\dot{z} = -l\dot{\theta}\cos\theta\sin\phi - l\dot{\phi}\sin\theta\cos\phi$ ⑥

$$T = \frac{1}{2}m(\dot{x}^2+\dot{y}^2+\dot{z}^2)$$

$$= \frac{1}{2}m\Big(l^2\dot{\theta}\cos^2\theta\cos^2\phi + l^2\dot{\phi}^2\sin^2\theta\sin^2\phi \cancel{-2l^2\dot{\theta}\dot{\phi}\cos\theta\cos\phi\sin\theta\sin\phi}$$

$$+ l^2\dot{\theta}^2\sin^2\theta$$

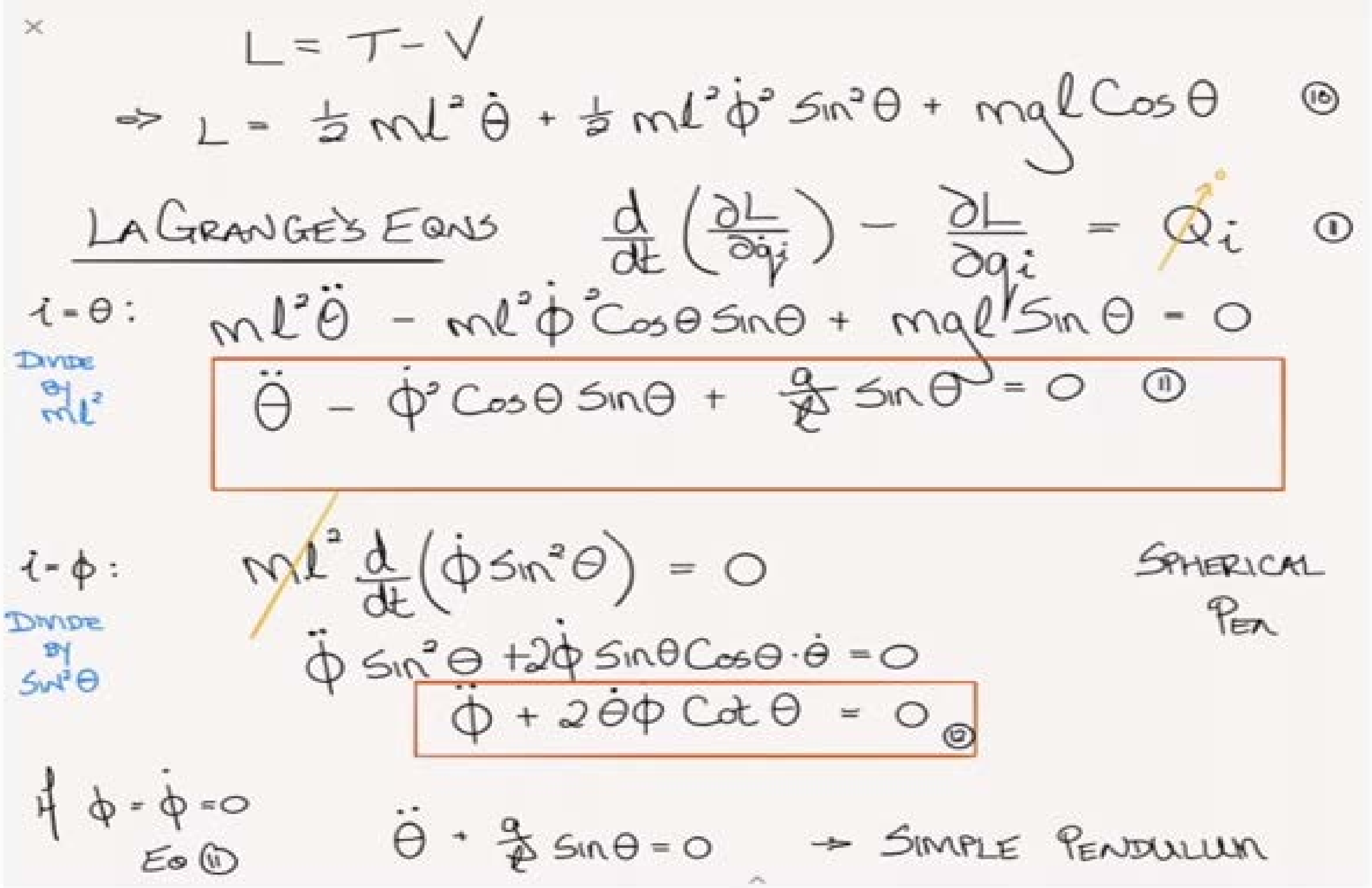
$$+ l^2\dot{\theta}^2\cos^2\theta\sin^2\phi + l^2\dot{\phi}^2\sin^2\theta\cos^2\phi \cancel{+2l^2\dot{\theta}\dot{\phi}\cos\theta\cos\phi\sin\theta\sin\phi}\Big)$$

$$= \frac{1}{2}ml^2\Big[\dot{\theta}^2\cos^2\theta\,\cancelto{1}{(\cos^2\phi+\sin^2\phi)} + \dot{\phi}^2\sin^2\theta\,\cancelto{1}{(\sin^2\phi+\cos^2\phi)}$$

$$+ \dot{\theta}^2\sin^2\theta\Big]$$

$$= \frac{1}{2}ml^2\Big[\dot{\theta}^2\,\cancelto{1}{(\cos^2\theta+\sin^2\theta)} + \dot{\phi}^2\sin^2\theta\Big]$$

$$\Rightarrow \boxed{T = \frac{1}{2}ml^2\left(\dot{\theta}^2+\dot{\phi}^2\sin^2\theta\right) \quad ⑨}$$



×

$$L = T - V$$

$$\Rightarrow L = \frac{1}{2}ml^2\dot{\theta} + \frac{1}{2}ml^2\dot{\phi}^2\sin^2\theta + mgl\cos\theta \quad ⑩$$

LAGRANGE'S EQNS $\quad \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{q}_i}\right) - \dfrac{\partial L}{\partial q_i} = \cancel{Q_i}$ ①

$i = \theta$: $\quad ml^2\ddot{\theta} - ml^2\dot{\phi}^2\cos\theta\sin\theta + mgl\sin\theta = 0$

DIVIDE BY ml^2

$$\boxed{\ddot{\theta} - \dot{\phi}^2\cos\theta\sin\theta + \frac{g}{l}\sin\theta = 0 \quad ⑪}$$

$i = \phi$: $\quad \cancel{ml^2}\dfrac{d}{dt}\left(\dot{\phi}\sin^2\theta\right) = 0$

SPHERICAL PEN

DIVIDE BY $\sin^2\theta$

$$\ddot{\phi}\sin^2\theta + 2\dot{\phi}\sin\theta\cos\theta\cdot\dot{\theta} = 0$$

$$\boxed{\ddot{\phi} + 2\dot{\theta}\dot{\phi}\cot\theta = 0 \quad ⑫}$$

If $\phi = \dot{\phi} = 0$ Eq ⑪ $\quad \ddot{\theta} + \frac{g}{l}\sin\theta = 0$ → SIMPLE PENDULUM

6.1 Simulation Overview

The simulation model is in file "LV_6DOF_Slosh.slx" and shown in Figure 6.1. It includes the 6DOF vehicle dynamics block which is implemented by a Simulink function "Variable Mass 6DoF ECEF (Quaternion)", the environmental model that produces the gravity forces, a block that produces: α, β, Mach number, and dynamic pressure, the block that generates the forces and moments on the vehicle, and the flight control system that also includes the guidance.

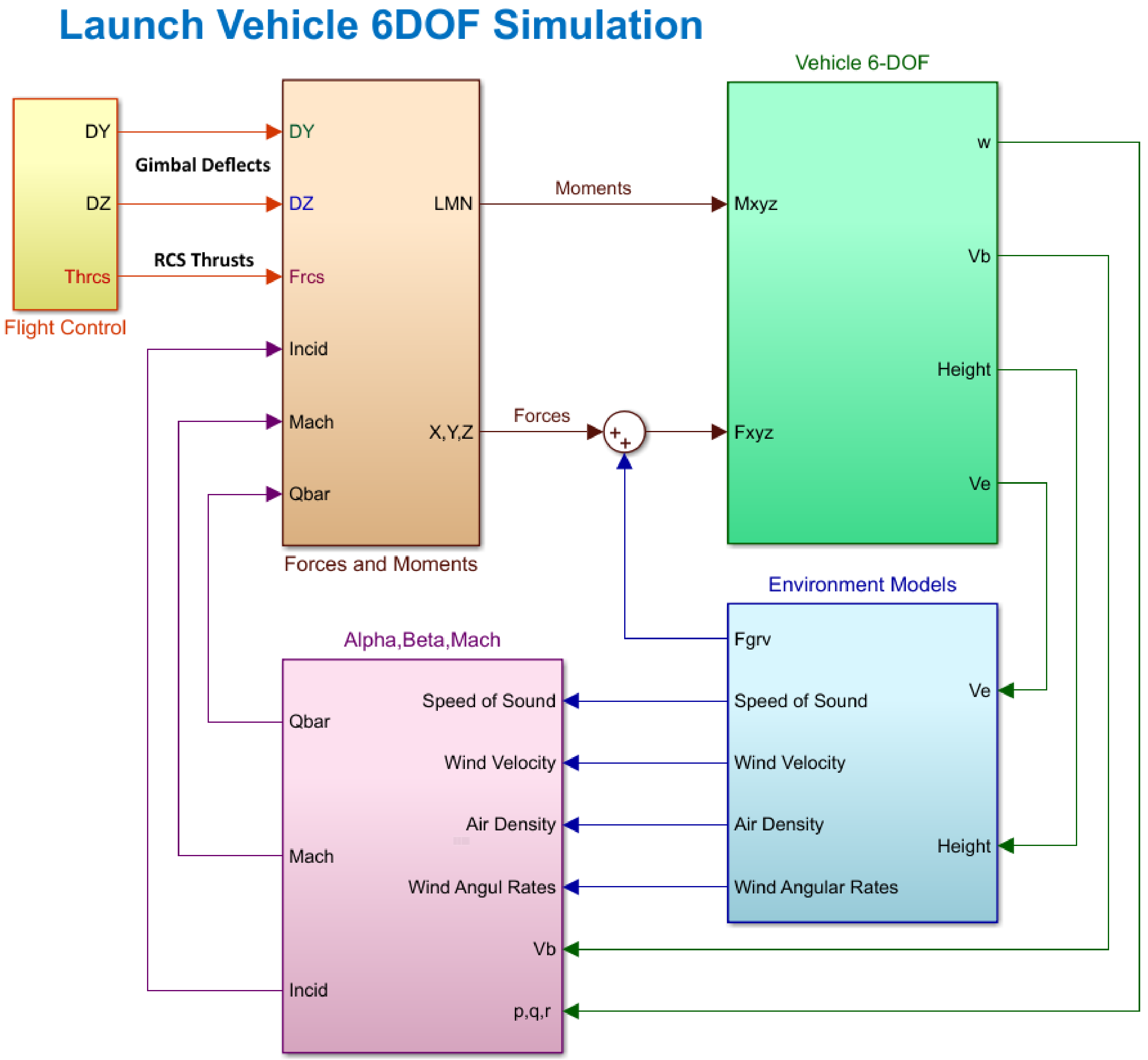


Figure 6.1 Launch Vehicle 6DOF Simulation Overview Block Diagram

The vehicle block is shown in detail in Figure 6.2. It receives external forces and moments and calculates the ECEF positions and velocities, accelerations, body rates, attitudes and quaternions. The vehicle mass and flow rate are calculated versus time and the moments of inertia are scheduled as a function of mass, see Figure 6.4.

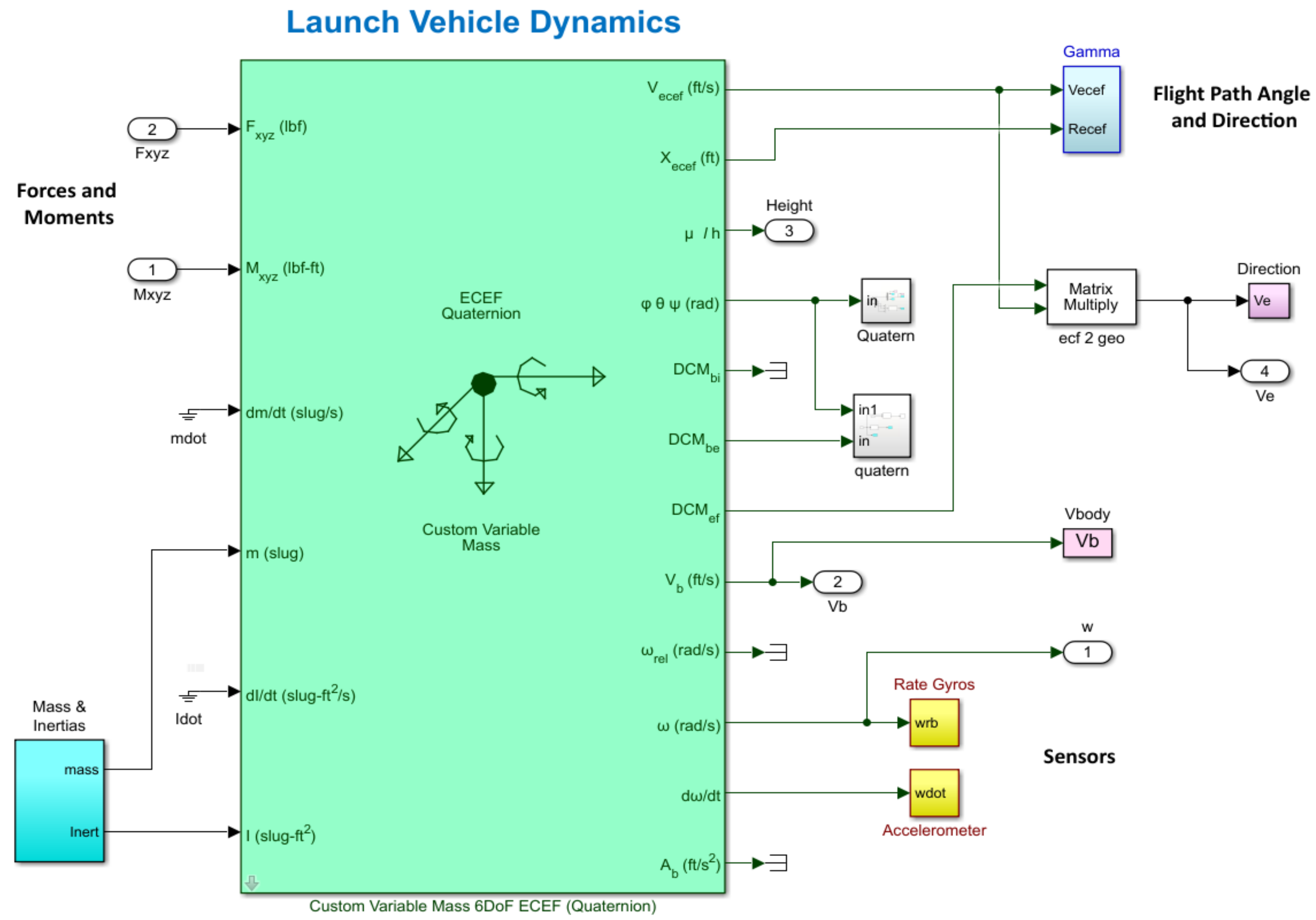


Figure 6.2 Vehicle Subsystem Uses a Simulink 6DOF Function

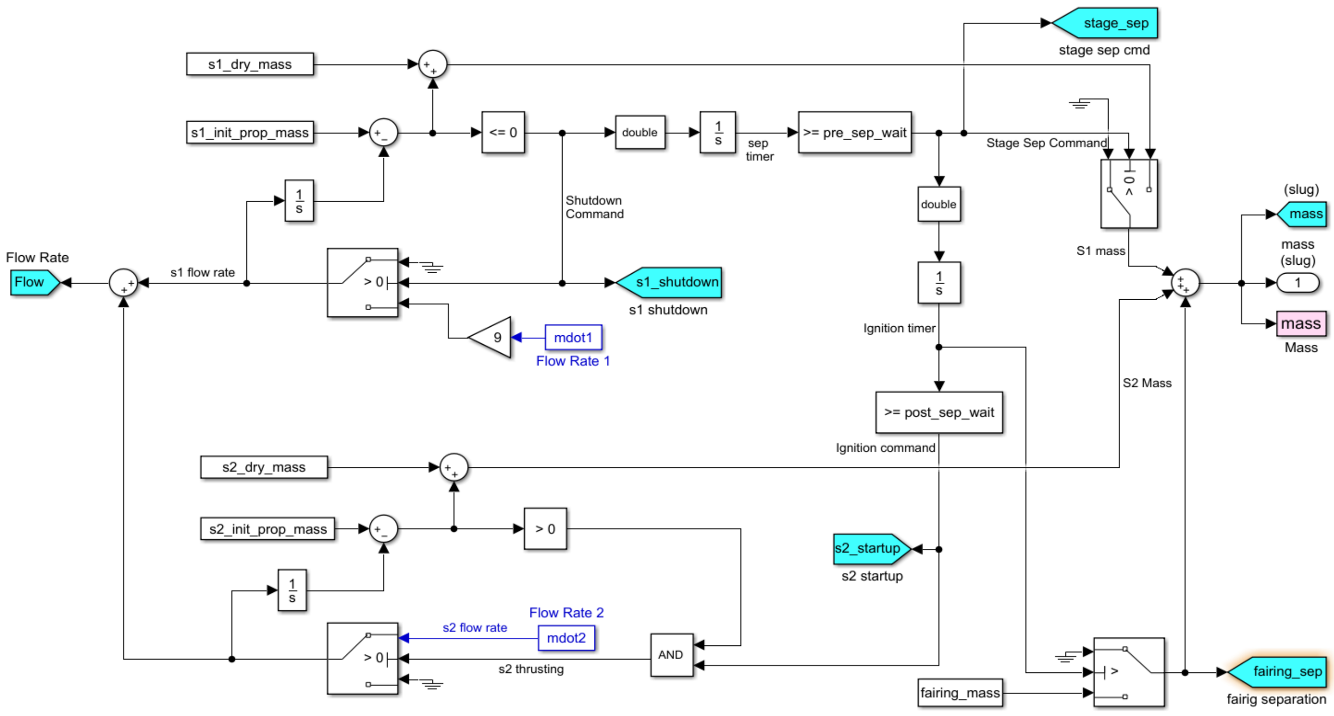


Figure 6.3 Vehicle Mass and Propellant Flow Rate Calculation

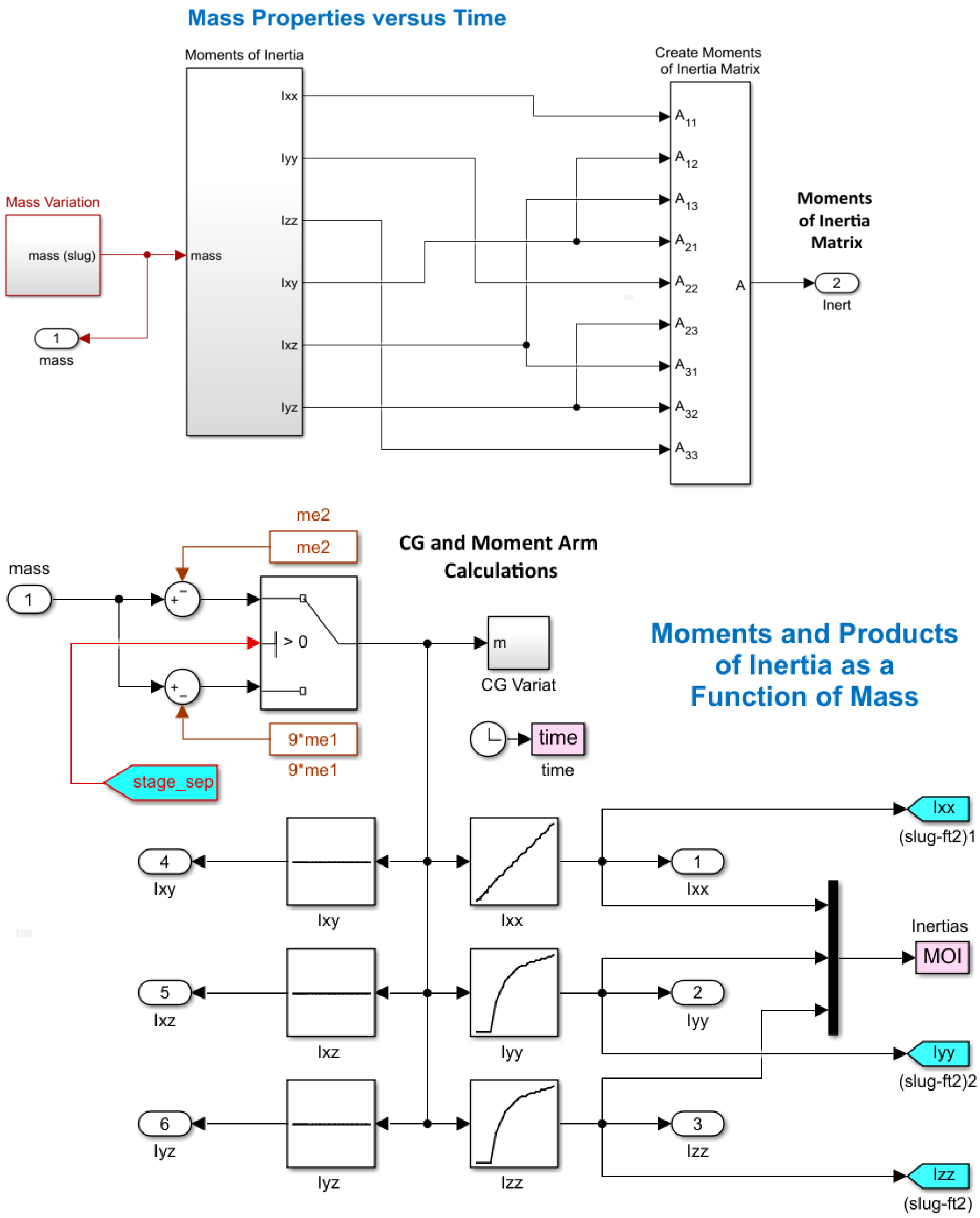


Figure 6.4 Moments of Inertia are Scheduled as a Function of Vehicle Mass

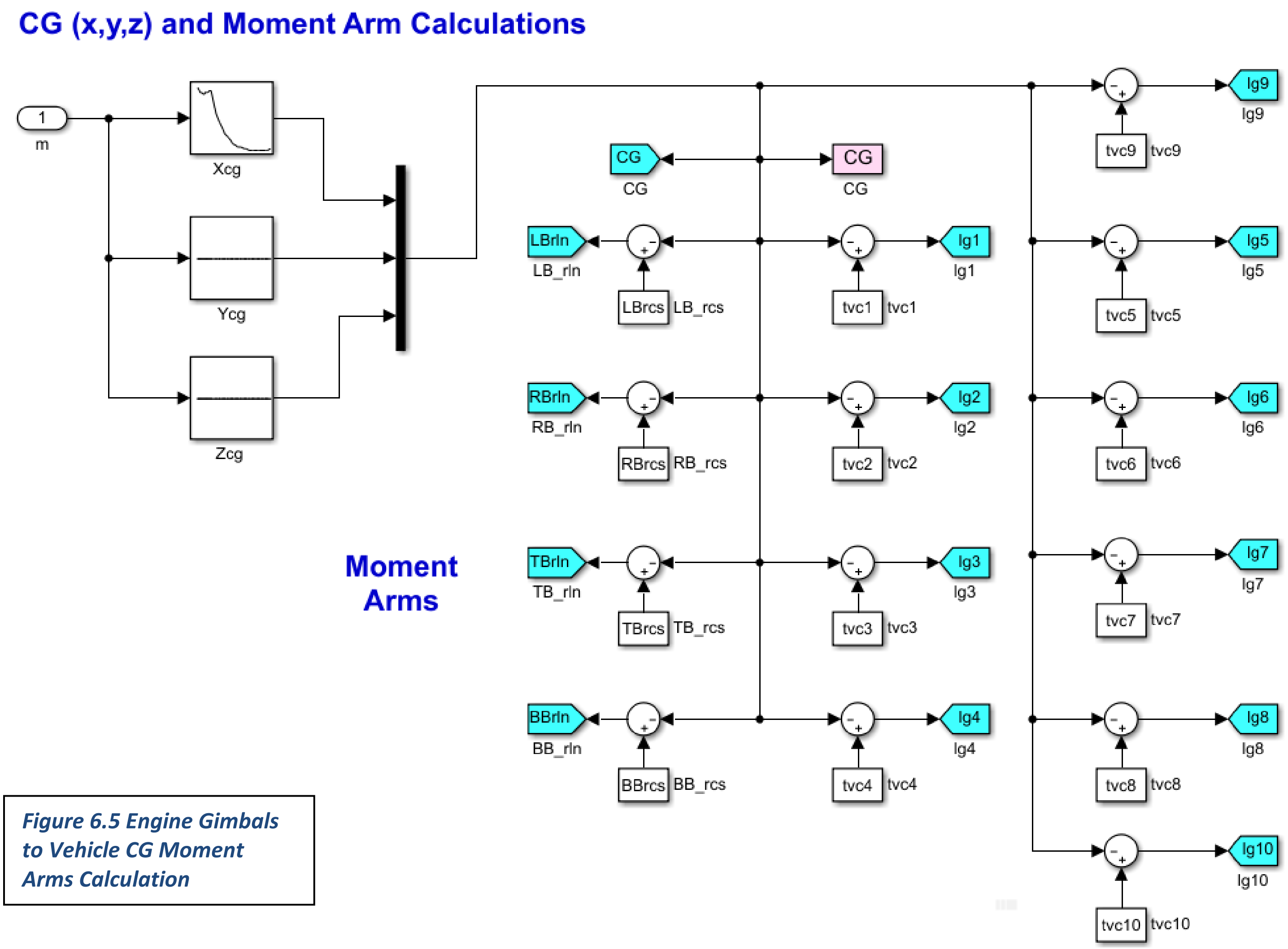


Figure 6.5 Engine Gimbals to Vehicle CG Moment Arms Calculation

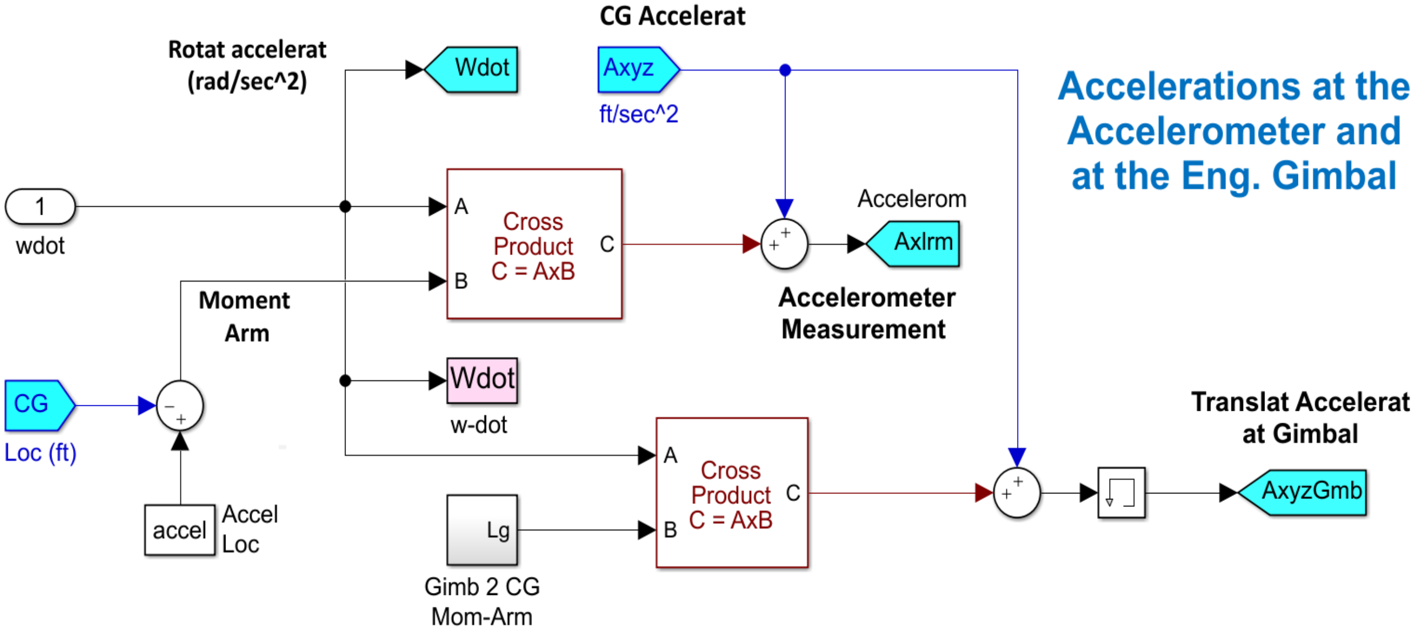


Figure 6.6 This Function Calculates the Acceleration at the Accelerometer Measurement and the Acceleration at the Engine Gimbals

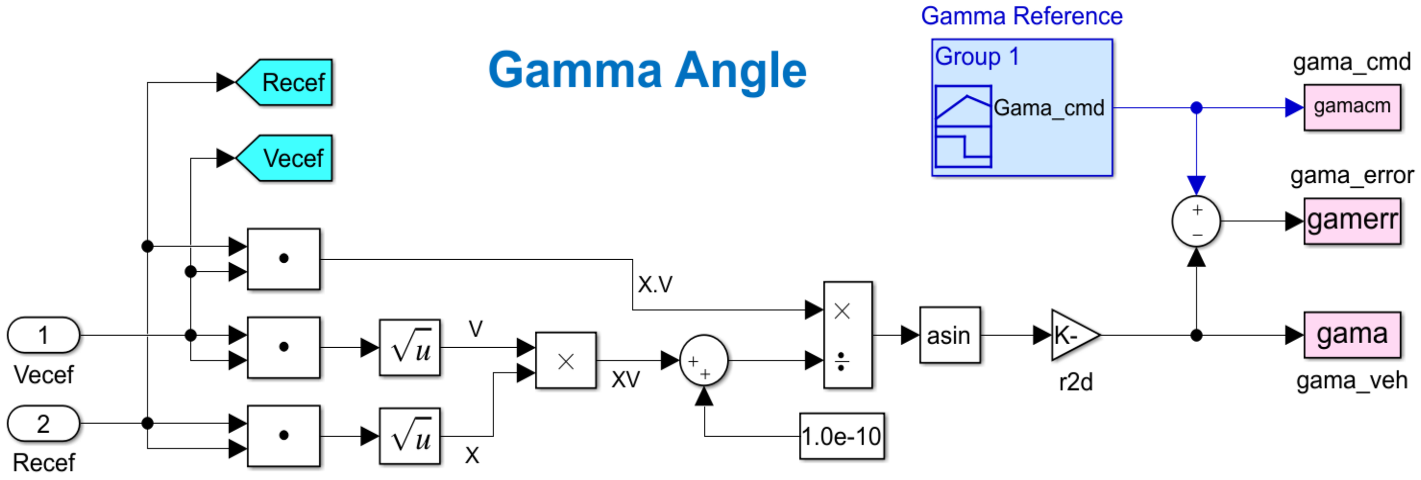


Figure 6.7 Flight-Path Angle Calculation

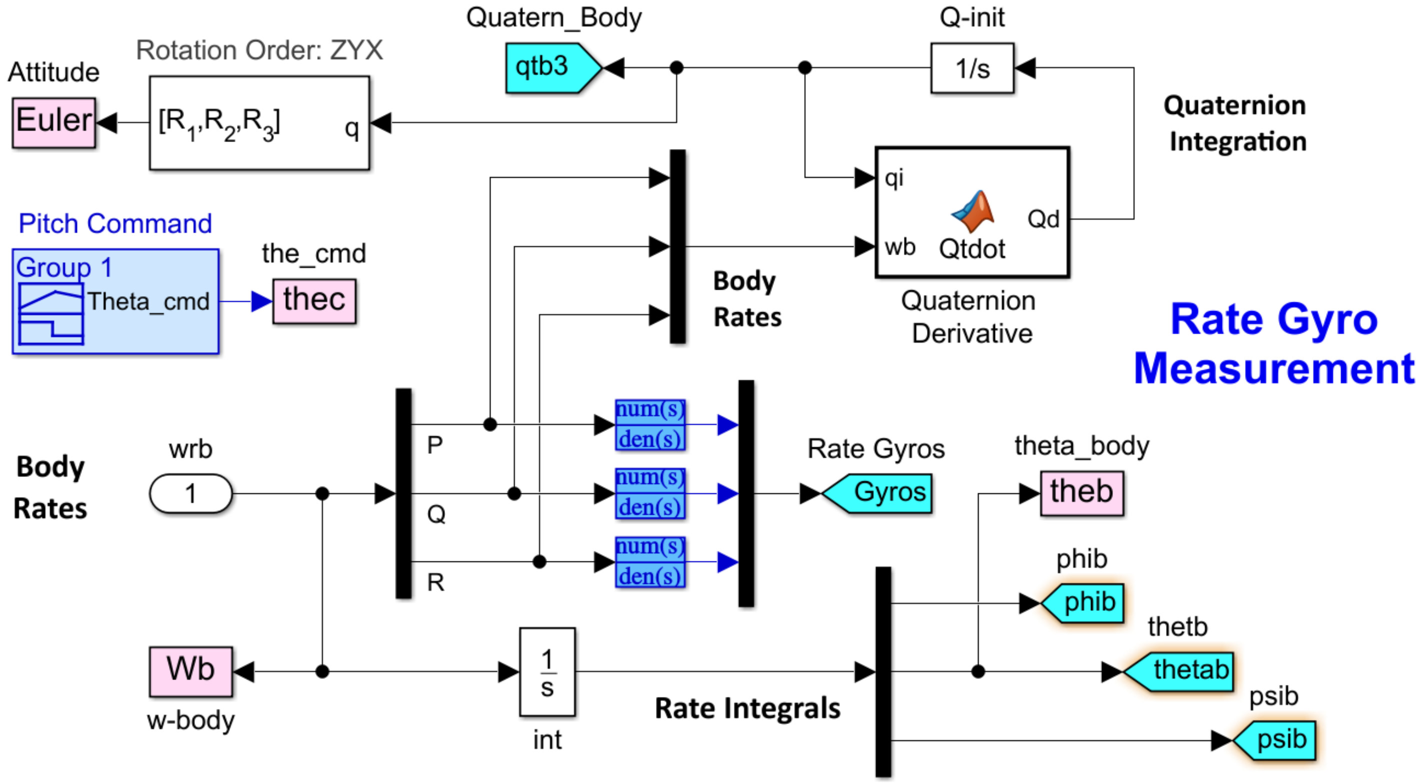


Figure 6.8 Rate Gyro Measurements and Attitude Quaternion Calculation

Figure 6.6 calculates the acceleration measured at the accelerometer location, and also the acceleration at the 1st stage and 2nd stage gimbals. The Gamma function in Figure 6.7 calculates the flight-path angle from the vehicle ECEF position and velocity. It compares it with the reference angle γ-ref and calculates the error. The rate-gyro function in Figure 6.8 calculates the gyro measurement and also the attitude quaternion. Figure 6.9 is the Environmental model which calculates the gravity force F_{grv} as a function of vehicle longitude, latitude, and altitude. It also calculates atmospheric parameters, such as pressure, air density and speed of sound which are used for calculating the dynamic pressure and Mach number.

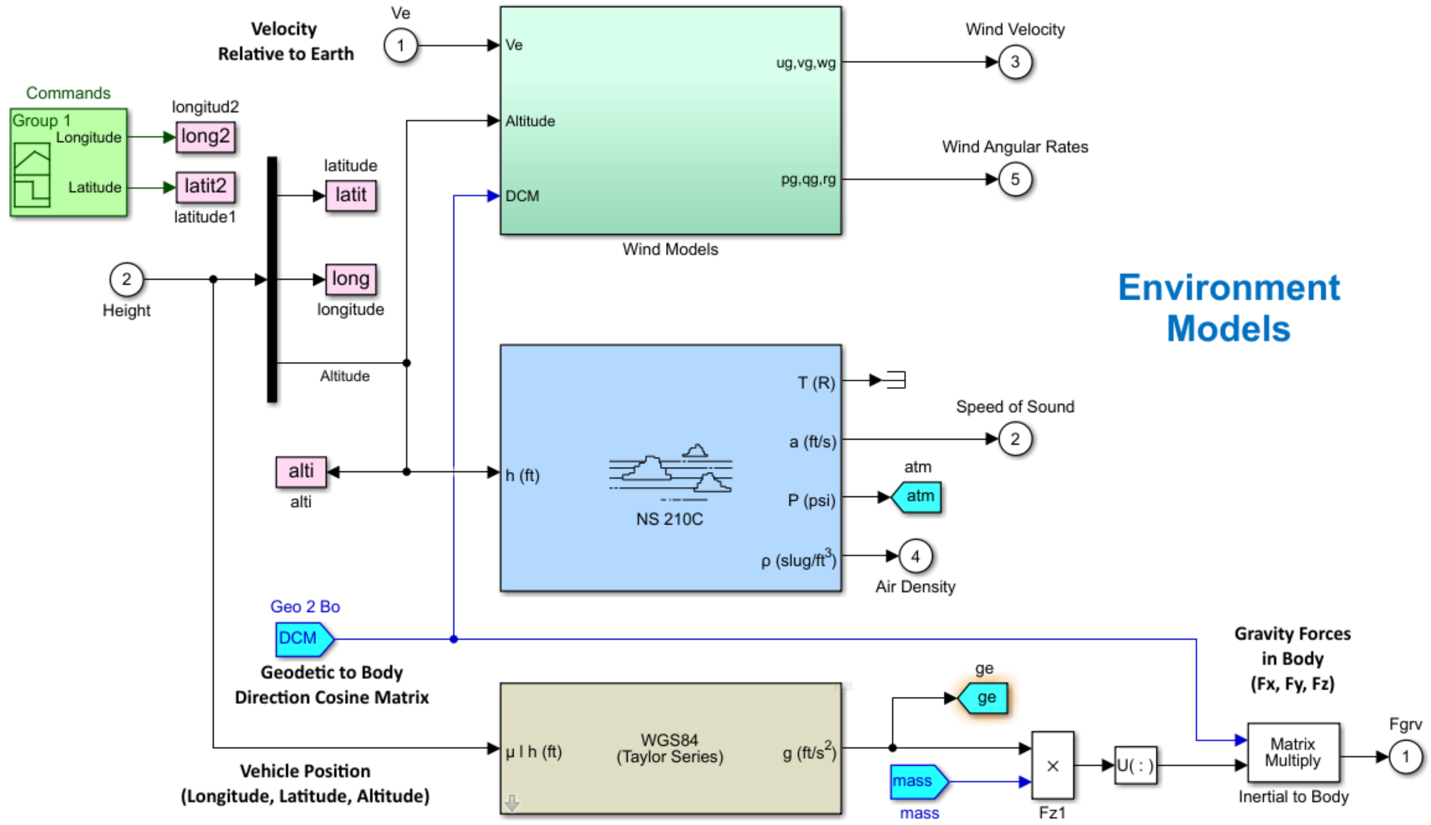


Figure 6.9 Environmental Model

There is also a winds disturbance model, shown in Figure 6.10 that produces additional wind velocities and wind rotational rates generated from the wind-shear and wind-gust disturbance models.

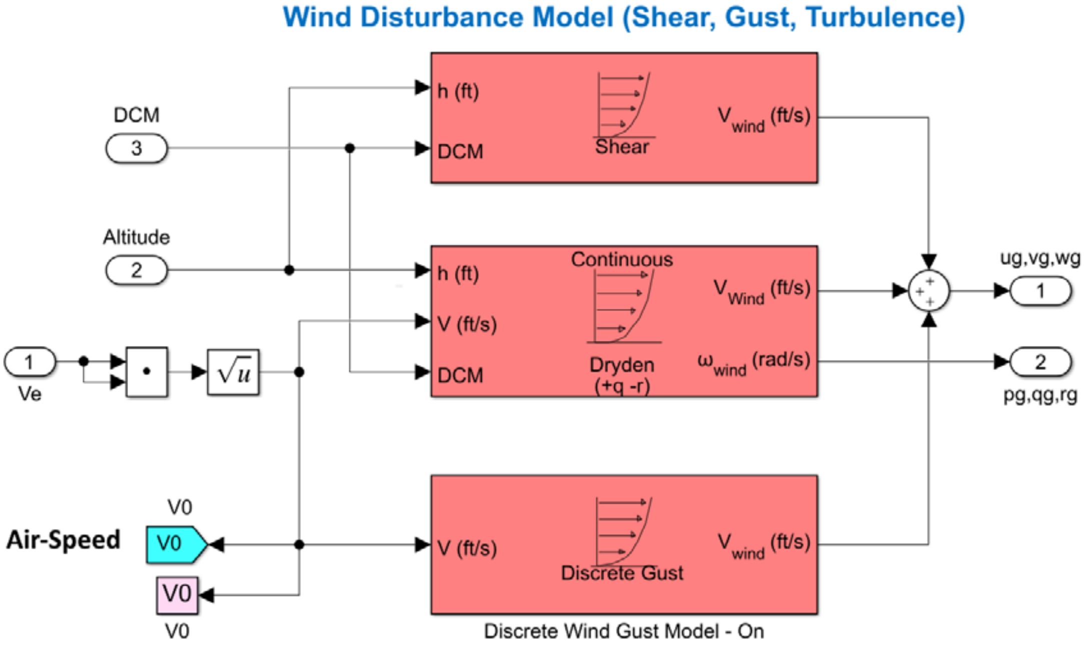


Figure 6.10 Wind Disturbance Models

The subsystem in Figure 6.11 calculates the angles of attack and sideslip (α, β), the dynamic pressure Qbar, the velocity relative to wind, the Mach number, and the vehicle rates relative to the wind (pqr). It also calculates the Q-alpha and Q-beta products which are used to measure the normal and lateral structural loads on the vehicle.

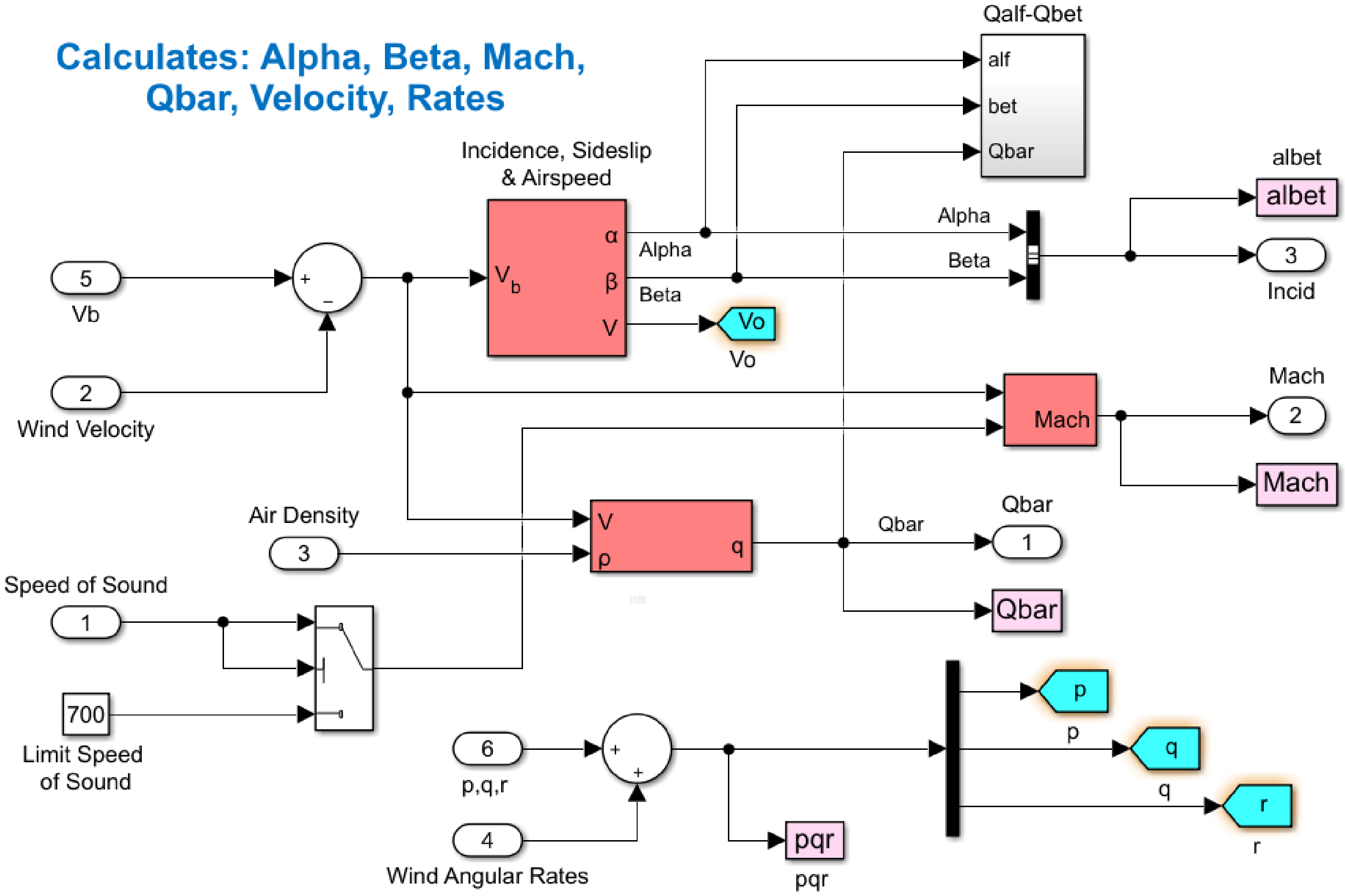


Figure 6.11 Alpha, Beta, Dynamic Pressure, Mach Number Calculation

6.2 Flight Control System Overview

Figure 6.12 shows an overview of the Flight Control System (FCS) which includes 1st and 2nd stages. The guidance block on the left side calculates the roll, pitch and yaw attitude errors that feed into the pitch and lateral FCS, and also into the reaction control system (RCS). The FCS calculates the roll, pitch and yaw acceleration commands that go into the mixing logic TVC matrices Kmix1 for 1st stage and Kmix2 for 2nd stage and they are converted to pitch and yaw engine deflection commands (δy and δz), as shown in Figures 6.13. The second stage TVC matrix controls only pitch and yaw because roll is handled by the RCS jets. The TVC matrices are fixed, but the commands $(DP, DQ, DR)_{TVC}$ are scaled as a function of the mass properties and vehicle geometry.

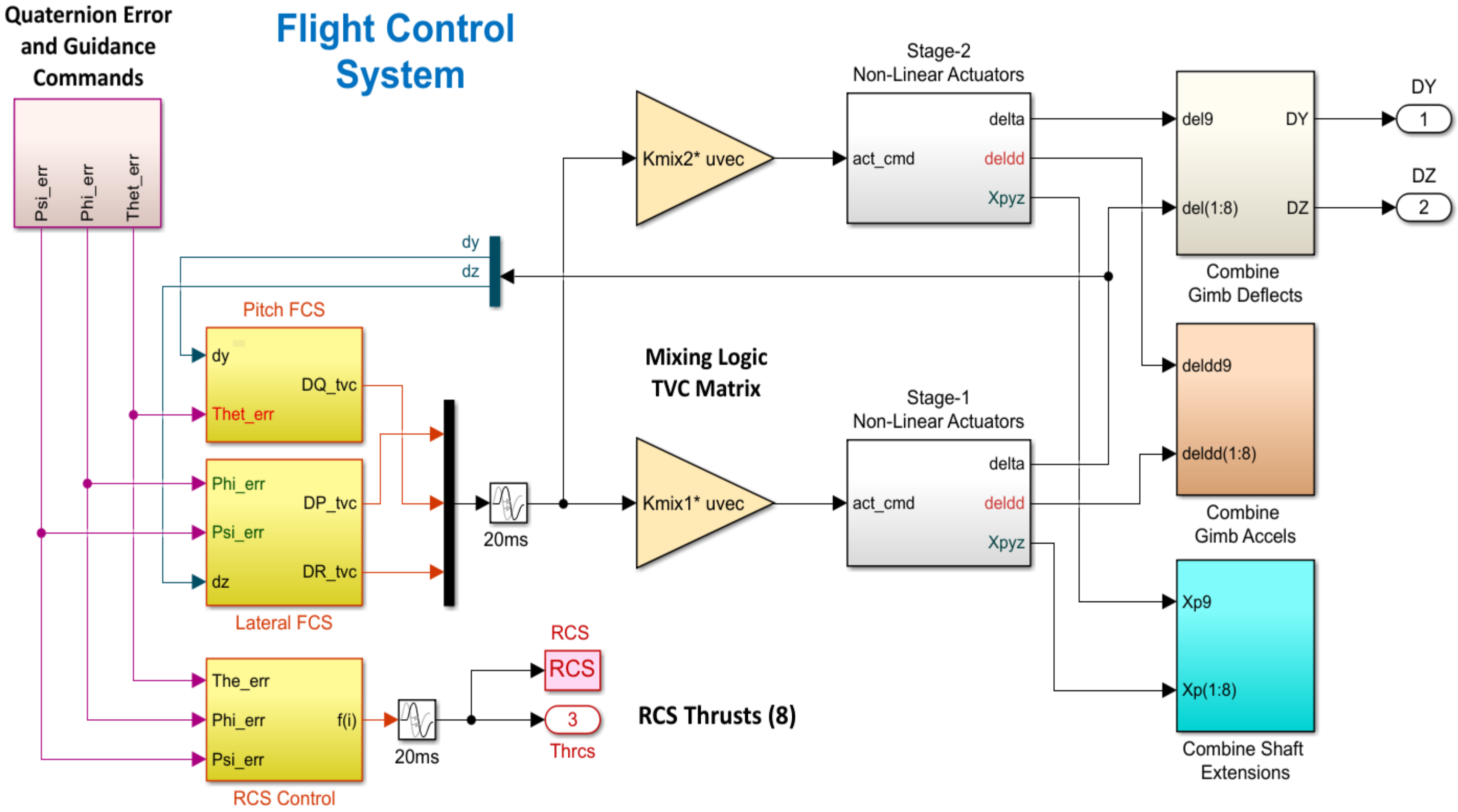
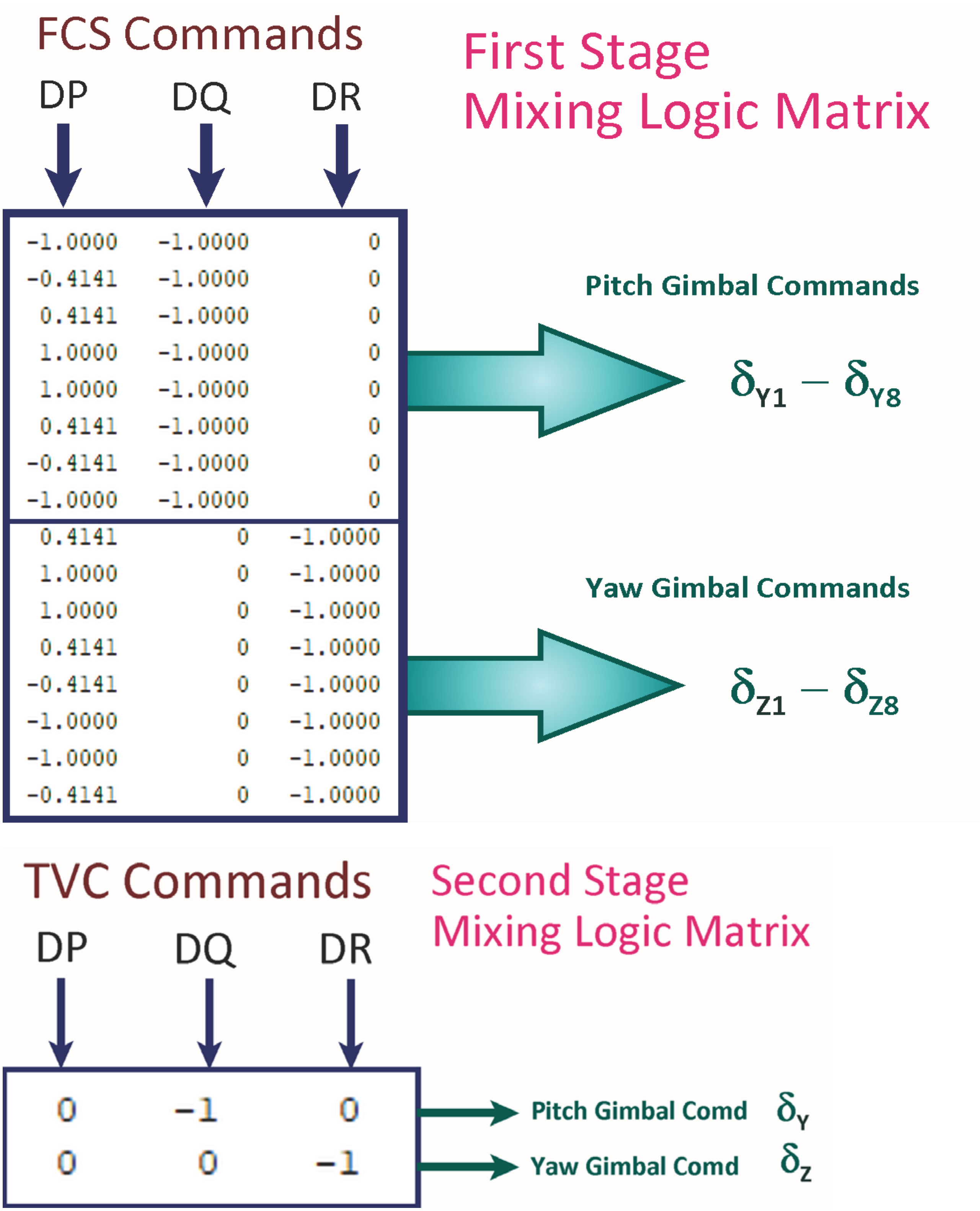


Figure 6.12 Flight Control System

The TVC matrices command the 1st and 2nd stage actuator subsystems, shown in detail in Figures 6.14 and 6.15. The 1st stage subsystem includes 8 pitch and 8 yaw actuators and the 2nd stage subsystem includes 1 pitch and 1 yaw actuator. The actuator subsystems are also driven by load-torques at the gimbals. These are reaction torques generated by vehicle motion due to from linear and angular accelerations at the gimbal. The load-torques counteract the control torques at each actuator subsystem. The flags “S1_shutdown” and “S2_startup” activate the proper actuator subsystem during 1st and 2nd stages.



$$\begin{array}{ccc} DP & DQ & DR \\ \hline -1.0000 & -1.0000 & 0 \\ -0.4141 & -1.0000 & 0 \\ 0.4141 & -1.0000 & 0 \\ 1.0000 & -1.0000 & 0 \\ 1.0000 & -1.0000 & 0 \\ 0.4141 & -1.0000 & 0 \\ -0.4141 & -1.0000 & 0 \\ -1.0000 & -1.0000 & 0 \\ \hline 0.4141 & 0 & -1.0000 \\ 1.0000 & 0 & -1.0000 \\ 1.0000 & 0 & -1.0000 \\ 0.4141 & 0 & -1.0000 \\ -0.4141 & 0 & -1.0000 \\ -1.0000 & 0 & -1.0000 \\ -1.0000 & 0 & -1.0000 \\ -0.4141 & 0 & -1.0000 \end{array}$$

$$\begin{array}{ccc} DP & DQ & DR \\ \hline 0 & -1 & 0 \\ 0 & 0 & -1 \end{array}$$

Figure 6.13 First and Second Stage TVC Matrices

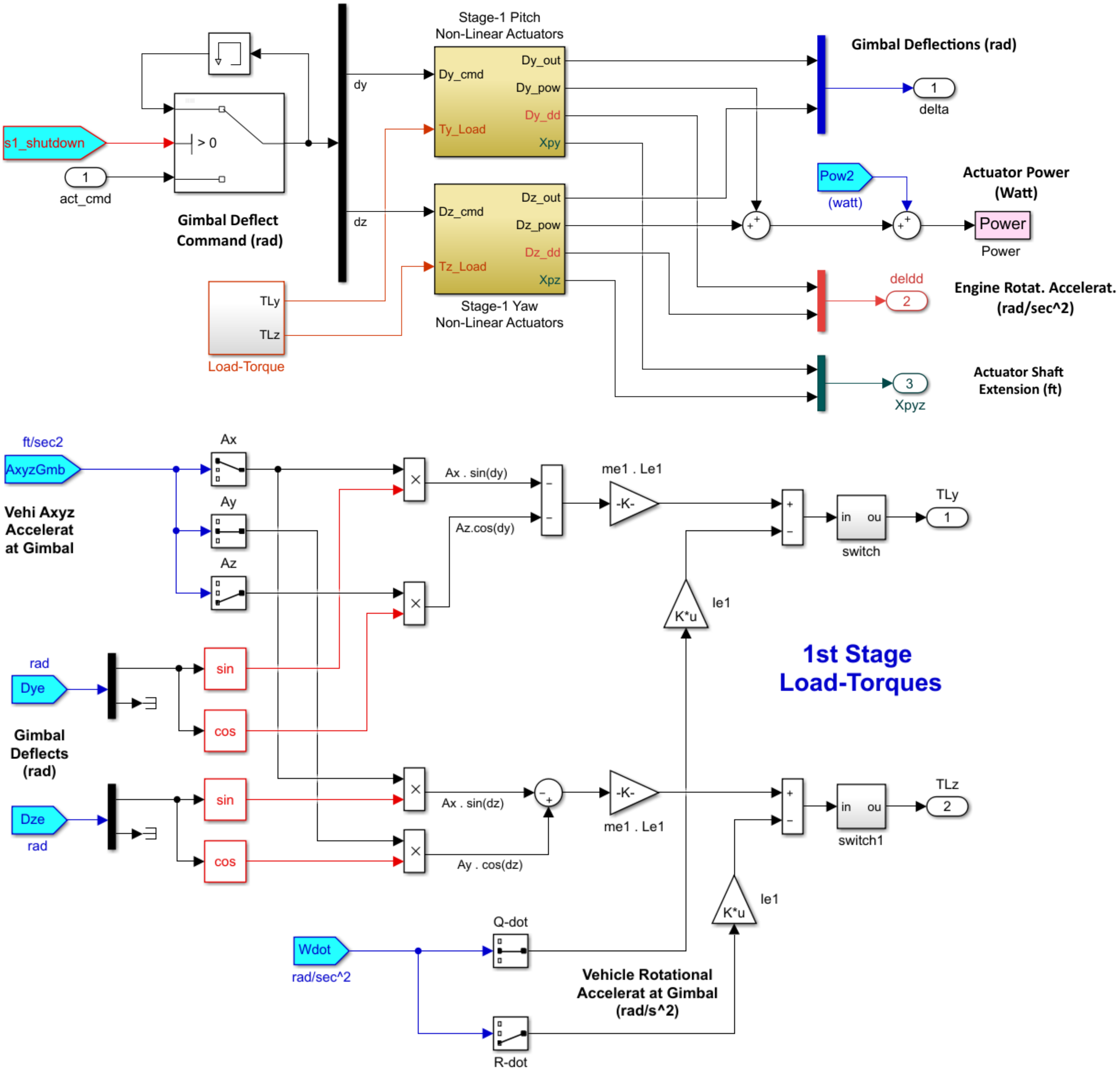


Figure 6.14 First Stage Pitch and Yaw Actuator Subsystem

The vectors Dye and Dze are the pitch and yaw gimbal deflections and they include 9 elements each. The first stage deflections are the first 8 elements in those vectors and the 9th element is grounded. The outputs TLy and TLz are the pitch and yaw load-torques at each gimbal. "AxyzGmb" are the linear accelerations and "Wdot" are the angular accelerations at the 1st stage gimbals. The parameters (me1, Le1, Ie1) are the engine mass, gimbal to engine CG moment arm, and engine inertia for 1st stage. The combined power from all engines is calculated "Power".

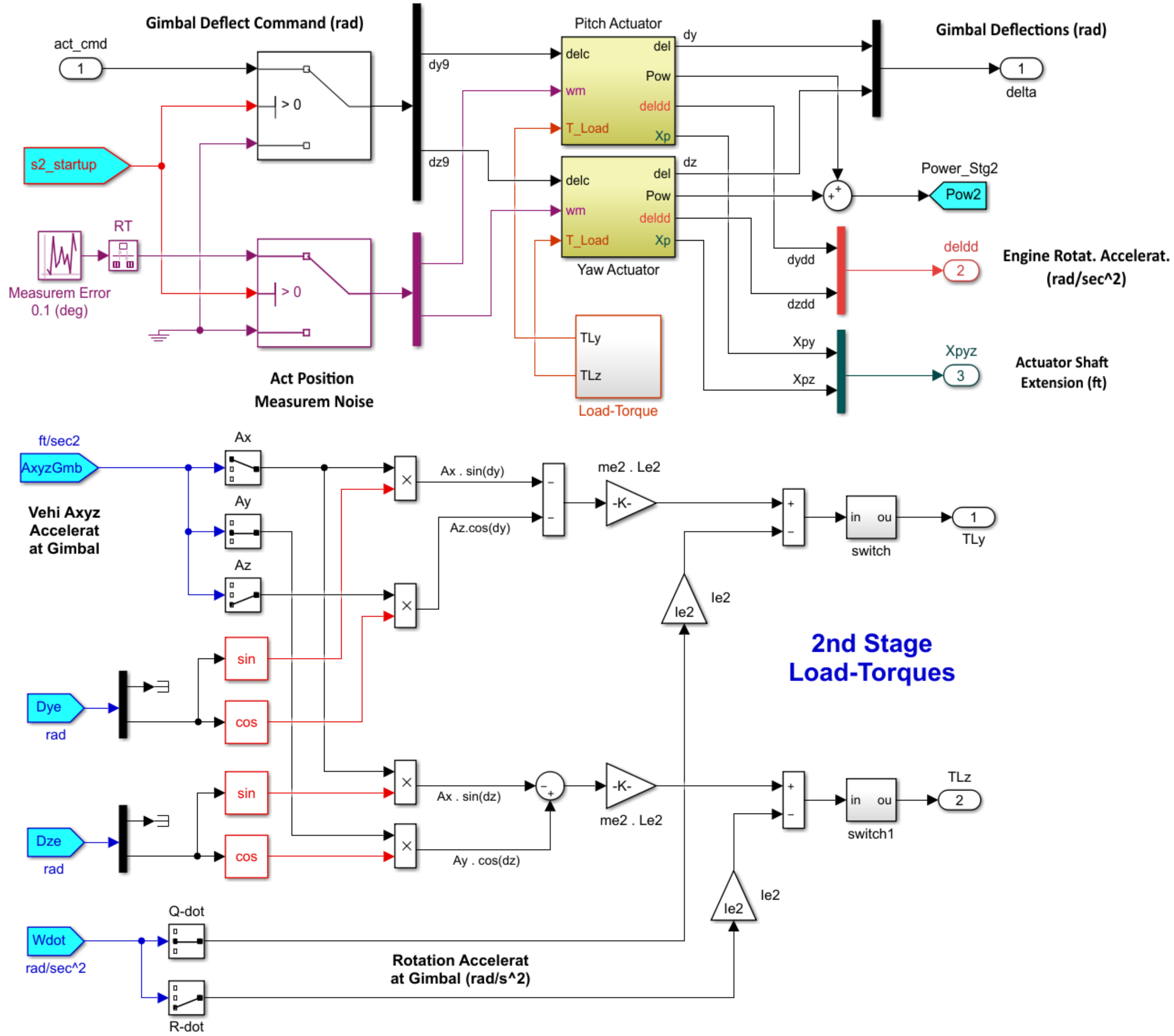


Figure 6.15 Second Stage Pitch and Yaw Actuator Subsystem

The pitch and yaw gimbal deflections vectors Dye and Dze include 9 elements. The 9th element in those vectors are the second stage deflections and the 1st stage deflections (1:8) are grounded. The outputs TLy and TLz are the pitch and yaw load-torques at the 2nd stage gimbal. “AxyzGmb” are the linear accelerations and “Wdot” are the angular accelerations at the 2nd stage gimbal. The parameters (me2, Le2, Ie2) are the engine mass, gimbal to engine CG moment arm and the engine inertia for 2nd stage. The position measurement error is included as noise in the actuator inputs.

The pitch and yaw gimbal deflections Dye and Dze and the actuator shaft positions Xpy and Xpz for the first and second stages are combined together into single 9-element vectors. The output is switched from 1st to 2nd stages by the stage separation flag “stage_sep”.

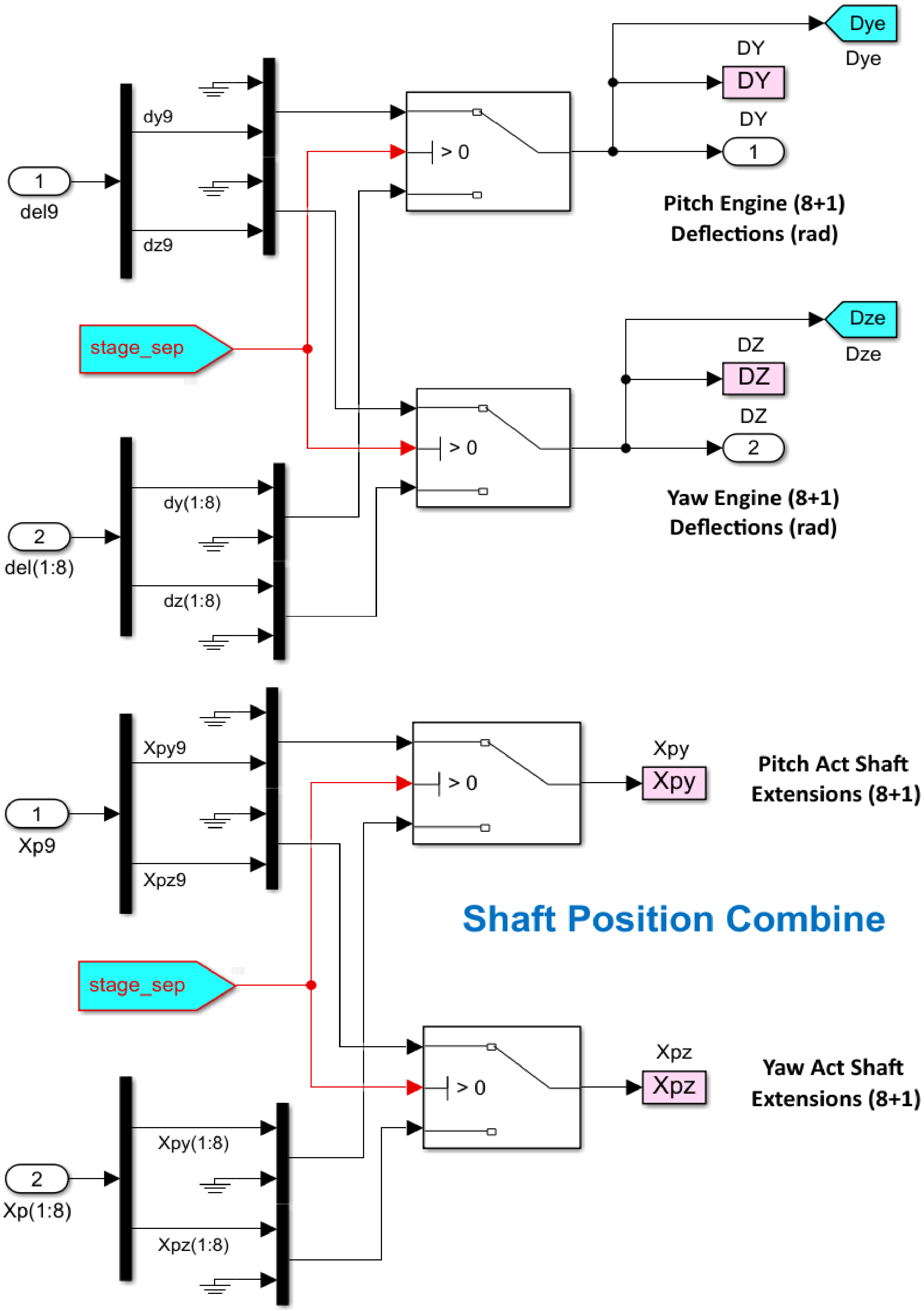
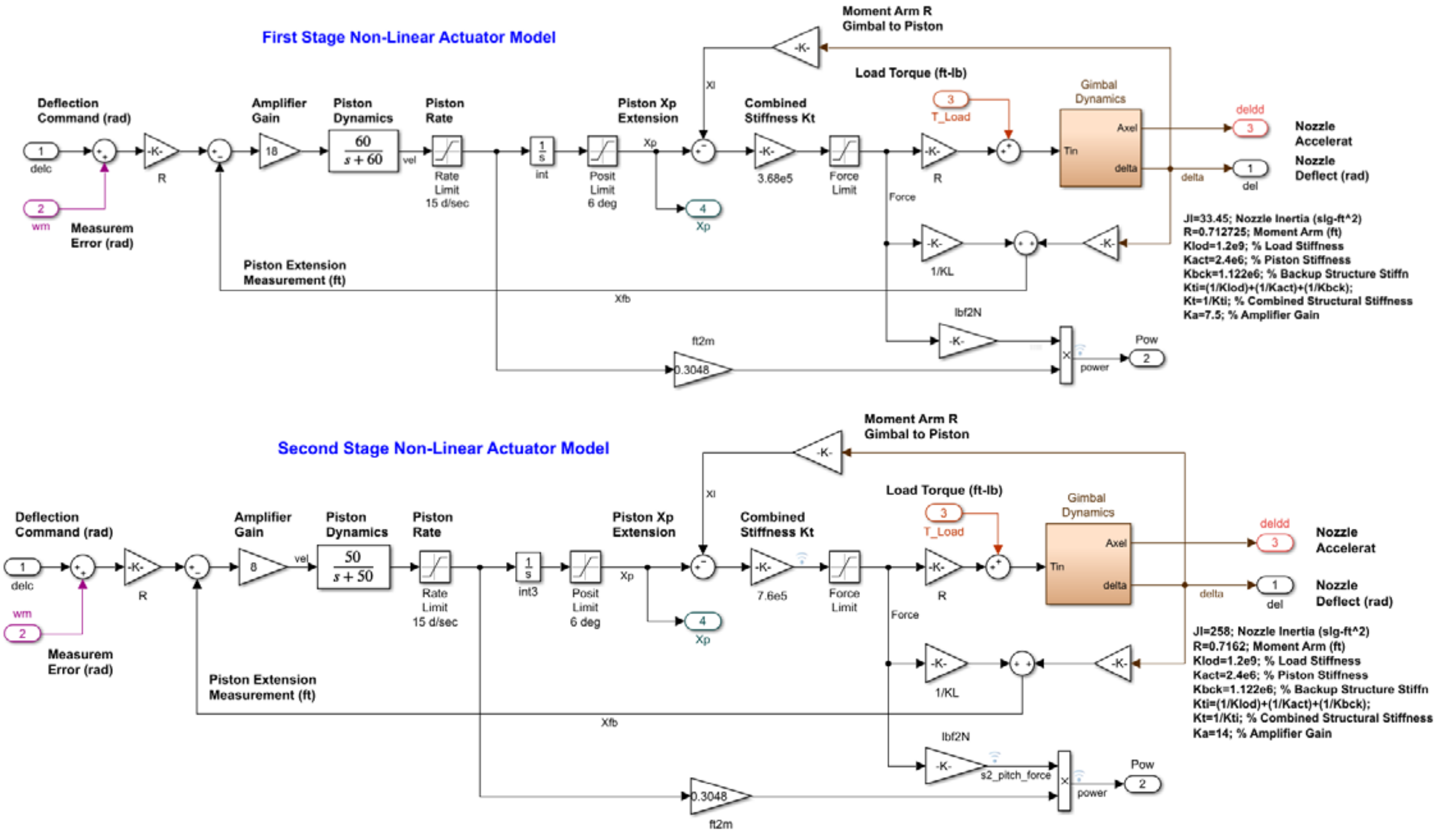


Figure 6.16 The 1st and 2nd Stage Engine Deflections are Combined in Single Vectors

6.3 Electro-Mechanical Actuator

The EM actuator models for 1st and 2nd stages are shown in Figure 6.17. The position error drives the shaft dynamics which is represented by a first order TF. The position error is pushing against the combined system stiffness K_T to generate the shaft force which is multiplied by the moment arm R to generate the control torque that rotates the engine. The control torque is counteracted by the load-torque T_{Load} that comes from the vehicle model. The stiffness K_T is calculated by combining 3 stiffnesses in series (adding the inverses): the actuator attachment to the backup structure, the actuator attachment to the nozzle, and the shaft stiffness. The actuator power is calculated as the product of shaft force times the shaft rate. The position measurement error is included as noise at the input. The shaft position output is X_P. The 1st and 2nd stage actuator models are a little different because the engine inertias and geometries are different. First stage includes 8 pitch and 8 yaw actuators, a total of 16, for the 8 gimbaling engines. Second stage has only one engine and therefore 2 actuators, one for pitch and one for yaw. The 1st and 2nd stage actuator models are implemented in library function "*Actuator_Lib.slx*".

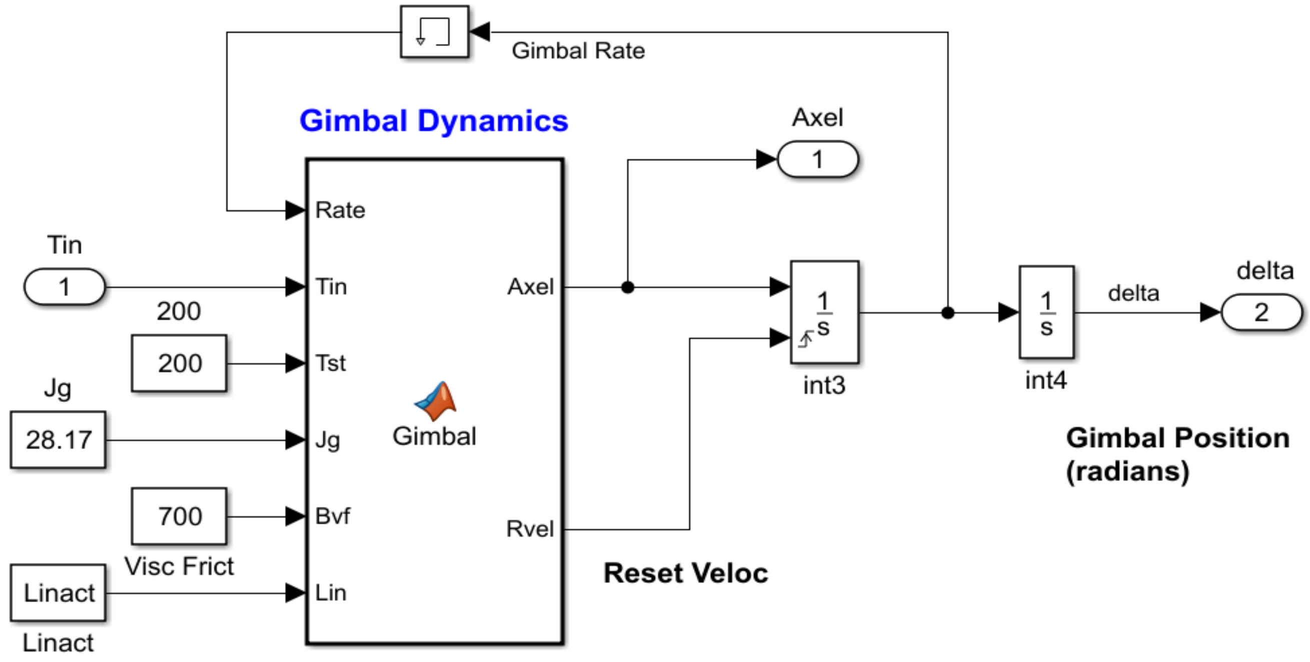


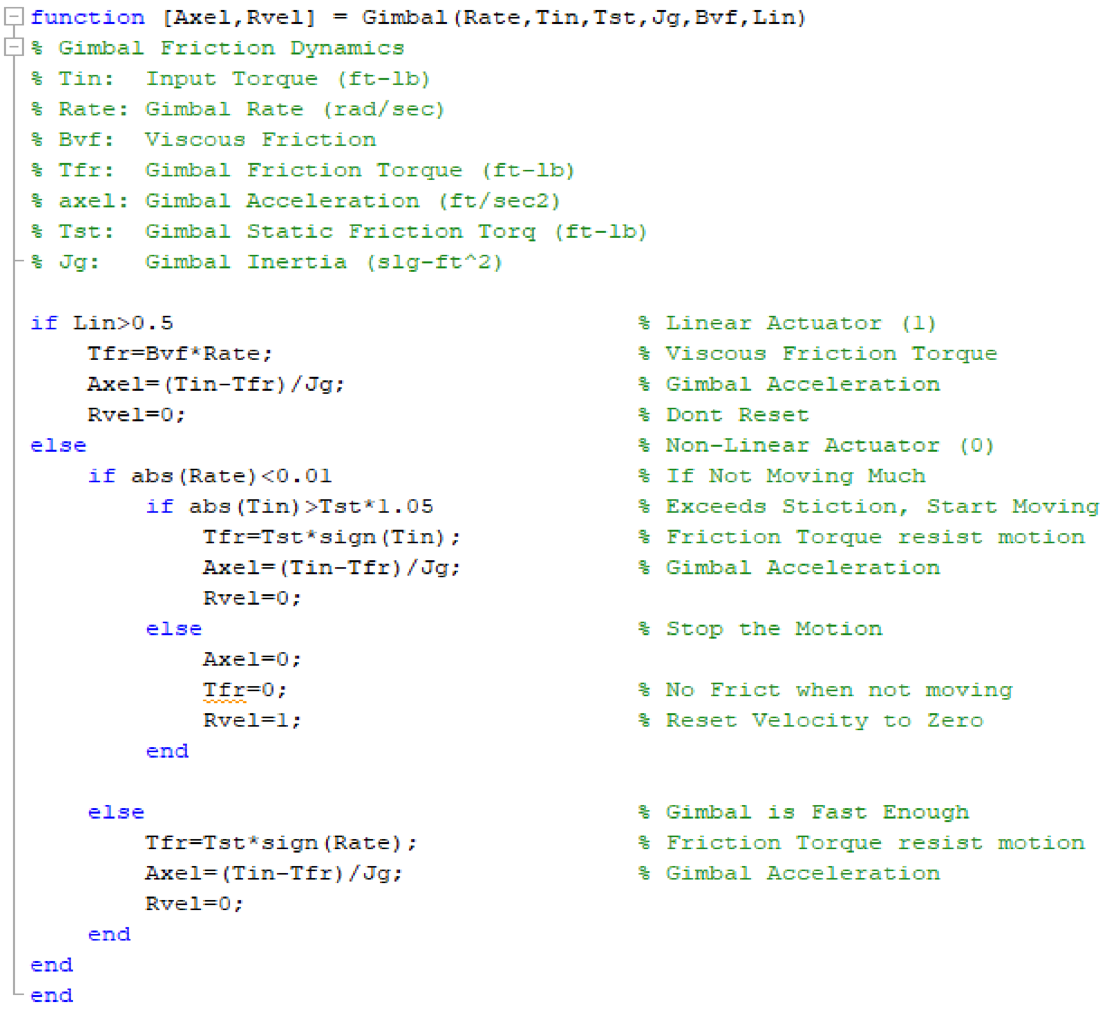
```
Jl=33.45; Nozzle Inertia (slg-ft^2)
R=0.712725; Moment Arm (ft)
Klod=1.2e9; % Load Stiffness
Kact=2.4e6; % Piston Stiffness
Kbck=1.122e6; % Backup Structure Stiffn
Kti=(1/Klod)+(1/Kact)+(1/Kbck);
Kt=1/Kti; % Combined Structural Stiffness
Ka=7.5; % Amplifier Gain
```

```
Jl=258; Nozzle Inertia (slg-ft^2)
R=0.7162; Moment Arm (ft)
Klod=1.2e9; % Load Stiffness
Kact=2.4e6; % Piston Stiffness
Kbck=1.122e6; % Backup Structure Stiffn
Kti=(1/Klod)+(1/Kact)+(1/Kbck);
Kt=1/Kti; % Combined Structural Stiffness
Ka=14; % Amplifier Gain
```

Figure 6.17 Engine Gimbal Actuator Model

Figure x shows the non-linear gimbal dynamics which is described by the function "gimbal". It calculates the gimbal angle and gimbal acceleration for each rotational direction. The input torque T_{in} must exceed the static friction torque T_{st} before the engine can move, and when it does the output acceleration is $(T_{in} - T_{st})/Jg$, where Jg is the engine inertia. When the gimbal is moving, the friction torque resisting motion is $T_{FR} = T_{st}$ sign(rate) and the output acceleration is $(T_{in} – T_{FR})/Jg$. The gimbal rate is obtained by integrating the acceleration. The rate is reset to zero by the signal R_{vel} every time the acceleration drops to zero. The rotation angle δ is the integral of the gimbal rate.





```
function [Axel,Rvel] = Gimbal(Rate,Tin,Tst,Jg,Bvf,Lin)
% Gimbal Friction Dynamics
% Tin:  Input Torque (ft-lb)
% Rate: Gimbal Rate (rad/sec)
% Bvf:  Viscous Friction
% Tfr:  Gimbal Friction Torque (ft-lb)
% axel: Gimbal Acceleration (ft/sec2)
% Tst:  Gimbal Static Friction Torq (ft-lb)
% Jg:   Gimbal Inertia (slg-ft^2)

if Lin>0.5                                % Linear Actuator (1)
    Tfr=Bvf*Rate;                         % Viscous Friction Torque
    Axel=(Tin-Tfr)/Jg;                    % Gimbal Acceleration
    Rvel=0;                               % Dont Reset
else                                      % Non-Linear Actuator (0)
    if abs(Rate)<0.01                     % If Not Moving Much
        if abs(Tin)>Tst*1.05              % Exceeds Stiction, Start Moving
            Tfr=Tst*sign(Tin);            % Friction Torque resist motion
            Axel=(Tin-Tfr)/Jg;            % Gimbal Acceleration
            Rvel=0;
        else                              % Stop the Motion
            Axel=0;
            Tfr=0;                        % No Frict when not moving
            Rvel=1;                       % Reset Velocity to Zero
        end

    else                                  % Gimbal is Fast Enough
        Tfr=Tst*sign(Rate);               % Friction Torque resist motion
        Axel=(Tin-Tfr)/Jg;                % Gimbal Acceleration
        Rvel=0;
    end
end
end
```

Figure 6.18 Engine Gimbal Dynamics

6.4 Guidance System

The Guidance block is shown in detail in Figure 6.19. In the pitch axis the guidance is open-loop, consisting of an attitude quaternion command that is compared against the vehicle attitude quaternion (qtb3). It calculates the attitude errors which command the flight control system.

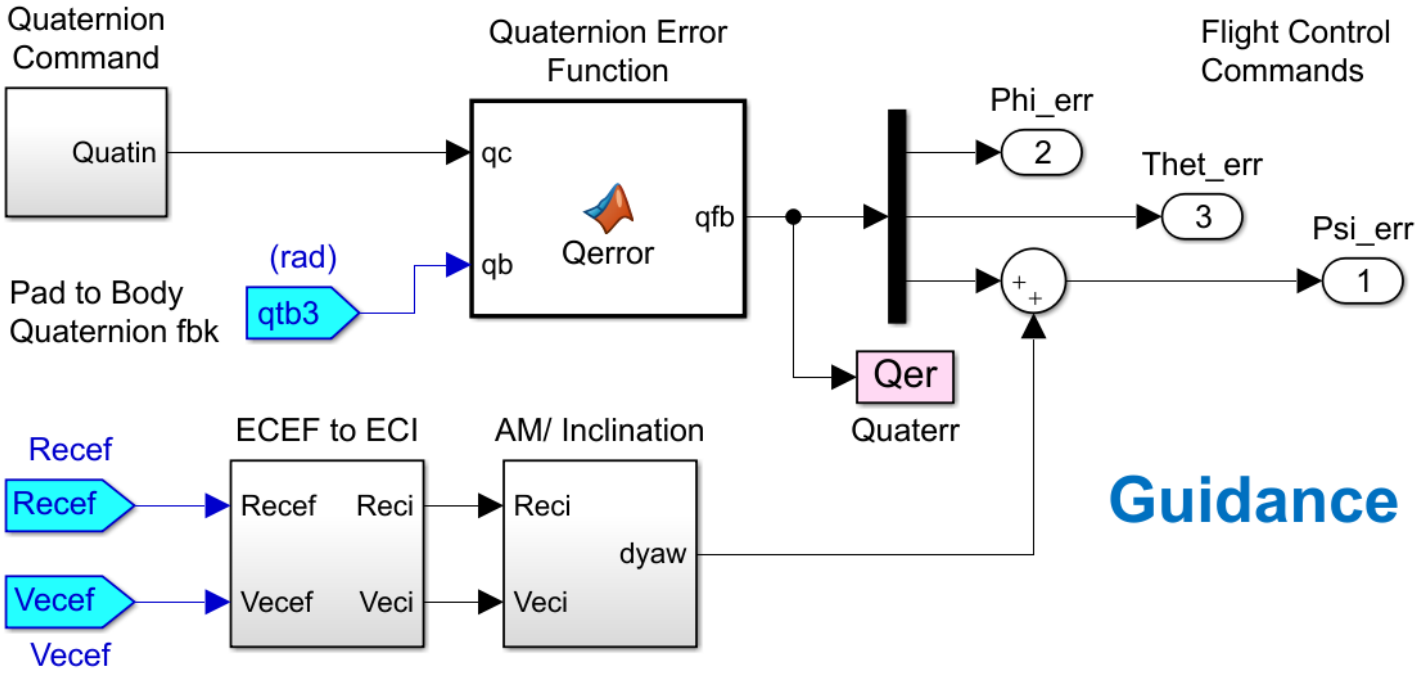
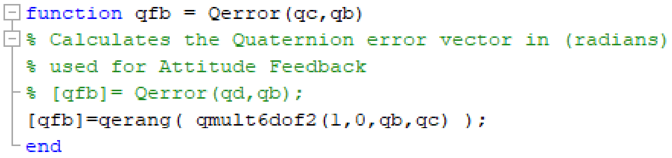
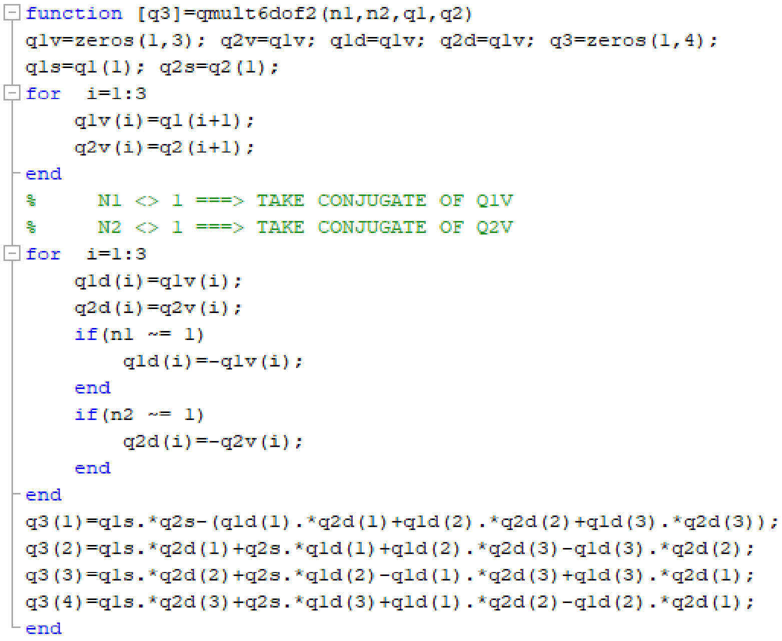


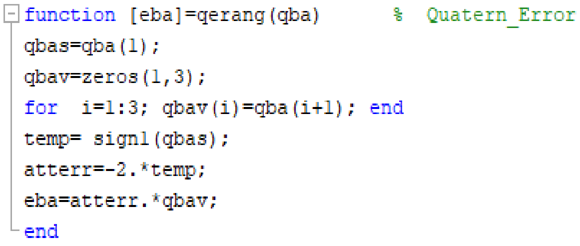
Figure 6.19 Guidance Subsystem



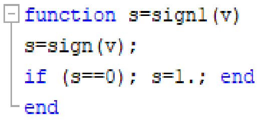
```
function qfb = Qerror(qc,qb)
% Calculates the Quaternion error vector in (radians)
% used for Attitude Feedback
% [qfb]= Qerror(qd,qb);
[qfb]=qerang( qmult6dof2(1,0,qb,qc) );
end
```



```
function [q3]=qmult6dof2(n1,n2,q1,q2)
q1v=zeros(1,3); q2v=q1v; q1d=q1v; q2d=q1v; q3=zeros(1,4);
q1s=q1(1); q2s=q2(1);
for  i=1:3
     q1v(i)=q1(i+1);
     q2v(i)=q2(i+1);
end
%     N1 <> 1 ===> TAKE CONJUGATE OF Q1V
%     N2 <> 1 ===> TAKE CONJUGATE OF Q2V
for  i=1:3
     q1d(i)=q1v(i);
     q2d(i)=q2v(i);
     if(n1 ~= 1)
         q1d(i)=-q1v(i);
     end
     if(n2 ~= 1)
         q2d(i)=-q2v(i);
     end
end
q3(1)=q1s.*q2s-(q1d(1).*q2d(1)+q1d(2).*q2d(2)+q1d(3).*q2d(3));
q3(2)=q1s.*q2d(1)+q2s.*q1d(1)+q1d(2).*q2d(3)-q1d(3).*q2d(2);
q3(3)=q1s.*q2d(2)+q2s.*q1d(2)-q1d(1).*q2d(3)+q1d(3).*q2d(1);
q3(4)=q1s.*q2d(3)+q2s.*q1d(3)+q1d(1).*q2d(2)-q1d(2).*q2d(1);
end
```



```
function [eba]=qerang(qba)      %  Quatern_Error
qbas=qba(1);
qbav=zeros(1,3);
for  i=1:3; qbav(i)=qba(i+1); end
temp= sign1(qbas);
aterr=-2.*temp;
eba=aterr.*qbav;
end
```



```
function s=sign1(v)
s=sign(v);
if (s==0); s=1.; end
end
```

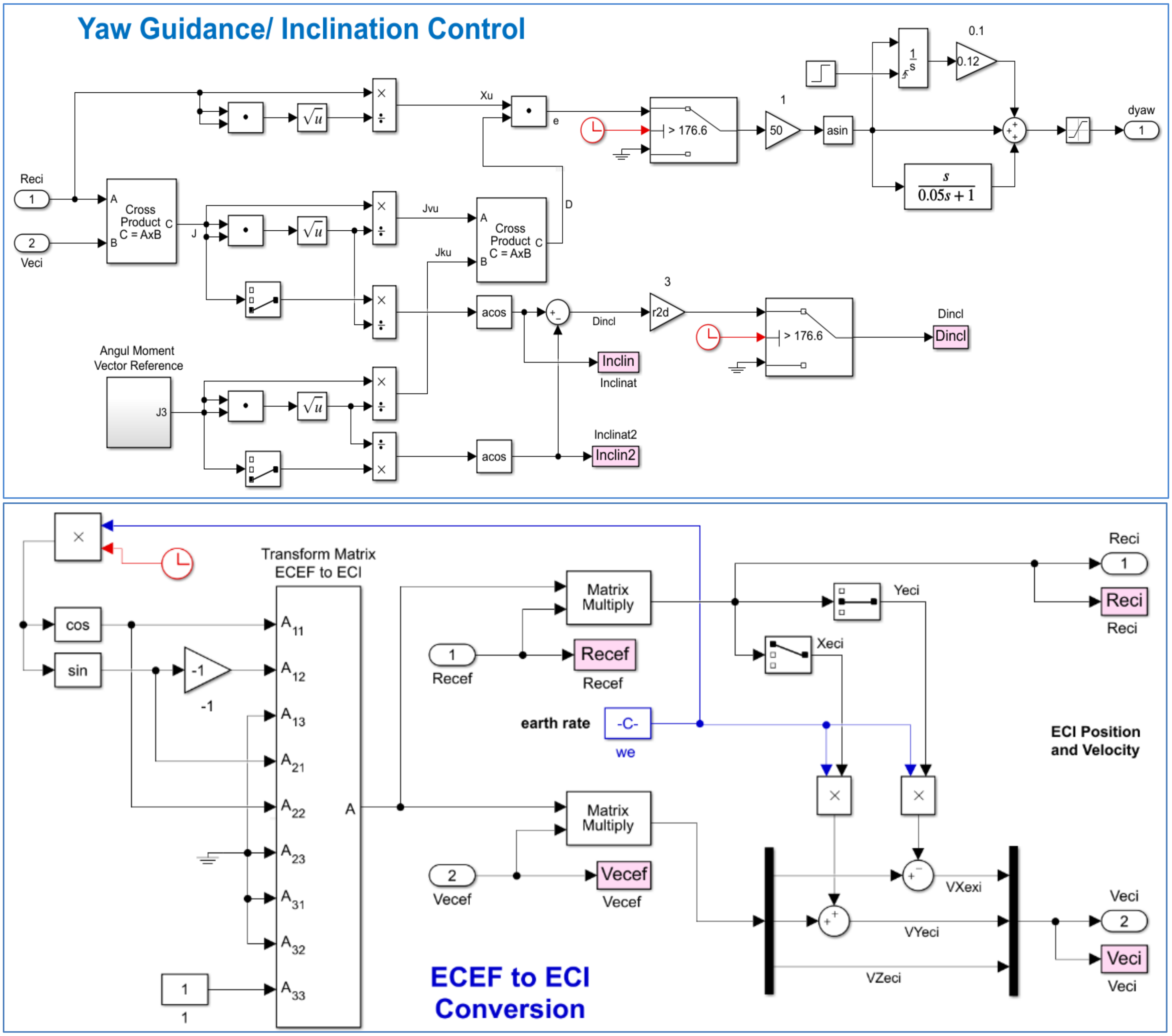


Figure 6.20 Inclination Control System

In lateral, the guidance system controls the vehicle inclination by comparing the angular momentum vector against the reference momentum. The cross-product produces an error which is proportional to the inclination error that generates the yaw command. The angular momentum reference is in ECI and there is a transformation included to convert the vehicle position and velocity from ECEF to ECI.

6.5 Pitch Flight Control System

The Pitch FCS block includes both 1st and 2nd stages and it is shown in detail in Figure 6.21. The LQR derived gains are scheduled as a function of the relative velocity V_0. The state-feedback during first stage is from: pitch attitude θ, pitch rate q, angle of attack α, θ-integral, and α-integral. The state-feedback for second stage is from: pitch attitude θ, pitch rate q, and θ-integral. An angle of attack estimator is used to estimate alpha. The FCS output is pitch acceleration demand “DQ_tvc” that goes to the TVC matrix. The flags “s1_shutdown” and “s2_startup” switch between the 1st and 2nd stage subsystems. The 1st and 2nd stage control demands are scaled by the loop-gains “Kml2-1” and “Kml2-2”. The loop-gains are scheduled as a function of the mass properties and geometry. The scaling factors are derived from the Flixan generated TVC matrices at each flight condition during the control design. By varying the loop-gains instead, it allows us to use constant TVC matrices throughout 1st or 2nd stages.

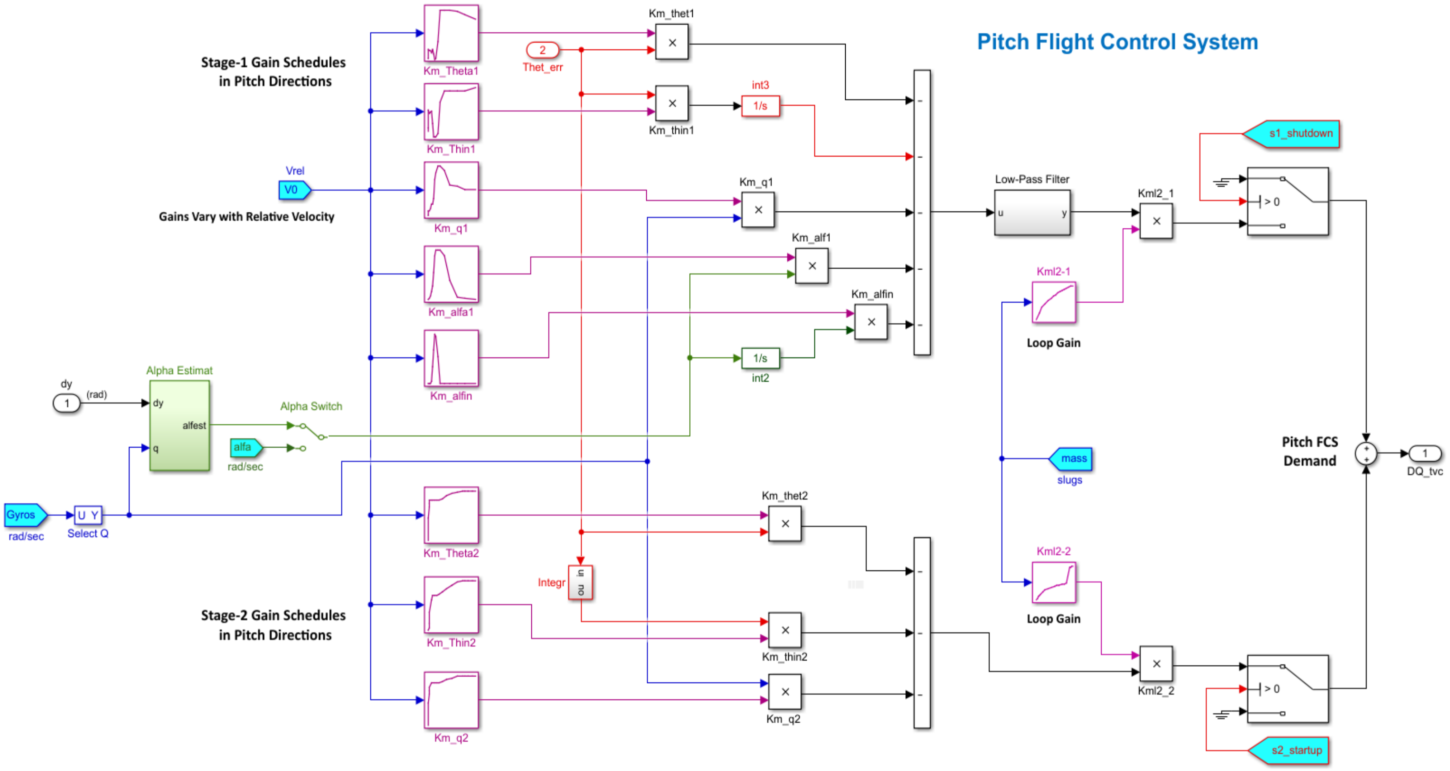


Figure 6.21 Pitch Flight Control System

The alpha-estimator is shown in detail in Figure 6.22. It estimates the angle of attack by solving the normal acceleration equation, real-time in closed-loop form including an integrator and a slosh notch filter. It uses the pitch rate, the normal accelerometer signal, the pitch gimbal deflections, the vehicle thrust and mass. It also uses the normal force aero coefficient $C_{Z\alpha}$ and the dynamic pressure Qbar which vary as a function of the relative velocity V_0. The slosh-notch/ integrator block inside the loop includes the estimator gain “K_estim1” and a slosh notch filter that has a variable notch frequency. The estimator gain and the notch frequency vary as a function of the relative velocity. Lookup tables are used to calculate the parameters as a function of the velocity V_0. Figure 6.23 shows the 1st stage low-pass filter. Its bandwidth varies as a function of the velocity V_0.

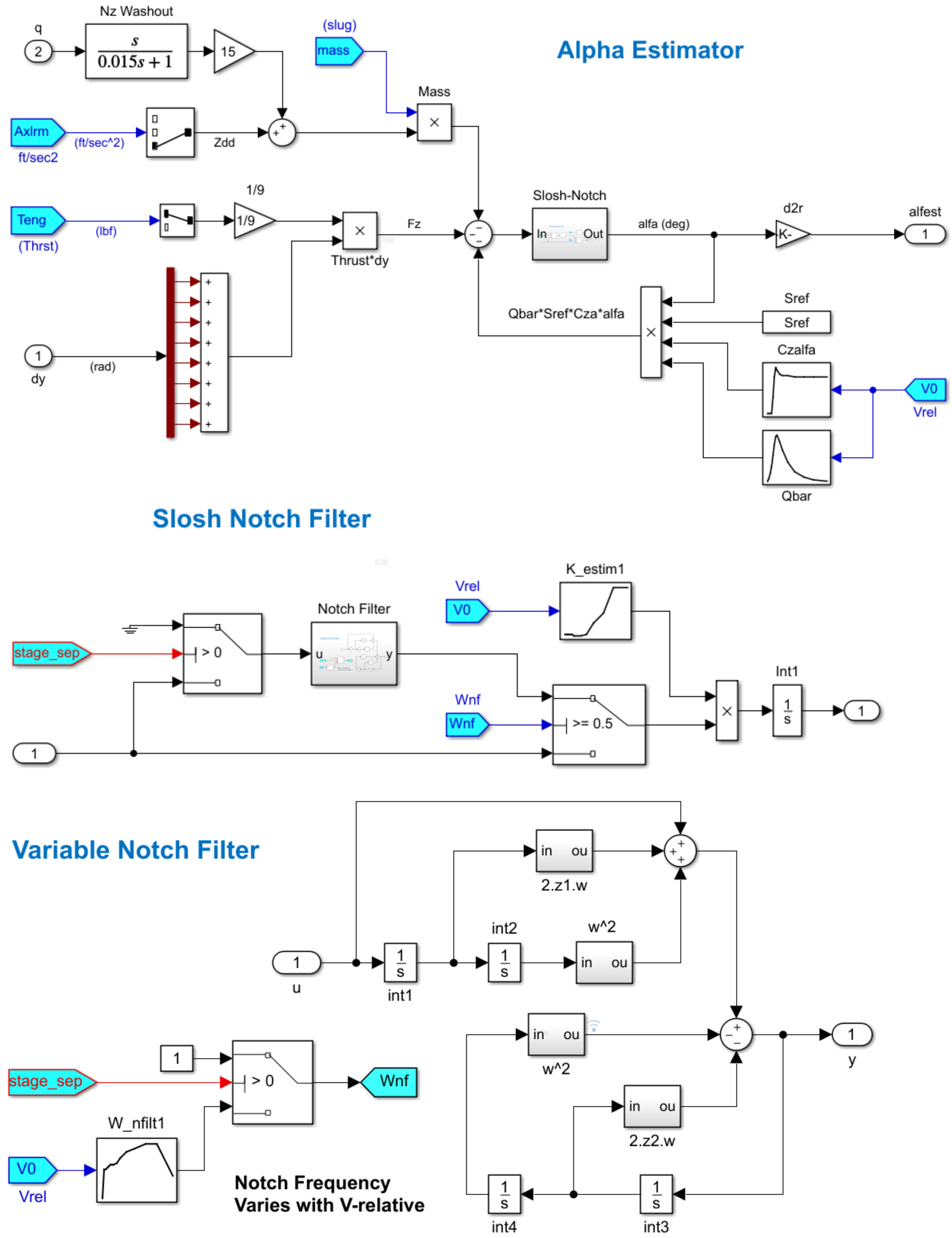


Figure 6.22 Alpha Estimator with Variable Slosh Notch Filter

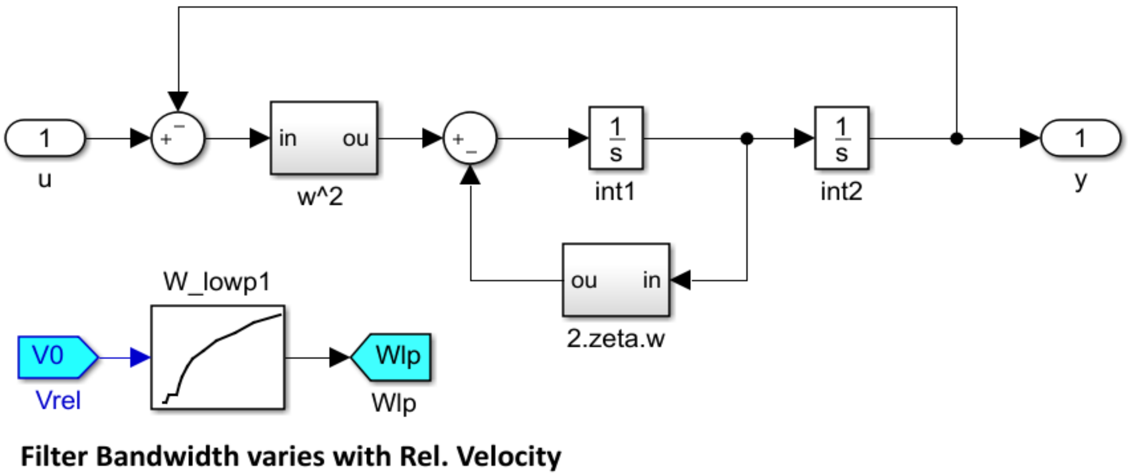


Figure 6.23 Variable Bandwidth Low-Pass Filter

6.6 Lateral Flight Control System

The Lateral FCS block consists of 1st stage and 2nd stage subsystems and it is shown in Figure 6.24. Its inputs are roll and yaw attitude errors and the outputs are roll and yaw acceleration demands $(DP, DR)_{TVC}$. Figure 6.25 shows the 1st stage FCS in detail. The state-feedback is from: roll and yaw attitudes (ϕ, ψ), roll and yaw rates (p, r), angle of sideslip β, ψ-integral, and β-integral. The LQR derived gains are scheduled as a function of the relative velocity V_0. The roll and yaw acceleration demand outputs are scaled by the loop-gains "Kml1-1" and "Kml2-2" which are scheduled as a function of the mass properties and geometry derived from the mixing-logic program. The β-estimator is shown in detail in Figure 6.26. It estimates the sideslip angle by solving the lateral acceleration equation, real-time in closed-loop form, including an integrator and a slosh notch filter. The estimator inputs are yaw rate (r), lateral acceleration ($\ddot{y}$), yaw gimbal deflections (δ_z), vehicle thrust and mass. It needs also the lateral aero force coefficient C_{Yb} and the dynamic pressure Qbar which vary with relative velocity V_0.

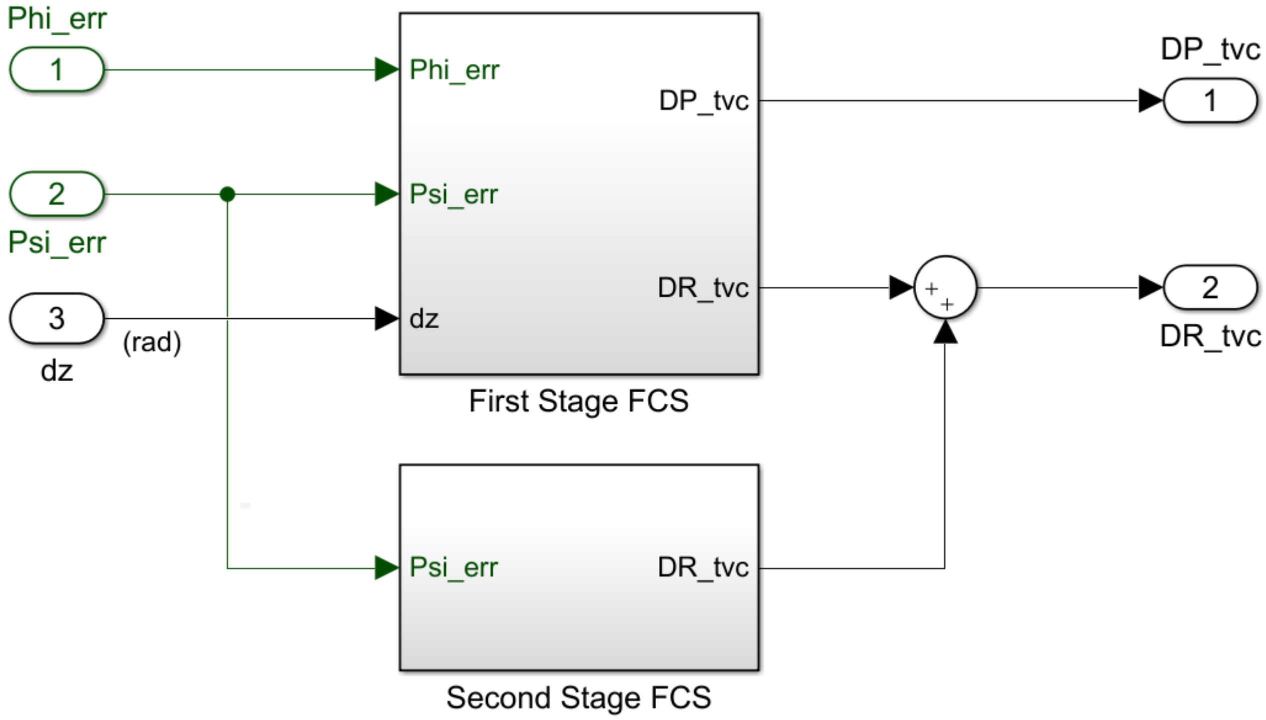


Figure 6.24 Lateral Flight Control System Consisting of 1st Stage and 2nd Stage Subsystems

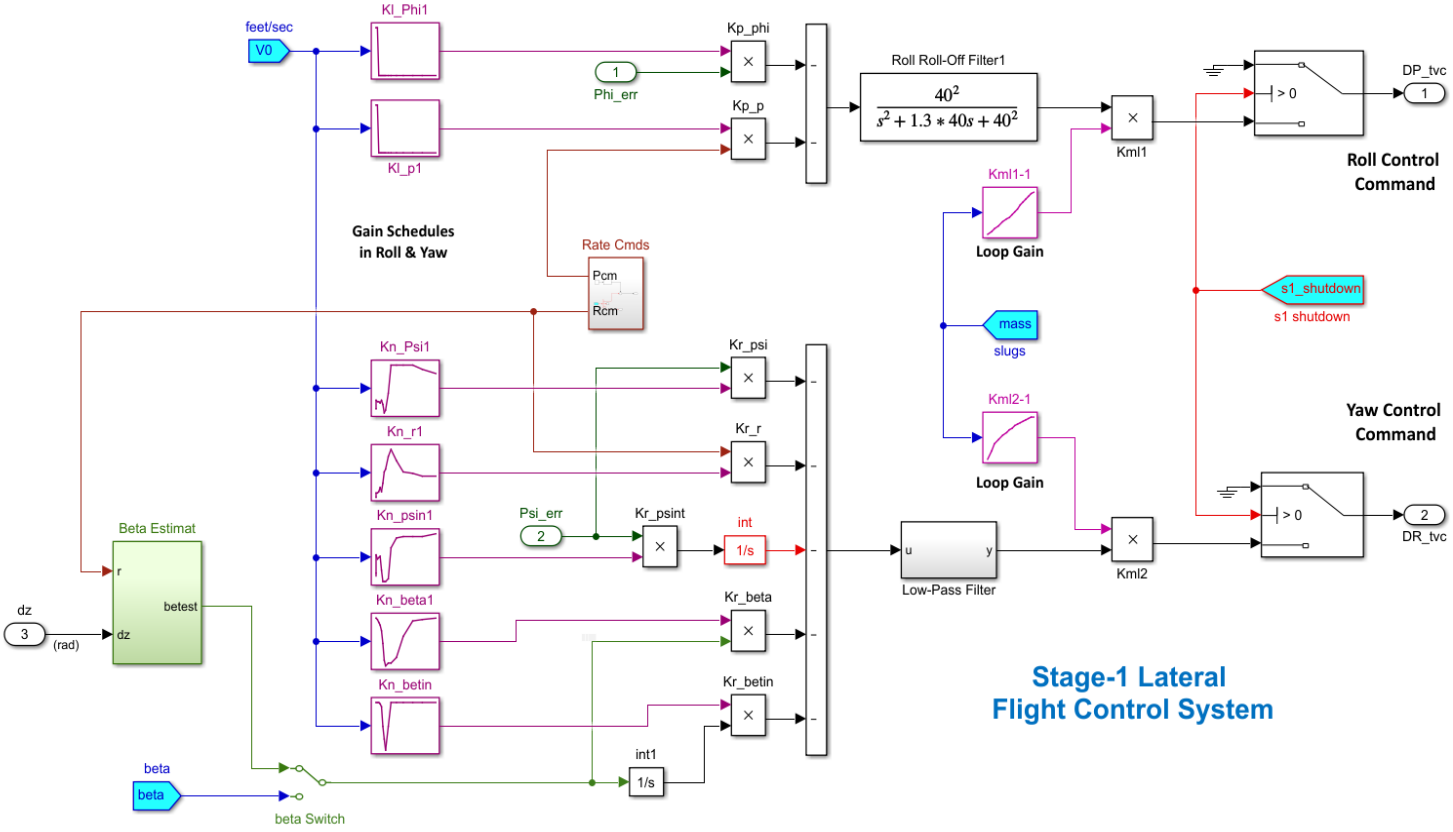


Figure 6.25 First Stage Lateral Flight Control System

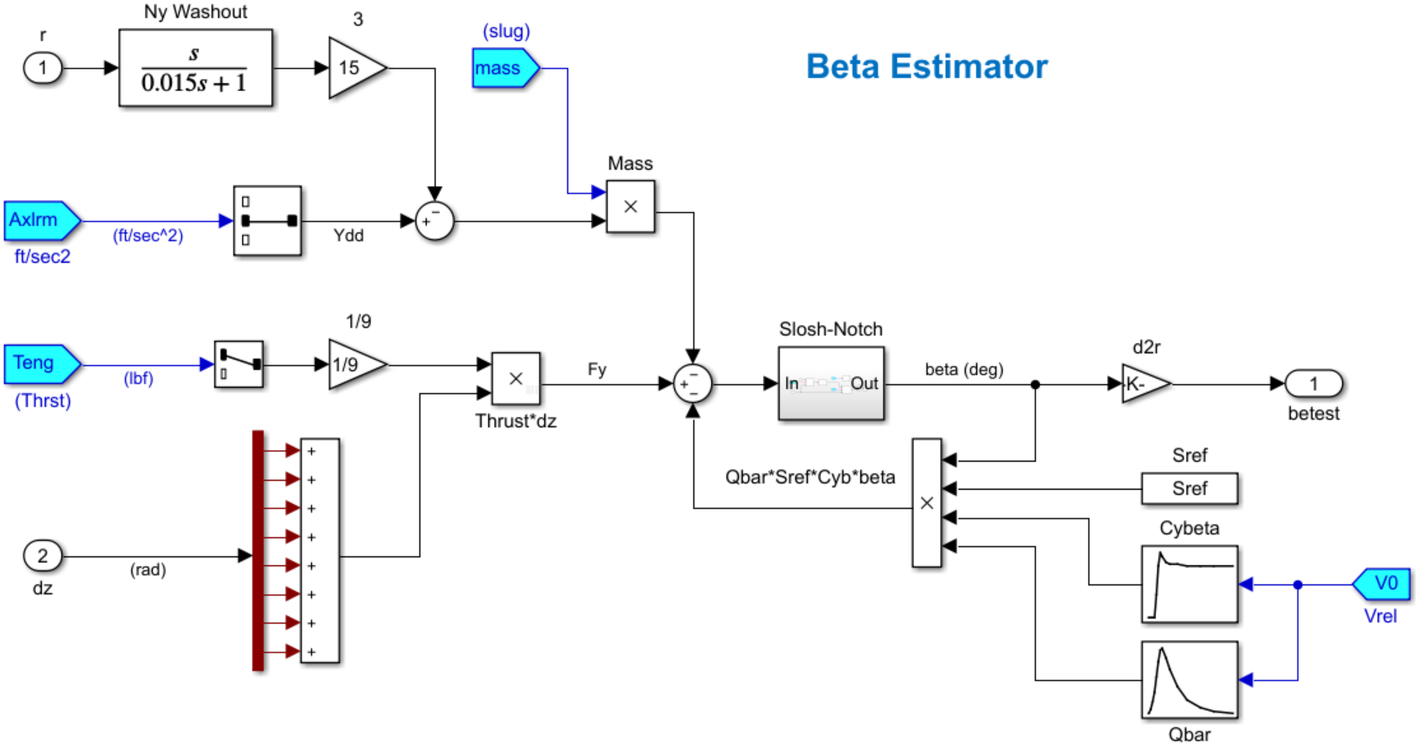


Figure 6.26 Angle of Sideslip Estimator

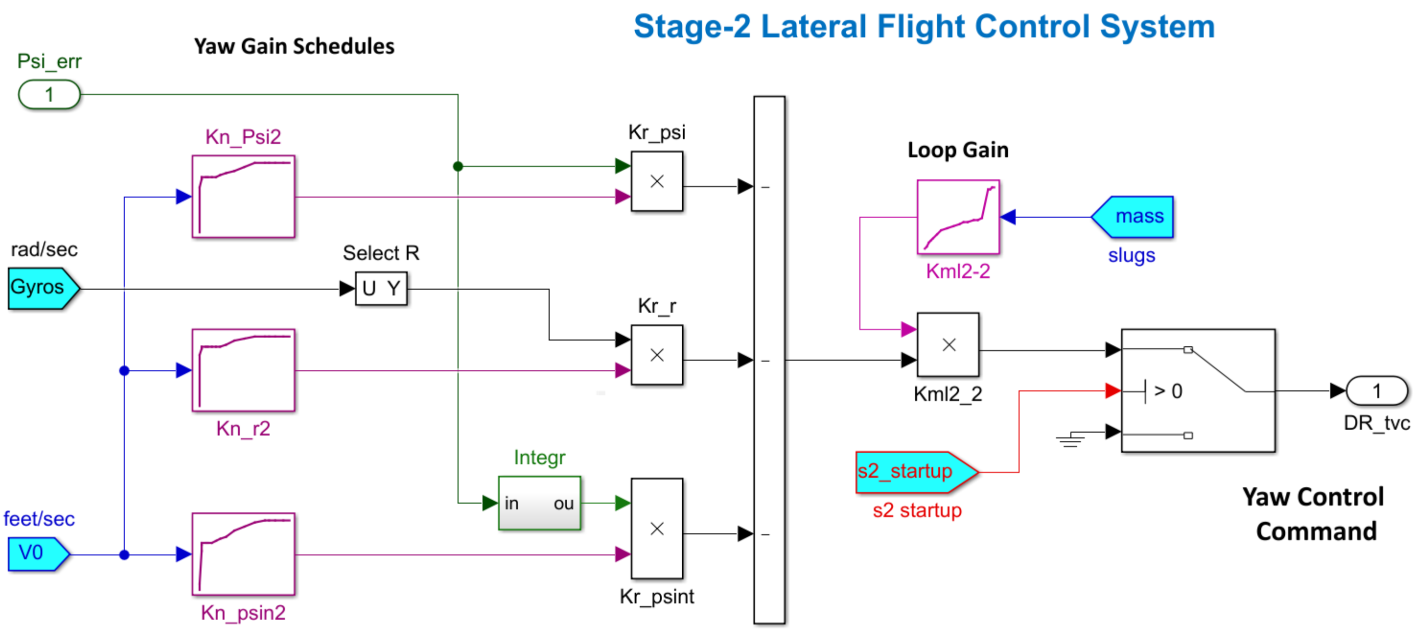


Figure 6.27 Second Stage Lateral FCS

The 2^{nd} stage FCS is shown in Figure 6.27. It controls only yaw (DR_tvc) because roll is controlled by the RCS during 2^{nd} stage. The state-feedback is from: yaw attitude ψ, yaw rate r, and ψ-integral. The LQR gains are scheduled as a function of the relative velocity V_0, and the yaw acceleration demand output is scheduled as a function of the loop-gain “Kml2-2” which varies with the mass properties and geometry.

6.7 Reaction Control System (RCS)

The RCS jets are used to control attitude during the coasting period between the first and second stage ignition. The jets are also used for roll control during stage-2. The RCS subsystem is shown in Figure 6.28. It includes a Phase-Plane logic and a Jet Selection logic that receives attitude errors and body rates.

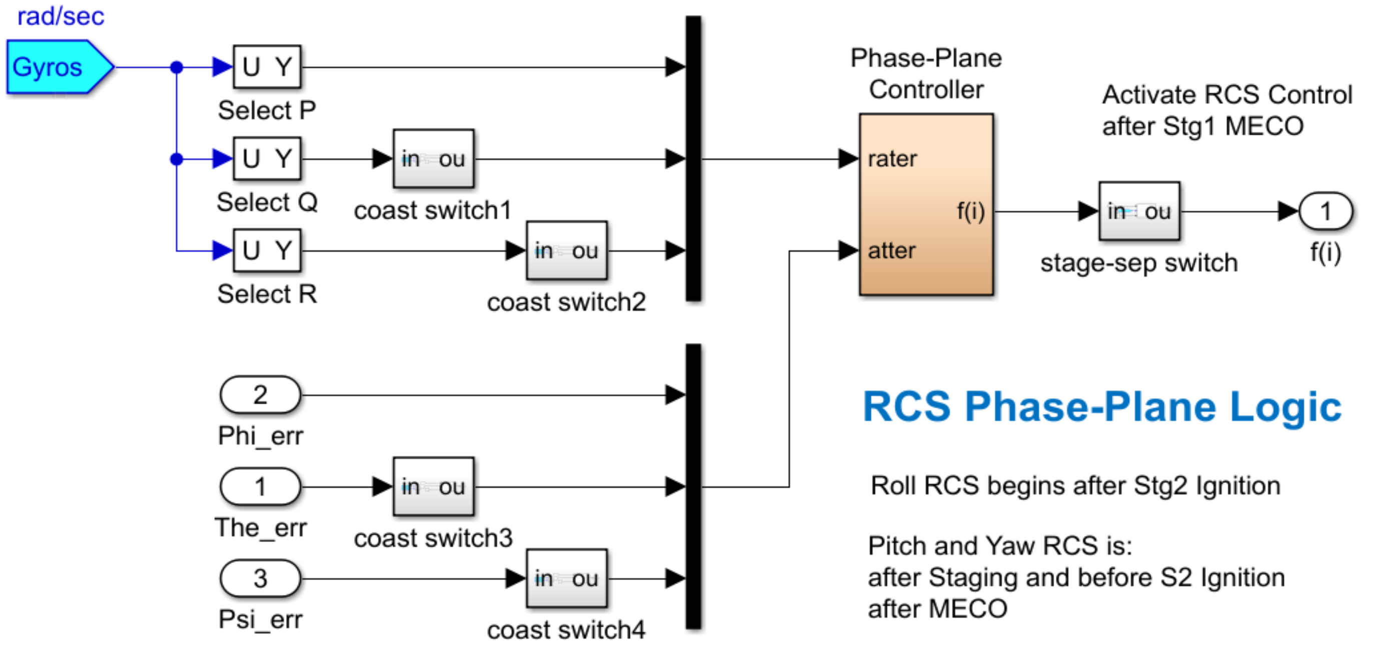
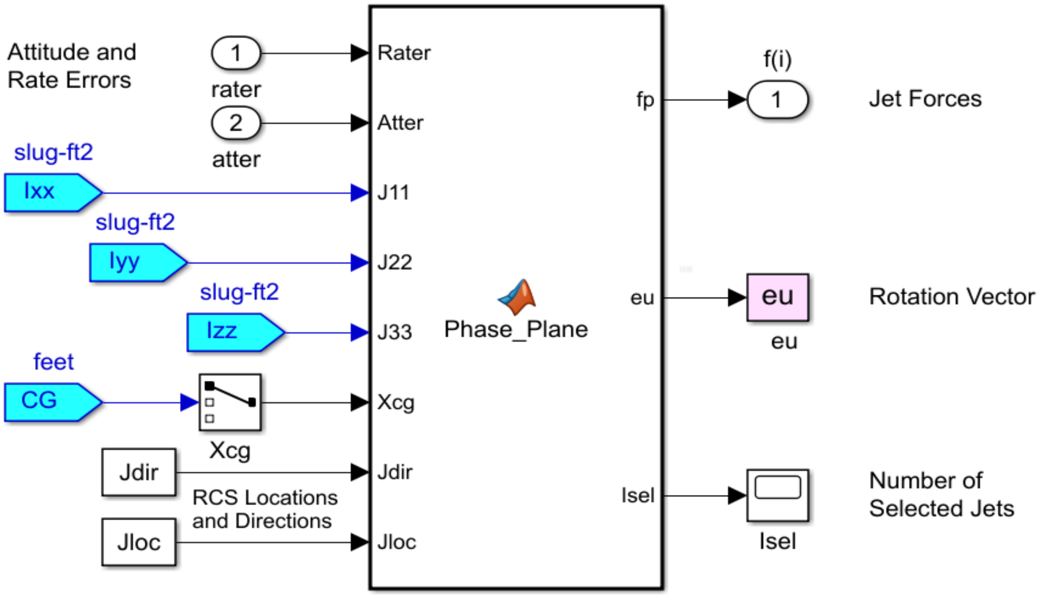
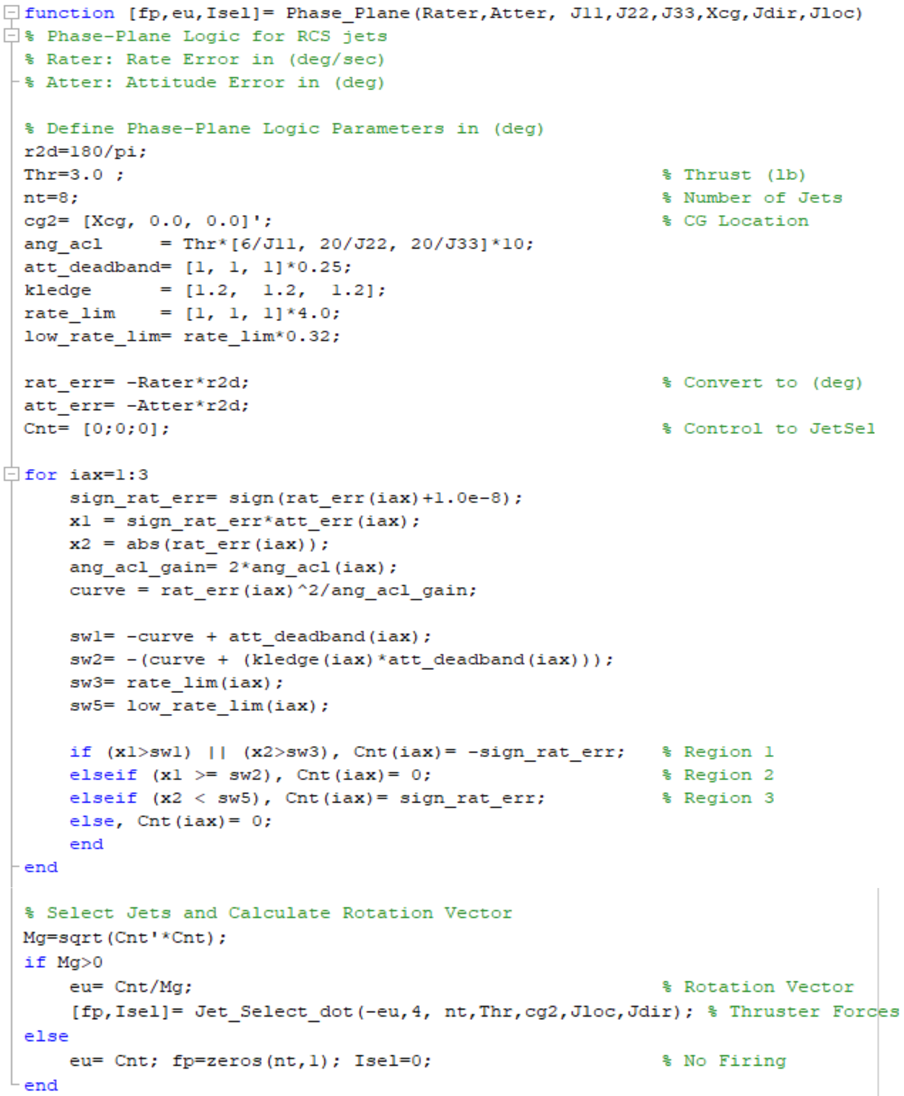


Figure 6.28 RCS Control System

Phase-Plane / Jet-Select Logic





```
function [fp,eu,Isel]= Phase_Plane(Rater,Atter, J11,J22,J33,Xcg,Jdir,Jloc)
% Phase-Plane Logic for RCS jets
% Rater: Rate Error in (deg/sec)
% Atter: Attitude Error in (deg)

% Define Phase-Plane Logic Parameters in (deg)
r2d=180/pi;
Thr=3.0 ;                                                % Thrust (lb)
nt=8;                                                    % Number of Jets
cg2= [Xcg, 0.0, 0.0]';                                   % CG Location
ang_acl     = Thr*[6/J11, 20/J22, 20/J33]*10;
att_deadband= [1, 1, 1]*0.25;
kledge      = [1.2,  1.2,  1.2];
rate_lim    = [1, 1, 1]*4.0;
low_rate_lim= rate_lim*0.32;

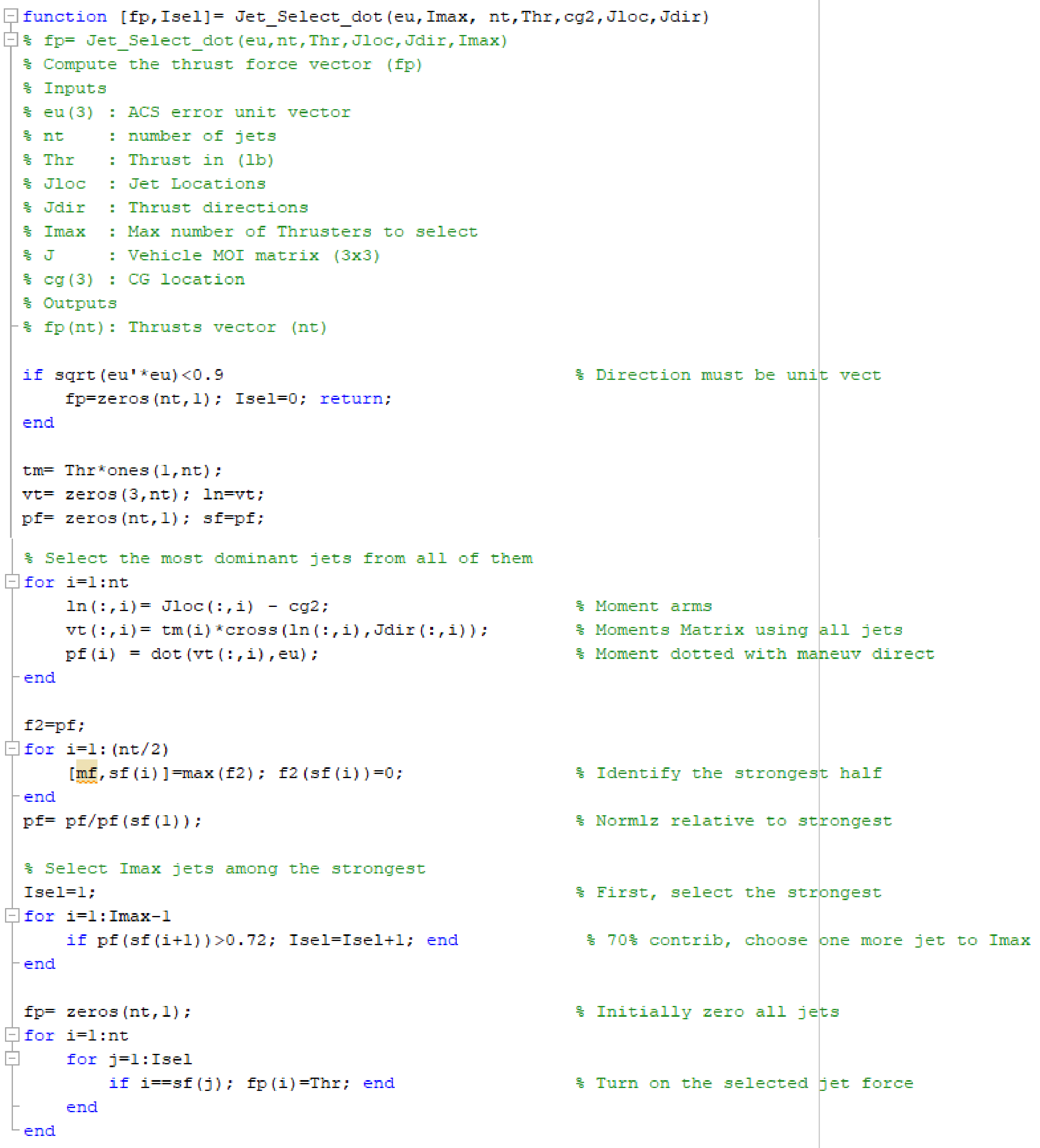
rat_err= -Rater*r2d;                                     % Convert to (deg)
att_err= -Atter*r2d;
Cnt= [0;0;0];                                            % Control to JetSel

for iax=1:3
    sign_rat_err= sign(rat_err(iax)+1.0e-8);
    x1 = sign_rat_err*att_err(iax);
    x2 = abs(rat_err(iax));
    ang_acl_gain= 2*ang_acl(iax);
    curve = rat_err(iax)^2/ang_acl_gain;

    sw1= -curve + att_deadband(iax);
    sw2= -(curve + (kledge(iax)*att_deadband(iax)));
    sw3= rate_lim(iax);
    sw5= low_rate_lim(iax);

    if (x1>sw1) || (x2>sw3), Cnt(iax)= -sign_rat_err;    % Region 1
    elseif (x1 >= sw2), Cnt(iax)= 0;                     % Region 2
    elseif (x2 < sw5), Cnt(iax)= sign_rat_err;           % Region 3
    else, Cnt(iax)= 0;
    end
end

% Select Jets and Calculate Rotation Vector
Mg=sqrt(Cnt'*Cnt);
if Mg>0
    eu= Cnt/Mg;                                          % Rotation Vector
    [fp,Isel]= Jet_Select_dot(-eu,4, nt,Thr,cg2,Jloc,Jdir); % Thruster Forces
else
    eu= Cnt; fp=zeros(nt,1); Isel=0;                     % No Firing
end
```



```
function [fp,Isel]= Jet_Select_dot(eu,Imax, nt,Thr,cg2,Jloc,Jdir)
% fp= Jet_Select_dot(eu,nt,Thr,Jloc,Jdir,Imax)
% Compute the thrust force vector (fp)
% Inputs
% eu(3)  : ACS error unit vector
% nt     : number of jets
% Thr    : Thrust in (lb)
% Jloc   : Jet Locations
% Jdir   : Thrust directions
% Imax   : Max number of Thrusters to select
% J      : Vehicle MOI matrix (3x3)
% cg(3)  : CG location
% Outputs
% fp(nt): Thrusts vector (nt)

if sqrt(eu'*eu)<0.9                                 % Direction must be unit vect
    fp=zeros(nt,1); Isel=0; return;
end

tm= Thr*ones(1,nt);
vt= zeros(3,nt); ln=vt;
pf= zeros(nt,1); sf=pf;

% Select the most dominant jets from all of them
for i=1:nt
    ln(:,i)= Jloc(:,i) - cg2;                       % Moment arms
    vt(:,i)= tm(i)*cross(ln(:,i),Jdir(:,i));        % Moments Matrix using all jets
    pf(i) = dot(vt(:,i),eu);                        % Moment dotted with maneuv direct
end

f2=pf;
for i=1:(nt/2)
    [mf,sf(i)]=max(f2); f2(sf(i))=0;                % Identify the strongest half
end
pf= pf/pf(sf(1));                                   % Normlz relative to strongest

% Select Imax jets among the strongest
Isel=1;                                             % First, select the strongest
for i=1:Imax-1
    if pf(sf(i+1))>0.72; Isel=Isel+1; end            % 70% contrib, choose one more jet to Imax
end

fp= zeros(nt,1);                                    % Initially zero all jets
for i=1:nt
    for j=1:Isel
        if i==sf(j); fp(i)=Thr; end                 % Turn on the selected jet force
    end
end
```

The phase-plane logic receives the vehicle attitude errors and the body rates, and calculates a unit vector (eu) that the vehicle must be rotated about. The RCS jet selection logic receives the rotational direction and the max number of jets to be selected (Imax) from the phase-plane logic. It uses the dot-product method to select a number of jets (Isel ≤ Imax) that contribute sufficiently in the required direction and it calculates the jet forces vector (fp). The unselected jet forces are set to zero. The logic selects the best contributing jet along (eu), plus other jets that can provide at least 72% of the best contributor torque.

6.8 Forces and Moments System

The forces and moments system in Figure 6.29 combines the propulsion forces and moments with the aerodynamic forces and moments and also the slosh forces and moments generated by separate subsystems and it applies them to the vehicle block. It also generates the linear acceleration vector along x, y, z. It consists of 3 subsystems: the propulsion, the aerodynamics, and the slosh models.

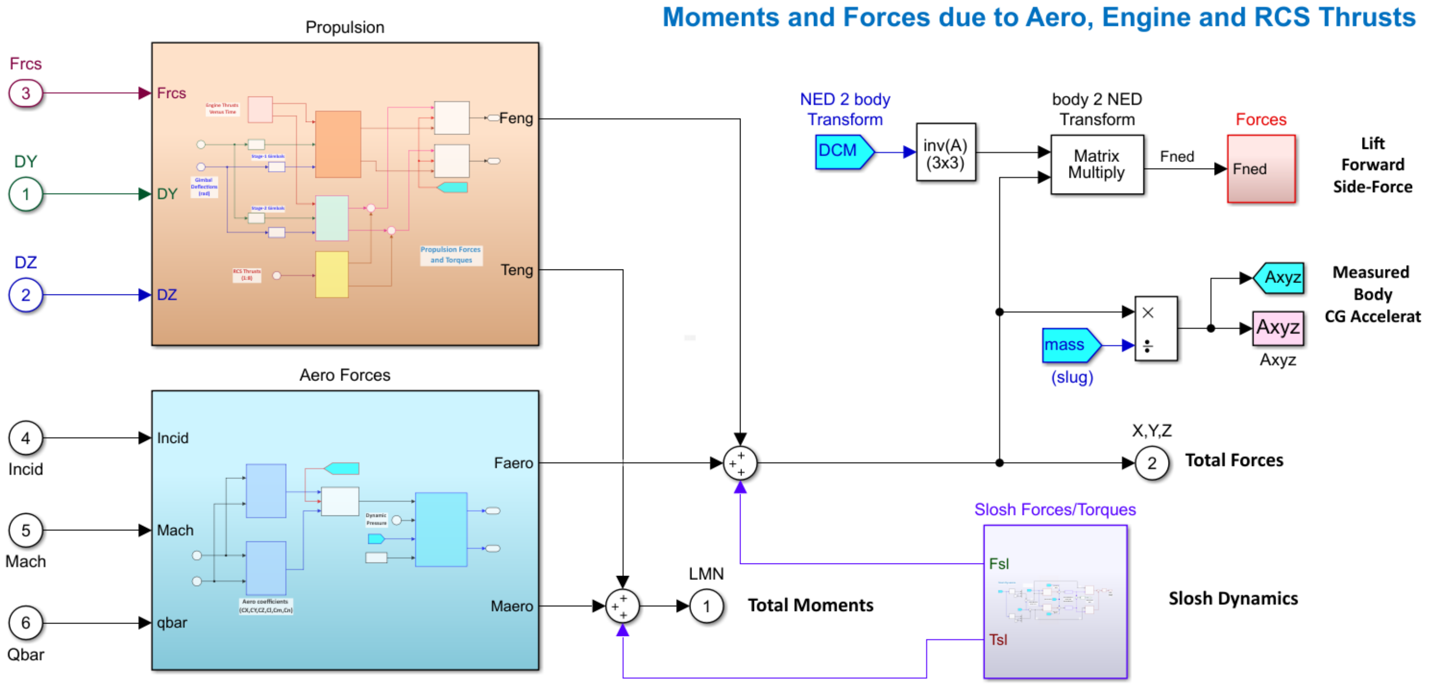


Figure 6.29 Forces and Moments System

6.8.1 Propulsion System

The propulsion block is shown in Figure 6.30. It generates the forces and torques on the vehicle generated by the TVC engines and the RCS jets. It includes several subsystems. A subsystem that calculates the engine thrusts for the 1st and 2nd stages and subsystems that calculate the engine and RCS x, y, z forces and torques. The force and torque vectors are combined and applied to the vehicle. It also includes logic that switches the forces and torque vectors from 1st to 2nd stage. The engine thrust calculation logic in Figure 6.31 calculates the 1st and 2nd stage TVC engines thrust as a function of the atmospheric pressure, the fuel flow rate and the ISP. It includes flags that switch between 1st and 2nd stages.

The subsystem in Figure 6.32 calculates the forces and moments applied to the vehicle by the 8 TVC engines and the fixed 9th engine which does not gimbal. The Forces and Moments of each engine are calculated by the 9 subsystems as a function of the corresponding gimbal angles (δy, δz). Similarly, the subsystem in Figure 6.33 calculates the forces and moments applied to the vehicle by the second stage TVC engine. The subsystem in Figure 6.34 calculates the forces and moments on the vehicle which are created by the 8 RCS jets located around the 2nd stage TVC engine and their thrusts are tangential to the circumference generating forces along the ±Y and the ±Z axis.

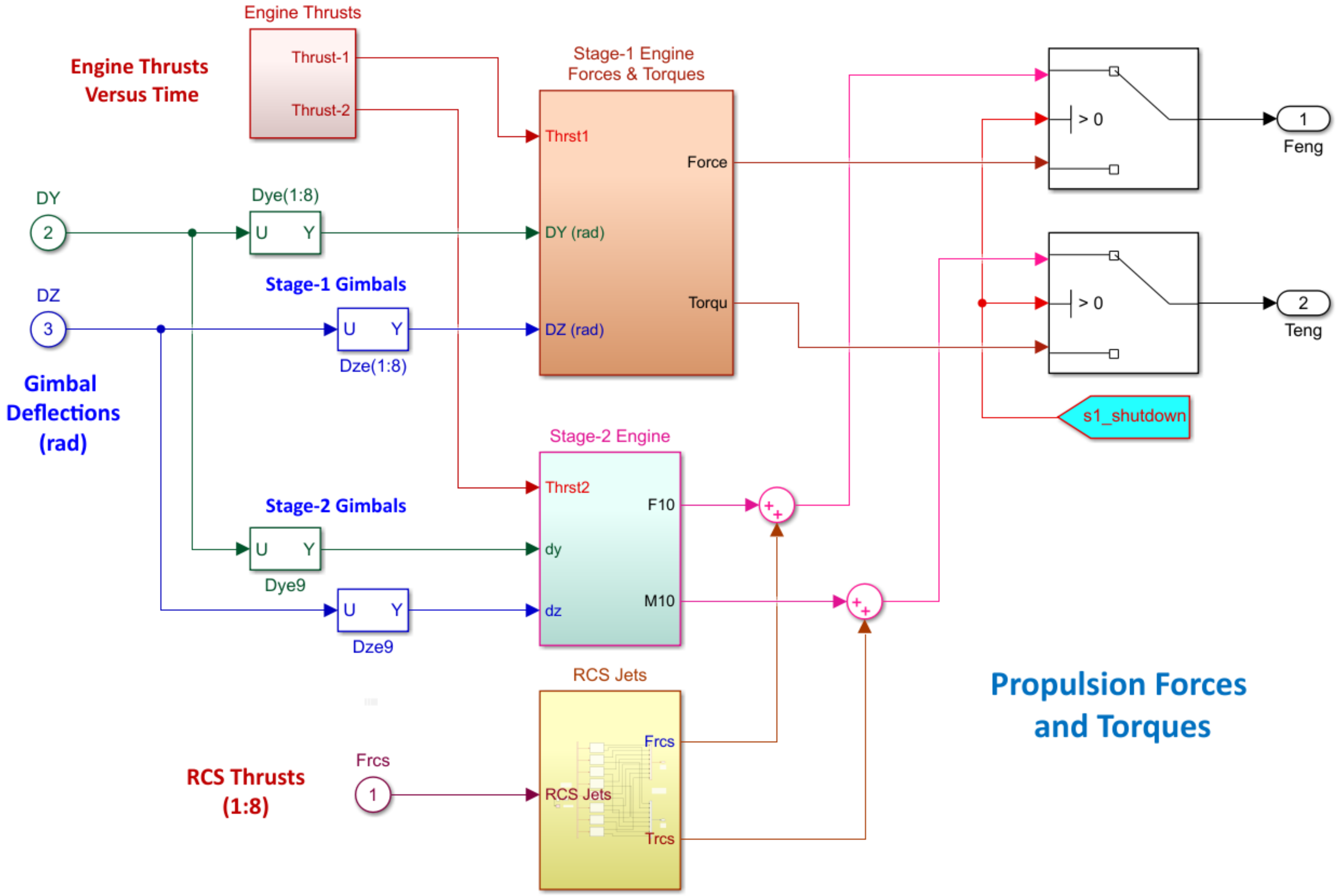


Figure 6.30 Propulsion System

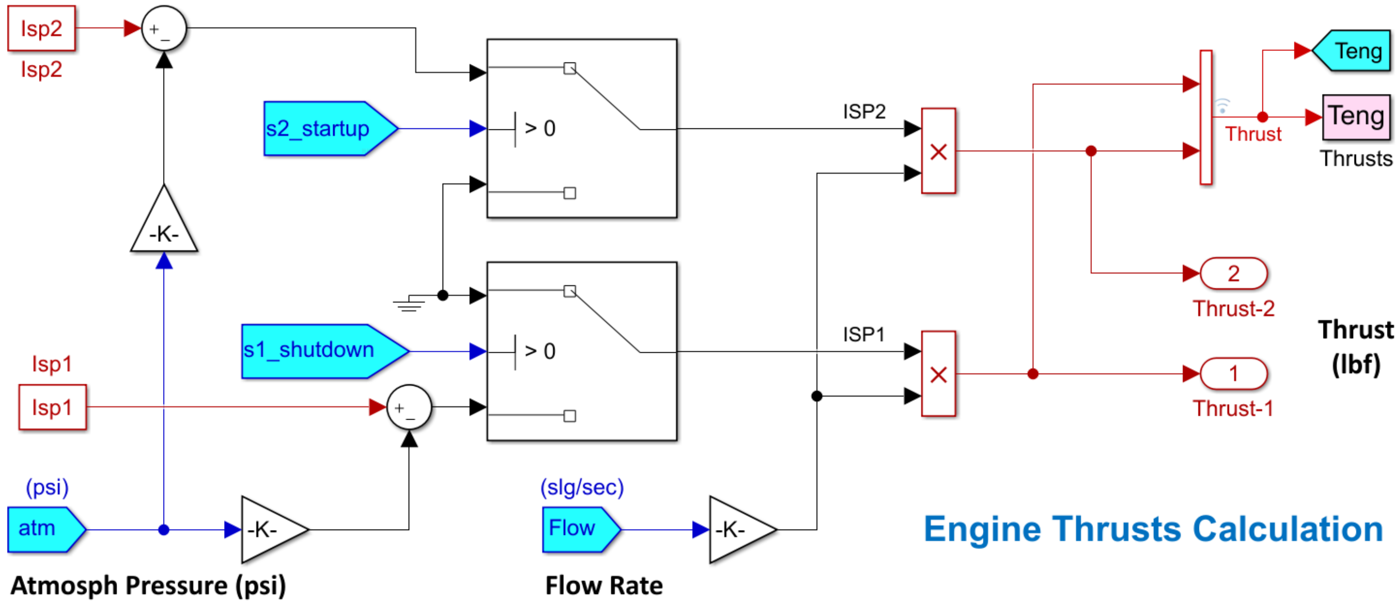


Figure 6.31 Engines Thrust Calculation Subsystem

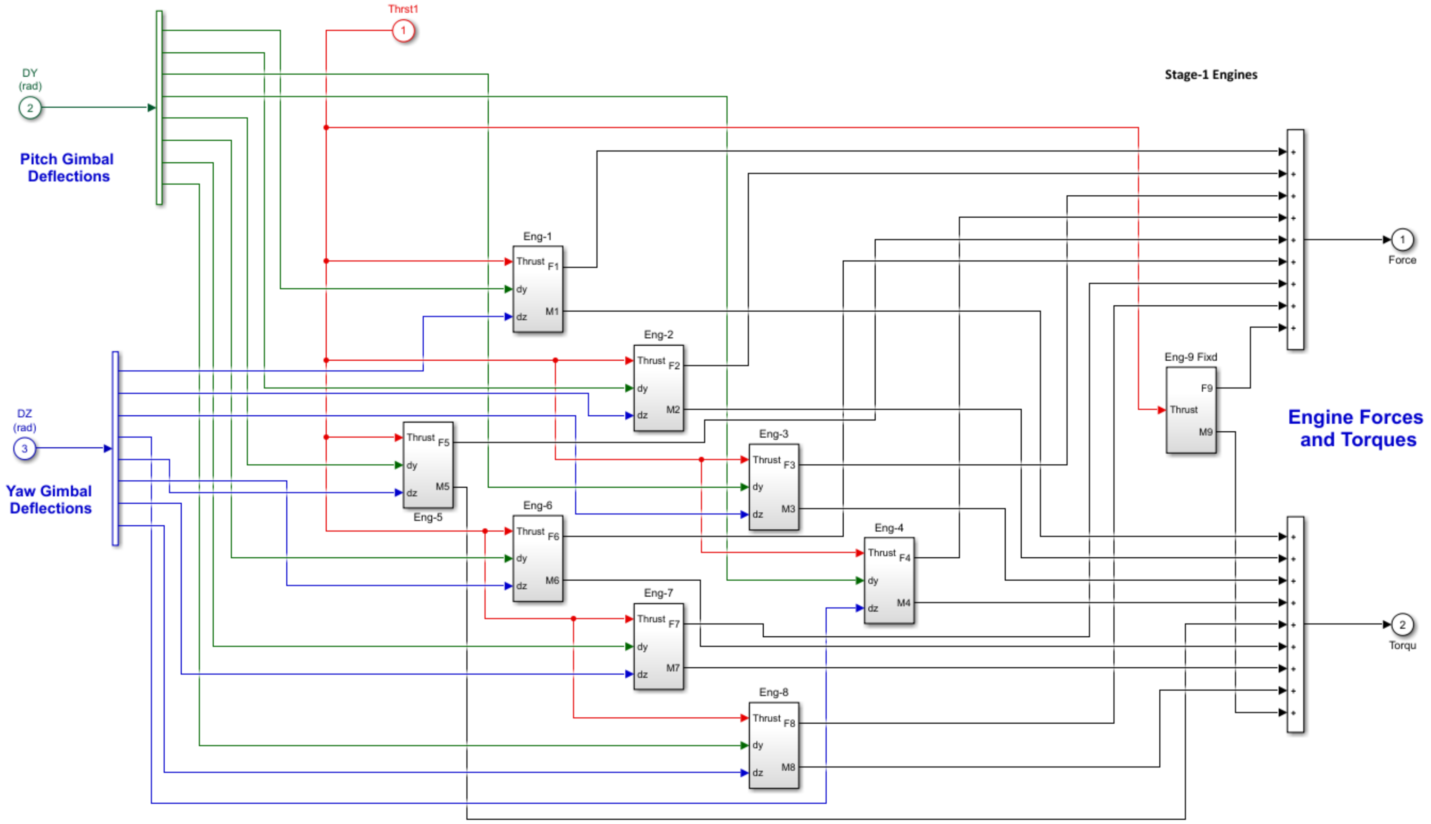
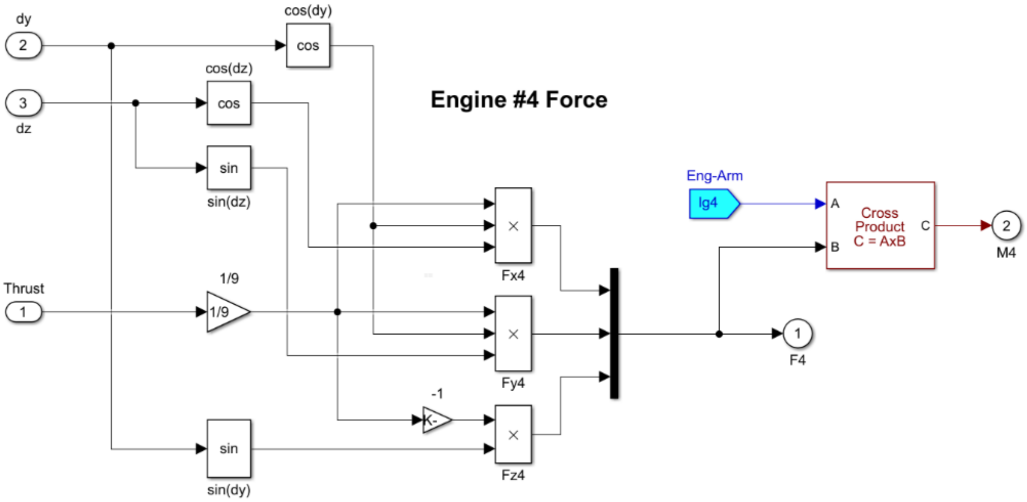
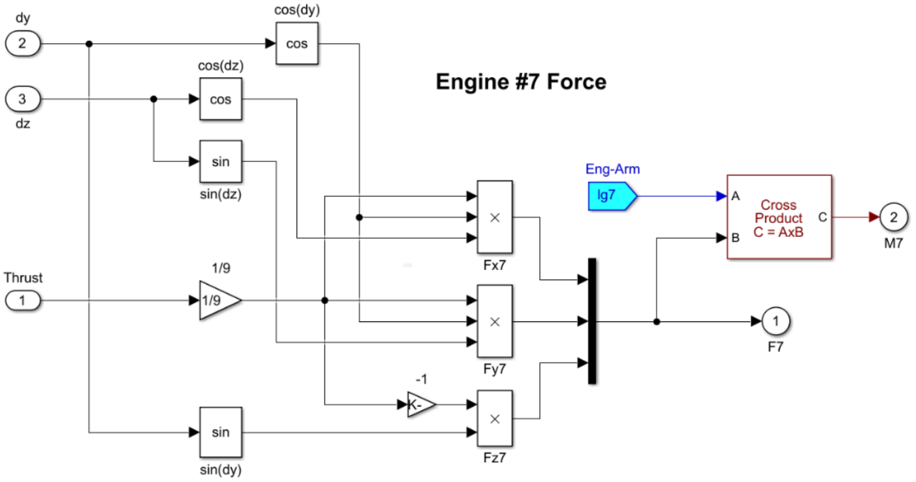


Figure 6.32 First Stage TVC Engine Forces and Moments



The Forces and Moments of each engine are calculated as a function of the Gimbal Angles (δy, δz)



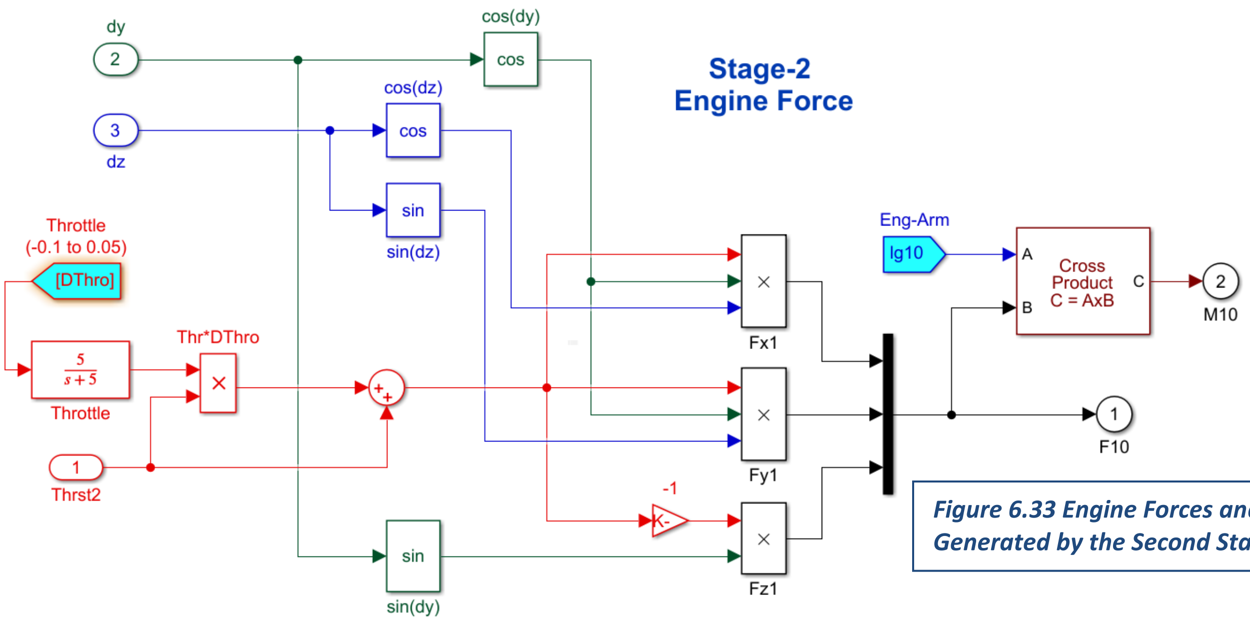


Figure 6.33 Engine Forces and Moments Generated by the Second Stage TVC

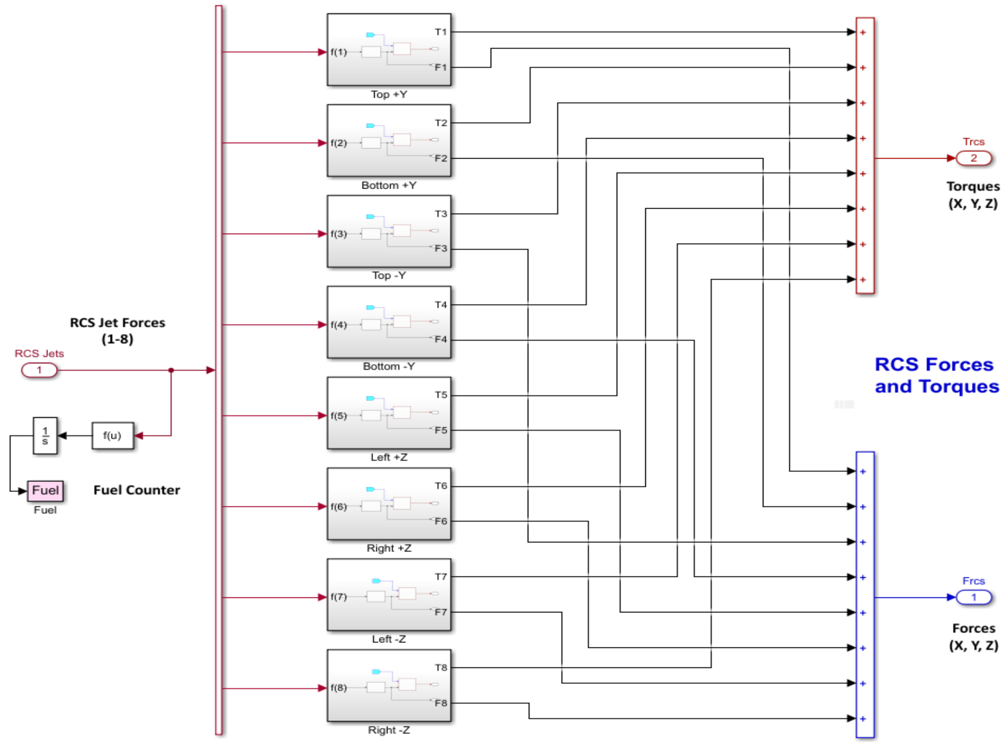
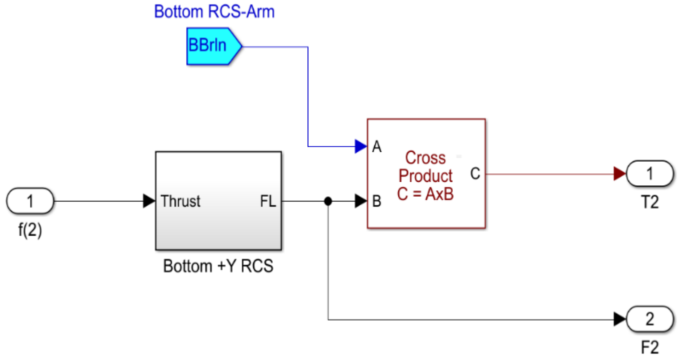
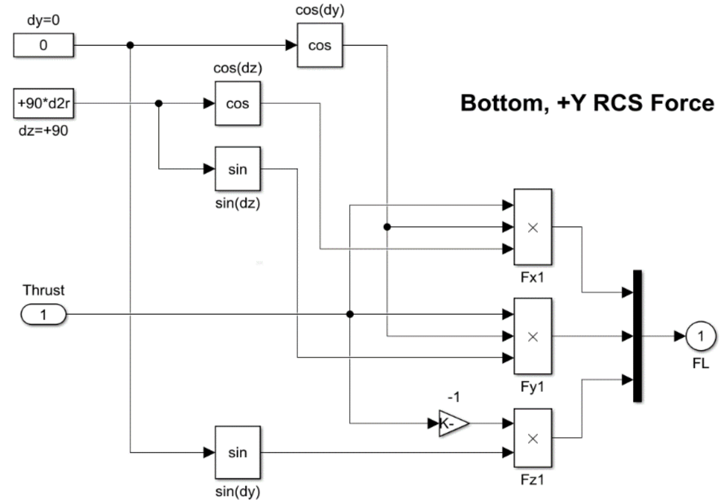


Figure 6.34, Forces and Moments on the Vehicle Generated by the RCS Jets

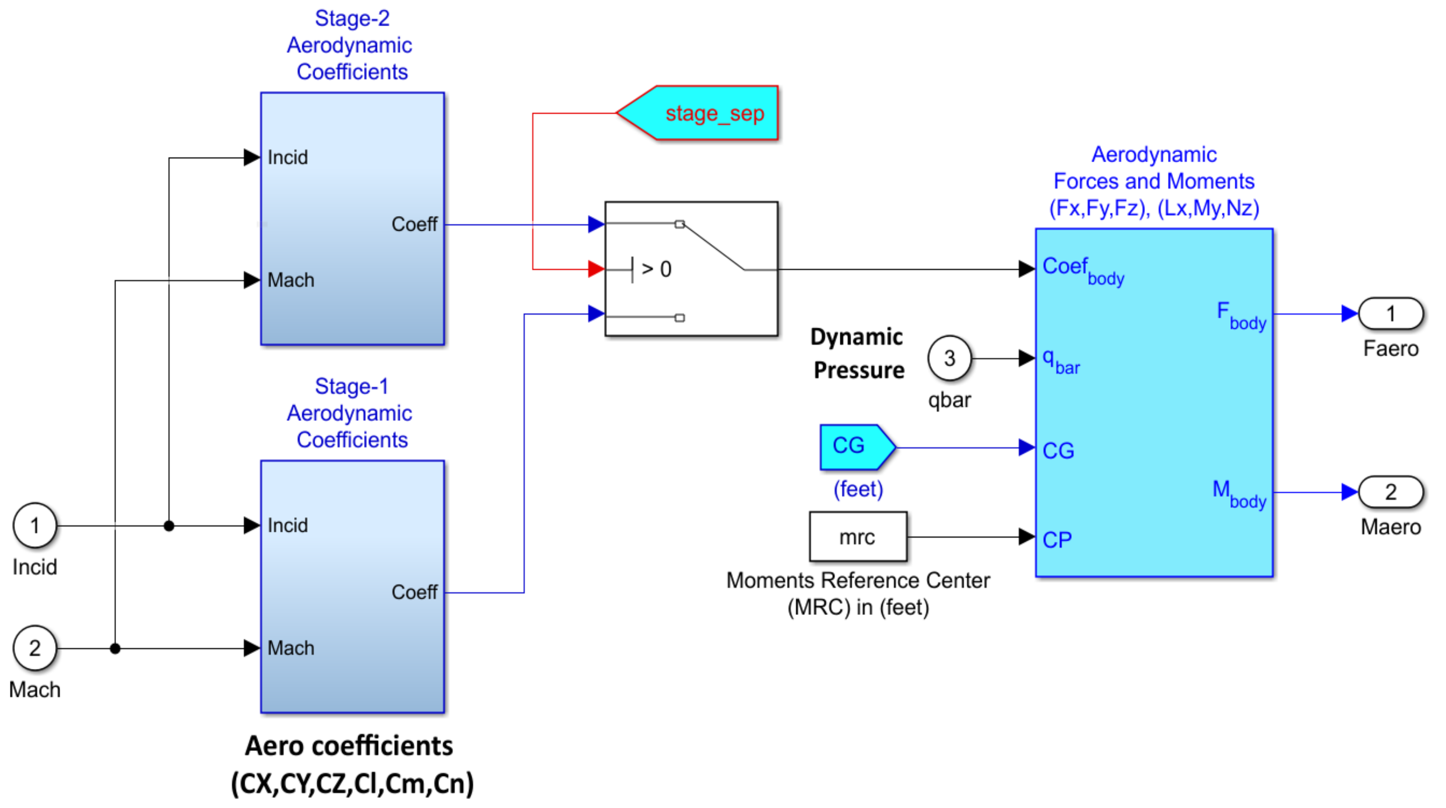
Each RCS Jet Produces a Force and a Torque Vector





6.8.2 Aerodynamic Forces and Moments

The subsystem in Figure 6.35 calculates the aerodynamic forces and moments for the 1st and 2nd stages. The aero coefficients are included in tables and they are interpolated as a function of the angles of attack and sideslip (α, β) and also as a function of the Mach number. The stage-separation flag switches the aero-data from 1st to 2nd stage look-up tables. The aero coefficients moments are computed relative to a fixed reference point, the MRC, and they are adjusted to be about the CG. A separate Simulink function on the RHS generates the forces and moments. It converts the coefficients from MRC to the vehicle CG and it uses the dynamic pressure, reference area and length to calculate the forces. The aero-data interpolation functions for 1st and 2nd stages are shown in detail in Figure 6.36.



Aero Coefficients funct. of: Alpha, Beta, Mach#

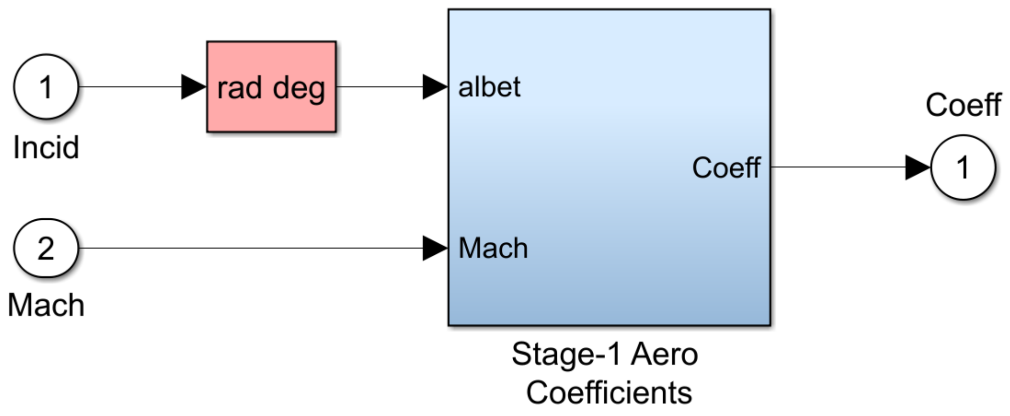
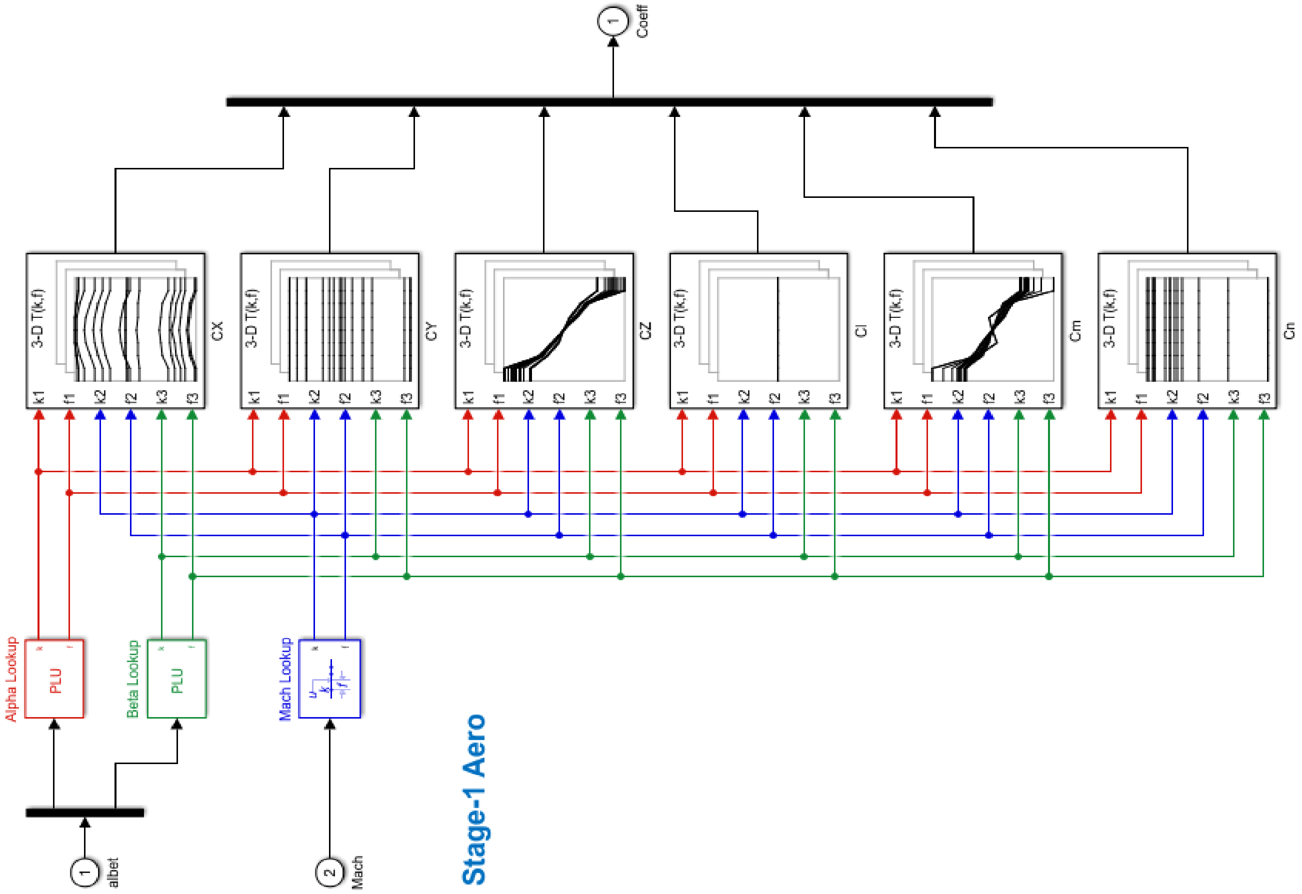


Figure 6.35 Aerodynamic Forces and Moments Block



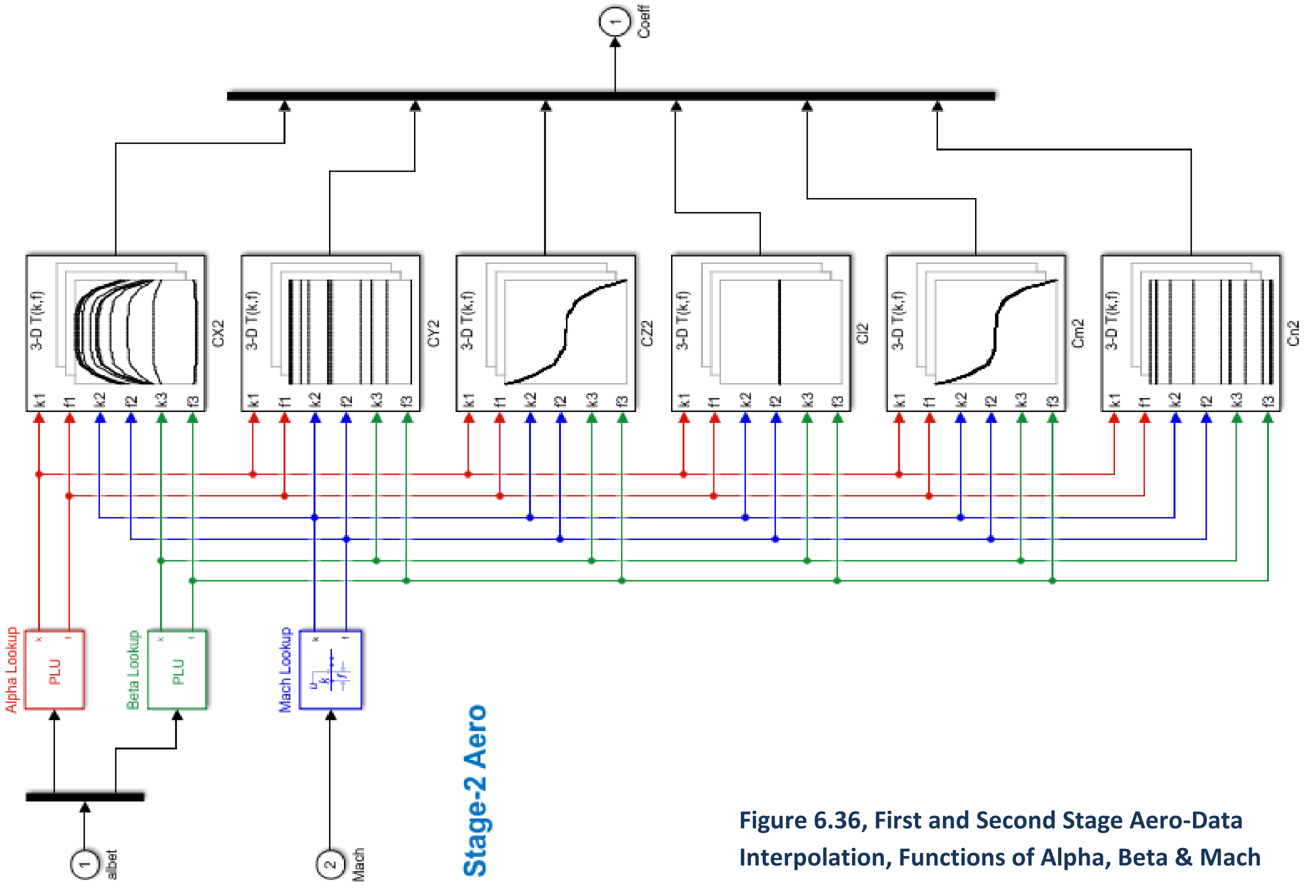


Figure 6.36, First and Second Stage Aero-Data Interpolation, Functions of Alpha, Beta & Mach

6.9 Propellant Sloshing Model

The non-linear, spherical, elastic pendulum, propellant sloshing model is included in this simulation. It is described in the vehicle equations chapter in Section 2.6, where the slosh mass is attached by an elastic string from the center of a sphere, see Figures (2.6.9 to 2.6.11) and equations for details. The mass can swing like a pendulum, spin around the tank, or bounce against the tank wall when its distance from the center exceeds the elastic pendulum length. The motion of the slosh mass relative to the tank is excited by vehicle accelerations and when it moves, it generates forces against the tank walls which are applied back to the vehicle at the center of the sphere. The slosh subsystem is shown in Figure 6.37, and it includes 2 tanks for the LOX and the LH2 propellants. The slosh mass locations (X_{lox}, X_{fuel}) are needed to calculate the moment arms and they are scheduled with vehicle mass. The inputs to the slosh models are vehicle accelerations at the slosh mass. The outputs are forces and moments produced by the motion of the mass. The non-linear dynamic motion of the slosh mass relative to the tank is implemented as a Matlab function, shown in Figures 6.38 and 6.39. The additional input parameters: slosh mass, pendulum length, and damping coefficient, are scheduled as a function of vehicle mass.

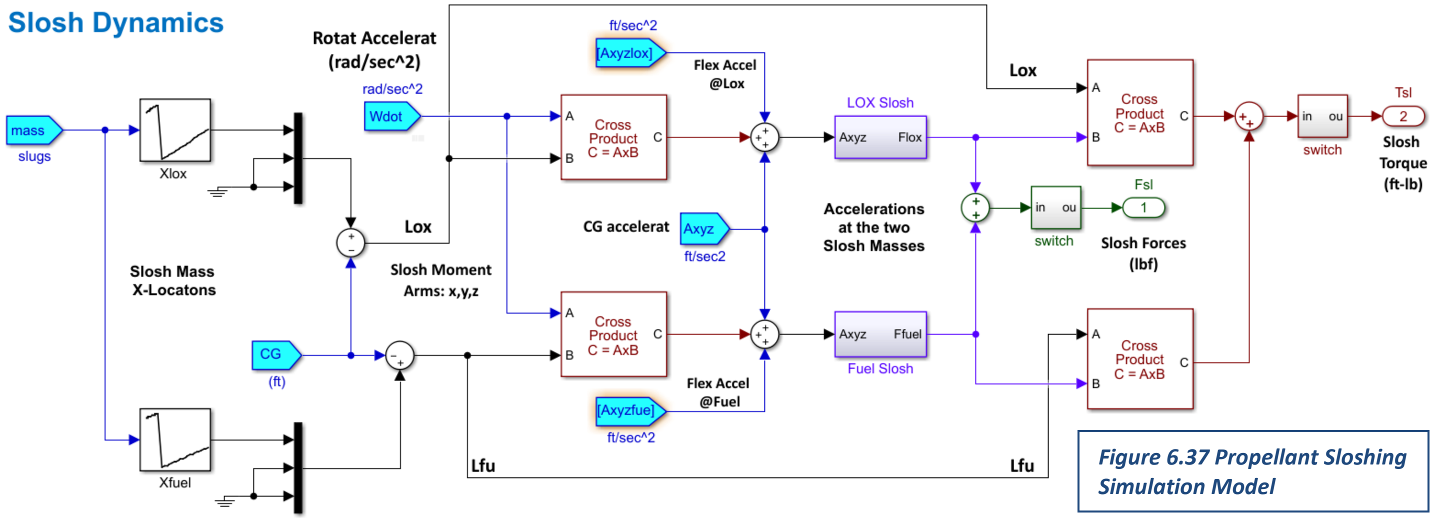


Figure 6.37 Propellant Sloshing Simulation Model

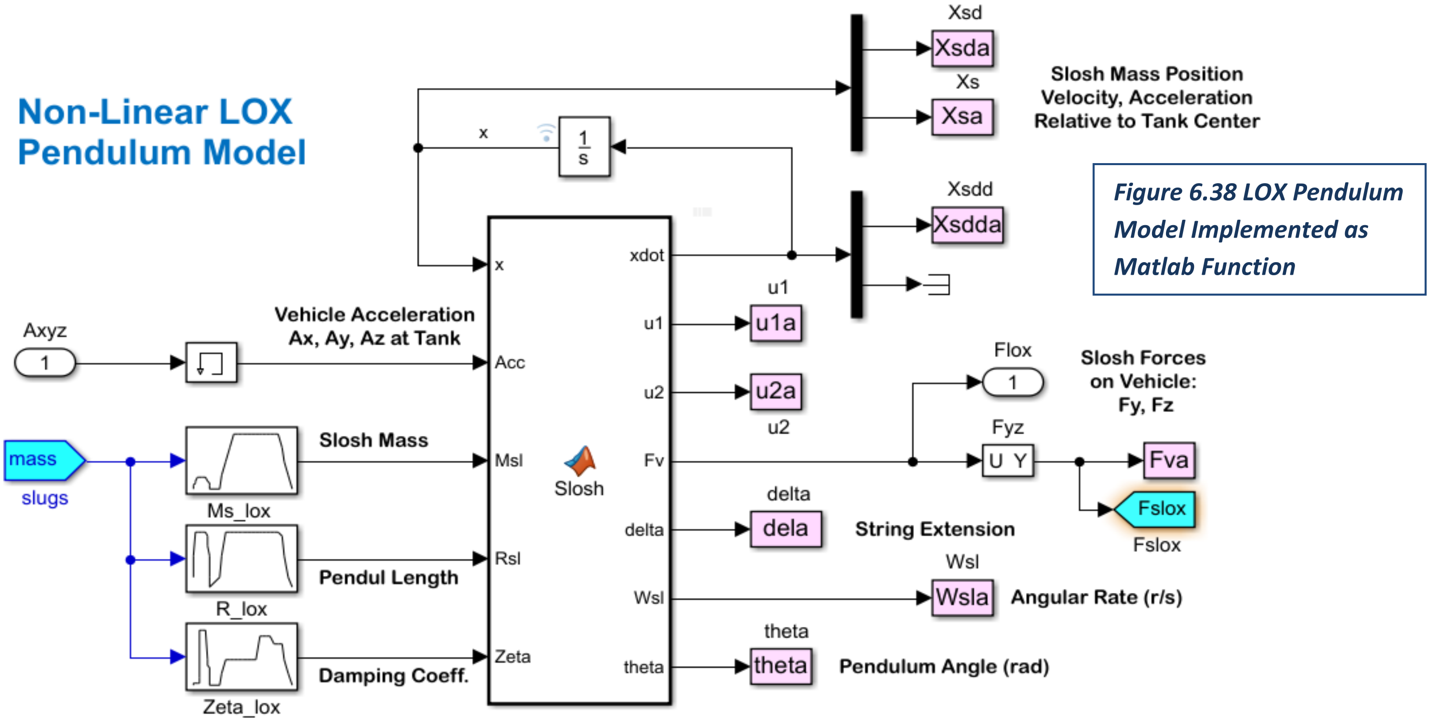
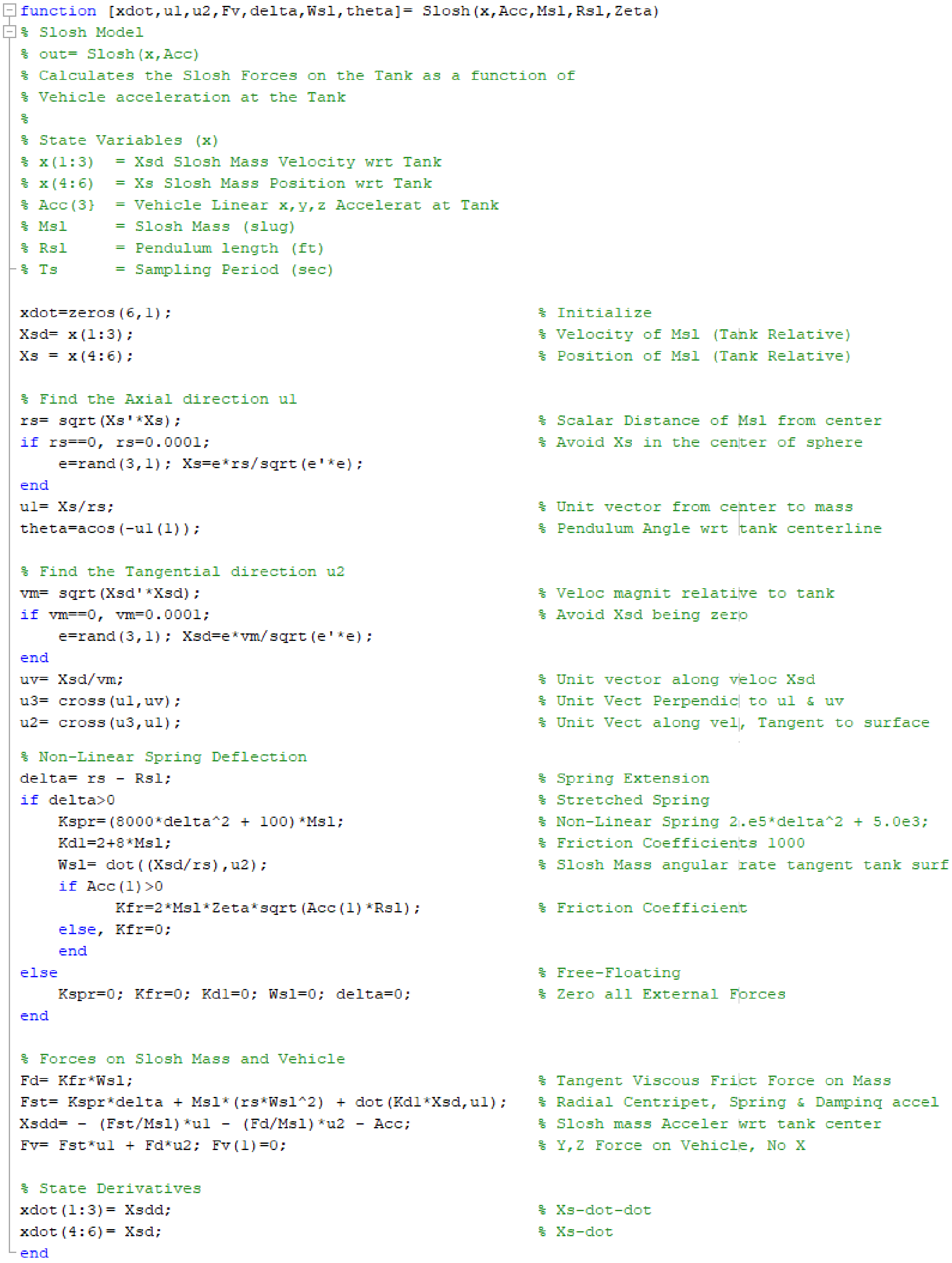


Figure 6.38 LOX Pendulum Model Implemented as Matlab Function

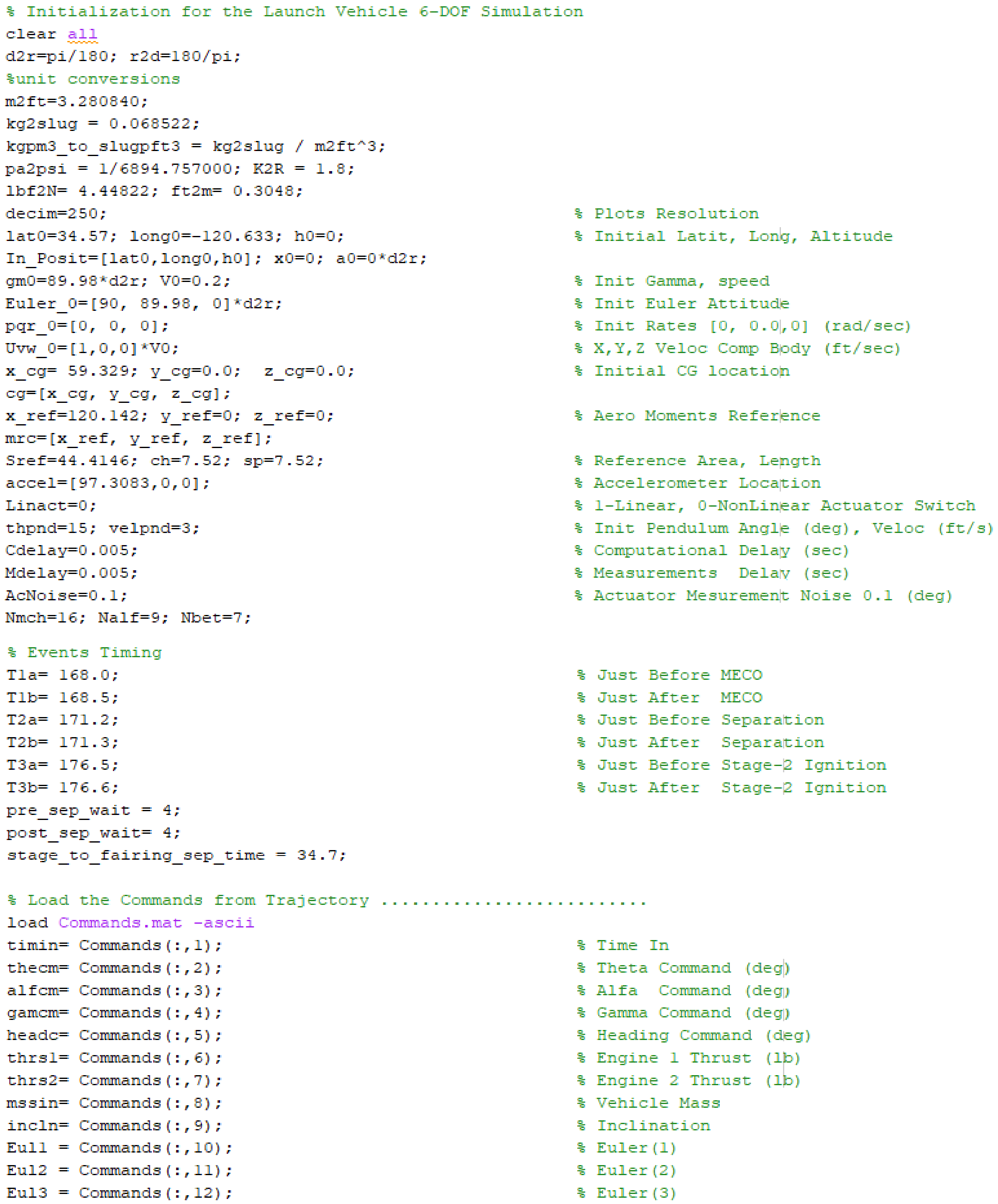


```matlab
function [xdot,u1,u2,Fv,delta,Wsl,theta]= Slosh(x,Acc,Msl,Rsl,Zeta)
% Slosh Model
% out= Slosh(x,Acc)
% Calculates the Slosh Forces on the Tank as a function of
% Vehicle acceleration at the Tank
%
% State Variables (x)
% x(1:3)  = Xsd Slosh Mass Velocity wrt Tank
% x(4:6)  = Xs Slosh Mass Position wrt Tank
% Acc(3}  = Vehicle Linear x,y,z Accelerat at Tank
% Msl     = Slosh Mass (slug)
% Rsl     = Pendulum length (ft)
% Ts      = Sampling Period (sec)

xdot=zeros(6,1);                                      % Initialize
Xsd= x(1:3);                                          % Velocity of Msl (Tank Relative)
Xs = x(4:6);                                          % Position of Msl (Tank Relative)

% Find the Axial direction u1
rs= sqrt(Xs'*Xs);                                     % Scalar Distance of Msl from center
if rs==0, rs=0.0001;                                  % Avoid Xs in the center of sphere
    e=rand(3,1); Xs=e*rs/sqrt(e'*e);
end
u1= Xs/rs;                                            % Unit vector from center to mass
theta=acos(-u1(1));                                   % Pendulum Angle wrt tank centerline

% Find the Tangential direction u2
vm= sqrt(Xsd'*Xsd);                                   % Veloc magnit relative to tank
if vm==0, vm=0.0001;                                  % Avoid Xsd being zero
    e=rand(3,1); Xsd=e*vm/sqrt(e'*e);
end
uv= Xsd/vm;                                           % Unit vector along veloc Xsd
u3= cross(u1,uv);                                     % Unit Vect Perpendic to u1 & uv
u2= cross(u3,u1);                                     % Unit Vect along vel, Tangent to surface

% Non-Linear Spring Deflection
delta= rs - Rsl;                                      % Spring Extension
if delta>0                                            % Stretched Spring
    Kspr=(8000*delta^2 + 100)*Msl;                    % Non-Linear Spring 2.e5*delta^2 + 5.0e3;
    Kdl=2+8*Msl;                                      % Friction Coefficients 1000
    Wsl= dot((Xsd/rs),u2);                            % Slosh Mass angular rate tangent tank surf
    if Acc(1)>0
          Kfr=2*Msl*Zeta*sqrt(Acc(1)*Rsl);            % Friction Coefficient
    else, Kfr=0;
    end
else                                                  % Free-Floating
    Kspr=0; Kfr=0; Kdl=0; Wsl=0; delta=0;             % Zero all External Forces
end

% Forces on Slosh Mass and Vehicle
Fd= Kfr*Wsl;                                          % Tangent Viscous Frict Force on Mass
Fst= Kspr*delta + Msl*(rs*Wsl^2) + dot(Kdl*Xsd,u1);   % Radial Centripet, Spring & Damping accel
Xsdd= - (Fst/Msl)*u1 - (Fd/Msl)*u2 - Acc;             % Slosh mass Acceler wrt tank center
Fv= Fst*u1 + Fd*u2; Fv(1)=0;                          % Y,Z Force on Vehicle, No X

% State Derivatives
xdot(1:3)= Xsdd;                                      % Xs-dot-dot
xdot(4:6)= Xsd;                                       % Xs-dot
end
```

Figure 6.39 Slosh Dynamics Implemented as a Matlab Function

6.10 Initialization and Other Data Files

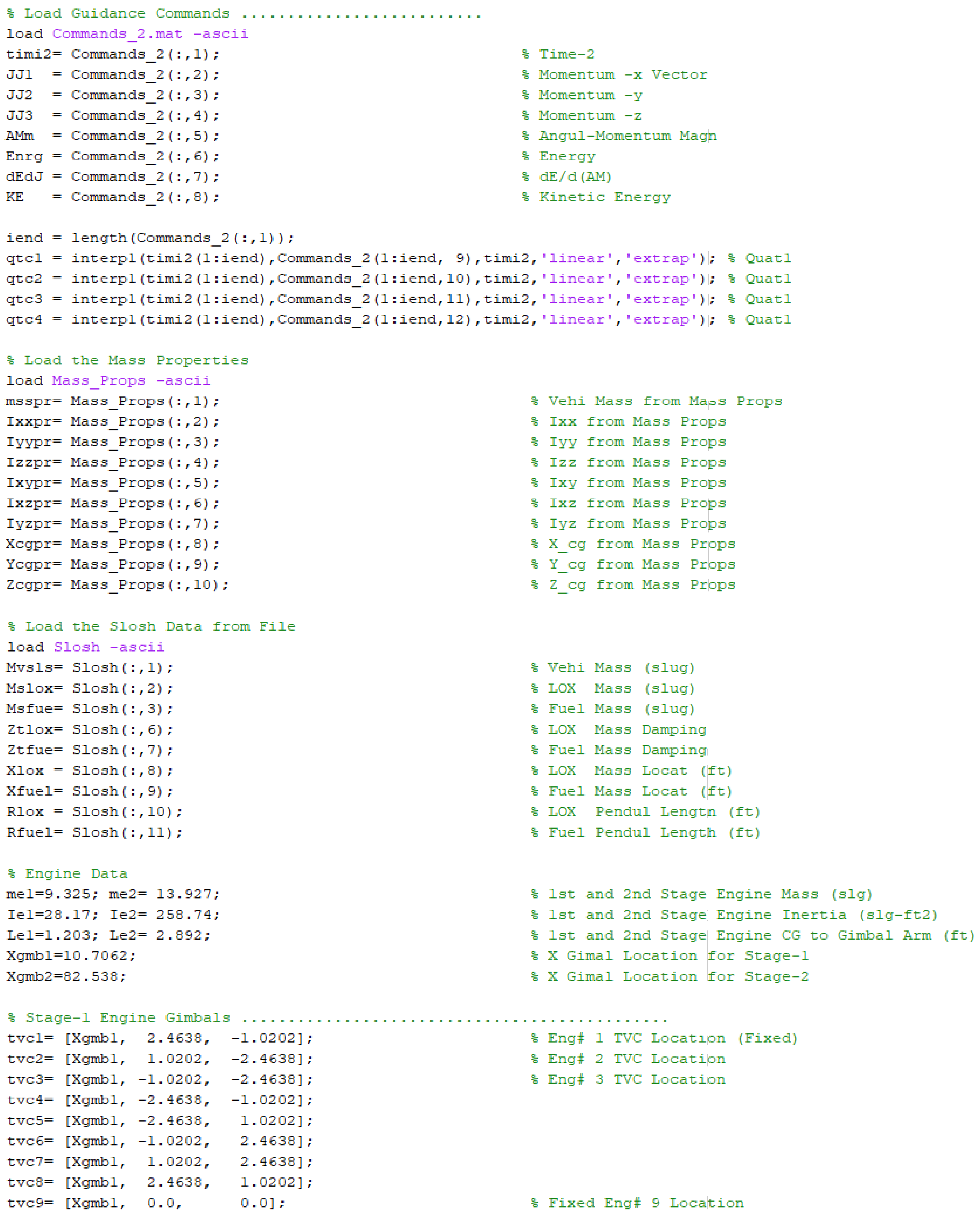
The simulation is initialized from file “init.m”.



```
% Initialization for the Launch Vehicle 6-DOF Simulation
clear all
d2r=pi/180; r2d=180/pi;
%unit conversions
m2ft=3.280840;
kg2slug = 0.068522;
kgpm3_to_slugpft3 = kg2slug / m2ft^3;
pa2psi = 1/6894.757000; K2R = 1.8;
lbf2N= 4.44822; ft2m= 0.3048;
decim=250;                                             % Plots Resolution
lat0=34.57; long0=-120.633; h0=0;                      % Initial Latit, Long, Altitude
In_Posit=[lat0,long0,h0]; x0=0; a0=0*d2r;
gm0=89.98*d2r; V0=0.2;                                 % Init Gamma, speed
Euler_0=[90, 89.98, 0]*d2r;                            % Init Euler Attitude
pqr_0=[0, 0, 0];                                       % Init Rates [0, 0.0,0] (rad/sec)
Uvw_0=[1,0,0]*V0;                                      % X,Y,Z Veloc Comp Body (ft/sec)
x_cg= 59.329; y_cg=0.0;  z_cg=0.0;                     % Initial CG location
cg=[x_cg, y_cg, z_cg];
x_ref=120.142; y_ref=0; z_ref=0;                       % Aero Moments Reference
mrc=[x_ref, y_ref, z_ref];
Sref=44.4146; ch=7.52; sp=7.52;                        % Reference Area, Length
accel=[97.3083,0,0];                                   % Accelerometer Location
Linact=0;                                              % 1-Linear, 0-NonLinear Actuator Switch
thpnd=15; velpnd=3;                                    % Init Pendulum Angle (deg), Veloc (ft/s)
Cdelay=0.005;                                          % Computational Delay (sec)
Mdelay=0.005;                                          % Measurements  Delay (sec)
AcNoise=0.1;                                           % Actuator Mesurement Noise 0.1 (deg)
Nmch=16; Nalf=9; Nbet=7;

% Events Timing
T1a= 168.0;                                            % Just Before MECO
T1b= 168.5;                                            % Just After   MECO
T2a= 171.2;                                            % Just Before Separation
T2b= 171.3;                                            % Just After   Separation
T3a= 176.5;                                            % Just Before Stage-2 Ignition
T3b= 176.6;                                            % Just After   Stage-2 Ignition
pre_sep_wait = 4;
post_sep_wait= 4;
stage_to_fairing_sep_time = 34.7;

% Load the Commands from Trajectory .........................
load Commands.mat -ascii
timin= Commands(:,1);                                  % Time In
thecm= Commands(:,2);                                  % Theta Command (deg)
alfcm= Commands(:,3);                                  % Alfa   Command (deg)
gamcm= Commands(:,4);                                  % Gamma Command (deg)
headc= Commands(:,5);                                  % Heading Command (deg)
thrs1= Commands(:,6);                                  % Engine 1 Thrust (lb)
thrs2= Commands(:,7);                                  % Engine 2 Thrust (lb)
mssin= Commands(:,8);                                  % Vehicle Mass
incln= Commands(:,9);                                  % Inclination
Eul1 = Commands(:,10);                                 % Euler(1)
Eul2 = Commands(:,11);                                 % Euler(2)
Eul3 = Commands(:,12);                                 % Euler(3)
```

- This file initializes the vehicle position and attitude (Euler angles) in the launch pad. Also, the CG, aerodynamic parameters, and some event parameters.

- Loads a “Commands” file that includes trajectory parameters used for reference, and a Guidance commands file “Commands_2” that includes the angular momentum vector command and the quaternion command.

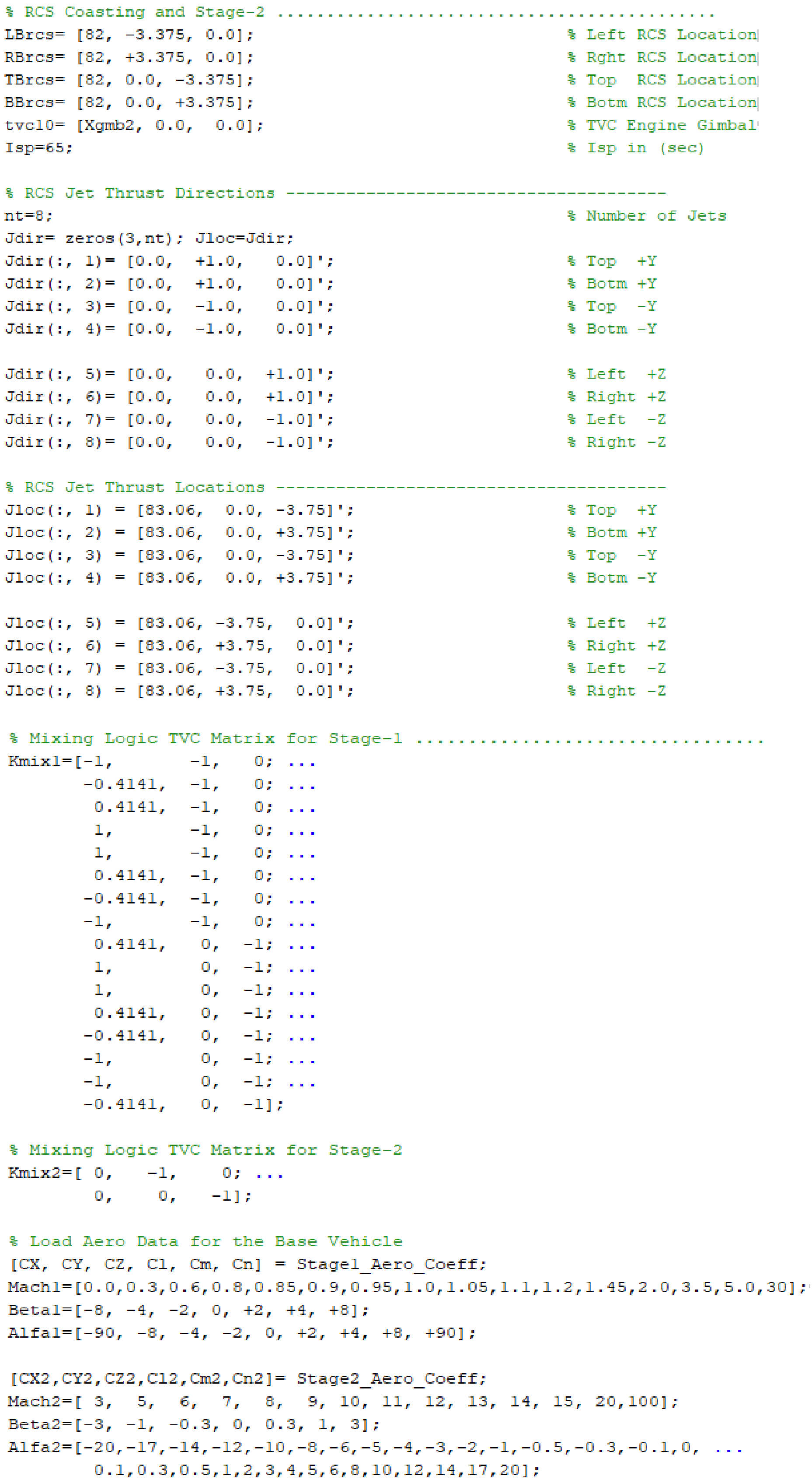


```
% Load Guidance Commands ..........................
load Commands_2.mat -ascii
timi2= Commands_2(:,1);                               % Time-2
JJ1   = Commands_2(:,2);                              % Momentum -x Vector
JJ2   = Commands_2(:,3);                              % Momentum -y
JJ3   = Commands_2(:,4);                              % Momentum -z
AMm   = Commands_2(:,5);                              % Angul-Momentum Magn
Enrg  = Commands_2(:,6);                              % Energy
dEdJ  = Commands_2(:,7);                              % dE/d(AM)
KE    = Commands_2(:,8);                              % Kinetic Energy

iend = length(Commands_2(:,1));
qtc1 = interp1(timi2(1:iend),Commands_2(1:iend, 9),timi2,'linear','extrap'); % Quat1
qtc2 = interp1(timi2(1:iend),Commands_2(1:iend,10),timi2,'linear','extrap'); % Quat1
qtc3 = interp1(timi2(1:iend),Commands_2(1:iend,11),timi2,'linear','extrap'); % Quat1
qtc4 = interp1(timi2(1:iend),Commands_2(1:iend,12),timi2,'linear','extrap'); % Quat1

% Load the Mass Properties
load Mass_Props -ascii
msspr= Mass_Props(:,1);                                % Vehi Mass from Mass Props
Ixxpr= Mass_Props(:,2);                                % Ixx from Mass Props
Iyypr= Mass_Props(:,3);                                % Iyy from Mass Props
Izzpr= Mass_Props(:,4);                                % Izz from Mass Props
Ixypr= Mass_Props(:,5);                                % Ixy from Mass Props
Ixzpr= Mass_Props(:,6);                                % Ixz from Mass Props
Iyzpr= Mass_Props(:,7);                                % Iyz from Mass Props
Xcgpr= Mass_Props(:,8);                                % X_cg from Mass Props
Ycgpr= Mass_Props(:,9);                                % Y_cg from Mass Props
Zcgpr= Mass_Props(:,10);                               % Z_cg from Mass Props

% Load the Slosh Data from File
load Slosh -ascii
Mvsls= Slosh(:,1);                                     % Vehi Mass (slug)
Mslox= Slosh(:,2);                                     % LOX  Mass (slug)
Msfue= Slosh(:,3);                                     % Fuel Mass (slug)
Ztlox= Slosh(:,6);                                     % LOX  Mass Damping
Ztfue= Slosh(:,7);                                     % Fuel Mass Damping
Xlox = Slosh(:,8);                                     % LOX  Mass Locat (ft)
Xfuel= Slosh(:,9);                                     % Fuel Mass Locat (ft)
Rlox = Slosh(:,10);                                    % LOX  Pendul Length (ft)
Rfuel= Slosh(:,11);                                    % Fuel Pendul Length (ft)

% Engine Data
me1=9.325; me2= 13.927;                                % 1st and 2nd Stage Engine Mass (slg)
Ie1=28.17; Ie2= 258.74;                                % 1st and 2nd Stage Engine Inertia (slg-ft2)
Le1=1.203; Le2= 2.892;                                 % 1st and 2nd Stage Engine CG to Gimbal Arm (ft)
Xgmb1=10.7062;                                         % X Gimal Location for Stage-1
Xgmb2=82.538;                                          % X Gimal Location for Stage-2

% Stage-1 Engine Gimbals ............................................
tvc1= [Xgmb1,  2.4638,  -1.0202];                      % Eng# 1 TVC Location (Fixed)
tvc2= [Xgmb1,  1.0202,  -2.4638];                      % Eng# 2 TVC Location
tvc3= [Xgmb1, -1.0202,  -2.4638];                      % Eng# 3 TVC Location
tvc4= [Xgmb1, -2.4638,  -1.0202];
tvc5= [Xgmb1, -2.4638,   1.0202];
tvc6= [Xgmb1, -1.0202,   2.4638];
tvc7= [Xgmb1,  1.0202,   2.4638];
tvc8= [Xgmb1,  2.4638,   1.0202];
tvc9= [Xgmb1,  0.0,      0.0];                         % Fixed Eng# 9 Location
```

- It also loads the vehicle mass and moments of inertia from the mass properties file “Mass-Props” and the slosh parameters from file “Slosh”, which includes: slosh masses, frequencies, damping and slosh locations versus vehicle mass.

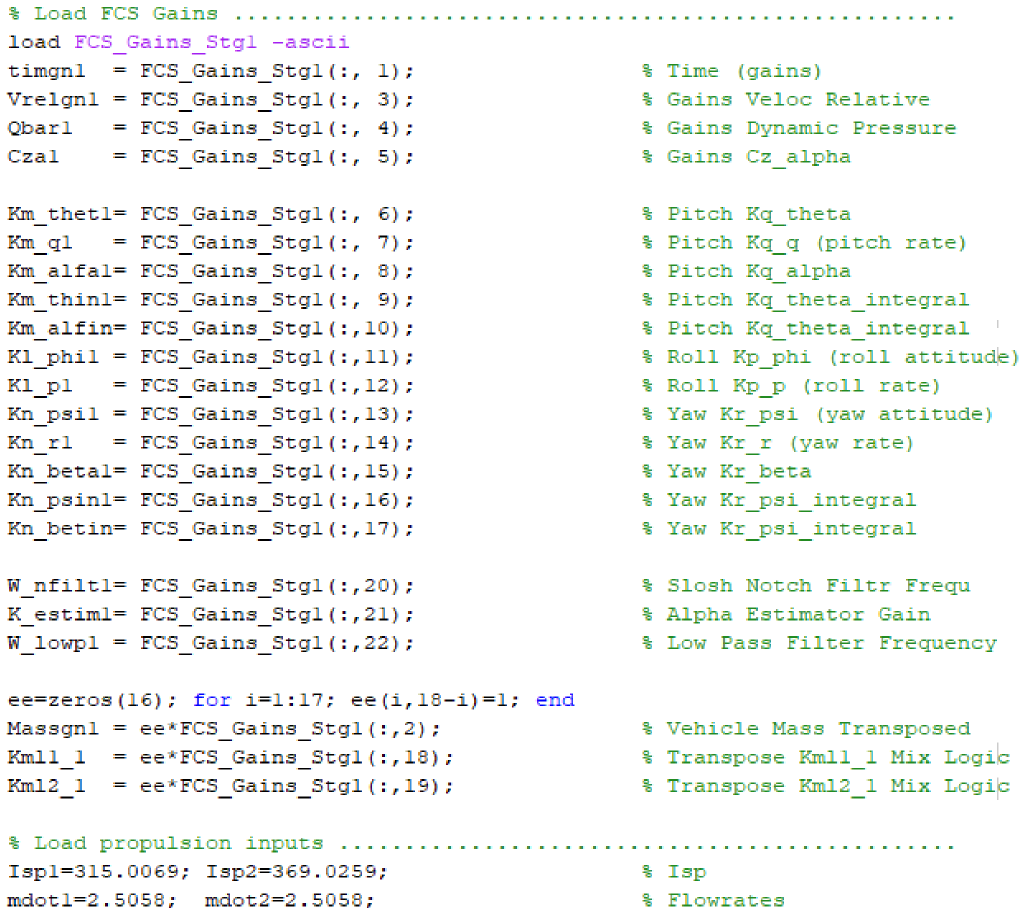
- There are TVC engine data, engine masses and inertias, gimbal locations, RCS jet locations and thrust directions.

- There are TVC matrices for first and second stages.

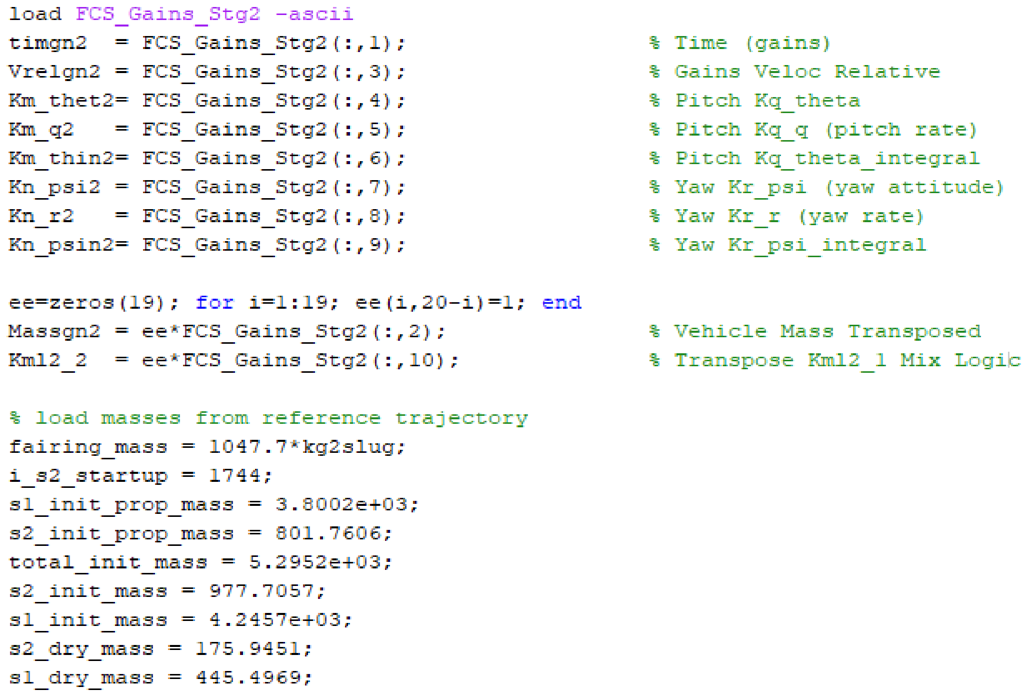


```
% RCS Coasting and Stage-2 ..........................................
LBrcs= [82, -3.375, 0.0];                          % Left RCS Location
RBrcs= [82, +3.375, 0.0];                          % Rght RCS Location
TBrcs= [82, 0.0, -3.375];                          % Top  RCS Location
BBrcs= [82, 0.0, +3.375];                          % Botm RCS Location
tvc10= [Xgmb2, 0.0,  0.0];                         % TVC Engine Gimbal
Isp=65;                                            % Isp in (sec)

% RCS Jet Thrust Directions -------------------------------------
nt=8;                                              % Number of Jets
Jdir= zeros(3,nt); Jloc=Jdir;
Jdir(:, 1)= [0.0,  +1.0,   0.0]';                  % Top  +Y
Jdir(:, 2)= [0.0,  +1.0,   0.0]';                  % Botm +Y
Jdir(:, 3)= [0.0,  -1.0,   0.0]';                  % Top  -Y
Jdir(:, 4)= [0.0,  -1.0,   0.0]';                  % Botm -Y

Jdir(:, 5)= [0.0,   0.0,  +1.0]';                  % Left  +Z
Jdir(:, 6)= [0.0,   0.0,  +1.0]';                  % Right +Z
Jdir(:, 7)= [0.0,   0.0,  -1.0]';                  % Left  -Z
Jdir(:, 8)= [0.0,   0.0,  -1.0]';                  % Right -Z

% RCS Jet Thrust Locations --------------------------------------
Jloc(:, 1) = [83.06,  0.0, -3.75]';                % Top  +Y
Jloc(:, 2) = [83.06,  0.0, +3.75]';                % Botm +Y
Jloc(:, 3) = [83.06,  0.0, -3.75]';                % Top  -Y
Jloc(:, 4) = [83.06,  0.0, +3.75]';                % Botm -Y

Jloc(:, 5) = [83.06, -3.75,  0.0]';                % Left  +Z
Jloc(:, 6) = [83.06, +3.75,  0.0]';                % Right +Z
Jloc(:, 7) = [83.06, -3.75,  0.0]';                % Left  -Z
Jloc(:, 8) = [83.06, +3.75,  0.0]';                % Right -Z

% Mixing Logic TVC Matrix for Stage-1 ................................
Kmix1=[-1,        -1,    0; ...
       -0.4141,   -1,    0; ...
        0.4141,   -1,    0; ...
        1,        -1,    0; ...
        1,        -1,    0; ...
        0.4141,   -1,    0; ...
       -0.4141,   -1,    0; ...
       -1,        -1,    0; ...
        0.4141,    0,   -1; ...
        1,         0,   -1; ...
        1,         0,   -1; ...
        0.4141,    0,   -1; ...
       -0.4141,    0,   -1; ...
       -1,         0,   -1; ...
       -1,         0,   -1; ...
       -0.4141,    0,   -1];

% Mixing Logic TVC Matrix for Stage-2
Kmix2=[ 0,    -1,     0; ...
        0,     0,    -1];

% Load Aero Data for the Base Vehicle
[CX, CY, CZ, Cl, Cm, Cn] = Stage1_Aero_Coeff;
Mach1=[0.0,0.3,0.6,0.8,0.85,0.9,0.95,1.0,1.05,1.1,1.2,1.45,2.0,3.5,5.0,30];
Beta1=[-8, -4, -2, 0, +2, +4, +8];
Alfa1=[-90, -8, -4, -2, 0, +2, +4, +8, +90];

[CX2,CY2,CZ2,Cl2,Cm2,Cn2]= Stage2_Aero_Coeff;
Mach2=[ 3,  5,  6,  7,  8,  9, 10, 11, 12, 13, 14, 15, 20,100];
Beta2=[-3, -1, -0.3, 0, 0.3, 1, 3];
Alfa2=[-20,-17,-14,-12,-10,-8,-6,-5,-4,-3,-2,-1,-0.5,-0.3,-0.1,0, ...
        0.1,0.3,0.5,1,2,3,4,5,6,8,10,12,14,17,20];
```

- The initialization file is also loading the aerodynamic coefficients for 1st and 2nd stages from files “Stage1_Aero_Coeff” and “Stage2_Aero_Coeff”. The 3 force and 3 moment coefficients are listed as 3-dimentional matrices of the variable against Mach number, alpha and beta angles.



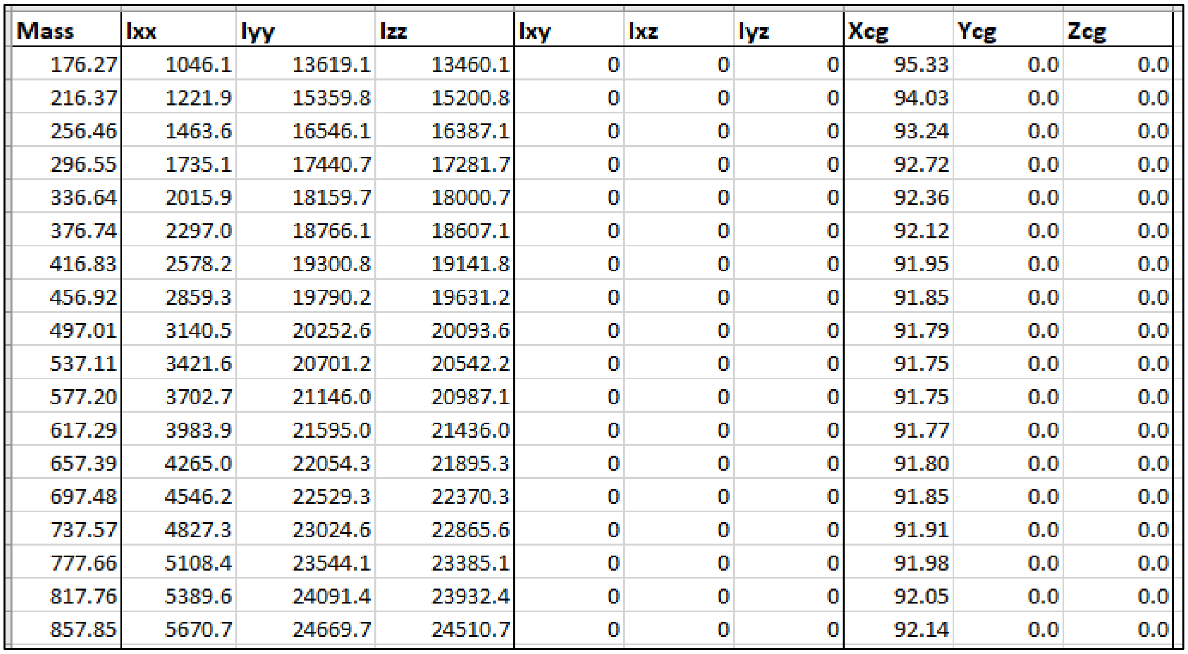
```
% Load FCS Gains ................................................
load FCS_Gains_Stg1 -ascii
timgn1   = FCS_Gains_Stg1(:, 1);               % Time (gains)
Vrelgn1  = FCS_Gains_Stg1(:, 3);               % Gains Veloc Relative
Qbar1    = FCS_Gains_Stg1(:, 4);               % Gains Dynamic Pressure
Cza1     = FCS_Gains_Stg1(:, 5);               % Gains Cz_alpha

Km_thet1= FCS_Gains_Stg1(:, 6);                % Pitch Kq_theta
Km_q1    = FCS_Gains_Stg1(:, 7);               % Pitch Kq_q (pitch rate)
Km_alfa1= FCS_Gains_Stg1(:, 8);                % Pitch Kq_alpha
Km_thin1= FCS_Gains_Stg1(:, 9);                % Pitch Kq_theta_integral
Km_alfin= FCS_Gains_Stg1(:,10);                % Pitch Kq_theta_integral
Kl_phi1 = FCS_Gains_Stg1(:,11);                % Roll Kp_phi (roll attitude)
Kl_p1    = FCS_Gains_Stg1(:,12);               % Roll Kp_p (roll rate)
Kn_psi1 = FCS_Gains_Stg1(:,13);                % Yaw Kr_psi (yaw attitude)
Kn_r1    = FCS_Gains_Stg1(:,14);               % Yaw Kr_r (yaw rate)
Kn_beta1= FCS_Gains_Stg1(:,15);                % Yaw Kr_beta
Kn_psin1= FCS_Gains_Stg1(:,16);                % Yaw Kr_psi_integral
Kn_betin= FCS_Gains_Stg1(:,17);                % Yaw Kr_psi_integral

W_nfilt1= FCS_Gains_Stg1(:,20);                % Slosh Notch Filtr Frequ
K_estim1= FCS_Gains_Stg1(:,21);                % Alpha Estimator Gain
W_lowp1 = FCS_Gains_Stg1(:,22);                % Low Pass Filter Frequency

ee=zeros(16); for i=1:17; ee(i,18-i)=1; end
Massgn1 = ee*FCS_Gains_Stg1(:,2);              % Vehicle Mass Transposed
Kml1_1   = ee*FCS_Gains_Stg1(:,18);            % Transpose Kml1_1 Mix Logic
Kml2_1   = ee*FCS_Gains_Stg1(:,19);            % Transpose Kml2_1 Mix Logic

% Load propulsion inputs ........................................
Isp1=315.0069; Isp2=369.0259;                  % Isp
mdot1=2.5058;  mdot2=2.5058;                   % Flowrates
```



```
load FCS_Gains_Stg2 -ascii
timgn2   = FCS_Gains_Stg2(:,1);                % Time (gains)
Vrelgn2  = FCS_Gains_Stg2(:,3);                % Gains Veloc Relative
Km_thet2= FCS_Gains_Stg2(:,4);                 % Pitch Kq_theta
Km_q2    = FCS_Gains_Stg2(:,5);                % Pitch Kq_q (pitch rate)
Km_thin2= FCS_Gains_Stg2(:,6);                 % Pitch Kq_theta_integral
Kn_psi2 = FCS_Gains_Stg2(:,7);                 % Yaw Kr_psi (yaw attitude)
Kn_r2    = FCS_Gains_Stg2(:,8);                % Yaw Kr_r (yaw rate)
Kn_psin2= FCS_Gains_Stg2(:,9);                 % Yaw Kr_psi_integral

ee=zeros(19); for i=1:19; ee(i,20-i)=1; end
Massgn2 = ee*FCS_Gains_Stg2(:,2);              % Vehicle Mass Transposed
Kml2_2   = ee*FCS_Gains_Stg2(:,10);            % Transpose Kml2_1 Mix Logic

% load masses from reference trajectory
fairing_mass = 1047.7*kg2slug;
i_s2_startup = 1744;
s1_init_prop_mass = 3.8002e+03;
s2_init_prop_mass = 801.7606;
total_init_mass = 5.2952e+03;
s2_init_mass = 977.7057;
s1_init_mass = 4.2457e+03;
s2_dry_mass = 175.9451;
s1_dry_mass = 445.4969;
```

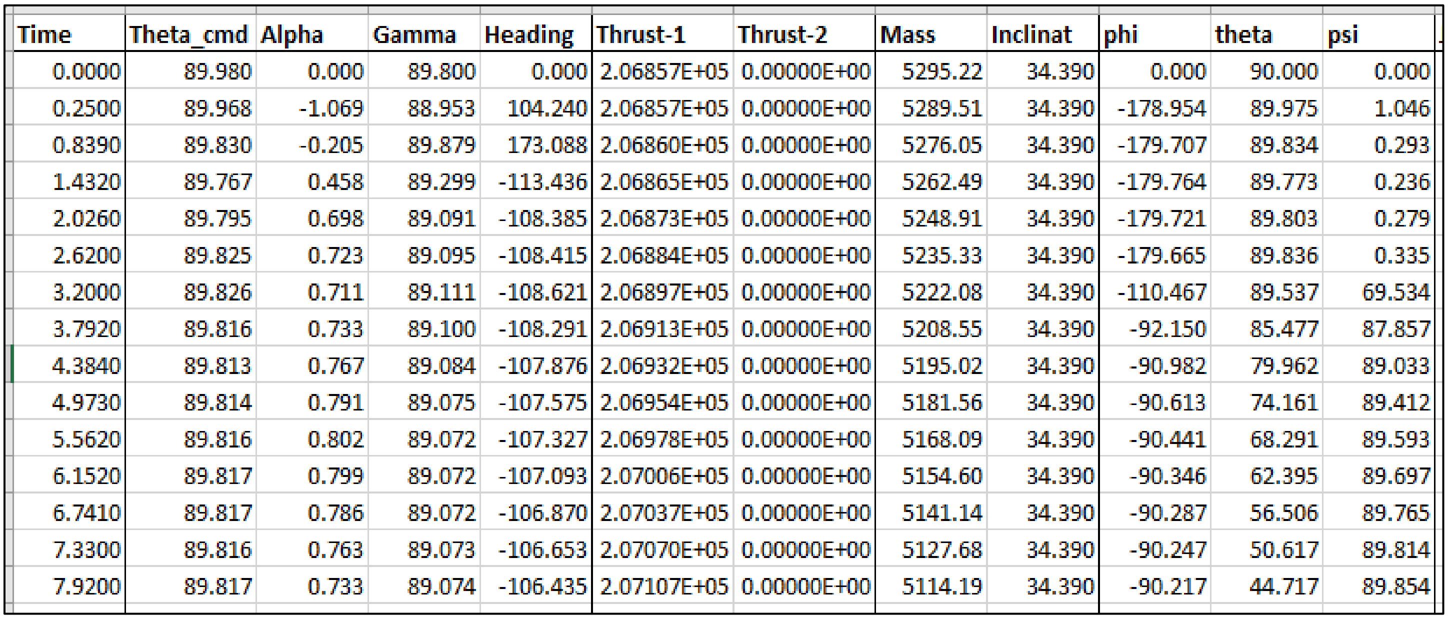
- Finally, it loads the 1st and 2nd stage flight control gains from files “FCS_Gains_Stg1” and “FCS_Gains_Stg2”, and other mass properties.

The following tables show part of the input data files which are loaded into Matlab



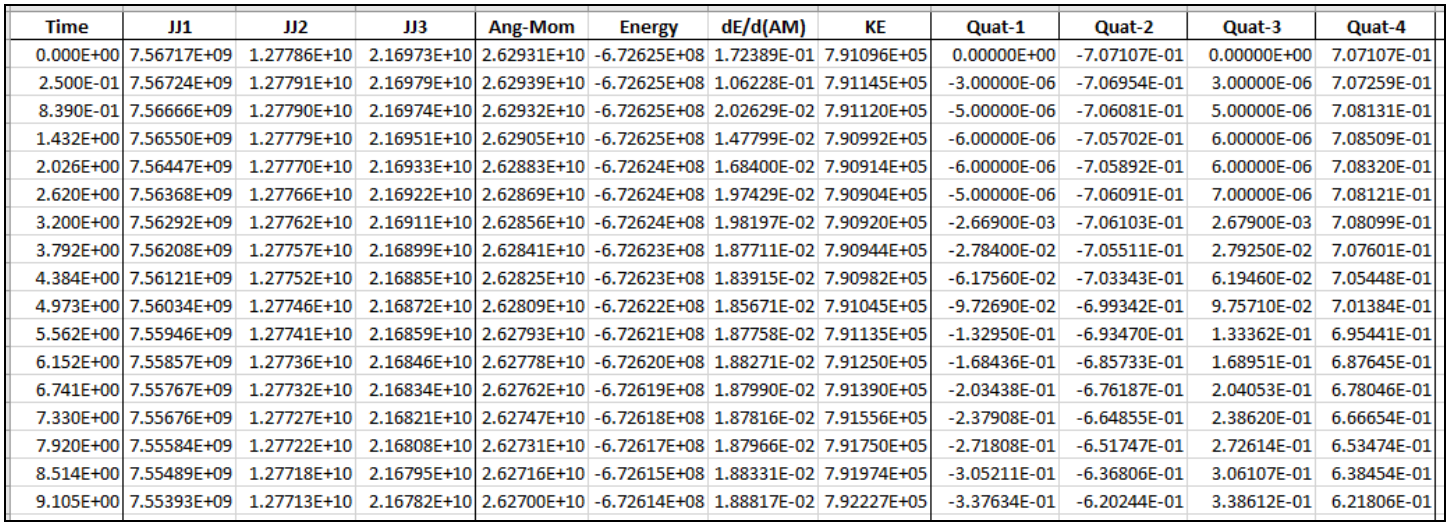
Mass	Ixx	Iyy	Izz	Ixy	Ixz	Iyz	Xcg	Ycg	Zcg
176.27	1046.1	13619.1	13460.1	0	0	0	95.33	0.0	0.0
216.37	1221.9	15359.8	15200.8	0	0	0	94.03	0.0	0.0
256.46	1463.6	16546.1	16387.1	0	0	0	93.24	0.0	0.0
296.55	1735.1	17440.7	17281.7	0	0	0	92.72	0.0	0.0
336.64	2015.9	18159.7	18000.7	0	0	0	92.36	0.0	0.0
376.74	2297.0	18766.1	18607.1	0	0	0	92.12	0.0	0.0
416.83	2578.2	19300.8	19141.8	0	0	0	91.95	0.0	0.0
456.92	2859.3	19790.2	19631.2	0	0	0	91.85	0.0	0.0
497.01	3140.5	20252.6	20093.6	0	0	0	91.79	0.0	0.0
537.11	3421.6	20701.2	20542.2	0	0	0	91.75	0.0	0.0
577.20	3702.7	21146.0	20987.1	0	0	0	91.75	0.0	0.0
617.29	3983.9	21595.0	21436.0	0	0	0	91.77	0.0	0.0
657.39	4265.0	22054.3	21895.3	0	0	0	91.80	0.0	0.0
697.48	4546.2	22529.3	22370.3	0	0	0	91.85	0.0	0.0
737.57	4827.3	23024.6	22865.6	0	0	0	91.91	0.0	0.0
777.66	5108.4	23544.1	23385.1	0	0	0	91.98	0.0	0.0
817.76	5389.6	24091.4	23932.4	0	0	0	92.05	0.0	0.0
857.85	5670.7	24669.7	24510.7	0	0	0	92.14	0.0	0.0

Figure 6.40 Mass Properties Table in File "Mass_Props"



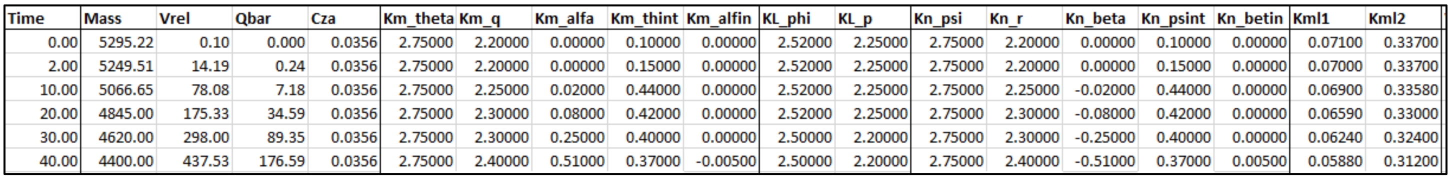
Time	Theta_cmd	Alpha	Gamma	Heading	Thrust-1	Thrust-2	Mass	Inclinat	phi	theta	psi
0.0000	89.980	0.000	89.800	0.000	2.06857E+05	0.00000E+00	5295.22	34.390	0.000	90.000	0.000
0.2500	89.968	-1.069	88.953	104.240	2.06857E+05	0.00000E+00	5289.51	34.390	-178.954	89.975	1.046
0.8390	89.830	-0.205	89.879	173.088	2.06860E+05	0.00000E+00	5276.05	34.390	-179.707	89.834	0.293
1.4320	89.767	0.458	89.299	-113.436	2.06865E+05	0.00000E+00	5262.49	34.390	-179.764	89.773	0.236
2.0260	89.795	0.698	89.091	-108.385	2.06873E+05	0.00000E+00	5248.91	34.390	-179.721	89.803	0.279
2.6200	89.825	0.723	89.095	-108.415	2.06884E+05	0.00000E+00	5235.33	34.390	-179.665	89.836	0.335
3.2000	89.826	0.711	89.111	-108.621	2.06897E+05	0.00000E+00	5222.08	34.390	-110.467	89.537	69.534
3.7920	89.816	0.733	89.100	-108.291	2.06913E+05	0.00000E+00	5208.55	34.390	-92.150	85.477	87.857
4.3840	89.813	0.767	89.084	-107.876	2.06932E+05	0.00000E+00	5195.02	34.390	-90.982	79.962	89.033
4.9730	89.814	0.791	89.075	-107.575	2.06954E+05	0.00000E+00	5181.56	34.390	-90.613	74.161	89.412
5.5620	89.816	0.802	89.072	-107.327	2.06978E+05	0.00000E+00	5168.09	34.390	-90.441	68.291	89.593
6.1520	89.817	0.799	89.072	-107.093	2.07006E+05	0.00000E+00	5154.60	34.390	-90.346	62.395	89.697
6.7410	89.817	0.786	89.072	-106.870	2.07037E+05	0.00000E+00	5141.14	34.390	-90.287	56.506	89.765
7.3300	89.816	0.763	89.073	-106.653	2.07070E+05	0.00000E+00	5127.68	34.390	-90.247	50.617	89.814
7.9200	89.817	0.733	89.074	-106.435	2.07107E+05	0.00000E+00	5114.19	34.390	-90.217	44.717	89.854

Figure 6.41 Table of Variables versus Time, File "Commands"



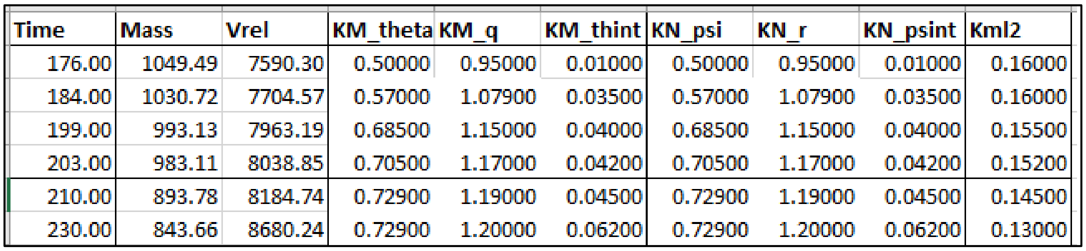
Time	JJ1	JJ2	JJ3	Ang-Mom	Energy	dE/d(AM)	KE	Quat-1	Quat-2	Quat-3	Quat-4
0.000E+00	7.56717E+09	1.27786E+10	2.16973E+10	2.62931E+10	-6.72625E+08	1.72389E-01	7.91096E+05	0.00000E+00	-7.07107E-01	0.00000E+00	7.07107E-01
2.500E-01	7.56724E+09	1.27791E+10	2.16979E+10	2.62939E+10	-6.72625E+08	1.06228E-01	7.91145E+05	-3.00000E-06	-7.06954E-01	3.00000E-06	7.07259E-01
8.390E-01	7.56666E+09	1.27790E+10	2.16974E+10	2.62932E+10	-6.72625E+08	2.02629E-02	7.91120E+05	-5.00000E-06	-7.06081E-01	5.00000E-06	7.08131E-01
1.432E+00	7.56550E+09	1.27779E+10	2.16951E+10	2.62905E+10	-6.72625E+08	1.47799E-02	7.90992E+05	-6.00000E-06	-7.05702E-01	6.00000E-06	7.08509E-01
2.026E+00	7.56447E+09	1.27770E+10	2.16933E+10	2.62883E+10	-6.72624E+08	1.68400E-02	7.90914E+05	-6.00000E-06	-7.05892E-01	6.00000E-06	7.08320E-01
2.620E+00	7.56368E+09	1.27766E+10	2.16922E+10	2.62869E+10	-6.72624E+08	1.97429E-02	7.90904E+05	-5.00000E-06	-7.06091E-01	7.00000E-06	7.08121E-01
3.200E+00	7.56292E+09	1.27762E+10	2.16911E+10	2.62856E+10	-6.72624E+08	1.98197E-02	7.90920E+05	-2.66900E-03	-7.06103E-01	2.67900E-03	7.08099E-01
3.792E+00	7.56208E+09	1.27757E+10	2.16899E+10	2.62841E+10	-6.72623E+08	1.87711E-02	7.90944E+05	-2.78400E-02	-7.05511E-01	2.79250E-02	7.07601E-01
4.384E+00	7.56121E+09	1.27752E+10	2.16885E+10	2.62825E+10	-6.72623E+08	1.83915E-02	7.90982E+05	-6.17560E-02	-7.03343E-01	6.19460E-02	7.05448E-01
4.973E+00	7.56034E+09	1.27746E+10	2.16872E+10	2.62809E+10	-6.72622E+08	1.85671E-02	7.91045E+05	-9.72690E-02	-6.99342E-01	9.75710E-02	7.01384E-01
5.562E+00	7.55946E+09	1.27741E+10	2.16859E+10	2.62793E+10	-6.72621E+08	1.87758E-02	7.91135E+05	-1.32950E-01	-6.93470E-01	1.33362E-01	6.95441E-01
6.152E+00	7.55857E+09	1.27736E+10	2.16846E+10	2.62778E+10	-6.72620E+08	1.88271E-02	7.91250E+05	-1.68436E-01	-6.85733E-01	1.68951E-01	6.87645E-01
6.741E+00	7.55767E+09	1.27732E+10	2.16834E+10	2.62762E+10	-6.72619E+08	1.87990E-02	7.91390E+05	-2.03438E-01	-6.76187E-01	2.04053E-01	6.78046E-01
7.330E+00	7.55676E+09	1.27727E+10	2.16821E+10	2.62747E+10	-6.72618E+08	1.87816E-02	7.91556E+05	-2.37908E-01	-6.64855E-01	2.38620E-01	6.66654E-01
7.920E+00	7.55584E+09	1.27722E+10	2.16808E+10	2.62731E+10	-6.72617E+08	1.87966E-02	7.91750E+05	-2.71808E-01	-6.51747E-01	2.72614E-01	6.53474E-01
8.514E+00	7.55489E+09	1.27718E+10	2.16795E+10	2.62716E+10	-6.72615E+08	1.88331E-02	7.91974E+05	-3.05211E-01	-6.36806E-01	3.06107E-01	6.38454E-01
9.105E+00	7.55393E+09	1.27713E+10	2.16782E+10	2.62700E+10	-6.72614E+08	1.88817E-02	7.92227E+05	-3.37634E-01	-6.20244E-01	3.38612E-01	6.21806E-01

Figure 6.42 Table of Lateral Guidance and Quaternion Commands, file “Commands_2”



Time	Mass	Vrel	Qbar	Cza	Km_theta	Km_q	Km_alfa	Km_thint	Km_alfin	KL_phi	KL_p	Kn_psi	Kn_r	Kn_beta	Kn_psint	Kn_betin	Kml1	Kml2
0.00	5295.22	0.10	0.000	0.0356	2.75000	2.20000	0.00000	0.10000	0.00000	2.52000	2.25000	2.75000	2.20000	0.00000	0.10000	0.00000	0.07100	0.33700
2.00	5249.51	14.19	0.24	0.0356	2.75000	2.20000	0.00000	0.15000	0.00000	2.52000	2.25000	2.75000	2.20000	0.00000	0.15000	0.00000	0.07000	0.33700
10.00	5066.65	78.08	7.18	0.0356	2.75000	2.25000	0.02000	0.44000	0.00000	2.52000	2.25000	2.75000	2.25000	-0.02000	0.44000	0.00000	0.06900	0.33580
20.00	4845.00	175.33	34.59	0.0356	2.75000	2.30000	0.08000	0.42000	0.00000	2.52000	2.25000	2.75000	2.30000	-0.08000	0.42000	0.00000	0.06590	0.33000
30.00	4620.00	298.00	89.35	0.0356	2.75000	2.30000	0.25000	0.40000	0.00000	2.50000	2.20000	2.75000	2.30000	-0.25000	0.40000	0.00000	0.06240	0.32400
40.00	4400.00	437.53	176.59	0.0356	2.75000	2.40000	0.51000	0.37000	-0.00500	2.50000	2.20000	2.75000	2.40000	-0.51000	0.37000	0.00500	0.05880	0.31200

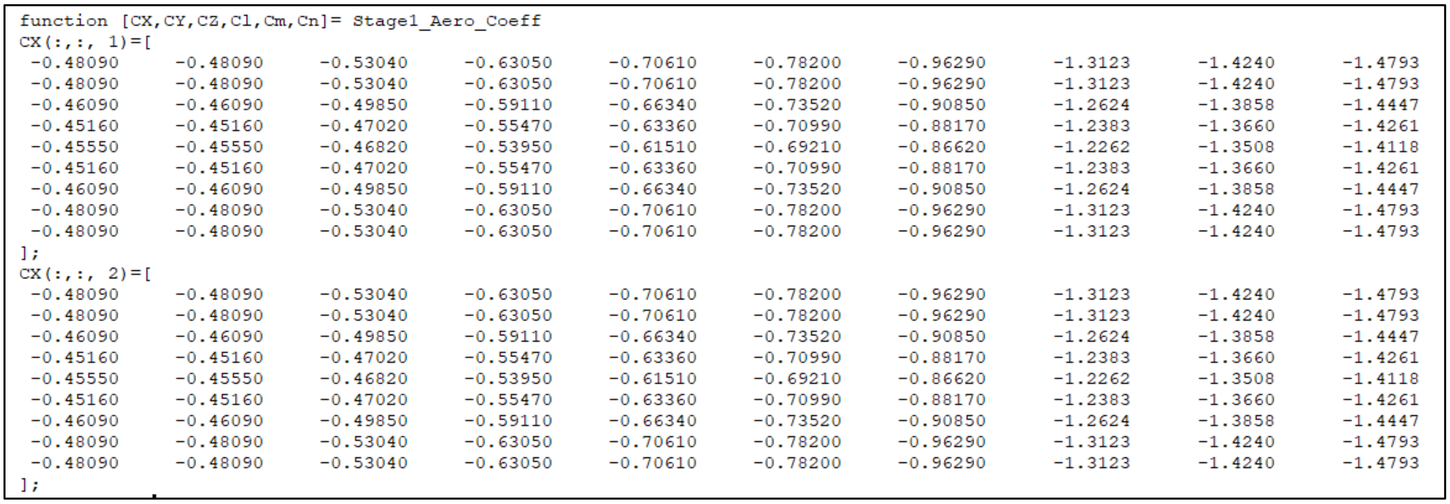
Figure 6.43 First Stage Flight Control Gains “FCS_Gains_Stg1”



Time	Mass	Vrel	KM_theta	KM_q	KM_thint	KN_psi	KN_r	KN_psint	Kml2
176.00	1049.49	7590.30	0.50000	0.95000	0.01000	0.50000	0.95000	0.01000	0.16000
184.00	1030.72	7704.57	0.57000	1.07900	0.03500	0.57000	1.07900	0.03500	0.16000
199.00	993.13	7963.19	0.68500	1.15000	0.04000	0.68500	1.15000	0.04000	0.15500
203.00	983.11	8038.85	0.70500	1.17000	0.04200	0.70500	1.17000	0.04200	0.15200
210.00	893.78	8184.74	0.72900	1.19000	0.04500	0.72900	1.19000	0.04500	0.14500
230.00	843.66	8680.24	0.72900	1.20000	0.06200	0.72900	1.20000	0.06200	0.13000

Figure 6.44 Second Stage Flight Control Gains “FCS_Gains_Stg2”

The aero coefficients are a function of Mach#, alpha, and beta. For example, CX(Mach, α, β)



```
function [CX,CY,CZ,Cl,Cm,Cn]= Stage1_Aero_Coeff
CX(:,:, 1)=[
 -0.48090     -0.48090     -0.53040     -0.63050     -0.70610     -0.78200     -0.96290       -1.3123      -1.4240      -1.4793
 -0.48090     -0.48090     -0.53040     -0.63050     -0.70610     -0.78200     -0.96290       -1.3123      -1.4240      -1.4793
 -0.46090     -0.46090     -0.49850     -0.59110     -0.66340     -0.73520     -0.90850       -1.2624      -1.3858      -1.4447
 -0.45160     -0.45160     -0.47020     -0.55470     -0.63360     -0.70990     -0.88170       -1.2383      -1.3660      -1.4261
 -0.45550     -0.45550     -0.46820     -0.53950     -0.61510     -0.69210     -0.86620       -1.2262      -1.3508      -1.4118
 -0.45160     -0.45160     -0.47020     -0.55470     -0.63360     -0.70990     -0.88170       -1.2383      -1.3660      -1.4261
 -0.46090     -0.46090     -0.49850     -0.59110     -0.66340     -0.73520     -0.90850       -1.2624      -1.3858      -1.4447
 -0.48090     -0.48090     -0.53040     -0.63050     -0.70610     -0.78200     -0.96290       -1.3123      -1.4240      -1.4793
 -0.48090     -0.48090     -0.53040     -0.63050     -0.70610     -0.78200     -0.96290       -1.3123      -1.4240      -1.4793
];
CX(:,:, 2)=[
 -0.48090     -0.48090     -0.53040     -0.63050     -0.70610     -0.78200     -0.96290       -1.3123      -1.4240      -1.4793
 -0.48090     -0.48090     -0.53040     -0.63050     -0.70610     -0.78200     -0.96290       -1.3123      -1.4240      -1.4793
 -0.46090     -0.46090     -0.49850     -0.59110     -0.66340     -0.73520     -0.90850       -1.2624      -1.3858      -1.4447
 -0.45160     -0.45160     -0.47020     -0.55470     -0.63360     -0.70990     -0.88170       -1.2383      -1.3660      -1.4261
 -0.45550     -0.45550     -0.46820     -0.53950     -0.61510     -0.69210     -0.86620       -1.2262      -1.3508      -1.4118
 -0.45160     -0.45160     -0.47020     -0.55470     -0.63360     -0.70990     -0.88170       -1.2383      -1.3660      -1.4261
 -0.46090     -0.46090     -0.49850     -0.59110     -0.66340     -0.73520     -0.90850       -1.2624      -1.3858      -1.4447
 -0.48090     -0.48090     -0.53040     -0.63050     -0.70610     -0.78200     -0.96290       -1.3123      -1.4240      -1.4793
 -0.48090     -0.48090     -0.53040     -0.63050     -0.70610     -0.78200     -0.96290       -1.3123      -1.4240      -1.4793
];
```

Figure 6.45 First Stage Aero-Data File “Stage1_Aero_Coeff.m”

Simulation Results

The simulation includes both first and second stages, beginning from the vertical position on the launch pad, all the way to orbit insertion. The vehicle performs a roll maneuver immediately after lift-off that rotates it vertically towards an azimuth attitude that will achieve the desired inclination. Figure-1 shows the vehicle flight-path angle gamma against the expected gamma. It begins with an almost vertical γ and ends up horizontal at the end of second stage.

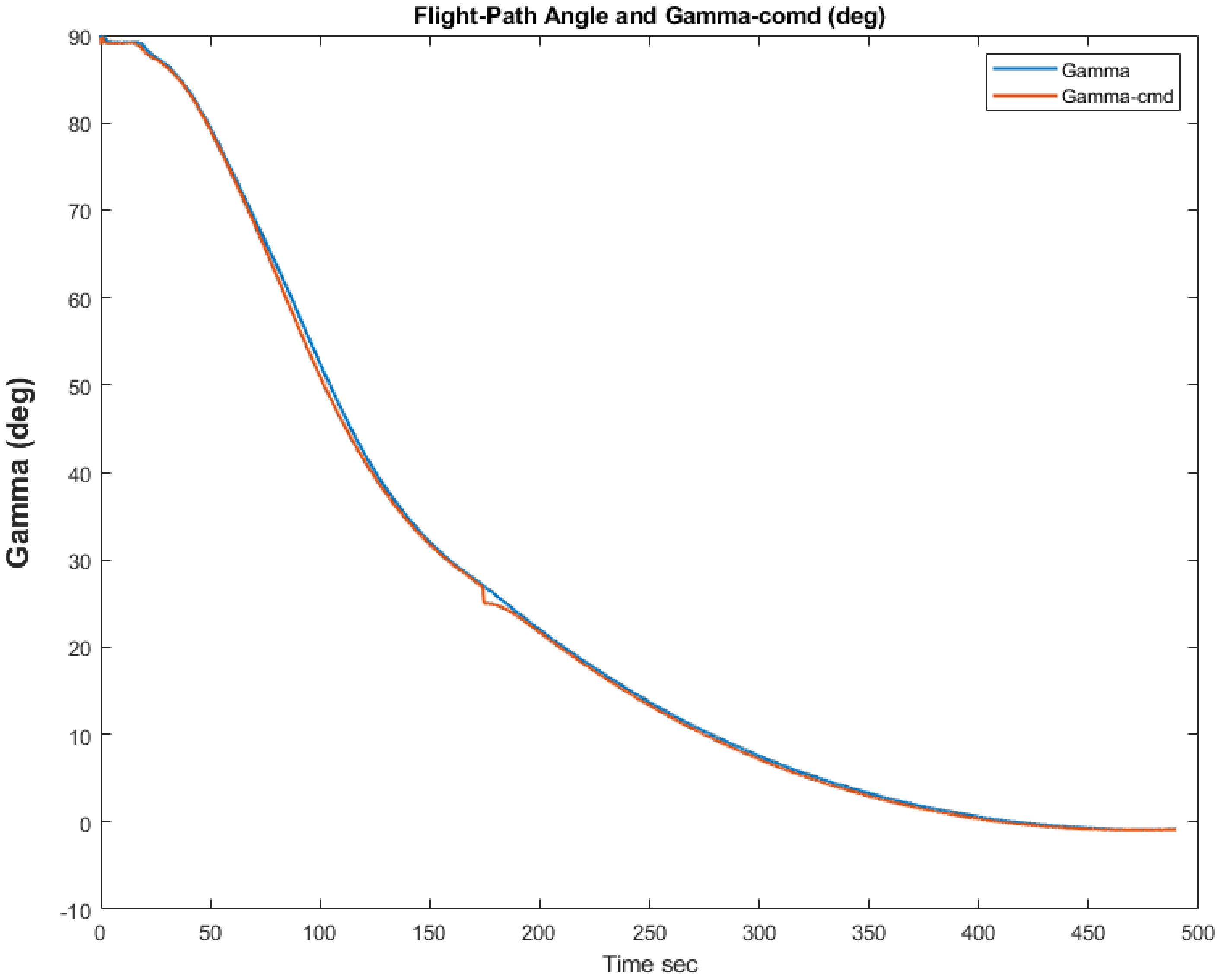


Figure 6.46 Flight-Path Angle Gamma

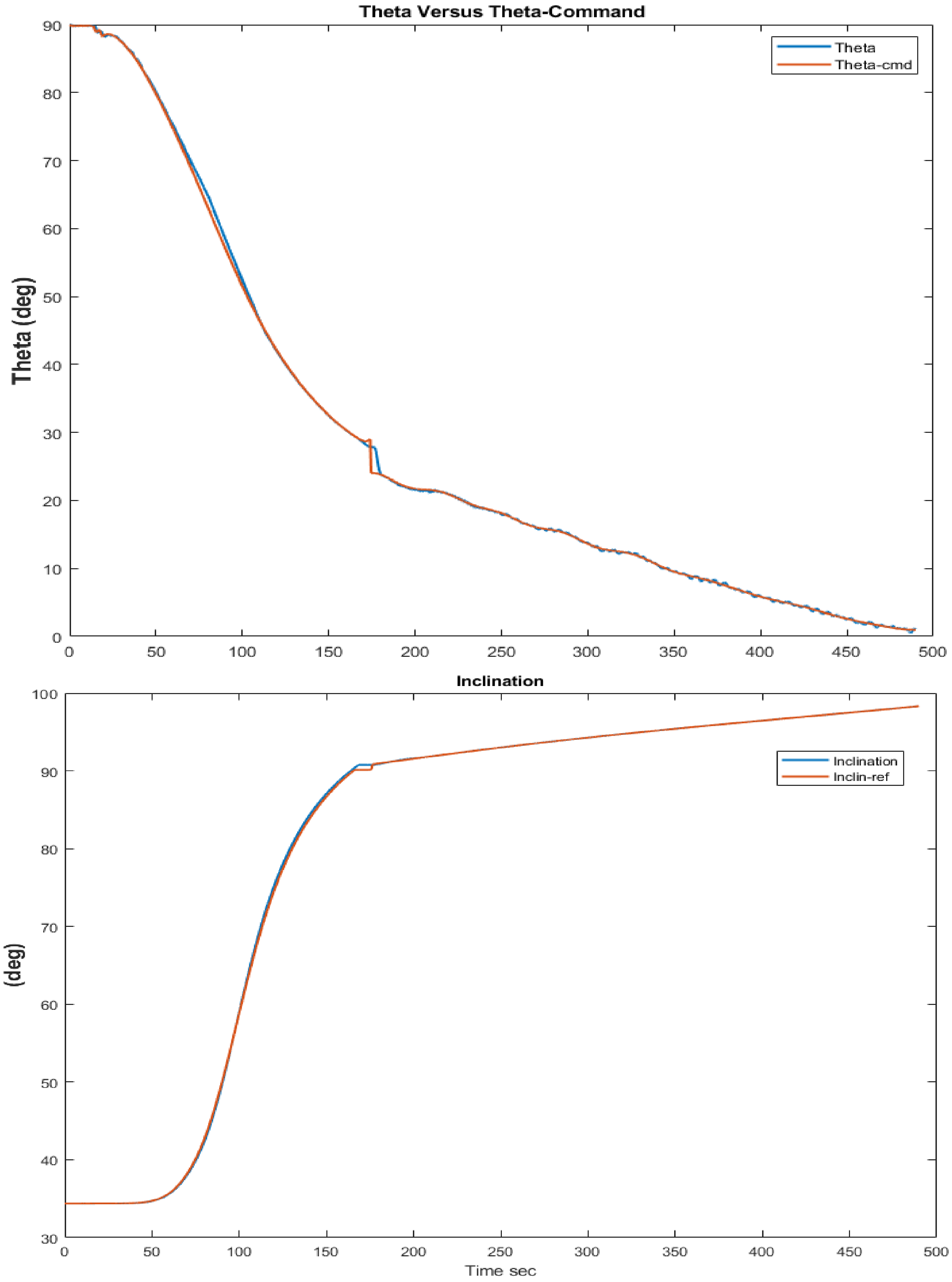


Figure 6.47 Pitch Attitude and Inclination Angles

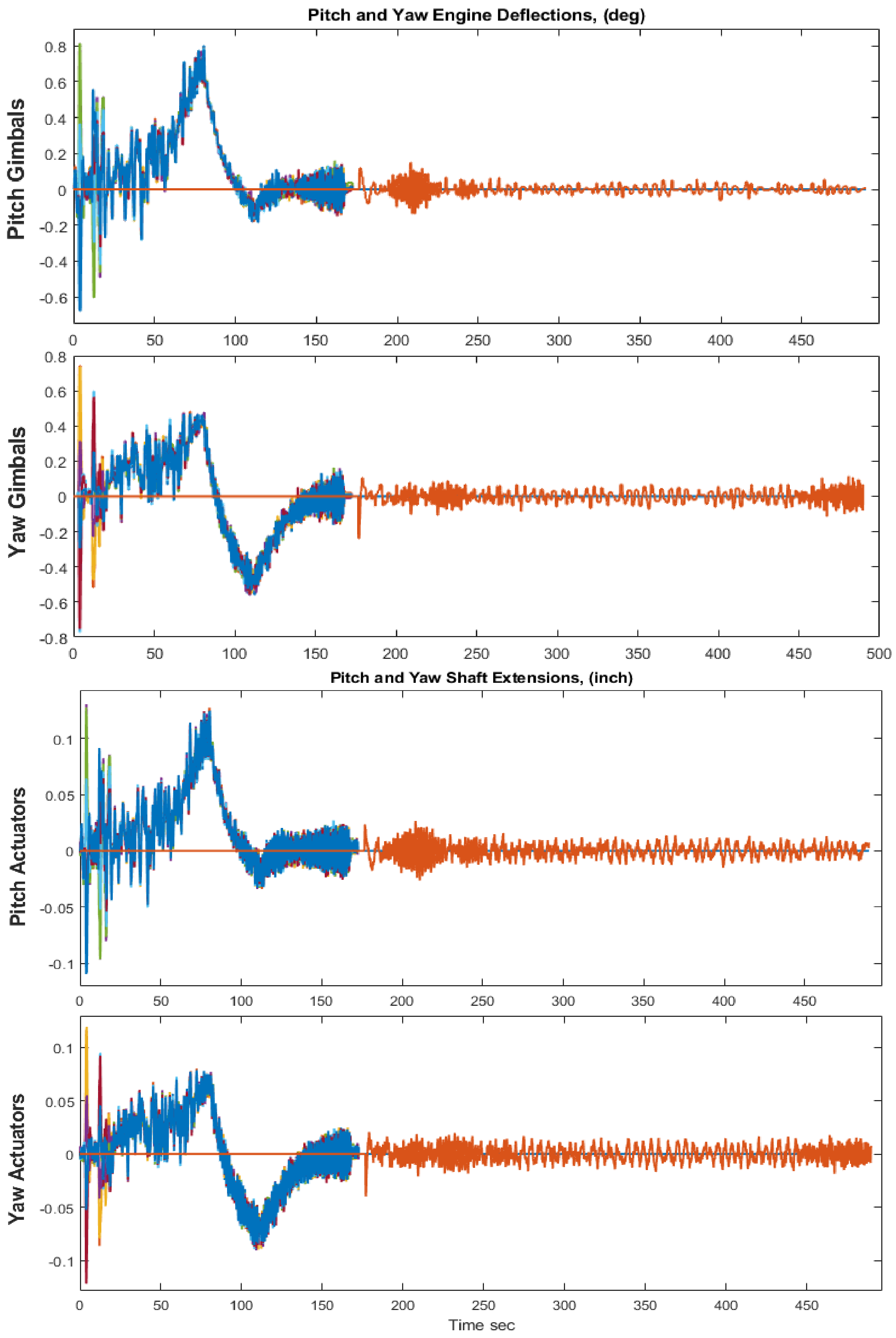
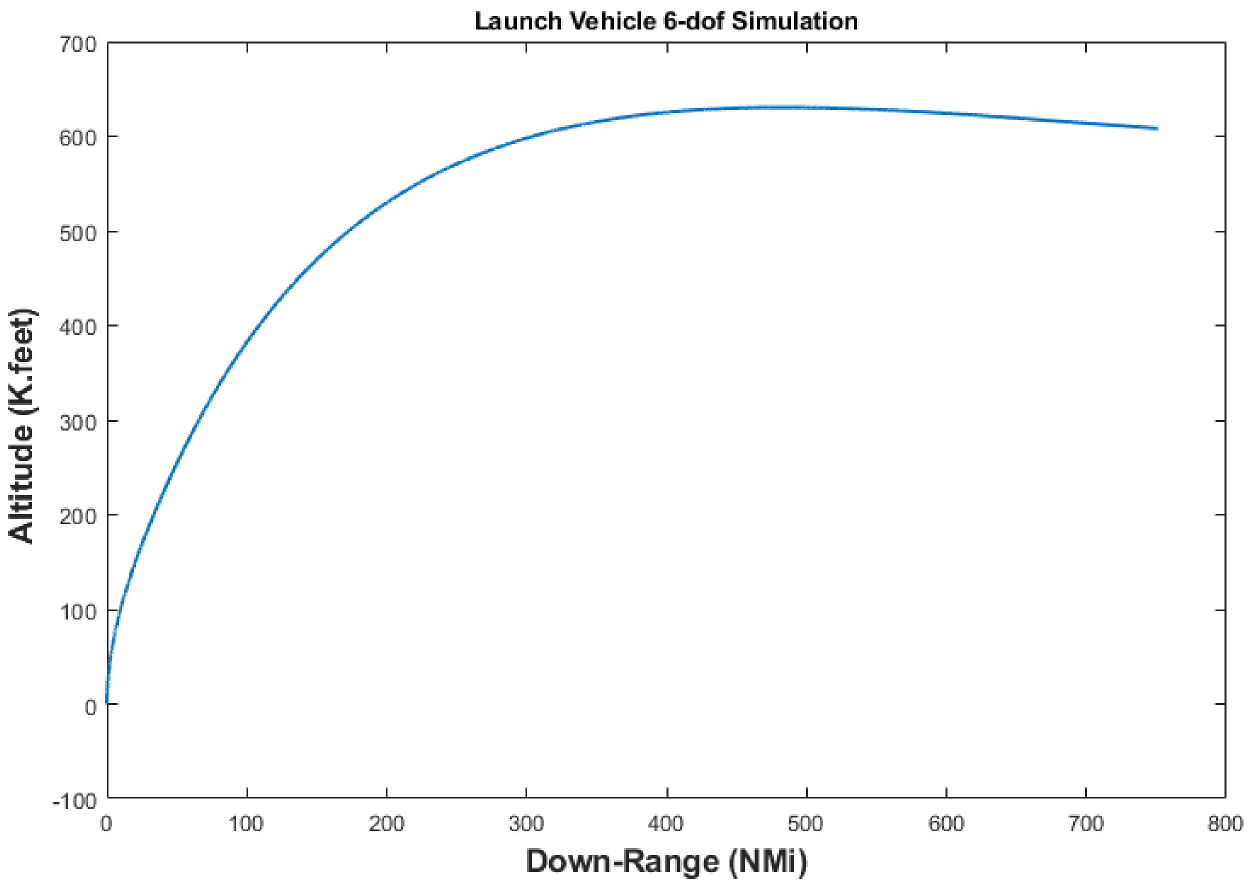
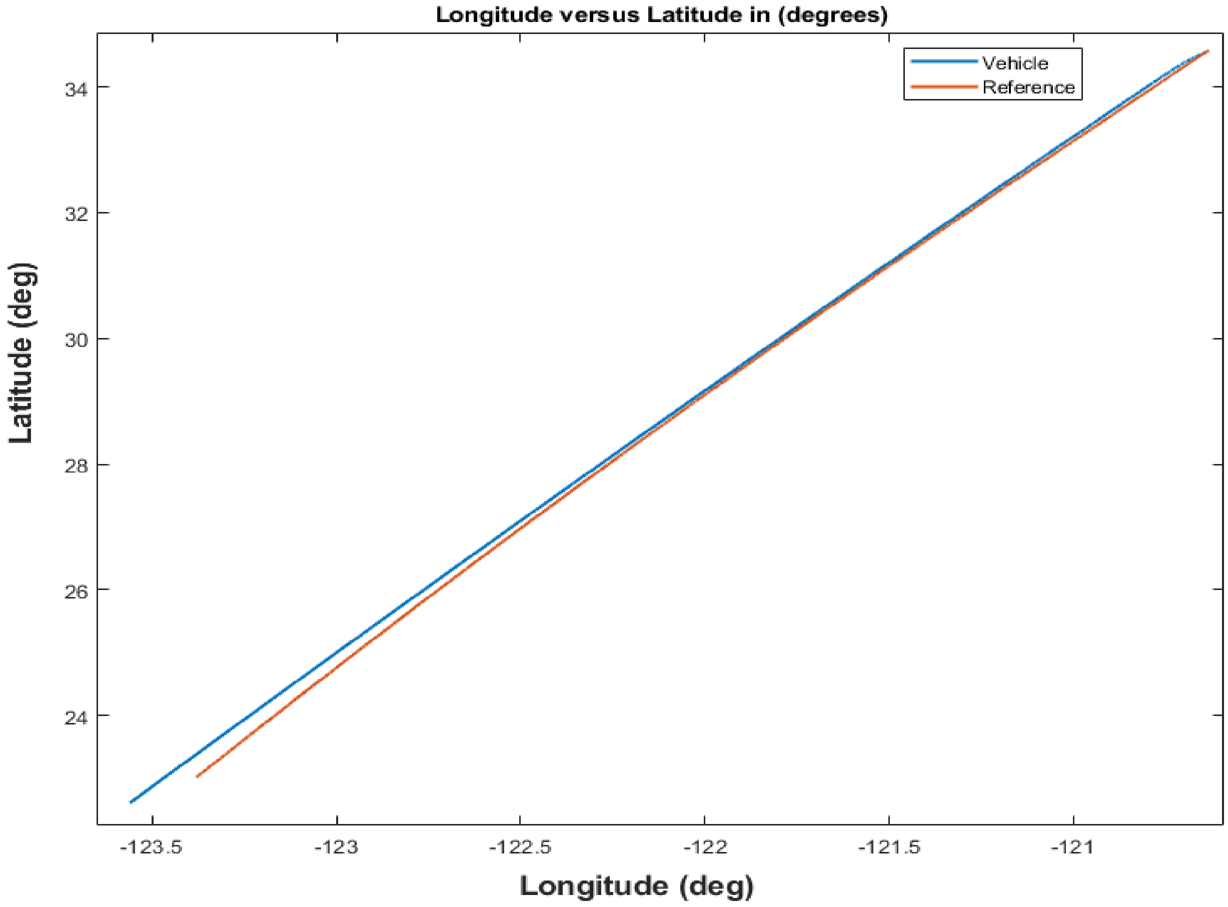
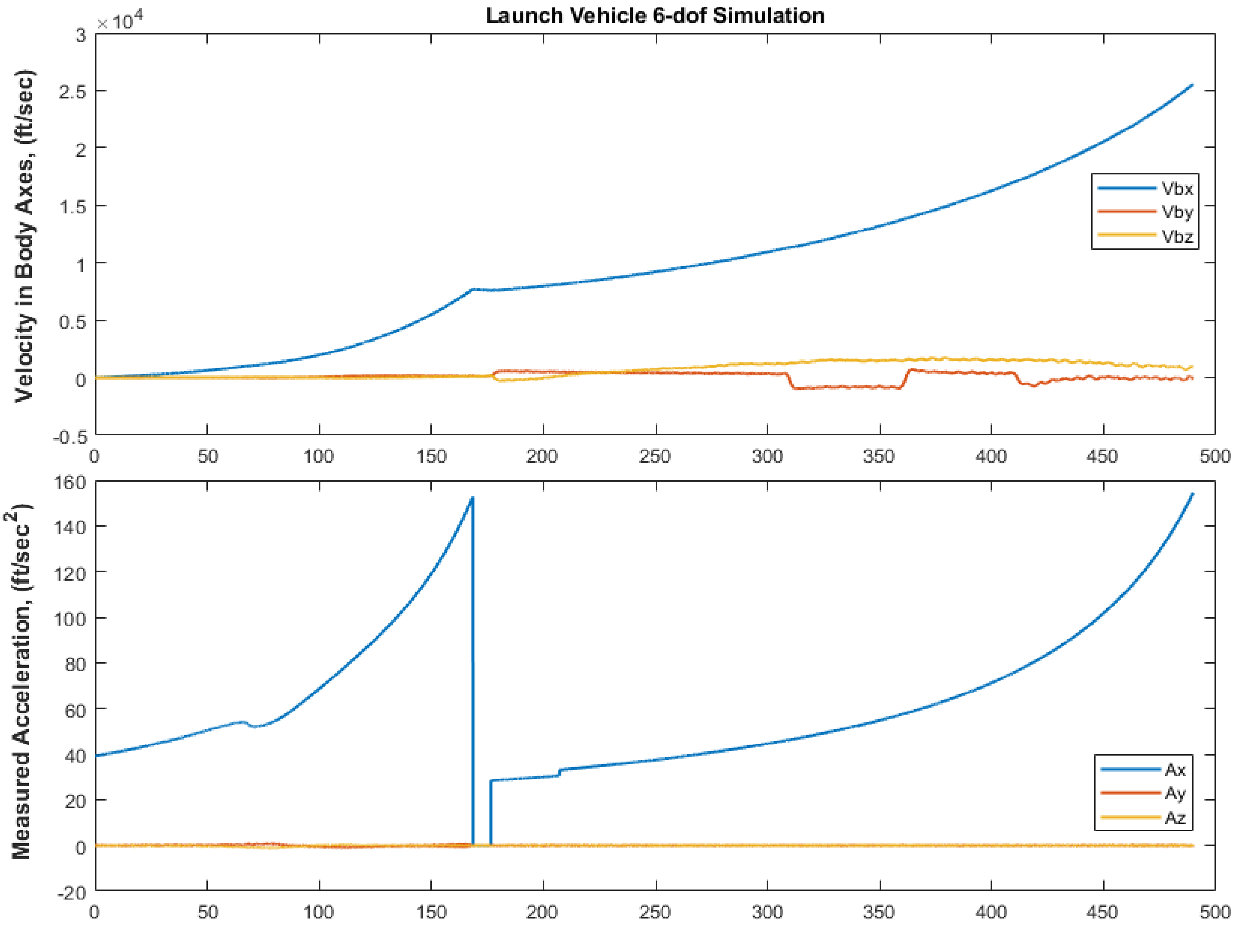
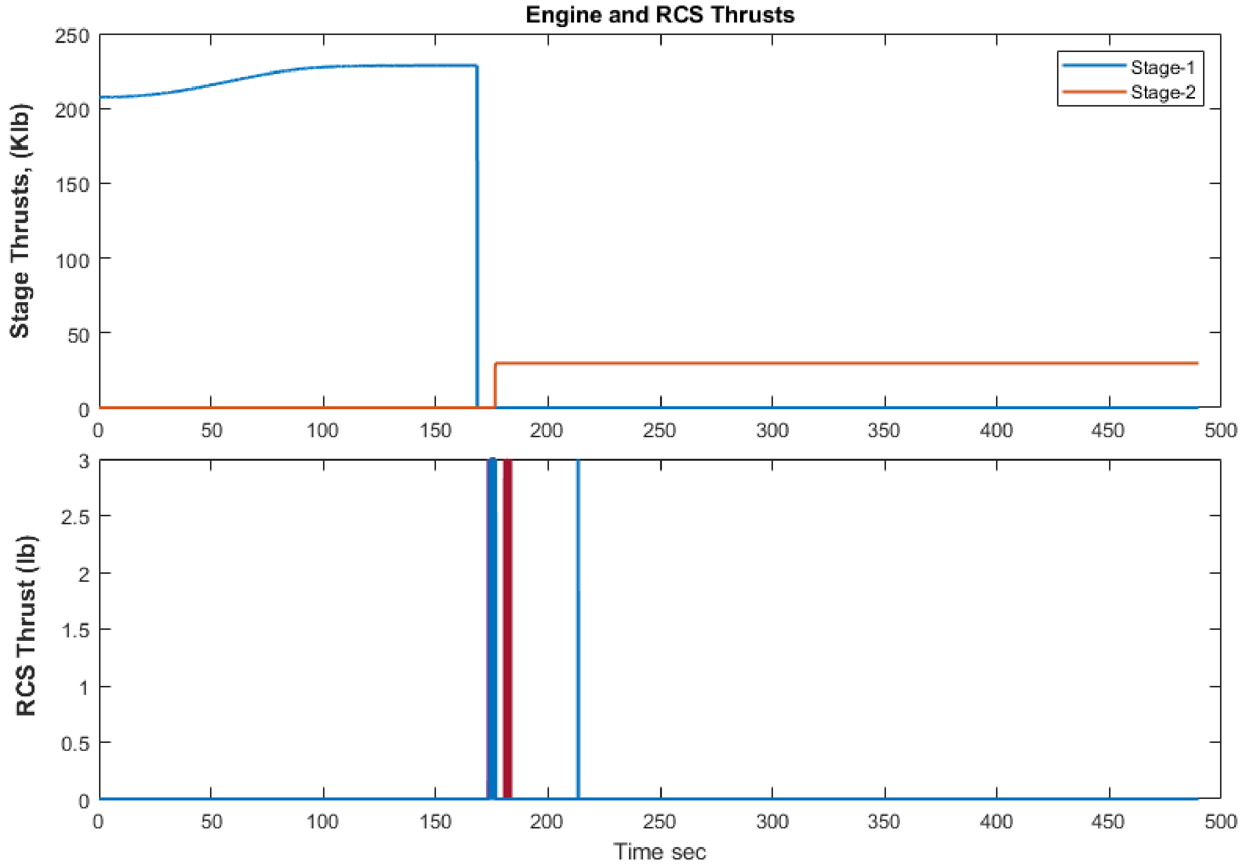


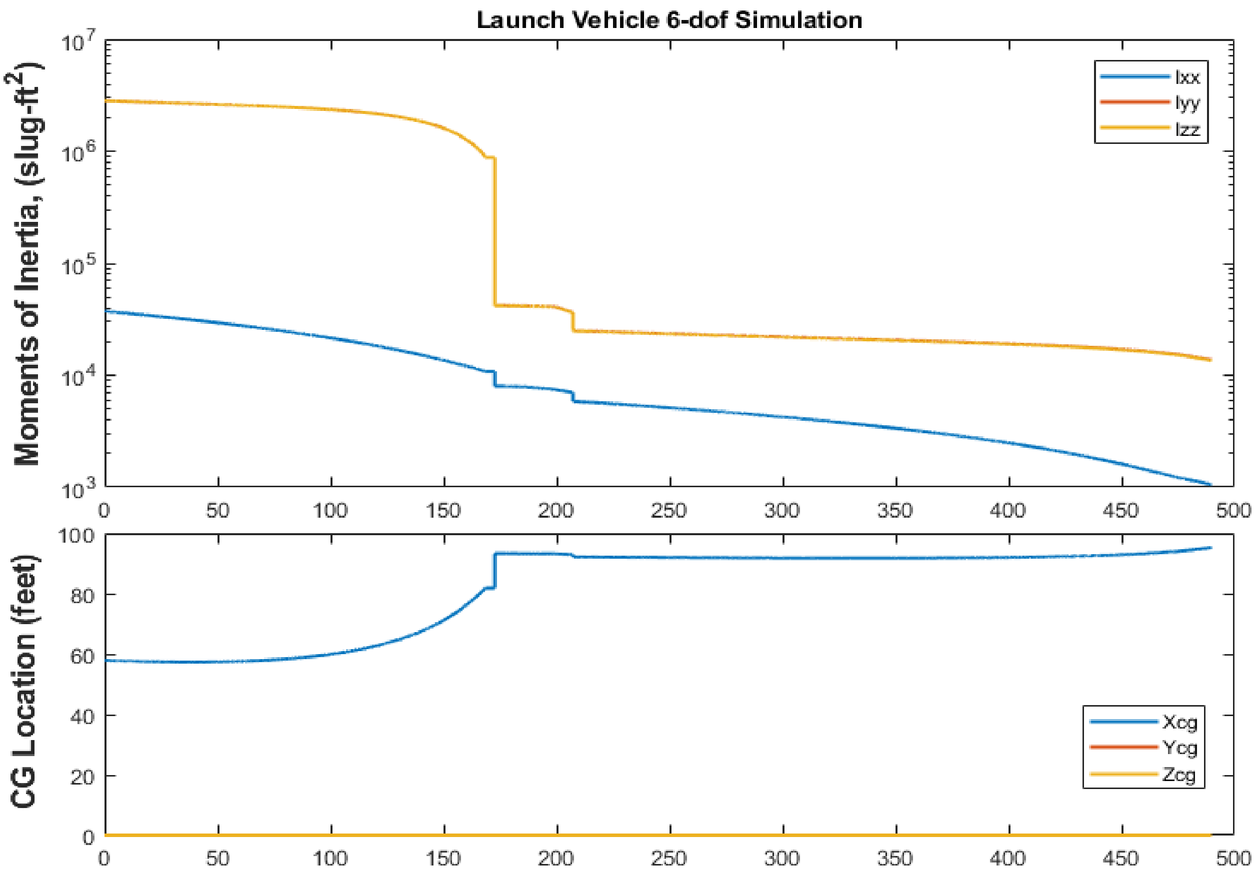
Figure 6.48 Gimbal Deflections and Actuator Shaft Extensions. Noise is Included in the Actuator Model

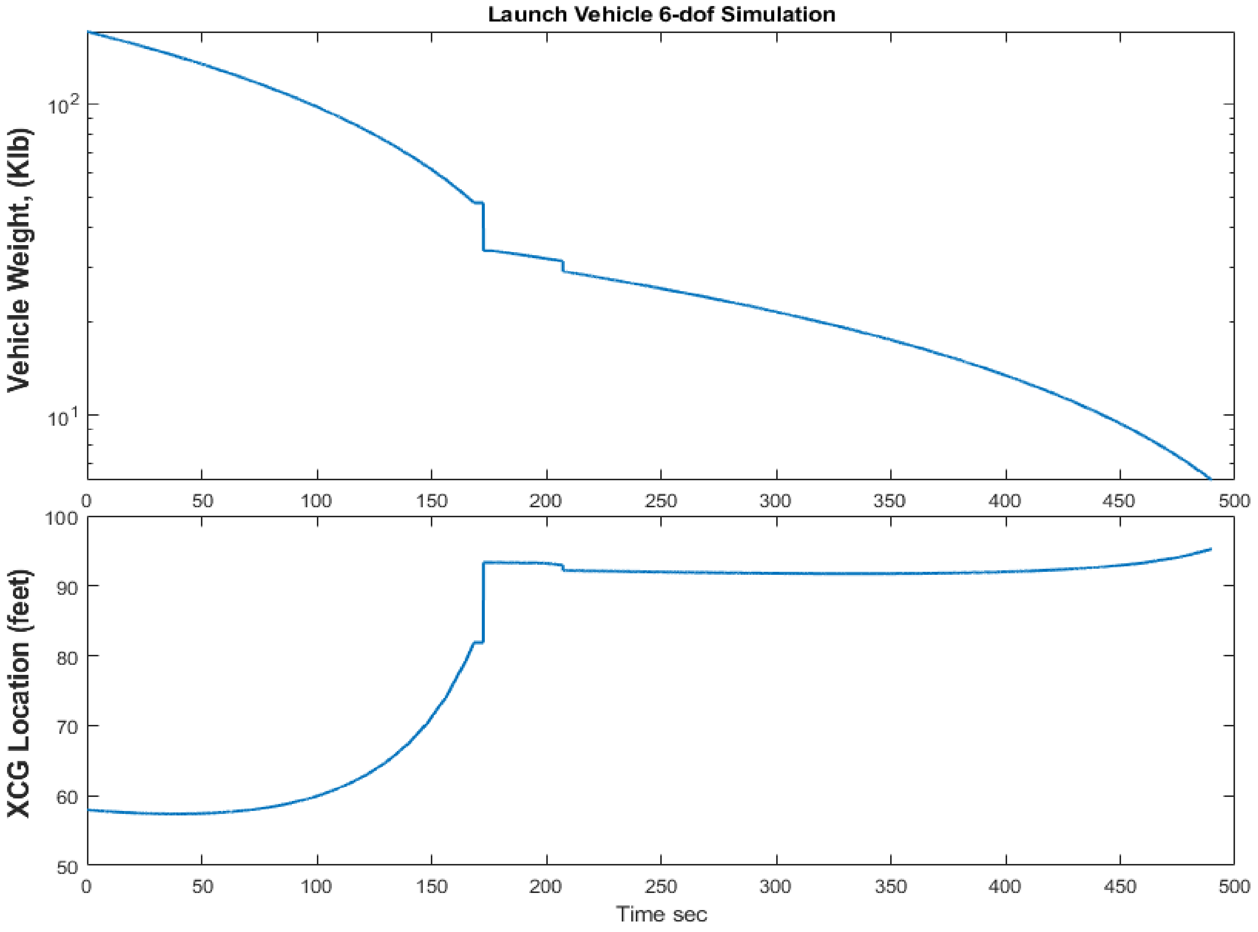

Launch Vehicle 6-dof Simulation
Altitude (K.feet)
700
600
500
400
300
200
100
0
-100
0
100
200
300
400
500
600
700
800
Down-Range (NMi)

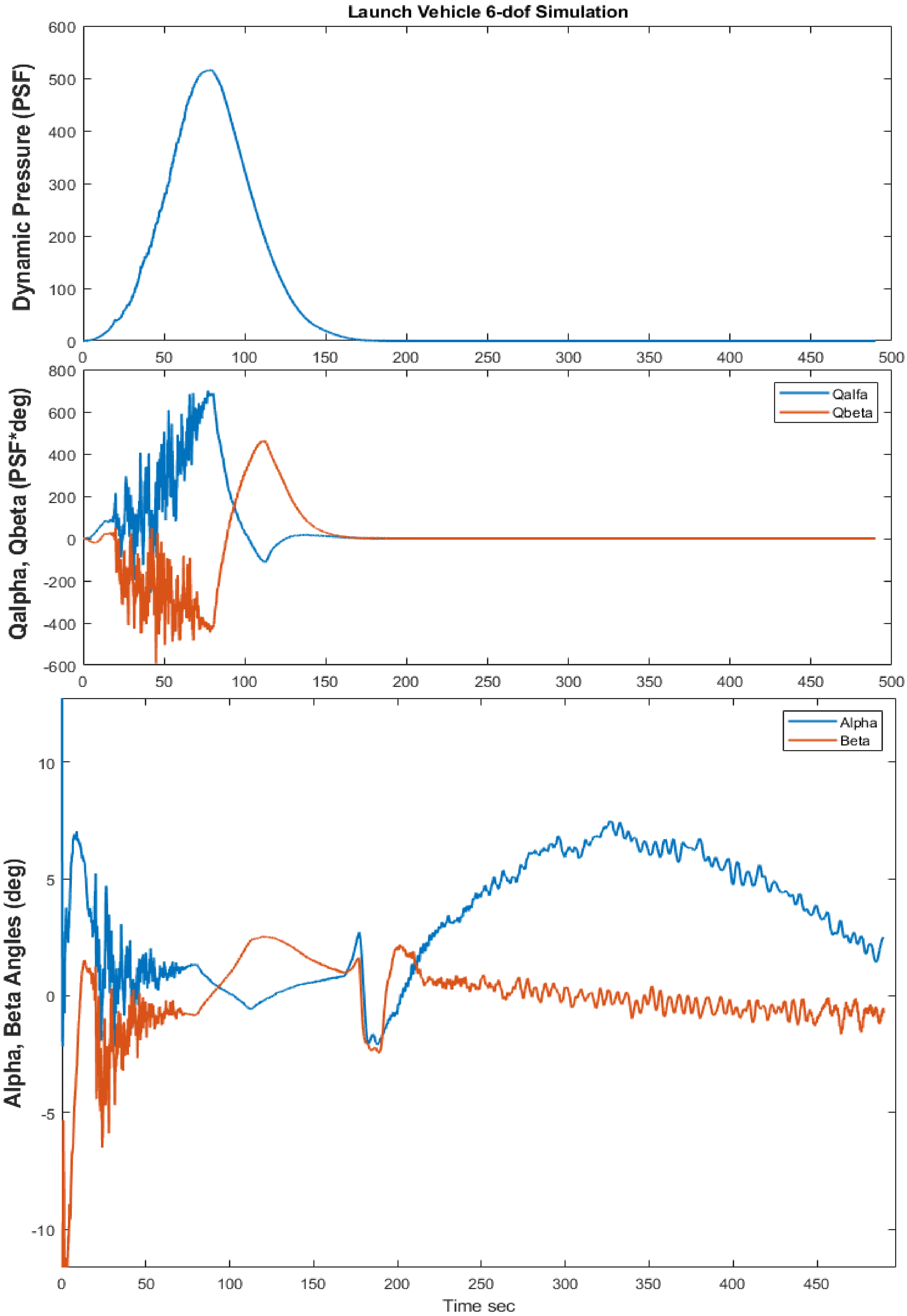

Longitude versus Latitude in (degrees)
Vehicle
Reference
Latitude (deg)
34
32
30
28
26
24
-123.5
-123
-122.5
-122
-121.5
-121
Longitude (deg)

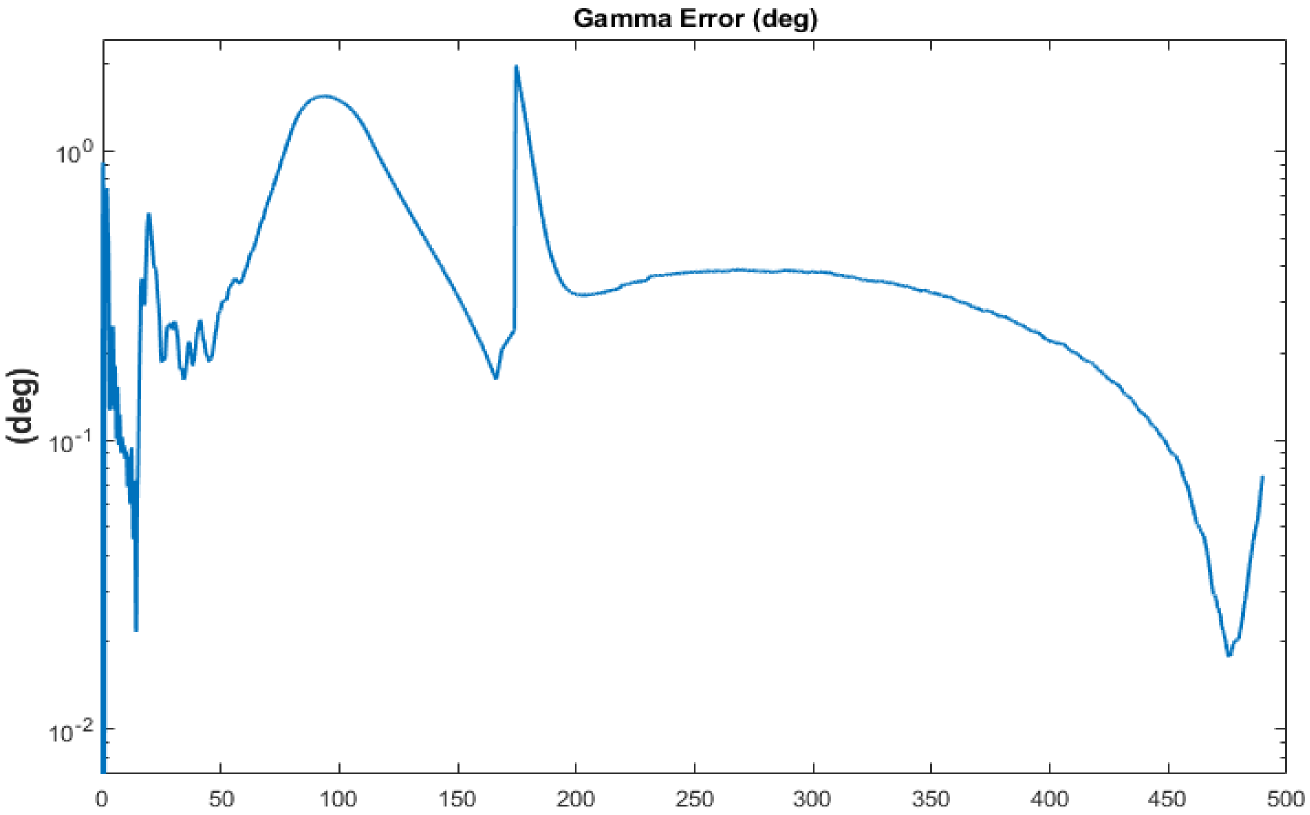

Launch Vehicle 6-dof Simulation
Velocity in Body Axes, (ft/sec)
$\times 10^4$
3
2.5
2
1.5
1
0.5
0
-0.5
Vbx
Vby
Vbz
Measured Acceleration, (ft/sec^2)
160
140
120
100
80
60
40
20
0
-20
Ax
Ay
Az
0
50
100
150
200
250
300
350
400
450
500


Engine and RCS Thrusts
Stage Thrusts, (Klb)
250
200
150
100
50
0
Stage-1
Stage-2
RCS Thrust (lb)
3
2.5
2
1.5
1
0.5
0
0
50
100
150
200
250
300
350
400
450
500
Time sec

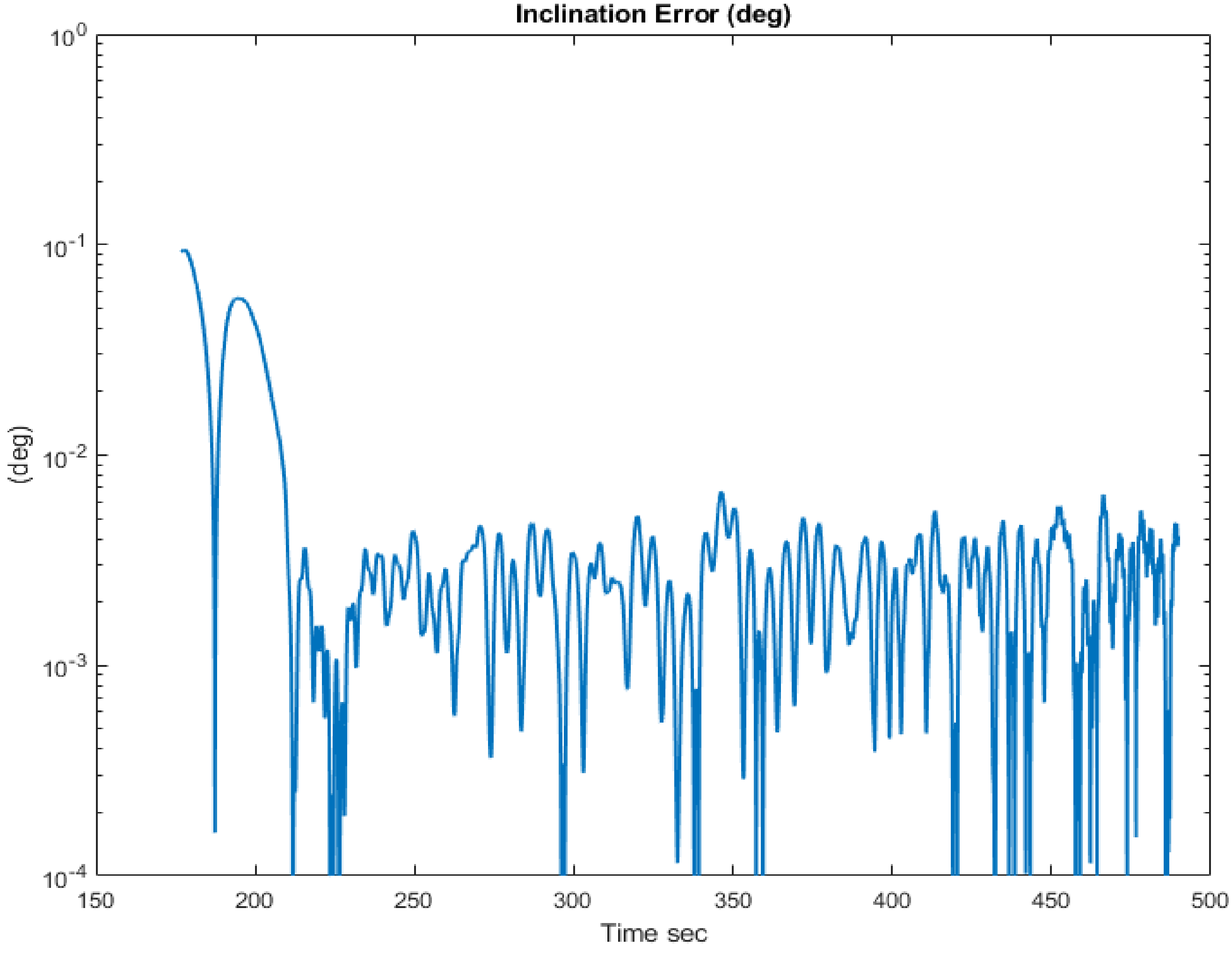

Launch Vehicle 6-dof Simulation
Moments of Inertia, (slug-ft^2)
10^7
10^6
10^5
10^4
10^3
Ixx
Iyy
Izz
0
50
100
150
200
250
300
350
400
450
500
CG Location (feet)
100
80
60
40
20
0
Xcg
Ycg
Zcg
0
50
100
150
200
250
300
350
400
450
500


Launch Vehicle 6-dof Simulation
Vehicle Weight, (Klb)
10^2
10^1
0
50
100
150
200
250
300
350
400
450
500
XCG Location (feet)
100
90
80
70
60
50
0
50
100
150
200
250
300
350
400
450
500
Time sec

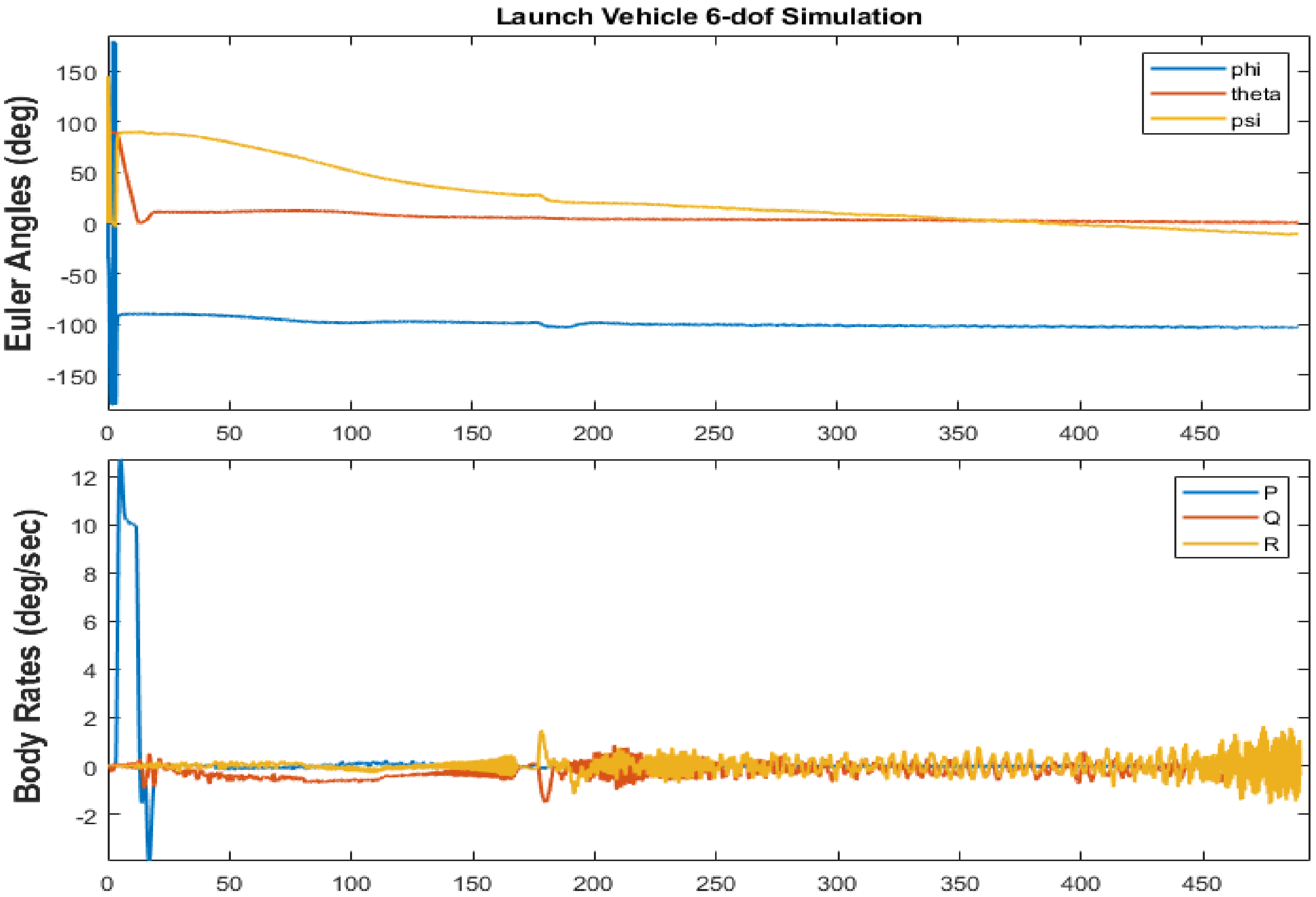

Launch Vehicle 6-dof Simulation
Dynamic Pressure (PSF)
600
500
400
300
200
100
0
0
50
100
150
200
250
300
350
400
450
500
Qalpha, Qbeta (PSF*deg)
Qalfa
Qbeta
800
600
400
200
0
-200
-400
-600
0
50
100
150
200
250
300
350
400
450
500
Alpha, Beta Angles (deg)
Alpha
Beta
10
5
0
-5
-10
0
50
100
150
200
250
300
350
400
450
Time sec

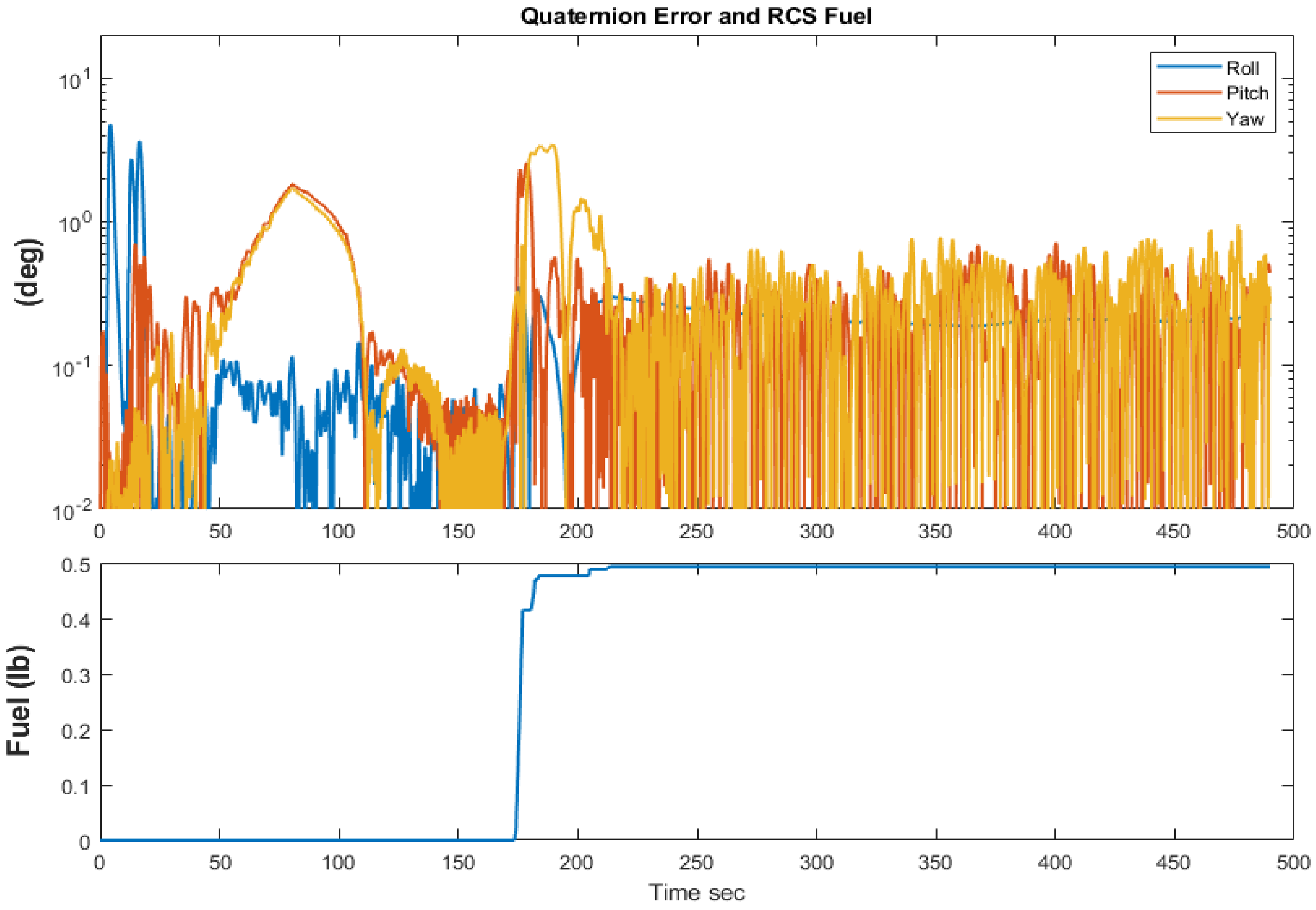


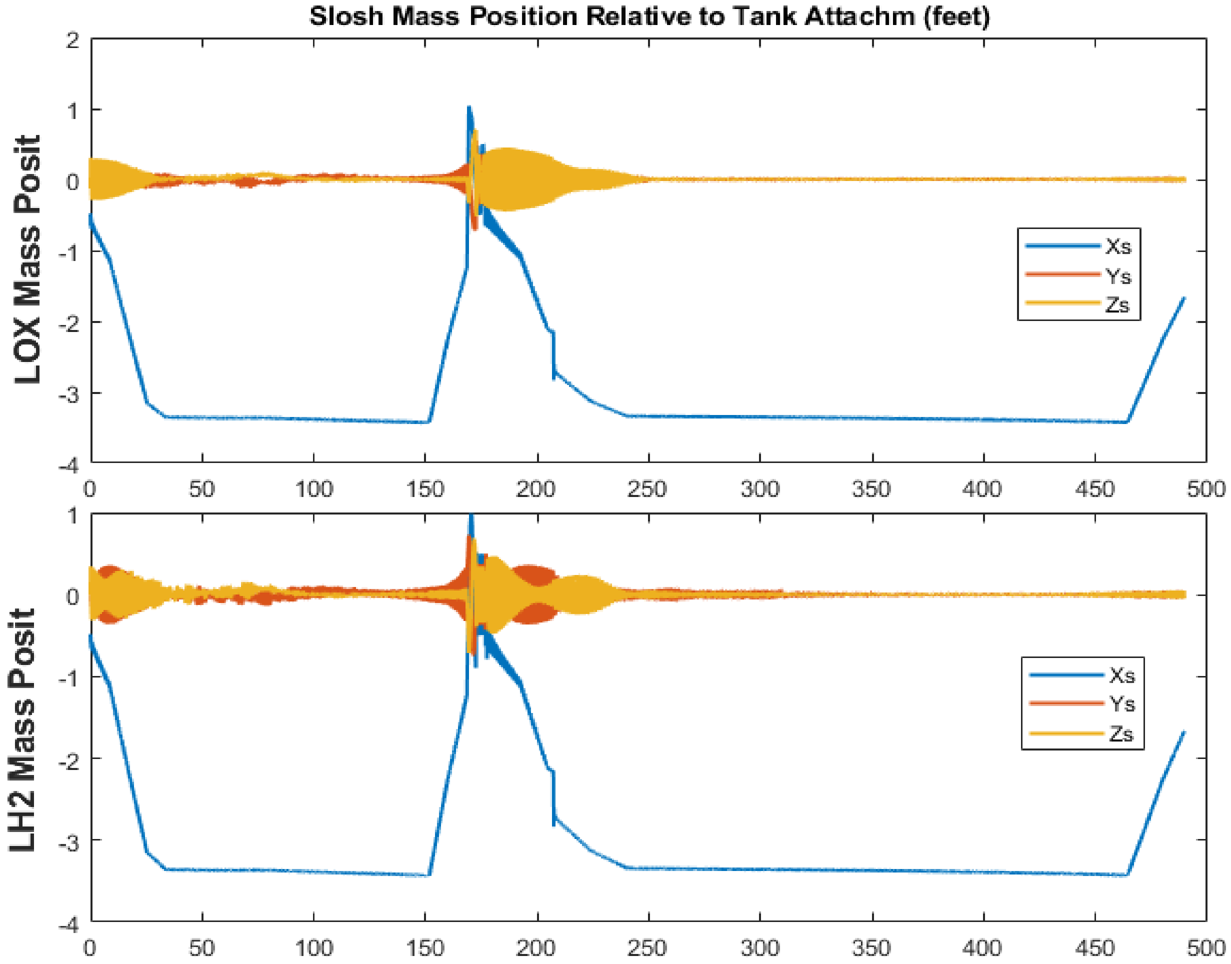
Gamma Error (deg)

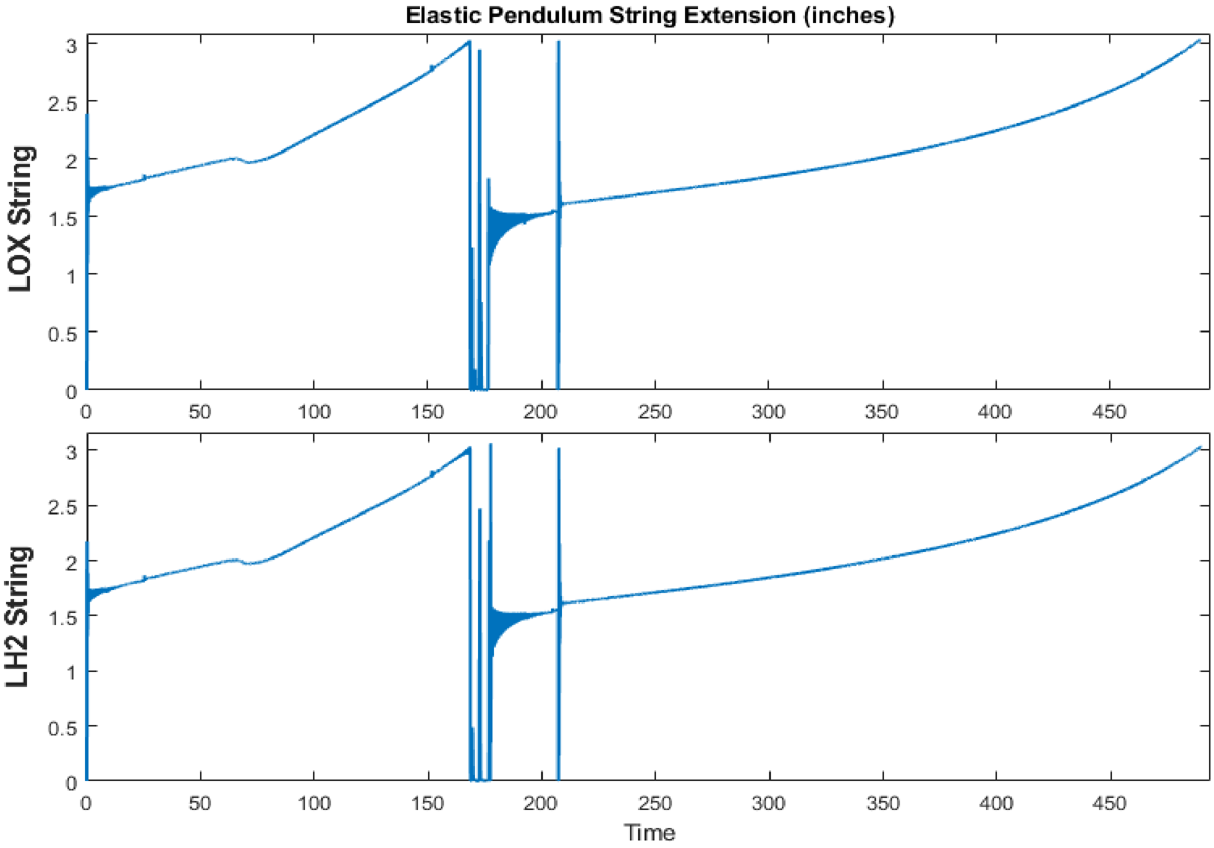


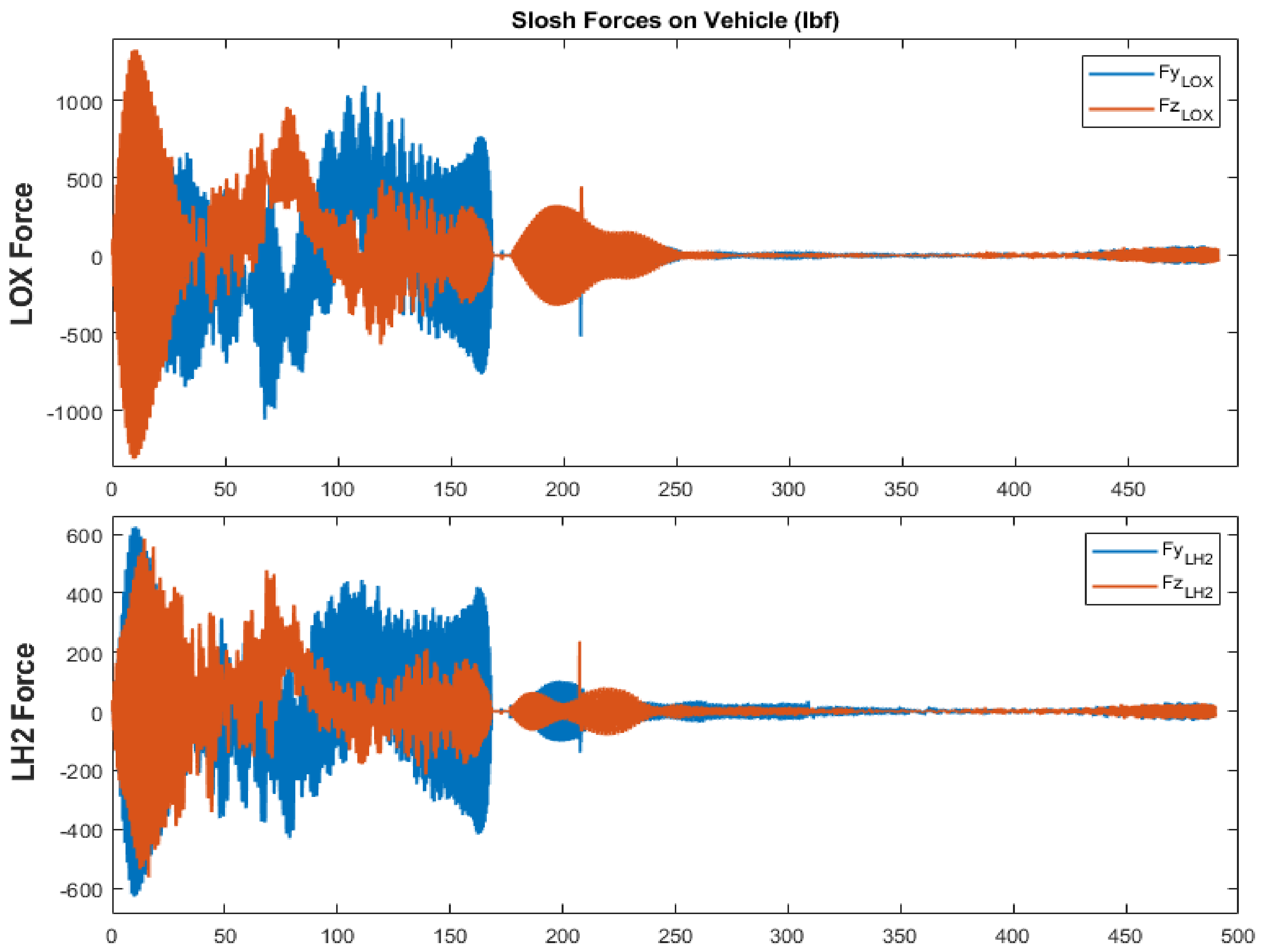
Inclination Error (deg)

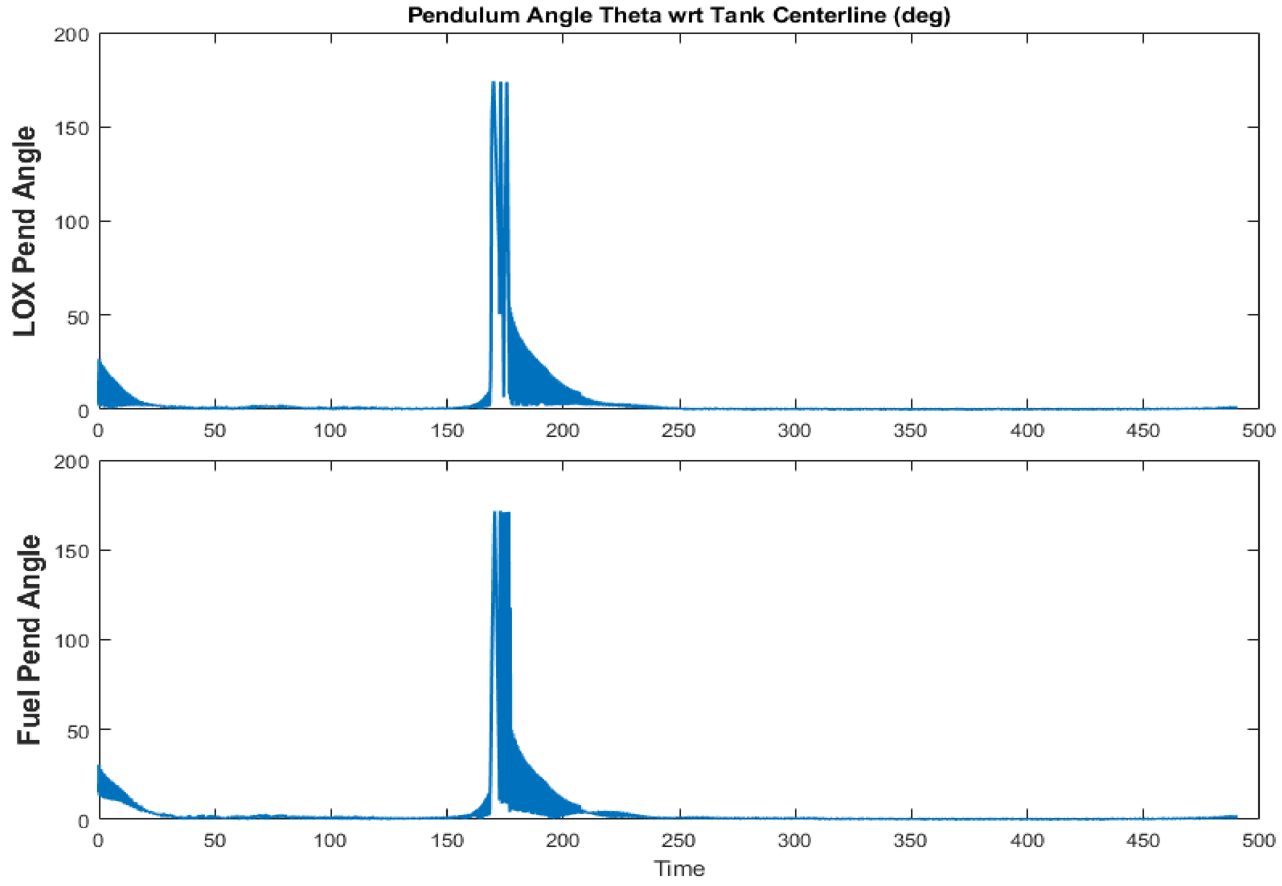

Launch Vehicle 6-dof Simulation
Euler Angles (deg)
150
100
50
0
-50
-100
-150
0
50
100
150
200
250
300
350
400
450
phi
theta
psi
Body Rates (deg/sec)
12
10
8
6
4
2
0
-2
0
50
100
150
200
250
300
350
400
450
P
Q
R

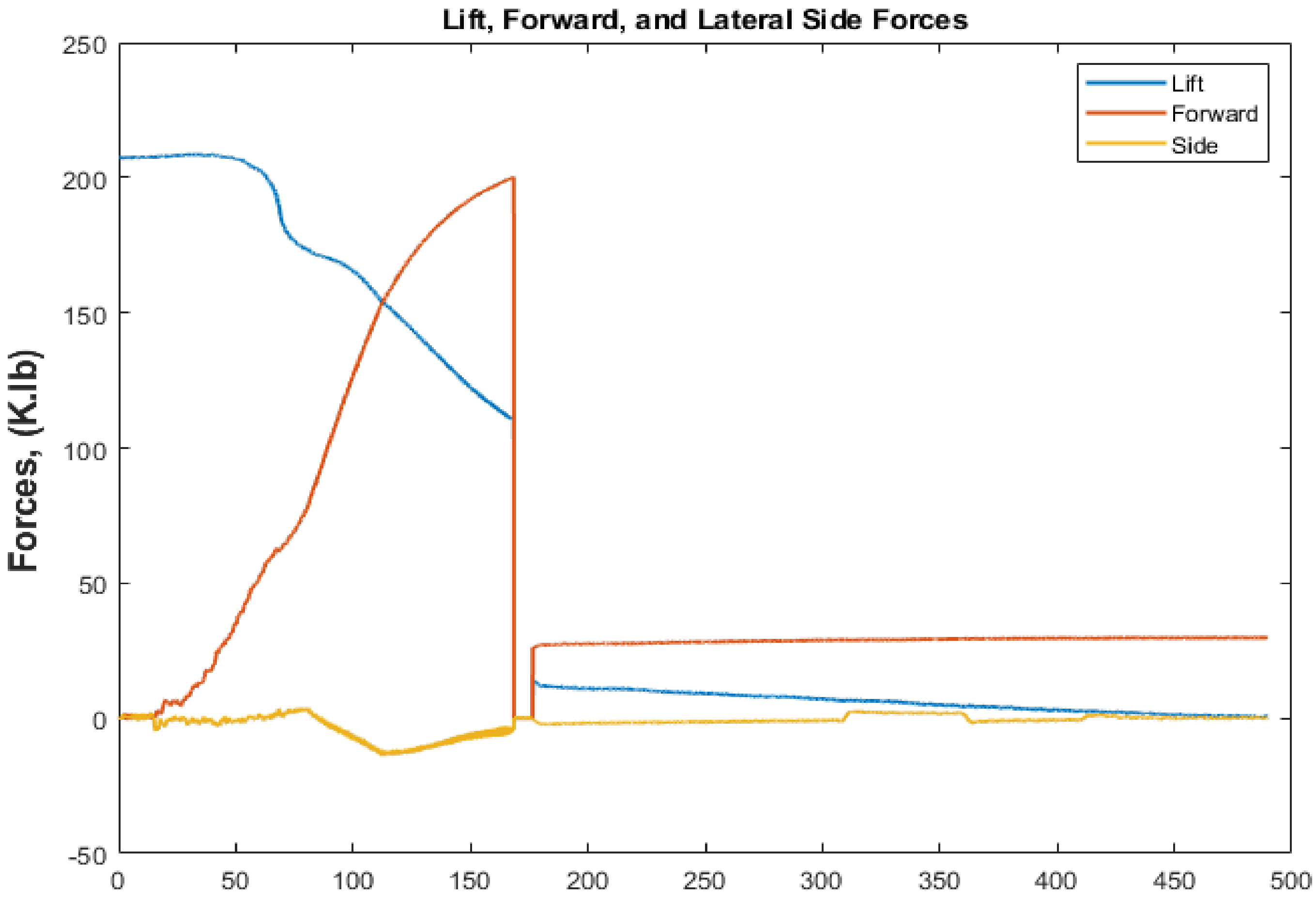

Quaternion Error and RCS Fuel
(deg)
10^1
10^0
10^{-1}
10^{-2}
0
50
100
150
200
250
300
350
400
450
500
Roll
Pitch
Yaw
Fuel (lb)
0.5
0.4
0.3
0.2
0.1
0
0
50
100
150
200
250
300
350
400
450
500
Time sec

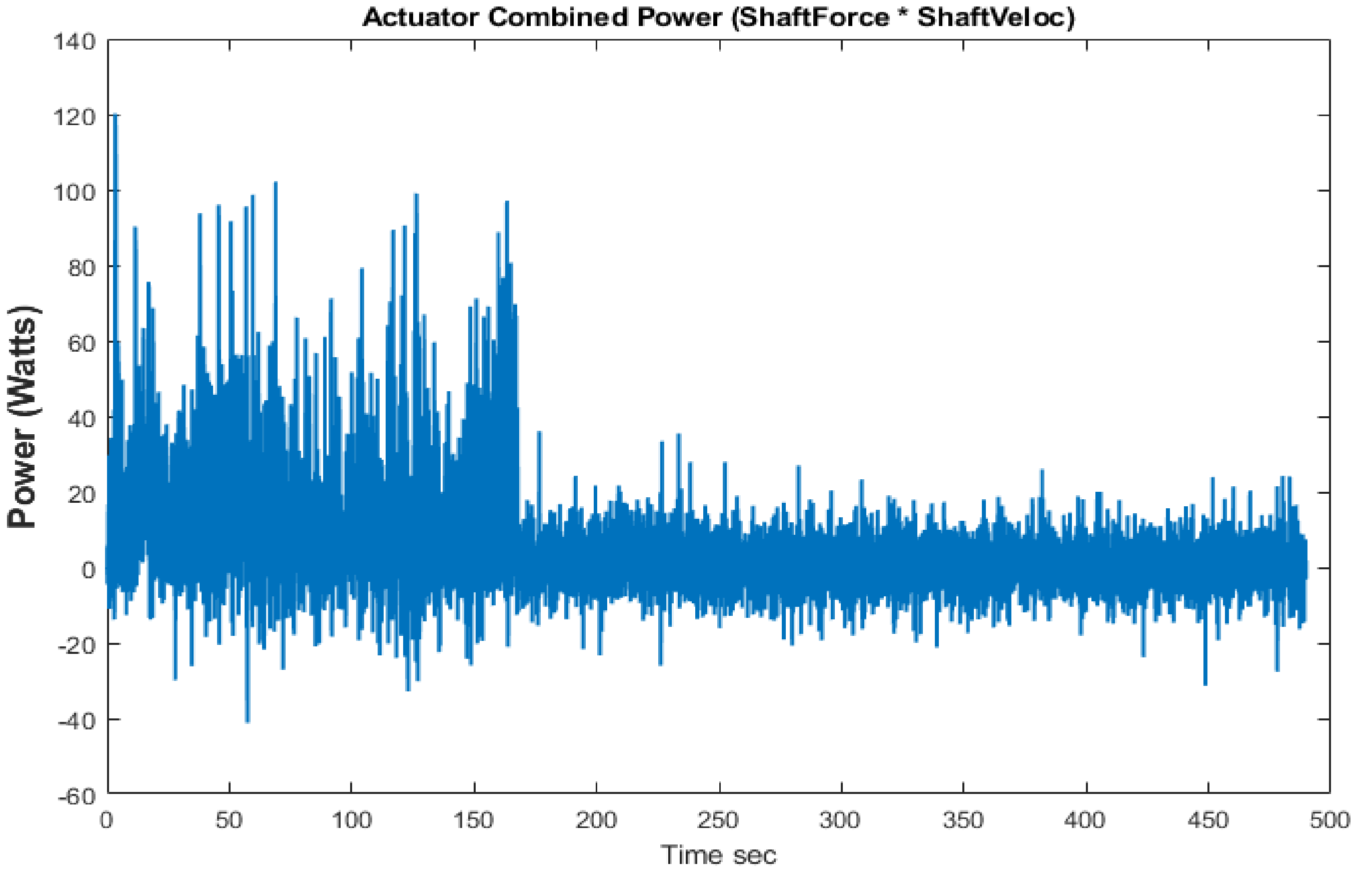

Slosh Mass Position Relative to Tank Attachm (feet)
LOX Mass Posit
LH2 Mass Posit
Xs
Ys
Zs
2
1
0
-1
-2
-3
-4
0
50
100
150
200
250
300
350
400
450
500


Elastic Pendulum String Extension (inches)
LOX String
LH2 String
Time
3
2.5
2
1.5
1
0.5
0
0
50
100
150
200
250
300
350
400
450


Slosh Forces on Vehicle (lbf)
LOX Force
1000
500
0
-500
-1000
0
50
100
150
200
250
300
350
400
450
Fy_{LOX}
Fz_{LOX}
LH2 Force
600
400
200
0
-200
-400
-600
0
50
100
150
200
250
300
350
400
450
500
Fy_{LH2}
Fz_{LH2}


Pendulum Angle Theta wrt Tank Centerline (deg)
LOX Pend Angle
200
150
100
50
0
0
50
100
150
200
250
300
350
400
450
500
Fuel Pend Angle
200
150
100
50
0
0
50
100
150
200
250
300
350
400
450
500
Time


Lift, Forward, and Lateral Side Forces
Forces, (K.lb)
250
200
150
100
50
0
-50
0
50
100
150
200
250
300
350
400
450
500
Lift
Forward
Side


Actuator Combined Power (ShaftForce * ShaftVeloc)
Power (Watts)
140
120
100
80
60
40
20
0
-20
-40
-60
0
50
100
150
200
250
300
350
400
450
500
Time sec